

# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

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# Chapter 1

## Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 375 ]. This is test number [ 25 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. [https://github.com/stblake/algebraic\\_integration](https://github.com/stblake/algebraic_integration). September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.



## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 375 )	0.00 ( 0 )
Mathematica	100.00 ( 375 )	0.00 ( 0 )
Maple	100.00 ( 375 )	0.00 ( 0 )
Giac	89.33 ( 335 )	10.67 ( 40 )
Fricas	88.00 ( 330 )	12.00 ( 45 )
Maxima	77.33 ( 290 )	22.67 ( 85 )
IntegrateAlgebraic	55.47 ( 208 )	44.53 ( 167 )
Mupad	52.00 ( 195 )	48.00 ( 180 )
Sympy	37.60 ( 141 )	% 62.40 ( 234 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

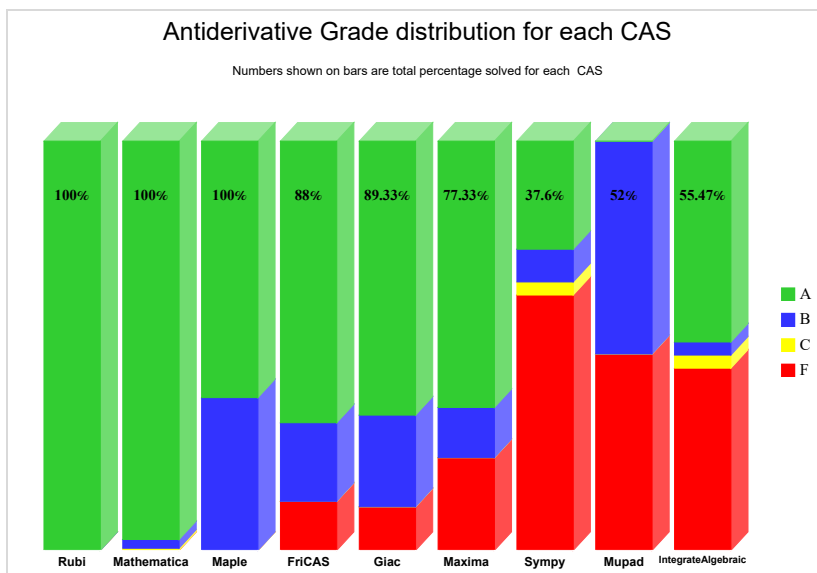
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

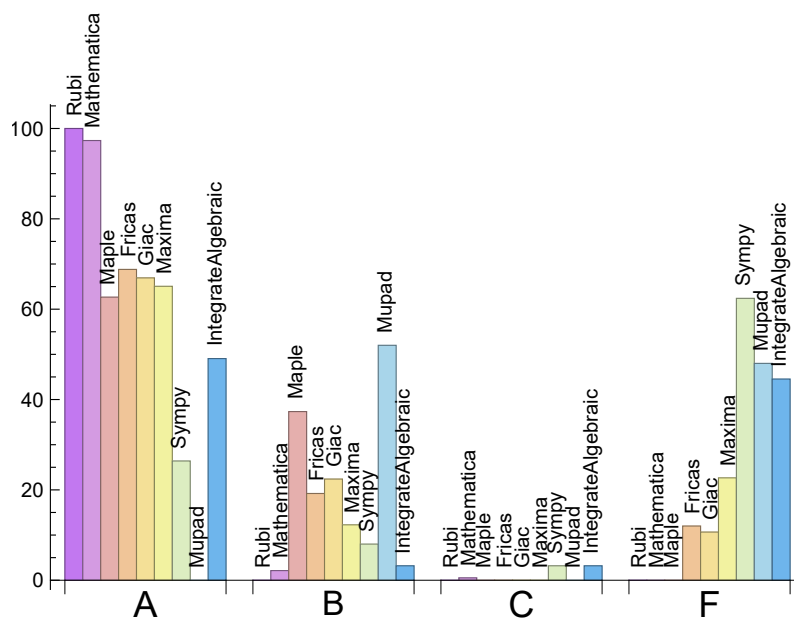
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	97.33	2.13	0.53	0.00
Fricas	68.80	19.20	0.00	12.00
Giac	66.93	22.40	0.00	10.67
Maxima	65.07	12.27	0.00	22.67
Maple	62.67	37.33	0.00	0.00
IntegrateAlgebraic	49.07	3.20	3.20	44.53
Sympy	26.40	8.00	3.20	62.40
Mupad	N/A	52.00	0.00	48.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**. The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	45	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	167	88.02 %	11.98 %	0.00 %
Giac	40	5.00 %	35.00 %	60.00 %
Maxima	85	11.76 %	2.35 %	85.88 %
Sympy	234	77.35 %	22.65 %	0.00 %
Mupad	180	99.44 %	0.56 %	0.00 %

Table 1.4: Failure statistics for each CAS

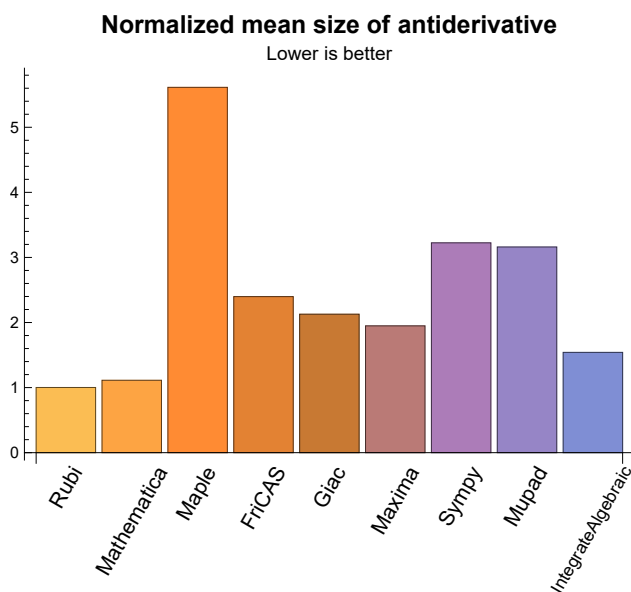
## 1.3 Performance

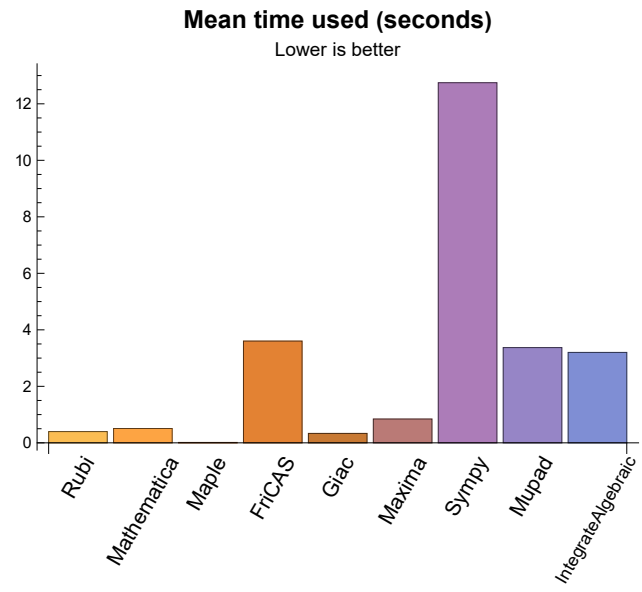
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.40	213.42	1.00	161.00	1.00
Mathematica	0.51	352.97	1.11	114.00	0.91
Maple	0.01	2315.63	5.61	200.00	1.42
Maxima	0.84	363.92	1.95	149.50	1.05
Fricas	3.60	436.93	2.40	171.50	1.40
Sympy	12.75	398.38	3.22	221.00	1.56
Giac	0.33	461.91	2.13	180.00	1.16
Mupad	3.37	773.44	3.16	185.00	1.29
IntegrateAlgebraic	3.20	523.97	1.54	124.00	0.91

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}



## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {361}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by

failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

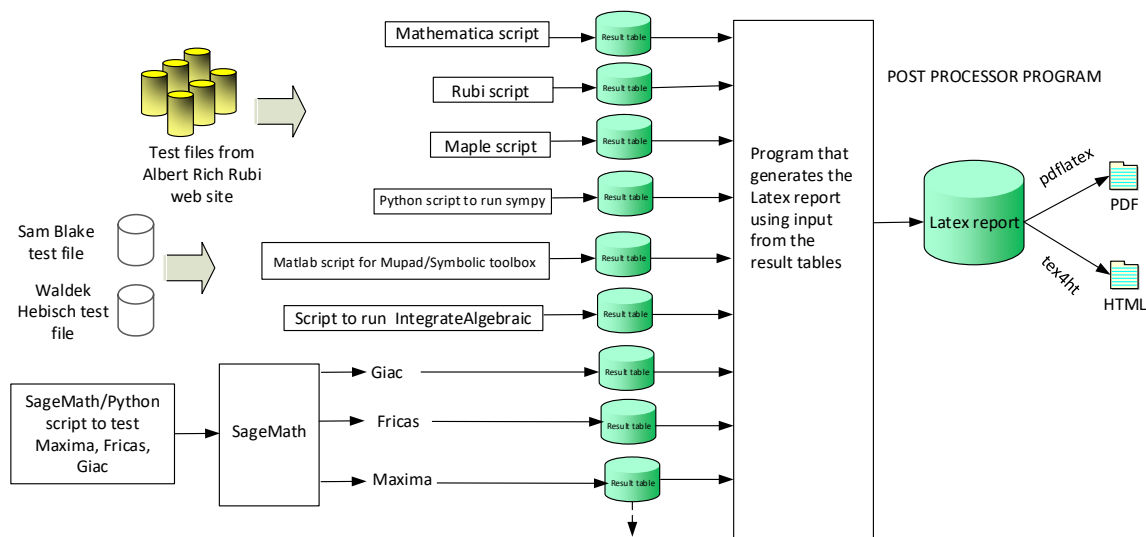
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. integer. 1 if result was verified or 0 if not verified.  
*The following field present only in Rubi and Mathematica Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

### High level overview of the CAS independent integration test build system



# Chapter 2

## detailed summary tables of results

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## 2.1 List of integrals sorted by grade for each CAS

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### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375 }

B grade: { 39, 40, 41, 42, 198, 199, 200, 258 }

C grade: { 255, 346 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 178, 179, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 250, 252, 253, 254, 255, 256, 257, 259, 263, 264, 265, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 291, 292, 293, 294, 298, 299, 300, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 343, 344, 352, 353, 354, 358, 359, 360, 364, 365, 366, 367, 370, 371, 372 }

B grade: { 4, 5, 6, 7, 15, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 56, 57, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 127, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 245, 248, 249, 251, 258, 260, 261, 262, 266, 267, 268, 289, 290, 295, 296, 297, 302, 303, 338, 339, 340, 341, 342, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 361, 362, 363, 368, 369, 373, 374, 375 }

C grade: { }

F grade: { }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 255, 256, 257, 259, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 344, 352, 353, 354, 358, 359, 360, 364, 365, 366, 370, 371, 372 }

B grade: { 8, 9, 15, 16, 17, 39, 40, 41, 42, 56, 62, 63, 64, 84, 85, 86, 87, 92, 94, 95, 96, 97, 98, 106, 107, 112, 113, 114, 131, 173, 248, 249, 258, 297, 303, 323, 338, 339, 340, 347, 348, 349, 355, 361, 367, 373 }

C grade: { }

F grade: { 6, 7, 99, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 254, 260, 261, 262, 263, 264, 265, 266, 267, 268, 345, 346, 350, 351, 356, 357, 362, 363, 368, 369, 374, 375 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 145, 146, 147, 148, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 182, 183, 184, 185, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 300, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 351, 352, 353, 354, 358, 359, 360, 364, 365, 366, 370, 371, 372 }

B grade: { 31, 38, 39, 40, 41, 42, 48, 50, 51, 54, 57, 58, 64, 65, 66, 67, 107, 112, 113, 114, 141, 142, 143, 151, 152, 153, 173, 174, 178, 179, 180, 181, 192, 193, 194, 229, 230, 231, 232, 233, 249, 254, 257, 258, 290, 291, 295, 296, 297, 298, 299, 302, 303, 340, 345, 346, 347, 348, 349, 350, 355, 356, 357, 361, 362, 363, 367, 368, 369, 373, 374, 375 }

C grade: { }

F grade: { 49, 55, 56, 61, 62, 63, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 149, 150, 154, 155, 186, 187, 188, 189, 190, 191, 196, 197, 198, 199, 200, 201, 202, 203, 226, 227, 228, 234, 235 }

### 2.1.6 Sympy

A grade: { 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 136, 137, 138, 139, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 256, 259, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 294, 301 }

B grade: { 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 58, 116, 117, 126, 132, 140, 141, 142, 143, 144, 145, 146, 147, 152, 153, 173, 255, 257, 258 }

C grade: { 1, 2, 3, 284, 285, 286, 291, 292, 293, 298, 299, 300 }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 47, 48, 49, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 82, 83, 84,

85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 148, 149, 150, 151, 154, 155, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 261, 262, 263, 264, 265, 266, 267, 268, 288, 289, 290, 295, 296, 297, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 88, 89, 90, 91, 92, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 182, 183, 184, 185, 195, 204, 205, 206, 207, 210, 211, 212, 213, 216, 217, 218, 219, 222, 223, 224, 225, 231, 232, 236, 237, 238, 239, 240, 242, 243, 244, 245, 248, 249, 250, 251, 259, 260, 261, 262, 264, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 314, 315, 316, 318, 324, 325, 326, 330, 331, 332, 333, 334, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 364, 365, 366, 367, 370, 371, 372, 373, 374 }

B grade: { 39, 40, 41, 42, 61, 63, 64, 65, 84, 85, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 123, 128, 129, 134, 143, 154, 155, 174, 180, 181, 192, 193, 194, 199, 208, 214, 220, 228, 229, 230, 233, 235, 241, 246, 247, 252, 253, 255, 256, 257, 258, 265, 266, 267, 268, 307, 308, 309, 310, 311, 312, 313, 317, 319, 320, 321, 322, 323, 327, 328, 329, 335, 336, 337, 342, 343, 344, 347, 348, 349, 357, 368, 369, 375 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 83, 86, 93, 96, 106, 113, 186, 187, 188, 189, 190, 191, 196, 197, 198, 200, 201, 202, 203, 209, 215, 221, 226, 227, 234, 254, 263, 356, 362, 363 }

### 2.1.8 Mupad

A grade: { }

B grade: { 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 178, 179, 180, 181, 182, 183, 184, 185, 204, 205, 206, 232, 250, 254, 255, 256, 257, 258, 259, 264, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, }

280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 112, 113, 114, 174, 175, 177, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 260, 261, 262, 263, 265, 266, 267, 268, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375 }

## 2.1.9 IntegrateAlgebraic

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 78, 79, 80, 81, 82, 83, 84, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 130, 131, 132, 135, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 260, 261, 262, 263, 264, 265, 266, 267, 268, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 352, 353, 354, 358, 359, 360, 364, 365, 366, 370, 371, 372 }

B grade: { 86, 97, 114, 181, 187, 192, 193, 194, 196, 197, 229, 234 }

C grade: { 355, 356, 357, 361, 362, 363, 367, 368, 369, 373, 374, 375 }

F grade: { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 85, 87, 95, 96, 98, 99, 127, 128, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 228, 235, 254, 255, 256, 257, 258, 259, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 346, 347, 348, 349, 350, 351 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	226	371	338	211	1231	197	-1	237
N.S.	1	1.00	0.96	1.57	1.43	0.89	5.22	0.83	-0.00	1.00
time (sec)	N/A	0.470	0.506	0.059	0.996	1.496	22.934	0.224	0.000	0.825
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	190	304	202	173	670	160	-1	199
N.S.	1	1.00	1.02	1.63	1.09	0.93	3.60	0.86	-0.01	1.07
time (sec)	N/A	0.227	0.299	0.017	0.976	1.105	12.771	0.213	0.000	0.694
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	121	154	116	108	343	85	-1	124
N.S.	1	1.00	0.97	1.23	0.93	0.86	2.74	0.68	-0.01	0.99
time (sec)	N/A	0.069	0.133	0.007	0.964	1.468	7.108	0.202	0.000	0.400

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	103	384	171	112	0	0	-1	130
N.S.	1	1.00	0.70	2.59	1.16	0.76	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.176	0.221	0.023	1.086	1.436	0.000	0.000	0.000	0.506

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	109	439	197	190	0	0	-1	134
N.S.	1	1.00	0.64	2.58	1.16	1.12	0.00	0.00	-0.01	0.79
time (sec)	N/A	0.203	0.227	0.028	1.014	1.413	0.000	0.000	0.000	0.567

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-1)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	114	318	0	258	0	0	-1	139
N.S.	1	1.00	0.77	2.13	0.00	1.73	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.189	0.221	0.018	0.000	1.001	0.000	0.000	0.000	0.748

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	112	453	0	304	0	0	-1	158
N.S.	1	1.00	0.57	2.31	0.00	1.55	0.00	0.00	-0.01	0.81
time (sec)	N/A	0.184	0.298	0.017	0.000	0.997	0.000	0.000	0.000	0.938

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	109	116	945	320	0	0	601	149
N.S.	1	1.00	0.61	0.64	5.25	1.78	0.00	0.00	3.34	0.83
time (sec)	N/A	0.210	0.203	0.009	0.542	0.785	0.000	0.000	4.670	0.913

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	144	152	1378	399	0	0	960	185
N.S.	1	1.00	0.62	0.65	5.89	1.71	0.00	0.00	4.10	0.79
time (sec)	N/A	0.264	0.225	0.011	0.575	1.018	0.000	0.000	5.243	1.068
Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	174	374	390	178	1268	166	-1	201
N.S.	1	1.00	0.74	1.58	1.65	0.75	5.37	0.70	-0.00	0.85
time (sec)	N/A	0.657	0.453	0.031	0.979	0.867	24.443	0.368	0.000	0.742
Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	139	301	253	145	891	131	-1	165
N.S.	1	1.00	0.73	1.58	1.32	0.76	4.66	0.69	-0.01	0.86
time (sec)	N/A	0.378	0.177	0.012	0.983	1.137	18.032	0.316	0.000	0.590
Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	103	234	150	109	484	97	270	129
N.S.	1	1.00	0.72	1.64	1.05	0.76	3.38	0.68	1.89	0.90
time (sec)	N/A	0.199	0.107	0.011	0.979	0.789	10.172	0.292	5.012	0.488



Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	67	108	70	71	262	52	148	90
N.S.	1	1.00	0.77	1.24	0.80	0.82	3.01	0.60	1.70	1.03
time (sec)	N/A	0.051	0.040	0.006	0.988	1.037	4.560	0.344	4.399	0.339
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	83	149	138	155	0	0	-1	105
N.S.	1	1.00	0.81	1.45	1.34	1.50	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.121	0.158	0.014	0.994	0.721	0.000	0.000	0.000	0.489
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	95	355	317	221	0	0	-1	122
N.S.	1	1.00	0.58	2.18	1.94	1.36	0.00	0.00	-0.01	0.75
time (sec)	N/A	0.169	0.218	0.016	0.995	0.985	0.000	0.000	0.000	0.748
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	103	116	608	244	0	0	109	113
N.S.	1	1.00	0.57	0.64	3.38	1.36	0.00	0.00	0.61	0.63
time (sec)	N/A	0.205	0.197	0.012	1.015	1.744	0.000	0.000	3.795	0.730

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	139	152	975	320	0	0	204	149
N.S.	1	1.00	0.59	0.65	4.17	1.37	0.00	0.00	0.87	0.64
time (sec)	N/A	0.249	0.221	0.010	1.061	1.184	0.000	0.000	3.776	0.850

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	173	208	217	202	248	257	242	206	0
N.S.	1	0.99	1.19	1.24	1.15	1.42	1.47	1.38	1.18	0.00
time (sec)	N/A	0.313	0.087	0.002	0.446	0.937	0.117	0.170	0.088	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	173	150	148	141	171	173	171	143	0
N.S.	1	0.99	0.86	0.85	0.81	0.98	0.99	0.98	0.82	0.00
time (sec)	N/A	0.216	0.060	0.001	0.448	0.966	0.095	0.152	3.614	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	79	80	94	97	100	80	0
N.S.	1	1.00	1.00	0.92	0.93	1.09	1.13	1.16	0.93	0.00
time (sec)	N/A	0.106	0.029	0.001	0.452	0.832	0.082	0.154	3.561	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	39	38	40	42	40	39	0
N.S.	1	1.00	1.00	0.85	0.83	0.87	0.91	0.87	0.85	0.00
time (sec)	N/A	0.029	0.013	0.002	0.440	1.262	0.072	0.157	0.025	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	143	136	210	159	161	148	170	175	0
N.S.	1	0.99	0.94	1.45	1.10	1.11	1.02	1.17	1.21	0.00
time (sec)	N/A	0.245	0.074	0.008	0.448	1.078	0.638	0.151	3.623	0.001

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	151	142	234	169	250	185	240	192	0
N.S.	1	0.99	0.93	1.53	1.10	1.63	1.21	1.57	1.25	0.00
time (sec)	N/A	0.205	0.154	0.010	0.451	0.826	1.257	0.163	0.090	0.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	154	176	257	177	273	206	167	185	0
N.S.	1	0.99	1.13	1.65	1.13	1.75	1.32	1.07	1.19	0.00
time (sec)	N/A	0.199	0.100	0.010	0.471	0.920	5.288	0.152	0.095	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	304	301	335	385	360	432	445	423	332	0
N.S.	1	0.99	1.10	1.27	1.18	1.42	1.46	1.39	1.09	0.00
time (sec)	N/A	0.535	0.131	0.002	0.450	0.751	0.134	0.159	0.140	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	216	241	268	257	302	311	302	244	0
N.S.	1	1.00	1.11	1.24	1.18	1.39	1.43	1.39	1.12	0.00
time (sec)	N/A	0.313	0.090	0.001	0.441	0.846	0.124	0.168	3.724	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	144	151	154	172	180	181	140	0
N.S.	1	1.00	1.12	1.18	1.20	1.34	1.41	1.41	1.09	0.00
time (sec)	N/A	0.159	0.051	0.001	0.447	0.882	0.097	0.152	3.695	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	69	75	74	76	83	76	74	0
N.S.	1	1.00	1.03	1.12	1.10	1.13	1.24	1.13	1.10	0.00
time (sec)	N/A	0.040	0.031	0.001	0.438	0.618	0.081	0.151	0.038	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	295	285	490	377	379	359	416	422	0
N.S.	1	0.99	0.96	1.65	1.27	1.28	1.21	1.40	1.42	0.00
time (sec)	N/A	0.640	0.169	0.008	0.485	1.015	0.937	0.158	3.685	0.001

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	289	272	527	392	553	416	497	575	0
N.S.	1	0.99	0.93	1.80	1.34	1.89	1.42	1.70	1.97	0.00
time (sec)	N/A	0.525	0.281	0.012	0.478	1.611	2.785	0.185	0.121	0.001

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	292	274	563	402	608	474	397	495	0
N.S.	1	0.99	0.93	1.91	1.36	2.06	1.61	1.35	1.68	0.00
time (sec)	N/A	0.495	0.122	0.014	0.494	0.628	14.200	0.160	3.825	0.001

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	404	400	459	553	512	618	646	606	490	0
N.S.	1	0.99	1.14	1.37	1.27	1.53	1.60	1.50	1.21	0.00
time (sec)	N/A	0.691	0.208	0.001	0.470	0.632	0.161	0.202	4.050	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	288	329	388	367	432	447	432	343	0
N.S.	1	1.00	1.14	1.34	1.27	1.49	1.55	1.49	1.19	0.00
time (sec)	N/A	0.424	0.130	0.001	0.454	0.906	0.147	0.155	3.938	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	196	223	222	249	265	261	187	0
N.S.	1	1.00	1.16	1.32	1.31	1.47	1.57	1.54	1.11	0.00
time (sec)	N/A	0.187	0.070	0.000	0.440	0.695	0.114	0.185	0.099	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	100	111	108	111	122	111	103	0
N.S.	1	1.00	1.15	1.28	1.24	1.28	1.40	1.28	1.18	0.00
time (sec)	N/A	0.058	0.031	0.001	0.432	0.689	0.086	0.186	0.057	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	490	487	498	880	672	674	685	764	741	0
N.S.	1	0.99	1.02	1.80	1.37	1.38	1.40	1.56	1.51	0.00
time (sec)	N/A	1.098	0.474	0.009	0.487	1.411	1.467	0.167	3.877	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	486	483	641	928	691	932	748	838	1511	0
N.S.	1	0.99	1.32	1.91	1.42	1.92	1.54	1.72	3.11	0.00
time (sec)	N/A	0.980	0.397	0.017	0.487	0.996	4.949	0.196	3.986	0.001

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	466	463	438	978	701	1025	816	727	1290	0
N.S.	1	0.99	0.94	2.10	1.50	2.20	1.75	1.56	2.77	0.00
time (sec)	N/A	0.967	0.225	0.017	0.530	0.874	25.279	0.174	3.936	0.001

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	62	76	82	78	73	111	85	0
N.S.	1	1.00	3.65	4.47	4.82	4.59	4.29	6.53	5.00	0.00
time (sec)	N/A	0.032	0.037	0.006	0.458	0.877	0.368	0.175	0.077	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	62	76	82	78	73	111	85	0
N.S.	1	1.00	3.65	4.47	4.82	4.59	4.29	6.53	5.00	0.00
time (sec)	N/A	0.019	0.016	0.005	0.466	0.778	0.367	0.155	3.836	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	90	157	160	120	153	216	252	0
N.S.	1	1.00	5.29	9.24	9.41	7.06	9.00	12.71	14.82	0.00
time (sec)	N/A	0.046	0.042	0.007	0.452	0.652	0.587	0.183	3.777	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	90	157	160	120	153	216	252	0
N.S.	1	1.00	5.29	9.24	9.41	7.06	9.00	12.71	14.82	0.00
time (sec)	N/A	0.025	0.025	0.006	0.444	1.000	0.610	0.199	0.049	0.001

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	237	223	399	244	592	1008	279	277	0
N.S.	1	0.99	0.93	1.66	1.02	2.47	4.20	1.16	1.15	0.00
time (sec)	N/A	0.471	0.243	0.011	0.982	1.069	5.462	0.170	3.990	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	166	155	256	161	404	638	176	181	0
N.S.	1	0.99	0.92	1.52	0.96	2.40	3.80	1.05	1.08	0.00
time (sec)	N/A	0.262	0.167	0.007	0.990	1.021	3.150	0.157	3.899	0.001



Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	86	133	86	206	337	91	97	0
N.S.	1	1.00	0.92	1.43	0.92	2.22	3.62	0.98	1.04	0.00
time (sec)	N/A	0.117	0.087	0.006	0.972	1.120	1.660	0.158	3.780	0.001
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	56	59	48	125	156	48	56	0
N.S.	1	1.00	1.02	1.07	0.87	2.27	2.84	0.87	1.02	0.00
time (sec)	N/A	0.053	0.041	0.003	0.966	1.162	0.486	0.159	3.731	0.001
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	120	247	123	262	0	125	840	0
N.S.	1	1.00	0.90	1.86	0.92	1.97	0.00	0.94	6.32	0.00
time (sec)	N/A	0.162	0.102	0.008	0.969	19.015	0.000	0.159	6.490	0.001
Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	214	214	188	462	255	904	0	270	1199	0
N.S.	1	1.00	0.88	2.16	1.19	4.22	0.00	1.26	5.60	0.00
time (sec)	N/A	0.355	0.312	0.010	1.035	72.027	0.000	0.170	6.773	0.001

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	305	277	754	495	0	0	489	2980	0
N.S.	1	1.00	0.91	2.47	1.62	0.00	0.00	1.60	9.77	0.00
time (sec)	N/A	0.651	0.309	0.015	1.052	0.000	0.000	0.197	9.186	0.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	233	484	287	931	952	289	303	0
N.S.	1	1.00	1.08	2.24	1.33	4.31	4.41	1.34	1.40	0.00
time (sec)	N/A	0.505	0.204	0.015	0.984	1.072	34.457	0.174	4.014	0.001

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	175	323	188	631	593	184	195	0
N.S.	1	1.00	1.20	2.21	1.29	4.32	4.06	1.26	1.34	0.00
time (sec)	N/A	0.245	0.138	0.013	0.965	0.836	18.398	0.170	0.229	0.001

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	102	134	113	337	318	112	191	0
N.S.	1	1.00	1.05	1.38	1.16	3.47	3.28	1.15	1.97	0.00
time (sec)	N/A	0.082	0.095	0.009	0.972	0.531	6.411	0.218	0.139	0.001

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	68	76	62	195	116	60	60	0
N.S.	1	1.00	0.99	1.10	0.90	2.83	1.68	0.87	0.87	0.00
time (sec)	N/A	0.043	0.052	0.008	0.965	2.999	0.654	0.183	0.100	0.001
Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	226	226	195	742	293	1024	0	350	1493	0
N.S.	1	1.00	0.86	3.28	1.30	4.53	0.00	1.55	6.61	0.00
time (sec)	N/A	0.435	0.224	0.017	1.002	83.522	0.000	0.193	7.675	0.001
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	374	371	320	1036	604	0	0	608	2094	0
N.S.	1	0.99	0.86	2.77	1.61	0.00	0.00	1.63	5.60	0.00
time (sec)	N/A	0.950	0.413	0.023	1.039	0.000	0.000	0.190	9.909	0.001
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	524	524	466	1588	1030	0	0	957	2828	0
N.S.	1	1.00	0.89	3.03	1.97	0.00	0.00	1.83	5.40	0.00
time (sec)	N/A	1.552	0.630	0.027	1.215	0.000	0.000	0.185	14.480	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	281	402	379	1138	0	348	920	0
N.S.	1	1.00	1.34	1.92	1.81	5.44	0.00	1.67	4.40	0.00
time (sec)	N/A	0.304	0.247	0.013	1.001	1.056	0.000	0.269	1.767	0.001

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	175	211	283	253	806	391	254	230	0
N.S.	1	1.12	1.35	1.81	1.62	5.17	2.51	1.63	1.47	0.00
time (sec)	N/A	0.231	0.139	0.010	0.995	1.022	141.179	0.160	3.959	0.001

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	137	157	160	470	240	152	128	0
N.S.	1	1.00	1.05	1.21	1.23	3.62	1.85	1.17	0.98	0.00
time (sec)	N/A	0.108	0.101	0.009	0.983	0.902	32.420	0.185	0.150	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	90	96	98	314	156	84	88	0
N.S.	1	1.00	0.92	0.98	1.00	3.20	1.59	0.86	0.90	0.00
time (sec)	N/A	0.063	0.070	0.007	0.971	1.038	1.234	0.162	3.843	0.001

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	321	1598	655	0	0	715	2392	0
N.S.	1	1.00	0.91	4.53	1.86	0.00	0.00	2.03	6.78	0.00
time (sec)	N/A	0.734	0.424	0.023	1.097	0.000	0.000	0.177	9.900	0.001
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	571	566	498	2159	1196	0	0	1107	6848	0
N.S.	1	0.99	0.87	3.78	2.09	0.00	0.00	1.94	11.99	0.00
time (sec)	N/A	1.925	0.764	0.031	1.239	0.000	0.000	0.235	6.657	0.001
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	753	753	672	2737	1835	0	0	1532	8774	0
N.S.	1	1.00	0.89	3.63	2.44	0.00	0.00	2.03	11.65	0.00
time (sec)	N/A	3.143	1.078	0.037	1.274	0.000	0.000	0.201	7.244	0.001
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	437	647	599	1864	0	636	669	0
N.S.	1	1.00	1.87	2.76	2.56	7.97	0.00	2.72	2.86	0.00
time (sec)	N/A	0.291	0.306	0.013	1.042	1.550	0.000	0.178	4.382	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	288	350	464	457	1378	0	475	402	0
N.S.	1	1.13	1.38	1.83	1.80	5.43	0.00	1.87	1.58	0.00
time (sec)	N/A	0.542	0.297	0.011	1.018	1.673	0.000	0.166	4.069	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	266	333	323	1062	0	328	287	0
N.S.	1	1.00	1.18	1.48	1.44	4.72	0.00	1.46	1.28	0.00
time (sec)	N/A	0.398	0.161	0.012	1.005	0.921	0.000	0.166	0.227	0.001

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	171	182	208	636	298	194	164	0
N.S.	1	1.00	1.04	1.10	1.26	3.85	1.81	1.18	0.99	0.00
time (sec)	N/A	0.136	0.134	0.010	0.981	0.595	139.971	0.157	3.936	0.001

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	112	113	133	430	196	109	116	0
N.S.	1	1.00	0.89	0.90	1.06	3.41	1.56	0.87	0.92	0.00
time (sec)	N/A	0.078	0.088	0.010	0.976	0.622	2.078	0.180	3.895	0.001

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	29	30	29	46	29	29	30	0
N.S.	1	1.00	0.67	0.70	0.67	1.07	0.67	0.67	0.70	0.00
time (sec)	N/A	0.050	0.019	0.008	0.953	0.917	0.129	0.148	0.035	0.001

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	27	24	23	40	20	23	23	0
N.S.	1	1.00	0.90	0.80	0.77	1.33	0.67	0.77	0.77	0.00
time (sec)	N/A	0.040	0.021	0.008	0.959	0.603	0.124	0.211	0.035	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	23	24	23	33	20	23	25	0
N.S.	1	1.00	0.79	0.83	0.79	1.14	0.69	0.79	0.86	0.00
time (sec)	N/A	0.025	0.009	0.005	0.963	0.591	0.127	0.153	0.031	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	20	10	12	14	0
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.71	0.86	1.00	0.00
time (sec)	N/A	0.011	0.007	0.005	0.955	0.732	0.114	0.148	3.797	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	28	26	25	41	24	26	32	0
N.S.	1	1.00	0.90	0.84	0.81	1.32	0.77	0.84	1.03	0.00
time (sec)	N/A	0.039	0.010	0.008	0.956	1.084	0.159	0.154	0.042	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	30	34	49	31	35	38	0
N.S.	1	1.00	1.00	0.91	1.03	1.48	0.94	1.06	1.15	0.00
time (sec)	N/A	0.045	0.016	0.009	0.952	2.965	0.161	0.184	3.808	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	39	38	41	61	42	43	47	0
N.S.	1	1.00	0.87	0.84	0.91	1.36	0.93	0.96	1.04	0.00
time (sec)	N/A	0.063	0.016	0.010	0.971	0.606	0.176	0.162	0.038	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	22	12	13	12	18	8	12	12	0
N.S.	1	1.83	1.00	1.08	1.00	1.50	0.67	1.00	1.00	0.00
time (sec)	N/A	0.007	0.012	0.004	0.951	1.557	0.115	0.150	0.031	0.000



Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	21	21	25	20	21	21	0
N.S.	1	1.00	1.00	0.78	0.78	0.93	0.74	0.78	0.78	0.00
time (sec)	N/A	0.014	0.011	0.006	0.965	0.957	0.127	0.183	3.830	0.000
Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	390	387	362	661	436	855	1088	475	-1	474
N.S.	1	0.99	0.93	1.69	1.12	2.19	2.79	1.22	-0.00	1.22
time (sec)	N/A	0.830	0.457	0.018	0.466	2.152	28.367	0.227	0.000	1.268
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	280	279	256	446	305	595	738	321	-1	302
N.S.	1	1.00	0.91	1.59	1.09	2.12	2.64	1.15	-0.00	1.08
time (sec)	N/A	0.498	0.646	0.009	0.463	0.800	21.001	0.206	0.000	0.912
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	153	230	169	329	384	180	-1	169
N.S.	1	1.00	0.87	1.31	0.97	1.88	2.19	1.03	-0.01	0.97
time (sec)	N/A	0.268	0.417	0.007	0.449	1.705	11.876	0.208	0.000	0.538

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	98	111	96	190	170	87	-1	89
N.S.	1	1.00	0.92	1.05	0.91	1.79	1.60	0.82	-0.01	0.84
time (sec)	N/A	0.064	0.204	0.005	0.451	1.051	6.903	0.184	0.000	0.356
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	224	1265	362	0	0	278	-1	233
N.S.	1	1.00	1.09	6.14	1.76	0.00	0.00	1.35	-0.00	1.13
time (sec)	N/A	0.392	0.436	0.018	0.606	0.000	0.000	0.224	0.000	0.853
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	308	303	264	2818	478	0	0	0	-1	316
N.S.	1	0.98	0.86	9.15	1.55	0.00	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.509	0.258	0.016	0.647	0.000	0.000	0.000	0.000	1.265
Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	296	295	318	4432	927	0	0	923	-1	411
N.S.	1	1.00	1.07	14.97	3.13	0.00	0.00	3.12	-0.00	1.39
time (sec)	N/A	0.546	0.561	0.016	0.703	0.000	0.000	0.353	0.000	2.375

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	314	314	382	5565	1772	0	0	1719	-1	0
N.S.	1	1.00	1.22	17.72	5.64	0.00	0.00	5.47	-0.00	0.00
time (sec)	N/A	0.505	0.825	0.015	0.841	0.000	0.000	0.503	0.000	180.124
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	312	439	7237	3404	0	0	0	-1	4080
N.S.	1	1.00	1.40	23.12	10.88	0.00	0.00	0.00	-0.00	13.04
time (sec)	N/A	0.429	1.305	0.020	1.140	0.000	0.000	0.000	0.000	122.079
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	433	432	583	8546	5793	0	0	4212	-1	0
N.S.	1	1.00	1.35	19.74	13.38	0.00	0.00	9.73	-0.00	0.00
time (sec)	N/A	0.743	1.619	0.025	1.525	0.000	0.000	0.656	0.000	180.011
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	462	462	481	794	525	1177	1916	652	-1	660
N.S.	1	1.00	1.04	1.72	1.14	2.55	4.15	1.41	-0.00	1.43
time (sec)	N/A	1.134	0.541	0.018	0.456	0.981	72.414	0.274	0.000	1.858

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	345	346	552	380	831	1304	452	-1	434
N.S.	1	1.00	1.00	1.60	1.10	2.40	3.77	1.31	-0.00	1.25
time (sec)	N/A	0.524	1.095	0.010	0.452	1.108	54.496	0.264	0.000	1.367
Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	212	209	287	211	477	768	264	-1	247
N.S.	1	1.00	0.98	1.35	0.99	2.24	3.61	1.24	-0.00	1.16
time (sec)	N/A	0.271	0.633	0.005	0.447	1.760	27.893	0.254	0.000	0.903
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	125	146	131	262	348	129	-1	125
N.S.	1	1.00	0.91	1.07	0.96	1.91	2.54	0.94	-0.01	0.91
time (sec)	N/A	0.083	0.257	0.006	0.448	1.284	17.013	0.234	0.000	0.447
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	326	326	348	2420	632	0	0	551	-1	551
N.S.	1	1.00	1.07	7.42	1.94	0.00	0.00	1.69	-0.00	1.69
time (sec)	N/A	0.766	1.211	0.013	0.789	0.000	0.000	0.293	0.000	1.724

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	432	428	392	5121	708	0	0	0	-1	478
N.S.	1	0.99	0.91	11.85	1.64	0.00	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.901	0.525	0.017	0.783	0.000	0.000	0.000	0.000	2.070
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	488	480	435	7817	1299	0	0	1036	-1	541
N.S.	1	0.98	0.89	16.02	2.66	0.00	0.00	2.12	-0.00	1.11
time (sec)	N/A	0.921	0.655	0.017	0.878	0.000	0.000	0.433	0.000	2.850
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	475	469	517	9835	2415	0	0	1900	-1	0
N.S.	1	0.99	1.09	20.71	5.08	0.00	0.00	4.00	-0.00	0.00
time (sec)	N/A	0.845	1.233	0.021	1.095	0.000	0.000	0.612	0.000	180.169
Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	511	511	575	12481	4326	0	0	0	-1	0
N.S.	1	1.00	1.13	24.42	8.47	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.092	2.154	0.025	1.503	0.000	0.000	0.000	0.000	180.026

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	507	507	639	14169	6650	0	0	4408	-1	4966
N.S.	1	1.00	1.26	27.95	13.12	0.00	0.00	8.69	-0.00	9.79
time (sec)	N/A	0.860	2.268	0.029	1.893	0.000	0.000	1.327	0.000	138.616
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	404	403	696	17026	10724	0	0	6122	-1	0
N.S.	1	1.00	1.72	42.14	26.54	0.00	0.00	15.15	-0.00	0.00
time (sec)	N/A	0.552	2.472	0.038	2.729	0.000	0.000	1.123	0.000	180.028
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	532	531	863	19093	0	0	0	7936	-1	0
N.S.	1	1.00	1.62	35.89	0.00	0.00	0.00	14.92	-0.00	0.00
time (sec)	N/A	0.889	2.495	0.051	0.000	0.000	0.000	1.118	0.000	180.081
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	150	181	166	333	510	168	-1	161
N.S.	1	1.00	0.89	1.08	0.99	1.98	3.04	1.00	-0.01	0.96
time (sec)	N/A	0.102	0.311	0.009	0.453	1.204	32.916	0.213	0.000	0.545

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	323	252	528	349	559	796	314	-1	296
N.S.	1	0.99	0.78	1.62	1.07	1.72	2.45	0.97	-0.00	0.91
time (sec)	N/A	0.664	0.355	0.015	0.452	0.897	22.204	0.255	0.000	0.866
Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	222	164	339	230	381	518	206	-1	178
N.S.	1	1.00	0.74	1.52	1.03	1.71	2.32	0.92	-0.00	0.80
time (sec)	N/A	0.372	0.233	0.010	0.453	0.829	15.782	0.230	0.000	0.623
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	135	96	172	126	199	282	110	227	99
N.S.	1	0.99	0.71	1.26	0.93	1.46	2.07	0.81	1.67	0.73
time (sec)	N/A	0.179	0.105	0.006	0.436	1.282	9.024	0.211	5.171	0.442
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	63	76	61	124	150	58	107	64
N.S.	1	1.00	0.85	1.03	0.82	1.68	2.03	0.78	1.45	0.86
time (sec)	N/A	0.048	0.038	0.006	0.435	0.868	3.496	0.197	4.559	0.319

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	125	453	218	0	0	138	-1	158
N.S.	1	1.00	0.96	3.48	1.68	0.00	0.00	1.06	-0.01	1.22
time (sec)	N/A	0.174	0.219	0.012	0.564	0.000	0.000	0.223	0.000	0.469

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	218	923	419	0	0	0	-1	250
N.S.	1	1.00	1.30	5.49	2.49	0.00	0.00	0.00	-0.01	1.49
time (sec)	N/A	0.235	0.363	0.015	0.578	0.000	0.000	0.000	0.000	0.938

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	224	254	1574	896	1088	0	848	-1	306
N.S.	1	1.00	1.13	7.00	3.98	4.84	0.00	3.77	-0.00	1.36
time (sec)	N/A	0.292	0.455	0.016	0.672	18.246	0.000	0.263	0.000	1.516

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	229	228	246	516	346	758	0	339	-1	316
N.S.	1	1.00	1.07	2.25	1.51	3.31	0.00	1.48	-0.00	1.38
time (sec)	N/A	0.325	0.450	0.016	0.457	1.004	0.000	0.253	0.000	1.089



Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	177	327	227	530	0	219	-1	190
N.S.	1	1.00	1.19	2.19	1.52	3.56	0.00	1.47	-0.01	1.28
time (sec)	N/A	0.184	0.284	0.010	0.448	0.861	0.000	0.247	0.000	0.730
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	102	163	126	278	209	116	151	104
N.S.	1	1.00	1.02	1.63	1.26	2.78	2.09	1.16	1.51	1.04
time (sec)	N/A	0.087	0.137	0.005	0.437	1.024	18.840	0.250	5.280	0.553
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	74	69	61	181	87	63	68	62
N.S.	1	1.00	1.21	1.13	1.00	2.97	1.43	1.03	1.11	1.02
time (sec)	N/A	0.036	0.062	0.006	0.430	0.930	8.859	0.201	4.332	0.368
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	137	862	453	721	0	294	-1	202
N.S.	1	1.00	0.99	6.25	3.28	5.22	0.00	2.13	-0.01	1.46
time (sec)	N/A	0.141	0.177	0.015	0.622	3.202	0.000	0.294	0.000	0.793

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	285	1663	1085	1573	0	0	-1	383
N.S.	1	1.00	1.19	6.96	4.54	6.58	0.00	0.00	-0.00	1.60
time (sec)	N/A	0.418	0.608	0.017	0.767	4.363	0.000	0.000	0.000	1.907

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	374	372	404	2584	2254	2853	0	1440	-1	2488
N.S.	1	0.99	1.08	6.91	6.03	7.63	0.00	3.85	-0.00	6.65
time (sec)	N/A	1.028	1.184	0.018	1.017	21.946	0.000	0.394	0.000	17.206

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	50	47	83	68	194	48	59	50
N.S.	1	1.00	0.75	0.70	1.24	1.01	2.90	0.72	0.88	0.75
time (sec)	N/A	0.042	0.032	0.004	0.435	0.746	17.131	0.211	4.222	0.499

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	71	72	118	103	638	80	93	75
N.S.	1	1.00	0.73	0.74	1.22	1.06	6.58	0.82	0.96	0.77
time (sec)	N/A	0.057	0.050	0.004	0.441	0.800	37.219	0.264	4.281	0.603

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	92	96	153	137	1880	112	115	99
N.S.	1	1.00	0.72	0.76	1.20	1.08	14.80	0.88	0.91	0.78
time (sec)	N/A	0.087	0.065	0.006	0.451	0.709	78.297	0.266	4.375	0.741
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	54	79	78	60	94	54	45	66
N.S.	1	1.00	0.51	0.75	0.74	0.57	0.89	0.51	0.42	0.62
time (sec)	N/A	0.114	0.065	0.013	0.948	0.579	2.203	0.226	0.050	0.337
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	48	65	64	54	75	48	40	58
N.S.	1	1.00	0.59	0.79	0.78	0.66	0.91	0.59	0.49	0.71
time (sec)	N/A	0.088	0.043	0.005	0.959	0.668	1.185	0.202	4.099	0.303
Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	44	51	50	49	63	44	35	56
N.S.	1	1.00	0.71	0.82	0.81	0.79	1.02	0.71	0.56	0.90
time (sec)	N/A	0.052	0.027	0.008	0.962	0.719	0.552	0.194	0.034	0.219

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	60	55	58	88	0	99	61	89
N.S.	1	1.00	0.90	0.82	0.87	1.31	0.00	1.48	0.91	1.33
time (sec)	N/A	0.081	0.028	0.009	0.962	0.753	0.000	0.235	0.189	0.393

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	64	65	65	106	0	48	68	97
N.S.	1	1.00	0.90	0.92	0.92	1.49	0.00	0.68	0.96	1.37
time (sec)	N/A	0.070	0.101	0.011	0.969	0.753	0.000	0.320	0.115	0.499

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	55	74	76	89	0	180	77	74
N.S.	1	1.00	0.71	0.96	0.99	1.16	0.00	2.34	1.00	0.96
time (sec)	N/A	0.067	0.064	0.012	0.971	0.643	0.000	0.257	0.111	0.599

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	58	79	78	76	0	54	110	66
N.S.	1	1.00	0.67	0.91	0.90	0.87	0.00	0.62	1.26	0.76
time (sec)	N/A	0.104	0.055	0.014	0.959	0.806	0.000	0.214	0.059	0.371

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	53	65	64	72	0	49	105	61
N.S.	1	1.00	0.75	0.92	0.90	1.01	0.00	0.69	1.48	0.86
time (sec)	N/A	0.084	0.050	0.006	0.963	0.516	0.000	0.196	4.067	0.365
Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	48	51	50	67	114	44	100	56
N.S.	1	1.00	0.87	0.93	0.91	1.22	2.07	0.80	1.82	1.02
time (sec)	N/A	0.044	0.029	0.005	0.959	0.976	15.959	0.235	0.037	0.313
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	51	88	58	83	0	82	106	0
N.S.	1	1.00	0.96	1.66	1.09	1.57	0.00	1.55	2.00	0.00
time (sec)	N/A	0.060	0.023	0.009	0.963	0.542	0.000	0.208	0.136	0.575
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	71	98	84	103	0	168	157	0
N.S.	1	1.00	0.95	1.31	1.12	1.37	0.00	2.24	2.09	0.00
time (sec)	N/A	0.077	0.042	0.012	0.965	0.838	0.000	0.274	4.145	0.683

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	78	107	124	119	0	196	180	84
N.S.	1	1.00	0.80	1.10	1.28	1.23	0.00	2.02	1.86	0.87
time (sec)	N/A	0.123	0.083	0.011	0.981	0.724	0.000	0.245	4.169	0.668

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	63	91	105	87	0	53	212	64
N.S.	1	1.00	0.86	1.25	1.44	1.19	0.00	0.73	2.90	0.88
time (sec)	N/A	0.085	0.068	0.014	0.951	0.465	0.000	0.184	0.057	0.449

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	58	77	91	83	0	48	200	61
N.S.	1	1.00	0.97	1.28	1.52	1.38	0.00	0.80	3.33	1.02
time (sec)	N/A	0.075	0.051	0.008	0.962	0.654	0.000	0.194	0.050	0.420

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	30	27	50	40	180	25	185	30
N.S.	1	1.00	0.73	0.66	1.22	0.98	4.39	0.61	4.51	0.73
time (sec)	N/A	0.053	0.017	0.005	0.425	0.954	77.502	0.336	4.107	0.378

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	58	133	81	103	0	91	218	0
N.S.	1	1.00	0.79	1.82	1.11	1.41	0.00	1.25	2.99	0.00
time (sec)	N/A	0.085	0.048	0.009	0.972	0.985	0.000	0.214	0.135	0.852
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	91	143	107	134	0	233	270	0
N.S.	1	1.00	0.96	1.51	1.13	1.41	0.00	2.45	2.84	0.00
time (sec)	N/A	0.169	0.055	0.013	0.977	0.562	0.000	0.517	4.312	1.024
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	75	140	147	149	0	183	301	94
N.S.	1	1.00	0.64	1.20	1.26	1.27	0.00	1.56	2.57	0.80
time (sec)	N/A	0.206	0.100	0.014	0.987	0.937	0.000	0.295	4.190	0.814
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	256	343	263	308	320	308	244	0
N.S.	1	1.00	1.01	1.35	1.04	1.21	1.26	1.21	0.96	0.00
time (sec)	N/A	0.326	0.094	0.001	0.436	0.542	0.136	0.155	0.126	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	163	223	165	187	197	187	149	0
N.S.	1	1.00	1.01	1.39	1.02	1.16	1.22	1.16	0.93	0.00
time (sec)	N/A	0.193	0.046	0.001	0.429	0.430	0.108	0.169	0.071	0.000
Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	90	87	99	102	99	88	0
N.S.	1	1.00	1.00	0.94	0.91	1.03	1.06	1.03	0.92	0.00
time (sec)	N/A	0.099	0.023	0.001	0.430	0.985	0.089	0.155	4.086	0.000
Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	39	38	40	42	40	39	0
N.S.	1	1.00	1.00	0.85	0.83	0.87	0.91	0.87	0.85	0.00
time (sec)	N/A	0.030	0.009	0.000	0.430	0.340	0.069	0.180	0.026	0.000
Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	84	140	0	265	413	78	224	0
N.S.	1	1.00	1.04	1.73	0.00	3.27	5.10	0.96	2.77	0.00
time (sec)	N/A	0.100	0.092	0.006	0.000	0.769	1.213	0.154	0.194	0.001



Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	98	146	0	511	376	108	172	0
N.S.	1	1.00	0.98	1.46	0.00	5.11	3.76	1.08	1.72	0.00
time (sec)	N/A	0.068	0.081	0.008	0.000	0.671	1.212	0.159	4.533	0.001
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	160	373	0	1199	774	217	401	0
N.S.	1	1.00	0.99	2.32	0.00	7.45	4.81	1.35	2.49	0.00
time (sec)	N/A	0.114	0.205	0.011	0.000	0.603	2.362	0.181	4.173	0.001
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	204	643	0	2103	1224	407	698	0
N.S.	1	1.00	0.99	3.12	0.00	10.21	5.94	1.98	3.39	0.00
time (sec)	N/A	0.190	0.408	0.015	0.000	0.866	4.220	0.172	4.358	0.001
Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	591	591	585	1738	0	2150	4972	771	967	0
N.S.	1	1.00	0.99	2.94	0.00	3.64	8.41	1.30	1.64	0.00
time (sec)	N/A	1.428	0.651	0.012	0.000	2.520	118.420	0.171	5.462	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	348	348	345	1028	0	1273	2839	426	557	0
N.S.	1	1.00	0.99	2.95	0.00	3.66	8.16	1.22	1.60	0.00
time (sec)	N/A	0.677	0.380	0.009	0.000	1.203	47.193	0.161	4.684	0.001
Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	173	510	0	654	1265	201	273	0
N.S.	1	1.00	0.98	2.88	0.00	3.69	7.15	1.14	1.54	0.00
time (sec)	N/A	0.350	0.205	0.006	0.000	0.752	14.462	0.194	0.530	0.001
Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	95	196	0	302	488	89	132	0
N.S.	1	1.00	1.03	2.13	0.00	3.28	5.30	0.97	1.43	0.00
time (sec)	N/A	0.156	0.073	0.004	0.000	0.743	2.139	0.156	0.253	0.001
Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	193	622	0	625	0	204	2467	0
N.S.	1	1.00	0.98	3.17	0.00	3.19	0.00	1.04	12.59	0.00
time (sec)	N/A	0.349	0.206	0.009	0.000	89.668	0.000	0.163	10.450	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	281	1125	0	0	0	449	3991	0
N.S.	1	1.00	0.89	3.56	0.00	0.00	0.00	1.42	12.63	0.00
time (sec)	N/A	0.761	0.536	0.013	0.000	0.000	0.000	0.196	14.713	0.001
Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	509	509	504	1945	0	0	0	1002	12784	0
N.S.	1	1.00	0.99	3.82	0.00	0.00	0.00	1.97	25.12	0.00
time (sec)	N/A	1.251	0.766	0.019	0.000	0.000	0.000	0.206	6.819	0.001
Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	398	1712	0	2771	0	540	742	0
N.S.	1	1.00	1.38	5.94	0.00	9.62	0.00	1.88	2.58	0.00
time (sec)	N/A	0.700	0.811	0.016	0.000	1.749	0.000	0.184	5.779	0.001
Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	225	500	0	1413	1535	285	376	0
N.S.	1	1.00	1.26	2.81	0.00	7.94	8.62	1.60	2.11	0.00
time (sec)	N/A	0.266	0.462	0.012	0.000	1.566	60.262	0.167	5.039	0.001

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	114	194	0	632	459	125	203	0
N.S.	1	1.00	0.97	1.64	0.00	5.36	3.89	1.06	1.72	0.00
time (sec)	N/A	0.098	0.106	0.008	0.000	1.063	2.236	0.160	3.896	0.001

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	407	407	405	3202	0	0	0	860	13698	0
N.S.	1	1.00	1.00	7.87	0.00	0.00	0.00	2.11	33.66	0.00
time (sec)	N/A	1.087	0.905	0.023	0.000	0.000	0.000	0.193	6.700	0.001

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	673	673	650	4716	0	0	0	1437	26278	0
N.S.	1	1.00	0.97	7.01	0.00	0.00	0.00	2.14	39.05	0.00
time (sec)	N/A	2.559	2.206	0.038	0.000	0.000	0.000	0.327	8.926	0.001

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	60	53	51	75	60	51	55	0
N.S.	1	1.00	0.97	0.85	0.82	1.21	0.97	0.82	0.89	0.00
time (sec)	N/A	0.074	0.036	0.008	0.955	1.928	0.158	0.158	0.041	0.001

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	46	46	70	54	46	48	0
N.S.	1	1.00	1.00	0.84	0.84	1.27	0.98	0.84	0.87	0.00
time (sec)	N/A	0.066	0.026	0.007	0.950	0.693	0.151	0.159	0.043	0.001
Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	52	45	43	60	53	43	59	0
N.S.	1	1.00	1.00	0.87	0.83	1.15	1.02	0.83	1.13	0.00
time (sec)	N/A	0.052	0.021	0.005	0.956	1.041	0.148	0.151	3.838	0.000
Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	39	34	32	41	41	32	35	0
N.S.	1	1.00	0.95	0.83	0.78	1.00	1.00	0.78	0.85	0.00
time (sec)	N/A	0.033	0.022	0.005	0.945	0.614	0.137	0.157	3.833	0.000
Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	48	47	72	54	48	58	0
N.S.	1	1.00	1.00	0.86	0.84	1.29	0.96	0.86	1.04	0.00
time (sec)	N/A	0.089	0.026	0.009	0.953	1.089	0.184	0.151	0.100	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	55	54	85	65	55	68	0
N.S.	1	1.00	1.00	0.90	0.89	1.39	1.07	0.90	1.11	0.00
time (sec)	N/A	0.128	0.023	0.011	0.958	1.250	0.198	0.150	4.132	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	66	60	63	98	71	63	75	0
N.S.	1	1.00	0.97	0.88	0.93	1.44	1.04	0.93	1.10	0.00
time (sec)	N/A	0.110	0.030	0.011	0.954	1.689	0.217	0.158	0.098	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	8	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	0.80	1.00	0.00
time (sec)	N/A	0.011	0.006	0.004	0.430	1.112	0.095	0.149	0.045	0.001

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	28	27	27	36	27	29	0
N.S.	1	1.00	1.00	0.90	0.87	0.87	1.16	0.87	0.94	0.00
time (sec)	N/A	0.032	0.008	0.003	0.953	1.954	0.117	0.173	0.032	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	21	21	22	21	21	0
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.96	0.91	0.91	0.00
time (sec)	N/A	0.035	0.005	0.005	0.953	1.581	0.110	0.151	0.042	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	18	17	25	14	18	17	0
N.S.	1	1.00	0.90	0.86	0.81	1.19	0.67	0.86	0.81	0.00
time (sec)	N/A	0.013	0.009	0.007	0.420	3.098	0.087	0.166	0.042	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	14	16	14	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.89	0.78	0.00
time (sec)	N/A	0.018	0.005	0.006	0.427	1.070	0.111	0.149	3.922	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	12	14	12	0
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	1.00	0.86	0.00
time (sec)	N/A	0.017	0.006	0.006	0.426	0.744	0.114	0.156	3.851	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	31	22	21	21	22	21	17	0
N.S.	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63	0.00
time (sec)	N/A	0.028	0.005	0.004	0.943	0.799	0.123	0.151	3.800	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	30	36	55	46	45	35	0
N.S.	1	1.00	1.00	0.62	0.75	1.15	0.96	0.94	0.73	0.00
time (sec)	N/A	0.045	0.075	0.003	0.956	0.742	0.123	0.190	0.111	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	17	19	19	15	19	17	0
N.S.	1	1.00	1.00	0.81	0.90	0.90	0.71	0.90	0.81	0.00
time (sec)	N/A	0.012	0.008	0.005	0.425	0.783	0.119	0.157	3.841	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	34	30	39	37	30	36	0
N.S.	1	1.00	1.00	0.87	0.77	1.00	0.95	0.77	0.92	0.00
time (sec)	N/A	0.023	0.027	0.006	0.958	0.863	0.136	0.165	3.835	0.000



Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	33	33	31	11	11	0
N.S.	1	1.00	1.00	1.09	3.00	3.00	2.82	1.00	1.00	0.00
time (sec)	N/A	0.008	0.006	0.005	0.436	0.816	0.132	0.151	3.800	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	344	997	0	953	0	482	-1	489
N.S.	1	1.00	1.29	3.73	0.00	3.57	0.00	1.81	-0.00	1.83
time (sec)	N/A	0.238	0.861	0.011	0.000	1.661	0.000	0.275	0.000	2.146

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	267	613	0	605	0	297	-1	295
N.S.	1	1.00	1.26	2.89	0.00	2.85	0.00	1.40	-0.00	1.39
time (sec)	N/A	0.183	0.581	0.010	0.000	0.925	0.000	0.282	0.000	1.125

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	144	327	0	355	0	160	240	161
N.S.	1	1.00	0.92	2.08	0.00	2.26	0.00	1.02	1.53	1.03
time (sec)	N/A	0.124	0.208	0.008	0.000	0.794	0.000	0.217	4.262	0.554

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	86	136	0	203	0	84	-1	92
N.S.	1	1.00	0.83	1.31	0.00	1.95	0.00	0.81	-0.01	0.88
time (sec)	N/A	0.080	0.146	0.010	0.000	0.942	0.000	0.247	0.000	0.453

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	104	169	0	403	0	110	108	100
N.S.	1	1.00	1.06	1.72	0.00	4.11	0.00	1.12	1.10	1.02
time (sec)	N/A	0.077	0.728	0.008	0.000	1.730	0.000	0.270	4.210	0.484

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	107	137	0	242	0	193	127	128
N.S.	1	1.00	0.94	1.20	0.00	2.12	0.00	1.69	1.11	1.12
time (sec)	N/A	0.093	0.894	0.008	0.000	2.068	0.000	0.258	4.142	0.970

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	148	316	0	563	0	452	578	295
N.S.	1	1.00	0.89	1.89	0.00	3.37	0.00	2.71	3.46	1.77
time (sec)	N/A	0.107	1.933	0.009	0.000	17.197	0.000	0.284	4.525	2.454

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	220	220	199	555	0	978	0	805	1018	525
N.S.	1	1.00	0.90	2.52	0.00	4.45	0.00	3.66	4.63	2.39
time (sec)	N/A	0.142	1.738	0.012	0.000	49.881	0.000	0.313	5.063	5.093

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	930	927	1093	3543	0	2817	0	1702	3262	1767
N.S.	1	1.00	1.18	3.81	0.00	3.03	0.00	1.83	3.51	1.90
time (sec)	N/A	3.013	2.418	0.024	0.000	3.620	0.000	0.306	14.702	13.411

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	584	581	436	2179	0	1791	0	1012	1881	1037
N.S.	1	0.99	0.75	3.73	0.00	3.07	0.00	1.73	3.22	1.78
time (sec)	N/A	1.440	0.966	0.016	0.000	2.059	0.000	0.312	7.911	5.516

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	258	1117	0	1009	0	495	877	498
N.S.	1	1.00	0.80	3.47	0.00	3.13	0.00	1.54	2.72	1.55
time (sec)	N/A	0.504	0.476	0.010	0.000	2.763	0.000	0.237	5.624	2.074



Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	603	601	439	19321	0	0	0	0	-1	0
N.S.	1	1.00	0.73	32.04	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.449	1.925	0.020	0.000	0.000	0.000	0.000	0.000	180.018
Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	497	499	447	29161	0	0	0	0	-1	0
N.S.	1	1.00	0.90	58.67	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.855	3.901	0.026	0.000	0.000	0.000	0.000	0.000	180.080
Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	824	826	1128	40336	0	0	0	0	-1	0
N.S.	1	1.00	1.37	48.95	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	2.335	6.333	0.036	0.000	0.000	0.000	0.000	0.000	180.045
Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1169	1166	721	5881	0	4751	0	2977	-1	3156
N.S.	1	1.00	0.62	5.03	0.00	4.06	0.00	2.55	-0.00	2.70
time (sec)	N/A	3.698	2.707	0.029	0.000	14.763	0.000	0.438	0.000	9.589





Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1226	1223	1111	76693	0	0	0	0	-1	0
N.S.	1	1.00	0.91	62.56	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	3.997	6.296	0.069	0.000	0.000	0.000	0.000	0.000	180.061
Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	657	660	766	100754	0	0	0	0	-1	0
N.S.	1	1.00	1.17	153.35	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.216	6.243	0.133	0.000	0.000	0.000	0.000	0.000	180.013
Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1062	1062	1221	126612	0	0	0	0	-1	0
N.S.	1	1.00	1.15	119.22	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	3.004	6.419	0.184	0.000	0.000	0.000	0.000	0.000	180.121
Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	70	115	126	83	0	78	170	85
N.S.	1	1.00	0.49	0.80	0.88	0.58	0.00	0.55	1.19	0.59
time (sec)	N/A	0.138	0.050	0.016	0.951	1.815	0.000	0.263	5.541	0.680



Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	65	98	109	78	0	73	153	80
N.S.	1	1.00	0.55	0.83	0.92	0.66	0.00	0.62	1.30	0.68
time (sec)	N/A	0.112	0.038	0.008	0.957	0.754	0.000	0.303	5.151	0.585
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	60	81	92	73	0	68	136	75
N.S.	1	1.00	0.65	0.87	0.99	0.78	0.00	0.73	1.46	0.81
time (sec)	N/A	0.066	0.028	0.007	0.957	0.858	0.000	0.263	4.879	0.453
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	86	95	96	115	0	126	-1	114
N.S.	1	1.00	0.85	0.94	0.95	1.14	0.00	1.25	-0.01	1.13
time (sec)	N/A	0.117	0.054	0.010	0.963	1.331	0.000	0.394	0.000	0.439
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	92	123	103	133	0	380	-1	121
N.S.	1	1.00	0.85	1.14	0.95	1.23	0.00	3.52	-0.01	1.12
time (sec)	N/A	0.116	0.086	0.014	0.974	0.873	0.000	0.719	0.000	0.625

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	93	125	114	149	0	0	-1	121
N.S.	1	1.00	0.81	1.09	0.99	1.30	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.117	0.085	0.013	0.986	1.392	0.000	0.000	0.000	0.657

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	80	134	155	93	0	88	-1	95
N.S.	1	1.00	0.51	0.85	0.98	0.59	0.00	0.56	-0.01	0.60
time (sec)	N/A	0.201	0.050	0.016	0.978	1.500	0.000	0.259	0.000	0.921

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	75	117	138	88	0	83	-1	90
N.S.	1	1.00	0.53	0.83	0.98	0.62	0.00	0.59	-0.01	0.64
time (sec)	N/A	0.121	0.046	0.006	0.927	1.264	0.000	0.270	0.000	0.826

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	70	100	121	83	0	78	-1	85
N.S.	1	1.00	0.60	0.86	1.04	0.72	0.00	0.67	-0.01	0.73
time (sec)	N/A	0.082	0.037	0.005	0.966	2.011	0.000	0.211	0.000	0.653

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	96	151	125	125	0	136	-1	124
N.S.	1	1.00	0.77	1.22	1.01	1.01	0.00	1.10	-0.01	1.00
time (sec)	N/A	0.144	0.062	0.008	0.988	0.958	0.000	0.283	0.000	0.556
Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	103	179	132	143	0	570	-1	131
N.S.	1	1.00	0.79	1.37	1.01	1.09	0.00	4.35	-0.01	1.00
time (sec)	N/A	0.140	0.092	0.011	0.978	1.403	0.000	0.841	0.000	0.627
Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	103	162	143	159	0	0	-1	131
N.S.	1	1.00	0.75	1.17	1.04	1.15	0.00	0.00	-0.01	0.95
time (sec)	N/A	0.139	0.107	0.013	0.984	0.808	0.000	0.000	0.000	0.788
Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	90	153	184	103	0	98	-1	105
N.S.	1	1.00	0.48	0.81	0.97	0.54	0.00	0.52	-0.01	0.56
time (sec)	N/A	0.156	0.062	0.017	0.978	1.616	0.000	0.201	0.000	1.113

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	85	136	167	98	0	93	-1	100
N.S.	1	1.00	0.52	0.83	1.02	0.60	0.00	0.57	-0.01	0.61
time (sec)	N/A	0.133	0.056	0.007	0.971	1.080	0.000	0.212	0.000	1.047

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	80	119	150	93	0	88	-1	95
N.S.	1	1.00	0.58	0.86	1.08	0.67	0.00	0.63	-0.01	0.68
time (sec)	N/A	0.092	0.048	0.006	0.967	1.092	0.000	0.187	0.000	0.910

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	106	207	154	135	0	146	-1	134
N.S.	1	1.00	0.72	1.41	1.05	0.92	0.00	0.99	-0.01	0.91
time (sec)	N/A	0.160	0.075	0.010	0.972	1.140	0.000	0.307	0.000	0.781

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	113	235	161	153	0	760	-1	141
N.S.	1	1.00	0.73	1.53	1.05	0.99	0.00	4.94	-0.01	0.92
time (sec)	N/A	0.163	0.116	0.012	0.999	0.622	0.000	1.097	0.000	0.866

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	113	199	172	169	0	0	-1	141
N.S.	1	1.00	0.70	1.24	1.07	1.05	0.00	0.00	-0.01	0.88
time (sec)	N/A	0.163	0.114	0.015	0.994	1.112	0.000	0.000	0.000	1.034

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	693	692	588	1869	0	1435	0	822	-1	820
N.S.	1	1.00	0.85	2.70	0.00	2.07	0.00	1.19	-0.00	1.18
time (sec)	N/A	2.102	1.251	0.019	0.000	2.304	0.000	0.323	0.000	3.585

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	420	418	343	1069	0	861	0	457	-1	447
N.S.	1	1.00	0.82	2.55	0.00	2.05	0.00	1.09	-0.00	1.06
time (sec)	N/A	1.011	0.649	0.013	0.000	1.983	0.000	0.296	0.000	1.638

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	215	505	0	461	0	210	-1	202
N.S.	1	1.00	0.96	2.26	0.00	2.07	0.00	0.94	-0.00	0.91
time (sec)	N/A	0.303	0.234	0.008	0.000	2.244	0.000	0.269	0.000	0.788

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	96	185	0	227	0	98	-1	102
N.S.	1	1.00	0.83	1.59	0.00	1.96	0.00	0.84	-0.01	0.88
time (sec)	N/A	0.106	0.152	0.007	0.000	2.725	0.000	0.249	0.000	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	172	599	0	0	0	0	-1	198
N.S.	1	1.00	0.96	3.35	0.00	0.00	0.00	0.00	-0.01	1.11
time (sec)	N/A	0.293	0.281	0.015	0.000	0.000	0.000	0.000	0.000	0.781

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	239	227	1671	0	0	0	0	-1	379
N.S.	1	0.99	0.94	6.93	0.00	0.00	0.00	0.00	-0.00	1.57
time (sec)	N/A	0.371	0.348	0.015	0.000	0.000	0.000	0.000	0.000	1.273

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	336	336	367	3615	0	0	0	2307	-1	0
N.S.	1	1.00	1.09	10.76	0.00	0.00	0.00	6.87	-0.00	0.00
time (sec)	N/A	0.656	1.100	0.017	0.000	0.000	0.000	0.545	0.000	180.264

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	504	502	715	2780	0	2937	0	1054	-1	1034
N.S.	1	1.00	1.42	5.52	0.00	5.83	0.00	2.09	-0.00	2.05
time (sec)	N/A	1.176	1.595	0.020	0.000	45.678	0.000	0.350	0.000	4.931
Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	288	412	1557	0	1769	0	580	-1	552
N.S.	1	1.00	1.43	5.39	0.00	6.12	0.00	2.01	-0.00	1.91
time (sec)	N/A	0.392	0.827	0.013	0.000	28.875	0.000	0.322	0.000	3.102
Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	205	735	0	905	0	271	-1	245
N.S.	1	1.00	1.10	3.95	0.00	4.87	0.00	1.46	-0.01	1.32
time (sec)	N/A	0.227	0.734	0.009	0.000	19.704	0.000	0.284	0.000	1.099
Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	113	249	0	429	0	122	143	111
N.S.	1	1.00	1.02	2.24	0.00	3.86	0.00	1.10	1.29	1.00
time (sec)	N/A	0.066	0.295	0.006	0.000	2.454	0.000	0.267	4.535	0.002

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	271	2079	0	1905	0	719	-1	293
N.S.	1	1.00	1.20	9.24	0.00	8.47	0.00	3.20	-0.00	1.30
time (sec)	N/A	0.266	0.493	0.014	0.000	53.931	0.000	0.286	0.000	1.215
Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	421	418	487	4930	0	0	0	0	-1	3385
N.S.	1	0.99	1.16	11.71	0.00	0.00	0.00	0.00	-0.00	8.04
time (sec)	N/A	0.797	2.460	0.019	0.000	0.000	0.000	0.000	0.000	11.493
Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	713	707	762	9126	0	0	0	5637	-1	0
N.S.	1	0.99	1.07	12.80	0.00	0.00	0.00	7.91	-0.00	0.00
time (sec)	N/A	2.670	5.291	0.024	0.000	0.000	0.000	0.882	0.000	180.037
Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	60	96	97	73	0	68	-1	75
N.S.	1	1.00	0.50	0.80	0.81	0.61	0.00	0.57	-0.01	0.62
time (sec)	N/A	0.135	0.045	0.015	0.954	0.680	0.000	0.244	0.000	0.594



Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	55	79	80	68	0	63	-1	70
N.S.	1	1.00	0.58	0.83	0.84	0.72	0.00	0.66	-0.01	0.74
time (sec)	N/A	0.099	0.031	0.008	0.972	1.494	0.000	0.221	0.000	0.508
Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	50	62	63	63	0	58	-1	65
N.S.	1	1.00	0.71	0.89	0.90	0.90	0.00	0.83	-0.01	0.93
time (sec)	N/A	0.058	0.021	0.007	0.960	0.691	0.000	0.264	0.000	0.360
Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	78	60	67	105	0	116	-1	101
N.S.	1	1.00	1.00	0.77	0.86	1.35	0.00	1.49	-0.01	1.29
time (sec)	N/A	0.097	0.046	0.009	0.973	1.529	0.000	0.572	0.000	0.396
Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	82	67	74	123	0	48	-1	109
N.S.	1	1.00	0.99	0.81	0.89	1.48	0.00	0.58	-0.01	1.31
time (sec)	N/A	0.095	0.053	0.012	0.979	0.991	0.000	0.277	0.000	0.421

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	69	74	82	96	0	204	-1	80
N.S.	1	1.00	0.78	0.83	0.92	1.08	0.00	2.29	-0.01	0.90
time (sec)	N/A	0.088	0.039	0.035	0.977	1.498	0.000	0.328	0.000	0.470

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	69	115	97	97	0	67	-1	75
N.S.	1	1.00	0.67	1.12	0.94	0.94	0.00	0.65	-0.01	0.73
time (sec)	N/A	0.124	0.041	0.014	0.965	0.866	0.000	0.210	0.000	0.643

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	61	98	80	92	0	62	-1	70
N.S.	1	1.00	0.74	1.20	0.98	1.12	0.00	0.76	-0.01	0.85
time (sec)	N/A	0.108	0.033	0.009	0.969	1.433	0.000	0.271	0.000	0.644

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	50	81	63	87	0	57	-1	65
N.S.	1	1.00	0.79	1.29	1.00	1.38	0.00	0.90	-0.02	1.03
time (sec)	N/A	0.060	0.119	0.006	0.943	0.990	0.000	0.272	0.000	0.561

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	73	102	64	96	0	91	-1	73
N.S.	1	1.00	1.18	1.65	1.03	1.55	0.00	1.47	-0.02	1.18
time (sec)	N/A	0.074	0.023	0.009	0.965	1.420	0.000	0.581	0.000	0.492
Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	74	109	96	106	0	168	-1	85
N.S.	1	1.00	0.85	1.25	1.10	1.22	0.00	1.93	-0.01	0.98
time (sec)	N/A	0.092	0.044	0.012	0.955	1.055	0.000	0.297	0.000	0.508
Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	79	111	145	126	0	223	-1	90
N.S.	1	1.00	0.71	0.99	1.29	1.12	0.00	1.99	-0.01	0.80
time (sec)	N/A	0.155	0.056	0.013	0.967	1.033	0.000	0.313	0.000	0.586
Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	71	163	202	117	0	67	-1	75
N.S.	1	1.00	0.83	1.90	2.35	1.36	0.00	0.78	-0.01	0.87
time (sec)	N/A	0.113	0.061	0.014	0.963	1.014	0.000	0.208	0.000	0.858

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	66	146	185	112	0	62	-1	70
N.S.	1	1.00	0.97	2.15	2.72	1.65	0.00	0.91	-0.01	1.03
time (sec)	N/A	0.095	0.126	0.009	0.967	0.936	0.000	0.237	0.000	0.719
Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	33	30	76	51	0	28	49	33
N.S.	1	1.00	0.70	0.64	1.62	1.09	0.00	0.60	1.04	0.70
time (sec)	N/A	0.048	0.059	0.005	0.437	1.196	0.000	0.196	4.196	0.550
Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	72	158	93	126	0	101	-1	83
N.S.	1	1.00	0.85	1.86	1.09	1.48	0.00	1.19	-0.01	0.98
time (sec)	N/A	0.094	0.051	0.010	0.962	0.954	0.000	0.434	0.000	0.624
Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	111	165	125	141	0	233	-1	95
N.S.	1	1.00	1.01	1.50	1.14	1.28	0.00	2.12	-0.01	0.86
time (sec)	N/A	0.151	0.064	0.011	0.974	0.742	0.000	0.363	0.000	0.717

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	89	148	174	156	0	233	-1	100
N.S.	1	1.00	0.66	1.10	1.29	1.16	0.00	1.73	-0.01	0.74
time (sec)	N/A	0.213	0.075	0.013	0.991	1.730	0.000	0.349	0.000	0.828

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	219	324	0	465	0	0	1089	0
N.S.	1	1.00	1.05	1.56	0.00	2.24	0.00	0.00	5.24	0.00
time (sec)	N/A	0.424	0.505	0.013	0.000	60.068	0.000	0.000	5.748	180.180

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	119	36	59	75	221	141	58	0
N.S.	1	1.00	2.90	0.88	1.44	1.83	5.39	3.44	1.41	0.00
time (sec)	N/A	0.052	0.106	0.005	0.576	0.893	13.248	0.181	4.246	0.293

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	34	39	66	83	280	191	78	0
N.S.	1	1.00	0.74	0.85	1.43	1.80	6.09	4.15	1.70	0.00
time (sec)	N/A	0.072	0.130	0.005	0.576	1.106	173.947	0.244	4.395	0.347

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	43	51	98	123	483	314	120	0
N.S.	1	1.00	0.75	0.89	1.72	2.16	8.47	5.51	2.11	0.00
time (sec)	N/A	0.121	0.306	0.006	0.598	0.808	171.168	0.306	4.457	0.313

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	167	8419	1779	2467	2281	2383	2026	0
N.S.	1	1.00	8.35	420.95	88.95	123.35	114.05	119.15	101.30	0.00
time (sec)	N/A	0.420	0.449	0.004	0.500	0.785	1.527	0.230	4.865	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	19	18	18	20	20	18	0
N.S.	1	1.00	1.00	0.73	0.69	0.69	0.77	0.77	0.69	0.00
time (sec)	N/A	0.025	0.005	0.005	0.426	0.597	0.244	0.165	0.051	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	282	783	0	701	0	330	-1	324
N.S.	1	1.00	0.82	2.26	0.00	2.03	0.00	0.95	-0.00	0.94
time (sec)	N/A	0.812	0.739	0.014	0.000	0.948	0.000	0.314	0.000	1.329

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	199	532	0	499	0	228	-1	220
N.S.	1	1.00	0.81	2.17	0.00	2.04	0.00	0.93	-0.00	0.90
time (sec)	N/A	0.438	0.443	0.010	0.000	0.764	0.000	0.386	0.000	0.864
Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	141	333	0	341	0	149	-1	144
N.S.	1	1.00	0.80	1.88	0.00	1.93	0.00	0.84	-0.01	0.81
time (sec)	N/A	0.235	0.264	0.009	0.000	1.041	0.000	0.268	0.000	0.595
Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	134	220	0	733	0	0	-1	145
N.S.	1	1.00	0.86	1.42	0.00	4.73	0.00	0.00	-0.01	0.94
time (sec)	N/A	0.256	0.369	0.010	0.000	6.925	0.000	0.000	0.000	0.641
Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	127	173	0	703	0	171	166	127
N.S.	1	1.00	0.91	1.24	0.00	5.06	0.00	1.23	1.19	0.91
time (sec)	N/A	0.235	0.399	0.012	0.000	6.164	0.000	0.372	4.460	0.656

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	137	241	0	783	0	352	-1	172
N.S.	1	1.00	0.86	1.52	0.00	4.92	0.00	2.21	-0.01	1.08
time (sec)	N/A	0.244	0.359	0.012	0.000	8.041	0.000	0.361	0.000	0.729

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	150	375	0	365	0	689	-1	193
N.S.	1	1.00	0.81	2.02	0.00	1.96	0.00	3.70	-0.01	1.04
time (sec)	N/A	0.320	0.310	0.014	0.000	7.645	0.000	0.292	0.000	1.167

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	212	591	0	525	0	1448	-1	278
N.S.	1	1.00	0.79	2.19	0.00	1.94	0.00	5.36	-0.00	1.03
time (sec)	N/A	0.488	0.522	0.013	0.000	15.353	0.000	0.300	0.000	2.178

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	371	371	299	859	0	727	0	2177	-1	391
N.S.	1	1.00	0.81	2.32	0.00	1.96	0.00	5.87	-0.00	1.05
time (sec)	N/A	0.817	0.731	0.018	0.000	40.333	0.000	0.508	0.000	2.972



Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	212	208	206	237	230	230	196	0
N.S.	1	1.00	0.82	0.81	0.80	0.92	0.89	0.89	0.76	0.00
time (sec)	N/A	0.257	0.040	0.002	0.434	0.742	0.518	0.168	4.204	0.000
Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	136	146	145	160	158	160	137	0
N.S.	1	1.00	0.87	0.93	0.92	1.02	1.01	1.02	0.87	0.00
time (sec)	N/A	0.166	0.032	0.002	0.432	0.881	0.153	0.157	4.108	0.000
Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	93	84	79	83	87	90	77	0
N.S.	1	1.00	1.00	0.90	0.85	0.89	0.94	0.97	0.83	0.00
time (sec)	N/A	0.108	0.013	0.001	0.428	0.762	1.911	0.151	0.047	0.000
Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	35	34	34	37	34	34	0
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.88	0.81	0.81	0.00
time (sec)	N/A	0.025	0.002	0.002	0.423	0.631	0.288	0.148	0.026	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	179	286	228	230	235	228	260	0
N.S.	1	1.00	0.79	1.25	1.00	1.01	1.03	1.00	1.14	0.00
time (sec)	N/A	0.193	0.059	0.006	0.433	0.734	1.174	0.155	4.139	0.001

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	223	313	234	319	238	308	363	0
N.S.	1	1.00	0.98	1.37	1.03	1.40	1.04	1.35	1.59	0.00
time (sec)	N/A	0.191	0.087	0.012	0.435	0.467	1.134	0.173	4.181	0.001

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	204	336	240	360	248	216	297	0
N.S.	1	1.00	0.88	1.45	1.04	1.56	1.07	0.94	1.29	0.00
time (sec)	N/A	0.204	0.066	0.011	0.444	0.768	2.627	0.180	0.092	0.001

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	391	391	277	264	263	305	298	296	251	0
N.S.	1	1.00	0.71	0.68	0.67	0.78	0.76	0.76	0.64	0.00
time (sec)	N/A	0.386	0.043	0.002	0.435	0.756	0.203	0.157	4.247	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	201	186	185	206	206	206	175	0
N.S.	1	1.00	1.00	0.93	0.92	1.02	1.02	1.02	0.87	0.00
time (sec)	N/A	0.240	0.026	0.001	0.429	0.460	0.146	0.161	0.108	0.000
Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	121	108	105	107	112	116	101	0
N.S.	1	1.00	1.00	0.89	0.87	0.88	0.93	0.96	0.83	0.00
time (sec)	N/A	0.160	0.016	0.001	0.431	0.806	0.136	0.156	4.173	0.000
Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	45	44	44	56	44	44	0
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.93	0.73	0.73	0.00
time (sec)	N/A	0.036	0.001	0.000	0.423	0.636	0.155	0.149	0.035	0.000
Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	352	352	262	465	366	368	372	378	434	0
N.S.	1	1.00	0.74	1.32	1.04	1.05	1.06	1.07	1.23	0.00
time (sec)	N/A	0.316	0.122	0.006	0.436	1.012	0.997	0.170	0.079	0.001

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	342	500	372	490	393	459	939	0
N.S.	1	1.00	0.97	1.42	1.05	1.39	1.11	1.30	2.66	0.00
time (sec)	N/A	0.328	0.139	0.012	0.438	0.881	2.311	0.180	4.217	0.001

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	354	354	311	531	378	545	394	354	771	0
N.S.	1	1.00	0.88	1.50	1.07	1.54	1.11	1.00	2.18	0.00
time (sec)	N/A	0.343	0.102	0.014	0.449	1.565	4.872	0.161	0.127	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	344	558	390	587	401	345	560	0
N.S.	1	1.00	0.96	1.55	1.08	1.63	1.11	0.96	1.56	0.00
time (sec)	N/A	0.358	0.123	0.014	0.462	0.932	8.089	0.159	4.284	0.001

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	178	291	206	206	450	212	397	0
N.S.	1	1.00	0.81	1.32	0.93	0.93	2.04	0.96	1.80	0.00
time (sec)	N/A	0.190	0.121	0.007	0.964	0.768	2.579	0.167	4.182	0.001

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	130	191	141	141	303	145	223	0
N.S.	1	1.00	0.83	1.22	0.90	0.90	1.94	0.93	1.43	0.00
time (sec)	N/A	0.162	0.085	0.006	0.961	0.776	1.719	0.159	0.099	0.001
Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	86	102	84	84	163	88	107	0
N.S.	1	1.00	0.87	1.03	0.85	0.85	1.65	0.89	1.08	0.00
time (sec)	N/A	0.108	0.054	0.005	0.956	0.800	0.849	0.155	0.070	0.001
Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	50	44	43	43	61	43	45	0
N.S.	1	1.00	0.89	0.79	0.77	0.77	1.09	0.77	0.80	0.00
time (sec)	N/A	0.049	0.018	0.003	0.967	0.754	0.232	0.162	0.042	0.000
Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	146	298	160	171	0	158	713	0
N.S.	1	1.00	0.87	1.77	0.95	1.02	0.00	0.94	4.24	0.00
time (sec)	N/A	0.194	0.110	0.012	0.964	0.800	0.000	0.216	6.389	0.001

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	233	538	294	416	0	355	312	0
N.S.	1	1.00	1.00	2.31	1.26	1.79	0.00	1.52	1.34	0.00
time (sec)	N/A	0.251	0.160	0.016	0.983	0.982	0.000	0.177	4.669	0.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	278	819	498	698	0	406	493	0
N.S.	1	1.00	0.88	2.58	1.57	2.20	0.00	1.28	1.56	0.00
time (sec)	N/A	0.290	0.434	0.015	0.998	1.265	0.000	0.179	4.763	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	209	283	212	350	444	206	333	0
N.S.	1	1.00	1.11	1.50	1.12	1.85	2.35	1.09	1.76	0.00
time (sec)	N/A	0.255	0.158	0.016	0.958	0.594	2.772	0.162	0.148	0.001

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	150	189	147	245	298	145	211	0
N.S.	1	1.00	1.07	1.35	1.05	1.75	2.13	1.04	1.51	0.00
time (sec)	N/A	0.208	0.113	0.012	0.960	0.825	1.961	0.156	0.114	0.001

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	96	106	90	147	165	94	115	0
N.S.	1	1.00	0.99	1.09	0.93	1.52	1.70	0.97	1.19	0.00
time (sec)	N/A	0.193	0.066	0.010	0.961	0.631	1.023	0.156	4.152	0.001
Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	51	52	78	65	52	52	0
N.S.	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.83	0.00
time (sec)	N/A	0.077	0.037	0.007	0.952	0.387	0.192	0.153	4.148	0.000
Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	186	691	289	479	0	284	330	0
N.S.	1	1.00	0.83	3.08	1.29	2.14	0.00	1.27	1.47	0.00
time (sec)	N/A	0.340	0.164	0.020	0.982	0.912	0.000	0.170	4.612	0.001
Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	313	313	270	986	548	910	0	571	601	0
N.S.	1	1.00	0.86	3.15	1.75	2.91	0.00	1.82	1.92	0.00
time (sec)	N/A	0.498	0.262	0.023	1.015	1.100	0.000	0.204	4.835	0.001

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	412	412	363	1314	851	1499	0	595	887	0
N.S.	1	1.00	0.88	3.19	2.07	3.64	0.00	1.44	2.15	0.00
time (sec)	N/A	0.715	0.396	0.027	1.090	1.439	0.000	0.204	4.940	0.001

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	209	267	222	441	469	201	299	0
N.S.	1	1.00	1.22	1.56	1.30	2.58	2.74	1.18	1.75	0.00
time (sec)	N/A	0.336	0.201	0.015	0.968	0.767	8.078	0.208	0.152	0.001

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	146	179	155	302	304	144	203	0
N.S.	1	1.00	1.09	1.34	1.16	2.25	2.27	1.07	1.51	0.00
time (sec)	N/A	0.239	0.182	0.013	0.961	0.807	3.963	0.165	4.214	0.001

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	107	102	101	172	163	97	125	0
N.S.	1	1.00	1.04	0.99	0.98	1.67	1.58	0.94	1.21	0.00
time (sec)	N/A	0.145	0.085	0.011	0.956	0.587	2.311	0.185	0.120	0.001



Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	53	47	56	75	61	46	55	0
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.95	0.72	0.86	0.00
time (sec)	N/A	0.050	0.039	0.007	0.958	0.809	0.201	0.155	0.049	0.001
Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	329	329	282	1437	571	1052	0	460	641	0
N.S.	1	1.00	0.86	4.37	1.74	3.20	0.00	1.40	1.95	0.00
time (sec)	N/A	0.496	0.305	0.025	1.030	1.545	0.000	0.247	4.793	0.001
Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	443	443	389	1850	916	1734	0	762	965	0
N.S.	1	1.00	0.88	4.18	2.07	3.91	0.00	1.72	2.18	0.00
time (sec)	N/A	0.893	0.526	0.031	1.134	1.589	0.000	0.256	4.989	0.001
Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	70	115	126	83	0	78	170	85
N.S.	1	1.00	0.49	0.80	0.88	0.58	0.00	0.55	1.19	0.59
time (sec)	N/A	0.155	0.154	0.017	0.965	0.864	0.000	0.204	1.716	1.289

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	98	109	78	0	73	153	80
N.S.	1	1.00	0.52	0.79	0.88	0.63	0.00	0.59	1.23	0.65
time (sec)	N/A	0.092	0.095	0.006	0.954	0.819	0.000	0.199	0.767	0.599
Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	91	127	128	125	0	129	-1	117
N.S.	1	1.00	0.61	0.85	0.86	0.84	0.00	0.87	-0.01	0.79
time (sec)	N/A	0.240	0.147	0.010	1.006	0.784	0.000	0.224	0.000	0.611
Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	98	152	132	143	0	531	-1	124
N.S.	1	1.00	0.66	1.02	0.89	0.96	0.00	3.56	-0.01	0.83
time (sec)	N/A	0.237	0.162	0.013	1.003	0.866	0.000	0.531	0.000	0.670
Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	98	158	143	159	0	258	-1	124
N.S.	1	1.00	0.65	1.05	0.95	1.05	0.00	1.71	-0.01	0.82
time (sec)	N/A	0.228	0.148	0.015	1.002	0.812	0.000	0.244	0.000	0.778

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	98	165	160	173	0	304	-1	124
N.S.	1	1.00	0.62	1.04	1.01	1.09	0.00	1.92	-0.01	0.78
time (sec)	N/A	0.226	0.159	0.013	0.980	0.834	0.000	0.258	0.000	0.876
Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	98	167	181	189	0	327	-1	124
N.S.	1	1.00	0.59	1.01	1.10	1.15	0.00	1.98	-0.01	0.75
time (sec)	N/A	0.234	0.176	0.013	1.022	1.214	0.000	0.373	0.000	0.827
Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	98	188	222	203	0	387	-1	124
N.S.	1	1.00	0.59	1.14	1.35	1.23	0.00	2.35	-0.01	0.75
time (sec)	N/A	0.229	0.200	0.014	1.044	1.073	0.000	0.278	0.000	0.955
Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	91	195	250	156	0	405	-1	93
N.S.	1	1.00	0.54	1.15	1.48	0.92	0.00	2.40	-0.01	0.55
time (sec)	N/A	0.218	0.169	0.016	1.050	0.530	0.000	0.264	0.000	1.005

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	96	216	301	171	0	456	-1	98
N.S.	1	1.00	0.49	1.11	1.55	0.88	0.00	2.35	-0.01	0.51
time (sec)	N/A	0.268	0.198	0.019	1.039	0.898	0.000	0.293	0.000	1.303
Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	80	134	155	93	0	88	-1	95
N.S.	1	1.00	0.48	0.81	0.93	0.56	0.00	0.53	-0.01	0.57
time (sec)	N/A	0.194	0.184	0.018	0.977	0.861	0.000	0.191	0.000	0.958
Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	75	117	138	88	0	83	-1	90
N.S.	1	1.00	0.51	0.80	0.94	0.60	0.00	0.56	-0.01	0.61
time (sec)	N/A	0.121	0.124	0.005	1.003	0.877	0.000	0.334	0.000	0.869
Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	101	183	157	135	0	139	-1	127
N.S.	1	1.00	0.59	1.06	0.91	0.78	0.00	0.81	-0.01	0.74
time (sec)	N/A	0.268	0.183	0.010	1.002	0.827	0.000	0.231	0.000	0.887

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	108	208	161	153	0	707	-1	134
N.S.	1	1.00	0.63	1.21	0.94	0.89	0.00	4.11	-0.01	0.78
time (sec)	N/A	0.282	0.210	0.013	1.013	1.002	0.000	0.451	0.000	1.057
Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	108	214	172	169	0	268	-1	134
N.S.	1	1.00	0.62	1.23	0.99	0.97	0.00	1.54	-0.01	0.77
time (sec)	N/A	0.273	0.216	0.015	1.001	0.885	0.000	0.303	0.000	0.996
Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	108	221	189	183	0	314	-1	134
N.S.	1	1.00	0.60	1.22	1.04	1.01	0.00	1.73	-0.01	0.74
time (sec)	N/A	0.267	0.219	0.017	1.036	0.816	0.000	0.269	0.000	1.078
Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	108	204	210	199	0	503	-1	134
N.S.	1	1.00	0.57	1.09	1.12	1.06	0.00	2.68	-0.01	0.71
time (sec)	N/A	0.265	0.228	0.017	1.042	1.303	0.000	0.421	0.000	0.974

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	108	225	251	213	0	406	-1	134
N.S.	1	1.00	0.55	1.15	1.29	1.09	0.00	2.08	-0.01	0.69
time (sec)	N/A	0.263	0.237	0.017	1.060	0.810	0.000	0.324	0.000	1.205

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	108	246	297	229	0	452	-1	134
N.S.	1	1.00	0.55	1.26	1.52	1.17	0.00	2.32	-0.01	0.69
time (sec)	N/A	0.268	0.253	0.017	1.072	0.793	0.000	0.345	0.000	1.356

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	108	267	348	243	0	489	-1	134
N.S.	1	1.00	0.55	1.37	1.78	1.25	0.00	2.51	-0.01	0.69
time (sec)	N/A	0.262	0.259	0.019	1.074	2.063	0.000	0.332	0.000	1.891

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	60	95	96	73	0	68	-1	75
N.S.	1	1.00	0.50	0.79	0.80	0.61	0.00	0.57	-0.01	0.62
time (sec)	N/A	0.136	0.117	0.012	0.974	0.710	0.000	0.194	0.000	0.598

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	55	79	80	68	0	63	-1	70
N.S.	1	1.00	0.54	0.78	0.79	0.67	0.00	0.62	-0.01	0.69
time (sec)	N/A	0.080	0.073	0.006	0.963	0.679	0.000	0.196	0.000	0.541
Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	81	92	99	115	0	119	-1	107
N.S.	1	1.00	0.64	0.73	0.79	0.91	0.00	0.94	-0.01	0.85
time (sec)	N/A	0.210	0.102	0.008	0.981	0.842	0.000	0.232	0.000	0.525
Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	88	96	103	133	0	339	-1	114
N.S.	1	1.00	0.70	0.76	0.82	1.06	0.00	2.69	-0.01	0.90
time (sec)	N/A	0.202	0.115	0.012	0.980	1.323	0.000	0.402	0.000	0.767
Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	88	102	114	149	0	248	-1	114
N.S.	1	1.00	0.69	0.80	0.89	1.16	0.00	1.94	-0.01	0.89
time (sec)	N/A	0.208	0.129	0.012	0.984	1.514	0.000	0.255	0.000	0.704

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	88	109	131	163	0	285	-1	114
N.S.	1	1.00	0.65	0.81	0.97	1.21	0.00	2.11	-0.01	0.84
time (sec)	N/A	0.205	0.149	0.014	1.003	1.696	0.000	0.257	0.000	0.763
Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	81	116	149	125	0	164	-1	83
N.S.	1	1.00	0.58	0.83	1.07	0.90	0.00	1.18	-0.01	0.60
time (sec)	N/A	0.191	0.135	0.013	1.024	0.860	0.000	0.283	0.000	0.783
Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	74	132	114	102	0	72	-1	80
N.S.	1	1.00	0.60	1.06	0.92	0.82	0.00	0.58	-0.01	0.65
time (sec)	N/A	0.152	0.470	0.017	0.972	0.936	0.000	0.225	0.000	0.647
Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	60	115	97	97	0	67	-1	75
N.S.	1	1.00	0.58	1.12	0.94	0.94	0.00	0.65	-0.01	0.73
time (sec)	N/A	0.102	0.185	0.007	0.970	1.442	0.000	0.216	0.000	0.600



Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	55	98	80	92	0	62	-1	70
N.S.	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01	0.85
time (sec)	N/A	0.055	0.131	0.008	0.952	1.647	0.000	0.219	0.000	0.599
Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	86	148	99	149	0	118	-1	107
N.S.	1	1.00	0.85	1.47	0.98	1.48	0.00	1.17	-0.01	1.06
time (sec)	N/A	0.151	0.354	0.011	0.977	1.167	0.000	0.396	0.000	0.528
Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	104	152	116	157	0	225	-1	114
N.S.	1	1.00	0.96	1.41	1.07	1.45	0.00	2.08	-0.01	1.06
time (sec)	N/A	0.153	0.323	0.011	1.007	1.153	0.000	0.423	0.000	0.630
Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	84	144	149	126	0	220	-1	83
N.S.	1	1.00	0.75	1.29	1.33	1.12	0.00	1.96	-0.01	0.74
time (sec)	N/A	0.146	0.298	0.013	0.985	1.198	0.000	0.247	0.000	0.588

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	95	151	217	141	0	271	-1	88
N.S.	1	1.00	0.69	1.10	1.58	1.03	0.00	1.98	-0.01	0.64
time (sec)	N/A	0.204	0.158	0.014	1.007	1.075	0.000	0.292	0.000	0.678

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	65	180	219	122	0	71	-1	80
N.S.	1	1.00	0.62	1.71	2.09	1.16	0.00	0.68	-0.01	0.76
time (sec)	N/A	0.131	0.687	0.023	0.983	1.081	0.000	0.216	0.000	0.675

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	60	163	202	117	0	66	-1	75
N.S.	1	1.00	0.70	1.90	2.35	1.36	0.00	0.77	-0.01	0.87
time (sec)	N/A	0.082	0.253	0.008	0.967	0.923	0.000	0.206	0.000	0.867

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	55	146	185	112	0	62	-1	70
N.S.	1	1.00	0.81	2.15	2.72	1.65	0.00	0.91	-0.01	1.03
time (sec)	N/A	0.052	0.229	0.007	0.983	0.933	0.000	0.304	0.000	0.614

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	80	190	110	126	0	92	-1	76
N.S.	1	1.00	0.94	2.24	1.29	1.48	0.00	1.08	-0.01	0.89
time (sec)	N/A	0.128	0.470	0.009	0.968	0.836	0.000	0.229	0.000	0.657
Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	92	194	127	141	0	206	-1	88
N.S.	1	1.00	0.84	1.76	1.15	1.28	0.00	1.87	-0.01	0.80
time (sec)	N/A	0.153	0.408	0.011	1.006	1.237	0.000	0.399	0.000	0.802
Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	97	200	178	155	0	228	-1	93
N.S.	1	1.00	0.72	1.48	1.32	1.15	0.00	1.69	-0.01	0.69
time (sec)	N/A	0.221	0.310	0.013	0.998	1.100	0.000	0.265	0.000	0.757
Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	89	207	246	170	0	279	-1	98
N.S.	1	1.00	0.56	1.29	1.54	1.06	0.00	1.74	-0.01	0.61
time (sec)	N/A	0.283	0.236	0.014	1.013	0.722	0.000	0.280	0.000	0.863

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	354	354	316	1406	0	1373	0	465	-1	425
N.S.	1	1.00	0.89	3.97	0.00	3.88	0.00	1.31	-0.00	1.20
time (sec)	N/A	0.377	1.145	0.015	0.000	95.075	0.000	0.306	0.000	2.611

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	353	353	319	1453	0	1385	0	488	-1	0
N.S.	1	1.00	0.90	4.12	0.00	3.92	0.00	1.38	-0.00	0.00
time (sec)	N/A	0.386	1.111	0.018	0.000	78.227	0.000	0.448	0.000	180.013

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	588	588	537	5924	2292	4795	0	10960	4341	0
N.S.	1	1.00	0.91	10.07	3.90	8.15	0.00	18.64	7.38	0.00
time (sec)	N/A	0.364	0.382	0.076	0.745	0.940	0.000	0.621	8.392	0.232

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	432	432	391	3222	1414	2796	0	6223	2625	0
N.S.	1	1.00	0.91	7.46	3.27	6.47	0.00	14.41	6.08	0.00
time (sec)	N/A	0.243	0.242	0.035	0.641	1.367	0.000	0.392	6.050	0.169

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	292	261	1504	788	1448	0	3098	1425	0
N.S.	1	1.00	0.89	5.15	2.70	4.96	0.00	10.61	4.88	0.00
time (sec)	N/A	0.189	0.169	0.016	0.548	0.783	0.000	0.266	5.087	0.124
Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	528	528	488	1244	0	3480	0	657	1027	0
N.S.	1	1.00	0.92	2.36	0.00	6.59	0.00	1.24	1.95	0.00
time (sec)	N/A	1.312	1.079	0.019	0.000	0.870	0.000	0.220	6.175	0.001
Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	765	765	754	1960	0	2643	0	982	2779	0
N.S.	1	1.00	0.99	2.56	0.00	3.45	0.00	1.28	3.63	0.00
time (sec)	N/A	5.825	0.731	0.013	0.000	1.750	0.000	0.177	7.261	0.001
Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	85	166	177	97	0	92	221	99
N.S.	1	1.00	0.41	0.80	0.85	0.47	0.00	0.44	1.06	0.48
time (sec)	N/A	0.352	0.305	0.033	0.990	0.760	0.000	0.210	6.306	1.342

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	75	132	143	87	0	82	187	89
N.S.	1	1.00	0.45	0.80	0.86	0.52	0.00	0.49	1.13	0.54
time (sec)	N/A	0.203	0.189	0.010	0.973	0.817	0.000	0.203	6.009	0.872
Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	98	109	77	0	72	153	79
N.S.	1	1.00	0.52	0.79	0.88	0.62	0.00	0.58	1.23	0.64
time (sec)	N/A	0.116	0.100	0.007	0.959	0.651	0.000	0.189	5.378	0.672
Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	189	403	500	304	0	144	-1	234
N.S.	1	1.00	1.01	2.16	2.67	1.63	0.00	0.77	-0.01	1.25
time (sec)	N/A	0.355	0.895	0.090	1.173	1.016	0.000	0.265	0.000	0.601
Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	354	1084	0	378	0	0	-1	431
N.S.	1	1.00	1.78	5.45	0.00	1.90	0.00	0.00	-0.01	2.17
time (sec)	N/A	0.250	1.276	0.032	0.000	0.958	0.000	0.000	0.000	0.743

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	334	2342	0	390	0	378	-1	608
N.S.	1	1.00	1.57	11.00	0.00	1.83	0.00	1.77	-0.00	2.85
time (sec)	N/A	0.237	1.397	0.031	0.000	0.993	0.000	0.262	0.000	1.056
Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	95	185	206	107	0	102	-1	109
N.S.	1	1.00	0.41	0.80	0.89	0.46	0.00	0.44	-0.00	0.47
time (sec)	N/A	0.364	0.418	0.036	1.005	0.865	0.000	0.235	0.000	1.288
Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	85	151	172	97	0	92	-1	99
N.S.	1	1.00	0.45	0.80	0.91	0.51	0.00	0.49	-0.01	0.52
time (sec)	N/A	0.227	0.251	0.010	0.984	0.819	0.000	0.216	0.000	1.046
Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	75	117	138	87	0	82	-1	89
N.S.	1	1.00	0.51	0.80	0.94	0.59	0.00	0.56	-0.01	0.61
time (sec)	N/A	0.130	0.148	0.008	0.957	0.817	0.000	0.204	0.000	0.900

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	210	210	202	730	535	326	0	154	-1	254
N.S.	1	1.00	0.96	3.48	2.55	1.55	0.00	0.73	-0.00	1.21
time (sec)	N/A	0.304	0.938	0.020	1.310	0.644	0.000	0.282	0.000	0.873

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	354	1828	0	378	0	0	-1	451
N.S.	1	1.00	1.59	8.23	0.00	1.70	0.00	0.00	-0.00	2.03
time (sec)	N/A	0.322	1.833	0.023	0.000	0.695	0.000	0.000	0.000	1.030

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	376	3828	0	447	0	0	-1	636
N.S.	1	1.00	1.61	16.36	0.00	1.91	0.00	0.00	-0.00	2.72
time (sec)	N/A	0.319	2.239	0.022	0.000	0.741	0.000	0.000	0.000	1.182

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	75	147	148	87	0	82	-1	89
N.S.	1	1.00	0.41	0.79	0.80	0.47	0.00	0.44	-0.01	0.48
time (sec)	N/A	0.312	0.252	0.026	0.994	0.991	0.000	0.245	0.000	0.942



Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	65	113	114	77	0	72	-1	79
N.S.	1	1.00	0.45	0.79	0.80	0.54	0.00	0.50	-0.01	0.55
time (sec)	N/A	0.203	0.143	0.010	0.974	0.694	0.000	0.392	0.000	0.674
Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	55	79	80	67	0	62	-1	69
N.S.	1	1.00	0.54	0.78	0.79	0.66	0.00	0.61	-0.01	0.68
time (sec)	N/A	0.145	0.080	0.009	0.964	0.785	0.000	0.220	0.000	0.561
Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	157	204	465	297	0	125	-1	211
N.S.	1	1.00	0.96	1.24	2.84	1.81	0.00	0.76	-0.01	1.29
time (sec)	N/A	0.230	0.467	0.019	1.138	0.415	0.000	0.275	0.000	0.464
Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	313	510	0	330	0	276	-1	352
N.S.	1	1.00	1.76	2.87	0.00	1.85	0.00	1.55	-0.01	1.98
time (sec)	N/A	0.196	0.999	0.021	0.000	0.576	0.000	0.279	0.000	0.621

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	371	1194	0	390	0	378	-1	437
N.S.	1	1.00	1.63	5.26	0.00	1.72	0.00	1.67	-0.00	1.93
time (sec)	N/A	0.271	1.175	0.021	0.000	0.851	0.000	0.322	0.000	0.945

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	75	166	148	112	0	81	-1	89
N.S.	1	1.00	0.45	1.00	0.89	0.67	0.00	0.49	-0.01	0.54
time (sec)	N/A	0.241	0.433	0.030	0.989	0.845	0.000	0.261	0.000	0.712

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	132	114	102	0	71	-1	79
N.S.	1	1.00	0.52	1.06	0.92	0.82	0.00	0.57	-0.01	0.64
time (sec)	N/A	0.161	0.266	0.008	0.968	0.872	0.000	0.248	0.000	0.951

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	55	98	80	92	0	62	-1	69
N.S.	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01	0.84
time (sec)	N/A	0.087	0.150	0.007	0.958	0.701	0.000	0.215	0.000	0.675

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	174	489	777	333	0	112	-1	199
N.S.	1	1.00	1.05	2.95	4.68	2.01	0.00	0.67	-0.01	1.20
time (sec)	N/A	0.216	1.135	0.018	1.155	0.776	0.000	0.248	0.000	0.621
Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	351	1214	0	392	0	295	-1	416
N.S.	1	1.00	1.63	5.65	0.00	1.82	0.00	1.37	-0.00	1.93
time (sec)	N/A	0.316	1.138	0.020	0.000	0.618	0.000	0.275	0.000	0.951
Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	381	2600	0	452	0	397	-1	611
N.S.	1	1.00	1.52	10.40	0.00	1.81	0.00	1.59	-0.00	2.44
time (sec)	N/A	0.324	1.586	0.024	0.000	1.170	0.000	0.323	0.000	1.319

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [164] had the largest ratio of [.3571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	34	0.147
2	A	6	5	1.00	32	0.156
3	A	5	5	1.00	27	0.185
4	A	5	5	1.00	34	0.147
5	A	5	5	1.00	34	0.147
6	A	5	5	1.00	34	0.147
7	A	8	6	1.00	34	0.176
8	A	4	4	1.00	34	0.118
9	A	5	4	1.00	34	0.118
10	A	7	4	1.00	34	0.118
11	A	6	4	1.00	34	0.118
12	A	5	4	1.00	32	0.125
13	A	4	4	1.00	27	0.148
14	A	4	4	1.00	34	0.118
15	A	7	5	1.00	34	0.147
16	A	4	4	1.00	34	0.118
17	A	5	4	1.00	34	0.118
18	A	2	1	0.99	25	0.040
19	A	2	1	0.99	25	0.040
20	A	2	1	1.00	23	0.043
21	A	2	1	1.00	18	0.056
22	A	2	1	0.99	25	0.040
23	A	2	1	0.99	25	0.040
24	A	2	1	0.99	25	0.040
25	A	3	2	0.99	27	0.074
26	A	3	2	1.00	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	3	2	1.00	25	0.080
28	A	3	2	1.00	20	0.100
29	A	2	1	0.99	27	0.037
30	A	2	1	0.99	27	0.037
31	A	2	1	0.99	27	0.037
32	A	3	2	0.99	27	0.074
33	A	3	2	1.00	27	0.074
34	A	3	2	1.00	25	0.080
35	A	3	2	1.00	20	0.100
36	A	2	1	0.99	27	0.037
37	A	2	1	0.99	27	0.037
38	A	2	1	0.99	27	0.037
39	A	1	1	1.00	32	0.031
40	A	1	1	1.00	31	0.032
41	A	1	1	1.00	34	0.029
42	A	1	1	1.00	33	0.030
43	A	5	4	0.99	27	0.148
44	A	5	4	0.99	27	0.148
45	A	5	4	1.00	25	0.160
46	A	5	4	1.00	20	0.200
47	A	5	4	1.00	27	0.148
48	A	5	4	1.00	27	0.148
49	A	5	4	1.00	27	0.148
50	A	6	5	1.00	27	0.185
51	A	5	5	1.00	27	0.185
52	A	4	4	1.00	25	0.160
53	A	3	3	1.00	20	0.150
54	A	6	5	1.00	27	0.185
55	A	6	5	0.99	27	0.185
56	A	6	5	1.00	27	0.185
57	A	5	5	1.00	27	0.185
58	A	3	3	1.12	27	0.111
59	A	3	3	1.00	25	0.120
60	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	27	0.222
62	A	7	5	0.99	27	0.185
63	A	7	5	1.00	27	0.185
64	A	4	4	1.00	27	0.148
65	A	4	4	1.13	27	0.148
66	A	4	4	1.00	27	0.148
67	A	4	4	1.00	25	0.160
68	A	5	4	1.00	20	0.200
69	A	6	5	1.00	17	0.294
70	A	5	5	1.00	17	0.294
71	A	4	4	1.00	15	0.267
72	A	3	3	1.00	14	0.214
73	A	6	5	1.00	17	0.294
74	A	6	5	1.00	17	0.294
75	A	6	5	1.00	17	0.294
76	A	3	3	1.83	16	0.188
77	A	3	3	1.00	18	0.167
78	A	7	6	0.99	29	0.207
79	A	6	6	1.00	29	0.207
80	A	5	5	1.00	27	0.185
81	A	5	5	1.00	22	0.227
82	A	7	6	1.00	29	0.207
83	A	7	6	0.98	29	0.207
84	A	7	6	1.00	29	0.207
85	A	7	6	1.00	29	0.207
86	A	5	5	1.00	29	0.172
87	A	6	6	1.00	29	0.207
88	A	8	6	1.00	29	0.207
89	A	7	6	1.00	29	0.207
90	A	6	5	1.00	27	0.185
91	A	6	5	1.00	22	0.227
92	A	8	6	1.00	29	0.207
93	A	8	6	0.99	29	0.207
94	A	8	7	0.98	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	6	0.99	29	0.207
96	A	8	7	1.00	29	0.241
97	A	8	6	1.00	29	0.207
98	A	6	5	1.00	29	0.172
99	A	7	6	1.00	29	0.207
100	A	7	5	1.00	22	0.227
101	A	6	5	0.99	29	0.172
102	A	5	5	1.00	29	0.172
103	A	4	4	0.99	27	0.148
104	A	4	4	1.00	22	0.182
105	A	6	5	1.00	29	0.172
106	A	6	5	1.00	29	0.172
107	A	4	4	1.00	29	0.138
108	A	5	5	1.00	29	0.172
109	A	4	4	1.00	29	0.138
110	A	4	4	1.00	27	0.148
111	A	4	4	1.00	22	0.182
112	A	4	4	1.00	29	0.138
113	A	4	4	1.00	29	0.138
114	A	5	5	0.99	29	0.172
115	A	3	3	1.00	22	0.136
116	A	4	4	1.00	22	0.182
117	A	5	4	1.00	22	0.182
118	A	5	4	1.00	29	0.138
119	A	4	4	1.00	29	0.138
120	A	3	3	1.00	27	0.111
121	A	5	5	1.00	29	0.172
122	A	5	5	1.00	29	0.172
123	A	4	4	1.00	29	0.138
124	A	5	4	1.00	29	0.138
125	A	4	4	1.00	29	0.138
126	A	3	3	1.00	27	0.111
127	A	4	4	1.00	29	0.138
128	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	5	5	1.00	29	0.172
130	A	4	3	1.00	29	0.103
131	A	4	3	1.00	29	0.103
132	A	2	2	1.00	27	0.074
133	A	5	5	1.00	29	0.172
134	A	5	4	1.00	29	0.138
135	A	6	5	1.00	29	0.172
136	A	2	1	1.00	20	0.050
137	A	2	1	1.00	20	0.050
138	A	2	1	1.00	20	0.050
139	A	2	1	1.00	18	0.056
140	A	6	5	1.00	20	0.250
141	A	4	4	1.00	20	0.200
142	A	5	5	1.00	20	0.250
143	A	6	5	1.00	20	0.250
144	A	6	5	1.00	30	0.167
145	A	6	5	1.00	30	0.167
146	A	6	5	1.00	28	0.179
147	A	6	5	1.00	23	0.217
148	A	6	5	1.00	30	0.167
149	A	6	5	1.00	30	0.167
150	A	6	5	1.00	30	0.167
151	A	6	6	1.00	30	0.200
152	A	5	5	1.00	28	0.179
153	A	4	4	1.00	23	0.174
154	A	7	6	1.00	30	0.200
155	A	7	6	1.00	30	0.200
156	A	7	6	1.00	20	0.300
157	A	7	6	1.00	20	0.300
158	A	5	5	1.00	18	0.278
159	A	4	4	1.00	17	0.235
160	A	7	6	1.00	20	0.300
161	A	7	6	1.00	20	0.300
162	A	7	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	1	1	1.00	16	0.062
164	A	6	5	1.00	14	0.357
165	A	6	5	1.00	16	0.312
166	A	3	2	1.00	18	0.111
167	A	5	3	1.00	16	0.188
168	A	5	3	1.00	19	0.158
169	A	6	5	1.00	23	0.217
170	A	5	3	1.00	19	0.158
171	A	2	2	1.00	23	0.087
172	A	4	4	1.00	17	0.235
173	A	1	1	1.00	19	0.053
174	A	7	5	1.00	22	0.227
175	A	6	5	1.00	22	0.227
176	A	5	5	1.00	22	0.227
177	A	4	4	1.00	22	0.182
178	A	4	4	1.00	22	0.182
179	A	3	3	1.00	22	0.136
180	A	4	4	1.00	22	0.182
181	A	5	4	1.00	22	0.182
182	A	7	6	1.00	32	0.188
183	A	6	6	0.99	32	0.188
184	A	5	5	1.00	30	0.167
185	A	5	5	1.00	25	0.200
186	A	7	6	1.00	32	0.188
187	A	7	6	0.99	32	0.188
188	A	7	6	1.00	32	0.188
189	A	7	6	1.00	32	0.188
190	A	5	5	1.00	32	0.156
191	A	6	6	1.00	32	0.188
192	A	8	6	1.00	32	0.188
193	A	7	6	0.99	32	0.188
194	A	6	5	1.00	30	0.167
195	A	6	5	1.00	25	0.200
196	A	8	6	1.00	32	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	8	6	0.99	32	0.188
198	A	8	7	0.99	32	0.219
199	A	8	6	1.00	32	0.188
200	A	8	7	1.00	32	0.219
201	A	8	6	1.00	32	0.188
202	A	6	5	1.00	32	0.156
203	A	7	6	1.00	32	0.188
204	A	7	6	1.00	32	0.188
205	A	6	6	1.00	32	0.188
206	A	5	5	1.00	30	0.167
207	A	7	7	1.00	32	0.219
208	A	7	7	1.00	32	0.219
209	A	7	7	1.00	32	0.219
210	A	9	6	1.00	32	0.188
211	A	7	6	1.00	32	0.188
212	A	6	5	1.00	30	0.167
213	A	8	7	1.00	32	0.219
214	A	8	7	1.00	32	0.219
215	A	8	8	1.00	32	0.250
216	A	9	6	1.00	32	0.188
217	A	8	6	1.00	32	0.188
218	A	7	5	1.00	30	0.167
219	A	9	7	1.00	32	0.219
220	A	9	7	1.00	32	0.219
221	A	9	8	1.00	32	0.250
222	A	6	5	1.00	32	0.156
223	A	5	5	1.00	32	0.156
224	A	4	4	1.00	30	0.133
225	A	4	4	1.00	25	0.160
226	A	6	5	1.00	32	0.156
227	A	6	5	0.99	32	0.156
228	A	4	4	1.00	32	0.125
229	A	5	5	1.00	32	0.156
230	A	4	4	1.00	32	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	4	4	1.00	30	0.133
232	A	4	4	1.00	25	0.160
233	A	4	4	1.00	32	0.125
234	A	4	4	0.99	32	0.125
235	A	5	5	0.99	32	0.156
236	A	6	5	1.00	32	0.156
237	A	5	5	1.00	32	0.156
238	A	4	4	1.00	30	0.133
239	A	6	6	1.00	32	0.188
240	A	6	6	1.00	32	0.188
241	A	4	4	1.00	32	0.125
242	A	6	5	1.00	32	0.156
243	A	5	5	1.00	32	0.156
244	A	4	4	1.00	30	0.133
245	A	4	4	1.00	32	0.125
246	A	4	4	1.00	32	0.125
247	A	5	5	1.00	32	0.156
248	A	5	4	1.00	32	0.125
249	A	5	4	1.00	32	0.125
250	A	2	2	1.00	30	0.067
251	A	5	5	1.00	32	0.156
252	A	5	4	1.00	32	0.125
253	A	6	5	1.00	32	0.156
254	A	3	3	1.00	47	0.064
255	A	3	3	1.00	42	0.071
256	A	2	2	1.00	46	0.043
257	A	2	2	1.00	69	0.029
258	A	2	2	1.00	75	0.027
259	A	6	4	1.00	16	0.250
260	A	6	5	1.00	33	0.152
261	A	5	4	1.00	31	0.129
262	A	5	4	1.00	30	0.133
263	A	7	5	1.00	33	0.152
264	A	7	6	1.00	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	7	5	1.00	33	0.152
266	A	5	4	1.00	33	0.121
267	A	6	5	1.00	33	0.152
268	A	7	5	1.00	33	0.152
269	A	2	1	1.00	36	0.028
270	A	2	1	1.00	36	0.028
271	A	2	1	1.00	34	0.029
272	A	2	1	1.00	29	0.034
273	A	2	1	1.00	36	0.028
274	A	2	1	1.00	36	0.028
275	A	2	1	1.00	36	0.028
276	A	2	1	1.00	38	0.026
277	A	2	1	1.00	38	0.026
278	A	2	1	1.00	36	0.028
279	A	2	1	1.00	31	0.032
280	A	2	1	1.00	38	0.026
281	A	2	1	1.00	38	0.026
282	A	2	1	1.00	38	0.026
283	A	2	1	1.00	38	0.026
284	A	6	5	1.00	38	0.132
285	A	6	5	1.00	38	0.132
286	A	6	5	1.00	36	0.139
287	A	6	5	1.00	31	0.161
288	A	6	5	1.00	38	0.132
289	A	6	5	1.00	38	0.132
290	A	6	5	1.00	38	0.132
291	A	7	6	1.00	38	0.158
292	A	7	6	1.00	38	0.158
293	A	7	6	1.00	36	0.167
294	A	7	6	1.00	31	0.194
295	A	7	6	1.00	38	0.158
296	A	7	6	1.00	38	0.158
297	A	7	6	1.00	38	0.158
298	A	8	6	1.00	38	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	8	6	1.00	38	0.158
300	A	6	6	1.00	36	0.167
301	A	5	4	1.00	31	0.129
302	A	8	6	1.00	38	0.158
303	A	8	6	1.00	38	0.158
304	A	7	5	1.00	38	0.132
305	A	7	5	1.00	33	0.152
306	A	9	7	1.00	40	0.175
307	A	9	8	1.00	40	0.200
308	A	9	8	1.00	40	0.200
309	A	9	7	1.00	40	0.175
310	A	9	7	1.00	40	0.175
311	A	9	7	1.00	40	0.175
312	A	7	5	1.00	40	0.125
313	A	8	6	1.00	40	0.150
314	A	8	5	1.00	38	0.132
315	A	8	5	1.00	33	0.152
316	A	10	7	1.00	40	0.175
317	A	10	8	1.00	40	0.200
318	A	10	8	1.00	40	0.200
319	A	10	7	1.00	40	0.175
320	A	10	8	1.00	40	0.200
321	A	10	7	1.00	40	0.175
322	A	10	8	1.00	40	0.200
323	A	10	7	1.00	40	0.175
324	A	6	4	1.00	38	0.105
325	A	6	4	1.00	33	0.121
326	A	8	6	1.00	40	0.150
327	A	8	7	1.00	40	0.175
328	A	8	7	1.00	40	0.175
329	A	8	6	1.00	40	0.150
330	A	6	4	1.00	40	0.100
331	A	7	5	1.00	40	0.125
332	A	6	5	1.00	38	0.132

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	5	5	1.00	33	0.152
334	A	7	7	1.00	40	0.175
335	A	7	7	1.00	40	0.175
336	A	5	5	1.00	40	0.125
337	A	6	5	1.00	40	0.125
338	A	6	5	1.00	40	0.125
339	A	5	4	1.00	38	0.105
340	A	5	4	1.00	33	0.121
341	A	5	4	1.00	40	0.100
342	A	5	4	1.00	40	0.100
343	A	6	5	1.00	40	0.125
344	A	7	5	1.00	40	0.125
345	A	5	4	1.00	35	0.114
346	A	5	4	1.00	36	0.111
347	A	2	1	1.00	38	0.026
348	A	2	1	1.00	38	0.026
349	A	2	1	1.00	36	0.028
350	A	6	5	1.00	38	0.132
351	A	6	5	1.00	53	0.094
352	A	11	5	1.00	35	0.143
353	A	9	5	1.00	35	0.143
354	A	7	5	1.00	33	0.152
355	A	9	7	1.00	35	0.200
356	A	9	7	1.00	35	0.200
357	A	7	5	1.00	35	0.143
358	A	12	5	1.00	35	0.143
359	A	10	5	1.00	35	0.143
360	A	8	5	1.00	33	0.152
361	A	10	7	1.00	35	0.200
362	A	10	8	1.00	35	0.229
363	A	10	7	1.00	35	0.200
364	A	10	4	1.00	35	0.114
365	A	8	4	1.00	35	0.114
366	A	6	4	1.00	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	8	6	1.00	35	0.171
368	A	6	4	1.00	35	0.114
369	A	7	4	1.00	35	0.114
370	A	9	5	1.00	35	0.143
371	A	7	5	1.00	35	0.143
372	A	5	5	1.00	33	0.152
373	A	6	4	1.00	35	0.114
374	A	7	4	1.00	35	0.114
375	A	8	4	1.00	35	0.114





# Chapter 3

## Listing of integrals

### Local contents

3.1	$\int (d + ex)^2 (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx$	150
3.2	$\int (d + ex) (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx$	155
3.3	$\int (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx$	160
3.4	$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex} dx$	164
3.5	$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2} dx$	169
3.6	$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3} dx$	174
3.7	$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$	179
3.8	$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5} dx$	186
3.9	$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6} dx$	191
3.10	$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} dx$	204
3.11	$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} dx$	209
3.12	$\int \frac{(d+ex)(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} dx$	214
3.13	$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}} dx$	219
3.14	$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}} dx$	223
3.15	$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}} dx$	227

3.16	$\int \frac{A+Bx+Cx^2}{(d+ex)^3 \sqrt{d^2-e^2x^2}} dx$	231
3.17	$\int \frac{A+Bx+Cx^2}{(d+ex)^4 \sqrt{d^2-e^2x^2}} dx$	236
3.18	$\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2) dx$	242
3.19	$\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2) dx$	246
3.20	$\int (d+ex) (a+cx^2) (A+Bx+Cx^2) dx$	249
3.21	$\int (a+cx^2) (A+Bx+Cx^2) dx$	252
3.22	$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex} dx$	255
3.23	$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2} dx$	259
3.24	$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} dx$	263
3.25	$\int (d+ex)^3 (a+cx^2)^2 (A+Bx+Cx^2) dx$	267
3.26	$\int (d+ex)^2 (a+cx^2)^2 (A+Bx+Cx^2) dx$	272
3.27	$\int (d+ex) (a+cx^2)^2 (A+Bx+Cx^2) dx$	276
3.28	$\int (a+cx^2)^2 (A+Bx+Cx^2) dx$	280
3.29	$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex} dx$	284
3.30	$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2} dx$	288
3.31	$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3} dx$	292
3.32	$\int (d+ex)^3 (a+cx^2)^3 (A+Bx+Cx^2) dx$	296
3.33	$\int (d+ex)^2 (a+cx^2)^3 (A+Bx+Cx^2) dx$	301
3.34	$\int (d+ex) (a+cx^2)^3 (A+Bx+Cx^2) dx$	306
3.35	$\int (a+cx^2)^3 (A+Bx+Cx^2) dx$	310
3.36	$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} dx$	314
3.37	$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} dx$	320
3.38	$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3} dx$	326
3.39	$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} dx$	332
3.40	$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$	335
3.41	$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$	338

3.42	$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$	342
3.43	$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2} dx$	346
3.44	$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2} dx$	351
3.45	$\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2} dx$	355
3.46	$\int \frac{A+Bx+Cx^2}{a+cx^2} dx$	359
3.47	$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)} dx$	363
3.48	$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)} dx$	367
3.49	$\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)} dx$	372
3.50	$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2} dx$	378
3.51	$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^2} dx$	383
3.52	$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2} dx$	388
3.53	$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2} dx$	392
3.54	$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^2} dx$	396
3.55	$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^2} dx$	402
3.56	$\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^2} dx$	408
3.57	$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3} dx$	415
3.58	$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3} dx$	421
3.59	$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3} dx$	426
3.60	$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3} dx$	430
3.61	$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^3} dx$	434
3.62	$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^3} dx$	441
3.63	$\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^3} dx$	451

3.64	$\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4} dx$	463
3.65	$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4} dx$	469
3.66	$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4} dx$	474
3.67	$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^4} dx$	479
3.68	$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} dx$	484
3.69	$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} dx$	488
3.70	$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$	492
3.71	$\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx$	496
3.72	$\int \frac{1+x+x^2}{(1+x^2)^2} dx$	500
3.73	$\int \frac{1+x+x^2}{x(1+x^2)^2} dx$	504
3.74	$\int \frac{1+x+x^2}{x^2(1+x^2)^2} dx$	508
3.75	$\int \frac{1+x+x^2}{x^3(1+x^2)^2} dx$	512
3.76	$\int \frac{1+2x+x^2}{(1+x^2)^2} dx$	516
3.77	$\int \frac{2+12x+3x^2}{(4+x^2)^2} dx$	519
3.78	$\int (g+hx)^3 \sqrt{a+cx^2} (d+ex+fx^2) dx$	523
3.79	$\int (g+hx)^2 \sqrt{a+cx^2} (d+ex+fx^2) dx$	530
3.80	$\int (g+hx) \sqrt{a+cx^2} (d+ex+fx^2) dx$	536
3.81	$\int \sqrt{a+cx^2} (d+ex+fx^2) dx$	541
3.82	$\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{g+hx} dx$	545
3.83	$\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^2} dx$	551
3.84	$\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^3} dx$	557
3.85	$\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^4} dx$	565

3.86	$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5} dx$	572
3.87	$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6} dx$	580
3.88	$\int (g+hx)^3 (a+cx^2)^{3/2} (d+ex+fx^2) dx$	590
3.89	$\int (g+hx)^2 (a+cx^2)^{3/2} (d+ex+fx^2) dx$	598
3.90	$\int (g+hx) (a+cx^2)^{3/2} (d+ex+fx^2) dx$	605
3.91	$\int (a+cx^2)^{3/2} (d+ex+fx^2) dx$	610
3.92	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx} dx$	614
3.93	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2} dx$	621
3.94	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3} dx$	627
3.95	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4} dx$	633
3.96	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5} dx$	640
3.97	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6} dx$	647
3.98	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7} dx$	660
3.99	$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8} dx$	673
3.100	$\int (a+cx^2)^{5/2} (A+Bx+Cx^2) dx$	682
3.101	$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$	687
3.102	$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$	693
3.103	$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$	698
3.104	$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}} dx$	703
3.105	$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}} dx$	707
3.106	$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}} dx$	712
3.107	$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}} dx$	717
3.108	$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}} dx$	723

3.109	$\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+cx^2)^{3/2}} dx$	728
3.110	$\int \frac{(g+hx)(d+ex+fx^2)}{(a+cx^2)^{3/2}} dx$	733
3.111	$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}} dx$	737
3.112	$\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}} dx$	741
3.113	$\int \frac{d+ex+fx^2}{(g+hx)^2(a+cx^2)^{3/2}} dx$	746
3.114	$\int \frac{d+ex+fx^2}{(g+hx)^3(a+cx^2)^{3/2}} dx$	752
3.115	$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}} dx$	762
3.116	$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}} dx$	766
3.117	$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}} dx$	770
3.118	$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}} dx$	775
3.119	$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}} dx$	779
3.120	$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}} dx$	783
3.121	$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}} dx$	787
3.122	$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}} dx$	791
3.123	$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}} dx$	795
3.124	$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx$	799
3.125	$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx$	803
3.126	$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx$	807
3.127	$\int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{3/2}} dx$	811
3.128	$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}} dx$	815
3.129	$\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}} dx$	819

3.130	$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$	824
3.131	$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$	828
3.132	$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$	832
3.133	$\int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{5/2}} dx$	836
3.134	$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}} dx$	841
3.135	$\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}} dx$	846
3.136	$\int (a+bx+cx^2)^4 (A+Cx^2) dx$	851
3.137	$\int (a+bx+cx^2)^3 (A+Cx^2) dx$	855
3.138	$\int (a+bx+cx^2)^2 (A+Cx^2) dx$	858
3.139	$\int (a+bx+cx^2) (A+Cx^2) dx$	861
3.140	$\int \frac{A+Cx^2}{a+bx+cx^2} dx$	864
3.141	$\int \frac{A+Cx^2}{(a+bx+cx^2)^2} dx$	868
3.142	$\int \frac{A+Cx^2}{(a+bx+cx^2)^3} dx$	873
3.143	$\int \frac{A+Cx^2}{(a+bx+cx^2)^4} dx$	879
3.144	$\int \frac{(d+ex)^3(f+gx+hx^2)}{a+bx+cx^2} dx$	886
3.145	$\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2} dx$	895
3.146	$\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2} dx$	902
3.147	$\int \frac{f+gx+hx^2}{a+bx+cx^2} dx$	907
3.148	$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)} dx$	911
3.149	$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)} dx$	917
3.150	$\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)} dx$	924
3.151	$\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$	935
3.152	$\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$	942

3.153	$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2} dx$	948
3.154	$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2} dx$	953
3.155	$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2} dx$	966
3.156	$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx$	986
3.157	$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx$	991
3.158	$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} dx$	996
3.159	$\int \frac{1+x+x^2}{(1-x+x^2)^2} dx$	1000
3.160	$\int \frac{1+x+x^2}{x(1-x+x^2)^2} dx$	1004
3.161	$\int \frac{1+x+x^2}{x^2(1-x+x^2)^2} dx$	1009
3.162	$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2} dx$	1014
3.163	$\int \frac{1-x^2}{(1+x+x^2)^2} dx$	1019
3.164	$\int \frac{1+x^2}{1+x+x^2} dx$	1022
3.165	$\int \frac{-1+x^2}{25-6x+x^2} dx$	1026
3.166	$\int \frac{-10+3x^2}{4-4x+x^2} dx$	1030
3.167	$\int \frac{8+x^2}{6-5x+x^2} dx$	1033
3.168	$\int \frac{-4+3x+x^2}{-8-2x+x^2} dx$	1036
3.169	$\int \frac{7+5x+4x^2}{5+4x+4x^2} dx$	1039
3.170	$\int \frac{2-x+x^2}{-5+2x+x^2} dx$	1043
3.171	$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2} dx$	1047
3.172	$\int \frac{1+x+x^2}{(3+2x+x^2)^2} dx$	1050
3.173	$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4} dx$	1054
3.174	$\int (a+bx+cx^2)^{5/2} (A+Cx^2) dx$	1057
3.175	$\int (a+bx+cx^2)^{3/2} (A+Cx^2) dx$	1063



3.176	$\int \sqrt{a+bx+cx^2} (A+Cx^2) dx$	.1068
3.177	$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}} dx$	.1073
3.178	$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}} dx$	.1077
3.179	$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}} dx$	.1082
3.180	$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}} dx$	.1086
3.181	$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}} dx$	.1091
3.182	$\int (g+hx)^3 \sqrt{a+bx+cx^2} (d+ex+fx^2) dx$	.1097
3.183	$\int (g+hx)^2 \sqrt{a+bx+cx^2} (d+ex+fx^2) dx$	.1109
3.184	$\int (g+hx) \sqrt{a+bx+cx^2} (d+ex+fx^2) dx$	.1118
3.185	$\int \sqrt{a+bx+cx^2} (d+ex+fx^2) dx$	.1124
3.186	$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{g+hx} dx$	.1129
3.187	$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^2} dx$	.1135
3.188	$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^3} dx$	.1140
3.189	$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^4} dx$	.1146
3.190	$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^5} dx$	.1152
3.191	$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^6} dx$	.1157
3.192	$\int (g+hx)^3 (a+bx+cx^2)^{3/2} (d+ex+fx^2) dx$	.1163
3.193	$\int (g+hx)^2 (a+bx+cx^2)^{3/2} (d+ex+fx^2) dx$	.1174
3.194	$\int (g+hx) (a+bx+cx^2)^{3/2} (d+ex+fx^2) dx$	.1185
3.195	$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2) dx$	.1193
3.196	$\int \frac{(a+bx+cx^2)^{3/2} (d+ex+fx^2)}{g+hx} dx$	.1198
3.197	$\int \frac{(a+bx+cx^2)^{3/2} (d+ex+fx^2)}{(g+hx)^2} dx$	.1203
3.198	$\int \frac{(a+bx+cx^2)^{3/2} (d+ex+fx^2)}{(g+hx)^3} dx$	.1208
3.199	$\int \frac{(a+bx+cx^2)^{3/2} (d+ex+fx^2)}{(g+hx)^4} dx$	.1216

3.200	$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5} dx$	.1225
3.201	$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6} dx$	.1231
3.202	$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7} dx$	.1237
3.203	$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8} dx$	.1242
3.204	$\int (1+2x)^3 \sqrt{2-x+3x^2} (1+3x+4x^2) dx$	.1248
3.205	$\int (1+2x)^2 \sqrt{2-x+3x^2} (1+3x+4x^2) dx$	.1253
3.206	$\int (1+2x) \sqrt{2-x+3x^2} (1+3x+4x^2) dx$	.1258
3.207	$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x} dx$	.1262
3.208	$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2} dx$	.1267
3.209	$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} dx$	.1272
3.210	$\int (1+2x)^3 (2-x+3x^2)^{3/2} (1+3x+4x^2) dx$	.1277
3.211	$\int (1+2x)^2 (2-x+3x^2)^{3/2} (1+3x+4x^2) dx$	.1282
3.212	$\int (1+2x) (2-x+3x^2)^{3/2} (1+3x+4x^2) dx$	.1287
3.213	$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x} dx$	.1291
3.214	$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} dx$	.1296
3.215	$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3} dx$	.1301
3.216	$\int (1+2x)^3 (2-x+3x^2)^{5/2} (1+3x+4x^2) dx$	.1307
3.217	$\int (1+2x)^2 (2-x+3x^2)^{5/2} (1+3x+4x^2) dx$	.1312
3.218	$\int (1+2x) (2-x+3x^2)^{5/2} (1+3x+4x^2) dx$	.1317
3.219	$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x} dx$	.1322
3.220	$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2} dx$	.1327
3.221	$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3} dx$	.1333
3.222	$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$	.1339
3.223	$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$	.1346
3.224	$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$	.1352

3.225	$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} dx$	. . . . .	.1357
3.226	$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}} dx$	. . . . .	.1361
3.227	$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}} dx$	. . . . .	.1366
3.228	$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}} dx$	. . . . .	.1371
3.229	$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1378
3.230	$\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1387
3.231	$\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1393
3.232	$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1398
3.233	$\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1403
3.234	$\int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1409
3.235	$\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}} dx$	. . . . .	.1417
3.236	$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$	. . . . .	.1425
3.237	$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$	. . . . .	.1429
3.238	$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$	. . . . .	.1433
3.239	$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}} dx$	. . . . .	.1437
3.240	$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}} dx$	. . . . .	.1442
3.241	$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}} dx$	. . . . .	.1446
3.242	$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx$	. . . . .	.1451
3.243	$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx$	. . . . .	.1456
3.244	$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx$	. . . . .	.1460
3.245	$\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{3/2}} dx$	. . . . .	.1464
3.246	$\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}} dx$	. . . . .	.1468

- 3.247  $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}} dx \dots\dots\dots .1472$
- 3.248  $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx \dots\dots\dots .1477$
- 3.249  $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx \dots\dots\dots .1481$
- 3.250  $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx \dots\dots\dots .1485$
- 3.251  $\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}} dx \dots\dots\dots .1489$
- 3.252  $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}} dx \dots\dots\dots .1494$
- 3.253  $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}} dx \dots\dots\dots .1499$
- 3.254  $\int \frac{d+ex+fx^2}{(g+hx)(-cg^2+bgh+bh^2x+ch^2x^2)^{3/2}} dx \dots\dots\dots .1504$
- 3.255  $\int (d+fx^2)^p (2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2) dx \dots\dots\dots .1509$
- 3.256  $\int (d+ex+fx^2)^p (-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2) dx \dots\dots\dots .1513$
- 3.257  $\int (d+ex+fx^2)^p (-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2) dx \dots\dots\dots .1517$
- 3.258  $\int (d+ex)^3 (a+bx+cx^2)^5 (d(6bd+5ae)+(12cd^2+17bde+5ae^2)x+e(29cd+11be)x^2+17ce^2x^3) dx \dots\dots\dots .1521$
- 3.259  $\int \frac{x^2+x^3}{-2+x+x^2} dx \dots\dots\dots .1529$
- 3.260  $\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1533$
- 3.261  $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1539$
- 3.262  $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1544$
- 3.263  $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1548$
- 3.264  $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1553$
- 3.265  $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1558$
- 3.266  $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1563$
- 3.267  $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1568$
- 3.268  $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}} dx \dots\dots\dots .1574$
- 3.269  $\int (d+ex)^3 (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4) dx \dots\dots\dots .1581$
- 3.270  $\int (d+ex)^2 (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4) dx \dots\dots\dots .1585$
- 3.271  $\int (d+ex)(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4) dx \dots\dots\dots .1588$

3.272	$\int (3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4) dx$	.1591
3.273	$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex} dx$	.1594
3.274	$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$	.1598
3.275	$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$	.1602
3.276	$\int (d+ex)^3 (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) dx$	.1606
3.277	$\int (d+ex)^2 (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) dx$	.1610
3.278	$\int (d+ex) (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) dx$	.1614
3.279	$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) dx$	.1617
3.280	$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex} dx$	.1620
3.281	$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$	.1625
3.282	$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$	.1630
3.283	$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} dx$	.1635
3.284	$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$	.1640
3.285	$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$	.1645
3.286	$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$	.1650
3.287	$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} dx$	.1654
3.288	$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)} dx$	.1658
3.289	$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)} dx$	.1663
3.290	$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)} dx$	.1668
3.291	$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$	.1674
3.292	$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$	.1680
3.293	$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$	.1686
3.294	$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx$	.1691
3.295	$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2} dx$	.1696

- 3.296  $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2} dx \dots\dots\dots .1702$
- 3.297  $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2} dx \dots\dots\dots .1709$
- 3.298  $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx \dots\dots\dots .1717$
- 3.299  $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx \dots\dots\dots .1723$
- 3.300  $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx \dots\dots\dots .1729$
- 3.301  $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3} dx \dots\dots\dots .1734$
- 3.302  $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3} dx \dots\dots\dots .1738$
- 3.303  $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3} dx \dots\dots\dots .1745$
- 3.304  $\int (5+2x)\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4) dx \dots\dots\dots .1754$
- 3.305  $\int \sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4) dx \dots\dots\dots .1759$
- 3.306  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{5+2x} dx \dots\dots\dots .1763$
- 3.307  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx \dots\dots\dots .1768$
- 3.308  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx \dots\dots\dots .1774$
- 3.309  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx \dots\dots\dots .1780$
- 3.310  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx \dots\dots\dots .1786$
- 3.311  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx \dots\dots\dots .1792$
- 3.312  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx \dots\dots\dots .1798$
- 3.313  $\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx \dots\dots\dots .1804$
- 3.314  $\int (5+2x) (3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4) dx \dots\dots\dots .1810$
- 3.315  $\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4) dx \dots\dots\dots .1815$
- 3.316  $\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{5+2x} dx \dots\dots\dots .1819$
- 3.317  $\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx \dots\dots\dots .1824$
- 3.318  $\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx \dots\dots\dots .1830$

- 3.319  $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx \dots\dots\dots .1836$
- 3.320  $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx \dots\dots\dots .1842$
- 3.321  $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx \dots\dots\dots .1848$
- 3.322  $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx \dots\dots\dots .1854$
- 3.323  $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx \dots\dots\dots .1860$
- 3.324  $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}} dx \dots\dots\dots .1866$
- 3.325  $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}} dx \dots\dots\dots .1870$
- 3.326  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}} dx \dots\dots\dots .1874$
- 3.327  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}} dx \dots\dots\dots .1879$
- 3.328  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}} dx \dots\dots\dots .1884$
- 3.329  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}} dx \dots\dots\dots .1889$
- 3.330  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}} dx \dots\dots\dots .1894$
- 3.331  $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1899$
- 3.332  $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1904$
- 3.333  $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1909$
- 3.334  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1913$
- 3.335  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1918$
- 3.336  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1923$
- 3.337  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}} dx \dots\dots\dots .1928$
- 3.338  $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1933$
- 3.339  $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1938$
- 3.340  $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1943$

- 3.341  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1947$
- 3.342  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1952$
- 3.343  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1957$
- 3.344  $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}} dx \dots\dots\dots .1963$
- 3.345  $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}} dx \dots\dots\dots .1969$
- 3.346  $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}} dx \dots\dots\dots .1975$
- 3.347  $\int (d+ex)^m (3+2x+5x^2)^3 (2+x+3x^2-5x^3+4x^4) dx \dots\dots\dots .1981$
- 3.348  $\int (d+ex)^m (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) dx \dots\dots\dots .1999$
- 3.349  $\int (d+ex)^m (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) dx \dots\dots\dots .2012$
- 3.350  $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3} dx \dots\dots\dots .2019$
- 3.351  $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2} dx \dots\dots\dots .2027$
- 3.352  $\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2} dx \dots\dots\dots .2035$
- 3.353  $\int (1+4x-7x^2)^2 (2+5x+x^2) \sqrt{3+2x+5x^2} dx \dots\dots\dots .2041$
- 3.354  $\int (1+4x-7x^2) (2+5x+x^2) \sqrt{3+2x+5x^2} dx \dots\dots\dots .2046$
- 3.355  $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2} dx \dots\dots\dots .2050$
- 3.356  $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2} dx \dots\dots\dots .2056$
- 3.357  $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3} dx \dots\dots\dots .2063$
- 3.358  $\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx \dots\dots\dots .2071$
- 3.359  $\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx \dots\dots\dots .2077$
- 3.360  $\int (1+4x-7x^2) (2+5x+x^2) (3+2x+5x^2)^{3/2} dx \dots\dots\dots .2082$
- 3.361  $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2} dx \dots\dots\dots .2087$
- 3.362  $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} dx \dots\dots\dots .2093$
- 3.363  $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3} dx \dots\dots\dots .2101$



3.364	$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$	. . . . .	.2110
3.365	$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$	. . . . .	.2115
3.366	$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$	. . . . .	.2119
3.367	$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}} dx$	. . . . .	.2123
3.368	$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}} dx$	. . . . .	.2128
3.369	$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}} dx$	. . . . .	.2134
3.370	$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx$	. . . . .	.2141
3.371	$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx$	. . . . .	.2147
3.372	$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx$	. . . . .	.2152
3.373	$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}} dx$	. . . . .	.2157
3.374	$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}} dx$	. . . . .	.2163
3.375	$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}} dx$	. . . . .	.2170

### 3.1 $\int (d + ex)^2 (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx$

**Optimal.** Leaf size=236

$$\frac{d^2x\sqrt{d^2 - e^2x^2} (10Ae^2 + 4Bde + 3Cd^2)}{16e^2} - \frac{x(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 3Cd^2)}{8e^2} - \frac{d(d^2 - e^2x^2)^{3/2} (e(10Ae + 7Bd) + 4Cd^2)}{15e^3}$$

**Rubi [A]** time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {1815, 641, 195, 217, 203}

$$\frac{d^2x\sqrt{d^2 - e^2x^2} (10Ae^2 + 4Bde + 3Cd^2)}{16e^2} - \frac{d(d^2 - e^2x^2)^{3/2} (e(10Ae + 7Bd) + 4Cd^2)}{15e^3} - \frac{x(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 3Cd^2)}{8e^2} + \frac{d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (10Ae^2 + 4Bde + 3Cd^2)}{16e^3} - \frac{x^2 (d^2 - e^2x^2)^{3/2} (Be + 2Cd)}{5e} - \frac{1}{6} Cx^3 (d^2 - e^2x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out] (d^2\*(3\*C\*d^2 + 4\*B\*d\*e + 10\*A\*e^2)\*x\*Sqrt[d^2 - e^2\*x^2])/(16\*e^2) - (d\*(4\*C\*d^2 + e\*(7\*B\*d + 10\*A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(15\*e^3) - ((3\*C\*d^2 + 2\*e\*(2\*B\*d + A\*e))\*x\*(d^2 - e^2\*x^2)^(3/2))/(8\*e^2) - ((2\*C\*d + B\*e)\*x^2\*(d^2 - e^2\*x^2)^(3/2))/(5\*e) - (C\*x^3\*(d^2 - e^2\*x^2)^(3/2))/6 + (d^4\*(3\*C\*d^2 + 4\*B\*d\*e + 10\*A\*e^2)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(16\*e^3)

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

### Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int (d + ex)^2 (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx &= -\frac{1}{6}Cx^3 (d^2 - e^2x^2)^{3/2} - \frac{\int \sqrt{d^2 - e^2x^2} (-6Ad^2e^2 - 6de^2(Bd + 2Ae) \\
&= -\frac{(2Cd + Be)x^2 (d^2 - e^2x^2)^{3/2}}{5e} - \frac{1}{6}Cx^3 (d^2 - e^2x^2)^{3/2} + \frac{\int \sqrt{d^2 - e^2x^2} \\
&= -\frac{(3Cd^2 + 2e(2Bd + Ae))x (d^2 - e^2x^2)^{3/2}}{8e^2} - \frac{(2Cd + Be)x^2 (d^2 - e^2x^2)^{3/2}}{5e} \\
&= -\frac{d(4Cd^2 + e(7Bd + 10Ae)) (d^2 - e^2x^2)^{3/2}}{15e^3} - \frac{(3Cd^2 + 2e(2Bd + Ae))x^2 (d^2 - e^2x^2)^{3/2}}{8e^2} \\
&= \frac{d^2(3Cd^2 + 4Bde + 10Ae^2)x\sqrt{d^2 - e^2x^2}}{16e^2} - \frac{d(4Cd^2 + e(7Bd + 10Ae))x^2\sqrt{d^2 - e^2x^2}}{15e^3} \\
&= \frac{d^2(3Cd^2 + 4Bde + 10Ae^2)x\sqrt{d^2 - e^2x^2}}{16e^2} - \frac{d(4Cd^2 + e(7Bd + 10Ae))x^2\sqrt{d^2 - e^2x^2}}{15e^3} \\
&= \frac{d^2(3Cd^2 + 4Bde + 10Ae^2)x\sqrt{d^2 - e^2x^2}}{16e^2} - \frac{d(4Cd^2 + e(7Bd + 10Ae))x^2\sqrt{d^2 - e^2x^2}}{15e^3}
\end{aligned}$$

**Mathematica [A]** time = 0.51, size = 226, normalized size = 0.96

$$\frac{\sqrt{d^2 - e^2x^2} \left( 15 \sin^{-1}\left(\frac{x}{\frac{d}{e}}\right) (2d^3e(5Ae + 2Bd) + 3Cd^3e) + \sqrt{1 - \frac{e^2x^2}{d^2}} (2e(5Ae(-16d^3 + 9d^2ex + 16de^2x^2 + 6e^3x^3) + B(-56d^4 - 30d^3ex + 32d^2e^2x^2 + 60de^3x^3 + 24e^4x^4)) + C(-64d^5 - 45d^4ex - 32d^3e^2x^2 + 50d^2e^3x^3 + 96de^4x^4 + 40e^5x^5)) \right)}{240e^3\sqrt{1 - \frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(\sqrt{d^2 - e^2 x^2}) * (\sqrt{1 - (e^2 x^2)/d^2}) * (C * (-64 d^5 - 45 d^4 e x - 32 d^3 e^2 x^2 + 50 d^2 e^3 x^3 + 96 d e^4 x^4 + 40 e^5 x^5) + 2 e * (5 A e * (-16 d^3 + 9 d^2 e x + 16 d e^2 x^2 + 6 e^3 x^3) + B * (-56 d^4 - 30 d^3 e x + 32 d^2 e^2 x^2 + 60 d e^3 x^3 + 24 e^4 x^4))) + 15 * (3 C d^5 + 2 d^3 e * (2 B d + 5 A e)) * \text{ArcSin}[(e x)/d]) / (240 e^3 \sqrt{1 - (e^2 x^2)/d^2})$

**IntegrateAlgebraic [A]** time = 0.82, size = 237, normalized size = 1.00

$$\frac{\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x\right) (10 A d^4 e^2 + 4 B d^5 e + 3 C d^6) + \sqrt{d^2 - e^2 x^2} (-160 A d^3 e^2 + 90 A d^2 e^3 x + 160 A d e^4 x^2 + 60 A e^5 x^3 - 112 B d^4 e - 60 B d^3 e^2 x + 64 B d^2 e^3 x^2 + 120 B d e^4 x^3 + 48 B e^5 x^4 - 64 C d^5 - 45 C d^4 e x - 32 C d^3 e^2 x^2 + 50 C d^2 e^3 x^3 + 96 C d e^4 x^4 + 40 C e^5 x^5)}{16 e^4} + \frac{\sqrt{d^2 - e^2 x^2} (-160 A d^3 e^2 + 90 A d^2 e^3 x + 160 A d e^4 x^2 + 60 A e^5 x^3 - 112 B d^4 e - 60 B d^3 e^2 x + 64 B d^2 e^3 x^2 + 120 B d e^4 x^3 + 48 B e^5 x^4 - 64 C d^5 - 45 C d^4 e x - 32 C d^3 e^2 x^2 + 50 C d^2 e^3 x^3 + 96 C d e^4 x^4 + 40 C e^5 x^5)}{240 e^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x)^2\*(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2],x]

[Out]  $(\sqrt{d^2 - e^2 x^2}) * (-64 C d^5 - 112 B d^4 e - 160 A d^3 e^2 - 45 C d^4 e x - 60 B d^3 e^2 x + 90 A d^2 e^3 x - 32 C d^3 e^2 x^2 + 64 B d^2 e^3 x^2 + 160 A d e^4 x^2 + 50 C d^2 e^3 x^3 + 120 B d e^4 x^3 + 60 A e^5 x^3 + 96 C d e^4 x^4 + 48 B e^5 x^4 + 40 C e^5 x^5) / (240 e^3) + (\sqrt{-e^2}) * (3 C d^6 + 4 B d^5 e + 10 A d^4 e^2) * \text{Log}[-(\sqrt{-e^2}) * x] + \sqrt{d^2 - e^2 x^2}) / (16 e^4)$

**fricas [A]** time = 1.50, size = 211, normalized size = 0.89

$$\frac{30(3 C d^6 + 4 B d^5 e + 10 A d^4 e^2) \arctan\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) - (40 C e^5 x^5 - 64 C d^5 - 112 B d^4 e - 160 A d^3 e^2 + 48(2 C d e^4 + B e^5) x^4 + 10(5 C d^2 e^3 + 12 B d e^4 + 6 A e^5) x^3 - 32(C d^3 e^2 - 2 B d^2 e^3 - 5 A d e^4) x^2 - 15(3 C d^4 e + 4 B d^3 e^2 - 6 A d^2 e^3) x) \sqrt{-e^2 x^2 + d^2}}{240 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out]  $-1/240 * (30 * (3 C d^6 + 4 B d^5 e + 10 A d^4 e^2) * \arctan(-(d - \sqrt{-e^2 x^2 + d^2}) / (e x)) - (40 C e^5 x^5 - 64 C d^5 - 112 B d^4 e - 160 A d^3 e^2 + 48 * (2 C d e^4 + B e^5) * x^4 + 10 * (5 C d^2 e^3 + 12 B d e^4 + 6 A e^5) * x^3 - 32 * (C d^3 e^2 - 2 B d^2 e^3 - 5 A d e^4) * x^2 - 15 * (3 C d^4 e + 4 B d^3 e^2 - 6 A d^2 e^3) * x) * \sqrt{-e^2 x^2 + d^2}) / e^3$

**giac [A]** time = 0.22, size = 197, normalized size = 0.83

$$\frac{1}{16} (3 C d^6 + 4 B d^5 e + 10 A d^4 e^2) \arcsin\left(\frac{x e}{d}\right) e^{(-3) \text{sgn}(d)} + \frac{1}{240} \sqrt{-e^2 x^2 + d^2} ((2 * ((4 * (5 C x^2 + 6 * (2 C d e^4 + B e^5)) e^{(-8)}) x + 5 * (5 C d^2 e^3 + 12 B d e^4 + 6 A e^5)) e^{(-8)}) x - 16 * (C d^3 e^2 - 2 B d^2 e^3 - 5 A d e^4) e^{(-8)}) x - 15 * (3 C d^4 e + 4 B d^3 e^2 - 6 A d^2 e^3) e^{(-8)}) x - 16 * (4 C d^5 e^2 + 7 B d^4 e^3 + 10 A d^3 e^4) e^{(-8)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out]  $1/16 * (3 C d^6 + 4 B d^5 e + 10 A d^4 e^2) * \arcsin(x e / d) * e^{(-3) \text{sgn}(d)} + 1/240 * \sqrt{-x^2 e^2 + d^2} * ((2 * ((4 * (5 C x^2 e^2 + 6 * (2 C d e^4 e^9 + B e^5 e^{10})) * e^{(-8)}) * x + 5 * (5 C d^2 e^8 + 12 B d e^9 + 6 A e^{10})) * e^{(-8)}) * x - 16 * (C d^3 e^7 - 2 * (2 C d^2 e^8 + B e^9) * e^{(-8)}) * x - 15 * (3 C d^4 e^9 + 4 B d^3 e^{10} - 6 A d^2 e^{11})) * e^{(-8)}) * x - 16 * (4 C d^5 e^{10} + 7 B d^4 e^{11} + 10 A d^3 e^{12})) * e^{(-8)})$

$B*d^2*e^8 - 5*A*d*e^9)*e^{(-8)}*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*e^8)*e^{(-8)}*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)*e^{(-8)}$

**maple [A]** time = 0.06, size = 371, normalized size = 1.57

$$\frac{5A^4 \arctan\left(\frac{\sqrt{x}}{\sqrt{d+ex}}\right)}{8\sqrt{d}} + \frac{B^4 \arctan\left(\frac{\sqrt{x}}{\sqrt{d+ex}}\right)}{4\sqrt{d}e} + \frac{3C^4 \arctan\left(\frac{\sqrt{x}}{\sqrt{d+ex}}\right)}{16\sqrt{d}e^2} + \frac{5\sqrt{-d^2+e^2} A^4 d^4 x}{8} + \frac{\sqrt{-d^2+e^2} B^4 d^4 x}{4e} + \frac{3\sqrt{-d^2+e^2} C^4 d^4 x}{16e^2} - \frac{(-d^2+e^2)^{\frac{3}{2}} C^2 x^3}{6} - \frac{(-d^2+e^2)^{\frac{3}{2}} B^2 x^2}{5} - \frac{2(-d^2+e^2)^{\frac{3}{2}} C d x^2}{5e} - \frac{(-d^2+e^2)^{\frac{3}{2}} A x}{4} - \frac{(-d^2+e^2)^{\frac{3}{2}} B d x}{2e} - \frac{3(-d^2+e^2)^{\frac{3}{2}} C d^2 x}{8e^2} - \frac{2(-d^2+e^2)^{\frac{3}{2}} A d}{3e} - \frac{7(-d^2+e^2)^{\frac{3}{2}} B d^2}{15e^2} - \frac{4(-d^2+e^2)^{\frac{3}{2}} C d^2}{15e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^{(1/2)}, x)$

[Out]  $-1/6*C*x^3*(-e^2*x^2+d^2)^{(3/2)} - 3/8/e^2*C*d^2*x*(-e^2*x^2+d^2)^{(3/2)} + 3/16/e^2*C*d^4*x*(-e^2*x^2+d^2)^{(1/2)} + 3/16/e^2*C*d^6/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)}) - 1/5*x^2*(-e^2*x^2+d^2)^{(3/2)}*B - 2/5*x^2*(-e^2*x^2+d^2)^{(3/2)}/e*d*C - 7/15*d^2/e^2*(-e^2*x^2+d^2)^{(3/2)}*B - 4/15*d^3/e^3*(-e^2*x^2+d^2)^{(3/2)}*C - 1/4*x*(-e^2*x^2+d^2)^{(3/2)}*A - 1/2*x*(-e^2*x^2+d^2)^{(3/2)}/e*B*d + 5/8*d^2*x*(-e^2*x^2+d^2)^{(1/2)}*A + 1/4*d^3/e*x*(-e^2*x^2+d^2)^{(1/2)}*B + 5/8*d^4/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})*A + 1/4*d^5/e/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})*B - 2/3*(-e^2*x^2+d^2)^{(3/2)}/e*A*d$

**maxima [A]** time = 1.00, size = 338, normalized size = 1.43

$$\frac{1}{6}(-d^2+e^2)^{\frac{3}{2}}C^2x^3 + \frac{C^4 \arcsin\left(\frac{x}{\sqrt{d+ex}}\right)}{16e^2} + \frac{A^4 \arcsin\left(\frac{x}{\sqrt{d+ex}}\right)}{2e} + \frac{1}{2}\sqrt{-d^2+e^2} A^4 d^4 x + \frac{\sqrt{-d^2+e^2} C^4 d^4 x}{16e^2} - \frac{(-d^2+e^2)^{\frac{3}{2}} C d^2 x}{8e^2} + \frac{(C^2+2Bd+Ad^2) \arcsin\left(\frac{x}{\sqrt{d+ex}}\right)}{8e^2} - \frac{(-d^2+e^2)^{\frac{3}{2}} B d^2}{3e^2} - \frac{2(-d^2+e^2)^{\frac{3}{2}} A d}{3e} + \frac{\sqrt{-d^2+e^2} (C^2+2Bd+Ad^2) d^2 x}{8e^2} - \frac{(-d^2+e^2)^{\frac{3}{2}} (2Cde+Be^2) x^2}{3e^2} - \frac{(-d^2+e^2)^{\frac{3}{2}} (C^2+2Bde+Ad^2) x}{4e^2} - \frac{2(-d^2+e^2)^{\frac{3}{2}} (2Cde+Be^2) d^2}{15e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $-1/6*(-e^2*x^2 + d^2)^{(3/2)}*C*x^3 + 1/16*C*d^6*\arcsin(e*x/d)/e^3 + 1/2*A*d^4*\arcsin(e*x/d)/e + 1/2*\sqrt{-e^2*x^2 + d^2}*A*d^2*x + 1/16*\sqrt{-e^2*x^2 + d^2}*C*d^4*x/e^2 - 1/8*(-e^2*x^2 + d^2)^{(3/2)}*C*d^2*x/e^2 + 1/8*(C*d^2 + 2*B*d*e + A*e^2)*d^4*\arcsin(e*x/d)/e^3 - 1/3*(-e^2*x^2 + d^2)^{(3/2)}*B*d^2/e^2 - 2/3*(-e^2*x^2 + d^2)^{(3/2)}*A*d/e + 1/8*\sqrt{-e^2*x^2 + d^2}*(C*d^2 + 2*B*d*e + A*e^2)*d^2*x/e^2 - 1/5*(-e^2*x^2 + d^2)^{(3/2)}*(2*C*d*e + B*e^2)*x^2/e^2 - 1/4*(-e^2*x^2 + d^2)^{(3/2)}*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/15*(-e^2*x^2 + d^2)^{(3/2)}*(2*C*d*e + B*e^2)*d^2/e^4$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d^2 - e^2 x^2} (d + e x)^2 (C x^2 + B x + A) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d^2 - e^2*x^2)^{(1/2)}*(d + e*x)^2*(A + B*x + C*x^2), x)$

[Out]  $\int ((d^2 - e^2 x^2)^{1/2} (d + e x)^2 (A + B x + C x^2), x)$

**sympy** [C] time = 22.93, size = 1231, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $A*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 2*A*d*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (- (d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + A*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2))), True)) + B*d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (- (d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + 2*B*d*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2))), True)) + B*e**2*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + C*d**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2))), True)) + 2*C*d*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + C*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2))), True))$

### 3.2 $\int (d + ex) (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx$

**Optimal.** Leaf size=186

$$\frac{dx\sqrt{d^2 - e^2x^2} (e(4Ae + Bd) + Cd^2)}{8e^2} - \frac{(d^2 - e^2x^2)^{3/2} (5e(Ae + Bd) + 2Cd^2)}{15e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (e(4Ae + Bd) + Cd^2)}{8e^3}$$

**Rubi [A]** time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1815, 641, 195, 217, 203}

$$\frac{dx\sqrt{d^2 - e^2x^2} (e(4Ae + Bd) + Cd^2)}{8e^2} - \frac{(d^2 - e^2x^2)^{3/2} (5e(Ae + Bd) + 2Cd^2)}{15e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (e(4Ae + Bd) + Cd^2)}{8e^3} - \frac{x (d^2 - e^2x^2)^{3/2} (Be + Cd)}{4e^2} - \frac{Cx^2 (d^2 - e^2x^2)^{3/2}}{5e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out] (d\*(C\*d^2 + e\*(B\*d + 4\*A\*e))\*x\*Sqrt[d^2 - e^2\*x^2])/(8\*e^2) - ((2\*C\*d^2 + 5\*e\*(B\*d + A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(15\*e^3) - ((C\*d + B\*e)\*x\*(d^2 - e^2\*x^2)^(3/2))/(4\*e^2) - (C\*x^2\*(d^2 - e^2\*x^2)^(3/2))/(5\*e) + (d^3\*(C\*d^2 + e\*(B\*d + 4\*A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e^3)

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /

; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int (d + ex) (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx &= -\frac{Cx^2 (d^2 - e^2x^2)^{3/2}}{5e} - \frac{\int \sqrt{d^2 - e^2x^2} (-5Ade^2 - e(2Cd^2 + 5e(Bd + Ae))) dx}{5e^2} \\
 &= -\frac{(Cd + Be)x (d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{Cx^2 (d^2 - e^2x^2)^{3/2}}{5e} + \frac{\int (5de^2 (Cd^2 + e(Bd + 4Ae))) \sqrt{d^2 - e^2x^2} dx}{5e^2} \\
 &= -\frac{(2Cd^2 + 5e(Bd + Ae)) (d^2 - e^2x^2)^{3/2}}{15e^3} - \frac{(Cd + Be)x (d^2 - e^2x^2)^{3/2}}{4e^2} \\
 &= \frac{d (Cd^2 + e(Bd + 4Ae)) x \sqrt{d^2 - e^2x^2}}{8e^2} - \frac{(2Cd^2 + 5e(Bd + Ae)) (d^2 - e^2x^2)^{3/2}}{15e^3} \\
 &= \frac{d (Cd^2 + e(Bd + 4Ae)) x \sqrt{d^2 - e^2x^2}}{8e^2} - \frac{(2Cd^2 + 5e(Bd + Ae)) (d^2 - e^2x^2)^{3/2}}{15e^3} \\
 &= \frac{d (Cd^2 + e(Bd + 4Ae)) x \sqrt{d^2 - e^2x^2}}{8e^2} - \frac{(2Cd^2 + 5e(Bd + Ae)) (d^2 - e^2x^2)^{3/2}}{15e^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.30, size = 190, normalized size = 1.02

$$\frac{\sqrt{d^2 - e^2x^2} \left( 15 \sin^{-1} \left( \frac{ex}{d} \right) (d^2 e(4Ae + Bd) + Cd^4) + \sqrt{1 - \frac{e^2x^2}{d^2}} (5e(4Ae(-2d^2 + 3dex + 2e^2x^2) + B(-8d^3 - 3d^2ex + 8de^2x^2 + 6e^3x^3)) + C(-16d^4 - 15d^3ex - 8d^2e^2x^2 + 30de^3x^3 + 24e^4x^4)) \right)}{120e^3 \sqrt{1 - \frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(Sqrt[1 - (e^2\*x^2)/d^2]\*(C\*(-16\*d^4 - 15\*d^3\*e\*x - 8\*d^2\*e^2\*x^2 + 30\*d\*e^3\*x^3 + 24\*e^4\*x^4) + 5\*e\*(4\*A\*e\*(-2\*d^2 + 3\*d\*e\*x + 2\*e^2\*x^2) + B\*(-8\*d^3 - 3\*d^2\*e\*x + 8\*d\*e^2\*x^2 + 6\*e^3\*x^3))) + 15\*(C\*d^4 + d^2\*e\*(B\*d + 4\*A\*e))\*ArcSin[(e\*x)/d])/(120\*e^3\*Sqrt[1 - (e^2\*x^2)/d^2])



**IntegrateAlgebraic [A]** time = 0.69, size = 199, normalized size = 1.07

$$\frac{\sqrt{d^2 - e^2 x^2} (-40Ad^2e^2 + 60Ade^3x + 40Ae^4x^2 - 40Bd^3e - 15Bd^2e^2x + 40Bde^3x^2 + 30Be^4x^3 - 16Cd^4 - 15Cd^3ex - 8Cd^2e^2x^2 + 30Cde^3x^3 + 24Ce^4x^4)}{120e^3} + \frac{\sqrt{-e^2} \log(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x) (4Ad^3e^2 + Bd^4e + Cd^5)}{8e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x)\*(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2],x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-16\*C\*d^4 - 40\*B\*d^3\*e - 40\*A\*d^2\*e^2 - 15\*C\*d^3\*e\*x - 15\*B\*d^2\*e^2\*x + 60\*A\*d\*e^3\*x - 8\*C\*d^2\*e^2\*x^2 + 40\*B\*d\*e^3\*x^2 + 40\*A\*e^4\*x^2 + 30\*C\*d\*e^3\*x^3 + 30\*B\*e^4\*x^3 + 24\*C\*e^4\*x^4))/(120\*e^3) + (Sqrt[-e^2]\*(C\*d^5 + B\*d^4\*e + 4\*A\*d^3\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(8\*e^4)

**fricas [A]** time = 1.11, size = 173, normalized size = 0.93

$$\frac{30(Cd^5 + Bd^4e + 4Ad^3e^2) \arctan\left(\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) - (24Ce^4x^4 - 16Cd^4 - 40Bd^3e - 40Ad^2e^2 + 30(Cde^3 + Be^4)x^3 - 8(Cd^2e^2 - 5Bde^3 - 5Ae^4)x^2 - 15(Cd^3e + Bd^2e^2 - 4Ade^3)x) \sqrt{-e^2x^2 + d^2}}{120e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/120\*(30\*(C\*d^5 + B\*d^4\*e + 4\*A\*d^3\*e^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (24\*C\*e^4\*x^4 - 16\*C\*d^4 - 40\*B\*d^3\*e - 40\*A\*d^2\*e^2 + 30\*(C\*d\*e^3 + B\*e^4)\*x^3 - 8\*(C\*d^2\*e^2 - 5\*B\*d\*e^3 - 5\*A\*e^4)\*x^2 - 15\*(C\*d^3\*e + B\*d^2\*e^2 - 4\*A\*d\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/e^3

**giac [A]** time = 0.21, size = 160, normalized size = 0.86

$$\frac{1}{8}(Cd^5 + Bd^4e + 4Ad^3e^2) \arcsin\left(\frac{xe}{d}\right) e^{(-3) \operatorname{sgn}(d)} + \frac{1}{120} \sqrt{-x^2e^2 + d^2} \left( (2(3(4Cxe + 5(Cde^3 + Be^4)e^{(-6)})x - 4(Cd^2e^5 - 5Bde^6 - 5Ae^7)e^{(-6)})x - 15(Cd^3e^4 + Bd^2e^5 - 4Ade^6)e^{(-6)})x - 8(2Cd^4e^3 + 5Bd^3e^4 + 5Ad^2e^5)e^{(-6)}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] 1/8\*(C\*d^5 + B\*d^4\*e + 4\*A\*d^3\*e^2)\*arcsin(x\*e/d)\*e^(-3)\*sgn(d) + 1/120\*sqrt(-x^2\*e^2 + d^2)\*((2\*(3\*(4\*C\*x\*e + 5\*(C\*d\*e^6 + B\*e^7)\*e^(-6))\*x - 4\*(C\*d^2\*e^5 - 5\*B\*d\*e^6 - 5\*A\*e^7)\*e^(-6))\*x - 15\*(C\*d^3\*e^4 + B\*d^2\*e^5 - 4\*A\*d\*e^6)\*e^(-6))\*x - 8\*(2\*C\*d^4\*e^3 + 5\*B\*d^3\*e^4 + 5\*A\*d^2\*e^5)\*e^(-6))

**maple [A]** time = 0.02, size = 304, normalized size = 1.63

$$\frac{Ad^3 \arctan\left(\frac{\sqrt{e}x}{\sqrt{-e^2x^2 + d^2}}\right)}{2\sqrt{e^2}} + \frac{Bd^4 \arctan\left(\frac{\sqrt{e}x}{\sqrt{-e^2x^2 + d^2}}\right)}{8\sqrt{e^2}} + \frac{Cd^5 \arctan\left(\frac{\sqrt{e}x}{\sqrt{-e^2x^2 + d^2}}\right)}{8\sqrt{e^2}} + \frac{\sqrt{-e^2x^2 + d^2} Adx}{2} + \frac{\sqrt{-e^2x^2 + d^2} Bdx}{8e} + \frac{\sqrt{-e^2x^2 + d^2} Cdx}{8e^2} - \frac{(-e^2x^2 + d^2)^{3/2} Cx^2}{5e} - \frac{(-e^2x^2 + d^2)^{3/2} Bx}{4e} - \frac{(-e^2x^2 + d^2)^{3/2} A}{4e^2} - \frac{(-e^2x^2 + d^2)^{3/2}}{3e} - \frac{(-e^2x^2 + d^2)^{3/2} Bd}{3e^2} - \frac{2(-e^2x^2 + d^2)^{3/2} Cd}{15e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2), x)`

[Out]  $-1/5*C*x^2*(-e^2*x^2+d^2)^{(3/2)}/e-2/15/e^3*C*d^2*(-e^2*x^2+d^2)^{(3/2)}-1/4*x*(-e^2*x^2+d^2)^{(3/2)}/e*B-1/4*x*(-e^2*x^2+d^2)^{(3/2)}/e^2*C*d+1/8*d^2/e*x*(-e^2*x^2+d^2)^{(1/2)*B+1/8*d^3/e^2*x*(-e^2*x^2+d^2)^{(1/2)*C+1/8*d^4/e/(e^2)^{(1/2)*arctan((e^2)^{(1/2)}/(-e^2*x^2+d^2)^{(1/2)*x)*B+1/8*d^5/e^2/(e^2)^{(1/2)*arctan((e^2)^{(1/2)}/(-e^2*x^2+d^2)^{(1/2)*x)*C-1/3*(-e^2*x^2+d^2)^{(3/2)}/e*A-1/3*(-e^2*x^2+d^2)^{(3/2)}/e^2*B*d+1/2*d*A*x*(-e^2*x^2+d^2)^{(1/2)+1/2*d^3*A/(e^2)^{(1/2)*arctan((e^2)^{(1/2)}/(-e^2*x^2+d^2)^{(1/2)*x)}$

**maxima** [A] time = 0.98, size = 202, normalized size = 1.09

$$\frac{Ad^3 \arcsin\left(\frac{x}{d}\right)}{2e} + \frac{1}{2} \sqrt{-e^2x^2 + d^2} Adx - \frac{(-e^2x^2 + d^2)^{3/2} Cx^2}{5e} + \frac{(Cd + Be)d^4 \arcsin\left(\frac{x}{d}\right)}{8e^3} + \frac{\sqrt{-e^2x^2 + d^2} (Cd + Be)d^2x}{8e^2} - \frac{2(-e^2x^2 + d^2)^{3/2} Cd^2}{15e^3} - \frac{(-e^2x^2 + d^2)^{3/2} Bd}{3e^2} - \frac{(-e^2x^2 + d^2)^{3/2} A}{3e} - \frac{(-e^2x^2 + d^2)^{3/2} (Cd + Be)x}{4e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2), x, algorithm="maxima")`

[Out]  $1/2*A*d^3*\arcsin(e*x/d)/e + 1/2*\sqrt{-e^2*x^2 + d^2}*A*d*x - 1/5*(-e^2*x^2 + d^2)^{(3/2)*C*x^2/e + 1/8*(C*d + B*e)*d^4*\arcsin(e*x/d)/e^3 + 1/8*\sqrt{-e^2*x^2 + d^2)*(C*d + B*e)*d^2*x/e^2 - 2/15*(-e^2*x^2 + d^2)^{(3/2)*C*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^{(3/2)*B*d/e^2 - 1/3*(-e^2*x^2 + d^2)^{(3/2)*A/e - 1/4*(-e^2*x^2 + d^2)^{(3/2)*(C*d + B*e)*x/e^2}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2 - e^2 x^2} (d + ex) (Cx^2 + Bx + A) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)`

[Out] `int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)`

**sympy** [C] time = 12.77, size = 670, normalized size = 3.60

$$Ad \left( \left( \frac{d^2 \operatorname{arcsinh}\left(\frac{x}{d}\right)}{2} - \frac{dx}{2\sqrt{-1+\frac{x^2}{d^2}}} + \frac{x^3}{2d\sqrt{-1+\frac{x^2}{d^2}}} \right) \text{ for } \left|\frac{x}{d}\right| > 1 \right) + Ar \left( \left( \frac{d^2 \sqrt{d^2 - e^2 x^2}}{2} \text{ for } e^2 = 0 \right) + Br \left( \left( \frac{d^2 \sqrt{d^2 - e^2 x^2}}{2} \text{ for } e^2 = 0 \right) + Br \left( \left( \frac{d^2 \operatorname{arcsinh}\left(\frac{x}{d}\right)}{2} + \frac{dx}{2\sqrt{-1+\frac{x^2}{d^2}}} - \frac{3dx^3}{4\sqrt{-1+\frac{x^2}{d^2}}} + \frac{x^3}{4d\sqrt{-1+\frac{x^2}{d^2}}} \right) \text{ for } \left|\frac{x}{d}\right| > 1 \right) + Cd \left( \left( \frac{d^2 \operatorname{arcsinh}\left(\frac{x}{d}\right)}{2} + \frac{dx}{2\sqrt{-1+\frac{x^2}{d^2}}} - \frac{3dx^3}{4\sqrt{-1+\frac{x^2}{d^2}}} + \frac{x^3}{4d\sqrt{-1+\frac{x^2}{d^2}}} \right) \text{ for } \left|\frac{x}{d}\right| > 1 \right) + Cr \left( \left( \frac{3d^2 \sqrt{d^2 - e^2 x^2}}{15d} - \frac{d^2 \sqrt{d^2 - e^2 x^2}}{15d} + \frac{d^4 \sqrt{d^2 - e^2 x^2}}{5} \right) \text{ for } e \neq 0 \right) \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2), x)`

[Out]  $A*d*\text{Piecewise}((-I*d**2*\operatorname{acosh}(e*x/d)/(2*e) - I*d*x/(2*\sqrt{-1 + e**2*x**2/d**2})) + I*e**2*x**3/(2*d*\sqrt{-1 + e**2*x**2/d**2}), \operatorname{Abs}(e**2*x**2/d**2) > 1), (d**2*\operatorname{asin}(e*x/d)/(2*e) + d*x*\sqrt{1 - e**2*x**2/d**2}/2, \text{True})) + A*e*\text{Piecewise}((x**2*\sqrt{d**2}/2, \operatorname{Eq}(e**2, 0)), (- (d**2 - e**2*x**2)**(3/2)/(3*e$

```

**2), True)) + B*d*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e
*2*x**2)**(3/2)/(3*e**2), True)) + B*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e
**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*
x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*
x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1
- e**2*x**2/d**2)), True)) + C*d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3)
+ I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/
d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/
d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e
*2*x**2/d**2)), True)) + C*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*
e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*
x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))

```

### 3.3 $\int (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx$

**Optimal.** Leaf size=125

$$\frac{1}{8}x\sqrt{d^2 - e^2x^2} \left(4A + \frac{Cd^2}{e^2}\right) + \frac{d^2(4Ae^2 + Cd^2) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} - \frac{B(d^2 - e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1815, 641, 195, 217, 203}

$$\frac{1}{8}x\sqrt{d^2 - e^2x^2} \left(4A + \frac{Cd^2}{e^2}\right) + \frac{d^2(4Ae^2 + Cd^2) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} - \frac{B(d^2 - e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out] ((4\*A + (C\*d^2)/e^2)\*x\*Sqrt[d^2 - e^2\*x^2])/8 - (B\*(d^2 - e^2\*x^2)^(3/2))/(3\*e^2) - (C\*x\*(d^2 - e^2\*x^2)^(3/2))/(4\*e^2) + (d^2\*(C\*d^2 + 4\*A\*e^2)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e^3)

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /

; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int (A + Bx + Cx^2) \sqrt{d^2 - e^2x^2} dx &= -\frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{\int (-Cd^2 - 4Ae^2 - 4Be^2x) \sqrt{d^2 - e^2x^2} dx}{4e^2} \\ &= -\frac{B(d^2 - e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{(-Cd^2 - 4Ae^2) \int \sqrt{d^2 - e^2x^2} dx}{4e^2} \\ &= \frac{(Cd^2 + 4Ae^2)x\sqrt{d^2 - e^2x^2}}{8e^2} - \frac{B(d^2 - e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{(d^2 - e^2x^2)^{3/2}}{4e^2} \\ &= \frac{(Cd^2 + 4Ae^2)x\sqrt{d^2 - e^2x^2}}{8e^2} - \frac{B(d^2 - e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{(d^2 - e^2x^2)^{3/2}}{4e^2} \\ &= \frac{(Cd^2 + 4Ae^2)x\sqrt{d^2 - e^2x^2}}{8e^2} - \frac{B(d^2 - e^2x^2)^{3/2}}{3e^2} - \frac{Cx(d^2 - e^2x^2)^{3/2}}{4e^2} + \frac{d^2(Cd^2 + 4Ae^2)}{8e^2} \end{aligned}$$

**Mathematica** [A] time = 0.13, size = 121, normalized size = 0.97

$$\frac{\sqrt{d^2 - e^2x^2} \left( e \sqrt{1 - \frac{e^2x^2}{d^2}} (12Ae^2x - 8Bd^2 + 8Be^2x^2 - 3Cd^2x + 6Ce^2x^3) + 3(4Ade^2 + Cd^3) \sin^{-1} \left( \frac{ex}{d} \right) \right)}{24e^3 \sqrt{1 - \frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(e\*Sqrt[1 - (e^2\*x^2)/d^2]\*(-8\*B\*d^2 - 3\*C\*d^2\*x + 12\*A\*e^2\*x + 8\*B\*e^2\*x^2 + 6\*C\*e^2\*x^3) + 3\*(C\*d^3 + 4\*A\*d\*e^2)\*ArcSin[(e\*x)/d]))/(24\*e^3\*Sqrt[1 - (e^2\*x^2)/d^2])

**IntegrateAlgebraic [A]** time = 0.40, size = 124, normalized size = 0.99

$$\frac{\sqrt{d^2 - e^2 x^2} (12Ae^2 x - 8Bd^2 + 8Be^2 x^2 - 3Cd^2 x + 6Ce^2 x^3)}{24e^2} + \frac{\sqrt{-e^2} (4Ad^2 e^2 + Cd^4) \log\left(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x\right)}{8e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2], x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-8\*B\*d^2 - 3\*C\*d^2\*x + 12\*A\*e^2\*x + 8\*B\*e^2\*x^2 + 6\*C\*e^2\*x^3))/(24\*e^2) + (Sqrt[-e^2]\*(C\*d^4 + 4\*A\*d^2\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(8\*e^4)

**fricas [A]** time = 1.47, size = 108, normalized size = 0.86

$$\frac{6(Cd^4 + 4Ad^2e^2) \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) - (6Ce^3x^3 + 8Be^3x^2 - 8Bd^2e - 3(Cd^2e - 4Ae^3)x)\sqrt{-e^2x^2 + d^2}}{24e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2), x, algorithm="fricas")

[Out] -1/24\*(6\*(C\*d^4 + 4\*A\*d^2\*e^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (6\*C\*e^3\*x^3 + 8\*B\*e^3\*x^2 - 8\*B\*d^2\*e - 3\*(C\*d^2\*e - 4\*A\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/e^3

**giac [A]** time = 0.20, size = 85, normalized size = 0.68

$$\frac{1}{8}(Cd^4 + 4Ad^2e^2) \arcsin\left(\frac{xe}{d}\right) e^{(-3)\operatorname{sgn}(d)} - \frac{1}{24}(8Bd^2e^{(-2)} - (2(3Cx + 4B)x - 3(Cd^2e^2 - 4Ae^4)e^{(-4)})x)\sqrt{-x^2e^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2), x, algorithm="giac")

[Out] 1/8\*(C\*d^4 + 4\*A\*d^2\*e^2)\*arcsin(x\*e/d)\*e^(-3)\*sgn(d) - 1/24\*(8\*B\*d^2\*e^(-2) - (2\*(3\*C\*x + 4\*B)\*x - 3\*(C\*d^2\*e^2 - 4\*A\*e^4)\*e^(-4))\*x)\*sqrt(-x^2\*e^2 + d^2)

**maple [A]** time = 0.01, size = 154, normalized size = 1.23

$$\frac{Ad^2 \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{2\sqrt{e^2}} + \frac{Cd^4 \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{8\sqrt{e^2} e^2} + \frac{\sqrt{-e^2x^2 + d^2} Ax}{2} + \frac{\sqrt{-e^2x^2 + d^2} Cd^2x}{8e^2} - \frac{(-e^2x^2 + d^2)^{\frac{3}{2}} Cx}{4e^2} - \frac{(-e^2x^2 + d^2)^{\frac{3}{2}} B}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2), x)

[Out]  $-1/4*C*x*(-e^2*x^2+d^2)^{(3/2)}/e^2+1/8*C*d^2/e^2*x*(-e^2*x^2+d^2)^{(1/2)}+1/8*C*d^4/e^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}/(-e^2*x^2+d^2)^{(1/2)}*x)-1/3*B*(-e^2*x^2+d^2)^{(3/2)}/e^2+1/2*A*x*(-e^2*x^2+d^2)^{(1/2)}+1/2*A*d^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}/(-e^2*x^2+d^2)^{(1/2)}*x)$

**maxima** [A] time = 0.96, size = 116, normalized size = 0.93

$$\frac{Cd^4 \arcsin\left(\frac{ex}{d}\right)}{8e^3} + \frac{Ad^2 \arcsin\left(\frac{ex}{d}\right)}{2e} + \frac{1}{2} \sqrt{-e^2x^2 + d^2} Ax + \frac{\sqrt{-e^2x^2 + d^2} Cd^2x}{8e^2} - \frac{(-e^2x^2 + d^2)^{\frac{3}{2}} Cx}{4e^2} - \frac{(-e^2x^2 + d^2)^{\frac{3}{2}} B}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")`

[Out]  $1/8*C*d^4*\arcsin(e*x/d)/e^3 + 1/2*A*d^2*\arcsin(e*x/d)/e + 1/2*\sqrt{-e^2*x^2 + d^2}*A*x + 1/8*\sqrt{-e^2*x^2 + d^2}*C*d^2*x/e^2 - 1/4*(-e^2*x^2 + d^2)^{(3/2)}*C*x/e^2 - 1/3*(-e^2*x^2 + d^2)^{(3/2)}*B/e^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2 - e^2 x^2} (C x^2 + B x + A) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2),x)`

[Out] `int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2), x)`

**sympy** [C] time = 7.11, size = 343, normalized size = 2.74

$$A \left( \begin{array}{l} \left( -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} \text{ for } \left|\frac{e^2x^2}{d^2}\right| > 1 \right) \\ \left( \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} \text{ otherwise} \right) \end{array} \right) + B \left( \begin{array}{l} \left( \frac{x^2\sqrt{d^2}}{2} \text{ for } e^2 = 0 \right) \\ \left( -\frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{3e^2} \text{ otherwise} \right) \end{array} \right) + C \left( \begin{array}{l} \left( -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} \text{ for } \left|\frac{e^2x^2}{d^2}\right| > 1 \right) \\ \left( \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} \text{ otherwise} \right) \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $A*\text{Piecewise}((-I*d**2*\operatorname{acosh}(e*x/d)/(2*e) - I*d*x/(2*\sqrt{-1 + e**2*x**2/d**2})) + I*e**2*x**3/(2*d*\sqrt{-1 + e**2*x**2/d**2}), \operatorname{Abs}(e**2*x**2/d**2) > 1), (d**2*\operatorname{asin}(e*x/d)/(2*e) + d*x*\sqrt{1 - e**2*x**2/d**2})/2, \operatorname{True})) + B*\text{Piecewise}((x**2*\sqrt{d**2})/2, \operatorname{Eq}(e**2, 0)), (- (d**2 - e**2*x**2)**(3/2)/(3*e**2), \operatorname{True})) + C*\text{Piecewise}((-I*d**4*\operatorname{acosh}(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*\sqrt{-1 + e**2*x**2/d**2})) - 3*I*d*x**3/(8*\sqrt{-1 + e**2*x**2/d**2})) + I*e**2*x**5/(4*d*\sqrt{-1 + e**2*x**2/d**2}), \operatorname{Abs}(e**2*x**2/d**2) > 1), (d**4*\operatorname{asin}(e*x/d)/(8*e**3) - d**3*x/(8*e**2*\sqrt{1 - e**2*x**2/d**2})) + 3*d*x**3/(8*\sqrt{1 - e**2*x**2/d**2}) - e**2*x**5/(4*d*\sqrt{1 - e**2*x**2/d**2}), \operatorname{True}))$

$$3.4 \quad \int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex} dx$$

**Optimal.** Leaf size=148

$$\frac{\sqrt{d^2 - e^2x^2} (Cd^2 - e(Bd - 2Ae))}{2e^3} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (Cd^2 - e(Bd - 2Ae))}{2e^3} + \frac{(d^2 - e^2x^2)^{3/2} (Cd - Be)}{2e^3(d + ex)} - \frac{C(d^2 - e^2x^2)^{3/2}}{3e^3}$$

**Rubi [A]** time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {1639, 795, 665, 217, 203}

$$\frac{\sqrt{d^2 - e^2x^2} (Cd^2 - e(Bd - 2Ae))}{2e^3} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (Cd^2 - e(Bd - 2Ae))}{2e^3} + \frac{(d^2 - e^2x^2)^{3/2} (Cd - Be)}{2e^3(d + ex)} - \frac{C(d^2 - e^2x^2)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] ((C\*d^2 - e\*(B\*d - 2\*A\*e))\*Sqrt[d^2 - e^2\*x^2])/(2\*e^3) - (C\*(d^2 - e^2\*x^2)^(3/2))/(3\*e^3) + ((C\*d - B\*e)\*(d^2 - e^2\*x^2)^(3/2))/(2\*e^3\*(d + e\*x)) + (d\*(C\*d^2 - e\*(B\*d - 2\*A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 665

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(2\*c\*d\*p)/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

### Rule 795



```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
+ a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

### Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{d + ex} dx &= -\frac{C(d^2 - e^2x^2)^{3/2}}{3e^3} - \frac{\int \frac{(-3Ae^4 + 3e^3(Cd - Be)x) \sqrt{d^2 - e^2x^2}}{d + ex} dx}{3e^4} \\ &= -\frac{C(d^2 - e^2x^2)^{3/2}}{3e^3} + \frac{(Cd - Be)(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)} + \frac{(Cd^2 - e(Bd - 2Ae)) \int \frac{\sqrt{d^2 - e^2x^2}}{d + ex} dx}{2e^2} \\ &= \frac{(Cd^2 - e(Bd - 2Ae)) \sqrt{d^2 - e^2x^2}}{2e^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{3e^3} + \frac{(Cd - Be)(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)} \\ &= \frac{(Cd^2 - e(Bd - 2Ae)) \sqrt{d^2 - e^2x^2}}{2e^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{3e^3} + \frac{(Cd - Be)(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)} \\ &= \frac{(Cd^2 - e(Bd - 2Ae)) \sqrt{d^2 - e^2x^2}}{2e^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{3e^3} + \frac{(Cd - Be)(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)} \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 103, normalized size = 0.70

$$\frac{\sqrt{d^2 - e^2x^2} (3e(2Ae - 2Bd + Bex) + C(4d^2 - 3dex + 2e^2x^2)) + 3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (e(2Ae - Bd) + Cd^2)}{6e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(3\*e\*(-2\*B\*d + 2\*A\*e + B\*e\*x) + C\*(4\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2)) + 3\*d\*(C\*d^2 + e\*(-(B\*d) + 2\*A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(6\*e^3)

**IntegrateAlgebraic [A]** time = 0.51, size = 130, normalized size = 0.88

$$\frac{\sqrt{d^2 - e^2 x^2} (6Ae^2 - 6Bde + 3Be^2 x + 4Cd^2 - 3Cdex + 2Ce^2 x^2)}{6e^3} + \frac{\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x\right) (2Ade^2 - Bd^2 e + Cd^3)}{2e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(4\*C\*d^2 - 6\*B\*d\*e + 6\*A\*e^2 - 3\*C\*d\*e\*x + 3\*B\*e^2\*x + 2\*C\*e^2\*x^2))/(6\*e^3) + (Sqrt[-e^2]\*(C\*d^3 - B\*d^2\*e + 2\*A\*d\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**fricas [A]** time = 1.44, size = 112, normalized size = 0.76

$$\frac{6(Cd^3 - Bd^2e + 2Ade^2) \arctan\left(-\frac{d - \sqrt{-e^2 x^2 + d^2}}{ex}\right) - (2Ce^2 x^2 + 4Cd^2 - 6Bde + 6Ae^2 - 3(Cde - Be^2)x)\sqrt{-e^2 x^2 + d^2}}{6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d), x, algorithm="fricas")

[Out] -1/6\*(6\*(C\*d^3 - B\*d^2\*e + 2\*A\*d\*e^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (2\*C\*e^2\*x^2 + 4\*C\*d^2 - 6\*B\*d\*e + 6\*A\*e^2 - 3\*(C\*d\*e - B\*e^2)\*x)\*sqrt(-e^2\*x^2 + d^2))/e^3

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2\*(4\*A\*d\*exp(2)^3-4\*A\*d\*exp(1)^4\*exp(2)+4\*B\*d^2\*exp(1)^3\*exp(2)-4\*B\*d^2\*exp(1)\*exp(2)^2)\*atan((-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)-1/4\*(-2\*C\*d^3-4\*A\*d\*exp(2)+2\*B\*d^2\*exp(1))\*sign(d)\*asin(x\*exp(2)/d/exp(1))/exp(1)/exp(2)+2\*((16\*exp(1)^4\*C/96/exp(1)^5\*x-(-24\*exp(1)^4\*B+24\*exp(1)^3\*C\*d)\*

/96/exp(1)^5)\*x-(-48\*exp(1)^4\*A+48\*exp(1)^3\*d\*B-32\*exp(1)^2\*C\*d^2)\*1/96/exp(1)^5)\*sqrt(d^2-x^2\*exp(2))

**maple [B]** time = 0.02, size = 384, normalized size = 2.59

$$\frac{Ad \arctan\left(\frac{\sqrt{x}}{\sqrt{(x+1/2)e-(x+1/2)^2}}\right)}{\sqrt{x}} - \frac{Bd^2 \arctan\left(\frac{\sqrt{x}}{\sqrt{(x+1/2)e-(x+1/2)^2}}\right)}{\sqrt{x}} + \frac{Bd^2 \arctan\left(\frac{\sqrt{x}}{\sqrt{(x+1/2)e-(x+1/2)^2}}\right)}{2\sqrt{x}} + \frac{Cd^2 \arctan\left(\frac{\sqrt{x}}{\sqrt{(x+1/2)e-(x+1/2)^2}}\right)}{\sqrt{x}} - \frac{Cd^2 \arctan\left(\frac{\sqrt{x}}{\sqrt{(x+1/2)e-(x+1/2)^2}}\right)}{2\sqrt{x}} + \frac{\sqrt{-x^2+d^2} Bx}{2e} - \frac{\sqrt{-x^2+d^2} Cd^2}{2e^2} + \frac{\sqrt{2(x+1/2)e-(x+1/2)^2}^2 A}{e} - \frac{\sqrt{2(x+1/2)e-(x+1/2)^2}^2 Bd}{e^2} + \frac{\sqrt{2(x+1/2)e-(x+1/2)^2}^2 Cd^2}{e^3} - \frac{(-x^2+d^2)^{3/2} C}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d), x)

[Out] -1/3\*C\*(-e^2\*x^2+d^2)^(3/2)/e^3+1/2/e\*B\*x\*(-e^2\*x^2+d^2)^(1/2)+1/2/e\*B\*d^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)/(-e^2\*x^2+d^2)^(1/2)\*x)-1/2/e^2\*C\*d\*x\*(-e^2\*x^2+d^2)^(1/2)-1/2/e^2\*C\*d^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)/(-e^2\*x^2+d^2)^(1/2)\*x)+1/e\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*A-1/e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*B\*d+1/e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*C\*d^2+d/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))\*A-1/e\*d^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))\*B+1/e^2\*d^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))\*C

**maxima [A]** time = 1.09, size = 171, normalized size = 1.16

$$\frac{Cd^3 \arcsin\left(\frac{ex}{d}\right)}{2e^3} - \frac{Bd^2 \arcsin\left(\frac{ex}{d}\right)}{2e^2} + \frac{Ad \arcsin\left(\frac{ex}{d}\right)}{e} - \frac{\sqrt{-e^2x^2+d^2} Cdx}{2e^2} + \frac{\sqrt{-e^2x^2+d^2} Bx}{2e} + \frac{\sqrt{-e^2x^2+d^2} Cd^2}{e^3} - \frac{\sqrt{-e^2x^2+d^2} Bd}{e^2} + \frac{\sqrt{-e^2x^2+d^2} A}{e} - \frac{(-e^2x^2+d^2)^{3/2} C}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d), x, algorithm="maxima")

[Out] 1/2\*C\*d^3\*arcsin(ex/d)/e^3 - 1/2\*B\*d^2\*arcsin(ex/d)/e^2 + A\*d\*arcsin(ex/d)/e - 1/2\*sqrt(-e^2\*x^2 + d^2)\*C\*d\*x/e^2 + 1/2\*sqrt(-e^2\*x^2 + d^2)\*B\*x/e + sqrt(-e^2\*x^2 + d^2)\*C\*d^2/e^3 - sqrt(-e^2\*x^2 + d^2)\*B\*d/e^2 + sqrt(-e^2\*x^2 + d^2)\*A/e - 1/3\*(-e^2\*x^2 + d^2)^(3/2)\*C/e^3

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2} (C x^2 + B x + A)}{d + e x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d^2 - e^2\*x^2)^(1/2)\*(A + B\*x + C\*x^2))/(d + e\*x), x)

[Out] int(((d^2 - e^2\*x^2)^(1/2)\*(A + B\*x + C\*x^2))/(d + e\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)} (A + Bx + Cx^2)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))\*(A + B\*x + C\*x\*\*2)/(d + e\*x), x)

$$3.5 \quad \int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=170

$$\frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{de^3(d + ex)^2} - \frac{\sqrt{d^2 - e^2x^2} (5Cd^2 - 2e(2Bd - Ae))}{2de^3} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (5Cd^2 - 2e(2Bd - Ae))}{2e^3}$$

**Rubi [A]** time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {1639, 793, 665, 217, 203}

$$\frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{de^3(d + ex)^2} - \frac{\sqrt{d^2 - e^2x^2} (5Cd^2 - 2e(2Bd - Ae))}{2de^3} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (5Cd^2 - 2e(2Bd - Ae))}{2e^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^2, x]

[Out] -((5\*C\*d^2 - 2\*e\*(2\*B\*d - A\*e))\*Sqrt[d^2 - e^2\*x^2])/(2\*d\*e^3) - ((C\*d^2 - B\*d\*e + A\*e^2)\*(d^2 - e^2\*x^2)^(3/2))/(d\*e^3\*(d + e\*x)^2) - (C\*(d^2 - e^2\*x^2)^(3/2))/(2\*e^3\*(d + e\*x)) - ((5\*C\*d^2 - 2\*e\*(2\*B\*d - A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 665

Int[((d\_) + (e\_.)\*(x\_)^2)^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + c\*x^2)^p)/(e\*(m + 2\*p + 1)), x] - Dist[(2\*c\*d\*p)/(e^2\*(m + 2\*p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2\*p + 1, 0] && IntegerQ[2\*p]

### Rule 793

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_
), x_Symbol] := Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

### Rule 1639

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^2} dx &= -\frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)} - \frac{\int \frac{(e^2(Cd^2 - 2Ae^2) + e^3(3Cd - 2Be)x) \sqrt{d^2 - e^2x^2}}{(d + ex)^2} dx}{2e^4} \\
&= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{de^3(d + ex)^2} - \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)} - \frac{(-3e^5(Cd^2 - 2Ae^2))}{2e^4} \\
&= -\frac{(5Cd^2 - 2e(2Bd - Ae)) \sqrt{d^2 - e^2x^2}}{2de^3} - \frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{de^3(d + ex)^2} \\
&= -\frac{(5Cd^2 - 2e(2Bd - Ae)) \sqrt{d^2 - e^2x^2}}{2de^3} - \frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{de^3(d + ex)^2} \\
&= -\frac{(5Cd^2 - 2e(2Bd - Ae)) \sqrt{d^2 - e^2x^2}}{2de^3} - \frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{de^3(d + ex)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 109, normalized size = 0.64

$$\frac{\sqrt{d^2 - e^2x^2} (2e(-2Ae + 3Bd + Bex) + C(-8d^2 - 3dex + e^2x^2))}{d + ex} - \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) (2e(Ae - 2Bd) + 5Cd^2)$$


---


$$2e^3$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^2,x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(2\*e\*(3\*B\*d - 2\*A\*e + B\*e\*x) + C\*(-8\*d^2 - 3\*d\*e\*x + e^2\*x^2)))/(d + e\*x) - (5\*C\*d^2 + 2\*e\*(-2\*B\*d + A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

**IntegrateAlgebraic [A]** time = 0.57, size = 134, normalized size = 0.79

$$\frac{\sqrt{d^2 - e^2 x^2} (-4Ae^2 + 6Bde + 2Be^2 x - 8Cd^2 - 3Cdex + Ce^2 x^2)}{2e^3(d + ex)} - \frac{\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x\right) (2Ae^2 - 4Bde + 5Cd^2)}{2e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^2,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-8\*C\*d^2 + 6\*B\*d\*e - 4\*A\*e^2 - 3\*C\*d\*e\*x + 2\*B\*e^2\*x + C\*e^2\*x^2))/(2\*e^3\*(d + e\*x)) - (Sqrt[-e^2]\*(5\*C\*d^2 - 4\*B\*d\*e + 2\*A\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**fricas [A]** time = 1.41, size = 190, normalized size = 1.12

$$\frac{8Cd^3 - 6Bd^2e + 4Ade^2 + 2(4Cd^2e - 3Bde^2 + 2Ae^3)x - 2(5Cd^3 - 4Bd^2e + 2Ade^2 + (5Cd^2e - 4Bde^2 + 2Ae^3)x) \arctan\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{ex}\right) - (Ce^2 x^2 - 8Cd^2 + 6Bde - 4Ae^2 - (3Cde - 2Be^2)x)\sqrt{-e^2 x^2 + d^2}}{2(e^4 x + de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^2,x, algorithm="fricas")

[Out] -1/2\*(8\*C\*d^3 - 6\*B\*d^2\*e + 4\*A\*d\*e^2 + 2\*(4\*C\*d^2\*e - 3\*B\*d\*e^2 + 2\*A\*e^3)\*x - 2\*(5\*C\*d^3 - 4\*B\*d^2\*e + 2\*A\*d\*e^2 + (5\*C\*d^2\*e - 4\*B\*d\*e^2 + 2\*A\*e^3)\*x)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (C\*e^2\*x^2 - 8\*C\*d^2 + 6\*B\*d\*e - 4\*A\*e^2 - (3\*C\*d\*e - 2\*B\*e^2)\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^4\*x + d\*e^3)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

*sage0x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^2,x, algorithm="giac")

[Out] sage0\*x

**maple [B]** time = 0.03, size = 439, normalized size = 2.58

$$\frac{A \arctan\left(\frac{\sqrt{e}}{\sqrt{(x+d)e - (x+d)^2}}\right)}{\sqrt{e}} + \frac{2Bd \arctan\left(\frac{\sqrt{e}}{\sqrt{(x+d)e - (x+d)^2}}\right)}{\sqrt{e^3}} + \frac{3Cd^2 \arctan\left(\frac{\sqrt{e}}{\sqrt{(x+d)e - (x+d)^2}}\right)}{\sqrt{e^5}} + \frac{C^2 d^3 \arctan\left(\frac{\sqrt{e}}{\sqrt{(x+d)e - (x+d)^2}}\right)}{2\sqrt{e^7}} + \frac{\sqrt{-e^2x^2 + d^2} Cx}{2e^2} + \frac{\sqrt{2(x+d)e - (x+d)^2} e^2 A}{de} + \frac{2\sqrt{2(x+d)e - (x+d)^2} e^2 B}{e^2} + \frac{3\sqrt{2(x+d)e - (x+d)^2} e^2 C d}{e^2} + \frac{2(x+d)e - (x+d)^2 e^2 A}{(x+d)^2 e^2} + \frac{2(x+d)e - (x+d)^2 e^2 B}{(x+d)^2 e^2} + \frac{2(x+d)e - (x+d)^2 e^2 C d}{(x+d)^2 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^2,x)

[Out]  $\frac{1}{2} C x^2 (-e^2 x^2 + d^2)^{1/2} / e^2 + \frac{1}{2} C / e^2 d^2 / (e^2)^{1/2} * \arctan((e^2)^{1/2} / (-e^2 x^2 + d^2)^{1/2} * x) - 1/e^3 d / (x+d/e)^2 * (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{3/2} * A + 1/e^4 / (x+d/e)^2 * (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{3/2} * B - 1/e^5 d / (x+d/e)^2 * (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{3/2} * C - 1/e d * (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{1/2} * A + 2/e^2 * (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{1/2} * B - 3/e^3 d * (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{1/2} * C - 1/(e^2)^{1/2} * \arctan((e^2)^{1/2} / (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{1/2} * x) * A + 2/e / (e^2)^{1/2} * \arctan((e^2)^{1/2} / (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{1/2} * x) * B * d - 3/e^2 / (e^2)^{1/2} * \arctan((e^2)^{1/2} / (2*(x+d/e)*d*e - (x+d/e)^2 * e^2)^{1/2} * x) * C * d^2$

**maxima [A]** time = 1.01, size = 197, normalized size = 1.16

$$-\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{e^4x+de^3} + \frac{2\sqrt{-e^2x^2+d^2}Bd}{e^3x+de^2} - \frac{5Cd^2\arcsin\left(\frac{ex}{d}\right)}{2e^3} + \frac{2Bd\arcsin\left(\frac{ex}{d}\right)}{e^2} - \frac{A\arcsin\left(\frac{ex}{d}\right)}{e} - \frac{2\sqrt{-e^2x^2+d^2}A}{e^2x+de} + \frac{\sqrt{-e^2x^2+d^2}Cx}{2e^2} - \frac{2\sqrt{-e^2x^2+d^2}Cd}{e^3} + \frac{\sqrt{-e^2x^2+d^2}B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-2*\sqrt{-e^2*x^2 + d^2} * C * d^2 / (e^4 * x + d * e^3) + 2*\sqrt{-e^2*x^2 + d^2} * B * d / (e^3 * x + d * e^2) - 5/2 * C * d^2 * \arcsin(ex/d) / e^3 + 2 * B * d * \arcsin(ex/d) / e^2 - A * \arcsin(ex/d) / e - 2*\sqrt{-e^2*x^2 + d^2} * A / (e^2 * x + d * e) + 1/2 * \sqrt{-e^2*x^2 + d^2} * C * x / e^2 - 2*\sqrt{-e^2*x^2 + d^2} * C * d / e^3 + \sqrt{-e^2*x^2 + d^2} * B / e^2$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2} (C x^2 + B x + A)}{(d + e x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d^2 - e^2\*x^2)^(1/2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2,x)

[Out] int(((d^2 - e^2\*x^2)^(1/2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)} (A + Bx + Cx^2)}{(d + ex)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

$$3.6 \quad \int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3} dx$$

**Optimal.** Leaf size=149

$$-\frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{3de^3(d + ex)^3} + \frac{2\sqrt{d^2 - e^2x^2} (3Cd - Be)}{e^3(d + ex)} + \frac{(3Cd - Be) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^2}$$

**Rubi [A]** time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {1639, 793, 663, 217, 203}

$$-\frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{3de^3(d + ex)^3} + \frac{2\sqrt{d^2 - e^2x^2} (3Cd - Be)}{e^3(d + ex)} + \frac{(3Cd - Be) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^3, x]

[Out] (2\*(3\*C\*d - B\*e)\*Sqrt[d^2 - e^2\*x^2])/(e^3\*(d + e\*x)) - ((C\*d^2 - B\*d\*e + A\*e^2)\*(d^2 - e^2\*x^2)^(3/2))/(3\*d\*e^3\*(d + e\*x)^3) - (C\*(d^2 - e^2\*x^2)^(3/2))/(e^3\*(d + e\*x)^2) + ((3\*C\*d - B\*e)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 663

Int[((d\_) + (e\_.)\*(x\_)^(m\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + c\*x^2)^p)/(e\*(m + p + 1)), x] - Dist[(c\*p)/(e^2\*(m + p + 1)), Int[(d + e\*x)^(m + 2)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 + a\*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2\*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2\*p]

### Rule 793

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_
), x_Symbol] := Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

### Rule 1639

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^3} dx &= -\frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^2} - \frac{\int \frac{(e^2(2Cd^2 - Ae^2) + e^3(3Cd - Be)x) \sqrt{d^2 - e^2x^2}}{(d + ex)^3} dx}{e^4} \\
&= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^2} - \frac{(3Cd - Be) \int \frac{\sqrt{d^2 - e^2x^2}}{d + ex} dx}{e^2} \\
&= \frac{2(3Cd - Be)\sqrt{d^2 - e^2x^2}}{e^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)} \\
&= \frac{2(3Cd - Be)\sqrt{d^2 - e^2x^2}}{e^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)} \\
&= \frac{2(3Cd - Be)\sqrt{d^2 - e^2x^2}}{e^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} - \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 114, normalized size = 0.77

$$\frac{\sqrt{d^2 - e^2x^2} (e(Ae(ex - d) - Bd(5d + 7ex)) + Cd(14d^2 + 19dex + 3e^2x^2))}{d(d + ex)^2} + 3(3Cd - Be) \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right)$$

$3e^3$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^3,x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(C\*d\*(14\*d^2 + 19\*d\*e\*x + 3\*e^2\*x^2) + e\*(A\*e\*(-d + e\*x) - B\*d\*(5\*d + 7\*e\*x))))/(d\*(d + e\*x)^2) + 3\*(3\*C\*d - B\*e)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]]/(3\*e^3)

**IntegrateAlgebraic [A]** time = 0.75, size = 139, normalized size = 0.93

$$\frac{\sqrt{d^2 - e^2 x^2} (-Ade^2 + Ae^3 x - 5Bd^2 e - 7Bde^2 x + 14Cd^3 + 19Cd^2 ex + 3Cde^2 x^2)}{3de^3(d + ex)^2} - \frac{\sqrt{-e^2} (Be - 3Cd) \log\left(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x\right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^3,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(14\*C\*d^3 - 5\*B\*d^2\*e - A\*d\*e^2 + 19\*C\*d^2\*e\*x - 7\*B\*d\*e^2\*x + A\*e^3\*x + 3\*C\*d\*e^2\*x^2))/(3\*d\*e^3\*(d + e\*x)^2) - (Sqrt[-e^2]\*(-3\*C\*d + B\*e)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/e^4

**fricas [A]** time = 1.00, size = 258, normalized size = 1.73

$$\frac{14Cd^4 - 5Bd^3e - Ad^2e^2 + (14Cd^2e^2 - 5Bd^2e - Ae^3)x^2 + 2(14Cd^3e - 5Bd^2e^2 - Ad^2e^2)x - 6(3Cd^4 - Bd^3e + (3Cd^2e^2 - Bde^3)x^2 + 2(3Cd^3e - Bd^2e^2)x) \arctan\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{ex}\right) + (3Cd^2e^2 + 14Cd^3 - 5Bd^2e - Ad^2e + (19Cd^2e - 7Bd^2e + Ae^3)x)\sqrt{-e^2 x^2 + d^2}}{3(d^2 x^2 + 2d^2 e x + d^3 e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/3\*(14\*C\*d^4 - 5\*B\*d^3\*e - A\*d^2\*e^2 + (14\*C\*d^2\*e^2 - 5\*B\*d\*e^3 - A\*e^4)\*x^2 + 2\*(14\*C\*d^3\*e - 5\*B\*d^2\*e^2 - A\*d\*e^3)\*x - 6\*(3\*C\*d^4 - B\*d^3\*e + (3\*C\*d^2\*e^2 - B\*d\*e^3)\*x^2 + 2\*(3\*C\*d^3\*e - B\*d^2\*e^2)\*x)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + (3\*C\*d\*e^2\*x^2 + 14\*C\*d^3 - 5\*B\*d^2\*e - A\*d\*e^2 + (19\*C\*d^2\*e - 7\*B\*d\*e^2 + A\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2)/(d\*e^5\*x^2 + 2\*d^2\*e^4\*x + d^3\*e^3)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-8\*A\*(-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x/exp(2))^2\*exp(1)^8\*exp

$$\begin{aligned}
& (2)^{-2} - 2A \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^{10} \cdot \exp(2) - 2A \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \\
& \cdot \exp(1)^8 \cdot \exp(2) - 4A \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^6 \cdot \exp(2) - 3A \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \\
& \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^6 \cdot \exp(2) + 3A \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^4 \cdot \exp(2) - 4A \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \\
& \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^4 \cdot \exp(2) - 4A \cdot \exp(1)^6 \cdot \exp(2) - 3B \cdot d \cdot \exp(1)^5 \cdot \exp(2) - 3A \cdot \exp(1)^4 \cdot \exp(2) + 4B \cdot d \cdot \exp(1)^3 \cdot \exp(2) - 8C \cdot d^2 \cdot \exp(2) + 2B \cdot d \cdot \exp(1) \cdot \exp(2) + 2C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^6 \cdot \exp(2) + 8B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^7 \cdot \exp(2) + 4B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^5 \cdot \exp(2) - 8C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^6 \cdot \exp(2) - 4C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^4 \cdot \exp(2) + 3B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^5 \cdot \exp(2) + 3B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^3 \cdot \exp(2) - 5C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^4 \cdot \exp(2) - 3C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(2) + 4B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^3 \cdot \exp(2) + 3C \cdot d^2 \cdot \exp(1)^4 \cdot \exp(2) - 8C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(2) + 1/2 \cdot A \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^4 \cdot \exp(2) + 4/x / \exp(2) + 6A \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^6 \cdot \exp(2) + 3/x / \exp(2) + A \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^8 \cdot \exp(2) + 2/x / \exp(2) - 2B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^9 \cdot \exp(2) + 2B \cdot d \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1) \cdot \exp(2) + 6C \cdot d^2 \cdot (-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(1)^8 \cdot \exp(2) + 13/2 \cdot C \cdot d^2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(2) + 5/x / \exp(2) - 7/2 \cdot B \cdot d \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^3 \cdot \exp(2) + 4/x / \exp(2) - 6B \cdot d \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^5 \cdot \exp(2) + 3/x / \exp(2) + 2B \cdot d \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^7 \cdot \exp(2) + 2/x / \exp(2) + 6C \cdot d^2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^4 \cdot \exp(2) + 3/x / \exp(2) - 5C \cdot d^2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1) \cdot \exp(1)^6 \cdot \exp(2) + 2/x / \exp(2) / ((-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x / \exp(2) \cdot \exp(2) - (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x + \exp(2) \cdot \exp(1)^9 + d \cdot \exp(1)^5 \cdot \exp(2) - d \cdot \exp(1)^7 \cdot \exp(2) + d \cdot \exp(1) \cdot \exp(2)^4 + 1/2 \cdot (4A \cdot \exp(1)^8 \cdot \exp(2) - 12B \cdot d \cdot \exp(1)^7 \cdot \exp(2) + 20C \cdot d^2 \cdot \exp(1)^6 \cdot \exp(2) + 2A \cdot \exp(1)^6 \cdot \exp(2) - 6B \cdot d \cdot \exp(1)^5 \cdot \exp(2) + 3 + 18C \cdot d^2 \cdot \exp(1)^4 \cdot \exp(2) + 3 + 4A \cdot \exp(1)^4 \cdot \exp(2) + 4 + 4B \cdot d \cdot \exp(1)^3 \cdot \exp(2) + 4 - 24C \cdot d^2 \cdot \exp(2) + 5 + 4B \cdot d \cdot \exp(1) \cdot \exp(2) - 5 - 4C \cdot d^2 \cdot \exp(1)^8 \cdot \exp(2)) \cdot \tan((-1/2 \cdot (-2d \cdot \exp(1) - 2\sqrt{d^2 - x^2 \exp(2)}) \cdot \exp(1)) / x + \exp(2)) / \sqrt{-\exp(1)^4 + \exp(2)^2} / \sqrt{-\exp(1)^4 + \exp(2)^2} / (d \cdot \exp(1)^{11} - d \cdot \exp(1)^7 \cdot \exp(2) - d \cdot \exp(1)^5 \cdot \exp(2)^3 + d \cdot \exp(1)^9 \cdot \exp(2)) - 1/4 \cdot (4B \cdot \exp(1) - 12C \cdot d) \cdot \text{sign}(d) \cdot \text{asin}(x \cdot \exp(2) / d / \exp(1)) / \exp(1) / \exp(2) + 4 \cdot \exp(1)^2 \cdot C \cdot 1/4 / \exp(1)^5 \cdot \sqrt{d^2 - x^2 \exp(2)} \\
& 2)
\end{aligned}$$

**maple [B]** time = 0.02, size = 318, normalized size = 2.13

$$\frac{B \arctan\left(\frac{\sqrt{e} x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e}} + \frac{3Cd \arctan\left(\frac{\sqrt{e} x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2 e^2}} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2} B}{d e^2} + \frac{3\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2} C}{e^3} - \frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} B}{\left(x+\frac{d}{e}\right)^2 d e^4} + \frac{2\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} C}{\left(x+\frac{d}{e}\right)^2 e^5} - \frac{(Ae^2 - Bde + C d^2)\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}}}{3\left(x+\frac{d}{e}\right)^3 d e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x)`

[Out]  $-1/e^4/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B+2/e^5/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/e^2/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*B+3/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*C-1/e/(e^2)^(1/2)*\arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*B+3/e^2/(e^2)^(1/2)*\arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*C*d-1/3*(A*e^2-B*d*e+C*d^2)/e^6/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)$

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima")`

[Out] Timed out

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2} (C x^2 + B x + A)}{(d + e x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3,x)`

[Out] `int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3, x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)} (A + Bx + Cx^2)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)`

[Out] `Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)`

$$3.7 \quad \int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4} dx$$

**Optimal.** Leaf size=196

$$-\frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{15d^2e^3(d+ex)^3} - \frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{5de^3(d+ex)^4} + \frac{(d^2 - e^2x^2)^{3/2} (2Cd - Be)}{3de^3(d+ex)^3} - \frac{2C\sqrt{d^2 - e^2x^2}}{e^3(d+ex)}$$

**Rubi [A]** time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {1637, 659, 651, 663, 217, 203}

$$-\frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{15d^2e^3(d+ex)^3} - \frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{5de^3(d+ex)^4} + \frac{(d^2 - e^2x^2)^{3/2} (2Cd - Be)}{3de^3(d+ex)^3} - \frac{2C\sqrt{d^2 - e^2x^2}}{e^3(d+ex)} - \frac{C \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^4,x]

[Out] (-2\*C\*Sqrt[d^2 - e^2\*x^2])/(e^3\*(d + e\*x)) - ((C\*d^2 - B\*d\*e + A\*e^2)\*(d^2 - e^2\*x^2)^(3/2))/(5\*d\*e^3\*(d + e\*x)^4) + ((2\*C\*d - B\*e)\*(d^2 - e^2\*x^2)^(3/2))/(3\*d\*e^3\*(d + e\*x)^3) - ((C\*d^2 - B\*d\*e + A\*e^2)\*(d^2 - e^2\*x^2)^(3/2))/(15\*d^2\*e^3\*(d + e\*x)^3) - (C\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 651

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

### Rule 659

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[Simplif

```
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

### Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

### Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^4} dx &= \int \left( \frac{(Cd^2 - Bde + Ae^2) \sqrt{d^2 - e^2x^2}}{e^2(d + ex)^4} + \frac{(-2Cd + Be) \sqrt{d^2 - e^2x^2}}{e^2(d + ex)^3} + \frac{C \sqrt{d^2 - e^2x^2}}{e^2(d + ex)^2} \right) dx \\ &= \frac{C \int \frac{\sqrt{d^2 - e^2x^2}}{(d + ex)^2} dx}{e^2} - \frac{(2Cd - Be) \int \frac{\sqrt{d^2 - e^2x^2}}{(d + ex)^3} dx}{e^2} + \frac{(Cd^2 - Bde + Ae^2) \int \frac{\sqrt{d^2 - e^2x^2}}{(d + ex)^4} dx}{e^2} \\ &= -\frac{2C \sqrt{d^2 - e^2x^2}}{e^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2) (d^2 - e^2x^2)^{3/2}}{5de^3(d + ex)^4} + \frac{(2Cd - Be) (d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} \\ &= -\frac{2C \sqrt{d^2 - e^2x^2}}{e^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2) (d^2 - e^2x^2)^{3/2}}{5de^3(d + ex)^4} + \frac{(2Cd - Be) (d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} \\ &= -\frac{2C \sqrt{d^2 - e^2x^2}}{e^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2) (d^2 - e^2x^2)^{3/2}}{5de^3(d + ex)^4} + \frac{(2Cd - Be) (d^2 - e^2x^2)^{3/2}}{3de^3(d + ex)^3} \end{aligned}$$

**Mathematica [A]** time = 0.30, size = 112, normalized size = 0.57

$$\frac{\sqrt{d^2 - e^2x^2} (e(d - ex)(Ae(4d + ex) + Bd(d + 4ex)) + 3Cd^2(8d^2 + 19dex + 13e^2x^2))}{d^2(d + ex)^3} + 15C \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right)$$


---

15e<sup>3</sup>



Antiderivative was successfully verified.

[In] Integrate[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^4,x]

[Out] 
$$-1/15 * ((\text{Sqrt}[d^2 - e^2*x^2] * (3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d - e*x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x)))) / (d^2*(d + e*x)^3) + 15*C*\text{ArcT an}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]) / e^3$$

**IntegrateAlgebraic [A]** time = 0.94, size = 158, normalized size = 0.81

$$\frac{\sqrt{d^2 - e^2x^2} \left( -4Ad^2e^2 + 3Ade^3x + Ae^4x^2 - Bd^3e - 3Bd^2e^2x + 4Bde^3x^2 - 24Cd^4 - 57Cd^3ex - 39Cd^2e^2x^2 \right)}{15d^2e^3(d + ex)^3} - \frac{C\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2x^2} - \sqrt{-e^2}x\right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^4,x]

[Out] 
$$\left( \text{Sqrt}[d^2 - e^2*x^2] * (-24*C*d^4 - B*d^3*e - 4*A*d^2*e^2 - 57*C*d^3*e*x - 3*B*d^2*e^2*x + 3*A*d*e^3*x - 39*C*d^2*e^2*x^2 + 4*B*d*e^3*x^2 + A*e^4*x^2) \right) / \left( 15*d^2*e^3*(d + e*x)^3 - (C*\text{Sqrt}[-e^2]*\text{Log}[-(\text{Sqrt}[-e^2]*x) + \text{Sqrt}[d^2 - e^2*x^2]]) \right) / e^4$$

**fricas [A]** time = 1.00, size = 304, normalized size = 1.55

$$\frac{24Cd^4 + Bd^3e + 4Ad^2e^2 + (24Cd^3e + Bde^4 + 4Ae^5)x^3 + 3(24Cd^2e^2 + Bd^2e^3 + 4Ade^4)x^2 + 3(24Cd^2e + Bd^2e^2 + 4Ade^3)x - 30(Cd^2e^3x^3 + 3Cd^2e^2x^2 + 3Cd^2ex + Cd^2) \arctan\left(\frac{d - \sqrt{-e^2x^2}}{ex}\right) + (24Cd^4 + Bd^3e + 4Ad^2e^2 + (39Cd^3e - 4Bde^4 - Ae^5)x^2 + 3(19Cd^2e + Bd^2e^2 - Ade^3)x) \sqrt{-e^2x^2 + d^2}}{15(d^2e^3 + 3d^2e^2x + 3d^2ex + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4,x, algorithm="fricas")

[Out] 
$$-1/15 * (24*C*d^5 + B*d^4*e + 4*A*d^3*e^2 + (24*C*d^2*e^3 + B*d*e^4 + 4*A*e^5) * x^3 + 3 * (24*C*d^3*e^2 + B*d^2*e^3 + 4*A*d*e^4) * x^2 + 3 * (24*C*d^4*e + B*d^3*e^2 + 4*A*d^2*e^3) * x - 30 * (C*d^2*e^3*x^3 + 3*C*d^3*e^2*x^2 + 3*C*d^4*e*x + C*d^5) * \arctan\left(\frac{d - \text{sqrt}(-e^2*x^2 + d^2)}{(e*x)}\right) + (24*C*d^4 + B*d^3*e + 4*A*d^2*e^2 + (39*C*d^2*e^2 - 4*B*d*e^3 - A*e^4) * x^2 + 3 * (19*C*d^3*e + B*d^2*e^2 - A*d*e^3) * x) * \text{sqrt}(-e^2*x^2 + d^2)) / (d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*e^3)$$

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output:  $(8*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{16}*exp(2)+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^{14}*exp(2)^2+6*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^{12}*exp(2)^3+3/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))^4*exp(2)^7/x/exp(2)+42*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))^6*exp(2)^6/x/exp(2)+9*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))^8*exp(2)^5/x/exp(2)+3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))^10*exp(2)^4/x/exp(2)-3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))^12*exp(2)^3/x/exp(2)+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^{13}*exp(2)^2+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^{12}*exp(2)^3+6*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^{10}*exp(2)^4+48*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{13}*exp(2)^2+36*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^{11}*exp(2)^3+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^9*exp(2)^4-96*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{12}*exp(2)^2-84*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^{10}*exp(2)^3-18*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^2*exp(1)^{14}*exp(2)^2-8*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{12}*exp(2)^3-24*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^{10}*exp(2)^4-12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^2*exp(1)^{13}*exp(2)^2+14*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{11}*exp(2)^3+3*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^7*exp(2)^5+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^2*exp(1)^{12}*exp(2)^2-32*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{10}*exp(2)^3-24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-6*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^2*exp(1)^{12}*exp(2)^3-72*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^{10}*exp(2)^4-84*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^5-24*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^6+108*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^2*exp(1)^{11}*exp(2)^3+96*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^4+48*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^5+12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5*exp(1)^5*exp(2)^6-204*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^2*exp(1)^{10}*exp(2)^3-120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*$



) \* exp(1) \* exp(1) ^ 8 \* exp(2) ^ 4 / x / exp(2) - 6 \* C \* d ^ 2 \* (-2 \* d \* exp(1) - 2 \* sqrt(d ^ 2 - x ^ 2 \* exp(2))) \* exp(1) \* exp(1) ^ 10 \* exp(2) ^ 3 / x / exp(2) / ((-1/2 \* (-2 \* d \* exp(1) - 2 \* sqrt(d ^ 2 - x ^ 2 \* exp(2))) \* exp(1)) / x / exp(2)) ^ 2 \* exp(2) - (-2 \* d \* exp(1) - 2 \* sqrt(d ^ 2 - x ^ 2 \* exp(2))) \* exp(1) / x + exp(2)) ^ 3 / (3 \* d ^ 2 \* exp(1) ^ 13 - 6 \* d ^ 2 \* exp(1) ^ 9 \* exp(2) ^ 2 - 6 \* d ^ 2 \* exp(1) ^ 7 \* exp(2) ^ 3 + 3 \* d ^ 2 \* exp(1) ^ 5 \* exp(2) ^ 4 + 3 \* d ^ 2 \* exp(1) ^ 11 \* exp(2) + 3 \* d ^ 2 \* exp(1) \* exp(2) ^ 6) + 1/2 \* (-4 \* B \* d \* exp(1) ^ 11 \* exp(2) ^ 2 + 16 \* C \* d ^ 2 \* exp(1) ^ 10 \* exp(2) ^ 2 - 2 \* B \* d \* exp(1) ^ 9 \* exp(2) ^ 3 + 8 \* C \* d ^ 2 \* exp(1) ^ 8 \* exp(2) ^ 3 + 8 \* A \* exp(1) ^ 8 \* exp(2) ^ 4 - 8 \* B \* d \* exp(1) ^ 7 \* exp(2) ^ 4 - 8 \* C \* d ^ 2 \* exp(1) ^ 6 \* exp(2) ^ 4 + 2 \* A \* exp(1) ^ 6 \* exp(2) ^ 5 - 10 \* C \* d ^ 2 \* exp(1) ^ 4 \* exp(2) ^ 5 + 4 \* A \* exp(1) ^ 4 \* exp(2) ^ 6 + 8 \* C \* d ^ 2 \* exp(2) ^ 7) \* atan((-1/2 \* (-2 \* d \* exp(1) - 2 \* sqrt(d ^ 2 - x ^ 2 \* exp(2))) \* exp(1)) / x + exp(2)) / sqrt(-exp(1) ^ 4 + exp(2) ^ 2)) / sqrt(-exp(1) ^ 4 + exp(2) ^ 2) / (-d ^ 2 \* exp(1) ^ 15 + 2 \* d ^ 2 \* exp(1) ^ 11 \* exp(2) ^ 2 + 2 \* d ^ 2 \* exp(1) ^ 9 \* exp(2) ^ 3 - d ^ 2 \* exp(1) ^ 7 \* exp(2) ^ 4 - d ^ 2 \* exp(1) ^ 5 \* exp(2) ^ 5 - d ^ 2 \* exp(1) ^ 13 \* exp(2)) - C \* sign(d) \* asin(x \* exp(2) / d / exp(1)) / exp(1) / exp(2)

**maple [B]** time = 0.02, size = 453, normalized size = 2.31

$$\frac{\operatorname{Catan}\left(\frac{\sqrt{d^2-x^2}}{\sqrt{d^2-x^2}\sqrt{d^2-x^2}}\right)}{\sqrt{d^2-x^2}} \sqrt{2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2}} \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{5\frac{(x+\frac{d}{2})^2}{d^2}} A \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{15\frac{(x+\frac{d}{2})^2}{d^2}} A \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{15\frac{(x+\frac{d}{2})^2}{d^2}} B \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{5\frac{(x+\frac{d}{2})^2}{d^2}} B \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{5\frac{(x+\frac{d}{2})^2}{d^2}} C \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{(x+\frac{d}{2})d} C \frac{(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{15\frac{(x+\frac{d}{2})^2}{d^2}} C \frac{(8x-2Cd)(2\frac{(x+\frac{d}{2})d-(x+\frac{d}{2})^2}{d^2})^{\frac{1}{2}}}{3\frac{(x+\frac{d}{2})^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4,x)

[Out] -C/e^5/d/(x+d/e)^2\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)-C/e^3/d\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(1/2)-C/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)/(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(1/2)\*x)-1/5/e^5/d/(x+d/e)^4\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)\*A+1/5/e^6/(x+d/e)^4\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)\*B-1/5/e^7\*d/(x+d/e)^4\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)\*C-1/15/e^4/d^2/(x+d/e)^3\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)\*A+1/15/e^5/d/(x+d/e)^3\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)\*B-1/15/e^6/(x+d/e)^3\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)\*C-1/3\*(B\*e-2\*C\*d)/e^6/d/(x+d/e)^3\*(2\*(x+d/e)\*d\*e-(x+d/e)^2\*e^2)^(3/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2 x^2 + d^2} (C x^2 + B x + A)}{(e x + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4,x, algorithm="maxima")

[Out] integrate(sqrt(-e^2\*x^2 + d^2)\*(C\*x^2 + B\*x + A)/(e\*x + d)^4, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2 x^2} (C x^2 + B x + A)}{(d + e x)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)`

[Out] `int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)} (A + Bx + Cx^2)}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4, x)`

[Out] `Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)`

$$3.8 \quad \int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5} dx$$

**Optimal.** Leaf size=180

$$\frac{(d^2 - e^2x^2)^{3/2} (e(2Ae + 5Bd) + 23Cd^2)}{35d^2e^3(d + ex)^4} - \frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{7de^3(d + ex)^5} - \frac{(d^2 - e^2x^2)^{3/2} (e(2Ae + 5Bd) + 23Cd^2)}{105d^3e^3(d + ex)^3}$$

**Rubi [A]** time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1639, 793, 659, 651}

$$\frac{(d^2 - e^2x^2)^{3/2} (e(2Ae + 5Bd) + 23Cd^2)}{105d^3e^3(d + ex)^3} - \frac{(d^2 - e^2x^2)^{3/2} (e(2Ae + 5Bd) + 23Cd^2)}{35d^2e^3(d + ex)^4} - \frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{7de^3(d + ex)^5} + \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^4}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^5, x]

[Out] -((C\*d^2 - B\*d\*e + A\*e^2)\*(d^2 - e^2\*x^2)^(3/2))/(7\*d\*e^3\*(d + e\*x)^5) + (C\*(d^2 - e^2\*x^2)^(3/2))/(e^3\*(d + e\*x)^4) - ((23\*C\*d^2 + e\*(5\*B\*d + 2\*A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(35\*d^2\*e^3\*(d + e\*x)^4) - ((23\*C\*d^2 + e\*(5\*B\*d + 2\*A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(105\*d^3\*e^3\*(d + e\*x)^3)

### Rule 651

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

### Rule 659

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[Simplify[m + 2\*p + 2]/(2\*d\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

### Rule 793

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[(m\*(g\*c\*d + c\*e\*f) + 2\*e\*c\*f\*(p + 1))/(e\*(2\*c\*d)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e

, f, g, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

### Rule 1639

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m + p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rubi steps

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^5} dx = \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^4} + \frac{\int \frac{(e^2(4Cd^2 + Ae^2) + e^3(3Cd + Be)x) \sqrt{d^2 - e^2x^2}}{(d + ex)^5} dx}{e^4}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{7de^3(d + ex)^5} + \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^4} + \frac{(23Cd^2 + e(5Bd - 7Cde + Ae^2)) \sqrt{d^2 - e^2x^2}}{35d^2e^3(d + ex)^4}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{7de^3(d + ex)^5} + \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^4} - \frac{(23Cd^2 + e(5Bd - 7Cde + Ae^2)) \sqrt{d^2 - e^2x^2}}{35d^2e^3(d + ex)^4}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{7de^3(d + ex)^5} + \frac{C(d^2 - e^2x^2)^{3/2}}{e^3(d + ex)^4} - \frac{(23Cd^2 + e(5Bd - 7Cde + Ae^2)) \sqrt{d^2 - e^2x^2}}{35d^2e^3(d + ex)^4}$$

**Mathematica [A]** time = 0.20, size = 109, normalized size = 0.61

$$\frac{(d - ex) \sqrt{d^2 - e^2x^2} (e(Ae(23d^2 + 10dex + 2e^2x^2) + 5Bd(d^2 + 5dex + e^2x^2)) + Cd^2(2d^2 + 10dex + 23e^2x^2))}{105d^3e^3(d + ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^5, x]

[Out] -1/105\*((d - e\*x)\*Sqrt[d^2 - e^2\*x^2]\*(C\*d^2\*(2\*d^2 + 10\*d\*e\*x + 23\*e^2\*x^2) + e\*(5\*B\*d\*(d^2 + 5\*d\*e\*x + e^2\*x^2) + A\*e\*(23\*d^2 + 10\*d\*e\*x + 2\*e^2\*x^2))))/(d^3\*e^3\*(d + e\*x)^4)

**IntegrateAlgebraic [A]** time = 0.91, size = 149, normalized size = 0.83

$$\frac{\sqrt{d^2 - e^2x^2} (-23Ad^3e^2 + 13Ad^2e^3x + 8Ade^4x^2 + 2Ae^5x^3 - 5Bd^4e - 20Bd^3e^2x + 20Bd^2e^3x^2 + 5Bde^4x^3 - 2Cd^5 - 8Cd^4ex - 13Cd^3e^2x^2 + 23Cd^2e^3x^3)}{105d^3e^3(d + ex)^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^5,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-2\*C\*d^5 - 5\*B\*d^4\*e - 23\*A\*d^3\*e^2 - 8\*C\*d^4\*e\*x - 20\*B\*d^3\*e^2\*x + 13\*A\*d^2\*e^3\*x - 13\*C\*d^3\*e^2\*x^2 + 20\*B\*d^2\*e^3\*x^2 + 8\*A\*d\*e^4\*x^2 + 23\*C\*d^2\*e^3\*x^3 + 5\*B\*d\*e^4\*x^3 + 2\*A\*e^5\*x^3))/(105\*d^3\*e^3\*(d + e\*x)^4)

**fricas [A]** time = 0.79, size = 320, normalized size = 1.78

$$\frac{2C^5e^6 + 5Bd^5e^5 + 23Ad^4e^4 + (2Cd^4e^5 + 5Bd^4e^4 + 23Ad^4e^3)x^4 + 4(2Cd^3e^5 + 5Bd^3e^4 + 23Ad^3e^3)x^3 + (2Cd^2e^5 + 5Bd^2e^4 + 23Ad^2e^3 - (23Cd^2e^3 + 5Bd^2e^2 + 2Ae^2)x^2 + (13Cd^2e^3 - 20Bd^2e^2 - 8Ad^2e^2) + (8Cd^2e^3 + 20Bd^2e^2 - 13Ad^2e^2)x - C^2e^2 + B^2e^2)}{105(d^2e^2 + 4d^2e^2 + 6d^2e^2 + 4d^2e^2 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^5,x, algorithm="fricas")

[Out] -1/105\*(2\*C\*d^6 + 5\*B\*d^5\*e + 23\*A\*d^4\*e^2 + (2\*C\*d^2\*e^4 + 5\*B\*d\*e^5 + 23\*A\*e^6)\*x^4 + 4\*(2\*C\*d^3\*e^3 + 5\*B\*d^2\*e^4 + 23\*A\*d\*e^5)\*x^3 + 6\*(2\*C\*d^4\*e^2 + 5\*B\*d^3\*e^3 + 23\*A\*d^2\*e^4)\*x^2 + 4\*(2\*C\*d^5\*e + 5\*B\*d^4\*e^2 + 23\*A\*d^3\*e^3)\*x + (2\*C\*d^5 + 5\*B\*d^4\*e + 23\*A\*d^3\*e^2 - (23\*C\*d^2\*e^3 + 5\*B\*d\*e^4 + 2\*A\*e^5)\*x^3 + (13\*C\*d^3\*e^2 - 20\*B\*d^2\*e^3 - 8\*A\*d\*e^4)\*x^2 + (8\*C\*d^4\*e + 20\*B\*d^3\*e^2 - 13\*A\*d^2\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/(d^3\*e^7\*x^4 + 4\*d^4\*e^6\*x^3 + 6\*d^5\*e^5\*x^2 + 4\*d^6\*e^4\*x + d^7\*e^3)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^5,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t\_n ostep)]Warning, choosing root of [1,0,%%{2,[2,0]%%},0,%%{1,[4,0]%%}+%%{-4,[2,1]%%}+%%{4,[0,2]%%}] at parameters values [86,-97]Limit: Max order reached or unable to make series expansion Error: Bad Argument Value



**maple [A]** time = 0.01, size = 116, normalized size = 0.64

$$\frac{(-ex + d)(2Ae^4x^2 + 5Bde^3x^2 + 23Cd^2e^2x^2 + 10Ad e^3x + 25Bd^2e^2x + 10Cd^3ex + 23Ad^2e^2 + 5Bd^3e + 2Cd^4)\sqrt{-e^2x^2 + d^2}}{105(ex + d)^4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^5,x)

[Out] -1/105\*(-e\*x+d)\*(2\*A\*e^4\*x^2+5\*B\*d\*e^3\*x^2+23\*C\*d^2\*e^2\*x^2+10\*A\*d\*e^3\*x+25\*B\*d^2\*e^2\*x+10\*C\*d^3\*e\*x+23\*A\*d^2\*e^2+5\*B\*d^3\*e+2\*C\*d^4)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4/d^3/e^3

**maxima [B]** time = 0.54, size = 945, normalized size = 5.25

1/105\*d^3/e^3\*(2\*A\*d^2\*e^4\*x^2+5\*B\*d^3\*e^3\*x^2+23\*C\*d^4\*e^2\*x^2+10\*A\*d^3\*e^3\*x+25\*B\*d^4\*e^2\*x+10\*C\*d^5\*e\*x+23\*A\*d^4\*e^2+5\*B\*d^5\*e+2\*C\*d^6)\*sqrt(-e^2\*x^2+d^2)/(e\*x+d)^4

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^5,x, algorithm="maxima")

[Out] -2/7\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(e^7\*x^4 + 4\*d\*e^6\*x^3 + 6\*d^2\*e^5\*x^2 + 4\*d^3\*e^4\*x + d^4\*e^3) + 1/35\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d\*e^6\*x^3 + 3\*d^2\*e^5\*x^2 + 3\*d^3\*e^4\*x + d^4\*e^3) + 2/105\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d^2\*e^5\*x^2 + 2\*d^3\*e^4\*x + d^4\*e^3) + 2/105\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d^3\*e^4\*x + d^4\*e^3) + 2/7\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(e^6\*x^4 + 4\*d\*e^5\*x^3 + 6\*d^2\*e^4\*x^2 + 4\*d^3\*e^3\*x + d^4\*e^2) - 1/35\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d\*e^5\*x^3 + 3\*d^2\*e^4\*x^2 + 3\*d^3\*e^3\*x + d^4\*e^2) - 2/105\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d^2\*e^4\*x^2 + 2\*d^3\*e^3\*x + d^4\*e^2) - 2/105\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d^3\*e^3\*x + d^4\*e^2) + 4/5\*sqrt(-e^2\*x^2 + d^2)\*C\*d/(e^6\*x^3 + 3\*d\*e^5\*x^2 + 3\*d^2\*e^4\*x + d^3\*e^3) - 2/15\*sqrt(-e^2\*x^2 + d^2)\*C\*d/(d\*e^5\*x^2 + 2\*d^2\*e^4\*x + d^3\*e^3) - 2/15\*sqrt(-e^2\*x^2 + d^2)\*C\*d/(d^2\*e^4\*x + d^3\*e^3) - 2/7\*sqrt(-e^2\*x^2 + d^2)\*A/(e^5\*x^4 + 4\*d\*e^4\*x^3 + 6\*d^2\*e^3\*x^2 + 4\*d^3\*e^2\*x + d^4\*e) + 1/35\*sqrt(-e^2\*x^2 + d^2)\*A/(d\*e^4\*x^3 + 3\*d^2\*e^3\*x^2 + 3\*d^3\*e^2\*x + d^4\*e) + 2/105\*sqrt(-e^2\*x^2 + d^2)\*A/(d^2\*e^3\*x^2 + 2\*d^3\*e^2\*x + d^4\*e) + 2/105\*sqrt(-e^2\*x^2 + d^2)\*A/(d^3\*e^2\*x + d^4\*e) - 2/5\*sqrt(-e^2\*x^2 + d^2)\*B/(e^5\*x^3 + 3\*d\*e^4\*x^2 + 3\*d^2\*e^3\*x + d^3\*e^2) + 1/15\*sqrt(-e^2\*x^2 + d^2)\*B/(d\*e^4\*x^2 + 2\*d^2\*e^3\*x + d^3\*e^2) + 1/15\*sqrt(-e^2\*x^2 + d^2)\*B/(d^2\*e^3\*x + d^3\*e^2) - 2/3\*sqrt(-e^2\*x^2 + d^2)\*C/(e^5\*x^2 + 2\*d\*e^4\*x + d^2\*e^3) + 1/3\*sqrt(-e^2\*x^2 + d^2)\*C/(d\*e^4\*x + d^2\*e^3)

**mupad [B]** time = 4.67, size = 601, normalized size = 3.34

1/105\*d^3/e^3\*(2\*A\*d^2\*e^4\*x^2+5\*B\*d^3\*e^3\*x^2+23\*C\*d^4\*e^2\*x^2+10\*A\*d^3\*e^3\*x+25\*B\*d^4\*e^2\*x+10\*C\*d^5\*e\*x+23\*A\*d^4\*e^2+5\*B\*d^5\*e+2\*C\*d^6)\*sqrt(-e^2\*x^2+d^2)/(e\*x+d)^4

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^5,x)`

[Out] 
$$\begin{aligned} & \frac{B(d^2 - e^2x^2)^{1/2}}{(21(d^3e^2 + d^2e^3x))} - \frac{(3B(d^2 - e^2x^2)^{1/2})}{(7(d^3e^2 + e^5x^3 + 3d^2e^3x + 3de^4x^2))} + \frac{(2A(d^2 - e^2x^2)^{1/2})}{(105(d^4e + 2d^3e^2x + d^2e^3x^2))} + \frac{(B(d^2 - e^2x^2)^{1/2})}{(21(d^3e^2 + 2d^2e^3x + de^4x^2))} - \frac{(82C(d^2 - e^2x^2)^{1/2})}{(105(d^2e^3 + e^5x^2 + 2de^4x))} + \frac{(2A(d^2 - e^2x^2)^{1/2})}{(105(d^4e + d^3e^2x))} + \frac{(23C(d^2 - e^2x^2)^{1/2})}{(105(d^2e^3 + de^4x))} - \frac{(2A(d^2 - e^2x^2)^{1/2})}{(7(d^4e + e^5x^4 + 4d^3e^2x + 4de^4x^3 + 6d^2e^3x^2))} + \frac{(A(d^2 - e^2x^2)^{1/2})}{(35(d^4e + 3d^3e^2x + de^4x^3 + 3d^2e^3x^2))} - \frac{(2Cd^2(d^2 - e^2x^2)^{1/2})}{(7(d^4e^3 + e^7x^4 + 4d^3e^4x + 4de^6x^3 + 6d^2e^5x^2))} + \frac{(2Bd(d^2 - e^2x^2)^{1/2})}{(7(d^4e^2 + e^6x^4 + 4d^3e^3x + 4de^5x^3 + 6d^2e^4x^2))} + \frac{(29Cd(d^2 - e^2x^2)^{1/2})}{(35(d^3e^3 + e^6x^3 + 3d^2e^4x + 3de^5x^2))} \end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)} (A + Bx + Cx^2)}{(d + ex)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)`

[Out] `Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)`

$$3.9 \quad \int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6} dx$$

**Optimal.** Leaf size=234

$$\frac{(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 11Cd^2)}{42d^2e^3(d + ex)^5} - \frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{9de^3(d + ex)^6} - \frac{(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 11Cd^2)}{315d^4e^3(d + ex)^3}$$

**Rubi [A]** time = 0.26, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1639, 793, 659, 651}

$$\frac{(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 11Cd^2)}{315d^4e^3(d + ex)^3} - \frac{(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 11Cd^2)}{105d^3e^3(d + ex)^4} - \frac{(d^2 - e^2x^2)^{3/2} (2e(Ae + 2Bd) + 11Cd^2)}{42d^2e^3(d + ex)^5} - \frac{(d^2 - e^2x^2)^{3/2} (Ae^2 - Bde + Cd^2)}{9de^3(d + ex)^6} + \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)^5}$$

Antiderivative was successfully verified.

[In] Int[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^6,x]

[Out] -((C\*d^2 - B\*d\*e + A\*e^2)\*(d^2 - e^2\*x^2)^(3/2))/(9\*d\*e^3\*(d + e\*x)^6) + (C\*(d^2 - e^2\*x^2)^(3/2))/(2\*e^3\*(d + e\*x)^5) - ((11\*C\*d^2 + 2\*e\*(2\*B\*d + A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(42\*d^2\*e^3\*(d + e\*x)^5) - ((11\*C\*d^2 + 2\*e\*(2\*B\*d + A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(105\*d^3\*e^3\*(d + e\*x)^4) - ((11\*C\*d^2 + 2\*e\*(2\*B\*d + A\*e))\*(d^2 - e^2\*x^2)^(3/2))/(315\*d^4\*e^3\*(d + e\*x)^3)

Rule 651

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

Rule 659

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[Simplify[m + 2\*p + 2]/(2\*d\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

Rule 793

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[(m\*(g\*c\*d + c\*e\*f) + 2\*e\*c\*f\*(p + 1))/(e\*(2\*c\*d)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e

, f, g, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

### Rule 1639

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

### Rubi steps

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^6} dx = \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)^5} + \frac{\int \frac{(e^2(5Cd^2 + 2Ae^2) + e^3(3Cd + 2Be)x) \sqrt{d^2 - e^2x^2}}{(d + ex)^6} dx}{2e^4}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{9de^3(d + ex)^6} + \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)^5} + \frac{(11Cd^2 + 2e(2Bd + 6e^2d))}{6e^4}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{9de^3(d + ex)^6} + \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)^5} - \frac{(11Cd^2 + 2e(2Bd + 6e^2d))}{42d^2e^3}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{9de^3(d + ex)^6} + \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)^5} - \frac{(11Cd^2 + 2e(2Bd + 6e^2d))}{42d^2e^3}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)(d^2 - e^2x^2)^{3/2}}{9de^3(d + ex)^6} + \frac{C(d^2 - e^2x^2)^{3/2}}{2e^3(d + ex)^5} - \frac{(11Cd^2 + 2e(2Bd + 6e^2d))}{42d^2e^3}$$

**Mathematica [A]** time = 0.22, size = 144, normalized size = 0.62

$$\frac{(d - ex)\sqrt{d^2 - e^2x^2} (e(Ae(58d^3 + 33d^2ex + 12de^2x^2 + 2e^3x^3) + Bd(11d^3 + 66d^2ex + 24de^2x^2 + 4e^3x^3)) + Cd^2(4d^3 + 24d^2ex + 66de^2x^2 + 11e^3x^3))}{315d^4e^3(d + ex)^5}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B\*x + C\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^6, x]

```
[Out] -1/315*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^2*x^2 + 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x^3) + B*d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(d^4*e^3*(d + e*x)^5)
```

**IntegrateAlgebraic [A]** time = 1.07, size = 185, normalized size = 0.79

$$\frac{\sqrt{d^2 - e^2 x^2} (-58Ad^4e^2 + 25Ad^3e^3x + 21Ad^2e^4x^2 + 10Ad^5e^3x^3 + 2Ae^6x^4 - 11Bd^5e - 55Bd^4e^2x + 42Bd^3e^3x^2 + 20Bd^2e^4x^3 + 4Bde^5x^4 - 4Cd^6 - 20Cd^5ex - 42Cd^4e^2x^2 + 55Cd^3e^3x^3 + 11Cd^2e^4x^4)}{315d^4e^3(d + ex)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-4*C*d^6 - 11*B*d^5*e - 58*A*d^4*e^2 - 20*C*d^5*e*e*x - 55*B*d^4*e^2*x + 25*A*d^3*e^3*x - 42*C*d^4*e^2*x^2 + 42*B*d^3*e^3*x^2 + 21*A*d^2*e^4*x^2 + 55*C*d^3*e^3*x^3 + 20*B*d^2*e^4*x^3 + 10*A*d*e^5*x^3 + 11*C*d^2*e^4*x^4 + 4*B*d*e^5*x^4 + 2*A*e^6*x^4))/(315*d^4*e^3*(d + e*x)^5)
```

**fricas [A]** time = 1.02, size = 399, normalized size = 1.71

$$\frac{4C^2d^6 + 11Bd^5e + 58Ad^4e^2 + (4C^2d^2 + 11Bd^2 + 58Ad^2)^2 + 5(4C^2d^4 + 11Bd^4 + 58Ad^4)^2 + 10(4C^2d^2 + 11Bd^2 + 58Ad^2)^2 + 5(4C^2d^4 + 11Bd^4 + 58Ad^4)^2 + (4C^2d^6 + 11Bd^5e + 58Ad^4e^2 - (11C^2d^2 + 4Bd^2 + 2Ad^2)^2 - 5(11C^2d^2 + 4Bd^2 + 2Ad^2)^2 + 21(2C^2d^2 - 2Bd^2 - Ad^2)^2 + 5(4C^2d^4 + 11Bd^4 - 5Ad^4)^2)^2}{315(4B^2d^4 + 5B^2d^2 + 10B^2d^2 + 5B^2d^2 + 5B^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas")
```

```
[Out] -1/315*(4*C*d^7 + 11*B*d^6*e + 58*A*d^5*e^2 + (4*C*d^2*e^5 + 11*B*d*e^6 + 5*8*A*e^7)*x^5 + 5*(4*C*d^3*e^4 + 11*B*d^2*e^5 + 58*A*d*e^6)*x^4 + 10*(4*C*d^4*e^3 + 11*B*d^3*e^4 + 58*A*d^2*e^5)*x^3 + 10*(4*C*d^5*e^2 + 11*B*d^4*e^3 + 58*A*d^3*e^4)*x^2 + 5*(4*C*d^6*e + 11*B*d^5*e^2 + 58*A*d^4*e^3)*x + (4*C*d^6 + 11*B*d^5*e + 58*A*d^4*e^2 - (11*C*d^2*e^4 + 4*B*d*e^5 + 2*A*e^6)*x^4 - 5*(11*C*d^3*e^3 + 4*B*d^2*e^4 + 2*A*d*e^5)*x^3 + 21*(2*C*d^4*e^2 - 2*B*d^3*e^3 - A*d^2*e^4)*x^2 + 5*(4*C*d^5*e + 11*B*d^4*e^2 - 5*A*d^3*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^4*e^8*x^5 + 5*d^5*e^7*x^4 + 10*d^6*e^6*x^3 + 10*d^7*e^5*x^2 + 5*d^8*e^4*x + d^9*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^24*
```





$$\begin{aligned}
& *A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(2)^{14} \\
& +4130*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1) \\
& ^8*exp(2)^{10}+4470*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp \\
& (2))^3*exp(1)^6*exp(2)^{11}+1200*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))* \\
& exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^{12}+27*B*d*exp(1)^{11}*exp(2)^8+90*A*exp(1) \\
& ^{10}*exp(2)^9+18040*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e \\
& xp(2))^2*exp(1)^6*exp(2)^{11}+15720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2) \\
& ))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^{12}-170*B*d*exp(1)^9*exp(2)^9-149*A*e \\
& xp(1)^8*exp(2)^{10}+3600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/ \\
& x/exp(2))^4*exp(2)^{14}+840*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1) \\
& ))/x/exp(2))^2*exp(1)^4*exp(2)^{12}-86*B*d*exp(1)^7*exp(2)^{10}+380*A*exp(1)^6* \\
& exp(2)^{11}+120*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)) \\
& ^3*exp(2)^{14}-30*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^{14}/x/e \\
& xp(2)-760*B*d*exp(1)^5*exp(2)^{11}+180*A*exp(1)^4*exp(2)^{12}+2400*A*(-1/2*(-2* \\
& d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^{14}-40*B*d*exp(1) \\
& ^3*exp(2)^{12}+600*A*exp(2)^{14}-120*B*d*exp(1)*exp(2)^{13}-2430*A*(-2*d*exp(1)-2 \\
& *sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^{12}/x/exp(2)-1275/2*A*(-2*d*ex \\
& p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^{11}/x/exp(2)-2125*A*(-2* \\
& d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^{10}/x/exp(2)+385*A*( \\
& -2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^{10}*exp(2)^9/x/exp(2)-210* \\
& A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^{12}*exp(2)^8/x/exp(2)-2 \\
& 50*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^{14}*exp(2)^7/x/exp(2) \\
& )+40*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^{16}*exp(2)^6/x/exp \\
& (2)+60*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^{18}*exp(2)^5/x/e \\
& xp(2)-480*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2 \\
& *exp(1)*exp(2)^{13}-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)) \\
& /x/exp(2))^6*exp(1)^{23}*exp(2)^2-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e \\
& xp(2))*exp(1))/x/exp(2))^7*exp(1)^{21}*exp(2)^3-120*B*d*(-1/2*(-2*d*exp(1)-2* \\
& sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^{19}*exp(2)^4-160*C*d^2*(-1/2 \\
& *(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^{22}*exp(2)^2 \\
& -160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*ex \\
& p(1)^{20}*exp(2)^3-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/ \\
& x/exp(2))^7*exp(1)^{19}*exp(2)^4-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex \\
& p(2))*exp(1))/x/exp(2))^8*exp(1)^{17}*exp(2)^5+320*C*d^2*(-1/2*(-2*d*exp(1)-2 \\
& *sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^{22}*exp(2)^2-160*C*d^2*(-1/ \\
& 2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^{20}*exp(2)^ \\
& 3-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*ex \\
& xp(1)^{18}*exp(2)^4-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)) \\
& /x/exp(2))^4*exp(1)^{23}*exp(2)^2-48*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex \\
& p(2))*exp(1))/x/exp(2))^5*exp(1)^{21}*exp(2)^3+720*B*d*(-1/2*(-2*d*exp(1)-2*s \\
& qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^{19}*exp(2)^4+960*B*d*(-1/2*(- \\
& 2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^{17}*exp(2)^5+48 \\
& 0*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^ \\
& 15*exp(2)^6-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e \\
& xp(2))^4*exp(1)^{22}*exp(2)^2+128*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
\end{aligned}$$



$$\begin{aligned}
& (2)) * \exp(1) / x / \exp(2)) ^5 * \exp(1) ^{20} * \exp(2) ^3 + 640 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \\
& \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^6 * \exp(1) ^{18} * \exp(2) ^4 + 640 * C * d^2 * (-1/2 \\
& * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^7 * \exp(1) ^{16} * \exp(2) ^5 \\
& - 2720 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^5 * \exp \\
& (1) ^{19} * \exp(2) ^4 - 3560 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / \\
& x / \exp(2)) ^6 * \exp(1) ^{17} * \exp(2) ^5 - 920 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp \\
& (2)) * \exp(1) / x / \exp(2)) ^7 * \exp(1) ^{15} * \exp(2) ^6 - 60 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{ \\
& d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^8 * \exp(1) ^{13} * \exp(2) ^7 - 60 * B * d * (-1/2 * (-2 * \\
& d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^9 * \exp(1) ^{11} * \exp(2) ^8 + 1760 \\
& * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^4 * \exp(1) \\
& ^{20} * \exp(2) ^3 + 11360 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / \\
& x / \exp(2)) ^5 * \exp(1) ^{18} * \exp(2) ^4 + 19120 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \\
& \exp(2)) * \exp(1) / x / \exp(2)) ^6 * \exp(1) ^{16} * \exp(2) ^5 + 11920 * C * d^2 * (-1/2 * (-2 * d * \exp \\
& (1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^7 * \exp(1) ^{14} * \exp(2) ^6 + 3240 * C * d \\
& ^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^8 * \exp(1) ^{12} * \\
& \exp(2) ^7 + 360 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp( \\
& 2)) ^9 * \exp(1) ^{10} * \exp(2) ^8 - 240 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \\
& \exp(1) / x / \exp(2)) ^3 * \exp(1) ^{21} * \exp(2) ^3 + 720 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \\
& \exp(2)) * \exp(1) / x / \exp(2)) ^4 * \exp(1) ^{19} * \exp(2) ^4 + 604 * B * d * (-1/2 * (-2 * d * \exp \\
& (1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^5 * \exp(1) ^{17} * \exp(2) ^5 - 1250 * B * d \\
& * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^6 * \exp(1) ^{15} * \exp \\
& (2) ^6 - 1910 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) \\
& ^7 * \exp(1) ^{13} * \exp(2) ^7 - 855 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp \\
& (1) / x / \exp(2)) ^8 * \exp(1) ^{11} * \exp(2) ^8 - 15 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \\
& \exp(2)) * \exp(1) / x / \exp(2)) ^9 * \exp(1) ^9 * \exp(2) ^9 - 160 * C * d^2 * (-1/2 * (-2 * d * \exp(1) \\
& ) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^3 * \exp(1) ^{20} * \exp(2) ^3 + 1120 * C * d^2 * \\
& (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^4 * \exp(1) ^{18} * \exp \\
& (2) ^4 + 3416 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) \\
& ) ^5 * \exp(1) ^{16} * \exp(2) ^5 + 3660 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) \\
& * \exp(1) / x / \exp(2)) ^6 * \exp(1) ^{14} * \exp(2) ^6 + 1860 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{ \\
& d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^7 * \exp(1) ^{12} * \exp(2) ^7 + 810 * C * d^2 * (-1/2 * (- \\
& 2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^8 * \exp(1) ^{10} * \exp(2) ^8 + 90 \\
& * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^9 * \exp(1) \\
& ^8 * \exp(2) ^9 + 240 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp \\
& (2)) ^3 * \exp(1) ^{19} * \exp(2) ^4 - 10040 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2) \\
& )) * \exp(1) / x / \exp(2)) ^4 * \exp(1) ^{17} * \exp(2) ^5 - 25760 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{ \\
& d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^5 * \exp(1) ^{15} * \exp(2) ^6 - 32380 * B * d * (-1/2 * (- \\
& 2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^6 * \exp(1) ^{13} * \exp(2) ^7 - 2 \\
& 1460 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^7 * \exp( \\
& 1) ^{11} * \exp(2) ^8 - 6390 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x \\
& / \exp(2)) ^8 * \exp(1) ^9 * \exp(2) ^9 - 630 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp( \\
& 2)) * \exp(1) / x / \exp(2)) ^9 * \exp(1) ^7 * \exp(2) ^{10} + 1760 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \\
& \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^3 * \exp(1) ^{18} * \exp(2) ^4 + 32080 * C * d^2 * (-1 \\
& /2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^4 * \exp(1) ^{16} * \exp(2) \\
& ^5 + 56120 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2)) ^
\end{aligned}$$

$$\begin{aligned}
& 5*\exp(1)^{14}*\exp(2)^6+43140*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}) * \\
& \exp(1))/x/\exp(2))^6*\exp(1)^{12}*\exp(2)^7+20460*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{ \\
& t(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^7*\exp(1)^{10}*\exp(2)^8+5670*C*d^2*(-1/2*( \\
& -2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^8*\exp(1)^8*\exp(2)^9+63 \\
& 0*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^9*\exp(1) \\
& )^6*\exp(2)^{10}-120*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/e \\
& xp(2))^2*\exp(1)^{19}*\exp(2)^4+960*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2) \\
& })*\exp(1))/x/\exp(2))^3*\exp(1)^{17}*\exp(2)^5-2450*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{ \\
& t(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^4*\exp(1)^{15}*\exp(2)^6-720*B*d*(-1/2*(-2* \\
& d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^4*\exp(1)*\exp(2)^{13}-6710*B \\
& *d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^5*\exp(1)^{13} * \\
& \exp(2)^7-96*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2)) \\
& ^5*\exp(1)^{25}*\exp(2)-5590*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp( \\
& 1))/x/\exp(2))^6*\exp(1)^{11}*\exp(2)^8-1930*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x \\
& ^2*\exp(2))*\exp(1))/x/\exp(2))^7*\exp(1)^9*\exp(2)^9-330*B*d*(-1/2*(-2*d*\exp(1) \\
& -2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^8*\exp(1)^7*\exp(2)^{10}-90*B*d*(-1/2 \\
& *(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^9*\exp(1)^5*\exp(2)^{11} \\
& -160*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^2*ex \\
& p(1)^{18}*\exp(2)^4+1440*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1) \\
& )/x/\exp(2))^3*\exp(1)^{16}*\exp(2)^5+6140*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2- \\
& x^2*\exp(2))*\exp(1))/x/\exp(2))^4*\exp(1)^{14}*\exp(2)^6+7320*C*d^2*(-1/2*(-2*d*e \\
& xp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^5*\exp(1)^{12}*\exp(2)^7+4350*C* \\
& d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^6*\exp(1)^{10} \\
& *\exp(2)^8+1530*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/ex \\
& p(2))^7*\exp(1)^8*\exp(2)^9+135*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2) \\
& })*\exp(1))/x/\exp(2))^8*\exp(1)^6*\exp(2)^{10}+15*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{ \\
& t(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^9*\exp(1)^4*\exp(2)^{11}+120*B*d*(-1/2*(-2* \\
& d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^2*\exp(1)^{17}*\exp(2)^5-1268 \\
& 0*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^3*\exp(1)^ \\
& 15*\exp(2)^6-48820*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/e \\
& xp(2))^4*\exp(1)^{13}*\exp(2)^7-67820*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp \\
& (2))*\exp(1))/x/\exp(2))^5*\exp(1)^{11}*\exp(2)^8-41940*B*d*(-1/2*(-2*d*\exp(1)-2* \\
& \sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^6*\exp(1)^9*\exp(2)^9-13260*B*d*(-1/2* \\
& (-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^7*\exp(1)^7*\exp(2)^{10}- \\
& 2760*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^8*\exp( \\
& 1)^5*\exp(2)^{11}-360*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/ \\
& exp(2))^9*\exp(1)^3*\exp(2)^{12}+800*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*ex \\
& p(2))*\exp(1))/x/\exp(2))^2*\exp(1)^{16}*\exp(2)^5+37680*C*d^2*(-1/2*(-2*d*\exp(1) \\
& -2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^3*\exp(1)^{14}*\exp(2)^6+63860*C*d^2* \\
& (-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^4*\exp(1)^{12}*\exp \\
& (2)^7+51900*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2) \\
& )^5*\exp(1)^{10}*\exp(2)^8+24300*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2) \\
& })*\exp(1))/x/\exp(2))^6*\exp(1)^8*\exp(2)^9+5460*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{ \\
& t(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^7*\exp(1)^6*\exp(2)^{10}+540*C*d^2*(-1/2*( \\
& -2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^8*\exp(1)^4*\exp(2)^{11}+6
\end{aligned}$$

$$\begin{aligned}
& 0 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^9 * \exp(2) \\
& ^{13} + 570 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \\
& \exp(1) ^{15} * \exp(2) ^6 - 3890 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1) \\
& ) / x / \exp(2) ^3 * \exp(1) ^{13} * \exp(2) ^7 - 7660 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^4 * \exp(1) ^{11} * \exp(2) ^8 - 5780 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^5 * \exp(1) ^9 * \exp(2) ^9 - 2220 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^6 * \exp(1) ^7 * \exp(2) ^{10} - 480 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^6 * \exp(1) * \exp(2) ^{13} - 660 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^7 * \exp(1) ^5 * \exp(2) ^{11} - 120 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^8 * \exp(1) ^3 * \exp(2) ^{12} - 6740 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^{13} * \exp(2) ^7 - 46140 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(1) ^{11} * \exp(2) ^8 - 60560 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^4 * \exp(1) ^9 * \exp(2) ^9 - 32400 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^5 * \exp(1) ^7 * \exp(2) ^{10} - 9840 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^6 * \exp(1) ^5 * \exp(2) ^{11} - 1920 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^7 * \exp(1) ^3 * \exp(2) ^{12} - 4 * C * d^2 * \exp(1) ^{14} * \exp(2) ^6 + 120 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^7 * \exp(2) ^{13} - 2450 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^{11} * \exp(2) ^8 - 4710 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(1) ^9 * \exp(2) ^9 - 3440 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^4 * \exp(1) ^7 * \exp(2) ^{10} - 1200 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^5 * \exp(1) ^5 * \exp(2) ^{11} - 240 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^6 * \exp(1) ^3 * \exp(2) ^{12} + 20 * C * d^2 * \exp(1) ^{12} * \exp(2) ^7 - 24860 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^9 * \exp(2) ^9 - 26740 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(1) ^7 * \exp(2) ^{10} - 12160 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^4 * \exp(1) ^5 * \exp(2) ^{11} - 3600 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^5 * \exp(1) ^3 * \exp(2) ^{12} + 28 * C * d^2 * \exp(1) ^{10} * \exp(2) ^8 - 1700 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^7 * \exp(2) ^{10} - 940 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(1) ^5 * \exp(2) ^{11} - 160 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^4 * \exp(1) ^3 * \exp(2) ^{12} + 610 * C * d^2 * \exp(1) ^8 * \exp(2) ^9 - 5840 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^5 * \exp(2) ^{11} - 2880 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(1) ^3 * \exp(2) ^{12} + 81 * C * d^2 * \exp(1) ^6 * \exp(2) ^{10} - 120 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(2) ^{13} - 80 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^3 * \exp(2) ^{12} + 420 * C * d^2 * \exp(1) ^4 * \exp(2) ^{11} - 120 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^8 * \exp(1) * \exp(2) ^{13} + 1100 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^2 * \exp(1) ^{14} * \exp(2) ^6 + 5740 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^3 * \exp(1) ^{12} * \exp(2) ^7 + 5650 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2 * \exp(2)}) * \exp(1)) / x / \exp(2) ^4 * \exp(1) ^{10} * \exp(2) ^8
\end{aligned}$$

$$\begin{aligned}
& +2430*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1))/x/exp(2))^5* \\
& xp(1)^8*exp(2)^9+330*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)))*exp(1) \\
& )/x/exp(2))^6*exp(1)^6*exp(2)^10-90*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2 \\
& *exp(2))*exp(1))/x/exp(2))^7*exp(1)^4*exp(2)^11+21260*C*d^2*(-1/2*(-2*d*exp \\
& (1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^7+37540*C*d \\
& ^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10* \\
& exp(2)^8+32200*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp \\
& (2))^4*exp(1)^8*exp(2)^9+12600*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp \\
& (2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^10+1800*C*d^2*(-1/2*(-2*d*exp(1)-2 \\
& *sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^4*exp(2)^11+2570*C*d^2*(-1 \\
& /2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2) \\
& ^8+1710*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3 \\
& *exp(1)^8*exp(2)^9+480*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp( \\
& 1))/x/exp(2))^4*exp(1)^6*exp(2)^10+14180*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^ \\
& 2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^9+11340*C*d^2*(-1/2*(-2*d \\
& *exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^10+2400* \\
& C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^ \\
& 4*exp(2)^11+270*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e \\
& xp(2))^2*exp(1)^6*exp(2)^10+90*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp( \\
& 2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^11+1560*C*d^2*(-1/2*(-2*d*exp(1)-2* \\
& sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^11-64*C*d^2*(-1/2* \\
& (-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^24*exp(2)+30 \\
& *C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^13/x/exp(2)+420*B \\
& *d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^12/x/exp(2)+ \\
& 155*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^11/x/exp \\
& (2)+3485*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^1 \\
& 0/x/exp(2)+845/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9* \\
& xp(2)^9/x/exp(2)+820*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1) \\
& ^11*exp(2)^8/x/exp(2)-135*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))* \\
& xp(1)^13*exp(2)^7/x/exp(2)+30*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1) \\
& )*exp(1)^17*exp(2)^5/x/exp(2)+15/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2) \\
& ))*exp(1))*exp(1)^4*exp(2)^11/x/exp(2)-1785*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x \\
& ^2*exp(2))*exp(1))*exp(1)^6*exp(2)^10/x/exp(2)-360*C*d^2*(-2*d*exp(1)-2*sqrt \\
& (d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^9/x/exp(2)-2870*C*d^2*(-2*d*exp(1) \\
& )-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^8/x/exp(2)-140*C*d^2*(-2* \\
& d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^7/x/exp(2)-100*C*d \\
& ^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^14*exp(2)^6/x/exp(2)+ \\
& 20*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^16*exp(2)^5/x/e \\
& xp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2) \\
& )-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^5/(-60*d^4*exp(1)^1 \\
& 9+240*d^4*exp(1)^15*exp(2)^2+240*d^4*exp(1)^13*exp(2)^3-360*d^4*exp(1)^11* \\
& xp(2)^4-360*d^4*exp(1)^9*exp(2)^5+240*d^4*exp(1)^7*exp(2)^6+240*d^4*exp(1)^ \\
& 5*exp(2)^7-60*d^4*exp(1)^17*exp(2)-120*d^4*exp(1)*exp(2)^9+1/2*(-4*B*d*exp \\
& (1)^11*exp(2)^4+24*C*d^2*exp(1)^10*exp(2)^4-B*d*exp(1)^9*exp(2)^5+6*C*d^2*e \\
& xp(1)^8*exp(2)^5+18*A*exp(1)^8*exp(2)^6-42*B*d*exp(1)^7*exp(2)^6+42*C*d^2*e
\end{aligned}$$

$$\begin{aligned} & xp(1)^6 \exp(2)^6 + 3A \exp(1)^6 \exp(2)^7 - 6Bd \exp(1)^5 \exp(2)^7 + Cd^2 \exp(1) \\ & \exp(2)^7 + 44A \exp(1)^4 \exp(2)^8 - 24Bd \exp(1)^3 \exp(2)^8 + 4C \exp(2)^9 + 12A \exp(2)^{10} \\ & \cdot \operatorname{atan}\left(\frac{-1/2(-2d \exp(1) - 2\sqrt{d^2 - x^2} \exp(2)) \exp(1)}{x + \exp(2)}\right) / \sqrt{-\exp(1)^4 + \exp(2)^2} / \sqrt{-\exp(1)^4 + \exp(2)^2} / (-2d^4 \exp(1)^{19} \\ & + 8d^4 \exp(1)^{15} \exp(2)^2 + 8d^4 \exp(1)^{13} \exp(2)^3 - 12d^4 \exp(1)^{11} \exp(2)^4 \\ & - 12d^4 \exp(1)^9 \exp(2)^5 + 8d^4 \exp(1)^7 \exp(2)^6 + 8d^4 \exp(1)^5 \exp(2)^7 - 2d^4 \exp(1)^{17} \exp(2) \\ & - 4d^4 \exp(1) \exp(2)^9 \end{aligned}$$

**maple [A]** time = 0.01, size = 152, normalized size = 0.65

$$\frac{(-ex + d)(2Ae^5x^3 + 4Bde^4x^3 + 11Cd^2e^3x^3 + 12Ad^4e^4x^2 + 24Bd^2e^3x^2 + 66Cd^3e^2x^2 + 33Ad^2e^3x + 66Bd^3e^2x + 24Cd^4ex + 58Ad^3e^2 + 11Bd^4e + 4Cd^5)\sqrt{-e^2x^2 + d^2}}{315(ex + d)^5d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^6,x)

[Out]  $-1/315(-ex+d)(2Ae^5x^3+4Bde^4x^3+11Cd^2e^3x^3+12Ad^4e^4x^2+24Bd^2e^3x^2+66Cd^3e^2x^2+33Ad^2e^3x+66Bd^3e^2x+24Cd^4ex+58Ad^3e^2+11Bd^4e+4Cd^5)(-e^2x^2+d^2)^{1/2}/(ex+d)^5/d^4/e^3$

**maxima [B]** time = 0.58, size = 1378, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^6,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -2/9\sqrt{-e^2x^2 + d^2}Cd^2/(e^8x^5 + 5d^7e^7x^4 + 10d^2e^6x^3 + 10d^3e^5x^2 + 5d^4e^4x + d^5e^3) + 1/63\sqrt{-e^2x^2 + d^2}Cd^2/(d \\ & e^7x^4 + 4d^2e^6x^3 + 6d^3e^5x^2 + 4d^4e^4x + d^5e^3) + 1/105\sqrt{-e^2x^2 + d^2}Cd^2/(d^2e^6x^3 + 3d^3e^5x^2 + 3d^4e^4x + d^5e^3) \\ & + 2/315\sqrt{-e^2x^2 + d^2}Cd^2/(d^3e^5x^2 + 2d^4e^4x + d^5e^3) + 2/315\sqrt{-e^2x^2 + d^2}Cd^2/(d^4e^4x + d^5e^3) + 2/9\sqrt{-e^2 \\ & x^2 + d^2}Bd/(e^7x^5 + 5d^7e^6x^4 + 10d^2e^5x^3 + 10d^3e^4x^2 + 5d^4e^3x + d^5e^2) - 1/63\sqrt{-e^2x^2 + d^2}Bd/(d^6e^6x^4 + 4d^2e^5x^3 \\ & + 6d^3e^4x^2 + 4d^4e^3x + d^5e^2) - 1/105\sqrt{-e^2x^2 + d^2}Bd/(d^2e^5x^3 + 3d^3e^4x^2 + 3d^4e^3x + d^5e^2) - 2/315\sqrt{-e^2x^2 + d^2}Bd/(d^3e^4x^2 \\ & + 2d^4e^3x + d^5e^2) - 2/315\sqrt{-e^2x^2 + d^2}Bd/(d^4e^3x + d^5e^2) + 4/7\sqrt{-e^2x^2 + d^2}Cd/(e^7x^4 + 4d^7e^6x^3 + 6d^2e^5x^2 + 4d^3e^4x + d^4e^3) \\ & - 2/35\sqrt{-e^2x^2 + d^2}Cd/(d^6e^6x^3 + 3d^2e^5x^2 + 3d^3e^4x + d^4e^3) - 4/105\sqrt{-e^2x^2 + d^2}Cd/(d^2e^5x^2 + 2d^3e^4x + d^4e^3) - 4/105\sqrt{-e^2x^2 + d^2}Cd/(d^3e^4x + d^4e^3) \\ & - 2/9\sqrt{-e^2x^2 + d^2}A/(e^6x^5 + 5d^7e^5x^4 + 10d^2e^4x^3 + 10d^3e^3x^2 + 5d^4e^2x + d^5e) + 1/63\sqrt{-e^2x^2 + d^2}A/(d^6e^5x^4 + 4d^2e^4x^3 + 6d^3e^3x^2 + \end{aligned}$$

$$4*d^4*e^2*x + d^5*e) + 1/105*\sqrt{-e^2*x^2 + d^2}*A/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) + 2/315*\sqrt{-e^2*x^2 + d^2}*A/(d^3*e^3*x^2 + 2*d^4*e^2*x + d^5*e) + 2/315*\sqrt{-e^2*x^2 + d^2}*A/(d^4*e^2*x + d^5*e) - 2/7*\sqrt{-e^2*x^2 + d^2}*B/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2 + 4*d^3*e^3*x + d^4*e^2) + 1/35*\sqrt{-e^2*x^2 + d^2}*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) + 2/105*\sqrt{-e^2*x^2 + d^2}*B/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) + 2/105*\sqrt{-e^2*x^2 + d^2}*B/(d^3*e^3*x + d^4*e^2) - 2/5*\sqrt{-e^2*x^2 + d^2}*C/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) + 1/15*\sqrt{-e^2*x^2 + d^2}*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 1/15*\sqrt{-e^2*x^2 + d^2}*C/(d^2*e^4*x + d^3*e^3)$$

**mupad [B]** time = 5.24, size = 960, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((d^2 - e^2*x^2)^{(1/2)}*(A + B*x + C*x^2))/(d + e*x)^6, x)$

[Out]  $(B*(d^2 - e^2*x^2)^{(1/2)})/(63*(d^4*e^2 + d^3*e^3*x)) + (C*(d^2 - e^2*x^2)^{(1/2)})/(135*(d^3*e^3 + d^2*e^4*x)) - (19*B*(d^2 - e^2*x^2)^{(1/2)})/(63*(d^4*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*d^2*e^4*x^2)) + (A*(d^2 - e^2*x^2)^{(1/2)})/(105*(d^5*e + 3*d^4*e^2*x + 3*d^3*e^3*x^2 + d^2*e^4*x^3)) + (2*B*(d^2 - e^2*x^2)^{(1/2)})/(105*(d^4*e^2 + 3*d^3*e^3*x + d*e^5*x^3 + 3*d^2*e^4*x^2)) - (47*C*(d^2 - e^2*x^2)^{(1/2)})/(105*(d^3*e^3 + e^6*x^3 + 3*d^2*e^4*x + 3*d*e^5*x^2)) + (2*A*(d^2 - e^2*x^2)^{(1/2)})/(315*(d^5*e + 2*d^4*e^2*x + d^3*e^3*x^2)) + (11*C*(d^2 - e^2*x^2)^{(1/2)})/(315*(d^3*e^3 + 2*d^2*e^4*x + d*e^5*x^2)) - (2*A*(d^2 - e^2*x^2)^{(1/2)})/(9*(d^5*e + e^6*x^5 + 5*d^4*e^2*x + 5*d*e^5*x^4 + 10*d^3*e^3*x^2 + 10*d^2*e^4*x^3)) + (A*(d^2 - e^2*x^2)^{(1/2)})/(63*(d^5*e + 4*d^4*e^2*x + d*e^5*x^4 + 6*d^3*e^3*x^2 + 4*d^2*e^4*x^3)) - (2*A*(d^2 - e^2*x^2)^{(1/2)})/(945*(d^5*e + d^4*e^2*x)) + (4*B*(d^2 - e^2*x^2)^{(1/2)})/(315*(d^4*e^2 + 2*d^3*e^3*x + d^2*e^4*x^2)) + (2*B*d*(d^2 - e^2*x^2)^{(1/2)})/(9*(d^5*e^2 + e^7*x^5 + 5*d^4*e^3*x + 5*d*e^6*x^4 + 10*d^3*e^4*x^2 + 10*d^2*e^5*x^3)) + (37*C*d*(d^2 - e^2*x^2)^{(1/2)})/(63*(d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) - (2*C*d^2*(d^2 - e^2*x^2)^{(1/2)})/(9*(d^5*e^3 + e^8*x^5 + 5*d^4*e^4*x + 5*d*e^7*x^4 + 10*d^3*e^5*x^2 + 10*d^2*e^6*x^3)) + (8*A*e^2*(d^2 - e^2*x^2)^{(1/2)})/(945*(d^5*e^3 + d^4*e^4*x)) + (26*C*d^2*(d^2 - e^2*x^2)^{(1/2)})/(945*(d^5*e^3 + d^4*e^4*x)) - (B*d*e*(d^2 - e^2*x^2)^{(1/2)})/(315*(d^5*e^3 + d^4*e^4*x))$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

$$3.10 \quad \int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=236

$$\frac{x^2\sqrt{d^2-e^2x^2} (5e(Ae+3Bd)+19Cd^2)}{15e} - \frac{dx\sqrt{d^2-e^2x^2} (12Ae^2+15Bde+13Cd^2)}{8e^2} - \frac{d^2\sqrt{d^2-e^2x^2} (55Ae^2+45Bde+38Cd^2)}{15e^3}$$

**Rubi [A]** time = 0.66, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1815, 641, 217, 203}

$$\frac{x^2\sqrt{d^2-e^2x^2} (5e(Ae+3Bd)+19Cd^2)}{15e} - \frac{dx\sqrt{d^2-e^2x^2} (12Ae^2+15Bde+13Cd^2)}{8e^2} - \frac{d^2\sqrt{d^2-e^2x^2} (55Ae^2+45Bde+38Cd^2)}{15e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) (20Ae^2+15Bde+13Cd^2)}{8e^3} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} (Be+3Cd) - \frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $-(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*\text{Sqrt}[d^2 - e^2*x^2])/(15*e^3) - (d*(13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*\text{Sqrt}[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2 + 5*e*(3*B*d + A*e))*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*\text{Sqrt}[d^2 - e^2*x^2])/4 - (C*e*x^4*\text{Sqrt}[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 + 15*B*d*e + 20*A*e^2)*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(8*e^3)$

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*



$(q + 2*p + 1), x] + \text{Dist}[1/(b*(q + 2*p + 1)), \text{Int}[(a + b*x^2)^p \text{ExpandToSum}[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x], x] /;$  FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex)^3 (A + Bx + Cx^2)}{\sqrt{d^2 - e^2x^2}} dx &= -\frac{1}{5} Cex^4 \sqrt{d^2 - e^2x^2} - \frac{\int \frac{-5Ad^3e^2 - 5d^2e^2(Bd + 3Ae)x - 5de^2(Cd^2 + 3e(Bd + Ae))x^2 - e^3(19Cd^2 + 5e(3Bd + Ae))x^3}{\sqrt{d^2 - e^2x^2}} dx}{5e^2} \\ &= -\frac{1}{4} (3Cd + Be)x^3 \sqrt{d^2 - e^2x^2} - \frac{1}{5} Cex^4 \sqrt{d^2 - e^2x^2} + \frac{\int \frac{20Ad^3e^4 + 20d^2e^4(Bd + 3Ae)x + 20de^4(Cd^2 + 3e(Bd + Ae))x^2 + 20e^4e^3(19Cd^2 + 5e(3Bd + Ae))x^3}{\sqrt{d^2 - e^2x^2}} dx}{15e^2} \\ &= -\frac{(19Cd^2 + 5e(3Bd + Ae))x^2 \sqrt{d^2 - e^2x^2}}{15e} - \frac{1}{4} (3Cd + Be)x^3 \sqrt{d^2 - e^2x^2} - \frac{1}{5} Cex^4 \sqrt{d^2 - e^2x^2} \\ &= -\frac{d(13Cd^2 + 15Bde + 12Ae^2)x \sqrt{d^2 - e^2x^2}}{8e^2} - \frac{(19Cd^2 + 5e(3Bd + Ae))x^2 \sqrt{d^2 - e^2x^2}}{15e} \\ &= -\frac{d^2(38Cd^2 + 45Bde + 55Ae^2) \sqrt{d^2 - e^2x^2}}{15e^3} - \frac{d(13Cd^2 + 15Bde + 12Ae^2)x \sqrt{d^2 - e^2x^2}}{8e^2} \\ &= -\frac{d^2(38Cd^2 + 45Bde + 55Ae^2) \sqrt{d^2 - e^2x^2}}{15e^3} - \frac{d(13Cd^2 + 15Bde + 12Ae^2)x \sqrt{d^2 - e^2x^2}}{8e^2} \\ &= -\frac{d^2(38Cd^2 + 45Bde + 55Ae^2) \sqrt{d^2 - e^2x^2}}{15e^3} - \frac{d(13Cd^2 + 15Bde + 12Ae^2)x \sqrt{d^2 - e^2x^2}}{8e^2} \end{aligned}$$

**Mathematica [A]** time = 0.45, size = 174, normalized size = 0.74

$$\frac{15d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (5e(4Ae + 3Bd) + 13Cd^2) - \sqrt{d^2 - e^2x^2} (5e(4Ae(22d^2 + 9dex + 2e^2x^2) + 3B(24d^3 + 15d^2ex + 8de^2x^2 + 2e^3x^3)) + C(304d^4 + 195d^3ex + 152d^2e^2x^2 + 90de^3x^3 + 24e^4x^4))}{120e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(-\text{Sqrt}[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 + 90*d*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*B*(24*d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3))) + 15*d^3*(13*C*d^2 + 5*e*(3*B*d + 4*A*e))*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(120*e^3)$

**IntegrateAlgebraic [A]** time = 0.74, size = 201, normalized size = 0.85

$$\frac{\sqrt{d^2 - e^2x^2} (-440Ad^2e^2 - 180Ade^3x - 40Ae^4x^2 - 360Bd^3e - 225Bde^2x - 120Bde^3x^2 - 30Be^4x^3 - 304Cd^4 - 195Cd^3ex - 152Cd^2e^2x^2 - 90Cde^3x^3 - 24Ce^4x^4) + \sqrt{-e^2} \log(\sqrt{d^2 - e^2x^2} - \sqrt{-e^2}x) (20Ad^3e^2 + 15Bd^4e + 13Cd^5)}{120e^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2],x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-304\*C\*d^4 - 360\*B\*d^3\*e - 440\*A\*d^2\*e^2 - 195\*C\*d^3\*e\*x - 225\*B\*d^2\*e^2\*x - 180\*A\*d\*e^3\*x - 152\*C\*d^2\*e^2\*x^2 - 120\*B\*d\*e^3\*x^2 - 40\*A\*e^4\*x^2 - 90\*C\*d\*e^3\*x^3 - 30\*B\*e^4\*x^3 - 24\*C\*e^4\*x^4))/(120\*e^3) + (Sqrt[-e^2]\*(13\*C\*d^5 + 15\*B\*d^4\*e + 20\*A\*d^3\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(8\*e^4)

**fricas** [A] time = 0.87, size = 178, normalized size = 0.75

$$\frac{30(13Cd^5 + 15Bd^4e + 20Ad^3e^2) \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + (24Ce^4x^4 + 304Cd^4 + 360Bd^3e + 440Ad^2e^2 + 30(3Cde^3 + Be^4)x^3 + 8(19Cd^2e^2 + 15Bde^3 + 5Ae^4)x^2 + 15(13Cd^2e + 15Bd^2e^2 + 12Ad^2e^3)x)\sqrt{-e^2x^2 + d^2}}{120e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/120\*(30\*(13\*C\*d^5 + 15\*B\*d^4\*e + 20\*A\*d^3\*e^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + (24\*C\*e^4\*x^4 + 304\*C\*d^4 + 360\*B\*d^3\*e + 440\*A\*d^2\*e^2 + 30\*(3\*C\*d\*e^3 + B\*e^4)\*x^3 + 8\*(19\*C\*d^2\*e^2 + 15\*B\*d\*e^3 + 5\*A\*e^4)\*x^2 + 15\*(13\*C\*d^3\*e + 15\*B\*d^2\*e^2 + 12\*A\*d\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/e^3

**giac** [A] time = 0.37, size = 166, normalized size = 0.70

$$\frac{1}{8}(13Cd^5 + 15Bd^4e + 20Ad^3e^2) \arcsin\left(\frac{xe}{d}\right) e^{-3} \operatorname{sgn}(d) - \frac{1}{120} \sqrt{-x^2e^2 + d^2} \left( (2(3(4Cxe + 5(3Cde^3 + Be^4)e^{-6})x + 4(19Cd^2e^2 + 15Bde^3 + 5Ae^4)e^{-6})x + 15(13Cd^3e^4 + 15Bd^2e^5 + 12Ad^2e^3)e^{-6})x + 8(38Cd^4e^3 + 45Bd^3e^4 + 55Ad^2e^5)e^{-6}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] 1/8\*(13\*C\*d^5 + 15\*B\*d^4\*e + 20\*A\*d^3\*e^2)\*arcsin(x\*e/d)\*e^(-3)\*sgn(d) - 1/120\*sqrt(-x^2\*e^2 + d^2)\*((2\*(3\*(4\*C\*x\*e + 5\*(3\*C\*d\*e^6 + B\*e^7)\*e^(-6))\*x + 4\*(19\*C\*d^2\*e^5 + 15\*B\*d\*e^6 + 5\*A\*e^7)\*e^(-6))\*x + 15\*(13\*C\*d^3\*e^4 + 15\*B\*d^2\*e^5 + 12\*A\*d\*e^6)\*e^(-6))\*x + 8\*(38\*C\*d^4\*e^3 + 45\*B\*d^3\*e^4 + 55\*A\*d^2\*e^5)\*e^(-6))

**maple** [A] time = 0.03, size = 374, normalized size = 1.58

$$\frac{\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{xe}{d}\right) + 5Ae^3 \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 15Bd^2 \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + \frac{\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 13Cde^3 \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 3\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + \sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + \frac{19\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 15\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 13\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 11\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 3\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right) + 38\sqrt{-e^2x^2 + d^2} \operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2}}{\sqrt{-e^2x^2 + d^2}}\right)}{8\sqrt{e^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/5\*C\*e\*x^4\*(-e^2\*x^2+d^2)^(1/2)-19/15/e\*C\*d^2\*x^2\*(-e^2\*x^2+d^2)^(1/2)-38/15/e^3\*C\*d^4\*(-e^2\*x^2+d^2)^(1/2)-1/4\*x^3\*e\*(-e^2\*x^2+d^2)^(1/2)\*B-3/4\*x^3

$(-e^2x^2+d^2)^{1/2} * d * C - 15/8 * (-e^2x^2+d^2)^{1/2} * B * d^2 / e * x - 13/8 * (-e^2x^2+d^2)^{1/2} * C * d^3 / e^2 * x + 15/8 / (e^2)^{1/2} * B * d^4 / e * \arctan((e^2)^{1/2} / (-e^2x^2+d^2)^{1/2} * x) + 13/8 / (e^2)^{1/2} * C * d^5 / e^2 * \arctan((e^2)^{1/2} / (-e^2x^2+d^2)^{1/2} * x) - 1/3 * x^2 * e * (-e^2x^2+d^2)^{1/2} * A - x^2 * (-e^2x^2+d^2)^{1/2} * d * B - 11/3 * d^2 / e * (-e^2x^2+d^2)^{1/2} * A - 3 * d^3 / e^2 * (-e^2x^2+d^2)^{1/2} * B - 3/2 * (-e^2x^2+d^2)^{1/2} * A * d * x + 5/2 / (e^2)^{1/2} * A * d^3 * \arctan((e^2)^{1/2} / (-e^2x^2+d^2)^{1/2} * x)$

**maxima** [A] time = 0.98, size = 390, normalized size = 1.65

$\frac{1}{5} \sqrt{-e^2x^2 + d^2} - \frac{4\sqrt{-e^2x^2 + d^2} C d^2}{15e} + \frac{A^2 \arcsin\left(\frac{x}{d}\right)}{e} + \frac{8\sqrt{-e^2x^2 + d^2} B C d^2}{15e^2} - \frac{\sqrt{-e^2x^2 + d^2} B d^2}{e^2} - \frac{3\sqrt{-e^2x^2 + d^2} A d^2}{e} + \frac{3(Cd^2 + B^2)\sqrt{-e^2x^2 + d^2}}{4e^2} - \frac{3(Cd^2 + 3Bd^2 + A^2)\sqrt{-e^2x^2 + d^2}}{3e^2} + \frac{3(3Cd^2 + B^2)^2 \arcsin\left(\frac{x}{d}\right)}{8e^2} + \frac{(C^2 + 3Bd^2 + 3Ad^2)^2 \arcsin\left(\frac{x}{d}\right)}{2e^2} - \frac{3(3Cd^2 + B^2)\sqrt{-e^2x^2 + d^2} d^2}{8e^4} - \frac{(C^2 + 3Bd^2 + 3Ad^2)\sqrt{-e^2x^2 + d^2} d^2}{2e^2} - \frac{2(3Cd^2 + 3Bd^2 + A^2)\sqrt{-e^2x^2 + d^2} d^2}{3e^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out]  $-1/5 * \sqrt{-e^2x^2 + d^2} * C * e * x^4 - 4/15 * \sqrt{-e^2x^2 + d^2} * C * d^2 * x^2 / e + A * d^3 * \arcsin(e * x / d) / e - 8/15 * \sqrt{-e^2x^2 + d^2} * C * d^4 / e^3 - \sqrt{-e^2x^2 + d^2} * B * d^3 / e^2 - 3 * \sqrt{-e^2x^2 + d^2} * A * d^2 / e - 1/4 * (3 * C * d * e^2 + B * e^3) * \sqrt{-e^2x^2 + d^2} * x^3 / e^2 - 1/3 * (3 * C * d^2 * e + 3 * B * d * e^2 + A * e^3) * \sqrt{-e^2x^2 + d^2} * x^2 / e^2 + 3/8 * (3 * C * d * e^2 + B * e^3) * d^4 * \arcsin(e * x / d) / e^5 + 1/2 * (C * d^3 + 3 * B * d^2 * e + 3 * A * d * e^2) * d^2 * \arcsin(e * x / d) / e^3 - 3/8 * (3 * C * d * e^2 + B * e^3) * \sqrt{-e^2x^2 + d^2} * d^2 * x / e^4 - 1/2 * (C * d^3 + 3 * B * d^2 * e + 3 * A * d * e^2) * \sqrt{-e^2x^2 + d^2} * x / e^2 - 2/3 * (3 * C * d^2 * e + 3 * B * d * e^2 + A * e^3) * \sqrt{-e^2x^2 + d^2} * d^2 / e^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + ex)^3 (Cx^2 + Bx + A)}{\sqrt{d^2 - e^2x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(A + B\*x + C\*x^2))/(d^2 - e^2\*x^2)^(1/2),x)

[Out] int(((d + e\*x)^3\*(A + B\*x + C\*x^2))/(d^2 - e^2\*x^2)^(1/2),x)

**sympy** [A] time = 24.44, size = 1268, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(C\*x\*\*2+B\*x+A)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out]  $A * d ** 3 * \text{Piecewise}((\sqrt{d ** 2 / e ** 2} * \text{asin}(x * \sqrt{e ** 2 / d ** 2})) / \sqrt{d ** 2}, (d ** 2 > 0) \& (e ** 2 > 0)), (\sqrt{-d ** 2 / e ** 2} * \text{asinh}(x * \sqrt{-e ** 2 / d ** 2})) / \sqrt{d ** 2})$

```

, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 3*A*d**2*e*Piecewise((x**2/(2*sqrt(d**2
)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*A*d*e**2*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**
2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt
(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2))), True)) + A*e**
3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**
2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2))), True)) + B*d**3*Piecewis
e((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
+ 3*B*d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e
**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3
) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d
**2))), True)) + 3*B*d*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4
) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2))),
True)) + B*e**3*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*
e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2
)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) > 1), (3*d
**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x
**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2))
), True)) + C*d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1
+ e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*
e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**
2/d**2))), True)) + 3*C*d**2*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*
e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2
))), True)) + 3*C*d*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d*
**3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x
**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2))), Abs(e**2*x**2/d**2) >
1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2
)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**
2/d**2))), True)) + C*e**3*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
6) - 4*d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x
**2)/(5*e**2), Ne(e, 0)), (x**6/(6*sqrt(d**2))), True))

```

$$3.11 \quad \int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=191

$$\frac{x\sqrt{d^2-e^2x^2}(4e(Ae+2Bd)+7Cd^2)}{8e^2} - \frac{d\sqrt{d^2-e^2x^2}(e(6Ae+5Bd)+4Cd^2)}{3e^3} + \frac{d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(12Ae^2+8Bde+7Cd^2)}{8e^3}$$

**Rubi [A]** time = 0.38, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1815, 641, 217, 203}

$$-\frac{x\sqrt{d^2-e^2x^2}(4e(Ae+2Bd)+7Cd^2)}{8e^2} - \frac{d\sqrt{d^2-e^2x^2}(e(6Ae+5Bd)+4Cd^2)}{3e^3} + \frac{d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(12Ae^2+8Bde+7Cd^2)}{8e^3} - \frac{x^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out] -(d\*(4\*C\*d^2 + e\*(5\*B\*d + 6\*A\*e))\*Sqrt[d^2 - e^2\*x^2])/(3\*e^3) - ((7\*C\*d^2 + 4\*e\*(2\*B\*d + A\*e))\*x\*Sqrt[d^2 - e^2\*x^2])/(8\*e^2) - ((2\*C\*d + B\*e)\*x^2\*Sqrt[d^2 - e^2\*x^2])/(3\*e) - (C\*x^3\*Sqrt[d^2 - e^2\*x^2])/4 + (d^2\*(7\*C\*d^2 + 8\*B\*d\*e + 12\*A\*e^2)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e^3)

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSu

m[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x  
 ], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(d + ex)^2 (A + Bx + Cx^2)}{\sqrt{d^2 - e^2x^2}} dx &= -\frac{1}{4}Cx^3\sqrt{d^2 - e^2x^2} - \frac{\int \frac{-4Ad^2e^2 - 4de^2(Bd + 2Ae)x - e^2(7Cd^2 + 4e(2Bd + Ae))x^2 - 4e^3(2Cd + Be)x^3}{\sqrt{d^2 - e^2x^2}} dx}{4e^2} \\
 &= -\frac{(2Cd + Be)x^2\sqrt{d^2 - e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2 - e^2x^2} + \frac{\int \frac{12Ad^2e^4 + 4de^3(4Cd^2 + e(5Bd + 6Ae))}{\sqrt{d^2 - e^2x^2}} dx}{12e^2} \\
 &= -\frac{(7Cd^2 + 4e(2Bd + Ae))x\sqrt{d^2 - e^2x^2}}{8e^2} - \frac{(2Cd + Be)x^2\sqrt{d^2 - e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2 - e^2x^2} \\
 &= -\frac{d(4Cd^2 + e(5Bd + 6Ae))\sqrt{d^2 - e^2x^2}}{3e^3} - \frac{(7Cd^2 + 4e(2Bd + Ae))x\sqrt{d^2 - e^2x^2}}{8e^2} \\
 &= -\frac{d(4Cd^2 + e(5Bd + 6Ae))\sqrt{d^2 - e^2x^2}}{3e^3} - \frac{(7Cd^2 + 4e(2Bd + Ae))x\sqrt{d^2 - e^2x^2}}{8e^2} \\
 &= -\frac{d(4Cd^2 + e(5Bd + 6Ae))\sqrt{d^2 - e^2x^2}}{3e^3} - \frac{(7Cd^2 + 4e(2Bd + Ae))x\sqrt{d^2 - e^2x^2}}{8e^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 139, normalized size = 0.73

$$\frac{3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (4e(3Ae + 2Bd) + 7Cd^2) - \sqrt{d^2 - e^2x^2} (4e(3Ae(4d + ex) + 2B(5d^2 + 3dex + e^2x^2)) + C(32d^3 + 21d^2ex + 16de^2x^2 + 6e^3x^3))}{24e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out] (-(Sqrt[d^2 - e^2\*x^2]\*(C\*(32\*d^3 + 21\*d^2\*e\*x + 16\*d\*e^2\*x^2 + 6\*e^3\*x^3) + 4\*e\*(3\*A\*e\*(4\*d + e\*x) + 2\*B\*(5\*d^2 + 3\*d\*e\*x + e^2\*x^2)))) + 3\*d^2\*(7\*C\*d^2 + 4\*e\*(2\*B\*d + 3\*A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(24\*e^3)

**IntegrateAlgebraic [A]** time = 0.59, size = 165, normalized size = 0.86

$$\frac{\sqrt{d^2 - e^2x^2} (-48Ade^2 - 12Ae^3x - 40Bd^2e - 24Bde^2x - 8Be^3x^2 - 32Cd^3 - 21Cd^2ex - 16Cde^2x^2 - 6Ce^3x^3)}{24e^3} + \frac{\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2x^2} - \sqrt{-e^2}x\right) (12Ad^2e^2 + 8Bd^3e + 7Cd^4)}{8e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2],x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-32\*C\*d^3 - 40\*B\*d^2\*e - 48\*A\*d\*e^2 - 21\*C\*d^2\*e\*x - 24\*B\*d\*e^2\*x - 12\*A\*e^3\*x - 16\*C\*d\*e^2\*x^2 - 8\*B\*e^3\*x^2 - 6\*C\*e^3\*x^3))/(24\*e^3) + (Sqrt[-e^2]\*(7\*C\*d^4 + 8\*B\*d^3\*e + 12\*A\*d^2\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(8\*e^4)

**fricas** [A] time = 1.14, size = 145, normalized size = 0.76

$$\frac{6(7Cd^4 + 8Bd^3e + 12Ad^2e^2) \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + (6Ce^3x^3 + 32Cd^3 + 40Bd^2e + 48Ade^2 + 8(2Cde^2 + Be^3)x^2 + 3(7Cd^2e + 8Bde^2 + 4Ae^3)x)\sqrt{-e^2x^2 + d^2}}{24e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/24\*(6\*(7\*C\*d^4 + 8\*B\*d^3\*e + 12\*A\*d^2\*e^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + (6\*C\*e^3\*x^3 + 32\*C\*d^3 + 40\*B\*d^2\*e + 48\*A\*d\*e^2 + 8\*(2\*C\*d\*e^2 + B\*e^3)\*x^2 + 3\*(7\*C\*d^2\*e + 8\*B\*d\*e^2 + 4\*A\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/e^3

**giac** [A] time = 0.32, size = 131, normalized size = 0.69

$$\frac{1}{8}(7Cd^4 + 8Bd^3e + 12Ad^2e^2) \arcsin\left(\frac{xe}{d}\right) e^{(-3)\operatorname{sgn}(d)} - \frac{1}{24}\sqrt{-x^2e^2 + d^2} \left( (2(3Cx + 4(2Cde^2 + Be^3)e^{(-5)})x + 3(7Cd^2e^3 + 8Bde^4 + 4Ae^5)e^{(-5)})x + 8(4Cd^3e^2 + 5Bd^2e^3 + 6Ade^4)e^{(-5)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] 1/8\*(7\*C\*d^4 + 8\*B\*d^3\*e + 12\*A\*d^2\*e^2)\*arcsin(x\*e/d)\*e^(-3)\*sgn(d) - 1/24\*sqrt(-x^2\*e^2 + d^2)\*((2\*(3\*C\*x + 4\*(2\*C\*d\*e^2 + B\*e^3)\*e^(-5))\*x + 3\*(7\*C\*d^2\*e^3 + 8\*B\*d\*e^4 + 4\*A\*e^5)\*e^(-5))\*x + 8\*(4\*C\*d^3\*e^2 + 5\*B\*d^2\*e^3 + 6\*A\*d\*e^4)\*e^(-5))

**maple** [A] time = 0.01, size = 301, normalized size = 1.58

$$\frac{3Ad^2 \arctan\left(\frac{\sqrt{e}x}{\sqrt{-e^2x^2 + d^2}}\right) + Bd^3 \arctan\left(\frac{\sqrt{e}x}{\sqrt{-e^2x^2 + d^2}}\right) + 7Cd^4 \arctan\left(\frac{\sqrt{e}x}{\sqrt{-e^2x^2 + d^2}}\right)}{2\sqrt{e^2}} - \frac{\sqrt{-e^2x^2 + d^2} C x^3}{4} - \frac{\sqrt{-e^2x^2 + d^2} B x^2}{3} - \frac{2\sqrt{-e^2x^2 + d^2} C d x^2}{3e} - \frac{\sqrt{-e^2x^2 + d^2} A x}{2} - \frac{\sqrt{-e^2x^2 + d^2} B d x}{e} - \frac{7\sqrt{-e^2x^2 + d^2} C d^2 x}{8e^2} - \frac{2\sqrt{-e^2x^2 + d^2} A d}{e} - \frac{5\sqrt{-e^2x^2 + d^2} B d^2}{3e^2} - \frac{4\sqrt{-e^2x^2 + d^2} C d^3}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/4\*C\*x^3\*(-e^2\*x^2+d^2)^(1/2)-7/8\*(-e^2\*x^2+d^2)^(1/2)\*C\*d^2/e^2\*x+7/8/(e^2)^(1/2)\*C\*d^4/e^2\*arctan((e^2)^(1/2)/(-e^2\*x^2+d^2)^(1/2)\*x)-1/3\*x^2\*(-e^2\*x^2+d^2)^(1/2)\*B-2/3\*x^2/e\*(-e^2\*x^2+d^2)^(1/2)\*d\*C-5/3\*d^2/e^2\*(-e^2\*x^2+d^2)^(1/2)\*B-4/3\*d^3/e^3\*(-e^2\*x^2+d^2)^(1/2)\*C-1/2\*(-e^2\*x^2+d^2)^(1/2)\*A\*x-x/e\*(-e^2\*x^2+d^2)^(1/2)\*B\*d+3/2/(e^2)^(1/2)\*A\*d^2\*arctan((e^2)^(1/2)/(-

$e^{2x^2+d^2} \sqrt{x} + d^3/e \arctan(e \sqrt{x} / (-e^{2x^2+d^2}) \sqrt{x}) - 2B/e \sqrt{x} + A d$

**maxima** [A] time = 0.98, size = 253, normalized size = 1.32

$$\frac{1}{4} \sqrt{-e^2 x^2 + d^2} C x^3 + \frac{3 C d^4 \arcsin\left(\frac{x}{d}\right)}{8 e^3} + \frac{A d^2 \arcsin\left(\frac{x}{d}\right)}{e} - \frac{3 \sqrt{-e^2 x^2 + d^2} C d^2 x}{8 e^2} - \frac{\sqrt{-e^2 x^2 + d^2} B d^2}{e^2} - \frac{2 \sqrt{-e^2 x^2 + d^2} A d}{e} - \frac{\sqrt{-e^2 x^2 + d^2} (2 C d e + B e^2) x^2}{3 e^2} + \frac{(C d^2 + 2 B d e + A e^2) d^2 \arcsin\left(\frac{x}{d}\right)}{2 e^3} - \frac{\sqrt{-e^2 x^2 + d^2} (C d^2 + 2 B d e + A e^2) x}{2 e^2} - \frac{2 \sqrt{-e^2 x^2 + d^2} (2 C d e + B e^2) d^2}{3 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out]  $-1/4 \sqrt{-e^2 x^2 + d^2} C x^3 + 3/8 C d^4 \arcsin(e x / d) / e^3 + A d^2 \arcsin(e x / d) / e - 3/8 \sqrt{-e^2 x^2 + d^2} C d^2 x / e^2 - \sqrt{-e^2 x^2 + d^2} B d^2 / e^2 - 2 \sqrt{-e^2 x^2 + d^2} A d / e - 1/3 \sqrt{-e^2 x^2 + d^2} (2 C d e + B e^2) x^2 / e^2 + 1/2 (C d^2 + 2 B d e + A e^2) d^2 \arcsin(e x / d) / e^3 - 1/2 \sqrt{-e^2 x^2 + d^2} (C d^2 + 2 B d e + A e^2) x / e^2 - 2/3 \sqrt{-e^2 x^2 + d^2} (2 C d e + B e^2) d^2 / e^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d + e x)^2 (C x^2 + B x + A)}{\sqrt{d^2 - e^2 x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^2\*(A + B\*x + C\*x^2))/(d^2 - e^2\*x^2)^(1/2),x)

[Out] int(((d + e\*x)^2\*(A + B\*x + C\*x^2))/(d^2 - e^2\*x^2)^(1/2), x)

**sympy** [A] time = 18.03, size = 891, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(C\*x\*\*2+B\*x+A)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out]  $A d^2 \text{Piecewise}(\left(\sqrt{d^2/e^2} \operatorname{asin}(x \sqrt{e^2/d^2}) / \sqrt{d^2}, (d^2 > 0) \ \& \ (e^2 > 0)\right), \left(\sqrt{-d^2/e^2} \operatorname{asinh}(x \sqrt{-e^2/d^2}) / \sqrt{d^2}, (d^2 > 0) \ \& \ (e^2 < 0)\right), \left(\sqrt{d^2/e^2} \operatorname{acosh}(x \sqrt{e^2/d^2}) / \sqrt{-d^2}, (d^2 < 0) \ \& \ (e^2 < 0)\right)) + 2 A d e \text{Piecewise}(\left(x^2 / (2 \sqrt{d^2}), \operatorname{Eq}(e^2, 0)\right), \left(-\sqrt{d^2 - e^2 x^2} / e^2, \operatorname{True}\right)) + A e^2 \text{Piecewise}(\left(-I d^2 \operatorname{acosh}(e x / d) / (2 e^3) - I d x \sqrt{-1 + e^2 x^2 / d^2} / (2 e^2), \operatorname{Abs}(e^2 x^2 / d^2) > 1\right), \left(d^2 \operatorname{asin}(e x / d) / (2 e^3) - d x / (2 e^2 \sqrt{1 - e^2 x^2 / d^2}) + x^3 / (2 d \sqrt{1 - e^2 x^2 / d^2}), \operatorname{True}\right)) + B d^2 \text{Piecewise}(\left(x^2 / (2 \sqrt{d^2}), \operatorname{Eq}(e^2, 0)\right), \left(-\sqrt{d^2 - e^2 x^2} / e^2, \operatorname{True}\right)) + 2 B d e \text{Piecewise}(\left(-I d^2 \operatorname{acosh}(e x / d) / (2 e^3) - I d x \sqrt{-1 + e^2 x^2 / d^2} / (2 e^2), \operatorname{Abs}(e^2 x^2 / d^2) > 1\right), \left(d^2 \operatorname{asinh}(e x / d) / (2 e^3) - d x / (2 e^2 \sqrt{1 - e^2 x^2 / d^2}) + x^3 / (2 d \sqrt{1 - e^2 x^2 / d^2}), \operatorname{True}\right))$



```

*2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3
) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d
**2)), True)) + B*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) -
x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True
)) + C*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2
*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3)
- d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2
)), True)) + 2*C*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x
**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)
) + C*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*
sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) -
I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*a
sin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(
8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), Tru
e))

```

$$3.12 \quad \int \frac{(d+ex)(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2} (3e(Ae+Bd)+2Cd^2)}{3e^3} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) (e(2Ae+Bd)+Cd^2)}{2e^3} - \frac{x\sqrt{d^2-e^2x^2} (Be+Cd)}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

**Rubi [A]** time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1815, 641, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2} (3e(Ae+Bd)+2Cd^2)}{3e^3} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) (e(2Ae+Bd)+Cd^2)}{2e^3} - \frac{x\sqrt{d^2-e^2x^2} (Be+Cd)}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out] -((2\*C\*d^2 + 3\*e\*(B\*d + A\*e))\*Sqrt[d^2 - e^2\*x^2])/(3\*e^3) - ((C\*d + B\*e)\*x\*Sqrt[d^2 - e^2\*x^2])/(2\*e^2) - (C\*x^2\*Sqrt[d^2 - e^2\*x^2])/(3\*e) + (d\*(C\*d^2 + e\*(B\*d + 2\*A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSu

m[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x  
 ], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex)(A + Bx + Cx^2)}{\sqrt{d^2 - e^2x^2}} dx &= -\frac{Cx^2\sqrt{d^2 - e^2x^2}}{3e} - \frac{\int \frac{-3Ade^2 - e(2Cd^2 + 3e(Bd + Ae))x - 3e^2(Cd + Be)x^2}{\sqrt{d^2 - e^2x^2}} dx}{3e^2} \\ &= -\frac{(Cd + Be)x\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2 - e^2x^2}}{3e} + \frac{\int \frac{3de^2(Cd^2 + e(Bd + 2Ae)) + 2e^3(2Cd^2 + 3e(Bd + Ae))x + e^4(Cd + Be)x^2}{\sqrt{d^2 - e^2x^2}} dx}{6e^4} \\ &= -\frac{(2Cd^2 + 3e(Bd + Ae))\sqrt{d^2 - e^2x^2}}{3e^3} - \frac{(Cd + Be)x\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2 - e^2x^2}}{3e} \\ &= -\frac{(2Cd^2 + 3e(Bd + Ae))\sqrt{d^2 - e^2x^2}}{3e^3} - \frac{(Cd + Be)x\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2 - e^2x^2}}{3e} \\ &= -\frac{(2Cd^2 + 3e(Bd + Ae))\sqrt{d^2 - e^2x^2}}{3e^3} - \frac{(Cd + Be)x\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2 - e^2x^2}}{3e} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 103, normalized size = 0.72

$$\frac{3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) (e(2Ae + Bd) + Cd^2) - \sqrt{d^2 - e^2x^2} (3e(2Ae + 2Bd + Bex) + C(4d^2 + 3dex + 2e^2x^2))}{6e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out] (-(Sqrt[d^2 - e^2\*x^2]\*(3\*e\*(2\*B\*d + 2\*A\*e + B\*e\*x) + C\*(4\*d^2 + 3\*d\*e\*x + 2\*e^2\*x^2))) + 3\*d\*(C\*d^2 + e\*(B\*d + 2\*A\*e))\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(6\*e^3)

**IntegrateAlgebraic [A]** time = 0.49, size = 129, normalized size = 0.90

$$\frac{\sqrt{d^2 - e^2x^2} (-6Ae^2 - 6Bde - 3Be^2x - 4Cd^2 - 3Cdex - 2Ce^2x^2)}{6e^3} + \frac{\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2x^2} - \sqrt{-e^2}x\right) (2Ade^2 + Bd^2e + Cd^3)}{2e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(\sqrt{d^2 - e^2 x^2}) * (-4 * C * d^2 - 6 * B * d * e - 6 * A * e^2 - 3 * C * d * e * x - 3 * B * e^2 * x - 2 * C * e^2 * x^2) / (6 * e^3) + (\sqrt{-e^2} * (C * d^3 + B * d^2 * e + 2 * A * d * e^2) * \text{Log}[-(\text{Sqrt}[-e^2] * x) + \text{Sqrt}[d^2 - e^2 * x^2]]) / (2 * e^4)$

**fricas** [A] time = 0.79, size = 109, normalized size = 0.76

$$\frac{6(Cd^3 + Bd^2e + 2Ade^2) \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + (2Ce^2x^2 + 4Cd^2 + 6Bde + 6Ae^2 + 3(Cde + Be^2)x)\sqrt{-e^2x^2 + d^2}}{6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out]  $-1/6 * (6 * (C * d^3 + B * d^2 * e + 2 * A * d * e^2) * \arctan(-(d - \text{sqrt}(-e^2 * x^2 + d^2)) / (e * x)) + (2 * C * e^2 * x^2 + 4 * C * d^2 + 6 * B * d * e + 6 * A * e^2 + 3 * (C * d * e + B * e^2) * x) * \text{sqrt}(-e^2 * x^2 + d^2)) / e^3$

**giac** [A] time = 0.29, size = 97, normalized size = 0.68

$$\frac{1}{2} (Cd^3 + Bd^2e + 2Ade^2) \arcsin\left(\frac{xe}{d}\right) e^{(-3) \text{sgn}(d)} - \frac{1}{6} \sqrt{-x^2e^2 + d^2} ((2Cxe^{(-1)} + 3(Cde^3 + Be^4)e^{(-5)})x + 2(2Cd^2e^2 + 3Bde^3 + 3Ae^4)e^{(-5)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out]  $1/2 * (C * d^3 + B * d^2 * e + 2 * A * d * e^2) * \arcsin(x * e / d) * e^{(-3) * \text{sgn}(d)} - 1/6 * \text{sqrt}(-x^2 * e^2 + d^2) * ((2 * C * x * e^{(-1)} + 3 * (C * d * e^3 + B * e^4) * e^{(-5)}) * x + 2 * (2 * C * d^2 * e^2 + 3 * B * d * e^3 + 3 * A * e^4) * e^{(-5)})$

**maple** [A] time = 0.01, size = 234, normalized size = 1.64

$$\frac{Ad \arctan\left(\frac{\sqrt{d^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{\sqrt{e^2}} + \frac{B d^2 \arctan\left(\frac{\sqrt{d^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{2\sqrt{e^2} e} + \frac{C d^3 \arctan\left(\frac{\sqrt{d^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{2\sqrt{e^2} e^2} - \frac{\sqrt{-e^2x^2 + d^2} C x^2}{3e} - \frac{\sqrt{-e^2x^2 + d^2} B x}{2e} - \frac{\sqrt{-e^2x^2 + d^2} C d x}{2e^2} - \frac{\sqrt{-e^2x^2 + d^2} A}{e} - \frac{\sqrt{-e^2x^2 + d^2} B d}{e^2} - \frac{2\sqrt{-e^2x^2 + d^2} C d^2}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x)

[Out]  $-1/3 * C * x^2 * (-e^2 * x^2 + d^2)^{(1/2)} / e - 2/3 * e^3 * C * d^2 * (-e^2 * x^2 + d^2)^{(1/2)} - 1/2 * (-e^2 * x^2 + d^2)^{(1/2)} * B / e * x - 1/2 * (-e^2 * x^2 + d^2)^{(1/2)} * C * d / e^2 * x + 1/2 / (e^2)^{(1/2)} * B * d^2 / e * \arctan((e^2)^{(1/2)} / (-e^2 * x^2 + d^2)^{(1/2)} * x) + 1/2 / (e^2)^{(1/2)} * C * d^3 / e^2 * \arctan((e^2)^{(1/2)} / (-e^2 * x^2 + d^2)^{(1/2)} * x) - 1/e * (-e^2 * x^2 + d^2)^{(1/2)} * A - 1/e^2 * (-e^2 * x^2 + d^2)^{(1/2)} * B * d + A * d / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} / (-e^2 * x^2 + d^2)^{(1/2)} * x)$

**maxima** [A] time = 0.98, size = 150, normalized size = 1.05

$$-\frac{\sqrt{-e^2x^2 + d^2} C x^2}{3e} + \frac{Ad \arcsin\left(\frac{ex}{d}\right)}{e} + \frac{(Cd + Be)d^2 \arcsin\left(\frac{ex}{d}\right)}{2e^3} - \frac{2\sqrt{-e^2x^2 + d^2} C d^2}{3e^3} - \frac{\sqrt{-e^2x^2 + d^2} B d}{e^2} - \frac{\sqrt{-e^2x^2 + d^2} A}{e} - \frac{\sqrt{-e^2x^2 + d^2} (Cd + Be)x}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out]  $-1/3\sqrt{-e^2x^2 + d^2}Cx^2/e + Ad\arcsin(e*x/d)/e + 1/2(C*d + B*e)d^2\arcsin(e*x/d)/e^3 - 2/3\sqrt{-e^2x^2 + d^2}C*d^2/e^3 - \sqrt{-e^2x^2 + d^2}B*d/e^2 - \sqrt{-e^2x^2 + d^2}A/e - 1/2\sqrt{-e^2x^2 + d^2}(C*d + B*e)*x/e^2$

**mupad [B]** time = 5.01, size = 270, normalized size = 1.89

$$\begin{cases} \frac{2Cdx^3+3Bdx^2+6Adx}{6\sqrt{d^2}} & \text{if } e = 0 \\ \frac{Ad \ln\left(x\sqrt{-e^2 + \sqrt{d^2 - e^2x^2}}\right)}{\sqrt{-e^2}} - \frac{A\sqrt{d^2 - e^2x^2}}{e} - \frac{Bd\sqrt{d^2 - e^2x^2}}{e^2} - \frac{Bx\sqrt{d^2 - e^2x^2}}{2e} - \frac{C\sqrt{d^2 - e^2x^2}(2d^2 + e^2x^2)}{3e^3} - \frac{Cd^3 \ln\left(2x\sqrt{-e^2 + 2\sqrt{d^2 - e^2x^2}}\right)}{2(-e^2)^{3/2}} - \frac{Bd^2e \ln\left(2x\sqrt{-e^2 + 2\sqrt{d^2 - e^2x^2}}\right)}{2(-e^2)^{3/2}} - \frac{Cdx\sqrt{d^2 - e^2x^2}}{2e^2} & \text{if } e \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(A + B\*x + C\*x^2))/(d^2 - e^2\*x^2)^(1/2),x)

[Out]  $\text{piecewise}(e == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*(d^2)^{(1/2)}), e \neq 0, - (A*(d^2 - e^2*x^2)^{(1/2)})/e + (A*d*\log(x*(-e^2)^{(1/2)} + (d^2 - e^2*x^2)^{(1/2)}))/(-e^2)^{(1/2)} - (B*d*(d^2 - e^2*x^2)^{(1/2)})/e^2 - (B*x*(d^2 - e^2*x^2)^{(1/2)})/(2*e) - (C*(d^2 - e^2*x^2)^{(1/2)}*(2*d^2 + e^2*x^2))/(3*e^3) - (C*d^3*\log(2*x*(-e^2)^{(1/2)} + 2*(d^2 - e^2*x^2)^{(1/2)}))/(2*(-e^2)^{(3/2)}) - (B*d^2*e*\log(2*x*(-e^2)^{(1/2)} + 2*(d^2 - e^2*x^2)^{(1/2)}))/(2*(-e^2)^{(3/2)}) - (C*d*x*(d^2 - e^2*x^2)^{(1/2)})/(2*e^2))$

**sympy [A]** time = 10.17, size = 484, normalized size = 3.38

$$Ad \begin{cases} \frac{\sqrt{\frac{d}{e}} \operatorname{asin}\left(\sqrt{\frac{d}{e}}\right)}{\sqrt{d}} & \text{for } d^2 > 0 \wedge e^2 > 0 \\ \frac{\sqrt{\frac{d}{e}} \operatorname{asinh}\left(\sqrt{\frac{d}{e}}\right)}{\sqrt{d}} & \text{for } d^2 > 0 \wedge e^2 < 0 \\ \frac{\sqrt{\frac{d}{e}} \operatorname{acosh}\left(\sqrt{\frac{d}{e}}\right)}{\sqrt{-d}} & \text{for } d^2 < 0 \wedge e^2 < 0 \end{cases} + Ae \begin{cases} \frac{x^2}{2\sqrt{d}} & \text{for } e^2 = 0 \\ \frac{x^2}{-\sqrt{d^2 - e^2x^2}} & \text{otherwise} \end{cases} + Bd \begin{cases} \frac{x^2}{2\sqrt{d}} & \text{for } e^2 = 0 \\ \frac{x^2}{-\sqrt{d^2 - e^2x^2}} & \text{otherwise} \end{cases} + Be \begin{cases} \frac{d^2 \operatorname{asin}\left(\frac{x}{d}\right)}{2d^3} - \frac{\operatorname{asin}\left(\sqrt{1 - \frac{d^2}{e^2}}\right)}{2d^2} & \text{for } \left|\frac{d^2}{e^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{x}{d}\right)}{2d^3} - \frac{dx}{2d^2\sqrt{1 - \frac{d^2}{e^2}}} + \frac{x^3}{2d\sqrt{1 - \frac{d^2}{e^2}}} & \text{otherwise} \end{cases} + Cd \begin{cases} \frac{d^2 \operatorname{asin}\left(\frac{x}{d}\right)}{2d^3} - \frac{\operatorname{asin}\left(\sqrt{1 - \frac{d^2}{e^2}}\right)}{2d^2} & \text{for } \left|\frac{d^2}{e^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{x}{d}\right)}{2d^3} - \frac{dx}{2d^2\sqrt{1 - \frac{d^2}{e^2}}} + \frac{x^3}{2d\sqrt{1 - \frac{d^2}{e^2}}} & \text{otherwise} \end{cases} + Ce \begin{cases} \frac{2d^2\sqrt{d^2 - e^2x^2}}{3d^4} - \frac{d^2\sqrt{d^2 - e^2x^2}}{3d^4} & \text{for } e \neq 0 \\ \frac{x^4}{4\sqrt{d}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x\*\*2+B\*x+A)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out]  $A*d*\text{Piecewise}((\sqrt{d**2/e**2})*\operatorname{asin}(x*\sqrt{e**2/d**2})/\sqrt{d**2}, (d**2 > 0) \& (e**2 > 0)), (\sqrt{-d**2/e**2})*\operatorname{asinh}(x*\sqrt{-e**2/d**2})/\sqrt{d**2}, (d**2 > 0) \& (e**2 < 0)), (\sqrt{d**2/e**2})*\operatorname{acosh}(x*\sqrt{e**2/d**2})/\sqrt{-d**2}, (d**2 < 0) \& (e**2 < 0))) + A*e*\text{Piecewise}((x**2/(2*\sqrt{d**2})), \text{Eq}(e**2, 0)), (-\sqrt{d**2 - e**2*x**2}/e**2, \text{True})) + B*d*\text{Piecewise}((x**2/(2*\sqrt{d**2})), \text{Eq}(e**2, 0)), (-\sqrt{d**2 - e**2*x**2}/e**2, \text{True})) + B*e*\text{Piecewise}((-I*d**2*\operatorname{acosh}(e*x/d)/(2*e**3) - I*d*x*\sqrt{-1 + e**2*x**2/d**2})/(2*e**2), \text{Abs}(e**2*x**2/d**2) > 1), (d**2*\operatorname{asin}(e*x/d)/(2*e**3) - d*x/(2*e**2*\sqrt{1 - e**2*x**2/d**2})) + x**3/(2*d*\sqrt{1 - e**2*x**2/d**2})), \text{True})) + C*d*\text{Piecewise}((-I*d**2*\operatorname{acosh}(e*x/d)/(2*e**3) - I*d*x*\sqrt{-1 + e**2*x**2/d**2})/(2*$

```

e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*s
qrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*
e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**
2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))

```

$$3.13 \quad \int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}} dx$$

Optimal. Leaf size=87

$$\frac{(2Ae^2 + Cd^2) \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} - \frac{B\sqrt{d^2-e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1815, 641, 217, 203}

$$\frac{(2Ae^2 + Cd^2) \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} - \frac{B\sqrt{d^2-e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out] -((B\*Sqrt[d^2 - e^2\*x^2])/e^2) - (C\*x\*Sqrt[d^2 - e^2\*x^2])/(2\*e^2) + ((C\*d^2 + 2\*A\*e^2)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x]

], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{\sqrt{d^2 - e^2x^2}} dx &= -\frac{Cx\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{\int \frac{-Cd^2 - 2Ae^2 - 2Be^2x}{\sqrt{d^2 - e^2x^2}} dx}{2e^2} \\
 &= -\frac{B\sqrt{d^2 - e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{(-Cd^2 - 2Ae^2) \int \frac{1}{\sqrt{d^2 - e^2x^2}} dx}{2e^2} \\
 &= -\frac{B\sqrt{d^2 - e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{(-Cd^2 - 2Ae^2) \text{Subst}\left(\int \frac{1}{1+e^2x^2} dx, x, \frac{x}{\sqrt{d^2 - e^2x^2}}\right)}{2e^2} \\
 &= -\frac{B\sqrt{d^2 - e^2x^2}}{e^2} - \frac{Cx\sqrt{d^2 - e^2x^2}}{2e^2} + \frac{(Cd^2 + 2Ae^2) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^3}
 \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 67, normalized size = 0.77

$$\frac{(2Ae^2 + Cd^2) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - e(2B + Cx)\sqrt{d^2 - e^2x^2}}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out] (-(e\*(2\*B + C\*x)\*Sqrt[d^2 - e^2\*x^2]) + (C\*d^2 + 2\*A\*e^2)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

**IntegrateAlgebraic** [A] time = 0.34, size = 90, normalized size = 1.03

$$\frac{\sqrt{-e^2} (2Ae^2 + Cd^2) \log\left(\sqrt{d^2 - e^2x^2} - \sqrt{-e^2} x\right)}{2e^4} + \frac{(-2B - Cx)\sqrt{d^2 - e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out] ((-2\*B - C\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*e^2) + (Sqrt[-e^2]\*(C\*d^2 + 2\*A\*e^2)\*Log[-(Sqrt[-e^2]\*x) + Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)



**fricas** [A] time = 1.04, size = 71, normalized size = 0.82

$$\frac{2(Cd^2 + 2Ae^2) \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + \sqrt{-e^2x^2 + d^2}(Cex + 2Be)}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/2\*(2\*(C\*d^2 + 2\*A\*e^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + sqrt(-e^2\*x^2 + d^2)\*(C\*e\*x + 2\*B\*e))/e^3

**giac** [A] time = 0.34, size = 52, normalized size = 0.60

$$\frac{1}{2}(Cd^2 + 2Ae^2) \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sgn}(d) - \frac{1}{2} \sqrt{-x^2e^2 + d^2} (Cxe^{(-2)} + 2Be^{(-2)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] 1/2\*(C\*d^2 + 2\*A\*e^2)\*arcsin(x\*e/d)\*e^(-3)\*sgn(d) - 1/2\*sqrt(-x^2\*e^2 + d^2)\*(C\*x\*e^(-2) + 2\*B\*e^(-2))

**maple** [A] time = 0.01, size = 108, normalized size = 1.24

$$\frac{A \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{C d^2 \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2} e^2} - \frac{\sqrt{-e^2x^2 + d^2} Cx}{2e^2} - \frac{\sqrt{-e^2x^2 + d^2} B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/2\*(-e^2\*x^2+d^2)^(1/2)\*C/e^2\*x+1/2/(e^2)^(1/2)\*C\*d^2/e^2\*arctan((e^2)^(1/2)/(-e^2\*x^2+d^2)^(1/2)\*x)-B\*(-e^2\*x^2+d^2)^(1/2)/e^2+A/(e^2)^(1/2)\*arctan((e^2)^(1/2)/(-e^2\*x^2+d^2)^(1/2)\*x)

**maxima** [A] time = 0.99, size = 70, normalized size = 0.80

$$\frac{Cd^2 \arcsin\left(\frac{ex}{d}\right)}{2e^3} + \frac{A \arcsin\left(\frac{ex}{d}\right)}{e} - \frac{\sqrt{-e^2x^2 + d^2} Cx}{2e^2} - \frac{\sqrt{-e^2x^2 + d^2} B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out]  $1/2*C*d^2*arcsin(e*x/d)/e^3 + A*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*C*x/e^2 - sqrt(-e^2*x^2 + d^2)*B/e^2$

mupad [B] time = 4.40, size = 148, normalized size = 1.70

$$\left\{ \begin{array}{ll} \frac{2Cx^3+3Bx^2+6Ax}{6\sqrt{d^2}} & \text{if } e = 0 \\ \frac{A \ln\left(x\sqrt{-e^2+\sqrt{d^2-e^2}x^2}\right)}{\sqrt{-e^2}} - \frac{B\sqrt{d^2-e^2}x^2}{e^2} - \frac{Cx\sqrt{d^2-e^2}x^2}{2e^2} - \frac{Cd^2 \ln\left(2x\sqrt{-e^2+2\sqrt{d^2-e^2}x^2}\right)}{2(-e^2)^{3/2}} & \text{if } e \neq 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(d^2 - e^2*x^2)^(1/2), x)`

[Out] `piecewise(e == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*(d^2)^(1/2)), e ~= 0, (A*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (B*(d^2 - e^2*x^2)^(1/2))/e^2 - (C*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2) - (C*d^2*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)))`

sympy [A] time = 4.56, size = 262, normalized size = 3.01

$$A \left( \begin{array}{l} \left( \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} \quad \text{for } d^2 > 0 \wedge e^2 > 0 \right. \\ \left. \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{asinh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} \quad \text{for } d^2 > 0 \wedge e^2 < 0 \right. \\ \left. \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} \quad \text{for } d^2 < 0 \wedge e^2 < 0 \right) \end{array} \right) + B \left( \begin{array}{l} \frac{x^2}{2\sqrt{d^2}} \quad \text{for } e^2 = 0 \\ -\frac{\sqrt{d^2-e^2}x^2}{e^2} \quad \text{otherwise} \end{array} \right) + C \left( \begin{array}{l} \left( -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} \quad \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \right. \\ \left. \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} \quad \text{otherwise} \right) \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2), x)`

[Out] `A*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0))) + B*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + C*Piecewise((-I*d**2*acosh(ex/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(ex/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))`

$$3.14 \quad \int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=103

$$-\frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{de^3(d + ex)} - \frac{(Cd - Be) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3} - \frac{C\sqrt{d^2 - e^2x^2}}{e^3}$$

**Rubi [A]** time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1639, 793, 217, 203}

$$-\frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{de^3(d + ex)} - \frac{(Cd - Be) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3} - \frac{C\sqrt{d^2 - e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]), x]

[Out] -((C\*Sqrt[d^2 - e^2\*x^2])/e^3) - ((C\*d^2 - B\*d\*e + A\*e^2)\*Sqrt[d^2 - e^2\*x^2])/(d\*e^3\*(d + e\*x)) - ((C\*d - B\*e)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 793

Int[((d\_) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[(m\*(g\*c\*d + c\*e\*f) + 2\*e\*c\*f\*(p + 1))/(e\*(2\*c\*d)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

Rule 1639

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{(d + ex)\sqrt{d^2 - e^2x^2}} dx &= -\frac{C\sqrt{d^2 - e^2x^2}}{e^3} - \frac{\int \frac{-Ae^4 + e^3(Cd - Be)x}{(d + ex)\sqrt{d^2 - e^2x^2}} dx}{e^4} \\
&= -\frac{C\sqrt{d^2 - e^2x^2}}{e^3} - \frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{de^3(d + ex)} - \frac{(Cd - Be) \int \frac{1}{\sqrt{d^2 - e^2x^2}} dx}{e^2} \\
&= -\frac{C\sqrt{d^2 - e^2x^2}}{e^3} - \frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{de^3(d + ex)} - \frac{(Cd - Be) \operatorname{Subst}\left(\int \frac{1}{1 + e^2x^2} dx, x, \frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^2} \\
&= -\frac{C\sqrt{d^2 - e^2x^2}}{e^3} - \frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{de^3(d + ex)} - \frac{(Cd - Be) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3}
\end{aligned}$$

**Mathematica** [A] time = 0.16, size = 83, normalized size = 0.81

$$\frac{(Be - Cd) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - \frac{\sqrt{d^2 - e^2x^2} (e(Ae - Bd) + Cd(2d + ex))}{d(d + ex)}}{e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]), x]

[Out] (-((Sqrt[d^2 - e^2\*x^2]\*(e\*(-(B\*d) + A\*e) + C\*d\*(2\*d + e\*x)))/(d\*(d + e\*x))) + (-((C\*d) + B\*e)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

**IntegrateAlgebraic** [A] time = 0.49, size = 105, normalized size = 1.02

$$\frac{\sqrt{d^2 - e^2x^2} (-Ae^2 + Bde - 2Cd^2 - Cdex)}{de^3(d + ex)} + \frac{\sqrt{-e^2} (Be - Cd) \log\left(\sqrt{d^2 - e^2x^2} - \sqrt{-e^2} x\right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out]  $((-2*C*d^2 + B*d*e - A*e^2 - C*d*e*x)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) + (Sqrt[-e^2]*(-(C*d) + B*e)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/e^4$

**fricas** [A] time = 0.72, size = 155, normalized size = 1.50

$$\frac{2Cd^3 - Bd^2e + Ade^2 + (2Cd^2e - Bde^2 + Ae^3)x - 2(Cd^3 - Bd^2e + (Cd^2e - Bde^2)x) \arctan\left(\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + (Cdex + 2Cd^2 - Bde + Ae^2)\sqrt{-e^2x^2 + d^2}}{d^4x + d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out]  $-(2*C*d^3 - B*d^2*e + A*d*e^2 + (2*C*d^2*e - B*d*e^2 + A*e^3)*x - 2*(C*d^3 - B*d^2*e + (C*d^2*e - B*d*e^2)*x)*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) + (C*d*e*x + 2*C*d^2 - B*d*e + A*e^2)*\sqrt{-e^2*x^2 + d^2})/(d*e^4*x + d^2*e^3)$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output:  $1/2*(-4*A*\exp(2)^2 - 4*C*d^2*\exp(2) + 4*B*d*\exp(1)*\exp(2))*\operatorname{atan}((-1/2*(-2*d*\exp(1) - 2*\sqrt{d^2 - x^2*\exp(2)})*\exp(1))/x + \exp(2))/\sqrt{-\exp(1)^4 + \exp(2)^2})/\sqrt{-\exp(1)^4 + \exp(2)^2}/d/\exp(1)/\exp(2) - 1/4*(-4*B*\exp(1) + 4*C*d)*\operatorname{sign}(d)*\operatorname{asin}(x*\exp(2)/d/\exp(1))/\exp(1)/\exp(2) - 4*\exp(1)^2*C/1/4/\exp(1)^5*\sqrt{-\exp(2)*x^2 + d^2}$

**maple** [A] time = 0.01, size = 149, normalized size = 1.45

$$\frac{B \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{\sqrt{e^2} e} - \frac{Cd \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2x^2 + d^2}}\right)}{\sqrt{e^2} e^2} - \frac{\sqrt{-e^2x^2 + d^2} C}{e^3} - \frac{(Ae^2 - Bde + Cd^2) \sqrt{2\left(x + \frac{d}{e}\right)de - \left(x + \frac{d}{e}\right)^2} e^2}{\left(x + \frac{d}{e}\right) d e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2),x)

[Out]  $-C*(-e^2*x^2+d^2)^(1/2)/e^3 + 1/e*B/(e^2)^(1/2)*\arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x) - 1/e^2*C*d/(e^2)^(1/2)*\arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x) - (A*e^2 - B*d*e + C*d^2)/e^4/d/(x+d/e)*(2*(x+d/e)*d*e - (x+d/e)^2*e^2)^(1/2)$

**maxima** [A] time = 0.99, size = 138, normalized size = 1.34

$$-\frac{\sqrt{-e^2x^2+d^2}Cd}{e^4x+de^3} - \frac{\sqrt{-e^2x^2+d^2}A}{de^2x+d^2e} + \frac{\sqrt{-e^2x^2+d^2}B}{e^3x+de^2} - \frac{Cd \arcsin\left(\frac{ex}{d}\right)}{e^3} + \frac{B \arcsin\left(\frac{ex}{d}\right)}{e^2} - \frac{\sqrt{-e^2x^2+d^2}C}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out] -sqrt(-e^2\*x^2 + d^2)\*C\*d/(e^4\*x + d\*e^3) - sqrt(-e^2\*x^2 + d^2)\*A/(d\*e^2\*x + d^2\*e) + sqrt(-e^2\*x^2 + d^2)\*B/(e^3\*x + d\*e^2) - C\*d\*arcsin(e\*x/d)/e^3 + B\*arcsin(e\*x/d)/e^2 - sqrt(-e^2\*x^2 + d^2)\*C/e^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Cx^2 + Bx + A}{\sqrt{d^2 - e^2x^2} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((d^2 - e^2\*x^2)^(1/2)\*(d + e\*x)),x)

[Out] int((A + B\*x + C\*x^2)/((d^2 - e^2\*x^2)^(1/2)\*(d + e\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{-(-d + ex)(d + ex)} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] Integral((A + B\*x + C\*x\*\*2)/(sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)), x)

$$3.15 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^2 \sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=163

$$\frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{3d^2e^3(d + ex)} - \frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{3de^3(d + ex)^2} + \frac{\sqrt{d^2 - e^2x^2} (2Cd - Be)}{de^3(d + ex)} + \frac{C \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3}$$

**Rubi [A]** time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$ , Rules used = {1637, 217, 203, 659, 651}

$$-\frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{3d^2e^3(d + ex)} - \frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{3de^3(d + ex)^2} + \frac{\sqrt{d^2 - e^2x^2} (2Cd - Be)}{de^3(d + ex)} + \frac{C \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^2\*sqrt[d^2 - e^2\*x^2]),x]

[Out] -(((C\*d^2 - B\*d\*e + A\*e^2)\*sqrt[d^2 - e^2\*x^2])/(3\*d\*e^3\*(d + e\*x)^2) + ((2\*C\*d - B\*e)\*sqrt[d^2 - e^2\*x^2])/(d\*e^3\*(d + e\*x)) - ((C\*d^2 - B\*d\*e + A\*e^2)\*sqrt[d^2 - e^2\*x^2])/(3\*d^2\*e^3\*(d + e\*x)) + (C\*ArcTan[(e\*x)/sqrt[d^2 - e^2\*x^2]])/e^3

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 651

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

### Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(d + ex)^2 \sqrt{d^2 - e^2x^2}} dx &= \int \left( \frac{C}{e^2 \sqrt{d^2 - e^2x^2}} + \frac{Cd^2 - Bde + Ae^2}{e^2(d + ex)^2 \sqrt{d^2 - e^2x^2}} + \frac{-2Cd + Be}{e^2(d + ex) \sqrt{d^2 - e^2x^2}} \right) dx \\ &= \frac{C \int \frac{1}{\sqrt{d^2 - e^2x^2}} dx}{e^2} - \frac{(2Cd - Be) \int \frac{1}{(d+ex) \sqrt{d^2 - e^2x^2}} dx}{e^2} + \frac{(Cd^2 - Bde + Ae^2) \int \frac{1}{(d+ex)^2 \sqrt{d^2 - e^2x^2}} dx}{e^2} \\ &= -\frac{(Cd^2 - Bde + Ae^2) \sqrt{d^2 - e^2x^2}}{3de^3(d + ex)^2} + \frac{(2Cd - Be) \sqrt{d^2 - e^2x^2}}{de^3(d + ex)} + \frac{C \operatorname{Subst} \left( \int \frac{1}{1+e^2x^2} dx, \frac{d+ex}{e} \right)}{e^2} \\ &= -\frac{(Cd^2 - Bde + Ae^2) \sqrt{d^2 - e^2x^2}}{3de^3(d + ex)^2} + \frac{(2Cd - Be) \sqrt{d^2 - e^2x^2}}{de^3(d + ex)} - \frac{(Cd^2 - Bde + Ae^2) \sqrt{d^2 - e^2x^2}}{3d^2e^3(d + ex)} \end{aligned}$$

**Mathematica** [A] time = 0.22, size = 95, normalized size = 0.58

$$\frac{\frac{\sqrt{d^2 - e^2x^2} (Cd^2(4d + 5ex) - e(Ae(2d + ex) + Bd(d + 2ex)))}{d^2(d + ex)^2} + 3C \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d + 5*e*x) - e*(A*e*(2*d + e*x) + B*d*(d +
2*e*x))))/(d^2*(d + e*x)^2) + 3*C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^3
)
```



**IntegrateAlgebraic [A]** time = 0.75, size = 122, normalized size = 0.75

$$\frac{\sqrt{d^2 - e^2 x^2} (-2Ade^2 - Ae^3 x - Bd^2 e - 2Bde^2 x + 4Cd^3 + 5Cd^2 ex)}{3d^2 e^3 (d + ex)^2} + \frac{C\sqrt{-e^2} \log\left(\sqrt{d^2 - e^2 x^2} - \sqrt{-e^2} x\right)}{e^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*sqrt[d^2 - e^2\*x^2]),x]

[Out] ((4\*C\*d^3 - B\*d^2\*e - 2\*A\*d\*e^2 + 5\*C\*d^2\*e\*x - 2\*B\*d\*e^2\*x - A\*e^3\*x)\*sqrt[d^2 - e^2\*x^2])/(3\*d^2\*e^3\*(d + e\*x)^2) + (C\*sqrt[-e^2]\*Log[-(sqrt[-e^2]\*x + sqrt[d^2 - e^2\*x^2])])/e^4

**fricas [A]** time = 0.98, size = 221, normalized size = 1.36

$$\frac{4Cd^4 - Bd^3e - 2Ad^2e^2 + (4Cd^2e^2 - Bde^3 - 2Ae^4)x^2 + 2(4Cd^3e - Bd^2e^2 - 2Ade^3)x - 6(Cd^2e^2x^2 + 2Cd^3ex + Cd^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (4Cd^3 - Bd^2e - 2Ade^2 + (5Cd^2e - 2Bde^2 - Ae^3)x)\sqrt{-e^2x^2+d^2}}{3(d^2e^2x^2 + 2d^3e^4x + d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] 1/3\*(4\*C\*d^4 - B\*d^3\*e - 2\*A\*d^2\*e^2 + (4\*C\*d^2\*e^2 - B\*d\*e^3 - 2\*A\*e^4)\*x^2 + 2\*(4\*C\*d^3\*e - B\*d^2\*e^2 - 2\*A\*d\*e^3)\*x - 6\*(C\*d^2\*e^2\*x^2 + 2\*C\*d^3\*e\*x + C\*d^4)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + (4\*C\*d^3 - B\*d^2\*e - 2\*A\*d\*e^2 + (5\*C\*d^2\*e - 2\*B\*d\*e^2 - A\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/(d^2\*e^5\*x^2 + 2\*d^3\*e^4\*x + d^4\*e^3)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t\_n ostep)]index.cc index\_m i\_lex\_is\_greater Error: Bad Argument Value

**maple [B]** time = 0.02, size = 355, normalized size = 2.18

$$\frac{\text{Arctan}\left(\frac{\sqrt{e^2}}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2A}{3\left(x+\frac{d}{e}\right)^2d^2} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2A}{3\left(x+\frac{d}{e}\right)^2d^2} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2B}{3\left(x+\frac{d}{e}\right)^2d^3} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2B}{3\left(x+\frac{d}{e}\right)^2e^4} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2C}{3\left(x+\frac{d}{e}\right)^2e^5} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2C}{3\left(x+\frac{d}{e}\right)^2e^4} - \frac{(Be-2Cd)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2}e^2}{\left(x+\frac{d}{e}\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2), x)`

[Out]  $C/e^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}/(-e^2*x^2+d^2)^{(1/2)}*x)-1/3/e^3/d/(x+d/e)^2*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}*A+1/3/e^4/(x+d/e)^2*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}*B-1/3/e^5*d/(x+d/e)^2*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}*C-1/3/e^2/d^2/(x+d/e)*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}*A+1/3/e^3/d/(x+d/e)*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}*B-1/3/e^4/(x+d/e)*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}*C-1/e^4*(B*e-2*C*d)/d/(x+d/e)*(2*(x+d/e)*d*e^{-(x+d/e)^2*e^2})^{(1/2)}$

**maxima** [B] time = 1.00, size = 317, normalized size = 1.94

$$-\frac{\sqrt{-e^2x^2+d^2}Cd^2}{3(d^5x^2+2d^2e^4x+d^3e^3)}-\frac{\sqrt{-e^2x^2+d^2}Cd^2}{3(d^2e^4x+d^3e^3)}+\frac{\sqrt{-e^2x^2+d^2}Bd}{3(d^4x^2+2d^2e^3x+d^3e^2)}+\frac{\sqrt{-e^2x^2+d^2}Bd}{3(d^2e^3x+d^3e^2)}-\frac{\sqrt{-e^2x^2+d^2}A}{3(d^3x^2+2d^2e^2x+d^3e)}-\frac{\sqrt{-e^2x^2+d^2}A}{3(d^2e^2x+d^3e)}-\frac{\sqrt{-e^2x^2+d^2}B}{de^3x+d^2e^2}+\frac{2\sqrt{-e^2x^2+d^2}C}{e^4x+d^3e}+\frac{\operatorname{Arcsin}\left(\frac{ex}{d}\right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2), x, algorithm="maxima")`

[Out]  $-1/3*\sqrt{-e^2*x^2+d^2}*C*d^2/(d*e^5*x^2+2*d^2*e^4*x+d^3*e^3)-1/3*\sqrt{-e^2*x^2+d^2}*C*d^2/(d^2*e^4*x+d^3*e^3)+1/3*\sqrt{-e^2*x^2+d^2}*B*d/(d*e^4*x^2+2*d^2*e^3*x+d^3*e^2)+1/3*\sqrt{-e^2*x^2+d^2}*B*d/(d^2*e^3*x+d^3*e^2)-1/3*\sqrt{-e^2*x^2+d^2}*A/(d*e^3*x^2+2*d^2*e^2*x+d^3*e)-1/3*\sqrt{-e^2*x^2+d^2}*A/(d^2*e^2*x+d^3*e)-\sqrt{-e^2*x^2+d^2}*B/(d*e^3*x+d^2*e^2)+2*\sqrt{-e^2*x^2+d^2}*C/(e^4*x+d^3e)+C*\arcsin(ex/d)/e^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{Cx^2+Bx+A}{\sqrt{d^2-e^2x^2}(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*x+C*x^2)/((d^2-e^2*x^2)^(1/2)*(d+e*x)^2), x)`

[Out] `int((A+B*x+C*x^2)/((d^2-e^2*x^2)^(1/2)*(d+e*x)^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2), x)`

[Out] `Integral((A+B*x+C*x**2)/(sqrt(-(-d+e*x)*(d+e*x))*(d+e*x)**2), x)`

$$3.16 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^3 \sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=180

$$\frac{\sqrt{d^2 - e^2x^2} (e(2Ae + 3Bd) + 7Cd^2)}{15d^2e^3(d + ex)^2} - \frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{5de^3(d + ex)^3} - \frac{\sqrt{d^2 - e^2x^2} (e(2Ae + 3Bd) + 7Cd^2)}{15d^3e^3(d + ex)} + \frac{C}{e}$$

**Rubi [A]** time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1639, 793, 659, 651}

$$\frac{\sqrt{d^2 - e^2x^2} (e(2Ae + 3Bd) + 7Cd^2)}{15d^3e^3(d + ex)} - \frac{\sqrt{d^2 - e^2x^2} (e(2Ae + 3Bd) + 7Cd^2)}{15d^2e^3(d + ex)^2} - \frac{\sqrt{d^2 - e^2x^2} (Ae^2 - Bde + Cd^2)}{5de^3(d + ex)^3} + \frac{C\sqrt{d^2 - e^2x^2}}{e^3(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out] -((C\*d^2 - B\*d\*e + A\*e^2)\*Sqrt[d^2 - e^2\*x^2])/(5\*d\*e^3\*(d + e\*x)^3) + (C\*Sqrt[d^2 - e^2\*x^2])/(e^3\*(d + e\*x)^2) - ((7\*C\*d^2 + e\*(3\*B\*d + 2\*A\*e))\*Sqrt[d^2 - e^2\*x^2])/(15\*d^2\*e^3\*(d + e\*x)^2) - ((7\*C\*d^2 + e\*(3\*B\*d + 2\*A\*e))\*Sqrt[d^2 - e^2\*x^2])/(15\*d^3\*e^3\*(d + e\*x))

Rule 651

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

Rule 659

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[Simplify[y[m + 2\*p + 2]/(2\*d\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

Rule 793

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[(m\*(g\*c\*d + c\*e\*f) + 2\*e\*c\*f\*(p + 1))/(e\*(2\*c\*d)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p

+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

### Rule 1639

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] && !IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(d + ex)^3 \sqrt{d^2 - e^2x^2}} dx &= \frac{C\sqrt{d^2 - e^2x^2}}{e^3(d + ex)^2} + \frac{\int \frac{e^2(2Cd^2 + Ae^2) + e^3(Cd + Be)x}{(d + ex)^3 \sqrt{d^2 - e^2x^2}} dx}{e^4} \\ &= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{5de^3(d + ex)^3} + \frac{C\sqrt{d^2 - e^2x^2}}{e^3(d + ex)^2} + \frac{(7Cd^2 + e(3Bd + 2Ae)) \int \frac{dx}{(d + ex)^3}}{5de^2} \\ &= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{5de^3(d + ex)^3} + \frac{C\sqrt{d^2 - e^2x^2}}{e^3(d + ex)^2} - \frac{(7Cd^2 + e(3Bd + 2Ae))\sqrt{d^2 - e^2x^2}}{15d^2e^3(d + ex)^2} \\ &= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{5de^3(d + ex)^3} + \frac{C\sqrt{d^2 - e^2x^2}}{e^3(d + ex)^2} - \frac{(7Cd^2 + e(3Bd + 2Ae))\sqrt{d^2 - e^2x^2}}{15d^2e^3(d + ex)^2} \end{aligned}$$

**Mathematica** [A] time = 0.20, size = 103, normalized size = 0.57

$$\frac{\sqrt{d^2 - e^2x^2} \left( e \left( Ae(7d^2 + 6dex + 2e^2x^2) + 3Bd(d^2 + 3dex + e^2x^2) \right) + Cd^2(2d^2 + 6dex + 7e^2x^2) \right)}{15d^3e^3(d + ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]), x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d*
(d^2 + 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(d^3*e^3*(
d + e*x)^3)
```

**IntegrateAlgebraic [A]** time = 0.73, size = 113, normalized size = 0.63

$$\frac{\sqrt{d^2 - e^2 x^2} (-7Ad^2e^2 - 6Ade^3x - 2Ae^4x^2 - 3Bd^3e - 9Bd^2e^2x - 3Bde^3x^2 - 2Cd^4 - 6Cd^3ex - 7Cd^2e^2x^2)}{15d^3e^3(d + ex)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^3\*sqrt[d^2 - e^2\*x^2]),x]

[Out] (sqrt[d^2 - e^2\*x^2]\*(-2\*C\*d^4 - 3\*B\*d^3\*e - 7\*A\*d^2\*e^2 - 6\*C\*d^3\*e\*x - 9\*B\*d^2\*e^2\*x - 6\*A\*d\*e^3\*x - 7\*C\*d^2\*e^2\*x^2 - 3\*B\*d\*e^3\*x^2 - 2\*A\*e^4\*x^2)) / (15\*d^3\*e^3\*(d + e\*x)^3)

**fricas [A]** time = 1.74, size = 244, normalized size = 1.36

$$\frac{2Cd^5 + 3Bd^4e + 7Ad^3e^2 + (2Cd^2e^3 + 3Bde^4 + 7Ae^5)x^3 + 3(2Cd^3e^2 + 3Bd^2e^3 + 7Ade^4)x^2 + 3(2Cd^4e + 3Bd^3e^2 + 7Ad^2e^3)x + (2Cd^4 + 3Bd^3e + 7Ad^2e^2 + (7Cd^2e^2 + 3Bde^3 + 2Ae^4)x^2 + 3(2Cd^3e + 3Bd^2e^2 + 2Ade^3)x)\sqrt{-e^2x^2 + d^2}}{15(d^3e^3x^3 + 3d^4e^3x^2 + 3d^5e^3x + d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/15\*(2\*C\*d^5 + 3\*B\*d^4\*e + 7\*A\*d^3\*e^2 + (2\*C\*d^2\*e^3 + 3\*B\*d\*e^4 + 7\*A\*e^5)\*x^3 + 3\*(2\*C\*d^3\*e^2 + 3\*B\*d^2\*e^3 + 7\*A\*d\*e^4)\*x^2 + 3\*(2\*C\*d^4\*e + 3\*B\*d^3\*e^2 + 7\*A\*d^2\*e^3)\*x + (2\*C\*d^4 + 3\*B\*d^3\*e + 7\*A\*d^2\*e^2 + (7\*C\*d^2\*e^2 + 3\*B\*d\*e^3 + 2\*A\*e^4)\*x^2 + 3\*(2\*C\*d^3\*e + 3\*B\*d^2\*e^2 + 2\*A\*d\*e^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/(d^3\*e^6\*x^3 + 3\*d^4\*e^5\*x^2 + 3\*d^5\*e^4\*x + d^6\*e^3)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-2\*A\*(-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x/exp(2))^3\*exp(1)^8\*exp(2)^2+7\*A\*(-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x/exp(2))^2\*exp(1)^6\*exp(2)^3-2\*A\*(-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x/exp(2))^2\*exp(1)^10\*exp(2)-2\*B\*d\*exp(1)\*exp(2)^5+2\*C\*d^2\*(-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x/exp(2))^3\*exp(1)^6\*exp(2)^2+5\*A\*(-1/2\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))/x/exp(2))^3\*exp(1)^4\*exp(2)^4-11/2\*A\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))\*exp(1)^4\*exp(2)^4/x/exp(2)+A\*(-2\*d\*exp(1)-2\*sqrt(d^2-x^2\*exp(2))\*exp(1))\*exp(1)^8\*exp(2)^2/x/exp(2)-5\*B\*d\*

$$\begin{aligned}
& -1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x/\exp(2))^2*\exp(1)^5*\exp(2) \\
& )^3-2*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x/\exp(2))^2*\exp \\
& (1)^9*\exp(2)-2*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x/\exp( \\
& 2))^2*\exp(1)*\exp(2)^5-3*B*d*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1) \\
& ))/x/\exp(2))^3*\exp(1)^3*\exp(2)^4+3*C*d^2*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2* \\
& \exp(2)}*\exp(1))/x/\exp(2))^2*\exp(1)^4*\exp(2)^3+C*d^2*(-1/2*(-2*d*\exp(1)-2*s \\
& \sqrt{d^2-x^2*\exp(2)}*\exp(1))/x/\exp(2))^3*\exp(2)^5-A*\exp(1)^6*\exp(2)^3-B*d*\exp \\
& (1)^5*\exp(2)^3+3*C*d^2*\exp(1)^4*\exp(2)^3+4*A*(-1/2*(-2*d*\exp(1)-2*\sqrt{d^2- \\
& x^2*\exp(2)}*\exp(1))/x/\exp(2))^2*\exp(2)^6+4*A*\exp(2)^6+6*C*d^2*(-1/2*(-2*d*e \\
& xp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x/\exp(2))^2*\exp(1)^8*\exp(2)+1/2*C*d^2* \\
& (-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))*\exp(2)^5/x/\exp(2)+5/2*B*d*(-2*d \\
& *\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))*\exp(1)^3*\exp(2)^4/x/\exp(2)+2*B*d*(-2 \\
& *d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))*\exp(1)^7*\exp(2)^2/x/\exp(2)-5*C*d^2 \\
& *(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))*\exp(1)^6*\exp(2)^2/x/\exp(2))/(( \\
& -1/2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x/\exp(2))^2*\exp(2)-(-2*d*e \\
& xp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x+\exp(2))^2/(-d^3*\exp(1)^9+2*d^3*\exp(1) \\
& )^5*\exp(2)^2-d^3*\exp(1)*\exp(2)^4)+1/2*(-2*A*\exp(1)^4*\exp(2)^3+6*B*d*\exp(1)^ \\
& 3*\exp(2)^3-2*C*d^2*\exp(2)^4-4*A*\exp(2)^5-4*C*d^2*\exp(1)^6*\exp(2))*\operatorname{atan}((-1/ \\
& 2*(-2*d*\exp(1)-2*\sqrt{d^2-x^2*\exp(2)}*\exp(1))/x+\exp(2))/\sqrt{-\exp(1)^4+\exp( \\
& 2)^2})/\sqrt{-\exp(1)^4+\exp(2)^2}/(d^3*\exp(1)^9-2*d^3*\exp(1)^5*\exp(2)^2+d^3*e \\
& xp(1)*\exp(2)^4)
\end{aligned}$$

**maple [A]** time = 0.01, size = 116, normalized size = 0.64

$$\frac{(-ex+d)(2Ae^4x^2+3Bde^3x^2+7Cd^2e^2x^2+6Ade^3x+9Bd^2e^2x+6Cd^3ex+7Ad^2e^2+3Bd^3e+2Cd^4)}{15(ex+d)^2\sqrt{-e^2x^2+d^2}d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2),x)

[Out]  $-1/15*(-e*x+d)*(2*A*e^4*x^2+3*B*d*e^3*x^2+7*C*d^2*e^2*x^2+6*A*d*e^3*x+9*B*d^2*e^2*x+6*C*d^3*e*x+7*A*d^2*e^2+3*B*d^3*e+2*C*d^4)/(e*x+d)^2/d^3/e^3/(-e^2*x^2+d^2)^(1/2)$

**maxima [B]** time = 1.02, size = 608, normalized size = 3.38

$$\frac{\sqrt{d^2+e^2x^2}}{5(d^2+3e^2x^2+3d^2x+e^2x^2)} - \frac{2\sqrt{d^2+e^2x^2}}{15(d^2+3e^2x^2+e^2x^2)} - \frac{2\sqrt{d^2+e^2x^2}}{15(d^2+e^2x^2)} + \frac{\sqrt{d^2+e^2x^2}}{5(d^2+3e^2x^2+3d^2x+e^2x^2)} + \frac{2\sqrt{d^2+e^2x^2}}{15(d^2+3e^2x^2+e^2x^2)} + \frac{2\sqrt{d^2+e^2x^2}}{15(d^2+e^2x^2)} + \frac{2\sqrt{d^2+e^2x^2}}{3(d^2+e^2x^2)} + \frac{2\sqrt{d^2+e^2x^2}}{3(d^2+e^2x^2)} + \frac{\sqrt{d^2+e^2x^2}}{5(d^2+3e^2x^2+3d^2x+e^2x^2)} - \frac{2\sqrt{d^2+e^2x^2}}{15(d^2+3e^2x^2+e^2x^2)} - \frac{2\sqrt{d^2+e^2x^2}}{15(d^2+e^2x^2)} + \frac{\sqrt{d^2+e^2x^2}}{3(d^2+3e^2x^2+e^2x^2)} + \frac{\sqrt{d^2+e^2x^2}}{3(d^2+e^2x^2)} + \frac{\sqrt{d^2+e^2x^2}}{3d^2+e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out]  $-1/5*\sqrt{-e^2*x^2+d^2}*C*d^2/(d*e^6*x^3+3*d^2*e^5*x^2+3*d^3*e^4*x+d^4*e^3)-2/15*\sqrt{-e^2*x^2+d^2}*C*d^2/(d^2*e^5*x^2+2*d^3*e^4*x+d^4$

$e^3) - 2/15\sqrt{-e^2x^2 + d^2}Cd^2/(d^3e^4x + d^4e^3) + 1/5\sqrt{-e^2x^2 + d^2}Bd/(d^5e^3x^3 + 3d^2e^4x^2 + 3d^3e^3x + d^4e^2) + 2/15\sqrt{-e^2x^2 + d^2}Bd/(d^2e^4x^2 + 2d^3e^3x + d^4e^2) + 2/15\sqrt{-e^2x^2 + d^2}Bd/(d^3e^3x + d^4e^2) + 2/3\sqrt{-e^2x^2 + d^2}Cd/(d^5e^3x^2 + 2d^2e^4x + d^3e^3) + 2/3\sqrt{-e^2x^2 + d^2}Cd/(d^2e^4x + d^3e^3) - 1/5\sqrt{-e^2x^2 + d^2}A/(d^4e^3x^3 + 3d^2e^3x^2 + 3d^3e^2x + d^4e) - 2/15\sqrt{-e^2x^2 + d^2}A/(d^2e^3x^2 + 2d^3e^2x + d^4e) - 2/15\sqrt{-e^2x^2 + d^2}A/(d^3e^2x + d^4e) - 1/3\sqrt{-e^2x^2 + d^2}B/(d^4e^3x^2 + 2d^2e^3x + d^3e^2) - 1/3\sqrt{-e^2x^2 + d^2}B/(d^2e^3x + d^3e^2) - \sqrt{-e^2x^2 + d^2}C/(d^4e^3x + d^2e^3)$

**mupad [B]** time = 3.80, size = 109, normalized size = 0.61

$$\frac{\sqrt{d^2 - e^2 x^2} (2 C d^4 + 6 C d^3 e x + 3 B d^3 e + 7 C d^2 e^2 x^2 + 9 B d^2 e^2 x + 7 A d^2 e^2 + 3 B d e^3 x^2 + 6 A d e^3 x + 2 A e^4 x^2)}{15 d^3 e^3 (d + e x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((d^2 - e^2\*x^2)^(1/2)\*(d + e\*x)^3),x)

[Out] -((d^2 - e^2\*x^2)^(1/2)\*(2\*C\*d^4 + 7\*A\*d^2\*e^2 + 2\*A\*e^4\*x^2 + 3\*B\*d^3\*e + 7\*C\*d^2\*e^2\*x^2 + 6\*A\*d\*e^3\*x + 6\*C\*d^3\*e\*x + 9\*B\*d^2\*e^2\*x + 3\*B\*d\*e^3\*x^2))/(15\*d^3\*e^3\*(d + e\*x)^3)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{-(-d + ex)(d + ex)}(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] Integral((A + B\*x + C\*x\*\*2)/(sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)\*\*3), x)

$$3.17 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^4 \sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=234

$$\frac{\sqrt{d^2-e^2x^2} (6Ae^2 + 8Bde + 13Cd^2)}{70d^2e^3(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2} (Ae^2 - Bde + Cd^2)}{7de^3(d+ex)^4} - \frac{\sqrt{d^2-e^2x^2} (6Ae^2 + 8Bde + 13Cd^2)}{105d^4e^3(d+ex)} - \frac{\sqrt{d^2-e^2x^2} (Ae^2 - Bde + Cd^2)}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

**Rubi [A]** time = 0.25, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {1639, 793, 659, 651}

$$\frac{\sqrt{d^2-e^2x^2} (6Ae^2 + 8Bde + 13Cd^2)}{105d^4e^3(d+ex)} - \frac{\sqrt{d^2-e^2x^2} (6Ae^2 + 8Bde + 13Cd^2)}{105d^3e^3(d+ex)^2} - \frac{\sqrt{d^2-e^2x^2} (6Ae^2 + 8Bde + 13Cd^2)}{70d^2e^3(d+ex)^3} - \frac{\sqrt{d^2-e^2x^2} (Ae^2 - Bde + Cd^2)}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^4\*sqrt[d^2 - e^2\*x^2]),x]

[Out] -((C\*d^2 - B\*d\*e + A\*e^2)\*sqrt[d^2 - e^2\*x^2])/(7\*d\*e^3\*(d + e\*x)^4) + (C\*sqrt[d^2 - e^2\*x^2])/(2\*e^3\*(d + e\*x)^3) - ((13\*C\*d^2 + 8\*B\*d\*e + 6\*A\*e^2)\*sqrt[d^2 - e^2\*x^2])/(70\*d^2\*e^3\*(d + e\*x)^3) - ((13\*C\*d^2 + 8\*B\*d\*e + 6\*A\*e^2)\*sqrt[d^2 - e^2\*x^2])/(105\*d^3\*e^3\*(d + e\*x)^2) - ((13\*C\*d^2 + 8\*B\*d\*e + 6\*A\*e^2)\*sqrt[d^2 - e^2\*x^2])/(105\*d^4\*e^3\*(d + e\*x))

**Rule 651**

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && EqQ[m + 2\*p + 2, 0]

**Rule 659**

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[(e\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[Simplify[m + 2\*p + 2]/(2\*d\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[p] && ILtQ[Simplify[m + 2\*p + 2], 0]

**Rule 793**

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(2\*c\*d\*(m + p + 1)), x] + Dist[(m\*(g\*c\*d + c\*e\*f) + 2\*e\*c\*f\*(p + 1))/(e\*(2\*c\*d)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e



, f, g, m, p}, x] && EqQ[c\*d^2 + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1])) || EqQ[m + 2\*p + 2, 0] && NeQ[m + p + 1, 0]

### Rule 1639

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m + p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rubi steps

$$\int \frac{A + Bx + Cx^2}{(d + ex)^4 \sqrt{d^2 - e^2x^2}} dx = \frac{C\sqrt{d^2 - e^2x^2}}{2e^3(d + ex)^3} + \frac{\int \frac{e^2(3Cd^2 + 2Ae^2) + e^3(Cd + 2Be)x}{(d + ex)^4 \sqrt{d^2 - e^2x^2}} dx}{2e^4}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{7de^3(d + ex)^4} + \frac{C\sqrt{d^2 - e^2x^2}}{2e^3(d + ex)^3} + \frac{(13Cd^2 + 8Bde + 6Ae^2) \int \frac{e^2(3Cd^2 + 2Ae^2) + e^3(Cd + 2Be)x}{(d + ex)^4 \sqrt{d^2 - e^2x^2}} dx}{14de^2}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{7de^3(d + ex)^4} + \frac{C\sqrt{d^2 - e^2x^2}}{2e^3(d + ex)^3} - \frac{(13Cd^2 + 8Bde + 6Ae^2)\sqrt{d^2 - e^2x^2}}{70d^2e^3(d + ex)^3}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{7de^3(d + ex)^4} + \frac{C\sqrt{d^2 - e^2x^2}}{2e^3(d + ex)^3} - \frac{(13Cd^2 + 8Bde + 6Ae^2)\sqrt{d^2 - e^2x^2}}{70d^2e^3(d + ex)^3}$$

$$= -\frac{(Cd^2 - Bde + Ae^2)\sqrt{d^2 - e^2x^2}}{7de^3(d + ex)^4} + \frac{C\sqrt{d^2 - e^2x^2}}{2e^3(d + ex)^3} - \frac{(13Cd^2 + 8Bde + 6Ae^2)\sqrt{d^2 - e^2x^2}}{70d^2e^3(d + ex)^3}$$

**Mathematica [A]** time = 0.22, size = 139, normalized size = 0.59

$$\frac{\sqrt{d^2 - e^2x^2} (e(3Ae(12d^3 + 13d^2ex + 8de^2x^2 + 2e^3x^3) + Bd(13d^3 + 52d^2ex + 32de^2x^2 + 8e^3x^3)) + Cd^2(8d^3 + 32d^2ex + 52de^2x^2 + 13e^3x^3))}{105d^4e^3(d + ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)^4\*Sqrt[d^2 - e^2\*x^2]),x]

[Out]  $-1/105*(\text{Sqrt}[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13*e^3*x^3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d*(13*d^3 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(d^4*e^3*(d + e*x)^4)$

**IntegrateAlgebraic [A]** time = 0.85, size = 149, normalized size = 0.64

$$\frac{\sqrt{d^2 - e^2x^2} (-36Ad^3e^2 - 39Ad^2e^3x - 24Ade^4x^2 - 6Ae^5x^3 - 13Bd^4e - 52Bd^3e^2x - 32Bd^2e^3x^2 - 8Bde^4x^3 - 8Cd^5 - 32Cd^4ex - 52Cd^3e^2x^2 - 13Cd^2e^3x^3)}{105d^4e^3(d + ex)^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^4\*Sqrt[d^2 - e^2\*x^2]),x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(-8*C*d^5 - 13*B*d^4*e - 36*A*d^3*e^2 - 32*C*d^4*e*x - 52*B*d^3*e^2*x - 39*A*d^2*e^3*x - 52*C*d^3*e^2*x^2 - 32*B*d^2*e^3*x^2 - 24*A*d*e^4*x^2 - 13*C*d^2*e^3*x^3 - 8*B*d*e^4*x^3 - 6*A*e^5*x^3))/(105*d^4*e^3*(d + e*x)^4)$

**fricas [A]** time = 1.18, size = 320, normalized size = 1.37

$$\frac{8C^2e^6 + 13B^2e^5 + 36A^2e^4 + (8C^2e^4 + 13B^2e^3 + 36A^2e^2)^2 + 4(8C^2e^3 + 13B^2e^2 + 36A^2e) + (8C^2e + 13B^2e + 36A^2e^2 + (13C^2e^2 + 8B^2e + 6A^2e^3)^2 + 4(13C^2e + 8B^2e + 6A^2e^2) + (32C^2e + 52B^2e + 39A^2e^2))\sqrt{-d^2 + e^2x^2}}{105(d^2x^2 + 4d^2ex^3 + 6d^2e^2x^2 + 4d^2e^3x + d^2e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^4/(-e^2\*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out]  $-1/105*(8*C*d^6 + 13*B*d^5*e + 36*A*d^4*e^2 + (8*C*d^2*e^4 + 13*B*d*e^5 + 36*A*e^6)*x^4 + 4*(8*C*d^3*e^3 + 13*B*d^2*e^4 + 36*A*d*e^5)*x^3 + 6*(8*C*d^4*e^2 + 13*B*d^3*e^3 + 36*A*d^2*e^4)*x^2 + 4*(8*C*d^5*e + 13*B*d^4*e^2 + 36*A*d^3*e^3)*x + (8*C*d^5 + 13*B*d^4*e + 36*A*d^3*e^2 + (13*C*d^2*e^3 + 8*B*d*e^4 + 6*A*e^5)*x^3 + 4*(13*C*d^3*e^2 + 8*B*d^2*e^3 + 6*A*d*e^4)*x^2 + (32*C*d^4*e + 52*B*d^3*e^2 + 39*A*d^2*e^3)*x)*\text{sqrt}(-e^2*x^2 + d^2))/(d^4*e^7*x^4 + 4*d^5*e^6*x^3 + 6*d^6*e^5*x^2 + 4*d^7*e^4*x + d^8*e^3)$

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^4/(-e^2\*x^2+d^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output:  $(64*C*d^2*(-1/2*(-2*d*\exp(1)-2*\text{sqrt}(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^3*\exp(1)^10*\exp(2)^3-18*A*(-1/2*(-2*d*\exp(1)-2*\text{sqrt}(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^2*\exp(1)^10*\exp(2)^4+8*A*(-1/2*(-2*d*\exp(1)-2*\text{sqrt}(d^2-x^2*\exp(2))*\exp(1))/x/\exp(2))^3*\exp(1)^16*\exp(2)+12*A*(-1/2*(-2*d*\exp(1)-2*\text{sqrt}(d^2-x^2*\exp(2))$

$$\begin{aligned}
& ) * \exp(1) / x / \exp(2) ^4 \exp(1) ^{14} \exp(2) ^2 + 6 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(1) ^{12} \exp(2) ^3 + 6 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^{13} \exp(2) ^2 + 12 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^{14} \exp(2) ^2 - 8 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^{12} \exp(2) ^3 - 36 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^{10} \exp(2) ^4 - 18 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(1) ^8 \exp(2) ^5 + 6 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^{13} \exp(2) ^2 - 34 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^{11} \exp(2) ^3 - 33 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^9 \exp(2) ^4 - 3 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(1) ^7 \exp(2) ^5 + 24 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^{12} \exp(2) ^2 + 60 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^8 \exp(2) ^4 + 12 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(1) ^6 \exp(2) ^5 + 42 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^8 \exp(2) ^5 + 81 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^6 \exp(2) ^6 + 27 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(1) ^4 \exp(2) ^7 - 84 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^9 \exp(2) ^4 - 84 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^7 \exp(2) ^5 - 42 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^5 \exp(2) ^6 - 12 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(1) ^3 \exp(2) ^7 + 102 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^8 \exp(2) ^4 + 78 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^6 \exp(2) ^5 + 15 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) ^4 \exp(2) ^6 + 3 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^5 \exp(2) ^8 + 2 * A * \exp(1) ^{10} \exp(2) ^4 + 120 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^6 \exp(2) ^6 + 108 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^4 \exp(2) ^7 + B * d * \exp(1) ^9 \exp(2) ^4 + 2 * C * d^2 * \exp(1) ^8 \exp(2) ^4 - 60 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^5 \exp(2) ^6 + 18 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(2) ^9 - 10 * B * d * \exp(1) ^5 \exp(2) ^6 - 6 * B * d * \exp(1) * \exp(2) ^8 - 36 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^3 \exp(2) ^7 + 24 * C * d^2 * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) ^4 \exp(2) ^6 - 5 * A * \exp(1) ^6 \exp(2) ^6 + 13 * C * d^2 * \exp(1) ^4 \exp(2) ^6 + 36 * A * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(2) ^9 + 18 * A * \exp(2) ^9 - 81/2 * A * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) * \exp(1) ^4 \exp(2) ^7 / x / \exp(2) + 6 * A * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) * \exp(1) ^8 \exp(2) ^5 / x / \exp(2) - 3 * A * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) * \exp(1) ^{12} \exp(2) ^3 / x / \exp(2) - 12 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^2 \exp(1) * \exp(2) ^8 + 4 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^3 \exp(1) ^{15} \exp(2) - 6 * B * d * (-1/2 * (-2 * d * \exp(1) - 2 * \sqrt{d^2 - x^2} * \exp(2)) * \exp(1) / x / \exp(2) ^4 \exp(1) * \exp(2) ^8 + 8 * C
\end{aligned}$$

```

*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
4*exp(2)+3/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/e
xp(2)+12*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^7/
x/exp(2)+57/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(
2)^5/x/exp(2)-3*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^11*e
xp(2)^3/x/exp(2)-33*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1
)^6*exp(2)^5/x/exp(2)-6*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
xp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)^3/(3*d^4*exp(1)^13-9*d^4*exp(1)^9*exp(2)^2+9*d^4*exp(1)^5*exp(2)^4-3*d^4*
exp(1)*exp(2)^6)+1/2*(2*B*d*exp(1)^7*exp(2)^3-8*C*d^2*exp(1)^6*exp(2)^3-6*A
*exp(1)^4*exp(2)^5+8*B*d*exp(1)^3*exp(2)^5-2*C*d^2*exp(2)^6-4*A*exp(2)^7)*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^4*exp(1)^13+3*d^4*exp(1)^9*exp(
2)^2-3*d^4*exp(1)^5*exp(2)^4+d^4*exp(1)*exp(2)^6)

```

**maple [A]** time = 0.01, size = 152, normalized size = 0.65

$$\frac{(-e x + d) (6 A e^5 x^3 + 8 B d e^4 x^3 + 13 C d^2 e^3 x^3 + 24 A d e^4 x^2 + 32 B d^2 e^3 x^2 + 52 C d^3 e^2 x^2 + 39 A d^2 e^3 x + 52 B d^3 e^2 x + 32 C d^4 e x + 36 A d^3 e^2 + 13 B d^4 e + 8 C d^5)}{105 (e x + d)^3 \sqrt{-e^2 x^2 + d^2} d^4 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)^4/(-e^2\*x^2+d^2)^(1/2), x)

[Out] -1/105\*(-e\*x+d)\*(6\*A\*e^5\*x^3+8\*B\*d\*e^4\*x^3+13\*C\*d^2\*e^3\*x^3+24\*A\*d\*e^4\*x^2+32\*B\*d^2\*e^3\*x^2+52\*C\*d^3\*e^2\*x^2+39\*A\*d^2\*e^3\*x+52\*B\*d^3\*e^2\*x+32\*C\*d^4\*e\*x+36\*A\*d^3\*e^2+13\*B\*d^4\*e+8\*C\*d^5)/(e\*x+d)^3/d^4/e^3/(-e^2\*x^2+d^2)^(1/2)

**maxima [B]** time = 1.06, size = 975, normalized size = 4.17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^4/(-e^2\*x^2+d^2)^(1/2), x, algorithm="maxima")

[Out] -1/7\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d\*e^7\*x^4 + 4\*d^2\*e^6\*x^3 + 6\*d^3\*e^5\*x^2 + 4\*d^4\*e^4\*x + d^5\*e^3) - 3/35\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d^2\*e^6\*x^3 + 3\*d^3\*e^5\*x^2 + 3\*d^4\*e^4\*x + d^5\*e^3) - 2/35\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d^3\*e^5\*x^2 + 2\*d^4\*e^4\*x + d^5\*e^3) - 2/35\*sqrt(-e^2\*x^2 + d^2)\*C\*d^2/(d^4\*e^4\*x + d^5\*e^3) + 1/7\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d\*e^6\*x^4 + 4\*d^2\*e^5\*x^3 + 6\*d^3\*e^4\*x^2 + 4\*d^4\*e^3\*x + d^5\*e^2) + 3/35\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d^2\*e^5\*x^3 + 3\*d^3\*e^4\*x^2 + 3\*d^4\*e^3\*x + d^5\*e^2) + 2/35\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d^3\*e^4\*x^2 + 2\*d^4\*e^3\*x + d^5\*e^2) + 2/35\*sqrt(-e^2\*x^2 + d^2)\*B\*d/(d^4\*e^3\*x + d^5\*e^2) + 2/5\*sqrt(-e^2\*x^2 + d^2)\*C\*d/(d\*e^6\*x^3 + 3\*d^2\*

$$e^5x^2 + 3d^3e^4x + d^4e^3) + 4/15\sqrt{-e^2x^2 + d^2}Cd/(d^2e^5x^2 + 2d^3e^4x + d^4e^3) + 4/15\sqrt{-e^2x^2 + d^2}Cd/(d^3e^4x + d^4e^3) - 1/7\sqrt{-e^2x^2 + d^2}A/(de^5x^4 + 4d^2e^4x^3 + 6d^3e^3x^2 + 4d^4e^2x + d^5e) - 3/35\sqrt{-e^2x^2 + d^2}A/(d^2e^4x^3 + 3d^3e^3x^2 + 3d^4e^2x + d^5e) - 2/35\sqrt{-e^2x^2 + d^2}A/(d^3e^3x^2 + 2d^4e^2x + d^5e) - 2/35\sqrt{-e^2x^2 + d^2}A/(d^4e^2x + d^5e) - 1/5\sqrt{-e^2x^2 + d^2}B/(de^5x^3 + 3d^2e^4x^2 + 3d^3e^3x + d^4e^2) - 2/15\sqrt{-e^2x^2 + d^2}B/(d^2e^4x^2 + 2d^3e^3x + d^4e^2) - 2/15\sqrt{-e^2x^2 + d^2}B/(d^3e^3x + d^4e^2) - 1/3\sqrt{-e^2x^2 + d^2}C/(de^5x^2 + 2d^2e^4x + d^3e^3) - 1/3\sqrt{-e^2x^2 + d^2}C/(d^2e^4x + d^3e^3)$$

**mupad [B]** time = 3.78, size = 204, normalized size = 0.87

$$\frac{\sqrt{d^2 - e^2x^2} \left( \frac{C}{5e^3} - \frac{-4Cd^2 + 4Bde + 3Ae^2}{35d^2e^3} \right)}{(d + ex)^3} - \frac{\sqrt{d^2 - e^2x^2} \left( \frac{A}{7de} + \frac{d \left( \frac{C}{7d^2} - \frac{B}{7de} \right)}{e} \right)}{(d + ex)^4} - \frac{\sqrt{d^2 - e^2x^2} (13Cd^2 + 8Bde + 6Ae^2)}{105d^3e^3(d + ex)^2} - \frac{\sqrt{d^2 - e^2x^2} (13Cd^2 + 8Bde + 6Ae^2)}{105d^4e^3(d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((d^2 - e^2\*x^2)^(1/2)\*(d + e\*x)^4), x)

[Out] ((d^2 - e^2\*x^2)^(1/2)\*(C/(5\*e^3) - (3\*A\*e^2 - 4\*C\*d^2 + 4\*B\*d\*e)/(35\*d^2\*e^3)))/(d + e\*x)^3 - ((d^2 - e^2\*x^2)^(1/2)\*(A/(7\*d\*e) + (d\*(C/(7\*e^2) - B/(7\*d\*e)))/e))/(d + e\*x)^4 - ((d^2 - e^2\*x^2)^(1/2)\*(6\*A\*e^2 + 13\*C\*d^2 + 8\*B\*d\*e))/(105\*d^3\*e^3\*(d + e\*x)^2) - ((d^2 - e^2\*x^2)^(1/2)\*(6\*A\*e^2 + 13\*C\*d^2 + 8\*B\*d\*e))/(105\*d^4\*e^3\*(d + e\*x))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{-(-d + ex)(d + ex)}(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out] Integral((A + B\*x + C\*x\*\*2)/(sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)\*\*4), x)

$$3.18 \quad \int (d + ex)^3 (a + cx^2) (A + Bx + Cx^2) dx$$

**Optimal.** Leaf size=175

$$\frac{(d + ex)^6 (aCe^2 + c(6Cd^2 - e(3Bd - Ae)))}{6e^5} - \frac{(d + ex)^5 (ae^2(2Cd - Be) + cd(4Cd^2 - e(3Bd - 2Ae)))}{5e^5} + \frac{(d + ex)^4 (ae^2(2Cd - Be) + cd(4Cd^2 - e(3Bd - 2Ae)))}{5e^5} + \dots$$

**Rubi [A]** time = 0.31, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1628}

$$\frac{(d + ex)^6 (aCe^2 - ce(3Bd - Ae) + 6cCd^2)}{6e^5} - \frac{(d + ex)^5 (ae^2(2Cd - Be) - cde(3Bd - 2Ae) + 4cCd^2)}{5e^5} + \frac{(d + ex)^4 (ae^2 + cd^2)(Ae^2 - Bde + Cd^2)}{4e^5} - \frac{c(d + ex)^7(4Cd - Be)}{7e^5} + \frac{cC(d + ex)^8}{8e^5}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] ((c\*d^2 + a\*e^2)\*(C\*d^2 - B\*d\*e + A\*e^2)\*(d + e\*x)^4)/(4\*e^5) - ((4\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 2\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))\*(d + e\*x)^5)/(5\*e^5) + ((6\*c\*C\*d^2 + a\*C\*e^2 - c\*e\*(3\*B\*d - A\*e))\*(d + e\*x)^6)/(6\*e^5) - (c\*(4\*C\*d - B\*e)\*(d + e\*x)^7)/(7\*e^5) + (c\*C\*(d + e\*x)^8)/(8\*e^5)

Rule 1628

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int (d + ex)^3 (a + cx^2) (A + Bx + Cx^2) dx = \int \left( \frac{(cd^2 + ae^2)(Cd^2 - Bde + Ae^2)(d + ex)^3}{e^4} + \frac{(-4cCd^3 + cde(3Bd - 2Ae))}{e^4} \right) dx$$

$$= \frac{(cd^2 + ae^2)(Cd^2 - Bde + Ae^2)(d + ex)^4}{4e^5} - \frac{(4cCd^3 - cde(3Bd - 2Ae))}{5e^5} + \dots$$

**Mathematica [A]** time = 0.09, size = 208, normalized size = 1.19

$$\frac{1}{5}x^5 (ae^2(Be + 3Cd) + 3cde(Ae + Bd) + cCd^2) + \frac{1}{6}ex^6 (aC^2 + ce(Ae + 3Bd) + 3cCd^2) + \frac{1}{3}dx^3 (A(3ae^2 + cd^2) + ad(3Be + Cd)) + \frac{1}{4}x^4 (aAe^3 + 3aBde^2 + 3aCd^2e + 3AcCd^2 + Bcd^3) + \frac{1}{2}ad^2x^2(3Ae + Bd) + aAd^2x + \frac{1}{7}ce^2x^7(Be + 3Cd) + \frac{1}{8}cC^2x^8$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] a\*A\*d^3\*x + (a\*d^2\*(B\*d + 3\*A\*e)\*x^2)/2 + (d\*(a\*d\*(C\*d + 3\*B\*e) + A\*(c\*d^2 + 3\*a\*e^2))\*x^3)/3 + ((B\*c\*d^3 + 3\*A\*c\*d^2\*e + 3\*a\*C\*d^2\*e + 3\*a\*B\*d\*e^2 + a\*A\*e^3)\*x^4)/4 + ((c\*C\*d^3 + 3\*c\*d\*e\*(B\*d + A\*e) + a\*e^2\*(3\*C\*d + B\*e))\*x^5)/5 + (e\*(3\*c\*C\*d^2 + a\*C\*e^2 + c\*e\*(3\*B\*d + A\*e))\*x^6)/6 + (c\*e^2\*(3\*C\*d + B\*e)\*x^7)/7 + (c\*C\*e^3\*x^8)/8

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3 (a + cx^2) (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^3\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)^3\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

**fricas** [A] time = 0.94, size = 248, normalized size = 1.42

$$\frac{1}{8}e^3c^2c + \frac{3}{7}e^2d^2c + \frac{1}{2}e^2c^2b + \frac{1}{2}e^2a^2c + \frac{1}{6}e^2c^2a + \frac{1}{2}e^2d^2b + \frac{1}{6}e^2c^2a + \frac{1}{3}e^2d^2c + \frac{3}{5}e^2d^2dc + \frac{3}{5}e^2c^2b + \frac{1}{5}e^2c^2ab + \frac{3}{5}e^2d^2ca + \frac{3}{4}e^2c^2a + \frac{1}{4}e^2d^2cb + \frac{3}{4}e^2d^2db + \frac{3}{4}e^2c^2ca + \frac{1}{4}e^2c^2aa + \frac{1}{3}e^2d^2c + e^2a^2ab + \frac{1}{3}e^2d^2da + \frac{1}{2}e^2d^2ab + \frac{3}{2}e^2a^2aa + xd^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/8\*x^8\*e^3\*c\*C + 3/7\*x^7\*e^2\*d\*c\*C + 1/7\*x^7\*e^3\*c\*B + 1/2\*x^6\*e\*d^2\*c\*C + 1/6\*x^6\*e^3\*a\*C + 1/2\*x^6\*e^2\*d\*c\*B + 1/6\*x^6\*e^3\*c\*A + 1/5\*x^5\*d^3\*c\*C + 3/5\*x^5\*e^2\*d\*a\*C + 3/5\*x^5\*e\*d^2\*c\*B + 1/5\*x^5\*e^3\*a\*B + 3/5\*x^5\*e^2\*d\*c\*A + 3/4\*x^4\*e\*d^2\*a\*C + 1/4\*x^4\*d^3\*c\*B + 3/4\*x^4\*e^2\*d\*a\*B + 3/4\*x^4\*e\*d^2\*c\*A + 1/4\*x^4\*e^3\*a\*A + 1/3\*x^3\*d^3\*a\*C + x^3\*e\*d^2\*a\*B + 1/3\*x^3\*d^3\*c\*A + x^3\*e^2\*d\*a\*A + 1/2\*x^2\*d^3\*a\*B + 3/2\*x^2\*e\*d^2\*a\*A + x\*d^3\*a\*A

**giac** [A] time = 0.17, size = 242, normalized size = 1.38

$$\frac{1}{8}Cae^3 + \frac{3}{7}Cads^2e + \frac{1}{2}Caf^2e + \frac{1}{5}Caf^3e + \frac{1}{7}Bce^2 + \frac{1}{2}Bcds^2e + \frac{3}{5}Baf^2s^2e + \frac{1}{4}Baf^3e + \frac{1}{6}Cae^3 + \frac{1}{6}Ace^3 + \frac{3}{5}Cads^2e + \frac{3}{5}Acbs^2e + \frac{3}{4}Caf^2s^2e + \frac{3}{4}Caf^3s^2e + \frac{1}{3}Caf^3e + \frac{1}{5}Bax^2e + \frac{3}{4}Baf^2s^2e + Baf^2s^2e + \frac{1}{2}Baf^2s^2e + \frac{1}{4}Aax^2e + Aad^2s^2e + \frac{3}{2}Aaf^2s^2e + Aaf^2s^2e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="giac")

[Out] 1/8\*C\*c\*x^8\*e^3 + 3/7\*C\*c\*d\*x^7\*e^2 + 1/2\*C\*c\*d^2\*x^6\*e + 1/5\*C\*c\*d^3\*x^5 + 1/7\*B\*c\*x^7\*e^3 + 1/2\*B\*c\*d\*x^6\*e^2 + 3/5\*B\*c\*d^2\*x^5\*e + 1/4\*B\*c\*d^3\*x^4 + 1/6\*C\*a\*x^6\*e^3 + 1/6\*A\*c\*x^6\*e^3 + 3/5\*C\*a\*d\*x^5\*e^2 + 3/5\*A\*c\*d\*x^5\*e^2 + 3/4\*C\*a\*d^2\*x^4\*e + 3/4\*A\*c\*d^2\*x^4\*e + 1/3\*C\*a\*d^3\*x^3 + 1/3\*A\*c\*d^3\*x^3 + 1/5\*B\*a\*x^5\*e^3 + 3/4\*B\*a\*d\*x^4\*e^2 + B\*a\*d^2\*x^3\*e + 1/2\*B\*a\*d^3\*x^2 + 1/4\*A\*a\*x^4\*e^3 + A\*a\*d\*x^3\*e^2 + 3/2\*A\*a\*d^2\*x^2\*e + A\*a\*d^3\*x

**maple [A]** time = 0.00, size = 217, normalized size = 1.24

$$\frac{Cce^3x^8}{8} + \frac{(e^3cB + 3d^2c^2C)x^7}{7} + AAd^3x + \frac{(Ac^3 + 3Bcd^2 + (e^3a + 3d^2ec)C)x^6}{6} + \frac{(3Acd^2 + (e^3a + 3d^2ec)B + (3d^2e + d^2c)C)x^5}{5} + \frac{(3Ca^2d^2 + (e^3a + 3d^2ec)A + (3d^2e + d^2c)B)x^4}{4} + \frac{(3Ba^2d^2 + Ca^3 + (3d^2e + d^2c)A)x^3}{3} + \frac{(3d^2ea + d^3aB)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x)

[Out]  $\frac{1}{8}e^3cC*x^8 + \frac{1}{7}(B*c*e^3 + 3*C*c*d*e^2)*x^7 + \frac{1}{6}((a*e^3 + 3*c*d^2*e)*C + 3*d*e^2*c*B + e^3*c*A)*x^6 + \frac{1}{5}((3*a*d*e^2 + c*d^3)*C + (a*e^3 + 3*c*d^2*e)*B + 3*d*e^2*c*A)*x^5 + \frac{1}{4}(3*d^2*e*a*C + (3*a*d*e^2 + c*d^3)*B + (a*e^3 + 3*c*d^2*e)*A)*x^4 + \frac{1}{3}(d^3*a*C + 3*d^2*e*a*B + (3*a*d*e^2 + c*d^3)*A)*x^3 + \frac{1}{2}(3*A*a*d^2*e + B*a*d^3)*x^2 + d^3*a*A*x$

**maxima [A]** time = 0.45, size = 202, normalized size = 1.15

$$\frac{1}{8}Cce^3x^8 + \frac{1}{7}(3Ccd^2 + Bce^3)x^7 + \frac{1}{6}(3Cd^2e + 3Bcd^2 + (Ca + Ac)e^3)x^6 + AAd^3x + \frac{1}{5}(Ccd^3 + 3Bcd^2e + Bae^3 + 3(Ca + Ac)d^2e)x^5 + \frac{1}{4}(Bcd^3 + 3Bade^2 + Aae^3 + 3(Ca + Ac)d^2e)x^4 + \frac{1}{3}(3Bade^2 + 3Aad^2 + (Ca + Ac)d^3)x^3 + \frac{1}{2}(Bad^3 + 3Aad^2e)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="maxima")

[Out]  $\frac{1}{8}C*c*e^3*x^8 + \frac{1}{7}(3*C*c*d*e^2 + B*c*e^3)*x^7 + \frac{1}{6}(3*C*c*d^2*e + 3*B*c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + \frac{1}{5}(C*c*d^3 + 3*B*c*d^2*e + B*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + \frac{1}{4}(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3 + 3*(C*a + A*c)*d^2*e)*x^4 + \frac{1}{3}(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)*d^3)*x^3 + \frac{1}{2}(B*a*d^3 + 3*A*a*d^2*e)*x^2$

**mupad [B]** time = 0.09, size = 206, normalized size = 1.18

$$x^3\left(\frac{Ac^3}{3} + \frac{Ccd^3}{3} + AAd^3 + BAd^2e\right) + x^6\left(\frac{Ac^3}{6} + \frac{Ccd^3}{6} + \frac{Bcd^2}{2} + \frac{Ccd^2e}{2}\right) + x^4\left(\frac{Aac^3}{4} + \frac{Bcd^3}{4} + \frac{3Bade^2}{4} + \frac{3Ac^2d^2}{4} + \frac{3Ccd^2e}{4}\right) + x^5\left(\frac{Bae^3}{5} + \frac{Ccd^3}{5} + \frac{3Ac^2d^2}{5} + \frac{3Ccd^2e}{5} + \frac{3Bcd^2e}{5}\right) + AAd^3x + \frac{Cce^3x^8}{8} + \frac{a^2x^2(3Ae + Bd)}{2} + \frac{ce^2x^2(Be + 3Cd)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)\*(d + e\*x)^3\*(A + B\*x + C\*x^2), x)

[Out]  $x^3*((A*c*d^3)/3 + (C*a*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^6*((A*c*e^3)/6 + (C*a*e^3)/6 + (B*c*d*e^2)/2 + (C*c*d^2*e)/2) + x^4*((A*a*e^3)/4 + (B*c*d^3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4 + (3*C*a*d^2*e)/4) + x^5*((B*a*e^3)/5 + (C*c*d^3)/5 + (3*A*c*d*e^2)/5 + (3*C*a*d*e^2)/5 + (3*B*c*d^2*e)/5) + A*a*d^3*x + (C*c*e^3*x^8)/8 + (a*d^2*x^2*(3*A*e + B*d))/2 + (c*e^2*x^7*(B*e + 3*C*d))/7$

**sympy [A]** time = 0.12, size = 257, normalized size = 1.47

$$AAd^3x + \frac{Cce^3x^8}{8} + x^7\left(\frac{Bce^3}{7} + \frac{3Ccd^2e}{7}\right) + x^6\left(\frac{Ac^3}{6} + \frac{Bcd^2}{2} + \frac{Cae^3}{6} + \frac{Ccd^2e}{2}\right) + x^5\left(\frac{3Acd^2}{5} + \frac{Bae^3}{5} + \frac{3Bcd^2e}{5} + \frac{3Ccd^2e}{5} + \frac{Ccd^3}{5}\right) + x^4\left(\frac{Aae^3}{4} + \frac{3Ac^2d^2}{4} + \frac{3Bade^2}{4} + \frac{Bcd^3}{4} + \frac{3Ccd^2e}{4}\right) + x^3\left(\frac{Aad^2}{3} + \frac{Ac^3}{3} + \frac{Bade^2}{3} + \frac{Ccd^3}{3}\right) + x^2\left(\frac{3Aad^2e}{2} + \frac{Bad^3}{2}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)`

[Out]  $A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(A*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**2/5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A*a*e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4) + x**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*d**2*e/2 + B*a*d**3/2)$

$$3.19 \quad \int (d + ex)^2 (a + cx^2) (A + Bx + Cx^2) dx$$

**Optimal.** Leaf size=175

$$\frac{(d + ex)^5 (aCe^2 + c(6Cd^2 - e(3Bd - Ae)))}{5e^5} - \frac{(d + ex)^4 (ae^2(2Cd - Be) + cd(4Cd^2 - e(3Bd - 2Ae)))}{4e^5} + \frac{(d + ex)^3 (ae^2(2Cd - Be) + cd(4Cd^2 - e(3Bd - 2Ae)))}{4e^5} + \dots$$

**Rubi [A]** time = 0.22, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1628}

$$\frac{(d + ex)^5 (aCe^2 - ce(3Bd - Ae) + 6cCd^2)}{5e^5} - \frac{(d + ex)^4 (ae^2(2Cd - Be) - cde(3Bd - 2Ae) + 4cCd^3)}{4e^5} + \frac{(d + ex)^3 (ae^2 + cd^2)(Ae^2 - Bde + Cd^2)}{3e^5} - \frac{c(d + ex)^6(4Cd - Be)}{6e^5} + \frac{cC(d + ex)^7}{7e^5}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] ((c\*d^2 + a\*e^2)\*(C\*d^2 - B\*d\*e + A\*e^2)\*(d + e\*x)^3)/(3\*e^5) - ((4\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 2\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))\*(d + e\*x)^4)/(4\*e^5) + ((6\*c\*C\*d^2 + a\*C\*e^2 - c\*e\*(3\*B\*d - A\*e))\*(d + e\*x)^5)/(5\*e^5) - (c\*(4\*C\*d - B\*e)\*(d + e\*x)^6)/(6\*e^5) + (c\*C\*(d + e\*x)^7)/(7\*e^5)

Rule 1628

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int (d + ex)^2 (a + cx^2) (A + Bx + Cx^2) dx = \int \left( \frac{(cd^2 + ae^2)(Cd^2 - Bde + Ae^2)(d + ex)^2}{e^4} + \frac{(-4cCd^3 + cde(3Bd - 2Ae))}{e^4} \right) dx$$

$$= \frac{(cd^2 + ae^2)(Cd^2 - Bde + Ae^2)(d + ex)^3}{3e^5} - \frac{(4cCd^3 - cde(3Bd - 2Ae))}{4e^5} + \dots$$

**Mathematica [A]** time = 0.06, size = 150, normalized size = 0.86

$$\frac{1}{5}x^5 (aCe^2 + Ace^2 + 2Bcde + cCd^2) + \frac{1}{4}x^4 (aBe^2 + 2aCde + 2Acde + Bcd^2) + \frac{1}{3}x^3 (aAe^2 + 2aBde + aCd^2 + Acd^2) + \frac{1}{2}adx^2(2Ae + Bd) + aAd^2x + \frac{1}{6}cex^6(Be + 2Cd) + \frac{1}{7}cCe^2x^7$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] a\*A\*d^2\*x + (a\*d\*(B\*d + 2\*A\*e))\*x^2)/2 + ((A\*c\*d^2 + a\*C\*d^2 + 2\*a\*B\*d\*e + a\*A\*e^2)\*x^3)/3 + ((B\*c\*d^2 + 2\*A\*c\*d\*e + 2\*a\*C\*d\*e + a\*B\*e^2)\*x^4)/4 + ((c\*C\*d^2 + 2\*B\*c\*d\*e + A\*c\*e^2 + a\*C\*e^2)\*x^5)/5 + (c\*e\*(2\*C\*d + B\*e)\*x^6)/6 + (c\*C\*e^2\*x^7)/7

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^2 (a + cx^2) (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^2\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)^2\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

**fricas** [A] time = 0.97, size = 171, normalized size = 0.98

$\frac{1}{7}x^7e^2c + \frac{1}{3}x^6edc + \frac{1}{6}x^6e^2cB + \frac{1}{5}x^5d^2c + \frac{1}{5}x^5e^2aC + \frac{2}{5}x^5edcB + \frac{1}{5}x^5e^2cA + \frac{1}{2}x^4edaC + \frac{1}{4}x^4d^2cB + \frac{1}{4}x^4e^2aB + \frac{1}{2}x^4edcA + \frac{1}{3}x^3d^2aC + \frac{2}{3}x^3edaB + \frac{1}{3}x^3d^2cA + \frac{1}{3}x^3e^2aA + \frac{1}{2}x^2d^2aB + x^2edaA + xd^2aA$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/7\*x^7\*e^2\*c\*C + 1/3\*x^6\*e\*d\*c\*C + 1/6\*x^6\*e^2\*c\*B + 1/5\*x^5\*d^2\*c\*C + 1/5\*x^5\*e^2\*a\*C + 2/5\*x^5\*e\*d\*c\*B + 1/5\*x^5\*e^2\*c\*A + 1/2\*x^4\*e\*d\*a\*C + 1/4\*x^4\*d^2\*c\*B + 1/4\*x^4\*e^2\*a\*B + 1/2\*x^4\*e\*d\*c\*A + 1/3\*x^3\*d^2\*a\*C + 2/3\*x^3\*e\*d\*a\*B + 1/3\*x^3\*d^2\*c\*A + 1/3\*x^3\*e^2\*a\*A + 1/2\*x^2\*d^2\*a\*B + x^2\*e\*d\*a\*A + x\*d^2\*a\*A

**giac** [A] time = 0.15, size = 171, normalized size = 0.98

$\frac{1}{7}Ccx^7e^2 + \frac{1}{3}Ccdx^6e + \frac{1}{6}Ccd^2x^5 + \frac{1}{5}Bcdx^5e + \frac{1}{5}Bcd^2x^4 + \frac{1}{5}Cax^5e^2 + \frac{1}{2}Acx^5e + \frac{1}{2}Acxd^4e + \frac{1}{2}Acxd^4e + \frac{1}{3}Cad^2x^3 + \frac{1}{3}Acdd^2x^3 + \frac{1}{4}Bax^4e^2 + \frac{2}{3}Badx^3e + \frac{1}{2}Bad^2x^2 + \frac{1}{3}Aax^3e^2 + Aaxd^2e + Aad^2x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="giac")

[Out] 1/7\*C\*c\*x^7\*e^2 + 1/3\*C\*c\*d\*x^6\*e + 1/5\*C\*c\*d^2\*x^5 + 1/6\*B\*c\*x^6\*e^2 + 2/5\*B\*c\*d\*x^5\*e + 1/4\*B\*c\*d^2\*x^4 + 1/5\*C\*a\*x^5\*e^2 + 1/5\*A\*c\*x^5\*e^2 + 1/2\*C\*a\*d\*x^4\*e + 1/2\*A\*c\*d\*x^4\*e + 1/3\*C\*a\*d^2\*x^3 + 1/3\*A\*c\*d^2\*x^3 + 1/4\*B\*a\*x^4\*e^2 + 2/3\*B\*a\*d\*x^3\*e + 1/2\*B\*a\*d^2\*x^2 + 1/3\*A\*a\*x^3\*e^2 + A\*a\*d\*x^2\*e + A\*a\*d^2\*x

**maple** [A] time = 0.00, size = 148, normalized size = 0.85

$\frac{Cc^2x^7}{7} + \frac{(c^2B + 2dec)x^6}{6} + Aa d^2 x + \frac{(Ac e^2 + 2Bcde + (a e^2 + c d^2) C) x^5}{5} + \frac{(2Acde + 2Cade + (a e^2 + c d^2) B) x^4}{4} + \frac{(2Bade + Ca d^2 + (a e^2 + c d^2) A) x^3}{3} + \frac{(2deaA + d^2aB) x^2}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x)`

[Out]  $1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*d*e*c*B+c*e^2*A)*x^5+1/4*(2*d*e*a*C+(a*e^2+c*d^2)*B+2*d*e*c*A)*x^4+1/3*(d^2*a*C+2*d*e*a*B+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+d^2*a*A*x$

**maxima** [A] time = 0.45, size = 141, normalized size = 0.81

$$\frac{1}{7}Cce^2x^7 + \frac{1}{6}(2Cde + Bce^2)x^6 + \frac{1}{5}(Ccd^2 + 2Bcde + (Ca + Ac)e^2)x^5 + Aad^2x + \frac{1}{4}(Bcd^2 + Bae^2 + 2(Ca + Ac)de)x^4 + \frac{1}{3}(2Bade + Aae^2 + (Ca + Ac)d^2)x^3 + \frac{1}{2}(Bad^2 + 2Aade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")`

[Out]  $1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e + (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*c)*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d^2 + 2*A*a*d*e)*x^2$

**mupad** [B] time = 3.61, size = 143, normalized size = 0.82

$$x^3\left(\frac{Aae^2}{3} + \frac{Acd^2}{3} + \frac{Cad^2}{3} + \frac{2Bade}{3}\right) + x^5\left(\frac{Ace^2}{5} + \frac{Ca e^2}{5} + \frac{Ccd^2}{5} + \frac{2Bcde}{5}\right) + x^4\left(\frac{Bae^2}{4} + \frac{Bcd^2}{4} + \frac{Acde}{2} + \frac{Cade}{2}\right) + Aad^2x + \frac{adx^2(2Ae+Bd)}{2} + \frac{cex^6(Be+2Cd)}{6} + \frac{Cce^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)*(d + e*x)^2*(A + B*x + C*x^2),x)`

[Out]  $x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (C*a*d^2)/3 + (2*B*a*d*e)/3) + x^5*((A*c*e^2)/5 + (C*a*e^2)/5 + (C*c*d^2)/5 + (2*B*c*d*e)/5) + x^4*((B*a*e^2)/4 + (B*c*d^2)/4 + (A*c*d*e)/2 + (C*a*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*d))/2 + (c*e*x^6*(B*e + 2*C*d))/6 + (C*c*e^2*x^7)/7$

**sympy** [A] time = 0.10, size = 173, normalized size = 0.99

$$Aad^2x + \frac{Cce^2x^7}{7} + x^6\left(\frac{Bce^2}{6} + \frac{Cde}{3}\right) + x^5\left(\frac{Ace^2}{5} + \frac{2Bcde}{5} + \frac{Ca e^2}{5} + \frac{Ccd^2}{5}\right) + x^4\left(\frac{Acde}{2} + \frac{Bae^2}{4} + \frac{Bcd^2}{4} + \frac{Cade}{2}\right) + x^3\left(\frac{Aae^2}{3} + \frac{Acd^2}{3} + \frac{2Bade}{3} + \frac{Cad^2}{3}\right) + x^2\left(\frac{Aade}{1} + \frac{Bad^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)`

[Out]  $A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e**2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**2/4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/3 + C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)$

$$3.20 \quad \int (d + ex) (a + cx^2) (A + Bx + Cx^2) dx$$

**Optimal.** Leaf size=86

$$\frac{1}{4}x^4(aCe + Ace + Bcd) + \frac{1}{3}x^3(aBe + aCd + Acd) + \frac{1}{2}ax^2(Ae + Bd) + aAdx + \frac{1}{5}cx^5(Be + Cd) + \frac{1}{6}cCex^6$$

**Rubi [A]** time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1628}

$$\frac{1}{4}x^4(aCe + Ace + Bcd) + \frac{1}{3}x^3(aBe + aCd + Acd) + \frac{1}{2}ax^2(Ae + Bd) + aAdx + \frac{1}{5}cx^5(Be + Cd) + \frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] a\*A\*d\*x + (a\*(B\*d + A\*e)\*x^2)/2 + ((A\*c\*d + a\*C\*d + a\*B\*e)\*x^3)/3 + ((B\*c\*d + A\*c\*e + a\*C\*e)\*x^4)/4 + (c\*(C\*d + B\*e)\*x^5)/5 + (c\*C\*e\*x^6)/6

Rule 1628

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (d + ex) (a + cx^2) (A + Bx + Cx^2) dx &= \int (aAd + a(Bd + Ae)x + (Acd + aCd + aBe)x^2 + (Bcd + Ace + aCe) \\ &= aAdx + \frac{1}{2}a(Bd + Ae)x^2 + \frac{1}{3}(Acd + aCd + aBe)x^3 + \frac{1}{4}(Bcd + Ace + aCe)x^4 + \frac{1}{5}cx^5(Be + Cd) + \frac{1}{6}cCex^6 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 86, normalized size = 1.00

$$\frac{1}{4}x^4(aCe + Ace + Bcd) + \frac{1}{3}x^3(aBe + aCd + Acd) + \frac{1}{2}ax^2(Ae + Bd) + aAdx + \frac{1}{5}cx^5(Be + Cd) + \frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] a\*A\*d\*x + (a\*(B\*d + A\*e)\*x^2)/2 + ((A\*c\*d + a\*C\*d + a\*B\*e)\*x^3)/3 + ((B\*c\*d + A\*c\*e + a\*C\*e)\*x^4)/4 + (c\*(C\*d + B\*e)\*x^5)/5 + (c\*C\*e\*x^6)/6

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)(a + cx^2)(A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)\*(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

**fricas** [A] time = 0.83, size = 94, normalized size = 1.09

$$\frac{1}{6}x^6ecC + \frac{1}{5}x^5dcC + \frac{1}{5}x^5ecB + \frac{1}{4}x^4eaC + \frac{1}{4}x^4dcB + \frac{1}{4}x^4ecA + \frac{1}{3}x^3daC + \frac{1}{3}x^3eaB + \frac{1}{3}x^3dcA + \frac{1}{2}x^2daB + \frac{1}{2}x^2eaA + xdaA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/6\*x^6\*e\*c\*C + 1/5\*x^5\*d\*c\*C + 1/5\*x^5\*e\*c\*B + 1/4\*x^4\*e\*a\*C + 1/4\*x^4\*d\*c\*B + 1/4\*x^4\*e\*c\*A + 1/3\*x^3\*d\*a\*C + 1/3\*x^3\*e\*a\*B + 1/3\*x^3\*d\*c\*A + 1/2\*x^2\*d\*a\*B + 1/2\*x^2\*e\*a\*A + x\*d\*a\*A

**giac** [A] time = 0.15, size = 100, normalized size = 1.16

$$\frac{1}{6}Ccx^6e + \frac{1}{5}Ccdx^5 + \frac{1}{5}Bcx^5e + \frac{1}{4}Bcdx^4 + \frac{1}{4}Cax^4e + \frac{1}{4}Acx^4e + \frac{1}{3}Cadx^3 + \frac{1}{3}Acx^3 + \frac{1}{3}Bax^3e + \frac{1}{2}Badx^2 + \frac{1}{2}Aax^2e + Aadx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="giac")

[Out] 1/6\*C\*c\*x^6\*e + 1/5\*C\*c\*d\*x^5 + 1/5\*B\*c\*x^5\*e + 1/4\*B\*c\*d\*x^4 + 1/4\*C\*a\*x^4\*e + 1/4\*A\*c\*x^4\*e + 1/3\*C\*a\*d\*x^3 + 1/3\*A\*c\*d\*x^3 + 1/3\*B\*a\*x^3\*e + 1/2\*B\*a\*d\*x^2 + 1/2\*A\*a\*x^2\*e + A\*a\*d\*x

**maple** [A] time = 0.00, size = 79, normalized size = 0.92

$$\frac{Cce x^6}{6} + \frac{(ecB + cdC) x^5}{5} + Aadx + \frac{(Ace + Bcd + aCe) x^4}{4} + \frac{(Ac d + Bae + Cad) x^3}{3} + \frac{(aeA + adB) x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(c\*x^2+a)\*(C\*x^2+B\*x+A), x)

[Out] 1/6\*c\*C\*e\*x^6+1/5\*(B\*c\*e+C\*c\*d)\*x^5+1/4\*(A\*c\*e+B\*c\*d+C\*a\*e)\*x^4+1/3\*(A\*c\*d+B\*a\*e+C\*a\*d)\*x^3+1/2\*(A\*a\*e+B\*a\*d)\*x^2+a\*A\*d\*x

**maxima [A]** time = 0.45, size = 80, normalized size = 0.93

$$\frac{1}{6} C c e x^6 + \frac{1}{5} (C c d + B c e) x^5 + \frac{1}{4} (B c d + (C a + A c) e) x^4 + A a d x + \frac{1}{3} (B a e + (C a + A c) d) x^3 + \frac{1}{2} (B a d + A a e) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out] 1/6\*C\*c\*e\*x^6 + 1/5\*(C\*c\*d + B\*c\*e)\*x^5 + 1/4\*(B\*c\*d + (C\*a + A\*c)\*e)\*x^4 + A\*a\*d\*x + 1/3\*(B\*a\*e + (C\*a + A\*c)\*d)\*x^3 + 1/2\*(B\*a\*d + A\*a\*e)\*x^2

**mupad [B]** time = 3.56, size = 80, normalized size = 0.93

$$\frac{C c e x^6}{6} + \frac{c (B e + C d) x^5}{5} + \left( \frac{A c e}{4} + \frac{B c d}{4} + \frac{C a e}{4} \right) x^4 + \left( \frac{A c d}{3} + \frac{B a e}{3} + \frac{C a d}{3} \right) x^3 + \frac{a (A e + B d) x^2}{2} + A a d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)\*(d + e\*x)\*(A + B\*x + C\*x^2),x)

[Out] x^3\*((A\*c\*d)/3 + (B\*a\*e)/3 + (C\*a\*d)/3) + x^4\*((A\*c\*e)/4 + (B\*c\*d)/4 + (C\*a\*e)/4) + (a\*x^2\*(A\*e + B\*d))/2 + (c\*x^5\*(B\*e + C\*d))/5 + (C\*c\*e\*x^6)/6 + A\*a\*d\*x

**sympy [A]** time = 0.08, size = 97, normalized size = 1.13

$$A a d x + \frac{C c e x^6}{6} + x^5 \left( \frac{B c e}{5} + \frac{C c d}{5} \right) + x^4 \left( \frac{A c e}{4} + \frac{B c d}{4} + \frac{C a e}{4} \right) + x^3 \left( \frac{A c d}{3} + \frac{B a e}{3} + \frac{C a d}{3} \right) + x^2 \left( \frac{A a e}{2} + \frac{B a d}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x\*\*2+a)\*(C\*x\*\*2+B\*x+A),x)

[Out] A\*a\*d\*x + C\*c\*e\*x\*\*6/6 + x\*\*5\*(B\*c\*e/5 + C\*c\*d/5) + x\*\*4\*(A\*c\*e/4 + B\*c\*d/4 + C\*a\*e/4) + x\*\*3\*(A\*c\*d/3 + B\*a\*e/3 + C\*a\*d/3) + x\*\*2\*(A\*a\*e/2 + B\*a\*d/2)

### 3.21 $\int (a + cx^2)(A + Bx + Cx^2) dx$

**Optimal.** Leaf size=46

$$\frac{1}{3}x^3(aC + Ac) + aAx + \frac{1}{2}aBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

**Rubi [A]** time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {1657}

$$\frac{1}{3}x^3(aC + Ac) + aAx + \frac{1}{2}aBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] a\*A\*x + (a\*B\*x^2)/2 + ((A\*c + a\*C)\*x^3)/3 + (B\*c\*x^4)/4 + (c\*C\*x^5)/5

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (a + cx^2)(A + Bx + Cx^2) dx &= \int (aA + aBx + (Ac + aC)x^2 + Bcx^3 + cCx^4) dx \\ &= aAx + \frac{1}{2}aBx^2 + \frac{1}{3}(Ac + aC)x^3 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{3}x^3(aC + Ac) + aAx + \frac{1}{2}aBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] a\*A\*x + (a\*B\*x^2)/2 + ((A\*c + a\*C)\*x^3)/3 + (B\*c\*x^4)/4 + (c\*C\*x^5)/5



**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + cx^2)(A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + c\*x^2)\*(A + B\*x + C\*x^2), x]

**fricas** [A] time = 1.26, size = 40, normalized size = 0.87

$$\frac{1}{5}x^5cC + \frac{1}{4}x^4cB + \frac{1}{3}x^3aC + \frac{1}{3}x^3cA + \frac{1}{2}x^2aB + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/5\*x^5\*c\*C + 1/4\*x^4\*c\*B + 1/3\*x^3\*a\*C + 1/3\*x^3\*c\*A + 1/2\*x^2\*a\*B + x\*a\*A

**giac** [A] time = 0.16, size = 40, normalized size = 0.87

$$\frac{1}{5} Ccx^5 + \frac{1}{4} Bcx^4 + \frac{1}{3} Cax^3 + \frac{1}{3} Acx^3 + \frac{1}{2} Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A), x, algorithm="giac")

[Out] 1/5\*C\*c\*x^5 + 1/4\*B\*c\*x^4 + 1/3\*C\*a\*x^3 + 1/3\*A\*c\*x^3 + 1/2\*B\*a\*x^2 + A\*a\*x

**maple** [A] time = 0.00, size = 39, normalized size = 0.85

$$\frac{Cc x^5}{5} + \frac{Bc x^4}{4} + \frac{Bax^2}{2} + Aax + \frac{(Ac + aC)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)\*(C\*x^2+B\*x+A), x)

[Out] a\*A\*x+1/2\*a\*B\*x^2+1/3\*(A\*c+C\*a)\*x^3+1/4\*B\*c\*x^4+1/5\*c\*C\*x^5

**maxima** [A] time = 0.44, size = 38, normalized size = 0.83

$$\frac{1}{5} Ccx^5 + \frac{1}{4} Bcx^4 + \frac{1}{2} Bax^2 + \frac{1}{3} (Ca + Ac)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out] 1/5\*C\*c\*x^5 + 1/4\*B\*c\*x^4 + 1/2\*B\*a\*x^2 + 1/3\*(C\*a + A\*c)\*x^3 + A\*a\*x

mupad [B] time = 0.03, size = 39, normalized size = 0.85

$$\frac{C c x^5}{5} + \frac{B c x^4}{4} + \left( \frac{A c}{3} + \frac{C a}{3} \right) x^3 + \frac{B a x^2}{2} + A a x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)\*(A + B\*x + C\*x^2),x)

[Out] x^3\*((A\*c)/3 + (C\*a)/3) + A\*a\*x + (B\*a\*x^2)/2 + (B\*c\*x^4)/4 + (C\*c\*x^5)/5

sympy [A] time = 0.07, size = 42, normalized size = 0.91

$$A a x + \frac{B a x^2}{2} + \frac{B c x^4}{4} + \frac{C c x^5}{5} + x^3 \left( \frac{A c}{3} + \frac{C a}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*(C\*x\*\*2+B\*x+A),x)

[Out] A\*a\*x + B\*a\*x\*\*2/2 + B\*c\*x\*\*4/4 + C\*c\*x\*\*5/5 + x\*\*3\*(A\*c/3 + C\*a/3)

$$3.22 \quad \int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex} dx$$

**Optimal.** Leaf size=145

$$\frac{(ae^2 + cd^2) \log(d + ex) (Ae^2 - Bde + Cd^2)}{e^5} - \frac{x (ae^2(Cd - Be) + cd(Cd^2 - e(Bd - Ae)))}{e^4} + \frac{x^2 (aCe^2 + c(Cd^2 - e(Bd - Ae)))}{2e^3}$$

**Rubi [A]** time = 0.25, antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1628}

$$\frac{x^2 (aCe^2 - ce(Bd - Ae) + cCd^2)}{2e^3} - \frac{x (ae^2(Cd - Be) - cde(Bd - Ae) + cCd^3)}{e^4} + \frac{(ae^2 + cd^2) \log(d + ex) (Ae^2 - Bde + Cd^2)}{e^5} - \frac{cx^3(Cd - Be)}{3e^2} + \frac{cCx^4}{4e}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x), x]

[Out] -(((c\*C\*d^3 - c\*d\*e\*(B\*d - A\*e) + a\*e^2\*(C\*d - B\*e))\*x)/e^4) + ((c\*C\*d^2 + a\*C\*e^2 - c\*e\*(B\*d - A\*e))\*x^2)/(2\*e^3) - (c\*(C\*d - B\*e)\*x^3)/(3\*e^2) + (c\*C\*x^4)/(4\*e) + (((c\*d^2 + a\*e^2)\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[d + e\*x])/e^5)

**Rule 1628**

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int \frac{(a + cx^2)(A + Bx + Cx^2)}{d + ex} dx &= \int \left( \frac{-ae^2(Cd - Be) - c(Cd^3 - de(Bd - Ae))}{e^4} + \frac{(cCd^2 + aCe^2 - ce(Bd - Ae))}{e^3} \right. \\ &= -\frac{(cCd^3 - cde(Bd - Ae) + ae^2(Cd - Be))x}{e^4} + \frac{(cCd^2 + aCe^2 - ce(Bd - Ae))x^2}{2e^3} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 136, normalized size = 0.94

$$\frac{12(ae^2 + cd^2) \log(d + ex) (e(Ae - Bd) + Cd^2) + ex(6ae^2(2Be - 2Cd + Cex) + 2ce(3Ae(ex - 2d) + B(6d^2 - 3dex + 2e^2x^2)) + cC(-12d^3 + 6d^2ex - 4de^2x^2 + 3e^3x^3))}{12e^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x),x]

[Out] (e\*x\*(6\*a\*e^2\*(-2\*C\*d + 2\*B\*e + C\*e\*x) + c\*C\*(-12\*d^3 + 6\*d^2\*e\*x - 4\*d\*e^2\*x^2 + 3\*e^3\*x^3) + 2\*c\*e\*(3\*A\*e\*(-2\*d + e\*x) + B\*(6\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2))) + 12\*(c\*d^2 + a\*e^2)\*(C\*d^2 + e\*(-(B\*d) + A\*e))\*Log[d + e\*x])/(12\*e^5)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)(A + Bx + Cx^2)}{d + ex} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x),x]

[Out] IntegrateAlgebraic[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x), x]

**fricas** [A] time = 1.08, size = 161, normalized size = 1.11

$$\frac{3Cce^4x^4 - 4(Ccd^2e^2 - Bce^4)x^3 + 6(Ccd^2e^2 - Bce^4 + (Ca + Ac)e^4)x^2 - 12(Ccd^3e - Bcd^2e^2 - Bae^4 + (Ca + Ac)de^3)x + 12(Ccd^4 - Bcd^3e - Bade^3 + Aae^4 + (Ca + Ac)d^2e^2)\log(ex + d)}{12e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d),x, algorithm="fricas")

[Out] 1/12\*(3\*C\*c\*e^4\*x^4 - 4\*(C\*c\*d\*e^3 - B\*c\*e^4)\*x^3 + 6\*(C\*c\*d^2\*e^2 - B\*c\*d\*e^3 + (C\*a + A\*c)\*e^4)\*x^2 - 12\*(C\*c\*d^3\*e - B\*c\*d^2\*e^2 - B\*a\*e^4 + (C\*a + A\*c)\*d\*e^3)\*x + 12\*(C\*c\*d^4 - B\*c\*d^3\*e - B\*a\*d\*e^3 + A\*a\*e^4 + (C\*a + A\*c)\*d^2\*e^2)\*log(e\*x + d))/e^5

**giac** [A] time = 0.15, size = 170, normalized size = 1.17

$$(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bae^4 + Aae^4)\log((xe + d)) + \frac{1}{12}(3Ccx^4e^3 - 4Ccd^3e^2 + 6Ccd^2e^2 - 12Ccd^3e + 4Bcx^3e^3 - 6Bcd^2e^2 + 12Bcd^2xe + 6Cax^2e^3 + 6Acx^2e^3 - 12Cadxe^2 - 12Acdx^2 + 12Baxe^3)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d),x, algorithm="giac")

[Out] (C\*c\*d^4 - B\*c\*d^3\*e + C\*a\*d^2\*e^2 + A\*c\*d^2\*e^2 - B\*a\*d\*e^3 + A\*a\*e^4)\*e^(-5)\*log(abs(x\*e + d)) + 1/12\*(3\*C\*c\*x^4\*e^3 - 4\*C\*c\*d\*x^3\*e^2 + 6\*C\*c\*d^2\*x^2\*e - 12\*C\*c\*d^3\*x + 4\*B\*c\*x^3\*e^3 - 6\*B\*c\*d\*x^2\*e^2 + 12\*B\*c\*d^2\*x\*e + 6\*C\*a\*x^2\*e^3 + 6\*A\*c\*x^2\*e^3 - 12\*C\*a\*d\*x\*e^2 - 12\*A\*c\*d\*x\*e^2 + 12\*B\*a\*x\*e^3)\*e^(-4)

**maple** [A] time = 0.01, size = 210, normalized size = 1.45

$$\frac{Ccx^4}{4e} + \frac{Bcx^3}{3e} - \frac{Ccdx^3}{3e^2} + \frac{Acx^2}{2e} - \frac{Bcdx^2}{2e^2} + \frac{Cax^2}{2e} + \frac{Ccd^2x^2}{2e^3} + \frac{Aa\ln(ex+d)}{e} + \frac{Ac d^2 \ln(ex+d)}{e^3} - \frac{Ac dx}{e^2} - \frac{Bad \ln(ex+d)}{e^2} + \frac{Bax}{e} - \frac{Bc d^3 \ln(ex+d)}{e^4} + \frac{Bc d^2 x}{e^3} + \frac{Ca d^2 \ln(ex+d)}{e^3} - \frac{Cad x}{e^2} + \frac{Ccd^4 \ln(ex+d)}{e^5} - \frac{Ccd^3 x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x)`

[Out]  $\frac{1}{4}cC^2x^4/e + 1/3/eB^3cx - 1/3/e^2C^3x^3cd + 1/2/eA^2x^2c - 1/2/e^2B^2x^2cd + 1/2/eC^2xa + 1/2/e^3C^2x^2cd^2 - 1/e^2A^2x^2cd + 1/eB^2xa + 1/e^3B^2x^2cd^2 - 1/e^2C^2x^2ad - 1/e^4C^2x^2cd^3 + 1/e \ln(e*x+d)A^2a + 1/e^3 \ln(e*x+d)A^2cd^2 - 1/e^2 \ln(e*x+d)B^2ad - 1/e^4 \ln(e*x+d)B^2cd^3 + 1/e^3 \ln(e*x+d)C^2ad^2 + 1/e^5 \ln(e*x+d)C^2cd^4$

**maxima [A]** time = 0.45, size = 159, normalized size = 1.10

$$\frac{3Cce^3x^4 - 4(Ccde^2 - Bce^3)x^3 + 6(Ccd^2e - Bcde^2 + (Ca + Ac)e^3)x^2 - 12(Ccd^3 - Bcd^2e - Bae^3 + (Ca + Ac)de^2)x + \frac{(Ccd^4 - Bcd^3e - Bae^4 + (Ca + Ac)d^2e^2) \log(ex + d)}{e^5}}{12e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")`

[Out]  $\frac{1}{12}*(3C^3c^3e^3x^4 - 4*(C^3cd^3e^2 - B^3c^3e^3)x^3 + 6*(C^3cd^2e^2 - B^3cd^2e^3 + (C^3a + A^3c)e^3)x^2 - 12*(C^3cd^3e - B^3cd^2e^2 - B^3a^3e^3 + (C^3a + A^3c)d^2e^2)x)/e^4 + (C^3cd^4 - B^3cd^3e - B^3ad^3e^3 + A^3a^3e^4 + (C^3a + A^3c)d^2e^2)*\log(e*x + d)/e^5$

**mupad [B]** time = 3.62, size = 175, normalized size = 1.21

$$x^3 \left( \frac{Bc}{3e} - \frac{Ccd}{3e^2} \right) - x \left( \frac{d \left( \frac{Ac+Ca}{e} - \frac{d \left( \frac{Bc}{e} - \frac{Ccd}{e^2} \right)}{e} \right)}{e} - \frac{Ba}{e} \right) + x^2 \left( \frac{Ac+Ca}{2e} - \frac{d \left( \frac{Bc}{e} - \frac{Ccd}{e^2} \right)}{2e} \right) + \frac{\ln(d+ex) (Aae^4 + Ccd^4 - Bad^3e - Bcd^3e + Acd^2e^2 + Ccd^2e^2)}{e^5} + \frac{Ccx^4}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x)`

[Out]  $x^3*((B*c)/(3*e) - (C*c*d)/(3*e^2)) - x*((d*((A*c + C*a)/e - (d*((B*c)/e - (C*c*d)/e^2))/e) - (B*a)/e) + x^2*((A*c + C*a)/(2*e) - (d*((B*c)/e - (C*c*d)/e^2))/(2*e)) + (\log(d + e*x)*(A*a*e^4 + C*c*d^4 - B*a*d^3e - B*c*d^3e + A*c*d^2e^2 + C*a*d^2e^2))/e^5 + (C*c*x^4)/(4*e)$

**sympy [A]** time = 0.64, size = 148, normalized size = 1.02

$$\frac{Ccx^4}{4e} + x^3 \left( \frac{Bc}{3e} - \frac{Ccd}{3e^2} \right) + x^2 \left( \frac{Ac}{2e} - \frac{Bcd}{2e^2} + \frac{Ca}{2e} + \frac{Ccd^2}{2e^3} \right) + x \left( -\frac{Acd}{e^2} + \frac{Ba}{e} + \frac{Bcd^2}{e^3} - \frac{Cad}{e^2} - \frac{Ccd^3}{e^4} \right) + \frac{(ae^2 + cd^2)(Ae^2 - Bde + Cd^2) \log(d + ex)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)`

[Out]  $C^2cx^4/(4*e) + x^3*(B^3c/(3*e) - C^2cd/(3*e^2)) + x^2*(A^2c/(2*e) - B^2cd/(2*e^2) + C^2a/(2*e) + C^2cd^2/(2*e^3)) + x*(-A^2cd/e^2 + B^2a/e + B^2c^2$

$$\frac{d^{**2}}{e^{**3}} - C*a*d/e^{**2} - C*c*d^{**3}/e^{**4} + (a*e^{**2} + c*d^{**2})*(A*e^{**2} - B*d*e + C*d^{**2})*\log(d + e*x)/e^{**5}$$

$$3.23 \quad \int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2} dx$$

**Optimal.** Leaf size=153

$$\frac{(ae^2 + cd^2)(Ae^2 - Bde + Cd^2)}{e^5(d+ex)} - \frac{\log(d+ex)(ae^2(2Cd - Be) + cd(4Cd^2 - e(3Bd - 2Ae)))}{e^5} + \frac{x(aCe^2 + c(3Cd^2 - Ae^2))}{e^4}$$

**Rubi [A]** time = 0.20, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1628}

$$\frac{x(aCe^2 - ce(2Bd - Ae) + 3cCd^2)}{e^4} - \frac{(ae^2 + cd^2)(Ae^2 - Bde + Cd^2)}{e^5(d+ex)} - \frac{\log(d+ex)(ae^2(2Cd - Be) - cde(3Bd - 2Ae) + 4cCd^3)}{e^5} - \frac{cx^2(2Cd - Be)}{2e^3} + \frac{cCx^3}{3e^2}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2, x]

[Out] ((3\*c\*C\*d^2 + a\*C\*e^2 - c\*e\*(2\*B\*d - A\*e))\*x)/e^4 - (c\*(2\*C\*d - B\*e)\*x^2)/(2\*e^3) + (c\*C\*x^3)/(3\*e^2) - ((c\*d^2 + a\*e^2)\*(C\*d^2 - B\*d\*e + A\*e^2))/(e^5\*(d + e\*x)) - ((4\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 2\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))\*Log[d + e\*x])/e^5

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2} dx &= \int \left( \frac{3cCd^2 + aCe^2 - ce(2Bd - Ae)}{e^4} + \frac{c(-2Cd + Be)x}{e^3} + \frac{cCx^2}{e^2} + \frac{(cd^2 + ae^2)}{e^4} \right) dx \\ &= \frac{(3cCd^2 + aCe^2 - ce(2Bd - Ae))x}{e^4} - \frac{c(2Cd - Be)x^2}{2e^3} + \frac{cCx^3}{3e^2} - \frac{(cd^2 + ae^2)(C(d+ex) - Ae)}{e^5} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 142, normalized size = 0.93

$$\frac{6 \log(d+ex)(ae^2(Be - 2Cd) + cde(3Bd - 2Ae) - 4cCd^3) + 6ex(aCe^2 + ce(Ae - 2Bd) + 3cCd^2) - \frac{6(ae^2 + cd^2)(c(Ae - Bd) + Cd^2)}{d+ex} + 3ce^2x^2(Be - 2Cd) + 2cCe^3x^3}{6e^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2,x]

[Out] (6\*e\*(3\*c\*C\*d^2 + a\*C\*e^2 + c\*e\*(-2\*B\*d + A\*e))\*x + 3\*c\*e^2\*(-2\*C\*d + B\*e)\*x^2 + 2\*c\*C\*e^3\*x^3 - (6\*(c\*d^2 + a\*e^2)\*(C\*d^2 + e\*(-B\*d + A\*e)))/(d + e\*x) + 6\*(-4\*c\*C\*d^3 + c\*d\*e\*(3\*B\*d - 2\*A\*e) + a\*e^2\*(-2\*C\*d + B\*e))\*Log[d + e\*x])/(6\*e^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)(A + Bx + Cx^2)}{(d + ex)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2,x]

[Out] IntegrateAlgebraic[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2, x]

**fricas [A]** time = 0.83, size = 250, normalized size = 1.63

$$\frac{2Ccd^4x^4 - 6Ccd^3e + 6Bcd^3e + 6Ade^3 - 6Aae^4 - 6(Ca + Ae)d^2e^2 - (4Ccd^3 - 3Bce^4)x^3 + 3(4Ccd^2e - 3Bcd^2e + 2(Ca + Ae)d^4)x^2 + 6(3Ccd^2e - 2Bcd^2e + (Ca + Ae)d^3)x - 6(4Ccd^4 - 3Bcd^3e - Bae^4 + 2(Ca + Ae)d^2e^2 + (4Ccd^3e - 3Bcd^2e - Bae^4 + 2(Ca + Ae)d^2e)x) \log(ex + d)}{6(e^5x + d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="fricas")

[Out] 1/6\*(2\*C\*c\*e^4\*x^4 - 6\*C\*c\*d^4 + 6\*B\*c\*d^3\*e + 6\*B\*a\*d\*e^3 - 6\*A\*a\*e^4 - 6\*(C\*a + A\*c)\*d^2\*e^2 - (4\*C\*c\*d\*e^3 - 3\*B\*c\*e^4)\*x^3 + 3\*(4\*C\*c\*d^2\*e^2 - 3\*B\*c\*d\*e^3 + 2\*(C\*a + A\*c)\*e^4)\*x^2 + 6\*(3\*C\*c\*d^3\*e - 2\*B\*c\*d^2\*e^2 + (C\*a + A\*c)\*d\*e^3)\*x - 6\*(4\*C\*c\*d^4 - 3\*B\*c\*d^3\*e - B\*a\*d\*e^3 + 2\*(C\*a + A\*c)\*d^2\*e^2 + (4\*C\*c\*d^3\*e - 3\*B\*c\*d^2\*e^2 - B\*a\*e^4 + 2\*(C\*a + A\*c)\*d\*e^3)\*x)\*log(e\*x + d))/(e^6\*x + d\*e^5)

**giac [A]** time = 0.16, size = 240, normalized size = 1.57

$$\frac{1}{6} \left( 2Cc - \frac{3(4Ccd e - Bce^2)e^{(-1)}}{xe + d} + \frac{6(6Ccd^2e^2 - 3Bcd^3 + Cae^4 + Ace^4)e^{(-2)}}{(xe + d)^2} \right) (xe + d)^3 e^{(-5)} + (4Ccd^3 - 3Bcd^2e + 2Ccd^2 + 2Acd^2 - Bae^3)e^{(-5)} \log\left(\frac{xe + d}{(xe + d)^2}\right) - \left( \frac{Ccd^4e^3}{xe + d} - \frac{Bcd^3e^4}{xe + d} + \frac{Cad^2e^5}{xe + d} + \frac{Acd^2e^5}{xe + d} - \frac{Bade^6}{xe + d} + \frac{Aae^7}{xe + d} \right) e^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="giac")

[Out] 1/6\*(2\*C\*c - 3\*(4\*C\*c\*d\*e - B\*c\*e^2))\*e^(-1)/(x\*e + d) + 6\*(6\*C\*c\*d^2\*e^2 - 3\*B\*c\*d\*e^3 + C\*a\*e^4 + A\*c\*e^4)\*e^(-2)/(x\*e + d)^2\*(x\*e + d)^3\*e^(-5) + (4\*C\*c\*d^3 - 3\*B\*c\*d^2\*e + 2\*C\*a\*d\*e^2 + 2\*A\*c\*d\*e^2 - B\*a\*e^3)\*e^(-5)\*log(abs(x\*e + d))\*e^(-1)/(x\*e + d)^2 - (C\*c\*d^4\*e^3/(x\*e + d) - B\*c\*d^3\*e^4/(x\*e



$$+ d) + C*a*d^2*e^5/(x*e + d) + A*c*d^2*e^5/(x*e + d) - B*a*d*e^6/(x*e + d) + A*a*e^7/(x*e + d))*e^{-8}$$

**maple [A]** time = 0.01, size = 234, normalized size = 1.53

$$\frac{Ccx^3}{3e^2} + \frac{Bcx^2}{2e^2} - \frac{Ccdx^2}{e^3} - \frac{Aa}{(ex+d)e} - \frac{Acd^2}{(ex+d)e^3} - \frac{2Acd \ln(ex+d)}{e^3} + \frac{Acx}{e^2} + \frac{Bad}{(ex+d)e^2} + \frac{Ba \ln(ex+d)}{e^2} + \frac{Bcd^3}{(ex+d)e^4} + \frac{3Bcd^2 \ln(ex+d)}{e^4} - \frac{2Bcdx}{e^3} - \frac{Cad^2}{(ex+d)e^3} - \frac{2Ccd \ln(ex+d)}{e^3} + \frac{Cax}{e^2} - \frac{Ccd^4}{(ex+d)e^5} - \frac{4Ccd^3 \ln(ex+d)}{e^5} + \frac{3Ccd^2x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x)

[Out]  $\frac{1}{3}cCx^3/e^2 + 1/2/e^2*B*x^2*c - 1/e^3*C*x^2*c*d + 1/e^2*A*c*x - 2/e^3*B*c*d*x + 1/e^2*a*C*x + 3/e^4*C*c*d^2*x - 1/e/(e*x+d)*A*a - 1/e^3/(e*x+d)*A*c*d^2 + 1/e^2/(e*x+d)*B*d*a + 1/e^4/(e*x+d)*B*c*d^3 - 1/e^3/(e*x+d)*C*a*d^2 - 1/e^5/(e*x+d)*C*c*d^4 - 2/e^3*\ln(e*x+d)*A*c*d + 1/e^2*\ln(e*x+d)*B*a + 3/e^4*\ln(e*x+d)*B*c*d^2 - 2/e^3*\ln(e*x+d)*C*a*d - 4/e^5*\ln(e*x+d)*C*c*d^3$

**maxima [A]** time = 0.45, size = 169, normalized size = 1.10

$$\frac{-Ccd^4 - Bcd^3e - Bade^3 + Aae^4 + (Ca + Ac)d^2e^2}{e^6x + de^5} + \frac{2Cce^2x^3 - 3(2Cde - Bce^2)x^2 + 6(3Ccd^2 - 2Bcde + (Ca + Ac)e^2)x}{6e^4} - \frac{(4Ccd^3 - 3Bcd^2e - Bae^3 + 2(Ca + Ac)de^2) \log(ex + d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^2*e^2)/(e^6*x + d*e^5) + 1/6*(2*C*c*e^2*x^3 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2 - 2*B*c*d*e + (C*a + A*c)*e^2)*x)/e^4 - (4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^3 + 2*(C*a + A*c)*d*e^2)*\log(e*x + d)/e^5$

**mupad [B]** time = 0.09, size = 192, normalized size = 1.25

$$x^2 \left( \frac{Bc}{2e^2} - \frac{Ccd}{e^3} \right) - x \left( \frac{2d \left( \frac{Bc}{e^2} - \frac{2Ccd}{e^3} \right) - Ac + Ca + Ccd^2}{e} - \frac{\ln(d + ex) (4Ccd^3 - Bae^3 + 2Acd^2 + 2Cade^2 - 3Bcd^2e)}{e^5} - \frac{Aae^4 + Ccd^4 - Bade^3 - Bcd^3e + Acd^2e^2 + Cad^2e^2}{e(xe^5 + de^4)} + \frac{Ccx^3}{3e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^2,x)

[Out]  $x^2*((B*c)/(2*e^2) - (C*c*d)/e^3) - x*((2*d*((B*c)/e^2 - (2*C*c*d)/e^3))/e - (A*c + C*a)/e^2 + (C*c*d^2)/e^4) - (\log(d + e*x)*(4*C*c*d^3 - B*a*e^3 + 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e))/e^5 - (A*a*e^4 + C*c*d^4 - B*a*d*e^3 - B*c*d^3*e + A*c*d^2*e^2 + C*a*d^2*e^2)/(e*(d*e^4 + e^5*x)) + (C*c*x^3)/(3*e^2)$

**sympy [A]** time = 1.26, size = 185, normalized size = 1.21

$$\frac{Ccx^3}{3e^2} + x^2 \left( \frac{Bc}{2e^2} - \frac{Ccd}{e^3} \right) + x \left( \frac{Ac}{e^2} - \frac{2Bcd}{e^3} + \frac{Ca}{e^2} + \frac{3Ccd^2}{e^4} \right) + \frac{-Aae^4 - Acd^2e^2 + Bade^3 + Bcd^3e - Cad^2e^2 - Ccd^4}{de^5 + e^6x} - \frac{(2Acd^2 - Bae^3 - 3Bcd^2e + 2Cade^2 + 4Ccd^3) \log(d + ex)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)`

[Out]  $C*c*x**3/(3*e**2) + x**2*(B*c/(2*e**2) - C*c*d/e**3) + x*(A*c/e**2 - 2*B*c*d/e**3 + C*a/e**2 + 3*C*c*d**2/e**4) + (-A*a*e**4 - A*c*d**2*e**2 + B*a*d*e**3 + B*c*d**3*e - C*a*d**2*e**2 - C*c*d**4)/(d*e**5 + e**6*x) - (2*A*c*d*e**2 - B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*\log(d + e*x)/e**5$

$$3.24 \quad \int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} dx$$

**Optimal.** Leaf size=156

$$\frac{ae^2(2Cd - Be) + cd(4Cd^2 - e(3Bd - 2Ae))}{e^5(d+ex)} - \frac{(ae^2 + cd^2)(Ae^2 - Bde + Cd^2)}{2e^5(d+ex)^2} + \frac{\log(d+ex)(aCe^2 + c(6Cd^2 - e(3Bd - 2Ae)))}{e^5}$$

**Rubi [A]** time = 0.20, antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {1628}

$$\frac{ae^2(2Cd - Be) - cde(3Bd - 2Ae) + 4cCd^3}{e^5(d+ex)} - \frac{(ae^2 + cd^2)(Ae^2 - Bde + Cd^2)}{2e^5(d+ex)^2} + \frac{\log(d+ex)(aCe^2 - ce(3Bd - Ae) + 6cCd^2)}{e^5} - \frac{cx(3Cd - Be)}{e^4} + \frac{cCx^2}{2e^3}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^3, x]

[Out] -((c\*(3\*C\*d - B\*e)\*x)/e^4) + (c\*C\*x^2)/(2\*e^3) - ((c\*d^2 + a\*e^2)\*(C\*d^2 - B\*d\*e + A\*e^2))/(2\*e^5\*(d + e\*x)^2) + (4\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 2\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))/(e^5\*(d + e\*x)) + (((6\*c\*C\*d^2 + a\*C\*e^2 - c\*e\*(3\*B\*d - A\*e))\*Log[d + e\*x])/e^5

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} dx &= \int \left( \frac{c(-3Cd + Be)}{e^4} + \frac{cCx}{e^3} + \frac{(cd^2 + ae^2)(Cd^2 - Bde + Ae^2)}{e^4(d+ex)^3} + \frac{-4cCd^3 + cde(3Bd - 2Ae)}{e^5} \right) dx \\ &= -\frac{c(3Cd - Be)x}{e^4} + \frac{cCx^2}{2e^3} - \frac{(cd^2 + ae^2)(Cd^2 - Bde + Ae^2)}{2e^5(d+ex)^2} + \frac{4cCd^3 - cde(3Bd - 2Ae)}{e^5} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 176, normalized size = 1.13

$$\frac{\log(d+ex)(aCe^2 + Ace^2 - 3Bcde + 6cCd^2)}{e^5} + \frac{-aBe^3 + 2aCde^2 + 2Acde^2 - 3Bcd^2e + 4cCd^3}{e^5(d+ex)} + \frac{-aAe^4 + aBde^3 - aCd^2e^2 - Acd^2e^2 + Bcd^3e - cCd^4}{2e^5(d+ex)^2} + \frac{cx(Be - 3Cd)}{e^4} + \frac{cCx^2}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] (c\*(-3\*C\*d + B\*e)\*x)/e^4 + (c\*C\*x^2)/(2\*e^3) + (-c\*C\*d^4 + B\*c\*d^3\*e - A\*c\*d^2\*e^2 - a\*C\*d^2\*e^2 + a\*B\*d\*e^3 - a\*A\*e^4)/(2\*e^5\*(d + e\*x)^2) + (4\*c\*C\*d^3 - 3\*B\*c\*d^2\*e + 2\*A\*c\*d\*e^2 + 2\*a\*C\*d\*e^2 - a\*B\*e^3)/(e^5\*(d + e\*x)) + ((6\*c\*C\*d^2 - 3\*B\*c\*d\*e + A\*c\*e^2 + a\*C\*e^2)\*Log[d + e\*x])/e^5

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)(A + Bx + Cx^2)}{(d + ex)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] IntegrateAlgebraic[((a + c\*x^2)\*(A + B\*x + C\*x^2))/(d + e\*x)^3, x]

**fricas** [A] time = 0.92, size = 273, normalized size = 1.75

$$\frac{Ccx^4 + 7Ccd^4 - 5Bcd^3e - Bde^3 - Aae^4 + 3(Ca + Acd)e^2 - 2(2Ccd^2 - Bce^2)x^2 - (11Ccd^2 - 4Bcd^2)x^2 + 2(Ccd^2 - 2Bcd^2 - Bae^4 + 2(Ca + Acd)e^2)x + 2(6Ccd^4 - 3Bcd^3e + (Ca + Acd)e^2 + (6Ccd^2 - 3Bcd^2 + (Ca + Acd)^2)x^2 + 2(6Ccd^2 - 3Bcd^2 + (Ca + Acd)^2)x) \log(x + d)}{2(e^2x^2 + 2Aex + d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/2\*(C\*c\*e^4\*x^4 + 7\*C\*c\*d^4 - 5\*B\*c\*d^3\*e - B\*a\*d\*e^3 - A\*a\*e^4 + 3\*(C\*a + A\*c)\*d^2\*e^2 - 2\*(2\*C\*c\*d\*e^3 - B\*c\*e^4)\*x^3 - (11\*C\*c\*d^2\*e^2 - 4\*B\*c\*d\*e^3)\*x^2 + 2\*(C\*c\*d^3\*e - 2\*B\*c\*d^2\*e^2 - B\*a\*e^4 + 2\*(C\*a + A\*c)\*d\*e^3)\*x + 2\*(6\*C\*c\*d^4 - 3\*B\*c\*d^3\*e + (C\*a + A\*c)\*d^2\*e^2 + (6\*C\*c\*d^2\*e^2 - 3\*B\*c\*d\*e^3 + (C\*a + A\*c)\*e^4)\*x^2 + 2\*(6\*C\*c\*d^3\*e - 3\*B\*c\*d^2\*e^2 + (C\*a + A\*c)\*d\*e^3)\*x)\*log(e\*x + d))/(e^7\*x^2 + 2\*d\*e^6\*x + d^2\*e^5)

**giac** [A] time = 0.15, size = 167, normalized size = 1.07

$$(6Ccd^2 - 3Bcd^2 + Cae^2 + Ace^2)e^{(-5)} \log(|xe + d|) + \frac{1}{2}(Ccx^2e^3 - 6Ccdxe^2 + 2Bcx^3)e^{(-6)} + \frac{(7Ccd^4 - 5Bcd^3e + 3Cad^2e^2 + 3Acd^2e^2 - Bde^3 - Aae^4 + 2(4Ccd^3e - 3Bcd^2e^2 + 2Cde^3 + 2Acde^3 - Bae^4)x)e^{(-5)}}{2(xe + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)\*(C\*x^2+B\*x+A)/(e\*x+d)^3,x, algorithm="giac")

[Out] (6\*C\*c\*d^2 - 3\*B\*c\*d\*e + C\*a\*e^2 + A\*c\*e^2)\*e^(-5)\*log(abs(x\*e + d)) + 1/2\*(C\*c\*x^2\*e^3 - 6\*C\*c\*d\*x\*e^2 + 2\*B\*c\*x\*e^3)\*e^(-6) + 1/2\*(7\*C\*c\*d^4 - 5\*B\*c\*d^3\*e + 3\*C\*a\*d^2\*e^2 + 3\*A\*c\*d^2\*e^2 - B\*a\*d\*e^3 - A\*a\*e^4 + 2\*(4\*C\*c\*d^3\*e - 3\*B\*c\*d^2\*e^2 + 2\*C\*a\*d\*e^3 + 2\*A\*c\*d\*e^3 - B\*a\*e^4)\*x)\*e^(-5)/(x\*e + d)^2

**maple [A]** time = 0.01, size = 257, normalized size = 1.65

$$\frac{-\frac{Aa}{2(ex+d)^2e} - \frac{Ac d^2}{2(ex+d)^2e^3} + \frac{Bad}{2(ex+d)^2e^2} + \frac{Bc d^3}{2(ex+d)^2e^4} - \frac{Ca d^2}{2(ex+d)^2e^3} - \frac{Cc d^4}{2(ex+d)^2e^5} + \frac{Ccx^2}{2e^5} + \frac{2Acd}{(ex+d)e^3} + \frac{Ac \ln(ex+d)}{e^3} - \frac{Ba}{(ex+d)e^2} - \frac{3Bcd^2}{(ex+d)e^4} - \frac{3Bcd \ln(ex+d)}{e^4} + \frac{Bcx}{e^3} + \frac{2Cad}{(ex+d)e^3} + \frac{Ca \ln(ex+d)}{e^3} + \frac{4Cc d^3}{(ex+d)e^5} + \frac{6Cc d^2 \ln(ex+d)}{e^5} - \frac{3Ccdx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x)`

[Out]  $\frac{1}{2}cCx^2/e^3 + c/e^3Bx - 3c/e^4Cd^2x + 2/e^3/(e*x+d)Acd - 1/e^2/(e*x+d)Ba - 3/e^4/(e*x+d)B^2cd^2 + 2/e^3/(e*x+d)C^2ad + 4/e^5/(e*x+d)C^2cd^3 + 1/e^3 \ln(e*x+d)A^2c - 3/e^4 \ln(e*x+d)B^2cd + 1/e^3 \ln(e*x+d)A^2c + 6/e^5 \ln(e*x+d)C^2cd^2 - 1/2/e/(e*x+d)^2A^2a - 1/2/e^3/(e*x+d)^2A^2d^2c + 1/2/e^2/(e*x+d)^2B^2da + 1/2/e^4/(e*x+d)^2B^2cd^3 - 1/2/e^3/(e*x+d)^2C^2d^2a - 1/2/e^5/(e*x+d)^2C^2cd^4$

**maxima [A]** time = 0.47, size = 177, normalized size = 1.13

$$\frac{7Ccd^4 - 5Bcd^3e - Bae^3 - Aae^4 + 3(Ca + Ac)d^2e^2 + 2(4Ccd^3e - 3Bcd^2e^2 - Bae^4 + 2(Ca + Ac)de^3)x}{2(e^7x^2 + 2de^6x + d^2e^5)} + \frac{Ccx^2 - 2(3Ccd - Bce)x}{2e^4} + \frac{(6Ccd^2 - 3Bcde + (Ca + Ac)e^2) \log(ex + d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2}*(7C^2cd^4 - 5B^2cd^3e - B^2a^2de^3 - A^2a^2e^4 + 3(C^2a + A^2c)d^2e^2 + 2*(4C^2cd^3e - 3B^2cd^2e^2 - B^2a^2e^4 + 2*(C^2a + A^2c)d^2e^3)*x)/(e^7x^2 + 2d^2e^6x + d^2e^5) + 1/2*(C^2ce^2x^2 - 2*(3C^2cd - B^2ce)*x)/e^4 + (6C^2cd^2 - 3B^2cd^2e + (C^2a + A^2c)e^2)*\log(ex + d)/e^5$

**mupad [B]** time = 0.09, size = 185, normalized size = 1.19

$$\frac{x(4Ccd^3 - Bae^3 + 2Acd^2e^2 + 2Cad^2e^2 - 3Bcd^2e) - \frac{Aae^4 - 7Ccd^4 + Bae^3 + 5Bcd^3e - 3Acd^2e^2 - 3Cad^2e^2}{2e}}{d^2e^4 + 2de^5x + e^6x^2} + x\left(\frac{Bc}{e^3} - \frac{3Ccd}{e^4}\right) + \frac{\ln(d + ex)(Ace^2 + Ca^2e^2 + 6Ccd^2 - 3Bcde)}{e^5} + \frac{Ccx^2}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x)`

[Out]  $\frac{(x*(4C^2cd^3 - B^2a^2e^3 + 2A^2acd^2e^2 + 2C^2aad^2e^2 - 3B^2cd^2e) - (A^2a^2e^4 - 7C^2cd^4 + B^2a^2de^3 + 5B^2cd^3e - 3A^2acd^2e^2 - 3C^2aad^2e^2))/(d^2e^4 + e^6x^2 + 2d^2e^5x) + x*((Bc)/e^3 - (3C^2cd)/e^4) + (\log(d + e*x)*(A^2ce^2 + C^2a^2e^2 + 6C^2cd^2 - 3B^2cd^2e))/e^5 + (C^2cx^2)/(2e^3)}$

**sympy [A]** time = 5.29, size = 206, normalized size = 1.32

$$\frac{Ccx^2}{2e^3} + x\left(\frac{Bc}{e^3} - \frac{3Ccd}{e^4}\right) + \frac{-Aae^4 + 3Acd^2e^2 - Bae^3 - 5Bcd^3e + 3Cad^2e^2 + 7Ccd^4 + x(4Acd^3e - 2Bae^4 - 6Bcd^2e^2 + 4Cade^3 + 8Ccd^3e)}{2d^2e^5 + 4de^6x + 2e^7x^2} + \frac{(Ace^2 - 3Bcde + Ca^2e^2 + 6Ccd^2) \log(d + ex)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*(C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*3,x)

[Out]  $C*c*x**2/(2*e**3) + x*(B*c/e**3 - 3*C*c*d/e**4) + (-A*a*e**4 + 3*A*c*d**2*e**2 - B*a*d*e**3 - 5*B*c*d**3*e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c*d*e**3 - 2*B*a*e**4 - 6*B*c*d**2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 + 6*C*c*d**2)*\log(d + e*x)/e**5$

$$3.25 \quad \int (d + ex)^3 (a + cx^2)^2 (A + Bx + Cx^2) dx$$

Optimal. Leaf size=304

$$\frac{1}{4}a^2ex^4(e(Ae + 3Bd) + 3Cd^2) + a^2Ad^3x + \frac{1}{8}cex^8(2aCe^2 + c(e(Ae + 3Bd) + 3Cd^2)) + \frac{1}{7}cx^7(2ae^2(Be + 3Cd) + cd)$$

**Rubi** [A] time = 0.53, antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1582, 1810}

$$\frac{1}{4}a^2ex^4(e(Ae + 3Bd) + 3Cd^2) + a^2Ad^3x + \frac{1}{8}cex^8(2aCe^2 + c(e(Ae + 3Bd) + 3Cd^2)) + \frac{1}{7}cx^7(2ae^2(Be + 3Cd) + cd)$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] a^2\*A\*d^3\*x + (a\*d\*(a\*d\*(C\*d + 3\*B\*e) + A\*(2\*c\*d^2 + 3\*a\*e^2))\*x^3)/3 + (a^2\*e\*(3\*C\*d^2 + e\*(3\*B\*d + A\*e))\*x^4)/4 + ((A\*c\*d\*(c\*d^2 + 6\*a\*e^2) + a\*(a\*e^2\*(3\*C\*d + B\*e) + 2\*c\*d^2\*(C\*d + 3\*B\*e)))\*x^5)/5 + (a\*e\*(6\*c\*C\*d^2 + a\*C\*e^2 + 2\*c\*e\*(3\*B\*d + A\*e))\*x^6)/6 + (c\*(c\*C\*d^3 + 3\*c\*d\*e\*(B\*d + A\*e) + 2\*a\*e^2\*(3\*C\*d + B\*e))\*x^7)/7 + (c\*e\*(3\*c\*C\*d^2 + 2\*a\*C\*e^2 + c\*e\*(3\*B\*d + A\*e))\*x^8)/8 + (c^2\*e^2\*(3\*C\*d + B\*e)\*x^9)/9 + (c^2\*C\*e^3\*x^10)/10 + (d^2\*(B\*d + 3\*A\*e)\*(a + c\*x^2)^3)/(6\*c)

Rule 1582

Int[(Px\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(Coeff[Px, x, n - 1]\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]\*x^(n - 1))\*(a + b\*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]\*x^(n - 1)] && !MatchQ[Px, (Qx\_)\*((c\_) + (d\_)\*x^(m\_))^(q\_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx\*(a + b\*x^n)^p, x, m - 1], 0] && GtQ[m\*q, n\*p]

Rule 1810

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int (d + ex)^3 (a + cx^2)^2 (A + Bx + Cx^2) dx &= \frac{d^2(Bd + 3Ae)(a + cx^2)^3}{6c} + \int (a + cx^2)^2 \left( -((Bd^3 + 3Ad^2e)x) + (d \right. \\
&= \frac{d^2(Bd + 3Ae)(a + cx^2)^3}{6c} + \int (a^2Ad^3 + ad(ad(Cd + 3Be) + A(2cd^2 + 3ae^2)) \\
&= a^2Ad^3x + \frac{1}{3}ad(ad(Cd + 3Be) + A(2cd^2 + 3ae^2))x^3 + \frac{1}{4}a^2e(3Cd^2 +
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 335, normalized size = 1.10

$$\frac{1}{2}e^2d^2(Ac + Bd) + e^2Ad^3 + \frac{1}{2}e^2(2a^2(Bc + 3Cd) + 3ad(Ac + Bd) + c^2d^2) + \frac{1}{2}e^2c^2(2aC^2 + ac(Ac + 3Bd) + 3cC^2) + \frac{1}{2}e^2(Ac(2a^2 + 3a^2) + Bd(6a^2 + c^2) + ac(a^2 + 6a^2)) + \frac{1}{2}e^2(Ac(6a^2 + c^2) + a(a^2(Bc + 3Cd) + 2a^2(Bc + Cd))) + \frac{1}{2}3ad^2(A(3a^2 + 2a^2) + ad(3Bc + Cd)) + \frac{1}{2}a^2(Ac^2 + 3aBd^2 + 3aC^2c + 6aC^2e + 2Bd^2) + \frac{1}{2}e^2d^2(Bc + 3Cd) + \frac{1}{10}e^2C^2d^3$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] a^2\*A\*d^3\*x + (a^2\*d^2\*(B\*d + 3\*A\*e)\*x^2)/2 + (a\*d\*(a\*d\*(C\*d + 3\*B\*e) + A\*(2\*c\*d^2 + 3\*a\*e^2))\*x^3)/3 + (a\*(2\*B\*c\*d^3 + 6\*A\*c\*d^2\*e + 3\*a\*C\*d^2\*e + 3\*a\*B\*d\*e^2 + a\*A\*e^3)\*x^4)/4 + ((A\*c\*d\*(c\*d^2 + 6\*a\*e^2) + a\*(a\*e^2\*(3\*C\*d + B\*e) + 2\*c\*d^2\*(C\*d + 3\*B\*e)))\*x^5)/5 + ((a\*C\*e\*(6\*c\*d^2 + a\*e^2) + A\*c\*e\*(3\*c\*d^2 + 2\*a\*e^2) + B\*c\*d\*(c\*d^2 + 6\*a\*e^2))\*x^6)/6 + (c\*(c\*C\*d^3 + 3\*c\*d\*e\*(B\*d + A\*e) + 2\*a\*e^2\*(3\*C\*d + B\*e))\*x^7)/7 + (c\*e\*(3\*c\*C\*d^2 + 2\*a\*C\*e^2 + c\*e\*(3\*B\*d + A\*e))\*x^8)/8 + (c^2\*e^2\*(3\*C\*d + B\*e)\*x^9)/9 + (c^2\*C\*e^3\*x^10)/10

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3 (a + cx^2)^2 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^3\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)^3\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.75, size = 432, normalized size = 1.42

$$\frac{1}{2}e^2d^2(Ac + Bd) + e^2Ad^3 + \frac{1}{2}e^2(2a^2(Bc + 3Cd) + 3ad(Ac + Bd) + c^2d^2) + \frac{1}{2}e^2c^2(2aC^2 + ac(Ac + 3Bd) + 3cC^2) + \frac{1}{2}e^2(Ac(2a^2 + 3a^2) + Bd(6a^2 + c^2) + ac(a^2 + 6a^2)) + \frac{1}{2}e^2(Ac(6a^2 + c^2) + a(a^2(Bc + 3Cd) + 2a^2(Bc + Cd))) + \frac{1}{2}3ad^2(A(3a^2 + 2a^2) + ad(3Bc + Cd)) + \frac{1}{2}a^2(Ac^2 + 3aBd^2 + 3aC^2c + 6aC^2e + 2Bd^2) + \frac{1}{2}e^2d^2(Bc + 3Cd) + \frac{1}{10}e^2C^2d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+a)^2\*(C\*x^2+B\*x+A), x, algorithm="fricas")



[Out]  $1/10*x^{10}*e^{3*c^2*C} + 1/3*x^9*e^{2*d*c^2*C} + 1/9*x^9*e^{3*c^2*B} + 3/8*x^8*e^{d^2*c^2*C} + 1/4*x^8*e^{3*c*a*C} + 3/8*x^8*e^{2*d*c^2*B} + 1/8*x^8*e^{3*c^2*A} + 1/7*x^7*d^3*c^2*C + 6/7*x^7*e^{2*d*c*a*C} + 3/7*x^7*e^{d^2*c^2*B} + 2/7*x^7*e^{3*c*a*B} + 3/7*x^7*e^{2*d*c^2*A} + x^6*e^{d^2*c*a*C} + 1/6*x^6*e^{3*a^2*C} + 1/6*x^6*d^3*c^2*B + x^6*e^{2*d*c*a*B} + 1/2*x^6*e^{d^2*c^2*A} + 1/3*x^6*e^{3*c*a*A} + 2/5*x^5*d^3*c*a*C + 3/5*x^5*e^{2*d*a^2*C} + 6/5*x^5*e^{d^2*c*a*B} + 1/5*x^5*e^{3*a^2*B} + 1/5*x^5*d^3*c^2*A + 6/5*x^5*e^{2*d*c*a*A} + 3/4*x^4*e^{d^2*a^2*C} + 1/2*x^4*d^3*c*a*B + 3/4*x^4*e^{2*d*a^2*B} + 3/2*x^4*e^{d^2*c*a*A} + 1/4*x^4*e^{3*a^2*A} + 1/3*x^3*d^3*a^2*C + x^3*e^{d^2*a^2*B} + 2/3*x^3*d^3*c*a*A + x^3*e^{2*d*a^2*A} + 1/2*x^2*d^3*a^2*B + 3/2*x^2*e^{d^2*a^2*A} + x*d^3*a^2*A$

**giac** [A] time = 0.16, size = 423, normalized size = 1.39

$\frac{1}{10} C^2 e^{3 c^2 C} x^{10} + \frac{1}{3} C e^{2 d c^2 C} x^9 + \frac{1}{9} B e^{3 c^2 B} x^9 + \frac{3}{8} C e^{d^2 c^2 C} x^8 + \frac{1}{4} C e^{3 c a C} x^8 + \frac{3}{8} B e^{2 d c^2 B} x^8 + \frac{1}{8} A e^{3 c^2 A} x^8 + \frac{6}{7} C e^{d^2 c^2 B} x^7 + \frac{6}{7} C e^{3 c a B} x^7 + \frac{3}{7} C e^{d^2 c^2 B} x^7 + \frac{2}{7} C e^{3 c a B} x^7 + \frac{3}{7} A e^{2 d c^2 A} x^7 + x^6 e^{d^2 c a C} + \frac{1}{6} C e^{3 a^2 C} x^6 + \frac{1}{6} C e^{d^2 c^2 B} x^6 + x^6 e^{2 d c a B} + \frac{1}{2} C e^{d^2 c^2 A} x^6 + \frac{1}{3} C e^{3 c a A} x^6 + \frac{2}{5} C e^{d^2 c^2 B} x^5 + \frac{3}{5} C e^{2 d a^2 C} x^5 + \frac{6}{5} C e^{d^2 c a B} x^5 + \frac{1}{5} C e^{3 a^2 B} x^5 + \frac{1}{5} C e^{d^2 c^2 A} x^5 + \frac{6}{5} C e^{2 d c a A} x^5 + \frac{3}{4} C e^{d^2 a^2 C} x^4 + \frac{1}{2} C e^{d^2 c a B} x^4 + \frac{3}{4} C e^{2 d a^2 B} x^4 + \frac{3}{2} C e^{d^2 c a A} x^4 + \frac{1}{4} C e^{3 a^2 A} x^4 + \frac{1}{3} C e^{d^2 a^2 C} x^3 + x^3 e^{d^2 a^2 B} + \frac{2}{3} C e^{d^2 c a A} x^3 + x^3 e^{2 d a^2 A} + \frac{1}{2} C e^{d^2 a^2 B} x^2 + \frac{3}{2} C e^{d^2 a^2 A} x^2 + x C e^{d^2 a^2 A}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")`

[Out]  $1/10*C*c^2*x^{10}*e^3 + 1/3*C*c^2*d*x^9*e^2 + 3/8*C*c^2*d^2*x^8*e + 1/7*C*c^2*d^3*x^7 + 1/9*B*c^2*x^9*e^3 + 3/8*B*c^2*d*x^8*e^2 + 3/7*B*c^2*d^2*x^7*e + 1/6*B*c^2*d^3*x^6 + 1/4*C*a*c*x^8*e^3 + 1/8*A*c^2*x^8*e^3 + 6/7*C*a*c*d*x^7*e^2 + 3/7*A*c^2*d*x^7*e^2 + C*a*c*d^2*x^6*e + 1/2*A*c^2*d^2*x^6*e + 2/5*C*a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 2/7*B*a*c*x^7*e^3 + B*a*c*d*x^6*e^2 + 6/5*B*a*c*d^2*x^5*e + 1/2*B*a*c*d^3*x^4 + 1/6*C*a^2*x^6*e^3 + 1/3*A*a*c*x^6*e^3 + 3/5*C*a^2*d*x^5*e^2 + 6/5*A*a*c*d*x^5*e^2 + 3/4*C*a^2*d^2*x^4*e + 3/2*A*a*c*d^2*x^4*e + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + 1/5*B*a^2*x^5*e^3 + 3/4*B*a^2*d*x^4*e^2 + B*a^2*d^2*x^3*e + 1/2*B*a^2*d^3*x^2 + 1/4*A*a^2*x^4*e^3 + A*a^2*d*x^3*e^2 + 3/2*A*a^2*d^2*x^2*e + A*a^2*d^3*x$

**maple** [A] time = 0.00, size = 385, normalized size = 1.27

$\frac{C^2 e^{3 c^2 C} x^{10}}{10} + \frac{C^2 e^{2 d c^2 C} x^9}{3} + \frac{B e^{3 c^2 B} x^9}{9} + \frac{3 C e^{d^2 c^2 C} x^8}{8} + \frac{C e^{3 c a C} x^8}{4} + \frac{3 B e^{2 d c^2 B} x^8}{8} + \frac{A e^{3 c^2 A} x^8}{8} + \frac{6 C e^{d^2 c^2 B} x^7}{7} + \frac{6 C e^{3 c a B} x^7}{7} + \frac{3 C e^{d^2 c^2 B} x^7}{7} + \frac{2 C e^{3 c a B} x^7}{7} + \frac{3 A e^{2 d c^2 A} x^7}{7} + x^6 e^{d^2 c a C} + \frac{C e^{3 a^2 C} x^6}{6} + \frac{C e^{d^2 c^2 B} x^6}{6} + x^6 e^{2 d c a B} + \frac{C e^{d^2 c^2 A} x^6}{2} + \frac{C e^{3 c a A} x^6}{3} + \frac{2 C e^{d^2 c^2 B} x^5}{5} + \frac{3 C e^{2 d a^2 C} x^5}{5} + \frac{6 C e^{d^2 c a B} x^5}{5} + \frac{C e^{3 a^2 B} x^5}{5} + \frac{C e^{d^2 c^2 A} x^5}{5} + \frac{6 C e^{2 d c a A} x^5}{5} + \frac{3 C e^{d^2 a^2 C} x^4}{4} + \frac{C e^{d^2 c a B} x^4}{2} + \frac{3 C e^{2 d a^2 B} x^4}{4} + \frac{3 C e^{d^2 c a A} x^4}{2} + \frac{C e^{3 a^2 A} x^4}{4} + \frac{C e^{d^2 a^2 C} x^3}{3} + x^3 e^{d^2 a^2 B} + \frac{2 C e^{d^2 c a A} x^3}{3} + x^3 e^{2 d a^2 A} + \frac{C e^{d^2 a^2 B} x^2}{2} + \frac{3 C e^{d^2 a^2 A} x^2}{2} + x C e^{d^2 a^2 A}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x)`

[Out]  $1/10*c^2*C*e^3*x^{10}+1/9*(B*c^2*e^3+3*C*c^2*d*e^2)*x^9+1/8*((2*a*c*e^3+3*c^2*d^2*e)*C+3*d*e^2*c^2*B+e^3*c^2*A)*x^8+1/7*((6*a*c*d*e^2+c^2*d^3)*C+(2*a*c*e^3+3*c^2*d^2*e)*B+3*d*e^2*c^2*A)*x^7+1/6*((a^2*e^3+6*a*c*d^2*e)*C+(6*a*c*d*e^2+c^2*d^3)*B+(2*a*c*e^3+3*c^2*d^2*e)*A)*x^6+1/5*((3*a^2*d*e^2+2*a*c*d^3)*C+(a^2*e^3+6*a*c*d^2*e)*B+(6*a*c*d*e^2+c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^2*C+(3*a^2*d*e^2+2*a*c*d^3)*B+(a^2*e^3+6*a*c*d^2*e)*A)*x^4+1/3*(d^3*a^2*C+3*d^2*e*a^2*B+(3*a^2*d*e^2+2*a*c*d^3)*A)*x^3+1/2*(3*A*a^2*d^2*e+B*a^2*d^3)*x^2+a^2*A*d^3*x$



$$\begin{aligned}
& e^{**2/5} + A*c^{**2}*d^{**3/5} + B*a^{**2}*e^{**3/5} + 6*B*a*c*d^{**2}*e/5 + 3*C*a^{**2}*d*e^{**2} \\
& /5 + 2*C*a*c*d^{**3/5}) + x^{**4}*(A*a^{**2}*e^{**3/4} + 3*A*a*c*d^{**2}*e/2 + 3*B*a^{**2}*d* \\
& e^{**2/4} + B*a*c*d^{**3/2} + 3*C*a^{**2}*d^{**2}*e/4) + x^{**3}*(A*a^{**2}*d*e^{**2} + 2*A*a*c* \\
& d^{**3/3} + B*a^{**2}*d^{**2}*e + C*a^{**2}*d^{**3/3}) + x^{**2}*(3*A*a^{**2}*d^{**2}*e/2 + B*a^{**2}* \\
& d^{**3/2})
\end{aligned}$$

$$3.26 \quad \int (d + ex)^2 (a + cx^2)^2 (A + Bx + Cx^2) dx$$

Optimal. Leaf size=217

$$a^2 Ad^2x + \frac{1}{4}a^2 ex^4(Be + 2Cd) + \frac{1}{7}cx^7(2aCe^2 + c(e(Ae + 2Bd) + Cd^2)) + \frac{1}{5}x^5(Ac(2ae^2 + cd^2) + a(aCe^2 + 2cd(2Be +$$

**Rubi [A]** time = 0.31, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1582, 1810}

$$a^2 Ad^2x + \frac{1}{4}a^2 ex^4(Be + 2Cd) + \frac{1}{7}cx^7(2aCe^2 + c(e(Ae + 2Bd) + Cd^2)) + \frac{1}{5}x^5(Ac(2ae^2 + cd^2) + a(aCe^2 + 2cd(2Be + Cd))) + \frac{d(a + cx^2)^3(2Ae + Bd)}{6c} + \frac{1}{3}acex^6(Be + 2Cd) + \frac{1}{8}c^2ex^8(Be + 2Cd) + \frac{1}{9}c^2Cx^9$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] a^2\*A\*d^2\*x + (a\*(a\*d\*(C\*d + 2\*B\*e) + A\*(2\*c\*d^2 + a\*e^2))\*x^3)/3 + (a^2\*e\*(2\*C\*d + B\*e)\*x^4)/4 + ((A\*c\*(c\*d^2 + 2\*a\*e^2) + a\*(a\*C\*e^2 + 2\*c\*d\*(C\*d + 2\*B\*e)))\*x^5)/5 + (a\*c\*e\*(2\*C\*d + B\*e)\*x^6)/3 + (c\*(c\*C\*d^2 + 2\*a\*C\*e^2 + c\*e\*(2\*B\*d + A\*e))\*x^7)/7 + (c^2\*e\*(2\*C\*d + B\*e)\*x^8)/8 + (c^2\*C\*e^2\*x^9)/9 + (d\*(B\*d + 2\*A\*e)\*(a + c\*x^2)^3)/(6\*c)

Rule 1582

Int[(Px\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(Coeff[Px, x, n - 1]\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]\*x^(n - 1))\*(a + b\*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]\*x^(n - 1)] && !MatchQ[Px, (Qx\_.)\*((c\_) + (d\_.)\*x^(m\_))^(q\_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx\*(a + b\*x^n)^p, x, m - 1], 0] && GtQ[m\*q, n\*p]]

Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps





$$2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c + A*c^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*a*c)*d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2$$

**mupad [B]** time = 3.72, size = 244, normalized size = 1.12

$$x^3 \left( \frac{C^2 d^2}{3} + \frac{2 B^2 d e}{3} + \frac{A^2 e^2}{3} + \frac{2 A C a d^2}{3} \right) + x^2 \left( \frac{C^2 d^2}{7} + \frac{2 B^2 d e}{7} + \frac{A^2 e^2}{7} + \frac{2 C a c d^2}{7} \right) + x \left( \frac{C^2 d^2}{5} + \frac{2 C a c d^2}{5} + \frac{4 B a e d e}{5} + \frac{2 A a c c^2}{5} + \frac{A^2 d^2}{5} \right) + \frac{A^4 (B a e^2 + 2 B c d^2 + 4 A c d e + 2 C a d e)}{4} + \frac{c^4 (2 B a e^2 + B c d^2 + 2 A c d e + 4 C a d e)}{6} + \frac{C^2 d^2 x^9}{9} + \frac{A^2 d^2 x}{A^2 d^2 x} + \frac{d^2 d^2 (2 A e + B d)}{2} + \frac{c^2 e^2 (B e + 2 C d)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^2\*(d + e\*x)^2\*(A + B\*x + C\*x^2), x)

$$[Out] x^3*((A*a^2*e^2)/3 + (C*a^2*d^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) + x^7*((A*c^2*e^2)/7 + (C*c^2*d^2)/7 + (2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7) + x^5*((A*c^2*d^2)/5 + (C*a^2*e^2)/5 + (2*A*a*c*e^2)/5 + (2*C*a*c*d^2)/5 + (4*B*a*c*d*e)/5) + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e + 2*C*a*d*e))/4 + (c*x^6*(2*B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 4*C*a*d*e))/6 + (C*c^2*e^2*x^9)/9 + A*a^2*d^2*x + (a^2*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^8*(B*e + 2*C*d))/8$$

**sympy [A]** time = 0.12, size = 311, normalized size = 1.43

$$A^2 d^2 x + \frac{C^2 d^2 x^9}{9} + x^3 \left( \frac{B^2 e^2}{8} + \frac{C^2 d e}{4} \right) + x^2 \left( \frac{A^2 e^2}{7} + \frac{2 B^2 d e}{7} + \frac{2 C a c d^2}{7} + \frac{C^2 d^2}{7} \right) + x^5 \left( \frac{A^2 d e}{3} + \frac{B a c^2}{3} + \frac{B^2 d^2}{6} + \frac{2 C a c d e}{3} \right) + x^7 \left( \frac{2 A a c c^2}{5} + \frac{A^2 d^2}{5} + \frac{4 B a e d e}{5} + \frac{C a^2 e^2}{5} + \frac{2 C a c d^2}{5} \right) + x^9 \left( \frac{A a c d e}{4} + \frac{B a^2 e^2}{4} + \frac{B a c d^2}{2} + \frac{C a^2 d e}{2} \right) + x^3 \left( \frac{A a^2 e^2}{3} + \frac{2 A a c d^2}{3} + \frac{2 B^2 d e}{3} + \frac{C a^2 d^2}{3} \right) + x^2 \left( \frac{A a^2 d e}{2} + \frac{B a^2 d^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+a)\*\*2\*(C\*x\*\*2+B\*x+A), x)

$$[Out] A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) + x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) + x**6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*(2*A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*d**2/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) + x**3*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) + x**2*(A*a**2*d*e + B*a**2*d**2/2)$$

$$3.27 \quad \int (d + ex) (a + cx^2)^2 (A + Bx + Cx^2) dx$$

Optimal. Leaf size=128

$$a^2 Adx + \frac{1}{4} a^2 Cex^4 + \frac{1}{5} cx^5 (2a(Be + Cd) + Acd) + \frac{1}{3} ax^3 (aBe + aCd + 2Acd) + \frac{(a + cx^2)^3 (Ae + Bd)}{6c} + \frac{1}{3} acCex^6 + \frac{1}{7} c^2 x^7 (Be + Cd) + \frac{1}{8} c^2 Cex^8$$

**Rubi [A]** time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {1582, 1810}

$$a^2 Adx + \frac{1}{4} a^2 Cex^4 + \frac{1}{5} cx^5 (2a(Be + Cd) + Acd) + \frac{1}{3} ax^3 (aBe + aCd + 2Acd) + \frac{(a + cx^2)^3 (Ae + Bd)}{6c} + \frac{1}{3} acCex^6 + \frac{1}{7} c^2 x^7 (Be + Cd) + \frac{1}{8} c^2 Cex^8$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2),x]

[Out] a^2\*A\*d\*x + (a\*(2\*A\*c\*d + a\*C\*d + a\*B\*e)\*x^3)/3 + (a^2\*C\*e\*x^4)/4 + (c\*(A\*c\*d + 2\*a\*(C\*d + B\*e))\*x^5)/5 + (a\*c\*C\*e\*x^6)/3 + (c^2\*(C\*d + B\*e)\*x^7)/7 + (c^2\*C\*e\*x^8)/8 + ((B\*d + A\*e)\*(a + c\*x^2)^3)/(6\*c)

### Rule 1582

Int[(Px\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(Coeff[Px, x, n - 1]\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]\*x^(n - 1))\*(a + b\*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]\*x^(n - 1)] && !MatchQ[Px, (Qx\_)\*((c\_) + (d\_)\*x^(m\_))^(q\_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx\*(a + b\*x^n)^p, x, m - 1], 0] && GtQ[m\*q, n\*p]]

### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps



$$\begin{aligned} \int (d + ex)(a + cx^2)^2 (A + Bx + Cx^2) dx &= \frac{(Bd + Ae)(a + cx^2)^3}{6c} + \int (a + cx^2)^2 (-(Bd + Ae)x + (d + ex)(A + Bx + Cx^2)) dx \\ &= \frac{(Bd + Ae)(a + cx^2)^3}{6c} + \int (a^2 Ad + a(2Acd + aCd + aBe)x^2 + a^2 Cex^3 - (Bd + Ae)x^2 + (d + ex)(A + Bx + Cx^2)) dx \\ &= a^2 Adx + \frac{1}{3}a(2Acd + aCd + aBe)x^3 + \frac{1}{4}a^2 Cex^4 + \frac{1}{5}c(Acd + 2a(Cd + Bd)x + dA)x^5 - \frac{1}{6}(Bd + Ae)x^6 + \frac{1}{7}(d + ex)(A + Bx + Cx^2)x^7 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 144, normalized size = 1.12

$$\frac{1}{2}a^2x^2(Ae + Bd) + a^2Adx + \frac{1}{6}cx^6(2aCe + Ace + Bcd) + \frac{1}{5}cx^5(2aBe + 2aCd + Acd) + \frac{1}{4}ax^4(aCe + 2Ace + 2Bcd) + \frac{1}{3}ax^3(aBe + aCd + 2Acd) + \frac{1}{7}c^2x^7(Be + Cd) + \frac{1}{8}c^2Cex^8$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] a^2\*A\*d\*x + (a^2\*(B\*d + A\*e)\*x^2)/2 + (a\*(2\*A\*c\*d + a\*C\*d + a\*B\*e)\*x^3)/3 + (a\*(2\*B\*c\*d + 2\*A\*c\*e + a\*C\*e)\*x^4)/4 + (c\*(A\*c\*d + 2\*a\*C\*d + 2\*a\*B\*e)\*x^5)/5 + (c\*(B\*c\*d + A\*c\*e + 2\*a\*C\*e)\*x^6)/6 + (c^2\*(C\*d + B\*e)\*x^7)/7 + (c^2\*C\*e\*x^8)/8

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)(a + cx^2)^2 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)\*(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.88, size = 172, normalized size = 1.34

$$\frac{1}{8}x^8ec^2C + \frac{1}{7}x^7dc^2C + \frac{1}{7}x^7ec^2B + \frac{1}{3}x^6ecaC + \frac{1}{6}x^6dc^2B + \frac{1}{6}x^6ec^2A + \frac{2}{5}x^5dcaC + \frac{2}{5}x^5ecaB + \frac{1}{5}x^5dc^2A + \frac{1}{4}x^4ea^2C + \frac{1}{2}x^4dcaB + \frac{1}{2}x^4ecaA + \frac{1}{3}x^3da^2C + \frac{1}{3}x^3ea^2B + \frac{2}{3}x^3dcaA + \frac{1}{2}x^2da^2B + \frac{1}{2}x^2ea^2A + xda^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)^2\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/8\*x^8\*e\*c^2\*C + 1/7\*x^7\*d\*c^2\*C + 1/7\*x^7\*e\*c^2\*B + 1/3\*x^6\*e\*c\*a\*C + 1/6\*x^6\*d\*c^2\*B + 1/6\*x^6\*e\*c^2\*A + 2/5\*x^5\*d\*c\*a\*C + 2/5\*x^5\*e\*c\*a\*B + 1/5\*x^5\*d\*c^2\*A + 1/4\*x^4\*e\*a^2\*C + 1/2\*x^4\*d\*c\*a\*B + 1/2\*x^4\*e\*c\*a\*A + 1/3\*x^3\*d

$$*a^2*C + 1/3*x^3*e*a^2*B + 2/3*x^3*d*c*a*A + 1/2*x^2*d*a^2*B + 1/2*x^2*e*a^2*A + x*d*a^2*A$$

**giac** [A] time = 0.15, size = 181, normalized size = 1.41

$$\frac{1}{8}C^2x^8e + \frac{1}{7}C^2dx^7 + \frac{1}{7}Bc^2x^7e + \frac{1}{6}Bc^2dx^6 + \frac{1}{3}Cacx^6e + \frac{1}{6}Ac^2x^6e + \frac{2}{5}Caedx^5 + \frac{1}{5}Ac^2dx^5 + \frac{2}{5}Bacx^5e + \frac{1}{2}Bacdx^4 + \frac{1}{4}Ca^2x^4e + \frac{1}{2}Aacx^4e + \frac{1}{3}Ca^2dx^3 + \frac{2}{3}Aacdx^3 + \frac{1}{3}Ba^2x^3e + \frac{1}{2}Ba^2dx^2 + \frac{1}{2}Aa^2x^2e + Aa^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)^2\*(C\*x^2+B\*x+A),x, algorithm="giac")

[Out]  $\frac{1}{8}C^2c^2x^8e + \frac{1}{7}C^2c^2d*x^7 + \frac{1}{7}B^2c^2x^7e + \frac{1}{6}B^2c^2d*x^6 + \frac{1}{3}C^2a*c*x^6e + \frac{1}{6}A^2c^2*x^6e + \frac{2}{5}C^2a*c*d*x^5 + \frac{1}{5}A^2c^2*d*x^5 + \frac{2}{5}B^2a*c*x^5e + \frac{1}{2}B^2a*c*d*x^4 + \frac{1}{4}C^2a^2*x^4e + \frac{1}{2}A^2a*c*x^4e + \frac{1}{3}C^2a^2*d*x^3 + \frac{2}{3}A^2a*c*d*x^3 + \frac{1}{3}B^2a^2*x^3e + \frac{1}{2}B^2a^2*d*x^2 + \frac{1}{2}A^2a^2*x^2e + A^2a^2*d*x$

**maple** [A] time = 0.00, size = 151, normalized size = 1.18

$$\frac{C^2ex^8}{8} + \frac{(c^2eB + c^2dC)x^7}{7} + \frac{(c^2eA + c^2dB + 2eacC)x^6}{6} + Aa^2dx + \frac{(c^2dA + 2eacB + 2dacC)x^5}{5} + \frac{(2eacA + 2dacB + e^2C)x^4}{4} + \frac{(2dacA + e^2B + d^2C)x^3}{3} + \frac{(e^2A + d^2B)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(c\*x^2+a)^2\*(C\*x^2+B\*x+A),x)

[Out]  $\frac{1}{8}c^2C^2ex^8 + \frac{1}{7}(B^2c^2e + C^2c^2d)x^7 + \frac{1}{6}(A^2c^2e + B^2c^2d + 2C^2a^2c^2e)x^6 + \frac{1}{5}(A^2c^2d + 2B^2a^2c^2e + 2C^2a^2c^2d)x^5 + \frac{1}{4}(2A^2a^2c^2e + 2B^2a^2c^2d + C^2a^2e)x^4 + \frac{1}{3}(2A^2a^2c^2d + B^2a^2e + C^2a^2d)x^3 + \frac{1}{2}(A^2a^2e + B^2a^2d)x^2 + a^2A^2dx$

**maxima** [A] time = 0.45, size = 154, normalized size = 1.20

$$\frac{1}{8}C^2ex^8 + \frac{1}{7}(C^2d + Bc^2e)x^7 + \frac{1}{6}(Bc^2d + (2Cac + Ac^2)e)x^6 + \frac{1}{5}(2Bace + (2Cac + Ac^2)d)x^5 + Aa^2dx + \frac{1}{4}(2Bacd + (Ca^2 + 2Aac)e)x^4 + \frac{1}{3}(Ba^2e + (Ca^2 + 2Aac)d)x^3 + \frac{1}{2}(Ba^2d + Aa^2e)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)^2\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out]  $\frac{1}{8}C^2c^2ex^8 + \frac{1}{7}(C^2c^2d + B^2c^2e)x^7 + \frac{1}{6}(B^2c^2d + (2C^2a^2c + A^2c^2)e)x^6 + \frac{1}{5}(2B^2a^2c^2e + (2C^2a^2c + A^2c^2)d)x^5 + A^2a^2d*x + \frac{1}{4}(2B^2a^2c^2d + (C^2a^2 + 2A^2a^2c^2)e)x^4 + \frac{1}{3}(B^2a^2e + (C^2a^2 + 2A^2a^2c^2)d)x^3 + \frac{1}{2}(B^2a^2d + A^2a^2e)x^2$

**mupad** [B] time = 3.69, size = 140, normalized size = 1.09

$$x^3 \left( \frac{Ba^2e}{3} + \frac{Ca^2d}{3} + \frac{2Aacd}{3} \right) + x^6 \left( \frac{Ac^2e}{6} + \frac{Bc^2d}{6} + \frac{Cace}{3} \right) + \frac{cx^5(Acd + 2Bae + 2Cad)}{5} + \frac{ax^4(2Ace + 2Bcd + Cae)}{4} + \frac{a^2x^2(Ae + Bd)}{2} + \frac{c^2x^7(Be + Cd)}{7} + Aa^2dx + \frac{C^2ex^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^2*(d + e*x)*(A + B*x + C*x^2), x)`

[Out]  $x^3*((B*a^2*e)/3 + (C*a^2*d)/3 + (2*A*a*c*d)/3) + x^6*((A*c^2*e)/6 + (B*c^2*d)/6 + (C*a*c*e)/3) + (c*x^5*(A*c*d + 2*B*a*e + 2*C*a*d))/5 + (a*x^4*(2*A*c*e + 2*B*c*d + C*a*e))/4 + (a^2*x^2*(A*e + B*d))/2 + (c^2*x^7*(B*e + C*d))/7 + A*a^2*d*x + (C*c^2*e*x^8)/8$

**sympy [A]** time = 0.10, size = 180, normalized size = 1.41

$$Aa^2dx + \frac{C^2ex^8}{8} + x^7\left(\frac{Bc^2e}{7} + \frac{Cc^2d}{7}\right) + x^6\left(\frac{Ac^2e}{6} + \frac{Bc^2d}{6} + \frac{Cace}{3}\right) + x^5\left(\frac{Ac^2d}{5} + \frac{2Bace}{5} + \frac{2Cacd}{5}\right) + x^4\left(\frac{Aace}{2} + \frac{Bacd}{2} + \frac{Ca^2e}{4}\right) + x^3\left(\frac{2Aacd}{3} + \frac{Ba^2e}{3} + \frac{Ca^2d}{3}\right) + x^2\left(\frac{Aa^2e}{2} + \frac{Ba^2d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A), x)`

[Out]  $A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c**2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*c*d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B*a**2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)$

$$3.28 \quad \int (a + cx^2)^2 (A + Bx + Cx^2) dx$$

Optimal. Leaf size=67

$$a^2Ax + \frac{1}{5}cx^5(2aC + Ac) + \frac{1}{3}ax^3(aC + 2Ac) + \frac{B(a + cx^2)^3}{6c} + \frac{1}{7}c^2Cx^7$$

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1582, 373}

$$a^2Ax + \frac{1}{5}cx^5(2aC + Ac) + \frac{1}{3}ax^3(aC + 2Ac) + \frac{B(a + cx^2)^3}{6c} + \frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] a^2\*A\*x + (a\*(2\*A\*c + a\*C)\*x^3)/3 + (c\*(A\*c + 2\*a\*C)\*x^5)/5 + (c^2\*C\*x^7)/7 + (B\*(a + c\*x^2)^3)/(6\*c)

### Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

### Rule 1582

Int[(Px\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(Coeff[Px, x, n - 1]\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]\*x^(n - 1))\*(a + b\*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]\*x^(n - 1)] && !MatchQ[Px, (Qx\_.)\*((c\_) + (d\_.)\*x^(m\_))^(q\_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx\*(a + b\*x^n)^p, x, m - 1], 0] && GtQ[m\*q, n\*p]]

### Rubi steps

$$\begin{aligned}
\int (a + cx^2)^2 (A + Bx + Cx^2) dx &= \frac{B(a + cx^2)^3}{6c} + \int (a + cx^2)^2 (A + Cx^2) dx \\
&= \frac{B(a + cx^2)^3}{6c} + \int (a^2A + a(2Ac + aC)x^2 + c(Ac + 2aC)x^4 + c^2Cx^6) dx \\
&= a^2Ax + \frac{1}{3}a(2Ac + aC)x^3 + \frac{1}{5}c(Ac + 2aC)x^5 + \frac{1}{7}c^2Cx^7 + \frac{B(a + cx^2)^3}{6c}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 69, normalized size = 1.03

$$\frac{1}{210}x(35a^2(6A + x(3B + 2Cx)) + 7acx^2(20A + 3x(5B + 4Cx)) + c^2x^4(42A + 5x(7B + 6Cx)))$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] (x\*(35\*a^2\*(6\*A + x\*(3\*B + 2\*C\*x)) + 7\*a\*c\*x^2\*(20\*A + 3\*x\*(5\*B + 4\*C\*x)) + c^2\*x^4\*(42\*A + 5\*x\*(7\*B + 6\*C\*x)))/210

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + cx^2)^2 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + c\*x^2)^2\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.62, size = 76, normalized size = 1.13

$$\frac{1}{7}x^7c^2C + \frac{1}{6}x^6c^2B + \frac{2}{5}x^5caC + \frac{1}{5}x^5c^2A + \frac{1}{2}x^4caB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3caA + \frac{1}{2}x^2a^2B + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^2\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/7\*x^7\*c^2\*C + 1/6\*x^6\*c^2\*B + 2/5\*x^5\*c\*a\*C + 1/5\*x^5\*c^2\*A + 1/2\*x^4\*c\*a\*B + 1/3\*x^3\*a^2\*C + 2/3\*x^3\*c\*a\*A + 1/2\*x^2\*a^2\*B + x\*a^2\*A

**giac [A]** time = 0.15, size = 76, normalized size = 1.13

$$\frac{1}{7} C c^2 x^7 + \frac{1}{6} B c^2 x^6 + \frac{2}{5} C a c x^5 + \frac{1}{5} A c^2 x^5 + \frac{1}{2} B a c x^4 + \frac{1}{3} C a^2 x^3 + \frac{2}{3} A a c x^3 + \frac{1}{2} B a^2 x^2 + A a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^2\*(C\*x^2+B\*x+A),x, algorithm="giac")

[Out] 1/7\*C\*c^2\*x^7 + 1/6\*B\*c^2\*x^6 + 2/5\*C\*a\*c\*x^5 + 1/5\*A\*c^2\*x^5 + 1/2\*B\*a\*c\*x^4 + 1/3\*C\*a^2\*x^3 + 2/3\*A\*a\*c\*x^3 + 1/2\*B\*a^2\*x^2 + A\*a^2\*x

**maple [A]** time = 0.00, size = 75, normalized size = 1.12

$$\frac{C c^2 x^7}{7} + \frac{B c^2 x^6}{6} + \frac{B a c x^4}{2} + \frac{B a^2 x^2}{2} + \frac{(A c^2 + 2 a c C) x^5}{5} + A a^2 x + \frac{(2 a c A + a^2 C) x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^2\*(C\*x^2+B\*x+A),x)

[Out] 1/7\*c^2\*C\*x^7+1/6\*c^2\*B\*x^6+1/5\*(A\*c^2+2\*C\*a\*c)\*x^5+1/2\*a\*c\*B\*x^4+1/3\*(2\*A\*a\*c+C\*a^2)\*x^3+1/2\*a^2\*B\*x^2+a^2\*A\*x

**maxima [A]** time = 0.44, size = 74, normalized size = 1.10

$$\frac{1}{7} C c^2 x^7 + \frac{1}{6} B c^2 x^6 + \frac{1}{2} B a c x^4 + \frac{1}{5} (2 C a c + A c^2) x^5 + \frac{1}{2} B a^2 x^2 + A a^2 x + \frac{1}{3} (C a^2 + 2 A a c) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^2\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out] 1/7\*C\*c^2\*x^7 + 1/6\*B\*c^2\*x^6 + 1/2\*B\*a\*c\*x^4 + 1/5\*(2\*C\*a\*c + A\*c^2)\*x^5 + 1/2\*B\*a^2\*x^2 + A\*a^2\*x + 1/3\*(C\*a^2 + 2\*A\*a\*c)\*x^3

**mupad [B]** time = 0.04, size = 74, normalized size = 1.10

$$x^3 \left( \frac{C a^2}{3} + \frac{2 A c a}{3} \right) + x^5 \left( \frac{A c^2}{5} + \frac{2 C a c}{5} \right) + \frac{B a^2 x^2}{2} + \frac{B c^2 x^6}{6} + \frac{C c^2 x^7}{7} + A a^2 x + \frac{B a c x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^2\*(A + B\*x + C\*x^2),x)

[Out] x^3\*((C\*a^2)/3 + (2\*A\*a\*c)/3) + x^5\*((A\*c^2)/5 + (2\*C\*a\*c)/5) + (B\*a^2\*x^2)/2 + (B\*c^2\*x^6)/6 + (C\*c^2\*x^7)/7 + A\*a^2\*x + (B\*a\*c\*x^4)/2

sympy [A] time = 0.08, size = 83, normalized size = 1.24

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Bacx^4}{2} + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + x^5 \left( \frac{Ac^2}{5} + \frac{2Cac}{5} \right) + x^3 \left( \frac{2Aac}{3} + \frac{Ca^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*2\*(C\*x\*\*2+B\*x+A),x)

[Out] A\*a\*\*2\*x + B\*a\*\*2\*x\*\*2/2 + B\*a\*c\*x\*\*4/2 + B\*c\*\*2\*x\*\*6/6 + C\*c\*\*2\*x\*\*7/7 + x\*\*5\*(A\*c\*\*2/5 + 2\*C\*a\*c/5) + x\*\*3\*(2\*A\*a\*c/3 + C\*a\*\*2/3)

$$3.29 \quad \int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex} dx$$

**Optimal.** Leaf size=297

$$\frac{x^2(a^2Ce^4 + 2ace^2(Cd^2 - e(Bd - Ae)) + c^2d^2(Cd^2 - e(Bd - Ae)))}{2e^5} - \frac{x(a^2e^4(Cd - Be) + 2acde^2(Cd^2 - e(Bd - Ae)))}{e^6}$$

**Rubi [A]** time = 0.64, antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$ , Rules used = {1628}

$$\frac{x^2(a^2Ce^4 + 2ace^2(Cd^2 - e(Bd - Ae)) + c^2(Cd^2 - e(Bd - Ae)))}{2e^5} - \frac{x(a^2e^4(Cd - Be) + 2acde^2(Cd^2 - e(Bd - Ae)))}{e^6} + \frac{c^2(2a^2Cd - ce(Bd - Ae) + cCd^2)}{3e^5} - \frac{c^3(2a^2(Cd - Be) - ce(Bd - Ae) + cCd^2)}{3e^5} + \frac{(a^2 + ce^2) \log(d + ex)(Ae^2 - Bde + Cd^2)}{e^6} - \frac{c^2x^2(Cd - Be)}{5e^5} + \frac{c^2Cx^2}{6e^6}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x), x]

[Out] -(((a^2\*e^4\*(C\*d - B\*e) + 2\*a\*c\*d\*e^2\*(C\*d^2 - e\*(B\*d - A\*e)) + c^2\*(C\*d^5 - d^3\*e\*(B\*d - A\*e)))\*x)/e^6 + ((a^2\*C\*e^4 + 2\*a\*c\*e^2\*(C\*d^2 - e\*(B\*d - A\*e)) + c^2\*(C\*d^4 - d^2\*e\*(B\*d - A\*e)))\*x^2)/(2\*e^5) - (c\*(c\*C\*d^3 - c\*d\*e\*(B\*d - A\*e) + 2\*a\*e^2\*(C\*d - B\*e))\*x^3)/(3\*e^4) + (c\*(c\*C\*d^2 + 2\*a\*C\*e^2 - c\*e\*(B\*d - A\*e))\*x^4)/(4\*e^3) - (c^2\*(C\*d - B\*e)\*x^5)/(5\*e^2) + (c^2\*C\*x^6)/(6\*e) + ((c\*d^2 + a\*e^2)^2\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[d + e\*x])/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex} dx = \int \left( \frac{-a^2e^4(Cd - Be) - 2acde^2(Cd^2 - e(Bd - Ae)) - c^2(Cd^5 - d^3e(Bd - Ae))}{e^6} \right. \\ \left. - \frac{(a^2e^4(Cd - Be) + 2acde^2(Cd^2 - e(Bd - Ae)) + c^2(Cd^5 - d^3e(Bd - Ae)))x}{e^6} \right) dx$$

**Mathematica [A]** time = 0.17, size = 285, normalized size = 0.96

$$\frac{cx(30a^2e^4(2Be - 2Cd + Cex) + 10ace^2(2(3Ae(ex - 2d) + B(6d^2 - 3dex + 2e^2x^2)) + C(-12d^3 + 6d^2ex - 4d^2x^2 + 3e^3x^3)) + c^2(e(5Ae(-12d^3 + 6d^2ex - 4d^2x^2 + 3e^3x^3)) + B(60d^4 - 30d^3ex + 20d^2e^2x^2 - 15d^2e^3x + 12e^4x^4)) + C(-60d^5 + 30d^4ex - 20d^3e^2x^2 + 15d^2e^3x^3 - 12d^2e^4x^4 + 10e^5x^5)) + 60(a^2 + ce^2) \log(d + ex)(Ae - Bd) + Cd^2)}{60e^6}$$







**mupad [B]** time = 3.68, size = 422, normalized size = 1.42

$$x^5 \left( \frac{Bc^2}{5e} - \frac{C^2d}{5e^2} \right) - x \left( \frac{d \left( \frac{A^2 + 2Cac}{4e} - \frac{Bc^2}{4e} \right)}{e} + \frac{Bc^2}{e} + x^4 \left( \frac{A^2 + 2Cac}{4e} - \frac{d \left( \frac{Bc^2}{4e} - \frac{C^2d}{4e} \right)}{3e} \right) - x^3 \left( \frac{d \left( \frac{A^2 + 2Cac}{4e} - \frac{Bc^2}{4e} \right)}{3e} - \frac{2Bac}{3e} \right) + x^2 \left( \frac{C^2d + 2Acs}{2e} + \frac{d \left( \frac{A^2 + 2Cac}{4e} - \frac{Bc^2}{4e} \right)}{2e} \right) + \frac{\ln(d+ex) (C^2d^2 - B^2d^2 + A^2d^2 + 2Cacd^2 - 2Bacd^2 + 2Aacd^2 + C^2d^2 - B^2d^2 + A^2d^2)}{e^2} + \frac{C^2d}{5e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x), x)`

[Out]  $x^5 * ((B*c^2)/(5*e) - (C*c^2*d)/(5*e^2)) - x * ((d * ((C*a^2 + 2*A*a*c)/e + (d * ((d * ((A*c^2 + 2*C*a*c)/e - (d * ((B*c^2)/e - (C*c^2*d)/e^2))/e)) / e - (2*B*a*c)/e)) / e) - (B*a^2)/e + x^4 * ((A*c^2 + 2*C*a*c)/(4*e) - (d * ((B*c^2)/e - (C*c^2*d)/e^2)) / (4*e)) - x^3 * ((d * ((A*c^2 + 2*C*a*c)/e - (d * ((B*c^2)/e - (C*c^2*d)/e^2)) / e) / (3*e) - (2*B*a*c)/(3*e)) + x^2 * ((C*a^2 + 2*A*a*c)/(2*e) + (d * ((d * ((A*c^2 + 2*C*a*c)/e - (d * ((B*c^2)/e - (C*c^2*d)/e^2)) / e)) / e - (2*B*a*c)/e) / (2*e)) + (\log(d + e*x) * (A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)) / e^7 + (C*c^2*x^6)/(6*e)$

**sympy [A]** time = 0.94, size = 359, normalized size = 1.21

$$\frac{C^2d^6}{6e} + x^5 \left( \frac{Bc^2}{5e} - \frac{C^2d}{5e^2} \right) + x^4 \left( \frac{A^2}{4e} - \frac{Bc^2}{4e^2} + \frac{Cac}{2e} + \frac{C^2d^2}{4e^3} \right) + x^3 \left( -\frac{A^2d}{3e^2} + \frac{2Bac}{3e} + \frac{Bc^2d}{3e^3} - \frac{2Cacd}{3e^2} - \frac{C^2d^2}{3e^3} \right) + x^2 \left( \frac{Aac}{e} + \frac{Ac^2d^2}{2e^2} - \frac{Bacd}{e^2} - \frac{Bc^2d^3}{2e^3} + \frac{C^2d}{2e} + \frac{Cacd^2}{e^3} + \frac{C^2d^4}{2e^3} \right) + x \left( -\frac{2Aacd}{e^2} - \frac{Ac^2d^3}{e^3} + \frac{Bd^2}{e} + \frac{2Bacd^2}{e^3} + \frac{Bc^2d^4}{e^3} - \frac{C^2d}{e^2} - \frac{2Cacd^3}{e^3} - \frac{C^2d^4}{e^3} \right) + \frac{(ac^2 + cd^2) (A^2 - Bde + Cd^2) \log(d + ex)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d), x)`

[Out]  $C*c**2*x**6/(6*e) + x**5*(B*c**2/(5*e) - C*c**2*d/(5*e**2)) + x**4*(A*c**2/(4*e) - B*c**2*d/(4*e**2) + C*a*c/(2*e) + C*c**2*d**2/(4*e**3)) + x**3*(-A*c**2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3) - 2*C*a*c*d/(3*e**2) - C*c**2*d**3/(3*e**4)) + x**2*(A*a*c/e + A*c**2*d**2/(2*e**3) - B*a*c*d/e**2 - B*c**2*d**3/(2*e**4) + C*a**2/(2*e) + C*a*c*d**2/e**3 + C*c**2*d**4/(2*e**5)) + x*(-2*A*a*c*d/e**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c*d**2/e**3 + B*c**2*d**4/e**5 - C*a**2*d/e**2 - 2*C*a*c*d**3/e**4 - C*c**2*d**5/e**6) + (a*e**2 + c*d**2)**2*(A*e**2 - B*d*e + C*d**2)*\log(d + e*x)/e**7$

$$3.30 \quad \int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2} dx$$

**Optimal.** Leaf size=292

$$\frac{x(a^2Ce^4 + 2ace^2(3Cd^2 - e(2Bd - Ae)) + c^2d^2(5Cd^2 - e(4Bd - 3Ae)))}{e^6} - \frac{(ae^2 + cd^2)^2(Ae^2 - Bde + Cd^2)}{e^7(d+ex)} - \frac{(ae^2 + cd^2)^2(Ae^2 - Bde + Cd^2)}{e^7(d+ex)}$$

**Rubi [A]** time = 0.53, antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$ , Rules used = {1628}

$$\frac{x(a^2Ce^4 + 2ace^2(3Cd^2 - e(2Bd - Ae)) + c^2d^2(5Cd^2 - e(4Bd - 3Ae)))}{e^6} - \frac{cx^2(2aC^2 - c(2Bd - Ae) + 3cCd^2)}{3e^4} - \frac{c^2(2a^2(2Cd - Be) - cd(3Bd - 2Ae) + 4cCd^2)}{2e^5} - \frac{(ae^2 + cd^2)^2(Ae^2 - Bde + Cd^2)}{e^7(d+ex)} - \frac{(ae^2 + cd^2) \log(d+ex)(ae^2(2Cd - Be) - cd(5Bd - 4Ae) + 6cCd^2)}{e^7} - \frac{c^2x^4(2Cd - Be)}{4e^5} + \frac{c^2Cx^5}{5e^2}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^2, x]

[Out] ((a^2\*C\*e^4 + c^2\*(5\*C\*d^4 - d^2\*e\*(4\*B\*d - 3\*A\*e)) + 2\*a\*c\*e^2\*(3\*C\*d^2 - e\*(2\*B\*d - A\*e)))\*x)/e^6 - (c\*(4\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 2\*A\*e) + 2\*a\*e^2\*(2\*C\*d - B\*e))\*x^2)/(2\*e^5) + (c\*(3\*c\*C\*d^2 + 2\*a\*C\*e^2 - c\*e\*(2\*B\*d - A\*e))\*x^3)/(3\*e^4) - (c^2\*(2\*C\*d - B\*e)\*x^4)/(4\*e^3) + (c^2\*C\*x^5)/(5\*e^2) - ((c\*d^2 + a\*e^2)^2\*(C\*d^2 - B\*d\*e + A\*e^2))/(e^7\*(d + e\*x)) - ((c\*d^2 + a\*e^2)\*(6\*c\*C\*d^3 - c\*d\*e\*(5\*B\*d - 4\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))\*Log[d + e\*x])/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2} dx = \int \left( \frac{a^2Ce^4 + c^2(5Cd^4 - d^2e(4Bd - 3Ae)) + 2ace^2(3Cd^2 - e(2Bd - Ae))}{e^6} + \frac{c}{e^6} \right) dx$$

$$= \frac{(a^2Ce^4 + c^2(5Cd^4 - d^2e(4Bd - 3Ae)) + 2ace^2(3Cd^2 - e(2Bd - Ae)))x}{e^6} - \frac{c}{e^6}$$

**Mathematica [A]** time = 0.28, size = 272, normalized size = 0.93

$$\frac{60cx(a^2Ce^4 + 2ace^2(e(Ae - 2Bd) + 3Cd^2) + c^2(d^2e(3Ae - 4Bd) + 5Cd^4)) - 30cx^2e^2(-2a^2(Be - 2Cd) + cd(2Ae - 3Bd) + 4cCd^2) - \frac{60(a^2 + c^2)^2(Ae - Bde + Cd^2)}{e^7} + 20cx^3e^3(2aC^2 + c(Ae - 2Bd) + 3cCd^2) - 60(a^2 + c^2) \log(d+ex)(ae^2(2Cd - Be) + cd(4Ae - 5Bd) + 6cCd^2) + 15c^2e^4(Be - 2Cd) + 12c^2Cx^5}{60e^7}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^2\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="giac")

[Out]  $\frac{1}{60}(12C^2c^2 - 15(6C^2cd^2e - B^2c^2e^2))e^{-1}/(xe + d) + 20(15C^2c^2d^2e^2 - 5B^2c^2d^2e^3 + 2C^2ac^2e^4 + A^2c^2e^4)e^{-2}/(xe + d)^2 - 60(10C^2c^2d^3e^3 - 5B^2c^2d^2e^4 + 4C^2ac^2d^2e^5 + 2A^2c^2d^2e^5 - B^2ac^2e^6)e^{-3}/(xe + d)^3 + 60(15C^2c^2d^4e^4 - 10B^2c^2d^3e^5 + 12C^2ac^2d^2e^6 + 6A^2c^2d^2e^6 - 6B^2ac^2d^2e^7 + C^2a^2e^8 + 2A^2ac^2e^8)e^{-4}/(xe + d)^4 * (xe + d)^5e^{-7} + (6C^2c^2d^5 - 5B^2c^2d^4e + 8C^2ac^2d^3e^2 + 4A^2c^2d^3e^2 - 6B^2ac^2d^2e^3 + 2C^2a^2d^2e^4 + 4A^2ac^2d^2e^4 - B^2a^2e^5)e^{-7} * \log(\text{abs}(xe + d))e^{-1}/(xe + d)^2 - (C^2c^2d^6e^5/(xe + d) - B^2c^2d^5e^6/(xe + d) + 2C^2ac^2d^4e^7/(xe + d) + A^2c^2d^4e^7/(xe + d) - 2B^2ac^2d^3e^8/(xe + d) + C^2a^2d^2e^9/(xe + d) + 2A^2ac^2d^2e^9/(xe + d) - B^2a^2d^2e^{10}/(xe + d) + A^2a^2e^{11}/(xe + d))e^{-12}$

**maple [A]** time = 0.01, size = 527, normalized size = 1.80

C^2d^6 - B^2c^2d^5e - 2B^2ac^2d^3e^2 + A^2c^2d^3e^2 + (2C^2c + A^2)d^2e^4 + (C^2 + 2Aa)d^2e^4, 12C^2c^2d^5 - 15(2C^2cd^3 - B^2c^2d^3) + 20(4C^2c^2d^2 - 2B^2cd^2 + (2C^2c + A^2)d^2) - 30(4C^2c^2d - 3B^2cd^2 - 2Baa^2 + 2(2C^2c + A^2)d^2) + 60(5C^2c^2d - 4B^2cd^2 - 4Baa^2 + 3(2C^2c + A^2)d^2 + (C^2 + 2Aa)d^2); (6C^2d^6 - 5B^2c^2d^5e - 6Baa^2d^3 - B^2d^2 + 4(2C^2c + A^2)d^2 + 2(C^2 + 2Aa)d^2) log(|xe + d|)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^2\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x)

[Out]  $-2/e^3C^2x^2a^2cd - 4/e^3d^2a^2cBx + 6/e^4C^2a^2cd^2x - 2/e^3/(e^2x+d)A^2a^2cd^2 + 2/e^4/(e^2x+d)B^2a^2cd^3 - 2/e^5/(e^2x+d)C^2a^2cd^4 - 4/e^3\ln(e^2x+d)A^2a^2cd + 6/e^4\ln(e^2x+d)B^2a^2cd^2 - 8/e^5\ln(e^2x+d)C^2a^2cd^3 + 1/5c^2C^2x^5/e^2 - 1/e/(e^2x+d)A^2a^2 + 1/e^2\ln(e^2x+d)B^2a^2 + 1/4/e^2B^2x^4c^2 + 1/3/e^2A^2x^3c^2 + 1/e^2a^2C^2x - 4/e^5\ln(e^2x+d)A^2c^2d^3 + 5/e^6\ln(e^2x+d)B^2c^2d^4 - 4/e^5B^2c^2d^3x + 5/e^6C^2c^2d^4x - 1/e^5/(e^2x+d)A^2c^2d^4 + 1/e^2/(e^2x+d)B^2d^2a^2 + 1/e^6/(e^2x+d)B^2c^2d^5 - 1/e^3/(e^2x+d)C^2a^2d^2 - 1/e^7/(e^2x+d)C^2c^2d^6 + 2/3/e^2C^2x^3a^2 + 1/e^4C^2x^3c^2d^2 - 1/e^3A^2x^2c^2d + 1/e^2B^2x^2a^2c + 3/2/e^4B^2x^2c^2d^2 - 2/e^5C^2x^2c^2d^3 + 2/e^2A^2a^2cx + 3/e^4A^2c^2d^2x - 1/2/e^3C^2x^4c^2d - 2/3/e^3B^2x^3c^2d - 2/e^3\ln(e^2x+d)C^2a^2d - 6/e^7\ln(e^2x+d)C^2c^2d^5$

**maxima [A]** time = 0.48, size = 392, normalized size = 1.34

C^2d^6 - B^2c^2d^5e - 2B^2ac^2d^3e^2 + A^2c^2d^3e^2 + (2C^2c + A^2)d^2e^4 + (C^2 + 2Aa)d^2e^4, 12C^2c^2d^5 - 15(2C^2cd^3 - B^2c^2d^3) + 20(4C^2c^2d^2 - 2B^2cd^2 + (2C^2c + A^2)d^2) - 30(4C^2c^2d - 3B^2cd^2 - 2Baa^2 + 2(2C^2c + A^2)d^2) + 60(5C^2c^2d - 4B^2cd^2 - 4Baa^2 + 3(2C^2c + A^2)d^2 + (C^2 + 2Aa)d^2); (6C^2d^6 - 5B^2c^2d^5e - 6Baa^2d^3 - B^2d^2 + 4(2C^2c + A^2)d^2 + 2(C^2 + 2Aa)d^2) log(|xe + d|)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^2\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-(C^2c^2d^6 - B^2c^2d^5e - 2B^2a^2cd^3e^3 - B^2a^2d^2e^5 + A^2a^2e^6 + (2C^2ac^2 + A^2c^2)d^4e^2 + (C^2a^2 + 2A^2ac^2)d^2e^4)/(e^8x + d^2e^7) + 1/60(12C^2c^2e^4x^5 - 15(2C^2c^2d^2e^3 - B^2c^2e^4)x^4 + 20(3C^2c^2d^2e^2$

$$2 - 2*B*c^2*d*e^3 + (2*C*a*c + A*c^2)*e^4)*x^3 - 30*(4*C*c^2*d^3*e - 3*B*c^2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c + A*c^2)*d*e^3)*x^2 + 60*(5*C*c^2*d^4 - 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + 3*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2*A*a*c)*e^4)*x)/e^6 - (6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3 - B*a^2*e^5 + 4*(2*C*a*c + A*c^2)*d^3*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e^4)*log(e*x + d)/e^7$$

**mupad [B]** time = 0.12, size = 575, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^2, x)

$$\begin{aligned} \text{[Out]} & x^4*((B*c^2)/(4*e^2) - (C*c^2*d)/(2*e^3)) + x*((C*a^2 + 2*A*a*c)/e^2 + (d^2 * ((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e^2 + (2*B*a*c)/e^2))/e - x^3*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(3*e) - (A*c^2 + 2*C*a*c)/(3*e^2) + (C*c^2*d^2)/(3*e^4)) + x^2*((d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(2*e^2) + (B*a*c)/e^2) - (A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)/(e*(d*e^6 + e^7*x)) - (log(d + e*x)*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 - 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a*c*d^3*e^2))/e^7 + (C*c^2*x^5)/(5*e^2) \end{aligned}$$

**sympy [A]** time = 2.79, size = 416, normalized size = 1.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*2\*(C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*2, x)

$$\begin{aligned} \text{[Out]} & C*c**2*x**5/(5*e**2) + x**4*(B*c**2/(4*e**2) - C*c**2*d/(2*e**3)) + x**3*(A*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3) + 2*C*a*c/(3*e**2) + C*c**2*d**2/e**4) + x**2*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4) - 2*C*a*c*d/e**3 - 2*C*c**2*d**3/e**5) + x*(2*A*a*c/e**2 + 3*A*c**2*d**2/e**4 - 4*B*a*c*d/e**3 - 4*B*c**2*d**3/e**5 + C*a**2/e**2 + 6*C*a*c*d**2/e**4 + 5*C*c**2*d**4/e**6) + (-A*a**2*e**6 - 2*A*a*c*d**2*e**4 - A*c**2*d**4*e**2 + B*a**2*d**e**5 + 2*B*a*c*d**3*e**3 + B*c**2*d**5*e - C*a**2*d**2*e**4 - 2*C*a*c*d**4*e**2 - C*c**2*d**6)/(d*e**7 + e**8*x) - (a*e**2 + c*d**2)*(4*A*c*d*e**2 - B*a*e**3 - 5*B*c*d**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e**7 \end{aligned}$$

$$3.31 \quad \int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3} dx$$

**Optimal.** Leaf size=295

$$\frac{\log(d+ex) \left( a^2 C e^4 + 2 a c e^2 (6 C d^2 - e(3 B d - A e)) + c^2 d^2 (15 C d^2 - 2 e(5 B d - 3 A e)) \right)}{e^7} + \frac{(a e^2 + c d^2) (a e^2 (2 C d - B e))}{e^7 (d+ex)^2}$$

**Rubi [A]** time = 0.49, antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$ , Rules used = {1628}

$$\frac{\log(d+ex) \left( a^2 C e^4 + 2 a c e^2 (e(3 B d - A e) + 6 C d^2) \right)}{e^7} + \frac{c^2 (2 a C e^2 - e(3 B d - A e) + 6 C d^2)}{2 e^6} - \frac{c x (2 a^2 (3 C d - B e) - 3 c d e (2 B d - A e) + 10 c C d^2)}{e^6} + \frac{(a^2 + c d^2) (a^2 (2 C d - B e) - c d e (5 B d - 4 A e) + 6 c C d^2)}{e^2 (d+ex)} + \frac{(a^2 + c d^2) (A e^2 - B d e + C d^2)}{2 e^2 (d+ex)^2} + \frac{e^2 x (3 C d - B e)}{3 e^4} + \frac{e^2 C d^4}{4 e^4}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] -((c\*(10\*c\*C\*d^3 - 3\*c\*d\*e\*(2\*B\*d - A\*e) + 2\*a\*e^2\*(3\*C\*d - B\*e))\*x)/e^6) + (c\*(6\*c\*C\*d^2 + 2\*a\*C\*e^2 - c\*e\*(3\*B\*d - A\*e))\*x^2)/(2\*e^5) - (c^2\*(3\*C\*d - B\*e)\*x^3)/(3\*e^4) + (c^2\*C\*x^4)/(4\*e^3) - ((c\*d^2 + a\*e^2)^2\*(C\*d^2 - B\*d\*e + A\*e^2))/(2\*e^7\*(d + e\*x)^2) + ((c\*d^2 + a\*e^2)\*(6\*c\*C\*d^3 - c\*d\*e\*(5\*B\*d - 4\*A\*e) + a\*e^2\*(2\*C\*d - B\*e)))/(e^7\*(d + e\*x)) + ((a^2\*C\*e^4 + c^2\*(15\*C\*d^4 - 2\*d^2\*e\*(5\*B\*d - 3\*A\*e)) + 2\*a\*c\*e^2\*(6\*C\*d^2 - e\*(3\*B\*d - A\*e)))\*Log[d + e\*x])/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_.))^m\_.]\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3} dx = \int \left( \frac{c(-10cCd^3 + 3cde(2Bd - Ae) - 2ae^2(3Cd - Be))}{e^6} + \frac{c(6cCd^2 + 2aCe^2 - ce^4)}{e^5} \right. \\ \left. - \frac{c(10cCd^3 - 3cde(2Bd - Ae) + 2ae^2(3Cd - Be))x}{e^6} + \frac{c(6cCd^2 + 2aCe^2 - ce^4)}{2e^5} \right) dx$$

**Mathematica [A]** time = 0.12, size = 274, normalized size = 0.93

$$\frac{12 \log(d+ex) \left( a^2 C e^4 + 2 a c e^2 (e(Ae - 3Bd) + 6 C d^2) \right) - 12 c x \left( -2 a^2 (Be - 3Cd) + 3 c d e (Ae - 2Bd) + 10 c C d^2 \right) + 6 c e^2 x^2 \left( 2 a C e^2 + e(Ae - 3Bd) + 6 C d^2 \right) - \frac{6(a^2 + c d^2) \left( c(Ae - B d) + C d^2 \right)}{(d+ex)^2} + \frac{12(a^2 + c d^2) \left( a^2 (2 C d - B e) + c d e (4 A e - 5 B d) + 6 C d^2 \right)}{d+ex} + 4 e^2 x^3 (Be - 3Cd) + 3 e^2 C d^4}{12 e^7}$$



Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] (-12\*c\*e\*(10\*c\*C\*d^3 + 3\*c\*d\*e\*(-2\*B\*d + A\*e) - 2\*a\*e^2\*(-3\*C\*d + B\*e))\*x + 6\*c\*e^2\*(6\*c\*C\*d^2 + 2\*a\*C\*e^2 + c\*e\*(-3\*B\*d + A\*e))\*x^2 + 4\*c^2\*e^3\*(-3\*C\*d + B\*e)\*x^3 + 3\*c^2\*C\*e^4\*x^4 - (6\*(c\*d^2 + a\*e^2)^2\*(C\*d^2 + e\*(-(B\*d) + A\*e)))/(d + e\*x)^2 + (12\*(c\*d^2 + a\*e^2)\*(6\*c\*C\*d^3 + c\*d\*e\*(-5\*B\*d + 4\*A\*e) + a\*e^2\*(2\*C\*d - B\*e)))/(d + e\*x) + 12\*(a^2\*C\*e^4 + 2\*a\*c\*e^2\*(6\*C\*d^2 + e\*(-3\*B\*d + A\*e)) + c^2\*(15\*C\*d^4 + 2\*d^2\*e\*(-5\*B\*d + 3\*A\*e)))\*Log[d + e\*x])/(12\*e^7)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^2 (A + Bx + Cx^2)}{(d + ex)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] IntegrateAlgebraic[((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^3, x]

**fricas** [B] time = 0.63, size = 608, normalized size = 2.06

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^2\*(C\*x^2+B\*x+A)/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/12\*(3\*C\*c^2\*e^6\*x^6 + 66\*C\*c^2\*d^6 - 54\*B\*c^2\*d^5\*e - 60\*B\*a\*c\*d^3\*e^3 - 6\*B\*a^2\*d\*e^5 - 6\*A\*a^2\*e^6 + 42\*(2\*C\*a\*c + A\*c^2)\*d^4\*e^2 + 18\*(C\*a^2 + 2\*A\*a\*c)\*d^2\*e^4 - 2\*(3\*C\*c^2\*d\*e^5 - 2\*B\*c^2\*e^6)\*x^5 + (15\*C\*c^2\*d^2\*e^4 - 10\*B\*c^2\*d\*e^5 + 6\*(2\*C\*a\*c + A\*c^2)\*e^6)\*x^4 - 4\*(15\*C\*c^2\*d^3\*e^3 - 10\*B\*c^2\*d^2\*e^4 - 6\*B\*a\*c\*e^6 + 6\*(2\*C\*a\*c + A\*c^2)\*d\*e^5)\*x^3 - 6\*(34\*C\*c^2\*d^4\*e^2 - 21\*B\*c^2\*d^3\*e^3 - 8\*B\*a\*c\*d\*e^5 + 11\*(2\*C\*a\*c + A\*c^2)\*d^2\*e^4)\*x^2 - 12\*(4\*C\*c^2\*d^5\*e - B\*c^2\*d^4\*e^2 + 4\*B\*a\*c\*d^2\*e^4 + B\*a^2\*e^6 - (2\*C\*a\*c + A\*c^2)\*d^3\*e^3 - 2\*(C\*a^2 + 2\*A\*a\*c)\*d\*e^5)\*x + 12\*(15\*C\*c^2\*d^6 - 10\*B\*c^2\*d^5\*e - 6\*B\*a\*c\*d^3\*e^3 + 6\*(2\*C\*a\*c + A\*c^2)\*d^4\*e^2 + (C\*a^2 + 2\*A\*a\*c)\*d^2\*e^4 + (15\*C\*c^2\*d^4\*e^2 - 10\*B\*c^2\*d^3\*e^3 - 6\*B\*a\*c\*d\*e^5 + 6\*(2\*C\*a\*c + A\*c^2)\*d^2\*e^4 + (C\*a^2 + 2\*A\*a\*c)\*e^6)\*x^2 + 2\*(15\*C\*c^2\*d^5\*e - 10\*B\*c^2\*d^4\*e^2 - 6\*B\*a\*c\*d^2\*e^4 + 6\*(2\*C\*a\*c + A\*c^2)\*d^3\*e^3 + (C\*a^2 + 2\*A\*a\*c)\*d\*e^5)\*x)\*log(e\*x + d))/(e^9\*x^2 + 2\*d\*e^8\*x + d^2\*e^7)

**giac** [A] time = 0.16, size = 397, normalized size = 1.35

(5 C^2 d^2 - 9 B^2 d^2 + 14 C a d^2 + 1 A^2 d^2 - 10 B a d^2 + 3 C^2 d^2 + 6 A a d^2 - 3 B^2 d^2 - A d^2 + 2) (C^2 d^2 - 5 B^2 d^2 + 9 C a d^2 + 4 A^2 d^2 - 9 B a d^2 + 3 C^2 d^2 + 4 A a d^2 - 3 B^2 d^2)^(1/2)



$$+ 1/12*(3*C*c^2*e^3*x^4 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c^2*d^2*e - 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^2 - 12*(10*C*c^2*d^3 - 6*B*c^2*d^2*e - 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x)/e^6 + (15*C*c^2*d^4 - 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + 6*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2*A*a*c)*e^4)*log(e*x + d)/e^7$$

**mupad [B]** time = 3.82, size = 495, normalized size = 1.68

$$\left( \frac{3d \left( \frac{15d^4 + 10d^3e + 6d^2e^2 + 3de^3 + 3e^4}{12} \right) + 3d^2 \left( \frac{15d^3 + 10d^2e + 6de^2 + 3e^3}{12} \right) + 3d^3 \left( \frac{15d^2 + 10de + 6e^2}{12} \right) + 3d^4 \left( \frac{15d + 10e}{12} \right) + 3d^5 \left( \frac{15}{12} \right)}{e^6} + \frac{15C^2d^4 + 10CB^2d^3 + 6C^2B^2d^2 + 3C^2B^3d + 3C^2B^4}{e^6} + \frac{15C^2d^4 + 10CB^2d^3 + 6C^2B^2d^2 + 3C^2B^3d + 3C^2B^4}{e^6} \right) \log(dx + d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^2\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x)

$$\begin{aligned} \text{[Out]} & x*((3*d*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e - (A*c^2 + 2*C*a*c)/e^3 + \\ & (3*C*c^2*d^2)/e^5))/e - (3*d^2*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e^2 + (2*B* \\ & a*c)/e^3 - (C*c^2*d^3)/e^6 + x^3*((B*c^2)/(3*e^3) - (C*c^2*d)/e^4) - x^2*( \\ & (3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/(2*e) - (A*c^2 + 2*C*a*c)/(2*e^3) + ( \\ & 3*C*c^2*d^2)/(2*e^5) + ((11*C*c^2*d^6 - A*a^2*e^6 - B*a^2*d*e^5 - 9*B*c^2* \\ & d^5*e + 7*A*c^2*d^4*e^2 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - 10*B*a*c*d^3* \\ & e^3 + 14*C*a*c*d^4*e^2)/(2*e) + x*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 \\ & - 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a \\ & *c*d^3*e^2))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + (log(d + e*x)*(C*a^2*e^4 + 1 \\ & 5*C*c^2*d^4 + 2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*B*a*c*d*e^ \\ & 3 + 12*C*a*c*d^2*e^2))/e^7 + (C*c^2*x^4)/(4*e^3) \end{aligned}$$

**sympy [A]** time = 14.20, size = 474, normalized size = 1.61

$$\frac{C^2d^4}{12e^6} + \frac{3d \left( \frac{15d^4 + 10d^3e + 6d^2e^2 + 3de^3 + 3e^4}{12} \right) + 3d^2 \left( \frac{15d^3 + 10d^2e + 6de^2 + 3e^3}{12} \right) + 3d^3 \left( \frac{15d^2 + 10de + 6e^2}{12} \right) + 3d^4 \left( \frac{15d + 10e}{12} \right) + 3d^5 \left( \frac{15}{12} \right)}{e^6} + \frac{15C^2d^4 + 10CB^2d^3 + 6C^2B^2d^2 + 3C^2B^3d + 3C^2B^4}{e^6} + \frac{15C^2d^4 + 10CB^2d^3 + 6C^2B^2d^2 + 3C^2B^3d + 3C^2B^4}{e^6} \log(dx + d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*2\*(C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*3,x)

$$\begin{aligned} \text{[Out]} & C*c**2*x**4/(4*e**3) + x**3*(B*c**2/(3*e**3) - C*c**2*d/e**4) + x**2*(A*c** \\ & 2/(2*e**3) - 3*B*c**2*d/(2*e**4) + C*a*c/e**3 + 3*C*c**2*d**2/e**5) + x*(-3 \\ & *A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5 - 6*C*a*c*d/e**4 - 10*C* \\ & c**2*d**3/e**6) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e**2 - \\ & B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e**4 + \\ & 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d**3*e* \\ & *3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*a**2*d* \\ & e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x + \\ & 2*e**9*x**2) + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3 - 10*B*c \\ & **2*d**3*e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*log(d + e*x \\ & )/e**7 \end{aligned}$$

$$3.32 \quad \int (d + ex)^3 (a + cx^2)^3 (A + Bx + Cx^2) dx$$

**Optimal.** Leaf size=404

$$\frac{1}{4}a^3ex^4(e(Ae + 3Bd) + 3Cd^2) + a^3Ad^3x + \frac{1}{6}a^2ex^6(aCe^2 + 3c(e(Ae + 3Bd) + 3Cd^2)) + \frac{1}{3}a^2dx^3(3A(ae^2 + cd^2) + ad^2)$$

**Rubi [A]** time = 0.69, antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1582, 1810}

$$\frac{1}{4}a^3ex^4(e(Ae + 3Bd) + 3Cd^2) + a^3Ad^3x + \frac{1}{6}a^2ex^6(aCe^2 + 3c(e(Ae + 3Bd) + 3Cd^2)) + \frac{1}{3}a^2dx^3(3A(ae^2 + cd^2) + ad^2)$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out]  $a^3A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^2 + a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12 + (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)$

**Rule 1582**

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

**Rule 1810**

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

**Rubi steps**

$$\begin{aligned}
\int (d + ex)^3 (a + cx^2)^3 (A + Bx + Cx^2) dx &= \frac{d^2(Bd + 3Ae)(a + cx^2)^4}{8c} + \int (a + cx^2)^3 \left( -((Bd^3 + 3Ad^2e)x) + \right. \\
&= \frac{d^2(Bd + 3Ae)(a + cx^2)^4}{8c} + \int (a^3Ad^3 + a^2d(ad(Cd + 3Be) + 3A \\
&= a^3Ad^3x + \frac{1}{3}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^3 + \frac{1}{4}a^3e(3Cd^2
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 459, normalized size = 1.14

$\frac{1}{12}a^3Ad^3x + \frac{1}{3}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^3 + \frac{1}{4}a^3e(3Cd^2 + 3Ae^2)x^4 + \frac{1}{5}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^5 + \frac{1}{6}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^6 + \frac{1}{7}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^7 + \frac{1}{8}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^8 + \frac{1}{9}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^9 + \frac{1}{10}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^{10} + \frac{1}{11}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^{11} + \frac{1}{12}a^2d(ad(Cd + 3Be) + 3A(cd^2 + ae^2))x^{12}$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out]  $a^3Ad^3x + (a^3d^2(Bd + 3Ae))x^2/2 + (a^2d(ad(Cd + 3Be) + 3Ae^2))x^3/3 + (a^2d^2(3Bcd^3 + 9Acd^2e + 3aCd^2e + 3aBde^2 + aAe^3))x^4/4 + (a(3Acd^2(c^2d^2 + 3ae^2) + a(ae^2(3Cd + Be) + 3c^2d^2(Cd + 3Be))))x^5/5 + (a(3Acd^2(3c^2d^2 + ae^2) + aC^2e(9c^2d^2 + ae^2) + 3Bcd^2(c^2d^2 + 3ae^2)))x^6/6 + (c(Acd^2(c^2d^2 + 9ae^2) + 3a(ae^2(3Cd + Be) + c^2d^2(Cd + 3Be))))x^7/7 + (c(Bcd^2(c^2d^2 + 9ae^2) + 3e(Acd^2 + ae^2) + aC(3c^2d^2 + ae^2)))x^8/8 + (c^2(cCd^3 + 3c^2de(Bd + Ae) + 3ae^2(3Cd + Be)))x^9/9 + (c^2e(3cCd^2 + 3aC^2e^2 + ce(3Bd + Ae)))x^{10}/10 + (c^3e^2(3Cd + Be))x^{11}/11 + (c^3C^2e^3)x^{12}/12$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3 (a + cx^2)^3 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^3\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)^3\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.63, size = 618, normalized size = 1.53

...

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="fricas")

[Out]  $\frac{1}{12}x^{12}e^3c^3C + \frac{3}{11}x^{11}e^2d^3c^3C + \frac{1}{11}x^{11}e^3c^3B + \frac{3}{10}x^{10}e^3d^2c^3C + \frac{3}{10}x^{10}e^3c^2a^3C + \frac{3}{10}x^{10}e^2d^3c^3B + \frac{1}{10}x^{10}e^3c^3A + \frac{1}{9}x^9d^3c^3C + x^9e^2d^3c^2a^3C + \frac{1}{3}x^9e^3d^2c^3B + \frac{1}{3}x^9e^3c^2a^3B + \frac{1}{3}x^9e^2d^3c^3A + \frac{9}{8}x^8e^2d^2c^2a^3C + \frac{3}{8}x^8e^3c^2a^2C + \frac{1}{8}x^8d^3c^3B + \frac{9}{8}x^8e^2d^2c^2a^3B + \frac{3}{8}x^8e^3d^2c^3A + \frac{3}{8}x^8e^3c^2a^3A + \frac{3}{7}x^7d^3c^2a^3C + \frac{9}{7}x^7e^2d^3c^2a^2C + \frac{9}{7}x^7e^3d^2c^2a^3B + \frac{3}{7}x^7e^3c^2a^2B + \frac{1}{7}x^7d^3c^3A + \frac{9}{7}x^7e^2d^3c^2a^3A + \frac{3}{2}x^6e^3d^2c^2a^2C + \frac{1}{6}x^6e^3a^3C + \frac{1}{2}x^6d^3c^2a^3B + \frac{3}{2}x^6e^2d^3c^2a^2B + \frac{3}{2}x^6e^2d^2c^2a^3A + \frac{1}{2}x^6e^3c^2a^2A + \frac{3}{5}x^5d^3c^2a^2C + \frac{3}{5}x^5e^2d^3a^3C + \frac{9}{5}x^5e^2d^2c^2a^2B + \frac{1}{5}x^5e^3a^3B + \frac{3}{5}x^5d^3c^2a^3A + \frac{9}{5}x^5e^2d^3c^2a^2A + \frac{3}{4}x^4e^3d^2a^3C + \frac{3}{4}x^4d^3c^2a^2B + \frac{3}{4}x^4e^2d^3a^3B + \frac{9}{4}x^4e^2d^2c^2a^2A + \frac{1}{4}x^4e^3a^3A + \frac{1}{3}x^3d^3a^3C + x^3e^2d^2a^3B + x^3d^3c^2a^2A + x^3e^2d^2a^3A + \frac{1}{2}x^2d^3a^3B + \frac{3}{2}x^2e^2d^2a^3A + xd^3a^3A$

**giac** [A] time = 0.20, size = 606, normalized size = 1.50

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="giac")

[Out]  $\frac{1}{12}C^3c^3x^{12}e^3 + \frac{3}{11}C^3c^3d^3x^{11}e^2 + \frac{3}{10}C^3c^3d^2x^{10}e + \frac{1}{9}C^3c^3d^3x^9 + \frac{1}{11}B^3c^3x^{11}e^3 + \frac{3}{10}B^3c^3d^3x^{10}e^2 + \frac{1}{3}B^3c^3d^2x^9e + \frac{1}{8}B^3c^3d^3x^8 + \frac{3}{10}C^3a^3c^2x^{10}e^3 + \frac{1}{10}A^3c^3x^{10}e^3 + C^3a^3c^2d^3x^9e^2 + \frac{1}{3}A^3c^3d^3x^9e^2 + \frac{9}{8}C^3a^3c^2d^2x^8e + \frac{3}{8}A^3c^3d^2x^8e + \frac{3}{7}C^3a^3c^2d^3x^7 + \frac{1}{7}A^3c^3d^3x^7 + \frac{1}{3}B^3a^3c^2x^9e^3 + \frac{9}{8}B^3a^3c^2d^3x^8e^2 + \frac{9}{7}B^3a^3c^2d^2x^7e + \frac{1}{2}B^3a^3c^2d^3x^6 + \frac{3}{8}C^3a^2c^3x^8e^3 + \frac{3}{8}A^3a^3c^2x^8e^3 + \frac{9}{7}C^3a^2c^3d^3x^7e^2 + \frac{9}{7}A^3a^3c^2d^3x^7e^2 + \frac{3}{2}C^3a^2c^3d^2x^6e + \frac{3}{2}A^3a^3c^2d^2x^6e + \frac{3}{5}C^3a^2c^3d^3x^5 + \frac{3}{5}A^3a^3c^2d^3x^5 + \frac{3}{7}B^3a^2c^3x^7e^3 + \frac{3}{2}B^3a^2c^3d^3x^6e^2 + \frac{9}{5}B^3a^2c^3d^2x^5e + \frac{3}{4}B^3a^2c^3d^3x^4 + \frac{1}{6}C^3a^3x^6e^3 + \frac{1}{2}A^3a^2c^3x^6e^3 + \frac{3}{5}C^3a^3d^3x^5e^2 + \frac{9}{5}A^3a^2c^3d^3x^5e^2 + \frac{3}{4}C^3a^3d^2x^4e + \frac{9}{4}A^3a^2c^3d^2x^4e + \frac{1}{3}C^3a^3d^3x^3 + A^3a^2c^3d^3x^3 + \frac{1}{5}B^3a^3x^5e^3 + \frac{3}{4}B^3a^3d^3x^4e^2 + B^3a^3d^2x^3e + \frac{1}{2}B^3a^3d^3x^2 + \frac{1}{4}A^3a^3x^4e^3 + A^3a^3d^3x^3e^2 + \frac{3}{2}A^3a^3d^2x^2e + A^3a^3d^3x$

**maple** [A] time = 0.00, size = 553, normalized size = 1.37

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x)

```
[Out] 1/12*c^3*C*e^3*x^12+1/11*(B*c^3*e^3+3*C*c^3*d*e^2)*x^11+1/10*((3*a*c^2*e^3+
3*c^3*d^2*e)*C+3*d*e^2*c^3*B+e^3*c^3*A)*x^10+1/9*((9*a*c^2*d*e^2+c^3*d^3)*C
+(3*a*c^2*e^3+3*c^3*d^2*e)*B+3*d*e^2*c^3*A)*x^9+1/8*((3*a^2*c*e^3+9*a*c^2*d
^2*e)*C+(9*a*c^2*d*e^2+c^3*d^3)*B+(3*a*c^2*e^3+3*c^3*d^2*e)*A)*x^8+1/7*((9*
a^2*c*d*e^2+3*a*c^2*d^3)*C+(3*a^2*c*e^3+9*a*c^2*d^2*e)*B+(9*a*c^2*d*e^2+c^3
*d^3)*A)*x^7+1/6*((a^3*e^3+9*a^2*c*d^2*e)*C+(9*a^2*c*d*e^2+3*a*c^2*d^3)*B+(
3*a^2*c*e^3+9*a*c^2*d^2*e)*A)*x^6+1/5*((3*a^3*d*e^2+3*a^2*c*d^3)*C+(a^3*e^3
+9*a^2*c*d^2*e)*B+(9*a^2*c*d*e^2+3*a*c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^3*C+(3*
a^3*d*e^2+3*a^2*c*d^3)*B+(a^3*e^3+9*a^2*c*d^2*e)*A)*x^4+1/3*(d^3*a^3*C+3*d^
2*e*a^3*B+(3*a^3*d*e^2+3*a^2*c*d^3)*A)*x^3+1/2*(3*A*a^3*d^2*e+B*a^3*d^3)*x^
2+a^3*A*d^3*x
```

**maxima [A]** time = 0.47, size = 512, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 + 3
*B*c^3*d^2*e + 3*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B*c^
3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*a*c^
2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*C*a*c
^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d^3 + 9
*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e^3)*x^6
+ 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3*(C*a^3
+ 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a^3*e^3 +
3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d*e^2 + (C*
a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

**mupad [B]** time = 4.05, size = 490, normalized size = 1.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (3*C*a^2*c*d^3)/5 + (3*C*a^3*d*e^2
)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*d^2*e)/5) + x^8*((B*c^3*d^3)/8 + (3*
A*a*c^2*e^3)/8 + (3*C*a^2*c*e^3)/8 + (3*A*c^3*d^2*e)/8 + (9*B*a*c^2*d*e^2)/
8 + (9*C*a*c^2*d^2*e)/8) + x^6*((C*a^3*e^3)/6 + (A*a^2*c*e^3)/2 + (B*a*c^2*
d^3)/2 + (3*A*a*c^2*d^2*e)/2 + (3*B*a^2*c*d*e^2)/2 + (3*C*a^2*c*d^2*e)/2) +
x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (3*C*a*c^2*d^3)/7 + (9*A*a*c^2*d*
e^2)/7 + (9*B*a*c^2*d^2*e)/7 + (9*C*a^2*c*d*e^2)/7) + (a^2*x^4*(A*a*e^3 + 3
```





$$3.33 \quad \int (d + ex)^2 (a + cx^2)^3 (A + Bx + Cx^2) dx$$

Optimal. Leaf size=289

$$a^3 A d^2 x + \frac{1}{4} a^3 e x^4 (Be + 2Cd) + \frac{1}{3} a^2 x^3 (A (ae^2 + 3cd^2) + ad(2Be + Cd)) + \frac{1}{2} a^2 c e x^6 (Be + 2Cd) + \frac{1}{9} c^2 x^9 (3aCe^2 + c(e(A$$

**Rubi** [A] time = 0.42, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1582, 1810}

$$\frac{1}{3} a^3 (A (ae^2 + 3cd^2) + ad(2Be + Cd)) + \frac{1}{4} a^3 e x^4 (Be + 2Cd) + \frac{1}{5} a^2 x^5 (3aC^2 + c(Ae + 2Bd) + cCd^2) + \frac{1}{6} a^2 x^6 (3Ac (ae^2 + cd^2) + 3a (aC^2 + cd(2Be + Cd))) + \frac{1}{7} a^2 x^7 (3Ac (ae^2 + cd^2) + a (aC^2 + 3cd(2Be + Cd))) + \frac{d(a + cx^2)^4 (2Ac + Bd)}{8c} + \frac{3}{8} a^2 c x^8 (Be + 2Cd) + \frac{1}{10} c^2 x^{10} (Be + 2Cd) + \frac{1}{11} c^3 x^{11} C^2$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] a^3\*A\*d^2\*x + (a^2\*(a\*d\*(C\*d + 2\*B\*e) + A\*(3\*c\*d^2 + a\*e^2))\*x^3)/3 + (a^3\*e\*(2\*C\*d + B\*e)\*x^4)/4 + (a\*(3\*A\*c\*(c\*d^2 + a\*e^2) + a\*(a\*C\*e^2 + 3\*c\*d\*(C\*d + 2\*B\*e)))\*x^5)/5 + (a^2\*c\*e\*(2\*C\*d + B\*e)\*x^6)/2 + (c\*(A\*c\*(c\*d^2 + 3\*a\*e^2) + 3\*a\*(a\*C\*e^2 + c\*d\*(C\*d + 2\*B\*e)))\*x^7)/7 + (3\*a\*c^2\*e\*(2\*C\*d + B\*e)\*x^8)/8 + (c^2\*(c\*C\*d^2 + 3\*a\*C\*e^2 + c\*e\*(2\*B\*d + A\*e))\*x^9)/9 + (c^3\*e\*(2\*C\*d + B\*e)\*x^10)/10 + (c^3\*C\*e^2\*x^11)/11 + (d\*(B\*d + 2\*A\*e)\*(a + c\*x^2)^4)/(8\*c)

Rule 1582

Int[(Px\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(Coeff[Px, x, n - 1]\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]\*x^(n - 1))\*(a + b\*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]\*x^(n - 1)] && !MatchQ[Px, (Qx\_)\*((c\_) + (d\_)\*x^(m\_))^(q\_)] /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx\*(a + b\*x^n)^p, x, m - 1], 0] && GtQ[m\*q, n\*p]]

Rule 1810

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int (d + ex)^2 (a + cx^2)^3 (A + Bx + Cx^2) dx &= \frac{d(Bd + 2Ae)(a + cx^2)^4}{8c} + \int (a + cx^2)^3 \left( -((Bd^2 + 2Ade)x) + (d + \right. \\
&= \frac{d(Bd + 2Ae)(a + cx^2)^4}{8c} + \int (a^3 Ad^2 + a^2(ad(Cd + 2Be) + A(3cd^2 \\
&= a^3 Ad^2 x + \frac{1}{3} a^2 (ad(Cd + 2Be) + A(3cd^2 + ae^2)) x^3 + \frac{1}{4} a^3 e(2Cd + B
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 329, normalized size = 1.14

$$\frac{1}{2} a^3 d^2 (2Ae + Bd) + a^2 A d^2 x + \frac{1}{2} a^2 e (2Bd^2 + 2Cd^2 + 6Ade + 3Bd^2) + \frac{1}{3} a^2 e^2 (A(a^2 + 3a^2) + ad(2Bc + Cd)) + \frac{1}{5} a^2 e^2 (3aC^2 + a(Ae + 2Bd) + cA^2) + \frac{1}{5} a^2 e^2 (3aBd^2 + 6aCd^2 + 2Ade + BcA^2) + \frac{1}{2} a^2 e^2 (Ac(3a^2 + a^2) + 3a(3c^2 + ad(2Bc + Cd))) + \frac{1}{2} a^2 e^2 (2Bc(Ac + A) + B(a^2 + a^2)) + \frac{1}{5} a^2 e^2 (3Ae(a^2 + a^2) + a(Ac^2 + 3a(2Bc + Cd))) + \frac{1}{10} a^2 e^2 (2Bc + 2Cd) + \frac{1}{11} a^2 e^2 A^2$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] a^3\*A\*d^2\*x + (a^3\*d\*(B\*d + 2\*A\*e)\*x^2)/2 + (a^2\*(a\*d\*(C\*d + 2\*B\*e) + A\*(3\*c\*d^2 + a\*e^2))\*x^3)/3 + (a^2\*(3\*B\*c\*d^2 + 6\*A\*c\*d\*e + 2\*a\*C\*d\*e + a\*B\*e^2)\*x^4)/4 + (a\*(3\*A\*c\*(c\*d^2 + a\*e^2) + a\*(a\*C\*e^2 + 3\*c\*d\*(C\*d + 2\*B\*e)))\*x^5)/5 + (a\*c\*(2\*(A\*c + a\*C)\*d\*e + B\*(c\*d^2 + a\*e^2))\*x^6)/2 + (c\*(A\*c\*(c\*d^2 + 3\*a\*e^2) + 3\*a\*(a\*C\*e^2 + c\*d\*(C\*d + 2\*B\*e)))\*x^7)/7 + (c^2\*(B\*c\*d^2 + 2\*A\*c\*d\*e + 6\*a\*C\*d\*e + 3\*a\*B\*e^2)\*x^8)/8 + (c^2\*(c\*C\*d^2 + 3\*a\*C\*e^2 + c\*e\*(2\*B\*d + A\*e))\*x^9)/9 + (c^3\*e\*(2\*C\*d + B\*e)\*x^10)/10 + (c^3\*C\*e^2\*x^11)/11

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^2 (a + cx^2)^3 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^2\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)^2\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.91, size = 432, normalized size = 1.49

$$\frac{1}{11} a^3 C e^2 x^{11} + \frac{1}{10} a^3 e (2 C d + B e) x^{10} + \frac{1}{9} a^2 c (3 A e^2 + c d (C d + 2 B e)) x^9 + \frac{1}{8} a^2 c (2 (A c + a C) d e + B (c d^2 + a e^2)) x^8 + \frac{1}{7} a^2 (a (a C e^2 + 3 c d (C d + 2 B e)) + a (3 A c (c d^2 + a e^2) + a (a C e^2 + 3 c d (C d + 2 B e)))) x^7 + \frac{1}{6} a^2 (a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2)) + a^2 d (B d + 2 A e)) x^6 + \frac{1}{5} a^2 (a^3 d (B d + 2 A e) + a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2))) x^5 + \frac{1}{4} a^2 (a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2)) + a^2 d (B d + 2 A e)) x^4 + \frac{1}{3} a^2 (a^3 A d^2 x + a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2))) x^3 + \frac{1}{2} a^2 (a^3 A d^2 x + a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2))) x^2 + \frac{1}{2} a^2 (a^3 A d^2 x + a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2))) x + \frac{1}{2} a^2 (a^3 A d^2 x + a^2 (a d (C d + 2 B e) + A (3 c d^2 + a e^2)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A), x, algorithm="fricas")

```
[Out] 1/11*x^11*e^2*c^3*C + 1/5*x^10*e*d*c^3*C + 1/10*x^10*e^2*c^3*B + 1/9*x^9*d^2*c^3*C + 1/3*x^9*e^2*c^2*a*C + 2/9*x^9*e*d*c^3*B + 1/9*x^9*e^2*c^3*A + 3/4*x^8*e*d*c^2*a*C + 1/8*x^8*d^2*c^3*B + 3/8*x^8*e^2*c^2*a*B + 1/4*x^8*e*d*c^3*A + 3/7*x^7*d^2*c^2*a*C + 3/7*x^7*e^2*c*a^2*C + 6/7*x^7*e*d*c^2*a*B + 1/7*x^7*d^2*c^3*A + 3/7*x^7*e^2*c^2*a*A + x^6*e*d*c*a^2*C + 1/2*x^6*d^2*c^2*a*B + 1/2*x^6*e^2*c*a^2*B + x^6*e*d*c^2*a*A + 3/5*x^5*d^2*c*a^2*C + 1/5*x^5*e^2*a^3*C + 6/5*x^5*e*d*c*a^2*B + 3/5*x^5*d^2*c^2*a*A + 3/5*x^5*e^2*c*a^2*A + 1/2*x^4*e*d*a^3*C + 3/4*x^4*d^2*c*a^2*B + 1/4*x^4*e^2*a^3*B + 3/2*x^4*e*d*c*a^2*A + 1/3*x^3*d^2*a^3*C + 2/3*x^3*e*d*a^3*B + x^3*d^2*c*a^2*A + 1/3*x^3*e^2*a^3*A + 1/2*x^2*d^2*a^3*B + x^2*e*d*a^3*A + x*d^2*a^3*A
```

**giac** [A] time = 0.15, size = 432, normalized size = 1.49

$\frac{1}{11}C^3e^2x^{11} + \frac{1}{5}C^3dex^{10} + \frac{1}{10}C^3d^2x^{10} + \frac{1}{9}B^3c^3x^9 + \frac{1}{3}C^2a^3x^9 + \frac{2}{9}B^3d^2x^9 + \frac{1}{9}C^2a^2e^2x^9 + \frac{3}{4}C^2a^2dex^8 + \frac{1}{8}B^3d^2x^8 + \frac{3}{8}C^2a^2e^2x^8 + \frac{1}{4}A^3c^3d^2x^8 + \frac{3}{7}C^2a^2d^2x^7 + \frac{1}{7}A^3c^3d^2x^7 + \frac{3}{7}B^3a^2c^2x^7 + \frac{3}{7}A^2a^2c^2x^7 + \frac{6}{7}B^2a^2c^2dex^7 + \frac{1}{2}B^2a^2c^2d^2x^6 + \frac{3}{5}C^2a^2c^2d^2x^5 + \frac{3}{5}A^2a^2c^2d^2x^5 + \frac{1}{2}B^2a^2c^2x^6 + \frac{6}{5}B^2a^2c^2dex^5 + \frac{3}{4}B^2a^2c^2d^2x^4 + \frac{1}{5}C^2a^3x^5 + \frac{3}{5}A^2a^2c^2x^5 + \frac{1}{2}C^2a^3d^2x^4 + \frac{3}{2}A^2a^2c^2dex^4 + \frac{1}{3}C^2a^3d^2x^3 + A^2a^2c^2d^2x^3 + \frac{1}{4}B^3a^3x^4 + \frac{2}{3}B^3a^3dex^3 + \frac{1}{2}B^3a^3d^2x^2 + \frac{1}{3}A^3a^3x^3 + A^3a^3dex^2 + A^3a^3d^2x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^3*x^11*e^2 + 1/5*C*c^3*d*x^10*e + 1/9*C*c^3*d^2*x^9 + 1/10*B*c^3*x^10*e^2 + 2/9*B*c^3*d*x^9*e + 1/8*B*c^3*d^2*x^8 + 1/3*C*a*c^2*x^9*e^2 + 1/9*A*c^3*x^9*e^2 + 3/4*C*a*c^2*d*x^8*e + 1/4*A*c^3*d*x^8*e + 3/7*C*a*c^2*d^2*x^7 + 1/7*A*c^3*d^2*x^7 + 3/8*B*a*c^2*x^8*e^2 + 6/7*B*a*c^2*d*x^7*e + 1/2*B*a*c^2*d^2*x^6 + 3/7*C*a^2*c*x^7*e^2 + 3/7*A*a*c^2*x^7*e^2 + C*a^2*c*d*x^6*e + A*a*c^2*d*x^6*e + 3/5*C*a^2*c*d^2*x^5 + 3/5*A*a*c^2*d^2*x^5 + 1/2*B*a^2*c*x^6*e^2 + 6/5*B*a^2*c*d*x^5*e + 3/4*B*a^2*c*d^2*x^4 + 1/5*C*a^3*x^5*e^2 + 3/5*A*a^2*c*x^5*e^2 + 1/2*C*a^3d^2*x^4*e + 3/2*A*a^2*c*d*x^4*e + 1/3*C*a^3*d^2*x^3 + A*a^2*c*d^2*x^3 + 1/4*B*a^3*x^4*e^2 + 2/3*B*a^3d^2*x^3*e + 1/2*B*a^3d^2*x^2 + 1/3*A*a^3x^3*e^2 + A*a^3d^2*x^2*e + A*a^3d^2*x
```

**maple** [A] time = 0.00, size = 388, normalized size = 1.34

$\frac{C^3e^2x^{11}}{11} + \frac{(C^2d+2Bd^2C^2)e^2x^{10}}{10} + \frac{(A^2d^2+2B^2d^2C^2+2C^2d^2)e^2x^9}{9} + \frac{(2A^2d^2+4C^2d^2C^2+2B^2d^2C^2)e^2x^8}{8} + \frac{(3B^2d^2+2C^2d^2C^2+2B^2d^2C^2)e^2x^7}{7} + \frac{(6B^2d^2+4C^2d^2C^2+2B^2d^2C^2)e^2x^6}{6} + \frac{(3B^2d^2+2C^2d^2C^2+2B^2d^2C^2)e^2x^5}{5} + \frac{(2B^2d^2+2C^2d^2C^2+2B^2d^2C^2)e^2x^4}{4} + \frac{(2B^2d^2+2C^2d^2C^2+2B^2d^2C^2)e^2x^3}{3} + \frac{(2B^2d^2+2C^2d^2C^2+2B^2d^2C^2)e^2x^2}{2} + \frac{(2B^2d^2+2C^2d^2C^2+2B^2d^2C^2)e^2x}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/11*c^3C*e^2*x^11+1/10*(B*c^3e^2+2*C*c^3d*e)*x^10+1/9*((3*a*c^2e^2+c^3d^2)*C+2*d*e*c^3B+e^2*c^3A)*x^9+1/8*(6*d*e*a*c^2C+(3*a*c^2e^2+c^3d^2)*B+2*d*e*c^3A)*x^8+1/7*((3*a^2c^2e^2+3*a*c^2d^2)*C+6*d*e*a*c^2B+(3*a*c^2e^2+c^3d^2)*A)*x^7+1/6*(6*d*e*a^2c^2C+(3*a^2c^2e^2+3*a*c^2d^2)*B+6*d*e*a*c^2A)*x^6+1/5*((a^3e^2+3*a^2c*d^2)*C+6*d*e*a^2c^2B+(3*a^2c^2e^2+3*a*c^2d^2)*A)*x^5+1/4*(2*d*e*a^3C+(a^3e^2+3*a^2c*d^2)*B+6*d*e*a^2c^2A)*x^4+1/3*(d^2a^3C+2*d*e*a^3B+(a^3e^2+3*a^2c*d^2)*A)*x^3+1/2*(2*A*a^3d^2e+B*a^3d^2)*x^2+a^3A*d^2*x
```

**maxima [A]** time = 0.45, size = 367, normalized size = 1.27

$$\frac{1}{11} C^2 d^{11} + \frac{1}{10} (C^2 d^9 + B^2 d^7) x^{10} + \frac{1}{9} (C^2 d^7 + 2 B C d^5 + (3 C d^2 + A^2) d^3) x^9 + \frac{1}{8} (B^2 d^5 + 3 B C d^3 + 2 (C^2 d + A^2) d) x^8 + \frac{1}{7} (6 B C d^3 + B^2 d + 2 (C^2 + 3 A^2) d) x^7 + \frac{1}{6} (6 B C d + B^2) x^6 + \frac{1}{5} (6 B C d + B^2) x^5 + \frac{1}{4} (6 B C d + B^2) x^4 + \frac{1}{3} (6 B C d + B^2) x^3 + \frac{1}{2} (6 B C d + B^2) x^2 + \frac{1}{2} (6 B C d + B^2) x + \frac{1}{2} (6 B C d + B^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out]  $\frac{1}{11} C^2 c^3 e^2 x^{11} + \frac{1}{10} (2 C^2 c^3 d e + B^2 c^3 e^2) x^{10} + \frac{1}{9} (C^2 c^3 d^2 + 2 B^2 c^3 d e + (3 C^2 a c^2 + A^2 c^3) e^2) x^9 + \frac{1}{8} (B^2 c^3 d^2 + 3 B^2 a c^2 e^2 + 2 (3 C^2 a c^2 + A^2 c^3) d e) x^8 + \frac{1}{7} (6 B^2 a c^2 d e + (3 C^2 a c^2 + A^2 c^3) d^2 + 3 (C^2 a^2 c + A^2 a c^2) e^2) x^7 + A^2 a^3 d^2 x + \frac{1}{2} (B^2 a c^2 d^2 + B^2 a^2 c e^2 + 2 (C^2 a^2 c + A^2 a c^2) d e) x^6 + \frac{1}{5} (6 B^2 a^2 c d e + 3 (C^2 a^2 c + A^2 a c^2) d^2 + (C^2 a^3 + 3 A^2 a^2 c) e^2) x^5 + \frac{1}{4} (3 B^2 a^2 c d^2 + B^2 a^3 e^2 + 2 (C^2 a^3 + 3 A^2 a^2 c) d e) x^4 + \frac{1}{3} (2 B^2 a^3 d e + A^2 a^3 e^2 + (C^2 a^3 + 3 A^2 a^2 c) d^2) x^3 + \frac{1}{2} (B^2 a^3 d^2 + 2 A^2 a^3 d e) x^2$

**mupad [B]** time = 3.94, size = 343, normalized size = 1.19

$$x \left( \frac{C^2 d^9}{9} + \frac{2 B C d^7}{9} + \frac{A^2 d^5}{9} + A^2 d^3 \right) + x^2 \left( \frac{C^2 d^7}{9} + \frac{2 B C d^5}{9} + \frac{A^2 d^3}{9} + \frac{C^2 d}{3} \right) + x^3 \left( \frac{C^2 d^5}{9} + \frac{2 B C d^3}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^4 \left( \frac{C^2 d^3}{9} + \frac{2 B C d}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^5 \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^6 \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^7 \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^8 \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^9 \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^{10} \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right) + x^{11} \left( \frac{C^2 d}{9} + \frac{2 B C}{9} + \frac{A^2 d}{3} + \frac{3 A^2 d^3}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^3\*(d + e\*x)^2\*(A + B\*x + C\*x^2),x)

[Out]  $x^3 \left( \frac{A^2 a^3 e^2}{3} + \frac{C^2 a^3 d^2}{3} + \frac{2 B^2 a^3 d e}{3} + A^2 a^2 c d^2 \right) + x^9 \left( \frac{A^2 c^3 e^2}{9} + \frac{C^2 c^3 d^2}{9} + \frac{2 B^2 c^3 d e}{9} + \frac{C^2 a c^2 e^2}{3} \right) + x^5 \left( \frac{C^2 a^3 e^2}{5} + \frac{3 A^2 a c^2 d^2}{5} + \frac{3 A^2 a^2 c e^2}{5} + \frac{3 C^2 a^2 c d^2}{5} + \frac{6 B^2 a^2 c d e}{5} \right) + x^7 \left( \frac{A^2 c^3 d^2}{7} + \frac{3 A^2 a c^2 e^2}{7} + \frac{3 C^2 a c^2 d^2}{7} + \frac{3 C^2 a^2 c e^2}{7} + \frac{6 B^2 a c^2 d e}{7} \right) + \frac{a^2 x^4 (B^2 a e^2 + 3 B^2 c d^2 + 6 A^2 c d e + 2 C^2 a d e)}{4} + \frac{c^2 x^8 (3 B^2 a e^2 + B^2 c d^2 + 2 A^2 c d e + 6 C^2 a d e)}{8} + \frac{C^2 c^3 e^2 x^{11}}{11} + \frac{a^2 c x^6 (B^2 a e^2 + B^2 c d^2 + 2 A^2 c d e + 2 C^2 a d e)}{2} + A^2 a^3 d^2 x + \frac{a^3 d x^2 (2 A^2 e + B^2 d)}{2} + \frac{c^3 e x^{10} (B^2 e + 2 C^2 d)}{10}$

**sympy [A]** time = 0.15, size = 447, normalized size = 1.55

$$A^2 a^3 d^2 x + \frac{C^2 c^3 e^2 x^{11}}{11} + x^{10} \left( \frac{B^2 c^3 e^2}{10} + \frac{C^2 c^3 d e}{5} \right) + x^9 \left( \frac{A^2 c^3 e^2}{9} + \frac{2 B^2 c^3 d e}{9} + \frac{C^2 a c^2 e^2}{3} + \frac{C^2 c^3 d^2}{9} \right) + x^8 \left( \frac{A^2 c^3 d e}{4} + \frac{3 B^2 a c^2 e^2}{8} + \frac{B^2 c^3 d^2}{8} + \frac{3 C^2 a c^2 d e}{4} \right) + x^7 \left( \frac{3 A^2 a c^2 e^2}{7} + \frac{A^2 c^3 d^2}{7} + \frac{6 B^2 a c^2 d e}{7} + \frac{3 C^2 a^2 c^2}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+a)\*\*3\*(C\*x\*\*2+B\*x+A),x)

[Out]  $A^2 a^3 d^2 x + C^2 c^3 e^2 x^{11}/11 + x^{10} (B^2 c^3 e^2/10 + C^2 c^3 d e/5) + x^9 (A^2 c^3 e^2/9 + 2 B^2 c^3 d e/9 + C^2 a c^2 e^2/3 + C^2 c^3 d^2/9) + x^8 (A^2 c^3 d e/4 + 3 B^2 a c^2 e^2/8 + B^2 c^3 d^2/8 + 3 C^2 a c^2 d e/4) + x^7 (3 A^2 a c^2 e^2/7 + A^2 c^3 d^2/7 + 6 B^2 a c^2 d e/7 + 3 C^2 a^2 c^2)$

$$\begin{aligned}
& c^{**2}/7 + 3*C*a*c^{**2}*d^{**2}/7) + x^{**6}*(A*a*c^{**2}*d*e + B*a^{**2}*c^{**2}/2 + B*a* \\
& c^{**2}*d^{**2}/2 + C*a^{**2}*c*d*e) + x^{**5}*(3*A*a^{**2}*c^{**2}/5 + 3*A*a*c^{**2}*d^{**2}/5 + \\
& 6*B*a^{**2}*c*d*e/5 + C*a^{**3}*e^{**2}/5 + 3*C*a^{**2}*c*d^{**2}/5) + x^{**4}*(3*A*a^{**2}*c*d \\
& *e/2 + B*a^{**3}*e^{**2}/4 + 3*B*a^{**2}*c*d^{**2}/4 + C*a^{**3}*d*e/2) + x^{**3}*(A*a^{**3}*e^{** \\
& 2/3 + A*a^{**2}*c*d^{**2} + 2*B*a^{**3}*d*e/3 + C*a^{**3}*d^{**2}/3) + x^{**2}*(A*a^{**3}*d*e + \\
& B*a^{**3}*d^{**2}/2)
\end{aligned}$$

$$3.34 \quad \int (d + ex) (a + cx^2)^3 (A + Bx + Cx^2) dx$$

Optimal. Leaf size=169

$$a^3 Adx + \frac{1}{4} a^3 Cex^4 + \frac{1}{3} a^2 x^3 (aBe + aCd + 3Acd) + \frac{1}{2} a^2 c Cex^6 + \frac{1}{7} c^2 x^7 (3a(Be + Cd) + Acd) + \frac{3}{5} acx^5 (aBe + aCd + Acd) + \frac{(a + cx^2)^4 (Ae + Bd)}{8c} + \frac{3}{8} ac^2 Cex^8 + \frac{1}{9} c^3 x^9 (Be + Cd) + \frac{1}{10} c^3 Cex^{10}$$

**Rubi [A]** time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {1582, 1810}

$$\frac{1}{3} a^2 x^3 (aBe + aCd + 3Acd) + a^3 Adx + \frac{1}{2} a^2 c Cex^6 + \frac{1}{4} a^3 Cex^4 + \frac{1}{7} c^2 x^7 (3a(Be + Cd) + Acd) + \frac{3}{5} acx^5 (aBe + aCd + Acd) + \frac{(a + cx^2)^4 (Ae + Bd)}{8c} + \frac{3}{8} ac^2 Cex^8 + \frac{1}{9} c^3 x^9 (Be + Cd) + \frac{1}{10} c^3 Cex^{10}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] a^3\*A\*d\*x + (a^2\*(3\*A\*c\*d + a\*C\*d + a\*B\*e)\*x^3)/3 + (a^3\*C\*e\*x^4)/4 + (3\*a\*c\*(A\*c\*d + a\*C\*d + a\*B\*e)\*x^5)/5 + (a^2\*c\*C\*e\*x^6)/2 + (c^2\*(A\*c\*d + 3\*a\*(C\*d + B\*e))\*x^7)/7 + (3\*a\*c^2\*C\*e\*x^8)/8 + (c^3\*(C\*d + B\*e)\*x^9)/9 + (c^3\*C\*e\*x^10)/10 + ((B\*d + A\*e)\*(a + c\*x^2)^4)/(8\*c)

### Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(Coeff[Px, x, n - 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a + b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

### Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned}
\int (d + ex) (a + cx^2)^3 (A + Bx + Cx^2) dx &= \frac{(Bd + Ae) (a + cx^2)^4}{8c} + \int (a + cx^2)^3 (-(Bd + Ae)x + (d + ex) (A + Bx + Cx^2)) dx \\
&= \frac{(Bd + Ae) (a + cx^2)^4}{8c} + \int (a^3 Ad + a^2(3Acd + aCd + aBe)x^2 + a^3 Cx^3 - (Bd + Ae)x(a + cx^2)^3 + (d + ex)(a + cx^2)^3(A + Bx + Cx^2)) dx \\
&= a^3 Adx + \frac{1}{3} a^2(3Acd + aCd + aBe)x^3 + \frac{1}{4} a^3 Cx^4 + \frac{3}{5} ac(Acd + aCd + aBe)x^5 - \frac{(Bd + Ae)x^4}{4} + \frac{(d + ex)(a + cx^2)^4}{4c}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 196, normalized size = 1.16

$$\frac{1}{2} a^3 x^2 (Ae + Bd) + a^3 Adx + \frac{1}{4} a^2 x^4 (aCe + 3Ace + 3Bcd) + \frac{1}{3} a^2 x^3 (aBe + aCd + 3Acd) + \frac{1}{8} c^2 x^6 (3aCe + Ace + Bcd) + \frac{1}{7} c^2 x^7 (3aBe + 3aCd + Acd) + \frac{1}{2} acx^6 (aCe + Ace + Bcd) + \frac{3}{5} acx^5 (aBe + aCd + Acd) + \frac{1}{9} c^3 x^9 (Be + Cd) + \frac{1}{10} c^3 Cx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] a^3\*A\*d\*x + (a^3\*(B\*d + A\*e)\*x^2)/2 + (a^2\*(3\*A\*c\*d + a\*C\*d + a\*B\*e)\*x^3)/3 + (a^2\*(3\*B\*c\*d + 3\*A\*c\*e + a\*C\*e)\*x^4)/4 + (3\*a\*c\*(A\*c\*d + a\*C\*d + a\*B\*e)\*x^5)/5 + (a\*c\*(B\*c\*d + A\*c\*e + a\*C\*e)\*x^6)/2 + (c^2\*(A\*c\*d + 3\*a\*C\*d + 3\*a\*B\*e)\*x^7)/7 + (c^2\*(B\*c\*d + A\*c\*e + 3\*a\*C\*e)\*x^8)/8 + (c^3\*(C\*d + B\*e)\*x^9)/9 + (c^3\*C\*e\*x^10)/10

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex) (a + cx^2)^3 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x)\*(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.70, size = 249, normalized size = 1.47

$$\frac{1}{10} x^{10} ec^3 C + \frac{1}{9} x^9 d c^3 C + \frac{1}{9} x^9 e c^3 B + \frac{3}{8} x^8 e c^2 a C + \frac{1}{8} x^8 d c^3 B + \frac{1}{8} x^8 ec^3 A + \frac{3}{7} x^7 d^2 a C + \frac{3}{7} x^7 ec^2 a B + \frac{1}{7} x^7 d c^3 A + \frac{1}{2} x^6 ec c^2 C + \frac{1}{2} x^6 d c^2 a B + \frac{1}{2} x^6 ec^2 a A + \frac{3}{5} x^5 d c a^2 C + \frac{3}{5} x^5 ec a^2 B + \frac{3}{5} x^5 d c^2 a A + \frac{1}{4} x^4 e a^3 C + \frac{3}{4} x^4 d c a^2 B + \frac{3}{4} x^4 ec a^2 A + \frac{1}{3} x^3 d a^3 C + \frac{1}{3} x^3 e a^3 B + x^3 d c a^2 A + \frac{1}{2} x^2 d a^3 B + \frac{1}{2} x^2 e a^3 A + x d a^3 A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/10\*x^10\*e\*c^3\*C + 1/9\*x^9\*d\*c^3\*C + 1/9\*x^9\*e\*c^3\*B + 3/8\*x^8\*e\*c^2\*a\*C + 1/8\*x^8\*d\*c^3\*B + 1/8\*x^8\*e\*c^3\*A + 3/7\*x^7\*d\*c^2\*a\*C + 3/7\*x^7\*e\*c^2\*a\*B

+ 1/7\*x^7\*d\*c^3\*A + 1/2\*x^6\*e\*c\*a^2\*C + 1/2\*x^6\*d\*c^2\*a\*B + 1/2\*x^6\*e\*c^2\*a\*A + 3/5\*x^5\*d\*c\*a^2\*C + 3/5\*x^5\*e\*c\*a^2\*B + 3/5\*x^5\*d\*c^2\*a\*A + 1/4\*x^4\*e\*a^3\*C + 3/4\*x^4\*d\*c\*a^2\*B + 3/4\*x^4\*e\*c\*a^2\*A + 1/3\*x^3\*d\*a^3\*C + 1/3\*x^3\*e\*a^3\*B + x^3\*d\*c\*a^2\*A + 1/2\*x^2\*d\*a^3\*B + 1/2\*x^2\*e\*a^3\*A + x\*d\*a^3\*A

**giac** [A] time = 0.19, size = 261, normalized size = 1.54

$$\frac{1}{10}C^3x^{10} + \frac{1}{9}C^2dx^9 + \frac{1}{8}Bc^3x^8 + \frac{3}{8}Ca^2x^8 + \frac{1}{8}A^3x^8 + \frac{3}{7}Ca^2dx^7 + \frac{1}{7}Ac^3dx^7 + \frac{3}{7}Ba^2x^7 + \frac{1}{2}Ba^2dx^6 + \frac{1}{2}Ca^2x^6 + \frac{1}{2}Aa^2x^6 + \frac{3}{5}Ca^2dx^5 + \frac{3}{5}Ba^2x^5 + \frac{3}{5}Aa^2x^5 + \frac{3}{4}Ba^2dx^4 + \frac{1}{4}Ca^3x^4 + \frac{3}{4}Aa^2x^4 + \frac{1}{3}Ca^3dx^3 + Aa^2x^3 + \frac{1}{3}Ba^3x^3 + \frac{1}{2}Ba^3dx^2 + \frac{1}{2}Aa^3x^2 + Aa^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="giac")

[Out] 1/10\*C\*c^3\*x^10\*e + 1/9\*C\*c^3\*d\*x^9 + 1/9\*B\*c^3\*x^9\*e + 1/8\*B\*c^3\*d\*x^8 + 3/8\*C\*a\*c^2\*x^8\*e + 1/8\*A\*c^3\*x^8\*e + 3/7\*C\*a\*c^2\*d\*x^7 + 1/7\*A\*c^3\*d\*x^7 + 3/7\*B\*a\*c^2\*x^7\*e + 1/2\*B\*a\*c^2\*d\*x^6 + 1/2\*C\*a^2\*c\*x^6\*e + 1/2\*A\*a\*c^2\*x^6\*e + 3/5\*C\*a^2\*c\*d\*x^5 + 3/5\*A\*a\*c^2\*d\*x^5 + 3/5\*B\*a^2\*c\*x^5\*e + 3/4\*B\*a^2\*c\*d\*x^4 + 1/4\*C\*a^3\*x^4\*e + 3/4\*A\*a^2\*c\*x^4\*e + 1/3\*C\*a^3\*d\*x^3 + A\*a^2\*c\*d\*x^3 + 1/3\*B\*a^3\*x^3\*e + 1/2\*B\*a^3\*d\*x^2 + 1/2\*A\*a^3\*x^2\*e + A\*a^3\*d\*x

**maple** [A] time = 0.00, size = 223, normalized size = 1.32

$$\frac{C^3e^{10}x^{10}}{10} + \frac{(e^2B + c^2dC)x^9}{9} + \frac{(e^2A + c^2dB + 3ea^2c^2C)x^8}{8} + \frac{(c^2dA + 3ea^2c^2B + 3da^2c^2C)x^7}{7} + Aa^3dx + \frac{(3ea^2c^2A + 3da^2c^2B + 3e^2a^2c^2C)x^6}{6} + \frac{(3da^2c^2A + 3e^2a^2c^2B + 3da^2c^2C)x^5}{5} + \frac{(3e^2a^2c^2A + 3da^2c^2B + ea^2c^2C)x^4}{4} + \frac{(3da^2c^2A + ea^2c^2B + da^2c^2C)x^3}{3} + \frac{(e^2a^3A + da^2c^2B)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x)

[Out] 1/10\*c^3\*C\*e\*x^10+1/9\*(B\*c^3\*e+C\*c^3\*d)\*x^9+1/8\*(A\*c^3\*e+B\*c^3\*d+3\*C\*a\*c^2\*e)\*x^8+1/7\*(A\*c^3\*d+3\*B\*a\*c^2\*e+3\*C\*a\*c^2\*d)\*x^7+1/6\*(3\*A\*a\*c^2\*e+3\*B\*a\*c^2\*d+3\*C\*a^2\*c\*e)\*x^6+1/5\*(3\*A\*a\*c^2\*d+3\*B\*a^2\*c\*e+3\*C\*a^2\*c\*d)\*x^5+1/4\*(3\*A\*a^2\*c\*e+3\*B\*a^2\*c\*d+C\*a^3\*e)\*x^4+1/3\*(3\*A\*a^2\*c\*d+B\*a^3\*e+C\*a^3\*d)\*x^3+1/2\*(A\*a^3\*e+B\*a^3\*d)\*x^2+a^3\*A\*d\*x

**maxima** [A] time = 0.44, size = 222, normalized size = 1.31

$$\frac{1}{10}C^3e^{10}x^{10} + \frac{1}{9}(C^2d + Bc^3)e^9 + \frac{1}{8}(Bc^3d + (3Ca^2 + Ac^3)e^8 + \frac{1}{7}(3Ba^2e + (3Ca^2 + Ac^3)d)^7 + \frac{1}{2}(Ba^2d + (Ca^2 + Aa^2c^2)e)^6 + Aa^3dx + \frac{1}{5}(3Ba^2e + (Ca^2 + Aa^2c^2)d)^5 + \frac{1}{4}(3Ba^2d + (Ca^3 + 3Aa^2c^2)e)^4 + \frac{1}{3}(Ba^3e + (Ca^3 + 3Aa^2c^2)d)^3 + \frac{1}{2}(Ba^3d + Aa^3e)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out] 1/10\*C\*c^3\*e\*x^10 + 1/9\*(C\*c^3\*d + B\*c^3\*e)\*x^9 + 1/8\*(B\*c^3\*d + (3\*C\*a\*c^2 + A\*c^3)\*e)\*x^8 + 1/7\*(3\*B\*a\*c^2\*e + (3\*C\*a\*c^2 + A\*c^3)\*d)\*x^7 + 1/2\*(B\*a\*c^2\*d + (C\*a^2\*c + A\*a\*c^2)\*e)\*x^6 + A\*a^3\*d\*x + 3/5\*(B\*a^2\*c\*e + (C\*a^2\*c + A\*a\*c^2)\*d)\*x^5 + 1/4\*(3\*B\*a^2\*c\*d + (C\*a^3 + 3\*A\*a^2\*c)\*e)\*x^4 + 1/3\*(B\*a^3\*e + (C\*a^3 + 3\*A\*a^2\*c)\*d)\*x^3 + 1/2\*(B\*a^3\*d + A\*a^3\*e)\*x^2



**mupad [B]** time = 0.10, size = 187, normalized size = 1.11

$$x^3 \left( \frac{Ba^2e}{3} + \frac{Ca^2d}{3} + Aa^2cd \right) + x^8 \left( \frac{Ac^3e}{8} + \frac{Bc^3d}{8} + \frac{3Ca^2e}{8} \right) + \frac{a^2x^2(Ae+Bd)}{2} + \frac{c^3x^9(Be+Cd)}{9} + \frac{c^2x^7(Acd+3Bae+3Cad)}{7} + \frac{a^2x^4(3Ace+3Bcd+Ca^2e)}{4} + Aa^3dx + \frac{3acx^5(Acd+Bae+Cad)}{5} + \frac{acx^6(Ace+Bcd+Ca^2e)}{2} + \frac{C^3ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^3\*(d + e\*x)\*(A + B\*x + C\*x^2), x)

[Out]  $x^3 \left( \frac{B*a^3*e}{3} + \frac{C*a^3*d}{3} + A*a^2*c*d \right) + x^8 \left( \frac{A*c^3*e}{8} + \frac{B*c^3*d}{8} + \frac{3*C*a*c^2*e}{8} \right) + \frac{a^3*x^2*(A*e + B*d)}{2} + \frac{c^3*x^9*(B*e + C*d)}{9} + \frac{c^2*x^7*(A*c*d + 3*B*a*e + 3*C*a*d)}{7} + \frac{a^2*x^4*(3*A*c*e + 3*B*c*d + C*a*e)}{4} + A*a^3*d*x + \frac{3*a*c*x^5*(A*c*d + B*a*e + C*a*d)}{5} + \frac{a*c*x^6*(A*c*e + B*c*d + C*a*e)}{2} + \frac{C*c^3*e*x^{10}}{10}$

**sympy [A]** time = 0.11, size = 265, normalized size = 1.57

$$Aa^3dx + \frac{C^3ex^{10}}{10} + x^9 \left( \frac{Bc^3e}{9} + \frac{C^3d}{9} \right) + x^8 \left( \frac{Ac^3e}{8} + \frac{Bc^3d}{8} + \frac{3Ca^2e}{8} \right) + x^7 \left( \frac{Ac^2d}{7} + \frac{3Bac^2e}{7} + \frac{3Ca^2d}{7} \right) + x^6 \left( \frac{Aac^2e}{2} + \frac{Bac^2d}{2} + \frac{Ca^2ce}{2} \right) + x^5 \left( \frac{3Aac^2d}{5} + \frac{3Ba^2ce}{5} + \frac{3Ca^2cd}{5} \right) + x^4 \left( \frac{3Aa^2ce}{4} + \frac{3Ba^2cd}{4} + \frac{Ca^2e}{4} \right) + x^3 \left( Aa^2cd + \frac{Ba^2e}{3} + \frac{Ca^2d}{3} \right) + x^2 \left( \frac{Aa^2e}{2} + \frac{Ba^2d}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x\*\*2+a)\*\*3\*(C\*x\*\*2+B\*x+A), x)

[Out]  $A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*c**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/7 + 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) + x**5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c*e/4 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a**3*d/3) + x**2*(A*a**3*e/2 + B*a**3*d/2)$

### 3.35 $\int (a + cx^2)^3 (A + Bx + Cx^2) dx$

Optimal. Leaf size=87

$$a^3 Ax + \frac{1}{3} a^2 x^3 (aC + 3Ac) + \frac{1}{7} c^2 x^7 (3aC + Ac) + \frac{3}{5} acx^5 (aC + Ac) + \frac{B(a + cx^2)^4}{8c} + \frac{1}{9} c^3 Cx^9$$

**Rubi [A]** time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1582, 373}

$$\frac{1}{3} a^2 x^3 (aC + 3Ac) + a^3 Ax + \frac{1}{7} c^2 x^7 (3aC + Ac) + \frac{3}{5} acx^5 (aC + Ac) + \frac{B(a + cx^2)^4}{8c} + \frac{1}{9} c^3 Cx^9$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] a^3\*A\*x + (a^2\*(3\*A\*c + a\*C)\*x^3)/3 + (3\*a\*c\*(A\*c + a\*C)\*x^5)/5 + (c^2\*(A\*c + 3\*a\*C)\*x^7)/7 + (c^3\*C\*x^9)/9 + (B\*(a + c\*x^2)^4)/(8\*c)

#### Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rule 1582

Int[(Px\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(Coeff[Px, x, n - 1]\*(a + b\*x^n)^(p + 1))/(b\*n\*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]\*x^(n - 1))\*(a + b\*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n - 1]\*x^(n - 1)] && !MatchQ[Px, (Qx\_.)\*((c\_) + (d\_.)\*x^(m\_))^(q\_) /; FreeQ[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx\*(a + b\*x^n)^p, x, m - 1], 0] && GtQ[m\*q, n\*p]]

#### Rubi steps

$$\begin{aligned}
\int (a + cx^2)^3 (A + Bx + Cx^2) dx &= \frac{B(a + cx^2)^4}{8c} + \int (a + cx^2)^3 (A + Cx^2) dx \\
&= \frac{B(a + cx^2)^4}{8c} + \int (a^3A + a^2(3Ac + aC)x^2 + 3ac(Ac + aC)x^4 + c^2(Ac + 3aC)x^6) dx \\
&= a^3Ax + \frac{1}{3}a^2(3Ac + aC)x^3 + \frac{3}{5}ac(Ac + aC)x^5 + \frac{1}{7}c^2(Ac + 3aC)x^7 + \frac{1}{9}c^3Cx^9
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 100, normalized size = 1.15

$$\frac{1}{6}a^3x(6A + x(3B + 2Cx)) + \frac{1}{20}a^2cx^3(20A + 3x(5B + 4Cx)) + \frac{1}{70}ac^2x^5(42A + 5x(7B + 6Cx)) + \frac{1}{504}c^3x^7(72A + 7x(9B + 8Cx))$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] (a^3\*x\*(6\*A + x\*(3\*B + 2\*C\*x)))/6 + (a^2\*c\*x^3\*(20\*A + 3\*x\*(5\*B + 4\*C\*x)))/20 + (a\*c^2\*x^5\*(42\*A + 5\*x\*(7\*B + 6\*C\*x)))/70 + (c^3\*x^7\*(72\*A + 7\*x\*(9\*B + 8\*C\*x)))/504

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + cx^2)^3 (A + Bx + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + c\*x^2)^3\*(A + B\*x + C\*x^2), x]

**fricas [A]** time = 0.69, size = 111, normalized size = 1.28

$$\frac{1}{9}x^9c^3C + \frac{1}{8}x^8c^3B + \frac{3}{7}x^7c^2aC + \frac{1}{7}x^7c^3A + \frac{1}{2}x^6c^2aB + \frac{3}{5}x^5ca^2C + \frac{3}{5}x^5c^2aA + \frac{3}{4}x^4ca^2B + \frac{1}{3}x^3a^3C + x^3ca^2A + \frac{1}{2}x^2a^3B + xa^3A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] 1/9\*x^9\*c^3\*C + 1/8\*x^8\*c^3\*B + 3/7\*x^7\*c^2\*a\*C + 1/7\*x^7\*c^3\*A + 1/2\*x^6\*c^2\*a\*B + 3/5\*x^5\*c\*a^2\*C + 3/5\*x^5\*c^2\*a\*A + 3/4\*x^4\*c\*a^2\*B + 1/3\*x^3\*a^3\*C + x^3\*c\*a^2\*A + 1/2\*x^2\*a^3\*B + x\*a^3\*A

**giac [A]** time = 0.19, size = 111, normalized size = 1.28

$$\frac{1}{9}C^3x^9 + \frac{1}{8}Bc^3x^8 + \frac{3}{7}Cac^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{1}{2}Bac^2x^6 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ba^2cx^4 + \frac{1}{3}Ca^3x^3 + Aa^2cx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="giac")

[Out] 1/9\*C\*c^3\*x^9 + 1/8\*B\*c^3\*x^8 + 3/7\*C\*a\*c^2\*x^7 + 1/7\*A\*c^3\*x^7 + 1/2\*B\*a\*c^2\*x^6 + 3/5\*C\*a^2\*c\*x^5 + 3/5\*A\*a\*c^2\*x^5 + 3/4\*B\*a^2\*c\*x^4 + 1/3\*C\*a^3\*x^3 + A\*a^2\*c\*x^3 + 1/2\*B\*a^3\*x^2 + A\*a^3\*x

**maple [A]** time = 0.00, size = 111, normalized size = 1.28

$$\frac{C^3x^9}{9} + \frac{Bc^3x^8}{8} + \frac{Bac^2x^6}{2} + \frac{3Ba^2cx^4}{4} + \frac{(c^3A + 3ac^2C)x^7}{7} + \frac{Ba^3x^2}{2} + Aa^3x + \frac{(3ac^2A + 3a^2cC)x^5}{5} + \frac{(3a^2cA + a^3C)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^3\*(C\*x^2+B\*x+A),x)

[Out] 1/9\*c^3\*C\*x^9+1/8\*c^3\*B\*x^8+1/7\*(A\*c^3+3\*C\*a\*c^2)\*x^7+1/2\*a\*c^2\*B\*x^6+1/5\*(3\*A\*a\*c^2+3\*C\*a^2\*c)\*x^5+3/4\*a^2\*c\*B\*x^4+1/3\*(3\*A\*a^2\*c+C\*a^3)\*x^3+1/2\*a^3\*B\*x^2+a^3\*A\*x

**maxima [A]** time = 0.43, size = 108, normalized size = 1.24

$$\frac{1}{9}C^3x^9 + \frac{1}{8}Bc^3x^8 + \frac{1}{2}Bac^2x^6 + \frac{3}{4}Ba^2cx^4 + \frac{1}{7}(3Cac^2 + Ac^3)x^7 + \frac{1}{2}Ba^3x^2 + \frac{3}{5}(Ca^2c + Aac^2)x^5 + Aa^3x + \frac{1}{3}(Ca^3 + 3Aa^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A),x, algorithm="maxima")

[Out] 1/9\*C\*c^3\*x^9 + 1/8\*B\*c^3\*x^8 + 1/2\*B\*a\*c^2\*x^6 + 3/4\*B\*a^2\*c\*x^4 + 1/7\*(3\*C\*a\*c^2 + A\*c^3)\*x^7 + 1/2\*B\*a^3\*x^2 + 3/5\*(C\*a^2\*c + A\*a\*c^2)\*x^5 + A\*a^3\*x + 1/3\*(C\*a^3 + 3\*A\*a^2\*c)\*x^3

**mupad [B]** time = 0.06, size = 103, normalized size = 1.18

$$x^3 \left( \frac{Ca^3}{3} + Aca^2 \right) + x^7 \left( \frac{Ac^3}{7} + \frac{3Cac^2}{7} \right) + \frac{Ba^3x^2}{2} + \frac{Bc^3x^8}{8} + \frac{C^3x^9}{9} + Aa^3x + \frac{3acx^5(Ac + Ca)}{5} + \frac{3Ba^2cx^4}{4} + \frac{Bac^2x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^3\*(A + B\*x + C\*x^2),x)

[Out] x^3\*((C\*a^3)/3 + A\*a^2\*c) + x^7\*((A\*c^3)/7 + (3\*C\*a\*c^2)/7) + (B\*a^3\*x^2)/2 + (B\*c^3\*x^8)/8 + (C\*c^3\*x^9)/9 + A\*a^3\*x + (3\*a\*c\*x^5\*(A\*c + C\*a))/5 + (3\*B\*a^2\*c\*x^4)/4 + (B\*a\*c^2\*x^6)/2

sympy [A] time = 0.09, size = 122, normalized size = 1.40

$$Aa^3x + \frac{Ba^3x^2}{2} + \frac{3Ba^2cx^4}{4} + \frac{Bac^2x^6}{2} + \frac{Bc^3x^8}{8} + \frac{Cc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Cac^2}{7}\right) + x^5\left(\frac{3Aac^2}{5} + \frac{3Ca^2c}{5}\right) + x^3\left(Aa^2c + \frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*3\*(C\*x\*\*2+B\*x+A),x)

[Out] A\*a\*\*3\*x + B\*a\*\*3\*x\*\*2/2 + 3\*B\*a\*\*2\*c\*x\*\*4/4 + B\*a\*c\*\*2\*x\*\*6/2 + B\*c\*\*3\*x\*\*8/8 + C\*c\*\*3\*x\*\*9/9 + x\*\*7\*(A\*c\*\*3/7 + 3\*C\*a\*c\*\*2/7) + x\*\*5\*(3\*A\*a\*c\*\*2/5 + 3\*C\*a\*\*2\*c/5) + x\*\*3\*(A\*a\*\*2\*c + C\*a\*\*3/3)

$$3.36 \quad \int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} dx$$

**Optimal.** Leaf size=490

$$\frac{c(d+ex)^4 (3a^2Ce^4 + 3ace^2 (15Cd^2 - e(5Bd - Ae)) + 5c^2d^2 (14Cd^2 - e(7Bd - 3Ae)))}{4e^9} + \frac{(d+ex)^2 (ae^2 + cd^2) (a^2C - B^2)}{e^8}$$

**Rubi [A]** time = 1.10, antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$ , Rules used = {1628}

[[1628]]

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x), x]

[Out] -(((c\*d^2 + a\*e^2)^2\*(8\*c\*C\*d^3 - c\*d\*e\*(7\*B\*d - 6\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))\*x)/e^8) + ((c\*d^2 + a\*e^2)\*(a^2\*C\*e^4 + c^2\*(28\*C\*d^4 - 3\*d^2\*e\*(7\*B\*d - 5\*A\*e)) + a\*c\*e^2\*(17\*C\*d^2 - 3\*e\*(3\*B\*d - A\*e)))\*(d + e\*x)^2)/(2\*e^9) - (c\*(3\*a^2\*e^4\*(4\*C\*d - B\*e) + c^2\*(56\*C\*d^5 - 5\*d^3\*e\*(7\*B\*d - 4\*A\*e)) + 6\*a\*c\*d\*e^2\*(10\*C\*d^2 - e\*(5\*B\*d - 2\*A\*e)))\*(d + e\*x)^3)/(3\*e^9) + (c\*(3\*a^2\*C\*e^4 + 5\*c^2\*(14\*C\*d^4 - d^2\*e\*(7\*B\*d - 3\*A\*e)) + 3\*a\*c\*e^2\*(15\*C\*d^2 - e\*(5\*B\*d - A\*e)))\*(d + e\*x)^4)/(4\*e^9) - (c^2\*(56\*c\*C\*d^3 - 3\*c\*d\*e\*(7\*B\*d - 2\*A\*e) + 3\*a\*e^2\*(6\*C\*d - B\*e))\*(d + e\*x)^5)/(5\*e^9) + (c^2\*(28\*c\*C\*d^2 + 3\*a\*C\*e^2 - c\*e\*(7\*B\*d - A\*e))\*(d + e\*x)^6)/(6\*e^9) - (c^3\*(8\*C\*d - B\*e)\*(d + e\*x)^7)/(7\*e^9) + (c^3\*C\*(d + e\*x)^8)/(8\*e^9) + ((c\*d^2 + a\*e^2)^3\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[d + e\*x])/e^9

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} dx = \int \left( \frac{(cd^2 + ae^2)^2 (-8cCd^3 + cde(7Bd - 6Ae) - ae^2(2Cd - Be))}{e^8} + \frac{(cd^2 + ae^2)^2}{e^8} \right) dx$$

$$= -\frac{(cd^2 + ae^2)^2 (8cCd^3 - cde(7Bd - 6Ae) + ae^2(2Cd - Be)) x}{e^8} + \frac{(cd^2 + ae^2)^2 (a^2C - B^2)}{e^8}$$

**Mathematica [A]** time = 0.47, size = 498, normalized size = 1.02

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x),x]

[Out] (x\*(420\*a^3\*e^6\*(-2\*C\*d + 2\*B\*e + C\*e\*x) + 210\*a^2\*c\*e^4\*(C\*(-12\*d^3 + 6\*d^2\*e\*x - 4\*d\*e^2\*x^2 + 3\*e^3\*x^3) + 2\*e\*(3\*A\*e\*(-2\*d + e\*x) + B\*(6\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2))) + 42\*a\*c^2\*e^2\*(C\*(-60\*d^5 + 30\*d^4\*e\*x - 20\*d^3\*e^2\*x^2 + 15\*d^2\*e^3\*x^3 - 12\*d\*e^4\*x^4 + 10\*e^5\*x^5) + e\*(5\*A\*e\*(-12\*d^3 + 6\*d^2\*e\*x - 4\*d\*e^2\*x^2 + 3\*e^3\*x^3) + B\*(60\*d^4 - 30\*d^3\*e\*x + 20\*d^2\*e^2\*x^2 - 15\*d\*e^3\*x^3 + 12\*e^4\*x^4))) + c^3\*(C\*(-840\*d^7 + 420\*d^6\*e\*x - 280\*d^5\*e^2\*x^2 + 210\*d^4\*e^3\*x^3 - 168\*d^3\*e^4\*x^4 + 140\*d^2\*e^5\*x^5 - 120\*d\*e^6\*x^6 + 105\*e^7\*x^7) + 2\*e\*(7\*A\*e\*(-60\*d^5 + 30\*d^4\*e\*x - 20\*d^3\*e^2\*x^2 + 15\*d^2\*e^3\*x^3 - 12\*d\*e^4\*x^4 + 10\*e^5\*x^5) + B\*(420\*d^6 - 210\*d^5\*e\*x + 140\*d^4\*e^2\*x^2 - 105\*d^3\*e^3\*x^3 + 84\*d^2\*e^4\*x^4 - 70\*d\*e^5\*x^5 + 60\*e^6\*x^6)))))/(840\*e^8) + ((c\*d^2 + a\*e^2)^3\*(C\*d^2 + e\*(-(B\*d) + A\*e))\*Log[d + e\*x])/e^9

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^3 (A + Bx + Cx^2)}{d + ex} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x),x]

[Out] IntegrateAlgebraic[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x), x]

**fricas [A]** time = 1.41, size = 674, normalized size = 1.38

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d),x, algorithm="fricas")

[Out] 1/840\*(105\*C\*c^3\*e^8\*x^8 - 120\*(C\*c^3\*d\*e^7 - B\*c^3\*e^8)\*x^7 + 140\*(C\*c^3\*d^2\*e^6 - B\*c^3\*d\*e^7 + (3\*C\*a\*c^2 + A\*c^3)\*e^8)\*x^6 - 168\*(C\*c^3\*d^3\*e^5 - B\*c^3\*d^2\*e^6 - 3\*B\*a\*c^2\*e^8 + (3\*C\*a\*c^2 + A\*c^3)\*d\*e^7)\*x^5 + 210\*(C\*c^3\*d^4\*e^4 - B\*c^3\*d^3\*e^5 - 3\*B\*a\*c^2\*d\*e^7 + (3\*C\*a\*c^2 + A\*c^3)\*d^2\*e^6 + 3\*(C\*a^2\*c + A\*a\*c^2)\*e^8)\*x^4 - 280\*(C\*c^3\*d^5\*e^3 - B\*c^3\*d^4\*e^4 - 3\*B\*a\*c^2\*d^2\*e^6 - 3\*B\*a^2\*c\*e^8 + (3\*C\*a\*c^2 + A\*c^3)\*d^3\*e^5 + 3\*(C\*a^2\*c + A\*a\*c^2)\*d\*e^7)\*x^3 + 420\*(C\*c^3\*d^6\*e^2 - B\*c^3\*d^5\*e^3 - 3\*B\*a\*c^2\*d^3\*e^5

$$- 3*B*a^2*c*d*e^7 + (3*C*a*c^2 + A*c^3)*d^4*e^4 + 3*(C*a^2*c + A*a*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 - 840*(C*c^3*d^7*e - B*c^3*d^6*e^2 - 3*B*a*c^2*d^4*e^4 - 3*B*a^2*c*d^2*e^6 - B*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^5*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)*x + 840*(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*\log(e*x + d))/e^9$$

**giac** [A] time = 0.17, size = 764, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d),x, algorithm="giac")

[Out]  $(C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d^6*e^2 - 3*B*a*c^2*d^5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a^2*c*d^3*e^5 + C*a^3*d^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8)*e^{(-9)}*\log(\text{abs}(x*e + d)) + 1/840*(105*C*c^3*x^8*e^7 - 120*C*c^3*d*x^7*e^6 + 140*C*c^3*d^2*x^6*e^5 - 168*C*c^3*d^3*x^5*e^4 + 210*C*c^3*d^4*x^4*e^3 - 280*C*c^3*d^5*x^3*e^2 + 420*C*c^3*d^6*x^2*e - 840*C*c^3*d^7*x + 120*B*c^3*x^7*e^7 - 140*B*c^3*d*x^6*e^6 + 168*B*c^3*d^2*x^5*e^5 - 210*B*c^3*d^3*x^4*e^4 + 280*B*c^3*d^4*x^3*e^3 - 420*B*c^3*d^5*x^2*e^2 + 840*B*c^3*d^6*x*e + 420*C*a*c^2*x^6*e^7 + 140*A*c^3*x^6*e^7 - 504*C*a*c^2*d*x^5*e^6 - 168*A*c^3*d*x^5*e^6 + 630*C*a*c^2*d^2*x^4*e^5 + 210*A*c^3*d^2*x^4*e^5 - 840*C*a*c^2*d^3*x^3*e^4 - 280*A*c^3*d^3*x^3*e^4 + 1260*C*a*c^2*d^4*x^2*e^3 - 2520*C*a*c^2*d^5*x*e^2 - 840*A*c^3*d^5*x*e^2 + 504*B*a*c^2*x^5*e^7 - 630*B*a*c^2*d*x^4*e^6 + 840*B*a*c^2*d^2*x^3*e^5 - 1260*B*a*c^2*d^3*x^2*e^4 + 2520*B*a*c^2*d^4*x*e^3 + 630*C*a^2*c*x^4*e^7 + 630*A*a*c^2*x^4*e^7 - 840*C*a^2*c*d*x^3*e^6 - 840*A*a*c^2*d*x^3*e^6 + 1260*C*a^2*c*d^2*x^2*e^5 + 1260*A*a*c^2*d^2*x^2*e^5 - 2520*C*a^2*c*d^3*x*e^4 - 2520*A*a*c^2*d^3*x*e^4 + 840*B*a^2*c*x^3*e^7 - 1260*B*a^2*c*d*x^2*e^6 + 2520*B*a^2*c*d^2*x*e^5 + 420*C*a^3*x^2*e^7 + 1260*A*a^2*c*x^2*e^7 - 840*C*a^3*d*x*e^6 - 2520*A*a^2*c*d*x*e^6 + 840*B*a^3*x*e^7)*e^{(-8)}$

**maple** [A] time = 0.01, size = 880, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d),x)

[Out]  $3/e^3*\ln(e*x+d)*A*a^2*c*d^2+3/e^5*\ln(e*x+d)*A*a*c^2*d^4-3/e^4*\ln(e*x+d)*B*a^2*c*d^3-3/e^4*A*x*a*c^2*d^3-3/e^6*\ln(e*x+d)*B*a*c^2*d^5+3/e^5*\ln(e*x+d)*C*$



$$\begin{aligned}
& a^2*c*d^4+3/e^7*\ln(e*x+d)*C*a*c^2*d^6+3/e^3*B*x*a^2*c*d^2+3/e^5*B*x*a*c^2*d \\
& ^4-3/e^2*A*x*a^2*c*d-3/e^4*C*x*a^2*c*d^3-3/e^6*C*x*a*c^2*d^5-3/5/e^2*C*x^5* \\
& a*c^2*d-3/4/e^2*B*x^4*a*c^2*d+3/4/e^3*C*x^4*a*c^2*d^2-1/e^2*A*x^3*a*c^2*d-1 \\
& /e^2*C*x^3*a^2*c*d-1/e^4*C*x^3*a*c^2*d^3+3/2/e^3*A*x^2*a*c^2*d^2-3/2/e^2*B* \\
& x^2*a^2*c*d-3/2/e^4*B*x^2*a*c^2*d^3+3/2/e^3*C*x^2*a^2*c*d^2+3/2/e^5*C*x^2*a \\
& *c^2*d^4+1/e^3*B*x^3*a*c^2*d^2+1/8/e*C*c^3*x^8+1/7/e*B*x^7*c^3+1/6/e*A*x^6* \\
& c^3+1/e*B*x*a^3+1/e*\ln(e*x+d)*A*a^3+1/2/e*C*x^2*a^3+1/2/e*C*x^6*a*c^2+1/4/e \\
& ^5*C*x^4*c^3*d^4-1/2/e^6*B*x^2*c^3*d^5+1/2/e^7*C*x^2*c^3*d^6+1/e*B*x^3*a^2* \\
& c+3/5/e*B*x^5*a*c^2+1/5/e^3*B*x^5*c^3*d^2-1/5/e^4*C*x^5*c^3*d^3+3/4/e*A*x^4 \\
& *a*c^2+1/4/e^3*A*x^4*c^3*d^2-1/4/e^4*B*x^4*c^3*d^3+3/4/e*C*x^4*a^2*c+1/e^7* \\
& \ln(e*x+d)*A*c^3*d^6-1/e^2*\ln(e*x+d)*B*a^3*d-1/e^8*\ln(e*x+d)*B*c^3*d^7+1/e^3 \\
& *\ln(e*x+d)*C*a^3*d^2+1/e^9*\ln(e*x+d)*C*c^3*d^8-1/e^6*A*x*c^3*d^5+1/e^7*B*x* \\
& c^3*d^6-1/6/e^2*B*x^6*c^3*d+1/6/e^3*C*x^6*c^3*d^2-1/5/e^2*A*x^5*c^3*d-1/7/e \\
& ^2*C*x^7*c^3*d-1/3/e^4*A*x^3*c^3*d^3+1/3/e^5*B*x^3*c^3*d^4-1/3/e^6*C*x^3*c^ \\
& ^3*d^5+3/2/e*A*x^2*a^2*c+1/2/e^5*A*x^2*c^3*d^4-1/e^2*C*x*a^3*d-1/e^8*C*x*c^3 \\
& *d^7
\end{aligned}$$

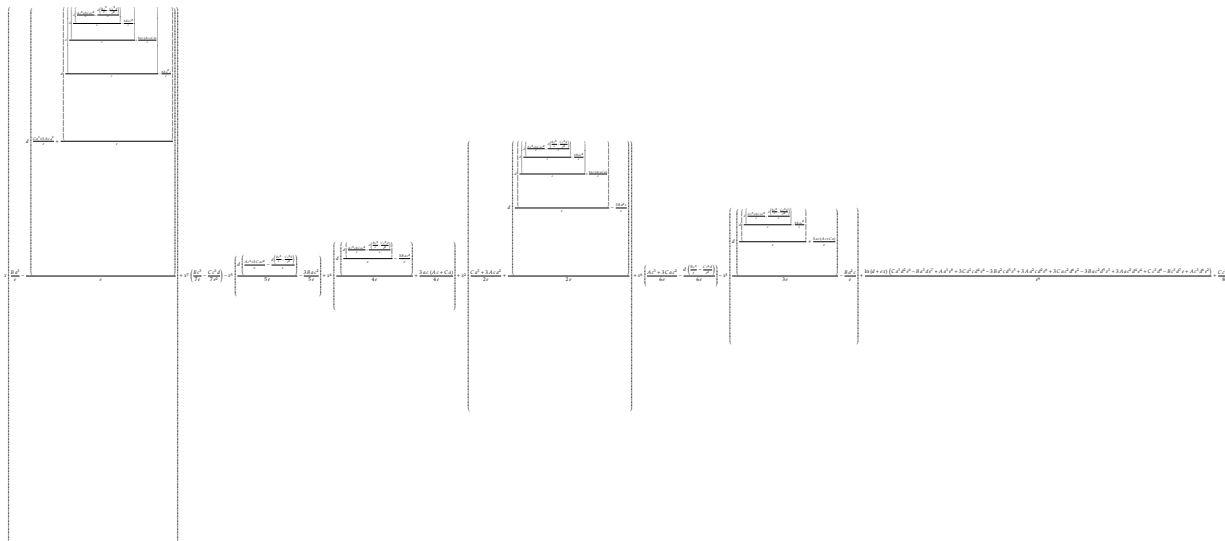
**maxima [A]** time = 0.49, size = 672, normalized size = 1.37

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d),x, algorithm="maxima")

[Out]  $1/840*(105*C*c^3*e^7*x^8 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*d^2*e^5 - B*c^3*d*e^6 + (3*C*a*c^2 + A*c^3)*e^7)*x^6 - 168*(C*c^3*d^3*e^4 - B*c^3*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2 + A*c^3)*d*e^6)*x^5 + 210*(C*c^3*d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + (3*C*a*c^2 + A*c^3)*d^2*e^5 + 3*(C*a^2*c + A*a*c^2)*e^7)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3 - 3*B*a*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2 + A*c^3)*d^3*e^4 + 3*(C*a^2*c + A*a*c^2)*d*e^6)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*a*c^2*d^3*e^4 - 3*B*a^2*c*d*e^6 + (3*C*a*c^2 + A*c^3)*d^4*e^3 + 3*(C*a^2*c + A*a*c^2)*d^2*e^5 + (C*a^3 + 3*A*a^2*c)*e^7)*x^2 - 840*(C*c^3*d^7 - B*c^3*d^6*e - 3*B*a*c^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 - B*a^3*e^7 + (3*C*a*c^2 + A*c^3)*d^5*e^2 + 3*(C*a^2*c + A*a*c^2)*d^3*e^4 + (C*a^3 + 3*A*a^2*c)*d*e^6)*x)/e^8 + (C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d)/e^9$

**mupad [B]** time = 3.88, size = 741, normalized size = 1.51



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x), x)$

[Out]  $x*((B*a^3)/e - (d*((C*a^3 + 3*A*a^2*c)/e + (d*((d*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/e) + x^7*((B*c^3)/(7*e) - (C*c^3*d)/(7*e^2)) - x^5*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/(5*e) - (3*B*a*c^2)/(5*e) + x^4*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/(4*e) + (3*a*c*(A*c + C*a))/(4*e) + x^2*((C*a^3 + 3*A*a^2*c)/(2*e) + (d*((d*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/(2*e) + x^6*((A*c^3 + 3*C*a*c^2)/(6*e) - (d*((B*c^3)/e - (C*c^3*d)/e^2))/(6*e)) - x^3*((d*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/(3*e) - (B*a^2*c)/e) + (\log(d + e*x)*(A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4))/e^9 + (C*c^3*x^8)/(8*e)$

**sympy [A]** time = 1.47, size = 685, normalized size = 1.40

sympy 0.7.6.1 (2013-03-08) Python 2.7.3 (2013-03-08) Linux (Ubuntu 12.04 LTS) (64-bit)

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d), x)$

[Out]  $C*c**3*x**8/(8*e) + x**7*(B*c**3/(7*e) - C*c**3*d/(7*e**2)) + x**6*(A*c**3/(6*e) - B*c**3*d/(6*e**2) + C*a*c**2/(2*e) + C*c**3*d**2/(6*e**3)) + x**5*($

$$\begin{aligned}
& -A*c**3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3) - 3*C*a*c**2*d \\
& / (5*e**2) - C*c**3*d**3/(5*e**4)) + x**4*(3*A*a*c**2/(4*e) + A*c**3*d**2/(4 \\
& *e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**4) + 3*C*a**2*c/(4*e) + \\
& 3*C*a*c**2*d**2/(4*e**3) + C*c**3*d**4/(4*e**5)) + x**3*(-A*a*c**2*d/e**2 - \\
& A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**2*d**2/e**3 + B*c**3*d**4/(3*e \\
& *5) - C*a**2*c*d/e**2 - C*a*c**2*d**3/e**4 - C*c**3*d**5/(3*e**6)) + x**2*( \\
& 3*A*a**2*c/(2*e) + 3*A*a*c**2*d**2/(2*e**3) + A*c**3*d**4/(2*e**5) - 3*B*a \\
& *2*c*d/(2*e**2) - 3*B*a*c**2*d**3/(2*e**4) - B*c**3*d**5/(2*e**6) + C*a**3/ \\
& (2*e) + 3*C*a**2*c*d**2/(2*e**3) + 3*C*a*c**2*d**4/(2*e**5) + C*c**3*d**6/( \\
& 2*e**7)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2*d**3/e**4 - A*c**3*d**5/e**6 \\
& + B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c**2*d**4/e**5 + B*c**3*d**6/e**7 \\
& - C*a**3*d/e**2 - 3*C*a**2*c*d**3/e**4 - 3*C*a*c**2*d**5/e**6 - C*c**3*d** \\
& 7/e**8) + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**9
\end{aligned}$$



**Mathematica [A]** time = 0.40, size = 641, normalized size = 1.32

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x)^2,x]

[Out] (420\*a^3\*e^6\*(e\*(B\*d - A\*e) + C\*(-d^2 + d\*e\*x + e^2\*x^2)) + 210\*a^2\*c\*e^4\*(2\*C\*(-3\*d^4 + 9\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 - 2\*d\*e^3\*x^3 + e^4\*x^4) + 3\*e\*(2\*A\*e\*(-d^2 + d\*e\*x + e^2\*x^2) + B\*(2\*d^3 - 4\*d^2\*e\*x - 3\*d\*e^2\*x^2 + e^3\*x^3))) + 21\*a\*c^2\*e^2\*(-6\*C\*(10\*d^6 - 50\*d^5\*e\*x - 30\*d^4\*e^2\*x^2 + 10\*d^3\*e^3\*x^3 - 5\*d^2\*e^4\*x^4 + 3\*d\*e^5\*x^5 - 2\*e^6\*x^6) + 5\*e\*(4\*A\*e\*(-3\*d^4 + 9\*d^3\*e\*x + 6\*d^2\*e^2\*x^2 - 2\*d\*e^3\*x^3 + e^4\*x^4) + B\*(12\*d^5 - 48\*d^4\*e\*x - 30\*d^3\*e^2\*x^2 + 10\*d^2\*e^3\*x^3 - 5\*d\*e^4\*x^4 + 3\*e^5\*x^5))) + c^3\*(-4\*C\*(105\*d^8 - 735\*d^7\*e\*x - 420\*d^6\*e^2\*x^2 + 140\*d^5\*e^3\*x^3 - 70\*d^4\*e^4\*x^4 + 42\*d^3\*e^5\*x^5 - 28\*d^2\*e^6\*x^6 + 20\*d\*e^7\*x^7 - 15\*e^8\*x^8) + 7\*e\*(6\*A\*e\*(-10\*d^6 + 50\*d^5\*e\*x + 30\*d^4\*e^2\*x^2 - 10\*d^3\*e^3\*x^3 + 5\*d^2\*e^4\*x^4 - 3\*d\*e^5\*x^5 + 2\*e^6\*x^6) + B\*(60\*d^7 - 360\*d^6\*e\*x - 210\*d^5\*e^2\*x^2 + 70\*d^4\*e^3\*x^3 - 35\*d^3\*e^4\*x^4 + 21\*d^2\*e^5\*x^5 - 14\*d\*e^6\*x^6 + 10\*e^7\*x^7))) - 420\*(c\*d^2 + a\*e^2)^2\*(8\*c\*C\*d^3 + c\*d\*e\*(-7\*B\*d + 6\*A\*e) + a\*e^2\*(2\*C\*d - B\*e))\*(d + e\*x)\*Log[d + e\*x])/(420\*e^9\*(d + e\*x))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^3 (A + Bx + Cx^2)}{(d + ex)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x)^2,x]

[Out] IntegrateAlgebraic[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x)^2, x]

**fricas [A]** time = 1.00, size = 932, normalized size = 1.92

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="fricas")

[Out] 1/420\*(60\*C\*c^3\*e^8\*x^8 - 420\*C\*c^3\*d^8 + 420\*B\*c^3\*d^7\*e + 1260\*B\*a\*c^2\*d^5\*e^3 + 1260\*B\*a^2\*c\*d^3\*e^5 + 420\*B\*a^3\*d\*e^7 - 420\*A\*a^3\*e^8 - 420\*(3\*C\*a\*c^2 + A\*c^3)\*d^6\*e^2 - 1260\*(C\*a^2\*c + A\*a\*c^2)\*d^4\*e^4 - 420\*(C\*a^3 + 3\*A\*a^2\*c)\*d^2\*e^6 - 10\*(8\*C\*c^3\*d\*e^7 - 7\*B\*c^3\*e^8)\*x^7 + 14\*(8\*C\*c^3\*d^2\*e^

$$6 - 7*B*c^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 21*(8*C*c^3*d^3*e^5 - 7*B*c^3*d^2*e^6 - 15*B*a*c^2*e^8 + 6*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 35*(8*C*c^3*d^4*e^4 - 7*B*c^3*d^3*e^5 - 15*B*a*c^2*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^6 + 12*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 70*(8*C*c^3*d^5*e^3 - 7*B*c^3*d^4*e^4 - 15*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^3*e^5 + 12*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 + 210*(8*C*c^3*d^6*e^2 - 7*B*c^3*d^5*e^3 - 15*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^4*e^4 + 12*(C*a^2*c + A*a*c^2)*d^2*e^6 + 2*(C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 420*(7*C*c^3*d^7*e - 6*B*c^3*d^6*e^2 - 12*B*a*c^2*d^4*e^4 - 6*B*a^2*c*d^2*e^6 + 5*(3*C*a*c^2 + A*c^3)*d^5*e^3 + 9*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)*x - 420*(8*C*c^3*d^8 - 7*B*c^3*d^7*e - 15*B*a*c^2*d^5*e^3 - 9*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 12*(C*a^2*c + A*a*c^2)*d^4*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + (8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^2*c)*d*e^7)*x)*log(e*x + d)/(e^10*x + d*e^9)$$

**giac [A]** time = 0.20, size = 838, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d)^2,x, algorithm="giac")

[Out]  $\frac{1}{420}*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)*e^{-1})/(x*e + d) + 84*(28*C*c^3*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)*e^{-2})/(x*e + d)^2 - 105*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*e^5 - 3*B*a*c^2*e^6)*e^{-3})/(x*e + d)^3 + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*d^3*e^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^2*c*e^8 + 3*A*a*c^2*e^8)*e^{-4})/(x*e + d)^4 - 210*(56*C*c^3*d^5*e^5 - 35*B*c^3*d^4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 + 12*C*a^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^{10})*e^{-5})/(x*e + d)^5 + 420*(28*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*d^4*e^8 - 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^{10} + 18*A*a*c^2*d^2*e^{10} - 9*B*a^2*c*d*e^{11} + C*a^3*e^{12} + 3*A*a^2*c*e^{12})*e^{-6})/(x*e + d)^6*(x*e + d)^7*e^{-9) + (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^5*e^2 - 15*B*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B*a^2*c*d^2*e^5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*e^{-9})*log(abs(x*e + d)*e^{-1})/(x*e + d)^2) - (C*c^3*d^8*e^7/(x*e + d) - B*c^3*d^7*e^8/(x*e + d) + 3*C*a*c^2*d^6*e^9/(x*e + d) + A*c^3*d^6*e^9/(x*e + d) - 3*B*a*c^2*d^5*e^{10}/(x*e + d) + 3*C*a^2*c*d^4*e^{11}/(x*e + d) + 3*A*a*c^2*d^4*e^{11}/(x*e + d) - 3*B*a^2*c*d^3*e^{12}/(x*e + d) + C*a^3*d^2*e^{13}/(x*e + d) + 3*A*a^2*c*d^2*e^{13}/(x*e + d) - B*a^3*d*e^{14}/(x*e + d) + A*a^3*e^{15}/(x*e + d))*e^{-16}$

**maple [A]** time = 0.02, size = 928, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2, x)$

[Out]  $\frac{1}{7}c^3Cx^7/e^2 - 1/e/(e*x+d)Aa^3 + 1/e^2 \ln(e*x+d)B^3a^3 + 1/e^2 a^3 Cx + 1/5/e^2 A^2x^5c^3 + 1/6/e^2 B^2x^6c^3 - 3/e^3 A^2x^2ac^2d + 9/2/e^4 B^2x^2ac^2d^2 - 3/e^3 Cx^2a^2cd - 3/e^3/(e*x+d)A^2c^2d^2 - 3/e^5/(e*x+d)A^2ac^2d^4 + 3/e^4/(e*x+d)B^2ac^2d^3 + 3/e^6/(e*x+d)B^2ac^2d^5 - 3/e^5/(e*x+d)C^2ac^2d^4 - 3/e^7/(e*x+d)C^2ac^2d^6 - 6/e^3 \ln(e*x+d)A^2cd - 12/e^5 \ln(e*x+d)A^2ac^2d^3 + 9/e^4 \ln(e*x+d)B^2ac^2d^2 + 15/e^6 \ln(e*x+d)B^2ac^2d^4 - 12/e^5 \ln(e*x+d)C^2ac^2d^3 - 18/e^7 \ln(e*x+d)C^2ac^2d^5 - 3/2/e^3 Cx^4ac^2d - 6/e^5 Cx^2ac^2d^3 + 9/e^4 A^2ac^2d^2x - 6/e^3 d^2a^2cBx - 12/e^5 B^2ac^2d^3x + 9/e^4 C^2ac^2d^2x + 15/e^6 C^2ac^2d^4x - 2/e^3 B^2x^3ac^2d + 3/e^4 Cx^3ac^2d^2 + 3/5/e^2 Cx^5ac^2 + 3/5/e^4 Cx^5c^3d^2 - 1/2/e^3 A^2x^4c^3d + 3/4/e^2 B^2x^4ac^2 + 3/4/e^4 B^2x^4c^3d^2 - 1/e^5 Cx^4c^3d^3 - 4/3/e^5 B^2x^3c^3d^3 + 5/3/e^6 Cx^3c^3d^4 - 2/e^5 A^2x^2c^3d^3 + 3/2/e^2 B^2x^2a^2c + 5/2/e^6 B^2x^2c^3d^4 - 3/e^7 Cx^2c^3d^5 + 3/e^2 A^2cx + 5/e^6 A^2c^3d^4x - 6/e^7 B^2c^3d^5x - 1/3/e^3 Cx^6c^3d + 1/e^2 Cx^3a^2c + 1/e^2 A^2x^3ac^2 + 1/e^4 A^2x^3c^3d^2 - 1/e^7/(e*x+d)A^2c^3d^6 + 1/e^2/(e*x+d)B^2da^3 + 7/e^8 C^2c^3d^6x - 2/5/e^3 B^2x^5c^3d + 1/e^8/(e*x+d)B^2c^3d^7 - 1/e^3/(e*x+d)C^2a^3d^2 - 1/e^9/(e*x+d)C^2c^3d^8 - 6/e^7 \ln(e*x+d)A^2c^3d^5 + 7/e^8 \ln(e*x+d)B^2c^3d^6 - 2/e^3 \ln(e*x+d)C^2a^3d^8 - 8/e^9 \ln(e*x+d)C^2c^3d^7$

**maxima [A]** time = 0.49, size = 691, normalized size = 1.42

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2, x, \text{algorithm}="maxima")$

[Out]  $-(C^2c^3d^8 - B^2c^3d^7e - 3B^2a^2c^2d^5e^3 - 3B^2a^2c^2d^3e^5 - B^2a^3de^7 + A^2a^3e^8 + (3C^2a^2c^2 + A^2c^3)d^6e^2 + 3(C^2a^2c + A^2ac^2)d^4e^4 + (C^2a^3 + 3A^2a^2c)d^2e^6)/(e^{10}x + de^9) + 1/420(60C^2c^3e^6x^7 - 70(2C^2c^3d^2e^5 - B^2c^3e^6)x^6 + 84(3C^2c^3d^2e^4 - 2B^2c^3d^2e^5 + (3C^2a^2c + A^2c^3)e^6)x^5 - 105(4C^2c^3d^3e^3 - 3B^2c^3d^2e^4 - 3B^2a^2c^2e^6 + 2(3C^2a^2c + A^2c^3)d^2e^5)x^4 + 140(5C^2c^3d^4e^2 - 4B^2c^3d^3e^3 - 6B^2a^2c^2de^5 + 3(3C^2a^2c + A^2c^3)d^2e^4 + 3(C^2a^2c + A^2ac^2)e^6)x^3 - 210(6C^2c^3d^5e - 5B^2c^3d^4e^2 - 9B^2a^2c^2d^2e^4 - 3B^2a^2c^2de^6 + 4(3C^2a^2c + A^2c^3)d^3e^3 + 6(C^2a^2c + A^2ac^2)d^2e^5)x^2 + 420(7C^2c^3d^6 - 6B^2c^3d^5e - 12B^2a^2c^2d^3e^3 - 6B^2a^2c^2de^5 + 5(3C^2a^2c + A^2c^3)d^4e^2 + 9(C^2a^2c + A^2ac^2)d^2$

$$\frac{*e^4 + (C*a^3 + 3*A*a^2*c)*e^6*x}{e^8} - (8*C*c^3*d^7 - 7*B*c^3*d^6*e - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 - B*a^3*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^5*e^2 + 12*(C*a^2*c + A*a*c^2)*d^3*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d*e^6)*\log(e*x + d)/e^9$$

**mupad [B]** time = 3.99, size = 1511, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x)`

[Out]  $x*((C*a^3 + 3*A*a^2*c)/e^2 + (2*d*((2*d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e^2 - (3*B*a^2*c)/e^2))/e - (d^2*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2))/e^2 + x^4*((d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/(2*e) - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/(4*e^2) + (3*B*a*c^2)/(4*e^2)) - x^2*((d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(2*e^2) - (3*B*a^2*c)/(2*e^2) + x^6*((B*c^3)/(6*e^2) - (C*c^3*d)/(3*e^3)) - x^5*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/(5*e) - (A*c^3 + 3*C*a*c^2)/(5*e^2) + (C*c^3*d^2)/(5*e^4)) + x^3*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/(3*e^2) - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(3*e) + (a*c*(A*c + C*a))/e^2 - (A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4)/(e*(d*e^8 + e^9*x)) - (\log(d + e*x)*(8*C*c^3*d^7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 12*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))/e^9 + (C*c^3*x^7)/(7*e^2)$







**Mathematica [A]** time = 0.23, size = 438, normalized size = 0.94

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Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] 
$$\begin{aligned} & (-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d + A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C*e^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3*a*e^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B*d + A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c*d^2 + a*e^2)^3*(C*d^2 + e*(-B*d) + A*e))/(d + e*x)^2 + (60*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 60*(c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)) + c^2*(28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x]/(60*e^9) \end{aligned}$$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^3 (A + Bx + Cx^2)}{(d + ex)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x)^3,x]

[Out] IntegrateAlgebraic[((a + c\*x^2)^3\*(A + B\*x + C\*x^2))/(d + e\*x)^3, x]

**fricas [B]** time = 0.87, size = 1025, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/60*(10*C*c^3*e^8*x^8 + 450*C*c^3*d^8 - 390*B*c^3*d^7*e - 810*B*a*c^2*d^5*e^3 - 450*B*a^2*c*d^3*e^5 - 30*B*a^3*d*e^7 - 30*A*a^3*e^8 + 330*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 630*(C*a^2*c + A*a*c^2)*d^4*e^4 + 90*(C*a^3 + 3*A*a^2*c)*d^2*e^6 - 4*(4*C*c^3*d*e^7 - 3*B*c^3*e^8)*x^7 + (28*C*c^3*d^2*e^6 - 21*B*c^3*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 2*(28*C*c^3*d^3*e^5 - 21*B*c^3*d^2*e^6 - 30*B*a*c^2*e^8 + 15*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 5*(28*C*c^3*d^4*e^4 - 21*B*c^3*d^3*e^5 - 30*B*a*c^2*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^2*e^6 + 18*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 20*(28*C*c^3*d^5*e^3 - 21*B*c^3*d^4 \end{aligned}$$

$$4e^4 - 30B^2ac^2d^2e^6 - 9B^2a^2c^2e^8 + 15(3C^2ac^2 + A^2c^3)d^3e^5 + 18(C^2ac + A^2ac^2)d^2e^7)x^3 - 30(69C^2c^3d^6e^2 - 50B^2c^3d^5e^3 - 63B^2ac^2d^3e^5 - 12B^2a^2c^2d^2e^7 + 34(3C^2ac^2 + A^2c^3)d^4e^4 + 33(C^2a^2c + A^2ac^2)d^2e^6)x^2 - 60(13C^2c^3d^7e - 8B^2c^3d^6e^2 - 3B^2ac^2d^4e^4 + 6B^2a^2c^2d^2e^6 + B^2a^3e^8 + 4(3C^2ac^2 + A^2c^3)d^5e^3 - 3(C^2a^2c + A^2ac^2)d^3e^5 - 2(C^2a^3 + 3A^2a^2c)d^2e^7)x + 60(28C^2c^3d^8 - 21B^2c^3d^7e - 30B^2ac^2d^5e^3 - 9B^2a^2c^2d^3e^5 + 15(3C^2ac^2 + A^2c^3)d^6e^2 + 18(C^2a^2c + A^2ac^2)d^4e^4 + (C^2a^3 + 3A^2a^2c)d^2e^6 + (28C^2c^3d^6e^2 - 21B^2c^3d^5e^3 - 30B^2ac^2d^3e^5 - 9B^2a^2c^2d^2e^7 + 15(3C^2ac^2 + A^2c^3)d^4e^4 + 18(C^2a^2c + A^2ac^2)d^2e^6 + (C^2a^3 + 3A^2a^2c)e^8)x^2 + 2(28C^2c^3d^7e - 21B^2c^3d^6e^2 - 30B^2ac^2d^4e^4 - 9B^2a^2c^2d^2e^6 + 15(3C^2ac^2 + A^2c^3)d^5e^3 + 18(C^2a^2c + A^2ac^2)d^3e^5 + (C^2a^3 + 3A^2a^2c)d^2e^7)x) \log(ex + d) / (e^{11}x^2 + 2d^2e^{10}x + d^2e^9)$$

**giac [A]** time = 0.17, size = 727, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^3\*(C\*x^2+B\*x+A)/(e\*x+d)^3,x, algorithm="giac")

[Out]  $(28C^2c^3d^6 - 21B^2c^3d^5e + 45C^2ac^2d^4e^2 + 15A^2c^3d^4e^2 - 30B^2ac^2d^3e^3 + 18C^2a^2c^2d^2e^4 + 18A^2ac^2d^2e^4 - 9B^2a^2c^2d^2e^5 + C^2a^3e^6 + 3A^2a^2c^2e^6)e^{(-9)} \log(\text{abs}(xe + d)) + 1/60(10C^2c^3x^6e^{15} - 36C^2c^3d^2x^5e^{14} + 90C^2c^3d^2x^4e^{13} - 200C^2c^3d^3x^3e^{12} + 450C^2c^3d^4x^2e^{11} - 1260C^2c^3d^5xe^{10} + 12B^2c^3x^5e^{15} - 45B^2c^3d^2x^4e^{14} + 120B^2c^3d^2x^3e^{13} - 300B^2c^3d^3x^2e^{12} + 900B^2c^3d^4xe^{11} + 45C^2ac^2x^4e^{15} + 15A^2c^3x^4e^{15} - 180C^2ac^2dx^3e^{14} - 60A^2c^3dx^3e^{14} + 540C^2ac^2d^2x^2e^{13} + 180A^2c^3d^2x^2e^{13} - 1800C^2ac^2d^3xe^{12} - 600A^2c^3d^3xe^{12} + 60B^2ac^2x^3e^{15} - 270B^2ac^2dx^2e^{14} + 1080B^2ac^2d^2xe^{13} + 90C^2a^2c^2x^2e^{15} + 90A^2ac^2x^2e^{15} - 540C^2a^2c^2dx^2e^{14} - 540A^2ac^2dx^2e^{14} + 180B^2a^2c^2xe^{15})e^{(-18)} + 1/2(15C^2c^3d^8 - 13B^2c^3d^7e + 33C^2ac^2d^6e^2 + 11A^2c^3d^6e^2 - 27B^2ac^2d^5e^3 + 21C^2a^2c^2d^4e^4 + 21A^2ac^2d^4e^4 - 15B^2a^2c^2d^3e^5 + 3C^2a^3d^2e^6 + 9A^2a^2c^2d^2e^6 - B^2a^3d^2e^7 - A^2a^3e^8 + 2(8C^2c^3d^7e - 7B^2c^3d^6e^2 + 18C^2ac^2d^5e^3 + 6A^2c^3d^5e^3 - 15B^2ac^2d^4e^4 + 12C^2a^2c^2d^3e^5 + 12A^2ac^2d^3e^5 - 9B^2a^2c^2d^2e^6 + 2C^2a^3d^2e^7 + 6A^2a^2c^2d^2e^7 - B^2a^3e^8)x)e^{(-9)}/(xe + d)^2$

**maple [B]** time = 0.02, size = 978, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3, x)$

[Out]  $\frac{1}{6}c^3C^3x^6/e^3 - 1/e^2/(e*x+d)*B*a^3 + 1/e^3*\ln(e*x+d)*a^3C - 1/2/e/(e*x+d)^2 * A*a^3 + 1/5*c^3/e^3*B*x^5 + 1/4*c^3/e^3*A*x^4 - 3/2/e^7/(e*x+d)^2*C*a*c^2*d^6 - 9*c^2/e^4*A*x*a*d + 18*c^2/e^5*B*x*a*d^2 - 30*c^2/e^6*C*x*a*d^3 - 3*c^2/e^4*C*x^3*a*d - 9/2*c^2/e^4*B*x^2*a*d - 9*c/e^4*C*x*a^2*d + 9*c^2/e^5*C*x^2*a*d^2 + 6/e^3/(e*x+d)*A*a^2*c*d + 12/e^5/(e*x+d)*A*a*c^2*d^3 - 9/e^4/(e*x+d)*B*a^2*c*d^2 - 15/e^6/(e*x+d)*B*a*c^2*d^4 + 12/e^5/(e*x+d)*C*a^2*c*d^3 + 18/e^7/(e*x+d)*C*a*c^2*d^5 + 18/e^5*\ln(e*x+d)*A*a*c^2*d^2 - 9/e^4*\ln(e*x+d)*B*a^2*c*d - 30/e^6*\ln(e*x+d)*B*a*c^2*d^3 + 18/e^5*\ln(e*x+d)*C*a^2*c*d^2 + 45/e^7*\ln(e*x+d)*C*a*c^2*d^4 - 3/2/e^3/(e*x+d)^2*A*d^2*a^2*c - 3/2/e^5/(e*x+d)^2*A*a*c^2*d^4 + 3/2/e^4/(e*x+d)^2*B*a^2*c*d^3 + 3/2/e^6/(e*x+d)^2*B*a*c^2*d^5 - 3/2/e^5/(e*x+d)^2*C*a^2*c*d^4 - 21/e^8*\ln(e*x+d)*B*c^3*d^5 + 28/e^9*\ln(e*x+d)*C*c^3*d^6 - 1/2/e^7/(e*x+d)^2*A*c^3*d^6 + 1/2/e^2/(e*x+d)^2*B*d*a^3 + 1/2/e^8/(e*x+d)^2*B*c^3*d^7 - 1/2/e^3/(e*x+d)^2*C*d^2*a^3 - 1/2/e^9/(e*x+d)^2*C*c^3*d^8 + 3/2*c/e^3*C*x^2*a^2 + 15*c^3/e^7*B*x*d^4 - 10*c^3/e^6*A*x*d^3 + 3/2*c^2/e^3*A*x^2*a + 3*c/e^3*B*x*a^2 - 21*c^3/e^8*C*x*d^5 + 3*c^3/e^5*A*x^2*d^2 - 5*c^3/e^6*B*x^2*d^3 + 15/2*c^3/e^7*C*x^2*d^4 + 3/4*c^2/e^3*C*x^4*a + 3/2*c^3/e^5*C*x^4*d^2 - c^3/e^4*A*x^3*d + c^2/e^3*B*x^3*a + 2*c^3/e^5*B*x^3*d^2 - 10/3*c^3/e^6*C*x^3*d^3 + 6/e^7/(e*x+d)*A*c^3*d^5 - 7/e^8/(e*x+d)*B*c^3*d^6 + 2/e^3/(e*x+d)*C*a^3*d + 8/e^9/(e*x+d)*C*c^3*d^7 - 3/5*c^3/e^4*C*x^5*d - 3/4*c^3/e^4*B*x^4*d + 3/e^3*\ln(e*x+d)*A*a^2*c + 15/e^7*\ln(e*x+d)*A*c^3*d^4$

**maxima** [A] time = 0.53, size = 701, normalized size = 1.50

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3, x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{2}*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 - A*a^3*e^8 + 11*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 21*(C*a^2*c + A*a*c^2)*d^4*e^4 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + 2*(8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^2*c)*d*e^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/60*(10*C*c^3*e^5*x^6 - 12*(3*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*d*e^4 + (3*C*a*c^2 + A*c^3)*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*e^3 - 3*B*a*c^2*e^5 + 3*(3*C*a*c^2 + A*c^3)*d*e^4)*x^3 + 30*(15*C*c^3*d^4*e - 10*B*c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^3 + 3*(C*a^2*c + A*a*c^2)*e^5)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a*c^2*d^2*e^3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2 + A*c^3)*d^3*e^2 + 9*(C*a^2*c + A*a*c^2)*d*e^4)*x)/e^8 + (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^3*e^3 - 9*B*a^2*c*d*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 18*(C*a^2*c + A*a*c^2)*d^2*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*log(e*x + d)/e^9$

**mupad [B]** time = 3.94, size = 1290, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a + c*x^2)^3*(A + B*x + C*x^2)/(d + e*x)^3, x)$

[Out]  $x^3*((d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (B*a*c^2)/e^3 - (C*c^3*d^3)/(3*e^6) + x*((3*d*((3*d*((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^6))/e - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^2 + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^3 - (3*a*c*(A*c + C*a))/e^3))/e + (d^3*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^3 - (3*d^2*((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^6))/e^2 + (3*B*a^2*c)/e^3 + x^5*((B*c^3)/(5*e^3) - (3*C*c^3*d)/(5*e^4)) - x^4*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(4*e) - (A*c^3 + 3*C*a*c^2)/(4*e^3) + (3*C*c^3*d^2)/(4*e^5)) - x^2*((3*d*((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^6))/(2*e) - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/(2*e^2) + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(2*e^3) - (3*a*c*(A*c + C*a))/(2*e^3) + ((15*C*c^3*d^8 - A*a^3*e^8 - B*a^3*d*e^7 - 13*B*c^3*d^7*e + 11*A*c^3*d^6*e^2 + 3*C*a^3*d^2*e^6 + 21*A*a*c^2*d^4*e^4 + 9*A*a^2*c*d^2*e^6 - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^5 + 33*C*a*c^2*d^6*e^2 + 21*C*a^2*c*d^4*e^4)/(2*e) + x*(8*C*c^3*d^7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 12*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (1*log(d + e*x)*(C*a^3*e^6 + 28*C*c^3*d^6 + 3*A*a^2*c*e^6 - 21*B*c^3*d^5*e + 15*A*c^3*d^4*e^2 + 18*A*a*c^2*d^2*e^4 - 30*B*a*c^2*d^3*e^3 + 45*C*a*c^2*d^4*e^2 + 18*C*a^2*c*d^2*e^4 - 9*B*a^2*c*d*e^5))/e^9 + (C*c^3*x^6)/(6*e^3)$

**sympy [A]** time = 25.28, size = 816, normalized size = 1.75

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3, x)$

[Out]  $C*c**3*x**6/(6*e**3) + x**5*(B*c**3/(5*e**3) - 3*C*c**3*d/(5*e**4)) + x**4*(A*c**3/(4*e**3) - 3*B*c**3*d/(4*e**4) + 3*C*a*c**2/(4*e**3) + 3*C*c**3*d**$

$$\begin{aligned}
& 2/(2e^{**5}) + x^{**3}*(-A^{**3}d/e^{**4} + B^{**2}a^{**2}/e^{**3} + 2B^{**3}c^{**2}d^{**2}/e^{**5} - 3 \\
& *C^{**2}a^{**2}d/e^{**4} - 10C^{**3}c^{**3}d^{**3}/(3e^{**6})) + x^{**2}*(3A^{**2}a^{**2}c^{**2}/(2e^{**3}) + 3 \\
& *A^{**3}c^{**3}d^{**2}/e^{**5} - 9B^{**2}a^{**2}c^{**2}d/(2e^{**4}) - 5B^{**3}c^{**3}d^{**3}/e^{**6} + 3C^{**2}a^{**2}c^{**2} \\
& /(2e^{**3}) + 9C^{**2}a^{**2}c^{**2}d^{**2}/e^{**5} + 15C^{**3}c^{**3}d^{**4}/(2e^{**7})) + x*(-9A^{**2}a^{**2}c^{**2} \\
& *d/e^{**4} - 10A^{**3}c^{**3}d^{**3}/e^{**6} + 3B^{**2}a^{**2}c^{**2}/e^{**3} + 18B^{**2}a^{**2}c^{**2}d^{**2}/e^{**5} + \\
& 15B^{**3}c^{**3}d^{**4}/e^{**7} - 9C^{**2}a^{**2}c^{**2}d/e^{**4} - 30C^{**2}a^{**2}c^{**2}d^{**3}/e^{**6} - 21C^{**3}c^{**3} \\
& *d^{**5}/e^{**8}) + (-A^{**3}a^{**3}e^{**8} + 9A^{**2}a^{**2}c^{**2}d^{**2}e^{**6} + 21A^{**2}a^{**2}c^{**2}d^{**4}e^{**4} \\
& + 11A^{**3}c^{**3}d^{**6}e^{**2} - B^{**3}a^{**3}d^{**7}e - 15B^{**2}a^{**2}c^{**2}d^{**3}e^{**5} - 27B^{**2}a^{**2}c^{**2} \\
& *d^{**5}e^{**3} - 13B^{**3}c^{**3}d^{**7}e + 3C^{**2}a^{**2}c^{**2}d^{**2}e^{**6} + 21C^{**2}a^{**2}c^{**2}d^{**4}e^{**4} \\
& + 33C^{**2}a^{**2}c^{**2}d^{**6}e^{**2} + 15C^{**3}c^{**3}d^{**8} + x*(12A^{**2}a^{**2}c^{**2}d^{**7}e^{**7} + 24A^{**2}a^{**2}c^{**2} \\
& *d^{**3}e^{**5} + 12A^{**3}c^{**3}d^{**5}e^{**3} - 2B^{**3}a^{**3}e^{**8} - 18B^{**2}a^{**2}c^{**2}d^{**2}e^{**6} \\
& - 30B^{**2}a^{**2}c^{**2}d^{**4}e^{**4} - 14B^{**3}c^{**3}d^{**6}e^{**2} + 4C^{**2}a^{**2}c^{**2}d^{**7}e^{**7} + 24C^{**2}a^{**2}c^{**2} \\
& *d^{**3}e^{**5} + 36C^{**2}a^{**2}c^{**2}d^{**5}e^{**3} + 16C^{**3}c^{**3}d^{**7}e^{**7}))/ (2d^{**2}e^{**9} + 4 \\
& *d^{**10}x + 2e^{**11}x^{**2}) + (a^{**2} + c^{**2}d^{**2})*(3A^{**2}a^{**2}c^{**2}e^{**4} + 15A^{**2}a^{**2}c^{**2}d^{**2} \\
& *e^{**2} - 9B^{**2}a^{**2}c^{**2}d^{**3}e^{**3} - 21B^{**2}c^{**2}d^{**3}e^{**3} + C^{**2}a^{**2}e^{**4} + 17C^{**2}a^{**2}c^{**2}d^{**2}e^{**2} \\
& + 28C^{**2}c^{**2}d^{**4})*\log(d + ex)/e^{**9}
\end{aligned}$$

$$3.39 \quad \int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

**Rubi [A]** time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$ , Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)\*(-(a\*d) + 4\*b\*c\*x + 3\*b\*d\*x^2))/(c + d\*x)^2,x]

[Out] (a + b\*x^2)^2/(c + d\*x)

**Rule 1590**

Int[(Pp\_)\*(Qq\_)^(m\_.)\*(Rr\_)^(n\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q - r + 1)\*Qq^(m + 1)\*Rr^(n + 1)]/((p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]), x] /; NeQ[p + m\*q + n\*r + 1, 0] && EqQ[(p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]\*Pp, Coeff[Pp, x, p]\*x^(p - q - r)\*((p - q - r + 1)\*Qq\*Rr + (m + 1)\*x\*Rr\*D[Qq, x] + (n + 1)\*x\*Qq\*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

**Rubi steps**

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

**Mathematica [B]** time = 0.04, size = 62, normalized size = 3.65

$$\frac{a^2d^4 + 2abd^2(c^2 + cdx + d^2x^2) + b^2(c^4 + c^3dx + d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.



[In] Integrate[((a + b\*x^2)\*(-(a\*d) + 4\*b\*c\*x + 3\*b\*d\*x^2))/(c + d\*x)^2,x]

[Out] (a^2\*d^4 + 2\*a\*b\*d^2\*(c^2 + c\*d\*x + d^2\*x^2) + b^2\*(c^4 + c^3\*d\*x + d^4\*x^4))/(d^4\*(c + d\*x))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(-ad + 4bcx + 3bdx^2)}{(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x^2)\*(-(a\*d) + 4\*b\*c\*x + 3\*b\*d\*x^2))/(c + d\*x)^2,x]

[Out] IntegrateAlgebraic[((a + b\*x^2)\*(-(a\*d) + 4\*b\*c\*x + 3\*b\*d\*x^2))/(c + d\*x)^2,x]

**fricas** [B] time = 0.88, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4 + 2abd^4x^2 + b^2c^4 + 2abc^2d^2 + a^2d^4 + (b^2c^3d + 2abcd^3)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(3\*b\*d\*x^2+4\*b\*c\*x-a\*d)/(d\*x+c)^2,x, algorithm="fricas")

[Out] (b^2\*d^4\*x^4 + 2\*a\*b\*d^4\*x^2 + b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4 + (b^2\*c^3\*d + 2\*a\*b\*c\*d^3)\*x)/(d^5\*x + c\*d^4)

**giac** [B] time = 0.17, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4b^2c}{dx+c} + \frac{6b^2c^2}{(dx+c)^2} + \frac{2abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(3\*b\*d\*x^2+4\*b\*c\*x-a\*d)/(d\*x+c)^2,x, algorithm="giac")

[Out] (b^2 - 4\*b^2\*c/(d\*x + c) + 6\*b^2\*c^2/(d\*x + c)^2 + 2\*a\*b\*d^2/(d\*x + c)^2)\*(d\*x + c)^3/d^4 + (b^2\*c^4\*d^3/(d\*x + c) + 2\*a\*b\*c^2\*d^5/(d\*x + c) + a^2\*d^7/(d\*x + c))/d^7

**maple** [B] time = 0.01, size = 76, normalized size = 4.47

$$\frac{(bd^2x^3 - bcdx^2 + 2ad^2x + bc^2x)b}{d^3} - \frac{-a^2d^4 - 2abc^2d^2 - b^2c^4}{(dx+c)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x)`

[Out]  $b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c^4)/d^4/(d*x+c)$

**maxima** [B] time = 0.46, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + (b^2c^2 + 2abd^2)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3$

**mupad** [B] time = 0.08, size = 85, normalized size = 5.00

$$x \left( \frac{b^2c^2}{d^3} + \frac{2ab}{d} \right) + \frac{b^2x^3}{d} + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{d(xd^4 + cd^3)} - \frac{b^2cx^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x^2)*(4*b*c*x - a*d + 3*b*d*x^2))/(c + d*x)^2,x)`

[Out]  $x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2$

**sympy** [B] time = 0.37, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + x \left( \frac{2ab}{d} + \frac{b^2c^2}{d^3} \right) + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)`

[Out]  $-b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)$

$$3.40 \quad \int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

**Rubi** [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$ , Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)\*(-(a\*d) + b\*x\*(4\*c + 3\*d\*x)))/(c + d\*x)^2,x]

[Out] (a + b\*x^2)^2/(c + d\*x)

Rule 1590

Int[(Pp\_)\*(Qq\_)^(m\_.)\*(Rr\_)^(n\_.), x\_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q - r + 1)\*Qq^(m + 1)\*Rr^(n + 1)]/((p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]), x] /; NeQ[p + m\*q + n\*r + 1, 0] && EqQ[(p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]\*Pp, Coeff[Pp, x, p]\*x^(p - q - r)\*((p - q - r + 1)\*Qq\*Rr + (m + 1)\*x\*Rr\*D[Qq, x] + (n + 1)\*x\*Qq\*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

**Mathematica** [B] time = 0.02, size = 62, normalized size = 3.65

$$\frac{a^2d^4 + 2abd^2(c^2 + cdx + d^2x^2) + b^2(c^4 + c^3dx + d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)\*(-(a\*d) + b\*x\*(4\*c + 3\*d\*x)))/(c + d\*x)^2,x]

[Out] (a^2\*d^4 + 2\*a\*b\*d^2\*(c^2 + c\*d\*x + d^2\*x^2) + b^2\*(c^4 + c^3\*d\*x + d^4\*x^4))/(d^4\*(c + d\*x))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(-ad + bx(4c + 3dx))}{(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x^2)\*(-(a\*d) + b\*x\*(4\*c + 3\*d\*x)))/(c + d\*x)^2,x]

[Out] IntegrateAlgebraic[((a + b\*x^2)\*(-(a\*d) + b\*x\*(4\*c + 3\*d\*x)))/(c + d\*x)^2,x]

**fricas** [B] time = 0.78, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4 + 2abd^4x^2 + b^2c^4 + 2abc^2d^2 + a^2d^4 + (b^2c^3d + 2abcd^3)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(-a\*d+b\*x\*(3\*d\*x+4\*c))/(d\*x+c)^2,x, algorithm="fricas")

[Out] (b^2\*d^4\*x^4 + 2\*a\*b\*d^4\*x^2 + b^2\*c^4 + 2\*a\*b\*c^2\*d^2 + a^2\*d^4 + (b^2\*c^3\*d + 2\*a\*b\*c\*d^3)\*x)/(d^5\*x + c\*d^4)

**giac** [B] time = 0.15, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4b^2c}{dx+c} + \frac{6b^2c^2}{(dx+c)^2} + \frac{2abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(-a\*d+b\*x\*(3\*d\*x+4\*c))/(d\*x+c)^2,x, algorithm="giac")

[Out] (b^2 - 4\*b^2\*c/(d\*x + c) + 6\*b^2\*c^2/(d\*x + c)^2 + 2\*a\*b\*d^2/(d\*x + c)^2)\*(d\*x + c)^3/d^4 + (b^2\*c^4\*d^3/(d\*x + c) + 2\*a\*b\*c^2\*d^5/(d\*x + c) + a^2\*d^7/(d\*x + c))/d^7

**maple** [B] time = 0.00, size = 76, normalized size = 4.47

$$\frac{(bd^2x^3 - bcdx^2 + 2ad^2x + bc^2x)b}{d^3} - \frac{-a^2d^4 - 2abc^2d^2 - b^2c^4}{(dx+c)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x)`

[Out]  $(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)*b/d^3-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c^4)/(d*x+c)/d^4$

**maxima** [B] time = 0.47, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + (b^2c^2 + 2abd^2)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3$

**mupad** [B] time = 3.84, size = 85, normalized size = 5.00

$$x \left( \frac{b^2c^2}{d^3} + \frac{2ab}{d} \right) + \frac{b^2x^3}{d} + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{d(xd^4 + cd^3)} - \frac{b^2cx^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-((a*d - b*x*(4*c + 3*d*x))*(a + b*x^2))/(c + d*x)^2,x)`

[Out]  $x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2$

**sympy** [B] time = 0.37, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + x \left( \frac{2ab}{d} + \frac{b^2c^2}{d^3} \right) + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)`

[Out]  $-b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)$

$$3.41 \quad \int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {1590}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*(-(a\*d) + 6\*b\*c\*x + 5\*b\*d\*x^2))/(c + d\*x)^2,x]

[Out] (a + b\*x^2)^3/(c + d\*x)

Rule 1590

Int[(Pp\_)\*(Qq\_)^(m\_.)\*(Rr\_)^(n\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q - r + 1)\*Qq^(m + 1)\*Rr^(n + 1)]/((p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]), x] /; NeQ[p + m\*q + n\*r + 1, 0] && EqQ[(p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]\*Pp, Coeff[Pp, x, p]\*x^(p - q - r)\*((p - q - r + 1)\*Qq\*Rr + (m + 1)\*x\*Rr\*D[Qq, x] + (n + 1)\*x\*Qq\*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] time = 0.04, size = 90, normalized size = 5.29

$$\frac{a^3 d^6 + 3a^2 b d^4 (c^2 + c d x + d^2 x^2) + 3a b^2 d^2 (c^4 + c^3 d x + d^4 x^4) + b^3 (c^6 + c^5 d x + d^6 x^6)}{d^6 (c + d x)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*(-(a\*d) + 6\*b\*c\*x + 5\*b\*d\*x^2))/(c + d\*x)^2,x]

[Out] (a^3\*d^6 + 3\*a^2\*b\*d^4\*(c^2 + c\*d\*x + d^2\*x^2) + 3\*a\*b^2\*d^2\*(c^4 + c^3\*d\*x + d^4\*x^4) + b^3\*(c^6 + c^5\*d\*x + d^6\*x^6))/(d^6\*(c + d\*x))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 (-ad + 6bcx + 5bdx^2)}{(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x^2)^2\*(-(a\*d) + 6\*b\*c\*x + 5\*b\*d\*x^2))/(c + d\*x)^2,x]

[Out] IntegrateAlgebraic[((a + b\*x^2)^2\*(-(a\*d) + 6\*b\*c\*x + 5\*b\*d\*x^2))/(c + d\*x)^2, x]

**fricas** [B] time = 0.65, size = 120, normalized size = 7.06

$$\frac{b^3 d^6 x^6 + 3 a b^2 d^6 x^4 + 3 a^2 b d^6 x^2 + b^3 c^6 + 3 a b^2 c^4 d^2 + 3 a^2 b c^2 d^4 + a^3 d^6 + (b^3 c^5 d + 3 a b^2 c^3 d^3 + 3 a^2 b c d^5) x}{d^7 x + c d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*(5\*b\*d\*x^2+6\*b\*c\*x-a\*d)/(d\*x+c)^2,x, algorithm="fricas")

[Out] (b^3\*d^6\*x^6 + 3\*a\*b^2\*d^6\*x^4 + 3\*a^2\*b\*d^6\*x^2 + b^3\*c^6 + 3\*a\*b^2\*c^4\*d^2 + 3\*a^2\*b\*c^2\*d^4 + a^3\*d^6 + (b^3\*c^5\*d + 3\*a\*b^2\*c^3\*d^3 + 3\*a^2\*b\*c\*d^5)\*x)/(d^7\*x + c\*d^6)

**giac** [B] time = 0.18, size = 216, normalized size = 12.71

$$\frac{\left(b^3 - \frac{6b^3c}{dx+c} + \frac{15b^3c^2}{(dx+c)^2} - \frac{20b^3c^3}{(dx+c)^3} + \frac{15b^3c^4}{(dx+c)^4} + \frac{3ab^2d^2}{(dx+c)^2} - \frac{12ab^2cd^2}{(dx+c)^3} + \frac{18ab^2c^2d^2}{(dx+c)^4} + \frac{3a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{b^3c^6d^5}{dx+c} + \frac{3ab^2c^4d^7}{dx+c} + \frac{3a^2bc^2d^9}{dx+c} + \frac{a^3d^{11}}{dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*(5\*b\*d\*x^2+6\*b\*c\*x-a\*d)/(d\*x+c)^2,x, algorithm="giac")

[Out] (b^3 - 6\*b^3\*c/(d\*x + c) + 15\*b^3\*c^2/(d\*x + c)^2 - 20\*b^3\*c^3/(d\*x + c)^3 + 15\*b^3\*c^4/(d\*x + c)^4 + 3\*a\*b^2\*d^2/(d\*x + c)^2 - 12\*a\*b^2\*c\*d^2/(d\*x + c)^3 + 18\*a\*b^2\*c^2\*d^2/(d\*x + c)^4 + 3\*a^2\*b\*d^4/(d\*x + c)^4)\*(d\*x + c)^5/d^6 + (b^3\*c^6\*d^5/(d\*x + c) + 3\*a\*b^2\*c^4\*d^7/(d\*x + c) + 3\*a^2\*b\*c^2\*d^9/(d\*x + c) + a^3\*d^11/(d\*x + c))/d^11

**maple [B]** time = 0.01, size = 157, normalized size = 9.24

$$\frac{(b^2d^4x^5 - b^2cd^3x^4 + 3abd^4x^3 + b^2c^2d^2x^3 - 3abc d^3x^2 - b^2c^3dx^2 + 3a^2d^4x + 3abc^2d^2x + b^2c^4x)b}{d^5} - \frac{-a^3d^6 - 3a^2bc^2d^4 - 3ab^2c^4d^2 - b^3c^6}{(dx + c)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*(5\*b\*d\*x^2+6\*b\*c\*x-a\*d)/(d\*x+c)^2,x)

[Out] b/d^5\*(b^2\*d^4\*x^5-b^2\*c\*d^3\*x^4+3\*a\*b\*d^4\*x^3+b^2\*c^2\*d^2\*x^3-3\*a\*b\*c\*d^3\*x^2-b^2\*c^3\*d\*x^2+3\*a^2\*d^4\*x+3\*a\*b\*c^2\*d^2\*x+b^2\*c^4\*x)-(-a^3\*d^6-3\*a^2\*b\*c^2\*d^4-3\*a\*b^2\*c^4\*d^2-b^3\*c^6)/d^6/(d\*x+c)

**maxima [B]** time = 0.45, size = 160, normalized size = 9.41

$$\frac{b^3c^6 + 3ab^2c^4d^2 + 3a^2bc^2d^4 + a^3d^6}{d^7x + cd^6} + \frac{b^3d^4x^5 - b^3cd^3x^4 + (b^3c^2d^2 + 3ab^2d^4)x^3 - (b^3c^3d + 3ab^2cd^3)x^2 + (b^3c^4 + 3ab^2c^2d^2 + 3a^2bd^4)x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*(5\*b\*d\*x^2+6\*b\*c\*x-a\*d)/(d\*x+c)^2,x, algorithm="maxima")

[Out] (b^3\*c^6 + 3\*a\*b^2\*c^4\*d^2 + 3\*a^2\*b\*c^2\*d^4 + a^3\*d^6)/(d^7\*x + c\*d^6) + (b^3\*d^4\*x^5 - b^3\*c\*d^3\*x^4 + (b^3\*c^2\*d^2 + 3\*a\*b^2\*d^4)\*x^3 - (b^3\*c^3\*d + 3\*a\*b^2\*c\*d^3)\*x^2 + (b^3\*c^4 + 3\*a\*b^2\*c^2\*d^2 + 3\*a^2\*b\*d^4)\*x)/d^5

**mupad [B]** time = 3.78, size = 252, normalized size = 14.82

$$x^3 \left( \frac{3ab^2}{d} + \frac{b^3c^2}{d^3} \right) - x \left( \frac{2c \left( \frac{4b^3c^3}{d^4} - \frac{2c \left( \frac{9ab^2}{d} + \frac{3b^3c^2}{d^3} \right)}{d} + \frac{12ab^2c}{d^2} \right)}{d} + \frac{c^2 \left( \frac{9ab^2}{d} + \frac{3b^3c^2}{d^3} \right) - 3a^2b}{d^2} - \frac{3a^2b}{d} \right) + x^2 \left( \frac{2b^3c^3}{d^4} - \frac{c \left( \frac{9ab^2}{d} + \frac{3b^3c^2}{d^3} \right) + 6ab^2c}{d^2} \right) + \frac{a^3d^6 + 3a^2bc^2d^4 + 3ab^2c^4d^2 + b^3c^6}{d(xd^6 + cd^5)} + \frac{b^3x^5}{d} - \frac{b^3cx^4}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x^2)^2\*(6\*b\*c\*x - a\*d + 5\*b\*d\*x^2))/(c + d\*x)^2,x)

[Out] x^3\*((3\*a\*b^2)/d + (b^3\*c^2)/d^3) - x\*((2\*c\*((4\*b^3\*c^3)/d^4 - (2\*c\*((9\*a\*b^2)/d + (3\*b^3\*c^2)/d^3))/d + (12\*a\*b^2\*c)/d^2))/d + (c^2\*((9\*a\*b^2)/d + (3\*b^3\*c^2)/d^3))/d^2 - (3\*a^2\*b)/d + x^2\*((2\*b^3\*c^3)/d^4 - (c\*((9\*a\*b^2)/d + (3\*b^3\*c^2)/d^3))/d + (6\*a\*b^2\*c)/d^2) + (a^3\*d^6 + b^3\*c^6 + 3\*a\*b^2\*c^4\*d^2 + 3\*a^2\*b\*c^2\*d^4)/(d\*(c\*d^5 + d^6\*x)) + (b^3\*x^5)/d - (b^3\*c\*x^4)/d^2

**sympy [B]** time = 0.59, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2} + \frac{b^3x^5}{d} + x^3 \left( \frac{3ab^2}{d} + \frac{b^3c^2}{d^3} \right) + x^2 \left( -\frac{3ab^2c}{d^2} - \frac{b^3c^3}{d^4} \right) + x \left( \frac{3a^2b}{d} + \frac{3ab^2c^2}{d^3} + \frac{b^3c^4}{d^5} \right) + \frac{a^3d^6 + 3a^2bc^2d^4 + 3ab^2c^4d^2 + b^3c^6}{cd^6 + d^7x}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)`

[Out] 
$$-b^3c x^4/d^2 + b^3x^5/d + x^3(3ab^2/d + b^3c^2/d^3) + x^2(-3ab^2c/d^2 - b^3c^3/d^4) + x(3a^2b/d + 3ab^2c^2/d^3 + b^3c^4/d^5) + (a^3d^6 + 3a^2b^2c^2d^4 + 3ab^2c^4d^2 + b^3c^6)/(cd^6 + d^7x)$$

$$3.42 \quad \int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$ , Rules used = {1590}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*(-(a\*d) + b\*x\*(6\*c + 5\*d\*x)))/(c + d\*x)^2,x]

[Out] (a + b\*x^2)^3/(c + d\*x)

Rule 1590

Int[(Pp\_)\*(Qq\_)^(m\_.)\*(Rr\_)^(n\_.), x\_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q - r + 1)\*Qq^(m + 1)\*Rr^(n + 1)]/((p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]), x] /; NeQ[p + m\*q + n\*r + 1, 0] && EqQ[(p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]\*Pp, Coeff[Pp, x, p]\*x^(p - q - r)\*((p - q - r + 1)\*Qq\*Rr + (m + 1)\*x\*Rr\*D[Qq, x] + (n + 1)\*x\*Qq\*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] time = 0.02, size = 90, normalized size = 5.29

$$\frac{a^3 d^6 + 3a^2 b d^4 (c^2 + c d x + d^2 x^2) + 3a b^2 d^2 (c^4 + c^3 d x + d^4 x^4) + b^3 (c^6 + c^5 d x + d^6 x^6)}{d^6 (c + d x)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*(-(a\*d) + b\*x\*(6\*c + 5\*d\*x)))/(c + d\*x)^2,x]

[Out] (a^3\*d^6 + 3\*a^2\*b\*d^4\*(c^2 + c\*d\*x + d^2\*x^2) + 3\*a\*b^2\*d^2\*(c^4 + c^3\*d\*x + d^4\*x^4) + b^3\*(c^6 + c^5\*d\*x + d^6\*x^6))/(d^6\*(c + d\*x))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^2 (-ad + bx(6c + 5dx))}{(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x^2)^2\*(-(a\*d) + b\*x\*(6\*c + 5\*d\*x)))/(c + d\*x)^2, x]

[Out] IntegrateAlgebraic[((a + b\*x^2)^2\*(-(a\*d) + b\*x\*(6\*c + 5\*d\*x)))/(c + d\*x)^2, x]

**fricas** [B] time = 1.00, size = 120, normalized size = 7.06

$$\frac{b^3 d^6 x^6 + 3 a b^2 d^6 x^4 + 3 a^2 b d^6 x^2 + b^3 c^6 + 3 a b^2 c^4 d^2 + 3 a^2 b c^2 d^4 + a^3 d^6 + (b^3 c^5 d + 3 a b^2 c^3 d^3 + 3 a^2 b c d^5) x}{d^7 x + c d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*(-a\*d+b\*x\*(5\*d\*x+6\*c))/(d\*x+c)^2,x, algorithm="fricas")

[Out] (b^3\*d^6\*x^6 + 3\*a\*b^2\*d^6\*x^4 + 3\*a^2\*b\*d^6\*x^2 + b^3\*c^6 + 3\*a\*b^2\*c^4\*d^2 + 3\*a^2\*b\*c^2\*d^4 + a^3\*d^6 + (b^3\*c^5\*d + 3\*a\*b^2\*c^3\*d^3 + 3\*a^2\*b\*c\*d^5)\*x)/(d^7\*x + c\*d^6)

**giac** [B] time = 0.20, size = 216, normalized size = 12.71

$$\frac{\left(b^3 - \frac{6b^3c}{dx+c} + \frac{15b^3c^2}{(dx+c)^2} - \frac{20b^3c^3}{(dx+c)^3} + \frac{15b^3c^4}{(dx+c)^4} + \frac{3ab^2d^2}{(dx+c)^2} - \frac{12ab^2cd^2}{(dx+c)^3} + \frac{18ab^2c^2d^2}{(dx+c)^4} + \frac{3a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c} + \frac{3ab^2c^4d^7}{dx+c} + \frac{3a^2bc^2d^9}{dx+c} + \frac{a^3d^{11}}{dx+c}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*(-a\*d+b\*x\*(5\*d\*x+6\*c))/(d\*x+c)^2,x, algorithm="giac")

[Out] (b^3 - 6\*b^3\*c/(d\*x + c) + 15\*b^3\*c^2/(d\*x + c)^2 - 20\*b^3\*c^3/(d\*x + c)^3 + 15\*b^3\*c^4/(d\*x + c)^4 + 3\*a\*b^2\*d^2/(d\*x + c)^2 - 12\*a\*b^2\*c\*d^2/(d\*x + c)^3 + 18\*a\*b^2\*c^2\*d^2/(d\*x + c)^4 + 3\*a^2\*b\*d^4/(d\*x + c)^4)\*(d\*x + c)^5/d^6 + (b^3\*c^6\*d^5/(d\*x + c) + 3\*a\*b^2\*c^4\*d^7/(d\*x + c) + 3\*a^2\*b\*c^2\*d^9/(d\*x + c) + a^3\*d^11/(d\*x + c))/d^11

**maple [B]** time = 0.01, size = 157, normalized size = 9.24

$$\frac{(b^2d^4x^5 - b^2cd^3x^4 + 3abd^4x^3 + b^2c^2d^2x^3 - 3abc d^3x^2 - b^2c^3dx^2 + 3a^2d^4x + 3abc^2d^2x + b^2c^4x)b}{d^5} - \frac{-a^3d^6 - 3a^2bc^2d^4 - 3ab^2c^4d^2 - b^3c^6}{(dx+c)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x)`

[Out]  $(b^2*d^4*x^5 - b^2*c*d^3*x^4 + 3*a*b*d^4*x^3 + b^2*c^2*d^2*x^3 - 3*a*b*c*d^3*x^2 - b^2*c^3*d*x^2 + 3*a^2*d^4*x + 3*a*b*c^2*d^2*x + b^2*c^4*x)*b/d^5 - (-a^3*d^6 - 3*a^2*b*c^2*d^4 - 3*a*b^2*c^4*d^2 - b^3*c^6)/(d*x+c)/d^6$

**maxima [B]** time = 0.44, size = 160, normalized size = 9.41

$$\frac{b^3c^6 + 3ab^2c^4d^2 + 3a^2bc^2d^4 + a^3d^6}{d^7x + cd^6} + \frac{b^3d^4x^5 - b^3cd^3x^4 + (b^3c^2d^2 + 3ab^2d^4)x^3 - (b^3c^3d + 3ab^2cd^3)x^2 + (b^3c^4 + 3ab^2c^2d^2 + 3a^2bd^4)x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d + 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5$

**mupad [B]** time = 0.05, size = 252, normalized size = 14.82

$$x^3 \left( \frac{3ab^2}{d} + \frac{b^3c^2}{d^3} \right) - x \left( \frac{2c \left( \frac{4b^3c^3}{d^4} - \frac{2c \left( \frac{9ab^2}{d} + \frac{3b^3c^2}{d^3} \right)}{d} + \frac{12ab^2c}{d^2} \right)}{d} + \frac{c^2 \left( \frac{9ab^2}{d} + \frac{3b^3c^2}{d^3} \right) - 3a^2b}{d^2} - \frac{3a^2b}{d} \right) + x^2 \left( \frac{2b^3c^3}{d^4} - \frac{c \left( \frac{9ab^2}{d} + \frac{3b^3c^2}{d^3} \right)}{d} + \frac{6ab^2c}{d^2} \right) + \frac{a^3d^6 + 3a^2bc^2d^4 + 3ab^2c^4d^2 + b^3c^6}{d(xd^6 + cd^6)} + \frac{b^3x^5}{d} - \frac{b^3cx^4}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-((a*d - b*x*(6*c + 5*d*x))*(a + b*x^2)^2)/(c + d*x)^2,x)`

[Out]  $x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^2$

**sympy [B]** time = 0.61, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2} + \frac{b^3x^5}{d} + x^3 \left( \frac{3ab^2}{d} + \frac{b^3c^2}{d^3} \right) + x^2 \left( -\frac{3ab^2c}{d^2} - \frac{b^3c^3}{d^4} \right) + x \left( \frac{3a^2b}{d} + \frac{3ab^2c^2}{d^3} + \frac{b^3c^4}{d^5} \right) + \frac{a^3d^6 + 3a^2bc^2d^4 + 3ab^2c^4d^2 + b^3c^6}{cd^6 + d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)`

[Out] 
$$-b^3c x^4/d^2 + b^3x^5/d + x^3(3ab^2/d + b^3c^2/d^3) + x^2(-3ab^2c/d^2 - b^3c^3/d^4) + x(3a^2b/d + 3ab^2c^2/d^3 + b^3c^4/d^5) + (a^3d^6 + 3a^2b^2c^2d^4 + 3ab^2c^4d^2 + b^3c^6)/(cd^6 + d^7x)$$

$$3.43 \quad \int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2} dx$$

**Optimal.** Leaf size=240

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Actd(cd^2 - 3ae^2) + a(ae^2(Be + 3Cd) - cd^2(3Be + Cd))\right)}{\sqrt{a}c^{5/2}} + \frac{\log(a + cx^2)(e(AC - aC)(3cd^2 - ae^2) + \dots)}{2c^3}$$

**Rubi [A]** time = 0.47, antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1629, 635, 205, 260}

$$\frac{ex^2(-aCx^2 + ce(Ac + 3Bd) + 3cCd^2)}{2c^2} + \frac{\log(a + cx^2)(e(AC - aC)(3cd^2 - ae^2) + Bcd(cd^2 - 3ae^2))}{2c^3} + \frac{x(-ae^2(Be + 3Cd) + 3cde(Ac + Bd) + cCd^2)}{c^2} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Actd(cd^2 - 3ae^2) + a(ae^2(Be + 3Cd) - cd^2(3Be + Cd))\right)}{\sqrt{a}c^{5/2}} + \frac{e^2x^3(Be + 3Cd)}{3c} + \frac{C^2x^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] ((c\*C\*d^3 + 3\*c\*d\*e\*(B\*d + A\*e) - a\*e^2\*(3\*C\*d + B\*e))\*x)/c^2 + (e\*(3\*c\*C\*d^2 - a\*C\*e^2 + c\*e\*(3\*B\*d + A\*e))\*x^2)/(2\*c^2) + (e^2\*(3\*C\*d + B\*e)\*x^3)/(3\*c) + (C\*e^3\*x^4)/(4\*c) + ((A\*c\*d\*(c\*d^2 - 3\*a\*e^2) + a\*(a\*e^2\*(3\*C\*d + B\*e) - c\*d^2\*(C\*d + 3\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(Sqrt[a]\*c^(5/2)) + ((B\*c\*d\*(c\*d^2 - 3\*a\*e^2) + (A\*c - a\*C)\*e\*(3\*c\*d^2 - a\*e^2))\*Log[a + c\*x^2]/(2\*c^3)

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 1629

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c,

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2} dx &= \int \left( \frac{cCd^3 + 3cde(Bd+ Ae) - ae^2(3Cd+ Be)}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd+ A))}{c^2} \right) dx \\ &= \frac{(cCd^3 + 3cde(Bd+ Ae) - ae^2(3Cd+ Be))x}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd+ A))}{2c^2} \\ &= \frac{(cCd^3 + 3cde(Bd+ Ae) - ae^2(3Cd+ Be))x}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd+ A))}{2c^2} \\ &= \frac{(cCd^3 + 3cde(Bd+ Ae) - ae^2(3Cd+ Be))x}{c^2} + \frac{e(3cCd^2 - aCe^2 + ce(3Bd+ A))}{2c^2} \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 223, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ACd(ac^2-3ae^2)+a(ac^2(Be+3Cd)-ce^2(3Be+Cd))\right)}{\sqrt{a}c^{5/2}} + \frac{6\log(a+cx^2)\left(e(Ac-aC)(3cd^2-ae^2)+Bcd(cd^2-3ae^2)\right)+cx(-6ae^2(2Be+6Cd+Ccx)+2ce(3Ae(6d+cx)+B(18d^2+9dex+2e^2x^2))+3cC(4d^3+6d^2ex+4de^2x^2+e^3x^3))}{12c^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] ((A\*c\*d\*(c\*d^2 - 3\*a\*e^2) + a\*(a\*e^2\*(3\*C\*d + B\*e) - c\*d^2\*(C\*d + 3\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(Sqrt[a]\*c^(5/2)) + (c\*x\*(-6\*a\*e^2\*(6\*C\*d + 2\*B\*e + C\*e\*x) + 3\*c\*C\*(4\*d^3 + 6\*d^2\*e\*x + 4\*d\*e^2\*x^2 + e^3\*x^3) + 2\*c\*e\*(3\*A\*e\*(6\*d + e\*x) + B\*(18\*d^2 + 9\*d\*e\*x + 2\*e^2\*x^2))) + 6\*(B\*c\*d\*(c\*d^2 - 3\*a\*e^2) + (A\*c - a\*C)\*e\*(3\*c\*d^2 - a\*e^2))\*Log[a + c\*x^2]/(12\*c^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

**fricas [A]** time = 1.07, size = 592, normalized size = 2.47

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ACd(ac^2-3ae^2)+a(ac^2(Be+3Cd)-ce^2(3Be+Cd))\right)}{\sqrt{a}c^{5/2}} + \frac{6\log(a+cx^2)\left(e(Ac-aC)(3cd^2-ae^2)+Bcd(cd^2-3ae^2)\right)+cx(-6ae^2(2Be+6Cd+Ccx)+2ce(3Ae(6d+cx)+B(18d^2+9dex+2e^2x^2))+3cC(4d^3+6d^2ex+4de^2x^2+e^3x^3))}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="fricas")

[Out] [1/12\*(3\*C\*a\*c^2\*e^3\*x^4 + 4\*(3\*C\*a\*c^2\*d\*e^2 + B\*a\*c^2\*e^3)\*x^3 + 6\*(3\*C\*a\*c^2\*d^2\*e + 3\*B\*a\*c^2\*d\*e^2 - (C\*a^2\*c - A\*a\*c^2)\*e^3)\*x^2 + 6\*(3\*B\*a\*c\*d^2\*e - B\*a^2\*e^3 + (C\*a\*c - A\*c^2)\*d^3 - 3\*(C\*a^2 - A\*a\*c)\*d\*e^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 12\*(C\*a\*c^2\*d^3 + 3\*B\*a\*c^2\*d^2\*e - B\*a^2\*c\*e^3 - 3\*(C\*a^2\*c - A\*a\*c^2)\*d\*e^2)\*x + 6\*(B\*a\*c^2\*d^3 - 3\*B\*a^2\*c\*d\*e^2 - 3\*(C\*a^2\*c - A\*a\*c^2)\*d^2\*e + (C\*a^3 - A\*a^2\*c)\*e^3)\*log(c\*x^2 + a)/(a\*c^3), 1/12\*(3\*C\*a\*c^2\*e^3\*x^4 + 4\*(3\*C\*a\*c^2\*d\*e^2 + B\*a\*c^2\*e^3)\*x^3 + 6\*(3\*C\*a\*c^2\*d^2\*e + 3\*B\*a\*c^2\*d\*e^2 - (C\*a^2\*c - A\*a\*c^2)\*e^3)\*x^2 - 12\*(3\*B\*a\*c\*d^2\*e - B\*a^2\*e^3 + (C\*a\*c - A\*c^2)\*d^3 - 3\*(C\*a^2 - A\*a\*c)\*d\*e^2)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + 12\*(C\*a\*c^2\*d^3 + 3\*B\*a\*c^2\*d^2\*e - B\*a^2\*c\*e^3 - 3\*(C\*a^2\*c - A\*a\*c^2)\*d\*e^2)\*x + 6\*(B\*a\*c^2\*d^3 - 3\*B\*a^2\*c\*d\*e^2 - 3\*(C\*a^2\*c - A\*a\*c^2)\*d^2\*e + (C\*a^3 - A\*a^2\*c)\*e^3)\*log(c\*x^2 + a)/(a\*c^3)]

**giac** [A] time = 0.17, size = 279, normalized size = 1.16

$$\frac{(Cac^3 - A^2c^3 + 3Bac^2e - 3C^2d^2 + 3Aacd^2 - Bc^2d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right) + (Bc^2d - 3Cac^2e + 3A^2d^2 - 3Bacd^2 + Cc^2d^2 - Aac^2)\log(cx^2 + a) + 3C^2c^3x^4 + 12C^2d^3x^3 + 18C^2d^2x^2 + 12C^2d^2x + 4Bc^3x^3 + 18Bc^3d^2x^2 + 36Bc^3d^2x + 6Cac^3x^2 - 36Cac^3dx^2 + 36A^2dx^2 - 12Bac^3x^2}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="giac")

[Out] -(C\*a\*c\*d^3 - A\*c^2\*d^3 + 3\*B\*a\*c\*d^2\*e - 3\*C\*a^2\*d\*e^2 + 3\*A\*a\*c\*d\*e^2 - B\*a^2\*e^3)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*c^2) + 1/2\*(B\*c^2\*d^3 - 3\*C\*a\*c\*d^2\*e + 3\*A\*c^2\*d^2\*e - 3\*B\*a\*c\*d\*e^2 + C\*a^2\*e^3 - A\*a\*c\*e^3)\*log(c\*x^2 + a)/c^3 + 1/12\*(3\*C\*c^3\*x^4\*e^3 + 12\*C\*c^3\*d\*x^3\*e^2 + 18\*C\*c^3\*d^2\*x^2\*e + 12\*C\*c^3\*d^3\*x + 4\*B\*c^3\*x^3\*e^3 + 18\*B\*c^3\*d\*x^2\*e^2 + 36\*B\*c^3\*d^2\*x\*e - 6\*C\*a\*c^2\*x^2\*e^3 + 6\*A\*c^3\*x^2\*e^3 - 36\*C\*a\*c^2\*d\*x\*e^2 + 36\*A\*c^3\*d\*x\*e^2 - 12\*B\*a\*c^2\*x\*e^3)/c^4

**maple** [A] time = 0.01, size = 399, normalized size = 1.66

$$\frac{C^2d^2}{4c} + \frac{B^2d^2}{3c} + \frac{C^2d^2}{c} + \frac{3Ad^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{A^2d^2}{2c} + \frac{A^2d\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{B^2d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{3Bcd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{3C^2d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{C^2d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{Cac^2}{2c^2} + \frac{3C^2d^2}{2c} + \frac{Aac^2\ln(cx^2+a)}{2c^2} + \frac{3A^2d\ln(cx^2+a)}{2c} + \frac{3Ad^2c}{c} + \frac{3Bcd^2\ln(cx^2+a)}{2c^2} + \frac{3Bc^2d}{c^2} + \frac{B^2d\ln(cx^2+a)}{2c} + \frac{3Bd^2c}{c} + \frac{C^2d^2\ln(cx^2+a)}{2c^2} + \frac{3Cac^2\ln(cx^2+a)}{2c^2} + \frac{3CAd^2c}{c^2} + \frac{C^2d^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a),x)

[Out] 1/4\*C\*e^3\*x^4/c+1/3/c\*B\*x^3\*e^3+1/c\*C\*x^3\*d\*e^2+1/2/c\*A\*x^2\*e^3+3/2/c\*B\*x^2\*d\*e^2-1/2/c^2\*C\*x^2\*a\*e^3+3/2/c\*C\*x^2\*d^2\*e+3/c\*A\*x\*d\*e^2-1/c^2\*B\*x\*a\*e^3+3/c\*B\*x\*d^2\*e-3/c^2\*C\*x\*a\*d\*e^2+1/c\*C\*x\*d^3-1/2/c^2\*ln(c\*x^2+a)\*A\*a\*e^3+3/2/c\*ln(c\*x^2+a)\*A\*d^2\*e-3/2/c^2\*ln(c\*x^2+a)\*B\*a\*d\*e^2+1/2/c\*ln(c\*x^2+a)\*B\*d^3+1/2/c^3\*ln(c\*x^2+a)\*C\*a^2\*e^3-3/2/c^2\*ln(c\*x^2+a)\*C\*a\*d^2\*e-3/c/(a\*c)^(1/



2)\*arctan(x\*c/(a\*c)^(1/2))\*A\*a\*d\*e^2+1/(a\*c)^(1/2)\*arctan(x\*c/(a\*c)^(1/2))\*  
 A\*d^3+1/c^2/(a\*c)^(1/2)\*arctan(x\*c/(a\*c)^(1/2))\*B\*a^2\*e^3-3/c/(a\*c)^(1/2)\*a  
 rctan(x\*c/(a\*c)^(1/2))\*B\*a\*d^2\*e+3/c^2/(a\*c)^(1/2)\*arctan(x\*c/(a\*c)^(1/2))\*  
 C\*a^2\*d\*e^2-1/c/(a\*c)^(1/2)\*arctan(x\*c/(a\*c)^(1/2))\*C\*a\*d^3

**maxima** [A] time = 0.98, size = 244, normalized size = 1.02

$$\frac{(3Bac^2e - B^2e^3 + (Cac - A^2)d^3 - 3(Ca^2 - Aac)de^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 3Cce^3x^4 + 4(3Ccd^2 + Bce^2)x^3 + 6(3Ccd^2e + 3Bcd^2 - (Ca - Ac)e^2)x^2 + 12(Ccd^3 + 3Bcd^2e - Bae^3 - 3(Ca - Ac)de^2)x + (Bc^2d^3 - 3Bacd^2 - 3(Cac - A^2)d^3e + (Ca^2 - Aac)e^3) \log(cx^2 + a)}{\sqrt{ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="maxima")

[Out]  $-(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2)$   
 $)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*c^2) + 1/12*(3*C*c*e^3*x^4 + 4*(3*C*c*d*$   
 $e^2 + B*c*e^3)*x^3 + 6*(3*C*c*d^2*e + 3*B*c*d*e^2 - (C*a - A*c)*e^3)*x^2 +$   
 $12*(C*c*d^3 + 3*B*c*d^2*e - B*a*e^3 - 3*(C*a - A*c)*d*e^2)*x/c^2 + 1/2*(B*$   
 $c^2*d^3 - 3*B*a*c*d*e^2 - 3*(C*a*c - A*c^2)*d^2*e + (C*a^2 - A*a*c)*e^3)*\log$   
 $(c*x^2 + a)/c^3$

**mupad** [B] time = 3.99, size = 277, normalized size = 1.15

$$x^2 \left( \frac{3C^2 d^2 e + 3Bd^2 + A^2}{2c} - \frac{Cae^2}{2c^2} \right) + x \left( \frac{Cd^3 + 3Bd^2 e + 3Ad^2}{c} - \frac{a(Bd^3 + 3Cde^2)}{c^2} \right) + \frac{x^3 (Bd^3 + 3Cde^2)}{3c} + \frac{C^2 d^4}{4c} + \frac{\operatorname{atan}\left(\frac{\sqrt{cx}}{\sqrt{ac}}\right) (3C^2 d^2 + B^2 d^2 - Cae d - 3Bac d^2 e - 3Aac d^2 + A^2 d^3)}{\sqrt{a} c^2} + \frac{\ln(cx^2 + a) (4C^2 d^3 e - 12C^2 d^2 d^2 e - 12B^2 d^2 d^2 e - 4A^2 d^2 d^3 + 4Bae d^3 + 12Aae d^3 e)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2),x)

[Out]  $x^2*((A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(2*c) - (C*a*e^3)/(2*c^2)) + x*((C*d^3$   
 $+ 3*A*d*e^2 + 3*B*d^2*e)/c - (a*(B*e^3 + 3*C*d*e^2))/c^2) + (x^3*(B*e^3 +$   
 $3*C*d*e^2))/(3*c) + (C*e^3*x^4)/(4*c) + (\operatorname{atan}((c^{1/2})*x)/a^{1/2})*(A*c^2*d$   
 $^3 + B*a^2*e^3 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e)$   
 $)/(a^{1/2}*c^{5/2}) + (\log(a + c*x^2)*(4*B*a*c^5*d^3 - 4*A*a^2*c^4*e^3 + 4*$   
 $C*a^3*c^3*e^3 - 12*B*a^2*c^4*d*e^2 - 12*C*a^2*c^4*d^2*e + 12*A*a*c^5*d^2*e)$   
 $)/(8*a*c^6)$

**sympy** [B] time = 5.46, size = 1008, normalized size = 4.20



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a),x)

[Out]  $C*e**3*x**4/(4*c) + x**3*(B*e**3/(3*c) + C*d*e**2/c) + x**2*(A*e**3/(2*c) +$   
 $3*B*d*e**2/(2*c) - C*a*e**3/(2*c**2) + 3*C*d**2*e/(2*c)) + x*(3*A*d*e**2/c$   
 $- B*a*e**3/c**2 + 3*B*d**2*e/c - 3*C*a*d*e**2/c**2 + C*d**3/c) + ((-A*a*c*$

$$\begin{aligned}
& e^{**3} + 3*A*c^{**2}*d^{**2}*e - 3*B*a*c*d*e^{**2} + B*c^{**2}*d^{**3} + C*a^{**2}*e^{**3} - 3*C*a \\
& *c*d^{**2}*e)/(2*c^{**3}) - \text{sqrt}(-a*c^{**7})*(-3*A*a*c*d*e^{**2} + A*c^{**2}*d^{**3} + B*a^{**2} \\
& *e^{**3} - 3*B*a*c*d^{**2}*e + 3*C*a^{**2}*d*e^{**2} - C*a*c*d^{**3})/(2*a*c^{**6}))*\log(x + \\
& (A*a^{**2}*c*e^{**3} - 3*A*a*c^{**2}*d^{**2}*e + 3*B*a^{**2}*c*d*e^{**2} - B*a*c^{**2}*d^{**3} - C* \\
& a^{**3}*e^{**3} + 3*C*a^{**2}*c*d^{**2}*e + 2*a*c^{**3}*(-A*a*c*e^{**3} + 3*A*c^{**2}*d^{**2}*e - \\
& 3*B*a*c*d*e^{**2} + B*c^{**2}*d^{**3} + C*a^{**2}*e^{**3} - 3*C*a*c*d^{**2}*e)/(2*c^{**3}) - \text{sq} \\
& \text{rt}(-a*c^{**7})*(-3*A*a*c*d*e^{**2} + A*c^{**2}*d^{**3} + B*a^{**2}*e^{**3} - 3*B*a*c*d^{**2}*e + \\
& 3*C*a^{**2}*d*e^{**2} - C*a*c*d^{**3})/(2*a*c^{**6}))/(-3*A*a*c^{**2}*d*e^{**2} + A*c^{**3}*d^{** \\
& 3 + B*a^{**2}*c*e^{**3} - 3*B*a*c^{**2}*d^{**2}*e + 3*C*a^{**2}*c*d*e^{**2} - C*a*c^{**2}*d^{**3})) \\
& + ((-A*a*c*e^{**3} + 3*A*c^{**2}*d^{**2}*e - 3*B*a*c*d*e^{**2} + B*c^{**2}*d^{**3} + C*a^{**2}* \\
& e^{**3} - 3*C*a*c*d^{**2}*e)/(2*c^{**3}) + \text{sqrt}(-a*c^{**7})*(-3*A*a*c*d*e^{**2} + A*c^{**2}*d \\
& ^{**3} + B*a^{**2}*e^{**3} - 3*B*a*c*d^{**2}*e + 3*C*a^{**2}*d*e^{**2} - C*a*c*d^{**3})/(2*a*c^{** \\
& 6}))*\log(x + (A*a^{**2}*c*e^{**3} - 3*A*a*c^{**2}*d^{**2}*e + 3*B*a^{**2}*c*d*e^{**2} - B*a*c \\
& ^{**2}*d^{**3} - C*a^{**3}*e^{**3} + 3*C*a^{**2}*c*d^{**2}*e + 2*a*c^{**3}*(-A*a*c*e^{**3} + 3*A*c \\
& ^{**2}*d^{**2}*e - 3*B*a*c*d*e^{**2} + B*c^{**2}*d^{**3} + C*a^{**2}*e^{**3} - 3*C*a*c*d^{**2}*e)/(2 \\
& *c^{**3}) + \text{sqrt}(-a*c^{**7})*(-3*A*a*c*d*e^{**2} + A*c^{**2}*d^{**3} + B*a^{**2}*e^{**3} - 3*B*a \\
& *c*d^{**2}*e + 3*C*a^{**2}*d*e^{**2} - C*a*c*d^{**3})/(2*a*c^{**6}))/(-3*A*a*c^{**2}*d*e^{**2} \\
& + A*c^{**3}*d^{**3} + B*a^{**2}*c*e^{**3} - 3*B*a*c^{**2}*d^{**2}*e + 3*C*a^{**2}*c*d*e^{**2} - C*a \\
& *c^{**2}*d^{**3}))
\end{aligned}$$

$$3.44 \quad \int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2} dx$$

**Optimal.** Leaf size=168

$$\frac{\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)\left(Ac(cd^2 - ae^2) + a(aCe^2 - cd(2Be + Cd))\right)}{\sqrt{a}c^{5/2}} + \frac{\log(a+cx^2)(-aBe^2 - 2aCde + 2Acde + Bcd^2)}{2c^2} - \frac{x(a+cx^2)}{2c^2}$$

**Rubi [A]** time = 0.26, antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1629, 635, 205, 260}

$$\frac{\log(a+cx^2)(-aBe^2 - 2aCde + 2Acde + Bcd^2)}{2c^2} + \frac{x(-aCe^2 + ce(Ae + 2Bd) + cCd^2)}{c^2} + \frac{\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)(Ac(cd^2 - ae^2) + a(aCe^2 - cd(2Be + Cd)))}{\sqrt{a}c^{5/2}} + \frac{ex^2(Be + 2Cd)}{2c} + \frac{Cx^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] ((c\*C\*d^2 - a\*C\*e^2 + c\*e\*(2\*B\*d + A\*e))\*x)/c^2 + (e\*(2\*C\*d + B\*e)\*x^2)/(2\*c) + (C\*e^2\*x^3)/(3\*c) + ((A\*c\*(c\*d^2 - a\*e^2) + a\*(a\*C\*e^2 - c\*d\*(C\*d + 2\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(Sqrt[a]\*c^(5/2)) + ((B\*c\*d^2 + 2\*A\*c\*d\*e - 2\*a\*C\*d\*e - a\*B\*e^2)\*Log[a + c\*x^2])/(2\*c^2)

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 1629

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_)^(m\_.))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2} dx &= \int \left( \frac{cCd^2 - aCe^2 + ce(2Bd+ Ae)}{c^2} + \frac{e(2Cd+ Be)x}{c} + \frac{Ce^2x^2}{c} + \frac{Ac(cd^2 - ae^2)}{c^2} + \frac{a(Ac^2 - ce^2)}{c^2} \right) dx \\
&= \frac{(cCd^2 - aCe^2 + ce(2Bd+ Ae))x}{c^2} + \frac{e(2Cd+ Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\int \frac{Ac(cd^2 - ae^2) + a(Ac^2 - ce^2)}{c^2} dx}{c^2} \\
&= \frac{(cCd^2 - aCe^2 + ce(2Bd+ Ae))x}{c^2} + \frac{e(2Cd+ Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{(Bcd^2 + 2Acde - a^2)}{c^2} \\
&= \frac{(cCd^2 - aCe^2 + ce(2Bd+ Ae))x}{c^2} + \frac{e(2Cd+ Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{(Ac(cd^2 - ae^2) + a(Ac^2 - ce^2))}{c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 155, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac(cd^2 - ae^2) + a(aCe^2 - cd(2Be + Cd))\right)}{\sqrt{a}c^{5/2}} + \frac{x(-6aCe^2 + 3ce(2Ae + 4Bd + Bex) + 2cC(3d^2 + 3dex + e^2x^2)) + 3\log(a+cx^2)(-aBe^2 - 2aCde + 2Acde + Bcd^2)}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] ((A\*c\*(c\*d^2 - a\*e^2) + a\*(a\*C\*e^2 - c\*d\*(C\*d + 2\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(Sqrt[a]\*c^(5/2)) + (x\*(-6\*a\*C\*e^2 + 3\*c\*e\*(4\*B\*d + 2\*A\*e + B\*e\*x) + 2\*c\*C\*(3\*d^2 + 3\*d\*e\*x + e^2\*x^2)) + 3\*(B\*c\*d^2 + 2\*A\*c\*d\*e - 2\*a\*C\*d\*e - a\*B\*e^2))\*Log[a + c\*x^2])/(6\*c^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

**fricas [A]** time = 1.02, size = 404, normalized size = 2.40

$$\frac{2[Cc^2d^2 + 3[2Ccd^2 + Bc^2]d^2 - 3[2Bcd + (Ck - A^2)e^2 - (C^2 - Ae^2)]\sqrt{a}\log\left(\frac{d^2 + 2dx + a}{2dx + a}\right) + 6[Cc^2d^2 + 2Bcd^2 - (C^2 - Ae^2)d^2] + 3[Bc^2d^2 - Bc^2d^2 - 2[C^2 - Ae^2]d]\log(c^2 + a)}{6c^2} + \frac{2Ccd^2d^2 + 3[2Ccd^2 + Bc^2]d^2 - 6[2Bcd + (Ck - A^2)e^2 - (C^2 - Ae^2)]\sqrt{a}\arctan\left(\frac{dx}{\sqrt{a}}\right) + 6[Cc^2d^2 + 2Bcd^2 - (C^2 - Ae^2)d^2] + 3[Bc^2d^2 - Bc^2d^2 - 2[C^2 - Ae^2]d]\log(c^2 + a)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="fricas")

[Out] [1/6\*(2\*C\*a\*c^2\*e^2\*x^3 + 3\*(2\*C\*a\*c^2\*d\*e + B\*a\*c^2\*e^2)\*x^2 - 3\*(2\*B\*a\*c\*d\*e + (C\*a\*c - A\*c^2)\*d^2 - (C\*a^2 - A\*a\*c)\*e^2)\*sqrt(-a\*c)\*log((c\*x^2 + 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 6\*(C\*a\*c^2\*d^2 + 2\*B\*a\*c^2\*d\*e - (C\*a^2\*c - A\*a\*c^2)\*e^2)\*x + 3\*(B\*a\*c^2\*d^2 - B\*a^2\*c\*e^2 - 2\*(C\*a^2\*c - A\*a\*c^2)\*d\*e)\*log(c\*x^2 + a))/(a\*c^3), 1/6\*(2\*C\*a\*c^2\*e^2\*x^3 + 3\*(2\*C\*a\*c^2\*d\*e + B\*a\*c^2\*e^2)\*x^2 - 6\*(2\*B\*a\*c\*d\*e + (C\*a\*c - A\*c^2)\*d^2 - (C\*a^2 - A\*a\*c)\*e^2)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + 6\*(C\*a\*c^2\*d^2 + 2\*B\*a\*c^2\*d\*e - (C\*a^2\*c - A\*a\*c^2)\*e^2)\*x + 3\*(B\*a\*c^2\*d^2 - B\*a^2\*c\*e^2 - 2\*(C\*a^2\*c - A\*a\*c^2)\*d\*e)\*log(c\*x^2 + a))/(a\*c^3)]

**giac** [A] time = 0.16, size = 176, normalized size = 1.05

$$\frac{(Bcd^2 - 2Cade + 2Acde - Ba^2e^2) \log(cx^2 + a)}{2c^2} - \frac{(Cacd^2 - Ac^2d^2 + 2Bacde - Ca^2e^2 + Aace^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2Cc^2x^3e^2 + 6Cc^2dx^2e + 6Cc^2d^2x + 3Bc^2x^2e^2 + 12Bc^2dxe - 6Cacxe^2 + 6Ac^2xe^2}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="giac")

[Out] 1/2\*(B\*c\*d^2 - 2\*C\*a\*d\*e + 2\*A\*c\*d\*e - B\*a\*e^2)\*log(c\*x^2 + a)/c^2 - (C\*a\*c\*d^2 - A\*c^2\*d^2 + 2\*B\*a\*c\*d\*e - C\*a^2\*e^2 + A\*a\*c\*e^2)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*c^2) + 1/6\*(2\*C\*c^2\*x^3\*e^2 + 6\*C\*c^2\*d\*x^2\*e + 6\*C\*c^2\*d^2\*x + 3\*B\*c^2\*x^2\*e^2 + 12\*B\*c^2\*d\*x\*e - 6\*C\*a\*c\*x\*e^2 + 6\*A\*c^2\*x\*e^2)/c^3

**maple** [A] time = 0.01, size = 256, normalized size = 1.52

$$\frac{C^2x^3}{3c} - \frac{Aae^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Ad^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{2Bade \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Bc^2x^2}{2c} + \frac{Ca^2e^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} - \frac{Ca^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Cde x^2}{c} + \frac{Ade \ln(cx^2 + a)}{c} + \frac{Ae^2 x}{c} - \frac{Ba^2 \ln(cx^2 + a)}{2c^2} + \frac{Bd^2 \ln(cx^2 + a)}{2c} + \frac{2Bdex}{c} - \frac{Cade \ln(cx^2 + a)}{c^2} - \frac{Ca^2 x}{c^2} + \frac{Cd^2 x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a),x)

[Out] 1/3\*C\*e^2\*x^3/c+1/2/c\*B\*x^2\*e^2+1/c\*C\*x^2\*d\*e+1/c\*A\*e^2\*x+2/c\*B\*d\*e\*x-1/c^2\*a\*C\*e^2\*x+1/c\*C\*d^2\*x+1/c\*ln(c\*x^2+a)\*A\*d\*e-1/2/c^2\*ln(c\*x^2+a)\*B\*a\*e^2+1/2/c\*ln(c\*x^2+a)\*B\*d^2-1/c^2\*ln(c\*x^2+a)\*C\*a\*d\*e-1/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*a\*e^2+1/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*d^2-2/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*B\*a\*d\*e+1/c^2/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*a^2\*C\*e^2-1/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C\*a\*d^2

**maxima** [A] time = 0.99, size = 161, normalized size = 0.96

$$\frac{(Bcd^2 - Ba^2e^2 - 2(Ca - Ac)de) \log(cx^2 + a)}{2c^2} - \frac{(2Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2Cce^2x^3 + 3(2Ccde + Bce^2)x^2 + 6(Ccd^2 + 2Bcde - (Ca - Ac)e^2)x}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="maxima")

[Out]  $1/2*(B*c*d^2 - B*a*e^2 - 2*(C*a - A*c)*d*e)*\log(c*x^2 + a)/c^2 - (2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*\arctan(c*x/\sqrt{a*c})/(\sqrt{(a*c)*c^2}) + 1/6*(2*C*c*e^2*x^3 + 3*(2*C*c*d*e + B*c*e^2)*x^2 + 6*(C*c*d^2 + 2*B*c*d*e - (C*a - A*c)*e^2)*x)/c^2$

**mupad [B]** time = 3.90, size = 181, normalized size = 1.08

$$x \left( \frac{C d^2 + 2 B d e + A e^2}{c} - \frac{C a e^2}{e^2} \right) + \frac{x^2 (B e^2 + 2 C d e)}{2 c} + \frac{C e^2 x^3}{3 c} - \frac{\operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right) (-C a^2 e^2 + C a c d^2 + 2 B a c d e + A a c e^2 - A c^2 d^2)}{\sqrt{a} c^{5/2}} + \frac{\ln(c x^2 + a) (-8 C a^2 c^3 d e - 4 B a^2 c^3 e^2 + 4 B a c^4 d^2 + 8 A a c^4 d e)}{8 a c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2), x)`

[Out]  $x*((A*e^2 + C*d^2 + 2*B*d*e)/c - (C*a*e^2)/c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (C*e^2*x^3)/(3*c) - (\operatorname{atan}((c^{1/2}*x)/a^{1/2}))*((A*a*c*e^2 - C*a^2*e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(a^{1/2}*c^{5/2}) + (\log(a + c*x^2)*(4*B*a*c^4*d^2 - 4*B*a^2*c^3*e^2 + 8*A*a*c^4*d*e - 8*C*a^2*c^3*d*e))/(8*a*c^5)$

**sympy [B]** time = 3.15, size = 638, normalized size = 3.80

$$\frac{C d^2}{c} + x \left( \frac{B e^2}{c} + \frac{C d^2}{c} \right) + \frac{C e^2 x^3}{3 c} - \frac{\operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right) (-C a^2 e^2 + C a c d^2 + 2 B a c d e + A a c e^2 - A c^2 d^2)}{\sqrt{a} c^{5/2}} + \frac{\ln(c x^2 + a) (-8 C a^2 c^3 d e - 4 B a^2 c^3 e^2 + 4 B a c^4 d^2 + 8 A a c^4 d e)}{8 a c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a), x)`

[Out]  $C*e**2*x**3/(3*c) + x**2*(B*e**2/(2*c) + C*d*e/c) + x*(A*e**2/c + 2*B*d*e/c - C*a*e**2/c**2 + C*d**2/c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e)/(2*c**2) - \sqrt{-a*c**5}*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*\log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e)/(2*c**2) - \sqrt{-a*c**5}*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e)/(2*c**2) + \sqrt{-a*c**5}*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*\log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*d*e)/(2*c**2) + \sqrt{-a*c**5}*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))$

$$3.45 \quad \int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2} dx$$

**Optimal.** Leaf size=93

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd - a(Be + Cd))}{\sqrt{a}c^{3/2}} + \frac{\log(a + cx^2)(-aCe + Ace + Bcd)}{2c^2} + \frac{x(Be + Cd)}{c} + \frac{Cex^2}{2c}$$

**Rubi [A]** time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1629, 635, 205, 260}

$$\frac{\log(a + cx^2)(-aCe + Ace + Bcd)}{2c^2} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd - a(Be + Cd))}{\sqrt{a}c^{3/2}} + \frac{x(Be + Cd)}{c} + \frac{Cex^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] ((C\*d + B\*e)\*x)/c + (C\*e\*x^2)/(2\*c) + ((A\*c\*d - a\*(C\*d + B\*e))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(Sqrt[a]\*c^(3/2)) + ((B\*c\*d + A\*c\*e - a\*C\*e)\*Log[a + c\*x^2])/(2\*c^2)

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 1629

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_)^(m\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c,

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2} dx &= \int \left( \frac{Cd+Be}{c} + \frac{Cex}{c} + \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{c(a+cx^2)} \right) dx \\ &= \frac{(Cd+Be)x}{c} + \frac{Cex^2}{2c} + \frac{\int \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{a+cx^2} dx}{c} \\ &= \frac{(Cd+Be)x}{c} + \frac{Cex^2}{2c} + \frac{(Bcd+Ace-aCe) \int \frac{x}{a+cx^2} dx}{c} + \frac{(Acd-a(Cd+Be)) \int \frac{1}{a+cx^2} dx}{c} \\ &= \frac{(Cd+Be)x}{c} + \frac{Cex^2}{2c} + \frac{(Acd-a(Cd+Be)) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{(Bcd+Ace-aCe) \log\left(\frac{a+cx^2}{a}\right)}{2c^2} \end{aligned}$$

**Mathematica** [A] time = 0.09, size = 86, normalized size = 0.92

$$\frac{\log(a+cx^2)(-aCe+Ace+Bcd) - \frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd-Acd)}{\sqrt{a}} + cx(2Be+2Cd+Cex)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] (c\*x\*(2\*C\*d + 2\*B\*e + C\*e\*x) - (2\*Sqrt[c]\*(-(A\*c\*d) + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/Sqrt[a] + (B\*c\*d + A\*c\*e - a\*C\*e)\*Log[a + c\*x^2])/(2\*c^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2), x]

**fricas** [A] time = 1.12, size = 206, normalized size = 2.22

$$\frac{\left[ \frac{Cacex^2 - (Bae + (Ca - Ac)d)\sqrt{-ac} \log\left(\frac{cx^2 + 2\sqrt{-ac}x - a}{cx^2 + a}\right) + 2(Cacd + Bace)x + (Bacd - (Ca^2 - Aac)e) \log(cx^2 + a)}{2ac^2}, \frac{Cacex^2 - 2(Bae + (Ca - Ac)d)\sqrt{ac} \arctan\left(\frac{\sqrt{ac}x}{a}\right) + 2(Cacd + Bace)x + (Bacd - (Ca^2 - Aac)e) \log(cx^2 + a)}{2ac^2} \right]}{2ac^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="fricas")

[Out]  $\left[ \frac{1}{2} * (C * a * c * e * x^2 - (B * a * e + (C * a - A * c) * d) * \sqrt{-a * c} * \log((c * x^2 + 2 * \sqrt{-a * c} * x - a) / (c * x^2 + a)) + 2 * (C * a * c * d + B * a * c * e) * x + (B * a * c * d - (C * a^2 - A * a * c) * e) * \log(c * x^2 + a)) / (a * c^2), \frac{1}{2} * (C * a * c * e * x^2 - 2 * (B * a * e + (C * a - A * c) * d) * \sqrt{a * c} * \arctan(\sqrt{a * c} * x / a) + 2 * (C * a * c * d + B * a * c * e) * x + (B * a * c * d - (C * a^2 - A * a * c) * e) * \log(c * x^2 + a)) / (a * c^2) \right]$

**giac** [A] time = 0.16, size = 91, normalized size = 0.98

$$\frac{(Cad - Acd + Bae) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{(Bcd - CAe + Ace) \log(cx^2 + a)}{2c^2} + \frac{Ccx^2e + 2Ccdx + 2Bcxe}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="giac")

[Out]  $-(C * a * d - A * c * d + B * a * e) * \arctan(c * x / \sqrt{a * c}) / (\sqrt{a * c} * c) + \frac{1}{2} * (B * c * d - C * a * e + A * c * e) * \log(c * x^2 + a) / c^2 + \frac{1}{2} * (C * c * x^2 * e + 2 * C * c * d * x + 2 * B * c * x * e) / c^2$

**maple** [A] time = 0.01, size = 133, normalized size = 1.43

$$\frac{Ad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{Bae \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} - \frac{Cad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Cex^2}{2c} + \frac{Ae \ln(cx^2 + a)}{2c} + \frac{Bd \ln(cx^2 + a)}{2c} + \frac{Bex}{c} - \frac{CAe \ln(cx^2 + a)}{2c^2} + \frac{Cdx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a),x)

[Out]  $\frac{1}{2} * C * e * x^2 / c + \frac{1}{c} * B * e * x + \frac{1}{c} * C * d * x + \frac{1}{2} * \frac{1}{c} * \ln(c * x^2 + a) * A * e + \frac{1}{2} * \frac{1}{c} * \ln(c * x^2 + a) * B * d - \frac{1}{2} * \frac{1}{c^2} * \ln(c * x^2 + a) * a * C * e + \frac{1}{(a * c)^{(1/2)}} * \arctan(1 / (a * c)^{(1/2)} * c * x) * A * d - \frac{1}{c} * \frac{1}{(a * c)^{(1/2)}} * \arctan(1 / (a * c)^{(1/2)} * c * x) * B * a * e - \frac{1}{c} * \frac{1}{(a * c)^{(1/2)}} * \arctan(1 / (a * c)^{(1/2)} * c * x) * C * a * d$

**maxima** [A] time = 0.97, size = 86, normalized size = 0.92

$$-\frac{(Bae + (Ca - Ac)d) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Cex^2 + 2(Cd + Be)x}{2c} + \frac{(Bcd - (Ca - Ac)e) \log(cx^2 + a)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="maxima")

[Out]  $-(B*a*e + (C*a - A*c)*d)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*c) + 1/2*(C*e*x^2 + 2*(C*d + B*e)*x)/c + 1/2*(B*c*d - (C*a - A*c)*e)*\log(c*x^2 + a)/c^2$

mupad [B] time = 3.78, size = 97, normalized size = 1.04

$$\frac{x(Be + Cd)}{c} - \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Bae - Acd + Cad)}{\sqrt{a}c^{3/2}} + \frac{Cex^2}{2c} + \frac{\ln(cx^2 + a)(4Aac^3e + 4Bac^3d - 4Ca^2c^2e)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2), x)`

[Out]  $(x*(B*e + C*d))/c - (\operatorname{atan}(c^{1/2}*x/a^{1/2})*(B*a*e - A*c*d + C*a*d))/(a^{1/2}*c^{3/2}) + (C*e*x^2)/(2*c) + (\log(a + c*x^2)*(4*A*a*c^3*e + 4*B*a*c^3*d - 4*C*a^2*c^2*e))/(8*a*c^4)$

sympy [B] time = 1.66, size = 337, normalized size = 3.62

$$\frac{Ccx^2}{2c} + x\left(\frac{Be}{c} + \frac{Cd}{c}\right) + \left(-\frac{Ace - Bcd + Cae}{2c^2} - \frac{\sqrt{-a^3}(-Acd + Bae + Cnd)}{2ac^4}\right) \log\left(x + \frac{Aace + Bacd - Ca^2e - 2ac^2\left(-\frac{Ace - Bcd + Cae}{2c^2} - \frac{\sqrt{-a^3}(-Acd + Bae + Cnd)}{2ac^4}\right)}{-Ac^2d + Bace + Cacd}\right) + \left(-\frac{Ace - Bcd + Cae}{2c^2} + \frac{\sqrt{-a^3}(-Acd + Bae + Cnd)}{2ac^4}\right) \log\left(x + \frac{Aace + Bacd - Ca^2e - 2ac^2\left(-\frac{Ace - Bcd + Cae}{2c^2} + \frac{\sqrt{-a^3}(-Acd + Bae + Cnd)}{2ac^4}\right)}{-Ac^2d + Bace + Cacd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a), x)`

[Out]  $C*e*x**2/(2*c) + x*(B*e/c + C*d/c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - \sqrt{-a*c**5}*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*\log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - \sqrt{-a*c**5}*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + \sqrt{-a*c**5}*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*\log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + \sqrt{-a*c**5}*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d))$

$$3.46 \quad \int \frac{A+Bx+Cx^2}{a+cx^2} dx$$

Optimal. Leaf size=55

$$\frac{(Ac - aC) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{B \log(a + cx^2)}{2c} + \frac{Cx}{c}$$

**Rubi [A]** time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1810, 635, 205, 260}

$$\frac{(Ac - aC) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{B \log(a + cx^2)}{2c} + \frac{Cx}{c}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2), x]

[Out] (C\*x)/c + ((A\*c - a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(Sqrt[a]\*c^(3/2)) + (B\*Log[a + c\*x^2])/(2\*c)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{a + cx^2} dx &= \int \left( \frac{C}{c} + \frac{Ac - aC + Bcx}{c(a + cx^2)} \right) dx \\
&= \frac{Cx}{c} + \frac{\int \frac{Ac - aC + Bcx}{a + cx^2} dx}{c} \\
&= \frac{Cx}{c} + B \int \frac{x}{a + cx^2} dx + \frac{(Ac - aC) \int \frac{1}{a + cx^2} dx}{c} \\
&= \frac{Cx}{c} + \frac{(Ac - aC) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{a}} \right)}{\sqrt{a} c^{3/2}} + \frac{B \log(a + cx^2)}{2c}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 56, normalized size = 1.02

$$-\frac{(aC - Ac) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{a}} \right)}{\sqrt{a} c^{3/2}} + \frac{B \log(a + cx^2)}{2c} + \frac{Cx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2), x]

[Out] (C\*x)/c - ((- (A\*c) + a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(Sqrt[a]\*c^(3/2)) + (B\*Log[a + c\*x^2])/(2\*c)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{a + cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2), x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2), x]

**fricas [A]** time = 1.16, size = 125, normalized size = 2.27

$$\left[ \frac{2Cacx + Bac \log(cx^2 + a) + (Ca - Ac)\sqrt{-ac} \log\left(\frac{cx^2 - 2\sqrt{-ac}x - a}{cx^2 + a}\right)}{2ac^2}, \frac{2Cacx + Bac \log(cx^2 + a) - 2(Ca - Ac)\sqrt{ac} \arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2ac^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="fricas")

[Out] [1/2\*(2\*C\*a\*c\*x + B\*a\*c\*log(c\*x^2 + a) + (C\*a - A\*c)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)))/(a\*c^2), 1/2\*(2\*C\*a\*c\*x + B\*a\*c\*log(c\*x^2 + a) - 2\*(C\*a - A\*c)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a))/(a\*c^2)]

**giac** [A] time = 0.16, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B \log(cx^2 + a)}{2c} - \frac{(Ca - Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="giac")

[Out] C\*x/c + 1/2\*B\*log(c\*x^2 + a)/c - (C\*a - A\*c)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*c)

**maple** [A] time = 0.00, size = 59, normalized size = 1.07

$$\frac{A \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{Ca \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{B \ln(cx^2 + a)}{2c} + \frac{Cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(c\*x^2+a),x)

[Out] C\*x/c+1/2\*B\*ln(c\*x^2+a)/c+1/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A-1/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*a\*C

**maxima** [A] time = 0.97, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B \log(cx^2 + a)}{2c} - \frac{(Ca - Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a),x, algorithm="maxima")

[Out] C\*x/c + 1/2\*B\*log(c\*x^2 + a)/c - (C\*a - A\*c)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*c)

**mupad** [B] time = 3.73, size = 56, normalized size = 1.02

$$\frac{B \ln(cx^2 + a)}{2c} + \frac{Cx}{c} + \frac{A \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{C\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(a + c*x^2),x)`

[Out]  $(B \cdot \log(a + c \cdot x^2)) / (2 \cdot c) + (C \cdot x) / c + (A \cdot \operatorname{atan}((c^{1/2} \cdot x) / a^{1/2})) / (a^{1/2} \cdot c^{1/2}) - (C \cdot a^{1/2} \cdot \operatorname{atan}((c^{1/2} \cdot x) / a^{1/2})) / c^{3/2}$

**sympy [B]** time = 0.49, size = 156, normalized size = 2.84

$$\frac{Cx}{c} + \left( \frac{B}{2c} - \frac{\sqrt{-ac^3}(-Ac + Ca)}{2ac^3} \right) \log \left( x + \frac{Ba - 2ac \left( \frac{B}{2c} - \frac{\sqrt{-ac^3}(-Ac + Ca)}{2ac^3} \right)}{-Ac + Ca} \right) + \left( \frac{B}{2c} + \frac{\sqrt{-ac^3}(-Ac + Ca)}{2ac^3} \right) \log \left( x + \frac{Ba - 2ac \left( \frac{B}{2c} + \frac{\sqrt{-ac^3}(-Ac + Ca)}{2ac^3} \right)}{-Ac + Ca} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/(c*x**2+a),x)`

[Out]  $C \cdot x / c + (B / (2 \cdot c) - \sqrt{-a \cdot c^3} \cdot (-A \cdot c + C \cdot a) / (2 \cdot a \cdot c^3)) \cdot \log(x + (B \cdot a - 2 \cdot a \cdot c \cdot (B / (2 \cdot c) - \sqrt{-a \cdot c^3} \cdot (-A \cdot c + C \cdot a) / (2 \cdot a \cdot c^3)))) / (-A \cdot c + C \cdot a) + (B / (2 \cdot c) + \sqrt{-a \cdot c^3} \cdot (-A \cdot c + C \cdot a) / (2 \cdot a \cdot c^3)) \cdot \log(x + (B \cdot a - 2 \cdot a \cdot c \cdot (B / (2 \cdot c) + \sqrt{-a \cdot c^3} \cdot (-A \cdot c + C \cdot a) / (2 \cdot a \cdot c^3)))) / (-A \cdot c + C \cdot a)$

$$3.47 \quad \int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)} dx$$

**Optimal.** Leaf size=133

$$\frac{\log(a+cx^2)(aCe - Ace + Bcd)}{2c(ae^2 + cd^2)} + \frac{\log(d+ex)(Ae^2 - Bde + Cd^2)}{e(ae^2 + cd^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe - aCd + Acd)}{\sqrt{a}\sqrt{c}(ae^2 + cd^2)}$$

**Rubi [A]** time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1629, 635, 205, 260}

$$\frac{\log(a+cx^2)(aCe - Ace + Bcd)}{2c(ae^2 + cd^2)} + \frac{\log(d+ex)(Ae^2 - Bde + Cd^2)}{e(ae^2 + cd^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe - aCd + Acd)}{\sqrt{a}\sqrt{c}(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)), x]
```

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) + ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

**Rule 205**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

**Rule 260**

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

**Rule 635**

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

**Rule 1629**

```
Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)} dx &= \int \left( \frac{Cd^2 - Bde + Ae^2}{(cd^2 + ae^2)(d + ex)} + \frac{Acd - aCd + aBe + (Bcd - Ace + aCe)x}{(cd^2 + ae^2)(a + cx^2)} \right) dx \\
 &= \frac{(Cd^2 - Bde + Ae^2) \log(d + ex)}{e(cd^2 + ae^2)} + \frac{\int \frac{Acd - aCd + aBe + (Bcd - Ace + aCe)x}{a + cx^2} dx}{cd^2 + ae^2} \\
 &= \frac{(Cd^2 - Bde + Ae^2) \log(d + ex)}{e(cd^2 + ae^2)} + \frac{(Acd - aCd + aBe) \int \frac{1}{a + cx^2} dx}{cd^2 + ae^2} + \frac{(Bcd - Ace + aCe)}{cd^2 + ae^2} \\
 &= \frac{(Acd - aCd + aBe) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}(cd^2 + ae^2)} + \frac{(Cd^2 - Bde + Ae^2) \log(d + ex)}{e(cd^2 + ae^2)} + \frac{(Bcd - Ace + aCe)}{2c(cd^2 + ae^2)}
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 120, normalized size = 0.90

$$\frac{\sqrt{a} \left( e \log(a + cx^2) (aCe - Ace + Bcd) + 2c \log(d + ex) (Ae^2 - Bde + Cd^2) \right) + 2\sqrt{c} e \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) (aBe - aCd + Acd)}{2\sqrt{a} ce (ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)), x]

[Out] (2\*Sqrt[c]\*e\*(A\*c\*d - a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]] + Sqrt[a]\*(2\*c\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[d + e\*x] + e\*(B\*c\*d - A\*c\*e + a\*C\*e)\*Log[a + c\*x^2])/((2\*Sqrt[a]\*c\*e\*(c\*d^2 + a\*e^2)))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)), x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)), x]



**fricas** [A] time = 19.02, size = 262, normalized size = 1.97

$$\frac{\left( \frac{(Bac^2 - (Ca - Ac)de)\sqrt{-ac} \log\left(\frac{cx^2 - 2\sqrt{-ac}x + a}{cx^2 + a}\right) - (Bacde + (Ca^2 - Aac)e^2) \log(cx^2 + a) - 2(Cacd^2 - Bacde + Aac^2) \log(ex + d)}{2(a^2de + a^2ce^3)} \right) - \frac{2(Bac^2 - (Ca - Ac)de)\sqrt{ac} \arctan\left(\frac{\sqrt{ac}x}{a}\right) + (Bacde + (Ca^2 - Aac)e^2) \log(cx^2 + a) + 2(Cacd^2 - Bacde + Aac^2) \log(ex + d)}{2(a^2de + a^2ce^3)}}{2(a^2d^2 + ace^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a),x, algorithm="fricas")

[Out]  $[-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*\sqrt{-a*c}*\log((c*x^2 - 2*\sqrt{-a*c})*x - a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*\log(c*x^2 + a) - 2*(C*a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*\log(e*x + d)]/(a*c^2*d^2*e + a^2*c*e^3), 1/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*\sqrt{a*c}*\arctan(\sqrt{a*c}*x/a) + (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*\log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*\log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)]$

**giac** [A] time = 0.16, size = 125, normalized size = 0.94

$$\frac{(Bcd + CAe - Ace) \log(cx^2 + a)}{2(c^2d^2 + ace^2)} + \frac{(Cd^2 - Bde + Ae^2) \log(|xe + d|)}{cd^2e + ae^3} - \frac{(Cad - Acd - Bae) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2 + ae^2)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a),x, algorithm="giac")

[Out]  $1/2*(B*c*d + C*a*e - A*c*e)*\log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B*d*e + A*e^2)*\log(\text{abs}(x*e + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*e)*\arctan(c*x/\sqrt{a*c})/((c*d^2 + a*e^2)*\sqrt{a*c})$

**maple** [A] time = 0.01, size = 247, normalized size = 1.86

$$\frac{Acd \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(a^2e + cd^2)\sqrt{ac}} + \frac{Bae \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(a^2e + cd^2)\sqrt{ac}} - \frac{Cad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(ae^2 + cd^2)\sqrt{ac}} - \frac{Ae \ln(cx^2 + a)}{2(ae^2 + cd^2)} + \frac{Ae \ln(ex + d)}{ae^2 + cd^2} + \frac{Bd \ln(cx^2 + a)}{2ae^2 + 2cd^2} - \frac{Bd \ln(ex + d)}{ae^2 + cd^2} + \frac{CAe \ln(cx^2 + a)}{2(ae^2 + cd^2)c} + \frac{Cd^2 \ln(ex + d)}{(ae^2 + cd^2)e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a),x)

[Out]  $1/(a*e^2+c*d^2)*e*\ln(e*x+d)*A-1/(a*e^2+c*d^2)*\ln(e*x+d)*B*d+1/(a*e^2+c*d^2)/e*\ln(e*x+d)*C*d^2-1/2/(a*e^2+c*d^2)*\ln(c*x^2+a)*A*e+1/2/(a*e^2+c*d^2)*\ln(c*x^2+a)*B*d+1/2/(a*e^2+c*d^2)/c*\ln(c*x^2+a)*a*C*e+1/(a*e^2+c*d^2)/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*A*c*d+1/(a*e^2+c*d^2)/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*B*a*e-1/(a*e^2+c*d^2)/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*a*d$

**maxima** [A] time = 0.97, size = 123, normalized size = 0.92

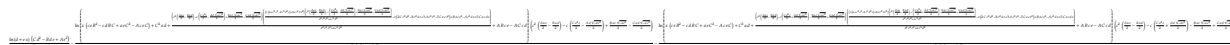
$$\frac{(Bcd + (Ca - Ac)e) \log(cx^2 + a)}{2(c^2d^2 + ace^2)} + \frac{(Cd^2 - Bde + Ae^2) \log(ex + d)}{cd^2e + ae^3} + \frac{(Bae - (Ca - Ac)d) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2 + ae^2)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(B*c*d + (C*a - A*c)*e)*\log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B*d*e + A*e^2)*\log(e*x + d)/(c*d^2*e + a*e^3) + (B*a*e - (C*a - A*c)*d)*\arctan(c*x/\sqrt{a*c})/((c*d^2 + a*e^2)*\sqrt{a*c})$

**mupad [B]** time = 6.49, size = 840, normalized size = 6.32



Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((a + c\*x^2)\*(d + e\*x)),x)

[Out]  $(\log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 + c*d^2*e) - (\log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^{(1/2)}))/2) + (B*a*e*(-a*c^3)^{(1/2)})/2 - (C*a*d*(-a*c^3)^{(1/2)})/2)*((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^{(1/2)}))/2) + (B*a*e*(-a*c^3)^{(1/2)})/2 - (C*a*d*(-a*c^3)^{(1/2)})/2))/(a*c^3*d^2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e)/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^{(1/2)}))/2) + (B*a*e*(-a*c^3)^{(1/2)})/2 - (C*a*d*(-a*c^3)^{(1/2)})/2))/(a*c^3*d^2 + a^2*c^2*e^2) - (\log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^{(1/2)}))/2) - (B*a*e*(-a*c^3)^{(1/2)})/2 + (C*a*d*(-a*c^3)^{(1/2)})/2)*((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^{(1/2)}))/2) - (B*a*e*(-a*c^3)^{(1/2)})/2 + (C*a*d*(-a*c^3)^{(1/2)})/2))/(a*c^3*d^2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e)/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^{(1/2)}))/2) - (B*a*e*(-a*c^3)^{(1/2)})/2 + (C*a*d*(-a*c^3)^{(1/2)})/2))/(a*c^3*d^2 + a^2*c^2*e^2)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)/(c\*x\*\*2+a),x)

[Out] Timed out

$$3.48 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)} dx$$

**Optimal.** Leaf size=214

$$\frac{\log(a+cx^2)(-aBe^2+2aCde-2Acde+Bcd^2)}{2(ae^2+cd^2)^2} - \frac{Ae^2-Bde+Cd^2}{e(d+ex)(ae^2+cd^2)} - \frac{\log(d+ex)(-aBe^2+2aCde-2Acde+Bcd^2)}{(ae^2+cd^2)^2}$$

**Rubi [A]** time = 0.36, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1629, 635, 205, 260}

$$\frac{\log(a+cx^2)(-aBe^2+2aCde-2Acde+Bcd^2)}{2(ae^2+cd^2)^2} - \frac{Ae^2-Bde+Cd^2}{e(d+ex)(ae^2+cd^2)} - \frac{\log(d+ex)(-aBe^2+2aCde-2Acde+Bcd^2)}{(ae^2+cd^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))}{\sqrt{a}\sqrt{c}(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)), x]

[Out] -((C\*d^2 - B\*d\*e + A\*e^2)/(e\*(c\*d^2 + a\*e^2)\*(d + e\*x))) + ((A\*c\*(c\*d^2 - a\*e^2) + a\*(a\*C\*e^2 - c\*d\*(C\*d - 2\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[c]\*(c\*d^2 + a\*e^2)^2) - ((B\*c\*d^2 - 2\*A\*c\*d\*e + 2\*a\*C\*d\*e - a\*B\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^2 + ((B\*c\*d^2 - 2\*A\*c\*d\*e + 2\*a\*C\*d\*e - a\*B\*e^2)\*Log[a + c\*x^2])/(2\*(c\*d^2 + a\*e^2)^2)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 1629

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_))^(m\_.)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c,

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)} dx &= \int \left( \frac{Cd^2 - Bde + Ae^2}{(cd^2 + ae^2)(d + ex)^2} + \frac{e(-Bcd^2 + 2Acde - 2aCde + aBe^2)}{(cd^2 + ae^2)^2 (d + ex)} + \frac{Ac(cd^2 - ae^2) + a}{(cd^2 + ae^2)(d + ex)} \right) dx \\
 &= -\frac{Cd^2 - Bde + Ae^2}{e(cd^2 + ae^2)(d + ex)} - \frac{(Bcd^2 - 2Acde + 2aCde - aBe^2) \log(d + ex)}{(cd^2 + ae^2)^2} + \int \frac{Ac(cd^2 - ae^2)}{(cd^2 + ae^2)(d + ex)} dx \\
 &= -\frac{Cd^2 - Bde + Ae^2}{e(cd^2 + ae^2)(d + ex)} - \frac{(Bcd^2 - 2Acde + 2aCde - aBe^2) \log(d + ex)}{(cd^2 + ae^2)^2} + \frac{c(Bcd^2 - 2Acde + 2aCde - aBe^2)}{(cd^2 + ae^2)(d + ex)} \\
 &= -\frac{Cd^2 - Bde + Ae^2}{e(cd^2 + ae^2)(d + ex)} + \frac{(Ac(cd^2 - ae^2) + a(aCe^2 - cd(Cd - 2Be))) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}(cd^2 + ae^2)^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 188, normalized size = 0.88

$$\frac{\log(a + cx^2)(-aBe^2 + 2aCde - 2Acde + Bcd^2) - \frac{2(ae^2 + cd^2)e(Ae - Bd + Cd^2)}{e(d + ex)} + \log(d + ex)(2aBe^2 - 4aCde + 4Acde - 2Bcd^2) + \frac{2 \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac(cd^2 - ae^2) + a(aCe^2 + cd(2Be - Cd)))}{\sqrt{a}\sqrt{c}}}{2(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)), x]

[Out] ((-2\*(c\*d^2 + a\*e^2)\*(C\*d^2 + e\*(-(B\*d) + A\*e)))/(e\*(d + e\*x)) + (2\*(A\*c\*(c\*d^2 - a\*e^2) + a\*(a\*C\*e^2 + c\*d\*(-(C\*d) + 2\*B\*e)))\*ArcTan[Sqrt[c]\*x]/Sqrt[a])/(Sqrt[a]\*Sqrt[c]) + (-2\*B\*c\*d^2 + 4\*A\*c\*d\*e - 4\*a\*C\*d\*e + 2\*a\*B\*e^2)\*Log[d + e\*x] + (B\*c\*d^2 - 2\*A\*c\*d\*e + 2\*a\*C\*d\*e - a\*B\*e^2)\*Log[a + c\*x^2]/(2\*(c\*d^2 + a\*e^2)^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)), x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)), x]

**fricas** [B] time = 72.03, size = 904, normalized size = 4.22

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*A*a^2*c*e^4 + \\ & 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - (2*B*a*c*d^2*e^2 - (C*a*c - A*c^2)*d^3*e + \\ & (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2)*d^2*e^2 + (C*a^2 - \\ & A*a*c)*e^4)*x)*\sqrt{-a*c}*\log((c*x^2 + 2*\sqrt{-a*c}*x - a)/(c*x^2 + a)) - \\ & (B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d \\ & ^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*\log(c*x^2 + a) + 2*( \\ & B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^ \\ & 2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*\log(e*x + d)]/(a*c^3* \\ & d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^ \\ & 4 + a^3*c*e^6)*x), -1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 \\ & + 2*A*a^2*c*e^4 + 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - 2*(2*B*a*c*d^2*e^2 - (C*a \\ & *c - A*c^2)*d^3*e + (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2 \\ & )*d^2*e^2 + (C*a^2 - A*a*c)*e^4)*x)*\sqrt{a*c}*\arctan(\sqrt{a*c}*x/a) - (B*a* \\ & c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^ \\ & 2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*\log(c*x^2 + a) + 2*(B*a*c \\ & ^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^2 \\ & - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*\log(e*x + d)]/(a*c^3*d^5*e \\ & + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^4 + a \\ & ^3*c*e^6)*x)] \end{aligned}$$

**giac** [A] time = 0.17, size = 270, normalized size = 1.26

$$-\frac{(Cacd^2e^2 - Ac^2d^2e^2 - 2Bacde^3 - Ca^2e^4 + Aace^4) \arctan\left(\frac{\left(\frac{cd}{xe+d} - \frac{ae^2}{xe+d}\right)^{(-1)}}{\sqrt{ac}}\right) e^{(-2)}}{(c^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{ac}} + \frac{(Bcd^2 + 2Cade - 2Acde - Bae^2) \log\left(c - \frac{2cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}\right) - \frac{Cd^2e}{xe+d} - \frac{Bde^2}{xe+d} + \frac{Ae^3}{xe+d}}{2(c^2d^4 + 2acd^2e^2 + a^2e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -(C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*c*e^4)*\ar \\ & \text{ctan}((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^{(-1)}/\sqrt{a*c})*e^{(-2)}/((c \\ & ^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\sqrt{a*c}) + 1/2*(B*c*d^2 + 2*C*a*d*e - 2 \\ & *A*c*d*e - B*a*e^2)*\log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x* \\ & e + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - (C*d^2*e/(x*e + d) - B*d*e^ \\ & 2/(x*e + d) + A*e^3/(x*e + d))/(c*d^2*e^2 + a*e^4) \end{aligned}$$

**maple [B]** time = 0.01, size = 462, normalized size = 2.16

$$\frac{Aa^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right) + A^2 d^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 2Bac d \arctan\left(\frac{cx}{\sqrt{ac}}\right) + C a^2 d \arctan\left(\frac{cx}{\sqrt{ac}}\right) + C a c d \arctan\left(\frac{cx}{\sqrt{ac}}\right) + A d c \ln(cx^2 + a) + 2A d c \ln(ex + d) + B a^2 \ln(cx^2 + a) + B a^2 \ln(ex + d) + B a^2 \ln(cx^2 + a) + B a^2 \ln(ex + d) + B c d^2 \ln(cx^2 + a) + B c d^2 \ln(ex + d) + C a d \ln(cx^2 + a) + 2C a d \ln(ex + d) + \frac{A c}{(a^2 + c d^2)(ex + d)} + \frac{B d}{(a^2 + c d^2)(ex + d)} + \frac{C d^2}{(a^2 + c d^2)(ex + d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a),x)

[Out] 
$$\begin{aligned} & -1/(a*e^2+c*d^2)*e/(e*x+d)*A+1/(a*e^2+c*d^2)/(e*x+d)*B*d-1/(a*e^2+c*d^2)/e/ \\ & (e*x+d)*C*d^2+2/(a*e^2+c*d^2)^2*\ln(e*x+d)*A*c*d*e+1/(a*e^2+c*d^2)^2*\ln(e*x+ \\ & d)*B*a*e^2-1/(a*e^2+c*d^2)^2*\ln(e*x+d)*B*c*d^2-2/(a*e^2+c*d^2)^2*\ln(e*x+d)* \\ & C*a*d*e-1/(a*e^2+c*d^2)^2*c*\ln(c*x^2+a)*A*d*e-1/2/(a*e^2+c*d^2)^2*\ln(c*x^2+ \\ & a)*e^2*B*a+1/2/(a*e^2+c*d^2)^2*c*\ln(c*x^2+a)*d^2*B+1/(a*e^2+c*d^2)^2*\ln(c*x \\ & ^2+a)*C*a*d*e-1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*A*a*c \\ & *e^2+1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2+2/(a \\ & *e^2+c*d^2)^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*B*a*c*d*e+1/(a*e^2+c*d^ \\ & 2)^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*a^2*C*e^2-1/(a*e^2+c*d^2)^2/(a*c \\ & )^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*a*c*d^2 \end{aligned}$$

**maxima [A]** time = 1.04, size = 255, normalized size = 1.19

$$\frac{(Bcd^2 - Bae^2 + 2(Ca - Ac)d)e \log(cx^2 + a)}{2(c^2d^4 + 2acd^2e^2 + a^2e^4)} - \frac{(Bcd^2 - Bae^2 + 2(Ca - Ac)d)e \log(ex + d)}{c^2d^4 + 2acd^2e^2 + a^2e^4} + \frac{(2Bacde - (Cac - Ac^2)d^2 + (Ca^2 - Aac)e^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^2d^4 + 2acd^2e^2 + a^2e^4)\sqrt{ac}} - \frac{Cd^2 - Bde + Ae^2}{cd^3e + ade^3 + (cd^2e^2 + ae^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 1/2*(B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*\log(c*x^2 + a)/(c^2*d^4 + 2*a*c \\ & *d^2*e^2 + a^2*e^4) - (B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*\log(e*x + d)/ \\ & (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (2*B*a*c*d*e - (C*a*c - A*c^2)*d^2 + \\ & (C*a^2 - A*a*c)*e^2)*\arctan(c*x/\sqrt{a*c})/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*c \\ & e^4)*\sqrt{a*c}) - (C*d^2 - B*d*e + A*e^2)/(c*d^3*e + a*d*e^3 + (c*d^2*e^2 + \\ & a*e^4)*x) \end{aligned}$$

**mupad [B]** time = 6.77, size = 1199, normalized size = 5.60

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((a + c\*x^2)\*(d + e\*x)^2),x)

[Out] 
$$\begin{aligned} & (\log(C*c*d^4*(-a*c)^(3/2) - A*a*e^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*c \\ & *e^4 + A*c^4*d^4*x + A*c^3*d^4*(-a*c)^(1/2) - C*a^3*e^4*(-a*c)^(1/2) - C*a \\ & *c^3*d^4*x - C*a^3*c*e^4*x + 14*A*c*d^2*e^2*(-a*c)^(3/2) - 14*C*a*d^2*e^2*( \\ & -a*c)^(3/2) - 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2* \end{aligned}$$

$$\begin{aligned}
& d^3e + Aa^2c^2e^4x - 10B^2a^2c^2d^2e^2 + 8B^2ad^3e^3(-ac)^{(3/2)} - \\
& 8B^2cd^3e^3(-ac)^{(3/2)} + 3B^2ae^4x(-ac)^{(3/2)} - 8A^2ac^3d^3e - 8C^2a^3cd^3e^3 + 14C^2a^2c^2d^2e^2x + 8A^2cd^3e^3x(-ac)^{(3/2)} - 8C^2a^2d^3e^3x(-ac)^{(3/2)} + 8C^2cd^3e^3x(-ac)^{(3/2)} + 8B^2ac^3d^3e^3x + 8A^2c^3d^3e^3x(-ac)^{(1/2)} - 10B^2cd^2e^2x(-ac)^{(3/2)} - 14A^2ac^3d^2e^2x - 8B^2a^2c^2d^2e^3x)(c^2(a((Bd^2)/2 - Ad^2e) + (Ad^2(-ac)^{(1/2)}/2) - c(a^2((Be^2)/2 - Cd^2e) + a((Ae^2(-ac)^{(1/2)}/2) + (Cd^2(-ac)^{(1/2)}/2 - Bd^2e(-ac)^{(1/2)}/2)) + (C^2a^2e^2(-ac)^{(1/2)}/2)))/(ac^3d^4 + a^3c^2e^4 + 2a^2c^2d^2e^2) - (\log(d + ex)(c(Bd^2 - 2Ad^2e) - a(Be^2 - 2Cd^2e)))/(a^2e^4 + c^2d^4 + 2ac^2d^2e^2) - (\log(A^2ae^4(-ac)^{(3/2)} - C^2cd^4(-ac)^{(3/2)} + 3B^2ac^3d^4 + 3B^2a^3c^2e^4 + A^2c^4d^4x - A^2c^3d^4(-ac)^{(1/2)} + C^2a^3e^4(-ac)^{(1/2)} - C^2ac^3d^4x - C^2a^3c^2e^4x - 14A^2cd^2e^2(-ac)^{(3/2)} + 14C^2ad^2e^2(-ac)^{(3/2)} + 3B^2c^3d^4x(-ac)^{(1/2)} + 8A^2a^2c^2d^2e^3 + 8C^2a^2c^2d^3e + A^2a^2c^2e^4x - 10B^2a^2c^2d^2e^2 - 8B^2ad^3e^3(-ac)^{(3/2)} + 8B^2cd^3e^3(-ac)^{(3/2)} - 3B^2ae^4x(-ac)^{(3/2)} - 8A^2ac^3d^3e - 8C^2a^3cd^3e^3 + 14C^2a^2c^2d^2e^2x - 8A^2cd^3e^3x(-ac)^{(3/2)} + 8C^2ad^3e^3x(-ac)^{(3/2)} - 8C^2cd^3e^3x(-ac)^{(3/2)} + 8B^2ac^3d^3e^3x - 8A^2c^3d^3e^3x(-ac)^{(1/2)} + 10B^2cd^2e^2x(-ac)^{(3/2)} - 14A^2ac^3d^2e^2x - 8B^2a^2c^2d^2e^3x)(c(a^2((Be^2)/2 - Cd^2e) - a((Ae^2(-ac)^{(1/2)}/2) + (Cd^2(-ac)^{(1/2)}/2 - Bd^2e(-ac)^{(1/2)}/2)) - c^2(a((Bd^2)/2 - Ad^2e) - (Ad^2(-ac)^{(1/2)}/2) + (C^2a^2e^2(-ac)^{(1/2)}/2)))/(ac^3d^4 + a^3c^2e^4 + 2a^2c^2d^2e^2) - (Ae^2 + Cd^2 - Bd^2e)/(e(ae^2 + cd^2)(d + ex))
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*2/(c\*x\*\*2+a),x)

[Out] Timed out

$$3.49 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)} dx$$

**Optimal.** Leaf size=305

$$\frac{\log(a+cx^2)(Bcd(cd^2-3ae^2)-e(Ac-aC)(3cd^2-ae^2))}{2(ae^2+cd^2)^3} - \frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2(ae^2+cd^2)} + \frac{-aBe^2+2aCde-2Acde+}{(d+ex)(ae^2+cd^2)^2}$$

**Rubi [A]** time = 0.65, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1629, 635, 205, 260}

$$\frac{\log(a+cx^2)(Bcd(cd^2-3ae^2)-e(Ac-aC)(3cd^2-ae^2))}{2(ae^2+cd^2)^3} - \frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2(ae^2+cd^2)} + \frac{-aBe^2+2aCde-2Acde+}{(d+ex)(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)), x]

[Out]  $-(C*d^2 - B*d*e + A*e^2)/(2*e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (\text{Sqrt}[c]*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]/(\text{Sqrt}[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*\text{Log}[d + e*x])/(c*d^2 + a*e^2)^3 + ((B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*\text{Log}[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)$

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 260**

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

**Rule 635**

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

**Rule 1629**



```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(d + ex)^3 (a + cx^2)} dx &= \int \left( \frac{Cd^2 - Bde + Ae^2}{(cd^2 + ae^2)(d + ex)^3} + \frac{e(-Bcd^2 + 2Acde - 2aCde + aBe^2)}{(cd^2 + ae^2)^2 (d + ex)^2} + \frac{e(-Bcd(cd^2 - 3ae^2))}{(cd^2 + ae^2)^3 (d + ex)} \right) dx \\ &= -\frac{Cd^2 - Bde + Ae^2}{2e(cd^2 + ae^2)(d + ex)^2} + \frac{Bcd^2 - 2Acde + 2aCde - aBe^2}{(cd^2 + ae^2)^2 (d + ex)} - \frac{(Bcd(cd^2 - 3ae^2))}{(cd^2 + ae^2)^3 (d + ex)} \\ &= -\frac{Cd^2 - Bde + Ae^2}{2e(cd^2 + ae^2)(d + ex)^2} + \frac{Bcd^2 - 2Acde + 2aCde - aBe^2}{(cd^2 + ae^2)^2 (d + ex)} - \frac{(Bcd(cd^2 - 3ae^2))}{(cd^2 + ae^2)^3 (d + ex)} \\ &= -\frac{Cd^2 - Bde + Ae^2}{2e(cd^2 + ae^2)(d + ex)^2} + \frac{Bcd^2 - 2Acde + 2aCde - aBe^2}{(cd^2 + ae^2)^2 (d + ex)} + \frac{\sqrt{c} (Acd(cd^2 - 3ae^2))}{(cd^2 + ae^2)^3 (d + ex)} \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 277, normalized size = 0.91

$$\frac{\log(a + cx^2)(Bcd(cd^2 - 3ae^2) - e(Ac - aC)(3cd^2 - ae^2)) - \frac{(a^2 + c^2)^2 (e(Ac - Bd) + Cd^2)}{e(d + ex)^2} + \frac{2(a^2 + c^2)(-aBe^2 + 2aCde - 2Acde + Bcd^2)}{d + ex} - 2 \log(d + ex)(Bcd(cd^2 - 3ae^2) - e(Ac - aC)(3cd^2 - ae^2)) + \frac{2\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(cd^2 - 3ae^2) + a(a^2(3Cd - B) + cd^2(3Be - Cd)))}{\sqrt{a}}}{2(a^2 + cd^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)),x]

[Out] (-(((c\*d^2 + a\*e^2)^2\*(C\*d^2 + e\*(-(B\*d) + A\*e)))/(e\*(d + e\*x)^2)) + (2\*(c\*d^2 + a\*e^2)\*(B\*c\*d^2 - 2\*A\*c\*d\*e + 2\*a\*C\*d\*e - a\*B\*e^2))/(d + e\*x) + (2\*sqrt[c]\*(A\*c\*d\*(c\*d^2 - 3\*a\*e^2) + a\*(a\*e^2\*(3\*C\*d - B\*e) + c\*d^2\*(-(C\*d) + 3\*B\*e)))\*ArcTan[(sqrt[c]\*x)/sqrt[a]])/sqrt[a] - 2\*(B\*c\*d\*(c\*d^2 - 3\*a\*e^2) - (A\*c - a\*C)\*e\*(3\*c\*d^2 - a\*e^2))\*Log[d + e\*x] + (B\*c\*d\*(c\*d^2 - 3\*a\*e^2) - (A\*c - a\*C)\*e\*(3\*c\*d^2 - a\*e^2))\*Log[a + c\*x^2]/(2\*(c\*d^2 + a\*e^2)^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3 (a + cx^2)} dx$$

Verification is not applicable to the result.



$$\frac{1}{(a^2e^2 + c^2d^2)^{3/2}} \frac{1}{(ac)^{1/2}} \arctan\left(\frac{1}{(ac)^{1/2}} cx\right) B^2 a^2 e^3 - c^2 \sqrt{a^2e^2 + c^2d^2} \frac{1}{(ac)^{1/2}} \arctan\left(\frac{1}{(ac)^{1/2}} cx\right) C^2 a^2 d^3 + 3 \sqrt{a^2e^2 + c^2d^2} \ln(e^2x + d) B^2 c^2 d e^2 a^3 - \sqrt{a^2e^2 + c^2d^2} \ln(e^2x + d) C^2 a^2 c^2 d^2 e^3 - 3 c^2 \sqrt{a^2e^2 + c^2d^2} \frac{1}{(ac)^{1/2}} \arctan\left(\frac{1}{(ac)^{1/2}} cx\right) A^2 a^2 d e^2 + 3 c^2 \sqrt{a^2e^2 + c^2d^2} \frac{1}{(ac)^{1/2}} \arctan\left(\frac{1}{(ac)^{1/2}} cx\right) B^2 a^2 d^2 e + 3 c^2 \sqrt{a^2e^2 + c^2d^2} \frac{1}{(ac)^{1/2}} \arctan\left(\frac{1}{(ac)^{1/2}} cx\right) C^2 a^2 d^2 e^2 + 1/2 c^2 \sqrt{a^2e^2 + c^2d^2} \ln(c^2x^2 + a) d^3 B - 1/2 \sqrt{a^2e^2 + c^2d^2} \ln(e^2x + d) d^3 c^2 B + 1/2 \sqrt{a^2e^2 + c^2d^2} \ln(e^2x + d) C^2 a^2 e^3 - 1/2 \sqrt{a^2e^2 + c^2d^2} / e / (e^2x + d)^2 C^2 d^2 - 1/2 \sqrt{a^2e^2 + c^2d^2} / (e^2x + d) B^2 a^2 e^2 + 1/2 \sqrt{a^2e^2 + c^2d^2} / (e^2x + d) B^2 c^2 d^2 - 1/2 \sqrt{a^2e^2 + c^2d^2} \ln(c^2x^2 + a) C^2 a^2 e^3$$

**maxima [A]** time = 1.05, size = 495, normalized size = 1.62

$$\frac{(Bc^2d^3 - 3Bacd^2 + 3(Cac - Ac^2)d^2e - (C^2d - Aac^2)e^2) \log(cx^2 + a)}{2(c^2d^3 + 3acd^2e^2 + 3a^2cd^2e + a^3e^3)} - \frac{(Bc^2d^3 - 3Bacd^2 + 3(Cac - Ac^2)d^2e - (C^2d - Aac^2)e^2) \log(ex + d)}{c^2d^3 + 3acd^2e^2 + 3a^2cd^2e + a^3e^3} + \frac{(3Bac^2d^2e - Bc^2cd^3 - (Cac^2 - Ac^3)d^3 + 3(C^2d^2e - Aac^2)d^2e^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^2d^3 + 3acd^2e^2 + 3a^2cd^2e + a^3e^3)\sqrt{ac}} - \frac{Ccd^4 - 3Bcd^3e + Bada^2 + Aae^4 - (3Ca - 5Ac)d^2e^2 - 2(Bcd^2d^2 - Ba^4 + 2(Ca - Ac)d^2e)}{2(c^2d^3e + 2acd^2e^2 + a^2cd^2e^2 + (c^2d^3e^2 + 2acd^2e^2 + a^2e^2)^2 + 2(c^2d^3e^2 + 2acd^2e^2 + a^2d^2e^2))\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a), x, algorithm="maxima")

[Out] 1/2\*(B\*c^2\*d^3 - 3\*B\*a\*c\*d\*e^2 + 3\*(C\*a\*c - A\*c^2)\*d^2\*e - (C\*a^2 - A\*a\*c)\*e^3)\*log(c\*x^2 + a)/(c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 + a^3\*e^6) - (B\*c^2\*d^3 - 3\*B\*a\*c\*d\*e^2 + 3\*(C\*a\*c - A\*c^2)\*d^2\*e - (C\*a^2 - A\*a\*c)\*e^3)\*log(e\*x + d)/(c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 + a^3\*e^6) + (3\*B\*a\*c^2\*d^2\*e - B\*a^2\*c\*e^3 - (C\*a\*c^2 - A\*c^3)\*d^3 + 3\*(C\*a^2\*c - A\*a\*c^2)\*d\*e^2)\*arctan(cx/sqrt(ac))/(c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 + a^3\*e^6)\*sqrt(ac) - 1/2\*(C\*c\*d^4 - 3\*B\*c\*d^3\*e + B\*a\*d\*e^3 + A\*a\*e^4 - (3\*C\*a - 5\*A\*c)\*d^2\*e^2 - 2\*(B\*c\*d^2\*e^2 - B\*a\*e^4 + 2\*(C\*a - A\*c)\*d\*e^3)\*x)/(c^2\*d^6\*e + 2\*a\*c\*d^4\*e^3 + a^2\*d^2\*e^5 + (c^2\*d^4\*e^3 + 2\*a\*c\*d^2\*e^5 + a^2\*e^7)\*x^2 + 2\*(c^2\*d^5\*e^2 + 2\*a\*c\*d^3\*e^4 + a^2\*d\*e^6)\*x)

**mupad [B]** time = 9.19, size = 2980, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((a + c\*x^2)\*(d + e\*x)^3), x)

[Out] (log(d + e\*x)\*(e^3\*(C\*a^2 - A\*a\*c) - B\*c^2\*d^3 + d^2\*e\*(3\*A\*c^2 - 3\*C\*a\*c) + 3\*B\*a\*c\*d\*e^2))/(a^3\*e^6 + c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4) - (log(9\*A^2\*a^5\*e^10\*(-a\*c)^(5/2) + A^2\*c^5\*d^10\*(-a\*c)^(5/2) - B^2\*a^7\*e^10\*(-a\*c)^(3/2) - 9\*B^2\*c^3\*d^10\*(-a\*c)^(7/2) + 9\*C^2\*a^9\*e^10\*(-a\*c)^(1/2) + C^2\*c\*d^10\*(-a\*c)^(9/2) + 9\*C^2\*a^9\*c\*e^10\*x - 6\*A^2\*a\*d^4\*e^6\*(-a\*c)^(9/2) - 6\*B^2\*a\*d^6\*e^4\*(-a\*c)^(9/2) + 106\*A^2\*c\*d^6\*e^4\*(-a\*c)^(9/2) + 77\*C^2\*a\*d^8\*e^2\*(-a\*c)^(9/2) - 27\*B^2\*c\*d^8\*e^2\*(-a\*c)^(9/2) + A^2\*a^2\*c^8\*d^10\*x + 9\*A^2\*a^7\*c^3\*e^10\*x + 9\*B^2\*a^3\*c^7\*d^10\*x + B^2\*a^8\*c^2\*e^10\*x + C^2\*a^4\*c^6\*d^10\*x + 27\*A^2\*a^3\*d^2\*e^8\*(-a\*c)^(7/2) - 106\*B^2\*a^3\*d^4\*e^6\*(-a\*c)^(7/2) + 77\*B^2\*a^5\*d^2\*e^8\*(-a\*c)^(5/2) - 77\*A^2\*c^3\*d^8\*e^2\*(-a\*c)^(7/2)

$$\begin{aligned}
& ) - 106*C^2*a^3*d^6*e^4*(-a*c)^{(7/2)} - 6*C^2*a^5*d^4*e^6*(-a*c)^{(5/2)} + 27* \\
& C^2*a^7*d^2*e^8*(-a*c)^{(3/2)} + 18*A*C*a^7*e^{10}*(-a*c)^{(3/2)} + 2*A*C*c^3*d^1 \\
& 0*(-a*c)^{(7/2)} + 224*A*B*a*d^5*e^5*(-a*c)^{(9/2)} - 48*A*B*a^5*d*e^9*(-a*c)^{(5/2)} - 212*A*C*a*d^6*e^4*(-a*c)^{(9/2)} + 64*A*B*c*d^7*e^3*(-a*c)^{(9/2)} + 48* \\
& A*B*c^3*d^9*e*(-a*c)^{(7/2)} - 64*B*C*a*d^7*e^3*(-a*c)^{(9/2)} - 48*B*C*a^7*d*e^9*(-a*c)^{(3/2)} - 154*A*C*c*d^8*e^2*(-a*c)^{(9/2)} + 77*A^2*a^3*c^7*d^8*e^2*x \\
& + 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2 \\
& *e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c^4 \\
& *d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^2 \\
& *a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x - \\
& 2*A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^{10}*x - 64*A*B*a^3*d^3*e^7*(-a*c)^{(7/2)} - 12*A*C*a^3*d^4*e^6*(-a*c)^{(7/2)} + 54*A*C*a^5*d^2*e^8*(-a*c)^{(5/2)} + 2 \\
& 24*B*C*a^3*d^5*e^5*(-a*c)^{(7/2)} - 64*B*C*a^5*d^3*e^7*(-a*c)^{(5/2)} + 48*B*C* \\
& c*d^9*e*(-a*c)^{(9/2)} - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48 \\
& *B*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x + \\
& 224*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8* \\
& e^2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c^3 \\
& *d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B* \\
& C*a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 - (3*C*a^2*d*(-a*c)^{(1/2)})/2) + ( \\
& 3*A*a*c*d*(-a*c)^{(1/2)})/2) + e^3*((C*a^3)/2 - (A*a^2*c)/2 + (B*a^2*(-a*c)^{(1/2)})/2) - e*((3*C*a^2*c*d^2)/2 - (3*A*a*c^2*d^2)/2 + (3*B*a*c*d^2*(-a*c)^{(1/2)})/2) - (B*a*c^2*d^3)/2 - (A*c^2*d^3*(-a*c)^{(1/2)})/2 + (C*a*c*d^3*(-a*c)^{(1/2)})/2)/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - ( \\
& \log(B^2*a^7*e^{10}*(-a*c)^{(3/2)} - A^2*c^5*d^{10}*(-a*c)^{(5/2)} - 9*A^2*a^5*e^{10} \\
& (-a*c)^{(5/2)} + 9*B^2*c^3*d^{10}*(-a*c)^{(7/2)} - 9*C^2*a^9*e^{10}*(-a*c)^{(1/2)} - \\
& C^2*c*d^{10}*(-a*c)^{(9/2)} + 9*C^2*a^9*c*e^{10}*x + 6*A^2*a*d^4*e^6*(-a*c)^{(9/2)} \\
& + 6*B^2*a*d^6*e^4*(-a*c)^{(9/2)} - 106*A^2*c*d^6*e^4*(-a*c)^{(9/2)} - 77*C^2*a \\
& *d^8*e^2*(-a*c)^{(9/2)} + 27*B^2*c*d^8*e^2*(-a*c)^{(9/2)} + A^2*a^2*c^8*d^{10}*x \\
& + 9*A^2*a^7*c^3*e^{10}*x + 9*B^2*a^3*c^7*d^{10}*x + B^2*a^8*c^2*e^{10}*x + C^2*a^4 \\
& *c^6*d^{10}*x - 27*A^2*a^3*d^2*e^8*(-a*c)^{(7/2)} + 106*B^2*a^3*d^4*e^6*(-a*c) \\
& ^{(7/2)} - 77*B^2*a^5*d^2*e^8*(-a*c)^{(5/2)} + 77*A^2*c^3*d^8*e^2*(-a*c)^{(7/2)} \\
& + 106*C^2*a^3*d^6*e^4*(-a*c)^{(7/2)} + 6*C^2*a^5*d^4*e^6*(-a*c)^{(5/2)} - 27*C^2 \\
& *a^7*d^2*e^8*(-a*c)^{(3/2)} - 18*A*C*a^7*e^{10}*(-a*c)^{(3/2)} - 2*A*C*c^3*d^{10} \\
& (-a*c)^{(7/2)} - 224*A*B*a*d^5*e^5*(-a*c)^{(9/2)} + 48*A*B*a^5*d*e^9*(-a*c)^{(5/2)} + 212*A*C*a*d^6*e^4*(-a*c)^{(9/2)} - 64*A*B*c*d^7*e^3*(-a*c)^{(9/2)} - 48*A* \\
& B*c^3*d^9*e*(-a*c)^{(7/2)} + 64*B*C*a*d^7*e^3*(-a*c)^{(9/2)} + 48*B*C*a^7*d*e^9 \\
& *(-a*c)^{(3/2)} + 154*A*C*c*d^8*e^2*(-a*c)^{(9/2)} + 77*A^2*a^3*c^7*d^8*e^2*x + \\
& 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2*e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c^4 \\
& *d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^2* \\
& a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x - 2* \\
& A*C*a^3*c^7*d^{10}*x - 18*A*C*a^8*c^2*e^{10}*x + 64*A*B*a^3*d^3*e^7*(-a*c)^{(7/2)} \\
& ) + 12*A*C*a^3*d^4*e^6*(-a*c)^{(7/2)} - 54*A*C*a^5*d^2*e^8*(-a*c)^{(5/2)} - 224 \\
& *B*C*a^3*d^5*e^5*(-a*c)^{(7/2)} + 64*B*C*a^5*d^3*e^7*(-a*c)^{(5/2)} - 48*B*C*c* \\
& d^9*e*(-a*c)^{(9/2)} - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48*B
\end{aligned}$$

$$\begin{aligned}
& *C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x + 22 \\
& 4*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*e^ \\
& 2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c^3 \\
& *d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*C* \\
& a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 + (3*C*a^2*d*(-a*c)^(1/2))/2 - (3* \\
& A*a*c*d*(-a*c)^(1/2))/2) - e^3*((A*a^2*c)/2 - (C*a^3)/2 + (B*a^2*(-a*c)^(1/ \\
& 2))/2) + e*((3*A*a*c^2*d^2)/2 - (3*C*a^2*c*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(1/ \\
& 2))/2) - (B*a*c^2*d^3)/2 + (A*c^2*d^3*(-a*c)^(1/2))/2 - (C*a*c*d^3*(-a*c)^( \\
& 1/2))/2)/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - ((A \\
& *a*e^4 + C*c*d^4 + B*a*d*e^3 - 3*B*c*d^3*e + 5*A*c*d^2*e^2 - 3*C*a*d^2*e^2) \\
& / (2*e*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(B*a*e^3 + 2*A*c*d*e^2 - 2* \\
& C*a*d*e^2 - B*c*d^2*e))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)/(d^2 + e^2*x^2 \\
& + 2*d*e*x)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*3/(c\*x\*\*2+a),x)

[Out] Timed out

$$3.50 \quad \int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=216

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ACd(3ae^2 + cd^2) - a(3ae^2(Be + 3Cd) - cd^2(3Be + Cd))\right) e \log(a + cx^2)\left(2aCe^2 - c(e(Ae + 3Bd) + 3Cd)\right)}{2a^{3/2}c^{5/2}} - \frac{e^3x^2(Ac - 2aC)}{2c^3}$$

**Rubi [A]** time = 0.50, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1645, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ACd(3ae^2 + cd^2) - a(3ae^2(Be + 3Cd) - cd^2(3Be + Cd))\right)}{2a^{3/2}c^{5/2}} - \frac{e \log(a + cx^2)\left(2aCe^2 - c(e(Ae + 3Bd) + 3Cd)\right)}{2c^3} - \frac{3e^2x(ACd - a(Be + 3Cd))}{2ac^2} - \frac{(d + ex)^3(aB - x(AC - aC))}{2ac(a + cx^2)} - \frac{e^3x^2(Ac - 2aC)}{2ac^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

[Out] (-3\*e^2\*(A\*c\*d - a\*(3\*C\*d + B\*e))\*x)/(2\*a\*c^2) - ((A\*c - 2\*a\*C)\*e^3\*x^2)/(2\*a\*c^2) - ((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^3)/(2\*a\*c\*(a + c\*x^2)) + ((A\*c\*d\*(c\*d^2 + 3\*a\*e^2) - a\*(3\*a\*e^2\*(3\*C\*d + B\*e) - c\*d^2\*(C\*d + 3\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(2\*a^(3/2)\*c^(5/2)) - (e\*(2\*a\*C\*e^2 - c\*(3\*C\*d^2 + e\*(3\*B\*d + A\*e)))\*Log[a + c\*x^2])/(2\*c^3)

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 801

Int[(((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x],

`x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]`

### Rule 1645

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2} dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} - \frac{\int \frac{(d+ex)^2(-Acd-aCd-3aBe+2(Ac-2aC)ex)}{a+cx^2} dx}{2ac} \\ &= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} - \frac{\int \left( \frac{3e^2(Acd-3aCd-aBe)}{c} + \frac{2(Ac-2aC)e^3x}{c} - \frac{Acd(cd^2+3)}{c} \right) dx}{2ac} \\ &= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} \\ &= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} \\ &= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 233, normalized size = 1.08

$$\frac{\sqrt{e} \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right) \left( Acd(3a^2+ca^2) + a(cd^2(3Be+Cd) - 3a^2(Be+3Cd)) \right)}{a^{3/2}} + \frac{-a^3C^3 + a^2ce(Ac+3Bd+Bcx) + 3Cd(d+ex) - ac^2d(3Ac(d+ex) + Bd(d+3cx) + Cd^2x) + Ac^3d^3x}{a(a+cx^2)} + e \log(a+cx^2) \left( -2aC^2 + ce(Ae+3Bd) + 3cCd^2 \right) + 2ce^2x(Be+3Cd) + cCe^3x^2}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

```
[Out] (2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 + 3*a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]]/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*Log[a + c*x^2]/(2*c^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3 (A + Bx + Cx^2)}{(a + cx^2)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2, x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2, x]
```

**fricas [B]** time = 1.07, size = 931, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^2*c^2*e^3*x^4 + 2*C*a^3*c*e^3*x^2 - 2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 + 6*(C*a^3*c - A*a^2*c^2)*d^2*e - 2*(C*a^4 - A*a^3*c)*e^3 + 4*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^2*c^2*d^2*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*d*e^2)*x + 2*(3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*log(c*x^2 + a)/(a^2*c^4*x^2 + a^3*c^3), 1/2*(C*a^2*c^2*e^3*x^4 + C*a^3*c*e^3*x^2 - B*a^2*c^2*d^3 + 3*B*a^3*c*d*e^2 + 3*(C*a^3*c - A*a^2*c^2)*d^2*e - (C*a^4 - A*a^3*c)*e^3 + 2*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^2*c^2*d^2*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*d*e^2)*x + (3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*log(c*x^2 + a)/(a^2*c^4*x^2 + a^3*c^3)]
```





$$3*C*d*e^2 + B*e^3)*x)/c^2 + 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - (2*C*a - A*c)*e^3)*\log(c*x^2 + a)/c^3 + 1/2*(3*B*a*c*d^2*e - 3*B*a^2*e^3 + (C*a*c + A*c^2)*d^3 - 3*(3*C*a^2 - A*a*c)*d*e^2)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a*c^2)$$

**mupad [B]** time = 4.01, size = 303, normalized size = 1.40

$$\frac{x(Bc^2 + 3Cd^2)}{c^2} - \frac{C^2d^2 - 3Ccd^2 - 3Bac^2 - Aac^2 + B^2d^2 + 3A^2d^2}{2c} - \frac{(3C^2d^2 + 3B^2d^2 - Ccd^2 - 3Bac^2 - 3Aac^2 + A^2d^2)}{2c} + \frac{C^2d^2}{2c^2} + \frac{\arctan\left(\frac{cx}{\sqrt{a}}\right)(-9C^2d^2 - 3B^2d^2 + Ccd^2 + 3Bac^2 + 3Aac^2 + A^2d^2)}{2a^{3/2}c^2} + \frac{\ln(cx^2 + a)(-32Ca^2c^2 + 48C^2d^2c + 48B^2c^2d^2 + 16A^3c^2)}{32a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x)

[Out] (x\*(B\*e^3 + 3\*C\*d\*e^2))/c^2 - ((B\*c^2\*d^3 + C\*a^2\*e^3 - A\*a\*c\*e^3 + 3\*A\*c^2\*d^2\*e - 3\*B\*a\*c\*d\*e^2 - 3\*C\*a\*c\*d^2\*e)/(2\*c) - (x\*(A\*c^2\*d^3 + B\*a^2\*e^3 - C\*a\*c\*d^3 + 3\*C\*a^2\*d\*e^2 - 3\*A\*a\*c\*d\*e^2 - 3\*B\*a\*c\*d^2\*e))/(2\*a))/(a\*c^2 + c^3\*x^2) + (C\*e^3\*x^2)/(2\*c^2) + (atan((c^(1/2)\*x)/a^(1/2))\*(A\*c^2\*d^3 - 3\*B\*a^2\*e^3 + C\*a\*c\*d^3 - 9\*C\*a^2\*d\*e^2 + 3\*A\*a\*c\*d\*e^2 + 3\*B\*a\*c\*d^2\*e))/(2\*a^(3/2)\*c^(5/2)) + (log(a + c\*x^2)\*(16\*A\*a^3\*c^4\*e^3 - 32\*C\*a^4\*c^3\*e^3 + 48\*B\*a^3\*c^4\*d\*e^2 + 48\*C\*a^3\*c^4\*d^2\*e))/(32\*a^3\*c^6)

**sympy [B]** time = 34.46, size = 952, normalized size = 4.41

$$\frac{x(Bc^2 + 3Cd^2)}{c^2} - \frac{C^2d^2 - 3Ccd^2 - 3Bac^2 - Aac^2 + B^2d^2 + 3A^2d^2}{2c} - \frac{(3C^2d^2 + 3B^2d^2 - Ccd^2 - 3Bac^2 - 3Aac^2 + A^2d^2)}{2c} + \frac{C^2d^2}{2c^2} + \frac{\arctan\left(\frac{cx}{\sqrt{a}}\right)(-9C^2d^2 - 3B^2d^2 + Ccd^2 + 3Bac^2 + 3Aac^2 + A^2d^2)}{2a^{3/2}c^2} + \frac{\ln(cx^2 + a)(-32Ca^2c^2 + 48C^2d^2c + 48B^2c^2d^2 + 16A^3c^2)}{32a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*2, x)

[Out] C\*e\*\*3\*x\*\*2/(2\*c\*\*2) + x\*(B\*e\*\*3/c\*\*2 + 3\*C\*d\*e\*\*2/c\*\*2) + (-e\*(-A\*c\*e\*\*2 - 3\*B\*c\*d\*e + 2\*C\*a\*e\*\*2 - 3\*C\*c\*d\*\*2)/(2\*c\*\*3) - sqrt(-a\*\*3\*c\*\*7)\*(-3\*A\*a\*c\*d\*e\*\*2 - A\*c\*\*2\*d\*\*3 + 3\*B\*a\*\*2\*e\*\*3 - 3\*B\*a\*c\*d\*\*2\*e + 9\*C\*a\*\*2\*d\*e\*\*2 - C\*a\*c\*d\*\*3)/(4\*a\*\*3\*c\*\*6))\*log(x + (2\*A\*a\*\*2\*c\*e\*\*3 + 6\*B\*a\*\*2\*c\*d\*e\*\*2 - 4\*C\*a\*\*3\*e\*\*3 + 6\*C\*a\*\*2\*c\*d\*\*2\*e - 4\*a\*\*2\*c\*\*3\*(-e\*(-A\*c\*e\*\*2 - 3\*B\*c\*d\*e + 2\*C\*a\*e\*\*2 - 3\*C\*c\*d\*\*2)/(2\*c\*\*3) - sqrt(-a\*\*3\*c\*\*7)\*(-3\*A\*a\*c\*d\*e\*\*2 - A\*c\*\*2\*d\*\*3 + 3\*B\*a\*\*2\*e\*\*3 - 3\*B\*a\*c\*d\*\*2\*e + 9\*C\*a\*\*2\*d\*e\*\*2 - C\*a\*c\*d\*\*3)/(4\*a\*\*3\*c\*\*6)))/(-3\*A\*a\*c\*\*2\*d\*e\*\*2 - A\*c\*\*3\*d\*\*3 + 3\*B\*a\*\*2\*c\*e\*\*3 - 3\*B\*a\*c\*\*2\*d\*\*2\*e + 9\*C\*a\*\*2\*c\*d\*e\*\*2 - C\*a\*c\*\*2\*d\*\*3)) + (-e\*(-A\*c\*e\*\*2 - 3\*B\*c\*d\*e + 2\*C\*a\*e\*\*2 - 3\*C\*c\*d\*\*2)/(2\*c\*\*3) + sqrt(-a\*\*3\*c\*\*7)\*(-3\*A\*a\*c\*d\*e\*\*2 - A\*c\*\*2\*d\*\*3 + 3\*B\*a\*\*2\*e\*\*3 - 3\*B\*a\*c\*d\*\*2\*e + 9\*C\*a\*\*2\*d\*e\*\*2 - C\*a\*c\*d\*\*3)/(4\*a\*\*3\*c\*\*6))\*log(x + (2\*A\*a\*\*2\*c\*e\*\*3 + 6\*B\*a\*\*2\*c\*d\*e\*\*2 - 4\*C\*a\*\*3\*e\*\*3 + 6\*C\*a\*\*2\*c\*d\*\*2\*e - 4\*a\*\*2\*c\*\*3\*(-e\*(-A\*c\*e\*\*2 - 3\*B\*c\*d\*e + 2\*C\*a\*e\*\*2 - 3\*C\*c\*d\*\*2)/(2\*c\*\*3) + sqrt(-a\*\*3\*c\*\*7)\*(-3\*A\*a\*c\*d\*e\*\*2 - A\*c\*\*2\*d\*\*3 + 3\*B\*a\*\*2\*e\*\*3 - 3\*B\*a\*c\*d\*\*2\*e + 9\*C\*a\*\*2\*d\*e\*\*2 - C\*a\*c\*d\*\*3)/(4\*a\*\*3\*c\*\*6)))/(-3\*A\*a\*c\*\*2\*d\*e\*\*2 - A\*c\*\*3\*d\*\*3 + 3\*B\*a\*\*2\*c\*e\*\*3 - 3\*B\*a\*c\*\*2\*d\*\*2\*e + 9\*C\*a\*\*2\*c\*d\*e\*\*2 - C\*a\*c\*\*2\*d\*\*3)) + (A\*a\*\*2\*c\*e\*\*3 - 3\*A\*a\*c\*\*2\*d\*\*2\*e + 3\*B\*a\*\*2\*c\*d\*e\*\*2 - B\*a\*c\*\*2\*d\*\*3 - C\*a\*\*3\*e\*\*3 + 3\*C\*a\*\*2\*c\*d\*\*2\*e + x\*(-3\*A\*a\*c\*\*2\*d\*e\*\*2 + A\*c\*\*3\*d\*\*3 + B\*a\*\*2\*c\*e\*\*3 - 3\*B\*a\*c\*\*2\*d\*\*2\*e + 3\*C\*a\*\*2\*c\*d\*e\*\*2 - C\*a\*c\*\*2\*d\*\*3))/(2\*a\*\*2\*c\*\*3 + 2\*a\*c\*\*4\*x\*\*2)

$$3.51 \quad \int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)(cd(2aBe + aCd + Acd) + ae^2(AC - 3aC))}{2a^{3/2}c^{5/2}} - \frac{(d+ex)^2(aB - x(AC - aC))}{2ac(a+cx^2)} - \frac{e^2x(AC - 3aC)}{2ac^2} + \frac{e \log(a+cx^2)}{2c^2}$$

**Rubi [A]** time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1645, 774, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)(cd(2aBe + aCd + Acd) + ae^2(AC - 3aC))}{2a^{3/2}c^{5/2}} - \frac{(d+ex)^2(aB - x(AC - aC))}{2ac(a+cx^2)} - \frac{e^2x(AC - 3aC)}{2ac^2} + \frac{e \log(a+cx^2)(Be + 2Cd)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

[Out] -((A\*c - 3\*a\*C)\*e^2\*x)/(2\*a\*c^2) - ((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^2)/(2\*a\*c\*(a + c\*x^2)) + ((a\*(A\*c - 3\*a\*C)\*e^2 + c\*d\*(A\*c\*d + a\*C\*d + 2\*a\*B\*e))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(2\*a^(3/2)\*c^(5/2)) + (e\*(2\*C\*d + B\*e)\*Log[a + c\*x^2])/(2\*c^2)

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 774

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(e\*g\*x)/c, x] + Dist[1/c, Int[(c\*d\*f - a\*e\*g + c\*(e\*f + d\*g))\*x

)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

### Rule 1645

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :  
 > With[{Q = PolynomialQuotient[Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 1]}, Simp[((d + e\*x)^m\*(a + c\*x^2)^(p + 1)\*(a\*g - c\*f\*x))/(2\*a\*c\*(p + 1)), x] + Dist[1/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1)\*ExpandToSum[2\*a\*c\*(p + 1)\*(d + e\*x)\*Q - a\*e\*g\*m + c\*d\*f\*(2\*p + 3) + c\*e\*f\*(m + 2\*p + 3)\*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && ! (IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex)^2 (A + Bx + Cx^2)}{(a + cx^2)^2} dx &= -\frac{(aB - (Ac - aC)x)(d + ex)^2}{2ac(a + cx^2)} - \frac{\int \frac{(d+ex)(-Acd - aCd - 2aBe + (Ac - 3aC)ex)}{a+cx^2} dx}{2ac} \\ &= -\frac{(Ac - 3aC)e^2x}{2ac^2} - \frac{(aB - (Ac - aC)x)(d + ex)^2}{2ac(a + cx^2)} - \frac{\int \frac{-a(Ac - 3aC)e^2 + cd(-Acd - aCd - 2aBe + (Ac - 3aC)ex)}{a+cx^2} dx}{2ac} \\ &= -\frac{(Ac - 3aC)e^2x}{2ac^2} - \frac{(aB - (Ac - aC)x)(d + ex)^2}{2ac(a + cx^2)} + \frac{(e(2Cd + Be)) \int \frac{x}{a+cx^2} dx}{c} + \frac{\int \frac{-a(Ac - 3aC)e^2 + cd(-Acd - aCd - 2aBe + (Ac - 3aC)ex)}{a+cx^2} dx}{2ac} \\ &= -\frac{(Ac - 3aC)e^2x}{2ac^2} - \frac{(aB - (Ac - aC)x)(d + ex)^2}{2ac(a + cx^2)} + \frac{(a(Ac - 3aC)e^2 + cd(Acd + aCd + 2aBe - Acd - aCd - 2aBe + (Ac - 3aC)ex))}{2a^{3/2}c} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 175, normalized size = 1.20

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac(ae^2+cd^2)+a(cd(2Be+Cd)-3aCe^2))}{a^{3/2}} + \frac{\sqrt{c}(a^2e(Be+2Cd+Cex)-ac(Ae(2d+ex)+Bd(d+2ex)+Cd^2x)+Ac^2d^2x)}{a(a+cx^2)} + \frac{\sqrt{c}e \log(a+cx^2)(Be+2Cd)+2\sqrt{c}Ce^2x}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

[Out] (2\*Sqrt[c]\*C\*e^2\*x + (Sqrt[c]\*(A\*c^2\*d^2\*x + a^2\*e\*(2\*C\*d + B\*e + C\*e\*x) - a\*c\*(C\*d^2\*x + A\*e\*(2\*d + e\*x) + B\*d\*(d + 2\*e\*x))))/(a\*(a + c\*x^2)) + ((A\*c

$(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e))$ )\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/a^(3/2) + Sqrt[c]\*e\*(2\*C\*d + B\*e)\*Log[a + c\*x^2]/(2\*c^(5/2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2 (A + Bx + Cx^2)}{(a + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2,x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

**fricas** [B] time = 0.84, size = 631, normalized size = 4.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="fricas")

[Out]  $[1/4*(4*C*a^2*c^2*e^2*x^3 - 2*B*a^2*c^2*d^2 + 2*B*a^3*c*e^2 + 4*(C*a^3*c - A*a^2*c^2)*d*e - (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*\sqrt{-a*c}*\log((c*x^2 - 2*\sqrt{-a*c}*x - a)/(c*x^2 + a)) - 2*(2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)*x + 2*(2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)*\log(c*x^2 + a)/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*C*a^2*c^2*e^2*x^3 - B*a^2*c^2*d^2 + B*a^3*c*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d*e + (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*\sqrt{a*c}*\arctan(\sqrt{a*c}*x/a) - (2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)*x + (2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)*\log(c*x^2 + a)/(a^2*c^4*x^2 + a^3*c^3)]$

**giac** [A] time = 0.17, size = 184, normalized size = 1.26

$$\frac{Cxe^2}{c^2} + \frac{(2Cde + B^2)\log(cx^2 + a)}{2c^2} + \frac{(Cacd^2 + Ac^2d^2 + 2Bacde - 3Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac^2} - \frac{Bacd^2 - 2Ca^2de + 2Aacde - Ba^2e^2 + (Cacd^2 - Ac^2d^2 + 2Bacde - Ca^2e^2 + Aace^2)x}{2(cx^2 + a)ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="giac")

[Out]  $C*x*e^2/c^2 + 1/2*(2*C*d*e + B*e^2)*\log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 + A*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*\arctan(c*x/\sqrt{a*c})/(sq$

rt(a\*c)\*a\*c^2) - 1/2\*(B\*a\*c\*d^2 - 2\*C\*a^2\*d\*e + 2\*A\*a\*c\*d\*e - B\*a^2\*e^2 + (C\*a\*c\*d^2 - A\*c^2\*d^2 + 2\*B\*a\*c\*d\*e - C\*a^2\*e^2 + A\*a\*c\*e^2)\*x)/((c\*x^2 + a)\*a\*c^2)

**maple [B]** time = 0.01, size = 323, normalized size = 2.21

$$\frac{A d^2 x}{2(c x^2+a) a} + \frac{A d^2 \arctan\left(\frac{c x}{\sqrt{a}}\right)}{2 \sqrt{a} c} - \frac{A c^2 x}{2(c x^2+a) c} + \frac{A c^2 \arctan\left(\frac{c x}{\sqrt{a}}\right)}{2 \sqrt{a} c} - \frac{B d e x}{(c x^2+a) c} + \frac{B d e \arctan\left(\frac{c x}{\sqrt{a}}\right)}{\sqrt{a} c} + \frac{C a c^2 x}{2(c x^2+a) c^2} - \frac{3 C a c^2 \arctan\left(\frac{c x}{\sqrt{a}}\right)}{2 \sqrt{a} c^2} - \frac{C d^2 x}{2(c x^2+a) c} + \frac{C d^2 \arctan\left(\frac{c x}{\sqrt{a}}\right)}{2 \sqrt{a} c} - \frac{A d e}{(c x^2+a) c} + \frac{B a e^2}{2(c x^2+a) c^2} - \frac{B d^2}{2(c x^2+a) c} + \frac{B e^2 \ln(c x^2+a)}{2 c^2} + \frac{C a d e}{(c x^2+a) c^2} + \frac{C d e \ln(c x^2+a)}{c^2} + \frac{C e^2 x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x)

[Out] C\*e^2/c^2\*x-1/2/c/(c\*x^2+a)\*A\*e^2\*x+1/2/(c\*x^2+a)/a\*x\*A\*d^2-1/c/(c\*x^2+a)\*B\*d\*e\*x+1/2/c^2/(c\*x^2+a)\*a\*C\*e^2\*x-1/2/c/(c\*x^2+a)\*C\*d^2\*x-1/c/(c\*x^2+a)\*A\*d\*e+1/2/c^2/(c\*x^2+a)\*B\*a\*e^2-1/2/c/(c\*x^2+a)\*B\*d^2+1/c^2/(c\*x^2+a)\*C\*a\*d\*e+1/2/c^2\*ln(c\*x^2+a)\*B\*e^2+1/c^2\*ln(c\*x^2+a)\*C\*d\*e+1/2/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*e^2+1/2/a/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*d^2+1/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*B\*d\*e-3/2/c^2\*a/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C\*e^2+1/2/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C\*d^2

**maxima [A]** time = 0.96, size = 188, normalized size = 1.29

$$\frac{C e^2 x}{c^2} - \frac{B a c d^2 - B a^2 e^2 - 2(C a^2 - A a c) d e + (2 B a c d e + (C a c - A c^2) d^2 - (C a^2 - A a c) e^2) x}{2(a c^3 x^2 + a^2 c^2)} + \frac{(2 C d e + B e^2) \log(c x^2 + a)}{c^2} + \frac{(2 B a c d e + (C a c + A c^2) d^2 - (3 C a^2 - A a c) e^2) \arctan\left(\frac{c x}{\sqrt{a}}\right)}{2 \sqrt{a} a c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="maxima")

[Out] C\*e^2\*x/c^2 - 1/2\*(B\*a\*c\*d^2 - B\*a^2\*e^2 - 2\*(C\*a^2 - A\*a\*c)\*d\*e + (2\*B\*a\*c\*d\*e + (C\*a\*c - A\*c^2)\*d^2 - (C\*a^2 - A\*a\*c)\*e^2)\*x)/(a\*c^3\*x^2 + a^2\*c^2) + 1/2\*(2\*C\*d\*e + B\*e^2)\*log(c\*x^2 + a)/c^2 + 1/2\*(2\*B\*a\*c\*d\*e + (C\*a\*c + A\*c^2)\*d^2 - (3\*C\*a^2 - A\*a\*c)\*e^2)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a\*c^2)

**mupad [B]** time = 0.23, size = 195, normalized size = 1.34

$$\frac{C e^2 x}{c^2} - \frac{x(-C a^2 d^2 + C a c d^2 + 2 B a c d e + A a c^2 - A c^2 d^2) - \frac{B a^2}{2} + \frac{B c d^2}{2} + A c d e - C a d e}{c^3 x^2 + a^2 c^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right) (-3 C a^2 e^2 + C a c d^2 + 2 B a c d e + A a c e^2 + A c^2 d^2)}{2 a^{3/2} c^{5/2}} + \frac{\ln(c x^2 + a) (16 B a^3 c^3 e^2 + 32 C d a^3 c^3 e)}{32 a^3 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2,x)

[Out] (C\*e^2\*x)/c^2 - ((x\*(A\*a\*c\*e^2 - C\*a^2\*e^2 - A\*c^2\*d^2 + C\*a\*c\*d^2 + 2\*B\*a\*c\*d\*e))/(2\*a) - (B\*a\*e^2)/2 + (B\*c\*d^2)/2 + A\*c\*d\*e - C\*a\*d\*e)/(a\*c^2 + c^3\*x^2) + (atan((c^(1/2)\*x)/a^(1/2))\*(A\*c^2\*d^2 - 3\*C\*a^2\*e^2 + A\*a\*c\*e^2 + C\*a\*c\*d^2 + 2\*B\*a\*c\*d\*e))/(2\*a^(3/2)\*c^(5/2)) + (log(a + c\*x^2)\*(16\*B\*a^3\*c^3\*e^2 + 32\*C\*a^3\*c^3\*d\*e))/(32\*a^3\*c^5)

sympy [B] time = 18.40, size = 593, normalized size = 4.06

$$\frac{C^2 d}{c^2} \left( \frac{(Bc + 2Cd) \sqrt{-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2}}{4d^2} \right) \log \left( \frac{2Bd^2 + 4Cd^2d - 4d^2 \left( \frac{Bcd^2d}{2d^2} + \frac{\sqrt{-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2}}{4d^2} \right)}{-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2} \right) + \frac{(Bc + 2Cd) \sqrt{-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2}}{2c^2} \log \left( \frac{2Bd^2 + 4Cd^2d - 4d^2 \left( \frac{Bcd^2d}{2d^2} + \frac{\sqrt{-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2}}{4d^2} \right)}{-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2} \right) - \frac{2Ade + B^2d^2 - Bcd^2 + 2C^2d^2d + (-Acd^2 - A^2d^2 - 2Bade + 3Cd^2d - Ccd^2)}{2d^2 + 2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*2,x)

[Out]  $C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - \text{sqrt}(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*\log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2) - \text{sqrt}(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + \text{sqrt}(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*\log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2) + \text{sqrt}(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a**2*e**2 - B*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)$

$$3.52 \quad \int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + Acd)}{2a^{3/2}c^{3/2}} - \frac{(d+ex)(aB - x(Ac - aC))}{2ac(a+cx^2)} + \frac{Ce \log(a+cx^2)}{2c^2}$$

**Rubi [A]** time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1645, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + Acd)}{2a^{3/2}c^{3/2}} - \frac{(d+ex)(aB - x(Ac - aC))}{2ac(a+cx^2)} + \frac{Ce \log(a+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2,x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x))/(2\*a\*c\*(a + c\*x^2)) + ((A\*c\*d + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(2\*a^(3/2)\*c^(3/2)) + (C\*e\*Log[a + c\*x^2])/(2\*c^2)

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 1645

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_))^(m\_.)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x]}]



```

nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

Rubi steps

$$\begin{aligned}
 \int \frac{(d + ex)(A + Bx + Cx^2)}{(a + cx^2)^2} dx &= -\frac{(aB - (Ac - aC)x)(d + ex)}{2ac(a + cx^2)} - \frac{\int \frac{-Acd - a(Cd + Be) - 2aCex}{a + cx^2} dx}{2ac} \\
 &= -\frac{(aB - (Ac - aC)x)(d + ex)}{2ac(a + cx^2)} + \frac{(Ce) \int \frac{x}{a + cx^2} dx}{c} + \frac{(Acd + aCd + aBe) \int \frac{1}{a + cx^2} dx}{2ac} \\
 &= -\frac{(aB - (Ac - aC)x)(d + ex)}{2ac(a + cx^2)} + \frac{(Acd + aCd + aBe) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{Ce \log(a + cx^2)}{2c^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 102, normalized size = 1.05

$$\frac{\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + Acd)}{a^{3/2}} + \frac{a^2Ce - ac(Ae + B(d + ex) + Cdx) + Ac^2dx}{a(a + cx^2)} + Ce \log(a + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

[Out] ((a^2\*C\*e + A\*c^2\*d\*x - a\*c\*(A\*e + C\*d\*x + B\*(d + e\*x)))/(a\*(a + c\*x^2)) + (Sqrt[c]\*(A\*c\*d + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/a^(3/2) + C\*e\*Log[a + c\*x^2]/(2\*c^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)(A + Bx + Cx^2)}{(a + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2, x]

**fricas** [A] time = 0.53, size = 337, normalized size = 3.47

$$\frac{2B^2cd + (B^2e + (Bae + (Cac + A^2d))x^2 + (C^2 + Aac)d)\sqrt{-a} \log\left(\frac{x^2 + \sqrt{-a}}{2ax}\right) - 2(C^2 - A^2c)e + 2(B^2ce + (C^2e - Aac^2)d)x - 2(C^2ex^2 + C^2d)\log(cx^2 + a) - B^2cd - (B^2e + (Bae + (Cac + A^2d))x^2 + (C^2 + Aac)d)\sqrt{a} \arctan\left(\frac{cx}{\sqrt{a}}\right) - (C^2 - A^2c)e + (B^2ce + (C^2e - Aac^2)d)x - (C^2ex^2 + C^2d)\log(cx^2 + a)}{4(a^2c^2 + a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/4\*(2\*B\*a^2\*c\*d + (B\*a^2\*e + (B\*a\*c\*e + (C\*a\*c + A\*c^2)\*d)\*x^2 + (C\*a^2 + A\*a\*c)\*d)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) - 2\*(C\*a^3 - A\*a^2\*c)\*e + 2\*(B\*a^2\*c\*e + (C\*a^2\*c - A\*a\*c^2)\*d)\*x - 2\*(C\*a^2\*c\*e\*x^2 + C\*a^3\*e)\*log(c\*x^2 + a)/(a^2\*c^3\*x^2 + a^3\*c^2), -1/2\*(B\*a^2\*c\*d - (B\*a^2\*e + (B\*a\*c\*e + (C\*a\*c + A\*c^2)\*d)\*x^2 + (C\*a^2 + A\*a\*c)\*d)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) - (C\*a^3 - A\*a^2\*c)\*e + (B\*a^2\*c\*e + (C\*a^2\*c - A\*a\*c^2)\*d)\*x - (C\*a^2\*c\*e\*x^2 + C\*a^3\*e)\*log(c\*x^2 + a)/(a^2\*c^3\*x^2 + a^3\*c^2)]

**giac** [A] time = 0.22, size = 112, normalized size = 1.15

$$\frac{Ce \log(cx^2 + a)}{2c^2} + \frac{(Cad + Acd + Bae) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac} - \frac{(Cad - Acd + Bae)x + \frac{Bacd - Ca^2e + Aace}{c}}{2(cx^2 + a)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="giac")

[Out] 1/2\*C\*e\*log(c\*x^2 + a)/c^2 + 1/2\*(C\*a\*d + A\*c\*d + B\*a\*e)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a\*c) - 1/2\*((C\*a\*d - A\*c\*d + B\*a\*e)\*x + (B\*a\*c\*d - C\*a^2\*e + A\*a\*c\*e)/c)/((c\*x^2 + a)\*a\*c)

**maple** [A] time = 0.01, size = 134, normalized size = 1.38

$$\frac{Ad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{Be \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}c} + \frac{Cd \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}c} + \frac{Ce \ln(cx^2 + a)}{2c^2} + \frac{(Acd - Bae - Cad)x - \frac{Ace + Bcd - aCe}{2c^2}}{cx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x)

[Out] (1/2\*(A\*c\*d-B\*a\*e-C\*a\*d)/a/c\*x-1/2\*(A\*c\*e+B\*c\*d-C\*a\*e)/c^2)/(c\*x^2+a)+1/2\*C\*e\*ln(c\*x^2+a)/c^2+1/2/a/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*d+1/2/c/(a

$(c \cdot x)^{1/2} \cdot \arctan(1/(a \cdot c)^{1/2} \cdot c \cdot x) \cdot B \cdot e^{1/2} / c / (a \cdot c)^{1/2} \cdot \arctan(1/(a \cdot c)^{1/2} \cdot c \cdot x) \cdot C \cdot d$

**maxima** [A] time = 0.97, size = 113, normalized size = 1.16

$$\frac{C e \log(cx^2 + a)}{2c^2} - \frac{Bacd - (Ca^2 - Aac)e + (Bace + (Cac - Ac^2)d)x}{2(ac^3x^2 + a^2c^2)} + \frac{(Bae + (Ca + Ac)d) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="maxima")

[Out]  $1/2 \cdot C \cdot e \cdot \log(c \cdot x^2 + a) / c^2 - 1/2 \cdot (B \cdot a \cdot c \cdot d - (C \cdot a^2 - A \cdot a \cdot c) \cdot e + (B \cdot a \cdot c \cdot e + (C \cdot a \cdot c - A \cdot c^2) \cdot d) \cdot x) / (a \cdot c^3 \cdot x^2 + a^2 \cdot c^2) + 1/2 \cdot (B \cdot a \cdot e + (C \cdot a + A \cdot c) \cdot d) \cdot a \cdot \arctan(c \cdot x / \sqrt{a \cdot c}) / (\sqrt{a \cdot c} \cdot a \cdot c)$

**mupad** [B] time = 0.14, size = 191, normalized size = 1.97

$$\frac{C e \ln(cx^2 + a)}{2c^2} - \frac{Bd}{2(c^2x^2 + ac)} - \frac{Bex}{2(c^2x^2 + ac)} - \frac{Cdx}{2(c^2x^2 + ac)} - \frac{Ae}{2(c^2x^2 + ac)} + \frac{Cae}{2(c^3x^2 + ac^2)} + \frac{Adx}{2(a^2 + cax^2)} + \frac{Ad \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{Be \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{a}c^{3/2}} + \frac{Cd \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{a}c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^2,x)

[Out]  $(C \cdot e \cdot \log(a + c \cdot x^2)) / (2 \cdot c^2) - (B \cdot d) / (2 \cdot (a \cdot c + c^2 \cdot x^2)) - (B \cdot e \cdot x) / (2 \cdot (a \cdot c + c^2 \cdot x^2)) - (C \cdot d \cdot x) / (2 \cdot (a \cdot c + c^2 \cdot x^2)) - (A \cdot e) / (2 \cdot (a \cdot c + c^2 \cdot x^2)) + (C \cdot a \cdot e) / (2 \cdot (a \cdot c^2 + c^3 \cdot x^2)) + (A \cdot d \cdot x) / (2 \cdot (a^2 + a \cdot c \cdot x^2)) + (A \cdot d \cdot \operatorname{atan}((c^{1/2} \cdot x) / a^{1/2})) / (2 \cdot a^{3/2} \cdot c^{1/2}) + (B \cdot e \cdot \operatorname{atan}((c^{1/2} \cdot x) / a^{1/2})) / (2 \cdot a^{1/2} \cdot c^{3/2}) + (C \cdot d \cdot \operatorname{atan}((c^{1/2} \cdot x) / a^{1/2})) / (2 \cdot a^{1/2} \cdot c^{3/2})$

**sympy** [B] time = 6.41, size = 318, normalized size = 3.28

$$\left(\frac{C e}{2c^2} - \frac{\sqrt{-a^3c^5}(Acd + Bae + Cad)}{4a^3c^4}\right) \log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{C e}{2c^2} - \frac{\sqrt{-a^3c^5}(Acd + Bae + Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \left(\frac{C e}{2c^2} + \frac{\sqrt{-a^3c^5}(Acd + Bae + Cad)}{4a^3c^4}\right) \log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{C e}{2c^2} + \frac{\sqrt{-a^3c^5}(Acd + Bae + Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \frac{-Aace - Bacd + Ca^2e + x(Ac^2d - Bace - Cacd)}{2a^2c^2 + 2ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*2,x)

[Out]  $(C \cdot e / (2 \cdot c^{**2}) - \sqrt{-a^{**3} \cdot c^{**5}} \cdot (A \cdot c \cdot d + B \cdot a \cdot e + C \cdot a \cdot d) / (4 \cdot a^{**3} \cdot c^{**4})) \cdot \log(x + (-2 \cdot C \cdot a^{**2} \cdot e + 4 \cdot a^{**2} \cdot c^{**2} \cdot (C \cdot e / (2 \cdot c^{**2}) - \sqrt{-a^{**3} \cdot c^{**5}} \cdot (A \cdot c \cdot d + B \cdot a \cdot e + C \cdot a \cdot d) / (4 \cdot a^{**3} \cdot c^{**4}))) / (A \cdot c^{**2} \cdot d + B \cdot a \cdot c \cdot e + C \cdot a \cdot c \cdot d)) + (C \cdot e / (2 \cdot c^{**2}) + \sqrt{-a^{**3} \cdot c^{**5}} \cdot (A \cdot c \cdot d + B \cdot a \cdot e + C \cdot a \cdot d) / (4 \cdot a^{**3} \cdot c^{**4})) \cdot \log(x + (-2 \cdot C \cdot a^{**2} \cdot e + 4 \cdot a^{**2} \cdot c^{**2} \cdot (C \cdot e / (2 \cdot c^{**2}) + \sqrt{-a^{**3} \cdot c^{**5}} \cdot (A \cdot c \cdot d + B \cdot a \cdot e + C \cdot a \cdot d) / (4 \cdot a^{**3} \cdot c^{**4}))) / (A \cdot c^{**2} \cdot d + B \cdot a \cdot c \cdot e + C \cdot a \cdot c \cdot d)) + (-A \cdot a \cdot c \cdot e - B \cdot a \cdot c \cdot d + C \cdot a^{**2} \cdot e + x \cdot (A \cdot c^{**2} \cdot d - B \cdot a \cdot c \cdot e - C \cdot a \cdot c \cdot d)) / (2 \cdot a^{**2} \cdot c^{**2} + 2 \cdot a \cdot c^{**3} \cdot x^{**2})$

$$3.53 \quad \int \frac{A+Bx+Cx^2}{(a+cx^2)^2} dx$$

Optimal. Leaf size=69

$$\frac{(aC + Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} - \frac{aB - x(Ac - aC)}{2ac(a + cx^2)}$$

**Rubi [A]** time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1814, 12, 205}

$$\frac{(aC + Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} - \frac{aB - x(Ac - aC)}{2ac(a + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2)^2,x]

[Out] -(a\*B - (A\*c - a\*C)\*x)/(2\*a\*c\*(a + c\*x^2)) + ((A\*c + a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(2\*a^(3/2)\*c^(3/2))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{(a + cx^2)^2} dx &= -\frac{aB - (Ac - aC)x}{2ac(a + cx^2)} - \frac{\int \frac{-A - \frac{aC}{c}}{a + cx^2} dx}{2a} \\
&= -\frac{aB - (Ac - aC)x}{2ac(a + cx^2)} + \frac{(Ac + aC) \int \frac{1}{a + cx^2} dx}{2ac} \\
&= -\frac{aB - (Ac - aC)x}{2ac(a + cx^2)} + \frac{(Ac + aC) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 68, normalized size = 0.99

$$\frac{(aC + Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{-aB - aCx + Acx}{2ac(a + cx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2)^2, x]

[Out]  $(-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^{(3/2)}*c^{(3/2)})$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^2, x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^2, x]

**fricas [A]** time = 3.00, size = 195, normalized size = 2.83

$$\left[ \frac{2Ba^2c + (Ca^2 + Aac + (Cac + Ac^2)x^2)\sqrt{-ac} \log\left(\frac{cx^2 - 2\sqrt{-ac}x - a}{cx^2 + a}\right) + 2(Ca^2c - Aac^2)x}{4(a^2c^3x^2 + a^3c^2)}, \frac{Ba^2c - (Ca^2 + Aac + (Cac + Ac^2)x^2)\sqrt{ac} \arctan\left(\frac{\sqrt{ac}x}{a}\right) + (Ca^2c - Aac^2)x}{2(a^2c^3x^2 + a^3c^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^2,x, algorithm="fricas")

[Out]  $[-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*\sqrt{-a*c}*\log((c*x^2 - 2*\sqrt{-a*c}*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*\sqrt{a*c}*\arctan(\sqrt{a*c}*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2)$   
]

**giac** [A] time = 0.18, size = 60, normalized size = 0.87

$$\frac{(Ca + Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac} - \frac{Cax - Acx + Ba}{2(cx^2 + a)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")`

[Out]  $1/2*(C*a + A*c)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a*c) - 1/2*(C*a*x - A*c*x + B*a)/((c*x^2 + a)*a*c)$

**maple** [A] time = 0.01, size = 76, normalized size = 1.10

$$\frac{A \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{C \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}c} + \frac{-\frac{B}{2c} + \frac{(Ac-aC)x}{2ac}}{cx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+B*x+A)/(c*x^2+a)^2,x)`

[Out]  $(1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2/a/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*A+1/2/c/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*C$

**maxima** [A] time = 0.96, size = 62, normalized size = 0.90

$$-\frac{Ba + (Ca - Ac)x}{2(ac^2x^2 + a^2c)} + \frac{(Ca + Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")`

[Out]  $-1/2*(B*a + (C*a - A*c)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(C*a + A*c)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a*c)$

**mapad** [B] time = 0.10, size = 60, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac + Ca)}{2a^{3/2}c^{3/2}} - \frac{\frac{B}{2c} - \frac{x(Ac-Ca)}{2ac}}{cx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(a + c*x^2)^2,x)`

[Out]  $(\operatorname{atan}((c^{1/2}x)/a^{1/2})*(A*c + C*a))/(2*a^{3/2}*c^{3/2}) - (B/(2*c) - (x*(A*c - C*a))/(2*a*c))/(a + c*x^2)$

**sympy [A]** time = 0.65, size = 116, normalized size = 1.68

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}(Ac + Ca)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c^3}}(Ac + Ca)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}} + x\right)}{4} + \frac{-Ba + x(Ac - Ca)}{2a^2c + 2ac^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)`

[Out]  $-\operatorname{sqrt}(-1/(a**3*c**3))*(A*c + C*a)*\log(-a**2*c*\operatorname{sqrt}(-1/(a**3*c**3)) + x)/4 + \operatorname{sqrt}(-1/(a**3*c**3))*(A*c + C*a)*\log(a**2*c*\operatorname{sqrt}(-1/(a**3*c**3)) + x)/4 + (-B*a + x*(A*c - C*a))/(2*a**2*c + 2*a*c**2*x**2)$

$$3.54 \quad \int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^2} dx$$

**Optimal.** Leaf size=226

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ACd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)\left(Cd-Be\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2} - \frac{a\left(aCe-Ace+Bcd\right)-cx\left(aBe-aCd+ACd\right)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)} - \frac{e\log\left(a\right)}{ae^2+cd^2}$$

**Rubi [A]** time = 0.43, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1647, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(ACd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)\left(Cd-Be\right)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2} - \frac{a\left(aCe-Ace+Bcd\right)-cx\left(aBe-aCd+ACd\right)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)} - \frac{e\log\left(a+cx^2\right)\left(Ae^2-Bde+Cd^2\right)}{2\left(ae^2+cd^2\right)^2} + \frac{e\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^2), x]

[Out] -(a\*(B\*c\*d - A\*c\*e + a\*C\*e) - c\*(A\*c\*d - a\*C\*d + a\*B\*e)\*x)/(2\*a\*c\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + ((a\*(C\*d - B\*e)\*(c\*d^2 - a\*e^2) + A\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(2\*a^(3/2)\*Sqrt[c]\*(c\*d^2 + a\*e^2)^2) + (e\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^2 - (e\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[a + c\*x^2])/(2\*(c\*d^2 + a\*e^2)^2)

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 801

Int[(((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x],



$x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

### Rule 1647

$\text{Int}[(\text{Pq}_*)*((d_) + (e_)*(x_))^{(m_)}*((a_) + (c_)*(x_)^2)^{(p_)}, x\_Symbol] :$   
 $> \text{With}[\{Q = \text{PolynomialQuotient}[(d + e*x)^m*\text{Pq}, a + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*\text{Pq}, a + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*\text{Pq}, a + c*x^2, x], x, 1]\}, \text{Simp}[(a*g - c*f*x)*(a + c*x^2)^{(p + 1)}/(2*a*c*(p + 1)), x] + \text{Dist}[1/(2*a*c*(p + 1)), \text{Int}[(d + e*x)^m*(a + c*x^2)^{(p + 1)}*\text{ExpandToSum}[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m, 0]$

### Rubi steps

$$\int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)^2} dx = -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{2ac(cd^2 + ae^2)(a + cx^2)} - \frac{\int \frac{c(ad(Cd - Be) + A(cd^2 + 2ae^2)) - ce(Acd - aCd + aBe)}{(d + ex)(a + cx^2)} dx}{2ac}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{2ac(cd^2 + ae^2)(a + cx^2)} - \frac{\int \left( -\frac{2ace^2(Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^2(d + ex)} + \frac{c(-a(Cd - Be))}{(cd^2 + ae^2)(d + ex)} \right) dx}{2ac}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{2ac(cd^2 + ae^2)(a + cx^2)} + \frac{e(Cd^2 - Bde + Ae^2) \log(d + ex)}{(cd^2 + ae^2)^2}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{2ac(cd^2 + ae^2)(a + cx^2)} + \frac{e(Cd^2 - Bde + Ae^2) \log(d + ex)}{(cd^2 + ae^2)^2}$$

$$= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{2ac(cd^2 + ae^2)(a + cx^2)} + \frac{(a(Cd - Be)(cd^2 - ae^2) + Acd)(cd^2 + ae^2)}{2a^{3/2}\sqrt{c}(cd^2 + ae^2)}$$

**Mathematica [A]** time = 0.22, size = 195, normalized size = 0.86

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(3ae^2 + cd^2) + a(cd^2 - ae^2)(Cd - Be))}{a^{3/2}\sqrt{c}} + \frac{(a^2 + cd^2)(a^2(-C)e + ac(Ae - Bd + Bex - Cdx) + Ac^2dx)}{ac(a + cx^2)} - \frac{e \log(a + cx^2)(e(Ae - Bd) + Cd^2) + 2e \log(d + ex)(e(Ae - Bd) + Cd^2)}{2(ae^2 + cd^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^2), x]

[Out] (((c\*d^2 + a\*e^2)\*(-a^2\*C\*e) + A\*c^2\*d\*x + a\*c\*(-(B\*d) + A\*e - C\*d\*x + B\*e\*x)))/(a\*c\*(a + c\*x^2)) + ((a\*(C\*d - B\*e)\*(c\*d^2 - a\*e^2) + A\*c\*d\*(c\*d^2 + 3\*a\*e^2))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(a^(3/2)\*Sqrt[c]) + 2\*e\*(C\*d^2 + e\*(-(B\*d) + A\*e))\*Log[d + e\*x] - e\*(C\*d^2 + e\*(-(B\*d) + A\*e))\*Log[a + c\*x^2]]/(2\*(c\*d^2 + a\*e^2)^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^2), x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^2), x]

**fricas [B]** time = 83.52, size = 1024, normalized size = 4.53

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a)^2,x, algorithm="fricas")

[Out] [-1/4\*(2\*B\*a^2\*c^2\*d^3 + 2\*B\*a^3\*c\*d\*e^2 + 2\*(C\*a^3\*c - A\*a^2\*c^2)\*d^2\*e + 2\*(C\*a^4 - A\*a^3\*c)\*e^3 - (B\*a^2\*c\*d^2\*e - B\*a^3\*e^3 - (C\*a^2\*c + A\*a\*c^2)\*d^3 + (C\*a^3 - 3\*A\*a^2\*c)\*d\*e^2 + (B\*a\*c^2\*d^2\*e - B\*a^2\*c\*e^3 - (C\*a\*c^2 + A\*c^3)\*d^3 + (C\*a^2\*c - 3\*A\*a\*c^2)\*d\*e^2)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) - 2\*(B\*a^2\*c^2\*d^2\*e + B\*a^3\*c\*e^3 - (C\*a^2\*c^2 - A\*a\*c^3)\*d^3 - (C\*a^3\*c - A\*a^2\*c^2)\*d\*e^2)\*x + 2\*(C\*a^3\*c\*d^2\*e - B\*a^3\*c\*d\*e^2 + A\*a^3\*c\*e^3 + (C\*a^2\*c^2\*d^2\*e - B\*a^2\*c^2\*d\*e^2 + A\*a^2\*c^2\*e^3)\*x^2)\*log(c\*x^2 + a) - 4\*(C\*a^3\*c\*d^2\*e - B\*a^3\*c\*d\*e^2 + A\*a^3\*c\*e^3 + (C\*a^2\*c^2\*d^2\*e - B\*a^2\*c^2\*d\*e^2 + A\*a^2\*c^2\*e^3)\*x^2)\*log(e\*x + d))/(a^3\*c^3\*d^4 + 2\*a^4\*c^2\*d^2\*e^2 + a^5\*c\*e^4 + (a^2\*c^4\*d^4 + 2\*a^3\*c^3\*d^2\*e^2 + a^4\*c^2\*e^4)\*x^2), -1/2\*(B\*a^2\*c^2\*d^3 + B\*a^3\*c\*d\*e^2 + (C\*a^3\*c - A\*a^2\*c^2)\*d^2\*e + (C\*a^4 - A\*a^3\*c)\*e^3 + (B\*a^2\*c\*d^2\*e - B\*a^3\*e^3 - (C\*a^2\*c + A\*a\*c^2)\*d^3 + (C\*a^3 - 3\*A\*a^2\*c)\*d\*e^2 + (B\*a\*c^2\*d^2\*e - B\*a^2\*c\*e^3 - (C\*a\*c^2 + A\*c^3)\*d^3 + (C\*a^2\*c - 3\*A\*a\*c^2)\*d\*e^2)\*x^2)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) - (B\*a^2\*c^2\*d^2\*e + B\*a^3\*c\*e^3 - (C\*a^2\*c^2 - A\*a\*c^3)\*d^3 - (C\*a^3\*c - A\*a^2\*c^2)\*d\*e^2)\*x + (C\*a^3\*c\*d^2\*e - B\*a^3\*c\*d\*e^2 + A\*a^3\*c\*e^3 + (C\*a^2\*c^2\*d^2\*e - B\*a^2\*c^2\*d\*e^2 + A\*a^2\*c^2\*e^3)\*x^2)\*log(c\*x^2 + a) - 2\*(C\*a^3\*c\*d^2\*e - B\*a^3\*c\*d\*e^2 + A\*a^3\*c\*e^3 + (C\*a^2\*c^2\*d^2\*e - B\*a^2\*c^2\*d\*e^2 + A\*a^2\*c^2\*e^3)\*x^2)\*log(e\*x + d))/(a^3\*c^3\*d^4 + 2\*a^4



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a)^2,x, algorithm="maxima")

[Out] 
$$-1/2*(C*d^2*e - B*d*e^2 + A*e^3)*\log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (C*d^2*e - B*d*e^2 + A*e^3)*\log(e*x + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*(B*a*c*d^2*e - B*a^2*e^3 - (C*a*c + A*c^2)*d^3 + (C*a^2 - 3*A*a*c)*d*e^2)*\arctan(c*x/\sqrt{a*c})/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*\sqrt{a*c}) - 1/2*(B*a*c*d + (C*a^2 - A*a*c)*e - (B*a*c*e - (C*a*c - A*c^2)*d)*x)/(a^2*c^2*d^2 + a^3*c*e^2 + (a*c^3*d^2 + a^2*c^2*e^2)*x^2)$$

mupad [B] time = 7.68, size = 1493, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((a + c\*x^2)^2\*(d + e\*x)),x)

[Out] 
$$\begin{aligned} & (\log(A*c^3*d^5*(-a^3*c)^{(1/2)} - B*a^3*e^5*(-a^3*c)^{(1/2)} + 6*A*a^4*c*e^5 - B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 8*C*a^3*c^2*d^4*e + 8*C*a^4*c*d^2*e^3 + C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^{(1/2)} + C*a^3*d*e^4*(-a^3*c)^{(1/2)} \\ & - 12*A*a^3*c^2*d^2*e^3 + 8*B*a^3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^4*d^5*x + 2*A*a^2*c^3*d^3*e^2*x + 14*B*a^3*c^2*d^2*e^3*x - 14*C*a^3*c^2*d^3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^{(1/2)} + 14*B*a^2*c*d^2*e^3*(-a^3*c)^{(1/2)} - 14*C*a^2*c*d^3*e^2*(-a^3*c)^{(1/2)} + C*a^4*c*d*e^4*x - 15*A*a^3*c^2*d*e^4*x - B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^{(1/2)} - B*a*c^2*d^4*e*(-a^3*c)^{(1/2)} - 6*A*a^2*c*e^5*x*(-a^3*c)^{(1/2)} + 2*A*c^3*d^4*e*x*(-a^3*c)^{(1/2)} \\ & ) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^{(1/2)} + 8*C*a*c^2*d^4*e*x*(-a^3*c)^{(1/2)} + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^{(1/2)} - 8*B*a*c^2*d^3*e^2*x*(-a^3*c)^{(1/2)} - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^{(1/2)}*(a^2*((B*e^3*(-a^3*c)^{(1/2)})/4 - (C*d*e^2*(-a^3*c)^{(1/2)})/4) - c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^2*e)/2) - a*((C*d^3*(-a^3*c)^{(1/2)})/4 + (3*A*d*e^2*(-a^3*c)^{(1/2)})/4 - (B*d^2*e*(-a^3*c)^{(1/2)})/4)) + (A*c^2*d^3*(-a^3*c)^{(1/2)})/4)/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - (\log(A*c^3*d^5*(-a^3*c)^{(1/2)} - B*a^3*e^5*(-a^3*c)^{(1/2)} \\ & - 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 8*C*a^3*c^2*d^4*e - 8*C*a^4*c*d^2*e^3 - C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^{(1/2)} + C*a^3*d*e^4*(-a^3*c)^{(1/2)} + 12*A*a^3*c^2*d^2*e^3 - 8*B*a^3*c^2*d^3*e^2 + 8*B*a^4*c*d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^3*e^2*x - 14*B*a^3*c^2*d^2*e^3*x + 14*C*a^3*c^2*d^3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^{(1/2)} + 14*B*a^2*c*d^2*e^3*(-a^3*c)^{(1/2)} - 14*C*a^2*c*d^3*e^2*(-a^3*c)^{(1/2)} - C*a^4*c*d*e^4*x + 15*A*a^3*c^2*d*e^4*x + B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^{(1/2)} - B*a*c^2*d^4*e*(-a^3*c)^{(1/2)} - 6*A*a^2*c*e^5*x*(-a^3*c)^{(1/2)} + 2*A*c^3*d^4*e*x*(-a^3*c)^{(1/2)} + 8*B*a^2*c*d*e^4*x*(-a^3*c)^{(1/2)} + 8*C*a*c^2*d^4*e*x*(-a^3*c)^{(1/2)} + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^{(1/2)} - 8*B*a*c^2*d^3*e^2*x*(-a^3*c)^{(1/2)} - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^{(1/2)}*(c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^2*e)/2) + a*((C*d^3*(-a^3*c)^{(1/2)})/4 + (3*A*d*e^2*(-a^3*c)^{(1/2)})/4 - (B*d^2*e*(-a^3*c)^{(1/2)})/4)) \\ & ) \\ & \end{aligned}$$

$$c)^{(1/2)}/4 - (B*d^2*e*(-a^3*c)^{(1/2)}/4) + a^2*((B*e^3*(-a^3*c)^{(1/2)}/4 - (C*d*e^2*(-a^3*c)^{(1/2)}/4) + (A*c^2*d^3*(-a^3*c)^{(1/2)}/4))/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - ((B*c*d - A*c*e + C*a*e)/(2*c*(a*e^2 + c*d^2)) - (x*(A*c*d + B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a + c*x^2) + (e*\log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^2$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)/(c\*x\*\*2+a)\*\*2,x)

[Out] Timed out

$$3.55 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^2} dx$$

**Optimal.** Leaf size=374

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4 + 6acd^2e^2 + c^2d^4\right) + a\left(a^2Ce^4 - 6acde^2(Cd - Be) + c^2d^3(Cd - 2Be)\right)\right) - a\left(-aBe^2 + 2aCd\right)}{2a^{3/2}\sqrt{c}\left(ae^2 + cd^2\right)^3}$$

**Rubi [A]** time = 0.95, antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1647, 1629, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4 + 6acd^2e^2 + c^2d^4\right) + a\left(a^2Ce^4 - 6acde^2(Cd - Be) + c^2d^3(Cd - 2Be)\right)\right) - a\left(-aBe^2 + 2aCd\right)}{2a^{3/2}\sqrt{c}\left(ae^2 + cd^2\right)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^2), x]

[Out] -((e\*(C\*d^2 - B\*d\*e + A\*e^2))/((c\*d^2 + a\*e^2)^2\*(d + e\*x))) - (a\*(B\*c\*d^2 - 2\*A\*c\*d\*e + 2\*a\*C\*d\*e - a\*B\*e^2) - (A\*c\*(c\*d^2 - a\*e^2) + a\*(a\*C\*e^2 - c\*d\*(C\*d - 2\*B\*e)))\*x)/(2\*a\*(c\*d^2 + a\*e^2)^2\*(a + c\*x^2)) + ((A\*c\*(c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4) + a\*(a^2\*C\*e^4 + c^2\*d^3\*(C\*d - 2\*B\*e) - 6\*a\*c\*d\*e^2\*(C\*d - B\*e)))\*ArcTan[ $\frac{\sqrt{c}x}{\sqrt{a}}$ ]/(2\*a^(3/2)\*sqrt[c]\*(c\*d^2 + a\*e^2)^3) + (e\*(2\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 4\*A\*e) - a\*e^2\*(2\*C\*d - B\*e))\*Log[d + e\*x]/(c\*d^2 + a\*e^2)^3 - (e\*(2\*c\*C\*d^3 - c\*d\*e\*(3\*B\*d - 4\*A\*e) - a\*e^2\*(2\*C\*d - B\*e))\*Log[a + c\*x^2])/(2\*(c\*d^2 + a\*e^2)^3)

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^2} dx &= -\frac{a(Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac(cd^2 - ae^2) + a(aCe^2 - cd(Cd - 2Be)))}{2a(cd^2 + ae^2)^2(a + cx^2)} \\ &= -\frac{a(Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac(cd^2 - ae^2) + a(aCe^2 - cd(Cd - 2Be)))}{2a(cd^2 + ae^2)^2(a + cx^2)} \\ &= -\frac{e(Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^2(d + ex)} - \frac{a(Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac(cd^2 - ae^2) + a(aCe^2 - cd(Cd - 2Be)))}{2a(cd^2 + ae^2)^2(a + cx^2)} \\ &= -\frac{e(Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^2(d + ex)} - \frac{a(Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac(cd^2 - ae^2) + a(aCe^2 - cd(Cd - 2Be)))}{2a(cd^2 + ae^2)^2(a + cx^2)} \\ &= -\frac{e(Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^2(d + ex)} - \frac{a(Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac(cd^2 - ae^2) + a(aCe^2 - cd(Cd - 2Be)))}{2a(cd^2 + ae^2)^2(a + cx^2)} \end{aligned}$$

**Mathematica [A]** time = 0.41, size = 320, normalized size = 0.86

$$\frac{\frac{(a^2 + cd^2)(d^2 + Bcd - 2Cd + Cx^2) - a(Ac(x - 2d) + B(d - 2a) + Cx^2) + A^2 d^2}{a^2(c + x^2)} + \frac{\tan^{-1}\left(\frac{2dx}{\sqrt{a^2 + cd^2}}\right)A(-3a^2d + 6acd^2 + 2d^3) + (d^2C^2 + 6acd^2(B - Cd) + 2d^2(Cd - 2Bd))}{a^2\sqrt{a^2 + cd^2}} - e \log(d + cx^2)(a^2(Be - 2Cd) + cde(4Ae - 3Bd) + 2cCd^2) + 2e \log(d + ex)(a^2(Be - 2Cd) + cde(4Ae - 3Bd) + 2cCd^2) - \frac{2(a^2 + cd^2)(c(Ae - Bd) + Cd^2)}{d^2cx}}{2(a^2 + cd^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^2), x]

[Out] 
$$\frac{((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-B*d) + A*e)))/(d + e*x) + ((c*d^2 + a*e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d - 2*e*x) + A*e*(-2*d + e*x)))}{(a*(a + c*x^2))} + ((A*c*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*d) + B*e)))*ArcTan[\frac{\sqrt{c}*x}{\sqrt{a}}]}{(a^{3/2}*\sqrt{c})} + 2*e*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2]}{(2*(c*d^2 + a*e^2)^3)}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^2), x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.19, size = 608, normalized size = 1.63

$$\frac{(Ca^2d^2 + A^2d^2 - 2Ba^2d^2 - 6C^2d^2e^4 + 6Aa^2d^2 + 6Ba^2d^2 + Ca^2d^2 - 3Aa^2d^2) \arctan\left(\frac{d + \frac{d}{c}}{\sqrt{a}}\right) e^{-2}}{2(a^2d^2 + 3a^2c^2d^2 + 3a^2c^2d^2 + a^2d^2)\sqrt{a}} - \frac{(2Ca^2d^2 - 3Ba^2d^2 - 2Ca^2d^2 + 4Aa^2d^2 + Ba^2d^2) \log\left(c - \frac{2d}{a} + \frac{d^2}{(a^2d^2 + a^2d^2)} + \frac{d^2}{(a^2d^2 + a^2d^2)}\right) - \frac{Ca^2d^2 - Ba^2d^2 + a^2d^2}{2a^2d^2 + 2a^2c^2d^2 + a^2d^2}}{2(c^2d^2 + 3a^2c^2d^2 + a^2d^2)} - \frac{Ca^2d^2 - Ba^2d^2 + a^2d^2}{2a^2d^2 + 2a^2c^2d^2 + a^2d^2} - \frac{Ca^2d^2 - Ba^2d^2 + a^2d^2}{2(c^2d^2 + 3a^2c^2d^2 + a^2d^2)} \sqrt{\frac{c - \frac{2d}{a} + \frac{d^2}{(a^2d^2 + a^2d^2)} + \frac{d^2}{(a^2d^2 + a^2d^2)}}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a)^2,x, algorithm="giac")

[Out] 
$$\frac{1}{2}*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*\arctan\left(\frac{(c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^{-1}/\sqrt{a*c}}{e^{-2}/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{a*c})}\right) - \frac{1}{2}*(2*C*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*\log(c - 2*c$$





```
[Out] -1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/2*(2*B*a*c^2*d^3*e - 6*B*a^2*c*d*e^3 - (C*a*c^2 + A*c^3)*d^4 + 6*(C*a^2*c - A*a*c^2)*d^2*e^2 - (C*a^3 - 3*A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 - 3*B*a^2*d*e^2 + 2*A*a^2*e^3 + 2*(2*C*a^2 - A*a*c)*d^2*e - (4*B*a*c*d*e^2 - (3*C*a*c - A*c^2)*d^2*e + (C*a^2 - 3*A*a*c)*e^3)*x^2 - (B*a*c*d^2*e + B*a^2*e^3 - (C*a*c - A*c^2)*d^3 - (C*a^2 - A*a*c)*d*e^2)*x)/(a^2*c^2*d^5 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)
```

**mupad [B]** time = 9.91, size = 2094, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^2), x)
```

```
[Out] ((x^2*(C*a^2*e^3 - 3*A*a*c*e^3 + A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*C*a*c*d^2*e))/((2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (2*A*a*e^3 + B*c*d^3 - 3*B*a*d*e^2 - 2*A*c*d^2*e + 4*C*a*d^2*e))/(2*(a*e^2 + c*d^2)^2) + (x*(A*c*d + B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a*d + a*e*x + c*d*x^2 + c*e*x^3) - (log(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a^3*c)^(1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) + 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^3*c)^(3/2) - 6*B*e^6*x*(-a^3*c)^(3/2) - C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*c)^(3/2) - 2*A*a^2*c^4*d^5*e + 30*A*a^4*c^2*d*e^5 - 14*C*a^3*c^3*d^5*e + 3*A*a^4*c^2*e^6*x + C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) - 36*A*a^3*c^3*d^3*e^3 + 22*B*a^3*c^3*d^4*e^2 - 36*B*a^4*c^2*d^2*e^4 + 36*C*a^4*c^2*d^3*e^3 - 14*C*a^5*c*d*e^5 + A*a*c^5*d^6*x + 5*A*a^2*c^4*d^4*e^2*x - 57*A*a^3*c^3*d^2*e^4*x + 44*B*a^3*c^3*d^3*e^3*x - 31*C*a^3*c^3*d^4*e^2*x + 31*C*a^4*c^2*d^2*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a^3*c)^(1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-a^3*c)^(1/2) - 2*B*a^2*c^4*d^5*e*x - 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e*(-a^3*c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(-a^3*c)^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*(-a^3*c)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x*(-a^3*c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^3*c)^(1/2)))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2) + a^3*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c)^(1/2))/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2) - a^4*((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(-a^3*c)^(1/2))/4)/((a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^2*e^
```

4) + (log(3\*A\*e^6\*(-a^3\*c)^(3/2) - A\*c^4\*d^6\*(-a^3\*c)^(1/2) + C\*a^4\*e^6\*(-a^3\*c)^(1/2) + 31\*C\*d^2\*e^4\*(-a^3\*c)^(3/2) - 6\*B\*a^5\*c\*e^6 - 18\*B\*d\*e^5\*(-a^3\*c)^(3/2) - 6\*B\*e^6\*x\*(-a^3\*c)^(3/2) + C\*a^5\*c\*e^6\*x + 14\*C\*d\*e^5\*x\*(-a^3\*c)^(3/2) + 2\*A\*a^2\*c^4\*d^5\*e - 30\*A\*a^4\*c^2\*d\*e^5 + 14\*C\*a^3\*c^3\*d^5\*e - 3\*A\*a^4\*c^2\*e^6\*x - C\*a^2\*c^4\*d^6\*x - C\*a\*c^3\*d^6\*(-a^3\*c)^(1/2) + 36\*A\*a^3\*c^3\*d^3\*e^3 - 22\*B\*a^3\*c^3\*d^4\*e^2 + 36\*B\*a^4\*c^2\*d^2\*e^4 - 36\*C\*a^4\*c^2\*d^3\*e^3 + 14\*C\*a^5\*c\*d\*e^5 - A\*a\*c^5\*d^6\*x - 5\*A\*a^2\*c^4\*d^4\*e^2\*x + 57\*A\*a^3\*c^3\*d^2\*e^4\*x - 44\*B\*a^3\*c^3\*d^3\*e^3\*x + 31\*C\*a^3\*c^3\*d^4\*e^2\*x - 31\*C\*a^4\*c^2\*d^2\*e^4\*x - 5\*A\*a\*c^3\*d^4\*e^2\*(-a^3\*c)^(1/2) + 57\*A\*a^2\*c^2\*d^2\*e^4\*(-a^3\*c)^(1/2) - 44\*B\*a^2\*c^2\*d^3\*e^3\*(-a^3\*c)^(1/2) + 31\*C\*a^2\*c^2\*d^4\*e^2\*(-a^3\*c)^(1/2) + 2\*B\*a^2\*c^4\*d^5\*e\*x + 18\*B\*a^4\*c^2\*d\*e^5\*x + 2\*B\*a\*c^3\*d^5\*e\*(-a^3\*c)^(1/2) - 2\*A\*c^4\*d^5\*e\*x\*(-a^3\*c)^(1/2) - 36\*B\*a^2\*c^2\*d^2\*e^4\*x\*(-a^3\*c)^(1/2) + 36\*C\*a^2\*c^2\*d^3\*e^3\*x\*(-a^3\*c)^(1/2) - 14\*C\*a\*c^3\*d^5\*e\*x\*(-a^3\*c)^(1/2) - 36\*A\*a\*c^3\*d^3\*e^3\*x\*(-a^3\*c)^(1/2) + 30\*A\*a^2\*c^2\*d\*e^5\*x\*(-a^3\*c)^(1/2) + 22\*B\*a\*c^3\*d^4\*e^2\*x\*(-a^3\*c)^(1/2))\*(c^2\*(a\*((C\*d^4\*(-a^3\*c)^(1/2))/4 + (3\*A\*d^2\*e^2\*(-a^3\*c)^(1/2))/2 - (B\*d^3\*e\*(-a^3\*c)^(1/2))/2) - a^3\*(2\*A\*d\*e^3 - (3\*B\*d^2\*e^2)/2 + C\*d^3\*e)) - c\*(a^2\*((3\*A\*e^4\*(-a^3\*c)^(1/2))/4 + (3\*C\*d^2\*e^2\*(-a^3\*c)^(1/2))/2 - (3\*B\*d\*e^3\*(-a^3\*c)^(1/2))/2) + a^4\*((B\*e^4)/2 - C\*d\*e^3)) + (A\*c^3\*d^4\*(-a^3\*c)^(1/2))/4 + (C\*a^3\*e^4\*(-a^3\*c)^(1/2))/4)/(a^6\*c\*e^6 + a^3\*c^4\*d^6 + 3\*a^4\*c^3\*d^4\*e^2 + 3\*a^5\*c^2\*d^2\*e^4) + (log(d + e\*x)\*(a\*(B\*e^4 - 2\*C\*d\*e^3) + c\*(4\*A\*d\*e^3 - 3\*B\*d^2\*e^2 + 2\*C\*d^3\*e)))/(a^3\*e^6 + c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*2,x)

[Out] Timed out

$$3.56 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^2} dx$$

Optimal. Leaf size=524

$$\frac{e \log(a+cx^2) \left( a^2 C e^4 - 2 a c e^2 \left( 4 C d^2 - e(3 B d - A e) \right) + c^2 d^2 \left( 3 C d^2 - 2 e(3 B d - 5 A e) \right) \right)}{2 \left( a e^2 + c d^2 \right)^4} + \frac{e \log(d+ex) \left( a^2 C e^4 - 2 a c e^2 \left( 4 C d^2 - e(3 B d - A e) \right) + c^2 d^2 \left( 3 C d^2 - 2 e(3 B d - 5 A e) \right) \right)}{2 \left( a e^2 + c d^2 \right)^4}$$

**Rubi [A]** time = 1.55, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1647, 1629, 635, 205, 260}

$$\frac{e \log(a+cx^2) \left( a^2 C e^4 - 2 a c e^2 \left( 4 C d^2 - e(3 B d - A e) \right) + c^2 d^2 \left( 3 C d^2 - 2 e(3 B d - 5 A e) \right) \right)}{2 \left( a e^2 + c d^2 \right)^4} + \frac{e \log(d+ex) \left( a^2 C e^4 - 2 a c e^2 \left( 4 C d^2 - e(3 B d - A e) \right) + c^2 d^2 \left( 3 C d^2 - 2 e(3 B d - 5 A e) \right) \right)}{2 \left( a e^2 + c d^2 \right)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)^2), x]

[Out]  $-(e*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (\text{Sqrt}[c]*(A*c*d*(c^2*d^4 + 10*a*c*d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d - 3*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]])/(2*a^(3/2)*(c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*\text{Log}[d + e*x])/(c*d^2 + a*e^2)^4 - (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*\text{Log}[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)$

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}

}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3 (a + cx^2)^2} dx = -\frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2)) - c (Acd (cd^2 - 3ae^2) - a (cd^2 (Cd^2 - 3ae^2) - a (cd^2 (Cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2))))}{2a (cd^2 + ae^2)^3 (a + cx^2)}$$

$$= -\frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2)) - c (Acd (cd^2 - 3ae^2) - a (cd^2 (Cd^2 - 3ae^2) - a (cd^2 (Cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2))))}{2a (cd^2 + ae^2)^3 (a + cx^2)}$$

$$= -\frac{e (Cd^2 - Bde + Ae^2)}{2 (cd^2 + ae^2)^2 (d + ex)^2} - \frac{e (2cCd^3 - cde(3Bd - 4Ae) - ae^2(2Cd - Be))}{(cd^2 + ae^2)^3 (d + ex)} - \frac{a (Bcd)}{(cd^2 + ae^2)^3 (d + ex)}$$

$$= -\frac{e (Cd^2 - Bde + Ae^2)}{2 (cd^2 + ae^2)^2 (d + ex)^2} - \frac{e (2cCd^3 - cde(3Bd - 4Ae) - ae^2(2Cd - Be))}{(cd^2 + ae^2)^3 (d + ex)} - \frac{a (Bcd)}{(cd^2 + ae^2)^3 (d + ex)}$$

$$= -\frac{e (Cd^2 - Bde + Ae^2)}{2 (cd^2 + ae^2)^2 (d + ex)^2} - \frac{e (2cCd^3 - cde(3Bd - 4Ae) - ae^2(2Cd - Be))}{(cd^2 + ae^2)^3 (d + ex)} - \frac{a (Bcd)}{(cd^2 + ae^2)^3 (d + ex)}$$



[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a)^2,x, algorithm="giac")

[Out] 
$$-1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3 + 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*\log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e^5 + C*a^2*e^6 - 2*A*a*c*e^6)*\log(\text{abs}(x*e + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 + 6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A*c^4*d^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 18*B*a^2*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5)*\arctan(c*x/\sqrt{a*c})/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*\sqrt{a*c}) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2*d^6*e - 3*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a^2*c^2*d^4*e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5 + B*a^4*d*e^6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3*d^4*e^3 - 2*C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5 - 7*C*a^3*c*d*e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d^6*e - 2*A*c^4*d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3*d^4*e^3 - 12*B*a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5 - C*a^4*e^7 + 2*A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6*e + 8*C*a^2*c^2*d^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^3*c*d^3*e^4 + 7*A*a^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 + 10*A*a^3*c*d*e^6 + 2*B*a^4*e^7)*x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)*(x*e + d)^2*a)$$

**maple [B]** time = 0.03, size = 1588, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a)^2,x)

[Out] 
$$-1/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*A*e^5*a^2-1/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*C*x*d^5+3/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*A*d^4*e-5/(a*e^2+c*d^2)^4*c^2*1n(c*x^2+a)*A*d^2*e^3+3/(a*e^2+c*d^2)^4*c^2*1n(c*x^2+a)*d^3*e^2*B-3/2/(a*e^2+c*d^2)^4*c^2*1n(c*x^2+a)*C*d^4*e+1/2/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*d^5+1/(a*e^2+c*d^2)^4*c*a*1n(c*x^2+a)*A*e^5+2*e^3/(a*e^2+c*d^2)^3/(e*x+d)*C*a*d-2*e/(a*e^2+c*d^2)^3/(e*x+d)*C*c*d^3-2*e^5/(a*e^2+c*d^2)^4*1n(e*x+d)*A*a*c+10*e^3/(a*e^2+c*d^2)^4*1n(e*x+d)*A*c^2*d^2-6*e^2/(a*e^2+c*d^2)^4*1n(e*x+d)*B*c^2*d^3+3*e/(a*e^2+c*d^2)^4*1n(e*x+d)*C*c^2*d^4-4*e^3/(a*e^2+c*d^2)^3/(e*x+d)*A*c*d+3*e^2/(a*e^2+c*d^2)^3/(e*x+d)*B*c*d^2-1/2*e^3/(a*e^2+c*d^2)^2/(e*x+d)^2*A+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*C*x*a*d^3*e^2+9/2/(a*e^2+c*d^2)^4*c*a^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*d*e^4-15/2/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*A*d*e^4+9/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e^3-7/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*d^3*e^2+3/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*C*x*a^2*d*e^4-3/2/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)$$

$$\begin{aligned} & ) * A * x * a * d * e^4 + 1 / (a * e^2 + c * d^2)^4 * c^2 / (c * x^2 + a) * B * x * a * d^2 * e^3 - 1 / (a * e^2 + c * d^2)^4 * c^3 / (c * x^2 + a) * A * x * d^3 * e^2 + 1 / 2 / (a * e^2 + c * d^2)^4 * c^4 / (c * x^2 + a) / a * x * A * d^5 + 3 / 2 / (a * e^2 + c * d^2)^4 * c^3 / (c * x^2 + a) * B * x * d^4 * e + 1 / (a * e^2 + c * d^2)^4 * c^2 / (c * x^2 + a) * A * d^2 * e^3 * a + 1 / (a * e^2 + c * d^2)^4 * c^2 / (c * x^2 + a) * d^3 * e^2 * B * a - 3 / 2 / (a * e^2 + c * d^2)^4 * c^2 / (c * x^2 + a) * C * a * d^4 * e + 6 * e^4 / (a * e^2 + c * d^2)^4 * \ln(e * x + d) * B * a * c * d - 8 * e^3 / (a * e^2 + c * d^2)^4 * \ln(e * x + d) * C * a * c * d^2 - 1 / 2 / (a * e^2 + c * d^2)^4 * c / (c * x^2 + a) * B * x * a^2 * e^5 + 3 / 2 / (a * e^2 + c * d^2)^4 * c / (c * x^2 + a) * d * e^4 * B * a^2 - 1 / (a * e^2 + c * d^2)^4 * c / (c * x^2 + a) * C * a^2 * d^2 * e^3 + 1 / 2 / (a * e^2 + c * d^2)^4 * c^4 / a / (a * c)^{(1/2)} * \arctan(1 / (a * c)^{(1/2)} * c * x) * A * d^5 + 5 / (a * e^2 + c * d^2)^4 * c^3 / (a * c)^{(1/2)} * \arctan(1 / (a * c)^{(1/2)} * c * x) * A * d^3 * e^2 - 3 / 2 / (a * e^2 + c * d^2)^4 * c^3 / (a * c)^{(1/2)} * \arctan(1 / (a * c)^{(1/2)} * c * x) * B * d^4 * e - 3 / (a * e^2 + c * d^2)^4 * c * a * \ln(c * x^2 + a) * d * e^4 * B + 4 / (a * e^2 + c * d^2)^4 * c * a * \ln(c * x^2 + a) * C * d^2 * e^3 - 3 / 2 / (a * e^2 + c * d^2)^4 * c * a^2 / (a * c)^{(1/2)} * \arctan(1 / (a * c)^{(1/2)} * c * x) * B * e^5 + 1 / 2 / (a * e^2 + c * d^2)^4 / (c * x^2 + a) * C * a^3 * e^5 - e^4 / (a * e^2 + c * d^2)^3 / (e * x + d) * B * a + 1 / 2 * e^2 / (a * e^2 + c * d^2)^2 / (e * x + d)^2 * B * d - 1 / 2 * e / (a * e^2 + c * d^2)^2 / (e * x + d)^2 * C * d^2 + e^5 / (a * e^2 + c * d^2)^4 * \ln(e * x + d) * a^2 * C - 1 / 2 / (a * e^2 + c * d^2)^4 * c^3 / (c * x^2 + a) * d^5 * B - 1 / 2 / (a * e^2 + c * d^2)^4 * a^2 * \ln(c * x^2 + a) * C * e^5 \end{aligned}$$

**maxima [B]** time = 1.22, size = 1030, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a)^2,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/2 * (3 * C * c^2 * d^4 * e - 6 * B * c^2 * d^3 * e^2 + 6 * B * a * c * d * e^4 - 2 * (4 * C * a * c - 5 * A * c^2) * d^2 * e^3 + (C * a^2 - 2 * A * a * c) * e^5) * \log(c * x^2 + a) / (c^4 * d^8 + 4 * a * c^3 * d^6 * e^2 + 6 * a^2 * c^2 * d^4 * e^4 + 4 * a^3 * c * d^2 * e^6 + a^4 * e^8) + (3 * C * c^2 * d^4 * e - 6 * B * c^2 * d^3 * e^2 + 6 * B * a * c * d * e^4 - 2 * (4 * C * a * c - 5 * A * c^2) * d^2 * e^3 + (C * a^2 - 2 * A * a * c) * e^5) * \log(e * x + d) / (c^4 * d^8 + 4 * a * c^3 * d^6 * e^2 + 6 * a^2 * c^2 * d^4 * e^4 + 4 * a^3 * c * d^2 * e^6 + a^4 * e^8) - 1/2 * (3 * B * a * c^3 * d^4 * e - 18 * B * a^2 * c^2 * d^2 * e^3 + 3 * B * a^3 * c * e^5 - (C * a * c^3 + A * c^4) * d^5 + 2 * (7 * C * a^2 * c^2 - 5 * A * a * c^3) * d^3 * e^2 - 3 * (3 * C * a^3 * c - 5 * A * a^2 * c^2) * d * e^4) * \arctan(c * x / \sqrt{a * c}) / ((a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8) * \sqrt{a * c}) - 1/2 * (B * a * c^2 * d^5 - 10 * B * a^2 * c * d^3 * e^2 + B * a^3 * d * e^4 + A * a^3 * e^5 + (8 * C * a^2 * c - 3 * A * a * c^2) * d^4 * e - 2 * (2 * C * a^3 - 5 * A * a^2 * c) * d^2 * e^3 - (9 * B * a * c^2 * d^2 * e^3 - 3 * B * a^2 * c * e^5 - (5 * C * a * c^2 - A * c^3) * d^3 * e^2 + (7 * C * a^2 * c - 11 * A * a * c^2) * d * e^4) * x^3 - (12 * B * a * c^2 * d^3 * e^2 - (7 * C * a * c^2 - 2 * A * c^3) * d^4 * e + 6 * (C * a^2 * c - 2 * A * a * c^2) * d^2 * e^3 + (C * a^3 - 2 * A * a^2 * c) * e^5) * x^2 - (B * a * c^2 * d^4 * e + 11 * B * a^2 * c * d^2 * e^3 - 2 * B * a^3 * e^5 - (C * a * c^2 - A * c^3) * d^5 - (7 * C * a^2 * c - 3 * A * a * c^2) * d^3 * e^2 + 2 * (3 * C * a^3 - 5 * A * a^2 * c) * d * e^4) * x) / (a^2 * c^3 * d^8 + 3 * a^3 * c^2 * d^6 * e^2 + 3 * a^4 * c * d^4 * e^4 + a^5 * d^2 * e^6 + (a * c^4 * d^6 * e^2 + 3 * a^2 * c^3 * d^4 * e^4 + 3 * a^3 * c^2 * d^2 * e^6 + a^4 * c * e^8) * x^4 + 2 * (a * c^4 * d^7 * e + 3 * a^2 * c^3 * d^5 * e^3 + 3 * a^3 * c^2 * d^3 * e^5 + a^4 * c * d * e^7) * x^3 + (a * c^4 * d^8 + 4 * a^2 * c^3 * d^6 * e^2 + 6 * a^3 * c^2 * d^4 * e^4 + 4 * a^4 * c * d^2 * e^6 + a^5 * e^8) * x^2 + 2 * (a^2 * c^3 * d^7 * e + 3 * a^3 * c^2 * d^5 * e^3 + 3 * a^4 * c * d^3 * e^5 + a^5 * d * e^7) * x) \end{aligned}$$





$$\begin{aligned}
& c^{(3/2)} + 42*B*a^2*d*e^6*x*(-a^3*c)^{(3/2)} - 20*C*c^2*d^6*e*x*(-a^3*c)^{(3/2)} \\
& + 102*A*a*c*d^2*e^5*x*(-a^3*c)^{(3/2)} - 108*B*a*c*d^3*e^4*x*(-a^3*c)^{(3/2)} \\
& + 94*C*a*c*d^4*e^3*x*(-a^3*c)^{(3/2)} + 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^{(1/2)} \\
& * (e^3*(5*A*a^3*c^2*d^2 - 4*C*a^4*c*d^2 + (9*B*a^2*c*d^2*(-a^3*c)^{(1/2)}))/2) - \\
& e^2*(3*B*a^3*c^2*d^3 - (5*A*a*c^2*d^3*(-a^3*c)^{(1/2)}))/2 + (7*C*a^2*c*d^3*(- \\
& -a^3*c)^{(1/2)}))/2 + e^4*(3*B*a^4*c*d + (9*C*a^3*d*(-a^3*c)^{(1/2)}))/4 - (15*A \\
& *a^2*c*d*(-a^3*c)^{(1/2)}))/4 + e*((3*C*a^3*c^2*d^4)/2 - (3*B*a*c^2*d^4*(-a^3 \\
& *c)^{(1/2)}))/4 - e^5*((3*B*a^3*(-a^3*c)^{(1/2)}))/4 - (C*a^5)/2 + A*a^4*c) + (A \\
& *c^3*d^5*(-a^3*c)^{(1/2)}))/4 + (C*a*c^2*d^5*(-a^3*c)^{(1/2)}))/4)/(a^7*e^8 + a^ \\
& 3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - ((A \\
& a^2*e^5 + B*c^2*d^5 + B*a^2*d*e^4 - 3*A*c^2*d^4*e - 4*C*a^2*d^2*e^3 + 8*C*a \\
& *c*d^4*e + 10*A*a*c*d^2*e^3 - 10*B*a*c*d^3*e^2)/(2*(a*e^2 + c*d^2)*(a^2*e^4 \\
& + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*B*a^2*c*e^5 - A*c^3*d^3*e^2 - 9*B*a \\
& c^2*d^2*e^3 + 5*C*a*c^2*d^3*e^2 + 11*A*a*c^2*d*e^4 - 7*C*a^2*c*d*e^4))/(2*a \\
& *(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(A*c^3*d^5 - \\
& 2*B*a^3*e^5 - C*a*c^2*d^5 + 6*C*a^3*d*e^4 + 3*A*a*c^2*d^3*e^2 + 11*B*a^2*c \\
& *d^2*e^3 - 7*C*a^2*c*d^3*e^2 - 10*A*a^2*c*d*e^4 + B*a*c^2*d^4*e))/(2*a*(a*e \\
& ^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x^2*(C*a^3*e^5 - 2*A*a^ \\
& 2*c*e^5 + 2*A*c^3*d^4*e - 12*A*a*c^2*d^2*e^3 + 12*B*a*c^2*d^3*e^2 + 6*C*a^2 \\
& *c*d^2*e^3 - 7*C*a*c^2*d^4*e))/(2*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2 \\
& a*c*d^2*e^2)))/(a*d^2 + x^2*(a*e^2 + c*d^2) + c*e^2*x^4 + 2*a*d*e*x + 2*c*d \\
& *e*x^3) + (log(d + e*x)*(c^2*(10*A*d^2*e^3 - 6*B*d^3*e^2 + 3*C*d^4*e) - c*( \\
& 2*A*a*e^5 - 6*B*a*d*e^4 + 8*C*a*d^2*e^3) + C*a^2*e^5))/(a^4*e^8 + c^4*d^8 + \\
& 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*3/(c\*x\*\*2+a)\*\*2,x)

[Out] Timed out

$$3.57 \quad \int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3} dx$$

**Optimal.** Leaf size=209

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2(3aBe + aCd + 3Acd) + 3ae^2(aBe + 3aCd + Acd))}{8a^{5/2}c^{5/2}} - \frac{(d+ex)(ae(3aBe + 5aCd + 3Acd) - x(3Ae^2 + 3aBe + 5aCd + 3Acd))}{8a^2c^2(a+cx^2)}$$

**Rubi [A]** time = 0.30, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1645, 819, 635, 205, 260}

$$\frac{(d+ex)(ae(3aBe + 5aCd + 3Acd) - x(3Ae^2 + 3aBe + 5aCd + 3Acd))}{8a^2c^2(a+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2(3aBe + aCd + 3Acd) + 3ae^2(aBe + 3aCd + Acd))}{8a^{5/2}c^{5/2}} - \frac{(d+ex)^3(aB - x(Ac - aC))}{4ac(a+cx^2)^2} + \frac{C^3 \log(a+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^3)/(4\*a\*c\*(a + c\*x^2)^2) - ((d + e\*x)\*(a\*e\*(3\*A\*c\*d + 5\*a\*C\*d + 3\*a\*B\*e) - (3\*A\*c^2\*d^2 - a\*(4\*a\*C\*e^2 - c\*d\*(C\*d + 3\*B\*e)))\*x)/(8\*a^2\*c^2\*(a + c\*x^2)) + ((3\*a\*e^2\*(A\*c\*d + 3\*a\*C\*d + a\*B\*e) + c\*d^2\*(3\*A\*c\*d + a\*C\*d + 3\*a\*B\*e))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(8\*a^(5/2)\*c^(5/2)) + (C\*e^3\*Log[a + c\*x^2])/(2\*c^3)

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 819

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1)\*(a\*(e\*f + d\*g

```
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a,
c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

### Rule 1645

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^(m)*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &&
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3} dx &= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{4ac(a+cx^2)^2} - \frac{\int \frac{(d+ex)^2(-3Acd - aCd - 3aBe - 4aCex)}{(a+cx^2)^2} dx}{4ac} \\ &= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{4ac(a+cx^2)^2} - \frac{(d+ex)(ae(3Acd + 5aCd + 3aBe) - (3Ac^2d^2)}{8a^2c^2(a+cx^2)} \\ &= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{4ac(a+cx^2)^2} - \frac{(d+ex)(ae(3Acd + 5aCd + 3aBe) - (3Ac^2d^2)}{8a^2c^2(a+cx^2)} \\ &= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{4ac(a+cx^2)^2} - \frac{(d+ex)(ae(3Acd + 5aCd + 3aBe) - (3Ac^2d^2)}{8a^2c^2(a+cx^2)} \end{aligned}$$

**Mathematica [A]** time = 0.25, size = 281, normalized size = 1.34

$$\frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3Acd(a^2+cx^2)+a(3ac^2(Be+3Cd)+c^2d^2(3Be+Cd)))}{a^5/2} + \frac{-2a^3C^2+2a^2c^2c(Ae+3Bd+Bcx)+3Cd(d+cx)-2ac^2d(3Ae(d+cx)+Bd(d+3cx)+Cd^2x)+2Ac^2d^2x}{a(a+cx^2)^2} + \frac{8a^3C^2-a^2c^2c(Ae+12Bd+5Bcx)+3Cd(d+5cx)+a^2dx(3e(Ae+Bd)+Cd^2)+3Ac^2d^2x}{a^2(a+cx^2)} + 4C^3 \log(a+cx^2)$$

8c<sup>3</sup>

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3,x]

[Out] ((-2\*a^3\*C\*e^3 + 2\*A\*c^3\*d^3\*x - 2\*a\*c^2\*d\*(C\*d^2\*x + 3\*A\*e\*(d + e\*x) + B\*d\*(d + 3\*e\*x)) + 2\*a^2\*c\*e\*(3\*C\*d\*(d + e\*x) + e\*(3\*B\*d + A\*e + B\*e\*x)))/(a\*(a + c\*x^2)^2) + (8\*a^3\*C\*e^3 + 3\*A\*c^3\*d^3\*x + a\*c^2\*d\*(C\*d^2 + 3\*e\*(B\*d + A\*e))\*x - a^2\*c\*e\*(3\*C\*d\*(4\*d + 5\*e\*x) + e\*(12\*B\*d + 4\*A\*e + 5\*B\*e\*x)))/(a^2\*(a + c\*x^2)) + (Sqrt[c]\*(3\*A\*c\*d\*(c\*d^2 + a\*e^2) + a\*(3\*a\*e^2\*(3\*C\*d + B\*e) + c\*d^2\*(C\*d + 3\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/a^(5/2) + 4\*C\*e^3\*Log[a + c\*x^2]/(8\*c^3)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3 (A + Bx + Cx^2)}{(a + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3,x]

[Out] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

**fricas** [B] time = 1.06, size = 1138, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/16\*(4\*B\*a^3\*c^2\*d^3 + 12\*B\*a^4\*c\*d\*e^2 + 12\*(C\*a^4\*c + A\*a^3\*c^2)\*d^2\*e - 4\*(3\*C\*a^5 - A\*a^4\*c)\*e^3 - 2\*(3\*B\*a^2\*c^3\*d^2\*e - 5\*B\*a^3\*c^2\*e^3 + (C\*a^2\*c^3 + 3\*A\*a\*c^4)\*d^3 - 3\*(5\*C\*a^3\*c^2 - A\*a^2\*c^3)\*d\*e^2)\*x^3 + 8\*(3\*C\*a^3\*c^2\*d^2\*e + 3\*B\*a^3\*c^2\*d\*e^2 - (2\*C\*a^4\*c - A\*a^3\*c^2)\*e^3)\*x^2 + (3\*B\*a^3\*c\*d^2\*e + 3\*B\*a^4\*e^3 + (3\*B\*a\*c^3\*d^2\*e + 3\*B\*a^2\*c^2\*e^3 + (C\*a\*c^3 + 3\*A\*c^4)\*d^3 + 3\*(3\*C\*a^2\*c^2 + A\*a\*c^3)\*d\*e^2)\*x^4 + (C\*a^3\*c + 3\*A\*a^2\*c^2)\*d^3 + 3\*(3\*C\*a^4 + A\*a^3\*c)\*d\*e^2 + 2\*(3\*B\*a^2\*c^2\*d^2\*e + 3\*B\*a^3\*c\*e^3 + (C\*a^2\*c^2 + 3\*A\*a\*c^3)\*d^3 + 3\*(3\*C\*a^3\*c + A\*a^2\*c^2)\*d\*e^2)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 2\*(3\*B\*a^3\*c^2\*d^2\*e + 3\*B\*a^4\*c\*e^3 + (C\*a^3\*c^2 - 5\*A\*a^2\*c^3)\*d^3 + 3\*(3\*C\*a^4\*c + A\*a^3\*c^2)\*d\*e^2)\*x - 8\*(C\*a^3\*c^2\*e^3\*x^4 + 2\*C\*a^4\*c\*e^3\*x^2 + C\*a^5\*e^3)\*log(c\*x^2 + a))/(a^3\*c^5\*x^4 + 2\*a^4\*c^4\*x^2 + a^5\*c^3), -1/8\*(2\*B\*a^3\*c^2\*d^3 + 6\*B\*a^4\*c\*d\*e^2 + 6\*(C\*a^4\*c + A\*a^3\*c^2)\*d^2\*e - 2\*(3\*C\*a^5 - A\*a^4\*c)\*e^3 - (3\*B\*a^2\*c^3\*d^2\*e - 5\*B\*a^3\*c^2\*e^3 + (C\*a^2\*c^3 + 3\*A\*a\*c^4)\*d^3 - 3\*(5\*C\*a^3\*c^2 - A\*a^2\*c^3)\*d\*e^2)\*x^3 + 4\*(3\*C\*a^3\*c^2\*d^2\*e + 3\*B\*a^3\*c^2\*d\*e^2 - (2\*C\*a^4\*c - A\*a^3\*c^2)\*e^3)\*x^2 - (3\*B\*a^3\*c\*d^2\*e + 3\*B\*a^4\*e^3 + (3\*B\*a\*c^3\*d^2\*e + 3\*B\*a^2\*c^2\*e^3 + (C\*a\*c^3 + 3\*A\*c^4)\*d^3 + 3\*(3\*C\*a^2\*c^2 + A\*a\*c^3)\*d\*e^2)\*x^4 + (C\*a^3\*c + 3\*A\*a^2\*c^2)\*d^3 + 3\*(3\*C\*a^4 + A\*a^3\*c



[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}C e^3 \log(c x^2 + a) / c^3 - \frac{1}{8}(2 B a^2 c^2 d^3 + 6 B a^3 c d e^2 + 6(C a^3 c + A a^2 c^2) d^2 e - 2(3 C a^4 - A a^3 c) e^3 - (3 B a^2 c^3 d^2 e - 5 B a^2 c^2 e^3 + (C a^2 c^3 + 3 A a^2 c^4) d^3 - 3(5 C a^2 c^2 - A a^2 c^3) d e^2) x^3 + 4(3 C a^2 c^2 d^2 e + 3 B a^2 c^2 d e^2 - (2 C a^3 c - A a^2 c^2) e^3) x^2 + (3 B a^2 c^2 d^2 e + 3 B a^3 c e^3 + (C a^2 c^2 - 5 A a^2 c^3) d^3 + 3(3 C a^3 c + A a^2 c^2) d e^2) x) / (a^2 c^5 x^4 + 2 a^3 c^4 x^2 + a^4 c^3) + \frac{1}{8}(3 B a^2 c d^2 e + 3 B a^2 e^3 + (C a^2 c + 3 A a^2 c^2) d^3 + 3(3 C a^2 + A a^2 c) d e^2) \arctan(c x / \sqrt{a c}) / (\sqrt{a c} a^2 c^2)$

**mupad** [B] time = 1.77, size = 920, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3,x)

[Out]  $\frac{(5 A d^3 x) / (8(a^3 + 2 a^2 c x^2 + a c^2 x^4)) - (B d^3) / (4(a^2 c + c^3 x^4 + 2 a c^2 x^2)) + (3 C a^2 e^3) / (4(a^2 c^3 + c^5 x^4 + 2 a c^4 x^2)) - (3 A d^2 e) / (4(a^2 c + c^3 x^4 + 2 a c^2 x^2)) + (C d^3 x^3) / (8(a^3 + 2 a^2 c x^2 + a c^2 x^4)) - (C d^3 x) / (8(a^2 c + c^3 x^4 + 2 a c^2 x^2)) - (A a^2 e^3) / (4(a^2 c^2 + c^4 x^4 + 2 a c^3 x^2)) - (A e^3 x^2) / (2(a^2 c + c^3 x^4 + 2 a c^2 x^2)) - (5 B e^3 x^3) / (8(a^2 c + c^3 x^4 + 2 a c^2 x^2)) + (C e^3 \log(a + c x^2)) / (2 c^3) - (3 B a d e^2) / (4(a^2 c^2 + c^4 x^4 + 2 a c^3 x^2)) - (3 C a d^2 e) / (4(a^2 c^2 + c^4 x^4 + 2 a c^3 x^2)) + (3 A c d^3 x^3) / (8(a^4 + 2 a^3 c x^2 + a^2 c^2 x^4)) - (3 B a e^3 x) / (8(a^2 c^2 + c^4 x^4 + 2 a c^3 x^2)) - (3 B d e^2 x^2) / (2(a^2 c + c^3 x^4 + 2 a c^2 x^2)) - (3 C d^2 e x^2) / (2(a^2 c + c^3 x^4 + 2 a c^2 x^2)) - (15 C d e^2 x^3) / (8(a^2 c + c^3 x^4 + 2 a c^2 x^2)) + (C a e^3 x^2) / (a^2 c^2 + c^4 x^4 + 2 a c^3 x^2) + (3 A d^3 \operatorname{atan}((c^{1/2} x) / a^{1/2})) / (8 a^{5/2} c^{1/2}) + (3 B e^3 \operatorname{atan}((c^{1/2} x) / a^{1/2})) / (8 a^{1/2} c^{5/2}) + (C d^3 \operatorname{atan}((c^{1/2} x) / a^{1/2})) / (8 a^{3/2} c^{3/2}) + (3 A d e^2 x^3) / (8(a^3 + 2 a^2 c x^2 + a c^2 x^4)) + (3 B d^2 e x^3) / (8(a^3 + 2 a^2 c x^2 + a c^2 x^4)) - (3 A d e^2 x) / (8(a^2 c + c^3 x^4 + 2 a c^2 x^2)) - (3 B d^2 e x) / (8(a^2 c + c^3 x^4 + 2 a c^2 x^2)) + (3 A d e^2 \operatorname{atan}((c^{1/2} x) / a^{1/2})) / (8 a^{3/2} c^{3/2}) + (3 B d^2 e \operatorname{atan}((c^{1/2} x) / a^{1/2})) / (8 a^{3/2} c^{3/2}) + (9 C d e^2 \operatorname{atan}((c^{1/2} x) / a^{1/2})) / (8 a^{1/2} c^{5/2}) - (9 C a d e^2 x) / (8(a^2 c^2 + c^4 x^4 + 2 a c^3 x^2))$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```



$$3.58 \quad \int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3} dx$$

**Optimal.** Leaf size=156

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(2aBe + aCd + 3Acd) + ae^2(3aC + Ac))}{8a^{5/2}c^{5/2}} - \frac{(d+ex)(ae(3aC + Ac) - cx(2aBe + aCd + 3Acd))}{8a^2c^2(a+cx^2)}$$

**Rubi [A]** time = 0.23, antiderivative size = 175, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1645, 778, 205}

$$\frac{x(ae^2(3aC + Ac) - cd(2aBe + aCd + 3Acd)) + 2ae(aBe + 2aCd + 2Acd)}{8a^2c^2(a+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(2aBe + aCd + 3Acd) + ae^2(3aC + Ac))}{8a^{5/2}c^{5/2}} - \frac{(d+ex)^2(aB - x(Ac - aC))}{4ac(a+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^2)/(4\*a\*c\*(a + c\*x^2)^2) - (2\*a\*e\*(2\*A\*c\*d + 2\*a\*C\*d + a\*B\*e) + (a\*(A\*c + 3\*a\*C)\*e^2 - c\*d\*(3\*A\*c\*d + a\*C\*d + 2\*a\*B\*e))\*x)/(8\*a^2\*c^2\*(a + c\*x^2)) + ((a\*(A\*c + 3\*a\*C)\*e^2 + c\*d\*(3\*A\*c\*d + a\*C\*d + 2\*a\*B\*e))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(8\*a^(5/2)\*c^(5/2))

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 778

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((a\*(e\*f + d\*g) - (c\*d\*f - a\*e\*g)\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(2\*a\*c\*(p + 1)), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]

### Rule 1645

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 1]}, Simp[((d + e\*x)^m\*(a + c\*x^2)^(p + 1)\*(a\*g - c\*f\*x))/(2\*a\*c\*(p + 1)), x] + Dist[1/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1)\*ExpandToSum[2\*a\*c\*(p + 1)\*(d + e\*x)\*Q - a\*e\*g\*m + c\*d\*f\*(2\*p + 3) + c\*e

```
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3} dx &= -\frac{(aB - (Ac - aC)x)(d+ex)^2}{4ac(a+cx^2)^2} - \frac{\int \frac{(d+ex)(-3Acd - aCd - 2aBe - (Ac+3aC)ex)}{(a+cx^2)^2} dx}{4ac} \\ &= -\frac{(aB - (Ac - aC)x)(d+ex)^2}{4ac(a+cx^2)^2} - \frac{2ae(2Acd + 2aCd + aBe) + (a(Ac + 3aC)e^2 - 8a^2c^2(a+cx^2))}{8a^2c^2(a+cx^2)} \\ &= -\frac{(aB - (Ac - aC)x)(d+ex)^2}{4ac(a+cx^2)^2} - \frac{2ae(2Acd + 2aCd + aBe) + (a(Ac + 3aC)e^2 - 8a^2c^2(a+cx^2))}{8a^2c^2(a+cx^2)} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 211, normalized size = 1.35

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac(ae^2 + 3cd^2) + a(3aC^2 + cd(2Be + Cd)))}{8a^{5/2}c^{5/2}} + \frac{a^2(-e)(4Be + 8Cd + 5Cex) + acx(e(Ae + 2Bd) + Cd^2) + 3Ac^2d^2x}{8a^2c^2(a+cx^2)} + \frac{a^2e(Be + 2Cd + Cex) - ac(Ae(2d + ex) + Bd(d + 2ex) + Cd^2x) + Ac^2d^2x}{4ac^2(a+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

[Out] (3\*A\*c^2\*d^2\*x + a\*c\*(C\*d^2 + e\*(2\*B\*d + A\*e))\*x - a^2\*e\*(8\*C\*d + 4\*B\*e + 5\*C\*e\*x))/(8\*a^2\*c^2\*(a + c\*x^2)) + (A\*c^2\*d^2\*x + a^2\*e\*(2\*C\*d + B\*e + C\*e\*x) - a\*c\*(C\*d^2\*x + A\*e\*(2\*d + e\*x) + B\*d\*(d + 2\*e\*x)))/(4\*a\*c^2\*(a + c\*x^2)^2) + ((A\*c\*(3\*c\*d^2 + a\*e^2) + a\*(3\*a\*C\*e^2 + c\*d\*(C\*d + 2\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(8\*a^(5/2)\*c^(5/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

**fricas** [B] time = 1.02, size = 806, normalized size = 5.17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/16*(4*B*a^3*c^2*d^2 + 4*B*a^4*c*e^2 - 2*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + \\ & 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 8*(C*a^4*c + A*a^3*c \\ & ^2)*d*e + 8*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 + (2*B*a^3*c*d*e + (2*B*a \\ & *c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*c^3)*e^2)*x^4 + (C \\ & a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(2*B*a^2*c^2*d*e + ( \\ & C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2)*x^2)*\sqrt{-a*c}* \\ & \log((c*x^2 - 2*\sqrt{-a*c}*x - a)/(c*x^2 + a)) + 2*(2*B*a^3*c^2*d*e + (C*a^3*c \\ & ^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x/(a^3*c^5*x^4 + 2*a \\ & ^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 + 2*B*a^4*c*e^2 - (2*B*a^2*c^3 \\ & *d*e + (C*a^2*c^3 + 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 4 \\ & *(C*a^4*c + A*a^3*c^2)*d*e + 4*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 - (2*B \\ & *a^3*c*d*e + (2*B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a \\ & c^3)*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*( \\ & 2*B*a^2*c^2*d*e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2 \\ & )*x^2)*\sqrt{a*c}*\arctan(\sqrt{a*c}*x/a) + (2*B*a^3*c^2*d*e + (C*a^3*c^2 - 5* \\ & A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x/(a^3*c^5*x^4 + 2*a^4*c^4*x \\ & ^2 + a^5*c^3)] \end{aligned}$$

**giac** [A] time = 0.16, size = 254, normalized size = 1.63

$$\frac{(Cac^2 + 3A^2d^2 + 2Bacde + 3C^2d^2 + Aac^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right) + \frac{Cac^2d^3x^3 + 3Ac^3d^2x^3 + 2Bac^2dx^3e - 5Ca^2cx^3e^2 + Aa^2x^3e^2 - 8Ca^2cdx^2e - Ca^2cd^2x + 5Aac^2d^2x - 4Ba^2cx^2e^2 - 2Ba^2cdxe - 2Ba^2cd^2 - 3Ca^3x^2e - Aa^2cxe^2 - 4Ca^3de - 4Aa^2cde - 2Ba^3e^2}{8\sqrt{ac}a^2c^2}}{8(cx^2 + a)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*\arctan \\ & (c*x/\sqrt{a*c})/(\sqrt{a*c}*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2*x \\ & ^3 + 2*B*a*c^2*d*x^3*e - 5*C*a^2*c*x^3*e^2 + A*a*c^2*x^3*e^2 - 8*C*a^2*c*d* \\ & x^2*e - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 4*B*a^2*c*x^2*e^2 - 2*B*a^2*c*d*x \\ & *e - 2*B*a^2*c*d^2 - 3*C*a^3*x*e^2 - A*a^2*c*x*e^2 - 4*C*a^3*d*e - 4*A*a^2* \\ & c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2) \end{aligned}$$

**maple** [A] time = 0.01, size = 283, normalized size = 1.81

$$\frac{Ae^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 3Ad^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right) + Bde \arctan\left(\frac{cx}{\sqrt{ac}}\right) + Cd^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 3C^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right) + \frac{(Be+2Cd)e^2}{2c} + \frac{(Aac^2+3A^2d^2+2Bacde-5a^2C^2+Cac^2d^2)x^3}{8a^2c} - \frac{(Aac^2-5A^2d^2+2Bacde+3a^2C^2+Cac^2d^2)x}{8a^2c} - \frac{2Acde+Ba^2e^2+Bc^2d^2+2Cade}{4c^2}}{8\sqrt{ac}ac} + \frac{3Ad^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2} + \frac{Bde \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}ac} + \frac{Cd^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}ac} + \frac{3C^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}c^2} + \frac{(Be+2Cd)e^2}{2c} + \frac{(Aac^2+3A^2d^2+2Bacde-5a^2C^2+Cac^2d^2)x^3}{8a^2c} - \frac{(Aac^2-5A^2d^2+2Bacde+3a^2C^2+Cac^2d^2)x}{8a^2c} - \frac{2Acde+Ba^2e^2+Bc^2d^2+2Cade}{4c^2} (cx^2 + a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x)`

[Out]  $(1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*a*c*d^2)/a/c^2*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^2+1/8/a/c/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*A*e^2+3/8/a^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*A*d^2+1/4/a/c/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*B*d*e+3/8/c^2/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*e^2+1/8/a/c/(a*c)^(1/2)*\arctan(1/(a*c)^(1/2)*c*x)*C*d^2$

**maxima** [A] time = 0.99, size = 253, normalized size = 1.62

$$\frac{2Ba^2cd^2 + 2Ba^3e^2 - (2Bacde + (Ca^2 + 3Ac^3)d^2 - (5Ca^2c - Aac^2)c^2)x^3 + 4(Ca^3 + Aa^2c)de + 4(2Ca^2cde + Ba^2cc^2)x^2 + (2Ba^2cde + (Ca^2c - 5Aac^2)d^2 + (3Ca^3 + Aa^2c)c^2)x}{8(a^2c^4x^4 + 2a^3c^3x^2 + a^4c^2)} + \frac{(2Bacde + (Ca^2 + 3Ac^3)d^2 + (3Ca^2 + Aac)c^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")`

[Out]  $-1/8*(2*B*a^2*c*d^2 + 2*B*a^3*e^2 - (2*B*a*c^2*d*e + (C*a*c^2 + 3*A*c^3)*d^2 - (5*C*a^2*c - A*a*c^2)*e^2)*x^3 + 4*(C*a^3 + A*a^2*c)*d*e + 4*(2*C*a^2*c*d*e + B*a^2*c*e^2)*x^2 + (2*B*a^2*c*d*e + (C*a^2*c - 5*A*a*c^2)*d^2 + (3*C*a^3 + A*a^2*c)*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(2*B*a*c*d*e + (C*a*c + 3*A*c^2)*d^2 + (3*C*a^2 + A*a*c)*e^2)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a^2*c^2)$

**mupad** [B] time = 3.96, size = 230, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) \left(3Ca^2e^2 + Cacd^2 + 2Bacde + Aace^2 + 3Aa^2d^2\right)}{8a^{5/2}c^{5/2}} - \frac{Ba^2+Bc^2+2Acde+2Cade}{4c^2} + \frac{x^2(Bc^2+2Cde)}{2c} + \frac{x(3Ca^2d^2+Cacd^2+2Bacde+Aac^2-5Aa^2d^2)}{8a^2} - \frac{x^3(-5Ca^2d^2+Cacd^2+2Bacde+Aac^2+3Aa^2d^2)}{8a^2c}}{a^2 + 2acx^2 + c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x)`

[Out]  $(\operatorname{atan}\left(\frac{c^{1/2}x}{a^{1/2}}\right)*(3*A*c^2*d^2 + 3*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^{5/2}*c^{5/2}) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 2*C*a*d*e)/(4*c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (x*(3*C*a^2*e^2 - 5*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a*c^2) - (x^3*(3*A*c^2*d^2 - 5*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^2*c))/(a^2 + c^2*x^4 + 2*a*c*x^2)$

**sympy** [B] time = 141.18, size = 391, normalized size = 2.51

$$\frac{\sqrt{\frac{c}{2a}} \left( Aa^2 + 3Aa^2d^2 + 2Bacde + 3Ca^2d^2 + Cacd^2 \right) \log\left(-\sqrt{\frac{c}{2a}} \sqrt{\frac{c}{2a}} + x\right) + \sqrt{\frac{c}{2a}} \left( Aa^2 + 3Aa^2d^2 + 2Bacde + 3Ca^2d^2 + Cacd^2 \right) \log\left(\sqrt{\frac{c}{2a}} \sqrt{\frac{c}{2a}} + x\right) - 4Aa^2d^2 - 2Ba^2d^2 - 2Ba^2cd^2 - 4Ca^2de + x^3 \left( Aa^2d^2 + 3Aa^2d^2 + 2Bacde - 5Ca^2d^2 + Cacd^2 \right) + x^2 \left( -4Ba^2d^2 - 8Ca^2de \right) + x \left( -Aa^2e^2 + 5Aa^2d^2 - 2Ba^2cd^2 - 3Ca^2d^2 - Cacd^2 \right)}{8a^2c^2 + 16a^2c^2x^2 + 8a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)`

```
[Out] -sqrt(-1/(a**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*
e**2 + C*a*c*d**2)*log(-a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + sqrt(-1/(a
**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*e**2 + C*a*
c*d**2)*log(a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + (-4*A*a**2*c*d*e - 2*B
*a**3*e**2 - 2*B*a**2*c*d**2 - 4*C*a**3*d*e + x**3*(A*a*c**2*e**2 + 3*A*c**
3*d**2 + 2*B*a*c**2*d*e - 5*C*a**2*c*e**2 + C*a*c**2*d**2) + x**2*(-4*B*a**
2*c*e**2 - 8*C*a**2*c*d*e) + x*(-A*a**2*c*e**2 + 5*A*a*c**2*d**2 - 2*B*a**2
*c*d*e - 3*C*a**3*e**2 - C*a**2*c*d**2))/(8*a**4*c**2 + 16*a**3*c**3*x**2 +
8*a**2*c**4*x**4)
```

$$3.59 \quad \int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3} dx$$

**Optimal.** Leaf size=130

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + 3Acd)}{8a^{5/2}c^{3/2}} - \frac{2ae(aC + Ac) - cx(aBe + aCd + 3Acd)}{8a^2c^2(a + cx^2)} - \frac{(d + ex)(aB - x(Ac - aC))}{4ac(a + cx^2)^2}$$

**Rubi [A]** time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {1645, 639, 205}

$$-\frac{2ae(aC + Ac) - cx(aBe + aCd + 3Acd)}{8a^2c^2(a + cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + 3Acd)}{8a^{5/2}c^{3/2}} - \frac{(d + ex)(aB - x(Ac - aC))}{4ac(a + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3,x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x))/(4\*a\*c\*(a + c\*x^2)^2) - (2\*a\*(A\*c + a\*C)\*e - c\*(3\*A\*c\*d + a\*C\*d + a\*B\*e)\*x)/(8\*a^2\*c^2\*(a + c\*x^2)) + ((3\*A\*c\*d + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(8\*a^(5/2)\*c^(3/2))

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 639

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((a\*e - c\*d\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] + Dist[(d\*(2\*p + 3))/(2\*a\*(p + 1)), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

### Rule 1645

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_))^(m\_.)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 1]}, Simp[((d + e\*x)^m\*(a + c\*x^2)^(p + 1)\*(a\*g - c\*f\*x))/(2\*a\*c\*(p + 1)), x] + Dist[1/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1)\*ExpandToSum[2\*a\*c\*(p + 1)\*(d + e\*x)\*Q - a\*e\*g\*m + c\*d\*f\*(2\*p + 3) + c\*e

\*f\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3} dx &= \frac{(aB - (Ac - aC)x)(d+ex)}{4ac(a+cx^2)^2} - \frac{\int \frac{-3Ac d - a(Cd+Be) - 2(Ac+aC)ex}{(a+cx^2)^2} dx}{4ac} \\ &= \frac{(aB - (Ac - aC)x)(d+ex)}{4ac(a+cx^2)^2} - \frac{2a(Ac+aC)e - c(3Ac d + aC d + aBe)x}{8a^2c^2(a+cx^2)} + \frac{(3A}{ \\ &= \frac{(aB - (Ac - aC)x)(d+ex)}{4ac(a+cx^2)^2} - \frac{2a(Ac+aC)e - c(3Ac d + aC d + aBe)x}{8a^2c^2(a+cx^2)} + \frac{(3A}{ \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 137, normalized size = 1.05

$$\frac{\sqrt{a}(-4a^2Ce+acx(Be+Cd)+3Ac^2dx)}{a+cx^2} + \frac{2a^{3/2}(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx)}{(a+cx^2)^2} + \sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+3Ac d)}{8a^{5/2}c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

[Out] ((Sqrt[a]\*(-4\*a^2\*C\*e + 3\*A\*c^2\*d\*x + a\*c\*(C\*d + B\*e)\*x))/(a + c\*x^2) + (2\*a^(3/2)\*(a^2\*C\*e + A\*c^2\*d\*x - a\*c\*(A\*e + C\*d\*x + B\*(d + e\*x)))/(a + c\*x^2)^2 + Sqrt[c]\*(3\*A\*c\*d + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(8\*a^(5/2)\*c^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3, x]

**fricas** [A] time = 0.90, size = 470, normalized size = 3.62

$$\frac{8C^2a^2 + 4Ba^2d - 2(Ba^2c + (Ca^2 + 3Aa^2)d)^2 + (Ba^2c + (Ca^2 + 3Aa^2)d)^2 + 2(Ba^2c + (Ca^2 + 3Aa^2)d)^2 + (Ca^2 + 3Aa^2)d\sqrt{-ac} \log\left(\frac{cx^2 + a}{2cx^2 + a}\right) - 4(Ca^2 + Aa^2)c + 2(Ba^2c + (Ca^2 + 3Aa^2)d)^2 - 4C^2a^2 + 2Ba^2d - (Ba^2c + (Ca^2 + 3Aa^2)d)^2 - (Ba^2c + (Ca^2 + 3Aa^2)d)^2 + 2(Ba^2c + (Ca^2 + 3Aa^2)d)^2 + (Ca^2 + 3Aa^2)d\sqrt{ac} \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 2(Ca^2 + Aa^2)c + (Ba^2c + (Ca^2 + 3Aa^2)d)^2}{16(Ba^2c + 2Aa^2d + A^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/16\*(8\*C\*a^3\*c\*e\*x^2 + 4\*B\*a^3\*c\*d - 2\*(B\*a^2\*c^2\*e + (C\*a^2\*c^2 + 3\*A\*a\*c^3)\*d)\*x^3 + (B\*a^3\*e + (B\*a\*c^2\*e + (C\*a\*c^2 + 3\*A\*c^3)\*d)\*x^4 + 2\*(B\*a^2\*c\*e + (C\*a^2\*c + 3\*A\*a\*c^2)\*d)\*x^2 + (C\*a^3 + 3\*A\*a^2\*c)\*d)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 4\*(C\*a^4 + A\*a^3\*c)\*e + 2\*(B\*a^3\*c\*e + (C\*a^3\*c - 5\*A\*a^2\*c^2)\*d)\*x)/(a^3\*c^4\*x^4 + 2\*a^4\*c^3\*x^2 + a^5\*c^2), -1/8\*(4\*C\*a^3\*c\*e\*x^2 + 2\*B\*a^3\*c\*d - (B\*a^2\*c^2\*e + (C\*a^2\*c^2 + 3\*A\*a\*c^3)\*d)\*x^3 - (B\*a^3\*e + (B\*a\*c^2\*e + (C\*a\*c^2 + 3\*A\*c^3)\*d)\*x^4 + 2\*(B\*a^2\*c\*e + (C\*a^2\*c + 3\*A\*a\*c^2)\*d)\*x^2 + (C\*a^3 + 3\*A\*a^2\*c)\*d)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + 2\*(C\*a^4 + A\*a^3\*c)\*e + (B\*a^3\*c\*e + (C\*a^3\*c - 5\*A\*a^2\*c^2)\*d)\*x)/(a^3\*c^4\*x^4 + 2\*a^4\*c^3\*x^2 + a^5\*c^2)]

**giac** [A] time = 0.19, size = 152, normalized size = 1.17

$$\frac{(Cad + 3Acd + Bae) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c} + \frac{Cac^2dx^3 + 3Ac^3dx^3 + Bac^2x^3e - 4Ca^2cx^2e - Ca^2cdx + 5Aac^2dx - Ba^2cxe - 2Ba^2cd - 2Ca^3e - 2Aa^2ce}{8(cx^2 + a)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="giac")

[Out] 1/8\*(C\*a\*d + 3\*A\*c\*d + B\*a\*e)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a^2\*c) + 1/8\*(C\*a\*c^2\*d\*x^3 + 3\*A\*c^3\*d\*x^3 + B\*a\*c^2\*x^3\*e - 4\*C\*a^2\*c\*x^2\*e - C\*a^2\*c\*d\*x + 5\*A\*a\*c^2\*d\*x - B\*a^2\*c\*x\*e - 2\*B\*a^2\*c\*d - 2\*C\*a^3\*e - 2\*A\*a^2\*c\*e)/(c\*x^2 + a)^2\*a^2\*c^2)

**maple** [A] time = 0.01, size = 157, normalized size = 1.21

$$\frac{3Ad \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2} + \frac{Be \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}ac} + \frac{Cd \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}ac} + \frac{-\frac{Ce x^2}{2c} + \frac{(3Acd+Bae+Cad)x^3}{8a^2} + \frac{(5Acd-Bae-Cad)x}{8ac} - \frac{Ace+Bcd+aCe}{4c^2}}{(cx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x)

[Out] (1/8\*(3\*A\*c\*d+B\*a\*e+C\*a\*d)/a^2\*x^3-1/2\*C/c\*e\*x^2+1/8\*(5\*A\*c\*d-B\*a\*e-C\*a\*d)/a/c\*x-1/4\*(A\*c\*e+B\*c\*d+C\*a\*e)/c^2)/(c\*x^2+a)^2+3/8/a^2/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*d+1/8/a/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*B\*e+1/8/a/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C\*d



**maxima [A]** time = 0.98, size = 160, normalized size = 1.23

$$\frac{4Ca^2cex^2 + 2Ba^2cd - (Bac^2e + (Cac^2 + 3Ac^3)d)x^3 + 2(Ca^3 + Aa^2c)e + (Ba^2ce + (Ca^2c - 5Aac^2)d)x}{8(a^2c^4x^4 + 2a^3c^3x^2 + a^4c^2)} + \frac{(Bae + (Ca + 3Ac)d) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="maxima")

[Out] 
$$-1/8*(4*C*a^2*c*e*x^2 + 2*B*a^2*c*d - (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^3 + 2*(C*a^3 + A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - 5*A*a*c^2)*d)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(B*a*e + (C*a + 3*A*c)*d)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a^2*c)$$

**mupad [B]** time = 0.15, size = 128, normalized size = 0.98

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3Acd + Bae + Cad)}{8a^{5/2}c^{3/2}} - \frac{\frac{Ace+Bcd+Ca e}{4c^2} - \frac{x^3(3Acd+Bae+Cad)}{8a^2} + \frac{Cex^2}{2c} + \frac{x(Bae-5Acd+Cad)}{8ac}}{a^2 + 2acx^2 + c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^3,x)

[Out] 
$$\left(\operatorname{atan}\left(\frac{c^{1/2}x}{a^{1/2}}\right)*(3A*c*d + B*a*e + C*a*d)\right)/(8*a^{5/2}*c^{3/2}) - \left(\frac{(A*c*e + B*c*d + C*a*e)/(4*c^2) - (x^3*(3*A*c*d + B*a*e + C*a*d))/(8*a^2) + (C*e*x^2)/(2*c) + (x*(B*a*e - 5*A*c*d + C*a*d))/(8*a*c)}{a^2 + c^2*x^2} + 2*a*c*x^2\right)$$

**sympy [A]** time = 32.42, size = 240, normalized size = 1.85

$$-\frac{\sqrt{-\frac{1}{a^2c^3}}(3Acd + Bae + Cad) \log\left(-a^2c\sqrt{-\frac{1}{a^2c^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^2c^3}}(3Acd + Bae + Cad) \log\left(a^2c\sqrt{-\frac{1}{a^2c^3}} + x\right)}{16} + \frac{-2Aa^2ce - 2Ba^2cd - 2Ca^3e - 4Ca^2cex^2 + x^3(3A^3d + Bac^2e + Cac^2d) + x(5Aac^2d - Ba^2ce - Ca^2cd)}{8a^4c^2 + 16a^3c^3x^2 + 8a^2c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*3,x)

[Out] 
$$-\sqrt{-1/(a**5*c**3)}*(3*A*c*d + B*a*e + C*a*d)*\log(-a**3*c*\sqrt{-1/(a**5*c**3)} + x)/16 + \sqrt{-1/(a**5*c**3)}*(3*A*c*d + B*a*e + C*a*d)*\log(a**3*c*\sqrt{-1/(a**5*c**3)} + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e - 4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*A*a**c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**2*c**4*x**4)$$

$$3.60 \quad \int \frac{A+Bx+Cx^2}{(a+cx^2)^3} dx$$

Optimal. Leaf size=98

$$\frac{(aC + 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}} + \frac{x(aC + 3Ac)}{8a^2c(a + cx^2)} - \frac{aB - x(Ac - aC)}{4ac(a + cx^2)^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1814, 12, 199, 205}

$$\frac{(aC + 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}} + \frac{x(aC + 3Ac)}{8a^2c(a + cx^2)} - \frac{aB - x(Ac - aC)}{4ac(a + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2)^3,x]

[Out] -(a\*B - (A\*c - a\*C)\*x)/(4\*a\*c\*(a + c\*x^2)^2) + ((3\*A\*c + a\*C)\*x)/(8\*a^2\*c\*(a + c\*x^2)) + ((3\*A\*c + a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(8\*a^(5/2)\*c^(3/2))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 1814

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(a + cx^2)^3} dx &= -\frac{aB - (Ac - aC)x}{4ac(a + cx^2)^2} - \frac{\int \frac{-3A - \frac{aC}{c}}{(a+cx^2)^2} dx}{4a} \\ &= -\frac{aB - (Ac - aC)x}{4ac(a + cx^2)^2} + \frac{(3Ac + aC) \int \frac{1}{(a+cx^2)^2} dx}{4ac} \\ &= -\frac{aB - (Ac - aC)x}{4ac(a + cx^2)^2} + \frac{(3Ac + aC)x}{8a^2c(a + cx^2)} + \frac{(3Ac + aC) \int \frac{1}{a+cx^2} dx}{8a^2c} \\ &= -\frac{aB - (Ac - aC)x}{4ac(a + cx^2)^2} + \frac{(3Ac + aC)x}{8a^2c(a + cx^2)} + \frac{(3Ac + aC) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 90, normalized size = 0.92

$$\frac{(aC + 3Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}} + \frac{-a^2(2B + Cx) + acx(5A + Cx^2) + 3Ac^2x^3}{8a^2c(a + cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2)^3, x]

[Out] (3\*A\*c^2\*x^3 - a^2\*(2\*B + C\*x) + a\*c\*x\*(5\*A + C\*x^2))/(8\*a^2\*c\*(a + c\*x^2)^2) + ((3\*A\*c + a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(8\*a^(5/2)\*c^(3/2))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^3, x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^3, x]

**fricas** [A] time = 1.04, size = 314, normalized size = 3.20

$$\frac{4 B a^2 c - 2 (C a^2 c + 3 A a c^2) x^3 + ((C a c^2 + 3 A c^3) x^4 + C a^3 + 3 A a^2 c + 2 (C a^2 c + 3 A a c^2) x^2) \sqrt{-a c} \log\left(\frac{c x^2 - 2 \sqrt{-a c} x + a}{c x^2 + a}\right) + 2 (C a^2 c - 5 A a^2 c^2) x}{16 (a^3 c^4 x^4 + 2 a^4 c^3 x^2 + a^5 c^2)} - \frac{2 B a^2 c - (C a^2 c + 3 A a c^2) x^3 - ((C a c^2 + 3 A c^3) x^4 + C a^3 + 3 A a^2 c + 2 (C a^2 c + 3 A a c^2) x^2) \sqrt{a c} \arctan\left(\frac{\sqrt{a c} x}{a}\right) + (C a^2 c - 5 A a^2 c^2) x}{8 (a^3 c^4 x^4 + 2 a^4 c^3 x^2 + a^5 c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/16\*(4\*B\*a^3\*c - 2\*(C\*a^2\*c^2 + 3\*A\*a\*c^3)\*x^3 + ((C\*a\*c^2 + 3\*A\*c^3)\*x^4 + C\*a^3 + 3\*A\*a^2\*c + 2\*(C\*a^2\*c + 3\*A\*a\*c^2)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 2\*(C\*a^3\*c - 5\*A\*a^2\*c^2)\*x)/(a^3\*c^4\*x^4 + 2\*a^4\*c^3\*x^2 + a^5\*c^2), -1/8\*(2\*B\*a^3\*c - (C\*a^2\*c^2 + 3\*A\*a\*c^3)\*x^3 - ((C\*a\*c^2 + 3\*A\*c^3)\*x^4 + C\*a^3 + 3\*A\*a^2\*c + 2\*(C\*a^2\*c + 3\*A\*a\*c^2)\*x^2)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + (C\*a^3\*c - 5\*A\*a^2\*c^2)\*x)/(a^3\*c^4\*x^4 + 2\*a^4\*c^3\*x^2 + a^5\*c^2)]

**giac** [A] time = 0.16, size = 84, normalized size = 0.86

$$\frac{(C a + 3 A c) \arctan\left(\frac{c x}{\sqrt{a c}}\right)}{8 \sqrt{a c} a^2 c} + \frac{C a c x^3 + 3 A c^2 x^3 - C a^2 x + 5 A a c x - 2 B a^2}{8 (c x^2 + a)^2 a^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="giac")

[Out] 1/8\*(C\*a + 3\*A\*c)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a^2\*c) + 1/8\*(C\*a\*c\*x^3 + 3\*A\*c^2\*x^3 - C\*a^2\*x + 5\*A\*a\*c\*x - 2\*B\*a^2)/((c\*x^2 + a)^2\*a^2\*c)

**maple** [A] time = 0.01, size = 96, normalized size = 0.98

$$\frac{3 A \arctan\left(\frac{c x}{\sqrt{a c}}\right)}{8 \sqrt{a c} a^2} + \frac{C \arctan\left(\frac{c x}{\sqrt{a c}}\right)}{8 \sqrt{a c} a c} + \frac{\frac{(3 A c + a C) x^3}{8 a^2} - \frac{B}{4 c} + \frac{(5 A c - a C) x}{8 a c}}{(c x^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(c\*x^2+a)^3,x)

[Out] (1/8\*(3\*A\*c+C\*a)/a^2\*x^3+1/8\*(5\*A\*c-C\*a)/a/c\*x-1/4\*B/c)/(c\*x^2+a)^2+3/8/a^2/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A+1/8/a/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C

**maxima [A]** time = 0.97, size = 98, normalized size = 1.00

$$\frac{(Cac + 3Ac^2)x^3 - 2Ba^2 - (Ca^2 - 5Aac)x}{8(a^2c^3x^4 + 2a^3c^2x^2 + a^4c)} + \frac{(Ca + 3Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^3,x, algorithm="maxima")

[Out] 1/8\*((C\*a\*c + 3\*A\*c^2)\*x^3 - 2\*B\*a^2 - (C\*a^2 - 5\*A\*a\*c)\*x)/(a^2\*c^3\*x^4 + 2\*a^3\*c^2\*x^2 + a^4\*c) + 1/8\*(C\*a + 3\*A\*c)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a^2\*c)

**mupad [B]** time = 3.84, size = 88, normalized size = 0.90

$$\frac{\frac{x^3(3Ac+Ca)}{8a^2} - \frac{B}{4c} + \frac{x(5Ac-Ca)}{8ac}}{a^2 + 2acx^2 + c^2x^4} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3Ac+Ca)}{8a^{5/2}c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/(a + c\*x^2)^3,x)

[Out] ((x^3\*(3\*A\*c + C\*a))/(8\*a^2) - B/(4\*c) + (x\*(5\*A\*c - C\*a))/(8\*a\*c))/(a^2 + c^2\*x^4 + 2\*a\*c\*x^2) + (atan((c^(1/2)\*x)/a^(1/2))\*(3\*A\*c + C\*a))/(8\*a^(5/2)\*c^(3/2))

**sympy [A]** time = 1.23, size = 156, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}(3Ac+Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}(3Ac+Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{-2Ba^2+x^3(3Ac^2+Ca)+x(5Aac-Ca^2)}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*3,x)

[Out] -sqrt(-1/(a\*\*5\*c\*\*3))\*(3\*A\*c + C\*a)\*log(-a\*\*3\*c\*sqrt(-1/(a\*\*5\*c\*\*3)) + x)/16 + sqrt(-1/(a\*\*5\*c\*\*3))\*(3\*A\*c + C\*a)\*log(a\*\*3\*c\*sqrt(-1/(a\*\*5\*c\*\*3)) + x)/16 + (-2\*B\*a\*\*2 + x\*\*3\*(3\*A\*c\*\*2 + C\*a\*c) + x\*(5\*A\*a\*c - C\*a\*\*2))/(8\*a\*\*4\*c + 16\*a\*\*3\*c\*\*2\*x\*\*2 + 8\*a\*\*2\*c\*\*3\*x\*\*4)

$$3.61 \quad \int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^3} dx$$

**Optimal.** Leaf size=353

$$\frac{4a^2e(Ae^2 - Bde + Cd^2) + x(Acd(7ae^2 + 3cd^2) + a(cd^2 - 3ae^2)(Cd - Be))}{8a^2(a + cx^2)(ae^2 + cd^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(15a^2e^4 + 10acd^2))}{8}$$

**Rubi [A]** time = 0.73, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {1647, 823, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(15a^2e^4 + 10acd^2 + 3c^2d^4) + a(-3a^2e^4 + 6acd^2e^2 + c^2d^4)(Cd - Be))}{8a^{5/2}\sqrt{c}(a^2 + cd^2)^3} + \frac{4a^2e(Ae^2 - Bde + Cd^2) + x(Acd(7ae^2 + 3cd^2) + a(cd^2 - 3ae^2)(Cd - Be))}{8a^2(a + cx^2)(ae^2 + cd^2)^2} - \frac{a(aCe - Ace + Bcd) - cx(Be - aCd + Acd)}{4ac(a + cx^2)^2(ae^2 + cd^2)} - \frac{e^3 \log(a + cx^2)(Ae^2 - Bde + Cd^2)}{2(ae^2 + cd^2)^3} + \frac{e^3 \log(d + cx)(Ae^2 - Bde + Cd^2)}{(ae^2 + cd^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^3), x]

[Out] -(a\*(B\*c\*d - A\*c\*e + a\*C\*e) - c\*(A\*c\*d - a\*C\*d + a\*B\*e)\*x)/(4\*a\*c\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)^2) + (4\*a^2\*e\*(C\*d^2 - B\*d\*e + A\*e^2) + (a\*(C\*d - B\*e)\*(c\*d^2 - 3\*a\*e^2) + A\*c\*d\*(3\*c\*d^2 + 7\*a\*e^2))\*x)/(8\*a^2\*(c\*d^2 + a\*e^2)^2\*(a + c\*x^2)) + ((a\*(C\*d - B\*e)\*(c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4) + A\*c\*d\*(3\*c^2\*d^4 + 10\*a\*c\*d^2\*e^2 + 15\*a^2\*e^4))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(8\*a^(5/2)\*Sqrt[c]\*(c\*d^2 + a\*e^2)^3) + (e^3\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^3 - (e^3\*(C\*d^2 - B\*d\*e + A\*e^2)\*Log[a + c\*x^2])/(2\*(c\*d^2 + a\*e^2)^3)

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

### Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

### Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)^3} dx &= \frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} - \int \frac{\frac{c(ad(Cd - Be) + A(3cd^2 + 4ae^2))}{cd^2 + ae^2} - \frac{3ce(Acd - aCd + aBe)x}{cd^2 + ae^2}}{(d + ex)(a + cx^2)^2} dx \\
&= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + (a(Cd - Bde + Ae^2))}{8a^2(cd^2 + ae^2)} \\
&= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + (a(Cd - Bde + Ae^2))}{8a^2(cd^2 + ae^2)} \\
&= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + (a(Cd - Bde + Ae^2))}{8a^2(cd^2 + ae^2)} \\
&= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + (a(Cd - Bde + Ae^2))}{8a^2(cd^2 + ae^2)} \\
&= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + (a(Cd - Bde + Ae^2))}{8a^2(cd^2 + ae^2)} \\
&= -\frac{a(Bcd - Ace + aCe) - c(Acd - aCd + aBe)x}{4ac(cd^2 + ae^2)(a + cx^2)^2} + \frac{4a^2e(Cd^2 - Bde + Ae^2) + (a(Cd - Bde + Ae^2))}{8a^2(cd^2 + ae^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.42, size = 321, normalized size = 0.91

$$\frac{\frac{2(a^2 + c^2)^2 (c^2(-Cx + ac(Ac - Bd + 3Bc - Cd) + A^2d))}{a(a + cx^2)^2} + \frac{(a^2 + c^2)(a^2c(4Ac - 4Bd + 3Bc) + Cd(M - 3c)) + acd(c(7Ac - Bd) + Cc^2 + 3A^2d^2)}{a^2(a + cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right) \left( Acd(15c^2d^2 + 10acd^2 + 3a^2d^4) + (-3c^2d^2 + 6acd^2 + 2a^2d^4)(Cd - Be) \right)}{8(a^2 + c^2)^3} - 4e^3 \log(a + cx^2)(c(Ac - Bd) + Cd^2) + 8e^3 \log(d + ex)(c(Ac - Bd) + Cd^2)}{8(a^2 + c^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^3), x]

[Out] ((2\*(c\*d^2 + a\*e^2)^2\*(-(a^2\*C\*e) + A\*c^2\*d\*x + a\*c\*(-(B\*d) + A\*e - C\*d\*x + B\*e\*x)))/(a\*c\*(a + c\*x^2)^2) + ((c\*d^2 + a\*e^2)\*(3\*A\*c^2\*d^3\*x + a\*c\*d\*(C\*d^2 + e\*(-(B\*d) + 7\*A\*e))\*x + a^2\*e\*(C\*d\*(4\*d - 3\*e\*x) + e\*(-4\*B\*d + 4\*A\*e + 3\*B\*e\*x))))/(a^2\*(a + c\*x^2)) + ((a\*(C\*d - B\*e)\*(c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4) + A\*c\*d\*(3\*c^2\*d^4 + 10\*a\*c\*d^2\*e^2 + 15\*a^2\*e^4))\*ArcTan[ $\sqrt{\frac{c*x}{a}}$ ]/(a^(5/2)\* $\sqrt{c}$ ) + 8\*e^3\*(C\*d^2 + e\*(-(B\*d) + A\*e))\*Log[d + e\*x] - 4\*e^3\*(C\*d^2 + e\*(-(B\*d) + A\*e))\*Log[a + c\*x^2]/(8\*(c\*d^2 + a\*e^2)^3)



**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)(a + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^3),x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)\*(a + c\*x^2)^3), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 0.18, size = 715, normalized size = 2.03

(C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a)^3

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)/(c\*x^2+a)^3,x, algorithm="giac")

[Out] 
$$-1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*\log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*\log(\text{abs}(x*e + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C*a*c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*d^3*e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*a \text{rctan}(c*x/\text{sqrt}(a*c))/(a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*\text{sqrt}(a*c) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A*a^2*c^3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e^4 + 2*C*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^4*e - 2*C*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C*a^3*c^2*d*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d^4*e - B*a^2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c^2*d*e^4 + A*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3*d^4*e + 6*C*a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 5*C*a^4*c*d*e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)^3*(c*x^2 + a)^2*a^2*c)$$

**maple [B]** time = 0.02, size = 1598, normalized size = 4.53



```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^3 - B*d*e^4 + A*e^5)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/8*(B*a*c^2*d^4*e + 6*B*a^2*c*d^2*e^3 - 3*B*a^3*e^5 - (C*a*c^2 + 3*A*c^3)*d^5 - 2*(3*C*a^2*c + 5*A*a*c^2)*d^3*e^2 + 3*(C*a^3 - 5*A*a^2*c)*d*e^4)*arctan(c*x/sqrt(a*c))/(a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sqrt(a*c) - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 - 2*(C*a^3*c + A*a^2*c^2)*d^2*e + 2*(C*a^4 - 3*A*a^3*c)*e^3 + (B*a*c^3*d^2*e - 3*B*a^2*c^2*e^3 - (C*a*c^3 + 3*A*c^4)*d^3 + (3*C*a^2*c^2 - 7*A*a*c^3)*d*e^2)*x^3 - 4*(C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2 - (B*a^2*c^2*d^2*e + 5*B*a^3*c*e^3 - (C*a^2*c^2 - 5*A*a*c^3)*d^3 - (5*C*a^3*c - 9*A*a^2*c^2)*d*e^2)*x)/(a^4*c^3*d^4 + 2*a^5*c^2*d^2*e^2 + a^6*c*e^4 + (a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4*c^3*e^4)*x^4 + 2*(a^3*c^4*d^4 + 2*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*x^2)
```

**mupad [B]** time = 9.90, size = 2392, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)),x)
```

```
[Out] ((x^2*(A*c*e^3 - B*c*d*e^2 + C*c*d^2*e))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (B*c^2*d^3 + C*a^2*e^3 - 3*A*a*c*e^3 - A*c^2*d^2*e + 3*B*a*c*d*e^2 - C*a*c*d^2*e)/(4*c*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3 + 5*B*a^2*e^3 - C*a*c*d^3 - 5*C*a^2*d*e^2 + 9*A*a*c*d*e^2 + B*a*c*d^2*e))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2*c*e^3 + C*a*c^2*d^3 + 7*A*a*c^2*d*e^2 - B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2))/(8*a^2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2) - (log(3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5*c)^(1/2) - 24*A*a^6*c*e^7 + 3*B*a^6*c*e^7*x + 6*A*a^3*c^4*d^6*e + 2*C*a^4*c^3*d^6*e - 30*C*a^6*c*d^2*e^5 - 3*A*a^2*c^5*d^7*x - C*a^3*c^4*d^7*x + C*a*c^3*d^7*(-a^5*c)^(1/2) + 3*C*a^4*d*e^6*(-a^5*c)^(1/2) + 20*A*a^4*c^3*d^4*e^3 + 54*A*a^5*c^2*d^2*e^5 - 2*B*a^4*c^3*d^5*e^2 - 36*B*a^5*c^2*d^3*e^4 + 36*C*a^5*c^2*d^4*e^3 + 30*B*a^6*c*d*e^6 - 7*A*a^3*c^4*d^5*e^2*x - 5*A*a^4*c^3*d^3*e^4*x + 5*B*a^4*c^3*d^4*e^3*x - 57*B*a^5*c^2*d^2*e^5*x - 5*C*a^4*c^3*d^5*e^2*x + 57*C*a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*a^3*c*d^2*e^5*(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))*(c*(a^2*((3*C*d^3*e^2*(-a^5*c)^(1/2) + 3*B*a^2*c^2*d^2*e^5*(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))
```

$$\begin{aligned}
& c^{(1/2)}/8 - (3*B*d^2*e^3*(-a^5*c)^{(1/2)})/8 + (15*A*d*e^4*(-a^5*c)^{(1/2)})/ \\
& 16) + a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^4)/2) + a^3*((3*B*e^5*(-a^5* \\
& c)^{(1/2)})/16 - (3*C*d*e^4*(-a^5*c)^{(1/2)})/16) + a*c^2*((C*d^5*(-a^5*c)^{(1/2)} \\
& )/16 + (5*A*d^3*e^2*(-a^5*c)^{(1/2)})/8 - (B*d^4*e*(-a^5*c)^{(1/2)})/16) + (3* \\
& A*c^3*d^5*(-a^5*c)^{(1/2)})/16)/(a^8*c*e^6 + a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 \\
& + 3*a^7*c^2*d^2*e^4) + (\log(3*A*c^4*d^7*(-a^5*c)^{(1/2)} - 3*B*a^4*e^7*(-a^5 \\
& *c)^{(1/2)} + 24*A*a^6*c*e^7 - 3*B*a^6*c*e^7*x - 6*A*a^3*c^4*d^6*e - 2*C*a^4* \\
& c^3*d^6*e + 30*C*a^6*c*d^2*e^5 + 3*A*a^2*c^5*d^7*x + C*a^3*c^4*d^7*x + C*a \\
& c^3*d^7*(-a^5*c)^{(1/2)} + 3*C*a^4*d*e^6*(-a^5*c)^{(1/2)} - 20*A*a^4*c^3*d^4*e^ \\
& 3 - 54*A*a^5*c^2*d^2*e^5 + 2*B*a^4*c^3*d^5*e^2 + 36*B*a^5*c^2*d^3*e^4 - 36* \\
& C*a^5*c^2*d^4*e^3 - 30*B*a^6*c*d*e^6 + 7*A*a^3*c^4*d^5*e^2*x + 5*A*a^4*c^3* \\
& d^3*e^4*x - 5*B*a^4*c^3*d^4*e^3*x + 57*B*a^5*c^2*d^2*e^5*x + 5*C*a^4*c^3*d^ \\
& 5*e^2*x - 57*C*a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^{(1/2)} + 57*B* \\
& a^3*c*d^2*e^5*(-a^5*c)^{(1/2)} - 57*C*a^3*c*d^3*e^4*(-a^5*c)^{(1/2)} + 3*C*a^6* \\
& c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^{(1/2)} - 5*B*a^2*c^2*d^4*e^3*(-a^5* \\
& c)^{(1/2)} + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^{(1/2)} - 63*A*a^5*c^2*d*e^6*x - B*a^ \\
& 3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^{(1/2)} - B*a*c^3*d^6*e*(-a^5*c)^{(1 \\
& /2)} - 24*A*a^3*c*e^7*x*(-a^5*c)^{(1/2)} + 6*A*c^4*d^6*e*x*(-a^5*c)^{(1/2)} + 54 \\
& *A*a^2*c^2*d^2*e^5*x*(-a^5*c)^{(1/2)} - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^{(1/2)} \\
& + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^{(1/2)} + 30*B*a^3*c*d*e^6*x*(-a^5*c)^{(1/2)} \\
& ) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^{(1/2)} + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^{(1/2)} \\
& - 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^{(1/2)} - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^{(1/2)}) \\
& *(c*(a^2*((3*C*d^3*e^2*(-a^5*c)^{(1/2)})/8 - (3*B*d^2*e^3*(-a^5*c)^{(1/2)})/8 + \\
& (15*A*d*e^4*(-a^5*c)^{(1/2)})/16) - a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^ \\
& 4)/2) + a^3*((3*B*e^5*(-a^5*c)^{(1/2)})/16 - (3*C*d*e^4*(-a^5*c)^{(1/2)})/16) \\
& + a*c^2*((C*d^5*(-a^5*c)^{(1/2)})/16 + (5*A*d^3*e^2*(-a^5*c)^{(1/2)})/8 - (B*d^ \\
& 4*e*(-a^5*c)^{(1/2)})/16) + (3*A*c^3*d^5*(-a^5*c)^{(1/2)})/16)/(a^8*c*e^6 + a^ \\
& 5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4) + (e^3*\log(d + e*x)*(A*e \\
& ^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^3
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)/(c\*x\*\*2+a)\*\*3,x)

[Out] Timed out

$$3.62 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^3} dx$$

**Optimal.** Leaf size=571

$$\frac{4a^2e \left( ae^2(2Cd - Be) - cd(2Cd^2 - e(3Bd - 4Ae)) \right) - x \left( Ac(-7a^2e^4 + 12acd^2e^2 + 3c^2d^4) + a(3a^2Ce^4 - 2acde^2) \right)}{8a^2(a+cx^2)(ae^2+cd^2)^3}$$

**Rubi [A]** time = 1.92, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1647, 1629, 635, 205, 260}

$$\frac{(c^2d^2 - 2cdBe + e^2Ae^2) \sqrt{a+cx^2} - (c^2d^2 + a^2e^2) \sqrt{d+ex} - (c^2d^2 + a^2e^2) \sqrt{a+cx^2} \operatorname{ArcTan}\left[\frac{\sqrt{d+ex}}{\sqrt{a+cx^2}}\right]}{8a^2(a+cx^2)(ae^2+cd^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^3), x]

[Out]  $-\frac{(e^3(Cd^2 - Bde + Ae^2))}{(c^2d^2 + a^2e^2)^3(d + ex)} - \frac{(a(Bcd^2 - 2Acd + 2A^2e))}{(4a^2(c^2d^2 + a^2e^2)^2(a + cx^2)^2)} + \frac{(4a^2e(2c^2Cd^3 - cd^2e(3Bd - 4Ae) - a^2e(2Cd - Be)) + (A^2c(3c^2d^4 + 12a^2cd^2e^2 - 7a^2e^4) + a(3a^2Cde^4 - 2a^2cd^2e(6Cd - 7Be) + c^2d^3(Cd - 2Be)))x)}{(8a^2(c^2d^2 + a^2e^2)^3(a + cx^2))} + \frac{((3A^2c^2d^6 + 5a^2c^2d^4e^2 + 15a^2cd^2e^4 - 5a^3e^6) + a(3a^3Cde^6 + a^2c^2d^3e^2(13Cd - 20Be) - 3a^2cde^4(11Cd - 10Be) + c^2d^5(Cd - 2Be))) \operatorname{ArcTan}\left[\frac{\sqrt{c}x}{\sqrt{a}}\right]}{(8a^{5/2}\sqrt{c}(c^2d^2 + a^2e^2)^4)} + \frac{(e^3(4cCd^3 - cde(5Bd - 6Ae) - a^2e(2Cd - Be)) \operatorname{Log}[d + ex])}{(c^2d^2 + a^2e^2)^4} - \frac{(e^3(4cCd^3 - cde(5Bd - 6Ae) - a^2e(2Cd - Be)) \operatorname{Log}[a + cx^2])}{(2(c^2d^2 + a^2e^2)^4)}$

**Rule 205**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 260**

Int[(x\_)^m/((a\_) + (b\_)\*(x\_)^n), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

**Rule 635**

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}

```
}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^3} dx &= -\frac{a (Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac (cd^2 - ae^2) + a (aCe^2 - cd(Cd - 2Be)))}{4a (cd^2 + ae^2)^2 (a + cx^2)^2} \\
&= -\frac{a (Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac (cd^2 - ae^2) + a (aCe^2 - cd(Cd - 2Be)))}{4a (cd^2 + ae^2)^2 (a + cx^2)^2} \\
&= -\frac{a (Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac (cd^2 - ae^2) + a (aCe^2 - cd(Cd - 2Be)))}{4a (cd^2 + ae^2)^2 (a + cx^2)^2} \\
&= -\frac{e^3 (Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^3 (d + ex)} - \frac{a (Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac (cd^2 - ae^2) + a (aCe^2 - cd(Cd - 2Be)))}{4a (cd^2 + ae^2)^2 (a + cx^2)^2} \\
&= -\frac{e^3 (Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^3 (d + ex)} - \frac{a (Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac (cd^2 - ae^2) + a (aCe^2 - cd(Cd - 2Be)))}{4a (cd^2 + ae^2)^2 (a + cx^2)^2} \\
&= -\frac{e^3 (Cd^2 - Bde + Ae^2)}{(cd^2 + ae^2)^3 (d + ex)} - \frac{a (Bcd^2 - 2Acde + 2aCde - aBe^2) - (Ac (cd^2 - ae^2) + a (aCe^2 - cd(Cd - 2Be)))}{4a (cd^2 + ae^2)^2 (a + cx^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.76, size = 498, normalized size = 0.87

$$\frac{3(a^2 + c^2) \sqrt{(A + Bx + Cx^2) / ((d + ex)^2 (a + cx^2)^3)} + (a^2 + c^2) \sqrt{(A + Bx + Cx^2) / ((d + ex)^2 (a + cx^2)^3)} - 4e^3 \log(e + cx) (a^2(Bc - 2Cd) + cd(6Ac - 5Bd) + 4c^2d^2) + 8e^3 \log(d + ex) (a^2(Bc - 2Cd) + cd(6Ac - 5Bd) + 4c^2d^2) - \frac{3(a^2 + c^2) \sqrt{(A + Bx + Cx^2) / ((d + ex)^2 (a + cx^2)^3)}}{8(a^2 + c^2)}}{8(a^2 + c^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^3), x]

[Out] ((-8\*e^3\*(c\*d^2 + a\*e^2)\*(C\*d^2 + e\*(-(B\*d) + A\*e)))/(d + e\*x) + (2\*(c\*d^2 + a\*e^2)^2\*(A\*c^2\*d^2\*x + a^2\*e\*(-2\*C\*d + B\*e + C\*e\*x) - a\*c\*(C\*d^2\*x + B\*d\*(d - 2\*e\*x) + A\*e\*(-2\*d + e\*x)))/(a\*(a + c\*x^2)^2) + ((c\*d^2 + a\*e^2)\*(3\*A\*c^3\*d^4\*x + a\*c^2\*d^2\*(C\*d^2 + 2\*e\*(-(B\*d) + 6\*A\*e)))\*x + a^3\*e^3\*(-8\*C\*d + 4\*B\*e + 3\*C\*e\*x) + a^2\*c\*e\*(4\*C\*d^2\*(2\*d - 3\*e\*x) + e\*(-2\*B\*d\*(6\*d - 7\*e\*x) + A\*e\*(16\*d - 7\*e\*x))))/(a^2\*(a + c\*x^2)) + ((3\*A\*c\*(c^3\*d^6 + 5\*a\*c^2\*d^4\*e^2 + 15\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6) + a\*(3\*a^3\*C\*e^6 + a\*c^2\*d^3\*e^2\*(13\*C\*d - 20\*B\*e) + c^3\*d^5\*(C\*d - 2\*B\*e) + 3\*a^2\*c\*d\*e^4\*(-11\*C\*d + 10\*B\*e))

)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(a^(5/2)\*Sqrt[c]) + 8\*e^3\*(4\*c\*C\*d^3 + c\*d\*e\*(-5\*B\*d + 6\*A\*e) + a\*e^2\*(-2\*C\*d + B\*e))\*Log[d + e\*x] - 4\*e^3\*(4\*c\*C\*d^3 + c\*d\*e\*(-5\*B\*d + 6\*A\*e) + a\*e^2\*(-2\*C\*d + B\*e))\*Log[a + c\*x^2]/(8\*(c\*d^2 + a\*e^2)^4)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^3),x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^2\*(a + c\*x^2)^3), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] Timed out

**giac [A]** time = 0.24, size = 1107, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a)^3,x, algorithm="giac")

[Out] 1/8\*(C\*a\*c^3\*d^6\*e^2 + 3\*A\*c^4\*d^6\*e^2 - 2\*B\*a\*c^3\*d^5\*e^3 + 13\*C\*a^2\*c^2\*d^4\*e^4 + 15\*A\*a\*c^3\*d^4\*e^4 - 20\*B\*a^2\*c^2\*d^3\*e^5 - 33\*C\*a^3\*c\*d^2\*e^6 + 45\*A\*a^2\*c^2\*d^2\*e^6 + 30\*B\*a^3\*c\*d\*e^7 + 3\*C\*a^4\*e^8 - 15\*A\*a^3\*c\*e^8)\*arctan((c\*d - c\*d^2/(x\*e + d) - a\*e^2/(x\*e + d))\*e^(-1)/sqrt(a\*c))\*e^(-2)/((a^2\*c^4\*d^8 + 4\*a^3\*c^3\*d^6\*e^2 + 6\*a^4\*c^2\*d^4\*e^4 + 4\*a^5\*c\*d^2\*e^6 + a^6\*e^8)\*sqrt(a\*c)) - 1/2\*(4\*C\*c\*d^3\*e^3 - 5\*B\*c\*d^2\*e^4 - 2\*C\*a\*d\*e^5 + 6\*A\*c\*d\*e^5 + B\*a\*e^6)\*log(c - 2\*c\*d/(x\*e + d) + c\*d^2/(x\*e + d)^2 + a\*e^2/(x\*e + d)^2)/(c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 6\*a^2\*c^2\*d^4\*e^4 + 4\*a^3\*c\*d^2\*e^6 + a^4\*e^8) - (C\*d^2\*e^9/(x\*e + d) - B\*d\*e^10/(x\*e + d) + A\*e^11/(x\*e + d))/(c^3\*d^6\*e^6 + 3\*a\*c^2\*d^4\*e^8 + 3\*a^2\*c\*d^2\*e^10 + a^3\*e^12) + 1/8\*(C\*a\*c^4\*d^5\*e + 3\*A\*c^5\*d^5\*e - 2\*B\*a\*c^4\*d^4\*e^2 - 22\*C\*a^2\*c^3\*d^3\*e^3 + 14\*A\*a\*c^4\*d^3\*e^3 + 32\*B\*a^2\*c^3\*d^2\*e^4 + 17\*C\*a^3\*c^2\*d\*e^5 - 29\*A\*a^2\*c^3\*d\*e^5 -



$$\begin{aligned}
& 6*B*a^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*d^5*e^3 \\
& - 77*C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 + 77*C* \\
& a^3*c^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*C*a^4*c*e^ \\
& 8 + 7*A*a^3*c^2*e^8)*e^{(-1)}/(x*e + d) + (3*C*a*c^4*d^7*e^3 + 9*A*c^5*d^7*e^ \\
& 3 - 6*B*a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5 + 140*B*a \\
& ^2*c^3*d^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^ \\
& 2*d^2*e^8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^10)*e^{(-2)}/ \\
& (x*e + d)^2 - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5 - 34*C \\
& *a^2*c^3*d^6*e^6 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 + 20*C*a^3*c^2 \\
& *d^4*e^8 - 60*A*a^2*c^3*d^4*e^8 + 26*B*a^3*c^2*d^3*e^9 + 50*C*a^4*c*d^2*e^1 \\
& 0 - 66*A*a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 - 5*C*a^5*e^12 + 9*A*a^4*c*e^ \\
& 12)*e^{(-3)}/(x*e + d)^3/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(x*e + d) + c*d^2 \\
& /((x*e + d)^2 + a*e^2/(x*e + d))^2)
\end{aligned}$$

**maple [B]** time = 0.03, size = 2159, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a)^3,x)

[Out] 
$$\begin{aligned}
& -3/(a*e^2+c*d^2)^4*c*\ln(c*x^2+a)*d*A*e^5+1/(a*e^2+c*d^2)^4*a*\ln(c*x^2+a)*C* \\
& d*e^5+3/8/(a*e^2+c*d^2)^4*a^2/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*C*e^6-3 \\
& /2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^3*d*e^5+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2* \\
& A*c^3*d^5*e+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*a^3*C*e^6*x-1/8/(a*e^2+c*d^2)^4 \\
& /(c*x^2+a)^2*C*c^3*d^6*x+5/2/(a*e^2+c*d^2)^4*c*\ln(c*x^2+a)*d^2*e^4*B-2/(a*e \\
& ^2+c*d^2)^4*c*\ln(c*x^2+a)*C*d^3*e^3+6*e^5/(a*e^2+c*d^2)^4*\ln(e*x+d)*A*c*d-5 \\
& *e^4/(a*e^2+c*d^2)^4*\ln(e*x+d)*B*c*d^2-2*e^5/(a*e^2+c*d^2)^4*\ln(e*x+d)*C*a \\
& d+4*e^3/(a*e^2+c*d^2)^4*\ln(e*x+d)*C*c*d^3-e^5/(a*e^2+c*d^2)^3/(e*x+d)*A-7/8 \\
& /((a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^2*e^4*x-13/8/(a*e^2+c*d^2)^4/(c*x^2+ \\
& a)^2*C*a*c^2*d^4*e^2*x+7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^3*a*c^2*d*e^5-9/ \\
& 8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*a*c^2*d^2*e^4+2/(a*e^2+c*d^2)^4/(c*x^2+ \\
& a)^2*A*x^2*a*c^2*d*e^5-1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*a*c^2*d^2*e^4-1/ \\
& (a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*a^2*c*d*e^5+15/8/(a*e^2+c*d^2)^4/a/(a*c)^ \\
& (1/2)*\arctan(1/(a*c)^{(1/2)}*c*x)*A*c^3*d^4*e^2+15/4/(a*e^2+c*d^2)^4*a/(a*c)^ \\
& (1/2)*\arctan(1/(a*c)^{(1/2)}*c*x)*B*c*d*e^5-1/4/(a*e^2+c*d^2)^4/a/(a*c)^{(1/2)} \\
& *\arctan(1/(a*c)^{(1/2)}*c*x)*B*c^3*d^5*e-33/8/(a*e^2+c*d^2)^4*a/(a*c)^{(1/2)* \\
& rctan(1/(a*c)^{(1/2)}*c*x)*C*c*d^2*e^4+15/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a \\
& *x^3*A*d^4*e^2-1/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*B*d^5*e+3/8/(a*e^2 \\
& +c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^2*e^4*x+9/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*d*a \\
& ^2*c*B*e^5*x+5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^3*e^3*x+e^6/(a*e^2+c \\
& *d^2)^4*\ln(e*x+d)*B*a+e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*d-e^3/(a*e^2+c*d^2)^3/( \\
& e*x+d)*C*d^2+3/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a^3*e^6-1/4/(a*e^2+c*d^2)^4/ \\
& (c*x^2+a)^2*B*c^3*d^6-1/2/(a*e^2+c*d^2)^4*a*\ln(c*x^2+a)*e^6*B+45/8/(a*e^2+c \\
& *d^2)^4/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*A*c^2*d^2*e^4-5/2/(a*e^2+c*d^
\end{aligned}$$

$$\begin{aligned}
& 2)^4/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*B*c^2*d^3*e^3+13/8/(a*e^2+c*d^2) \\
& ^4/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*C*c^2*d^4*e^2+3/2/(a*e^2+c*d^2)^4/ \\
& (c*x^2+a)^2*B*x^3*c^3*d^3*e^3-11/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*c^3*d^ \\
& 4*e^2+2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^2*c^3*d^3*e^3+1/2/(a*e^2+c*d^2)^4/( \\
& c*x^2+a)^2*B*x^2*a^2*c*e^6-3/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*c^3*d^4*e^ \\
& 2+1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*c^3*d^5*e-9/8/(a*e^2+c*d^2)^4/(c*x^2+ \\
& a)^2*A*a^2*c*e^6*x+17/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*c^3*d^4*e^2*x+1/4/(a* \\
& e^2+c*d^2)^4/(c*x^2+a)^2*B*c^3*d^5*e*x+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^ \\
& 3*a^2*c*e^6-7/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^3*a*c^2*e^6+5/8/(a*e^2+c*d^ \\
& 2)^4/(c*x^2+a)^2*A*x^3*c^3*d^2*e^4+5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*a^2*c* \\
& d*e^5+3/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^3*e^3-3/4/(a*e^2+c*d^2)^4/(c* \\
& x^2+a)^2*B*a^2*c*d^2*e^4-7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^4*e^2-1/ \\
& (a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^3*e^3+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2 \\
& *C*a*c^2*d^5*e+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^5/a^2*x^3*A*d^6+1/8/(a*e^2 \\
& +c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*C*d^6+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2/a*x*A \\
& *c^4*d^6-15/8/(a*e^2+c*d^2)^4*a/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*A*c*e \\
& ^6+3/8/(a*e^2+c*d^2)^4/a^2/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*A*c^4*d^6+ \\
& 1/8/(a*e^2+c*d^2)^4/a/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*C*c^3*d^6
\end{aligned}$$

**maxima** [B] time = 1.24, size = 1196, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^2/(c\*x^2+a)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*e^5)*\log( \\
& c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 \\
& + a^4*e^8) + (4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d* \\
& e^5)*\log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c* \\
& d^2*e^6 + a^4*e^8) - 1/8*(2*B*a*c^3*d^5*e + 20*B*a^2*c^2*d^3*e^3 - 30*B*a^3 \\
& *c*d*e^5 - (C*a*c^3 + 3*A*c^4)*d^6 - (13*C*a^2*c^2 + 15*A*a*c^3)*d^4*e^2 + \\
& 3*(11*C*a^3*c - 15*A*a^2*c^2)*d^2*e^4 - 3*(C*a^4 - 5*A*a^3*c)*e^6)*\arctan(c \\
& *x/\sqrt{a*c})/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5 \\
& *c*d^2*e^6 + a^6*e^8)*\sqrt{a*c}) - 1/8*(2*B*a^2*c^2*d^5 + 12*B*a^3*c*d^3*e^ \\
& 2 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - 4*(C*a^3*c + A*a^2*c^2)*d^4*e + 20*(C*a^ \\
& 4 - A*a^3*c)*d^2*e^3 + (2*B*a*c^3*d^3*e^2 - 22*B*a^2*c^2*d*e^4 - (C*a*c^3 + \\
& 3*A*c^4)*d^4*e + 4*(5*C*a^2*c^2 - 3*A*a*c^3)*d^2*e^3 - 3*(C*a^3*c - 5*A*a^ \\
& 2*c^2)*e^5)*x^4 + (2*B*a*c^3*d^4*e - 2*B*a^2*c^2*d^2*e^3 - 4*B*a^3*c*e^5 - \\
& (C*a*c^3 + 3*A*c^4)*d^5 + 4*(C*a^2*c^2 - 3*A*a*c^3)*d^3*e^2 + (5*C*a^3*c - \\
& 9*A*a^2*c^2)*d*e^4)*x^3 + (10*B*a^2*c^2*d^3*e^2 - 38*B*a^3*c*d*e^4 - (7*C*a \\
& ^2*c^2 + 5*A*a*c^3)*d^4*e + 4*(9*C*a^3*c - 7*A*a^2*c^2)*d^2*e^3 - 5*(C*a^4 \\
& - 5*A*a^3*c)*e^5)*x^2 - (6*B*a^3*c*d^2*e^3 + 6*B*a^4*e^5 - (C*a^2*c^2 - 5*A \\
& *a*c^3)*d^5 - 8*(C*a^3*c - 2*A*a^2*c^2)*d^3*e^2 - (7*C*a^4 - 11*A*a^3*c)*d* \\
& e^4)*x)/(a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e^4 + a^7*d*e^6 + (a \\
& ^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^5 + (
\end{aligned}$$

$$a^2c^5d^7 + 3a^3c^4d^5e^2 + 3a^4c^3d^3e^4 + a^5c^2de^6)x^4 + 2(a^3c^4d^6e + 3a^4c^3d^4e^3 + 3a^5c^2d^2e^5 + a^6c^2e^7)x^3 + 2(a^3c^4d^7 + 3a^4c^3d^5e^2 + 3a^5c^2d^3e^4 + a^6c^2de^6)x^2 + (a^4c^3d^6e + 3a^5c^2d^4e^3 + 3a^6c^2d^2e^5 + a^7e^7)x$$

**mupad [B]** time = 6.66, size = 6848, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + Bx + Cx^2)/((a + cx^2)^3(d + ex)^2), x)$

[Out]  $\text{symsum}(\log(\text{root}(17920a^9c^5d^8e^8z^3 + 14336a^{10}c^4d^6e^{10}z^3 + 14336a^8c^6d^{10}e^6z^3 + 7168a^{11}c^3d^4e^{12}z^3 + 7168a^7c^7d^{12}e^4z^3 + 2048a^{12}c^2d^2e^{14}z^3 + 2048a^6c^8d^{14}e^2z^3 + 256a^5c^9d^{16}z^3 + 256a^{13}c^2e^{16}z^3 + 948B^2C^2a^7c^2de^{11}z - 12AB^2a^7c^2d^{11}e^z + 9768B^2C^2a^5c^3d^5e^7z - 7476B^2C^2a^6c^2d^3e^9z - 328B^2C^2a^4c^4d^7e^5z - 92B^2C^2a^3c^5d^9e^3z - 12486A^2C^2a^5c^3d^4e^8z + 5868A^2C^2a^6c^2d^2e^{10}z + 282A^2C^2a^3c^5d^8e^4z + 168A^2C^2a^4c^4d^6e^6z + 108A^2C^2a^2c^6d^{10}e^2z + 14820A^2B^2a^5c^3d^3e^9z - 840A^2B^2a^4c^4d^5e^7z - 600A^2B^2a^3c^5d^7e^5z - 180A^2B^2a^2c^6d^9e^3z - 4B^2C^2a^2c^6d^{11}e^z - 3204A^2B^2a^6c^2de^{11}z + 4239C^2a^6c^2d^4e^8z - 3924C^2a^5c^3d^6e^6z + 103C^2a^4c^4d^8e^4z + 26C^2a^3c^5d^{10}e^2z - 6000B^2a^5c^3d^4e^8z + 2820B^2a^6c^2d^2e^{10}z + 280B^2a^4c^4d^6e^6z + 80B^2a^3c^5d^8e^4z + 4B^2a^2c^6d^{10}e^2z - 8262A^2a^5c^3d^2e^{10}z + 1575A^2a^4c^4d^4e^8z + 1260A^2a^3c^5d^6e^6z + 495A^2a^2c^6d^8e^4z - 90A^2C^2a^7c^2e^{12}z + 6A^2C^2a^7c^2d^{12}z - 966C^2a^7c^2d^2e^{10}z + 90A^2a^2c^7d^{10}e^2z + C^2a^2c^6d^{12}z + 225A^2a^6c^2e^{12}z - 192B^2a^7c^2e^{12}z + 9A^2c^8d^{12}z + 9C^2a^8e^{12}z + 78A^2B^2C^2a^4c^4d^6e^4 + 942A^2B^2C^2a^3c^3d^4e^6 - 342A^2B^2C^2a^3c^2d^2e^8 - 129B^2C^2a^4c^2d^2e^8 + 990A^2C^2a^3c^2de^9 - 234A^2C^2a^4d^5e^5 - 24A^2C^2a^4d^7e^3 + 333A^2B^2a^4c^4d^4e^6 - 252A^2B^2a^3c^2de^9 - 60A^2B^2a^4d^5e^5 + 204B^2C^2a^4c^2de^9 - 234A^2C^2a^4c^2de^9 - 624B^2C^2a^3c^2d^3e^7 + 405B^2C^2a^3c^2d^4e^6 - 36B^2C^2a^2c^3d^5e^5 + 21B^2C^2a^2c^3d^6e^4 - 1296A^2C^2a^2c^3d^3e^7 + 396A^2C^2a^3c^2d^3e^7 - 330A^2C^2a^2c^3d^5e^5 + 1863A^2B^2a^2c^3d^2e^8 - 672A^2B^2a^2c^3d^3e^7 + 90A^2B^2C^2a^4c^2e^{10} + 8C^3a^4c^2d^3e^7 - 1350A^3a^2c^3de^9 - 324A^3a^2c^4d^3e^7 - 36A^2C^2c^5d^7e^3 + 45A^2B^2c^5d^6e^4 - 225A^2B^2a^3c^2e^{10} - 86C^3a^3c^2d^5e^5 - 4C^3a^2c^3d^7e^3 + 316B^3a^3c^2d^2e^8 + 20B^3a^2c^3d^4e^6 + 18C^3a^5de^9 - 64B^3a^4c^2e^{10} - 9B^2C^2a^5e^{10} - 54A^3c^5d^5e^5, z, k)*((120A^2a^8c^2e^{13} - 24C^2a^9c^2e^{13} + 24A^2a^2c^8d^{12}e - 112B^2a^8c^2de^{12} + 8C^2a^3c^7d^{12}e + 144A^2a^3c^7d^{10}e^3 + 456A^2a^4c^6d^8e^5 + 864A^2a^5c^5d^6e^7 + 936A^2a^6c^4d^4e^9 + 528A^2a^7c^3d^2e^{11} - 16B^2a^3c^7d^{11}e^2 - 176$

$$\begin{aligned}
& *B*a^4*c^6*d^9*e^4 - 544*B*a^5*c^5*d^7*e^6 - 736*B*a^6*c^4*d^5*e^8 - 464*B* \\
& a^7*c^3*d^3*e^{10} + 112*C*a^4*c^6*d^{10}*e^3 + 344*C*a^5*c^5*d^8*e^5 + 416*C*a \\
& ^6*c^4*d^6*e^7 + 184*C*a^7*c^3*d^4*e^9 - 16*C*a^8*c^2*d^2*e^{11})/(64*(a^{10}*e \\
& ^{12} + a^4*c^6*d^{12} + 6*a^9*c*d^2*e^{10} + 6*a^5*c^5*d^{10}*e^2 + 15*a^6*c^4*d^8 \\
& *e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) + \text{root}(17920*a^9*c^5*d^8*e \\
& ^8*z^3 + 14336*a^{10}*c^4*d^6*e^{10}*z^3 + 14336*a^8*c^6*d^{10}*e^6*z^3 + 7168*a^ \\
& 11*c^3*d^4*e^{12}*z^3 + 7168*a^7*c^7*d^{12}*e^4*z^3 + 2048*a^{12}*c^2*d^2*e^{14}*z^ \\
& 3 + 2048*a^6*c^8*d^{14}*e^2*z^3 + 256*a^5*c^9*d^{16}*z^3 + 256*a^{13}*c*e^{16}*z^3 \\
& + 948*B*C*a^7*c*d*e^{11}*z - 12*A*B*a*c^7*d^{11}*e*z + 9768*B*C*a^5*c^3*d^5*e^7 \\
& *z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^ \\
& 5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^{10}*z + 2 \\
& 82*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^{10} \\
& *e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B* \\
& a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^{11}*e*z - 32 \\
& 04*A*B*a^6*c^2*d*e^{11}*z + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6 \\
& *e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*C^2*a^3*c^5*d^{10}*e^2*z - 6000*B^2*a \\
& ^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*e^{10}*z + 280*B^2*a^4*c^4*d^6*e^6*z \\
& + 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c^6*d^{10}*e^2*z - 8262*A^2*a^5*c^3*d^ \\
& 2*e^{10}*z + 1575*A^2*a^4*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^ \\
& 2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^{12}*z + 6*A*C*a*c^7*d^{12}*z - 966*C^2*a^ \\
& 7*c*d^2*e^{10}*z + 90*A^2*a*c^7*d^{10}*e^2*z + C^2*a^2*c^6*d^{12}*z + 225*A^2*a^6 \\
& *c^2*e^{12}*z - 192*B^2*a^7*c*e^{12}*z + 9*A^2*c^8*d^{12}*z + 9*C^2*a^8*e^{12}*z + \\
& 78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2* \\
& e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d \\
& ^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c \\
& ^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c \\
& *d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a \\
& ^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 39 \\
& 6*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^ \\
& 2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A*B*C*a^4*c*e^{10} + 8*C^3*a^4*c*d^3*e \\
& ^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 \\
& + 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c^2*e^{10} - 86*C^3*a^3*c^2*d^5*e^5 - \\
& 4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + \\
& 18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^{10} - 9*B*C^2*a^5*e^{10} - 54*A^3*c^5*d^5*e^ \\
& 5, z, k)*((512*a^{11}*c^2*d*e^{14} + 512*a^5*c^8*d^{13}*e^2 + 3072*a^6*c^7*d^{11}*e \\
& ^4 + 7680*a^7*c^6*d^9*e^6 + 10240*a^8*c^5*d^7*e^8 + 7680*a^9*c^4*d^5*e^{10} + \\
& 3072*a^{10}*c^3*d^3*e^{12})/(64*(a^{10}*e^{12} + a^4*c^6*d^{12} + 6*a^9*c*d^2*e^{10} + \\
& 6*a^5*c^5*d^{10}*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2* \\
& d^4*e^8)) + (x*(384*a^{11}*c^2*e^{15} - 128*a^4*c^9*d^{14}*e - 384*a^5*c^8*d^{12}*e \\
& ^3 + 384*a^6*c^7*d^{10}*e^5 + 3200*a^7*c^6*d^8*e^7 + 5760*a^8*c^5*d^6*e^9 + 4 \\
& 992*a^9*c^4*d^4*e^{11} + 2176*a^{10}*c^3*d^2*e^{13}))/((64*(a^{10}*e^{12} + a^4*c^6*d^ \\
& 12 + 6*a^9*c*d^2*e^{10} + 6*a^5*c^5*d^{10}*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^ \\
& 3*d^6*e^6 + 15*a^8*c^2*d^4*e^8))) + (x*(192*B*a^8*c^2*e^{13} + 912*A*a^7*c^3* \\
& d*e^{12} - 336*C*a^8*c^2*d*e^{12} + 48*A*a^2*c^8*d^{11}*e^2 + 336*A*a^3*c^7*d^9*e \\
& ^4 + 1632*A*a^4*c^6*d^7*e^6 + 3360*A*a^5*c^5*d^5*e^8 + 2928*A*a^6*c^4*d^3*e
\end{aligned}$$

$$\begin{aligned}
& ^{10} - 32*B*a^3*c^7*d^{10}*e^3 - 704*B*a^4*c^6*d^8*e^5 - 1728*B*a^5*c^5*d^6*e^7 - 1280*B*a^6*c^4*d^4*e^9 - 32*B*a^7*c^3*d^2*e^{11} + 16*C*a^3*c^7*d^{11}*e^2 \\
& + 496*C*a^4*c^6*d^9*e^4 + 1056*C*a^5*c^5*d^7*e^6 + 352*C*a^6*c^4*d^5*e^8 - 560*C*a^7*c^3*d^3*e^{10}) / (64*(a^{10}*e^{12} + a^4*c^6*d^{12} + 6*a^9*c*d^2*e^{10} + \\
& 6*a^5*c^5*d^{10}*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) + (9*A^2*c^7*d^9*e^2 + 198*A^2*a^2*c^5*d^5*e^6 + 216*A^2*a^3*c^4 \\
& *d^3*e^8 + 4*B^2*a^2*c^5*d^7*e^4 - 8*B^2*a^3*c^4*d^5*e^6 - 412*B^2*a^4*c^3*d^3*e^8 + C^2*a^2*c^5*d^9*e^2 - 8*C^2*a^3*c^4*d^7*e^4 - 250*C^2*a^4*c^3*d^5 \\
& *e^6 + 296*C^2*a^5*c^2*d^3*e^8 - 120*A*B*a^5*c^2*e^{11} - 39*C^2*a^6*c*d*e^{10} + 72*A^2*a*c^6*d^7*e^4 - 495*A^2*a^4*c^3*d*e^{10} + 176*B^2*a^5*c^2*d*e^{10} + \\
& 24*B*C*a^6*c*e^{11} - 12*A*B*a*c^6*d^8*e^3 + 6*A*C*a*c^6*d^9*e^2 + 294*A*C*a^5*c^2*d*e^{10} - 36*A*B*a^2*c^5*d^6*e^5 + 36*A*B*a^3*c^4*d^4*e^7 + 1092*A*B* \\
& a^4*c^3*d^2*e^9 - 108*A*C*a^3*c^4*d^5*e^6 - 960*A*C*a^4*c^3*d^3*e^8 - 4*B*C*a^2*c^5*d^8*e^3 + 20*B*C*a^3*c^4*d^6*e^5 + 652*B*C*a^4*c^3*d^4*e^7 - 500*B \\
& *C*a^5*c^2*d^2*e^9) / (64*(a^{10}*e^{12} + a^4*c^6*d^{12} + 6*a^9*c*d^2*e^{10} + 6*a^5*c^5*d^{10}*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8 \\
& ^8)) + (x*(225*A^2*a^4*c^3*e^{11} + 9*A^2*c^7*d^8*e^3 + 9*C^2*a^6*c*e^{11} + 54 \\
& *A^2*a^2*c^5*d^4*e^7 - 360*A^2*a^3*c^4*d^2*e^9 + 4*B^2*a^2*c^5*d^6*e^5 - 88 \\
& *B^2*a^3*c^4*d^4*e^7 + 484*B^2*a^4*c^3*d^2*e^9 + C^2*a^2*c^5*d^8*e^3 - 40*C^2*a^3*c^4*d^6*e^5 + 406*C^2*a^4*c^3*d^4*e^7 - 120*C^2*a^5*c^2*d^2*e^9 - 90 \\
& *A*C*a^5*c^2*e^{11} + 72*A^2*a*c^6*d^6*e^5 - 12*A*B*a*c^6*d^7*e^4 - 660*A*B*a^4*c^3*d*e^{10} + 6*A*C*a*c^6*d^8*e^3 + 132*B*C*a^5*c^2*d*e^{10} + 84*A*B*a^2*c^5 \\
& *d^5*e^6 + 588*A*B*a^3*c^4*d^3*e^8 - 96*A*C*a^2*c^5*d^6*e^5 - 492*A*C*a^3*c^4*d^4*e^7 + 672*A*C*a^4*c^3*d^2*e^9 - 4*B*C*a^2*c^5*d^7*e^4 + 124*B*C*a^3 \\
& *c^4*d^5*e^6 - 892*B*C*a^4*c^3*d^3*e^8)) / (64*(a^{10}*e^{12} + a^4*c^6*d^{12} + 6 \\
& *a^9*c*d^2*e^{10} + 6*a^5*c^5*d^{10}*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) *root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^{10}*c^4 \\
& *d^6*e^{10}*z^3 + 14336*a^8*c^6*d^{10}*e^6*z^3 + 7168*a^{11}*c^3*d^4*e^{12}*z^3 + 7 \\
& 168*a^7*c^7*d^{12}*e^4*z^3 + 2048*a^{12}*c^2*d^2*e^{14}*z^3 + 2048*a^6*c^8*d^{14}*e^2*z^3 + 256*a^5*c^9*d^{16}*z^3 + 256*a^{13}*c*e^{16}*z^3 + 948*B*C*a^7*c*d*e^{11}* \\
& z - 12*A*B*a*c^7*d^{11}*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d^3 \\
& *e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C \\
& *a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^{10}*z + 282*A*C*a^3*c^5*d^8*e^4*z \\
& + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^{10}*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180 \\
& *A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^{11}*e*z - 3204*A*B*a^6*c^2*d*e^{11}*z \\
& + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^4 \\
& *d^8*e^4*z + 26*C^2*a^3*c^5*d^{10}*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820 \\
& *B^2*a^6*c^2*d^2*e^{10}*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^4*z \\
& + 4*B^2*a^2*c^6*d^{10}*e^2*z - 8262*A^2*a^5*c^3*d^2*e^{10}*z + 1575*A^2*a^4*c^4 \\
& *d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 90 \\
& *A*C*a^7*c*e^{12}*z + 6*A*C*a*c^7*d^{12}*z - 966*C^2*a^7*c*d^2*e^{10}*z + 90*A^2 \\
& *a*c^7*d^{10}*e^2*z + C^2*a^2*c^6*d^{12}*z + 225*A^2*a^6*c^2*e^{12}*z - 192*B^2*a^7 \\
& *c*e^{12}*z + 9*A^2*c^8*d^{12}*z + 9*C^2*a^8*e^{12}*z + 78*A*B*C*a*c^4*d^6*e^4 \\
& + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d
\end{aligned}$$

$$\begin{aligned}
&^2e^8 + 990A^2C^2a^3c^2d^2e^9 - 234A^2C^2a^4d^5e^5 - 24AC^2a^4 \\
&*d^7e^3 + 333A^2B^2a^4d^4e^6 - 252AB^2a^3c^2d^2e^9 - 60AB^2a^4 \\
&^4d^5e^5 + 204B^2C^2a^4c^2d^2e^9 - 234AC^2a^4c^2d^2e^9 - 624B^2C^2a^3 \\
&c^2d^3e^7 + 405B^2C^2a^3c^2d^4e^6 - 36B^2C^2a^2c^3d^5e^5 + 21B^2C \\
&^2a^2c^3d^6e^4 - 1296A^2C^2a^2c^3d^3e^7 + 396AC^2a^3c^2d^3e^7 \\
&- 330AC^2a^2c^3d^5e^5 + 1863A^2B^2a^2c^3d^2e^8 - 672AB^2a^2c \\
&^3d^3e^7 + 90AB^2C^2a^4c^2e^10 + 8C^3a^4c^2d^3e^7 - 1350A^3a^2c^3d \\
&e^9 - 324A^3a^4c^2d^3e^7 - 36A^2C^2c^5d^7e^3 + 45A^2B^2c^5d^6e^4 \\
&- 225A^2B^2a^3c^2e^10 - 86C^3a^3c^2d^5e^5 - 4C^3a^2c^3d^7e^3 + \\
&316B^3a^3c^2d^2e^8 + 20B^3a^2c^3d^4e^6 + 18C^3a^5d^2e^9 - 64B \\
&^3a^4c^2e^10 - 9B^2C^2a^5e^10 - 54A^3c^5d^5e^5, z, k), k, 1, 3) + (( \\
&x^4(3C^2a^3c^2e^5 + 3A^2c^4d^4e - 15A^2a^2c^2e^5 + 12A^2a^3c^2d^2e^3 \\
&- 2B^2a^3c^3d^3e^2 + 22B^2a^2c^2d^2e^4 - 20C^2a^2c^2d^2e^3 + C^2a^3c^3d \\
&^4e)))/(8a^2(a^3e^6 + c^3d^6 + 3a^2c^2d^4e^2 + 3a^2c^2d^2e^4)) - (4 \\
&A^2a^2e^5 + B^2c^2d^5 - 7B^2a^2d^2e^4 - 2A^2c^2d^4e + 10C^2a^2d^2e^3 - \\
&2C^2a^2c^2d^4e - 10A^2a^2c^2d^2e^3 + 6B^2a^2c^2d^3e^2)/(4(a^2e^2 + c^2d^2)*(a^ \\
&2e^4 + c^2d^4 + 2a^2c^2d^2e^2)) + (x^3(3A^2c^3d^3 + 4B^2a^2c^2e^3 + C^2a \\
&c^2d^3 + 9A^2a^2c^2d^2e^2 - 2B^2a^2c^2d^2e - 5C^2a^2c^2d^2e^2))/(8a^2(a^ \\
&2e^4 + c^2d^4 + 2a^2c^2d^2e^2)) + (x*(5A^2c^2d^3 + 6B^2a^2e^3 - C^2a^2c^2d \\
&^3 - 7C^2a^2d^2e^2 + 11A^2a^2c^2d^2e^2))/(8a^2(a^2e^4 + c^2d^4 + 2a^2c^2d^2e \\
&^2)) + (x^2(5C^2a^3e^5 - 25A^2a^2c^2e^5 + 5A^2c^3d^4e + 28A^2a^2c^2d^2e \\
&^3 - 10B^2a^2c^2d^3e^2 - 36C^2a^2c^2d^2e^3 + 38B^2a^2c^2d^2e^4 + 7C^2a^2c^ \\
&2d^4e))/(8a^2(a^2e^2 + c^2d^2)*(a^2e^4 + c^2d^4 + 2a^2c^2d^2e^2)))/(a^2d \\
&+ c^2d^2x^4 + c^2e^2x^5 + a^2e^2x + 2a^2c^2d^2x^2 + 2a^2c^2e^2x^3)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*3,x)

[Out] Timed out

$$3.63 \quad \int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^3} dx$$

**Optimal.** Leaf size=753

$$\frac{e^3 \log(a+cx^2) \left( a^2 C e^4 - a c e^2 (3 A e^2 - 9 B d e + 13 C d^2) + c^2 d^2 (10 C d^2 - 3 e (5 B d - 7 A e)) \right)}{2 (a e^2 + c d^2)^5} + \frac{e^3 \log(d+ex) (a^2 C d^2 - a c d^2 (3 A e^2 - 9 B d e + 13 C d^2) + c^2 d^2 (10 C d^2 - 3 e (5 B d - 7 A e)))}{2 (a e^2 + c d^2)^5}$$

**Rubi [A]** time = 3.14, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1647, 1629, 635, 205, 260}

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)^3), x]

[Out]  $-(e^3*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^3*(d + e*x)^2) - (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^4*(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e))) + c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(13*C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(8*a^(5/2)*(c*d^2 + a*e^2)^5) + (e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d - 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)$

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 260**

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{(d + ex)^3 (a + cx^2)^3} dx &= -\frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2)) - c (Acd (cd^2 - 3ae^2) - a (cd^2(Cd^2 - Bde + Ae^2)))}{4a (cd^2 + ae^2)^3 (a + cx^2)^2} \\
&= -\frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2)) - c (Acd (cd^2 - 3ae^2) - a (cd^2(Cd^2 - Bde + Ae^2)))}{4a (cd^2 + ae^2)^3 (a + cx^2)^2} \\
&= -\frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2)) - c (Acd (cd^2 - 3ae^2) - a (cd^2(Cd^2 - Bde + Ae^2)))}{4a (cd^2 + ae^2)^3 (a + cx^2)^2} \\
&= -\frac{e^3 (Cd^2 - Bde + Ae^2)}{2 (cd^2 + ae^2)^3 (d + ex)^2} - \frac{e^3 (4cCd^3 - cde(5Bd - 6Ae) - ae^2(2Cd - Be))}{(cd^2 + ae^2)^4 (d + ex)} - \frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2))}{(cd^2 + ae^2)^3 (a + cx^2)^2} \\
&= -\frac{e^3 (Cd^2 - Bde + Ae^2)}{2 (cd^2 + ae^2)^3 (d + ex)^2} - \frac{e^3 (4cCd^3 - cde(5Bd - 6Ae) - ae^2(2Cd - Be))}{(cd^2 + ae^2)^4 (d + ex)} - \frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2))}{(cd^2 + ae^2)^3 (a + cx^2)^2} \\
&= -\frac{e^3 (Cd^2 - Bde + Ae^2)}{2 (cd^2 + ae^2)^3 (d + ex)^2} - \frac{e^3 (4cCd^3 - cde(5Bd - 6Ae) - ae^2(2Cd - Be))}{(cd^2 + ae^2)^4 (d + ex)} - \frac{a (Bcd (cd^2 - 3ae^2) - (Ac - aC)e (3cd^2 - ae^2))}{(cd^2 + ae^2)^3 (a + cx^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 1.08, size = 672, normalized size = 0.89

---

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)^3), x]

[Out] 
$$\begin{aligned}
&((-4e^3(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8e^3 \\
&*(c*d^2 + a*e^2)*(4c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e) \\
&))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d \\
&^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*( \\
&-3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C*e^5 \\
&+ 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*e^3*(C \\
&*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2*C*d^2* \\
&(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x)))/(a^2
\end{aligned}$$

\*(a + c\*x^2)) + (Sqrt[c]\*(3\*A\*c\*d\*(c^3\*d^6 + 7\*a\*c^2\*d^4\*e^2 + 35\*a^2\*c\*d^2\*e^4 - 35\*a^3\*e^6) + a\*(a\*c^2\*d^4\*e^2\*(23\*C\*d - 45\*B\*e) - 5\*a^2\*c\*d^2\*e^4\*(25\*C\*d - 27\*B\*e) + c^3\*d^6\*(C\*d - 3\*B\*e) - 15\*a^3\*e^6\*(-3\*C\*d + B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/a^(5/2) + 8\*(a^2\*C\*e^7 + a\*c\*e^5\*(-13\*C\*d^2 + 9\*B\*d\*e - 3\*A\*e^2) + c^2\*d^2\*e^3\*(10\*C\*d^2 + 3\*e\*(-5\*B\*d + 7\*A\*e)))\*Log[d + e\*x] - 4\*(a^2\*C\*e^7 + a\*c\*e^5\*(-13\*C\*d^2 + 9\*B\*d\*e - 3\*A\*e^2) + c^2\*d^2\*e^3\*(10\*C\*d^2 + 3\*e\*(-5\*B\*d + 7\*A\*e)))\*Log[a + c\*x^2])/(8\*(c\*d^2 + a\*e^2)^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3 (a + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)^3),x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/((d + e\*x)^3\*(a + c\*x^2)^3), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a)^3,x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 0.20, size = 1532, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a)^3,x, algorithm="giac")

[Out] 
$$-1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^2*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*\log(c*x^2 + a)/(c^5*d^10 + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6 + 21*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*\log(\text{abs}(x*e + d)) / (c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*e^7 + 5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a*c^4*d^6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*e^3 - 12*5*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^5 + 45*C*a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*\arctan(c*x/\text{sqrt}(a*c))/(a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6$$

$$\begin{aligned}
& + 5a^6cd^2e^8 + a^7e^{10})\sqrt{ac}) + 1/8*(C^2ac^4d^5x^5e^2 + 3A^2c^5d^5x^5e^2 + 2C^2ac^4d^6x^4e + 6A^2c^5d^6x^4e + C^2ac^4d^7x^3 \\
& + 3A^2c^5d^7x^3 - 3B^2ac^4d^4x^5e^3 - 6B^2ac^4d^5x^4e^2 - 3B^2ac^4d^6x^3e - 58C^2ac^2c^3d^3x^5e^4 + 18A^2ac^4d^3x^5e^4 - 76C^2ac^2c^3d^4x^4e^3 \\
& + 36A^2ac^4d^4x^4e^3 - 3C^2ac^2c^3d^5x^3e^2 + 23A^2ac^4d^5x^3e^2 + 10C^2ac^2c^3d^6x^2e + 10A^2ac^4d^6x^2e - C^2ac^2c^3d^7x \\
& + 5A^2ac^4d^7x + 78B^2ac^2c^3d^2x^5e^5 + 96B^2ac^2c^3d^3x^4e^4 - 7B^2ac^2c^3d^4x^3e^3 - 20B^2ac^2c^3d^5x^2e^2 - B^2ac^2c^3d^6x \\
& *xe - 2B^2ac^2c^3d^7 + 37C^2ac^3c^2d*x^5e^6 - 81A^2ac^2c^3d*x^5e^6 + 22C^2ac^3c^2d^2x^4e^5 - 78A^2ac^2c^3d^2x^4e^5 - 129C^2ac^3c^2d^3x^3e^4 \\
& + 61A^2ac^2c^3d^3x^3e^4 - 142C^2ac^3c^2d^4x^2e^3 + 74A^2ac^2c^3d^4x^2e^3 - 10C^2ac^3c^2d^5x*e^2 + 26A^2ac^2c^3d^5x*e^2 + 6C^2ac^3c^2d^6e \\
& + 6A^2ac^2c^3d^6e - 15B^2ac^3c^2x^5e^7 + 6B^2ac^3c^2d*x^4e^6 + 163B^2ac^3c^2d^2x^3e^5 + 176B^2ac^3c^2d^3x^2e^4 + 2B^2ac^3c^2d^4x*e^3 \\
& - 20B^2ac^3c^2d^5e^2 + 4C^2ac^4c*x^4e^7 - 12A^2ac^3c^2x^4e^7 + 67C^2ac^4c*d*x^3e^6 - 151A^2ac^3c^2d*x^3e^6 + 46C^2ac^4c*d^2x^2e^5 - 146A^2ac^3c^2d^2x^2e^5 \\
& - 77C^2ac^4c*d^3x*e^4 + 49A^2ac^3c^2d^3x*e^4 - 72C^2ac^4c*d^4e^3 + 44A^2ac^3c^2d^4e^3 - 25B^2ac^4c*x^3e^7 + 4B^2ac^4c*d*x^2e^6 + 91B^2ac^4c*d^2x*e^5 \\
& + 74B^2ac^4c*d^3e^4 + 6C^2ac^5x^2e^7 - 18A^2ac^4c*x^2e^7 + 28C^2ac^5d*x*e^6 - 68A^2ac^4c*d*x*e^6 + 18C^2ac^5d^2e^5 - 62A^2ac^4c*d^2e^5 \\
& - 8B^2ac^5x*e^7 - 4B^2ac^5d*e^6 - 4A^2ac^5e^7)/(a^2c^4d^8 + 4a^3c^3d^6e^2 + 6a^4c^2d^4e^4 + 4a^5cd^2e^6 + a^6e^8)*(c*x^3e + c*d*x^2 + a*x*e + a*d)^2)
\end{aligned}$$

**maple [B]** time = 0.04, size = 2737, normalized size = 3.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((C^2x^2+Bx+A)/(ex+d)^3/(cx^2+a)^3, x)$

[Out]  $15/2/(a^2e^2+c^2d^2)^5c^2\ln(cx^2+a)d^3e^4B-5/(a^2e^2+c^2d^2)^5c^2\ln(cx^2+a)C^2d^4e^3+3/2/(a^2e^2+c^2d^2)^5c^2a\ln(cx^2+a)Ae^7+2e^5/(a^2e^2+c^2d^2)^4/(ex+d)C^2ad-4e^3/(a^2e^2+c^2d^2)^4/(ex+d)C^2c^3d-3e^7/(a^2e^2+c^2d^2)^5\ln(ex+d)A^2ac+21e^5/(a^2e^2+c^2d^2)^5\ln(ex+d)A^2c^2d^2-15e^4/(a^2e^2+c^2d^2)^5\ln(ex+d)B^2c^2d^3+10e^3/(a^2e^2+c^2d^2)^5\ln(ex+d)C^2c^2d^4-6e^5/(a^2e^2+c^2d^2)^4/(ex+d)A^2c^2d+5e^4/(a^2e^2+c^2d^2)^4/(ex+d)B^2c^2d^2-5/4/(a^2e^2+c^2d^2)^5c/(cx^2+a)^2A^2e^7a^3+3/4/(a^2e^2+c^2d^2)^5c^4/(cx^2+a)^2A^2d^6e-1/8/(a^2e^2+c^2d^2)^5c^4/(cx^2+a)^2C^2x^2d^7-21/2/(a^2e^2+c^2d^2)^5c^2\ln(cx^2+a)A^2d^2e^5-1/2e^5/(a^2e^2+c^2d^2)^3/(ex+d)^2A-7/2/(a^2e^2+c^2d^2)^5c^2/(cx^2+a)^2C^2x^2a^2d^2e^5-5/2/(a^2e^2+c^2d^2)^5c^3/(cx^2+a)^2C^2x^2ad^4e^3+21/8/(a^2e^2+c^2d^2)^5c^5/(cx^2+a)^2/a^3A^2d^5e^2-3/8/(a^2e^2+c^2d^2)^5c^5/(cx^2+a)^2/a^3B^2d^6e+31/8/(a^2e^2+c^2d^2)^5c^3/(cx^2+a)^2B^2x^3ad^2e^5-39/8/(a^2e^2+c^2d^2)^5c^2/(cx^2+a)^2A^2x^2ad^2e^6-25/8/(a^2e^2+c^2d^2)^5c^3/(cx^2+a)^2A^2x^2ad^3e^4+33/8/(a^2e^2+c^2d^2)^5c^$

$$\begin{aligned}
& 2/(c*x^2+a)^2*B*x*a^2*d^2*e^5+45/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*B*x*a*d^4 \\
& e^3+5/8/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*C*x*a^2*d^3*e^4-23/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2 \\
& C*x*a*d^5*e^2-105/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x) \\
& *A*d*e^6+21/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x) \\
& *A*d^5*e^2+135/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x) \\
& *B*d^2*e^5-3/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x) \\
& *B*d^6*e-125/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x) \\
& *C*d^3*e^4+45/8/(a*e^2+c*d^2)^5*c*a^2/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x) \\
& *C*d*e^6+27/8/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*x*a^3*d*e^6-33/8/(a*e^2+c*d^2)^5*c^3 \\
& /((c*x^2+a)^2*A*x^3*a*d*e^6+21/8/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*C*x^3*a*d^3*e^4 \\
& +4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x^2*a*d^2*e^5+3/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2 \\
& *B*x^2*a^2*d*e^6-1/2/(a*e^2+c*d^2)^5*a^2*ln(c*x^2+a)*C*e^7-e^6/(a*e^2+c*d^2)^4 \\
& /((e*x+d)*B*a+1/2*e^4/(a*e^2+c*d^2)^3/(e*x+d)^2*B*d-1/2*e^3/(a*e^2+c*d^2)^3 \\
& /((e*x+d)^2*C*d^2+e^7/(a*e^2+c*d^2)^5*ln(e*x+d)*a^2*C+3/4/(a*e^2+c*d^2)^5 \\
& /((c*x^2+a)^2*C*a^4*e^7-1/4/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*d^7*B-15/8/(a*e^2+c*d^2)^5 \\
& *c^4/(c*x^2+a)^2*A*x^3*d^3*e^4+35/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*B*x^3*d^4*e^3 \\
& -25/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x^3*d^5*e^2-1/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2 \\
& *A*x^2*a^2*e^7+5/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*A*x^2*d^4*e^3-3/(a*e^2+c*d^2)^5*c^4 \\
& /((c*x^2+a)^2*B*x^2*d^5*e^2+3/2/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x^2*d^6*e-7/8/(a*e^2+c*d^2)^5 \\
& *c^2/(c*x^2+a)^2*B*x^3*a^2*e^7+3/8/(a*e^2+c*d^2)^5*c^6/(c*x^2+a)^2/a^2*x^3*A*d^7 \\
& +1/8/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*C*d^7+5/8/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2 \\
& /a*x*A*d^7+17/4/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*d^2*e^5*a^2+25/4/(a*e^2+c*d^2)^5 \\
& *c^3/(c*x^2+a)^2*A*d^4*e^3*a+5/4/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*d^3*e^4*B*a^2-11/4 \\
& /((a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*d^5*e^2*B*a-15/4/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2 \\
& *C*a^2*d^4*e^3+3/4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*C*a*d^6*e+9*e^6/(a*e^2+c*d^2)^5 \\
& *ln(e*x+d)*B*a*c*d-13*e^5/(a*e^2+c*d^2)^5*ln(e*x+d)*C*a*c*d^2+1/2/(a*e^2+c*d^2)^5 \\
& *c/(c*x^2+a)^2*C*x^2*a^3*e^7-9/8/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*B*x*a^3*e^7+15/4 \\
& /((a*e^2+c*d^2)^5*c/(c*x^2+a)^2*d*e^6*B*a^3-15/4/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*a^3*d^2 \\
& *e^5+1/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x)*C*d^7-9/2/(a*e^2+c*d^2)^5 \\
& *c*a*ln(c*x^2+a)*d*e^6*B+13/2/(a*e^2+c*d^2)^5*c*a*ln(c*x^2+a)*C*d^2*e^5-15/8/(a*e^2+c*d^2)^5 \\
& *c*a^2/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x)*B*e^7+105/8/(a*e^2+c*d^2)^5*c^3/(a*c)^{(1/2)} \\
& *arctan(1/(a*c)^{(1/2)}*c*x)*A*d^3*e^4-45/8/(a*e^2+c*d^2)^5*c^3/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)} \\
& *c*x)*B*d^4*e^3+23/8/(a*e^2+c*d^2)^5*c^3/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x)*C*d^5*e^2+3/8 \\
& /((a*e^2+c*d^2)^5*c^5/a^2/(a*c)^{(1/2)}*arctan(1/(a*c)^{(1/2)}*c*x)*A*d^7+19/8/(a*e^2+c*d^2)^5 \\
& *c^4/(c*x^2+a)^2*A*x*d^5*e^2+3/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*B*x*d^6*e
\end{aligned}$$

**maxima** [B] time = 1.27, size = 1835, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(e\*x+d)^3/(c\*x^2+a)^3,x, algorithm="maxima")

[Out] 
$$-1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 21*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*\log(c*x^2 + a)/(c^5*d^{10} + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e^{10}) + (10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 21*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*\log(e*x + d)/(c^5*d^{10} + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e^{10}) - 1/8*(3*B*a*c^4*d^6*e + 45*B*a^2*c^3*d^4*e^3 - 135*B*a^3*c^2*d^2*e^5 + 15*B*a^4*c*e^7 - (C*a*c^4 + 3*A*c^5)*d^7 - (23*C*a^2*c^3 + 21*A*a*c^4)*d^5*e^2 + 5*(25*C*a^3*c^2 - 21*A*a^2*c^3)*d^3*e^4 - 15*(3*C*a^4*c - 7*A*a^3*c^2)*d*e^6)*\arctan(c*x/\sqrt{a*c})/((a^2*c^5*d^{10} + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6 + 5*a^6*c*d^2*e^8 + a^7*e^{10})*\sqrt{a*c}) - 1/8*(2*B*a^2*c^3*d^7 + 20*B*a^3*c^2*d^5*e^2 - 74*B*a^4*c*d^3*e^4 + 4*B*a^5*d*e^6 + 4*A*a^5*e^7 - 6*(C*a^3*c^2 + A*a^2*c^3)*d^6*e + 4*(18*C*a^4*c - 11*A*a^3*c^2)*d^4*e^3 - 2*(9*C*a^5 - 31*A*a^4*c)*d^2*e^5 + (3*B*a*c^4*d^4*e^3 - 78*B*a^2*c^3*d^2*e^5 + 15*B*a^3*c^2*e^7 - (C*a*c^4 + 3*A*c^5)*d^5*e^2 + 2*(29*C*a^2*c^3 - 9*A*a*c^4)*d^3*e^4 - (37*C*a^3*c^2 - 81*A*a^2*c^3)*d*e^6)*x^5 + 2*(3*B*a*c^4*d^5*e^2 - 48*B*a^2*c^3*d^3*e^4 - 3*B*a^3*c^2*d*e^6 - (C*a*c^4 + 3*A*c^5)*d^6*e + 2*(19*C*a^2*c^3 - 9*A*a*c^4)*d^4*e^3 - (11*C*a^3*c^2 - 39*A*a^2*c^3)*d^2*e^5 - 2*(C*a^4*c - 3*A*a^3*c^2)*e^7)*x^4 + (3*B*a*c^4*d^6*e + 7*B*a^2*c^3*d^4*e^3 - 163*B*a^3*c^2*d^2*e^5 + 25*B*a^4*c*e^7 - (C*a*c^4 + 3*A*c^5)*d^7 + (3*C*a^2*c^3 - 23*A*a*c^4)*d^5*e^2 + (129*C*a^3*c^2 - 61*A*a^2*c^3)*d^3*e^4 - (67*C*a^4*c - 151*A*a^3*c^2)*d*e^6)*x^3 + 2*(10*B*a^2*c^3*d^5*e^2 - 88*B*a^3*c^2*d^3*e^4 - 2*B*a^4*c*d*e^6 - 5*(C*a^2*c^3 + A*a*c^4)*d^6*e + (71*C*a^3*c^2 - 37*A*a^2*c^3)*d^4*e^3 - (23*C*a^4*c - 73*A*a^3*c^2)*d^2*e^5 - 3*(C*a^5 - 3*A*a^4*c)*e^7)*x^2 + (B*a^2*c^3*d^6*e - 2*B*a^3*c^2*d^4*e^3 - 91*B*a^4*c*d^2*e^5 + 8*B*a^5*e^7 + (C*a^2*c^3 - 5*A*a*c^4)*d^7 + 2*(5*C*a^3*c^2 - 13*A*a^2*c^3)*d^5*e^2 + 7*(11*C*a^4*c - 7*A*a^3*c^2)*d^3*e^4 - 4*(7*C*a^5 - 17*A*a^4*c)*d*e^6)*x)/(a^4*c^4*d^{10} + 4*a^5*c^3*d^8*e^2 + 6*a^6*c^2*d^6*e^4 + 4*a^7*c*d^4*e^6 + a^8*d^2*e^8 + (a^2*c^6*d^8*e^2 + 4*a^3*c^5*d^6*e^4 + 6*a^4*c^4*d^4*e^6 + 4*a^5*c^3*d^2*e^8 + a^6*c^2*e^{10})*x^6 + 2*(a^2*c^6*d^9*e + 4*a^3*c^5*d^7*e^3 + 6*a^4*c^4*d^5*e^5 + 4*a^5*c^3*d^3*e^7 + a^6*c^2*d*e^9)*x^5 + (a^2*c^6*d^{10} + 6*a^3*c^5*d^8*e^2 + 14*a^4*c^4*d^6*e^4 + 16*a^5*c^3*d^4*e^6 + 9*a^6*c^2*d^2*e^8 + 2*a^7*c*e^{10})*x^4 + 4*(a^3*c^5*d^9*e + 4*a^4*c^4*d^7*e^3 + 6*a^5*c^3*d^5*e^5 + 4*a^6*c^2*d^3*e^7 + a^7*c*d*e^9)*x^3 + (2*a^3*c^5*d^{10} + 9*a^4*c^4*d^8*e^2 + 16*a^5*c^3*d^6*e^4 + 14*a^6*c^2*d^4*e^6 + 6*a^7*c*d^2*e^8 + a^8*e^{10})*x^2 + 2*(a^4*c^4*d^9*e + 4*a^5*c^3*d^7*e^3 + 6*a^6*c^2*d^5*e^5 + 4*a^7*c*d^3*e^7 + a^8*d*e^9)*x)$$

**mupad [B]** time = 7.24, size = 8774, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/((a + c\*x^2)^3\*(d + e\*x)^3),x)

[Out] ((x^5\*(3\*A\*c^5\*d^5\*e^2 - 15\*B\*a^3\*c^2\*e^7 + 18\*A\*a\*c^4\*d^3\*e^4 - 81\*A\*a^2\*c^3\*d\*e^6 - 3\*B\*a\*c^4\*d^4\*e^3 + C\*a\*c^4\*d^5\*e^2 + 37\*C\*a^3\*c^2\*d\*e^6 + 78\*B\*a^2\*c^3\*d^2\*e^5 - 58\*C\*a^2\*c^3\*d^3\*e^4))/(8\*a^2\*(a^4\*e^8 + c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 4\*a^3\*c\*d^2\*e^6 + 6\*a^2\*c^2\*d^4\*e^4)) - (2\*A\*a^3\*e^7 + B\*c^3\*d^7 + 2\*B\*a^3\*d\*e^6 - 3\*A\*c^3\*d^6\*e - 9\*C\*a^3\*d^2\*e^5 - 22\*A\*a\*c^2\*d^4\*e^3 + 31\*A\*a^2\*c\*d^2\*e^5 + 10\*B\*a\*c^2\*d^5\*e^2 - 37\*B\*a^2\*c\*d^3\*e^4 + 36\*C\*a^2\*c\*d^4\*e^3 - 3\*C\*a\*c^2\*d^6\*e)/(4\*(a^4\*e^8 + c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 4\*a^3\*c\*d^2\*e^6 + 6\*a^2\*c^2\*d^4\*e^4)) + (x\*(5\*A\*c^4\*d^7 - 8\*B\*a^4\*e^7 - C\*a\*c^3\*d^7 + 28\*C\*a^4\*d\*e^6 + 26\*A\*a\*c^3\*d^5\*e^2 + 91\*B\*a^3\*c\*d^2\*e^5 - 77\*C\*a^3\*c\*d^3\*e^4 + 49\*A\*a^2\*c^2\*d^3\*e^4 + 2\*B\*a^2\*c^2\*d^4\*e^3 - 10\*C\*a^2\*c^2\*d^5\*e^2 - 68\*A\*a^3\*c\*d\*e^6 - B\*a\*c^3\*d^6\*e))/(8\*a\*(a^4\*e^8 + c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 4\*a^3\*c\*d^2\*e^6 + 6\*a^2\*c^2\*d^4\*e^4)) + (x^2\*(3\*C\*a^4\*e^7 - 9\*A\*a^3\*c\*e^7 + 5\*A\*c^4\*d^6\*e + 37\*A\*a\*c^3\*d^4\*e^3 - 10\*B\*a\*c^3\*d^5\*e^2 + 23\*C\*a^3\*c\*d^2\*e^5 - 73\*A\*a^2\*c^2\*d^2\*e^5 + 88\*B\*a^2\*c^2\*d^3\*e^4 - 71\*C\*a^2\*c^2\*d^4\*e^3 + 2\*B\*a^3\*c\*d\*e^6 + 5\*C\*a\*c^3\*d^6\*e))/(4\*a\*(a^4\*e^8 + c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 4\*a^3\*c\*d^2\*e^6 + 6\*a^2\*c^2\*d^4\*e^4)) + (x^3\*(3\*A\*c^5\*d^7 - 25\*B\*a^4\*c\*e^7 + C\*a\*c^4\*d^7 + 23\*A\*a\*c^4\*d^5\*e^2 - 151\*A\*a^3\*c^2\*d\*e^6 + 61\*A\*a^2\*c^3\*d^3\*e^4 - 7\*B\*a^2\*c^3\*d^4\*e^3 + 163\*B\*a^3\*c^2\*d^2\*e^5 - 3\*C\*a^2\*c^3\*d^5\*e^2 - 129\*C\*a^3\*c^2\*d^3\*e^4 - 3\*B\*a\*c^4\*d^6\*e + 67\*C\*a^4\*c\*d\*e^6))/(8\*a^2\*(a^4\*e^8 + c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 4\*a^3\*c\*d^2\*e^6 + 6\*a^2\*c^2\*d^4\*e^4)) + (x^4\*(2\*C\*a^4\*c\*e^7 + 3\*A\*c^5\*d^6\*e - 6\*A\*a^3\*c^2\*e^7 + 18\*A\*a\*c^4\*d^4\*e^3 - 3\*B\*a\*c^4\*d^5\*e^2 + 3\*B\*a^3\*c^2\*d\*e^6 - 39\*A\*a^2\*c^3\*d^2\*e^5 + 48\*B\*a^2\*c^3\*d^3\*e^4 - 38\*C\*a^2\*c^3\*d^4\*e^3 + 11\*C\*a^3\*c^2\*d^2\*e^5 + C\*a\*c^4\*d^6\*e))/(4\*a^2\*(a^4\*e^8 + c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 4\*a^3\*c\*d^2\*e^6 + 6\*a^2\*c^2\*d^4\*e^4)))/(x^2\*(a^2\*e^2 + 2\*a\*c\*d^2) + x^4\*(c^2\*d^2 + 2\*a\*c\*e^2) + a^2\*d^2 + c^2\*e^2\*x^6 + 2\*a^2\*d\*e\*x + 2\*c^2\*d\*e\*x^5 + 4\*a\*c\*d\*e\*x^3) + symsum(log(root(2560\*a^14\*c\*d^2\*e^18\*z^3 + 64512\*a^10\*c^5\*d^10\*e^10\*z^3 + 53760\*a^11\*c^4\*d^8\*e^12\*z^3 + 53760\*a^9\*c^6\*d^12\*e^8\*z^3 + 30720\*a^12\*c^3\*d^6\*e^14\*z^3 + 30720\*a^8\*c^7\*d^14\*e^6\*z^3 + 11520\*a^13\*c^2\*d^4\*e^16\*z^3 + 11520\*a^7\*c^8\*d^16\*e^4\*z^3 + 2560\*a^6\*c^9\*d^18\*e^2\*z^3 + 256\*a^5\*c^10\*d^20\*z^3 + 256\*a^15\*e^20\*z^3 - 4806\*B\*C\*a^8\*c\*d\*e^13\*z - 18\*A\*B\*a\*c^8\*d^13\*e\*z - 147930\*B\*C\*a^6\*c^3\*d^5\*e^9\*z + 74760\*B\*C\*a^5\*c^4\*d^7\*e^7\*z + 66588\*B\*C\*a^7\*c^2\*d^3\*e^11\*z - 1050\*B\*C\*a^4\*c^5\*d^9\*e^5\*z - 228\*B\*C\*a^3\*c^6\*d^11\*e^3\*z + 152052\*A\*C\*a^6\*c^3\*d^4\*e^10\*z - 109830\*A\*C\*a^5\*c^4\*d^6\*e^8\*z - 32490\*A\*C\*a^7\*c^2\*d^2\*e^12\*z + 426\*A\*C\*a^3\*c^6\*d^10\*e^4\*z - 360\*A\*C\*a^4\*c^5\*d^8\*e^6\*z + 180\*A\*C\*a^2\*c^7\*d^12\*e^2\*z + 158130\*A\*B\*a^5\*c^4\*d^5\*e^9\*z - 121356\*A\*B\*a^6\*c^3\*d^3\*e^11\*z - 3240\*A\*B\*a^4\*c^5\*d^7\*e^7\*z - 1710\*A\*B\*a^3\*c^6\*d^9\*e^5\*z - 396\*A\*B\*a^2\*c^7\*d^11\*e^3\*z - 6\*B\*C\*a^2\*c^7\*d^13\*e\*z + 13518\*A\*B\*a^7\*c^2\*d\*e^13\*z + 67615\*C^2\*a^6\*c^3\*d^6\*e^8\*z - 47538\*C^2\*a^7\*c^2\*d^4\*e^10\*z - 24860\*C^2\*a^5\*c^4\*d^8\*e^6\*z + 279\*C^2\*a^4\*c^5\*d^10\*e^4\*z + 46\*C^2\*a^3\*c^6\*d^12\*e^2\*z + 71415\*B^2\*a^6\*c^3\*d^4\*e^10\*z - 55260\*B^2\*a^5\*c^4\*d^6\*e^8\*z - 19602\*B^2\*a^7\*c^2\*d^2\*e^12\*z + 1215\*B^2\*a^4\*c^5\*d^8\*e^6\*z + 270\*B^2\*a^3\*c^6\*d^10\*e^4\*z + 9\*B^2\*a^2\*c^7\*d^12\*e^2\*z - 106722\*A^2\*a^5\*c^4\*d^4\*e^10\*z + 35217\*A^2\*a^6\*c^

$$\begin{aligned}
& 3*d^2*e^{12}*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 10 \\
& 71*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^{14}*z + 6*A*C*a*c^8*d^14*z + 70 \\
& 17*C^2*a^8*c*d^2*e^{12}*z + 126*A^2*a*c^8*d^{12}*e^2*z + C^2*a^2*c^7*d^{14}*z - 1 \\
& 728*A^2*a^7*c^2*e^{14}*z + 225*B^2*a^8*c*e^{14}*z + 9*A^2*c^9*d^{14}*z - 192*C^2* \\
& a^9*e^{14}*z + 3168*A*B*C*a^4*c^2*d*e^{10} + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B \\
& *C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 - \\
& 60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^{10} + 1512*A^2*B*a*c^5*d^5* \\
& e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^{10} - 5049*B^2*C*a^3*c^3 \\
& *d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B \\
& *C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4 \\
& + 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^ \\
& 4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 1 \\
& 4283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^ \\
& 3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^{10} - 1674*A^3*a*c^ \\
& 5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c \\
& ^2*e^{11} + 675*A*B^2*a^4*c^2*e^{11} - 225*B^2*C*a^5*c*e^{11} + 576*A*C^2*a^5*c*e \\
& ^{11} - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^ \\
& 8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2* \\
& c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^{11} - 64*C^3*a^6*e^{11} \\
& , z, k)*(root(2560*a^{14}*c*d^2*e^{18}*z^3 + 64512*a^{10}*c^5*d^{10}*e^{10}*z^3 + 537 \\
& 60*a^{11}*c^4*d^8*e^{12}*z^3 + 53760*a^9*c^6*d^{12}*e^8*z^3 + 30720*a^{12}*c^3*d^6* \\
& e^{14}*z^3 + 30720*a^8*c^7*d^{14}*e^6*z^3 + 11520*a^{13}*c^2*d^4*e^{16}*z^3 + 11520 \\
& *a^7*c^8*d^{16}*e^4*z^3 + 2560*a^6*c^9*d^{18}*e^2*z^3 + 256*a^5*c^{10}*d^{20}*z^3 + \\
& 256*a^{15}*e^{20}*z^3 - 4806*B*C*a^8*c*d*e^{13}*z - 18*A*B*a*c^8*d^{13}*e*z - 1479 \\
& 30*B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2* \\
& d^3*e^{11}*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^{11}*e^3*z + 1520 \\
& 52*A*C*a^6*c^3*d^4*e^{10}*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^ \\
& 2*d^2*e^{12}*z + 426*A*C*a^3*c^6*d^{10}*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180 \\
& *A*C*a^2*c^7*d^{12}*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3 \\
& *d^3*e^{11}*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396 \\
& *A*B*a^2*c^7*d^{11}*e^3*z - 6*B*C*a^2*c^7*d^{13}*e*z + 13518*A*B*a^7*c^2*d*e^{13} \\
& *z + 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^{10}*z - 24860*C^2 \\
& *a^5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^{10}*e^4*z + 46*C^2*a^3*c^6*d^{12}*e^2*z \\
& + 71415*B^2*a^6*c^3*d^4*e^{10}*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a \\
& ^7*c^2*d^2*e^{12}*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^{10}*e^4*z \\
& + 9*B^2*a^2*c^7*d^{12}*e^2*z - 106722*A^2*a^5*c^4*d^4*e^{10}*z + 35217*A^2*a^6 \\
& *c^3*d^2*e^{12}*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + \\
& 1071*A^2*a^2*c^7*d^{10}*e^4*z + 1152*A*C*a^8*c*e^{14}*z + 6*A*C*a*c^8*d^{14}*z + \\
& 7017*C^2*a^8*c*d^2*e^{12}*z + 126*A^2*a*c^8*d^{12}*e^2*z + C^2*a^2*c^7*d^{14}*z \\
& - 1728*A^2*a^7*c^2*e^{14}*z + 225*B^2*a^8*c*e^{14}*z + 9*A^2*c^9*d^{14}*z - 192*C \\
& ^2*a^9*e^{14}*z + 3168*A*B*C*a^4*c^2*d*e^{10} + 270*A*B*C*a*c^5*d^7*e^4 - 6930* \\
& A*B*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^ \\
& 5 - 60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^{10} + 1512*A^2*B*a*c^5*d \\
& ^5*e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^{10} - 5049*B^2*C*a^3* \\
& c^3*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 197
\end{aligned}$$

$$\begin{aligned}
& 7*B*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7* \\
& e^4 + 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2 \\
& *a^4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 \\
& + 14283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3 \\
& *c^3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a \\
& *c^5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^ \\
& 4*c^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5* \\
& c*e^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4 \\
& *d^8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a \\
& ^2*c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e \\
& ^11, z, k)*((512*a^13*c^2*d*e^18 + 512*a^5*c^10*d^17*e^2 + 4096*a^6*c^9*d^1 \\
& 5*e^4 + 14336*a^7*c^8*d^13*e^6 + 28672*a^8*c^7*d^11*e^8 + 35840*a^9*c^6*d^9 \\
& *e^10 + 28672*a^10*c^5*d^7*e^12 + 14336*a^11*c^4*d^5*e^14 + 4096*a^12*c^3*d \\
& ^3*e^16)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14 \\
& *e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56* \\
& a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(384*a^13*c^2*e^19 - 128*a^4 \\
& *c^11*d^18*e - 640*a^5*c^10*d^16*e^3 - 512*a^6*c^9*d^14*e^5 + 3584*a^7*c^8* \\
& d^12*e^7 + 12544*a^8*c^7*d^10*e^9 + 19712*a^9*c^6*d^8*e^11 + 17920*a^10*c^5 \\
& *d^6*e^13 + 9728*a^11*c^4*d^4*e^15 + 2944*a^12*c^3*d^2*e^17))/(64*(a^12*e^1 \\
& 6 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12 \\
& *e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28* \\
& a^10*c^2*d^4*e^12))) + (120*B*a^10*c^2*e^16 + 24*A*a^2*c^10*d^15*e + 456*A* \\
& a^9*c^3*d*e^15 + 8*C*a^3*c^9*d^15*e - 232*C*a^10*c^2*d*e^15 + 216*A*a^3*c^9 \\
& *d^13*e^3 + 1176*A*a^4*c^8*d^11*e^5 + 3480*A*a^5*c^7*d^9*e^7 + 5640*A*a^6*c \\
& ^6*d^7*e^9 + 5064*A*a^7*c^5*d^5*e^11 + 2376*A*a^8*c^4*d^3*e^13 - 24*B*a^3*c \\
& ^9*d^14*e^2 - 408*B*a^4*c^8*d^12*e^4 - 1560*B*a^5*c^7*d^10*e^6 - 2520*B*a^6 \\
& *c^6*d^8*e^8 - 1800*B*a^7*c^5*d^6*e^10 - 264*B*a^8*c^4*d^4*e^12 + 312*B*a^9 \\
& *c^3*d^2*e^14 + 200*C*a^4*c^8*d^13*e^3 + 648*C*a^5*c^7*d^11*e^5 + 520*C*a^6 \\
& *c^6*d^9*e^7 - 680*C*a^7*c^5*d^7*e^9 - 1512*C*a^8*c^4*d^5*e^11 - 1000*C*a^9 \\
& *c^3*d^3*e^13)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^ \\
& 7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 \\
& + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(192*C*a^10*c^2*e^16 - \\
& 576*A*a^9*c^3*e^16 + 1488*B*a^9*c^3*d*e^15 + 48*A*a^2*c^10*d^14*e^2 + 480* \\
& A*a^3*c^9*d^12*e^4 + 4176*A*a^4*c^8*d^10*e^6 + 12288*A*a^5*c^7*d^8*e^8 + 15 \\
& 312*A*a^6*c^6*d^6*e^10 + 7776*A*a^7*c^5*d^4*e^12 + 432*A*a^8*c^4*d^2*e^14 - \\
& 48*B*a^3*c^9*d^13*e^3 - 1824*B*a^4*c^8*d^11*e^5 - 5328*B*a^5*c^7*d^9*e^7 - \\
& 4032*B*a^6*c^6*d^7*e^9 + 2352*B*a^7*c^5*d^5*e^11 + 4320*B*a^8*c^4*d^3*e^13 \\
& + 16*C*a^3*c^9*d^14*e^2 + 1056*C*a^4*c^8*d^12*e^4 + 2160*C*a^5*c^7*d^10*e^ \\
& 6 - 1408*C*a^6*c^6*d^8*e^8 - 6672*C*a^7*c^5*d^6*e^10 - 5472*C*a^8*c^4*d^4*e \\
& ^12 - 1136*C*a^9*c^3*d^2*e^14))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^ \\
& 2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 7 \\
& 0*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12))) + (9*A^2* \\
& c^9*d^11*e^2 + 342*A^2*a^2*c^7*d^7*e^6 + 36*A^2*a^3*c^6*d^5*e^8 - 7479*A^2* \\
& a^4*c^5*d^3*e^10 + 9*B^2*a^2*c^7*d^9*e^4 - 108*B^2*a^3*c^6*d^7*e^6 - 3402*B \\
& ^2*a^4*c^5*d^5*e^8 + 5076*B^2*a^5*c^4*d^3*e^10 + C^2*a^2*c^7*d^11*e^2 - 36*
\end{aligned}$$



$$\begin{aligned}
& C^2 a^3 c^6 d^9 e^4 - 1306 C^2 a^4 c^5 d^7 e^6 + 4708 C^2 a^5 c^4 d^5 e^8 - \\
& 2943 C^2 a^6 c^3 d^3 e^{10} + 360 A B a^6 c^3 e^{13} - 120 B C a^7 c^2 e^{13} + \\
& 108 A^2 a c^8 d^9 e^4 + 1944 A^2 a^5 c^4 d e^{12} - 855 B^2 a^6 c^3 d e^{12} + \\
& 296 C^2 a^7 c^2 d e^{12} - 18 A B a^8 c^8 d^{10} e^3 + 6 A C a^8 c^8 d^{11} e^2 - 153 \\
& 6 A C a^6 c^3 d e^{12} + 756 A B a^3 c^6 d^6 e^7 + 11016 A B a^4 c^5 d^4 e^9 \\
& - 7794 A B a^5 c^4 d^2 e^{11} - 72 A C a^2 c^7 d^9 e^4 - 732 A C a^3 c^6 d^7 e^6 - \\
& 7368 A C a^4 c^5 d^5 e^8 + 10182 A C a^5 c^4 d^3 e^{10} - 6 B C a^2 c^7 \\
& d^{10} e^3 + 144 B C a^3 c^6 d^8 e^5 + 4284 B C a^4 c^5 d^6 e^7 - 10440 B C a^5 \\
& c^4 d^4 e^9 + 3738 B C a^6 c^3 d^2 e^{11}) / (64 (a^{12} e^{16} + a^4 c^8 d^{16} \\
& + 8 a^{11} c d^2 e^{14} + 8 a^5 c^7 d^{14} e^2 + 28 a^6 c^6 d^{12} e^4 + 56 a^7 c^5 \\
& d^{10} e^6 + 70 a^8 c^4 d^8 e^8 + 56 a^9 c^3 d^6 e^{10} + 28 a^{10} c^2 d^4 e^{12} \\
& )) + (x (225 B^2 a^6 c^3 e^{13} + 9 A^2 c^9 d^{10} e^3 - 162 A^2 a^2 c^7 d^6 e^7 \\
& - 2916 A^2 a^3 c^6 d^4 e^9 + 6561 A^2 a^4 c^5 d^2 e^{11} + 9 B^2 a^2 c^7 d^8 \\
& e^5 - 468 B^2 a^3 c^6 d^6 e^7 + 6174 B^2 a^4 c^5 d^4 e^9 - 2340 B^2 a^5 c^4 \\
& d^2 e^{11} + C^2 a^2 c^7 d^{10} e^3 - 116 C^2 a^3 c^6 d^8 e^5 + 3438 C^2 a^4 \\
& c^5 d^6 e^7 - 4292 C^2 a^5 c^4 d^4 e^9 + 1369 C^2 a^6 c^3 d^2 e^{11} + 108 A \\
& ^2 a c^8 d^8 e^5 - 18 A B a^8 c^8 d^9 e^4 + 2430 A B a^5 c^4 d e^{12} + 6 A C a^8 \\
& c^8 d^{10} e^3 - 1110 B C a^6 c^3 d e^{12} + 360 A B a^2 c^7 d^7 e^6 + 3204 A B \\
& a^3 c^6 d^5 e^8 - 13176 A B a^4 c^5 d^3 e^{10} - 312 A C a^2 c^7 d^8 e^5 - \\
& 2028 A C a^3 c^6 d^6 e^7 + 10728 A C a^4 c^5 d^4 e^9 - 5994 A C a^5 c^4 d^2 \\
& e^{11} - 6 B C a^2 c^7 d^9 e^4 + 504 B C a^3 c^6 d^7 e^6 - 9300 B C a^4 c^5 \\
& d^5 e^8 + 7512 B C a^5 c^4 d^3 e^{10})) / (64 (a^{12} e^{16} + a^4 c^8 d^{16} + 8 a^4 \\
& c^8 d^{16} + 8 a^{11} c d^2 e^{14} + 8 a^5 c^7 d^{14} e^2 + 28 a^6 c^6 d^{12} e^4 + 56 a^7 c^5 \\
& d^{10} e^6 + 70 a^8 c^4 d^8 e^8 + 56 a^9 c^3 d^6 e^{10} + 28 a^{10} c^2 d^4 e^{12}))) * \text{root} \\
& (2560 a^{14} c d^2 e^{18} z^3 + 64512 a^{10} c^5 d^{10} e^{10} z^3 + 53760 a^{11} c^4 \\
& d^8 e^{12} z^3 + 53760 a^9 c^6 d^{12} e^8 z^3 + 30720 a^{12} c^3 d^6 e^{14} z^3 + 3 \\
& 0720 a^8 c^7 d^{14} e^6 z^3 + 11520 a^{13} c^2 d^4 e^{16} z^3 + 11520 a^7 c^8 d^{16} \\
& e^4 z^3 + 2560 a^6 c^9 d^{18} e^2 z^3 + 256 a^5 c^{10} d^{20} z^3 + 256 a^{15} e^{20} z^3 \\
& - 4806 B C a^8 c d e^{13} z - 18 A B a^8 c^8 d^{13} e z - 147930 B C a^6 c^3 \\
& d^5 e^9 z + 74760 B C a^5 c^4 d^7 e^7 z + 66588 B C a^7 c^2 d^3 e^{11} z - \\
& 1050 B C a^4 c^5 d^9 e^5 z - 228 B C a^3 c^6 d^{11} e^3 z + 152052 A C a^6 c^3 \\
& d^4 e^{10} z - 109830 A C a^5 c^4 d^6 e^8 z - 32490 A C a^7 c^2 d^2 e^{12} z \\
& + 426 A C a^3 c^6 d^{10} e^4 z - 360 A C a^4 c^5 d^8 e^6 z + 180 A C a^2 c^7 \\
& d^{12} e^2 z + 158130 A B a^5 c^4 d^5 e^9 z - 121356 A B a^6 c^3 d^3 e^{11} z \\
& - 3240 A B a^4 c^5 d^7 e^7 z - 1710 A B a^3 c^6 d^9 e^5 z - 396 A B a^2 c^7 \\
& d^{11} e^3 z - 6 B C a^2 c^7 d^{13} e z + 13518 A B a^7 c^2 d e^{13} z + 67615 C \\
& ^2 a^6 c^3 d^6 e^8 z - 47538 C^2 a^7 c^2 d^4 e^{10} z - 24860 C^2 a^5 c^4 d^8 \\
& e^6 z + 279 C^2 a^4 c^5 d^{10} e^4 z + 46 C^2 a^3 c^6 d^{12} e^2 z + 71415 B^2 \\
& a^6 c^3 d^4 e^{10} z - 55260 B^2 a^5 c^4 d^6 e^8 z - 19602 B^2 a^7 c^2 d^2 e^{12} z \\
& + 1215 B^2 a^4 c^5 d^8 e^6 z + 270 B^2 a^3 c^6 d^{10} e^4 z + 9 B^2 a^2 \\
& c^7 d^{12} e^2 z - 106722 A^2 a^5 c^4 d^4 e^{10} z + 35217 A^2 a^6 c^3 d^2 e^{12} z \\
& + 6615 A^2 a^4 c^5 d^6 e^8 z + 3780 A^2 a^3 c^6 d^8 e^6 z + 1071 A^2 a^2 \\
& c^7 d^{10} e^4 z + 1152 A C a^8 c e^{14} z + 6 A C a^8 c^8 d^{14} z + 7017 C^2 a^8 \\
& c^8 d^2 e^{12} z + 126 A^2 a^8 c^8 d^{12} e^2 z + C^2 a^2 c^7 d^{14} z - 1728 A^2 a^7 \\
& c^2 e^{14} z + 225 B^2 a^8 c e^{14} z + 9 A^2 c^9 d^{14} z - 192 C^2 a^9 e^{14} z
\end{aligned}$$

$$\begin{aligned}
& z + 3168*A*B*C*a^4*c^2*d*e^{10} + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 - 60*A*C^2 \\
& *a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^{10} + 1512*A^2*B*a*c^5*d^5*e^6 - 270 \\
& *A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^{10} - 5049*B^2*C*a^3*c^3*d^4*e^7 \\
& + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B*C^2*a^4* \\
& c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4 + 15921* \\
& A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^4*c^2*d^2 \\
& *e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 14283*A^2* \\
& B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^3*d^2*e^9 \\
& + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^{10} - 1674*A^3*a*c^5*d^4*e^7 \\
& - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c^2*e^{11} + \\
& 675*A*B^2*a^4*c^2*e^{11} - 225*B^2*C*a^5*c*e^{11} + 576*A*C^2*a^5*c*e^{11} - 397 \\
& *C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^8*e^3 + 3 \\
& 294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*c^4*d^2*e \\
& ^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^{11} - 64*C^3*a^6*e^{11}, z, k), \\
& k, 1, 3)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(e\*x+d)\*\*3/(c\*x\*\*2+a)\*\*3,x)

[Out] Timed out

$$3.64 \quad \int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4} dx$$

Optimal. Leaf size=234

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) (cd(4aBe + aCd + 5Acd) + ae^2(5aC + Ac))}{16a^{7/2}c^{7/2}} - \frac{(d+ex)(ae-cdx) (cd(4aBe + aCd + 5Acd) + ae^2(5aC + Ac))}{16a^3c^3(a+cx^2)}$$

Rubi [A] time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1645, 805, 723, 205}

$$\frac{(ae^2 + cd^2) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) (cd(4aBe + aCd + 5Acd) + ae^2(5aC + Ac))}{16a^{7/2}c^{7/2}} - \frac{(d+ex)(ae-cdx) (cd(4aBe + aCd + 5Acd) + ae^2(5aC + Ac))}{16a^3c^3(a+cx^2)} - \frac{(d+ex)^3(ae(5aC + Ac) - cx(4aBe + aCd + 5Acd))}{24a^2c^2(a+cx^2)^2} - \frac{(d+ex)^4(aB - x(Ac - aC))}{6ac(a+cx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^4\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^4)/(6\*a\*c\*(a + c\*x^2)^3) - ((d + e\*x)^3\*(a\*(A\*c + 5\*a\*C)\*e - c\*(5\*A\*c\*d + a\*C\*d + 4\*a\*B\*e)\*x))/(24\*a^2\*c^2\*(a + c\*x^2)^2) - ((a\*(A\*c + 5\*a\*C)\*e^2 + c\*d\*(5\*A\*c\*d + a\*C\*d + 4\*a\*B\*e))\*(a\*e - c\*d\*x)\*(d + e\*x))/(16\*a^3\*c^3\*(a + c\*x^2)) + ((c\*d^2 + a\*e^2)\*(a\*(A\*c + 5\*a\*C)\*e^2 + c\*d\*(5\*A\*c\*d + a\*C\*d + 4\*a\*B\*e))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(16\*a^(7/2)\*c^(7/2))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 723

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m-1)\*(a\*e - c\*d\*x)\*(a + c\*x^2)^(p+1))/(2\*a\*c\*(p+1)), x] + Dist[((2\*p+3)\*(c\*d^2 + a\*e^2))/(2\*a\*c\*(p+1)), Int[(d + e\*x)^(m-2)\*(a + c\*x^2)^(p+1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 2, 0] && LtQ[p, -1]

Rule 805

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m\*(a + c\*x^2)^(p+1)\*(a\*g - c\*f\*x))/(2\*a\*c\*(p+1)), x] - Dist[(m\*(c\*d\*f + a\*e\*g))/(2\*a\*c\*(p+1)), Int[(d + e\*x)^(m

- 1)\*(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0] && LtQ[p, -1]

### Rule 1645

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\int \frac{(d + ex)^4 (A + Bx + Cx^2)}{(a + cx^2)^4} dx = -\frac{(aB - (Ac - aC)x)(d + ex)^4}{6ac(a + cx^2)^3} - \frac{\int \frac{(d+ex)^3(-5Acd - aCd - 4aBe - (Ac+5aC)ex)}{(a+cx^2)^3} dx}{6ac}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)^4}{6ac(a + cx^2)^3} - \frac{(d + ex)^3(a(Ac + 5aC)e - c(5Acd + aCd + 4aBe))}{24a^2c^2(a + cx^2)^2}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)^4}{6ac(a + cx^2)^3} - \frac{(d + ex)^3(a(Ac + 5aC)e - c(5Acd + aCd + 4aBe))}{24a^2c^2(a + cx^2)^2}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)^4}{6ac(a + cx^2)^3} - \frac{(d + ex)^3(a(Ac + 5aC)e - c(5Acd + aCd + 4aBe))}{24a^2c^2(a + cx^2)^2}$$

**Mathematica [A]** time = 0.31, size = 437, normalized size = 1.87

$\frac{(a^2 + c^2) \operatorname{atan}\left(\frac{d+ex}{\sqrt{a+cx^2}}\right) (Ac(a^2+5aC) + a(5Ac^2+4aBe+4aC))}{16a^2c^2} - \frac{a^2c^2(Bd + 5Cd + 11C^2) + a^2c^2(Ac + 5aC)(4aBe + 4aC^2) + a^2c^2(Acd^2 + 4aBe + 4aC^2) + 5a^2c^2Ae}{16a^2c^2(a+cx^2)^3} - \frac{a^2c^2(Bd + 4Cd + Cx) + a^2c^2(Acd + ex) + 2Bd(d+2cx) + 2C^2(d+3cx)}{6a^2c^2(a+cx^2)^2} - \frac{a^2c^2(4aBe + 4aC^2) + B(d+4cx) + C(Ac) + A^2c^2}{24a^2c^2(a+cx^2)^2} - \frac{a^2c^2(5Acd + 4aCd + 11C^2) + a^2c^2(Ac + 5aC)(4aBe + 4aC^2) + a^2c^2(Acd^2 + 4aBe + 4aC^2) + 5a^2c^2Ae}{24a^2c^2(a+cx^2)^2}$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^4\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] (5\*A\*c^3\*d^4\*x + a\*c^2\*d^2\*(C\*d^2 + 4\*B\*d\*e + 6\*A\*e^2)\*x + a^2\*c\*e^2\*(6\*C\*d^2 + e\*(4\*B\*d + A\*e))\*x - a^3\*e^3\*(32\*C\*d + 8\*B\*e + 11\*C\*e\*x))/(16\*a^3\*c^3\*

$$\begin{aligned} & (a + cx^2)) + (A^3c^3d^4x - a^3e^3(4Cd + Be + Cex) - a^2c^2d^2(4Ade + C^2d^2x + 6Ae^2x + B^2d(d + 4ex)) + a^2c^2e(2C^2d^2(2d + 3ex) + e(Ae(4d + ex) + 2B^2d(3d + 2ex))))/(6a^3c^3(a + cx^2)^3) \\ & + (5A^3c^3d^4x + a^2c^2d^2(C^2d^2 + 4B^2de + 6Ae^2)x + a^3e^3(48Cd + 12Be + 13Cex) - a^2c^2e(6C^2d^2(4d + 7ex) + e(4B^2d(9d + 7ex) + Ae(24d + 7ex))))/(24a^2c^3(a + cx^2)^2) + ((c^2d^2 + ae^2) * (A^2c^2(5c^2d^2 + ae^2) + a^2(5a^2C^2e^2 + cd(Cd + 4Be))) * ArcTan[(Sqrt[c] * x) / Sqrt[a]]) / (16a^(7/2) * c^(7/2)) \end{aligned}$$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^4 (A + Bx + Cx^2)}{(a + cx^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + ex)^4\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] IntegrateAlgebraic[((d + ex)^4\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

**fricas [B]** time = 1.55, size = 1864, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((ex+d)^4\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="fricas")

[Out] [-1/96\*(16\*B\*a^4\*c^3\*d^4 + 48\*B\*a^5\*c^2\*d^2\*e^2 + 16\*B\*a^6\*c\*e^4 - 6\*(4\*B\*a^2\*c^5\*d^3\*e + 4\*B\*a^3\*c^4\*d\*e^3 + (C\*a^2\*c^5 + 5\*A\*a\*c^6)\*d^4 + 6\*(C\*a^3\*c^4 + A\*a^2\*c^5)\*d^2\*e^2 - (11\*C\*a^4\*c^3 - A\*a^3\*c^4)\*e^4)\*x^5 + 32\*(C\*a^5\*c^2 + 2\*A\*a^4\*c^3)\*d^3\*e + 32\*(2\*C\*a^6\*c + A\*a^5\*c^2)\*d\*e^3 + 48\*(4\*C\*a^4\*c^3\*d\*e^3 + B\*a^4\*c^3\*e^4)\*x^4 - 16\*(4\*B\*a^3\*c^4\*d^3\*e - 4\*B\*a^4\*c^3\*d\*e^3 + (C\*a^3\*c^4 + 5\*A\*a^2\*c^5)\*d^4 - 6\*(C\*a^4\*c^3 - A\*a^3\*c^4)\*d^2\*e^2 - (5\*C\*a^5\*c^2 + A\*a^4\*c^3)\*e^4)\*x^3 + 48\*(2\*C\*a^4\*c^3\*d^3\*e + 3\*B\*a^4\*c^3\*d^2\*e^2 + B\*a^5\*c^2\*e^4 + 2\*(2\*C\*a^5\*c^2 + A\*a^4\*c^3)\*d\*e^3)\*x^2 + 3\*(4\*B\*a^4\*c^2\*d^3\*e + 4\*B\*a^5\*c\*d\*e^3 + (4\*B\*a\*c^5\*d^3\*e + 4\*B\*a^2\*c^4\*d\*e^3 + (C\*a\*c^5 + 5\*A\*c^6)\*d^4 + 6\*(C\*a^2\*c^4 + A\*a\*c^5)\*d^2\*e^2 + (5\*C\*a^3\*c^3 + A\*a^2\*c^4)\*e^4)\*x^6 + (C\*a^4\*c^2 + 5\*A\*a^3\*c^3)\*d^4 + 6\*(C\*a^5\*c + A\*a^4\*c^2)\*d^2\*e^2 + (5\*C\*a^6 + A\*a^5\*c)\*e^4 + 3\*(4\*B\*a^2\*c^4\*d^3\*e + 4\*B\*a^3\*c^3\*d\*e^3 + (C\*a^2\*c^4 + 5\*A\*a\*c^5)\*d^4 + 6\*(C\*a^3\*c^3 + A\*a^2\*c^4)\*d^2\*e^2 + (5\*C\*a^4\*c^2 + A\*a^3\*c^3)\*e^4)\*x^4 + 3\*(4\*B\*a^3\*c^3\*d^3\*e + 4\*B\*a^4\*c^2\*d\*e^3 + (C\*a^3\*c^3 + 5\*A\*a^2\*c^4)\*d^4 + 6\*(C\*a^4\*c^2 + A\*a^3\*c^3)\*d^2\*e^2 + (5\*C\*a^5\*c + A\*a^4\*c^2)\*e^4)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 6\*(4\*B\*a^4\*c^3\*d^3\*e + 4\*B\*a^5\*c^2\*d\*e^3 + (C\*a^4\*c^3 - 11\*A\*a^3\*c^4)\*d^4 + 6\*(C\*a^5\*c^2 + A\*a^4\*c^3)\*d^2\*e^2 + (5\*C\*a^6\*c + A\*a^5\*c^2)\*e^4)\*x)/(a^4\*c^7\*x^6 + 3\*a^5\*c^6\*x^4 + 3\*a^6\*c^5\*x^2 + a^7\*c^4), -1/48\*(8\*B\*a^4\*c^3\*d^4

```

+ 24*B*a^5*c^2*d^2*e^2 + 8*B*a^6*c*e^4 - 3*(4*B*a^2*c^5*d^3*e + 4*B*a^3*c^
4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c^4 + A*a^2*c^5)*d^2*e^2 -
(11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 16*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3*e +
16*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 24*(4*C*a^4*c^3*d*e^3 + B*a^4*c^3*e^4)*
x^4 - 8*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 + (C*a^3*c^4 + 5*A*a^2*c^5)*
d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^5*c^2 + A*a^4*c^3)*e^4)*x^
3 + 24*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 + B*a^5*c^2*e^4 + 2*(2*C*a^
5*c^2 + A*a^4*c^3)*d*e^3)*x^2 - 3*(4*B*a^4*c^2*d^3*e + 4*B*a^5*c*d*e^3 + (4
*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5*A*c^6)*d^4 + 6*(C*a^2*c^4
+ A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e^4)*x^6 + (C*a^4*c^2 + 5*A
*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 + (5*C*a^6 + A*a^5*c)*e^4 +
3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^4 + 6
*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 + A*a^3*c^3)*e^4)*x^4 + 3*(
4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^4 + 6*(
C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a^4*c^2)*e^4)*x^2)*sqrt(a*c
)*arctan(sqrt(a*c)*x/a) + 3*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4
*c^3 - 11*A*a^3*c^4)*d^4 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c +
A*a^5*c^2)*e^4)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4)
]

```

**giac [B]** time = 0.18, size = 636, normalized size = 2.72

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^4\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="giac")

```

[Out] 1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*A
*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^5 +
12*B*a*c^4*d^3*x^5*e + 18*C*a^2*c^3*d^2*x^5*e^2 + 18*A*a*c^4*d^2*x^5*e^2 +
8*C*a^2*c^3*d^4*x^3 + 40*A*a*c^4*d^4*x^3 + 12*B*a^2*c^3*d*x^5*e^3 + 32*B*a
^2*c^3*d^3*x^3*e - 33*C*a^3*c^2*x^5*e^4 + 3*A*a^2*c^3*x^5*e^4 - 96*C*a^3*c^
2*d*x^4*e^3 - 48*C*a^3*c^2*d^2*x^3*e^2 + 48*A*a^2*c^3*d^2*x^3*e^2 - 48*C*a^
3*c^2*d^3*x^2*e - 3*C*a^3*c^2*d^4*x + 33*A*a^2*c^3*d^4*x - 24*B*a^3*c^2*x^4
*e^4 - 32*B*a^3*c^2*d*x^3*e^3 - 72*B*a^3*c^2*d^2*x^2*e^2 - 12*B*a^3*c^2*d^3
*x*e - 8*B*a^3*c^2*d^4 - 40*C*a^4*c*x^3*e^4 - 8*A*a^3*c^2*x^3*e^4 - 96*C*a^
4*c*d*x^2*e^3 - 48*A*a^3*c^2*d*x^2*e^3 - 18*C*a^4*c*d^2*x*e^2 - 18*A*a^3*c^
2*d^2*x*e^2 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a^4*c*x^2*e^4 -
12*B*a^4*c*d*x*e^3 - 24*B*a^4*c*d^2*e^2 - 15*C*a^5*x*e^4 - 3*A*a^4*c*x*e^4
- 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 + a)^3*a^3*c^3)

```

**maple [B]** time = 0.01, size = 647, normalized size = 2.76

---

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4, x)$

[Out]  $(1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+4*C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^3-4*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1/2*e*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*A*c^2*d^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*e)/c^3)/(c*x^2+a)^3+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^4+3/8/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2*e^2+5/16/a^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^4+1/4/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d*e^3+1/4/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^3*e+5/16/c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^4+3/8/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^2*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^4$

**maxima [B]** time = 1.04, size = 599, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4, x, \text{algorithm}="maxima")$

[Out]  $-1/48*(8*B*a^3*c^2*d^4 + 24*B*a^4*c*d^2*e^2 + 8*B*a^5*e^4 - 3*(4*B*a*c^4*d^3*e + 4*B*a^2*c^3*d*e^3 + (C*a*c^4 + 5*A*c^5)*d^4 + 6*(C*a^2*c^3 + A*a*c^4)*d^2*e^2 - (11*C*a^3*c^2 - A*a^2*c^3)*e^4)*x^5 + 16*(C*a^4*c + 2*A*a^3*c^2)*d^3*e + 16*(2*C*a^5 + A*a^4*c)*d*e^3 + 24*(4*C*a^3*c^2*d*e^3 + B*a^3*c^2*e^4)*x^4 - 8*(4*B*a^2*c^3*d^3*e - 4*B*a^3*c^2*d*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^4 - 6*(C*a^3*c^2 - A*a^2*c^3)*d^2*e^2 - (5*C*a^4*c + A*a^3*c^2)*e^4)*x^3 + 24*(2*C*a^3*c^2*d^3*e + 3*B*a^3*c^2*d^2*e^2 + B*a^4*c*e^4 + 2*(2*C*a^4*c + A*a^3*c^2)*d*e^3)*x^2 + 3*(4*B*a^3*c^2*d^3*e + 4*B*a^4*c*d*e^3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^4 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e^2 + (5*C*a^5 + A*a^4*c)*e^4)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/16*(4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3 + (C*a*c^2 + 5*A*c^3)*d^4 + 6*(C*a^2*c + A*a*c^2)*d^2*e^2 + (5*C*a^3 + A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^3)$

**mupad [B]** time = 4.38, size = 669, normalized size = 2.86

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x)`

[Out] 
$$\left( \operatorname{atan}\left(\frac{c^{1/2} x (a e^2 + c d^2) (5 A c^2 d^2 + 5 C a^2 e^2 + A a c e^2 + C a c d^2 + 4 B a c d e)}{a^{1/2} (5 A c^3 d^4 + 5 C a^3 e^4 + A a^2 c e^4 + C a c^2 d^4 + 6 A a c^2 d^2 e^2 + 6 C a^2 c d^2 e^2 + 4 B a c^2 d^3 e + 4 B a^2 c d e^3)}\right) \right) \cdot (a e^2 + c d^2) \cdot (5 A c^2 d^2 + 5 C a^2 e^2 + A a c e^2 + C a c d^2 + 4 B a c d e) \Big/ (16 a^{7/2} c^{7/2}) - \left( (B a^2 e^4 + B c^2 d^4 + 4 A c^2 d^3 e + 4 C a^2 d e^3 + 2 A a c d e^3 + 2 C a c d^3 e + 3 B a c d^2 e^2) / (6 c^3) + (x^2 (B a e^4 + 2 A c d e^3 + 4 C a d e^3 + 2 C c d^3 e + 3 B c d^2 e^2)) / (2 c^2) + (x^4 (B e^4 + 4 C d e^3)) / (2 c) + (x (5 C a^3 e^4 - 11 A c^3 d^4 + A a^2 c e^4 + C a c^2 d^4 + 6 A a c^2 d^2 e^2 + 6 C a^2 c d^2 e^2 + 4 B a c^2 d^3 e + 4 B a^2 c d e^3)) / (16 a c^3) - (x^3 (5 A c^3 d^4 - 5 C a^3 e^4 - A a^2 c e^4 + C a c^2 d^4 + 6 A a c^2 d^2 e^2 - 6 C a^2 c d^2 e^2 + 4 B a c^2 d^3 e - 4 B a^2 c d e^3)) / (6 a^2 c^2) - (x^5 (5 A c^3 d^4 - 11 C a^3 e^4 + A a^2 c e^4 + C a c^2 d^4 + 6 A a c^2 d^2 e^2 + 6 C a^2 c d^2 e^2 + 4 B a c^2 d^3 e + 4 B a^2 c d e^3)) / (16 a^3 c) \right) \Big/ (a^3 + c^3 x^6 + 3 a^2 c x^2 + 3 a c^2 x^4)$$

sympy [F(-1)]    time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)`

[Out] Timed out



$$3.65 \quad \int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4} dx$$

**Optimal.** Leaf size=254

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(3ae^2+5cd^2)+a(ae^2(Be+3Cd)+cd^2(3Be+Cd))\right)}{16a^{7/2}c^{5/2}} - \frac{(d+ex)(ae(3aBe+aCd+5Acd)-x)}{48a^3}$$

**Rubi [A]** time = 0.54, antiderivative size = 288, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1645, 821, 778, 205}

$$\frac{4ac(Ac(ae^2+5cd^2)+a(2ac^2+cd(3Be+Cd)))-cx(Acd(15cd^2-ae^2)+a(ae^2(7Cd-3Be)+3cd^2(3Be+Cd))}{48a^3c^3(a+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(3ae^2+5cd^2)+a(ae^2(Be+3Cd)+cd^2(3Be+Cd))\right)}{16a^{7/2}c^{5/2}} - \frac{(d+ex)^2(2ac(2aC+Ac)-cx(3aBe+aCd+5Acd))}{24a^2c^2(a+cx^2)^2} - \frac{(d+cx)^2(aB-x(Ac-aC))}{6ac(a+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^3)/(6\*a\*c\*(a + c\*x^2)^3) - ((d + e\*x)^2\*(2\*a\*(A\*c + 2\*a\*C)\*e - c\*(5\*A\*c\*d + a\*C\*d + 3\*a\*B\*e)\*x)/(24\*a^2\*c^2\*(a + c\*x^2)^2) - (4\*a\*e\*(A\*c\*(5\*c\*d^2 + a\*e^2) + a\*(2\*a\*C\*e^2 + c\*d\*(C\*d + 3\*B\*e))) - c\*(A\*c\*d\*(15\*c\*d^2 - a\*e^2) + a\*(a\*e^2\*(7\*C\*d - 3\*B\*e) + 3\*c\*d^2\*(C\*d + 3\*B\*e)))\*x)/(48\*a^3\*c^3\*(a + c\*x^2)) + ((A\*c\*d\*(5\*c\*d^2 + 3\*a\*e^2) + a\*(a\*e^2\*(3\*C\*d + B\*e) + c\*d^2\*(C\*d + 3\*B\*e)))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(16\*a^(7/2)\*c^(5/2))

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 778

Int[((d\_) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((a\*(e\*f + d\*g) - (c\*d\*f - a\*e\*g)\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(2\*a\*c\*(p + 1)), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]

### Rule 821

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^m\*(a + c\*x^2)^(p + 1)\*(a\*g - c\*f\*x))/(2\*a\*c\*(p + 1)), x] - Dist[1/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1)\*Simp[a\*e\*g\*m - c\*d\*f\*(2\*p + 3) - c\*e\*f\*(m + 2\*p + 3)\*x, x], x], x]

```

/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

### Rule 1645

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x)/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4} dx &= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{6ac(a+cx^2)^3} - \frac{\int \frac{(d+ex)^2(-5Acd - aCd - 3aBe - 2(Ac+2aC)ex)}{(a+cx^2)^3} dx}{6ac} \\
&= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{6ac(a+cx^2)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e - c(5Acd + aCd + 3aBe))}{24a^2c^2(a+cx^2)^2} \\
&= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{6ac(a+cx^2)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e - c(5Acd + aCd + 3aBe))}{24a^2c^2(a+cx^2)^2} \\
&= -\frac{(aB - (Ac - aC)x)(d+ex)^3}{6ac(a+cx^2)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e - c(5Acd + aCd + 3aBe))}{24a^2c^2(a+cx^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 350, normalized size = 1.38

$$\frac{3\sqrt{c} \sqrt{a^2c^2 - 2c^2d(Bc+3Cd) - a^2d(3c(A+Bd+Cd^2) - 5Ac^2d^2)} \sqrt{a^2c^2 - 2c^2d(Ac+3Bd+3Cd+3c)} + a^2(3A(d+cx) + B(d+3cx) + Cd^2) - Ac^2d^2}{48a^2c^3} + \frac{2a^2(12a^2c^2 - a^2c(6Ac+18Bd+7Bc)+3C(6d+7cx)+a^2d(3c(A+Bd+Cd^2)+5Ac^2d^2)}{(a+cx^2)^2} + 3\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right) (Acd(3ae^2 + 5cd^2) + a(a^2(Bc+3Cd) + cd^2(3Bc+Cd)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4, x]
```

```
[Out] ((-3*sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e))*x - a*c
^2*d*(C*d^2 + 3*e*(B*d + A*e))*x)/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 - A*
```

$$c^3 d^3 x + a c^2 d (C d^2 x + 3 A e (d + e x) + B d (d + 3 e x)) - a^2 c e (3 C d (d + e x) + e (3 B d + A e + B e x)) / (a + c x^2)^3 + (2 a^{3/2}) (12 a^3 C e^3 + 5 A c^3 d^3 x + a c^2 d (C d^2 + 3 e (B d + A e)) x - a^2 c e (3 C d (6 d + 7 e x) + e (18 B d + 6 A e + 7 B e x))) / (a + c x^2)^2 + 3 \sqrt{c} (A c d (5 c d^2 + 3 a e^2) + a (a e^2 (3 C d + B e) + c d^2 (C d + 3 B e))) \operatorname{ArcTan}[\sqrt{c} x / \sqrt{a}] / (48 a^{7/2} c^3)$$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3 (A + Bx + Cx^2)}{(a + cx^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4,x]

[Out] IntegrateAlgebraic[((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

**fricas** [B] time = 1.67, size = 1378, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="fricas")

[Out] [-1/96\*(48\*C\*a^4\*c^2\*e^3\*x^4 + 16\*B\*a^4\*c^2\*d^3 + 24\*B\*a^5\*c\*d\*e^2 - 6\*(3\*B\*a^2\*c^4\*d^2\*e + B\*a^3\*c^3\*e^3 + (C\*a^2\*c^4 + 5\*A\*a\*c^5)\*d^3 + 3\*(C\*a^3\*c^3 + A\*a^2\*c^4)\*d\*e^2)\*x^5 + 24\*(C\*a^5\*c + 2\*A\*a^4\*c^2)\*d^2\*e + 8\*(2\*C\*a^6 + A\*a^5\*c)\*e^3 - 16\*(3\*B\*a^3\*c^3\*d^2\*e - B\*a^4\*c^2\*e^3 + (C\*a^3\*c^3 + 5\*A\*a^2\*c^4)\*d^3 - 3\*(C\*a^4\*c^2 - A\*a^3\*c^3)\*d\*e^2)\*x^3 + 24\*(3\*C\*a^4\*c^2\*d^2\*e + 3\*B\*a^4\*c^2\*d\*e^2 + (2\*C\*a^5\*c + A\*a^4\*c^2)\*e^3)\*x^2 + 3\*(3\*B\*a^4\*c\*d^2\*e + B\*a^5\*e^3 + (3\*B\*a\*c^4\*d^2\*e + B\*a^2\*c^3\*e^3 + (C\*a\*c^4 + 5\*A\*c^5)\*d^3 + 3\*(C\*a^2\*c^3 + A\*a\*c^4)\*d\*e^2)\*x^6 + 3\*(3\*B\*a^2\*c^3\*d^2\*e + B\*a^3\*c^2\*e^3 + (C\*a^2\*c^3 + 5\*A\*a\*c^4)\*d^3 + 3\*(C\*a^3\*c^2 + A\*a^2\*c^3)\*d\*e^2)\*x^4 + (C\*a^4\*c + 5\*A\*a^3\*c^2)\*d^3 + 3\*(C\*a^5 + A\*a^4\*c)\*d\*e^2 + 3\*(3\*B\*a^3\*c^2\*d^2\*e + B\*a^4\*c\*e^3 + (C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*d^3 + 3\*(C\*a^4\*c + A\*a^3\*c^2)\*d\*e^2)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 6\*(3\*B\*a^4\*c^2\*d^2\*e + B\*a^5\*c\*e^3 + (C\*a^4\*c^2 - 11\*A\*a^3\*c^3)\*d^3 + 3\*(C\*a^5\*c + A\*a^4\*c^2)\*d\*e^2)\*x)/(a^4\*c^6\*x^6 + 3\*a^5\*c^5\*x^4 + 3\*a^6\*c^4\*x^2 + a^7\*c^3), -1/48\*(24\*C\*a^4\*c^2\*e^3\*x^4 + 8\*B\*a^4\*c^2\*d^3 + 12\*B\*a^5\*c\*d\*e^2 - 3\*(3\*B\*a^2\*c^4\*d^2\*e + B\*a^3\*c^3\*e^3 + (C\*a^2\*c^4 + 5\*A\*a\*c^5)\*d^3 + 3\*(C\*a^3\*c^3 + A\*a^2\*c^4)\*d\*e^2)\*x^5 + 12\*(C\*a^5\*c + 2\*A\*a^4\*c^2)\*d^2\*e + 4\*(2\*C\*a^6 + A\*a^5\*c)\*e^3 - 8\*(3\*B\*a^3\*c^3\*d^2\*e - B\*a^4\*c^2\*e^3 + (C\*a^3\*c^3 + 5\*A\*a^2\*c^4)\*d^3 - 3\*(C\*a^4\*c^2 - A\*a^3\*c^3)\*d\*e^2)\*x^3 + 12\*(3\*C\*a^4\*c^2\*d^2\*e + 3\*B\*a^4\*c^2\*d\*e^2 + (2\*C\*a^5\*c + A\*a^4\*c^2)\*e^3)\*x^2 - 3\*(3\*B\*a^4\*c\*d^2\*e + B\*a^5\*e^3 + (3\*B\*a\*c^4\*d^2\*e + B\*a^2\*c^3\*e^3 + (C\*a\*c^4 + 5\*A\*c^5)\*d^3 + 3\*



**maxima [A]** time = 1.02, size = 457, normalized size = 1.80

$$\frac{24C^2d^2e^4 + 8Bd^2e^2 + 12Bd^2e^2 - 3(8Bd^2e^2 + Bd^2e^2 + (C^2 + 5A^2)d^2 - 3(C^2d^2 + A^2d^2)^2 + 12(C^2d^2 + 2Ad^2)e^2 + 4(2C^2d^2 + Ad^2)^2 - 8(3Bd^2e^2 - Bd^2e^2 + (C^2d^2 + 5A^2d^2)^2 - 3(C^2d^2 - Ad^2)e^2)^2 + 12(2C^2d^2e^2 + 3Bd^2e^2 + (2C^2d^2 + Ad^2)e^2)^2 + 3(3Bd^2e^2 + Bd^2e^2 + (C^2d^2 - 11Ad^2)e^2 + 3(C^2d^2 + Ad^2)e^2)^2}{48(d^2e^2 + 3A^2d^2 + 3d^2e^2 + e^4)} \cdot \frac{(3Bd^2e^2 + Bd^2e^2 + (C^2 + 5A^2)d^2 + 3(C^2 + A^2)d^2) \operatorname{atan}\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="maxima")

[Out] 
$$-1/48*(24*C*a^3*c^2*e^3*x^4 + 8*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 - 3*(3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*(C*a^2*c^3 + A*a*c^4)*d^2*e^2)*x^5 + 12*(C*a^4*c + 2*A*a^3*c^2)*d^2*e^2 + 4*(2*C*a^5 + A*a^4*c)*e^3 - 8*(3*B*a^2*c^3*d^2*e - B*a^3*c^2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3 - 3*(C*a^3*c^2 - A*a^2*c^3)*d^2*e^2)*x^3 + 12*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d^2*e^2 + (2*C*a^4*c + A*a^3*c^2)*e^3)*x^2 + 3*(3*B*a^3*c^2*d^2*e + B*a^4*c*e^3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d^2*e^2)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/16*(3*B*a*c*d^2*e + B*a^2*e^3 + (C*a*c + 5*A*c^2)*d^3 + 3*(C*a^2 + A*a*c)*d^2*e^2)*\operatorname{arctan}(c*x/\operatorname{sqrt}(a*c))/(\operatorname{sqrt}(a*c)*a^3*c^2)$$

**mupad [B]** time = 4.07, size = 402, normalized size = 1.58

$$\operatorname{atan}\left(\frac{cx}{\sqrt{ac}}\right) \frac{(3C^2d^2e^2 + B^2d^2 + C^2ac^2 + 3B^2ac^2 + 3A^2cd^2 + 5A^2d^2)}{16B^2c^2d^2} - \frac{2(C^2d^2 + 2C^2d^2 + 3B^2d^2 + 2B^2d^2 + 2A^2d^2)}{32} + \frac{d^2(A^2c^2 + 2C^2d^2 + 3B^2d^2 + 2C^2d^2)}{4d^2} - \frac{d^2(3C^2d^2 + 2B^2d^2 + C^2d^2 + 3B^2d^2 + 3A^2d^2)}{16} + \frac{C^2d^4}{2d^2} - \frac{d^2(3C^2d^2 + 2B^2d^2 + C^2d^2 + 3B^2d^2 + 3A^2d^2)}{8d^2} + \frac{d^2(3C^2d^2 + 2B^2d^2 + C^2d^2 + 3B^2d^2 + 3A^2d^2)}{16d^2} + \frac{d^2(3C^2d^2 + 2B^2d^2 + C^2d^2 + 3B^2d^2 + 3A^2d^2)}{16d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4,x)

[Out] 
$$\operatorname{atan}\left(\frac{c^{1/2}*x}{a^{1/2}}\right)*\left(\frac{5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d^2*e^2 + 3*A*a*c*d^2*e^2 + 3*B*a*c*d^2*e^2}{16*a^{7/2}*c^{5/2}} - \frac{((2*B*c^2*d^3 + 2*C*a^2*e^3 + A*a*c*e^3 + 6*A*c^2*d^2*e^2 + 3*B*a*c*d^2*e^2 + 3*C*a*c*d^2*e^2)/(12*c^3) + (x^2*(A*c*e^3 + 2*C*a*e^3 + 3*B*c*d^2*e^2 + 3*C*c*d^2*e^2))/(4*c^2) - (x^5*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d^2*e^2 + 3*A*a*c*d^2*e^2 + 3*B*a*c*d^2*e^2))/(16*a^3) + (C*e^3*x^4)/(2*c) - (x^3*(5*A*c^2*d^3 - B*a^2*e^3 + C*a*c*d^3 - 3*C*a^2*d^2*e^2 + 3*A*a*c*d^2*e^2 + 3*B*a*c*d^2*e^2))/(6*a^2*c) + (x*(B*a^2*e^3 - 11*A*c^2*d^3 + C*a*c*d^3 + 3*C*a^2*d^2*e^2 + 3*A*a*c*d^2*e^2 + 3*B*a*c*d^2*e^2))/(16*a*c^2)\right)/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*4,x)

[Out] Timed out

$$3.66 \quad \int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4} dx$$

**Optimal.** Leaf size=225

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(2aBe + aCd + 5Acd) + ae^2(aC + Ac))}{16a^{7/2}c^{5/2}} + \frac{x(cd(2aBe + aCd + 5Acd) + ae^2(aC + Ac))}{16a^3c^2(a + cx^2)} - \frac{x(3ae^2(aC + Ac))}{6ac(a + cx^2)^3}$$

**Rubi [A]** time = 0.40, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1645, 778, 199, 205}

$$\frac{x(cd(2aBe + aCd + 5Acd) + ae^2(aC + Ac))}{16a^3c^2(a + cx^2)} - \frac{x(3ae^2(aC + Ac) - cd(2aBe + aCd + 5Acd)) + 2ae(aBe + 2aCd + 4Acd)}{24a^2c^2(a + cx^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(2aBe + aCd + 5Acd) + ae^2(aC + Ac))}{16a^{7/2}c^{5/2}} - \frac{(d+ex)^2(aB - x(Ac - aC))}{6ac(a + cx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x)^2)/(6\*a\*c\*(a + c\*x^2)^3) - (2\*a\*e\*(4\*A\*c\*d + 2\*a\*C\*d + a\*B\*e) + (3\*a\*(A\*c + a\*C)\*e^2 - c\*d\*(5\*A\*c\*d + a\*C\*d + 2\*a\*B\*e))\*x)/(24\*a^2\*c^2\*(a + c\*x^2)^2) + ((a\*(A\*c + a\*C)\*e^2 + c\*d\*(5\*A\*c\*d + a\*C\*d + 2\*a\*B\*e))\*x)/(16\*a^3\*c^2\*(a + c\*x^2)) + ((a\*(A\*c + a\*C)\*e^2 + c\*d\*(5\*A\*c\*d + a\*C\*d + 2\*a\*B\*e))\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(16\*a^(7/2)\*c^(5/2))

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 778

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((a\*(e\*f + d\*g) - (c\*d\*f - a\*e\*g)\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(2\*a\*c\*(p + 1)), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]

Rule 1645

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :  
 > With[{Q = PolynomialQuotient[Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x, 1]}, Simp[((d + e\*x)^m\*(a + c\*x^2)^(p + 1)\*(a\*g - c\*f\*x))/(2\*a\*c\*(p + 1)), x] + Dist[1/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1)\*ExpandToSum[2\*a\*c\*(p + 1)\*(d + e\*x)\*Q - a\*e\*g\*m + c\*d\*f\*(2\*p + 3) + c\*e\*f\*(m + 2\*p + 3)\*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rubi steps

$$\int \frac{(d + ex)^2 (A + Bx + Cx^2)}{(a + cx^2)^4} dx = -\frac{(aB - (Ac - aC)x)(d + ex)^2}{6ac(a + cx^2)^3} - \frac{\int \frac{(d+ex)(-5Acd - aCd - 2aBe - 3(Ac + aC)ex)}{(a+cx^2)^3} dx}{6ac}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)^2}{6ac(a + cx^2)^3} - \frac{2ae(4Acd + 2aCd + aBe) + (3a(Ac + aC)e^2)}{24a^2c^2(a + cx^2)^2}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)^2}{6ac(a + cx^2)^3} - \frac{2ae(4Acd + 2aCd + aBe) + (3a(Ac + aC)e^2)}{24a^2c^2(a + cx^2)^2}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)^2}{6ac(a + cx^2)^3} - \frac{2ae(4Acd + 2aCd + aBe) + (3a(Ac + aC)e^2)}{24a^2c^2(a + cx^2)^2}$$

**Mathematica [A]** time = 0.16, size = 266, normalized size = 1.18

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Ac(a^2 + 5cd^2) + a(aC^2 + cd(2Be + Cd)))}{16a^{7/2}c^{5/2}} + \frac{x(Ac(a^2 + 5cd^2) + a(aC^2 + cd(2Be + Cd)))}{16a^3c^2(a + cx^2)} + \frac{a^2(-e)(6Be + 12Cd + 7Cex) + acx(e(Ae + 2Bd) + Cd^2) + 5Ac^2d^2x}{24a^2c^2(a + cx^2)^2} + \frac{a^2e(Be + 2Cd + Cex) - ac(Ae(2d + ex) + Bd(d + 2ex) + Cd^2x) + Ac^2d^2x}{6ac^2(a + cx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] ((A\*c\*(5\*c\*d^2 + a\*e^2) + a\*(a\*C\*e^2 + c\*d\*(C\*d + 2\*B\*e)))\*x)/(16\*a^3\*c^2\*(a + c\*x^2)) + (5\*A\*c^2\*d^2\*x + a\*c\*(C\*d^2 + e\*(2\*B\*d + A\*e))\*x - a^2\*e\*(12\*C\*d + 6\*B\*e + 7\*C\*e\*x))/(24\*a^2\*c^2\*(a + c\*x^2)^2) + (A\*c^2\*d^2\*x + a^2\*e\*(2\*C\*d + B\*e + C\*e\*x) - a\*c\*(C\*d^2\*x + A\*e\*(2\*d + e\*x) + B\*d\*(d + 2\*e\*x)))/(

$6*a*c^2*(a + c*x^2)^3 + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]])/(16*a^{(7/2)}*c^{(5/2)})$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2 (A + Bx + Cx^2)}{(a + cx^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4,x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

**fricas** [B] time = 0.92, size = 1062, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="fricas")

[Out] [-1/96\*(16\*B\*a^4\*c^2\*d^2 + 8\*B\*a^5\*c\*e^2 - 6\*(2\*B\*a^2\*c^4\*d\*e + (C\*a^2\*c^4 + 5\*A\*a\*c^5)\*d^2 + (C\*a^3\*c^3 + A\*a^2\*c^4)\*e^2)\*x^5 - 16\*(2\*B\*a^3\*c^3\*d\*e + (C\*a^3\*c^3 + 5\*A\*a^2\*c^4)\*d^2 - (C\*a^4\*c^2 - A\*a^3\*c^3)\*e^2)\*x^3 + 16\*(C\*a^5\*c + 2\*A\*a^4\*c^2)\*d\*e + 24\*(2\*C\*a^4\*c^2\*d\*e + B\*a^4\*c^2\*e^2)\*x^2 + 3\*(2\*B\*a^4\*c\*d\*e + (2\*B\*a\*c^4\*d\*e + (C\*a\*c^4 + 5\*A\*c^5)\*d^2 + (C\*a^2\*c^3 + A\*a\*c^4)\*e^2)\*x^6 + 3\*(2\*B\*a^2\*c^3\*d\*e + (C\*a^2\*c^3 + 5\*A\*a\*c^4)\*d^2 + (C\*a^3\*c^2 + A\*a^2\*c^3)\*e^2)\*x^4 + (C\*a^4\*c + 5\*A\*a^3\*c^2)\*d^2 + (C\*a^5 + A\*a^4\*c)\*e^2 + 3\*(2\*B\*a^3\*c^2\*d\*e + (C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*d^2 + (C\*a^4\*c + A\*a^3\*c^2)\*e^2)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 6\*(2\*B\*a^4\*c^2\*d\*e + (C\*a^4\*c^2 - 11\*A\*a^3\*c^3)\*d^2 + (C\*a^5\*c + A\*a^4\*c^2)\*e^2)\*x)/(a^4\*c^6\*x^6 + 3\*a^5\*c^5\*x^4 + 3\*a^6\*c^4\*x^2 + a^7\*c^3), -1/48\*(8\*B\*a^4\*c^2\*d^2 + 4\*B\*a^5\*c\*e^2 - 3\*(2\*B\*a^2\*c^4\*d\*e + (C\*a^2\*c^4 + 5\*A\*a\*c^5)\*d^2 + (C\*a^3\*c^3 + A\*a^2\*c^4)\*e^2)\*x^5 - 8\*(2\*B\*a^3\*c^3\*d\*e + (C\*a^3\*c^3 + 5\*A\*a^2\*c^4)\*d^2 - (C\*a^4\*c^2 - A\*a^3\*c^3)\*e^2)\*x^3 + 8\*(C\*a^5\*c + 2\*A\*a^4\*c^2)\*d\*e + 12\*(2\*C\*a^4\*c^2\*d\*e + B\*a^4\*c^2\*e^2)\*x^2 - 3\*(2\*B\*a^4\*c\*d\*e + (2\*B\*a\*c^4\*d\*e + (C\*a\*c^4 + 5\*A\*c^5)\*d^2 + (C\*a^2\*c^3 + A\*a\*c^4)\*e^2)\*x^6 + 3\*(2\*B\*a^2\*c^3\*d\*e + (C\*a^2\*c^3 + 5\*A\*a\*c^4)\*d^2 + (C\*a^3\*c^2 + A\*a^2\*c^3)\*e^2)\*x^4 + (C\*a^4\*c + 5\*A\*a^3\*c^2)\*d^2 + (C\*a^5 + A\*a^4\*c)\*e^2 + 3\*(2\*B\*a^3\*c^2\*d\*e + (C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*d^2 + (C\*a^4\*c + A\*a^3\*c^2)\*e^2)\*x^2)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + 3\*(2\*B\*a^4\*c^2\*d\*e + (C\*a^4\*c^2 - 11\*A\*a^3\*c^3)\*d^2 + (C\*a^5\*c + A\*a^4\*c^2)\*e^2)\*x)/(a^4\*c^6\*x^6 + 3\*a^5\*c^5\*x^4 + 3\*a^6\*c^4\*x^2 + a^7\*c^3)]



**giac [A]** time = 0.17, size = 328, normalized size = 1.46

$$\frac{(Cax^2 + 5A^2d^2 + 2Bade + Ca^2 + Aac^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}a^2c} + \frac{3Cac^2d^2x^3 + 15A^4d^2x^3 + 6Bac^2d^2x^3 + 3Ca^2c^2x^3 + 3Aac^2x^3 + 8Ca^2c^2d^2x^3 + 40Aac^2d^2x^3 + 16Ba^2c^2d^2x^3 - 8Ca^2c^2x^3 + 8Aa^2c^2d^2x^3 - 24Ca^2d^2x^3 - 3Ca^2d^2x^3 + 33Aa^2c^2d^2x^3 - 12Ba^2c^2d^2x^3 - 6Ba^2d^2x^3 - 8Ba^2c^2d^2x^3 - 3Ca^2c^2d^2x^3 - 3Aa^2c^2d^2x^3 - 8Ca^2d^2x^3 - 16Aa^2c^2d^2x^3 - 4Ba^2c^2d^2x^3}{48(c^2 + a)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="giac")

[Out] 1/16\*(C\*a\*c\*d^2 + 5\*A\*c^2\*d^2 + 2\*B\*a\*c\*d\*e + C\*a^2\*e^2 + A\*a\*c\*e^2)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a^3\*c^2) + 1/48\*(3\*C\*a\*c^3\*d^2\*x^5 + 15\*A\*c^4\*d^2\*x^5 + 6\*B\*a\*c^3\*d\*x^5\*e + 3\*C\*a^2\*c^2\*x^5\*e^2 + 3\*A\*a\*c^3\*x^5\*e^2 + 8\*C\*a^2\*c^2\*d^2\*x^3 + 40\*A\*a\*c^3\*d^2\*x^3 + 16\*B\*a^2\*c^2\*d\*x^3\*e - 8\*C\*a^3\*c\*x^3\*e^2 + 8\*A\*a^2\*c^2\*x^3\*e^2 - 24\*C\*a^3\*c\*d\*x^2\*e - 3\*C\*a^3\*c\*d^2\*x + 33\*A\*a^2\*c^2\*d^2\*x - 12\*B\*a^3\*c\*x^2\*e^2 - 6\*B\*a^3\*c\*d\*x\*e - 8\*B\*a^3\*c\*d^2 - 3\*C\*a^4\*x\*e^2 - 3\*A\*a^3\*c\*x\*e^2 - 8\*C\*a^4\*d\*e - 16\*A\*a^3\*c\*d\*e - 4\*B\*a^4\*e^2)/((c\*x^2 + a)^3\*a^3\*c^2)

**maple [A]** time = 0.01, size = 333, normalized size = 1.48

$$\frac{A^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}a^2c} + \frac{5A^2d^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}a^2} + \frac{Bde \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c} + \frac{C^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}a^2c} + \frac{Cd^2 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}a^2c} + \frac{(Aac^2+5A^2d^2+2Bade+c^2+Ca^2d^2)^2}{16a^3} - \frac{(Be+2Cd)^2x^2}{4c} + \frac{(Aac^2+5A^2d^2+2Bade-c^2+Ca^2d^2)^2}{6a^2c} - \frac{(Aac^2-11A^2d^2+2Bade+c^2+Ca^2d^2)^2}{16a^2} - \frac{4Acde+Ba^2+2Bc^2d^2+2Cade}{12c^2} \frac{1}{(cx^2+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x)

[Out] (1/16\*(A\*a\*c\*e^2+5\*A\*c^2\*d^2+2\*B\*a\*c\*d\*e+C\*a^2\*e^2+C\*a\*c\*d^2)/a^3\*x^5+1/6\*(A\*a\*c\*e^2+5\*A\*c^2\*d^2+2\*B\*a\*c\*d\*e-C\*a^2\*e^2+C\*a\*c\*d^2)/a^2/c\*x^3-1/4\*(B\*e+2\*C\*d)/c\*e\*x^2-1/16\*(A\*a\*c\*e^2-11\*A\*c^2\*d^2+2\*B\*a\*c\*d\*e+C\*a^2\*e^2+C\*a\*c\*d^2)/a/c^2\*x-1/12\*(4\*A\*c\*d\*e+B\*a\*e^2+2\*B\*c\*d^2+2\*C\*a\*d\*e)/c^2)/(c\*x^2+a)^3+1/16/a^2/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*e^2+5/16/a^3/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*A\*d^2+1/8/a^2/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*B\*d\*e+1/16/a/c^2/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C\*e^2+1/16/a^2/c/(a\*c)^(1/2)\*arctan(1/(a\*c)^(1/2)\*c\*x)\*C\*d^2

**maxima [A]** time = 1.01, size = 323, normalized size = 1.44

$$\frac{8Ba^2d^2 + 4Ba^2e^2 - 3(2Ba^2de + (Ca^2 + 5A^2d^2 + Aac^2)^2)x^5 - 8(2Ba^2d^2e + (Ca^2 + 5A^2d^2 + Aac^2)^2)x^3 - 8(Ca^2 + 2Aa^2d^2 + 12(2Ca^2d^2e + Ba^2c^2)^2 + 3(2Ba^2d^2e + (Ca^2 + 5A^2d^2 + Aac^2)^2)x^2 + (Ca^2 + Aa^2d^2)^2)}{48(a^2c^2x^2 + 3a^2c^2x + a^2c^2)} + \frac{(2Bade + (Ca^2 + 5A^2d^2 + Aac^2)^2)x^5 + (Ca^2 + Aa^2d^2)^2}{16\sqrt{ac}a^2c} \arctan\left(\frac{cx}{\sqrt{ac}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="maxima")

[Out] -1/48\*(8\*B\*a^3\*c\*d^2 + 4\*B\*a^4\*e^2 - 3\*(2\*B\*a\*c^3\*d\*e + (C\*a\*c^3 + 5\*A\*c^4)\*d^2 + (C\*a^2\*c^2 + A\*a\*c^3)\*e^2)\*x^5 - 8\*(2\*B\*a^2\*c^2\*d\*e + (C\*a^2\*c^2 + 5\*A\*a\*c^3)\*d^2 - (C\*a^3\*c - A\*a^2\*c^2)\*e^2)\*x^3 + 8\*(C\*a^4 + 2\*A\*a^3\*c)\*d\*e + 12\*(2\*C\*a^3\*c\*d\*e + B\*a^3\*c\*e^2)\*x^2 + 3\*(2\*B\*a^3\*c\*d\*e + (C\*a^3\*c - 11\*A

$$*a^2*c^2)*d^2 + (C*a^4 + A*a^3*c)*e^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(2*B*a*c*d*e + (C*a*c + 5*A*c^2)*d^2 + (C*a^2 + A*a*c)*e^2)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a^3*c^2)$$

**mupad [B]** time = 0.23, size = 287, normalized size = 1.28

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) (Ca^2c^2 + Cacd^2 + 2Bacde + Aace^2 + 5Ac^2d^2)}{16a^{7/2}c^{5/2}} - \frac{Ba^2 + 2Bcd^2 + 4Acde + 2Cade}{12c^2} - \frac{x^5(Ca^2c^2 + Cacd^2 + 2Bacde + Aace^2 + 5Ac^2d^2)}{16a^3} + \frac{x^2(B^2 + 2Cde)}{4c} + \frac{x(Ca^2c^2 + Cacd^2 + 2Bacde + Aace^2 - 11A^2d^2)}{16ac^2} - \frac{x^3(-Ca^2c^2 + Cacd^2 + 2Bacde + Aace^2 + 5Ac^2d^2)}{6a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^2\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x)

[Out] (atan((c^(1/2)\*x)/a^(1/2))\*(5\*A\*c^2\*d^2 + C\*a^2\*e^2 + A\*a\*c\*e^2 + C\*a\*c\*d^2 + 2\*B\*a\*c\*d\*e))/(16\*a^(7/2)\*c^(5/2)) - ((B\*a\*e^2 + 2\*B\*c\*d^2 + 4\*A\*c\*d\*e + 2\*C\*a\*d\*e)/(12\*c^2) - (x^5\*(5\*A\*c^2\*d^2 + C\*a^2\*e^2 + A\*a\*c\*e^2 + C\*a\*c\*d^2 + 2\*B\*a\*c\*d\*e))/(16\*a^3) + (x^2\*(B\*e^2 + 2\*C\*d\*e))/(4\*c) + (x\*(C\*a^2\*e^2 - 11\*A\*c^2\*d^2 + A\*a\*c\*e^2 + C\*a\*c\*d^2 + 2\*B\*a\*c\*d\*e))/(16\*a\*c^2) - (x^3\*(5\*A\*c^2\*d^2 - C\*a^2\*e^2 + A\*a\*c\*e^2 + C\*a\*c\*d^2 + 2\*B\*a\*c\*d\*e))/(6\*a^2\*c))/(a^3 + c^3\*x^6 + 3\*a^2\*c\*x^2 + 3\*a\*c^2\*x^4)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*4, x)

[Out] Timed out

$$3.67 \quad \int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^4} dx$$

**Optimal.** Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + 5Acd)}{16a^{7/2}c^{3/2}} + \frac{x(aBe + aCd + 5Acd)}{16a^3c(a + cx^2)} - \frac{2ae(aC + 2Ac) - cx(aBe + aCd + 5Acd)}{24a^2c^2(a + cx^2)^2} - \frac{(d + ex)(aB - x(Ac - aC))}{6ac}$$

**Rubi [A]** time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1645, 639, 199, 205}

$$-\frac{2ae(aC + 2Ac) - cx(aBe + aCd + 5Acd)}{24a^2c^2(a + cx^2)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + 5Acd)}{16a^{7/2}c^{3/2}} + \frac{x(aBe + aCd + 5Acd)}{16a^3c(a + cx^2)} - \frac{(d + ex)(aB - x(Ac - aC))}{6ac(a + cx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] -((a\*B - (A\*c - a\*C)\*x)\*(d + e\*x))/(6\*a\*c\*(a + c\*x^2)^3) - (2\*a\*(2\*A\*c + a\*C)\*e - c\*(5\*A\*c\*d + a\*C\*d + a\*B\*e)\*x)/(24\*a^2\*c^2\*(a + c\*x^2)^2) + ((5\*A\*c\*d + a\*C\*d + a\*B\*e)\*x)/(16\*a^3\*c\*(a + c\*x^2)) + ((5\*A\*c\*d + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(16\*a^(7/2)\*c^(3/2))

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 639

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((a\*e - c\*d\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] + Dist[(d\*(2\*p + 3))/(2\*a\*(p + 1)), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

## Rule 1645

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x)/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

## Rubi steps

$$\int \frac{(d + ex)(A + Bx + Cx^2)}{(a + cx^2)^4} dx = -\frac{(aB - (Ac - aC)x)(d + ex)}{6ac(a + cx^2)^3} - \frac{\int \frac{-5Acd - a(Cd + Be) - 2(2Ac + aC)ex}{(a + cx^2)^3} dx}{6ac}$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)}{6ac(a + cx^2)^3} - \frac{2a(2Ac + aC)e - c(5Acd + aCd + aBe)x}{24a^2c^2(a + cx^2)^2} + \frac{(5Acd - a(Cd + Be) - 2(2Ac + aC)e)x}{24a^2c^2(a + cx^2)^2} \quad (5A)$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)}{6ac(a + cx^2)^3} - \frac{2a(2Ac + aC)e - c(5Acd + aCd + aBe)x}{24a^2c^2(a + cx^2)^2} + \frac{(5Acd - a(Cd + Be) - 2(2Ac + aC)e)x}{24a^2c^2(a + cx^2)^2} \quad (5A)$$

$$= -\frac{(aB - (Ac - aC)x)(d + ex)}{6ac(a + cx^2)^3} - \frac{2a(2Ac + aC)e - c(5Acd + aCd + aBe)x}{24a^2c^2(a + cx^2)^2} + \frac{(5Acd - a(Cd + Be) - 2(2Ac + aC)e)x}{24a^2c^2(a + cx^2)^2} \quad (5A)$$

**Mathematica** [A] time = 0.13, size = 171, normalized size = 1.04

$$\frac{8a^{5/2}(a^2Ce - ac(Ae + B(d + ex) + Cdx) + Ac^2dx)}{(a + cx^2)^3} + \frac{2a^{3/2}(-6a^2Ce + acx(Be + Cd) + 5Ac^2dx)}{(a + cx^2)^2} + \frac{3\sqrt{a}cx(aBe + aCd + 5Acd)}{a + cx^2} + 3\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe + aCd + 5Acd)$$


---


$$48a^{7/2}c^2$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

[Out] ((2\*a^(3/2)\*(-6\*a^2\*C\*e + 5\*A\*c^2\*d\*x + a\*c\*(C\*d + B\*e)\*x))/(a + c\*x^2)^2 + (3\*Sqrt[a]\*c\*(5\*A\*c\*d + a\*C\*d + a\*B\*e)\*x)/(a + c\*x^2) + (8\*a^(5/2)\*(a^2\*C\*e + A\*c^2\*d\*x - a\*c\*(A\*e + C\*d\*x + B\*(d + e\*x))))/(a + c\*x^2)^3 + 3\*Sqrt[c]\*(5\*A\*c\*d + a\*C\*d + a\*B\*e)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]]/(48\*a^(7/2)\*c^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)(A + Bx + Cx^2)}{(a + cx^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4,x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(A + B\*x + C\*x^2))/(a + c\*x^2)^4, x]

**fricas** [B] time = 0.60, size = 636, normalized size = 3.85

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="fricas")

[Out] [-1/96\*(24\*C\*a^4\*c\*e\*x^2 + 16\*B\*a^4\*c\*d - 6\*(B\*a^2\*c^3\*e + (C\*a^2\*c^3 + 5\*A\*a\*c^4)\*d)\*x^5 - 16\*(B\*a^3\*c^2\*e + (C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*d)\*x^3 + 3\*((B\*a\*c^3\*e + (C\*a\*c^3 + 5\*A\*c^4)\*d)\*x^6 + B\*a^4\*e + 3\*(B\*a^2\*c^2\*e + (C\*a^2\*c^2 + 5\*A\*a\*c^3)\*d)\*x^4 + 3\*(B\*a^3\*c\*e + (C\*a^3\*c + 5\*A\*a^2\*c^2)\*d)\*x^2 + (C\*a^4 + 5\*A\*a^3\*c)\*d)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 8\*(C\*a^5 + 2\*A\*a^4\*c)\*e + 6\*(B\*a^4\*c\*e + (C\*a^4\*c - 11\*A\*a^3\*c^2)\*d)\*x)/(a^4\*c^5\*x^6 + 3\*a^5\*c^4\*x^4 + 3\*a^6\*c^3\*x^2 + a^7\*c^2), -1/48\*(12\*C\*a^4\*c\*e\*x^2 + 8\*B\*a^4\*c\*d - 3\*(B\*a^2\*c^3\*e + (C\*a^2\*c^3 + 5\*A\*a\*c^4)\*d)\*x^5 - 8\*(B\*a^3\*c^2\*e + (C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*d)\*x^3 - 3\*((B\*a\*c^3\*e + (C\*a\*c^3 + 5\*A\*c^4)\*d)\*x^6 + B\*a^4\*e + 3\*(B\*a^2\*c^2\*e + (C\*a^2\*c^2 + 5\*A\*a\*c^3)\*d)\*x^4 + 3\*(B\*a^3\*c\*e + (C\*a^3\*c + 5\*A\*a^2\*c^2)\*d)\*x^2 + (C\*a^4 + 5\*A\*a^3\*c)\*d)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + 4\*(C\*a^5 + 2\*A\*a^4\*c)\*e + 3\*(B\*a^4\*c\*e + (C\*a^4\*c - 11\*A\*a^3\*c^2)\*d)\*x)/(a^4\*c^5\*x^6 + 3\*a^5\*c^4\*x^4 + 3\*a^6\*c^3\*x^2 + a^7\*c^2)]

**giac** [A] time = 0.16, size = 194, normalized size = 1.18

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$$\frac{(Cad + 5Acd + Bae) \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 3Cac^3dx^5 + 15Ac^4dx^5 + 3Bac^3x^5e + 8Ca^2c^2dx^3 + 40Aac^3dx^3 + 8Ba^2c^2x^3e - 12Ca^3cx^2e - 3Ca^3cdx + 33Aa^2c^2dx - 3Ba^3cxe - 8Ba^3cd - 4Ca^4e - 8Aa^3ce}{16\sqrt{ac}a^3c} + \frac{3Cac^3dx^5 + 15Ac^4dx^5 + 3Bac^3x^5e + 8Ca^2c^2dx^3 + 40Aac^3dx^3 + 8Ba^2c^2x^3e - 12Ca^3cx^2e - 3Ca^3cdx + 33Aa^2c^2dx - 3Ba^3cxe - 8Ba^3cd - 4Ca^4e - 8Aa^3ce}{48(cx^2 + a)^3a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="giac")

[Out] 1/16\*(C\*a\*d + 5\*A\*c\*d + B\*a\*e)\*arctan(c\*x/sqrt(a\*c))/(sqrt(a\*c)\*a^3\*c) + 1/48\*(3\*C\*a\*c^3\*d\*x^5 + 15\*A\*c^4\*d\*x^5 + 3\*B\*a\*c^3\*x^5\*e + 8\*C\*a^2\*c^2\*d\*x^3 + 40\*A\*a\*c^3\*d\*x^3 + 8\*B\*a^2\*c^2\*x^3\*e - 12\*C\*a^3\*c\*x^2\*e - 3\*C\*a^3\*c\*d\*x +



sympy [A] time = 139.97, size = 298, normalized size = 1.81

$$\frac{\sqrt{-\frac{1}{a^2c}}(5Acd + Bae + Cad) \log\left(-a^4c\sqrt{-\frac{1}{a^2c}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{a^2c}}(5Acd + Bae + Cad) \log\left(a^4c\sqrt{-\frac{1}{a^2c}} + x\right)}{32} + \frac{-8Aa^3ce - 8Ba^3cd - 4Ca^4e - 12Ca^3ce^2 + x^5(15Ac^4d + 3Bac^3e + 3Ca^3cd) + x^3(40Aac^3d + 8Ba^2c^2e + 8Ca^2c^2d) + x(33Aa^2c^2d - 3Ba^3ce - 3Ca^3cd)}{48a^6c^2 + 144a^5c^3x^2 + 144a^4c^4x^4 + 48a^3c^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*4,x)

[Out]  $-\sqrt{-1/(a**7*c**3)}*(5*A*c*d + B*a*e + C*a*d)*\log(-a**4*c*\sqrt{-1/(a**7*c**3)} + x)/32 + \sqrt{-1/(a**7*c**3)}*(5*A*c*d + B*a*e + C*a*d)*\log(a**4*c*\sqrt{-1/(a**7*c**3)} + x)/32 + (-8*A*a**3*c*e - 8*B*a**3*c*d - 4*C*a**4*e - 12*C*a**3*c*e*x**2 + x**5*(15*A*c**4*d + 3*B*a*c**3*e + 3*C*a*c**3*d) + x**3*(40*A*a*c**3*d + 8*B*a**2*c**2*e + 8*C*a**2*c**2*d) + x*(33*A*a**2*c**2*d - 3*B*a**3*c*e - 3*C*a**3*c*d))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a**4*c**4*x**4 + 48*a**3*c**5*x**6)$

$$3.68 \quad \int \frac{A+Bx+Cx^2}{(a+cx^2)^4} dx$$

Optimal. Leaf size=126

$$\frac{(aC + 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}} + \frac{x(aC + 5Ac)}{16a^3c(a + cx^2)} + \frac{x(aC + 5Ac)}{24a^2c(a + cx^2)^2} - \frac{aB - x(AC - aC)}{6ac(a + cx^2)^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1814, 12, 199, 205}

$$\frac{(aC + 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}} + \frac{x(aC + 5Ac)}{16a^3c(a + cx^2)} + \frac{x(aC + 5Ac)}{24a^2c(a + cx^2)^2} - \frac{aB - x(AC - aC)}{6ac(a + cx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2)^4, x]

[Out] -(a\*B - (A\*c - a\*C)\*x)/(6\*a\*c\*(a + c\*x^2)^3) + ((5\*A\*c + a\*C)\*x)/(24\*a^2\*c\*(a + c\*x^2)^2) + ((5\*A\*c + a\*C)\*x)/(16\*a^3\*c\*(a + c\*x^2)) + ((5\*A\*c + a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(16\*a^(7/2)\*c^(3/2))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 1814



```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{(a + cx^2)^4} dx &= -\frac{aB - (Ac - aC)x}{6ac(a + cx^2)^3} - \frac{\int \frac{-5A - \frac{aC}{c}}{(a + cx^2)^3} dx}{6a} \\
 &= -\frac{aB - (Ac - aC)x}{6ac(a + cx^2)^3} + \frac{(5Ac + aC) \int \frac{1}{(a + cx^2)^3} dx}{6ac} \\
 &= -\frac{aB - (Ac - aC)x}{6ac(a + cx^2)^3} + \frac{(5Ac + aC)x}{24a^2c(a + cx^2)^2} + \frac{(5Ac + aC) \int \frac{1}{(a + cx^2)^2} dx}{8a^2c} \\
 &= -\frac{aB - (Ac - aC)x}{6ac(a + cx^2)^3} + \frac{(5Ac + aC)x}{24a^2c(a + cx^2)^2} + \frac{(5Ac + aC)x}{16a^3c(a + cx^2)} + \frac{(5Ac + aC) \int \frac{1}{a + cx^2} dx}{16a^3c} \\
 &= -\frac{aB - (Ac - aC)x}{6ac(a + cx^2)^3} + \frac{(5Ac + aC)x}{24a^2c(a + cx^2)^2} + \frac{(5Ac + aC)x}{16a^3c(a + cx^2)} + \frac{(5Ac + aC) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}
 \end{aligned}$$

**Mathematica** [A] time = 0.09, size = 112, normalized size = 0.89

$$\frac{(aC + 5Ac) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}} + \frac{-a^3(8B + 3Cx) + a^2cx(33A + 8Cx^2) + ac^2x^3(40A + 3Cx^2) + 15Ac^3x^5}{48a^3c(a + cx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2)^4, x]

[Out] (15\*A\*c^3\*x^5 - a^3\*(8\*B + 3\*C\*x) + a\*c^2\*x^3\*(40\*A + 3\*C\*x^2) + a^2\*c\*x\*(3\*3\*A + 8\*C\*x^2))/(48\*a^3\*c\*(a + c\*x^2)^3 + ((5\*A\*c + a\*C)\*ArcTan[(Sqrt[c]\*x)/Sqrt[a]])/(16\*a^(7/2)\*c^(3/2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + cx^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^4, x]

[Out] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^4, x]

**fricas** [A] time = 0.62, size = 430, normalized size = 3.41

$$\frac{16Ba^4c - 6(Ca^2c^2 + 5Aac^2)c^2 - 16(Ca^2c^2 + 5Aac^2)c^2 + 3((Ca^2c^2 + 5Aac^2)c^2 + Ca^4 + 5Aa^2c + 3((Ca^2c^2 + 5Aac^2)c^2 + 3(Ca^2c^2 + 5Aac^2)c^2))\sqrt{ac} \log\left(\frac{cx^2 + a}{cx^2 - a}\right) + 6(Ca^4c - 11Aa^3c^2)c}{96(a^4c^2 + 3a^2c^4 + 3a^2c^2 + a^2c^2)} - \frac{8Ba^4c - 3(Ca^2c^2 + 5Aac^2)c^2 - 8(Ca^2c^2 + 5Aac^2)c^2 - 3((Ca^2c^2 + 5Aac^2)c^2 + Ca^4 + 5Aa^2c + 3((Ca^2c^2 + 5Aac^2)c^2 + 3(Ca^2c^2 + 5Aac^2)c^2))\sqrt{ac} \arctan\left(\frac{cx}{\sqrt{ac}}\right) + 3(Ca^4c - 11Aa^3c^2)c}{48(a^4c^2 + 3a^2c^4 + 3a^2c^2 + a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="fricas")

[Out] [-1/96\*(16\*B\*a^4\*c - 6\*(C\*a^2\*c^3 + 5\*A\*a\*c^4)\*x^5 - 16\*(C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*x^3 + 3\*((C\*a\*c^3 + 5\*A\*c^4)\*x^6 + C\*a^4 + 5\*A\*a^3\*c + 3\*(C\*a^2\*c^2 + 5\*A\*a\*c^3)\*x^4 + 3\*(C\*a^3\*c + 5\*A\*a^2\*c^2)\*x^2)\*sqrt(-a\*c)\*log((c\*x^2 - 2\*sqrt(-a\*c)\*x - a)/(c\*x^2 + a)) + 6\*(C\*a^4\*c - 11\*A\*a^3\*c^2)\*x)/(a^4\*c^5\*x^6 + 3\*a^5\*c^4\*x^4 + 3\*a^6\*c^3\*x^2 + a^7\*c^2), -1/48\*(8\*B\*a^4\*c - 3\*(C\*a^2\*c^3 + 5\*A\*a\*c^4)\*x^5 - 8\*(C\*a^3\*c^2 + 5\*A\*a^2\*c^3)\*x^3 - 3\*((C\*a\*c^3 + 5\*A\*c^4)\*x^6 + C\*a^4 + 5\*A\*a^3\*c + 3\*(C\*a^2\*c^2 + 5\*A\*a\*c^3)\*x^4 + 3\*(C\*a^3\*c + 5\*A\*a^2\*c^2)\*x^2)\*sqrt(a\*c)\*arctan(sqrt(a\*c)\*x/a) + 3\*(C\*a^4\*c - 11\*A\*a^3\*c^2)\*x)/(a^4\*c^5\*x^6 + 3\*a^5\*c^4\*x^4 + 3\*a^6\*c^3\*x^2 + a^7\*c^2)]

**giac** [A] time = 0.18, size = 109, normalized size = 0.87

$$\frac{(Ca + 5Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac} a^3c} + \frac{3Cac^2x^5 + 15Ac^3x^5 + 8Ca^2cx^3 + 40Aac^2x^3 - 3Ca^3x + 33Aa^2cx - 8Ba^3}{48(cx^2 + a)^3 a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^4,x, algorithm="giac")

[Out] 1/16\*(C\*a + 5\*A\*c)\*arctan(cx/sqrt(a\*c))/(sqrt(a\*c)\*a^3\*c) + 1/48\*(3\*C\*a\*c^2\*x^5 + 15\*A\*c^3\*x^5 + 8\*C\*a^2\*c\*x^3 + 40\*A\*a\*c^2\*x^3 - 3\*C\*a^3\*x + 33\*A\*a^2\*c\*x - 8\*B\*a^3)/((c\*x^2 + a)^3\*a^3\*c)

**maple** [A] time = 0.01, size = 113, normalized size = 0.90

$$\frac{5A \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac} a^3} + \frac{C \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac} a^2c} + \frac{(5Ac+aC)c x^5}{16a^3} + \frac{(5Ac+aC)x^3}{6a^2} - \frac{B}{6c} + \frac{(11Ac-aC)x}{16ac} \frac{1}{(cx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+B*x+A)/(c*x^2+a)^4,x)`

[Out]  $(1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*x-1/6*B/c)/(c*x^2+a)^3+5/16/a^3/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*A+1/16/a^2/c/(a*c)^{(1/2)}*\arctan(1/(a*c)^{(1/2)}*c*x)*C$

**maxima** [A] time = 0.98, size = 133, normalized size = 1.06

$$\frac{3(Cac^2 + 5Ac^3)x^5 - 8Ba^3 + 8(Ca^2c + 5Aac^2)x^3 - 3(Ca^3 - 11Aa^2c)x}{48(a^3c^4x^6 + 3a^4c^3x^4 + 3a^5c^2x^2 + a^6c)} + \frac{(Ca + 5Ac) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")`

[Out]  $1/48*(3*(C*a*c^2 + 5*A*c^3)*x^5 - 8*B*a^3 + 8*(C*a^2*c + 5*A*a*c^2)*x^3 - 3*(C*a^3 - 11*A*a^2*c)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6*c) + 1/16*(C*a + 5*A*c)*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a^3*c)$

**mupad** [B] time = 3.89, size = 116, normalized size = 0.92

$$\frac{\frac{x^5(5Ac+Ca)}{6a^2} - \frac{B}{6c} + \frac{cx^5(5Ac+Ca)}{16a^3} + \frac{x(11Ac-Ca)}{16ac}}{a^3 + 3a^2cx^2 + 3a^2c^2x^4 + c^3x^6} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(5Ac+Ca)}{16a^{7/2}c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(a + c*x^2)^4,x)`

[Out]  $((x^3*(5*A*c + C*a))/(6*a^2) - B/(6*c) + (c*x^5*(5*A*c + C*a))/(16*a^3) + (x*(11*A*c - C*a))/(16*a*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) + (\operatorname{atan}((c^{(1/2)}*x)/a^{(1/2)})*(5*A*c + C*a))/(16*a^{(7/2)}*c^{(3/2)})$

**sympy** [A] time = 2.08, size = 196, normalized size = 1.56

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}(5Ac+Ca)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7c^3}}(5Ac+Ca)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32} + \frac{-8Ba^3+x^5(15Ac^3+3Cac^2)+x^3(40Aac^2+8Ca^2c)+x(33Aa^2c-3Ca^3)}{48a^6c+144a^5c^2x^2+144a^4c^3x^4+48a^3c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)`

[Out]  $-\sqrt{-1/(a**7*c**3)}*(5*A*c + C*a)*\log(-a**4*c*\sqrt{-1/(a**7*c**3)} + x)/32 + \sqrt{-1/(a**7*c**3)}*(5*A*c + C*a)*\log(a**4*c*\sqrt{-1/(a**7*c**3)} + x)/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8*C*a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 + 144*a**4*c**3*x**4 + 48*a**3*c**4*x**6)$

$$3.69 \quad \int \frac{x^3(1+x+x^2)}{(1+x^2)^2} dx$$

Optimal. Leaf size=43

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 + 1) - \frac{x^3}{2(x^2 + 1)} + \frac{3x}{2} - \frac{3}{2} \tan^{-1}(x)$$

**Rubi [A]** time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {1804, 801, 635, 203, 260}

$$-\frac{x^3}{2(x^2 + 1)} + \frac{x^2}{2} - \frac{1}{2} \log(x^2 + 1) + \frac{3x}{2} - \frac{3}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] (3\*x)/2 + x^2/2 - x^3/(2\*(1 + x^2)) - (3\*ArcTan[x])/2 - Log[1 + x^2]/2

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 801

Int[(((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

Rule 1804

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq,
a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} dx &= -\frac{x^3}{2(1+x^2)} - \frac{1}{2} \int \frac{(-3-2x)x^2}{1+x^2} dx \\
&= -\frac{x^3}{2(1+x^2)} - \frac{1}{2} \int \left(-3-2x + \frac{3+2x}{1+x^2}\right) dx \\
&= \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2(1+x^2)} - \frac{1}{2} \int \frac{3+2x}{1+x^2} dx \\
&= \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2(1+x^2)} - \frac{3}{2} \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= \frac{3x}{2} + \frac{x^2}{2} - \frac{x^3}{2(1+x^2)} - \frac{3}{2} \tan^{-1}(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 29, normalized size = 0.67

$$\frac{1}{2} \left( x \left( \frac{1}{x^2+1} + x + 2 \right) - \log(x^2+1) - 3 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] (x\*(2 + x + (1 + x^2)^(-1)) - 3\*ArcTan[x] - Log[1 + x^2])/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] IntegrateAlgebraic[(x^3\*(1 + x + x^2))/(1 + x^2)^2, x]

**fricas** [A] time = 0.92, size = 46, normalized size = 1.07

$$\frac{x^4 + 2x^3 + x^2 - 3(x^2 + 1)\arctan(x) - (x^2 + 1)\log(x^2 + 1) + 3x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2\*(x^4 + 2\*x^3 + x^2 - 3\*(x^2 + 1)\*arctan(x) - (x^2 + 1)\*log(x^2 + 1) + 3\*x)/(x^2 + 1)

**giac** [A] time = 0.15, size = 29, normalized size = 0.67

$$\frac{1}{2}x^2 + x + \frac{x}{2(x^2 + 1)} - \frac{3}{2}\arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")

[Out] 1/2\*x^2 + x + 1/2\*x/(x^2 + 1) - 3/2\*arctan(x) - 1/2\*log(x^2 + 1)

**maple** [A] time = 0.01, size = 30, normalized size = 0.70

$$\frac{x^2}{2} + x + \frac{x}{2x^2 + 2} - \frac{3\arctan(x)}{2} - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(x^2+x+1)/(x^2+1)^2,x)

[Out] 1/2\*x^2+x+1/2\*x/(x^2+1)-1/2\*ln(x^2+1)-3/2\*arctan(x)

**maxima** [A] time = 0.95, size = 29, normalized size = 0.67

$$\frac{1}{2}x^2 + x + \frac{x}{2(x^2 + 1)} - \frac{3}{2}\arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")

[Out] 1/2\*x^2 + x + 1/2\*x/(x^2 + 1) - 3/2\*arctan(x) - 1/2\*log(x^2 + 1)

**mupad [B]** time = 0.04, size = 30, normalized size = 0.70

$$x - \frac{\ln(x^2 + 1)}{2} - \frac{3 \operatorname{atan}(x)}{2} + \frac{x}{2(x^2 + 1)} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(x + x^2 + 1))/(x^2 + 1)^2,x)

[Out] x - log(x^2 + 1)/2 - (3\*atan(x))/2 + x/(2\*(x^2 + 1)) + x^2/2

**sympy [A]** time = 0.13, size = 29, normalized size = 0.67

$$\frac{x^2}{2} + x + \frac{x}{2x^2 + 2} - \frac{\log(x^2 + 1)}{2} - \frac{3 \operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(x\*\*2+x+1)/(x\*\*2+1)\*\*2,x)

[Out] x\*\*2/2 + x + x/(2\*x\*\*2 + 2) - log(x\*\*2 + 1)/2 - 3\*atan(x)/2

$$3.70 \quad \int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$$

Optimal. Leaf size=30

$$-\frac{x^2}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + x - \tan^{-1}(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {1804, 774, 635, 203, 260}

$$-\frac{x^2}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] x - x^2/(2\*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 774

Int[(((d\_.) + (e\_.)\*(x\_))\*((f\_) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*g\*x)/c, x] + Dist[1/c, Int[(c\*d\*f - a\*e\*g + c\*(e\*f + d\*g)\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]



Rule 1804

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq,
a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
1]}, Simp[((c*x)^(m)*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx &= -\frac{x^2}{2(1+x^2)} - \frac{1}{2} \int \frac{(-2-2x)x}{1+x^2} dx \\ &= x - \frac{x^2}{2(1+x^2)} - \frac{1}{2} \int \frac{2-2x}{1+x^2} dx \\ &= x - \frac{x^2}{2(1+x^2)} - \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\ &= x - \frac{x^2}{2(1+x^2)} - \tan^{-1}(x) + \frac{1}{2} \log(1+x^2) \end{aligned}$$

**Mathematica** [A]    time = 0.02, size = 27, normalized size = 0.90

$$\frac{1}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(1 + x + x^2))/(1 + x^2)^2, x]

[Out] x + 1/(2\*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2

**IntegrateAlgebraic** [F]    time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] IntegrateAlgebraic[(x^2\*(1 + x + x^2))/(1 + x^2)^2, x]

**fricas** [A] time = 0.60, size = 40, normalized size = 1.33

$$\frac{2x^3 - 2(x^2 + 1)\arctan(x) + (x^2 + 1)\log(x^2 + 1) + 2x + 1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2\*(2\*x^3 - 2\*(x^2 + 1)\*arctan(x) + (x^2 + 1)\*log(x^2 + 1) + 2\*x + 1)/(x^2 + 1)

**giac** [A] time = 0.21, size = 23, normalized size = 0.77

$$x + \frac{1}{2(x^2 + 1)} - \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")

[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2\*log(x^2 + 1)

**maple** [A] time = 0.01, size = 24, normalized size = 0.80

$$x - \arctan(x) + \frac{\ln(x^2 + 1)}{2} + \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(x^2+x+1)/(x^2+1)^2,x)

[Out] x+1/2/(x^2+1)+1/2\*ln(x^2+1)-arctan(x)

**maxima** [A] time = 0.96, size = 23, normalized size = 0.77

$$x + \frac{1}{2(x^2 + 1)} - \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")

[Out]  $x + 1/2/(x^2 + 1) - \arctan(x) + 1/2*\log(x^2 + 1)$

**mupad [B]** time = 0.03, size = 23, normalized size = 0.77

$$x + \frac{\ln(x^2 + 1)}{2} - \operatorname{atan}(x) + \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)`

[Out]  $x + \log(x^2 + 1)/2 - \operatorname{atan}(x) + 1/(2*(x^2 + 1))$

**sympy [A]** time = 0.12, size = 20, normalized size = 0.67

$$x + \frac{\log(x^2 + 1)}{2} - \operatorname{atan}(x) + \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)`

[Out]  $x + \log(x**2 + 1)/2 - \operatorname{atan}(x) + 1/(2*x**2 + 2)$

$$3.71 \quad \int \frac{x(1+x+x^2)}{(1+x^2)^2} dx$$

Optimal. Leaf size=29

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + \frac{1}{2} \tan^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {1804, 635, 203, 260}

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] -x/(2\*(1 + x^2)) + ArcTan[x]/2 + Log[1 + x^2]/2

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 1804

Int[(Pq\_)\*((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((c\*x)^m\*(a + b\*x^2)^(p + 1)\*(a\*g - b\*f\*x))/(2\*a\*b\*(p + 1)), x] + Dist[c/(2\*a\*b\*(p + 1)), Int[(c\*x)^(m - 1)\*(a + b\*x^2)^(p + 1)\*ExpandToSum

`[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]`

Rubi steps

$$\begin{aligned} \int \frac{x(1+x+x^2)}{(1+x^2)^2} dx &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{-1-2x}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2} \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x^2) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.79

$$\frac{1}{2} \left( -\frac{x}{x^2+1} + \log(x^2+1) + \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] (-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x\*(1 + x + x^2))/(1 + x^2)^2,x]

[Out] IntegrateAlgebraic[(x\*(1 + x + x^2))/(1 + x^2)^2, x]

**fricas [A]** time = 0.59, size = 33, normalized size = 1.14

$$\frac{(x^2+1) \arctan(x) + (x^2+1) \log(x^2+1) - x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2\*((x^2 + 1)\*arctan(x) + (x^2 + 1)\*log(x^2 + 1) - x)/(x^2 + 1)

**giac** [A] time = 0.15, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x) + \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2\*x/(x^2 + 1) + 1/2\*arctan(x) + 1/2\*log(x^2 + 1)

**maple** [A] time = 0.00, size = 24, normalized size = 0.83

$$-\frac{x}{2(x^2+1)} + \frac{\arctan(x)}{2} + \frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(x^2+x+1)/(x^2+1)^2,x)

[Out] -1/2/(x^2+1)\*x+1/2\*arctan(x)+1/2\*ln(x^2+1)

**maxima** [A] time = 0.96, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x) + \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")

[Out] -1/2\*x/(x^2 + 1) + 1/2\*arctan(x) + 1/2\*log(x^2 + 1)

**mupad** [B] time = 0.03, size = 25, normalized size = 0.86

$$\frac{\ln(x^2+1)}{2} + \frac{\operatorname{atan}(x)}{2} - \frac{x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(x + x^2 + 1))/(x^2 + 1)^2,x)

[Out] log(x^2 + 1)/2 + atan(x)/2 - x/(2\*(x^2 + 1))

sympy [A] time = 0.13, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2 + 2} + \frac{\log(x^2 + 1)}{2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(x\*\*2+x+1)/(x\*\*2+1)\*\*2,x)

[Out] -x/(2\*x\*\*2 + 2) + log(x\*\*2 + 1)/2 + atan(x)/2

$$3.72 \quad \int \frac{1+x+x^2}{(1+x^2)^2} dx$$

Optimal. Leaf size=14

$$\tan^{-1}(x) - \frac{1}{2(x^2 + 1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {1814, 12, 203}

$$\tan^{-1}(x) - \frac{1}{2(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(1 + x^2)^2, x]

[Out] -1/(2\*(1 + x^2)) + ArcTan[x]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :=> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :=> With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rubi steps



$$\begin{aligned} \int \frac{1+x+x^2}{(1+x^2)^2} dx &= -\frac{1}{2(1+x^2)} - \frac{1}{2} \int -\frac{2}{1+x^2} dx \\ &= -\frac{1}{2(1+x^2)} + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{2(1+x^2)} + \tan^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(x) - \frac{1}{2(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(1 + x^2)^2,x]

[Out] -1/2\*1/(1 + x^2) + ArcTan[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(1 + x^2)^2,x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(1 + x^2)^2, x]

**fricas [A]** time = 0.73, size = 20, normalized size = 1.43

$$\frac{2(x^2 + 1) \arctan(x) - 1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2\*(2\*(x^2 + 1)\*arctan(x) - 1)/(x^2 + 1)

**giac** [A] time = 0.15, size = 12, normalized size = 0.86

$$-\frac{1}{2(x^2 + 1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2/(x^2 + 1) + arctan(x)

**maple** [A] time = 0.00, size = 13, normalized size = 0.93

$$\arctan(x) - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/(x^2+1)^2,x)

[Out] -1/2/(x^2+1)+arctan(x)

**maxima** [A] time = 0.96, size = 12, normalized size = 0.86

$$-\frac{1}{2(x^2 + 1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")

[Out] -1/2/(x^2 + 1) + arctan(x)

**mupad** [B] time = 3.80, size = 14, normalized size = 1.00

$$\operatorname{atan}(x) - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + 1)/(x^2 + 1)^2,x)

[Out] atan(x) - 1/(2\*(x^2 + 1))

**sympy** [A] time = 0.11, size = 10, normalized size = 0.71

$$\operatorname{atan}(x) - \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] atan(x) - 1/(2*x**2 + 2)
```

$$3.73 \quad \int \frac{1+x+x^2}{x(1+x^2)^2} dx$$

Optimal. Leaf size=31

$$\frac{x}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) + \frac{1}{2} \tan^{-1}(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {1805, 801, 635, 203, 260}

$$\frac{x}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(x\*(1 + x^2)^2), x]

[Out] x/(2\*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

Rule 801

Int[(((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

Rule 1805

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x(1+x^2)^2} dx &= \frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{-2-x}{x(1+x^2)} dx \\
&= \frac{x}{2(1+x^2)} - \frac{1}{2} \int \left( -\frac{2}{x} + \frac{-1+2x}{1+x^2} \right) dx \\
&= \frac{x}{2(1+x^2)} + \log(x) - \frac{1}{2} \int \frac{-1+2x}{1+x^2} dx \\
&= \frac{x}{2(1+x^2)} + \log(x) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= \frac{x}{2(1+x^2)} + \frac{1}{2} \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.90

$$\frac{1}{2} \left( \frac{x}{x^2+1} - \log(x^2+1) + 2\log(x) + \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(x\*(1 + x^2)^2), x]

[Out] (x/(1 + x^2) + ArcTan[x] + 2\*Log[x] - Log[1 + x^2])/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{x(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(x\*(1 + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(x\*(1 + x^2)^2), x]

**fricas** [A] time = 1.08, size = 41, normalized size = 1.32

$$\frac{(x^2 + 1) \arctan(x) - (x^2 + 1) \log(x^2 + 1) + 2(x^2 + 1) \log(x) + x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2\*((x^2 + 1)\*arctan(x) - (x^2 + 1)\*log(x^2 + 1) + 2\*(x^2 + 1)\*log(x) + x)/(x^2 + 1)

**giac** [A] time = 0.15, size = 26, normalized size = 0.84

$$\frac{x}{2(x^2 + 1)} + \frac{1}{2} \arctan(x) - \frac{1}{2} \log(x^2 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")

[Out] 1/2\*x/(x^2 + 1) + 1/2\*arctan(x) - 1/2\*log(x^2 + 1) + log(abs(x))

**maple** [A] time = 0.01, size = 26, normalized size = 0.84

$$\frac{x}{2x^2 + 2} + \frac{\arctan(x)}{2} + \ln(x) - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/x/(x^2+1)^2,x)

[Out] 1/2/(x^2+1)\*x+1/2\*arctan(x)+ln(x)-1/2\*ln(x^2+1)

**maxima** [A] time = 0.96, size = 25, normalized size = 0.81

$$\frac{x}{2(x^2 + 1)} + \frac{1}{2} \arctan(x) - \frac{1}{2} \log(x^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")

[Out]  $1/2*x/(x^2 + 1) + 1/2*\arctan(x) - 1/2*\log(x^2 + 1) + \log(x)$

**mupad [B]** time = 0.04, size = 32, normalized size = 1.03

$$\ln(x) + \frac{x}{2(x^2 + 1)} + \ln(x - i) \left( -\frac{1}{2} - \frac{1}{4}i \right) + \ln(x + i) \left( -\frac{1}{2} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 + 1)/(x*(x^2 + 1)^2), x)`

[Out]  $\log(x) - \log(x + 1i)*(1/2 - 1i/4) - \log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 + 1))$

**sympy [A]** time = 0.16, size = 24, normalized size = 0.77

$$\frac{x}{2x^2 + 2} + \log(x) - \frac{\log(x^2 + 1)}{2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x+1)/x/(x**2+1)**2, x)`

[Out]  $x/(2*x**2 + 2) + \log(x) - \log(x**2 + 1)/2 + \operatorname{atan}(x)/2$

$$3.74 \quad \int \frac{1+x+x^2}{x^2(1+x^2)^2} dx$$

Optimal. Leaf size=33

$$\frac{1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) - \frac{1}{x} + \log(x) - \tan^{-1}(x)$$

**Rubi [A]** time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {1805, 801, 635, 203, 260}

$$\frac{1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) - \frac{1}{x} + \log(x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(x^2\*(1 + x^2)^2), x]

[Out] -x^(-1) + 1/(2\*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

#### Rule 801

Int[(((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

#### Rule 1805



```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^2(1+x^2)^2} dx &= \frac{1}{2(1+x^2)} - \frac{1}{2} \int \frac{-2-2x}{x^2(1+x^2)} dx \\
&= \frac{1}{2(1+x^2)} - \frac{1}{2} \int \left( -\frac{2}{x^2} - \frac{2}{x} + \frac{2(1+x)}{1+x^2} \right) dx \\
&= -\frac{1}{x} + \frac{1}{2(1+x^2)} + \log(x) - \int \frac{1+x}{1+x^2} dx \\
&= -\frac{1}{x} + \frac{1}{2(1+x^2)} + \log(x) - \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= -\frac{1}{x} + \frac{1}{2(1+x^2)} - \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 1.00

$$\frac{1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) - \frac{1}{x} + \log(x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(x^2\*(1 + x^2)^2), x]

[Out] -x^(-1) + 1/(2\*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{x^2(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(x^2\*(1 + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(x^2\*(1 + x^2)^2), x]

**fricas** [A] time = 2.97, size = 49, normalized size = 1.48

$$\frac{2x^2 + 2(x^3 + x)\arctan(x) + (x^3 + x)\log(x^2 + 1) - 2(x^3 + x)\log(x) - x + 2}{2(x^3 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/2\*(2\*x^2 + 2\*(x^3 + x)\*arctan(x) + (x^3 + x)\*log(x^2 + 1) - 2\*(x^3 + x)\*log(x) - x + 2)/(x^3 + x)

**giac** [A] time = 0.18, size = 35, normalized size = 1.06

$$-\frac{2x^2 - x + 2}{2(x^3 + x)} - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2\*(2\*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2\*log(x^2 + 1) + log(abs(x))

**maple** [A] time = 0.01, size = 30, normalized size = 0.91

$$-\arctan(x) + \ln(x) - \frac{\ln(x^2 + 1)}{2} - \frac{1}{x} + \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/x^2/(x^2+1)^2,x)

[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2\*ln(x^2+1)

**maxima** [A] time = 0.95, size = 34, normalized size = 1.03

$$-\frac{2x^2 - x + 2}{2(x^3 + x)} - \arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")

[Out]  $-1/2*(2*x^2 - x + 2)/(x^3 + x) - \arctan(x) - 1/2*\log(x^2 + 1) + \log(x)$

**mupad [B]** time = 3.81, size = 38, normalized size = 1.15

$$\ln(x) - \frac{x^2 - \frac{x}{2} + 1}{x^3 + x} + \ln(x - i) \left( -\frac{1}{2} + \frac{1}{2}i \right) + \ln(x + i) \left( -\frac{1}{2} - \frac{1}{2}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 + 1)/(x^2*(x^2 + 1)^2), x)`

[Out]  $\log(x) - \log(x + 1i)*(1/2 + 1i/2) - \log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2 + 1)/(x + x^3)$

**sympy [A]** time = 0.16, size = 31, normalized size = 0.94

$$\log(x) - \frac{\log(x^2 + 1)}{2} - \operatorname{atan}(x) + \frac{-2x^2 + x - 2}{2x^3 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x+1)/x**2/(x**2+1)**2, x)`

[Out]  $\log(x) - \log(x^2 + 1)/2 - \operatorname{atan}(x) + (-2*x^2 + x - 2)/(2*x^3 + 2*x)$

$$3.75 \quad \int \frac{1+x+x^2}{x^3(1+x^2)^2} dx$$

Optimal. Leaf size=45

$$-\frac{x}{2(x^2+1)} - \frac{1}{2x^2} + \frac{1}{2} \log(x^2+1) - \frac{1}{x} - \log(x) - \frac{3}{2} \tan^{-1}(x)$$

**Rubi [A]** time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {1805, 1802, 635, 203, 260}

$$-\frac{x}{2(x^2+1)} - \frac{1}{2x^2} + \frac{1}{2} \log(x^2+1) - \frac{1}{x} - \log(x) - \frac{3}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(x^3\*(1 + x^2)^2), x]

[Out] -1/(2\*x^2) - x^(-1) - x/(2\*(1 + x^2)) - (3\*ArcTan[x])/2 - Log[x] + Log[1 + x^2]/2

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 1802

Int[(Pq\_)\*((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1805

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^3(1+x^2)^2} dx &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{-2-2x+x^3}{x^3(1+x^2)} dx \\
&= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \left( -\frac{2}{x^3} - \frac{2}{x^2} + \frac{2}{x} + \frac{3-2x}{1+x^2} \right) dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \log(x) - \frac{1}{2} \int \frac{3-2x}{1+x^2} dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \log(x) - \frac{3}{2} \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \frac{3}{2} \tan^{-1}(x) - \log(x) + \frac{1}{2} \log(1+x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.87

$$\frac{1}{2} \left( -\frac{x}{x^2+1} - \frac{1}{x^2} + \log(x^2+1) - \frac{2}{x} - 2\log(x) - 3\tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(x^3\*(1 + x^2)^2), x]

[Out] (-x^(-2) - 2/x - x/(1 + x^2) - 3\*ArcTan[x] - 2\*Log[x] + Log[1 + x^2])/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(x^3\*(1 + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(x^3\*(1 + x^2)^2), x]

**fricas** [A] time = 0.61, size = 61, normalized size = 1.36

$$\frac{3x^3 + x^2 + 3(x^4 + x^2) \arctan(x) - (x^4 + x^2) \log(x^2 + 1) + 2(x^4 + x^2) \log(x) + 2x + 1}{2(x^4 + x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/2\*(3\*x^3 + x^2 + 3\*(x^4 + x^2)\*arctan(x) - (x^4 + x^2)\*log(x^2 + 1) + 2\*(x^4 + x^2)\*log(x) + 2\*x + 1)/(x^4 + x^2)

**giac** [A] time = 0.16, size = 43, normalized size = 0.96

$$-\frac{3x^3 + x^2 + 2x + 1}{2(x^2 + 1)x^2} - \frac{3}{2} \arctan(x) + \frac{1}{2} \log(x^2 + 1) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2\*(3\*x^3 + x^2 + 2\*x + 1)/((x^2 + 1)\*x^2) - 3/2\*arctan(x) + 1/2\*log(x^2 + 1) - log(abs(x))

**maple** [A] time = 0.01, size = 38, normalized size = 0.84

$$-\frac{x}{2(x^2 + 1)} - \frac{3 \arctan(x)}{2} - \ln(x) + \frac{\ln(x^2 + 1)}{2} - \frac{1}{x} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/x^3/(x^2+1)^2,x)

[Out] -1/2/x^2-1/x-1/2/(x^2+1)\*x-3/2\*arctan(x)-ln(x)+1/2\*ln(x^2+1)

**maxima** [A] time = 0.97, size = 41, normalized size = 0.91

$$-\frac{3x^3 + x^2 + 2x + 1}{2(x^4 + x^2)} - \frac{3}{2} \arctan(x) + \frac{1}{2} \log(x^2 + 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")

[Out]  $-1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*\arctan(x) + 1/2*\log(x^2 + 1) - \log(x)$

**mupad [B]** time = 0.04, size = 47, normalized size = 1.04

$$-\ln(x) - \frac{\frac{3x^3}{2} + \frac{x^2}{2} + x + \frac{1}{2}}{x^4 + x^2} + \ln(x - i) \left( \frac{1}{2} + \frac{3i}{4} \right) + \ln(x + i) \left( \frac{1}{2} - \frac{3i}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + 1)/(x^3\*(x^2 + 1)^2),x)

[Out]  $\log(x - 1i)*(1/2 + 3i/4) + \log(x + 1i)*(1/2 - 3i/4) - \log(x) - (x + x^2/2 + (3*x^3)/2 + 1/2)/(x^2 + x^4)$

**sympy [A]** time = 0.18, size = 42, normalized size = 0.93

$$-\log(x) + \frac{\log(x^2 + 1)}{2} - \frac{3 \operatorname{atan}(x)}{2} + \frac{-3x^3 - x^2 - 2x - 1}{2x^4 + 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+x+1)/x\*\*3/(x\*\*2+1)\*\*2,x)

[Out]  $-\log(x) + \log(x**2 + 1)/2 - 3*\operatorname{atan}(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x**4 + 2*x**2)$

$$3.76 \quad \int \frac{1+2x+x^2}{(1+x^2)^2} dx$$

Optimal. Leaf size=12

$$\tan^{-1}(x) - \frac{1}{x^2 + 1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {27, 723, 203}

$$\tan^{-1}(x) - \frac{(1-x)(x+1)}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x + x^2)/(1 + x^2)^2, x]

[Out] -((1 - x)\*(1 + x))/(2\*(1 + x^2)) + ArcTan[x]

Rule 27

Int[(u\_.)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[u\*Cancel[(b/2 + c\*x)^(2\*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 723

Int[((d\_) + (e\_.)\*(x\_)^(m\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m - 1)\*(a\*e - c\*d\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] + Dist[((2\*p + 3)\*(c\*d^2 + a\*e^2))/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^(m - 2)\*(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 2, 0] && LtQ[p, -1]

Rubi steps



$$\begin{aligned} \int \frac{1+2x+x^2}{(1+x^2)^2} dx &= \int \frac{(1+x)^2}{(1+x^2)^2} dx \\ &= -\frac{(1-x)(1+x)}{2(1+x^2)} + \int \frac{1}{1+x^2} dx \\ &= -\frac{(1-x)(1+x)}{2(1+x^2)} + \tan^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 12, normalized size = 1.00

$$\tan^{-1}(x) - \frac{1}{x^2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x + x^2)/(1 + x^2)^2, x]

[Out] -(1 + x^2)^(-1) + ArcTan[x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2\*x + x^2)/(1 + x^2)^2, x]

[Out] IntegrateAlgebraic[(1 + 2\*x + x^2)/(1 + x^2)^2, x]

**fricas [A]** time = 1.56, size = 18, normalized size = 1.50

$$\frac{(x^2 + 1) \arctan(x) - 1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2\*x+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] ((x^2 + 1)\*arctan(x) - 1)/(x^2 + 1)

**giac [A]** time = 0.15, size = 12, normalized size = 1.00

$$-\frac{1}{x^2 + 1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2\*x+1)/(x^2+1)^2,x, algorithm="giac")

[Out] -1/(x^2 + 1) + arctan(x)

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$\arctan(x) - \frac{1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2\*x+1)/(x^2+1)^2,x)

[Out] -1/(x^2+1)+arctan(x)

maxima [A] time = 0.95, size = 12, normalized size = 1.00

$$-\frac{1}{x^2 + 1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2\*x+1)/(x^2+1)^2,x, algorithm="maxima")

[Out] -1/(x^2 + 1) + arctan(x)

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$\operatorname{atan}(x) - \frac{1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + x^2 + 1)/(x^2 + 1)^2,x)

[Out] atan(x) - 1/(x^2 + 1)

sympy [A] time = 0.12, size = 8, normalized size = 0.67

$$\operatorname{atan}(x) - \frac{1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+2\*x+1)/(x\*\*2+1)\*\*2,x)

[Out] atan(x) - 1/(x\*\*2 + 1)

$$3.77 \quad \int \frac{2+12x+3x^2}{(4+x^2)^2} dx$$

Optimal. Leaf size=27

$$\frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right) - \frac{5x+24}{4(x^2+4)}$$

**Rubi** [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1814, 12, 203}

$$\frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right) - \frac{5x+24}{4(x^2+4)}$$

Antiderivative was successfully verified.

[In] Int[(2 + 12\*x + 3\*x^2)/(4 + x^2)^2,x]

[Out] -(24 + 5\*x)/(4\*(4 + x^2)) + (7\*ArcTan[x/2])/8

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{2 + 12x + 3x^2}{(4 + x^2)^2} dx &= -\frac{24 + 5x}{4(4 + x^2)} - \frac{1}{8} \int -\frac{14}{4 + x^2} dx \\
&= -\frac{24 + 5x}{4(4 + x^2)} + \frac{7}{4} \int \frac{1}{4 + x^2} dx \\
&= -\frac{24 + 5x}{4(4 + x^2)} + \frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x - 24}{4(x^2 + 4)} + \frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 12\*x + 3\*x^2)/(4 + x^2)^2, x]

[Out] (-24 - 5\*x)/(4\*(4 + x^2)) + (7\*ArcTan[x/2])/8

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + 12x + 3x^2}{(4 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 12\*x + 3\*x^2)/(4 + x^2)^2, x]

[Out] IntegrateAlgebraic[(2 + 12\*x + 3\*x^2)/(4 + x^2)^2, x]

**fricas** [A] time = 0.96, size = 25, normalized size = 0.93

$$\frac{7(x^2 + 4) \arctan\left(\frac{1}{2}x\right) - 10x - 48}{8(x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+12\*x+2)/(x^2+4)^2,x, algorithm="fricas")

[Out] 1/8\*(7\*(x^2 + 4)\*arctan(1/2\*x) - 10\*x - 48)/(x^2 + 4)

**giac** [A] time = 0.18, size = 21, normalized size = 0.78

$$-\frac{5x + 24}{4(x^2 + 4)} + \frac{7}{8} \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+12\*x+2)/(x^2+4)^2,x, algorithm="giac")

[Out] -1/4\*(5\*x + 24)/(x^2 + 4) + 7/8\*arctan(1/2\*x)

**maple** [A] time = 0.01, size = 21, normalized size = 0.78

$$\frac{7 \arctan\left(\frac{x}{2}\right)}{8} + \frac{-\frac{5x}{4} - 6}{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2+12\*x+2)/(x^2+4)^2,x)

[Out] (-5/4\*x-6)/(x^2+4)+7/8\*arctan(1/2\*x)

**maxima** [A] time = 0.97, size = 21, normalized size = 0.78

$$-\frac{5x + 24}{4(x^2 + 4)} + \frac{7}{8} \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+12\*x+2)/(x^2+4)^2,x, algorithm="maxima")

[Out] -1/4\*(5\*x + 24)/(x^2 + 4) + 7/8\*arctan(1/2\*x)

**mupad** [B] time = 3.83, size = 21, normalized size = 0.78

$$\frac{7 \operatorname{atan}\left(\frac{x}{2}\right)}{8} - \frac{\frac{5x}{4} + 6}{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12\*x + 3\*x^2 + 2)/(x^2 + 4)^2,x)

[Out] (7\*atan(x/2))/8 - ((5\*x)/4 + 6)/(x^2 + 4)

**sympy** [A] time = 0.13, size = 20, normalized size = 0.74

$$\frac{-5x - 24}{4x^2 + 16} + \frac{7 \operatorname{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

```
[Out] (-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

$$3.78 \quad \int (g + hx)^3 \sqrt{a + cx^2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=390

$$\frac{x\sqrt{a + cx^2} (a^2h^2(eh + 3fg) - 2acg(3h(dh + eg) + fg^2) + 8c^2dg^3)}{16c^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right) (a^2h^2(eh + 3fg) - 2acg)}{16c^{5/2}}$$

**Rubi [A]** time = 0.83, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{(a + cx^2)^{3/2} (8(h^2f)^2 - 2ac^2(3hdh + 3g) + 15f^2) - c^2(3f^2 - 2f^2(2dh + eg)) - 3ah(a^2(3hd + 4f)g) - 14gh(2dh + eg) + 8c^2d^2g^3}{84c^6} - \frac{\sqrt{a + cx^2} (a^2h^2(eh + 3fg) - 2acg(3hdh + eg) + fg^2) + 8c^2dg^3}{16c^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right) (a^2h^2(eh + 3fg) - 2acg(3hdh + eg) + fg^2) + 8c^2dg^3}{16c^{5/2}} + \frac{(a + cx^2)^{3/2} (g + hx)^2 (8(h^2f)^2 - 2ac^2(3hdh + 3g) + 15f^2)}{70c^6} - \frac{(a + cx^2)^{3/2} (g + hx)^2 (8(h^2f)^2 - 2ac^2(3hdh + 3g) + 15f^2)}{42c^6} - \frac{f(a + cx^2)^{3/2} (g + hx)^2}{7c^6}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^3\*Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] ((8\*c^2\*d\*g^3 + a^2\*h^2\*(3\*f\*g + e\*h) - 2\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*x\*Sqrt[a + c\*x^2])/(16\*c^2) - ((3\*c\*f\*g^2 + 8\*a\*f\*h^2 - 7\*c\*h\*(e\*g + 2\*d\*h))\*(g + h\*x)^2\*(a + c\*x^2)^(3/2))/(70\*c^2\*h) - ((3\*f\*g - 7\*e\*h)\*(g + h\*x)^3\*(a + c\*x^2)^(3/2))/(42\*c\*h) + (f\*(g + h\*x)^4\*(a + c\*x^2)^(3/2))/(7\*c\*h) + ((8\*(8\*a^2\*f\*h^4 - 2\*a\*c\*h^2\*(15\*f\*g^2 + 7\*h\*(3\*e\*g + d\*h)) - c^2\*(3\*f\*g^4 - 7\*g^2\*h\*(e\*g + 12\*d\*h))) - 3\*c\*h\*(6\*c\*f\*g^3 - 14\*c\*g\*h\*(e\*g + 7\*d\*h) + a\*h^2\*(41\*f\*g + 35\*e\*h))\*x\*(a + c\*x^2)^(3/2))/(840\*c^3\*h) + (a\*(8\*c^2\*d\*g^3 + a^2\*h^2\*(3\*f\*g + e\*h) - 2\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(16\*c^(5/2))

### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps



$$\begin{aligned}
\int (g + hx)^3 \sqrt{a + cx^2} (d + ex + fx^2) dx &= \frac{f(g + hx)^4 (a + cx^2)^{3/2}}{7ch} + \frac{\int (g + hx)^3 ((7cd - 4af)h^2 - ch(3fg - 7eh))}{7ch^2} \\
&= -\frac{(3fg - 7eh)(g + hx)^3 (a + cx^2)^{3/2}}{42ch} + \frac{f(g + hx)^4 (a + cx^2)^{3/2}}{7ch} + \frac{\int (g + hx)^2 ((3cf g^2 + 8afh^2 - 7ch(eg + 2dh)) (g + hx)^2 (a + cx^2)^{3/2}}{70c^2h} \\
&= -\frac{(3cf g^2 + 8afh^2 - 7ch(eg + 2dh)) (g + hx)^2 (a + cx^2)^{3/2}}{70c^2h} - \frac{(3fg - 7eh)(g + hx)^3 (a + cx^2)^{3/2}}{42ch} \\
&= -\frac{(3cf g^2 + 8afh^2 - 7ch(eg + 2dh)) (g + hx)^2 (a + cx^2)^{3/2}}{70c^2h} - \frac{(3fg - 7eh)(g + hx)^3 (a + cx^2)^{3/2}}{42ch} \\
&= \frac{(8c^2dg^3 + a^2h^2(3fg + eh) - 2acg(fg^2 + 3h(eg + dh))) x \sqrt{a + cx^2}}{16c^2} \\
&= \frac{(8c^2dg^3 + a^2h^2(3fg + eh) - 2acg(fg^2 + 3h(eg + dh))) x \sqrt{a + cx^2}}{16c^2} \\
&= \frac{(8c^2dg^3 + a^2h^2(3fg + eh) - 2acg(fg^2 + 3h(eg + dh))) x \sqrt{a + cx^2}}{16c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.46, size = 362, normalized size = 0.93

$\frac{105x^6 \log(\sqrt{a+cx^2} + c)}{1680c^3} + \frac{c^2(3fg + eh)(g + hx)^3 + 3fg(g + hx)^3 - 2acg(3fg + eh)(g + hx)^2 + 8c^2fg^2}{1680c^3} + \frac{\sqrt{a+cx^2}(4d^2fg^2 + 7adh(4d^2 + 3g) + 3fg^2) + 35c^2d^2(3h + eg) + 16a(8cd^2fg^2 - 14adh(4d^2 + 3g) + 3fg^2) + 35c^2d^2(3h + eg) + 35cx(-d^2fg^2 + 3fg) + 2ac(3h(eg + dh) + fg^2) + 8c^2fg^2}{1680c^3} + \frac{70c^2(a^2fg + 3fg) + 6c(3h(eg + dh) + fg^2) + 48c^2h^2(fg^2 + 7h(4d^2 + 3g) + 3fg^2) + 280c^2d^2(3h + eg) + 240c^2fg^2}{1680c^3}$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(16\*a\*(8\*a^2\*f\*h^3 + 35\*c^2\*g^2\*(e\*g + 3\*d\*h) - 14\*a\*c\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))) + 105\*c\*(8\*c^2\*d\*g^3 - a^2\*h^2\*(3\*f\*g + e\*h) + 2\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*x + 16\*c\*(-4\*a^2\*f\*h^3 + 35\*c^2\*g^2\*(e\*g + 3\*d\*h) + 7\*a\*c\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*x^2 + 70\*c^2\*(a\*h^2\*(3\*f\*g + e\*h) + 6\*c\*(f\*g^3 + 3\*g\*h\*(e\*g + d\*h)))\*x^3 + 48\*c^2\*h\*(a\*f\*h^2 + 7\*c\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*x^4 + 280\*c^3\*h^2\*(3\*f\*g + e\*h)\*x^5 + 240\*c^3\*f\*h^3\*x^6) + 105\*a\*Sqrt[c]\*(8\*c^2\*d\*g^3 + a^2\*h^2\*(3\*f\*g + e\*h) - 2\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]]/(1680\*c^3)

**IntegrateAlgebraic [A]** time = 1.27, size = 474, normalized size = 1.22

$\frac{105x^6 \log(\sqrt{a+cx^2} + c)}{1680c^3} + \frac{c^2(3fg + eh)(g + hx)^3 + 3fg(g + hx)^3 - 2acg(3fg + eh)(g + hx)^2 + 8c^2fg^2}{1680c^3} + \frac{\sqrt{a+cx^2}(4d^2fg^2 + 7adh(4d^2 + 3g) + 3fg^2) + 35c^2d^2(3h + eg) + 16a(8cd^2fg^2 - 14adh(4d^2 + 3g) + 3fg^2) + 35c^2d^2(3h + eg) + 35cx(-d^2fg^2 + 3fg) + 2ac(3h(eg + dh) + fg^2) + 8c^2fg^2}{1680c^3} + \frac{70c^2(a^2fg + 3fg) + 6c(3h(eg + dh) + fg^2) + 48c^2h^2(fg^2 + 7h(4d^2 + 3g) + 3fg^2) + 280c^2d^2(3h + eg) + 240c^2fg^2}{1680c^3}$

Antiderivative was successfully verified.



[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{1680}\sqrt{c x^2 + a} \left( (2 \left( (4 (5 (6 f h^3 x + 7 (3 c^5 f g h^2 + c^5 h^3 e) / c^5) x + 6 (21 c^5 f g^2 h + 7 c^5 d h^3 + a c^4 f h^3 + 21 c^5 g h^2 e) / c^5) x + 35 (6 c^5 f g^3 + 18 c^5 d g h^2 + 3 a c^4 f g h^2 + 18 c^5 g^2 h e + a c^4 h^3 e) / c^5) x + 8 (105 c^5 d g^2 h + 21 a c^4 f g^2 h + 7 a c^4 d h^3 - 4 a^2 c^3 f h^3 + 35 c^5 g^3 e + 21 a c^4 g h^2 e) / c^5) x + 105 (8 c^5 d g^3 + 2 a c^4 f g^3 + 6 a c^4 d g h^2 - 3 a^2 c^3 f g h^2 + 6 a c^4 g^2 h e - a^2 c^3 h^3 e) / c^5) x + 16 (105 a c^4 d g^2 h - 42 a^2 c^3 f g^2 h - 14 a^2 c^3 d h^3 + 8 a^3 c^2 f h^3 + 35 a c^4 g^3 e - 42 a^2 c^3 g h^2 e) / c^5 - \frac{1}{16} (8 a c^2 d g^3 - 2 a^2 c f g^3 - 6 a^2 c d g h^2 + 3 a^3 f g h^2 - 6 a^2 c g^2 h e + a^3 h^3 e) \log(\text{abs}(-\sqrt{c} x + \sqrt{c x^2 + a})) \right) / c^{5/2}$

**maple [A]** time = 0.02, size = 661, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^3\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2),x)

[Out]  $\frac{1}{3} (c x^2 + a)^{3/2} / c e g^3 + \frac{1}{2} d g^3 x (c x^2 + a)^{1/2} + \frac{3}{5} x^2 (c x^2 + a)^{3/2} / c e g h^2 - \frac{2}{5} a / c^2 (c x^2 + a)^{3/2} f g^2 h + \frac{3}{5} x^2 (c x^2 + a)^{3/2} / c f g^2 h - \frac{2}{5} a / c^2 (c x^2 + a)^{3/2} e g h^2 + \frac{3}{4} x (c x^2 + a)^{3/2} / c d g h^2 + \frac{3}{4} x (c x^2 + a)^{3/2} / c e g^2 h - \frac{1}{8} a / c x (c x^2 + a)^{1/2} f g^3 - \frac{3}{8} a^2 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) e g^2 h - \frac{4}{35} f h^3 a / c^2 x^2 (c x^2 + a)^{3/2} + \frac{1}{2} x^3 (c x^2 + a)^{3/2} / c f g h^2 - \frac{1}{8} a / c^2 x (c x^2 + a)^{3/2} e h^3 + \frac{1}{16} a^2 / c^2 x (c x^2 + a)^{1/2} e h^3 + \frac{3}{16} a^3 / c^{5/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) f g h^2 - \frac{3}{8} a^2 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) d g h^2 - \frac{3}{8} a / c^2 x (c x^2 + a)^{3/2} f g h^2 - \frac{3}{8} a / c x (c x^2 + a)^{1/2} e g^2 h + \frac{3}{16} a^2 / c^2 x (c x^2 + a)^{1/2} f g h^2 - \frac{3}{8} a / c x (c x^2 + a)^{1/2} d g h^2 + \frac{1}{6} x^3 (c x^2 + a)^{3/2} / c e h^3 + \frac{1}{16} a^3 / c^{5/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) e h^3 + \frac{1}{5} x^2 (c x^2 + a)^{3/2} / c d h^3 - \frac{2}{15} a / c^2 (c x^2 + a)^{3/2} d h^3 + \frac{1}{4} x (c x^2 + a)^{3/2} / c f g^3 - \frac{1}{8} a^2 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) f g^3 + \frac{1}{7} f h^3 x^4 (c x^2 + a)^{3/2} / c + \frac{8}{105} f h^3 a^2 / c^3 (c x^2 + a)^{3/2} + (c x^2 + a)^{3/2} / c d g^2 h + \frac{1}{2} d g^3 a / c^{1/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2})$

**maxima [A]** time = 0.47, size = 436, normalized size = 1.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{7} (c x^2 + a)^{3/2} f h^3 x^4 / c - \frac{4}{35} (c x^2 + a)^{3/2} a f h^3 x^2 / c^2 + \frac{1}{2} \sqrt{c x^2 + a} d g^3 x + \frac{1}{2} a d g^3 \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c}$

+ 1/3\*(c\*x^2 + a)^(3/2)\*e\*g^3/c + (c\*x^2 + a)^(3/2)\*d\*g^2\*h/c + 8/105\*(c\*x^2 + a)^(3/2)\*a^2\*f\*h^3/c^3 + 1/6\*(3\*f\*g\*h^2 + e\*h^3)\*(c\*x^2 + a)^(3/2)\*x^3/c + 1/5\*(3\*f\*g^2\*h + 3\*e\*g\*h^2 + d\*h^3)\*(c\*x^2 + a)^(3/2)\*x^2/c - 1/8\*(3\*f\*g\*h^2 + e\*h^3)\*(c\*x^2 + a)^(3/2)\*a\*x/c^2 + 1/16\*(3\*f\*g\*h^2 + e\*h^3)\*sqrt(c\*x^2 + a)\*a^2\*x/c^2 + 1/4\*(f\*g^3 + 3\*e\*g^2\*h + 3\*d\*g\*h^2)\*(c\*x^2 + a)^(3/2)\*x/c - 1/8\*(f\*g^3 + 3\*e\*g^2\*h + 3\*d\*g\*h^2)\*sqrt(c\*x^2 + a)\*a\*x/c + 1/16\*(3\*f\*g\*h^2 + e\*h^3)\*a^3\*arcsinh(c\*x/sqrt(a\*c))/c^(5/2) - 1/8\*(f\*g^3 + 3\*e\*g^2\*h + 3\*d\*g\*h^2)\*a^2\*arcsinh(c\*x/sqrt(a\*c))/c^(3/2) - 2/15\*(3\*f\*g^2\*h + 3\*e\*g\*h^2 + d\*h^3)\*(c\*x^2 + a)^(3/2)\*a/c^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + hx)^3 \sqrt{cx^2 + a} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h\*x)^3\*(a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2), x)

[Out] int((g + h\*x)^3\*(a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2), x)

**sympy** [A] time = 28.37, size = 1088, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*(f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+a)\*\*(1/2), x)

[Out] -a\*\*(5/2)\*e\*h\*\*3\*x/(16\*c\*\*2\*sqrt(1 + c\*x\*\*2/a)) - 3\*a\*\*(5/2)\*f\*g\*h\*\*2\*x/(16\*c\*\*2\*sqrt(1 + c\*x\*\*2/a)) + 3\*a\*\*(3/2)\*d\*g\*h\*\*2\*x/(8\*c\*sqrt(1 + c\*x\*\*2/a)) + 3\*a\*\*(3/2)\*e\*g\*\*2\*h\*x/(8\*c\*sqrt(1 + c\*x\*\*2/a)) - a\*\*(3/2)\*e\*h\*\*3\*x\*\*3/(48\*c\*sqrt(1 + c\*x\*\*2/a)) + a\*\*(3/2)\*f\*g\*\*3\*x/(8\*c\*sqrt(1 + c\*x\*\*2/a)) - a\*\*(3/2)\*f\*g\*h\*\*2\*x\*\*3/(16\*c\*sqrt(1 + c\*x\*\*2/a)) + sqrt(a)\*d\*g\*\*3\*x\*sqrt(1 + c\*x\*\*2/a)/2 + 9\*sqrt(a)\*d\*g\*h\*\*2\*x\*\*3/(8\*sqrt(1 + c\*x\*\*2/a)) + 9\*sqrt(a)\*e\*g\*\*2\*h\*x\*\*3/(8\*sqrt(1 + c\*x\*\*2/a)) + 5\*sqrt(a)\*e\*h\*\*3\*x\*\*5/(24\*sqrt(1 + c\*x\*\*2/a)) + 3\*sqrt(a)\*f\*g\*\*3\*x\*\*3/(8\*sqrt(1 + c\*x\*\*2/a)) + 5\*sqrt(a)\*f\*g\*h\*\*2\*x\*\*5/(8\*sqrt(1 + c\*x\*\*2/a)) + a\*\*3\*e\*h\*\*3\*asinh(sqrt(c)\*x/sqrt(a))/(16\*c\*\*(5/2)) + 3\*a\*\*3\*f\*g\*h\*\*2\*asinh(sqrt(c)\*x/sqrt(a))/(16\*c\*\*(5/2)) - 3\*a\*\*2\*d\*g\*h\*\*2\*asinh(sqrt(c)\*x/sqrt(a))/(8\*c\*\*(3/2)) - 3\*a\*\*2\*e\*g\*\*2\*h\*asinh(sqrt(c)\*x/sqrt(a))/(8\*c\*\*(3/2)) - a\*\*2\*f\*g\*\*3\*asinh(sqrt(c)\*x/sqrt(a))/(8\*c\*\*(3/2)) + a\*d\*g\*\*3\*asinh(sqrt(c)\*x/sqrt(a))/(2\*sqrt(c)) + 3\*d\*g\*\*2\*h\*Piecewise((sqrt(a)\*x\*\*2/2, Eq(c, 0)), ((a + c\*x\*\*2)\*\*(3/2)/(3\*c), True)) + d\*h\*\*3\*Piecewise((-2\*a\*\*2\*sqrt(a + c\*x\*\*2)/(15\*c\*\*2) + a\*x\*\*2\*sqrt(a + c\*x\*\*2)/(15\*c) + x\*\*4\*sqrt(a + c\*x\*\*2)/5, Ne(c, 0)), (sqrt(a)\*x\*\*4/4, True)) + e\*g\*\*3\*Piecewise((sqrt(a)\*x\*\*2/2, Eq(c, 0)), ((a + c\*x\*\*2)\*\*(3/2)/(3\*c), True)) + 3\*e\*g\*h\*\*2\*Piecewise((-2\*a\*\*2\*sqrt(a + c\*x\*\*2)/(15\*c\*\*2) + a\*x\*\*2\*sqrt(a + c\*x\*\*2)/(15\*c) + x\*\*4\*sqrt(a + c\*x\*\*2)/5, Ne(c, 0)), (sqrt(a)\*x\*\*4/4, True)) + 3\*f

```

*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x
**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) +
f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g*h**2*x**5/(4*sqrt(a)*sqr
t(1 + c*x**2/a)) + 3*c*e*g**2*h*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*h
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g**3*x**5/(4*sqrt(a)*sqrt(1 +
c*x**2/a)) + c*f*g*h**2*x**7/(2*sqrt(a)*sqrt(1 + c*x**2/a))

```

$$3.79 \quad \int (g + hx)^2 \sqrt{a + cx^2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=280

$$\frac{x\sqrt{a+cx^2} (a^2fh^2 - 2ac(h(dh+2eg) + fg^2) + 8c^2dg^2)}{16c^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) (a^2fh^2 - 2ac(h(dh+2eg) + fg^2) + 8c^2dg^2)}{16c^{5/2}}$$

**Rubi [A]** time = 0.50, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\sqrt{a+cx^2} (a^2fh^2 - 2ac(h(dh+2eg) + fg^2) + 8c^2dg^2)}{16c^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) (a^2fh^2 - 2ac(h(dh+2eg) + fg^2) + 8c^2dg^2)}{16c^{5/2}} - \frac{(a+cx^2)^{3/2} (8(2ah^2(eh+2fg) - 2egh(5dh+eg) + cfg^2) - 3hx(5h^2(2hd-af) - 2eg(fg-2dh)))}{120c^2h} - \frac{(a+cx^2)^{3/2} (g+hx)^2 (fg-2dh)}{10ch} + \frac{f(a+cx^2)^{3/2} (g+hx)^3}{6ch}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^2\*sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((8\*c^2\*d\*g^2 + a^2\*f\*h^2 - 2\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*x\*sqrt[a + c\*x^2])/(16\*c^2) - ((f\*g - 2\*e\*h)\*(g + h\*x)^2\*(a + c\*x^2)^(3/2))/(10\*c\*h) + (f\*(g + h\*x)^3\*(a + c\*x^2)^(3/2))/(6\*c\*h) - ((8\*(c\*f\*g^3 - 2\*c\*g\*h\*(e\*g + 5\*d\*h) + 2\*a\*h^2\*(2\*f\*g + e\*h)) - 3\*h\*(5\*(2\*c\*d - a\*f)\*h^2 - 2\*c\*g\*(f\*g - 2\*e\*h)))\*x\*(a + c\*x^2)^(3/2))/(120\*c^2\*h) + (a\*(8\*c^2\*d\*g^2 + a^2\*f\*h^2 - 2\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*ArcTanh[(sqrt[c]\*x)/sqrt[a + c\*x^2]])/(16\*c^(5/2))

### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int (g + hx)^2 \sqrt{a + cx^2} (d + ex + fx^2) dx &= \frac{f(g + hx)^3 (a + cx^2)^{3/2}}{6ch} + \frac{\int (g + hx)^2 (3(2cd - af)h^2 - 3ch(fg - 2eh)x)}{6ch^2} \\
&= -\frac{(fg - 2eh)(g + hx)^2 (a + cx^2)^{3/2}}{10ch} + \frac{f(g + hx)^3 (a + cx^2)^{3/2}}{6ch} + \frac{\int (g + hx)}{6ch} \\
&= -\frac{(fg - 2eh)(g + hx)^2 (a + cx^2)^{3/2}}{10ch} + \frac{f(g + hx)^3 (a + cx^2)^{3/2}}{6ch} - \frac{(8(cfg - 2eh)h)}{6ch} \\
&= \frac{(8c^2dg^2 + a^2fh^2 - 2ac(fg^2 + h(2eg + dh)))x\sqrt{a + cx^2}}{16c^2} - \frac{(fg - 2eh)(g + hx)^2 (a + cx^2)^{3/2}}{10ch} \\
&= \frac{(8c^2dg^2 + a^2fh^2 - 2ac(fg^2 + h(2eg + dh)))x\sqrt{a + cx^2}}{16c^2} - \frac{(fg - 2eh)(g + hx)^2 (a + cx^2)^{3/2}}{10ch} \\
&= \frac{(8c^2dg^2 + a^2fh^2 - 2ac(fg^2 + h(2eg + dh)))x\sqrt{a + cx^2}}{16c^2} - \frac{(fg - 2eh)(g + hx)^2 (a + cx^2)^{3/2}}{10ch}
\end{aligned}$$

**Mathematica [A]** time = 0.65, size = 256, normalized size = 0.91

$$\frac{\sqrt{a + cx^2} \left( \sqrt{c} (a^2(-h)(32dh + 64fg + 15f^2hx) + 2ac(5dh(16g + 3hx) + e(40g^2 + 30ghx + 8h^2x^2) + fx(15g^2 + 16ghx + 5h^2x^2)) + 4c^2x(5d(6g^2 + 8ghx + 3h^2x^2) + x(2e(10g^2 + 15ghx + 6h^2x^2) + fx(15g^2 + 24ghx + 10h^2x^2)))) + \frac{15\sqrt{c} \operatorname{sinh}^{-1}\left(\frac{cx}{\sqrt{a}}\right) (c^2fg^2 - 2ac(6dh + 2eg) + f^2g^2 + 8c^2dg^2)}{\sqrt{\frac{a}{c^2} + 1}} \right)}{240c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^2\*sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (sqrt[a + c\*x^2]\*(sqrt[c]\*(-a^2\*h\*(64\*f\*g + 32\*e\*h + 15\*f\*h\*x)) + 2\*a\*c\*(5\*d\*h\*(16\*g + 3\*h\*x) + f\*x\*(15\*g^2 + 16\*g\*h\*x + 5\*h^2\*x^2) + e\*(40\*g^2 + 30\*g\*h\*x + 8\*h^2\*x^2)) + 4\*c^2\*x\*(5\*d\*(6\*g^2 + 8\*g\*h\*x + 3\*h^2\*x^2) + x\*(2\*e\*(10\*g^2 + 15\*g\*h\*x + 6\*h^2\*x^2) + f\*x\*(15\*g^2 + 24\*g\*h\*x + 10\*h^2\*x^2)))) + (15\*sqrt[a]\*(8\*c^2\*d\*g^2 + a^2\*f\*h^2 - 2\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*ArcSinh[(sqrt[c]\*x)/sqrt[a]])/sqrt[1 + (c\*x^2)/a])/(240\*c^(5/2))

**IntegrateAlgebraic [A]** time = 0.91, size = 302, normalized size = 1.08

$$\frac{\sqrt{a + cx^2} (-32c^2dh^2 - 64c^2fgh - 15c^2f^2x + 160acgh + 30acd^2h^2 + 80ac^2e + 60ac^2ghx + 16ac^2h^2x^2 + 30ac^2fg^2 + 32ac^2fghx + 10ac^2f^2x^2 + 120c^2dg^2x + 160c^2dghx^2 + 60c^2dh^2x^2 + 80c^2cg^2x^2 + 120c^2cghx^2 + 48c^2d^2x^2 + 60c^2f^2x^2 + 96c^2fghx^2 + 40c^2f^2x^2)}{240c^2} + \frac{\log(\sqrt{a + cx^2} - \sqrt{c}) (c^2(-f)h^2 + 2a^2dh^2 + 4c^2cgh + 2a^2fg^2 - 8a^2dg^2)}{16c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)^2\*sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (sqrt[a + c\*x^2]\*(80\*a\*c\*e\*g^2 + 160\*a\*c\*d\*g\*h - 64\*a^2\*f\*g\*h - 32\*a^2\*e\*h^2 + 120\*c^2\*d\*g^2\*x + 30\*a\*c\*f\*g^2\*x + 60\*a\*c\*e\*g\*h\*x + 30\*a\*c\*d\*h^2\*x - 15





Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^{(1/2)}, x)$

[Out]  $\frac{1}{6}f*h^2*x^3*(c*x^2+a)^{(3/2)}/c - \frac{1}{8}f*h^2*a/c^2*x*(c*x^2+a)^{(3/2)} + \frac{1}{16}f*h^2*a^2/c^2*x*(c*x^2+a)^{(1/2)} + \frac{1}{16}f*h^2*a^3/c^{(5/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)}) + \frac{1}{5}e*h^2*(c*x^2+a)^{(3/2)}/c - \frac{1}{2}e*h^2*d/c^2*x*(c*x^2+a)^{(3/2)}/c + \frac{1}{15}a/c^2*(c*x^2+a)^{(3/2)}*e*h^2 - \frac{4}{15}a/c^2*(c*x^2+a)^{(3/2)}*f*g*h + \frac{1}{4}x*(c*x^2+a)^{(3/2)}/c*d*h^2 + \frac{1}{2}x*(c*x^2+a)^{(3/2)}/c*e*g*h + \frac{1}{4}x*(c*x^2+a)^{(3/2)}/c*f*g^2 - \frac{1}{8}a/c*x*(c*x^2+a)^{(1/2)}*d*h^2 - \frac{1}{4}a/c*x*(c*x^2+a)^{(1/2)}*e*g*h - \frac{1}{8}a/c*x*(c*x^2+a)^{(1/2)}*f*g^2 - \frac{1}{8}a^2/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*d*h^2 - \frac{1}{4}a^2/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*e*g*h - \frac{1}{8}a^2/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*f*g^2 + \frac{2}{3}*(c*x^2+a)^{(3/2)}/c*d*g*h + \frac{1}{3}*(c*x^2+a)^{(3/2)}/c*e*g^2 + \frac{1}{2}d*g^2*x*(c*x^2+a)^{(1/2)} + \frac{1}{2}d*g^2*a/c^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})$

**maxima** [A] time = 0.46, size = 305, normalized size = 1.09

$$\frac{(c^2+a)^{\frac{3}{2}}fh^2x^3}{6c} + \frac{1}{2}\sqrt{cx^2+a}d^2x - \frac{(c^2+a)^{\frac{3}{2}}afh^2x}{8c^2} + \frac{\sqrt{cx^2+a}d^2fh^2x}{16c^2} + \frac{adg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{a^2fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{a}}\right)}{16c^{\frac{5}{2}}} + \frac{(c^2+a)^{\frac{3}{2}}e^2x^2}{3c} + \frac{2(c^2+a)^{\frac{3}{2}}dgh}{3c} + \frac{(2fgh+dh^2)(c^2+a)^{\frac{3}{2}}x}{5c} + \frac{(fg^2+2egh+dh^2)(c^2+a)^{\frac{3}{2}}x}{4c} - \frac{(fg^2+2egh+dh^2)\sqrt{cx^2+a}x}{8c} - \frac{(fg^2+2egh+dh^2)^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{a}}\right)}{8c^{\frac{5}{2}}} - \frac{2(2fgh+dh^2)(c^2+a)^{\frac{3}{2}}d}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{6}(c*x^2+a)^{(3/2)}*f*h^2*x^3/c + \frac{1}{2}*\sqrt{c*x^2+a}*d*g^2*x - \frac{1}{8}(c*x^2+a)^{(3/2)}*a*f*h^2*x/c^2 + \frac{1}{16}*\sqrt{c*x^2+a}*a^2*f*h^2*x/c^2 + \frac{1}{2}*a*d*g^2*\operatorname{arcsinh}(c*x/\sqrt{a*c})/\sqrt{c} + \frac{1}{16}*a^3*f*h^2*\operatorname{arcsinh}(c*x/\sqrt{a*c})/c^{(5/2)} + \frac{1}{3}*(c*x^2+a)^{(3/2)}*e*g^2/c + \frac{2}{3}*(c*x^2+a)^{(3/2)}*d*g*h/c + \frac{1}{5}*(2*f*g*h+e*h^2)*(c*x^2+a)^{(3/2)}*x^2/c + \frac{1}{4}*(f*g^2+2*e*g*h+d*h^2)*(c*x^2+a)^{(3/2)}*x/c - \frac{1}{8}*(f*g^2+2*e*g*h+d*h^2)*\sqrt{c*x^2+a}*a*x/c - \frac{1}{8}*(f*g^2+2*e*g*h+d*h^2)*a^2*\operatorname{arcsinh}(c*x/\sqrt{a*c})/c^{(3/2)} - \frac{2}{15}*(2*f*g*h+e*h^2)*(c*x^2+a)^{(3/2)}*a/c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g+hx)^2 \sqrt{cx^2+a} (fx^2+ex+d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g+h*x)^2*(a+c*x^2)^{(1/2)}*(d+e*x+f*x^2), x)$

[Out]  $\text{int}((g+h*x)^2*(a+c*x^2)^{(1/2)}*(d+e*x+f*x^2), x)$

**sympy** [A] time = 21.00, size = 738, normalized size = 2.64

$$\frac{d}{16c\sqrt{c}} - \frac{d}{8c\sqrt{c}} + \frac{d}{4c\sqrt{c}} - \frac{d}{2c\sqrt{c}} + \frac{d}{4c\sqrt{c}} - \frac{d}{8c\sqrt{c}} + \frac{d}{16c\sqrt{c}} - \frac{d}{32c\sqrt{c}} + \frac{d}{64c\sqrt{c}} - \frac{d}{128c\sqrt{c}} + \frac{d}{256c\sqrt{c}} - \frac{d}{512c\sqrt{c}} + \frac{d}{1024c\sqrt{c}} - \frac{d}{2048c\sqrt{c}} + \frac{d}{4096c\sqrt{c}} - \frac{d}{8192c\sqrt{c}} + \frac{d}{16384c\sqrt{c}} - \frac{d}{32768c\sqrt{c}} + \frac{d}{65536c\sqrt{c}} - \frac{d}{131072c\sqrt{c}} + \frac{d}{262144c\sqrt{c}} - \frac{d}{524288c\sqrt{c}} + \frac{d}{1048576c\sqrt{c}} - \frac{d}{2097152c\sqrt{c}} + \frac{d}{4194304c\sqrt{c}} - \frac{d}{8388608c\sqrt{c}} + \frac{d}{16777216c\sqrt{c}} - \frac{d}{33554432c\sqrt{c}} + \frac{d}{67108864c\sqrt{c}} - \frac{d}{134217728c\sqrt{c}} + \frac{d}{268435456c\sqrt{c}} - \frac{d}{536870912c\sqrt{c}} + \frac{d}{1073741824c\sqrt{c}} - \frac{d}{2147483648c\sqrt{c}} + \frac{d}{4294967296c\sqrt{c}} - \frac{d}{8589934592c\sqrt{c}} + \frac{d}{17179869184c\sqrt{c}} - \frac{d}{34359738368c\sqrt{c}} + \frac{d}{68719476736c\sqrt{c}} - \frac{d}{137438953472c\sqrt{c}} + \frac{d}{274877906944c\sqrt{c}} - \frac{d}{549755813888c\sqrt{c}} + \frac{d}{1099511627776c\sqrt{c}} - \frac{d}{2199023255552c\sqrt{c}} + \frac{d}{4398046511104c\sqrt{c}} - \frac{d}{8796093022208c\sqrt{c}} + \frac{d}{17592186044416c\sqrt{c}} - \frac{d}{35184372088832c\sqrt{c}} + \frac{d}{70368744177664c\sqrt{c}} - \frac{d}{140737488355328c\sqrt{c}} + \frac{d}{281474976710656c\sqrt{c}} - \frac{d}{562949953421312c\sqrt{c}} + \frac{d}{1125899906842624c\sqrt{c}} - \frac{d}{2251799813685248c\sqrt{c}} + \frac{d}{4503599627370496c\sqrt{c}} - \frac{d}{9007199254740992c\sqrt{c}} + \frac{d}{18014398509481984c\sqrt{c}} - \frac{d}{36028797018963968c\sqrt{c}} + \frac{d}{72057594037927936c\sqrt{c}} - \frac{d}{144115188075855872c\sqrt{c}} + \frac{d}{288230376151711744c\sqrt{c}} - \frac{d}{576460752303423488c\sqrt{c}} + \frac{d}{1152921504606846976c\sqrt{c}} - \frac{d}{2305843009213693952c\sqrt{c}} + \frac{d}{4611686018427387904c\sqrt{c}} - \frac{d}{9223372036854775808c\sqrt{c}} + \frac{d}{18446744073709551616c\sqrt{c}} - \frac{d}{36893488147419103232c\sqrt{c}} + \frac{d}{73786976294838206464c\sqrt{c}} - \frac{d}{147573952589676412928c\sqrt{c}} + \frac{d}{295147905179352825856c\sqrt{c}} - \frac{d}{590295810358705651712c\sqrt{c}} + \frac{d}{1180591620717411303424c\sqrt{c}} - \frac{d}{2361183241434822606848c\sqrt{c}} + \frac{d}{4722366482869645213696c\sqrt{c}} - \frac{d}{9444732965739290427392c\sqrt{c}} + \frac{d}{18889465931478580854784c\sqrt{c}} - \frac{d}{37778931862957161709568c\sqrt{c}} + \frac{d}{75557863725914323419136c\sqrt{c}} - \frac{d}{151115727451828646838272c\sqrt{c}} + \frac{d}{302231454903657293676544c\sqrt{c}} - \frac{d}{604462909807314587353088c\sqrt{c}} + \frac{d}{1208925819614629174706176c\sqrt{c}} - \frac{d}{2417851639229258349412352c\sqrt{c}} + \frac{d}{4835703278458516698824704c\sqrt{c}} - \frac{d}{9671406556917033397649408c\sqrt{c}} + \frac{d}{19342813113834066795298816c\sqrt{c}} - \frac{d}{38685626227668133590597632c\sqrt{c}} + \frac{d}{77371252455336267181195264c\sqrt{c}} - \frac{d}{154742504910672534362390528c\sqrt{c}} + \frac{d}{309485009821345068724781056c\sqrt{c}} - \frac{d}{618970019642690137449562112c\sqrt{c}} + \frac{d}{1237940039285380274899124224c\sqrt{c}} - \frac{d}{2475880078570760549798248448c\sqrt{c}} + \frac{d}{4951760157141521099596496896c\sqrt{c}} - \frac{d}{9903520314283042199192993792c\sqrt{c}} + \frac{d}{19807040628566084398385987584c\sqrt{c}} - \frac{d}{39614081257132168796771975168c\sqrt{c}} + \frac{d}{79228162514264337593543950336c\sqrt{c}} - \frac{d}{158456325028528675187087900672c\sqrt{c}} + \frac{d}{316912650057057350374175801344c\sqrt{c}} - \frac{d}{633825300114114700748351602688c\sqrt{c}} + \frac{d}{1267650600228229401496703205376c\sqrt{c}} - \frac{d}{2535301200456458802993406410752c\sqrt{c}} + \frac{d}{5070602400912917605986812821504c\sqrt{c}} - \frac{d}{10141204801825835211973625643008c\sqrt{c}} + \frac{d}{20282409603651670423947251286016c\sqrt{c}} - \frac{d}{40564819207303340847894502572032c\sqrt{c}} + \frac{d}{81129638414606681695789005144064c\sqrt{c}} - \frac{d}{162259276829213363391578010288128c\sqrt{c}} + \frac{d}{324518553658426726783156020576256c\sqrt{c}} - \frac{d}{649037107316853453566312041152512c\sqrt{c}} + \frac{d}{1298074214633706907132624082305024c\sqrt{c}} - \frac{d}{2596148429267413814265248164610048c\sqrt{c}} + \frac{d}{5192296858534827628530496329220096c\sqrt{c}} - \frac{d}{10384593717069655257060992658440192c\sqrt{c}} + \frac{d}{20769187434139310514121985316880384c\sqrt{c}} - \frac{d}{41538374868278621028243970633760768c\sqrt{c}} + \frac{d}{83076749736557242056487941267521536c\sqrt{c}} - \frac{d}{166153499473114484112975882535043072c\sqrt{c}} + \frac{d}{332306998946228968225951765070086144c\sqrt{c}} - \frac{d}{664613997892457936451903530140172288c\sqrt{c}} + \frac{d}{1329227995784915872903807060280344576c\sqrt{c}} - \frac{d}{2658455991569831745807614120560689152c\sqrt{c}} + \frac{d}{5316911983139663491615228241121378304c\sqrt{c}} - \frac{d}{10633823966279326983230456482242756608c\sqrt{c}} + \frac{d}{21267647932558653966460912964485513216c\sqrt{c}} - \frac{d}{42535295865117307932921825928971026432c\sqrt{c}} + \frac{d}{85070591730234615865843651857942052864c\sqrt{c}} - \frac{d}{170141183460469231731687303715884105728c\sqrt{c}} + \frac{d}{340282366920938463463374607431768211456c\sqrt{c}} - \frac{d}{680564733841876926926749214863536422912c\sqrt{c}} + \frac{d}{1361129467683753853853498429727072845824c\sqrt{c}} - \frac{d}{2722258935367507707706996859454145691648c\sqrt{c}} + \frac{d}{5444517870735015415413993718908291383296c\sqrt{c}} - \frac{d}{10889035741470030830827987437816582766592c\sqrt{c}} + \frac{d}{21778071482940061661655974875633165533184c\sqrt{c}} - \frac{d}{43556142965880123323311949751266331066368c\sqrt{c}} + \frac{d}{87112285931760246646623899502532662132736c\sqrt{c}} - \frac{d}{174224571863520493293247799005065324265472c\sqrt{c}} + \frac{d}{348449143727040986586495598010130648530944c\sqrt{c}} - \frac{d}{696898287454081973172991196020261297061888c\sqrt{c}} + \frac{d}{1393796574908163946345982392040522594123776c\sqrt{c}} - \frac{d}{2787593149816327892691964784081045188247552c\sqrt{c}} + \frac{d}{5575186299632655785383929568162090376495104c\sqrt{c}} - \frac{d}{11150372599265311570767859136324180752990208c\sqrt{c}} + \frac{d}{22300745198530623141535718272648361505980416c\sqrt{c}} - \frac{d}{44601490397061246283071436545296723011960832c\sqrt{c}} + \frac{d}{89202980794122492566142873090593446023921664c\sqrt{c}} - \frac{d}{178405961588244985132285746181186892047843328c\sqrt{c}} + \frac{d}{356811923176489970264571492362373784095686656c\sqrt{c}} - \frac{d}{713623846352979940529142984724747568191373312c\sqrt{c}} + \frac{d}{1427247692705959881058285969449495136382746624c\sqrt{c}} - \frac{d}{2854495385411919762116571938898990272765493248c\sqrt{c}} + \frac{d}{570899077082383952423314387779798054553098656c\sqrt{c}} - \frac{d}{1141798154164767904846628775559596109106197312c\sqrt{c}} + \frac{d}{2283596308329535809693257551119192218212394624c\sqrt{c}} - \frac{d}{4567192616659071619386515102238384436424789248c\sqrt{c}} + \frac{d}{9134385233318143238773030204476768872849578496c\sqrt{c}} - \frac{d}{18268770466636286477546060408953537745699156992c\sqrt{c}} + \frac{d}{36537540933272572955092120817907075491398313984c\sqrt{c}} - \frac{d}{73075081866545145910184241635814150982796627968c\sqrt{c}} + \frac{d}{146150163733090291820368483271628301965593255936c\sqrt{c}} - \frac{d}{292300327466180583640736966543256603931186511872c\sqrt{c}} + \frac{d}{584600654932361167281473933086513207862373023744c\sqrt{c}} - \frac{d}{1169201309864722334562947866173026415724746047488c\sqrt{c}} + \frac{d}{2338402619729444669125895732346052831449492094976c\sqrt{c}} - \frac{d}{4676805239458889338251791464692105662898984189952c\sqrt{c}} + \frac{d}{9353610478917778676503582929384211325797968379904c\sqrt{c}} - \frac{d}{18707220957835557353007165858768422651595936759808c\sqrt{c}} + \frac{d}{37414441915671114706014331717536845303191873519616c\sqrt{c}} - \frac{d}{74828883831342229412028663435073690606383747039232c\sqrt{c}} + \frac{d}{149657767662684458824057326870147381212767494078464c\sqrt{c}} - \frac{d}{299315535325368917648114653740294762425534988156928c\sqrt{c}} + \frac{d}{598631070650737835296229307480589524851069976313856c\sqrt{c}} - \frac{d}{1197262141301475670592458614961179049702139952627136c\sqrt{c}} + \frac{d}{2394524282602951341184917229922358099404279905254272c\sqrt{c}} - \frac{d}{4789048565205902682369834459844716198808559810508544c\sqrt{c}} + \frac{d}{9578097130411805364739668919689432397617119621017088c\sqrt{c}} - \frac{d}{19156194260823610729479337839378864795234239242034176c\sqrt{c}} + \frac{d}{38312388521647221458958675678757729590468478484068352c\sqrt{c}} - \frac{d}{76624777043294442917917351357515459180936956968136704c\sqrt{c}} + \frac{d}{153249554086588885835834702715030918361873913936273408c\sqrt{c}} - \frac{d}{306499108173177771671669405430061836723747827872546816c\sqrt{c}} + \frac{d}{612998216346355543343338810860123673447495655745136832c\sqrt{c}} - \frac{d}{12259964326927110866866776217202473468949911154902656c\sqrt{c}} + \frac{d}{24519928653854221733733552434404946937899822309805312c\sqrt{c}} - \frac{d}{49039857307708443467467104868809893875799644619610624c\sqrt{c}} + \frac{d}{98079714615416886934934209737619787751599289239221248c\sqrt{c}} - \frac{d}{196159429230833773869868419475239575503198578478442496c\sqrt{c}} + \frac{d}{392318858461667547739736838950479151006397156956884992c\sqrt{c}} - \frac{d}{784637716923335095479473677900958302012794313913769984c\sqrt{c}} + \frac{d}{1569275433846670190958947355801916604025588627827539968c\sqrt{c}} - \frac{d}{3138550867693340381917894711603833208051177255655079936c\sqrt{c}} + \frac{d}{6277101735386680763835789423207666416102354511310159872c\sqrt{c}} - \frac{d}{12554203470773361527671578846415332832204709022620319744c\sqrt{c}} + \frac{d}{25108406941546723055343157692830665664409418045240639488c\sqrt{c}} - \frac{d}{50216813883093446110686315385661331328818836090481279968c\sqrt{c}} + \frac{d}{100433627766186892221372630771322662657637672180962559936c\sqrt{c}} - \frac{d}{200867255532373784442745261542645325315275344361925119872c\sqrt{c}} + \frac{d}{401734511064747568885490523085290650630550688723850239744c\sqrt{c}} - \frac{d}{803469022129495137770981046170581301261101377447700479488c\sqrt{c}} + \frac{d}{160693804425899027554196209234116260252220275489540098976c\sqrt{c}} - \frac{d}{321387608851798055108392418468232520504440550979080197952c\sqrt{c}} + \frac{d}{642775217703596110216784836936465041008881101958160395904c\sqrt{c}} - \frac{d}{1285550435407192220433569673872930082017762203916320791808c\sqrt{c}} + \frac{d}{25711008708143844408671393477458601640355244078326415376c\sqrt{c}} - \frac{d}{51422017416287688817342786954917203280710488156652830752c\sqrt{c}} + \frac{d}{102844034832575377634685573909834406561420976313305661504c\sqrt{c}} - \frac{d}{2056880696651507552693711478196688131228419526266113088c\sqrt{c}} + \frac{d}{4113761393303015105387422956393376262456839052532226176c\sqrt{c}} - \frac{d}{8227522786606030210774845912786752524913678105064452352c\sqrt{c}} + \frac{d}{16455045573212060421549691825573505049827356210128904704c\sqrt{c}} - \frac{d}{32910091146424120843099383651147010099654712420257808c\sqrt{c}} + \frac{d}{65820182292848241686198767302294020199309424840515616c\sqrt{c}} - \frac{d}{131640364585696483372397534604588040398618849681031232c\sqrt{c}} + \frac{d}{263280729171392966744795069209176080797237699362062464c\sqrt{c}} - \frac{d}{526561458342785933489590138418352161594475398724124928c\sqrt{c}} + \frac{d}{1053122916685571866979180276836704323188950797448249856c\sqrt{c}} - \frac{d}{2106245833371143733958360553673408646377901594896499712c\sqrt{c}} + \frac{d}{4212491666742287467916721107346817292755803189792999424c\sqrt{c}} - \frac{d}{8424983333484574935833442214693634585511606379585998848c\sqrt{c}} + \frac{d}{16849966666969149871666884429387269171023212759171997696c\sqrt{c}} - \frac{d}{33699933333938299743333768858774538342046425518343995392c\sqrt{c}} + \frac{d}{6739986666787659948666753$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)`

[Out] `-a**(5/2)*f*h**2*x/(16*c**2*sqrt(1 + c*x**2/a)) + a**(3/2)*d*h**2*x/(8*c*sqrt(1 + c*x**2/a)) + a**(3/2)*e*g*h*x/(4*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g**2*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*f*h**2*x**3/(48*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*d*h**2*x**3/(8*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*e*g*h*x**3/(4*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*f*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*h**2*x**5/(24*sqrt(1 + c*x**2/a)) + a**3*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - a**2*d*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*e*g*h*asinh(sqrt(c)*x/sqrt(a))/(4*c**(3/2)) - a**2*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d*g**2*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 2*d*g*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g**2*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*f*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*g*h*x**5/(2*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*h**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))`

$$3.80 \quad \int (g + hx) \sqrt{a + cx^2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=175

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh - afg + 4cdg)}{8c^{3/2}} - \frac{(a + cx^2)^{3/2} (4(2afh^2 + c(3fg^2 - 5h(dh + eg))) + 3chx(3fg - 5eh))}{60c^2h} + \dots$$

**Rubi [A]** time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1654, 780, 195, 217, 206}

$$\frac{(a + cx^2)^{3/2} (4(2afh^2 + c(3fg^2 - 5h(dh + eg))) + 3chx(3fg - 5eh))}{60c^2h} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh - afg + 4cdg)}{8c^{3/2}} + \frac{x\sqrt{a+cx^2}(4cdg - a(eh + fg))}{8c} + \frac{f(a + cx^2)^{3/2}(g + hx)^2}{5ch}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)\*Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] ((4\*c\*d\*g - a\*(f\*g + e\*h))\*x\*Sqrt[a + c\*x^2])/(8\*c) + (f\*(g + h\*x)^2\*(a + c\*x^2)^(3/2))/(5\*c\*h) - ((4\*(2\*a\*f\*h^2 + c\*(3\*f\*g^2 - 5\*h\*(e\*g + d\*h))) + 3\*c\*h\*(3\*f\*g - 5\*e\*h)\*x)\*(a + c\*x^2)^(3/2))/(60\*c^2\*h) + (a\*(4\*c\*d\*g - a\*f\*g - a\*e\*h)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*c^(3/2))

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 780

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^p

+ 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

### Rule 1654

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && True) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int (g + hx)\sqrt{a + cx^2} (d + ex + fx^2) dx &= \frac{f(g + hx)^2 (a + cx^2)^{3/2}}{5ch} + \frac{\int (g + hx) ((5cd - 2af)h^2 - ch(3fg - 5eh)x)}{5ch^2} \\ &= \frac{f(g + hx)^2 (a + cx^2)^{3/2}}{5ch} - \frac{(4(2afh^2 + c(3fg^2 - 5h(eg + dh))) + 3ch)}{60c^2h} \\ &= \frac{(4cdg - a(fg + eh))x\sqrt{a + cx^2}}{8c} + \frac{f(g + hx)^2 (a + cx^2)^{3/2}}{5ch} - \frac{(4(2afh^2))}{60c^2h} \\ &= \frac{(4cdg - a(fg + eh))x\sqrt{a + cx^2}}{8c} + \frac{f(g + hx)^2 (a + cx^2)^{3/2}}{5ch} - \frac{(4(2afh^2))}{60c^2h} \\ &= \frac{(4cdg - a(fg + eh))x\sqrt{a + cx^2}}{8c} + \frac{f(g + hx)^2 (a + cx^2)^{3/2}}{5ch} - \frac{(4(2afh^2))}{60c^2h} \end{aligned}$$

**Mathematica [A]** time = 0.42, size = 153, normalized size = 0.87

$$\frac{\sqrt{a + cx^2} \left( -16a^2fh - \frac{15\sqrt{a}\sqrt{c}\sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)(aeh + afg - 4cdg)}{\sqrt{\frac{cx^2}{a} + 1}} + ac(40dh + 5e(8g + 3hx)) + fx(15g + 8hx) + 2c^2x(10d(3g + 2hx) + x(5e(4g + 3hx) + 3fx(5g + 4hx))) \right)}{120c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)\*Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(-16\*a^2\*f\*h + a\*c\*(40\*d\*h + 5\*e\*(8\*g + 3\*h\*x) + f\*x\*(15\*g + 8\*h\*x)) + 2\*c^2\*x\*(10\*d\*(3\*g + 2\*h\*x) + x\*(5\*e\*(4\*g + 3\*h\*x) + 3\*f\*x\*(5\*g + 4\*h\*x))) - (15\*Sqrt[a]\*Sqrt[c]\*(-4\*c\*d\*g + a\*f\*g + a\*e\*h)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]])/Sqrt[1 + (c\*x^2)/a))/(120\*c^2)

**IntegrateAlgebraic [A]** time = 0.54, size = 169, normalized size = 0.97

$$\frac{\log\left(\frac{\sqrt{a+cx^2}-\sqrt{cx}}{8c^{3/2}}\right)(a^2eh+a^2fg-4acd g)}{8c^{3/2}} + \frac{\sqrt{a+cx^2}(-16a^2fh+40acd h+40aceg+15acelx+15acfgx+8acfhx^2+60c^2d gx+40c^2d hx^2+40c^2egx^2+30c^2ehx^3+30c^2fgx^3+24c^2fhx^4)}{120c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)\*Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(40\*a\*c\*e\*g + 40\*a\*c\*d\*h - 16\*a^2\*f\*h + 60\*c^2\*d\*g\*x + 15\*a\*c\*f\*g\*x + 15\*a\*c\*e\*h\*x + 40\*c^2\*e\*g\*x^2 + 40\*c^2\*d\*h\*x^2 + 8\*a\*c\*f\*h\*x^2 + 30\*c^2\*f\*g\*x^3 + 30\*c^2\*e\*h\*x^3 + 24\*c^2\*f\*h\*x^4))/(120\*c^2) + ((-4\*a\*c\*d\*g + a^2\*f\*g + a^2\*e\*h)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(8\*c^(3/2))

**fricas [A]** time = 1.70, size = 329, normalized size = 1.88

$$\frac{15(\rho h - (4ad - \rho^2 f)g)\sqrt{c}\log\left(-2cx^2 + 2\sqrt{cx^2 + a}\sqrt{cx - a}\right) + 2(24\rho^2 fh^4 + 40aeg + 30(\rho^2 fg + c^2 h)^2 + 8(5c^2 g + (5\rho d + acf)h)^2 + 8(5ad - 2\rho^2 f)h + 15(aoh + (4c^2 d + acf)g))\sqrt{cx^2 + a} + 15(\rho h - (4ad - \rho^2 f)g)\sqrt{c}\arctan\left(\frac{\sqrt{cx^2 + a}}{\sqrt{c}x}\right) + (24\rho^2 fh^4 + 40aeg + 30(\rho^2 fg + c^2 h)^2 + 8(5c^2 g + (5\rho d + acf)h)^2 + 8(5ad - 2\rho^2 f)h + 15(aoh + (4c^2 d + acf)g))\sqrt{cx^2 + a}}{120c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/240\*(15\*(a^2\*e\*h - (4\*a\*c\*d - a^2\*f)\*g)\*sqrt(c)\*log(-2\*c\*x^2 + 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) + 2\*(24\*c^2\*f\*h\*x^4 + 40\*a\*c\*e\*g + 30\*(c^2\*f\*g + c^2\*e\*h)\*x^3 + 8\*(5\*c^2\*e\*g + (5\*c^2\*d + a\*c\*f)\*h)\*x^2 + 8\*(5\*a\*c\*d - 2\*a^2\*f)\*h + 15\*(a\*c\*e\*h + (4\*c^2\*d + a\*c\*f)\*g)\*x)\*sqrt(c\*x^2 + a))/c^2, 1/120\*(15\*(a^2\*e\*h - (4\*a\*c\*d - a^2\*f)\*g)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + (24\*c^2\*f\*h\*x^4 + 40\*a\*c\*e\*g + 30\*(c^2\*f\*g + c^2\*e\*h)\*x^3 + 8\*(5\*c^2\*e\*g + (5\*c^2\*d + a\*c\*f)\*h)\*x^2 + 8\*(5\*a\*c\*d - 2\*a^2\*f)\*h + 15\*(a\*c\*e\*h + (4\*c^2\*d + a\*c\*f)\*g)\*x)\*sqrt(c\*x^2 + a))/c^2]

**giac [A]** time = 0.21, size = 180, normalized size = 1.03

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4fhx+\frac{5(c^3fg+c^3he)}{c^3}\right)\right)x+\frac{4(5c^3dh+ac^2fh+5c^3ge)}{c^3}\right)x+\frac{15(4c^3dg+ac^2fg+ac^2he)}{c^3}\right)+\frac{8(5ac^2dh-2a^2cfh+5ac^2ge)}{c^3}\right)-\frac{(4acd g-a^2fg-a^2he)\log\left(-\sqrt{cx^2+a}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2), x, algorithm="giac")

[Out] 1/120\*sqrt(c\*x^2 + a)\*((2\*(3\*(4\*f\*h\*x + 5\*(c^3\*f\*g + c^3\*h\*e)/c^3)\*x + 4\*(5\*c^3\*d\*h + a\*c^2\*f\*h + 5\*c^3\*g\*e)/c^3)\*x + 15\*(4\*c^3\*d\*g + a\*c^2\*f\*g + a\*c^2\*h\*e)/c^3)\*x + 8\*(5\*a\*c^2\*d\*h - 2\*a^2\*c\*f\*h + 5\*a\*c^2\*g\*e)/c^3 - 1/8\*(4\*a\*c\*d\*g - a^2\*f\*g - a^2\*h\*e)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

**maple [A]** time = 0.01, size = 230, normalized size = 1.31

$$\frac{a^2 e h \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{8c^{\frac{3}{2}}} - \frac{a^2 f g \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{8c^{\frac{3}{2}}} + \frac{a d g \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{2\sqrt{c}} - \frac{\sqrt{c x^2 + a} a d h x}{8c} - \frac{\sqrt{c x^2 + a} a f g x}{8c} + \frac{(c x^2 + a)^{\frac{3}{2}} f h x^2}{5c} + \frac{\sqrt{c x^2 + a} d g x}{2} + \frac{(c x^2 + a)^{\frac{3}{2}} e h x}{4c} + \frac{(c x^2 + a)^{\frac{3}{2}} f g x}{4c} - \frac{2(c x^2 + a)^{\frac{3}{2}} a f h}{15c^2} + \frac{(c x^2 + a)^{\frac{3}{2}} d h}{3c} + \frac{(c x^2 + a)^{\frac{3}{2}} e g}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2), x)

[Out]  $\frac{1}{5} h f x^2 (c x^2 + a)^{3/2} / c - \frac{2}{15} h f a / c^2 (c x^2 + a)^{3/2} + \frac{1}{4} h x (c x^2 + a)^{3/2} / c e h + \frac{1}{4} h x (c x^2 + a)^{3/2} / c f g - \frac{1}{8} a / c x (c x^2 + a)^{1/2} e h - \frac{1}{8} a / c x (c x^2 + a)^{1/2} f g - \frac{1}{8} a^2 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) e h - \frac{1}{8} a^2 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) f g + \frac{1}{3} (c x^2 + a)^{3/2} / c d h + \frac{1}{3} (c x^2 + a)^{3/2} / c e g + \frac{1}{2} d g x (c x^2 + a)^{1/2} + \frac{1}{2} d g a / c^{1/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2})$

**maxima [A]** time = 0.45, size = 169, normalized size = 0.97

$$\frac{(c x^2 + a)^{\frac{3}{2}} f h x^2}{5c} + \frac{1}{2} \sqrt{c x^2 + a} d g x + \frac{a d g \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{2\sqrt{c}} + \frac{(c x^2 + a)^{\frac{3}{2}} e g}{3c} + \frac{(c x^2 + a)^{\frac{3}{2}} d h}{3c} - \frac{2(c x^2 + a)^{\frac{3}{2}} a f h}{15c^2} + \frac{(c x^2 + a)^{\frac{3}{2}} (f g + e h) x}{4c} - \frac{\sqrt{c x^2 + a} (f g + e h) a x}{8c} - \frac{(f g + e h) a^2 \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2), x, algorithm="maxima")

[Out]  $\frac{1}{5} (c x^2 + a)^{3/2} f h x^2 / c + \frac{1}{2} \sqrt{c x^2 + a} d g x + \frac{1}{2} a d g \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c} + \frac{1}{3} (c x^2 + a)^{3/2} e g / c + \frac{1}{3} (c x^2 + a)^{3/2} d h / c - \frac{2}{15} (c x^2 + a)^{3/2} a f h / c^2 + \frac{1}{4} (c x^2 + a)^{3/2} (f g + e h) x / c - \frac{1}{8} \sqrt{c x^2 + a} (f g + e h) a x / c - \frac{1}{8} (f g + e h) a^2 \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2}$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (g + h x) \sqrt{c x^2 + a} (f x^2 + e x + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h\*x)\*(a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2), x)

[Out] int((g + h\*x)\*(a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2), x)

**sympy [A]** time = 11.88, size = 384, normalized size = 2.19

$$\frac{a^2 d h x}{8c\sqrt{1+\frac{c x^2}{a}}} + \frac{a^2 f g x}{8c\sqrt{1+\frac{c x^2}{a}}} + \frac{\sqrt{a} d g x \sqrt{1+\frac{c x^2}{a}}}{2} + \frac{3\sqrt{a} e h x^3}{8\sqrt{1+\frac{c x^2}{a}}} + \frac{3\sqrt{a} f g x^3}{8\sqrt{1+\frac{c x^2}{a}}} - \frac{a^2 e h \operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}} - \frac{a^2 f g \operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}} + \frac{a d g \operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{2\sqrt{c}} + d h \left( \begin{cases} \frac{\sqrt{a} a^2}{2} & \text{for } c = 0 \\ \frac{(a+c x^2)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases} \right) + e g \left( \begin{cases} \frac{\sqrt{a} a^2}{2} & \text{for } c = 0 \\ \frac{(a+c x^2)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases} \right) + f h \left( \begin{cases} \frac{2a^2\sqrt{c x^2+a}}{15c^2} + \frac{a^2\sqrt{c x^2+a}}{15c} + \frac{x^4\sqrt{c x^2+a}}{5} & \text{for } c \neq 0 \\ \frac{\sqrt{a} a^4}{4} & \text{otherwise} \end{cases} \right) + \frac{c d h x^5}{4\sqrt{a}\sqrt{1+\frac{c x^2}{a}}} + \frac{c f g x^5}{4\sqrt{a}\sqrt{1+\frac{c x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+a)\*\*(1/2), x)

```
[Out] a**(3/2)*e*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*g*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*e*h*x**3/(8*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*f*g*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*e*h*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*f*g*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d*g*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + d*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + f*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*h*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```



### 3.81 $\int \sqrt{a + cx^2} (d + ex + fx^2) dx$

**Optimal.** Leaf size=106

$$\frac{a(4cd - af) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}} + \frac{x\sqrt{a+cx^2}(4cd - af)}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c}$$

**Rubi [A]** time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1815, 641, 195, 217, 206}

$$\frac{a(4cd - af) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}} + \frac{x\sqrt{a+cx^2}(4cd - af)}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((4\*c\*d - a\*f)\*x\*Sqrt[a + c\*x^2])/(8\*c) + (e\*(a + c\*x^2)^(3/2))/(3\*c) + (f\*x\*(a + c\*x^2)^(3/2))/(4\*c) + (a\*(4\*c\*d - a\*f)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*c^(3/2))

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /

; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :=> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \sqrt{a+cx^2} (d+ex+fx^2) dx &= \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{\int(4cd-af+4cex)\sqrt{a+cx^2} dx}{4c} \\ &= \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{(4cd-af)\int\sqrt{a+cx^2} dx}{4c} \\ &= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{(a(4cd-af))\int\frac{1}{\sqrt{a+cx^2}} dx}{8c} \\ &= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{(a(4cd-af))\text{Subst}\left[\int\frac{1}{\sqrt{a+cx^2}} dx, \frac{\sqrt{a+cx^2}}{c}\right]}{8c} \\ &= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8c^{3/2}} \end{aligned}$$

**Mathematica** [A] time = 0.20, size = 98, normalized size = 0.92

$$\frac{\sqrt{a+cx^2} \left( \sqrt{c} (a(8e+3fx) + 2cx(6d+x(4e+3fx))) - \frac{3\sqrt{a}(af-4cd)\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}} \right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(Sqrt[c]\*(a\*(8\*e + 3\*f\*x) + 2\*c\*x\*(6\*d + x\*(4\*e + 3\*f\*x))) - (3\*Sqrt[a]\*(-4\*c\*d + a\*f)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]])/Sqrt[1 + (c\*x^2)/a]))/(24\*c^(3/2))

**IntegrateAlgebraic [A]** time = 0.36, size = 89, normalized size = 0.84

$$\frac{(a^2 f - 4acd) \log\left(\sqrt{a + cx^2} - \sqrt{c}x\right)}{8c^{3/2}} + \frac{\sqrt{a + cx^2} (8ae + 3afx + 12cdx + 8cex^2 + 6cfx^3)}{24c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(8\*a\*e + 12\*c\*d\*x + 3\*a\*f\*x + 8\*c\*e\*x^2 + 6\*c\*f\*x^3))/(24\*c) + ((-4\*a\*c\*d + a^2\*f)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(8\*c^(3/2))

**fricas [A]** time = 1.05, size = 190, normalized size = 1.79

$$\left[ \frac{3(4acd - a^2f)\sqrt{c} \log\left(-2cx^2 + 2\sqrt{cx^2 + a}\sqrt{c}x - a\right) - 2(6c^2fx^3 + 8c^2ex^2 + 8ace + 3(4c^2d + acf)x)\sqrt{cx^2 + a}}{48c^2}, \frac{3(4acd - a^2f)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2 + a}}\right) - (6c^2fx^3 + 8c^2ex^2 + 8ace + 3(4c^2d + acf)x)\sqrt{cx^2 + a}}{24c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [-1/48\*(3\*(4\*a\*c\*d - a^2\*f)\*sqrt(c)\*log(-2\*c\*x^2 + 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(6\*c^2\*f\*x^3 + 8\*c^2\*e\*x^2 + 8\*a\*c\*e + 3\*(4\*c^2\*d + a\*c\*f)\*x)\*sqrt(c\*x^2 + a))/c^2, -1/24\*(3\*(4\*a\*c\*d - a^2\*f)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) - (6\*c^2\*f\*x^3 + 8\*c^2\*e\*x^2 + 8\*a\*c\*e + 3\*(4\*c^2\*d + a\*c\*f)\*x)\*sqrt(c\*x^2 + a))/c^2]

**giac [A]** time = 0.18, size = 87, normalized size = 0.82

$$\frac{1}{24} \sqrt{cx^2 + a} \left( \left( 2(3fx + 4e)x + \frac{3(4c^2d + acf)}{c^2} \right) x + \frac{8ae}{c} \right) - \frac{(4acd - a^2f) \log\left(\left| -\sqrt{c}x + \sqrt{cx^2 + a} \right|\right)}{8c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2), x, algorithm="giac")

[Out] 1/24\*sqrt(c\*x^2 + a)\*((2\*(3\*f\*x + 4\*e)\*x + 3\*(4\*c^2\*d + a\*c\*f)/c^2)\*x + 8\*a\*e/c) - 1/8\*(4\*a\*c\*d - a^2\*f)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

**maple [A]** time = 0.00, size = 111, normalized size = 1.05

$$-\frac{a^2 f \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{8c^{3/2}} + \frac{ad \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{2\sqrt{c}} - \frac{\sqrt{cx^2 + a} afx}{8c} + \frac{\sqrt{cx^2 + a} dx}{2} + \frac{(cx^2 + a)^{3/2} fx}{4c} + \frac{(cx^2 + a)^{3/2} e}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2),x)

[Out]  $\frac{1}{4}f*x*(c*x^2+a)^{(3/2)}/c - 1/8*f*a/c*x*(c*x^2+a)^{(1/2)} - 1/8*f*a^2/c^{(3/2)}*\ln(c^{(1/2)*x+(c*x^2+a)^{(1/2)})} + 1/3*e*(c*x^2+a)^{(3/2)}/c + 1/2*d*x*(c*x^2+a)^{(1/2)} + 1/2*d*a/c^{(1/2)}*\ln(c^{(1/2)*x+(c*x^2+a)^{(1/2)})}$

**maxima** [A] time = 0.45, size = 96, normalized size = 0.91

$$\frac{1}{2}\sqrt{cx^2+a}dx + \frac{(cx^2+a)^{\frac{3}{2}}fx}{4c} - \frac{\sqrt{cx^2+a}afx}{8c} + \frac{ad \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}} - \frac{a^2f \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8c^{\frac{3}{2}}} + \frac{(cx^2+a)^{\frac{3}{2}}e}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{2}*\sqrt{c*x^2+a}*d*x + \frac{1}{4}*(c*x^2+a)^{(3/2)}*f*x/c - \frac{1}{8}*\sqrt{c*x^2+a}*a*f*x/c + \frac{1}{2}*a*d*\operatorname{arcsinh}(c*x/\sqrt{a*c})/\sqrt{c} - \frac{1}{8}*a^2*f*\operatorname{arcsinh}(c*x/\sqrt{a*c})/c^{(3/2)} + \frac{1}{3}*(c*x^2+a)^{(3/2)}*e/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{cx^2+a} (fx^2+ex+d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2),x)

[Out] int((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2), x)

**sympy** [A] time = 6.90, size = 170, normalized size = 1.60

$$\frac{a^{\frac{3}{2}}fx}{8c\sqrt{1+\frac{cx^2}{a}}} + \frac{\sqrt{a}dx\sqrt{1+\frac{cx^2}{a}}}{2} + \frac{3\sqrt{a}fx^3}{8\sqrt{1+\frac{cx^2}{a}}} - \frac{a^2f \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}} + \frac{ad \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}} + e \left\{ \begin{array}{ll} \frac{\sqrt{a}x^2}{2} & \text{for } c = 0 \\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{array} \right\} + \frac{cfx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $a^{(3/2)}*f*x/(8*c*\sqrt{1+c*x**2/a}) + \sqrt{a}*d*x*\sqrt{1+c*x**2/a}/2 + 3*\sqrt{a}*f*x**3/(8*\sqrt{1+c*x**2/a}) - a**2*f*\operatorname{asinh}(\sqrt{c}*x/\sqrt{a})/(8*c^{(3/2)}) + a*d*\operatorname{asinh}(\sqrt{c}*x/\sqrt{a})/(2*\sqrt{c}) + e*\operatorname{Piecewise}((\sqrt{a}*x**2/2, \operatorname{Eq}(c, 0)), ((a+c*x**2)**(3/2)/(3*c), \operatorname{True})) + c*f*x**5/(4*\sqrt{a}*\sqrt{1+c*x**2/a})$

$$3.82 \quad \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx} dx$$

**Optimal.** Leaf size=206

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left((ah^2+2cg^2)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4} - \frac{\sqrt{ah^2+cg^2}(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^4}$$

**Rubi [A]** time = 0.39, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}(2(dh^2-egh+fg^2)-hx(fg-eh))}{2h^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left((ah^2+2cg^2)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}h^4} - \frac{\sqrt{ah^2+cg^2}(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^4} + \frac{f(a+cx^2)^{3/2}}{3ch}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] ((2\*(f\*g^2 - e\*g\*h + d\*h^2) - h\*(f\*g - e\*h)\*x)\*Sqrt[a + c\*x^2])/(2\*h^3) + (f\*(a + c\*x^2)^(3/2))/(3\*c\*h) - ((2\*c\*d\*g\*h^2 + (f\*g - e\*h)\*(2\*c\*g^2 + a\*h^2))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*Sqrt[c]\*h^4) - (Sqrt[c\*g^2 + a\*h^2]\*(f\*g^2 - e\*g\*h + d\*h^2)\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]))/h^4

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

### Rule 815

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1))]*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

### Rule 844

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

```

### Rule 1654

```

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{g+hx} dx &= \frac{f(a+cx^2)^{3/2}}{3ch} + \frac{\int \frac{(3cdh^2-3ch(fg-eh)x)\sqrt{a+cx^2}}{g+hx} dx}{3ch^2} \\
&= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} + \frac{\int \frac{3ac^2h^2(fg^2-egh+dh^2)}{g+hx} dx}{3ch^2} \\
&= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} + \frac{((cg^2+ah^2)(fg^2-egh+dh^2)-h(fg-eh)x)}{3ch^2} \\
&= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} - \frac{((cg^2+ah^2)(fg^2-egh+dh^2)-h(fg-eh)x)}{3ch^2} \\
&= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} - \frac{(2cdgh^2+(cg^2+ah^2)(fg^2-egh+dh^2)-h(fg-eh)x)}{3ch^2}
\end{aligned}$$

**Mathematica [A]** time = 0.44, size = 224, normalized size = 1.09

$$\frac{(h(dh-eg)+fg^2)\left(-\sqrt{ah^2+cg^2}\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)-\sqrt{c}g\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+h\sqrt{a+cx^2}\right)}{h^4} + \frac{\sqrt{a+cx^2}\left(\sqrt{c}x\sqrt{\frac{cx^2}{a}+1}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\right)(eh-fg)}{2\sqrt{c}h^2\sqrt{\frac{cx^2}{a}+1}} + \frac{f(a+cx^2)^{3/2}}{3ch}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] (f\*(a + c\*x^2)^(3/2))/(3\*c\*h) + ((-(f\*g) + e\*h)\*Sqrt[a + c\*x^2]\*(Sqrt[c]\*x\*Sqrt[1 + (c\*x^2)/a] + Sqrt[a]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]]))/(2\*Sqrt[c]\*h^2\*Sqrt[1 + (c\*x^2)/a]) + ((f\*g^2 + h\*(-e\*g) + d\*h)\*(h\*Sqrt[a + c\*x^2] - Sqrt[c]\*g\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]] - Sqrt[c\*g^2 + a\*h^2]\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])]))/h^4

**IntegrateAlgebraic [A]** time = 0.85, size = 233, normalized size = 1.13

$$\frac{2\sqrt{-ah^2-cg^2}(dh^2-egh+fg^2)\tan^{-1}\left(\frac{-h\sqrt{a+cx^2}+\sqrt{c}g+\sqrt{c}hx}{\sqrt{-ah^2-cg^2}}\right)}{h^4} + \frac{\sqrt{a+cx^2}(2afh^2+6cdh^2-6cegh+3celh^2x+6cfhg^2-3cfxghx+2cfxh^2x^2)}{6ch^3} + \frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)(-ah^3+afgh^2+2cdgh^2-2ceg^2h+2cfxg^3)}{2\sqrt{c}h^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] (Sqrt[a + c\*x^2]\*(6\*c\*f\*g^2 - 6\*c\*e\*g\*h + 6\*c\*d\*h^2 + 2\*a\*f\*h^2 - 3\*c\*f\*g\*h\*x + 3\*c\*e\*h^2\*x + 2\*c\*f\*h^2\*x^2))/(6\*c\*h^3) + (2\*Sqrt[-(c\*g^2) - a\*h^2]\*(f

$$\frac{g^2 - e*g*h + d*h^2}{\sqrt{-(c*g^2 - a*h^2)}} * \text{ArcTan}[\frac{\sqrt{c}*g + \sqrt{c}*h*x - h*\sqrt{a + c*x^2}}{h^4} + \frac{((2*c*f*g^3 - 2*c*e*g^2*h + 2*c*d*g*h^2 + a*f*g*h^2 - a*e*h^3)*\text{Log}[-(\sqrt{c}*x + \sqrt{a + c*x^2})])}{(2*\sqrt{c}*h^4)}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g),x, algorithm="fricas")

[Out] Timed out

**giac** [A] time = 0.22, size = 278, normalized size = 1.35

$$\frac{1}{6\sqrt{cx^2+a}} \left( \frac{2fx}{h} - \frac{3(cfg^2h^2 - ch^2e)}{ch^{10}} \right) x + \frac{2(3cf^2g^2h^7 + 3cdh^9 + afh^9 - 3cgh^9e)}{ch^{10}} + \frac{2(cfg^4 + cdg^2h^2 + afg^2h^2 + adh^4 - cg^2he - agh^2e) \arctan\left(\frac{\sqrt{cx^2+a}h + \sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}h^4} + \frac{(2c^{\frac{3}{2}}fg^3 + 2c^{\frac{3}{2}}dgh^2 + a\sqrt{c}fg^2h^2 - 2c^{\frac{3}{2}}g^2he - a\sqrt{c}h^2e) \log\left(\frac{-\sqrt{cx^2+a}}{2ch^4}\right)}{2ch^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g),x, algorithm="giac")

[Out]  $\frac{1}{6}\sqrt{c*x^2 + a} * ((2*f*x/h - 3*(c*f*g*h^8 - c*h^9*e)/(c*h^{10}))*x + 2*(3*c*f*g^2*h^7 + 3*c*d*h^9 + a*f*h^9 - 3*c*g*h^8*e)/(c*h^{10})) + 2*(c*f*g^4 + c*d*g^2*h^2 + a*f*g^2*h^2 + a*d*h^4 - c*g^3*h*e - a*g*h^3*e) * \arctan(-(\sqrt{c}*x - \sqrt{c*x^2 + a})*h + \sqrt{c}*g)/\sqrt{-c*g^2 - a*h^2}) / (\sqrt{-c*g^2 - a*h^2})*h^4 + 1/2*(2*c^(3/2)*f*g^3 + 2*c^(3/2)*d*g*h^2 + a*\sqrt{c}*f*g*h^2 - 2*c^(3/2)*g^2*h*e - a*\sqrt{c}*h^3*e) * \log(\text{abs}(-\sqrt{c}*x + \sqrt{c*x^2 + a}))) / (c*h^4)$

**maple** [B] time = 0.02, size = 1265, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g),x)

[Out]  $\frac{1}{3}f*(c*x^2+a)^{(3/2)}/c/h+1/2/h*e*x*(c*x^2+a)^{(1/2)}+1/2/h*e*a/c^{(1/2)}*\ln(c^{(1/2)*x+(c*x^2+a)^{(1/2)})}-1/2/h^2*f*g*x*(c*x^2+a)^{(1/2)}-1/2/h^2*f*g*a/c^{(1/2)}*\ln(c^{(1/2)*x+(c*x^2+a)^{(1/2)})}+1/h*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}*d-1/h^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}*e*g+1/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}*f*g^2-1/h^2*c^{(1/2)}*g*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)})*d+1/h^3*c^{(1/2)}*g^2*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)})*e-1/h^4*c^{(1/2)}*g^3*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)})*f-1/h/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x$



$g/h)+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h)*a*d+1/h^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h))*a*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h))*a*f*g^2-1/h^3/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h))*c*g^2*d+1/h^4/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h))*c*g^3*e-1/h^5/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h))*c*g^4*f$

**maxima** [A] time = 0.61, size = 362, normalized size = 1.76

$$\frac{\sqrt{cx^2+a} f g}{2 h^3} - \frac{\sqrt{cx^2+a} e x}{2 h} - \frac{\sqrt{f} g^2 \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{h^4} + \frac{\sqrt{c} g \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{h^3} - \frac{\sqrt{c} d g \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{h^2} - \frac{a / g \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{2 \sqrt{c} h^2} + \frac{a \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{2 \sqrt{c} h} + \frac{\sqrt{a+c^2} f g^2 \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{h^3} - \frac{\sqrt{a+c^2} f g \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{h^2} - \frac{\sqrt{a+c^2} d \operatorname{arcsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{h} + \frac{\sqrt{cx^2+a} f g^2}{h^3} - \frac{\sqrt{cx^2+a} e x}{h^2} + \frac{\sqrt{cx^2+a} d}{h} + \frac{(cx^2+a)^{3/2} f}{3ch}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g), x, algorithm="maxima")

[Out]  $-1/2*\sqrt{c*x^2 + a}*f*g*x/h^2 + 1/2*\sqrt{c*x^2 + a}*e*x/h - \sqrt{c}*f*g^3*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^4 + \sqrt{c}*e*g^2*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^3 - \sqrt{c}*d*g*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^2 - 1/2*a*f*g*\operatorname{arcsinh}(c*x/\sqrt{a*c})/(\sqrt{c}*h^2) + 1/2*a*e*\operatorname{arcsinh}(c*x/\sqrt{a*c})/(\sqrt{c}*h) + \sqrt{a + c*g^2/h^2}*f*g^2*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/h^3 - \sqrt{a + c*g^2/h^2}*e*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/h^2 + \sqrt{a + c*g^2/h^2}*d*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/h + \sqrt{c*x^2 + a}*f*g^2/h^3 - \sqrt{c*x^2 + a}*e*g/h^2 + \sqrt{c*x^2 + a}*d/h + 1/3*(c*x^2 + a)^{(3/2)}*f/(c*h)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2+a} (fx^2+ex+d)}{g+hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x)

[Out] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{g+hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

$$3.83 \quad \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2} dx$$

Optimal. Leaf size=308

$$\frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(afh^2+2c(3fg^2-h(2eg-dh)))}{2\sqrt{c}h^4} + \frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^4\sqrt{ah^2+cg^2}}$$

**Rubi [A]** time = 0.51, antiderivative size = 303, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 815, 844, 217, 206, 725}

$$\frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)} - \frac{\sqrt{a+cx^2}(2(ah^2(2fg-dh)-cgh(2eg-dh)+3cfg^3)-hx(afh^2-2ch(eg-dh)+3cfg^2))}{2h^3(ah^2+cg^2)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(afh^2-2ch(2eg-dh)+6cfg^2)}{2\sqrt{c}h^4} + \frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-dh)-cgh(2eg-dh)+3cfg^3)}{h^4\sqrt{ah^2+cg^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] -((2\*(3\*c\*f\*g^3 - c\*g\*h\*(2\*e\*g - d\*h) + a\*h^2\*(2\*f\*g - e\*h)) - h\*(3\*c\*f\*g^2 + a\*f\*h^2 - 2\*c\*h\*(e\*g - d\*h))\*x)\*Sqrt[a + c\*x^2])/(2\*h^3\*(c\*g^2 + a\*h^2)) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(3/2))/(h\*(c\*g^2 + a\*h^2)\*(g + h\*x)) + (((6\*c\*f\*g^2 + a\*f\*h^2 - 2\*c\*h\*(2\*e\*g - d\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*Sqrt[c]\*h^4) + ((3\*c\*f\*g^3 - c\*g\*h\*(2\*e\*g - d\*h) + a\*h^2\*(2\*f\*g - e\*h))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(h^4\*Sqrt[c\*g^2 + a\*h^2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^2} dx &= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{h(cg^2 + ah^2)(g+hx)} - \frac{\int \frac{(-cdg+afg-ae h - (afh-c(2eg-\frac{3fg^2}{h}-2dh))x)\sqrt{a+cx^2}}{g+hx}}{cg^2 + ah^2} \\
&= -\frac{(2(3cfg^3 - cgh(2eg - dh) + ah^2(2fg - eh)) - h(3cfg^2 + afh^2 - 2ch(eg - d))}{2h^3(cg^2 + ah^2)} \\
&= -\frac{(2(3cfg^3 - cgh(2eg - dh) + ah^2(2fg - eh)) - h(3cfg^2 + afh^2 - 2ch(eg - d))}{2h^3(cg^2 + ah^2)} \\
&= -\frac{(2(3cfg^3 - cgh(2eg - dh) + ah^2(2fg - eh)) - h(3cfg^2 + afh^2 - 2ch(eg - d))}{2h^3(cg^2 + ah^2)} \\
&= -\frac{(2(3cfg^3 - cgh(2eg - dh) + ah^2(2fg - eh)) - h(3cfg^2 + afh^2 - 2ch(eg - d))}{2h^3(cg^2 + ah^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 264, normalized size = 0.86

$$\frac{h\sqrt{a+cx^2}(2h(-dh+2eg+ehx)+f(-6g^2-3ghx+h^2x^2))}{g+hx} + \frac{\log(\sqrt{c}\sqrt{a+cx^2}+cx)(afh^2+2ch(dh-2eg)+6cfg^2)}{\sqrt{c}} + \frac{2\log(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx)(ah^2(2fg-eh)+cgh(dh-2eg)+3cfg^3)}{\sqrt{ah^2+cg^2}} - \frac{2\log(g+hx)(ah^2(2fg-eh)+cgh(dh-2eg)+3cfg^3)}{\sqrt{ah^2+cg^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] ((h\*Sqrt[a + c\*x^2]\*(2\*h\*(2\*e\*g - d\*h + e\*h\*x) + f\*(-6\*g^2 - 3\*g\*h\*x + h^2\*x^2)))/(g + h\*x) - (2\*(3\*c\*f\*g^3 + c\*g\*h\*(-2\*e\*g + d\*h) + a\*h^2\*(2\*f\*g - e\*h))\*Log[g + h\*x])/Sqrt[c\*g^2 + a\*h^2] + ((6\*c\*f\*g^2 + a\*f\*h^2 + 2\*c\*h\*(-2\*e\*g + d\*h))\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/Sqrt[c] + (2\*(3\*c\*f\*g^3 + c\*g\*h\*(-2\*e\*g + d\*h) + a\*h^2\*(2\*f\*g - e\*h))\*Log[a\*h - c\*g\*x + Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/Sqrt[c\*g^2 + a\*h^2])/(2\*h^4)

**IntegrateAlgebraic [A]** time = 1.27, size = 316, normalized size = 1.03

$$\frac{\log(\sqrt{a+cx^2}-\sqrt{c}x)(-afh^2-2cdh^2+4cgh-6cfs^2)}{2\sqrt{c}h^4} + \frac{\sqrt{a+cx^2}(-2dh^2+4gh+2dh^2x-6fg^2-3fghx+f^2x^2)}{2h^3(g+hx)} - \frac{2\tan^{-1}\left(\frac{h\sqrt{a+cx^2}+\sqrt{c}g+\sqrt{c}hx}{\sqrt{ah^2+cg^2}}\right)(cdgh^2\sqrt{-ah^2-cg^2}-2cgh^2h\sqrt{-ah^2-cg^2}-ach^3\sqrt{-ah^2-cg^2}+2afgh^2\sqrt{-ah^2-cg^2}+3cfs^2\sqrt{-ah^2-cg^2})}{h^4(ah^2+cg^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

```
[Out] (Sqrt[a + c*x^2]*(-6*f*g^2 + 4*e*g*h - 2*d*h^2 - 3*f*g*h*x + 2*e*h^2*x + f*
h^2*x^2))/(2*h^3*(g + h*x)) - (2*(3*c*f*g^3*Sqrt[-(c*g^2) - a*h^2] - 2*c*e*
g^2*h*Sqrt[-(c*g^2) - a*h^2] + c*d*g*h^2*Sqrt[-(c*g^2) - a*h^2] + 2*a*f*g*h
^2*Sqrt[-(c*g^2) - a*h^2] - a*e*h^3*Sqrt[-(c*g^2) - a*h^2])*ArcTan[(Sqrt[c]
*g + Sqrt[c]*h*x - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(h^4*(c*g^2
+ a*h^2)) + ((-6*c*f*g^2 + 4*c*e*g*h - 2*c*d*h^2 - a*f*h^2)*Log[-(Sqrt[c]*x
) + Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
```

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

[Out] Timed out

**maple** [B] time = 0.02, size = 2818, normalized size = 9.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] 1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
3/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(3/2)*f*g^2-1/h*c*g/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h^2*c*g^2/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-1/h^3*c*g^3/(a*h^2+c*g^2)*(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+1/h^2*c^(3/2)*g^2/(a*h^2+c*g^2)
*ln((-c*g/h+(x+g/h)*c)/c^(1/2))+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2)*d-1/h^3*c^(3/2)*g^3/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)
+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e+1/h^4*c^(3/2)*g^
4/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2))+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2)*f+2/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
```

$$\begin{aligned}
& *c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}/(x+g/h))*a*f*g-1/h^4/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*c*g^2*e+2/h^5/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*c*g^3*f-1/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(3/2)}*d-2/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*f*g+1/2*f/h^2*x*(c*x^2+a)^{(1/2)}+1/h^2*c^(1/2)/(a*h^2+c*g^2)*\ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*a*f*g^2+1/h^3*c^2*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*d-1/h^4*c^2*g^4/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*e+1/h^5*c^2*g^5/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*f-1/h*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*e*g+1/h^2*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*f*g^2-1/h*c^(1/2)/(a*h^2+c*g^2)*\ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*a*e*g+1/h^3*c*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*a*f+1/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*a*d-1/h^2*c*g^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*a*e+1/2*f/h^2*a/c^(1/2)*\ln(c^(1/2)*x+(c*x^2+a)^{(1/2)})+c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*d+1/h^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e+c^(1/2)/(a*h^2+c*g^2)*\ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*a*d-1/h^3*c^(1/2)*g*\ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*e+2/h^4*c^(1/2)*g^2*\ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*f-1/h^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*a*e
\end{aligned}$$

**maxima [A]** time = 0.65, size = 478, normalized size = 1.55

$$\frac{\sqrt{c^2+a} f^2}{h^2 x + g h^2}, \frac{\sqrt{c^2+a} g}{h^2 x + g h^2}, \frac{\sqrt{c^2+a} d}{h^2 x + g h^2}, \frac{\sqrt{c^2+a} f g}{2 h^2}, \frac{3 \sqrt{c} f^2 \operatorname{arsinh}\left(\frac{a}{g}\right)}{h^3}, \frac{2 \sqrt{c} g \operatorname{arsinh}\left(\frac{a}{g}\right)}{h^3}, \frac{\sqrt{c} d \operatorname{arsinh}\left(\frac{a}{g}\right)}{h^3}, \frac{a f \operatorname{arsinh}\left(\frac{a}{g}\right)}{2 \sqrt{c} h^2}, \frac{c f^2 \operatorname{arsinh}\left(\frac{a^2}{g^2+h^2}\right)}{\sqrt{c+a} h^2}, \frac{c g^2 \operatorname{arsinh}\left(\frac{a^2}{g^2+h^2}\right)}{\sqrt{c+a} h^2}, \frac{c f g \operatorname{arsinh}\left(\frac{a^2}{g^2+h^2}\right)}{\sqrt{c+a} h^2}, \frac{2 \sqrt{c+a} f g \operatorname{arsinh}\left(\frac{a^2}{g^2+h^2}\right)}{h^3}, \frac{\sqrt{c+a} f^2 \operatorname{arsinh}\left(\frac{a^2}{g^2+h^2}\right)}{h^3}, \frac{2 \sqrt{c^2+a} f g}{h^3}, \frac{\sqrt{c^2+a} d}{h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^2,x, algorithm="maxima")

[Out] 
$$-\sqrt{c x^2 + a} f g^2 / (h^4 x + g h^3) + \sqrt{c x^2 + a} e g / (h^3 x + g h^2) - \sqrt{c x^2 + a} d / (h^2 x + g h) + 1/2 \sqrt{c x^2 + a} f x / h^2 + 3 \sqrt{c} f g^2 \operatorname{arcsinh}(c x / \sqrt{a c}) / h^4 - 2 \sqrt{c} e g \operatorname{arcsinh}(c x / \sqrt{a c}) / h^3 + \sqrt{c} d \operatorname{arcsinh}(c x / \sqrt{a c}) / h^2 + 1/2 a f \operatorname{arcsinh}(c x / \sqrt{a c}) / (\sqrt{c} h^2) - c f g^3 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / (\sqrt{a + c g^2 / h^2} h^5) + c e g^2 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / (\sqrt{a + c g^2 / h^2} h^4) - c d g \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / (\sqrt{a + c g^2 / h^2} h^3) - 2 \sqrt{a + c g^2 / h^2} f g \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / h^3 + \sqrt{a + c g^2 / h^2} e \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / h^2 - 2 \sqrt{c x^2 + a} f g / h^3 + \sqrt{c x^2 + a} e / h^2$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c x^2 + a} (f x^2 + e x + d)}{(g + h x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x)

[Out] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + c x^2} (d + e x + f x^2)}{(g + h x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+a)\*\*(1/2)/(h\*x+g)\*\*2,x)

[Out] Integral(sqrt(a + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*2, x)



$$3.84 \quad \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3} dx$$

Optimal. Leaf size=296

$$\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2(9fg^2-h(3eg-dh))+2c^2g^3(3fg-eh)\right)}{2h^4(ah^2+cg^2)^{3/2}} - \frac{(a+cx^2)^{3/2}(dh^2-egh+)}{2h(g+hx)^2(ah^2+cg^2)}$$

Rubi [A] time = 0.55, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 813, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2(9fg^2-h(3eg-dh))+2c^2g^3(3fg-eh)\right)}{2h^4(ah^2+cg^2)^{3/2}} - \frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{2h(g+hx)^2(ah^2+cg^2)} + \frac{\sqrt{a+cx^2}(hx(2afh^2-ch(eg-dh))+3cfg^2)+2(ah^2+cg^2)(3fg-eh)}{2h^2(g+hx)(ah^2+cg^2)} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3fg-eh)}{h^4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] ((2\*(3\*f\*g - e\*h)\*(c\*g^2 + a\*h^2) + h\*(3\*c\*f\*g^2 + 2\*a\*f\*h^2 - c\*h\*(e\*g - d\*h))\*x)\*Sqrt[a + c\*x^2])/(2\*h^3\*(c\*g^2 + a\*h^2)\*(g + h\*x)) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(3/2))/(2\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^2) - (Sqrt[c]\*(3\*f\*g - e\*h)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/h^4 - ((2\*a^2\*f\*h^4 + 2\*c^2\*g^3\*(3\*f\*g - e\*h) + a\*c\*h^2\*(9\*f\*g^2 - h\*(3\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(2\*h^4\*(c\*g^2 + a\*h^2)^(3/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^3} dx &= \frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{2h(cg^2 + ah^2)(g+hx)^2} - \frac{\int \frac{(-2(cdg - afg + aeh) - (2afh - c(eg - \frac{3fg^2}{h} - dh))x) \sqrt{a+cx^2}}{(g+hx)^2}}{2(cg^2 + ah^2)} \\
&= \frac{(2(3fg - eh)(cg^2 + ah^2) + h(3cfg^2 + 2afh^2 - ch(eg - dh))x) \sqrt{a+cx^2}}{2h^3(cg^2 + ah^2)(g+hx)} \\
&= \frac{(2(3fg - eh)(cg^2 + ah^2) + h(3cfg^2 + 2afh^2 - ch(eg - dh))x) \sqrt{a+cx^2}}{2h^3(cg^2 + ah^2)(g+hx)} \\
&= \frac{(2(3fg - eh)(cg^2 + ah^2) + h(3cfg^2 + 2afh^2 - ch(eg - dh))x) \sqrt{a+cx^2}}{2h^3(cg^2 + ah^2)(g+hx)} \\
&= \frac{(2(3fg - eh)(cg^2 + ah^2) + h(3cfg^2 + 2afh^2 - ch(eg - dh))x) \sqrt{a+cx^2}}{2h^3(cg^2 + ah^2)(g+hx)}
\end{aligned}$$

**Mathematica [A]** time = 0.56, size = 318, normalized size = 1.07

$$\frac{-\frac{\log(\sqrt{a+cx^2}\sqrt{ah^2+cg^2+ah-cg})}{(ah^2+cg^2)^{3/2}} + \frac{\log(g+hx)(2a^2fh^4+acdh^2(h(dh-3cg)+9fg^2)+2c^2g^3(3fg-dh))}{(ah^2+cg^2)^{3/2}}}{2h^4} + h\sqrt{a+cx^2} \left( \frac{-2ah^2(eh-2fg)+cg(h(dh-3cg)+5fg^3)}{(g+hx)(ah^2+cg^2)} + \frac{h(eg-dh)-fg^2}{(g+hx)^2} + 2f \right) + 2\sqrt{c} \log(\sqrt{c}\sqrt{a+cx^2} + cx)(eh-3fg)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x]

[Out] (h\*Sqrt[a + c\*x^2]\*(2\*f + (-f\*g^2) + h\*(e\*g - d\*h))/(g + h\*x)^2 + (5\*c\*f\*g^3 + c\*g\*h\*(-3\*e\*g + d\*h) - 2\*a\*h^2\*(-2\*f\*g + e\*h))/((c\*g^2 + a\*h^2)\*(g + h\*x))) + ((2\*a^2\*f\*h^4 + 2\*c^2\*g^3\*(3\*f\*g - e\*h) + a\*c\*h^2\*(9\*f\*g^2 + h\*(-3\*e\*g + d\*h)))\*Log[g + h\*x])/((c\*g^2 + a\*h^2)^(3/2)) + 2\*Sqrt[c]\*(-3\*f\*g + e\*h)\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]] - ((2\*a^2\*f\*h^4 + 2\*c^2\*g^3\*(3\*f\*g - e\*h) + a\*c\*h^2\*(9\*f\*g^2 + h\*(-3\*e\*g + d\*h)))\*Log[a\*h - c\*g\*x + Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/((c\*g^2 + a\*h^2)^(3/2))/(2\*h^4)

**IntegrateAlgebraic [A]** time = 2.37, size = 411, normalized size = 1.39

$$\frac{\tan^{-1}\left(\frac{h\sqrt{a+cx^2}\sqrt{ah^2+cg^2+ah-cg}}{\sqrt{ah^2+cg^2}}\right)(2a^2fh^4\sqrt{ah^2+cg^2}-2c^2g^3h\sqrt{ah^2+cg^2}+6c^2fg^4\sqrt{ah^2+cg^2}+acdh^4\sqrt{ah^2+cg^2}-3acg^3h\sqrt{ah^2+cg^2}+9acfg^2h^2\sqrt{ah^2+cg^2})}{h^4(ah^2+cg^2)^{3/2}} + \frac{\sqrt{a+cx^2}(-ah^4-ahg^3-2ah^3a+5afg^2h^2+8afgh^3a+2afh^2+ahg^3h+cdgh^3x-2cg^2h-3cg^2h^2x+6c^2fg^4+9c^2fg^2hx+2c^2fg^2h^2x)}{2h^4(g+hx)^2(ah^2+cg^2)} + \frac{\sqrt{c}\log(\sqrt{a+cx^2}-\sqrt{c})}{h^4}(3fg-dh)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] (Sqrt[a + c\*x^2]\*(6\*c\*f\*g^4 - 2\*c\*e\*g^3\*h + 5\*a\*f\*g^2\*h^2 - a\*e\*g\*h^3 - a\*d\*h^4 + 9\*c\*f\*g^3\*h\*x - 3\*c\*e\*g^2\*h^2\*x + c\*d\*g\*h^3\*x + 8\*a\*f\*g\*h^3\*x - 2\*a\*e\*h^4\*x + 2\*c\*f\*g^2\*h^2\*x^2 + 2\*a\*f\*h^4\*x^2))/(2\*h^3\*(c\*g^2 + a\*h^2)\*(g + h\*x)^2) + ((6\*c^2\*f\*g^4\*Sqrt[-(c\*g^2) - a\*h^2] - 2\*c^2\*e\*g^3\*h\*Sqrt[-(c\*g^2) - a\*h^2] + 9\*a\*c\*f\*g^2\*h^2\*Sqrt[-(c\*g^2) - a\*h^2] - 3\*a\*c\*e\*g\*h^3\*Sqrt[-(c\*g^2) - a\*h^2] + a\*c\*d\*h^4\*Sqrt[-(c\*g^2) - a\*h^2] + 2\*a^2\*f\*h^4\*Sqrt[-(c\*g^2) - a\*h^2])\*ArcTan[(Sqrt[c]\*g + Sqrt[c]\*h\*x - h\*Sqrt[a + c\*x^2])/Sqrt[-(c\*g^2) - a\*h^2]])/(h^4\*(c\*g^2 + a\*h^2)^2) + (Sqrt[c]\*(3\*f\*g - e\*h)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/h^4

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [B] time = 0.35, size = 923, normalized size = 3.12

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^3,x, algorithm="giac")

[Out] -(6\*c^2\*f\*g^4 + 9\*a\*c\*f\*g^2\*h^2 + a\*c\*d\*h^4 + 2\*a^2\*f\*h^4 - 2\*c^2\*g^3\*h\*e - 3\*a\*c\*g\*h^3\*e)\*arctan(((sqrt(c)\*x - sqrt(c\*x^2 + a))\*h + sqrt(c)\*g)/sqrt(-c\*g^2 - a\*h^2))/((c\*g^2\*h^4 + a\*h^6)\*sqrt(-c\*g^2 - a\*h^2)) + sqrt(c\*x^2 + a)\*f/h^3 + (6\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^3\*c^2\*f\*g^4\*h + 2\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^3\*c^2\*d\*g^2\*h^3 + 5\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^3\*a\*c\*f\*g^2\*h^3 + (sqrt(c)\*x - sqrt(c\*x^2 + a))^3\*a\*c\*d\*h^5 - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^3\*c^2\*g^3\*h^2\*e - 3\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^3\*a\*c\*g\*h^4\*e + 10\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*c^(5/2)\*f\*g^5 + 2\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*c^(5/2)\*d\*g^3\*h^2 + 3\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*a\*c^(3/2)\*f\*g^3\*h^2 - (sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*a\*c^(3/2)\*d\*g\*h^4 - 4\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*a^2\*sqrt(c)\*f\*g\*h^4 - 6\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*c^(5/2)\*g^4\*h\*e - (sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*a\*c^(3/2)\*g^2\*h^3\*e + 2\*(sqrt(c)\*x - sqrt(c\*x^2 + a))^2\*a^2\*sqrt(c)\*h^5\*e - 14\*(sqrt(c)\*x - sqrt(c\*x^2 + a))\*a\*c^2\*f\*g^4\*h - 2\*(sqrt(c)\*x - sqrt(c\*x^2 + a))\*a\*c^2\*d\*g^2\*h^3 - 11\*(sqrt(c)\*x - sqrt(c\*x^2 + a))\*a^2\*c\*f\*g^2\*h^3 + (sqrt(c)\*x - sqrt(c\*x^2 + a))\*a^2\*c\*d\*h^5 + 8\*(sqrt(c)\*x - sqrt(c\*x^2 + a))\*a\*c^2\*g^3\*h^2\*e + 5\*(sqrt(c)\*x - sqrt(c\*x^2 + a))\*a^2\*c\*g\*h^4\*e + 5\*a^2\*c^(3/2)\*f\*g^3\*h^2 + a^



$$\begin{aligned}
& )/(x+g/h) * a * d - 5/2/h^3 * c * g^2 / (a * h^2 + c * g^2) / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((- \\
& 2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x+g/h) \\
& * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * a * f + 3/2/h^2 * c * g / (a * h^ \\
& 2 + c * g^2) / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 \\
& + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^ \\
& 2)^{(1/2)}) / (x+g/h) * a * e + 1/2/h^3 * c^2 * g^4 / (a * h^2 + c * g^2)^2 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} \\
& * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \\
& (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * a * f + f / h^3 * \\
& (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} + 3/2/h^4 * c^2 * g^3 / (a * h \\
& ^2 + c * g^2) / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^ \\
& 2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h \\
& ^2)^{(1/2)}) / (x+g/h) * e - 5/2/h^5 * c^2 * g^4 / (a * h^2 + c * g^2) / ((a * h^2 + c * g^2) / h^2)^{(1/2)} \\
& * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 \\
& * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * f - 2/h^2 * c / (a * \\
& h^2 + c * g^2) * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * x * f * g - 2/h \\
& ^2 * c^{(1/2)} / (a * h^2 + c * g^2) * \ln((-c * g / h + (x+g/h) * c) / c^{(1/2)} + (-2 * (x+g/h) * c * g / h + (x \\
& + g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) * a * f * g - 1/2/h * c / (a * h^2 + c * g^2) / ((a * h^2 + c * g \\
& ^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^ \\
& 2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * a \\
& * d - 1/2/h^3 * c^2 / (a * h^2 + c * g^2) / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h \\
& + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^ \\
& 2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * g^2 * d + 1/2/h^2 * c^2 * g^3 / (a * h^2 + c * g^2)^2 \\
& * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * x * f - 1/2/h * c^{(3/2)} * \\
& g^2 / (a * h^2 + c * g^2)^2 * \ln((-c * g / h + (x+g/h) * c) / c^{(1/2)} + (-2 * (x+g/h) * c * g / h + (x+g/h) \\
& ^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) * a * e + 1/2/h^5 * c^3 * g^6 / (a * h^2 + c * g^2)^2 / ((a * h^2 + \\
& c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) \\
& / h^2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) \\
& ) * f - 1/2/h^2 * c * g^3 / (a * h^2 + c * g^2)^2 / (x+g/h) * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a \\
& h^2 + c * g^2) / h^2)^{(3/2)} * f + 1/2/h^3 * c^3 * g^4 / (a * h^2 + c * g^2)^2 / ((a * h^2 + c * g^2) / h^2) \\
& ^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} \\
& * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * d - 1/2/h^4 \\
& * c^3 * g^5 / (a * h^2 + c * g^2)^2 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * ( \\
& a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + \\
& (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x+g/h) * e - 1/2/h * c^2 * g^2 / (a * h^2 + c * g^2)^2 * (-2 * (x+g \\
& / h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * x * e + 1/2/h * c * g^2 / (a * h^2 + c * g^2 \\
& )^2 / (x+g/h) * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(3/2)} * e + 1/2/h^ \\
& 2 * c^{(3/2)} * g^3 / (a * h^2 + c * g^2)^2 * \ln((-c * g / h + (x+g/h) * c) / c^{(1/2)} + (-2 * (x+g/h) * c * g \\
& / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) * a * f - 1/h / (a * h^2 + c * g^2) / (x+g/h) * (-2 * \\
& (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(3/2)} * e - f / h^4 * c^{(1/2)} * g * \ln((-c \\
& * g / h + (x+g/h) * c) / c^{(1/2)} + (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1 \\
& / 2)}) - f / h^3 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x+g/h) * c * g / h + 2 * (a * h^2 + c * g^2) / h \\
& ^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + (a * h^2 + c * g^2) / \\
& h^2)^{(1/2)}) / (x+g/h) * a + 1/2/h * c / (a * h^2 + c * g^2) * (-2 * (x+g/h) * c * g / h + (x+g/h)^2 * c + \\
& (a * h^2 + c * g^2) / h^2)^{(1/2)} * d - 1/2/h / (a * h^2 + c * g^2) / (x+g/h)^2 * (-2 * (x+g/h) * c * g / h + \\
& (x+g/h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(3/2)} * d
\end{aligned}$$

**maxima** [B] time = 0.70, size = 927, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/2*\sqrt{c*x^2 + a}*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) \\ & + 1/2*\sqrt{c*x^2 + a}*c*e*g^2/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) \\ & - 1/2*(c*x^2 + a)^{(3/2)}*f*g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + \\ & 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*\sqrt{c*x^2 + a}*c*f*g^2/(c*g^2*h^3 + \\ & a*h^5) - 1/2*\sqrt{c*x^2 + a}*c*d*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a \\ & *g*h^3) + 1/2*(c*x^2 + a)^{(3/2)}*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x \\ & x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*\sqrt{c*x^2 + a}*c*e*g/(c*g^2*h^2 \\ & + a*h^4) - 1/2*(c*x^2 + a)^{(3/2)}*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x + \\ & 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*\sqrt{c*x^2 + a}*c*d/(c*g^2*h + a*h^3 \\ & ) + 2*\sqrt{c*x^2 + a}*f*g/(h^4*x + g*h^3) - \sqrt{c*x^2 + a}*e/(h^3*x + g*h^2) \\ & - 3*\sqrt{c}*f*g*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^4 + \sqrt{c}*e*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^3 \\ & - 1/2*c^2*f*g^4*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & /((a + c*g^2/h^2)^{(3/2)}*h^7) + 1/2*c^2*e*g^3*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) /((a + c*g^2/h^2)^{(3/2)}*h^6) - 1/2*c^2*d*g^2*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) /((a + c*g^2/h^2)^{(3/2)}*h^5) + 5/2*c*f*g^2*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) /(\sqrt{a + c*g^2/h^2}*h^5) - 3/2*c*e*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) /(\sqrt{a + c*g^2/h^2}*h^4) + 1/2*c*d*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) /(\sqrt{a + c*g^2/h^2}*h^3) + \sqrt{a + c*g^2/h^2}*f*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) \\ & - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) /h^3 + \sqrt{c*x^2 + a}*f/h^3 \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a} (fx^2 + ex + d)}{(g + hx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x)

[Out] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2} (d + ex + fx^2)}{(g + hx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```



$$3.85 \quad \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4} dx$$

**Optimal.** Leaf size=314

$$\frac{c \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right) \left(a^2h^4(4fg-eh) + acgh^2(5fg^2-dh^2) + 2c^2fg^5\right) \sqrt{a+cx^2} \left(hx(2a^2fh^4 + acgh^2(6f\right)}{2h^4(ah^2 + cg^2)^{5/2}}$$

**Rubi [A]** time = 0.51, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 811, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2} \left(hx(2a^2fh^4 + acgh^2(6fg-dh) + c^2(3fg^4-dg^2h^2)) + a^2eh^5 + acgh^2(dh^2+3fg^2) + 2c^2fg^5\right) + \frac{c \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right) \left(a^2h^4(4fg-eh) + acgh^2(5fg^2-dh^2) + 2c^2fg^5\right)}{2h^4(ah^2 + cg^2)^{5/2}} - \frac{(a+cx^2)^{3/2} (dh^2-cgh+fg^2)}{3h(g+hx)^3(ah^2 + cg^2)} + \frac{\sqrt{c} f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x]

[Out] -((2\*c^2\*f\*g^5 + a^2\*e\*h^5 + a\*c\*g\*h^2\*(3\*f\*g^2 + d\*h^2) + h\*(2\*a^2\*f\*h^4 + a\*c\*g\*h^2\*(6\*f\*g - e\*h) + c^2\*(3\*f\*g^4 - d\*g^2\*h^2))\*x)\*Sqrt[a + c\*x^2])/((2\*h^3\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)^2) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(3/2))/(3\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^3) + (Sqrt[c]\*f\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/h^4 + (c\*(2\*c^2\*f\*g^5 + a^2\*h^4\*(4\*f\*g - e\*h) + a\*c\*g\*h^2\*(5\*f\*g^2 - d\*h^2))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(2\*h^4\*(c\*g^2 + a\*h^2)^(5/2))

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
+ 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
&& !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^4} dx &= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{3h(cg^2 + ah^2)(g+hx)^3} - \frac{\int \frac{(-3(cdg-afg+afh)-3f(\frac{cg^2}{h}+ah)x)\sqrt{a+cx^2}}{(g+hx)^3} dx}{3(cg^2 + ah^2)} \\
&= -\frac{(2c^2fg^5 + a^2eh^5 + acgh^2(3fg^2 + dh^2) + h(2a^2fh^4 + acgh^2(6fg - eh) + c^2)}{2h^3(cg^2 + ah^2)^2(g+hx)^2} \\
&= -\frac{(2c^2fg^5 + a^2eh^5 + acgh^2(3fg^2 + dh^2) + h(2a^2fh^4 + acgh^2(6fg - eh) + c^2)}{2h^3(cg^2 + ah^2)^2(g+hx)^2} \\
&= -\frac{(2c^2fg^5 + a^2eh^5 + acgh^2(3fg^2 + dh^2) + h(2a^2fh^4 + acgh^2(6fg - eh) + c^2)}{2h^3(cg^2 + ah^2)^2(g+hx)^2} \\
&= -\frac{(2c^2fg^5 + a^2eh^5 + acgh^2(3fg^2 + dh^2) + h(2a^2fh^4 + acgh^2(6fg - eh) + c^2)}{2h^3(cg^2 + ah^2)^2(g+hx)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.82, size = 382, normalized size = 1.22

$$\frac{3c \log\left(\frac{\sqrt{a+cx^2} \sqrt{ah^2+cg^2} + ah - c^2x}{(ah^2+cg^2)^{3/2}}\right) (d^2h^4(4fg-eh) + acgh^2(5fg^2-dh^2) + 2c^2fg^2) - \frac{3c \log(g+hx) (d^2h^4(4fg-eh) + acgh^2(5fg^2-dh^2) + 2c^2fg^2)}{(ah^2+cg^2)^{3/2}} + \frac{h \sqrt{a+cx^2} \left( \frac{(g+hx)^2 (a^2fg^4 + aah^2(12ah-5cg) + 20fg^2) + 2(11fg^4 - d^2h^4 + 2cg)}{(ah^2+cg^2)^2} \right)}{(g+hx)^3} + 6\sqrt{c} f \log\left(\sqrt{c} \sqrt{a+cx^2} + cx\right)}{6h^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x]

[Out] ((h\*Sqrt[a + c\*x^2]\*(-2\*(f\*g^2 + h\*(-(e\*g) + d\*h)) + ((7\*c\*f\*g^3 + c\*g\*h\*(-4\*e\*g + d\*h) - 3\*a\*h^2\*(-2\*f\*g + e\*h))\*(g + h\*x))/(c\*g^2 + a\*h^2) - ((6\*a^2\*f\*h^4 + c^2\*(11\*f\*g^4 - g^2\*h\*(2\*e\*g + d\*h)) + a\*c\*h^2\*(20\*f\*g^2 + h\*(-5\*e\*g + 2\*d\*h)))\*(g + h\*x)^2)/(c\*g^2 + a\*h^2)^2))/(g + h\*x)^3 - (3\*c\*(2\*c^2\*f\*g^5 + a^2\*h^4\*(4\*f\*g - e\*h) + a\*c\*g\*h^2\*(5\*f\*g^2 - d\*h^2))\*Log[g + h\*x])/(c\*g^2 + a\*h^2)^(5/2) + 6\*Sqrt[c]\*f\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]] + (3\*c\*(2\*c^2\*f\*g^5 + a^2\*h^4\*(4\*f\*g - e\*h) + a\*c\*g\*h^2\*(5\*f\*g^2 - d\*h^2))\*Log[a\*h - c\*g\*x + Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/(c\*g^2 + a\*h^2)^(5/2))/(6\*h^4)

**IntegrateAlgebraic [F]** time = 180.12, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] $Aborted
```

```
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [B] time = 0.50, size = 1719, normalized size = 5.47
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] -(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*c
*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 -
a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2)) - s
qrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2
*f*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^
3*g^4*h^3*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
c^(7/2)*f*g^6*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 + 3*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 - 12*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*g^5*h^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2
)*g^3*h^4*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 44*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^4*d*g^5*h^2 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 + 14*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 96*(sqrt(c)*x - sqrt(c*x^2
+ a))^3*a^2*c^2*f*g^3*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*
h^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - squ
rt(c*x^2 + a))^3*c^4*g^6*h*e - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h
^3*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 78*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(7/2)*f*g^6*h + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2
```

$$\begin{aligned} & *a*c^{(7/2)}*d*g^4*h^3 - 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*a^2*c^{(5/2)}}*f*g^4*h^3 - 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*a^2*c^{(5/2)}}*d*g^2*h^5 + 12*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*a^4*\text{sqrt}(c)}*f*h^7 + 12*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*a*c^{(7/2)}}*g^5*h^2*e + 30*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*a^2*c^{(5/2)}}*g^3*h^4*e - 12*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*a^3*c^{(3/2)}}*g*h^6*e + 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^2*c^3*d*g^3*h^4 + 87*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^3*c^2*f*g^3*h^4 + 9*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^4*c*f*g*h^6 - 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^2*c^3*g^4*h^3*e - 18*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^3*c^2*g^2*h^5*e + 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^4*c*h^7*e - 11*a^3*c^{(5/2)}*f*g^4*h^3 + a^3*c^{(5/2)}*d*g^2*h^5 - 20*a^4*c^{(3/2)}*f*g^2*h^5 - 2*a^4*c^{(3/2)}*d*h^7 - 6*a^5*\text{sqrt}(c)*f*h^7 + 2*a^3*c^{(5/2)}*g^3*h^4*e + 5*a^4*c^{(3/2)}*g*h^6*e)/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^{2*h} + 2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*\text{sqrt}(c)*g - a*h)^3) \end{aligned}$$

**maple [B]** time = 0.02, size = 5565, normalized size = 17.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x)`

[Out] result too large to display

**maxima [B]** time = 0.84, size = 1772, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -1/2*\text{sqrt}(c*x^2 + a)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x \\ & + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*\text{sqrt}(c*x^2 + a)*c^2*e*g^3 \\ & / (c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 \\ & + a^2*g*h^6) - 1/2*(c*x^2 + a)^{(3/2)}*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2* \\ & h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + \\ & c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/2*\text{sqrt}(c*x^2 + a)*c^2*f*g^3/( \\ & c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*\text{sqrt}(c*x^2 + a)*c^2*d*g^2/(c^2* \\ & g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2* \\ & g*h^5) + 1/2*(c*x^2 + a)^{(3/2)}*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 \\ & + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 \\ & + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/2*\text{sqrt}(c*x^2 + a)*c^2*e*g^2/(c^2*g^4*h^2 \\ & + 2*a*c*g^2*h^4 + a^2*h^6) - 1/2*(c*x^2 + a)^{(3/2)}*c*d*g/(c^2*g^4*h*x^2 + \\ & 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h \\ & ^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/2*\text{sqrt}(c*x^2 + a)*c^2*d*g \end{aligned}$$

$$\begin{aligned} & / (c^2 g^4 h + 2 a c g^2 h^3 + a^2 h^5) - 1/3 (c x^2 + a)^{3/2} f g^2 / (c g^2 h^4 x^3 + a h^6 x^3 + 3 c g^3 h^3 x^2 + 3 a g h^5 x^2 + 3 c g^4 h^2 x + 3 a g^2 h^4 x + c g^5 h + a g^3 h^3) + \sqrt{c x^2 + a} c f g^2 / (c g^2 h^4 x + a h^6 x + c g^3 h^3 + a g h^5) + 1/3 (c x^2 + a)^{3/2} e g / (c g^2 h^3 x^3 + a h^5 x^3 + 3 c g^3 h^2 x^2 + 3 a g h^4 x^2 + 3 c g^4 h x + 3 a g^2 h^3 x + c g^5 + a g^3 h^2) - 1/2 \sqrt{c x^2 + a} c e g / (c g^2 h^3 x + a h^5 x + c g^3 h^2 + a g h^4) + (c x^2 + a)^{3/2} f g / (c g^2 h^3 x^2 + a h^5 x^2 + 2 c g^3 h^2 x + 2 a g h^4 x + c g^4 h + a g^2 h^3) - \sqrt{c x^2 + a} c f g / (c g^2 h^3 + a h^5) - 1/3 (c x^2 + a)^{3/2} d / (c g^2 h^2 x^3 + a h^4 x^3 + 3 c g^3 h x^2 + 3 a g h^3 x^2 + 3 c g^4 x + 3 a g^2 h^2 x + c g^5 / h + a g^3 h) - 1/2 (c x^2 + a)^{3/2} e / (c g^2 h^2 x^2 + a h^4 x^2 + 2 c g^3 h x + 2 a g h^3 x + c g^4 + a g^2 h^2) + 1/2 \sqrt{c x^2 + a} c e / (c g^2 h^2 + a h^4) - \sqrt{c x^2 + a} f / (h^4 x + g h^3) + \sqrt{c} f \operatorname{arcsinh}(c x / \sqrt{a c}) / h^4 - 1/2 c^3 f g^5 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / ((a + c g^2 / h^2)^{5/2} h^9) + 1/2 c^3 e g^4 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / ((a + c g^2 / h^2)^{5/2} h^8) - 1/2 c^3 d g^3 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / ((a + c g^2 / h^2)^{5/2} h^7) + 3/2 c^2 f g^3 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / ((a + c g^2 / h^2)^{3/2} h^7) - c^2 e g^2 \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / ((a + c g^2 / h^2)^{3/2} h^6) + 1/2 c^2 d g \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / ((a + c g^2 / h^2)^{3/2} h^5) - 2 c f g \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / (\sqrt{a + c g^2 / h^2} h^5) + 1/2 c e \operatorname{arcsinh}(c g x / (\sqrt{a c} \operatorname{abs}(h x + g))) - a h / (\sqrt{a c} \operatorname{abs}(h x + g)) / (\sqrt{a + c g^2 / h^2} h^4) \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c x^2 + a} (f x^2 + e x + d)}{(g + h x)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x)

[Out] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + c x^2} (d + e x + f x^2)}{(g + h x)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

$$3.86 \quad \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5} dx$$

**Optimal.** Leaf size=313

$$\frac{\sqrt{a+cx^2}(ah-cgx)(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8(g+hx)^2(ah^2+cg^2)^3} - \frac{ac \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8(ah^2+cg^2)^{7/2}}$$

**Rubi [A]** time = 0.43, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1651, 807, 721, 725, 206}

$$\frac{\sqrt{a+cx^2}(ah-cgx)(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8(g+hx)^2(ah^2+cg^2)^3} - \frac{ac \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8(ah^2+cg^2)^{7/2}} + \frac{(a+cx^2)^{3/2}(4ah^2(2fg-eh)+cgh(eg-5dh)+3cfg^2)}{12h(g+hx)^3(ah^2+cg^2)^2} - \frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{4h(g+hx)^4(ah^2+cg^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x]

[Out] -(((4\*c^2\*d\*g^2 + 4\*a^2\*f\*h^2 - a\*c\*(f\*g^2 - h\*(5\*e\*g - d\*h)))\*(a\*h - c\*g\*x)\*Sqrt[a + c\*x^2])/(8\*(c\*g^2 + a\*h^2)^3\*(g + h\*x)^2) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(3/2))/(4\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^4) + ((3\*c\*f\*g^3 + c\*g\*h\*(e\*g - 5\*d\*h) + 4\*a\*h^2\*(2\*f\*g - e\*h))\*(a + c\*x^2)^(3/2))/(12\*h\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)^3) - (a\*c\*(4\*c^2\*d\*g^2 + 4\*a^2\*f\*h^2 - a\*c\*(f\*g^2 - h\*(5\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])]/(8\*(c\*g^2 + a\*h^2)^(7/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 721

Int[((d\_) + (e\_)\*(x\_)^(m\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(-2\*a\*e + (2\*c\*d)\*x)\*(a + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), x] - Dist[(4\*a\*c\*p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 2, 0] && GtQ[p, 0]

### Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ



[{a, c, d, e}, x]

### Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^5} dx &= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{4h(cg^2 + ah^2)(g+hx)^4} - \frac{\int \frac{(-4(cdg - afg + aeh) - (4afh + c(eg + \frac{3fg^2}{h} - dh))x)\sqrt{a+cx^2}}{(g+hx)^4}}{4(cg^2 + ah^2)} \\
 &= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{4h(cg^2 + ah^2)(g+hx)^4} + \frac{(3cfg^3 + cgh(eg - 5dh) + 4ah^2(2fg - eh))}{12h(cg^2 + ah^2)^2(g+hx)^3} \\
 &= -\frac{(4c^2dg^2 + 4a^2fh^2 - ac(fg^2 - h(5eg - dh)))(ah - cgx)\sqrt{a+cx^2}}{8(cg^2 + ah^2)^3(g+hx)^2} - \frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{4h(cg^2 + ah^2)(g+hx)^4} \\
 &= -\frac{(4c^2dg^2 + 4a^2fh^2 - ac(fg^2 - h(5eg - dh)))(ah - cgx)\sqrt{a+cx^2}}{8(cg^2 + ah^2)^3(g+hx)^2} - \frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{4h(cg^2 + ah^2)(g+hx)^4} \\
 &= -\frac{(4c^2dg^2 + 4a^2fh^2 - ac(fg^2 - h(5eg - dh)))(ah - cgx)\sqrt{a+cx^2}}{8(cg^2 + ah^2)^3(g+hx)^2} - \frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{4h(cg^2 + ah^2)(g+hx)^4}
 \end{aligned}$$

**Mathematica [A]** time = 1.31, size = 439, normalized size = 1.40

$$\frac{\frac{1}{24} \left( \frac{3ac \log(\sqrt{c^2 x^2 + a}) \sqrt{c^2 x^2 + a} - ab - c^2 x}{(a^2 + c^2 x^2)^2} + \frac{3ac \log(x + h) (4d^2 f^2 - a(4bd - 5c) + f^2) + 4^2 d^2 f^2}{(a^2 + c^2 x^2)^2} + \frac{\sqrt{c^2 x^2 + a} (c + h) (4d^2 f^2 - a(4bd - 5c) + f^2) + a^2 (4bd - 5c) + 35 f^2}{(a^2 + c^2 x^2)^2} + 2^2 (9 f^2 - c^2 (4bd + c)) - 4c + h^2 (4d^2 f^2 - 2ab) + c^2 (4bd - 13b) + 19 f^2}{(c + h)^2 (a^2 + c^2 x^2)} + 2^2 (2^2 (4bd + c) + 3 f^2) + 4 (a^2 + c^2 x^2) (4bd - c) + f^2}{(c + h)^2 (a^2 + c^2 x^2)} \right)}{24}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] 
$$\begin{aligned} & -((\text{Sqrt}[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-e*g) + d*h)) - 2*(c*g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h)) * \\ & (g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g + d*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h))) * (g + h*x)^2 - c*(4*a^2*h^4 * \\ & (7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3*f*g^5 + g^3*h*(e*g + d*h))) * (g + h*x)^3) / ((c*g^2*h + a*h^3)^3*(g + h*x)^4) + \\ & (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h))) * \text{Log}[g + h*x]) / (c*g^2 + a*h^2)^{(7/2)} - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h))) * \text{Log}[a*h - c*g*x + \text{Sqrt}[c*g^2 + a*h^2]*\text{Sqrt}[a + c*x^2]]) / (c*g^2 + a*h^2)^{(7/2))} / 24 \end{aligned}$$

**IntegrateAlgebraic [B]** time = 122.08, size = 4080, normalized size = 13.04

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] 
$$\begin{aligned} & (2*a^6*f*g^2*h^7 + 2*a^6*e*g*h^8 + 6*a^6*d*h^9 + 8*a^6*f*g*h^8*x + 8*a^6*e*h^9*x + 12*a^6*f*h^9*x^2 + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]*(-8*a^5*f*g^2*h^7*x - 8*a^5*e*g*h^8*x - 24*a^5*d*h^9*x - 32*a^5*f*g*h^8*x^2 - 32*a^5*e*h^9*x^2 - 48*a^5*f*h^9*x^3) + c*(-13*a^5*f*g^4*h^5 + 9*a^5*e*g^3*h^6 + 19*a^5*d*g^2*h^7 - 40*a^5*f*g^3*h^6*x + 36*a^5*e*g^2*h^7*x + 4*a^5*d*g*h^8*x - 19*a^5*f*g^2*h^7*x^2 + 35*a^5*e*g*h^8*x^2 + 57*a^5*d*h^9*x^2 + 44*a^5*f*g*h^8*x^3 + 80*a^5*e*h^9*x^3 + 108*a^5*f*h^9*x^4) + c^{(3/2)}*\text{Sqrt}[a + c*x^2]*(-28*a^4*f*g^5*h^4 + 8*a^4*e*g^4*h^5 - 60*a^4*f*g^4*h^5*x - 4*a^4*e*g^3*h^6*x - 76*a^4*d*g^2*h^7*x - 8*a^4*f*g^3*h^6*x^2 - 96*a^4*e*g^2*h^7*x^2 - 16*a^4*d*g*h^8*x^2 + 12*a^4*f*g^2*h^7*x^3 - 60*a^4*e*g*h^8*x^3 - 84*a^4*d*h^9*x^3 - 12*a^4*f*g*h^8*x^4 - 120*a^4*e*h^9*x^4 - 144*a^4*f*h^9*x^5) + c^2*(-8*a^4*e*g^5*h^4 + 28*a^4*d*g^4*h^5 + 109*a^4*f*g^5*h^4*x - 49*a^4*e*g^4*h^5*x + 37*a^4*d*g^3*h^6*x + 327*a^4*f*g^4*h^5*x^2 - 83*a^4*e*g^3*h^6*x^2 + 211*a^4*d*g^2*h^7*x^2 + 293*a^4*f*g^3*h^6*x^3 + 123*a^4*e*g^2*h^7*x^3 + 49*a^4*d*g*h^8*x^3 + 147*a^4*f*g^2*h^7*x^4 + 57*a^4*e*g*h^8*x^4 + 123*a^4*d*h^9*x^4 - 12*a^4*f*g*h^8*x^5 + 168*a^4*e*h^9*x^5 + 192*a^4*f*h^9*x^6) + c^{(11/2)}*\text{Sqrt}[a + c*x^2]*(-48*f*g^9*x^4 - 16*e*g^8*h*x^4 - 16*d*g^7*h^2*x^4 - 192*f*g^8*h*x^5 - 64*e*g^7*h^2*x^5 - 64*d*g^6*h^3*x^5 - 288*f*g^7*h^2*x^6 - 96*e*g^6*h^3*x^6 - 192*f*g^6*h^3*x^7) + c^{(9/2)}*\text{Sqrt}[a + c*x^2]*(-48*a*f*g^9*x^2 - 16*a*e*g^8* \end{aligned}$$

$$\begin{aligned}
& h^2x^2 - 16adg^7h^2x^2 - 192afg^8hx^3 - 64aeg^7h^2x^3 - 64adg^6h^3x^3 - 440afg^7h^2x^4 - 168aeg^6h^3x^4 + 152adg^5h^4x^4 - 800afg^6h^3x^5 - 192aeg^5h^4x^5 + 224adg^4h^5x^5 - 864afg^5h^4x^6 - 288aeg^4h^5x^6 + 336adg^3h^6x^6 - 576afg^4h^5x^7 + 96adg^2h^7x^7) + c^{(7/2)}\sqrt{a + cx^2} * (-6a^2fg^9 - 2a^2eg^8h - 2a^2dg^7h^2 - 24a^2fg^8hx - 8a^2eg^7h^2x - 8a^2dg^6h^3x - 188a^2fg^7h^2x^2 - 84a^2eg^6h^3x^2 + 140a^2dg^5h^4x^2 - 632a^2fg^6h^3x^3 - 168a^2eg^5h^4x^3 + 104a^2dg^4h^5x^3 - 1082a^2fg^5h^4x^4 - 158a^2eg^4h^5x^4 + 186a^2dg^3h^6x^4 - 1352a^2fg^4h^5x^5 + 328a^2eg^3h^6x^5 - 216a^2dg^2h^7x^5 - 948a^2fg^3h^6x^6 + 132a^2eg^2h^7x^6 - 84a^2dgh^8x^6 - 600a^2fg^2h^7x^7 + 120a^2eggh^8x^7 - 24a^2dh^9x^7) + c^{(5/2)}\sqrt{a + cx^2} * (-19a^3fg^7h^2 - 9a^3eg^6h^3 + 13a^3dg^5h^4 - 76a^3fg^6h^3x - 4a^3eg^5h^4x - 60a^3dg^4h^5x - 326a^3fg^5h^4x^2 + 78a^3eg^4h^5x^2 - 70a^3dg^3h^6x^2 - 800a^3fg^4h^5x^3 + 256a^3eg^3h^6x^3 - 336a^3dg^2h^7x^3 - 807a^3fg^3h^6x^4 - 21a^3eg^2h^7x^4 - 87a^3dgh^8x^4 - 468a^3fg^2h^7x^5 + 36a^3eggh^8x^5 - 84a^3dh^9x^5 + 48a^3fgh^8x^6 - 96a^3eh^9x^6 - 96a^3fh^9x^7) + c^6(48fg^9x^5 + 16eg^8hx^5 + 16dg^7h^2x^5 + 192fg^8hx^6 + 64eg^7h^2x^6 + 64dg^6h^3x^6 + 288fg^7h^2x^7 + 96eg^6h^3x^7 + 192fg^6h^3x^8) + c^5(72afg^9x^3 + 24aeg^8hx^3 + 24adg^7h^2x^3 + 288afg^8hx^4 + 96aeg^7h^2x^4 + 96adg^6h^3x^4 + 584afg^7h^2x^5 + 216aeg^6h^3x^5 - 152adg^5h^4x^5 + 896afg^6h^3x^6 + 192aeg^5h^4x^6 - 224adg^4h^5x^6 + 864afg^5h^4x^7 + 288aeg^4h^5x^7 - 336adg^3h^6x^7 + 576afg^4h^5x^8 - 96adg^2h^7x^8) + c^4(24a^2fg^9x + 8a^2eg^8hx + 8a^2dg^7h^2x + 96a^2fg^8hx^2 + 32a^2eg^7h^2x^2 + 32a^2dg^6h^3x^2 + 372a^2fg^7h^2x^3 + 156a^2eg^6h^3x^3 - 216a^2dg^5h^4x^3 + 1008a^2fg^6h^3x^4 + 264a^2eg^5h^4x^4 - 216a^2dg^4h^5x^4 + 1514a^2fg^5h^4x^5 + 302a^2eg^4h^5x^5 - 354a^2dg^3h^6x^5 + 1640a^2fg^4h^5x^6 - 328a^2eg^3h^6x^6 + 168a^2dg^2h^7x^6 + 948a^2fg^3h^6x^7 - 132a^2eg^2h^7x^7 + 84a^2dgh^8x^7 + 600a^2fg^2h^7x^8 - 120a^2eggh^8x^8 + 24a^2dh^9x^8) + c^3(76a^3fg^7h^2x + 36a^3eg^6h^3x - 64a^3dg^5h^4x + 304a^3fg^6h^3x^2 + 64a^3eg^5h^4x^2 + 36a^3dg^4h^5x^2 + 759a^3fg^5h^4x^3 - 35a^3eg^4h^5x^3 + 19a^3dg^3h^6x^3 + 1404a^3fg^4h^5x^4 - 420a^3eg^3h^6x^4 + 456a^3dg^2h^7x^4 + 1281a^3fg^3h^6x^5 - 45a^3eg^2h^7x^5 + 129a^3dgh^8x^5 + 768a^3fg^2h^7x^6 - 96a^3eggh^8x^6 + 96a^3dh^9x^6 - 48a^3fgh^8x^7 + 96a^3eh^9x^7 + 96a^3fh^9x^8) / (96a^5\sqrt{c}h^{10}x(g + hx)^4 + 192c^{(11/2)}g^6h^4x^5(g + hx)^4 - 24a^5h^{10}(g + hx)^4\sqrt{a + cx^2} - 192c^5g^6h^4x^4(g + hx)^4\sqrt{a + cx^2} + 24c^4h^4(g + hx)^4\sqrt{a + cx^2} * (-3a^4g^2h^4 - 8a^4h^6x^2) + 24c^{(3/2)}h^4(g + hx)^4(12a^4g^2h^4x + 12a^4h^6x^3) + 24c^4h^4(g + hx)^4\sqrt{a + cx^2} * (-8ag^6x^2 - 24ag^4h^2x^4) + 24c^3h^4(g + hx)^4\sqrt{a + cx^2} * (-a^2g^6) - 24a^2g^
\end{aligned}$$

$$\begin{aligned}
& 4h^2x^2 - 24a^2g^2h^4x^4) + 24c^2h^4(g + hx)^4\sqrt{a + cx^2} * (- \\
& 3a^3g^4h^2 - 24a^3g^2h^4x^2 - 8a^3h^6x^4) + 24c^{(9/2)}h^4(g + h \\
& x)^4(12a^3g^6x^3 + 24a^3g^4h^2x^5) + 24c^{(7/2)}h^4(g + hx)^4(4a^2 \\
& g^6x + 36a^2g^4h^2x^3 + 24a^2g^2h^4x^5) + 24c^{(5/2)}h^4(g + hx \\
& )^4(12a^3g^4h^2x + 36a^3g^2h^4x^3 + 8a^3h^6x^5) + (3a^3fh^2 \\
& * \text{ArcTan}[-(\sqrt{c}*g) - \sqrt{c}*hx + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a* \\
& h^2}])/(2*g^2*\sqrt{-(c*g^2) - a*h^2}*(c*g^2 + a*h^2)^2) - (5*a^3*e*h^3*\text{ArcT} \\
& \text{an}[-(\sqrt{c}*g) - \sqrt{c}*hx + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a*h^2}] \\
& )/(2*g^3*\sqrt{-(c*g^2) - a*h^2}*(c*g^2 + a*h^2)^2) + (7*a^3*d*h^4*\text{ArcTan}[( - \\
& (\sqrt{c}*g) - \sqrt{c}*hx + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a*h^2}])/(2* \\
& g^4*\sqrt{-(c*g^2) - a*h^2}*(c*g^2 + a*h^2)^2) - (a^2*f*\text{ArcTan}[( - (\sqrt{c}*g) \\
& - \sqrt{c}*hx + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a*h^2}])/(4*g^2*\sqrt{-(c* \\
& g^2) - a*h^2}*(c*g^2 + a*h^2)) + (5*a^2*e*h*\text{ArcTan}[( - (\sqrt{c}*g) - \sqrt{c} \\
& ]*hx + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a*h^2}])/(4*g^3*\sqrt{-(c*g^2) - \\
& a*h^2}*(c*g^2 + a*h^2)) - (13*a^2*d*h^2*\text{ArcTan}[( - (\sqrt{c}*g) - \sqrt{c}*hx \\
& + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a*h^2}])/(4*g^4*\sqrt{-(c*g^2) - a*h^2} \\
& *(c*g^2 + a*h^2)) + ((-5*a^4*f*h^4)/(4*g^2*\sqrt{-(c*g^2) - a*h^2}*(c*g^2 + \\
& a*h^2)^3) + (5*a^4*e*h^5)/(4*g^3*\sqrt{-(c*g^2) - a*h^2}*(c*g^2 + a*h^2)^3) \\
& - (5*a^4*d*h^6)/(4*g^4*\sqrt{-(c*g^2) - a*h^2}*(c*g^2 + a*h^2)^3))*\text{ArcTan}[( - \\
& (\sqrt{c}*g) - \sqrt{c}*hx + h*\sqrt{a + cx^2}]/\sqrt{-(c*g^2) - a*h^2}] - (a \\
& *d*\text{ArcTan}[(\sqrt{c}*g)/\sqrt{-(c*g^2) - a*h^2} + (\sqrt{c}*hx)/\sqrt{-(c*g^2) \\
& - a*h^2}] - (h*\sqrt{a + cx^2})/\sqrt{-(c*g^2) - a*h^2}])/(g^4*\sqrt{-(c*g^2) \\
& - a*h^2})
\end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^5,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^5,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.02, size = 7237, normalized size = 23.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)*(c*x^2+a)^{(1/2)}/(h*x+g)^5,x)$

[Out] result too large to display

**maxima [B]** time = 1.14, size = 3404, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x^2+e*x+d)*(c*x^2+a)^{(1/2)}/(h*x+g)^5,x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & -5/8*\sqrt{c*x^2 + a}*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c \\ & *g^2*h^8*x + a^3*h^{10}*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + \\ & a^3*g*h^9) + 5/8*\sqrt{c*x^2 + a}*c^3*e*g^4/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x \\ & + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2 \\ & *c*g^3*h^6 + a^3*g*h^8) - 5/8*(c*x^2 + a)^{(3/2)}*c^2*f*g^4/(c^3*g^6*h^3*x^2 \\ & + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x \\ & + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a \\ & c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 5/8*\sqrt{c*x^2 + a}*c^3*f*g^4 \\ & / (c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 5/8*\sqrt{c*x \\ & x^2 + a}*c^3*d*g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + \\ & a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 5 \\ & /8*(c*x^2 + a)^{(3/2)}*c^2*e*g^3/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a \\ & ^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*c \\ & *g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + \\ & a^3*g^2*h^6) - 5/8*\sqrt{c*x^2 + a}*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 \\ & + 3*a^2*c*g^2*h^6 + a^3*h^8) - 5/8*(c*x^2 + a)^{(3/2)}*c^2*d*g^2/(c^3*g^6*h \\ & *x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7* \\ & x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a \\ & *c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) + 5/8*\sqrt{c*x^2 + a}*c^3*d*g^2 \\ & / (c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 5/12*(c*x^2 + \\ & a)^{(3/2)}*c*f*g^3/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2 \\ & *g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a* \\ & c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) + \\ & 9/8*\sqrt{c*x^2 + a}*c^2*f*g^3/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x \\ & + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 5/12*(c*x^2 + a)^{(3/2)}*c*e*g^2 \\ & / (c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6 \\ & *a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^ \\ & 2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 5/8*\sqrt{c*x^2 + a}* \\ & c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a* \\ & c*g^3*h^4 + a^2*g*h^6) + 9/8*(c*x^2 + a)^{(3/2)}*c*f*g^2/(c^2*g^4*h^3*x^2 + 2 \\ & *a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2* \\ & g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 9/8*\sqrt{c*x^2 + a}*c^ \\ & 2*f*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 5/12*(c*x^2 + a)^{(3/2)}*c \end{aligned}$$

$$\begin{aligned}
& d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + \\
& 6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2 \\
& *g^2*h^4*x + c^2*g^7/h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*sqrt(c*x^2 + a)*c \\
& ^2*d*g/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3 \\
& *h^3 + a^2*g*h^5) - 5/8*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^2 \\
& *h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + \\
& c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) + 5/8*sqrt(c*x^2 + a)*c^2*e*g/(c^2* \\
& g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^5 \\
& *x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a* \\
& g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 1/8*(c \\
& *x^2 + a)^(3/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^ \\
& 2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g \\
& ^2*h^3) - 1/8*sqrt(c*x^2 + a)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) + \\
& 1/4*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4 \\
& *a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^ \\
& 3*x + c*g^6 + a*g^4*h^2) + 2/3*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^4*x^3 + a*h^6 \\
& *x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c* \\
& g^5*h + a*g^3*h^3) - 1/2*sqrt(c*x^2 + a)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g \\
& ^3*h^3 + a*g*h^5) - 1/4*(c*x^2 + a)^(3/2)*d/(c*g^2*h^3*x^4 + a*h^5*x^4 + 4* \\
& c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x \\
& + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) - 1/3*(c*x^2 + a)^(3/2)*e/(c*g^2*h^3* \\
& x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h \\
& ^3*x + c*g^5 + a*g^3*h^2) - 1/2*(c*x^2 + a)^(3/2)*f/(c*g^2*h^3*x^2 + a*h^5* \\
& x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 + \\
& a)*c*f/(c*g^2*h^3 + a*h^5) - 5/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h* \\
& x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) + 5/8* \\
& c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + \\
& g)))/((a + c*g^2/h^2)^(7/2)*h^10) - 5/8*c^4*d*g^4*arcsinh(c*g*x/(sqrt(a*c) \\
& *abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^9) \\
& + 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs \\
& (h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) - 5/4*c^3*e*g^3*arcsinh(c*g*x/(sqrt \\
& (a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)* \\
& h^8) + 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c) \\
& *abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 13/8*c^2*f*g^2*arcsinh(c*g*x \\
& /(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^( \\
& 3/2)*h^7) + 5/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt \\
& (a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) - 1/8*c^2*d*arcsinh(c*g*x/ \\
& (sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^( \\
& 3/2)*h^5) + 1/2*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c) \\
& *abs(h*x + g)))/sqrt(a + c*g^2/h^2)*h^5)
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a} (fx^2 + ex + d)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

[Out] int(((a + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2} (d + ex + fx^2)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+a)\*\*(1/2)/(h\*x+g)\*\*5, x)

[Out] Integral(sqrt(a + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*5, x)

$$3.87 \quad \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6} dx$$

**Optimal.** Leaf size=433

$$\frac{(a+cx^2)^{3/2} (20a^2fh^4 - ach^2(18fg^2 - h(33eg - 8dh)) - c^2g^2(h(2eg - 27dh) + 3fg^2)) - c\sqrt{a+cx^2}(ah - cgx)(a - cgx)}{60h(g+hx)^3(ah^2 + cg^2)^3}$$

**Rubi [A]** time = 0.74, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 835, 807, 721, 725, 206}

$$\frac{c\sqrt{a+cx^2}(ah - cgx)(a^2h^2(6fg - eh) - acg(fg^2 - 3h(2eg - dh)) + 4c^2dg^2)}{8g(h+hx)^3(ah^2 + cg^2)^3} - \frac{(a+cx^2)^{3/2}(20a^2fh^4 - ach^2(18fg^2 - h(33eg - 8dh)) - c^2g^2(h(2eg - 27dh) + 3fg^2))}{60h(g+hx)^3(ah^2 + cg^2)^3} - \frac{a^2 \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(a^2h^2(6fg-eh) - acg(fg^2 - 3h(2eg-dh)) + 4c^2dg^2)}{8(ah^2 + cg^2)^{3/2}} + \frac{(a+cx^2)^{3/2}(5ah^2(2fg-eh) + cgh(2eg-7dh) + 3c^2fg^2)}{20h(g+hx)^3(ah^2 + cg^2)^3} - \frac{(a+cx^2)^{3/2}(dh^2 - cgh + fg^2)}{5h(g+hx)^3(ah^2 + cg^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^6,x]

[Out] -(c\*(4\*c^2\*d\*g^3 + a^2\*h^2\*(6\*f\*g - e\*h) - a\*c\*g\*(f\*g^2 - 3\*h\*(2\*e\*g - d\*h)))\*(a\*h - c\*g\*x)\*Sqrt[a + c\*x^2])/(8\*(c\*g^2 + a\*h^2)^4\*(g + h\*x)^2) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(3/2))/(5\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^5) + ((3\*c\*f\*g^3 + c\*g\*h\*(2\*e\*g - 7\*d\*h) + 5\*a\*h^2\*(2\*f\*g - e\*h))\*(a + c\*x^2)^(3/2))/(20\*h\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)^4) - ((20\*a^2\*f\*h^4 - c^2\*(3\*f\*g^4 + g^2\*h\*(2\*e\*g - 27\*d\*h)) - a\*c\*h^2\*(18\*f\*g^2 - h\*(33\*e\*g - 8\*d\*h)))\*(a + c\*x^2)^(3/2))/(60\*h\*(c\*g^2 + a\*h^2)^3\*(g + h\*x)^3) - (a\*c^2\*(4\*c^2\*d\*g^3 + a^2\*h^2\*(6\*f\*g - e\*h) - a\*c\*g\*(f\*g^2 - 3\*h\*(2\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(8\*(c\*g^2 + a\*h^2)^(9/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 721

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(-2\*a\*e + (2\*c\*d)\*x)\*(a + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), x] - Dist[(4\*a\*c\*p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 2, 0] && GtQ[p, 0]

Rule 725



```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 807

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

### Rule 835

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2} (d+ex+fx^2)}{(g+hx)^6} dx &= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{5h(cg^2 + ah^2)(g+hx)^5} - \frac{\int \frac{(-5(cdg-afg+afh) - (5afh+c(2eg+\frac{3fg^2}{h}-2dh)))x\sqrt{a+cx^2}}{(g+hx)^5}}{5(cg^2 + ah^2)} \\
&= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{5h(cg^2 + ah^2)(g+hx)^5} + \frac{(3cfg^3 + cgh(2eg - 7dh) + 5ah^2(2fg - eh))}{20h(cg^2 + ah^2)^2(g+hx)^4} \\
&= -\frac{(fg^2 - egh + dh^2)(a+cx^2)^{3/2}}{5h(cg^2 + ah^2)(g+hx)^5} + \frac{(3cfg^3 + cgh(2eg - 7dh) + 5ah^2(2fg - eh))}{20h(cg^2 + ah^2)^2(g+hx)^4} \\
&= -\frac{c(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 - 3h(2eg - dh)))(ah - cgx)\sqrt{a+cx^2}}{8(cg^2 + ah^2)^4(g+hx)^2} \\
&= -\frac{c(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 - 3h(2eg - dh)))(ah - cgx)\sqrt{a+cx^2}}{8(cg^2 + ah^2)^4(g+hx)^2} \\
&= -\frac{c(4c^2dg^3 + a^2h^2(6fg - eh) - acg(fg^2 - 3h(2eg - dh)))(ah - cgx)\sqrt{a+cx^2}}{8(cg^2 + ah^2)^4(g+hx)^2}
\end{aligned}$$

**Mathematica [A]** time = 1.62, size = 583, normalized size = 1.35

Integrate[Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2)/(g + h\*x)^6, x]

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x]

[Out] 
$$\begin{aligned}
& -1/120*(\text{Sqrt}[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - \\
& 6*(c*g^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g + \\
& e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^2*h \\
& *(2*e*g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^2 - \\
& c*(c*g^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 + h*(24 \\
& *e*g - 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^3 - c \\
& (-40*a^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^3*(6*f* \\
& g^6 + 2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g + 16*d*h) \\
& ))*(g + h*x)^4)/(h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c^2*d*g^3 \\
& + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*\text{Log}[g + h*x])
\end{aligned}$$

$$\frac{/(8*(c*g^2 + a*h^2)^{(9/2)}) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*\text{Log}[a*h - c*g*x + \text{Sqrt}[c*g^2 + a*h^2]*\text{Sqrt}[a + c*x^2]]}{(8*(c*g^2 + a*h^2)^{(9/2)})}$$

**IntegrateAlgebraic [F]** time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^6,x]

[Out] \$Aborted

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^6,x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 0.66, size = 4212, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^6,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 - 3*a^2*c^3*d*g*h^2 + 6*a^3*c^2*f*g*h^2 \\ & + 6*a^2*c^3*g^2*h*e - a^3*c^2*h^3*e)*\arctan(((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & *h + \text{sqrt}(c)*g)/\text{sqrt}(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c \\ & ^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*\text{sqrt}(-c*g^2 - a*h^2)) - 1/60*(60*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^9*a^2*c^3*f*g^3*h^8 - 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^9*a^2*c^3*d*g*h^10 + 90*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^9*a^3*c^2*f*g*h^10 + 90*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^9*a^2*c^3*g^2*h^9*e - 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^9*a^3*c^2*h^11*e - 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*c^{(11/2)}*f*g^8*h^3 - 480*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a*c^{(9/2)}*f*g^6*h^5 + 540*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a*c^{(9/2)}*d*g^4*h^7 - 855*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a^2*c^{(7/2)}*f*g^4*h^7 - 405*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a^2*c^{(7/2)}*d*g^2*h^9 + 330*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a^3*c^{(5/2)}*f*g^2*h^9 - 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a^4*c^{(3/2)}*f*h^11 + 810*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a^2*c^{(7/2)}*g^3*h^8*e - 135*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^8*a^3*c^{(5/2)}*g*h^10*e - 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\ & ^7*c^6*f*g^9*h^2 - 9 \end{aligned}$$

$$\begin{aligned}
& 60*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^5*f*g^7*h^4 + 1880*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^5*d*g^5*h^6 - 1910*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^4*d*g^3*h^8 + 1930*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^3*f*g^3*h^8 + 210*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^3*d*g*h^10 - 660*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^4*c^2*f*g*h^10 - 160*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*c^6*g^8*h^3*e - 640*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^5*g^6*h^5*e + 1860*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^4*g^4*h^7*e - 1530*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^3*g^2*h^9*e - 90*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^4*c^2*h^11*e - 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*c^(13/2)*f*g^10*h - 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*c^(13/2)*d*g^8*h^3 - 720*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a*c^(11/2)*f*g^8*h^3 + 2120*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a*c^(11/2)*d*g^6*h^5 - 1250*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^2*c^(9/2)*f*g^6*h^5 - 5710*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^2*c^(9/2)*d*g^4*h^7 + 5590*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^3*c^(7/2)*f*g^4*h^7 + 510*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^3*c^(7/2)*d*g^2*h^9 - 2220*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^4*c^(5/2)*f*g^2*h^9 - 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^4*c^(5/2)*d*h^11 + 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^5*c^(3/2)*f*h^11 - 160*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*c^(13/2)*g^9*h^2*e - 640*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a*c^(11/2)*g^7*h^4*e + 3660*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^2*c^(9/2)*g^5*h^6*e - 4350*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^3*c^(7/2)*g^3*h^8*e + 330*(\sqrt{c}*x - \sqrt{c*x^2 + a})^6*a^4*c^(5/2)*g*h^10*e - 96*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*c^7*f*g^11 - 96*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*c^7*d*g^9*h^2 + 48*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a*c^6*f*g^9*h^2 + 1808*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a*c^6*d*g^7*h^4 + 604*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^2*c^5*f*g^7*h^4 - 7076*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^2*c^5*d*g^5*h^6 + 6710*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^3*c^4*f*g^5*h^6 + 3770*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^3*c^4*d*g^3*h^8 - 5780*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^4*c^3*f*g^3*h^8 - 480*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^4*c^3*d*g*h^10 + 1200*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^5*c^2*f*g*h^10 - 64*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*c^7*g^10*h*e - 128*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a*c^6*g^8*h^3*e + 3416*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^2*c^5*g^6*h^5*e - 7320*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^3*c^4*g^4*h^7*e + 2430*(\sqrt{c}*x - \sqrt{c*x^2 + a})^5*a^4*c^3*g^2*h^9*e + 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a*c^(13/2)*f*g^10*h + 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a*c^(13/2)*d*g^8*h^3 + 720*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^2*c^(11/2)*f*g^8*h^3 - 5240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^2*c^(11/2)*d*g^6*h^5 + 2450*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^3*c^(9/2)*f*g^6*h^5 + 5590*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^3*c^(9/2)*d*g^4*h^7 - 7660*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^4*c^(7/2)*f*g^4*h^7 - 2240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^4*c^(7/2)*d*g^2*h^9 + 3440*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^5*c^(5/2)*f*g^2*h^9 - 80*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^5*c^(5/2)*d*h^11 - 160*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^6*c^(3/2)*f*h^11 + 160*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a*c^(13/2)*g^9*h^2*e + 1120*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^2*c^(11/2)*g^7*h^4*e - 6140*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^3*c^(9/2)*g^5*h^6*e + 5650*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^4*c^(7/2)*g
\end{aligned}$$

$$\begin{aligned}
&^3h^8e - 480*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^5*c^{(5/2)}*g*h^{10}e - 240*( \\
&\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 480*(\text{sqrt}(c)*x - \text{sqrt}(c* \\
&x^2 + a))^3*a^2*c^6*d*g^7*h^4 - 960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^3*c^5 \\
&*f*g^7*h^4 + 5000*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^3*c^5*d*g^5*h^6 - 3890* \\
&(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 - 2910*(\text{sqrt}(c)*x - \text{sqrt}( \\
&c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 + 4710*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^5* \\
&c^3*f*g^3*h^8 + 430*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^5*c^3*d*g*h^{10} - 940* \\
&(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^6*c^2*f*g*h^{10} - 160*(\text{sqrt}(c)*x - \text{sqrt}(c* \\
&x^2 + a))^3*a^2*c^6*g^8*h^3e - 1440*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^3*c^ \\
&5*g^6*h^5e + 5740*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^4*c^4*g^4*h^7e - 1710 \\
&*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^5*c^3*g^2*h^9e + 90*(\text{sqrt}(c)*x - \text{sqrt}(c \\
&*x^2 + a))^3*a^6*c^2*h^{11}e + 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^3*c^{(11 \\
&/2)}*f*g^8*h^3 + 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^3*c^{(11/2)}*d*g^6*h^5 \\
&+ 570*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^4*c^{(9/2)}*f*g^6*h^5 - 2810*(\text{sqrt}(c) \\
&*x - \text{sqrt}(c*x^2 + a))^2*a^4*c^{(9/2)}*d*g^4*h^7 + 2450*(\text{sqrt}(c)*x - \text{sqrt}(c*x^ \\
&2 + a))^2*a^5*c^{(7/2)}*f*g^4*h^7 + 650*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^5*c \\
&^{(7/2)}*d*g^2*h^9 - 1700*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^6*c^{(5/2)}*f*g^2*h \\
&^9 - 80*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^6*c^{(5/2)}*d*h^{11} + 80*(\text{sqrt}(c)*x \\
&- \text{sqrt}(c*x^2 + a))^2*a^7*c^{(3/2)}*f*h^{11} + 160*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\
&^2*a^3*c^{(11/2)}*g^7*h^4e + 1100*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^4*c^{(9/2)} \\
&)*g^5*h^6e - 2570*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^5*c^{(7/2)}*g^3*h^8e + \\
&270*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^6*c^{(5/2)}*g*h^{10}e - 60*(\text{sqrt}(c)*x - \\
&\text{sqrt}(c*x^2 + a))*a^4*c^5*f*g^7*h^4 - 60*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^4*c \\
&^5*d*g^5*h^6 - 270*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^5*c^4*f*g^5*h^6 + 770*(s \\
&\text{qrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^5*c^4*d*g^3*h^8 - 845*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 \\
&+ a))*a^6*c^3*f*g^3*h^8 - 115*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^6*c^3*d*g*h^ \\
&10 + 310*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^7*c^2*f*g*h^{10} - 40*(\text{sqrt}(c)*x - s \\
&\text{qrt}(c*x^2 + a))*a^4*c^5*g^6*h^5e - 280*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^5*c \\
&^4*g^4*h^7e + 720*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^6*c^3*g^2*h^9e + 15*(sq \\
&\text{rt}(c)*x - \text{sqrt}(c*x^2 + a))*a^7*c^2*h^{11}e + 6*a^5*c^{(9/2)}*f*g^6*h^5 + 6*a^5 \\
&*c^{(9/2)}*d*g^4*h^7 + 27*a^6*c^{(7/2)}*f*g^4*h^7 - 83*a^6*c^{(7/2)}*d*g^2*h^9 + \\
&86*a^7*c^{(5/2)}*f*g^2*h^9 + 16*a^7*c^{(5/2)}*d*h^{11} - 40*a^8*c^{(3/2)}*f*h^{11} + \\
&4*a^5*c^{(9/2)}*g^5*h^6e + 28*a^6*c^{(7/2)}*g^3*h^8e - 81*a^7*c^{(5/2)}*g*h^{10}e \\
&)/((c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^{10} + \\
&a^4*h^{12})*((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*h + 2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
&a))*\text{sqrt}(c)*g - a*h)^5)
\end{aligned}$$

**maple [B]** time = 0.02, size = 8546, normalized size = 19.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)*(c*x^2+a)^{(1/2)}/(h*x+g)^6,x)$

[Out] result too large to display

maxima [B] time = 1.53, size = 5793, normalized size = 13.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+a)^(1/2)/(h\*x+g)^6,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -7/8*\sqrt{c*x^2 + a}*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^{10}*x + a^4*h^{12}*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^{11}) + 7/8*\sqrt{c*x^2 + a}*c^4*e*g^5/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*h^{11}*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^{10}) - 7/8*(c*x^2 + a)^{(3/2)}*c^3*f*g^5/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^{11}*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^{10}*x + c^4*g^{10}*h + 4*a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 7/8*\sqrt{c*x^2 + a}*c^4*f*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^{11}) - 7/8*\sqrt{c*x^2 + a}*c^4*d*g^4/(c^4*g^8*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^{10}*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*g*h^9) + 7/8*(c*x^2 + a)^{(3/2)}*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^{10}*x^2 + 2*c^4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x + 2*a^4*g*h^9*x + c^4*g^{10} + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*g^4*h^6 + a^4*g^2*h^8) - 7/8*\sqrt{c*x^2 + a}*c^4*e*g^4/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^{10}) - 7/8*(c*x^2 + a)^{(3/2)}*c^3*d*g^3/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^{10}/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 7/8*\sqrt{c*x^2 + a}*c^4*d*g^3/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) - 7/12*(c*x^2 + a)^{(3/2)}*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^{10}*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) + 13/8*\sqrt{c*x^2 + a}*c^3*f*g^4/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^{10}*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/12*(c*x^2 + a)^{(3/2)}*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) - \sqrt{c*x^2 + a}*c^3*e*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6) \end{aligned}$$

$$\begin{aligned}
& 6 + a^3 g^* h^8) + 13/8 * (c * x^2 + a)^{(3/2)} * c^2 * f * g^3 / (c^3 * g^6 * h^3 * x^2 + 3 * a * c^2 * g^4 * h^5 * x^2 + 3 * a^2 * c * g^2 * h^7 * x^2 + a^3 * h^9 * x^2 + 2 * c^3 * g^7 * h^2 * x + 6 * a * c^2 * g^5 * h^4 * x + 6 * a^2 * c * g^3 * h^6 * x + 2 * a^3 * g * h^8 * x + c^3 * g^8 * h + 3 * a * c^2 * g^6 * h^3 + 3 * a^2 * c * g^4 * h^5 + a^3 * g^2 * h^7) - 13/8 * \text{sqrt}(c * x^2 + a) * c^3 * f * g^3 / (c^3 * g^6 * h^3 + 3 * a * c^2 * g^4 * h^5 + 3 * a^2 * c * g^2 * h^7 + a^3 * h^9) - 7/12 * (c * x^2 + a)^{(3/2)} * c^2 * d * g^2 / (c^3 * g^6 * h^2 * x^3 + 3 * a * c^2 * g^4 * h^4 * x^3 + 3 * a^2 * c * g^2 * h^6 * x^3 + a^3 * h^8 * x^3 + 3 * c^3 * g^7 * h * x^2 + 9 * a * c^2 * g^5 * h^3 * x^2 + 9 * a^2 * c * g^3 * h^5 * x^2 + 3 * a^3 * g * h^7 * x^2 + 3 * c^3 * g^8 * x + 9 * a * c^2 * g^6 * h^2 * x + 9 * a^2 * c * g^4 * h^4 * x + 3 * a^3 * g^2 * h^6 * x + c^3 * g^9 / h + 3 * a * c^2 * g^7 * h + 3 * a^2 * c * g^5 * h^3 + a^3 * g^3 * h^5) + 3/8 * \text{sqrt}(c * x^2 + a) * c^3 * d * g^2 / (c^3 * g^6 * h^2 * x + 3 * a * c^2 * g^4 * h^4 * x + 3 * a^2 * c * g^2 * h^6 * x + a^3 * h^8 * x + c^3 * g^7 * h + 3 * a * c^2 * g^5 * h^3 + 3 * a^2 * c * g^3 * h^5 + a^3 * g * h^7) - (c * x^2 + a)^{(3/2)} * c^2 * e * g^2 / (c^3 * g^6 * h^2 * x^2 + 3 * a * c^2 * g^4 * h^4 * x^2 + 3 * a^2 * c * g^2 * h^6 * x^2 + a^3 * h^8 * x^2 + 2 * c^3 * g^7 * h * x + 6 * a * c^2 * g^5 * h^3 * x + 6 * a^2 * c * g^3 * h^5 * x + 2 * a^3 * g * h^7 * x + c^3 * g^8 + 3 * a * c^2 * g^6 * h^2 + 3 * a^2 * c * g^4 * h^4 + a^3 * g^2 * h^6) + \text{sqrt}(c * x^2 + a) * c^3 * e * g^2 / (c^3 * g^6 * h^2 + 3 * a * c^2 * g^4 * h^4 + 3 * a^2 * c * g^2 * h^6 + a^3 * h^8) - 7/20 * (c * x^2 + a)^{(3/2)} * c * f * g^3 / (c^2 * g^4 * h^5 * x^4 + 2 * a * c * g^2 * h^7 * x^4 + a^2 * h^9 * x^4 + 4 * c^2 * g^5 * h^4 * x^3 + 8 * a * c * g^3 * h^6 * x^3 + 4 * a^2 * g * h^8 * x^3 + 6 * c^2 * g^6 * h^3 * x^2 + 12 * a * c * g^4 * h^5 * x^2 + 6 * a^2 * g^2 * h^7 * x^2 + 4 * c^2 * g^7 * h^2 * x + 8 * a * c * g^5 * h^4 * x + 4 * a^2 * g^3 * h^6 * x + c^2 * g^8 * h + 2 * a * c * g^6 * h^3 + a^2 * g^4 * h^5) + 3/8 * (c * x^2 + a)^{(3/2)} * c^2 * d * g / (c^3 * g^6 * h * x^2 + 3 * a * c^2 * g^4 * h^3 * x^2 + 3 * a^2 * c * g^2 * h^5 * x^2 + a^3 * h^7 * x^2 + 2 * c^3 * g^7 * x + 6 * a * c^2 * g^5 * h^2 * x + 6 * a^2 * c * g^3 * h^4 * x + 2 * a^3 * g * h^6 * x + c^3 * g^8 / h + 3 * a * c^2 * g^6 * h + 3 * a^2 * c * g^4 * h^3 + a^3 * g^2 * h^5) - 3/8 * \text{sqrt}(c * x^2 + a) * c^3 * d * g / (c^3 * g^6 * h + 3 * a * c^2 * g^4 * h^3 + 3 * a^2 * c * g^2 * h^5 + a^3 * h^7) + 7/20 * (c * x^2 + a)^{(3/2)} * c * e * g^2 / (c^2 * g^4 * h^4 * x^4 + 2 * a * c * g^2 * h^6 * x^4 + a^2 * h^8 * x^4 + 4 * c^2 * g^5 * h^3 * x^3 + 8 * a * c * g^3 * h^5 * x^3 + 4 * a^2 * g * h^7 * x^3 + 6 * c^2 * g^6 * h^2 * x^2 + 12 * a * c * g^4 * h^4 * x^2 + 6 * a^2 * g^2 * h^6 * x^2 + 4 * c^2 * g^7 * h * x + 8 * a * c * g^5 * h^3 * x + 4 * a^2 * g^3 * h^5 * x + c^2 * g^8 + 2 * a * c * g^6 * h^2 + a^2 * g^4 * h^4) + 29/30 * (c * x^2 + a)^{(3/2)} * c * f * g^2 / (c^2 * g^4 * h^4 * x^3 + 2 * a * c * g^2 * h^6 * x^3 + a^2 * h^8 * x^3 + 3 * c^2 * g^5 * h^3 * x^2 + 6 * a * c * g^3 * h^5 * x^2 + 3 * a^2 * g * h^7 * x^2 + 3 * c^2 * g^6 * h^2 * x + 6 * a * c * g^4 * h^4 * x + 3 * a^2 * g^2 * h^6 * x + c^2 * g^7 * h + 2 * a * c * g^5 * h^3 + a^2 * g^3 * h^5) - 3/4 * \text{sqrt}(c * x^2 + a) * c^2 * f * g^2 / (c^2 * g^4 * h^4 * x + 2 * a * c * g^2 * h^6 * x + a^2 * h^8 * x + c^2 * g^5 * h^3 + 2 * a * c * g^3 * h^5 + a^2 * g * h^7) - 7/20 * (c * x^2 + a)^{(3/2)} * c * d * g / (c^2 * g^4 * h^3 * x^4 + 2 * a * c * g^2 * h^5 * x^4 + a^2 * h^7 * x^4 + 4 * c^2 * g^5 * h^2 * x^3 + 8 * a * c * g^3 * h^4 * x^3 + 4 * a^2 * g * h^6 * x^3 + 6 * c^2 * g^6 * h * x^2 + 12 * a * c * g^4 * h^3 * x^2 + 6 * a^2 * g^2 * h^5 * x^2 + 4 * c^2 * g^7 * x + 8 * a * c * g^5 * h^2 * x + 4 * a^2 * g^3 * h^4 * x + c^2 * g^8 / h + 2 * a * c * g^6 * h + a^2 * g^4 * h^3) - 11/20 * (c * x^2 + a)^{(3/2)} * c * e * g / (c^2 * g^4 * h^3 * x^3 + 2 * a * c * g^2 * h^5 * x^3 + a^2 * h^7 * x^3 + 3 * c^2 * g^5 * h^2 * x^2 + 6 * a * c * g^3 * h^4 * x^2 + 3 * a^2 * g * h^6 * x^2 + 3 * c^2 * g^6 * h * x + 6 * a * c * g^4 * h^3 * x + 3 * a^2 * g^2 * h^5 * x + c^2 * g^7 + 2 * a * c * g^5 * h^2 + a^2 * g^3 * h^4) + 1/8 * \text{sqrt}(c * x^2 + a) * c^2 * e * g / (c^2 * g^4 * h^3 * x + 2 * a * c * g^2 * h^5 * x + a^2 * h^7 * x + c^2 * g^5 * h^2 + 2 * a * c * g^3 * h^4 + a^2 * g * h^6) - 3/4 * (c * x^2 + a)^{(3/2)} * c * f * g / (c^2 * g^4 * h^3 * x^2 + 2 * a * c * g^2 * h^5 * x^2 + a^2 * h^7 * x^2 + 2 * c^2 * g^5 * h^2 * x + 4 * a * c * g^3 * h^4 * x + 2 * a^2 * g * h^6 * x + c^2 * g^6 * h + 2 * a * c * g^4 * h^3 + a^2 * g^2 * h^5) + 3/4 * \text{sqrt}(c * x^2 + a) * c^2 * f * g / (c^2 * g^4 * h^3 + 2 * a * c * g^2 * h^5 + a^2 * h^7) - 1/5 * (c * x^2 + a)^{(3/2)} * f * g^2 / (c * g^2 * h^6 * x^
\end{aligned}$$

$$\begin{aligned}
& 5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) + 2/15*(c*x^2 + a)^{(3/2)}*c*d/(c^2*g^4*h^2*x^3 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*(c*x^2 + a)^{(3/2)}*c*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/8*sqrt(c*x^2 + a)*c^2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/5*(c*x^2 + a)^{(3/2)}*e*g/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x^2 + a)^{(3/2)}*f*g/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) - 1/5*(c*x^2 + a)^{(3/2)}*d/(c*g^2*h^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^4 + 10*c*g^4*h^2*x^3 + 10*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2 + 5*c*g^6*x + 5*a*g^4*h^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^{(3/2)}*e/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) - 1/3*(c*x^2 + a)^{(3/2)}*f/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 7/8*c^5*f*g^7*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^13) + 7/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^12) - 7/8*c^5*d*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^11) + 5/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) - 15/8*c^4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^10) + 5/4*c^4*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^9) - 19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 9/8*c^3*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^8) - 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/4*c^2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) - 1/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6)
\end{aligned}$$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + a} (fx^2 + ex + d)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

$$3.88 \quad \int (g + hx)^3 (a + cx^2)^{3/2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=462

$$\frac{x(a + cx^2)^{3/2} (3a^2h^2(eh + 3fg) - 8acg(3h(dh + eg) + fg^2) + 48c^2dg^3)}{192c^2} + \frac{ax\sqrt{a + cx^2} (3a^2h^2(eh + 3fg) - 8acg(3h(dh + eg) + fg^2) + 48c^2dg^3)}{128c^2}$$

**Rubi [A]** time = 1.13, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1654, 833, 780, 195, 217, 206}

$\frac{(c+cx^2)^{3/2} (4(2af^2h - 8af^2(h^2h + 3g) + 12f^2g) - f^2(h^2g^2 - 9gh^2h + 3g^2)) - 8a^2c^2(h^2h + 3g) + 48c^2dg^3}{192c^2} + \frac{(c+cx^2)^{3/2} (4(2af^2h - 8af^2(h^2h + 3g) + 12f^2g) - f^2(h^2g^2 - 9gh^2h + 3g^2)) - 8a^2c^2(h^2h + 3g) + 48c^2dg^3}{128c^2}$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^3\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (a\*(48\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) - 8\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*x\*sqrt[a + c\*x^2])/(128\*c^2) + ((48\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) - 8\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*x\*(a + c\*x^2)^(3/2))/(192\*c^2) + ((8\*(9\*c\*d - 4\*a\*f)\*h^2 - 3\*c\*g\*(5\*f\*g - 9\*e\*h))\*(g + h\*x)^2\*(a + c\*x^2)^(5/2))/(504\*c^2\*h) - ((5\*f\*g - 9\*e\*h)\*(g + h\*x)^3\*(a + c\*x^2)^(5/2))/(72\*c\*h) + (f\*(g + h\*x)^4\*(a + c\*x^2)^(5/2))/(9\*c\*h) + ((4\*(32\*a^2\*f\*h^4 - 8\*a\*c\*h^2\*(17\*f\*g^2 + 9\*h\*(3\*e\*g + d\*h)) - c^2\*(15\*f\*g^4 - 9\*g^2\*h\*(3\*e\*g + 64\*d\*h))) - 5\*c\*h\*(a\*h^2\*(61\*f\*g + 63\*e\*h) + 2\*c\*(5\*f\*g^3 - 9\*g\*h\*(e\*g + 12\*d\*h)))\*x\*(a + c\*x^2)^(5/2))/(5040\*c^3\*h) + (a^2\*(48\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) - 8\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*ArcTanh[(sqrt[c]\*x)/sqrt[a + c\*x^2]])/(128\*c^(5/2))

### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 780

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x
_Symbol] := Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

### Rule 833

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(GtQ[m, 0] && EqQ[f, 0])
```

### Rule 1654

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(GtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int (g + hx)^3 (a + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{f(g + hx)^4 (a + cx^2)^{5/2}}{9ch} + \frac{\int (g + hx)^3 ((9cd - 4af)h^2 - ch(5fg - 9eh)) dx}{9ch^2} \\
&= -\frac{(5fg - 9eh)(g + hx)^3 (a + cx^2)^{5/2}}{72ch} + \frac{f(g + hx)^4 (a + cx^2)^{5/2}}{9ch} + \frac{\int (g + hx)^2 ((8(9cd - 4af)h^2 - 3cg(5fg - 9eh)) (g + hx)^2 (a + cx^2)^{5/2} - (5fg - 9eh)(g + hx)^3 (a + cx^2)^{5/2}) dx}{504c^2h} \\
&= \frac{(8(9cd - 4af)h^2 - 3cg(5fg - 9eh)) (g + hx)^2 (a + cx^2)^{5/2}}{504c^2h} - \frac{(5fg - 9eh)(g + hx)^3 (a + cx^2)^{5/2}}{504c^2h} \\
&= \frac{(48c^2dg^3 + 3a^2h^2(3fg + eh) - 8acg(fg^2 + 3h(eg + dh))) x (a + cx^2)^{5/2}}{192c^2} \\
&= \frac{a(48c^2dg^3 + 3a^2h^2(3fg + eh) - 8acg(fg^2 + 3h(eg + dh))) x \sqrt{a + cx^2}}{128c^2} \\
&= \frac{a(48c^2dg^3 + 3a^2h^2(3fg + eh) - 8acg(fg^2 + 3h(eg + dh))) x \sqrt{a + cx^2}}{128c^2} \\
&= \frac{a(48c^2dg^3 + 3a^2h^2(3fg + eh) - 8acg(fg^2 + 3h(eg + dh))) x \sqrt{a + cx^2}}{128c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.54, size = 481, normalized size = 1.04

Integrate[(g + hx)^3 (a + cx^2)^(3/2) (d + ex + fx^2), x]

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(128\*a^2\*(8\*a^2\*f\*h^3 + 63\*c^2\*g^2\*(e\*g + 3\*d\*h) - 18\*a\*c\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))) + 315\*a\*c\*(80\*c^2\*d\*g^3 - 3\*a^2\*h^2\*(3\*f\*g + e\*h) + 8\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*x + 128\*a\*c\*(-4\*a^2\*f\*h^3 + 126\*c^2\*g^2\*(e\*g + 3\*d\*h) + 9\*a\*c\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*x^2 + 210\*c^2\*(48\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) + 56\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*x^3 + 384\*c^2\*(a^2\*f\*h^3 + 21\*c^2\*g^2\*(e\*g + 3\*d\*h) + 24\*a\*c\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*x^4 + 840\*c^3\*(9\*a\*h^2\*(3\*f\*g + e\*h) + 8\*c\*(f\*g^3 + 3\*g\*h\*(e\*g + d\*h)))\*x^5 + 640\*c^3\*h\*(10\*a\*f\*h^2 + 9\*c\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*x^6 + 5040\*c^4\*h^2\*(3\*f\*g + e\*h)\*x^7 + 4480\*c^4\*f\*h^3\*x^8) + 315\*a^2\*Sqrt[c]\*(48\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) - 8\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]]/(40320\*c^3)



$$c*f)*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*\sqrt{-c}*\arctan(\sqrt{-c}*x/\sqrt{c*x^2 + a}) + (4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d - 4*a^4*f)*h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*c^3*f)*g^3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3 + 27*a^2*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d - 4*a^3*c*f)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*\sqrt{c*x^2 + a})/c^3]$$

**giac** [A] time = 0.27, size = 652, normalized size = 1.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out]  $\frac{1}{40320}\sqrt{c*x^2 + a}*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 + c^8*h^3*e)/c^7)*x + 8*(27*c^8*f*g^2*h + 9*c^8*d*h^3 + 10*a*c^7*f*h^3 + 27*c^8*g*h^2*e)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*d*g*h^2 + 27*a*c^7*f*g*h^2 + 24*c^8*g^2*h*e + 9*a*c^7*h^3*e)/c^7)*x + 48*(63*c^8*d*g^2*h + 72*a*c^7*f*g^2*h + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3 + 21*c^8*g^3*e + 72*a*c^7*g*h^2*e)/c^7)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*d*g*h^2 + 9*a^2*c^6*f*g*h^2 + 168*a*c^7*g^2*h*e + 3*a^2*c^6*h^3*e)/c^7)*x + 64*(378*a*c^7*d*g^2*h + 27*a^2*c^6*f*g^2*h + 9*a^2*c^6*d*h^3 - 4*a^3*c^5*f*h^3 + 126*a*c^7*g^3*e + 27*a^2*c^6*g*h^2*e)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*g^3 + 24*a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 + 24*a^2*c^6*g^2*h*e - 3*a^3*c^5*h^3*e)/c^7)*x + 128*(189*a^2*c^6*d*g^2*h - 54*a^3*c^5*f*g^2*h - 18*a^3*c^5*d*h^3 + 8*a^4*c^4*f*h^3 + 63*a^2*c^6*g^3*e - 54*a^3*c^5*g*h^2*e)/c^7) - 1/128*(48*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*d*g*h^2 + 9*a^4*f*g*h^2 - 24*a^3*c*g^2*h*e + 3*a^4*h^3*e)*\log(\text{abs}(-\sqrt{c}*x + \sqrt{c*x^2 + a}))/c^{(5/2)}$

**maple** [A] time = 0.02, size = 794, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^3\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x)

[Out]  $-3/16*a^3/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*d*g*h^2-3/16*a^3/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*e*g^2*h+3/7*x^2*(c*x^2+a)^{(5/2)}/c*e*g*h^2+3/7*x$



```
[Out] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

```
sympy [A] time = 72.41, size = 1916, normalized size = 4.15
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d), x)
```

```
[Out] -3*a**(7/2)*e*h**3*x/(128*c**2*sqrt(1 + c*x**2/a)) - 9*a**(7/2)*f*g*h**2*x/
(128*c**2*sqrt(1 + c*x**2/a)) + 3*a**(5/2)*d*g*h**2*x/(16*c*sqrt(1 + c*x**2
/a)) + 3*a**(5/2)*e*g**2*h*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*e*h**3*x*
*3/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g**3*x/(16*c*sqrt(1 + c*x**2/a))
- 3*a**(5/2)*f*g*h**2*x**3/(128*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*g**3*x*
sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**3*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/
2)*d*g*h**2*x**3/(16*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e*g**2*h*x**3/(16*sq
rt(1 + c*x**2/a)) + 13*a**(3/2)*e*h**3*x**5/(64*sqrt(1 + c*x**2/a)) + 17*a*
*(3/2)*f*g**3*x**3/(48*sqrt(1 + c*x**2/a)) + 39*a**(3/2)*f*g*h**2*x**5/(64*
sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 11*s
qrt(a)*c*d*g*h**2*x**5/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g**2*h*x**5/
(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*e*h**3*x**7/(16*sqrt(1 + c*x**2/a)) +
11*sqrt(a)*c*f*g**3*x**5/(24*sqrt(1 + c*x**2/a)) + 15*sqrt(a)*c*f*g*h**2*x*
*7/(16*sqrt(1 + c*x**2/a)) + 3*a**4*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(128*c*
*(5/2)) + 9*a**4*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)) - 3*a**3*
d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - 3*a**3*e*g**2*h*asinh(sqr
t(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(16*c*
*(3/2)) + 3*a**2*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 3*a*d*g**2*h
*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) +
a*d*h**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c
*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True))
+ a*e*g**3*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c
), True)) + 3*a*e*g*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + 3*a*f*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2
) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (s
qrt(a)*x**4/4, True)) + a*f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c*
*3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35
*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g
**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**
2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c
*d*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*e*g**3*Piecewise((-2*a**2*sqrt
(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x
**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*c*e*g*h**2*Piecewise((8*a**3
```



```

*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*
x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)
*x**6/6, True)) + 3*c*f*g**2*h*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3
) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c
) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*f*h**3*
Piecewise((-16*a**4*sqrt(a + c*x**2)/(315*c**4) + 8*a**3*x**2*sqrt(a + c*x*
*2)/(315*c**3) - 2*a**2*x**4*sqrt(a + c*x**2)/(105*c**2) + a*x**6*sqrt(a +
c*x**2)/(63*c) + x**8*sqrt(a + c*x**2)/9, Ne(c, 0)), (sqrt(a)*x**8/8, True)
) + c**2*d*g**3*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*d*g*h**2*x**7/(2
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g**2*h*x**7/(2*sqrt(a)*sqrt(1 + c*x**
2/a)) + c**2*e*h**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**3*x**7/
(6*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*c**2*f*g*h**2*x**9/(8*sqrt(a)*sqrt(1 + c
*x**2/a))

```

$$3.89 \quad \int (g + hx)^2 (a + cx^2)^{3/2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=346

$$\frac{x(a + cx^2)^{3/2} (3a^2fh^2 - 8ac(h(dh + 2eg) + fg^2) + 48c^2dg^2)}{192c^2} + \frac{ax\sqrt{a + cx^2} (3a^2fh^2 - 8ac(h(dh + 2eg) + fg^2) + 48c^2dg^2)}{128c^2}$$

**Rubi [A]** time = 0.52, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x(a + cx^2)^{3/2} (3a^2fh^2 - 8ac(h(dh + 2eg) + fg^2) + 48c^2dg^2)}{192c^2} + \frac{ax\sqrt{a + cx^2} (3a^2fh^2 - 8ac(h(dh + 2eg) + fg^2) + 48c^2dg^2)}{128c^2} + \frac{c^2 \tanh^{-1}\left(\frac{x\sqrt{a + cx^2}}{\sqrt{a}}\right) (3a^2fh^2 - 8ac(h(dh + 2eg) + fg^2) + 48c^2dg^2)}{128c^2} + \frac{(a + cx^2)^{5/2} (12(8a^2bh + 2fg) - 8c^2g(2dh + eg) + 5c^2fg^2) - 51a(7d^2(8cd - 3af) - 2c^2(5fg - 8bh))}{1680c^2h} + \frac{(a + cx^2)^{5/2} (g + h^2(5fg - 8bh))}{56ch} + \frac{(a + cx^2)^{5/2} (g + h^2)}{8ch}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^2\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (a\*(48\*c^2\*d\*g^2 + 3\*a^2\*f\*h^2 - 8\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*x\*sqrt[a + c\*x^2])/(128\*c^2) + ((48\*c^2\*d\*g^2 + 3\*a^2\*f\*h^2 - 8\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*x\*(a + c\*x^2)^(3/2))/(192\*c^2) - ((5\*f\*g - 8\*e\*h)\*(g + h\*x)^2\*(a + c\*x^2)^(5/2))/(56\*c\*h) + (f\*(g + h\*x)^3\*(a + c\*x^2)^(5/2))/(8\*c\*h) - ((12\*(5\*c\*f\*g^3 - 8\*c\*g\*h\*(e\*g + 7\*d\*h) + 8\*a\*h^2\*(2\*f\*g + e\*h)) - 5\*h\*(7\*(8\*c\*d - 3\*a\*f)\*h^2 - 2\*c\*g\*(5\*f\*g - 8\*e\*h))\*x\*(a + c\*x^2)^(5/2))/(1680\*c^2\*h) + (a^2\*(48\*c^2\*d\*g^2 + 3\*a^2\*f\*h^2 - 8\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*ArcTanh[(sqrt[c]\*x)/sqrt[a + c\*x^2]])/(128\*c^(5/2))

Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)^2\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (Sqrt[a + c\*x^2]\*(2688\*a^2\*c\*e\*g^2 + 5376\*a^2\*c\*d\*g\*h - 1536\*a^3\*f\*g\*h - 76  
8\*a^3\*e\*h^2 + 8400\*a\*c^2\*d\*g^2\*x + 840\*a^2\*c\*f\*g^2\*x + 1680\*a^2\*c\*e\*g\*h\*x +  
840\*a^2\*c\*d\*h^2\*x - 315\*a^3\*f\*h^2\*x + 5376\*a\*c^2\*e\*g^2\*x^2 + 10752\*a\*c^2\*d  
\*g\*h\*x^2 + 768\*a^2\*c\*f\*g\*h\*x^2 + 384\*a^2\*c\*e\*h^2\*x^2 + 3360\*c^3\*d\*g^2\*x^3 +  
3920\*a\*c^2\*f\*g^2\*x^3 + 7840\*a\*c^2\*e\*g\*h\*x^3 + 3920\*a\*c^2\*d\*h^2\*x^3 + 210\*a  
^2\*c\*f\*h^2\*x^3 + 2688\*c^3\*e\*g^2\*x^4 + 5376\*c^3\*d\*g\*h\*x^4 + 6144\*a\*c^2\*f\*g\*h  
\*x^4 + 3072\*a\*c^2\*e\*h^2\*x^4 + 2240\*c^3\*f\*g^2\*x^5 + 4480\*c^3\*e\*g\*h\*x^5 + 224  
0\*c^3\*d\*h^2\*x^5 + 2520\*a\*c^2\*f\*h^2\*x^5 + 3840\*c^3\*f\*g\*h\*x^6 + 1920\*c^3\*e\*h^2  
\*x^6 + 1680\*c^3\*f\*h^2\*x^7))/(13440\*c^2) + ((-48\*a^2\*c^2\*d\*g^2 + 8\*a^3\*c\*f\*  
g^2 + 16\*a^3\*c\*e\*g\*h + 8\*a^3\*c\*d\*h^2 - 3\*a^4\*f\*h^2)\*Log[-(Sqrt[c]\*x) + Sqrt  
[a + c\*x^2]])/(128\*c^(5/2))

**fricas** [A] time = 1.11, size = 831, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] [-1/26880\*(105\*(16\*a^3\*c\*e\*g\*h - 8\*(6\*a^2\*c^2\*d - a^3\*c\*f)\*g^2 + (8\*a^3\*c\*d  
- 3\*a^4\*f)\*h^2)\*sqrt(c)\*log(-2\*c\*x^2 - 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) -  
2\*(1680\*c^4\*f\*h^2\*x^7 + 2688\*a^2\*c^2\*e\*g^2 - 768\*a^3\*c\*e\*h^2 + 1920\*(2\*c^4\*  
f\*g\*h + c^4\*e\*h^2)\*x^6 + 280\*(8\*c^4\*f\*g^2 + 16\*c^4\*e\*g\*h + (8\*c^4\*d + 9\*a\*c  
^3\*f)\*h^2)\*x^5 + 384\*(7\*c^4\*e\*g^2 + 8\*a\*c^3\*e\*h^2 + 2\*(7\*c^4\*d + 8\*a\*c^3\*f)  
\*g\*h)\*x^4 + 70\*(112\*a\*c^3\*e\*g\*h + 8\*(6\*c^4\*d + 7\*a\*c^3\*f)\*g^2 + (56\*a\*c^3\*d  
+ 3\*a^2\*c^2\*f)\*h^2)\*x^3 + 768\*(7\*a^2\*c^2\*d - 2\*a^3\*c\*f)\*g\*h + 384\*(14\*a\*c^3  
\*e\*g^2 + a^2\*c^2\*e\*h^2 + 2\*(14\*a\*c^3\*d + a^2\*c^2\*f)\*g\*h)\*x^2 + 105\*(16\*a^2  
\*c^2\*e\*g\*h + 8\*(10\*a\*c^3\*d + a^2\*c^2\*f)\*g^2 + (8\*a^2\*c^2\*d - 3\*a^3\*c\*f)\*h^2  
)\*x)\*sqrt(c\*x^2 + a)/c^3, 1/13440\*(105\*(16\*a^3\*c\*e\*g\*h - 8\*(6\*a^2\*c^2\*d -  
a^3\*c\*f)\*g^2 + (8\*a^3\*c\*d - 3\*a^4\*f)\*h^2)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c  
\*x^2 + a)) + (1680\*c^4\*f\*h^2\*x^7 + 2688\*a^2\*c^2\*e\*g^2 - 768\*a^3\*c\*e\*h^2 + 1  
920\*(2\*c^4\*f\*g\*h + c^4\*e\*h^2)\*x^6 + 280\*(8\*c^4\*f\*g^2 + 16\*c^4\*e\*g\*h + (8\*c^4  
\*d + 9\*a\*c^3\*f)\*h^2)\*x^5 + 384\*(7\*c^4\*e\*g^2 + 8\*a\*c^3\*e\*h^2 + 2\*(7\*c^4\*d +  
8\*a\*c^3\*f)\*g\*h)\*x^4 + 70\*(112\*a\*c^3\*e\*g\*h + 8\*(6\*c^4\*d + 7\*a\*c^3\*f)\*g^2 +  
(56\*a\*c^3\*d + 3\*a^2\*c^2\*f)\*h^2)\*x^3 + 768\*(7\*a^2\*c^2\*d - 2\*a^3\*c\*f)\*g\*h + 3  
84\*(14\*a\*c^3\*e\*g^2 + a^2\*c^2\*e\*h^2 + 2\*(14\*a\*c^3\*d + a^2\*c^2\*f)\*g\*h)\*x^2 +  
105\*(16\*a^2\*c^2\*e\*g\*h + 8\*(10\*a\*c^3\*d + a^2\*c^2\*f)\*g^2 + (8\*a^2\*c^2\*d - 3\*a  
^3\*c\*f)\*h^2)\*x)\*sqrt(c\*x^2 + a))/c^3]

**giac** [A] time = 0.26, size = 452, normalized size = 1.31



$+ a^{(3/2)} * a^2 * f * h^2 * x / c^2 + 3/128 * \sqrt{c * x^2 + a} * a^3 * f * h^2 * x / c^2 + 3/8 * a^2 * d * g^2 * \operatorname{arcsinh}(c * x / \sqrt{a * c}) / \sqrt{c} + 3/128 * a^4 * f * h^2 * \operatorname{arcsinh}(c * x / \sqrt{a * c}) / c^{(5/2)} + 1/5 * (c * x^2 + a)^{(5/2)} * e * g^2 / c + 2/5 * (c * x^2 + a)^{(5/2)} * d * g * h / c + 1/7 * (2 * f * g * h + e * h^2) * (c * x^2 + a)^{(5/2)} * x^2 / c + 1/6 * (f * g^2 + 2 * e * g * h + d * h^2) * (c * x^2 + a)^{(5/2)} * x / c - 1/24 * (f * g^2 + 2 * e * g * h + d * h^2) * (c * x^2 + a)^{(3/2)} * a * x / c - 1/16 * (f * g^2 + 2 * e * g * h + d * h^2) * \sqrt{c * x^2 + a} * a^2 * x / c - 1/16 * (f * g^2 + 2 * e * g * h + d * h^2) * a^3 * \operatorname{arcsinh}(c * x / \sqrt{a * c}) / c^{(3/2)} - 2/35 * (2 * f * g * h + e * h^2) * (c * x^2 + a)^{(5/2)} * a / c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + hx)^2 (cx^2 + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)`

[Out] `int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)`

**sympy** [A] time = 54.50, size = 1304, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)`

[Out] `-3*a**(7/2)*f*h**2*x/(128*c**2*sqrt(1 + c*x**2/a)) + a**(5/2)*d*h**2*x/(16*c*sqrt(1 + c*x**2/a)) + a**(5/2)*e*g*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g**2*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*f*h**2*x**3/(128*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**2*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*d*h**2*x**3/(48*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e*g*h*x**3/(24*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*g**2*x**3/(48*sqrt(1 + c*x**2/a)) + 13*a**(3/2)*f*h**2*x**5/(64*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*d*h**2*x**5/(24*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g*h*x**5/(12*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*g**2*x**5/(24*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*f*h**2*x**7/(16*sqrt(1 + c*x**2/a)) + 3*a**4*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)) - a**3*d*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*e*g*h*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**3*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*a**2*d*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 2*a*d*g*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a*e*g**2*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a*e*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*a*f*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4`

$$\begin{aligned}
&/4, \text{True})) + 2*c*d*g*h*\text{Piecewise}((-2*a**2*\text{sqrt}(a + c*x**2)/(15*c**2) + a*x* \\
&*2*\text{sqrt}(a + c*x**2)/(15*c) + x**4*\text{sqrt}(a + c*x**2)/5, \text{Ne}(c, 0)), (\text{sqrt}(a)*x \\
&**4/4, \text{True})) + c*e*g**2*\text{Piecewise}((-2*a**2*\text{sqrt}(a + c*x**2)/(15*c**2) + a* \\
&x**2*\text{sqrt}(a + c*x**2)/(15*c) + x**4*\text{sqrt}(a + c*x**2)/5, \text{Ne}(c, 0)), (\text{sqrt}(a) \\
&*x**4/4, \text{True})) + c*e*h**2*\text{Piecewise}((8*a**3*\text{sqrt}(a + c*x**2)/(105*c**3) - \\
&4*a**2*x**2*\text{sqrt}(a + c*x**2)/(105*c**2) + a*x**4*\text{sqrt}(a + c*x**2)/(35*c) + \\
&x**6*\text{sqrt}(a + c*x**2)/7, \text{Ne}(c, 0)), (\text{sqrt}(a)*x**6/6, \text{True})) + 2*c*f*g*h*\text{Pie} \\
&\text{cewise}((8*a**3*\text{sqrt}(a + c*x**2)/(105*c**3) - 4*a**2*x**2*\text{sqrt}(a + c*x**2)/( \\
&105*c**2) + a*x**4*\text{sqrt}(a + c*x**2)/(35*c) + x**6*\text{sqrt}(a + c*x**2)/7, \text{Ne}(c, \\
&0)), (\text{sqrt}(a)*x**6/6, \text{True})) + c**2*d*g**2*x**5/(4*\text{sqrt}(a)*\text{sqrt}(1 + c*x**2 \\
&/a)) + c**2*d*h**2*x**7/(6*\text{sqrt}(a)*\text{sqrt}(1 + c*x**2/a)) + c**2*e*g*h*x**7/(3 \\
&*\text{sqrt}(a)*\text{sqrt}(1 + c*x**2/a)) + c**2*f*g**2*x**7/(6*\text{sqrt}(a)*\text{sqrt}(1 + c*x**2/ \\
&a)) + c**2*f*h**2*x**9/(8*\text{sqrt}(a)*\text{sqrt}(1 + c*x**2/a))
\end{aligned}$$



$$3.90 \quad \int (g + hx) (a + cx^2)^{3/2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=213

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) (-aeh - afg + 6cdg)}{16c^{3/2}} - \frac{(a + cx^2)^{5/2} (6(2afh^2 + c(5fg^2 - 7h(dh + eg))) + 5chx(5fg - 7eh))}{210c^2h}$$

**Rubi [A]** time = 0.27, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {1654, 780, 195, 217, 206}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) (-aeh - afg + 6cdg)}{16c^{3/2}} - \frac{(a + cx^2)^{5/2} (6(2afh^2 - 7ch(dh + eg) + 5c^2fg^2) + 5chx(5fg - 7eh))}{210c^2h} + \frac{x(a + cx^2)^{3/2} (6cdg - a(eh + fg))}{24c} + \frac{ax\sqrt{a + cx^2} (-aeh - afg + 6cdg)}{16c} + \frac{f(a + cx^2)^{5/2} (g + hx)^2}{7ch}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (a\*(6\*c\*d\*g - a\*f\*g - a\*e\*h)\*x\*sqrt[a + c\*x^2])/(16\*c) + ((6\*c\*d\*g - a\*(f\*g + e\*h))\*x\*(a + c\*x^2)^(3/2))/(24\*c) + (f\*(g + h\*x)^2\*(a + c\*x^2)^(5/2))/(7\*c\*h) - (((6\*(5\*c\*f\*g^2 + 2\*a\*f\*h^2 - 7\*c\*h\*(e\*g + d\*h)) + 5\*c\*h\*(5\*f\*g - 7\*e\*h)\*x)\*(a + c\*x^2)^(5/2))/(210\*c^2\*h) + (a^2\*(6\*c\*d\*g - a\*f\*g - a\*e\*h)\*ArcTanh[(sqrt[c]\*x)/sqrt[a + c\*x^2]])/(16\*c^(3/2))

### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

### Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int (g + hx) (a + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{f(g + hx)^2 (a + cx^2)^{5/2}}{7ch} + \frac{\int (g + hx) ((7cd - 2af)h^2 - ch(5fg - 7eh))}{7ch^2} \\
&= \frac{f(g + hx)^2 (a + cx^2)^{5/2}}{7ch} - \frac{(6(5c fg^2 + 2afh^2 - 7ch(eg + dh)) + 5ch(5fg - 7eh))}{210c^2h} \\
&= \frac{(6cdg - a(fg + eh))x (a + cx^2)^{3/2}}{24c} + \frac{f(g + hx)^2 (a + cx^2)^{5/2}}{7ch} - \frac{(6(5c fg^2 + 2afh^2 - 7ch(eg + dh)) + 5ch(5fg - 7eh))}{210c^2h} \\
&= \frac{a(6cdg - afg - aeh)x\sqrt{a + cx^2}}{16c} + \frac{(6cdg - a(fg + eh))x (a + cx^2)^{3/2}}{24c} - \frac{(6(5c fg^2 + 2afh^2 - 7ch(eg + dh)) + 5ch(5fg - 7eh))}{210c^2h} \\
&= \frac{a(6cdg - afg - aeh)x\sqrt{a + cx^2}}{16c} + \frac{(6cdg - a(fg + eh))x (a + cx^2)^{3/2}}{24c} - \frac{(6(5c fg^2 + 2afh^2 - 7ch(eg + dh)) + 5ch(5fg - 7eh))}{210c^2h} \\
&= \frac{a(6cdg - afg - aeh)x\sqrt{a + cx^2}}{16c} + \frac{(6cdg - a(fg + eh))x (a + cx^2)^{3/2}}{24c} - \frac{(6(5c fg^2 + 2afh^2 - 7ch(eg + dh)) + 5ch(5fg - 7eh))}{210c^2h}
\end{aligned}$$

**Mathematica** [A] time = 0.63, size = 209, normalized size = 0.98

$$\sqrt{a + cx^2} \left( -\frac{105a^{5/2} \sqrt{\frac{a^2}{c^2} + 1} \sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right) (ah + afg - 6cdg)}{c^{3/2}(a + cx^2)} - \frac{96a^3 fh}{c^2} + \frac{3a^2(112fh + 7c(16g + 5fx) + f(35g + 16fx))}{c} + 2ax(21d(25g + 16fx) + x(7e(48g + 35fx) + fx(245g + 192fx))) + 4cx^3(21d(5g + 4fx) + 2x(7e(6g + 5fx) + 5fx(7g + 6fx))) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (Sqrt[a + c\*x^2]\*((-96\*a^3\*f\*h)/c^2 + (3\*a^2\*(112\*d\*h + 7\*e\*(16\*g + 5\*h\*x) + f\*x\*(35\*g + 16\*h\*x)))/c + 4\*c\*x^3\*(21\*d\*(5\*g + 4\*h\*x) + 2\*x\*(7\*e\*(6\*g + 5\*h\*x) + 5\*f\*x\*(7\*g + 6\*h\*x))) + 2\*a\*x\*(21\*d\*(25\*g + 16\*h\*x) + x\*(7\*e\*(48\*g + 35\*h\*x) + f\*x\*(245\*g + 192\*h\*x))) - (105\*a^(5/2)\*(-6\*c\*d\*g + a\*f\*g + a\*e\*h)\*Sqrt[1 + (c\*x^2)/a]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]]/(c^(3/2)\*(a + c\*x^2)))/1680

**IntegrateAlgebraic [A]** time = 0.90, size = 247, normalized size = 1.16

$\frac{\log\left(\frac{\sqrt{a+cx^2}-\sqrt{c}x}{16c^{3/2}}\right)\left(a^2h+afg-6a^2cdg\right)+\sqrt{a+cx^2}\left(-96a^3fh+336a^2cdh+336a^2ceg+105a^2cdh+105a^2cfx+48a^2cfx^2+1050a^2dgh+672a^2dix^2+672a^2cfx^2+490a^2cfx^3+384a^2cfx^4+420c^3dix^3+336c^3dix^4+336c^3cfx^4+280c^3dix^5+280c^3fgx^5+240c^3fhx^6\right)}{1680c^2}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)\*(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (Sqrt[a + c\*x^2]\*(336\*a^2\*c\*e\*g + 336\*a^2\*c\*d\*h - 96\*a^3\*f\*h + 1050\*a\*c^2\*d\*g\*x + 105\*a^2\*c\*f\*g\*x + 105\*a^2\*c\*e\*h\*x + 672\*a\*c^2\*e\*g\*x^2 + 672\*a\*c^2\*d\*h\*x^2 + 48\*a^2\*c\*f\*h\*x^2 + 420\*c^3\*d\*g\*x^3 + 490\*a\*c^2\*f\*g\*x^3 + 490\*a\*c^2\*e\*h\*x^3 + 336\*c^3\*e\*g\*x^4 + 336\*c^3\*d\*h\*x^4 + 384\*a\*c^2\*f\*h\*x^4 + 280\*c^3\*f\*g\*x^5 + 280\*c^3\*e\*h\*x^5 + 240\*c^3\*f\*h\*x^6))/(1680\*c^2) + ((-6\*a^2\*c\*d\*g + a^3\*f\*g + a^3\*e\*h)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(16\*c^(3/2))

**fricas [A]** time = 1.76, size = 477, normalized size = 2.24

$\frac{105(a^2h+afg-6a^2cdg)\sqrt{a+cx^2}+336a^2cdh+336a^2ceg+105a^2cdh+105a^2cfx+48a^2cfx^2+1050a^2dgh+672a^2dix^2+672a^2cfx^2+490a^2cfx^3+384a^2cfx^4+420c^3dix^3+336c^3dix^4+336c^3cfx^4+280c^3dix^5+280c^3fgx^5+240c^3fhx^6}{1680c^2}+\frac{(-6a^2cdg+a^3fg+a^3eh)\log(-\sqrt{c}x+\sqrt{a+cx^2})}{16c^{3/2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] [1/3360\*(105\*(a^3\*e\*h - (6\*a^2\*c\*d - a^3\*f)\*g)\*sqrt(c)\*log(-2\*c\*x^2 + 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) + 2\*(240\*c^3\*f\*h\*x^6 + 280\*(c^3\*f\*g + c^3\*e\*h)\*x^5 + 336\*a^2\*c\*e\*g + 48\*(7\*c^3\*e\*g + (7\*c^3\*d + 8\*a\*c^2\*f)\*h)\*x^4 + 70\*(7\*a\*c^2\*e\*h + (6\*c^3\*d + 7\*a\*c^2\*f)\*g)\*x^3 + 48\*(14\*a\*c^2\*e\*g + (14\*a\*c^2\*d + a^2\*c\*f)\*h)\*x^2 + 48\*(7\*a^2\*c\*d - 2\*a^3\*f)\*h + 105\*(a^2\*c\*e\*h + (10\*a\*c^2\*d + a^2\*c\*f)\*g)\*x)\*sqrt(c\*x^2 + a))/c^2, 1/1680\*(105\*(a^3\*e\*h - (6\*a^2\*c\*d - a^3\*f)\*g)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + (240\*c^3\*f\*h\*x^6 + 280\*(c^3\*f\*g + c^3\*e\*h)\*x^5 + 336\*a^2\*c\*e\*g + 48\*(7\*c^3\*e\*g + (7\*c^3\*d + 8\*a\*c^2\*f)\*h)\*x^4 + 70\*(7\*a\*c^2\*e\*h + (6\*c^3\*d + 7\*a\*c^2\*f)\*g)\*x^3 + 48\*(14\*a\*c^2\*e\*g + (14\*a\*c^2\*d + a^2\*c\*f)\*h)\*x^2 + 48\*(7\*a^2\*c\*d - 2\*a^3\*f)\*h + 105\*(a^2\*c\*e\*h + (10\*a\*c^2\*d + a^2\*c\*f)\*g)\*x)\*sqrt(c\*x^2 + a))/c^2]

**giac [A]** time = 0.25, size = 264, normalized size = 1.24

$\frac{1}{1680}\sqrt{cx^2+a}\left(2\left(\frac{1}{5}\left(6cfhx+\frac{7(c^2fg+e^2h)}{c^2}\right)\right)+\frac{6(7c^2dh+8ac^2fh+7c^2ge)}{c^3}\right)+\frac{35(6c^2dg+7ac^2he)}{c^3}+\frac{24(14ac^2dh+a^2c^2fh+14ac^2ge)}{c^3}+\frac{105(10ac^2dg+a^2c^2fg+a^2c^2he)}{c^3}+\frac{48(7a^2c^2dh-2a^2c^2fh+7a^2c^2ge)}{c^3}\right)\log\left(\frac{-\sqrt{c}x+\sqrt{cx^2+a}}{16c^{3/2}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out]  $\frac{1}{1680}\sqrt{c x^2 + a} \left( (2 \left( (4 (5 (6 c f h x + 7 (c^6 f g + c^6 h e) / c^5) x + 6 (7 c^6 d h + 8 a c^5 f h + 7 c^6 g e) / c^5) x + 35 (6 c^6 d g + 7 a c^5 f g + 7 a c^5 h e) / c^5) x + 24 (14 a c^5 d h + a^2 c^4 f h + 14 a c^5 g e) / c^5) x + 105 (10 a c^5 d g + a^2 c^4 f g + a^2 c^4 h e) / c^5) x + 48 (7 a^2 c^4 d h - 2 a^3 c^3 f h + 7 a^2 c^4 g e) / c^5 - \frac{1}{16} (6 a^2 c d g - a^3 f g - a^3 h e) \log(\operatorname{abs}(-\sqrt{c} x + \sqrt{c x^2 + a})) \right) / c^{3/2}$

**maple** [A] time = 0.00, size = 287, normalized size = 1.35

$$\frac{a^3 h \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{16 c^{\frac{3}{2}}} - \frac{a^3 f g \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{16 c^{\frac{3}{2}}} + \frac{3 a^2 d g \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{8 \sqrt{c}} - \frac{\sqrt{c x^2 + a} a^2 e h x}{16 c} - \frac{\sqrt{c x^2 + a} a^2 f g x}{16 c} + \frac{3 \sqrt{c x^2 + a} a d g x}{8} - \frac{(c x^2 + a)^{\frac{5}{2}} a h x}{24 c} - \frac{(c x^2 + a)^{\frac{5}{2}} a f g x}{24 c} + \frac{(c x^2 + a)^{\frac{5}{2}} f h x^2}{7 c} + \frac{(c x^2 + a)^{\frac{5}{2}} d g x}{4} + \frac{(c x^2 + a)^{\frac{5}{2}} e h x}{6 c} + \frac{(c x^2 + a)^{\frac{5}{2}} f g x}{6 c} + \frac{2 (c x^2 + a)^{\frac{5}{2}} a f h}{35 c^2} + \frac{(c x^2 + a)^{\frac{5}{2}} d h}{5 c} + \frac{(c x^2 + a)^{\frac{5}{2}} e g}{5 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x)

[Out]  $\frac{1}{7} h f x^2 (c x^2 + a)^{5/2} / c - \frac{2}{35} h f a (c x^2 + a)^{5/2} + \frac{1}{6} x (c x^2 + a)^{5/2} / c e h + \frac{1}{6} x (c x^2 + a)^{5/2} / c f g - \frac{1}{24} a / c x (c x^2 + a)^{3/2} e h - \frac{1}{24} a / c x (c x^2 + a)^{3/2} f g - \frac{1}{16} a^2 / c x (c x^2 + a)^{1/2} e h - \frac{1}{16} a^2 / c x (c x^2 + a)^{1/2} f g - \frac{1}{16} a^3 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) e h - \frac{1}{16} a^3 / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) f g + \frac{1}{5} (c x^2 + a)^{5/2} / c d h + \frac{1}{5} (c x^2 + a)^{5/2} / c e g + \frac{1}{4} d g x (c x^2 + a)^{3/2} + \frac{3}{8} d g a x (c x^2 + a)^{1/2} + \frac{3}{8} d g a^2 / c^{1/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2})$

**maxima** [A] time = 0.45, size = 211, normalized size = 0.99

$$\frac{(c x^2 + a)^{\frac{5}{2}} f h x^2}{7 c} + \frac{1}{4} (c x^2 + a)^{\frac{5}{2}} d g x + \frac{3}{8} \sqrt{c x^2 + a} a d g x + \frac{3 a^2 d g \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{8 \sqrt{c}} + \frac{(c x^2 + a)^{\frac{5}{2}} e g}{5 c} + \frac{(c x^2 + a)^{\frac{5}{2}} d h}{5 c} - \frac{2 (c x^2 + a)^{\frac{5}{2}} a f h}{35 c^2} + \frac{(c x^2 + a)^{\frac{5}{2}} (f g + e h) x}{6 c} - \frac{(c x^2 + a)^{\frac{5}{2}} (f g + e h) a x}{24 c} - \frac{\sqrt{c x^2 + a} (f g + e h) a^2 x}{16 c} - \frac{(f g + e h) a^3 \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{16 c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out]  $\frac{1}{7} (c x^2 + a)^{5/2} f h x^2 / c + \frac{1}{4} (c x^2 + a)^{3/2} d g x + \frac{3}{8} \sqrt{c x^2 + a} a d g x + \frac{3}{8} a^2 d g \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c} + \frac{1}{5} (c x^2 + a)^{5/2} e g / c + \frac{1}{5} (c x^2 + a)^{5/2} d h / c - \frac{2}{35} (c x^2 + a)^{5/2} a f h / c^2 + \frac{1}{6} (c x^2 + a)^{5/2} (f g + e h) x / c - \frac{1}{24} (c x^2 + a)^{3/2} (f g + e h) a x / c - \frac{1}{16} \sqrt{c x^2 + a} (f g + e h) a^2 x / c - \frac{1}{16} (f g + e h) a^3 \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + h x) (c x^2 + a)^{3/2} (f x^2 + e x + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.



### 3.91 $\int (a + cx^2)^{3/2} (d + ex + fx^2) dx$

**Optimal.** Leaf size=137

$$\frac{a^2(6cd - af) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}} + \frac{x(a + cx^2)^{3/2} (6cd - af)}{24c} + \frac{ax\sqrt{a + cx^2} (6cd - af)}{16c} + \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c}$$

**Rubi [A]** time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1815, 641, 195, 217, 206}

$$\frac{a^2(6cd - af) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}} + \frac{x(a + cx^2)^{3/2} (6cd - af)}{24c} + \frac{ax\sqrt{a + cx^2} (6cd - af)}{16c} + \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (a\*(6\*c\*d - a\*f)\*x\*Sqrt[a + c\*x^2])/(16\*c) + ((6\*c\*d - a\*f)\*x\*(a + c\*x^2)^(3/2))/(24\*c) + (e\*(a + c\*x^2)^(5/2))/(5\*c) + (f\*x\*(a + c\*x^2)^(5/2))/(6\*c) + (a^2\*(6\*c\*d - a\*f)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(16\*c^(3/2))

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /

; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :=> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int (a + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{fx(a + cx^2)^{5/2}}{6c} + \frac{\int (6cd - af + 6cex)(a + cx^2)^{3/2} dx}{6c} \\
 &= \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c} + \frac{(6cd - af) \int (a + cx^2)^{3/2} dx}{6c} \\
 &= \frac{(6cd - af)x(a + cx^2)^{3/2}}{24c} + \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c} + \frac{a(6cd - af)}{8c} \\
 &= \frac{a(6cd - af)x\sqrt{a + cx^2}}{16c} + \frac{(6cd - af)x(a + cx^2)^{3/2}}{24c} + \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c} \\
 &= \frac{a(6cd - af)x\sqrt{a + cx^2}}{16c} + \frac{(6cd - af)x(a + cx^2)^{3/2}}{24c} + \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c} \\
 &= \frac{a(6cd - af)x\sqrt{a + cx^2}}{16c} + \frac{(6cd - af)x(a + cx^2)^{3/2}}{24c} + \frac{e(a + cx^2)^{5/2}}{5c} + \frac{fx(a + cx^2)^{5/2}}{6c}
 \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 125, normalized size = 0.91

$$\frac{\sqrt{a + cx^2} \left( \sqrt{c} (3a^2(16e + 5fx) + 2acx(75d + x(48e + 35fx)) + 4c^2x^3(15d + 2x(6e + 5fx))) - \frac{15a^{3/2}(af - 6cd) \sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a} + 1}} \right)}{240c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(Sqrt[c]\*(3\*a^2\*(16\*e + 5\*f\*x) + 4\*c^2\*x^3\*(15\*d + 2\*x\*(6\*e + 5\*f\*x)) + 2\*a\*c\*x\*(75\*d + x\*(48\*e + 35\*f\*x))) - (15\*a^(3/2)\*(-6\*c\*d + a\*f)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]])/Sqrt[1 + (c\*x^2)/a])/(240\*c^(3/2))

**IntegrateAlgebraic [A]** time = 0.45, size = 125, normalized size = 0.91

$$\frac{\sqrt{a+cx^2} (48a^2e + 15a^2fx + 150acdx + 96acex^2 + 70acfx^3 + 60c^2dx^3 + 48c^2ex^4 + 40c^2fx^5)}{240c} + \frac{(a^3f - 6a^2cd) \log(\sqrt{a+cx^2} - \sqrt{cx})}{16c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(48\*a^2\*e + 150\*a\*c\*d\*x + 15\*a^2\*f\*x + 96\*a\*c\*e\*x^2 + 60\*c^2\*d\*x^3 + 70\*a\*c\*f\*x^3 + 48\*c^2\*e\*x^4 + 40\*c^2\*f\*x^5))/(240\*c) + ((-6\*a^2\*c\*d + a^3\*f)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(16\*c^(3/2))

**fricas [A]** time = 1.28, size = 262, normalized size = 1.91

$$\left[ \frac{15(6a^2cd - a^3f)\sqrt{c} \log(-2cx^2 + 2\sqrt{cx^2+a}\sqrt{c}x - a) - 2(40c^3fx^5 + 48c^3e*x^4 + 96a^2c^2e*x^2 + 48a^2c*e + 10(6c^3d + 7a^2c^2f)*x^3 + 15(10ac^2d + a^2c^2f)*x)\sqrt{cx^2+a}}{480c^2}, \frac{15(6a^2cd - a^3f)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}}{\sqrt{cx^2+a}}\right) - (40c^3fx^5 + 48c^3e*x^4 + 96a^2c^2e*x^2 + 48a^2c*e + 10(6c^3d + 7a^2c^2f)*x^3 + 15(10ac^2d + a^2c^2f)*x)\sqrt{cx^2+a}}{240c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d), x, algorithm="fricas")

[Out] [-1/480\*(15\*(6\*a^2\*c\*d - a^3\*f)\*sqrt(c)\*log(-2\*c\*x^2 + 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(40\*c^3\*f\*x^5 + 48\*c^3\*e\*x^4 + 96\*a\*c^2\*e\*x^2 + 48\*a^2\*c\*e + 10\*(6\*c^3\*d + 7\*a\*c^2\*f)\*x^3 + 15\*(10\*a\*c^2\*d + a^2\*c^2\*f)\*x)\*sqrt(c\*x^2 + a))/c^2, -1/240\*(15\*(6\*a^2\*c\*d - a^3\*f)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) - (40\*c^3\*f\*x^5 + 48\*c^3\*e\*x^4 + 96\*a\*c^2\*e\*x^2 + 48\*a^2\*c\*e + 10\*(6\*c^3\*d + 7\*a\*c^2\*f)\*x^3 + 15\*(10\*a\*c^2\*d + a^2\*c^2\*f)\*x)\*sqrt(c\*x^2 + a))/c^2]

**giac [A]** time = 0.23, size = 129, normalized size = 0.94

$$\frac{1}{240} \sqrt{cx^2+a} \left( \left( \left( 4(5cfx + 6ce)x + \frac{5(6c^5d + 7ac^4f)}{c^4} \right) x + 48ae \right) x + \frac{15(10ac^4d + a^2c^3f)}{c^4} \right) x + \frac{48a^2e}{c} - \frac{(6a^2cd - a^3f) \log(|-\sqrt{cx} + \sqrt{cx^2+a}|)}{16c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d), x, algorithm="giac")

[Out] 1/240\*sqrt(c\*x^2 + a)\*((2\*((4\*(5\*c\*f\*x + 6\*c\*e)\*x + 5\*(6\*c^5\*d + 7\*a\*c^4\*f)/c^4)\*x + 48\*a\*e)\*x + 15\*(10\*a\*c^4\*d + a^2\*c^3\*f)/c^4)\*x + 48\*a^2\*e/c) - 1/16\*(6\*a^2\*c\*d - a^3\*f)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

**maple [A]** time = 0.01, size = 146, normalized size = 1.07

$$-\frac{a^3f \ln(\sqrt{c}x + \sqrt{cx^2+a})}{16c^{3/2}} + \frac{3a^2d \ln(\sqrt{c}x + \sqrt{cx^2+a})}{8\sqrt{c}} - \frac{\sqrt{cx^2+a}afx}{16c} + \frac{3\sqrt{cx^2+a}adx}{8} - \frac{(cx^2+a)^{3/2}afx}{24c} + \frac{(cx^2+a)^{3/2}dx}{4} + \frac{(cx^2+a)^{5/2}fx}{6c} + \frac{(cx^2+a)^{5/2}e}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)`

[Out]  $\frac{1}{6}f*x*(c*x^2+a)^{5/2}/c - \frac{1}{24}f*a/c*x*(c*x^2+a)^{3/2} - \frac{1}{16}f*a^2/c*x*(c*x^2+a)^{1/2} - \frac{1}{16}f*a^3/c^{3/2}*\ln(c^{1/2}*x+(c*x^2+a)^{1/2}) + \frac{1}{5}e*(c*x^2+a)^{5/2}/c + \frac{1}{4}d*x*(c*x^2+a)^{3/2} + \frac{3}{8}d*a*x*(c*x^2+a)^{1/2} + \frac{3}{8}d*a^2/c^{1/2}*\ln(c^{1/2}*x+(c*x^2+a)^{1/2})$

**maxima** [A] time = 0.45, size = 131, normalized size = 0.96

$$\frac{1}{4}(cx^2+a)^{\frac{3}{2}}dx + \frac{3}{8}\sqrt{cx^2+a}adx + \frac{(cx^2+a)^{\frac{5}{2}}fx}{6c} - \frac{(cx^2+a)^{\frac{3}{2}}afx}{24c} - \frac{\sqrt{cx^2+a}a^2fx}{16c} + \frac{3a^2d \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}} - \frac{a^3f \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16c^{\frac{3}{2}}} + \frac{(cx^2+a)^{\frac{5}{2}}e}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")`

[Out]  $\frac{1}{4}*(c*x^2 + a)^{3/2}*d*x + \frac{3}{8}*sqrt(c*x^2 + a)*a*d*x + \frac{1}{6}*(c*x^2 + a)^{5/2}*f*x/c - \frac{1}{24}*(c*x^2 + a)^{3/2}*a*f*x/c - \frac{1}{16}*sqrt(c*x^2 + a)*a^2*f*x/c + \frac{3}{8}*a^2*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - \frac{1}{16}*a^3*f*arcsinh(c*x/sqrt(a*c))/c^{3/2} + \frac{1}{5}*(c*x^2 + a)^{5/2}*e/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (cx^2 + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)`

[Out] `int((a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)`

**sympy** [A] time = 17.01, size = 348, normalized size = 2.54

$$\frac{\frac{a^{\frac{5}{2}}fx}{16c\sqrt{1+\frac{cx^2}{a}}} + \frac{a^{\frac{3}{2}}dx\sqrt{1+\frac{cx^2}{a}}}{2} + \frac{a^{\frac{3}{2}}dx}{8\sqrt{1+\frac{cx^2}{a}}} + \frac{17a^{\frac{3}{2}}fx^3}{48\sqrt{1+\frac{cx^2}{a}}} + \frac{3\sqrt{a}cdx^3}{8\sqrt{1+\frac{cx^2}{a}}} + \frac{11\sqrt{a}cfx^5}{24\sqrt{1+\frac{cx^2}{a}}} - \frac{a^{\frac{3}{2}}f \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}} + \frac{3a^{\frac{3}{2}}d \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8\sqrt{c}} + ae \left( \begin{cases} \frac{\sqrt{a}x^2}{2} & \text{for } c = 0 \\ \frac{(cx^2)^{\frac{3}{2}}}{c} & \text{otherwise} \end{cases} \right) + ce \left( \begin{cases} \frac{-2a^{\frac{3}{2}}\sqrt{a+cx^2}}{15c^{\frac{3}{2}}} + \frac{a^{\frac{3}{2}}\sqrt{a+cx^2}}{15c} + \frac{a^{\frac{3}{2}}\sqrt{a+cx^2}}{5} & \text{for } c \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases} \right) + \frac{c^2dx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}} + \frac{c^2fx^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)`

[Out]  $a^{5/2}f*x/(16*c*sqrt(1 + c*x**2/a)) + a^{3/2}d*x*sqrt(1 + c*x**2/a)/2 + a^{3/2}d*x/(8*sqrt(1 + c*x**2/a)) + 17*a^{3/2}f*x**3/(48*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*x**5/(24*sqrt(1 + c*x**2/a)) - a^{3/2}f*a*asinh(sqrt(c)*x/sqrt(a))/(16*c^{3/2}) + 3*a^{3/2}d*a*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + a*e*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*e*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c**2*d*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))$

$$3.92 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx} dx$$

**Optimal.** Leaf size=326

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh) + 12acgh^2(fg^2-h(eg-dh)) + 8c^2g^3(fg^2-h(eg-dh))\right) (ah^2+cg^2)^{3/2} (dh^2)}{8\sqrt{c}h^6}$$

**Rubi [A]** time = 0.77, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh) + 12acgh^2(fg^2-h(eg-dh)) + 8c^2g^3(fg^2-h(eg-dh))\right) (ah^2+cg^2)^{3/2} (dh^2)}{8\sqrt{c}h^6} + \frac{(a+cx^2)^{3/2}\left(4(dh^2-egh+fg^2)-3ah(fg-eh)\right)}{12h^3} + \frac{\sqrt{a+cx^2}\left(8(ah^2+cg^2)(dh^2-egh+fg^2)-8h(3ah^2+4cg^2)(fg-eh)+4cdg^2\right)}{8h^3} + \frac{(ah^2+cg^2)^{3/2}(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{dh-cx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^6} + \frac{f(a+cx^2)^{5/2}}{5ch}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] ((8\*(c\*g^2 + a\*h^2)\*(f\*g^2 - e\*g\*h + d\*h^2) - h\*(4\*c\*d\*g\*h^2 + (f\*g - e\*h)\*(4\*c\*g^2 + 3\*a\*h^2))\*x)\*Sqrt[a + c\*x^2])/(8\*h^5) + ((4\*(f\*g^2 - e\*g\*h + d\*h^2) - 3\*h\*(f\*g - e\*h)\*x)\*(a + c\*x^2)^(3/2))/(12\*h^3) + (f\*(a + c\*x^2)^(5/2))/(5\*c\*h) - ((3\*a^2\*h^4\*(f\*g - e\*h) + 12\*a\*c\*g\*h^2\*(f\*g^2 - h\*(e\*g - d\*h)) + 8\*c^2\*(f\*g^5 - g^3\*h\*(e\*g - d\*h)))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*Sqrt[c]\*h^6) - ((c\*g^2 + a\*h^2)^(3/2)\*(f\*g^2 - e\*g\*h + d\*h^2)\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/h^6

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

### Rule 815

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1))]*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

### Rule 844

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

```

### Rule 1654

```

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))

```

### Rubi steps



[In] IntegrateAlgebraic[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x),x]

[Out] (Sqrt[a + c\*x^2]\*(120\*c^2\*f\*g^4 - 120\*c^2\*e\*g^3\*h + 120\*c^2\*d\*g^2\*h^2 + 160\*a\*c\*f\*g^2\*h^2 - 160\*a\*c\*e\*g\*h^3 + 160\*a\*c\*d\*h^4 + 24\*a^2\*f\*h^4 - 60\*c^2\*f\*g^3\*h\*x + 60\*c^2\*e\*g^2\*h^2\*x - 60\*c^2\*d\*g\*h^3\*x - 75\*a\*c\*f\*g\*h^3\*x + 75\*a\*c\*e\*h^4\*x + 40\*c^2\*f\*g^2\*h^2\*x^2 - 40\*c^2\*e\*g\*h^3\*x^2 + 40\*c^2\*d\*h^4\*x^2 + 48\*a\*c\*f\*h^4\*x^2 - 30\*c^2\*f\*g\*h^3\*x^3 + 30\*c^2\*e\*h^4\*x^3 + 24\*c^2\*f\*h^4\*x^4)/(120\*c\*h^5) + (2\*(c\*f\*g^4\*Sqrt[-(c\*g^2) - a\*h^2] - c\*e\*g^3\*h\*Sqrt[-(c\*g^2) - a\*h^2] + c\*d\*g^2\*h^2\*Sqrt[-(c\*g^2) - a\*h^2] + a\*f\*g^2\*h^2\*Sqrt[-(c\*g^2) - a\*h^2] - a\*e\*g\*h^3\*Sqrt[-(c\*g^2) - a\*h^2] + a\*d\*h^4\*Sqrt[-(c\*g^2) - a\*h^2]))\*ArcTan[(Sqrt[c]\*g + Sqrt[c]\*h\*x - h\*Sqrt[a + c\*x^2])/Sqrt[-(c\*g^2) - a\*h^2]]/h^6 + ((8\*c^2\*f\*g^5 - 8\*c^2\*e\*g^4\*h + 8\*c^2\*d\*g^3\*h^2 + 12\*a\*c\*f\*g^3\*h^2 - 12\*a\*c\*e\*g^2\*h^3 + 12\*a\*c\*d\*g\*h^4 + 3\*a^2\*f\*g\*h^4 - 3\*a^2\*e\*h^5)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(8\*Sqrt[c]\*h^6)

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x, algorithm="fricas")

[Out] Timed out

**giac** [A] time = 0.29, size = 551, normalized size = 1.69

1/120\*sqrt(c\*x^2+a)\*((2\*(3\*(4\*c\*f\*x/h - 5\*(c^4\*f\*g\*h^19 - c^4\*h^20\*e))/(c^3\*h^21))\*x + 4\*(5\*c^4\*f\*g^2\*h^18 + 5\*c^4\*d\*h^20 + 6\*a\*c^3\*f\*h^20 - 5\*c^4\*g\*h^19\*e)/(c^3\*h^21))\*x - 15\*(4\*c^4\*f\*g^3\*h^17 + 4\*c^4\*d\*g\*h^19 + 5\*a\*c^3\*f\*g\*h^19 - 4\*c^4\*g^2\*h^18\*e - 5\*a\*c^3\*h^20\*e)/(c^3\*h^21))\*x + 8\*(15\*c^4\*f\*g^4\*h^16 + 15\*c^4\*d\*g^2\*h^18 + 20\*a\*c^3\*f\*g^2\*h^18 + 20\*a\*c^3\*d\*h^20 + 3\*a^2\*c^2\*f\*h^20 - 15\*c^4\*g^3\*h^17\*e - 20\*a\*c^3\*g\*h^19\*e)/(c^3\*h^21) + 2\*(c^2\*f\*g^6 + c^2\*d\*g^4\*h^2 + 2\*a\*c\*f\*g^4\*h^2 + 2\*a\*c\*d\*g^2\*h^4 + a^2\*f\*g^2\*h^4 + a^2\*d\*h^6 - c^2\*g^5\*h\*e - 2\*a\*c\*g^3\*h^3\*e - a^2\*g\*h^5\*e)\*arctan(-(sqrt(c)\*x - sqrt(c\*x^2+a))\*h + sqrt(c)\*g)/sqrt(-c\*g^2 - a\*h^2))/sqrt(-c\*g^2 - a\*h^2)\*h^6 + 1/8\*(8\*c^(5/2)\*f\*g^5 + 8\*c^(5/2)\*d\*g^3\*h^2 + 12\*a\*c^(3/2)\*f\*g^3\*h^2 + 12\*a\*c^(3/2)\*d\*g\*h^4 + 3\*a^2\*sqrt(c)\*f\*g\*h^4 - 8\*c^(5/2)\*g^4\*h\*e - 12\*a\*c^(3/2)\*g^2\*h^3\*e - 3\*a^2\*sqrt(c)\*h^5\*e)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2+a)))/(c\*h^6)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x, algorithm="giac")

[Out] 1/120\*sqrt(c\*x^2 + a)\*((2\*(3\*(4\*c\*f\*x/h - 5\*(c^4\*f\*g\*h^19 - c^4\*h^20\*e))/(c^3\*h^21))\*x + 4\*(5\*c^4\*f\*g^2\*h^18 + 5\*c^4\*d\*h^20 + 6\*a\*c^3\*f\*h^20 - 5\*c^4\*g\*h^19\*e)/(c^3\*h^21))\*x - 15\*(4\*c^4\*f\*g^3\*h^17 + 4\*c^4\*d\*g\*h^19 + 5\*a\*c^3\*f\*g\*h^19 - 4\*c^4\*g^2\*h^18\*e - 5\*a\*c^3\*h^20\*e)/(c^3\*h^21))\*x + 8\*(15\*c^4\*f\*g^4\*h^16 + 15\*c^4\*d\*g^2\*h^18 + 20\*a\*c^3\*f\*g^2\*h^18 + 20\*a\*c^3\*d\*h^20 + 3\*a^2\*c^2\*f\*h^20 - 15\*c^4\*g^3\*h^17\*e - 20\*a\*c^3\*g\*h^19\*e)/(c^3\*h^21) + 2\*(c^2\*f\*g^6 + c^2\*d\*g^4\*h^2 + 2\*a\*c\*f\*g^4\*h^2 + 2\*a\*c\*d\*g^2\*h^4 + a^2\*f\*g^2\*h^4 + a^2\*d\*h^6 - c^2\*g^5\*h\*e - 2\*a\*c\*g^3\*h^3\*e - a^2\*g\*h^5\*e)\*arctan(-(sqrt(c)\*x - sqrt(c\*x^2+a))\*h + sqrt(c)\*g)/sqrt(-c\*g^2 - a\*h^2))/sqrt(-c\*g^2 - a\*h^2)\*h^6 + 1/8\*(8\*c^(5/2)\*f\*g^5 + 8\*c^(5/2)\*d\*g^3\*h^2 + 12\*a\*c^(3/2)\*f\*g^3\*h^2 + 12\*a\*c^(3/2)\*d\*g\*h^4 + 3\*a^2\*sqrt(c)\*f\*g\*h^4 - 8\*c^(5/2)\*g^4\*h\*e - 12\*a\*c^(3/2)\*g^2\*h^3\*e - 3\*a^2\*sqrt(c)\*h^5\*e)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/(c\*h^6)

**maple** [B] time = 0.01, size = 2420, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g), x)$

[Out]  $\frac{1}{5}f*(c*x^2+a)^{(5/2)}/c/h+1/2/h^3*c*g^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*e-1/2/h^4*c*g^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*f-3/2/h^2*c^{(1/2)}*g*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*a*d-3/8/h^2*f*g*a*x*(c*x^2+a)^{(1/2)}-3/8/h^2*f*g*a^2/c^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})+3/2/h^3*c^{(1/2)}*g^2*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*a*e-3/2/h^4*c^{(1/2)}*g^3*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*a*f+1/h^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*a^2*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*a^2*f*g^2+1/h^6/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*c^2*g^5*e-1/h^7/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*c^2*g^6*f-1/h^5/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*c^2*g^4*d-1/2/h^2*c*g*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*d-1/3/h^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(3/2)}*e*g+1/h*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*a*d+1/3/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(3/2)}*f*g^2+1/4/h*e*x*(c*x^2+a)^{(3/2)}-2/h^5/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*a*c*g^4*f-2/h^3/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*a*c*g^2*d+2/h^4/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)*a*c*g^3*e+1/3/h*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(3/2)}*d+3/8/h*e*a*x*(c*x^2+a)^{(1/2)}+3/8/h*e*a^2/c^{(1/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})-1/4/h^2*f*g*x*(c*x^2+a)^{(3/2)}+1/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*a*f*g^2-1/h^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*a*e*g+1/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*c*g^2*d-1/h^4*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*c*g^3*e+1/h^5*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*c*g^4*f-1/h^4*c^{(3/2)}*g^3*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})*d+1/h^5*c^{(3/2)}*g^4*\ln((-c*g/h+(x+g/h)*c)/c^{(1/2)}+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})$

$$h^2)^{(1/2)} * e^{-1/h^6 * c^{(3/2)} * g^5 * \ln((-c * g/h + (x + g/h) * c) / c^{(1/2)} + (-2 * (x + g/h) * c * g/h + (x + g/h)^2 * c + (a * h^2 + c * g^2) / h^2))^{(1/2)}} * f - 1/h / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x + g/h) * c * g/h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g/h) * c * g/h + (x + g/h)^2 * c + (a * h^2 + c * g^2) / h^2))^{(1/2)}) / (x + g/h) * a^2 * d$$

**maxima** [B] time = 0.79, size = 632, normalized size = 1.94

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x, algorithm="maxima")

[Out] 
$$-1/2 * \sqrt{c*x^2 + a} * c * f * g^3 * x / h^4 + 1/2 * \sqrt{c*x^2 + a} * c * e * g^2 * x / h^3 - 1/2 * \sqrt{c*x^2 + a} * c * d * g * x / h^2 - 1/4 * (c*x^2 + a)^{(3/2)} * f * g * x / h^2 - 3/8 * \sqrt{c*x^2 + a} * a * f * g * x / h^2 + 1/4 * (c*x^2 + a)^{(3/2)} * e * x / h + 3/8 * \sqrt{c*x^2 + a} * a * e * x / h - c^{(3/2)} * f * g^5 * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / h^6 + c^{(3/2)} * e * g^4 * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / h^5 - c^{(3/2)} * d * g^3 * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / h^4 - 3/2 * a * \sqrt{c} * f * g^3 * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / h^4 + 3/2 * a * \sqrt{c} * e * g^2 * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / h^3 - 3/2 * a * \sqrt{c} * d * g * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / h^2 - 3/8 * a^2 * f * g * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / (\sqrt{c} * h^2) + 3/8 * a^2 * e * \operatorname{arcsinh}(c*x/\sqrt{a*c}) / (\sqrt{c} * h) + (a + c * g^2 / h^2)^{(3/2)} * f * g^2 * \operatorname{arcsinh}(c * g * x / (\sqrt{a * c} * \operatorname{abs}(h * x + g))) - a * h / (\sqrt{a * c} * \operatorname{abs}(h * x + g)) / h^3 - (a + c * g^2 / h^2)^{(3/2)} * e * g * \operatorname{arcsinh}(c * g * x / (\sqrt{a * c} * \operatorname{abs}(h * x + g))) - a * h / (\sqrt{a * c} * \operatorname{abs}(h * x + g)) / h^2 + (a + c * g^2 / h^2)^{(3/2)} * d * \operatorname{arcsinh}(c * g * x / (\sqrt{a * c} * \operatorname{abs}(h * x + g))) - a * h / (\sqrt{a * c} * \operatorname{abs}(h * x + g)) / h + \sqrt{c * x^2 + a} * c * f * g^4 / h^5 - \sqrt{c * x^2 + a} * c * e * g^3 / h^4 + \sqrt{c * x^2 + a} * c * d * g^2 / h^3 + 1/3 * (c * x^2 + a)^{(3/2)} * f * g^2 / h^3 + \sqrt{c * x^2 + a} * a * f * g^2 / h^3 - 1/3 * (c * x^2 + a)^{(3/2)} * e * g / h^2 - \sqrt{c * x^2 + a} * a * e * g / h^2 + 1/3 * (c * x^2 + a)^{(3/2)} * d / h + \sqrt{c * x^2 + a} * a * d / h + 1/5 * (c * x^2 + a)^{(5/2)} * f / (c * h)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{g + hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x),x)

[Out] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{g + hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```



$$3.93 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2} dx$$

Optimal. Leaf size=432

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4 + 12ach^2(3fg^2 - h(2eg - dh)) + 8c^2g^2(5fg^2 - h(4eg - 3dh))\right)}{8\sqrt{c}h^6} - \frac{(a+cx^2)^{5/2}(dh^2 - eg)}{h(g+hx)(ah^2 +$$

Rubi [A] time = 0.90, antiderivative size = 428, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 815, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4 + 12ach^2(3fg^2 - h(2eg - dh)) + 8c^2g^2(5fg^2 - h(4eg - 3dh))\right)}{8\sqrt{c}h^6} - \frac{(a+cx^2)^{5/2}(dh^2 - eg)}{h(g+hx)(ah^2 +$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] -((8\*(5\*c\*f\*g^3 - c\*g\*h\*(4\*e\*g - 3\*d\*h) + a\*h^2\*(2\*f\*g - e\*h)) - h\*(20\*c\*f\*g^2 - 16\*c\*e\*g\*h + 12\*c\*d\*h^2 + 3\*a\*f\*h^2)\*x)\*Sqrt[a + c\*x^2])/(8\*h^5) - ((4\*(5\*c\*f\*g^3 - c\*g\*h\*(4\*e\*g - 3\*d\*h) + a\*h^2\*(2\*f\*g - e\*h)) - 3\*h\*(5\*c\*f\*g^2 + a\*f\*h^2 - 4\*c\*h\*(e\*g - d\*h))\*x)\*(a + c\*x^2)^(3/2))/(12\*h^3\*(c\*g^2 + a\*h^2)) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(5/2))/(h\*(c\*g^2 + a\*h^2)\*(g + h\*x)) + ((3\*a^2\*f\*h^4 + 8\*c^2\*(5\*f\*g^4 - g^2\*h\*(4\*e\*g - 3\*d\*h)) + 12\*a\*c\*h^2\*(3\*f\*g^2 - h\*(2\*e\*g - d\*h)))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*Sqrt[c]\*h^6) + (Sqrt[c\*g^2 + a\*h^2]\*(5\*c\*f\*g^3 - c\*g\*h\*(4\*e\*g - 3\*d\*h) + a\*h^2\*(2\*f\*g - e\*h))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/h^6

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ

[{a, c, d, e}, x]

### Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1))]*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^2} dx &= -\frac{(fg^2 - egh + dh^2)(a + cx^2)^{5/2}}{h(cg^2 + ah^2)(g + hx)} - \frac{\int \frac{\left(-cdg + afg - aeh - \left(afh - c\left(4eg - \frac{5fg^2}{h} - 4dh\right)\right)x\right)(a + cx^2)}{g + hx} dx}{cg^2 + ah^2} \\
&= -\frac{\left(4(5cfg^3 - cgh(4eg - 3dh) + ah^2(2fg - eh)) - 3h(5cfg^2 + afh^2 - 4ch(eg - dh))\right)}{12h^3(cg^2 + ah^2)} \\
&= -\frac{\left(8(5cfg^3 - cgh(4eg - 3dh) + ah^2(2fg - eh)) - h(20cfg^2 - 16cegh + 12ca^2)\right)}{8h^5} \\
&= -\frac{\left(8(5cfg^3 - cgh(4eg - 3dh) + ah^2(2fg - eh)) - h(20cfg^2 - 16cegh + 12ca^2)\right)}{8h^5} \\
&= -\frac{\left(8(5cfg^3 - cgh(4eg - 3dh) + ah^2(2fg - eh)) - h(20cfg^2 - 16cegh + 12ca^2)\right)}{8h^5} \\
&= -\frac{\left(8(5cfg^3 - cgh(4eg - 3dh) + ah^2(2fg - eh)) - h(20cfg^2 - 16cegh + 12ca^2)\right)}{8h^5}
\end{aligned}$$

**Mathematica [A]** time = 0.53, size = 392, normalized size = 0.91

$$\frac{3 \sqrt{c} \sqrt{a + cx^2} \left( (2d^2 + 2d(ah + 2ah^2) + 2ah^2) \sqrt{a + cx^2} + (2d^2 + 2d(ah + 2ah^2) + 2ah^2) \sqrt{a + cx^2} \right) + 24 \sqrt{ah^2 + cg^2} \log \left( \sqrt{a + cx^2} \sqrt{ah^2 + cg^2} + ah - cg \right) (ah^2(2fg - dh) + cgh(3dh - 4cg) + 5fg^2) - 24 \sqrt{ah^2 + cg^2} \log \left( \sqrt{a + cx^2} \sqrt{ah^2 + cg^2} + ah - cg \right) (ah^2(2fg - dh) + cgh(3dh - 4cg) + 5fg^2) + 3 \sqrt{a + cx^2} \left( 8(ah^2(ah - 2fg) - 3c(ah(2dh - 3cg) + 4fg^2)) + 3h(5ah^2 + 4c(h(ah - 2cg) + 3fg^2)) \right) - \frac{24ah^2 \sqrt{ah^2 + cg^2}}{24h^5} + 8ah^2(ah - 2fg) + 6c(ah^2)^2}{24h^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] (h\*sqrt[a + c\*x^2]\*(8\*(4\*a\*h^2\*(-2\*f\*g + e\*h) - 3\*c\*(4\*f\*g^3 + g\*h\*(-3\*e\*g + 2\*d\*h))) + 3\*h\*(5\*a\*f\*h^2 + 4\*c\*(3\*f\*g^2 + h\*(-2\*e\*g + d\*h)))\*x + 8\*c\*h^2\*(-2\*f\*g + e\*h)\*x^2 + 6\*c\*f\*h^3\*x^3 - (24\*(c\*g^2 + a\*h^2)\*(f\*g^2 + h\*(-(e\*g) + d\*h)))/(g + h\*x)) - 24\*sqrt[c\*g^2 + a\*h^2]\*(5\*c\*f\*g^3 + c\*g\*h\*(-4\*e\*g + 3\*d\*h) + a\*h^2\*(2\*f\*g - e\*h))\*Log[g + h\*x] + (3\*(3\*a^2\*f\*h^4 + 12\*a\*c\*h^2\*(3\*f\*g^2 + h\*(-2\*e\*g + d\*h)) + 8\*c^2\*(5\*f\*g^4 + g^2\*h\*(-4\*e\*g + 3\*d\*h)))\*Log[c\*x + sqrt[c]\*sqrt[a + c\*x^2]]/sqrt[c] + 24\*sqrt[c\*g^2 + a\*h^2]\*(5\*c\*f\*g^3 + c\*g\*h\*(-4\*e\*g + 3\*d\*h) + a\*h^2\*(2\*f\*g - e\*h))\*Log[a\*h - c\*g\*x + sqrt[c\*g^2 + a\*h^2]\*sqrt[a + c\*x^2]]/(24\*h^6)

**IntegrateAlgebraic [A]** time = 2.07, size = 478, normalized size = 1.11

$$\frac{\sqrt{a+cx^2} \left( -3d^2h^2 + 2acgh^2 - 3ae^2gh - 2a^2d^2h + 12c^2gh^2 - 4a^2d^2g \right) \sqrt{a+cx^2} \left( 5d^2g^2\sqrt{-c^2g^2-ah^2} - 4a^2d^2g\sqrt{-c^2g^2-ah^2} - a^2d^2\sqrt{-c^2g^2-ah^2} + 2d^2g^2\sqrt{-c^2g^2-ah^2} + 5d^2g\sqrt{-c^2g^2-ah^2} \right) \sqrt{a+cx^2} \left( -2abd^2 + 5acgh^2 + 12abd^2h - 8a^2d^2g - 4a^2d^2h + 15d^2gh^2 - 3a^2d^2g^2 + 12abd^2h^2 + 12acgh^2 + 4acgh^2h - 12a^2d^2g - 6a^2d^2h + 20d^2gh^2 - 10d^2gh^2 + 4d^2gh^2 \right)}{8d^2g^2h^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x)

[Out] (Sqrt[a + c\*x^2]\*(-120\*c\*f\*g^4 + 96\*c\*e\*g^3\*h - 72\*c\*d\*g^2\*h^2 - 88\*a\*f\*g^2\*h^2 + 56\*a\*e\*g\*h^3 - 24\*a\*d\*h^4 - 60\*c\*f\*g^3\*h\*x + 48\*c\*e\*g^2\*h^2\*x - 36\*c\*d\*g\*h^3\*x - 49\*a\*f\*g\*h^3\*x + 32\*a\*e\*h^4\*x + 20\*c\*f\*g^2\*h^2\*x^2 - 16\*c\*e\*g\*h^3\*x^2 + 12\*c\*d\*h^4\*x^2 + 15\*a\*f\*h^4\*x^2 - 10\*c\*f\*g\*h^3\*x^3 + 8\*c\*e\*h^4\*x^3 + 6\*c\*f\*h^4\*x^4))/(24\*h^5\*(g + h\*x)) - (2\*(5\*c\*f\*g^3\*Sqrt[-(c\*g^2) - a\*h^2] - 4\*c\*e\*g^2\*h\*Sqrt[-(c\*g^2) - a\*h^2] + 3\*c\*d\*g\*h^2\*Sqrt[-(c\*g^2) - a\*h^2] + 2\*a\*f\*g\*h^2\*Sqrt[-(c\*g^2) - a\*h^2] - a\*e\*h^3\*Sqrt[-(c\*g^2) - a\*h^2])\*ArcTan[(Sqrt[c]\*g + Sqrt[c]\*h\*x - h\*Sqrt[a + c\*x^2])/Sqrt[-(c\*g^2) - a\*h^2]])/h^6 + ((-40\*c^2\*f\*g^4 + 32\*c^2\*e\*g^3\*h - 24\*c^2\*d\*g^2\*h^2 - 36\*a\*c\*f\*g^2\*h^2 + 24\*a\*c\*e\*g\*h^3 - 12\*a\*c\*d\*h^4 - 3\*a^2\*f\*h^4)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(8\*Sqrt[c]\*h^6)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^2,x, algorithm="giac")

[Out] Timed out

**maple [B]** time = 0.02, size = 5121, normalized size = 11.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^2,x)

[Out] result too large to display

**maxima** [A] time = 0.78, size = 708, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^2,x, algorithm="maxima")

[Out] 
$$-(c*x^2 + a)^{(3/2)}*f*g^2/(h^4*x + g*h^3) + (c*x^2 + a)^{(3/2)}*e*g/(h^3*x + g*h^2) - (c*x^2 + a)^{(3/2)}*d/(h^2*x + g*h) + 5/2*\sqrt{c*x^2 + a}*c*f*g^2*x/h^4 - 2*\sqrt{c*x^2 + a}*c*e*g*x/h^3 + 3/2*\sqrt{c*x^2 + a}*c*d*x/h^2 + 1/4*(c*x^2 + a)^{(3/2)}*f*x/h^2 + 3/8*\sqrt{c*x^2 + a}*a*f*x/h^2 + 5*c^{(3/2)}*f*g^4*a*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^6 - 4*c^{(3/2)}*e*g^3*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^5 + 3*c^{(3/2)}*d*g^2*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^4 + 9/2*a*\sqrt{c}*f*g^2*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^4 - 3*a*\sqrt{c}*e*g*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^3 + 3/2*a*\sqrt{c}*d*\operatorname{arcsinh}(c*x/\sqrt{a*c})/h^2 + 3/8*a^2*f*\operatorname{arcsinh}(c*x/\sqrt{a*c})/(\sqrt{c}*h^2) - 3*\sqrt{a + c*g^2/h^2}*c*f*g^3*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c}*\operatorname{abs}(h*x + g))/h^5 + 3*\sqrt{a + c*g^2/h^2}*c*e*g^2*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c}*\operatorname{abs}(h*x + g))/h^4 - 3*\sqrt{a + c*g^2/h^2}*c*d*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c}*\operatorname{abs}(h*x + g))/h^3 - 2*(a + c*g^2/h^2)^{(3/2)}*f*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c}*\operatorname{abs}(h*x + g))/h^3 + (a + c*g^2/h^2)^{(3/2)}*e*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c}*\operatorname{abs}(h*x + g))/h^2 - 5*\sqrt{c*x^2 + a}*c*f*g^3/h^5 + 4*\sqrt{c*x^2 + a}*c*e*g^2/h^4 - 3*\sqrt{c*x^2 + a}*c*d*g/h^3 - 2/3*(c*x^2 + a)^{(3/2)}*f*g/h^3 - 2*\sqrt{c*x^2 + a}*a*f*g/h^3 + 1/3*(c*x^2 + a)^{(3/2)}*e/h^2 + \sqrt{c*x^2 + a}*a*e/h^2$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x)

[Out] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

$$3.94 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3} dx$$

**Optimal.** Leaf size=488

$$\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4 + ach^2(19fg^2 - 3h(3eg - dh)) + 2c^2g^2(10fg^2 - 3h(2eg - dh))\right) + \sqrt{a+cx^2}}{2h^6\sqrt{ah^2+cg^2}}$$

**Rubi [A]** time = 0.92, antiderivative size = 480, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {1651, 813, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}(2f^2g^2 - c(a^2f^2g - 3ah)(2fg - dh) + ah(19f^2g - 3h(3eg - dh)) - 2c^2g^2(-3ah + ceg - \frac{ah^2c}{cg^2})) + \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(2a^2fh^4 + ach^2(19fg^2 - 3h(3eg - dh)) + 2c^2g^2(10fg^2 - 3h(2eg - dh)))}{2h^6\sqrt{ah^2+cg^2}}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] ((2\*a^2\*f\*h^3 - 2\*c^2\*g^2\*(6\*e\*g - (10\*f\*g^2)/h - 3\*d\*h) + a\*c\*h\*(19\*f\*g^2 - 3\*h\*(3\*e\*g - d\*h)) - c\*(10\*c\*f\*g^3 - 3\*c\*g\*h\*(2\*e\*g - d\*h) + a\*h^2\*(7\*f\*g - 3\*e\*h))\*x)\*Sqrt[a + c\*x^2])/(2\*h^4\*(c\*g^2 + a\*h^2)) - ((2\*(c\*g\*(6\*e\*g - (10\*f\*g^2)/h - 3\*d\*h) - a\*h\*(7\*f\*g - 3\*e\*h)) - (5\*c\*f\*g^2 + 2\*a\*f\*h^2 - 3\*c\*h\*(e\*g - d\*h))\*x)\*(a + c\*x^2)^(3/2))/(6\*h^2\*(c\*g^2 + a\*h^2)\*(g + h\*x)) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(5/2))/(2\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^2) - (Sqrt[c]\*(20\*c\*f\*g^3 - 6\*c\*g\*h\*(2\*e\*g - d\*h) + 3\*a\*h^2\*(3\*f\*g - e\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*h^6) - ((2\*a^2\*f\*h^4 + 2\*c^2\*(10\*f\*g^4 - 3\*g^2\*h\*(2\*e\*g - d\*h)) + a\*c\*h^2\*(19\*f\*g^2 - 3\*h\*(3\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(2\*h^6\*Sqrt[c\*g^2 + a\*h^2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ

[{a, c, d, e}, x]

### Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

### Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^3} dx &= -\frac{(fg^2 - egh + dh^2)(a + cx^2)^{5/2}}{2h(cg^2 + ah^2)(g + hx)^2} - \frac{\int \frac{\left(-2(cdg - afg + aeh) - \left(2afh - c\left(3eg - \frac{5fg^2}{h} - 3dh\right)\right)x\right)}{(g + hx)^2} dx}{2(cg^2 + ah^2)} \\
&= -\frac{\left(2\left(cg\left(6eg - \frac{10fg^2}{h} - 3dh\right) - ah(7fg - 3eh)\right) - (5cfg^2 + 2afh^2 - 3ch(eg - dh))\right)}{6h^2(cg^2 + ah^2)(g + hx)} \\
&= \frac{\left(2a^2fh^3 - 2c^2g^2\left(6eg - \frac{10fg^2}{h} - 3dh\right) + ach(19fg^2 - 3h(3eg - dh)) - c(10c^2g^2h - 3ah^2)\right)}{2h^4(cg^2 + ah^2)} \\
&= \frac{\left(2a^2fh^3 - 2c^2g^2\left(6eg - \frac{10fg^2}{h} - 3dh\right) + ach(19fg^2 - 3h(3eg - dh)) - c(10c^2g^2h - 3ah^2)\right)}{2h^4(cg^2 + ah^2)} \\
&= \frac{\left(2a^2fh^3 - 2c^2g^2\left(6eg - \frac{10fg^2}{h} - 3dh\right) + ach(19fg^2 - 3h(3eg - dh)) - c(10c^2g^2h - 3ah^2)\right)}{2h^4(cg^2 + ah^2)} \\
&= \frac{\left(2a^2fh^3 - 2c^2g^2\left(6eg - \frac{10fg^2}{h} - 3dh\right) + ach(19fg^2 - 3h(3eg - dh)) - c(10c^2g^2h - 3ah^2)\right)}{2h^4(cg^2 + ah^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.65, size = 435, normalized size = 0.89

$$\frac{3h\sqrt{a+cx^2}\sqrt{ah^2+cg^2}\sqrt{d+ex+fx^2}\sqrt{g+hx} + 3h\sqrt{a+cx^2}\sqrt{ah^2+cg^2}\sqrt{d+ex+fx^2}\sqrt{g+hx} - 3\sqrt{c}\log\left(\sqrt{a+cx^2}\sqrt{g+hx}\right)\left(-3ah^2(ah-3fg)+6cg(ah-2cg)+20c^2fg\right) + \sqrt{a+cx^2}\sqrt{ah^2+cg^2}\sqrt{d+ex+fx^2}\sqrt{g+hx}\sqrt{a+cx^2}\sqrt{g+hx}}{6h^6}$$

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x]

[Out] ((h\*sqrt[a + c\*x^2]\*(a\*h^2\*(-3\*h\*(e\*g + d\*h + 2\*e\*h\*x) + f\*(17\*g^2 + 28\*g\*h\*x + 8\*h^2\*x^2)) + c\*(f\*(60\*g^4 + 90\*g^3\*h\*x + 20\*g^2\*h^2\*x^2 - 5\*g\*h^3\*x^3 + 2\*h^4\*x^4) + 3\*h\*(d\*h\*(6\*g^2 + 9\*g\*h\*x + 2\*h^2\*x^2) + e\*(-12\*g^3 - 18\*g^2\*h\*x - 4\*g\*h^2\*x^2 + h^3\*x^3)))))/(g + h\*x)^2 + (3\*(2\*a^2\*f\*h^4 + a\*c\*h^2\*(19\*f\*g^2 + 3\*h\*(-3\*e\*g + d\*h)) + 2\*c^2\*(10\*f\*g^4 + 3\*g^2\*h\*(-2\*e\*g + d\*h)))\*Log[g + h\*x])/sqrt[c\*g^2 + a\*h^2] - 3\*sqrt[c]\*(20\*c\*f\*g^3 + 6\*c\*g\*h\*(-2\*e\*g + d\*h) - 3\*a\*h^2\*(-3\*f\*g + e\*h))\*Log[c\*x + sqrt[c]\*sqrt[a + c\*x^2]] - (3\*(2\*a^2\*f\*h^4 + a\*c\*h^2\*(19\*f\*g^2 + 3\*h\*(-3\*e\*g + d\*h)) + 2\*c^2\*(10\*f\*g^4 + 3\*g^2\*h\*(-2\*e\*g + d\*h)))\*Log[a\*h - c\*g\*x + sqrt[c\*g^2 + a\*h^2]\*sqrt[a + c\*x^2]])/sqrt[c\*g^2 + a\*h^2])/(6\*h^6)



$$\begin{aligned} & t(c*x^2 + a)^3*c^2*d*g^2*h^3 + 5*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a*c*f*g^2 \\ & *h^3 + (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a*c*d*h^5 - 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 \\ & + a))^3*c^2*g^3*h^2*e - 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a*c*g*h^4*e + 1 \\ & 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*c^{(5/2)}*f*g^5 + 10*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 \\ & + a))^2*c^{(5/2)}*d*g^3*h^2 - (\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a*c^{(3/2)}*f*g \\ & ^3*h^2 - 5*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a*c^{(3/2)}*d*g*h^4 - 4*(\text{sqrt}(c)*x \\ & - \text{sqrt}(c*x^2 + a))^2*a^2*\text{sqrt}(c)*f*g*h^4 - 14*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a) \\ & )^2*c^{(5/2)}*g^4*h^e + 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a*c^{(3/2)}*g^2*h^3*e \\ & + 2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^2*\text{sqrt}(c)*h^5*e - 26*(\text{sqrt}(c)*x - \text{sq} \\ & \text{rt}(c*x^2 + a))*a*c^2*f*g^4*h - 14*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a*c^2*d*g^2 \\ & *h^3 - 11*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^2*c*f*g^2*h^3 + (\text{sqrt}(c)*x - \text{sqrt} \\ & (c*x^2 + a))*a^2*c*d*h^5 + 20*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a*c^2*g^3*h^2*e \\ & + 5*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^2*c*g*h^4*e + 9*a^2*c^{(3/2)}*f*g^3*h^2 \\ & + 5*a^2*c^{(3/2)}*d*g*h^4 + 4*a^3*\text{sqrt}(c)*f*g*h^4 - 7*a^2*c^{(3/2)}*g^2*h^3*e - \\ & 2*a^3*\text{sqrt}(c)*h^5*e)/(((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*h + 2*(\text{sqrt}(c)*x - \\ & \text{sqrt}(c*x^2 + a))*\text{sqrt}(c)*g - a*h)^2*h^6) \end{aligned}$$

**maple [B]** time = 0.02, size = 7817, normalized size = 16.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g)^3,x)$

[Out] result too large to display

**maxima [B]** time = 0.88, size = 1299, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g)^3,x, \text{algorithm}=\text{"maxima"})$

[Out] 
$$\begin{aligned} & 3/2*\text{sqrt}(c*x^2 + a)*c^2*f*g^4/(c*g^2*h^5 + a*h^7) - 3/2*\text{sqrt}(c*x^2 + a)*c^2 \\ & *f*g^3*x/(c*g^2*h^4 + a*h^6) - 3/2*\text{sqrt}(c*x^2 + a)*c^2*e*g^3/(c*g^2*h^4 + a \\ & *h^6) + 1/2*(c*x^2 + a)^{(3/2)}*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + \\ & a*g*h^5) + 3/2*\text{sqrt}(c*x^2 + a)*c^2*e*g^2*x/(c*g^2*h^3 + a*h^5) + 3/2*\text{sqrt}(c \\ & *x^2 + a)*c^2*d*g^2/(c*g^2*h^3 + a*h^5) - 1/2*(c*x^2 + a)^{(3/2)}*c*e*g^2/(c* \\ & g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^{(5/2)}*f*g^2/(c \\ & *g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^ \\ & 3) + 1/2*(c*x^2 + a)^{(3/2)}*c*f*g^2/(c*g^2*h^3 + a*h^5) - 3/2*\text{sqrt}(c*x^2 + a \\ & )*c^2*d*g*x/(c*g^2*h^2 + a*h^4) + 1/2*(c*x^2 + a)^{(3/2)}*c*d*g/(c*g^2*h^2*x \\ & + a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*(c*x^2 + a)^{(5/2)}*e*g/(c*g^2*h^2*x^2 + \\ & a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*(c*x^2 + \\ & a)^{(3/2)}*c*e*g/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^{(5/2)}*d/(c*g^2*h*x^2 + \\ & a*h^3*x^2 + 2*c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*(c*x^2 + a) \end{aligned}$$

$$\begin{aligned} & \frac{1}{h^3} \left( \frac{(c^2 x^2 + a)^{3/2} (d + e x + f x^2)}{(g + h x)^3} - \frac{c^2 x^2 + a}{h^3} \frac{d + e x + f x^2}{g + h x} \right) \\ & - \frac{7}{2} \frac{\sqrt{c^2 x^2 + a} (d + e x + f x^2)}{h^4} + \frac{3}{2} \frac{\sqrt{c^2 x^2 + a} (d + e x + f x^2)}{h^4} \\ & - \frac{10 c^2 x^2 + a}{h^3} \frac{d + e x + f x^2}{g + h x} + \frac{6 c^2 x^2 + a}{h^5} \frac{d + e x + f x^2}{g + h x} \\ & - \frac{3 c^2 x^2 + a}{h^4} \frac{d + e x + f x^2}{g + h x} + \frac{3}{2} \frac{c^2 x^2 + a}{h^3} \frac{d + e x + f x^2}{g + h x} \\ & - \frac{a h}{\sqrt{a c} \operatorname{abs}(h x + g)} \left( \frac{d + e x + f x^2}{\sqrt{a + c g^2/h^2} h^7} - \frac{3}{2} \frac{c^2 x^2 + a}{h^6} \frac{d + e x + f x^2}{g + h x} \right) \\ & - \frac{a h}{\sqrt{a c} \operatorname{abs}(h x + g)} \left( \frac{d + e x + f x^2}{\sqrt{a + c g^2/h^2} h^5} + \frac{15}{2} \frac{\sqrt{a + c g^2/h^2} (d + e x + f x^2)}{h^5} \right) \\ & - \frac{9}{2} \frac{\sqrt{a + c g^2/h^2} (d + e x + f x^2)}{h^4} + \frac{3}{2} \frac{\sqrt{a + c g^2/h^2} (d + e x + f x^2)}{h^4} \\ & - \frac{a h}{\sqrt{a c} \operatorname{abs}(h x + g)} \left( \frac{d + e x + f x^2}{\sqrt{a + c g^2/h^2} h^3} + \frac{17}{2} \frac{\sqrt{c^2 x^2 + a} (d + e x + f x^2)}{h^5} \right) \\ & - \frac{9}{2} \frac{\sqrt{c^2 x^2 + a} (d + e x + f x^2)}{h^4} + \frac{3}{2} \frac{\sqrt{c^2 x^2 + a} (d + e x + f x^2)}{h^3} + \frac{1}{3} \frac{(c^2 x^2 + a)^{3/2} (d + e x + f x^2)}{h^3} \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c x^2 + a)^{3/2} (f x^2 + e x + d)}{(g + h x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x)

[Out] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + c x^2)^{3/2} (d + e x + f x^2)}{(g + h x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*3,x)

[Out] Integral((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*3, x)

$$3.95 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4} dx$$

**Optimal.** Leaf size=475

$$\frac{c \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right) \left(3a^2h^4(4fg-eh) + 3acgh^2(11fg^2-h(4eg-dh)) + 2c^2g^3(10fg^2-h(4eg-dh))\right)}{2h^6(ah^2+cg^2)^{3/2}}$$

**Rubi [A]** time = 0.84, antiderivative size = 469, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 813, 844, 217, 206, 725}

$$\frac{c \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right) \left(3a^2h^4(4fg-eh) + 3acgh^2(11fg^2-h(4eg-dh)) + 2c^2g^3(10fg^2-h(4eg-dh))\right)}{2h^6(ah^2+cg^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x]

[Out] -(((c\*g^2 + a\*h^2)\*(20\*c\*f\*g^2 + 3\*a\*f\*h^2 - 2\*c\*h\*(4\*e\*g - d\*h)) + c\*h\*(10\*c\*f\*g^3 - c\*g\*h\*(4\*e\*g - d\*h) + 3\*a\*h^2\*(3\*f\*g - e\*h))\*x)\*Sqrt[a + c\*x^2])/(2\*h^5\*(c\*g^2 + a\*h^2)\*(g + h\*x)) - ((c\*g\*(4\*e\*g - (10\*f\*g^2)/h - d\*h) - 3\*a\*h\*(3\*f\*g - e\*h) - (5\*c\*f\*g^2 + 3\*a\*f\*h^2 - 2\*c\*h\*(e\*g - d\*h))\*x)\*(a + c\*x^2)^(3/2))/(6\*h^2\*(c\*g^2 + a\*h^2)\*(g + h\*x)^2) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(5/2))/(3\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^3) + (Sqrt[c]\*(20\*c\*f\*g^2 + 3\*a\*f\*h^2 - 2\*c\*h\*(4\*e\*g - d\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*h^6) + (c\*(3\*a^2\*h^4\*(4\*f\*g - e\*h) + 3\*a\*c\*g\*h^2\*(11\*f\*g^2 - h\*(4\*e\*g - d\*h)) + 2\*c^2\*(10\*f\*g^5 - g^3\*h\*(4\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(2\*h^6\*(c\*g^2 + a\*h^2)^(3/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ

[{a, c, d, e}, x]

### Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^4} dx &= -\frac{(fg^2 - egh + dh^2)(a + cx^2)^{5/2}}{3h(cg^2 + ah^2)(g + hx)^3} - \frac{\int \frac{\left(-3(cdg - afg + aeh) - \left(3afh - c\left(2eg - \frac{5fg^2}{h} - 2dh\right)\right)x\right)}{(g + hx)^3} dx}{3(cg^2 + ah^2)} \\
&= -\frac{\left(cg\left(4eg - \frac{10fg^2}{h} - dh\right) - 3ah(3fg - eh) - (5cfg^2 + 3afh^2 - 2ch(eg - dh))\right)}{6h^2(cg^2 + ah^2)(g + hx)^2} \\
&= -\frac{\left((cg^2 + ah^2)(20cfg^2 + 3afh^2 - 2ch(4eg - dh)) + ch(10cfg^3 - cgh(4eg - dh))\right)}{2h^5(cg^2 + ah^2)(g + hx)} \\
&= -\frac{\left((cg^2 + ah^2)(20cfg^2 + 3afh^2 - 2ch(4eg - dh)) + ch(10cfg^3 - cgh(4eg - dh))\right)}{2h^5(cg^2 + ah^2)(g + hx)} \\
&= -\frac{\left((cg^2 + ah^2)(20cfg^2 + 3afh^2 - 2ch(4eg - dh)) + ch(10cfg^3 - cgh(4eg - dh))\right)}{2h^5(cg^2 + ah^2)(g + hx)} \\
&= -\frac{\left((cg^2 + ah^2)(20cfg^2 + 3afh^2 - 2ch(4eg - dh)) + ch(10cfg^3 - cgh(4eg - dh))\right)}{2h^5(cg^2 + ah^2)(g + hx)}
\end{aligned}$$

**Mathematica [A]** time = 1.23, size = 517, normalized size = 1.09

$$\frac{3h\sqrt{cg^2 + ah^2}\sqrt{a + cx^2}(d + ex + fx^2) - 3h^2(cg^2 + ah^2)(d + ex + fx^2) - 3h^3(cg^2 + ah^2)(d + ex + fx^2) - 3h^4(cg^2 + ah^2)(d + ex + fx^2) - 3h^5(cg^2 + ah^2)(d + ex + fx^2) + 3h^6\log\left(\sqrt{c}\sqrt{a + cx^2} + c\right)\sqrt{a + cx^2}}{(g + hx)^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x]

[Out] 
$$\begin{aligned}
& -\left(\frac{h\sqrt{a + cx^2}(2(cg^2 + ah^2)^2(fg^2 + h(-eg) + dh)) - (cg^2 + ah^2)(13c*fg^3 + c*gh(-10*eg + 7*d*h) - 3*ah^2(-2*fg + eh))}{(g + hx)} + (6*a^2*f*h^4 + a*c*h^2(50*f*g^2 + h(-23*eg + 8*d*h)) + c^2*(47*f*g^4 + g^2*h(-26*eg + 11*d*h))\right) * (g + hx)^2 + 6*c*(4*f*g - e*h)*(c*g^2 + a*h^2)*(g + hx)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + hx)^3\right) / ((c*g^2 + a*h^2)*(g + hx)^3) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h(-4*eg + d*h)) + 2*c^2*(10*f*g^5 + g^3*h(-4*eg + d*h))) * \text{Log}[g + h*x]) / (c*g^2 + a*h^2)^(3/2) + 3*sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 + 2*c*h*(-4*eg + d*h)) * \text{Log}[c*x + sqrt[c]*sqrt[a + c*x^2]] + (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h(-4*eg + d*h)) + 2*c^2*(10*f*g^5 + g^3*h(-4*eg + d*h))) * \text{Log}[g + h*x]) / (c*g^2 + a*h^2)^(3/2)
\end{aligned}$$

$3*h*(-4*e*g + d*h))*\text{Log}[a*h - c*g*x + \text{Sqrt}[c*g^2 + a*h^2]*\text{Sqrt}[a + c*x^2]]$   
 $)/(c*g^2 + a*h^2)^{(3/2)})/(6*h^6)$

**IntegrateAlgebraic [F]** time = 180.17, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x]

[Out] \$Aborted

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^4,x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 0.61, size = 1900, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^4,x, algorithm="giac")

[Out]  $\frac{1}{2}\sqrt{c*x^2 + a}*(c*f*x/h^4 - 2*(4*c*f*g*h^{10} - c*h^{11}*e)/h^{15}) - (20*c^3*f*g^5 + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 + 3*a*c^2*d*g*h^4 + 12*a^2*c*f*g*h^4 - 8*c^3*g^4*h*e - 12*a*c^2*g^2*h^3*e - 3*a^2*c*h^5*e)*\arctan(-(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*h + \text{sqrt}(c)*g)/\text{sqrt}(-c*g^2 - a*h^2))/((c*g^2*h^6 + a*h^8)*\text{sqrt}(-c*g^2 - a*h^2)) - 1/3*(60*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*c^3*f*g^5*h^2 + 18*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^2*f*g^3*h^4 + 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c*f*g*h^6 - 36*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*c^3*g^4*h^3*e - 36*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^2*g^2*h^5*e - 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c*h^7*e + 210*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*c^{(7/2)}*f*g^6*h + 54*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*c^{(7/2)}*d*g^4*h^3 + 183*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a*c^{(5/2)}*f*g^4*h^3 + 27*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a*c^{(5/2)}*d*g^2*h^5 - 18*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^2*c^{(3/2)}*f*g^2*h^5 - 12*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^2*c^{(3/2)}*d*h^7 - 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*\text{sqrt}(c)*f*h^7 - 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*c^{(7/2)}*g^5*h^2*e - 84*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a*c^{(5/2)}*g^3*h^4*e + 21*(\text{sqrt}(c)*x - \text{sqrt}(c$



$$\begin{aligned} & x^2 + a)^4 a^2 c^{(3/2)} g h^6 e + 188 (\sqrt{c} x - \sqrt{c x^2 + a})^3 c^4 d g^5 h^2 - 82 (\sqrt{c} x - \\ & \sqrt{c x^2 + a})^3 a c^3 f g^5 h^2 - 34 (\sqrt{c} x - \sqrt{c x^2 + a})^3 a c^3 d g^3 h^4 - 276 (\sqrt{c} x - \sqrt{c x^2 + a})^3 a^2 c^2 f g^3 h^4 - 48 ( \\ & \sqrt{c} x - \sqrt{c x^2 + a})^3 a^2 c^2 d g h^6 - 36 (\sqrt{c} x - \sqrt{c x^2 + a})^3 a^3 c f g h^6 - 104 (\sqrt{c} x - \sqrt{c x^2 + a})^3 c^4 g^6 h e + \\ & 64 (\sqrt{c} x - \sqrt{c x^2 + a})^3 a c^3 g^4 h^3 e + 138 (\sqrt{c} x - \sqrt{c x^2 + a})^3 a^2 c^2 g^2 h^5 e - 354 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a c^{(7/2)} f g^6 h - \\ & 78 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a c^{(7/2)} d g^4 h^3 - 276 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^2 c^{(5/2)} f g^4 h^3 - 36 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^2 c^{(5/2)} d g^2 h^5 + \\ & 60 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^3 c^{(3/2)} f g^2 h^5 + 12 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^3 c^{(3/2)} d h^7 + 12 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^4 \sqrt{c} f h^7 + \\ & 192 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a c^{(7/2)} g^5 h^2 e + 114 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^2 c^{(5/2)} g^3 h^4 e - 48 (\sqrt{c} x - \sqrt{c x^2 + a})^2 a^3 c^{(3/2)} g h^6 e + \\ & 222 (\sqrt{c} x - \sqrt{c x^2 + a}) a^2 c^3 f g^5 h^2 + 48 (\sqrt{c} x - \sqrt{c x^2 + a}) a^2 c^3 d g^3 h^4 + 231 (\sqrt{c} x - \sqrt{c x^2 + a}) a^3 c^2 f g^3 h^4 + \\ & 33 (\sqrt{c} x - \sqrt{c x^2 + a}) a^3 c^2 d g h^6 + 24 (\sqrt{c} x - \sqrt{c x^2 + a}) a^4 c f g h^6 - 120 (\sqrt{c} x - \sqrt{c x^2 + a}) a^2 c^3 g^4 h^3 e - \\ & 102 (\sqrt{c} x - \sqrt{c x^2 + a}) a^3 c^2 g^2 h^5 e + 3 (\sqrt{c} x - \sqrt{c x^2 + a}) a^4 c h^7 e - 47 a^3 c^{(5/2)} f g^4 h^3 - 11 a^3 c^{(5/2)} d g^2 h^5 - \\ & 50 a^4 c^{(3/2)} f g^2 h^5 - 8 a^4 c^{(3/2)} d h^7 - 6 a^5 \sqrt{c} f h^7 + 26 a^3 c^{(5/2)} g^3 h^4 e + 23 a^4 c^{(3/2)} g h^6 e \\ & / ((c g^2 h^6 + a h^8) (\sqrt{c} x - \sqrt{c x^2 + a})^2 h + 2 (\sqrt{c} x - \sqrt{c x^2 + a}) \sqrt{c} g - a h)^3 - 1/2 (20 c^{(3/2)} f g^2 + 2 c^{(3/2)} d h^2 + \\ & 3 a \sqrt{c} f h^2 - 8 c^{(3/2)} g h e) \log(\text{abs}(-\sqrt{c} x + \sqrt{c x^2 + a})) / h^6 \end{aligned}$$

**maple [B]** time = 0.02, size = 9835, normalized size = 20.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c x^2 + a)^{(3/2)} (f x^2 + e x + d) / (h x + g)^4, x)$

[Out] result too large to display

**maxima [B]** time = 1.10, size = 2415, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c x^2 + a)^{(3/2)} (f x^2 + e x + d) / (h x + g)^4, x, \text{algorithm} = \text{"maxima"})$

[Out]  $1/2 \sqrt{c x^2 + a} c^3 f g^5 / (c^2 g^4 h^5 + 2 a c g^2 h^7 + a^2 h^9) - 1/2 \sqrt{c x^2 + a} c^3 f g^4 x / (c^2 g^4 h^4 + 2 a c g^2 h^6 + a^2 h^8) - 1/2$

$$\begin{aligned}
& \sqrt{c^2x^2 + a}c^3e^*g^4/(c^2g^4h^4 + 2a^*c^*g^2h^6 + a^2h^8) + 1/6*(c^2x^2 + a)^{(3/2)}c^2f^*g^4/(c^2g^4h^4x + 2a^*c^*g^2h^6x + a^2h^8x + c^2g^5h^3 + 2a^*c^*g^3h^5 + a^2g^*h^7) + 1/2*\sqrt{c^2x^2 + a}c^3e^*g^3x/(c^2g^4h^3 + 2a^*c^*g^2h^5 + a^2h^7) + 1/2*\sqrt{c^2x^2 + a}c^3d^*g^3/(c^2g^4h^3x + 2a^*c^*g^2h^5x + a^2h^7x + c^2g^5h^2 + 2a^*c^*g^3h^4 + a^2g^*h^6) - 1/6*(c^2x^2 + a)^{(5/2)}c^2f^*g^3/(c^2g^4h^3x^2 + 2a^*c^*g^2h^5x^2 + a^2h^7x^2 + 2c^2g^5h^2x + 4a^*c^*g^3h^4x + 2a^2g^*h^6x + c^2g^6h + 2a^*c^*g^4h^3 + a^2g^2h^5) + 1/6*(c^2x^2 + a)^{(3/2)}c^2f^*g^3/(c^2g^4h^3 + 2a^*c^*g^2h^5 + a^2h^7) - 1/2*\sqrt{c^2x^2 + a}c^3d^*g^2x/(c^2g^4h^2 + 2a^*c^*g^2h^4 + a^2h^6) + 1/6*(c^2x^2 + a)^{(3/2)}c^2d^*g^2/(c^2g^4h^2x + 2a^*c^*g^2h^4x + a^2h^6x + c^2g^5h + 2a^*c^*g^3h^3 + a^2g^*h^5) + 1/6*(c^2x^2 + a)^{(5/2)}c^2e^*g^2/(c^2g^4h^2x^2 + 2a^*c^*g^2h^4x^2 + a^2h^6x^2 + 2c^2g^5h^2x + 4a^*c^*g^3h^3x + 2a^2g^*h^5x + c^2g^6 + 2a^*c^*g^4h^2 + a^2g^2h^4) - 1/6*(c^2x^2 + a)^{(3/2)}c^2e^*g^2/(c^2g^4h^2 + 2a^*c^*g^2h^4 + a^2h^6) - 9/2*\sqrt{c^2x^2 + a}c^2f^*g^3/(c^2g^2h^5 + ah^7) + 4*\sqrt{c^2x^2 + a}c^2f^*g^2x/(c^2g^2h^4 + ah^6) - 1/6*(c^2x^2 + a)^{(5/2)}c^2d^*g/(c^2g^4h^2x^2 + 2a^*c^*g^2h^3x^2 + a^2h^5x^2 + 2c^2g^5x + 4a^*c^*g^3h^2x + 2a^2g^*h^4x + c^2g^6/h + 2a^*c^*g^4h + a^2g^2h^3) + 1/6*(c^2x^2 + a)^{(3/2)}c^2d^*g/(c^2g^4h + 2a^*c^*g^2h^3 + a^2h^5) + 3*\sqrt{c^2x^2 + a}c^2e^*g^2/(c^2g^2h^4 + ah^6) - 1/3*(c^2x^2 + a)^{(5/2)}f^*g^2/(c^2g^2h^4x^3 + ah^6x^3 + 3c^2g^3h^3x^2 + 3a^2g^*h^5x^2 + 3c^2g^4h^2x + 3a^2g^2h^4x + c^2g^5h + a^2g^3h^3) - 5/3*(c^2x^2 + a)^{(3/2)}c^2f^*g^2/(c^2g^2h^4x + ah^6x + c^2g^3h^3 + a^2g^*h^5) - 5/2*\sqrt{c^2x^2 + a}c^2e^*g^*x/(c^2g^2h^3 + ah^5) - 3/2*\sqrt{c^2x^2 + a}c^2d^*g/(c^2g^2h^3 + ah^5) + 1/3*(c^2x^2 + a)^{(5/2)}e^*g/(c^2g^2h^3x^3 + ah^5x^3 + 3c^2g^3h^2x^2 + 3a^2g^*h^4x^2 + 3c^2g^4h^2x + 3a^2g^2h^3x + c^2g^5 + a^2g^3h^2) + 7/6*(c^2x^2 + a)^{(3/2)}c^2e^*g/(c^2g^2h^3x + ah^5x + c^2g^3h^2 + a^2g^*h^4) + (c^2x^2 + a)^{(5/2)}f^*g/(c^2g^2h^3x^2 + ah^5x^2 + 2c^2g^3h^2x + 2a^2g^*h^4x + c^2g^4h + a^2g^2h^3) - (c^2x^2 + a)^{(3/2)}c^2f^*g/(c^2g^2h^3 + ah^5) + \sqrt{c^2x^2 + a}c^2d^*x/(c^2g^2h^2 + ah^4) - 1/3*(c^2x^2 + a)^{(5/2)}d/(c^2g^2h^2x^3 + ah^4x^3 + 3c^2g^3h^2x^2 + 3a^2g^*h^3x^2 + 3c^2g^4x + 3a^2g^2h^2x + c^2g^5/h + a^2g^3h) - 2/3*(c^2x^2 + a)^{(3/2)}c^2d/(c^2g^2h^2x + ah^4x + c^2g^3h + a^2g^*h^3) - 1/2*(c^2x^2 + a)^{(5/2)}e/(c^2g^2h^2x^2 + ah^4x^2 + 2c^2g^3h^2x + 2a^2g^*h^3x + c^2g^4 + a^2g^2h^2) + 1/2*(c^2x^2 + a)^{(3/2)}c^2e/(c^2g^2h^2 + ah^4) - (c^2x^2 + a)^{(3/2)}f/(h^4x + g^*h^3) + 3/2*\sqrt{c^2x^2 + a}c^2f^*x/h^4 + 10c^{(3/2)}f^*g^2*arcsinh(c^2x/\sqrt{a^*c})/h^6 - 4c^{(3/2)}e^*g^*arcsinh(c^2x/\sqrt{a^*c})/h^5 + c^{(3/2)}d^*arcsinh(c^2x/\sqrt{a^*c})/h^4 + 3/2a^*sqrt(c^2)*f^*arcsinh(c^2x/\sqrt{a^*c})/h^4 + 1/2c^3f^*g^5*arcsinh(c^2g^*x/(\sqrt{a^*c})*abs(h^*x + g)) - a^*h/(\sqrt{a^*c})*abs(h^*x + g))/((a + c^2g^2/h^2)^{(3/2)}*h^9) - 1/2c^3e^*g^4*arcsinh(c^2g^*x/(\sqrt{a^*c})*abs(h^*x + g)) - a^*h/(\sqrt{a^*c})*abs(h^*x + g))/((a + c^2g^2/h^2)^{(3/2)}*h^8) + 1/2c^3d^*g^3*arcsinh(c^2g^*x/(\sqrt{a^*c})*abs(h^*x + g)) - a^*h/(\sqrt{a^*c})*abs(h^*x + g))/((a + c^2g^2/h^2)^{(3/2)}*h^7) - 9/2c^2f^*g^3*arcsinh(c^2g^*x/(\sqrt{a^*c})*abs(h^*x + g)) - a^*h/(\sqrt{a^*c})*abs(h^*x + g))/(\sqrt{a + c^2g^2/h^2}*h^7) + 3c^2e^*g^2*arcsinh(c^2g^*x/(\sqrt{a^*c})
\end{aligned}$$

\*abs(h\*x + g)) - a\*h/(sqrt(a\*c)\*abs(h\*x + g)))/(sqrt(a + c\*g^2/h^2)\*h^6) - 3/2\*c^2\*d\*g\*arcsinh(c\*g\*x/(sqrt(a\*c)\*abs(h\*x + g)) - a\*h/(sqrt(a\*c)\*abs(h\*x + g)))/(sqrt(a + c\*g^2/h^2)\*h^5) - 6\*sqrt(a + c\*g^2/h^2)\*c\*f\*g\*arcsinh(c\*g\*x/(sqrt(a\*c)\*abs(h\*x + g)) - a\*h/(sqrt(a\*c)\*abs(h\*x + g)))/h^5 + 3/2\*sqrt(a + c\*g^2/h^2)\*c\*e\*arcsinh(c\*g\*x/(sqrt(a\*c)\*abs(h\*x + g)) - a\*h/(sqrt(a\*c)\*abs(h\*x + g)))/h^4 - 6\*sqrt(c\*x^2 + a)\*c\*f\*g/h^5 + 3/2\*sqrt(c\*x^2 + a)\*c\*e/h^4

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x)

[Out] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*4, x)

[Out] Integral((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*4, x)

$$3.96 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5} dx$$

**Optimal.** Leaf size=511

$$\frac{(a+cx^2)^{3/2}(-3hx(4a^2fh^4+ach^2(17fg^2-h(5eg-dh))+2c^2g^2(5fg^2-h(dh+eg)))+4a^2h^4(fg-2eh)-acgh)}{24h^3(g+hx)^3(ah^2+cg^2)^2}$$

**Rubi [A]** time = 1.09, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {1651, 811, 813, 844, 217, 206, 725}

$$\frac{(c+cx^2)^{3/2}(-3h(4a^2fh^4+ach^2(17fg^2-h(5eg-dh))+2c^2g^2(5fg^2-h(dh+eg)))+4a^2h^4(fg-2eh)-acgh)}{24h^3(g+hx)^3(ah^2+cg^2)^2} + \frac{c\sqrt{a+cx^2}(h(12af^3h^4+ah^2(25fg^2-3dh))+4c^2g^2(5fg-ah))+8(a^2+cg^2)^2(5fg-ah)}{8h^2g+4h^2(ah^2+cg^2)} + \frac{c\sqrt{a+cx^2}\left(\frac{ah^2+cg^2}{2ah^2g+4h^2(ah^2+cg^2)}\right)(h^2(25fg^2-3dh)+12af^3h^4+20a^2c^2g^2(5fg-ah)+8c^2g^2(5fg-ah))}{ah^2g+4h^2(ah^2+cg^2)} + \frac{c^2\sqrt{a+cx^2}\left(\frac{ah^2+cg^2}{g^2}\right)(5fg-ah)}{4h^2g+4h^2(ah^2+cg^2)} + \frac{(c+cx^2)^{3/2}(ah^2+cg^2)}{4h^2g+4h^2(ah^2+cg^2)}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] (c\*(8\*(5\*f\*g - e\*h)\*(c\*g^2 + a\*h^2)^2 + h\*(12\*a^2\*f\*h^4 + 4\*c^2\*g^3\*(5\*f\*g - e\*h) + a\*c\*h^2\*(35\*f\*g^2 - h\*(7\*e\*g - 3\*d\*h)))\*x)\*Sqrt[a + c\*x^2])/(8\*h^5\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)) + ((4\*a^2\*h^3\*(f\*g - 2\*e\*h) - (4\*c^2\*g^4\*(5\*f\*g - e\*h))/h - a\*c\*g\*h\*(25\*f\*g^2 - h\*(5\*e\*g - 9\*d\*h)) - 3\*(4\*a^2\*f\*h^4 + a\*c\*h^2\*(17\*f\*g^2 - h\*(5\*e\*g - d\*h)) + 2\*c^2\*(5\*f\*g^4 - g^2\*h\*(e\*g + d\*h)))\*x)\*(a + c\*x^2)^(3/2))/(24\*h^2\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)^3) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(5/2))/(4\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^4) - (c^(3/2)\*(5\*f\*g - e\*h)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/h^6 - (c\*(12\*a^3\*f\*h^6 + 8\*c^3\*g^5\*(5\*f\*g - e\*h) + 20\*a\*c^2\*g^3\*h^2\*(5\*f\*g - e\*h) + 3\*a^2\*c\*h^4\*(25\*f\*g^2 - h\*(5\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/(8\*h^6\*(c\*g^2 + a\*h^2)^(5/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 217**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

**Rule 725**

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 811

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
2*c*d*p*(e*f - d*g)*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
+ 2*p + 2)) - 2*a*e^2*g*(m + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
&& !ILtQ[m + 2*p + 3, 0]
```

### Rule 813

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

### Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^5} dx &= -\frac{(fg^2 - egh + dh^2)(a + cx^2)^{5/2}}{4h(cg^2 + ah^2)(g + hx)^4} - \int \frac{\left(-4(cdg - afg + aeh) - \left(4afh - c\left(eg - \frac{5fg^2}{h} - dh\right)\right)x\right)(a + cx^2)}{(g + hx)^4}{4(cg^2 + ah^2)} \\
&= \frac{\left(4a^2h^3(fg - 2eh) - \frac{4c^2g^4(5fg - eh)}{h} - acgh(25fg^2 - h(5eg - 9dh)) - 3(4a^2fh^4 + \dots)\right)}{24h^2(cg^2 + ah^2)} \\
&= \frac{c(8(5fg - eh)(cg^2 + ah^2)^2 + h(12a^2fh^4 + 4c^2g^3(5fg - eh) + ach^2(35fg^2 - \dots))}{8h^5(cg^2 + ah^2)^2(g + hx)} \\
&= \frac{c(8(5fg - eh)(cg^2 + ah^2)^2 + h(12a^2fh^4 + 4c^2g^3(5fg - eh) + ach^2(35fg^2 - \dots))}{8h^5(cg^2 + ah^2)^2(g + hx)} \\
&= \frac{c(8(5fg - eh)(cg^2 + ah^2)^2 + h(12a^2fh^4 + 4c^2g^3(5fg - eh) + ach^2(35fg^2 - \dots))}{8h^5(cg^2 + ah^2)^2(g + hx)} \\
&= \frac{c(8(5fg - eh)(cg^2 + ah^2)^2 + h(12a^2fh^4 + 4c^2g^3(5fg - eh) + ach^2(35fg^2 - \dots))}{8h^5(cg^2 + ah^2)^2(g + hx)}
\end{aligned}$$

**Mathematica [A]** time = 2.15, size = 575, normalized size = 1.13

Integrate[(a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2)/(g + h\*x)^5, x]

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x]

[Out] 
$$\begin{aligned}
& -1/24*((h*\text{Sqrt}[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - \\
& 2*(c*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*f* \\
& g + e*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 + g^2 \\
& *h*(-23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g + h*x \\
& )^2 - c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g + 3* \\
& d*h)) + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 24*c*f* \\
& (c*g^2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3*c*(12*
\end{aligned}$$

$$a^3 f h^6 + 8 c^3 g^5 (5 f g - e h) + 20 a c^2 g^3 h^2 (5 f g - e h) + 3 a^2 c h^4 (25 f g^2 + h(-5 e g + d h)) \operatorname{Log}[g + h x] / (c g^2 + a h^2)^{5/2} + 24 c^{3/2} (5 f g - e h) \operatorname{Log}[c x + \operatorname{Sqrt}[c] \operatorname{Sqrt}[a + c x^2]] + (3 c (12 a^3 f h^6 + 8 c^3 g^5 (5 f g - e h) + 20 a c^2 g^3 h^2 (5 f g - e h) + 3 a^2 c h^4 (25 f g^2 + h(-5 e g + d h))) \operatorname{Log}[a h - c g x + \operatorname{Sqrt}[c g^2 + a h^2] \operatorname{Sqrt}[a + c x^2]]) / (c g^2 + a h^2)^{5/2} / h^6$$

**IntegrateAlgebraic** [F] time = 180.03, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.02, size = 12481, normalized size = 24.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x)

[Out] result too large to display

**maxima** [B] time = 1.50, size = 4326, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x, algorithm="maxima")

[Out]  $\frac{3}{8}\sqrt{c x^2 + a} c^4 f g^6 / (c^3 g^6 h^5 + 3 a c^2 g^4 h^7 + 3 a^2 c g^2 h^9 + a^3 h^{11}) - \frac{3}{8}\sqrt{c x^2 + a} c^4 f g^5 x / (c^3 g^6 h^4 + 3 a c^2 g^4 h^6 + 3 a^2 c g^2 h^8 + a^3 h^{10}) - \frac{3}{8}\sqrt{c x^2 + a} c^4 e g^5 / (c^3 g^6 h^4 + 3 a c^2 g^4 h^6 + 3 a^2 c g^2 h^8 + a^3 h^{10}) + \frac{1}{8}(c x^2 + a)^{3/2} c^3 f g^5 / (c^3 g^6 h^4 x + 3 a c^2 g^4 h^6 x + 3 a^2 c g^2 h^8 x + a^3 h^{10} x + c^3 g^7 h^3 + 3 a c^2 g^5 h^5 + 3 a^2 c g^3 h^7 + a^3 g h^9) + \frac{3}{8}\sqrt{c x^2 + a} c^4 e g^4 x / (c^3 g^6 h^3 + 3 a c^2 g^4 h^5 + 3 a^2 c g^2 h^7 + a^3 h^9) + \frac{3}{8}\sqrt{c x^2 + a} c^4 d g^4 / (c^3 g^6 h^3 + 3 a c^2 g^4 h^5 + 3 a^2 c g^2 h^7 + a^3 h^9) - \frac{1}{8}(c x^2 + a)^{3/2} c^3 e g^4 / (c^3 g^6 h^3 x + 3 a c^2 g^4 h^5 x + 3 a^2 c g^2 h^7 x + a^3 h^9 x + c^3 g^7 h^2 + 3 a c^2 g^5 h^4 + 3 a^2 c g^3 h^6 + a^3 g h^8) - \frac{1}{8}(c x^2 + a)^{5/2} c^2 f g^4 / (c^3 g^6 h^3 x^2 + 3 a c^2 g^4 h^5 x^2 + 3 a^2 c g^2 h^7 x^2 + a^3 h^9 x^2 + 2 c^3 g^7 h^2 x + 6 a c^2 g^5 h^4 x + 6 a^2 c g^3 h^6 x + 2 a^3 g h^8 x + c^3 g^8 h + 3 a c^2 g^6 h^3 + 3 a^2 c g^4 h^5 + a^3 g^2 h^7) + \frac{1}{8}(c x^2 + a)^{3/2} c^3 f g^4 / (c^3 g^6 h^3 + 3 a c^2 g^4 h^5 + 3 a^2 c g^2 h^7 + a^3 h^9) - \frac{3}{8}\sqrt{c x^2 + a} c^4 d g^3 x / (c^3 g^6 h^2 + 3 a c^2 g^4 h^4 + 3 a^2 c g^2 h^6 + a^3 h^8) + \frac{1}{8}(c x^2 + a)^{3/2} c^3 d g^3 / (c^3 g^6 h^2 x + 3 a c^2 g^4 h^4 x + 3 a^2 c g^2 h^6 x + a^3 h^8 x + c^3 g^7 h + 3 a c^2 g^5 h^3 + 3 a^2 c g^3 h^5 + a^3 g h^7) + \frac{1}{8}(c x^2 + a)^{5/2} c^2 e g^3 / (c^3 g^6 h^2 x^2 + 3 a c^2 g^4 h^4 x^2 + 3 a^2 c g^2 h^6 x^2 + a^3 h^8 x^2 + 2 c^3 g^7 h x + 6 a c^2 g^5 h^3 x + 6 a^2 c g^3 h^5 x + 2 a^3 g h^7 x + c^3 g^8 + 3 a c^2 g^6 h^2 + 3 a^2 c g^4 h^4 + a^3 g^2 h^6) - \frac{1}{8}(c x^2 + a)^{3/2} c^3 e g^3 / (c^3 g^6 h^2 + 3 a c^2 g^4 h^4 + 3 a^2 c g^2 h^6 + a^3 h^8) - \frac{7}{4}\sqrt{c x^2 + a} c^3 f g^4 / (c^2 g^4 h^5 + 2 a c g^2 h^7 + a^2 h^9) + \frac{11}{8}\sqrt{c x^2 + a} c^3 f g^3 x / (c^2 g^4 h^4 + 2 a c g^2 h^6 + a^2 h^8) - \frac{1}{8}(c x^2 + a)^{5/2} c^2 d g^2 / (c^3 g^6 h x^2 + 3 a c^2 g^4 h^3 x^2 + 3 a^2 c g^2 h^5 x^2 + a^3 h^7 x^2 + 2 c^3 g^7 x + 6 a c^2 g^5 h^2 x + 6 a^2 c g^3 h^4 x + 2 a^3 g h^6 x + c^3 g^8 / h + 3 a c^2 g^6 h + 3 a^2 c g^4 h^3 + a^3 g^2 h^5) + \frac{1}{8}(c x^2 + a)^{3/2} c^3 d g^2 / (c^3 g^6 h + 3 a c^2 g^4 h^3 + 3 a^2 c g^2 h^5 + a^3 h^7) + \frac{5}{4}\sqrt{c x^2 + a} c^3 e g^3 / (c^2 g^4 h^4 + 2 a c g^2 h^6 + a^2 h^8) - \frac{1}{4}(c x^2 + a)^{5/2} c^2 f g^3 / (c^2 g^4 h^4 x^3 + 2 a c g^2 h^6 x^3 + a^2 h^8 x^3 + 3 c^2 g^5 h^3 x^2 + 6 a c g^3 h^5 x^2 + 3 a^2 g h^7 x^2 + 3 c^2 g^6 h^2 x + 6 a c g^4 h^4 x + 3 a^2 g^2 h^6 x + c^2 g^7 h + 2 a c g^5 h^3 + a^2 g^3 h^5) - \frac{17}{24}(c x^2 + a)^{3/2} c^2 f g^3 / (c^2 g^4 h^4 x + 2 a c g^2 h^6 x + a^2 h^8 x + c^2 g^5 h^3 + 2 a c g^3 h^5 + a^2 g h^7) - \frac{7}{8}\sqrt{c x^2 + a} c^3 e g^2 x / (c^2 g^4 h^3 + 2 a c g^2 h^5 + a^2 h^7) - \frac{3}{4}\sqrt{c x^2 + a} c^3 d g^2 / (c^2 g^4 h^3 + 2 a c g^2 h^5 + a^2 h^7) + \frac{1}{4}(c x^2 + a)^{5/2} c^2 e g^2 / (c^2 g^4 h^3 x^3 + 2 a c g^2 h^5 x^3 + a^2 h^7 x^3 + 3 c^2 g^5 h^2 x^2 + 6 a c g^3 h^4 x^2 + 3 a^2 g h^6 x^2 + 3 c^2 g^6 h x + 6 a c g^4 h^3 x + 3 a^2 g^2 h^5 x + c^2 g^7 + 2 a c g^5 h^3$



$$\begin{aligned}
& 2 + a^2 g^3 h^4) + 13/24 * (c x^2 + a)^{(3/2)} * c^2 * e * g^2 / (c^2 * g^4 * h^3 * x + 2 * a * c * \\
& * g^2 * h^5 * x + a^2 * h^7 * x + c^2 * g^5 * h^2 + 2 * a * c * g^3 * h^4 + a^2 * g * h^6) + 5/24 * (c \\
& * x^2 + a)^{(5/2)} * c * f * g^2 / (c^2 * g^4 * h^3 * x^2 + 2 * a * c * g^2 * h^5 * x^2 + a^2 * h^7 * x^2 \\
& + 2 * c^2 * g^5 * h^2 * x + 4 * a * c * g^3 * h^4 * x + 2 * a^2 * g * h^6 * x + c^2 * g^6 * h + 2 * a * c * g^4 \\
& * h^3 + a^2 * g^2 * h^5) - 5/24 * (c x^2 + a)^{(3/2)} * c^2 * f * g^2 / (c^2 * g^4 * h^3 + 2 * a * c * \\
& * g^2 * h^5 + a^2 * h^7) + 3/8 * \text{sqrt}(c x^2 + a) * c^3 * d * g * x / (c^2 * g^4 * h^2 + 2 * a * c * g^2 \\
& * h^4 + a^2 * h^6) - 1/4 * (c x^2 + a)^{(5/2)} * c * d * g / (c^2 * g^4 * h^2 * x^3 + 2 * a * c * g^2 \\
& * h^4 * x^3 + a^2 * h^6 * x^3 + 3 * c^2 * g^5 * h * x^2 + 6 * a * c * g^3 * h^3 * x^2 + 3 * a^2 * g * h^5 * \\
& x^2 + 3 * c^2 * g^6 * x + 6 * a * c * g^4 * h^2 * x + 3 * a^2 * g^2 * h^4 * x + c^2 * g^7 / h + 2 * a * c * g \\
& ^5 * h + a^2 * g^3 * h^3) - 3/8 * (c x^2 + a)^{(3/2)} * c^2 * d * g / (c^2 * g^4 * h^2 * x + 2 * a * c * \\
& g^2 * h^4 * x + a^2 * h^6 * x + c^2 * g^5 * h + 2 * a * c * g^3 * h^3 + a^2 * g * h^5) - 1/24 * (c x^2 \\
& + a)^{(5/2)} * c * e * g / (c^2 * g^4 * h^2 * x^2 + 2 * a * c * g^2 * h^4 * x^2 + a^2 * h^6 * x^2 + 2 * c \\
& ^2 * g^5 * h * x + 4 * a * c * g^3 * h^3 * x + 2 * a^2 * g * h^5 * x + c^2 * g^6 + 2 * a * c * g^4 * h^2 + a^2 \\
& * g^2 * h^4) + 1/24 * (c x^2 + a)^{(3/2)} * c^2 * e * g / (c^2 * g^4 * h^2 + 2 * a * c * g^2 * h^4 + \\
& a^2 * h^6) - 1/4 * (c x^2 + a)^{(5/2)} * f * g^2 / (c * g^2 * h^5 * x^4 + a * h^7 * x^4 + 4 * c * g^3 * \\
& * h^4 * x^3 + 4 * a * g * h^6 * x^3 + 6 * c * g^4 * h^3 * x^2 + 6 * a * g^2 * h^5 * x^2 + 4 * c * g^5 * h^2 * \\
& x + 4 * a * g^3 * h^4 * x + c * g^6 * h + a * g^4 * h^3) + 39/8 * \text{sqrt}(c x^2 + a) * c^2 * f * g^2 / ( \\
& c * g^2 * h^5 + a * h^7) - 7/2 * \text{sqrt}(c x^2 + a) * c^2 * f * g * x / (c * g^2 * h^4 + a * h^6) - 1/ \\
& 8 * (c x^2 + a)^{(5/2)} * c * d / (c^2 * g^4 * h * x^2 + 2 * a * c * g^2 * h^3 * x^2 + a^2 * h^5 * x^2 + \\
& 2 * c^2 * g^5 * x + 4 * a * c * g^3 * h^2 * x + 2 * a^2 * g * h^4 * x + c^2 * g^6 / h + 2 * a * c * g^4 * h + a \\
& ^2 * g^2 * h^3) + 1/8 * (c x^2 + a)^{(3/2)} * c^2 * d / (c^2 * g^4 * h + 2 * a * c * g^2 * h^3 + a^2 * \\
& h^5) + 1/4 * (c x^2 + a)^{(5/2)} * e * g / (c * g^2 * h^4 * x^4 + a * h^6 * x^4 + 4 * c * g^3 * h^3 * x \\
& ^3 + 4 * a * g * h^5 * x^3 + 6 * c * g^4 * h^2 * x^2 + 6 * a * g^2 * h^4 * x^2 + 4 * c * g^5 * h * x + 4 * a * \\
& g^3 * h^3 * x + c * g^6 + a * g^4 * h^2) - 15/8 * \text{sqrt}(c x^2 + a) * c^2 * e * g / (c * g^2 * h^4 + \\
& a * h^6) + 2/3 * (c x^2 + a)^{(5/2)} * f * g / (c * g^2 * h^4 * x^3 + a * h^6 * x^3 + 3 * c * g^3 * h^3 * \\
& x^2 + 3 * a * g * h^5 * x^2 + 3 * c * g^4 * h^2 * x + 3 * a * g^2 * h^4 * x + c * g^5 * h + a * g^3 * h^3) \\
& + 11/6 * (c x^2 + a)^{(3/2)} * c * f * g / (c * g^2 * h^4 * x + a * h^6 * x + c * g^3 * h^3 + a * g * h^5) \\
& + \text{sqrt}(c x^2 + a) * c^2 * e * x / (c * g^2 * h^3 + a * h^5) - 1/4 * (c x^2 + a)^{(5/2)} * d / \\
& (c * g^2 * h^3 * x^4 + a * h^5 * x^4 + 4 * c * g^3 * h^2 * x^3 + 4 * a * g * h^4 * x^3 + 6 * c * g^4 * h * x^2 \\
& + 6 * a * g^2 * h^3 * x^2 + 4 * c * g^5 * x + 4 * a * g^3 * h^2 * x + c * g^6 / h + a * g^4 * h) + 3/8 * \\
& \text{sqrt}(c x^2 + a) * c^2 * d / (c * g^2 * h^3 + a * h^5) - 1/3 * (c x^2 + a)^{(5/2)} * e / (c * g^2 * \\
& h^3 * x^3 + a * h^5 * x^3 + 3 * c * g^3 * h^2 * x^2 + 3 * a * g * h^4 * x^2 + 3 * c * g^4 * h * x + 3 * a * g \\
& ^2 * h^3 * x + c * g^5 + a * g^3 * h^2) - 2/3 * (c x^2 + a)^{(3/2)} * c * e / (c * g^2 * h^3 * x + a * \\
& h^5 * x + c * g^3 * h^2 + a * g * h^4) - 1/2 * (c x^2 + a)^{(5/2)} * f / (c * g^2 * h^3 * x^2 + a * h \\
& ^5 * x^2 + 2 * c * g^3 * h^2 * x + 2 * a * g * h^4 * x + c * g^4 * h + a * g^2 * h^3) + 1/2 * (c x^2 + \\
& a)^{(3/2)} * c * f / (c * g^2 * h^3 + a * h^5) - 5 * c^{(3/2)} * f * g * \text{arcsinh}(c x / \text{sqrt}(a * c)) / h^6 \\
& + c^{(3/2)} * e * \text{arcsinh}(c x / \text{sqrt}(a * c)) / h^5 + 3/8 * c^4 * f * g^6 * \text{arcsinh}(c * g * x / (\text{sqrt} \\
& (a * c) * \text{abs}(h * x + g)) - a * h / (\text{sqrt}(a * c) * \text{abs}(h * x + g))) / ((a + c * g^2 / h^2)^{(5/2)} * \\
& h^{11}) - 3/8 * c^4 * e * g^5 * \text{arcsinh}(c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) - a * h / (\text{sqrt}(a * \\
& c) * \text{abs}(h * x + g))) / ((a + c * g^2 / h^2)^{(5/2)} * h^{10}) + 3/8 * c^4 * d * g^4 * \text{arcsinh}(c * g * \\
& x / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) - a * h / (\text{sqrt}(a * c) * \text{abs}(h * x + g))) / ((a + c * g^2 / h^2) \\
& ^{(5/2)} * h^9) - 7/4 * c^3 * f * g^4 * \text{arcsinh}(c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) - a * h / (\text{s} \\
& \text{qrt}(a * c) * \text{abs}(h * x + g))) / ((a + c * g^2 / h^2)^{(3/2)} * h^9) + 5/4 * c^3 * e * g^3 * \text{arcsinh} \\
& (c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) - a * h / (\text{sqrt}(a * c) * \text{abs}(h * x + g))) / ((a + c * g^2 \\
& / h^2)^{(3/2)} * h^8) - 3/4 * c^3 * d * g^2 * \text{arcsinh}(c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) - a
\end{aligned}$$

$$\frac{h}{\sqrt{ac} \operatorname{abs}(hx + g)} \left( \frac{1}{(a + c \frac{g^2}{h^2})^{3/2} h^7} + \frac{39}{8} c^2 f g^2 \operatorname{arcsinh}\left(\frac{c g x}{\sqrt{ac} \operatorname{abs}(hx + g)}\right) - \frac{a h}{\sqrt{ac} \operatorname{abs}(hx + g)} \right) / \sqrt{a + c \frac{g^2}{h^2} h^7} - \frac{15}{8} c^2 e g \operatorname{arcsinh}\left(\frac{c g x}{\sqrt{ac} \operatorname{abs}(hx + g)}\right) - \frac{a h}{\sqrt{ac} \operatorname{abs}(hx + g)} \left( \frac{1}{\sqrt{a + c \frac{g^2}{h^2} h^6}} + \frac{3}{8} c^2 d \operatorname{arcsinh}\left(\frac{c g x}{\sqrt{ac} \operatorname{abs}(hx + g)}\right) - \frac{a h}{\sqrt{ac} \operatorname{abs}(hx + g)} \right) / \sqrt{a + c \frac{g^2}{h^2} h^5} + \frac{3}{2} \sqrt{a + c \frac{g^2}{h^2}} c f \operatorname{arcsinh}\left(\frac{c g x}{\sqrt{ac} \operatorname{abs}(hx + g)}\right) - \frac{a h}{\sqrt{ac} \operatorname{abs}(hx + g)} / h^5 + \frac{3}{2} \sqrt{c x^2 + a} c f / h^5$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

[Out] int(((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*5, x)

[Out] Integral((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*5, x)

$$3.97 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6} dx$$

Optimal. Leaf size=507

$$\frac{(a+cx^2)^{3/2} \left( hx(4a^2fh^4 + acgh^2(14fg - 3eh)) + c^2(7fg^4 - 3dg^2h^2) \right) - a^2h^4(2fg - 3eh) + acgh^2(3dh^2 + 5fg^2)}{12h^3(g+hx)^4(ah^2 + cg^2)^2}$$

**Rubi [A]** time = 0.86, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 811, 844, 217, 206, 725}

$$\frac{(a+cx^2)^{3/2} \left( hx(4a^2fh^4 + acgh^2(14fg - 3eh)) + c^2(7fg^4 - 3dg^2h^2) \right) - a^2h^4(2fg - 3eh) + acgh^2(3dh^2 + 5fg^2)}{12h^3(g+hx)^4(ah^2 + cg^2)^2}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6,x]

[Out]  $-(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h)) + a^2*c*g*h^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(34*f*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*\text{Sqrt}[a + c*x^2])/(8*h^5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h) + a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*e*h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*\text{ArcTanh}[(a*h - c*g*x)/(\text{Sqrt}[c*g^2 + a*h^2]*\text{Sqrt}[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))$

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 811

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
+ 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
&& !ILtQ[m + 2*p + 3, 0]
```

### Rule 844

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^6} dx &= -\frac{(fg^2 - egh + dh^2)(a + cx^2)^{5/2}}{5h(cg^2 + ah^2)(g + hx)^5} - \frac{\int \frac{\left(-5(cdg - afg + aeh) - 5f\left(\frac{cg^2}{h} + ah\right)x\right)(a + cx^2)^{3/2}}{(g + hx)^5} dx}{5(cg^2 + ah^2)} \\
&= -\frac{(4c^2fg^5 - a^2h^4(2fg - 3eh) + acgh^2(5fg^2 + 3dh^2) + h(4a^2fh^4 + acgh^2(13fg^2 + 3dh^2) + h^2(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h^5(cg^2 - ah^2)(2fg - 3eh))))}{12h^3(cg^2 + ah^2)^2(g + hx)} \\
&= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h^5(cg^2 - ah^2)(2fg - 3eh)))}{8h^5(cg^2 - ah^2)(g + hx)} \\
&= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h^5(cg^2 - ah^2)(2fg - 3eh)))}{8h^5(cg^2 - ah^2)(g + hx)} \\
&= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h^5(cg^2 - ah^2)(2fg - 3eh)))}{8h^5(cg^2 - ah^2)(g + hx)} \\
&= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg - 3eh) + a^2cgh^4(13fg^2 + 3dh^2) + h^5(cg^2 - ah^2)(2fg - 3eh)))}{8h^5(cg^2 - ah^2)(g + hx)}
\end{aligned}$$

**Mathematica [A]** time = 2.27, size = 639, normalized size = 1.26

---

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x]

[Out] 
$$\begin{aligned}
& -\left(\frac{h\sqrt{a + cx^2}(24(cg^2 + ah^2)^4(fg^2 + h(-eg) + dh)) - 6(cg^2 + ah^2)^3(21cf^3g^3 + cgh^2(-16eg + 11dh) - 5ah^2(-2fgh + eh))(g + hx) + 2(cg^2 + ah^2)^2(20a^2f^4h^4 + c^2(137f^4g^4 + 9g^2h(-8eg + 3dh)) + acgh^2(154f^2g^2 + 3h(-23eg + 8dh)))(g + hx)^2 - c(cg^2 + ah^2)(5a^2h^4(58fg - 15eh) + c^2(326f^5g^5 + 6g^3h(-16eg + dh)) + acgh^2(631f^2g^2 + 3h(-62eg + 7dh)))(g + hx)^3 + c(160a^3f^6h^6 + c^3(274f^6g^6 - 6g^4h(4eg + dh)) + 3a^2c^2h^4(238f^2g^2 + h(-33eg + 8dh)) + 3ac^2g^2h^2(261f^2g^2}
\end{aligned}$$

$$- h*(26*e*g + 9*d*h)))*(g + h*x)^4)/((c*g^2 + a*h^2)^3*(g + h*x)^5) - (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^{(7/2)} + 120*c^{(3/2)}*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])]/(c*g^2 + a*h^2)^{(7/2)}/(120*h^6)$$

**IntegrateAlgebraic [B]** time = 138.62, size = 4966, normalized size = 9.79

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6,x]

[Out] (a\*(4\*a^6\*f\*g^2\*h^8 + 6\*a^6\*e\*g\*h^9 + 24\*a^6\*d\*h^10 + 20\*a^6\*f\*g\*h^9\*x + 30\*a^6\*e\*h^10\*x + 40\*a^6\*f\*h^10\*x^2) + a\*Sqrt[c]\*Sqrt[a + c\*x^2]\*(-20\*a^5\*f\*g^2\*h^8\*x - 30\*a^5\*e\*g\*h^9\*x - 120\*a^5\*d\*h^10\*x - 100\*a^5\*f\*g\*h^9\*x^2 - 150\*a^5\*e\*h^10\*x^2 - 200\*a^5\*f\*h^10\*x^3) + a\*c\*(48\*a^5\*f\*g^4\*h^6 + 27\*a^5\*e\*g^3\*h^7 + 78\*a^5\*d\*g^2\*h^8 + 240\*a^5\*f\*g^3\*h^7\*x + 135\*a^5\*e\*g^2\*h^8\*x + 30\*a^5\*d\*g\*h^9\*x + 530\*a^5\*f\*g^2\*h^8\*x^2 + 165\*a^5\*e\*g\*h^9\*x^2 + 360\*a^5\*d\*h^10\*x^2 + 610\*a^5\*f\*g\*h^9\*x^3 + 465\*a^5\*e\*h^10\*x^3 + 680\*a^5\*f\*h^10\*x^4) + a\*c^(3/2)\*Sqrt[a + c\*x^2]\*(-114\*a^4\*f\*g^5\*h^5 + 24\*a^4\*e\*g^4\*h^6 + 6\*a^4\*d\*g^3\*h^7 - 690\*a^4\*f\*g^4\*h^6\*x - 15\*a^4\*e\*g^3\*h^7\*x - 360\*a^4\*d\*g^2\*h^8\*x - 1800\*a^4\*f\*g^3\*h^7\*x^2 - 435\*a^4\*e\*g^2\*h^8\*x^2 - 90\*a^4\*d\*g\*h^9\*x^2 - 2670\*a^4\*f\*g^2\*h^8\*x^3 - 315\*a^4\*e\*g\*h^9\*x^3 - 660\*a^4\*d\*h^10\*x^3 - 1950\*a^4\*f\*g\*h^9\*x^4 - 855\*a^4\*e\*h^10\*x^4 - 1440\*a^4\*f\*h^10\*x^5) + a\*c^2\*(309\*a^4\*f\*g^6\*h^4 - 24\*a^4\*e\*g^5\*h^5 + 99\*a^4\*d\*g^4\*h^6 + 1965\*a^4\*f\*g^5\*h^5\*x - 195\*a^4\*e\*g^4\*h^6\*x + 105\*a^4\*d\*g^3\*h^7\*x + 5325\*a^4\*f\*g^4\*h^6\*x^2 - 480\*a^4\*e\*g^3\*h^7\*x^2 + 1095\*a^4\*d\*g^2\*h^8\*x^2 + 8055\*a^4\*f\*g^3\*h^7\*x^3 + 420\*a^4\*e\*g^2\*h^8\*x^3 + 165\*a^4\*d\*g\*h^9\*x^3 + 8040\*a^4\*f\*g^2\*h^8\*x^4 + 1020\*a^4\*d\*h^10\*x^4 + 4110\*a^4\*f\*g\*h^9\*x^5 + 1215\*a^4\*e\*h^10\*x^5 + 2400\*a^4\*f\*h^10\*x^6) + a\*c^(5/2)\*Sqrt[a + c\*x^2]\*(-108\*a^3\*f\*g^7\*h^3 - 27\*a^3\*e\*g^6\*h^4 + 42\*a^3\*d\*g^5\*h^5 - 1725\*a^3\*f\*g^6\*h^4\*x - 15\*a^3\*e\*g^5\*h^5\*x - 285\*a^3\*d\*g^4\*h^6\*x - 7803\*a^3\*f\*g^5\*h^5\*x^2 + 393\*a^3\*e\*g^4\*h^6\*x^2 - 183\*a^3\*d\*g^3\*h^7\*x^2 - 16875\*a^3\*f\*g^4\*h^6\*x^3 + 1785\*a^3\*e\*g^3\*h^7\*x^3 - 1935\*a^3\*d\*g^2\*h^8\*x^3 - 19905\*a^3\*f\*g^3\*h^7\*x^4 + 720\*a^3\*e\*g^2\*h^8\*x^4 - 135\*a^3\*d\*g\*h^9\*x^4 - 14880\*a^3\*f\*g^2\*h^8\*x^5 + 1440\*a^3\*e\*g\*h^9\*x^5 - 720\*a^3\*d\*h^10\*x^5 - 4920\*a^3\*f\*g\*h^9\*x^6 - 540\*a^3\*e\*h^10\*x^6 - 1920\*a^3\*f\*h^10\*x^7) + a\*c^(11/2)\*Sqrt[a + c\*x^2]\*(-960\*f\*g^10\*x^3 - 4944\*f\*g^9\*h\*x^4 - 96\*e\*g^8\*h^2\*x^4 - 144\*d\*g^7\*h^3\*x^4 - 10320\*f\*g^8\*h^2\*x^5 - 480\*e\*g^7\*h^3\*x^5 - 720\*d\*g^6\*h^4\*x^5 - 11040\*f\*g^7\*h^3\*x^6 - 960\*e\*g^6\*h^4\*x^6 - 6240\*f\*g^6\*h^4\*x^7 - 960\*f\*g^5\*h^5\*x^8) + a\*c^(9/2)\*Sqrt[a + c\*x^2]\*(-480\*a\*f\*g^10\*x - 2628\*a\*f\*g^9\*h\*x^2 - 72\*a\*e\*g^8\*h^2\*x^2 - 108\*a\*d\*g^7\*h^3\*x^2 - 8580\*a\*f\*g^8\*h^2\*x^3 - 360\*a\*e\*g^7\*h^3\*x^3 - 540\*a\*d\*g^6\*h^4\*x^3 - 20748\*a\*f\*g^7\*h^3\*x^4 - 1152\*a\*e\*g^6\*h^4\*x^4 +

$$\begin{aligned}
& 1212*a*d*g^5*h^5*x^4 - 33420*a*f*g^6*h^4*x^5 - 1440*a*e*g^5*h^5*x^5 + 2460 \\
& *a*d*g^4*h^6*x^5 - 31800*a*f*g^5*h^5*x^6 - 2880*a*e*g^4*h^6*x^6 + 5640*a*d* \\
& g^3*h^7*x^6 - 17640*a*f*g^4*h^6*x^7 + 3240*a*d*g^2*h^8*x^7 - 2640*a*f*g^3*h \\
& ^7*x^8 + 720*a*d*g*h^9*x^8) + a*c^{(7/2)}*Sqrt[a + c*x^2]*(-39*a^2*f*g^9*h - \\
& 6*a^2*e*g^8*h^2 - 9*a^2*d*g^7*h^3 - 1535*a^2*f*g^8*h^2*x - 30*a^2*e*g^7*h^3 \\
& *x - 45*a^2*d*g^6*h^4*x - 7741*a^2*f*g^7*h^3*x^2 - 384*a^2*e*g^6*h^4*x^2 + \\
& 789*a^2*d*g^5*h^5*x^2 - 19325*a^2*f*g^6*h^4*x^3 - 960*a^2*e*g^5*h^5*x^3 + 8 \\
& 85*a^2*d*g^4*h^6*x^3 - 32004*a^2*f*g^5*h^5*x^4 - 1236*a^2*e*g^4*h^6*x^4 + 2 \\
& 976*a^2*d*g^3*h^7*x^4 - 39240*a^2*f*g^4*h^6*x^5 + 3180*a^2*e*g^3*h^7*x^5 - \\
& 240*a^2*d*g^2*h^8*x^5 - 30900*a^2*f*g^3*h^7*x^6 + 2760*a^2*e*g^2*h^8*x^6 + \\
& 420*a^2*d*g*h^9*x^6 - 16560*a^2*f*g^2*h^8*x^7 + 3240*a^2*e*g*h^9*x^7 - 2400 \\
& *a^2*f*g*h^9*x^8 + 720*a^2*e*h^10*x^8) + a*c^3*(340*a^3*f*g^8*h^2 + 2075*a^ \\
& 3*f*g^7*h^3*x + 135*a^3*e*g^6*h^4*x - 285*a^3*d*g^5*h^5*x + 7600*a^3*f*g^6* \\
& h^4*x^2 + 315*a^3*e*g^5*h^5*x^2 + 150*a^3*d*g^4*h^6*x^2 + 19890*a^3*f*g^5*h \\
& ^5*x^3 - 135*a^3*e*g^4*h^6*x^3 - 600*a^3*d*g^3*h^7*x^3 + 34290*a^3*f*g^4*h^ \\
& 6*x^4 - 3375*a^3*e*g^3*h^7*x^4 + 2460*a^3*d*g^2*h^8*x^4 + 35025*a^3*f*g^3*h \\
& ^7*x^5 - 2100*a^3*e*g^2*h^8*x^5 + 15*a^3*d*g*h^9*x^5 + 23160*a^3*f*g^2*h^8* \\
& x^6 - 3060*a^3*e*g*h^9*x^6 + 720*a^3*d*h^10*x^6 + 6120*a^3*f*g*h^9*x^7 + 18 \\
& 0*a^3*e*h^10*x^7 + 1920*a^3*f*h^10*x^8) + a*c^6*(960*f*g^10*x^4 + 4944*f*g^ \\
& 9*h*x^5 + 96*e*g^8*h^2*x^5 + 144*d*g^7*h^3*x^5 + 10320*f*g^8*h^2*x^6 + 480* \\
& e*g^7*h^3*x^6 + 720*d*g^6*h^4*x^6 + 11040*f*g^7*h^3*x^7 + 960*e*g^6*h^4*x^7 \\
& + 6240*f*g^6*h^4*x^8 + 960*f*g^5*h^5*x^9) + a*c^5*(960*a*f*g^10*x^2 + 5100 \\
& *a*f*g^9*h*x^3 + 120*a*e*g^8*h^2*x^3 + 180*a*d*g^7*h^3*x^3 + 13740*a*f*g^8* \\
& h^2*x^4 + 600*a*e*g^7*h^3*x^4 + 900*a*d*g^6*h^4*x^4 + 26268*a*f*g^7*h^3*x^5 \\
& + 1632*a*e*g^6*h^4*x^5 - 1212*a*d*g^5*h^5*x^5 + 36540*a*f*g^6*h^4*x^6 + 14 \\
& 40*a*e*g^5*h^5*x^6 - 2460*a*d*g^4*h^6*x^6 + 32280*a*f*g^5*h^5*x^7 + 2880*a* \\
& e*g^4*h^6*x^7 - 5640*a*d*g^3*h^7*x^7 + 17640*a*f*g^4*h^6*x^8 - 3240*a*d*g^2 \\
& *h^8*x^8 + 2640*a*f*g^3*h^7*x^9 - 720*a*d*g*h^9*x^9) + a*c^4*(120*a^2*f*g^1 \\
& 0 + 735*a^2*f*g^9*h*x + 30*a^2*e*g^8*h^2*x + 45*a^2*d*g^7*h^3*x + 4535*a^2* \\
& f*g^8*h^2*x^2 + 150*a^2*e*g^7*h^3*x^2 + 225*a^2*d*g^6*h^4*x^2 + 16735*a^2*f \\
& *g^7*h^3*x^3 + 840*a^2*e*g^6*h^4*x^3 - 1395*a^2*d*g^5*h^5*x^3 + 35255*a^2*f \\
& *g^6*h^4*x^4 + 1680*a^2*e*g^5*h^5*x^4 - 2115*a^2*d*g^4*h^6*x^4 + 47784*a^2* \\
& f*g^5*h^5*x^5 + 2676*a^2*e*g^4*h^6*x^5 - 5796*a^2*d*g^3*h^7*x^5 + 48060*a^2 \\
& *f*g^4*h^6*x^6 - 3180*a^2*e*g^3*h^7*x^6 - 1380*a^2*d*g^2*h^8*x^6 + 32220*a^ \\
& 2*f*g^3*h^7*x^7 - 2760*a^2*e*g^2*h^8*x^7 - 780*a^2*d*g*h^9*x^7 + 16560*a^2* \\
& f*g^2*h^8*x^8 - 3240*a^2*e*g*h^9*x^8 + 2400*a^2*f*g*h^9*x^9 - 720*a^2*e*h^1 \\
& 0*x^9))/(600*a^5*Sqrt[c]*h^11*x*(g + h*x)^5 + 1920*c^(11/2)*g^6*h^5*x^5*(g \\
& + h*x)^5 - 120*a^5*h^11*(g + h*x)^5*Sqrt[a + c*x^2] - 1920*c^5*g^6*h^5*x^4* \\
& (g + h*x)^5*Sqrt[a + c*x^2] + 120*c*h^5*(g + h*x)^5*Sqrt[a + c*x^2]*(-3*a^4 \\
& *g^2*h^4 - 12*a^4*h^6*x^2) + 120*c^(3/2)*h^5*(g + h*x)^5*(15*a^4*g^2*h^4*x \\
& + 20*a^4*h^6*x^3) + 120*c^4*h^5*(g + h*x)^5*Sqrt[a + c*x^2]*(-12*a*g^6*x^2 \\
& - 48*a*g^4*h^2*x^4) + 120*c^3*h^5*(g + h*x)^5*Sqrt[a + c*x^2]*(-(a^2*g^6) - \\
& 36*a^2*g^4*h^2*x^2 - 48*a^2*g^2*h^4*x^4) + 120*c^2*h^5*(g + h*x)^5*Sqrt[a \\
& + c*x^2]*(-3*a^3*g^4*h^2 - 36*a^3*g^2*h^4*x^2 - 16*a^3*h^6*x^4) + 120*c^(9/ \\
& 2)*h^5*(g + h*x)^5*(20*a*g^6*x^3 + 48*a*g^4*h^2*x^5) + 120*c^(7/2)*h^5*(g +
\end{aligned}$$

$$\begin{aligned}
& h^5 x^5 (5 a^2 g^6 x + 60 a^2 g^4 h^2 x^3 + 48 a^2 g^2 h^4 x^5) + 120 c^{5/2} h^5 (g + h x)^5 (15 a^3 g^4 h^2 x + 60 a^3 g^2 h^4 x^3 + 16 a^3 h^6 x^5) \\
& + (3 a^4 f h^2 \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}]) / \sqrt{-(c g^2) - a h^2} \\
& + (3 a^4 e h^3 \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}]) / \sqrt{-(c g^2) - a h^2} \\
& + (9 a^4 d h^4 \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}]) / \sqrt{-(c g^2) - a h^2} \\
& + (a^3 f \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}]) / \sqrt{-(c g^2) - a h^2} \\
& + (3 a^3 e h \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}]) / \sqrt{-(c g^2) - a h^2} \\
& + (4 a^3 d h^2 \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}]) / \sqrt{-(c g^2) - a h^2} \\
& + (a^2 f) / (4 g^3 h^2 \sqrt{-(c g^2) - a h^2}) \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}] / \sqrt{-(c g^2) - a h^2} \\
& + ((-3 a^5 f h^4) / (4 g^3 \sqrt{-(c g^2) - a h^2} (c g^2 + a h^2)^3) + (3 a^5 e h^5) / (4 g^4 \sqrt{-(c g^2) - a h^2} (c g^2 + a h^2)^3) \\
& - (3 a^5 d h^6) / (4 g^5 \sqrt{-(c g^2) - a h^2} (c g^2 + a h^2)^3)) \operatorname{ArcTan}[(\sqrt{c} g) - \sqrt{c} h x + h \sqrt{a + c x^2}] / \sqrt{-(c g^2) - a h^2} \\
& + (2 c^2 f g \operatorname{ArcTan}[(\sqrt{c} g) / \sqrt{-(c g^2) - a h^2} + (\sqrt{c} h x) / \sqrt{-(c g^2) - a h^2} - (h \sqrt{a + c x^2}) / \sqrt{-(c g^2) - a h^2}]) \\
& + (\sqrt{c} h x) / \sqrt{-(c g^2) - a h^2} - (h \sqrt{a + c x^2}) / \sqrt{-(c g^2) - a h^2} \\
& + (a c f \operatorname{ArcTan}[(\sqrt{c} g) / \sqrt{-(c g^2) - a h^2} + (\sqrt{c} h x) / \sqrt{-(c g^2) - a h^2} - (h \sqrt{a + c x^2}) / \sqrt{-(c g^2) - a h^2}]) \\
& + (g h^4 \sqrt{-(c g^2) - a h^2}) + ((-137 c^{3/2} f g^5) / (60 h^6) + (c^{3/2} e g^4) / (5 h^5) + (c^{3/2} d g^3) / (20 h^4) \\
& - (125 c^{3/2} f g^4 x) / (12 h^5) + (c^{3/2} e g^3 x) / h^4 + (c^{3/2} d g^2 x) / (4 h^3) - (55 c^{3/2} f g^3 x^2) / (3 h^4) \\
& + (2 c^{3/2} e g^2 x^2) / h^3 + (c^{3/2} d g x^2) / (2 h^2) - (15 c^{3/2} f g^2 x^3) / h^3 + (2 c^{3/2} e g x^3) / h^2 \\
& + (c^{3/2} d x^3) / (2 h) - (5 c^{3/2} f g x^4) / h^2 + (c^{3/2} e x^4) / h - (c^{3/2} f g^5 \operatorname{Log}[-(\sqrt{c} x) + \sqrt{a + c x^2}]) / h^6 \\
& - (5 c^{3/2} f g^4 x \operatorname{Log}[-(\sqrt{c} x) + \sqrt{a + c x^2}]) / h^5 - (10 c^{3/2} f g^3 x^2 \operatorname{Log}[-(\sqrt{c} x) + \sqrt{a + c x^2}]) / h^4 \\
& - (10 c^{3/2} f g^2 x^3 \operatorname{Log}[-(\sqrt{c} x) + \sqrt{a + c x^2}]) / h^3 - (5 c^{3/2} f g x^4 \operatorname{Log}[-(\sqrt{c} x) + \sqrt{a + c x^2}]) / h^2 \\
& - (c^{3/2} f x^5 \operatorname{Log}[-(\sqrt{c} x) + \sqrt{a + c x^2}]) / h) / (g + h x)^5
\end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^6,x, algorithm="fricas")

[Out] Timed out

**giac** [B] time = 1.33, size = 4408, normalized size = 8.69



result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*d
*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*h^7*e)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*g^4
*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*log(a
bs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*x^2 +
a))^9*c^5*f*g^7*h^4 + 1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6
+ 1635*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*f*g^3*h^8 + 45*(sqrt(c)*x -
sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^3*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*g^6*h^5*e - 360*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*g^4*h^7*e - 360*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^3*g^2*h^9*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*h
^11*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 - 120*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a*c^(9/2)*f*g^6*h^5 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9
/2)*d*g^4*h^7 + 8595*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7
+ 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1530*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*(sqrt(c)*x - sqrt(c*x^2
+ a))^8*a^3*c^(5/2)*d*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2
)*f*h^11 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*g^7*h^4*e - 1440*(s
qrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*g^5*h^6*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^
3*c^(5/2)*g*h^10*e + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 2
40*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*d*g^7*h^4 + 21240*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a*c^5*f*g^7*h^4 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d
*g^5*h^6 + 11670*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 690*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 - 4970*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a^3*c^3*f*g^3*h^8 - 450*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3
*d*g*h^10 - 2580*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqrt(c*x^2 +
a))^7*a*c^5*g^6*h^5*e - 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*g^4*h
^7*e + 1170*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2*h^9*e + 30*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e + 10000*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*
h^3 + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 720*(sqr
t(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 14430*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 + 1590*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^2*c^(9/2)*d*g^4*h^7 - 28790*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f
*g^4*h^7 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*g^2*h^9 - 582
0*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 + 720*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^6
```

$$\begin{aligned}
& *c^{(13/2)}*g^9*h^2*e - 1680*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a*c^{(11/2)}*g^7*h \\
& ^4*e + 720*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^2*c^{(9/2)}*g^5*h^6*e + 4950*(\text{sq} \\
& \text{rt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^3*c^{(7/2)}*g^3*h^8*e - 270*(\text{sqrt}(c)*x - \text{sqrt}( \\
& c*x^2 + a))^6*a^4*c^{(5/2)}*g*h^{10}*e + 4384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*c \\
& ^7*f*g^{11} - 96*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*c^7*d*g^9*h^2 - 9392*(\text{sqrt}(c \\
& )*x - \text{sqrt}(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\
& ^5*a*c^6*d*g^7*h^4 - 42996*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c^5*f*g^7*h^ \\
& 4 + 2364*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 - 31070*(\text{sqrt}(c) \\
& *x - \text{sqrt}(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 - 2730*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
& a))^5*a^3*c^4*d*g^3*h^8 + 8620*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^4*c^3*f*g^ \\
& 3*h^8 + 720*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^4*c^3*d*g*h^{10} + 4800*(\text{sqrt}(c \\
& )*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^2*f*g*h^{10} - 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a \\
& ))^5*c^7*g^{10}*h^e + 672*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3 \\
& 936*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e + 5580*(\text{sqrt}(c)*x - s \\
& \text{qrt}(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e - 2970*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5* \\
& a^4*c^3*g^2*h^9*e - 11920*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a*c^{(13/2)}*f*g^{10} \\
& *h + 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a*c^{(13/2)}*d*g^8*h^3 - 15720*(\text{sqrt} \\
& (c)*x - \text{sqrt}(c*x^2 + a))^4*a^2*c^{(11/2)}*f*g^8*h^3 + 720*(\text{sqrt}(c)*x - \text{sqrt}(c \\
& *x^2 + a))^4*a^2*c^{(11/2)}*d*g^6*h^5 + 19670*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4 \\
& *a^3*c^{(9/2)}*f*g^6*h^5 - 3510*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*c^{(9/2)}*d \\
& *g^4*h^7 + 36260*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^4*c^{(7/2)}*f*g^4*h^7 + 14 \\
& 40*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^4*c^{(7/2)}*d*g^2*h^9 + 6240*(\text{sqrt}(c)*x \\
& - \text{sqrt}(c*x^2 + a))^4*a^5*c^{(5/2)}*f*g^2*h^9 - 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
& a))^4*a^5*c^{(5/2)}*d*h^{11} - 880*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^6*c^{(3/2)}* \\
& f*h^{11} + 960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a*c^{(13/2)}*g^9*h^2*e + 1680*(s \\
& \text{qrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^2*c^{(11/2)}*g^7*h^4*e - 480*(\text{sqrt}(c)*x - sqr \\
& t(c*x^2 + a))^4*a^3*c^{(9/2)}*g^5*h^6*e - 6150*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^ \\
& 4*a^4*c^{(7/2)}*g^3*h^8*e + 720*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^5*c^{(5/2)}*g \\
& *h^{10}*e + 13120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 240*(sq \\
& \text{rt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 + 30440*(\text{sqrt}(c)*x - \text{sqrt}(c* \\
& x^2 + a))^3*a^3*c^5*f*g^7*h^4 - 1200*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^3*c^ \\
& 5*d*g^5*h^6 + 14130*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 + 231 \\
& 0*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 - 10790*(\text{sqrt}(c)*x - sq \\
& \text{rt}(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 - 510*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^ \\
& 5*c^3*d*g*h^{10} - 3820*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^6*c^2*f*g*h^{10} - 96 \\
& 0*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^2*c^6*g^8*h^3*e - 2640*(\text{sqrt}(c)*x - sqr \\
& t(c*x^2 + a))^3*a^3*c^5*g^6*h^5*e - 2640*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^ \\
& 4*c^4*g^4*h^7*e + 2790*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e - \\
& 30*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^6*c^2*h^{11}*e - 7360*(\text{sqrt}(c)*x - \text{sqrt}( \\
& c*x^2 + a))^2*a^3*c^{(11/2)}*f*g^8*h^3 + 120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2* \\
& a^3*c^{(11/2)}*d*g^6*h^5 - 19930*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^4*c^{(9/2)}* \\
& f*g^6*h^5 + 690*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^4*c^{(9/2)}*d*g^4*h^7 - 160 \\
& 50*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^5*c^{(7/2)}*f*g^4*h^7 - 1050*(\text{sqrt}(c)*x \\
& - \text{sqrt}(c*x^2 + a))^2*a^5*c^{(7/2)}*d*g^2*h^9 - 1300*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
& a))^2*a^6*c^{(5/2)}*f*g^2*h^9 + 560*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^7*c^{(3}
\end{aligned}$$

$$\begin{aligned} & /2)*f*h^{11} + 480*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^3*c^{(11/2)}*g^7*h^4*e + 1 \\ & 560*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^4*c^{(9/2)}*g^5*h^6*e + 2130*(\sqrt{c}*x \\ & - \sqrt{c*x^2 + a})^2*a^5*c^{(7/2)}*g^3*h^8*e - 570*(\sqrt{c}*x - \sqrt{c*x^2 + \\ & a})^2*a^6*c^{(5/2)}*g*h^{10}*e + 2140*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^4*c^5*f* \\ & g^7*h^4 - 60*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^4*c^5*d*g^5*h^6 + 6090*(\sqrt{c} \\ & )*x - \sqrt{c*x^2 + a})*a^5*c^4*f*g^5*h^6 - 270*(\sqrt{c}*x - \sqrt{c*x^2 + a} \\ & )*a^5*c^4*d*g^3*h^8 + 5505*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^6*c^3*f*g^3*h^8 \\ & + 195*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^6*c^3*d*g*h^{10} + 1150*(\sqrt{c}*x - \sqrt{c} \\ & )*x - \sqrt{c*x^2 + a})*a^7*c^2*f*g*h^{10} - 120*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^4*c^5 \\ & *g^6*h^5*e - 420*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^5*c^4*g^4*h^7*e - 630*(\sqrt{c} \\ & )*x - \sqrt{c*x^2 + a})*a^6*c^3*g^2*h^9*e + 75*(\sqrt{c}*x - \sqrt{c*x^2 + \\ & a})*a^7*c^2*h^{11}*e - 274*a^5*c^{(9/2)}*f*g^6*h^5 + 6*a^5*c^{(9/2)}*d*g^4*h^7 - \\ & 783*a^6*c^{(7/2)}*f*g^4*h^7 + 27*a^6*c^{(7/2)}*d*g^2*h^9 - 714*a^7*c^{(5/2)}*f*g^ \\ & 2*h^9 - 24*a^7*c^{(5/2)}*d*h^{11} - 160*a^8*c^{(3/2)}*f*h^{11} + 24*a^5*c^{(9/2)}*g^5 \\ & *h^6*e + 78*a^6*c^{(7/2)}*g^3*h^8*e + 99*a^7*c^{(5/2)}*g*h^{10}*e)/((c^3*g^6*h^6 \\ & + 3*a*c^2*g^4*h^8 + 3*a^2*c*g^2*h^{10} + a^3*h^{12})*((\sqrt{c}*x - \sqrt{c*x^2 + \\ & a})^2*h + 2*(\sqrt{c}*x - \sqrt{c*x^2 + a}))*\sqrt{c}*g - a*h)^5) \end{aligned}$$

**maple [B]** time = 0.03, size = 14169, normalized size = 27.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g)^6,x)$

[Out] result too large to display

**maxima [B]** time = 1.89, size = 6650, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g)^6,x, \text{algorithm}=\text{"maxima"})$

[Out] 
$$\begin{aligned} & 3/8*\sqrt{c*x^2 + a}*c^5*f*g^7/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^ \\ & 4*h^9 + 4*a^3*c*g^2*h^{11} + a^4*h^{13}) - 3/8*\sqrt{c*x^2 + a}*c^5*f*g^6*x/(c^4 \\ & *g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^{10} + a^4*h^{1 \\ & 2}) - 3/8*\sqrt{c*x^2 + a}*c^5*e*g^6/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c \\ & ^2*g^4*h^8 + 4*a^3*c*g^2*h^{10} + a^4*h^{12}) + 1/8*(c*x^2 + a)^{(3/2)}*c^4*f*g^6 \\ & /((c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^1 \\ & 0*x + a^4*h^{12}*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^ \\ & 3*c*g^3*h^9 + a^4*g*h^{11}) + 3/8*\sqrt{c*x^2 + a}*c^5*e*g^5*x/(c^4*g^8*h^3 + \\ & 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^{11}) + 3/8*\sqrt{c} \\ & )*x - \sqrt{c*x^2 + a})*c^5*d*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + \\ & 4*a^3*c*g^2*h^9 + a^4*h^{11}) - 1/8*(c*x^2 + a)^{(3/2)}*c^4*e*g^5/(c^4*g^8*h^3 \\ & *x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*h^{11} \end{aligned}$$

$$\begin{aligned}
& *x + c^4g^9h^2 + 4a^3c^3g^7h^4 + 6a^2c^2g^5h^6 + 4a^3c^3g^3h^8 + \\
& a^4g^h^{10}) - 1/8*(c^2x^2 + a)^{(5/2)}c^3f^5g^5/(c^4g^8h^3x^2 + 4a^3c^3g^6h^5x^2 + 6a^2c^2g^4h^7x^2 + 4a^3c^3g^2h^9x^2 + a^4h^{11}x^2 + 2c^4g^9h^2x + 8a^3c^3g^7h^4x + 12a^2c^2g^5h^6x + 8a^3c^3g^3h^8x + 2a^4g^h^{10}x + c^4g^{10}h + 4a^3c^3g^8h^3 + 6a^2c^2g^6h^5 + 4a^3c^3g^4h^7 + a^4g^2h^9) + 1/8*(c^2x^2 + a)^{(3/2)}c^4f^5g^5/(c^4g^8h^3 + 4a^3c^3g^6h^5 + 6a^2c^2g^4h^7 + 4a^3c^3g^2h^9 + a^4h^{11}) - 3/8sqr(c^2x^2 + a)c^5d^4g^4x/(c^4g^8h^2 + 4a^3c^3g^6h^4 + 6a^2c^2g^4h^6 + 4a^3c^3g^2h^8 + a^4h^{10}) + 1/8*(c^2x^2 + a)^{(3/2)}c^4d^4g^4/(c^4g^8h^2x + 4a^3c^3g^6h^4x + 6a^2c^2g^4h^6x + 4a^3c^3g^2h^8x + a^4h^{10}x + c^4g^9h + 4a^3c^3g^7h^3 + 6a^2c^2g^5h^5 + 4a^3c^3g^3h^7 + a^4g^h^9) + 1/8*(c^2x^2 + a)^{(5/2)}c^3e^4g^4/(c^4g^8h^2x^2 + 4a^3c^3g^6h^4x^2 + 6a^2c^2g^4h^6x^2 + 4a^3c^3g^2h^8x^2 + a^4h^{10}x^2 + 2c^4g^9hx + 8a^3c^3g^7h^3x + 12a^2c^2g^5h^5x + 8a^3c^3g^3h^7x + 2a^4g^h^9x + c^4g^{10} + 4a^3c^3g^8h^2 + 6a^2c^2g^6h^4 + 4a^3c^3g^4h^6 + a^4g^2h^8) - 1/8*(c^2x^2 + a)^{(3/2)}c^4e^4g^4/(c^4g^8h^2 + 4a^3c^3g^6h^4 + 6a^2c^2g^4h^6 + 4a^3c^3g^2h^8 + a^4h^{10}) - 3/2*sqrt(c^2x^2 + a)c^4f^5g^5/(c^3g^6h^5 + 3a^2c^2g^4h^7 + 3a^2c^2g^2h^9 + a^3h^{11}) + 9/8*sqrt(c^2x^2 + a)c^4f^4g^4x/(c^3g^6h^4 + 3a^2c^2g^4h^6 + 3a^2c^2g^2h^8 + a^3h^{10}) - 1/8*(c^2x^2 + a)^{(5/2)}c^3d^3g^3/(c^4g^8hx^2 + 4a^3c^3g^6h^3x^2 + 6a^2c^2g^4h^5x^2 + 4a^3c^3g^2h^7x^2 + a^4h^9x^2 + 2c^4g^9x + 8a^3c^3g^7h^2x + 12a^2c^2g^5h^4x + 8a^3c^3g^3h^6x + 2a^4g^h^8x + c^4g^{10}/h + 4a^3c^3g^8h + 6a^2c^2g^6h^3 + 4a^3c^3g^4h^5 + a^4g^2h^7) + 1/8*(c^2x^2 + a)^{(3/2)}c^4d^3g^3/(c^4g^8h + 4a^3c^3g^6h^3 + 6a^2c^2g^4h^5 + 4a^3c^3g^2h^7 + a^4h^9) + 9/8*sqrt(c^2x^2 + a)c^4e^4g^4/(c^3g^6h^4 + 3a^2c^2g^4h^6 + 3a^2c^2g^2h^8 + a^3h^{10}) - 1/4*(c^2x^2 + a)^{(5/2)}c^2f^4g^4/(c^3g^6h^4x^3 + 3a^2c^2g^4h^6x^3 + 3a^2c^2g^2h^8x^3 + a^3h^{10}x^3 + 3c^3g^7h^3x^2 + 9a^2c^2g^5h^5x^2 + 9a^2c^2g^3h^7x^2 + 3a^3g^h^9x^2 + 3c^3g^8h^2x + 9a^2c^2g^6h^4x + 9a^2c^2g^4h^6x + 3a^3g^2h^8x + c^3g^9h + 3a^2c^2g^7h^3 + 3a^2c^2g^5h^5 + a^3g^3h^7) - 5/8*(c^2x^2 + a)^{(3/2)}c^3f^4g^4/(c^3g^6h^4x + 3a^2c^2g^4h^6x + 3a^2c^2g^2h^8x + a^3h^{10}x + c^3g^7h^3 + 3a^2c^2g^5h^5 + 3a^2c^2g^3h^7 + a^3g^h^9) - 3/4*sqrt(c^2x^2 + a)c^4e^3g^3x/(c^3g^6h^3 + 3a^2c^2g^4h^5 + 3a^2c^2g^2h^7 + a^3h^9) - 3/4*sqrt(c^2x^2 + a)c^4d^3g^3/(c^3g^6h^3 + 3a^2c^2g^4h^5 + 3a^2c^2g^2h^7 + a^3h^9) + 1/4*(c^2x^2 + a)^{(5/2)}c^2e^3g^3/(c^3g^6h^3x^3 + 3a^2c^2g^4h^5x^3 + 3a^2c^2g^2h^7x^3 + a^3h^9x^3 + 3c^3g^7h^2x^2 + 9a^2c^2g^5h^4x^2 + 9a^2c^2g^3h^6x^2 + 3a^3g^h^8x^2 + 3c^3g^8hx + 9a^2c^2g^6h^3x + 9a^2c^2g^4h^5x + 3a^3g^2h^7x + c^3g^9 + 3a^2c^2g^7h^2 + 3a^2c^2g^5h^4 + a^3g^3h^6) + 1/2*(c^2x^2 + a)^{(3/2)}c^3e^3g^3/(c^3g^6h^3x + 3a^2c^2g^4h^5x + 3a^2c^2g^2h^7x + a^3h^9x + c^3g^7h^2 + 3a^2c^2g^5h^4 + 3a^2c^2g^3h^6 + a^3g^h^8) + 1/8*(c^2x^2 + a)^{(5/2)}c^2f^3g^3/(c^3g^6h^3x^2 + 3a^2c^2g^4h^5x^2 + 3a^2c^2g^2h^7x^2 + a^3h^9x^2 + 2c^3g^7h^2x + 6a^2c^2g^5h^4x + 6a^2c^2g^3h^6x + 2a^3g^h^8x + c^3g^8h + 3a^2c^2g^6h^3 + 3a^2c^2g^4h^5 + a^3g^2
\end{aligned}$$

$$\begin{aligned}
& *h^7) - 1/8*(c*x^2 + a)^{(3/2)}*c^3*f*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3* \\
& a^2*c*g^2*h^7 + a^3*h^9) + 3/8*\text{sqrt}(c*x^2 + a)*c^4*d*g^2*x/(c^3*g^6*h^2 + 3 \\
& *a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 1/4*(c*x^2 + a)^{(5/2)}*c^2*d*g \\
& ^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^3*h^8*x \\
& ^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3*a^3*g* \\
& h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^3*g^2*h \\
& ^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) - 3/8*(c* \\
& x^2 + a)^{(3/2)}*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h \\
& ^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^ \\
& 7) - 1/4*(c*x^2 + a)^{(5/2)}*c*f*g^3/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a \\
& ^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^ \\
& 2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x + \\
& 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5 \\
& ) + 19/8*\text{sqrt}(c*x^2 + a)*c^3*f*g^3/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) \\
& - 5/4*\text{sqrt}(c*x^2 + a)*c^3*f*g^2*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) - \\
& 1/8*(c*x^2 + a)^{(5/2)}*c^2*d*g/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2 \\
& *c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^ \\
& 3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3 \\
& *g^2*h^5) + 1/8*(c*x^2 + a)^{(3/2)}*c^3*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3* \\
& a^2*c*g^2*h^5 + a^3*h^7) + 1/4*(c*x^2 + a)^{(5/2)}*c*e*g^2/(c^2*g^4*h^4*x^4 + \\
& 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 + \\
& 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^ \\
& 2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6 \\
& *h^2 + a^2*g^4*h^4) - 9/8*\text{sqrt}(c*x^2 + a)*c^3*e*g^2/(c^2*g^4*h^4 + 2*a*c*g^ \\
& 2*h^6 + a^2*h^8) + 1/2*(c*x^2 + a)^{(5/2)}*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g \\
& ^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g* \\
& h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + \\
& 2*a*c*g^5*h^3 + a^2*g^3*h^5) + 11/12*(c*x^2 + a)^{(3/2)}*c^2*f*g^2/(c^2*g^4* \\
& h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h \\
& ^7) + 3/8*\text{sqrt}(c*x^2 + a)*c^3*e*g*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) \\
& - 1/4*(c*x^2 + a)^{(5/2)}*c*d*g/(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h \\
& ^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^ \\
& 6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5* \\
& h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) + 3/8*\text{sqrt} \\
& (c*x^2 + a)*c^3*d*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/4*(c*x^2 + \\
& a)^{(5/2)}*c*e*g/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g \\
& ^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^ \\
& 4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 3/8*(c \\
& *x^2 + a)^{(3/2)}*c^2*e*g/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2* \\
& g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 1/12*(c*x^2 + a)^{(5/2)}*c*f*g/(c^2*g^ \\
& 4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h \\
& ^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 1/12*(c*x \\
& ^2 + a)^{(3/2)}*c^2*f*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2 \\
& + a)^{(5/2)}*f*g^2/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x \\
& ^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*
\end{aligned}$$

$$\begin{aligned}
& x^2 + 5c^2g^6h^2x + 5a^2g^4h^4x + c^2g^7h + ag^5h^3) - 1/8(c^2x^2 + a \\
& )^{(5/2)}c^2e/(c^2g^4h^2x^2 + 2a^2c^2g^2h^4x^2 + a^2h^6x^2 + 2c^2g^5h \\
& hx + 4a^2c^2g^3h^3x + 2a^2g^2h^5x + c^2g^6 + 2a^2c^2g^4h^2 + a^2g^2h \\
& ^4) + 1/8(c^2x^2 + a)^{(3/2)}c^2e/(c^2g^4h^2 + 2a^2c^2g^2h^4 + a^2h^6) + \\
& 1/5(c^2x^2 + a)^{(5/2)}e^2g/(c^2g^2h^5x^5 + ah^7x^5 + 5c^2g^3h^4x^4 + 5 \\
& a^2g^2h^6x^4 + 10c^2g^4h^3x^3 + 10a^2g^2h^5x^3 + 10c^2g^5h^2x^2 + 10 \\
& a^2g^3h^4x^2 + 5c^2g^6hx + 5a^2g^4h^3x + c^2g^7 + ag^5h^2) + 1/2(c^2x \\
& ^2 + a)^{(5/2)}fg/(c^2g^2h^5x^4 + ah^7x^4 + 4c^2g^3h^4x^3 + 4a^2g^2h^6x \\
& x^3 + 6c^2g^4h^3x^2 + 6a^2g^2h^5x^2 + 4c^2g^5h^2x + 4a^2g^3h^4x + c \\
& ^2g^6h + ag^4h^3) - 9/4\sqrt{c^2x^2 + a}c^2fg/(c^2g^2h^5 + ah^7) + \sqrt{ \\
& c^2x^2 + a}c^2fx/(c^2g^2h^4 + ah^6) - 1/5(c^2x^2 + a)^{(5/2)}d/(c^2g^2h \\
& ^4x^5 + ah^6x^5 + 5c^2g^3h^3x^4 + 5a^2g^2h^5x^4 + 10c^2g^4h^2x^3 + 1 \\
& 0a^2g^2h^4x^3 + 10c^2g^5hx^2 + 10a^2g^3h^3x^2 + 5c^2g^6x + 5a^2g^4h \\
& ^2x + c^2g^7/h + ag^5h) - 1/4(c^2x^2 + a)^{(5/2)}e/(c^2g^2h^4x^4 + ah^6x \\
& x^4 + 4c^2g^3h^3x^3 + 4a^2g^2h^5x^3 + 6c^2g^4h^2x^2 + 6a^2g^2h^4x^2 + \\
& 4c^2g^5hx + 4a^2g^3h^3x + c^2g^6 + ag^4h^2) + 3/8\sqrt{c^2x^2 + a}c^2 \\
& e/(c^2g^2h^4 + ah^6) - 1/3(c^2x^2 + a)^{(5/2)}f/(c^2g^2h^4x^3 + ah^6x^3 \\
& + 3c^2g^3h^3x^2 + 3a^2g^2h^5x^2 + 3c^2g^4h^2x + 3a^2g^2h^4x + c^2g^5h \\
& h + ag^3h^3) - 2/3(c^2x^2 + a)^{(3/2)}cf/(c^2g^2h^4x + ah^6x + c^2g^3h \\
& ^3 + ag^2h^5) + c^{(3/2)}f\operatorname{arcsinh}(cx/\sqrt{ac})/h^6 + 3/8c^5fg^7\operatorname{arcsin} \\
& h(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g))/((a + c^2g^2/h^2)^{(7/2)}h^{13}) - 3/8c^5e^2g^6\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - \\
& ah/(\sqrt{ac})\operatorname{abs}(hx + g))/((a + c^2g^2/h^2)^{(7/2)}h^{12}) + 3/8c^5d^2g^5 \\
& \operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g))/((a \\
& + c^2g^2/h^2)^{(7/2)}h^{11}) - 3/2c^4fg^5\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx \\
& + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g))/((a + c^2g^2/h^2)^{(5/2)}h^{11}) + 9/8c^4 \\
& e^2g^4\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g \\
& ))/((a + c^2g^2/h^2)^{(5/2)}h^{10}) - 3/4c^4d^2g^3\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs} \\
& (hx + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g))/((a + c^2g^2/h^2)^{(5/2)}h^9) + \\
& 19/8c^3fg^3\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}( \\
& hx + g))/((a + c^2g^2/h^2)^{(3/2)}h^9) - 9/8c^3e^2g^2\operatorname{arcsinh}(cgx/(\sqrt{ac} \\
& )\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g))/((a + c^2g^2/h^2)^{(3/2)}h \\
& ^8) + 3/8c^3d^2g\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs} \\
& (hx + g))/((a + c^2g^2/h^2)^{(3/2)}h^7) - 9/4c^2fg\operatorname{arcsinh}(cgx/(\sqrt{ac} \\
& )\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}(hx + g))/(\sqrt{a + c^2g^2/h^2}h^7) + 3/8c^2e^2\operatorname{arcsinh}(cgx/(\sqrt{ac})\operatorname{abs}(hx + g)) - ah/(\sqrt{ac})\operatorname{abs}( \\
& hx + g))/(\sqrt{a + c^2g^2/h^2}h^6)
\end{aligned}$$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

$$3.98 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7} dx$$

**Optimal.** Leaf size=404

$$\frac{(a+cx^2)^{3/2}(ah-cgx)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{24(g+hx)^4(ah^2+cg^2)^3} - \frac{ac\sqrt{a+cx^2}(ah-cgx)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16(g+hx)^2(ah^2+cg^2)^3}$$

**Rubi [A]** time = 0.55, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1651, 807, 721, 725, 206}

$$\frac{(a+cx^2)^{3/2}(ah-cgx)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{24(g+hx)^4(ah^2+cg^2)^3} - \frac{ac\sqrt{a+cx^2}(ah-cgx)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16(g+hx)^2(ah^2+cg^2)^3} - \frac{a^2c^2 \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16(ah^2+cg^2)^{3/2}} + \frac{(a+cx^2)^{3/2}(6ah^2(2fg-dh)+cg^2(eg-7dh)+5c^2fg^2)}{30h(g+hx)^3(ah^2+cg^2)^2} - \frac{(a+cx^2)^{3/2}(dh^2-cgh+fg^2)}{6h(g+hx)^2(ah^2+cg^2)}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^7, x]

[Out]  $-(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*\text{Sqrt}[a + c*x^2])/(16*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((5*c*f*g^3 + c*g*h*(e*g - 7*d*h) + 6*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(g + h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*\text{ArcTanh}[(a*h - c*g*x)/(\text{Sqrt}[c*g^2 + a*h^2]*\text{Sqrt}[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(9/2))$

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 721

Int[((d\_) + (e\_.)\*(x\_)^(m\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[((d + e\*x)^(m + 1)\*(-2\*a\*e + (2\*c\*d)\*x)\*(a + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), x] - Dist[(4\*a\*c\*p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m + 2\*p + 2, 0] && GtQ[p, 0]

### Rule 725



```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 807

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p
_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps



$$7 + g^5 h (2 e g + d h) + 2 a c^2 g^3 h^2 (83 f g^2 + h (19 e g + 14 d h)) (g + h x)^5 / (h^5 (c g^2 + a h^2)^4 (g + h x)^6) + (15 a^2 c^2 (6 c^2 d g^2 + 6 a^2 f h^2 - a c (f g^2 + h (-7 e g + d h))) \operatorname{Log}[g + h x]) / (c g^2 + a h^2)^{9/2} - (15 a^2 c^2 (6 c^2 d g^2 + 6 a^2 f h^2 - a c (f g^2 + h (-7 e g + d h))) \operatorname{Log}[a h - c g x + \sqrt{c g^2 + a h^2}] \sqrt{a + c x^2}) / (c g^2 + a h^2)^{9/2} / 240$$

**IntegrateAlgebraic [F]** time = 180.03, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^7,x]

[Out] \$Aborted

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^7,x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 1.12, size = 6122, normalized size = 15.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^7,x, algorithm="giac")

[Out]  $1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 - a^3*c^3*d*h^2 + 6*a^4*c^2*f*h^2 + 7*a^3*c^3*g*h*e)*\arctan(-(\sqrt{c}*x - \sqrt{c*x^2 + a})*h + \sqrt{c}*g)/\sqrt{-c*g^2 - a*h^2})/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*\sqrt{-c*g^2 - a*h^2}) + 1/120*(240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*c^6*f*g^8*h^5 + 960*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a*c^5*f*g^6*h^7 + 1440*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a^2*c^4*f*g^4*h^9 - 90*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a^2*c^4*d*g^2*h^{11} + 975*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a^3*c^3*f*g^2*h^{11} + 15*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a^3*c^3*d*h^{13} + 150*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a^4*c^2*f*h^{13} - 105*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{11}*a^3*c^3*g*h^{12}*e + 1200*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*c^{13/2}*f*g^9*h^4 + 4800*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a*c^{11/2}*f*g^7*h^6 + 7200*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a^2*c^{9/2}*f*g^5*h^8 - 990*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a^2*c^{9/2}*d*g^3*h^{10} + 4965*(\sqrt{c}($

$$\begin{aligned}
& *x - \sqrt{c*x^2 + a})^{10}*a^3*c^{(7/2)}*f*g^3*h^{10} + 165*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a^3*c^{(7/2)}*d*g*h^{12} + 210*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a^4 \\
& *c^{(5/2)}*f*g*h^{12} + 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*c^{(13/2)}*g^8*h^5*e + 960*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a*c^{(11/2)}*g^6*h^7*e + 1440*(\sqrt{c} \\
& )*x - \sqrt{c*x^2 + a})^{10}*a^2*c^{(9/2)}*g^4*h^9*e - 195*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a^3*c^{(7/2)}*g^2*h^{11}*e + 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^{10}*a \\
& ^4*c^{(5/2)}*h^{13}*e + 3200*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*c^7*f*g^{10}*h^3 + 3 \\
& 20*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*c^7*d*g^8*h^5 + 12080*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a*c^6*f*g^8*h^5 + 1280*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a*c^6* \\
& d*g^6*h^7 + 16320*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^2*c^5*f*g^6*h^7 - 2520* \\
& (\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^2*c^5*d*g^4*h^9 + 9220*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^3*c^4*f*g^4*h^9 + 2530*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^3* \\
& c^4*d*g^2*h^{11} - 4205*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^4*c^3*f*g^2*h^{11} + \\
& 235*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^4*c^3*d*h^{13} - 210*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^5*c^2*f*h^{13} + 640*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*c^7*g^9* \\
& h^4*e + 2560*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a*c^6*g^7*h^6*e + 3840*(\sqrt{c} \\
& )*x - \sqrt{c*x^2 + a})^9*a^2*c^5*g^5*h^8*e - 2620*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^3*c^4*g^3*h^{10}*e + 1235*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^4*c^3*g* \\
& h^{12}*e + 4800*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*c^{(15/2)}*f*g^{11}*h^2 + 480*(\sqrt{c} \\
& )*x - \sqrt{c*x^2 + a})^8*c^{(15/2)}*d*g^9*h^4 + 15120*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a*c^{(13/2)}*f*g^9*h^4 + 1920*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a* \\
& c^{(13/2)}*d*g^7*h^6 + 12480*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^2*c^{(11/2)}*f*g \\
& ^7*h^6 - 7380*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^2*c^{(11/2)}*d*g^5*h^8 - 3570 \\
& *( \sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^3*c^{(9/2)}*f*g^5*h^8 + 8220*(\sqrt{c}*x - \\
& \sqrt{c*x^2 + a})^8*a^3*c^{(9/2)}*d*g^3*h^{10} - 22545*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^4*c^{(7/2)}*f*g^3*h^{10} - 285*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^4*c^{( \\
& 7/2)}*d*g*h^{12} + 510*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^5*c^{(5/2)}*f*g*h^{12} + \\
& 960*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*c^{(15/2)}*g^{10}*h^3*e + 3600*(\sqrt{c}*x - \\
& \sqrt{c*x^2 + a})^8*a*c^{(13/2)}*g^8*h^5*e + 4800*(\sqrt{c}*x - \sqrt{c*x^2 + a} \\
& )^8*a^2*c^{(11/2)}*g^6*h^7*e - 9570*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^3*c^{(9 \\
& /2)}*g^4*h^9*e + 5355*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^4*c^{(7/2)}*g^2*h^{11}*e \\
& - 240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^5*c^{(5/2)}*h^{13}*e + 3840*(\sqrt{c}*x \\
& - \sqrt{c*x^2 + a})^7*c^8*f*g^{12}*h + 384*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*c^ \\
& 8*d*g^{10}*h^3 + 6336*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^7*f*g^{10}*h^3 + 1728 \\
& *( \sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^7*d*g^8*h^5 - 11808*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^6*f*g^8*h^5 - 9456*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2* \\
& c^6*d*g^6*h^7 - 31704*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^5*f*g^6*h^7 + 2 \\
& 0760*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^5*d*g^4*h^9 - 39960*(\sqrt{c}*x - \\
& \sqrt{c*x^2 + a})^7*a^4*c^4*f*g^4*h^9 - 2700*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7 \\
& *a^4*c^4*d*g^2*h^{11} + 12150*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^5*c^3*f*g^2* \\
& h^{11} + 390*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^5*c^3*d*h^{13} + 60*(\sqrt{c}*x - \\
& \sqrt{c*x^2 + a})^7*a^6*c^2*f*h^{13} + 768*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*c^ \\
& 8*g^{11}*h^2*e + 1728*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^7*g^9*h^4*e - 768*( \\
& \sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^6*g^7*h^6*e - 19608*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^5*g^5*h^8*e + 14040*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^4
\end{aligned}$$

$$\begin{aligned}
& *c^4*g^3*h^{10}*e - 2730*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^7*a^5*c^3*g*h^{12}*e + 1 \\
& 280*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*c^{(17/2)}*f*g^{13} + 128*(\text{sqrt}(c)*x - \text{sqrt} \\
& (c*x^2 + a))^6*c^{(17/2)}*d*g^{11}*h^2 - 4288*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a \\
& *c^{(15/2)}*f*g^{11}*h^2 - 64*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a*c^{(15/2)}*d*g^9* \\
& h^4 - 24096*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^2*c^{(13/2)}*f*g^9*h^4 - 8592*( \\
& \text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^2*c^{(13/2)}*d*g^7*h^6 - 26728*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^6*a^3*c^{(11/2)}*f*g^7*h^6 + 24440*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
& a))^6*a^3*c^{(11/2)}*d*g^5*h^8 - 12640*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^4*c \\
& ^{(9/2)}*f*g^5*h^8 - 14860*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^4*c^{(9/2)}*d*g^3* \\
& h^{10} + 41610*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^5*c^{(7/2)}*f*g^3*h^{10} + 810*( \\
& \text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^5*c^{(7/2)}*d*g*h^{12} - 2460*(\text{sqrt}(c)*x - \text{sqr} \\
& t(c*x^2 + a))^6*a^6*c^{(5/2)}*f*g*h^{12} + 256*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6* \\
& c^{(17/2)}*g^{12}*h*e - 704*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a*c^{(15/2)}*g^{10}*h^3 \\
& *e - 4896*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^2*c^{(13/2)}*g^8*h^5*e - 15656*(s \\
& \text{qrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^3*c^{(11/2)}*g^6*h^7*e + 26800*(\text{sqrt}(c)*x - s \\
& \text{qrt}(c*x^2 + a))^6*a^4*c^{(9/2)}*g^4*h^9*e - 9510*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a) \\
& )^6*a^5*c^{(7/2)}*g^2*h^{11}*e + 480*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^6*c^{(5/2} \\
& )*h^{13}*e - 3840*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^8*f*g^{12}*h - 384*(\text{sqrt}( \\
& c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^8*d*g^{10}*h^3 - 6336*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
& a))^5*a^2*c^7*f*g^{10}*h^3 - 1728*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c^7*d* \\
& g^8*h^5 + 11808*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^3*c^6*f*g^8*h^5 + 19056*( \\
& \text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^3*c^6*d*g^6*h^7 + 29304*(\text{sqrt}(c)*x - \text{sqrt}( \\
& c*x^2 + a))^5*a^4*c^5*f*g^6*h^7 - 21480*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^4 \\
& *c^5*d*g^4*h^9 + 46080*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^4*f*g^4*h^9 + \\
& 7020*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^4*d*g^2*h^{11} - 17370*(\text{sqrt}(c)*x \\
& - \text{sqrt}(c*x^2 + a))^5*a^6*c^3*f*g^2*h^{11} + 390*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a)) \\
& ^5*a^6*c^3*d*h^{13} + 60*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^7*c^2*f*h^{13} - 768 \\
& *(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a*c^8*g^{11}*h^2*e - 1728*(\text{sqrt}(c)*x - \text{sqrt}( \\
& c*x^2 + a))^5*a^2*c^7*g^9*h^4*e - 192*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^3*c \\
& ^6*g^7*h^6*e + 26808*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^4*c^5*g^5*h^8*e - 19 \\
& 440*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^4*g^3*h^{10}*e + 3030*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^5*a^6*c^3*g*h^{12}*e + 4800*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a \\
& ^2*c^{(15/2)}*f*g^{11}*h^2 + 480*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^2*c^{(15/2)}* \\
& d*g^9*h^4 + 15120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*c^{(13/2)}*f*g^9*h^4 + \\
& 3840*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*c^{(13/2)}*d*g^7*h^6 + 12360*(\text{sqrt}(c \\
& )*x - \text{sqrt}(c*x^2 + a))^4*a^4*c^{(11/2)}*f*g^7*h^6 - 18720*(\text{sqrt}(c)*x - \text{sqrt}(c \\
& *x^2 + a))^4*a^4*c^{(11/2)}*d*g^5*h^8 + 1020*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a \\
& ^5*c^{(9/2)}*f*g^5*h^8 + 11640*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^5*c^{(9/2)}*d \\
& *g^3*h^{10} - 32490*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^6*c^{(7/2)}*f*g^3*h^{10} - \\
& 930*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^6*c^{(7/2)}*d*g*h^{12} + 3180*(\text{sqrt}(c)*x \\
& - \text{sqrt}(c*x^2 + a))^4*a^7*c^{(5/2)}*f*g*h^{12} + 960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a \\
& ))^4*a^2*c^{(15/2)}*g^{10}*h^3*e + 3600*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*c^{( \\
& 13/2)}*g^8*h^5*e + 7080*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^4*c^{(11/2)}*g^6*h^7 \\
& *e - 22260*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^5*c^{(9/2)}*g^4*h^9*e + 7470*(sq \\
& rt(c)*x - \text{sqrt}(c*x^2 + a))^4*a^6*c^{(7/2)}*g^2*h^{11}*e - 480*(\text{sqrt}(c)*x - \text{sqrt}
\end{aligned}$$

$$\begin{aligned}
& (c*x^2 + a)^4*a^7*c^{(5/2)}*h^{13}*e - 3200*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^3*c^7*d*g^8*h^5 - \\
& 12080*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^4*c^6*f*g^8*h^5 - 2960*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^4*c^6*d*g^6*h^7 - 16440*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^5*c^5*f*g^6*h^7 + 12120*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^5*c^5*d*g^4*h^9 - 14120*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^6*c^4*f*g^4*h^9 - 2330*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^6*c^4*d*g^2*h^{11} + 10555*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^7*c^3*f*g^2*h^{11} + 235*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^7*c^3*d*h^{13} - 210*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^8*c^2*f*h^{13} - 640*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^3*c^7*g^9*h^4*e - 3040*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^4*c^6*g^7*h^6*e - 7800*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^5*c^5*g^5*h^8*e + 10280*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^3*a^6*c^4*g^3*h^{10}*e - 1645*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^3*a^7*c^3*g*h^{12}*e + 1200*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^4*c^{(13/2)}*f*g^9*h^4 + 240*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^4*c^{(13/2)}*d*g^7*h^6 + 4920*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^5*c^{(11/2)}*f*g^7*h^6 + 1656*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^5*c^{(11/2)}*d*g^5*h^8 + 7824*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^6*c^{(9/2)}*f*g^5*h^8 - 4038*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^6*c^{(9/2)}*d*g^3*h^{10} + 8193*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^7*c^{(7/2)}*f*g^3*h^{10} + 321*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^7*c^{(7/2)}*d*g*h^{12} - 1686*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^8*c^{(5/2)}*f*g*h^{12} + 2400*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^4*c^{(13/2)}*g^8*h^5*e + 1272*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^5*c^{(11/2)}*g^6*h^7*e + 3552*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^6*c^{(9/2)}*g^4*h^9*e - 3207*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*a^7*c^{(7/2)}*g^2*h^{11}*e + 48*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))^2*a^8*c^{(5/2)}*h^{13}*e - 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^5*c^6*f*g^8*h^5 - 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^5*c^6*d*g^6*h^7 - \\
& 1032*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^6*c^5*f*g^6*h^7 - 336*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^6*c^5*d*g^4*h^9 - 1764*(\text{sqrt}(c)*x - \\
& \text{sqrt}(c*x^2 + a))*a^7*c^4*f*g^4*h^9 + 882*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^7*c^4*d*g^2*h^{11} - 1977*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^8*c^3*f*g^2*h^{11} + \\
& 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^8*c^3*d*h^{13} + 150*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^9*c^2*f*h^{13} - 96*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^5*c^6*g^7*h^6*e - \\
& 456*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^6*c^5*g^5*h^8*e - 1044*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^7*c^4*g^3*h^{10}*e + 471*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*a^8*c^3*g*h^{12}*e + \\
& 40*a^6*c^{(11/2)}*f*g^7*h^6 + 4*a^6*c^{(11/2)}*d*g^5*h^8 + 166*a^7*c^{(9/2)}*f*g^5*h^8 + 28*a^7*c^{(9/2)}*d*g^3*h^{10} + 267*a^8*c^{(7/2)}*f*g^3*h^{10} - 81*a^8*c^{(7/2)}*d*g*h^{12} + \\
& 246*a^9*c^{(5/2)}*f*g*h^{12} + 8*a^6*c^{(11/2)}*g^6*h^7*e + 38*a^7*c^{(9/2)}*g^4*h^9*e + 87*a^8*c^{(7/2)}*g^2*h^{11}*e - 48*a^9*c^{(5/2)}*h^{13}*e)/((c^4*g^8*h^6 + 4*a*c^3*g^6*h^8 + 6*a^2*c^2*g^4*h^{10} + 4*a^3*c*g^2*h^{12} + a^4*h^{14})*((\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^2*h + 2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))*\text{sqrt}(c)*g - a*h)^6)
\end{aligned}$$

**maple [B]** time = 0.04, size = 17026, normalized size = 42.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g)^7,x)$

[Out] result too large to display

**maxima** [B] time = 2.73, size = 10724, normalized size = 26.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2+a)^{(3/2)}*(f*x^2+e*x+d)/(h*x+g)^7,x, \text{algorithm}=\text{"maxima"})$

[Out] 
$$\begin{aligned} & \frac{7}{16}\sqrt{c*x^2 + a}*c^6*f*g^8/(c^5*g^{10}*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^3*g^6*h^9 \\ & + 10*a^3*c^2*g^4*h^{11} + 5*a^4*c*g^2*h^{13} + a^5*h^{15}) - \frac{7}{16}\sqrt{c*x^2 + a}*c^6*f*g^7*x/(c^5*g^{10}*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 \\ & + 10*a^3*c^2*g^4*h^{10} + 5*a^4*c*g^2*h^{12} + a^5*h^{14}) - \frac{7}{16}\sqrt{c*x^2 + a}*c^6*e*g^7/(c^5*g^{10}*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^2*g^4*h^{10} \\ & + 5*a^4*c*g^2*h^{12} + a^5*h^{14}) + \frac{7}{48}(c*x^2 + a)^{(3/2)}*c^5*f*g^7/(c^5*g^{10}*h^4*x + 5*a*c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 10*a^3*c^2*g^4*h^{10}*x \\ & + 5*a^4*c*g^2*h^{12}*x + a^5*h^{14}*x + c^5*g^{11}*h^3 + 5*a*c^4*g^9*h^5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^{11} + a^5*g*h^{13}) \\ & + \frac{7}{16}\sqrt{c*x^2 + a}*c^6*e*g^6*x/(c^5*g^{10}*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^{11} + a^5*h^{13}) + \frac{7}{16}\sqrt{c*x^2 + a}*c^6*d*g^6/(c^5*g^{10}*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^{11} + a^5*h^{13}) - \frac{7}{48}(c*x^2 + a)^{(3/2)}*c^5*e*g^6/(c^5*g^{10}*h^3*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c^3*g^6*h^7*x + 10*a^3*c^2*g^4*h^9*x + 5*a^4*c*g^2*h^{11}*x + a^5*h^{13}*x + c^5*g^{11}*h^2 + 5*a*c^4*g^9*h^4 + 10*a^2*c^3*g^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5*a^4*c*g^3*h^{10} + a^5*g*h^{12}) - \frac{7}{48}(c*x^2 + a)^{(5/2)}*c^4*f*g^6/(c^5*g^{10}*h^3*x^2 + 5*a*c^4*g^8*h^5*x^2 + 10*a^2*c^3*g^6*h^7*x^2 + 10*a^3*c^2*g^4*h^9*x^2 + 5*a^4*c*g^2*h^{11}*x^2 + a^5*h^{13}*x^2 + 2*c^5*g^{11}*h^2*x + 10*a*c^4*g^9*h^4*x + 20*a^2*c^3*g^7*h^6*x + 20*a^3*c^2*g^5*h^8*x + 10*a^4*c*g^3*h^{10}*x + 2*a^5*g*h^{12}*x + c^5*g^{12}*h + 5*a*c^4*g^{10}*h^3 + 10*a^2*c^3*g^8*h^5 + 10*a^3*c^2*g^6*h^7 + 5*a^4*c*g^4*h^9 + a^5*g^2*h^{11}) + \frac{7}{48}(c*x^2 + a)^{(3/2)}*c^5*f*g^6/(c^5*g^{10}*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^{11} + a^5*h^{13}) - \frac{7}{16}\sqrt{c*x^2 + a}*c^6*d*g^5*x/(c^5*g^{10}*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^{10} + a^5*h^{12}) + \frac{7}{48}(c*x^2 + a)^{(3/2)}*c^5*d*g^5/(c^5*g^{10}*h^2*x + 5*a*c^4*g^8*h^4*x + 10*a^2*c^3*g^6*h^6*x + 10*a^3*c^2*g^4*h^8*x + 5*a^4*c*g^2*h^{10}*x + a^5*h^{12}*x + c^5*g^{11}*h + 5*a*c^4*g^9*h^3 + 10*a^2*c^3*g^7*h^5 + 10*a^3*c^2*g^5*h^7 + 5*a^4*c*g^3*h^9 + a^5*g*h^{11}) + \frac{7}{48}(c*x^2 + a)^{(5/2)}*c^4*e*g^5/(c^5*g^{10}*h^2*x^2 + 5*a*c^4*g^8*h^4*x^2 + 10*a^2*c^3*g^6*h^6*x^2 + 10*a^3*c^2*g^4*h^8*x^2 + 5*a^4*c*g^2*h^{10}*x^2 + a^5*h^{12}*x^2 + 2*c^5*g^{11}*h*x + 10*a*c^4*g^9*h^3*x + 20*a^2*c^3*g^7*h^5*x + 20*a^3*c^2*g^5*h^7*x + 10*a^4*c*g^3*h^9*x + 2*a^5*g*h^{11}*x + c^5*g^{12} + 5*a*c^4*g^{10}*h^2 + 10*a^2*c^3*g^8*h^4 + 10*a^3*c^2*g^6*h^6 + 5*a^4*c*g^4*h^8 + a^5*g^2*h^{10}) - \frac{7}{48}(c*x^2 + a)^{(3/2)}*c^5*e*g^5/(c^5*g^{10}*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6) \end{aligned}$$

$$\begin{aligned}
& + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^{10} + a^5*h^{12}) - 27/16*\sqrt{c*x^2 + a} \\
& *c^5*f*g^6/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^4*h^9 + 4*a^3*c*g^2 \\
& *h^{11} + a^4*h^{13}) + 5/4*\sqrt{c*x^2 + a}*c^5*f*g^5*x/(c^4*g^8*h^4 + 4*a*c^3* \\
& g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^{10} + a^4*h^{12}) - 7/48*(c*x^2 + \\
& a)^{(5/2)}*c^4*d*g^4/(c^5*g^{10}*h*x^2 + 5*a*c^4*g^8*h^3*x^2 + 10*a^2*c^3*g^6*h \\
& ^5*x^2 + 10*a^3*c^2*g^4*h^7*x^2 + 5*a^4*c*g^2*h^9*x^2 + a^5*h^{11}*x^2 + 2*c^ \\
& 5*g^{11}*x + 10*a*c^4*g^9*h^2*x + 20*a^2*c^3*g^7*h^4*x + 20*a^3*c^2*g^5*h^6*x \\
& + 10*a^4*c*g^3*h^8*x + 2*a^5*g*h^{10}*x + c^5*g^{12}/h + 5*a*c^4*g^{10}*h + 10*a \\
& ^2*c^3*g^8*h^3 + 10*a^3*c^2*g^6*h^5 + 5*a^4*c*g^4*h^7 + a^5*g^2*h^9) + 7/48 \\
& *(c*x^2 + a)^{(3/2)}*c^5*d*g^4/(c^5*g^{10}*h + 5*a*c^4*g^8*h^3 + 10*a^2*c^3*g^6 \\
& *h^5 + 10*a^3*c^2*g^4*h^7 + 5*a^4*c*g^2*h^9 + a^5*h^{11}) + 21/16*\sqrt{c*x^2 \\
& + a}*c^5*e*g^5/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c \\
& *g^2*h^{10} + a^4*h^{12}) - 7/24*(c*x^2 + a)^{(5/2)}*c^3*f*g^5/(c^4*g^8*h^4*x^3 + \\
& 4*a*c^3*g^6*h^6*x^3 + 6*a^2*c^2*g^4*h^8*x^3 + 4*a^3*c*g^2*h^{10}*x^3 + a^4*h \\
& ^{12}*x^3 + 3*c^4*g^9*h^3*x^2 + 12*a*c^3*g^7*h^5*x^2 + 18*a^2*c^2*g^5*h^7*x^2 \\
& + 12*a^3*c*g^3*h^9*x^2 + 3*a^4*g*h^{11}*x^2 + 3*c^4*g^{10}*h^2*x + 12*a*c^3*g^ \\
& 8*h^4*x + 18*a^2*c^2*g^6*h^6*x + 12*a^3*c*g^4*h^8*x + 3*a^4*g^2*h^{10}*x + c^ \\
& 4*g^{11}*h + 4*a*c^3*g^9*h^3 + 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3* \\
& h^9) - 17/24*(c*x^2 + a)^{(3/2)}*c^4*f*g^5/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x \\
& + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^{10}*x + a^4*h^{12}*x + c^4*g^9*h^3 + 4* \\
& a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^{11}) - 7/8*\sqrt{ \\
& t(c*x^2 + a)*c^5*e*g^4*x/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 \\
& + 4*a^3*c*g^2*h^9 + a^4*h^{11}) - 15/16*\sqrt{c*x^2 + a}*c^5*d*g^4/(c^4*g^8*h \\
& ^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^{11}) + 7/ \\
& 24*(c*x^2 + a)^{(5/2)}*c^3*e*g^4/(c^4*g^8*h^3*x^3 + 4*a*c^3*g^6*h^5*x^3 + 6*a \\
& ^2*c^2*g^4*h^7*x^3 + 4*a^3*c*g^2*h^9*x^3 + a^4*h^{11}*x^3 + 3*c^4*g^9*h^2*x^2 \\
& + 12*a*c^3*g^7*h^4*x^2 + 18*a^2*c^2*g^5*h^6*x^2 + 12*a^3*c*g^3*h^8*x^2 + 3 \\
& *a^4*g*h^{10}*x^2 + 3*c^4*g^{10}*h*x + 12*a*c^3*g^8*h^3*x + 18*a^2*c^2*g^6*h^5* \\
& x + 12*a^3*c*g^4*h^7*x + 3*a^4*g^2*h^9*x + c^4*g^{11} + 4*a*c^3*g^9*h^2 + 6*a \\
& ^2*c^2*g^7*h^4 + 4*a^3*c*g^5*h^6 + a^4*g^3*h^8) + 7/12*(c*x^2 + a)^{(3/2)}*c^ \\
& 4*e*g^4/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c* \\
& g^2*h^9*x + a^4*h^{11}*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 \\
& + 4*a^3*c*g^3*h^8 + a^4*g*h^{10}) + 1/8*(c*x^2 + a)^{(5/2)}*c^3*f*g^4/(c^4*g^8* \\
& h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 \\
& + a^4*h^{11}*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6* \\
& x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^{10}*x + c^4*g^{10}*h + 4*a*c^3*g^8*h^3 + 6*a \\
& ^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) - 1/8*(c*x^2 + a)^{(3/2)}*c^4 \\
& *f*g^4/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 \\
& + a^4*h^{11}) + 1/2*\sqrt{c*x^2 + a}*c^5*d*g^3*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h \\
& ^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^{10}) - 7/24*(c*x^2 + a)^{(5/ \\
& 2)}*c^3*d*g^3/(c^4*g^8*h^2*x^3 + 4*a*c^3*g^6*h^4*x^3 + 6*a^2*c^2*g^4*h^6*x^3 \\
& + 4*a^3*c*g^2*h^8*x^3 + a^4*h^{10}*x^3 + 3*c^4*g^9*h*x^2 + 12*a*c^3*g^7*h^3* \\
& x^2 + 18*a^2*c^2*g^5*h^5*x^2 + 12*a^3*c*g^3*h^7*x^2 + 3*a^4*g*h^9*x^2 + 3*c \\
& ^4*g^{10}*x + 12*a*c^3*g^8*h^2*x + 18*a^2*c^2*g^6*h^4*x + 12*a^3*c*g^4*h^6*x \\
& + 3*a^4*g^2*h^8*x + c^4*g^{11}/h + 4*a*c^3*g^9*h + 6*a^2*c^2*g^7*h^3 + 4*a^3*
\end{aligned}$$



$$\begin{aligned}
& c^4 g^5 h^5 + a^4 g^3 h^7) - 11/24 * (c^2 x^2 + a)^{3/2} * c^4 d g^3 / (c^4 g^8 h^2 x \\
& + 4 a^3 c^3 g^6 h^4 x + 6 a^2 c^2 g^4 h^6 x + 4 a^3 c^3 g^2 h^8 x + a^4 h^{10} x \\
& + c^4 g^9 h + 4 a^3 c^3 g^7 h^3 + 6 a^2 c^2 g^5 h^5 + 4 a^3 c^3 g^3 h^7 + a^4 g^9 h) - 7/24 * (c^2 x^2 + a)^{5/2} * c^2 f g^4 / (c^3 g^6 h^5 x^4 + 3 a^3 c^2 g^4 h^7 \\
& x^4 + 3 a^2 c^2 g^2 h^9 x^4 + a^3 h^{11} x^4 + 4 c^3 g^7 h^4 x^3 + 12 a^2 c^2 g^5 h^6 x^3 + 12 a^2 c^2 g^3 h^8 x^3 + 4 a^3 g^2 h^{10} x^3 + 6 c^3 g^8 h^3 x^2 + \\
& 18 a^2 c^2 g^6 h^5 x^2 + 18 a^2 c^2 g^4 h^7 x^2 + 6 a^3 g^2 h^9 x^2 + 4 c^3 g^9 h^2 x + 12 a^2 c^2 g^7 h^4 x + 12 a^2 c^2 g^5 h^6 x + 4 a^3 g^3 h^8 x + c^3 g^ \\
& 10 h + 3 a^2 c^2 g^8 h^3 + 3 a^2 c^2 g^6 h^5 + a^3 g^4 h^7) + 39/16 * \text{sqrt}(c^2 x^2 + a) * c^4 f g^4 / (c^3 g^6 h^5 + 3 a^3 c^2 g^4 h^7 + 3 a^2 c^2 g^2 h^9 + a^3 h^{11}) \\
& - 19/16 * \text{sqrt}(c^2 x^2 + a) * c^4 f g^3 x / (c^3 g^6 h^4 + 3 a^3 c^2 g^4 h^6 + 3 a^2 c^2 g^2 h^8 + a^3 h^{10}) - 1/8 * (c^2 x^2 + a)^{5/2} * c^3 d g^2 / (c^4 g^8 h x^2 + 4 \\
& a^3 c^3 g^6 h^3 x^2 + 6 a^2 c^2 g^4 h^5 x^2 + 4 a^3 c^3 g^2 h^7 x^2 + a^4 h^9 x^2 + 2 c^4 g^9 x + 8 a^3 c^3 g^7 h^2 x + 12 a^2 c^2 g^5 h^4 x + 8 a^3 c^3 g^3 h^6 x \\
& + 2 a^4 g^8 x + c^4 g^{10} / h + 4 a^3 c^3 g^8 h + 6 a^2 c^2 g^6 h^3 + 4 a^3 c^3 g^4 h^5 + a^4 g^2 h^7) + 1/8 * (c^2 x^2 + a)^{3/2} * c^4 d g^2 / (c^4 g^8 h + \\
& 4 a^3 c^3 g^6 h^3 + 6 a^2 c^2 g^4 h^5 + 4 a^3 c^3 g^2 h^7 + a^4 h^9) + 7/24 * (c^2 x^2 + a)^{5/2} * c^2 e g^3 / (c^3 g^6 h^4 x^4 + 3 a^3 c^2 g^4 h^6 x^4 + 3 a^2 c^2 g^2 h^8 x^4 \\
& + a^3 h^{10} x^4 + 4 c^3 g^7 h^3 x^3 + 12 a^2 c^2 g^5 h^5 x^3 + 12 a^2 c^2 g^3 h^7 x^3 + 4 a^3 g^2 h^9 x^3 + 6 c^3 g^8 h^2 x^2 + 18 a^2 c^2 g^6 h^4 x^2 + 18 a^2 c^2 g^4 h^6 x^2 \\
& + 6 a^3 g^2 h^8 x^2 + 4 c^3 g^9 h x + 12 a^2 c^2 g^7 h^3 x + 12 a^2 c^2 g^5 h^5 x + 4 a^3 g^3 h^7 x + c^3 g^{10} + 3 a^2 c^2 g^8 h^2 + 3 a^2 c^2 g^6 h^4 + a^3 g^4 h^6) - 21/16 * \text{sqrt}(c^2 x^2 + a) * c^4 e g^3 / (c^3 g^6 h^4 + 3 a^3 c^2 g^4 h^6 + 3 a^2 c^2 g^2 h^8 + a^3 h^{10}) + 13/24 * (c^2 x^2 + a)^{5/2} * c^2 f g^3 / (c^3 g^6 h^4 x^3 + 3 a^3 c^2 g^4 h^6 x^3 + 3 a^2 c^2 g^2 h^8 x^3 \\
& + a^3 h^{10} x^3 + 3 c^3 g^7 h^3 x^2 + 9 a^2 c^2 g^5 h^5 x^2 + 9 a^2 c^2 g^3 h^7 x^2 + 3 a^3 g^2 h^9 x^2 + 3 c^3 g^8 h^2 x + 9 a^2 c^2 g^6 h^4 x + 9 a^2 c^2 g^4 h^6 x + 3 a^3 g^2 h^8 x + c^3 g^9 h + 3 a^2 c^2 g^7 h^3 + 3 a^2 c^2 g^5 h^5 + \\
& a^3 g^3 h^7) + 15/16 * (c^2 x^2 + a)^{3/2} * c^3 f g^3 / (c^3 g^6 h^4 x + 3 a^3 c^2 g^4 h^6 x + 3 a^2 c^2 g^2 h^8 x + a^3 h^{10} x + c^3 g^7 h^3 + 3 a^2 c^2 g^5 h^5 + \\
& 3 a^2 c^2 g^3 h^7 + a^3 g^2 h^9) + 7/16 * \text{sqrt}(c^2 x^2 + a) * c^4 e g^2 x / (c^3 g^6 h^3 + 3 a^3 c^2 g^4 h^5 + 3 a^2 c^2 g^2 h^7 + a^3 h^9) - 7/24 * (c^2 x^2 + a)^{5/2} * c^2 d g^2 / (c^3 g^6 h^3 x^4 + 3 a^3 c^2 g^4 h^5 x^4 + 3 a^2 c^2 g^2 h^7 x^4 + a^3 h^9 x^4 + 4 c^3 g^7 h^2 x^3 + 12 a^2 c^2 g^5 h^4 x^3 + 12 a^2 c^2 g^3 h^6 x^3 + 4 a^3 g^2 h^8 x^3 + 6 c^3 g^8 h x^2 + 18 a^2 c^2 g^6 h^3 x^2 + 18 a^2 c^2 g^4 h^5 x^2 + 6 a^3 g^2 h^7 x^2 + 4 c^3 g^9 x + 12 a^2 c^2 g^7 h^2 x + 12 a^2 c^2 g^5 h^4 x + 4 a^3 g^3 h^6 x + c^3 g^{10} / h + 3 a^2 c^2 g^8 h + 3 a^2 c^2 g^6 h^3 + a^3 g^4 h^5) + 9/16 * \text{sqrt}(c^2 x^2 + a) * c^4 d g^2 / (c^3 g^6 h^3 + 3 a^3 c^2 g^4 h^5 + 3 a^2 c^2 g^2 h^7 + a^3 h^9) - 7/24 * (c^2 x^2 + a)^{5/2} * c^2 e g^2 / (c^3 g^6 h^3 x^3 + 3 a^3 c^2 g^4 h^5 x^3 + 3 a^2 c^2 g^2 h^7 x^3 + a^3 h^9 x^3 + 3 c^3 g^7 h^2 x^2 + 9 a^2 c^2 g^5 h^4 x^2 + 9 a^2 c^2 g^3 h^6 x^2 + 3 a^3 g^2 h^8 x^2 + 3 c^3 g^8 h x + 9 a^2 c^2 g^6 h^3 x + 9 a^2 c^2 g^4 h^5 x + 3 a^3 g^2 h^7 x + c^3 g^9 + 3 a^2 c^2 g^7 h^2 + 3 a^2 c^2 g^5 h^4 + a^3 g^3 h^6) - 7/16 * (c^2 x^2 + a)^{3/2} * c^3 e g^2 / (c^3 g^6 h^3 x + 3 a^3 c^2 g^4 h^5 x + 3 a^2 c^2 g^2 h^7 x + a^3 h^9 x + c^3 g^7 h^2 + 3 a^2 c^2 g^5 h^4 + 3 a^2 c^2 g^3 h^6 + a^3 g^2 h^8) +
\end{aligned}$$

$$\begin{aligned}
& 7/48*(c*x^2 + a)^{(5/2)}*c^2*f*g^2/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + \\
& 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6 \\
& *a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^ \\
& 4*h^5 + a^3*g^2*h^7) - 7/48*(c*x^2 + a)^{(3/2)}*c^3*f*g^2/(c^3*g^6*h^3 + 3*a* \\
& c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/30*(c*x^2 + a)^{(5/2)}*c*f*g^3/( \\
& c^2*g^4*h^6*x^5 + 2*a*c*g^2*h^8*x^5 + a^2*h^10*x^5 + 5*c^2*g^5*h^5*x^4 + 10 \\
& *a*c*g^3*h^7*x^4 + 5*a^2*g*h^9*x^4 + 10*c^2*g^6*h^4*x^3 + 20*a*c*g^4*h^6*x^ \\
& 3 + 10*a^2*g^2*h^8*x^3 + 10*c^2*g^7*h^3*x^2 + 20*a*c*g^5*h^5*x^2 + 10*a^2*g \\
& ^3*h^7*x^2 + 5*c^2*g^8*h^2*x + 10*a*c*g^6*h^4*x + 5*a^2*g^4*h^6*x + c^2*g^9 \\
& *h + 2*a*c*g^7*h^3 + a^2*g^5*h^5) - 1/16*sqrt(c*x^2 + a)*c^4*d*g*x/(c^3*g^6 \\
& *h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/24*(c*x^2 + a)^{(5/2)} \\
& )*c^2*d/g/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^ \\
& 3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3 \\
& *a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^ \\
& 3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) + \\
& 1/16*(c*x^2 + a)^{(3/2)}*c^3*d/g/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c \\
& *g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^ \\
& 3*g*h^7) - 7/48*(c*x^2 + a)^{(5/2)}*c^2*e*g/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^ \\
& 4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3 \\
& *x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2* \\
& c*g^4*h^4 + a^3*g^2*h^6) + 7/48*(c*x^2 + a)^{(3/2)}*c^3*e*g/(c^3*g^6*h^2 + 3* \\
& a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 7/30*(c*x^2 + a)^{(5/2)}*c*e*g^2 \\
& /(c^2*g^4*h^5*x^5 + 2*a*c*g^2*h^7*x^5 + a^2*h^9*x^5 + 5*c^2*g^5*h^4*x^4 + 1 \\
& 0*a*c*g^3*h^6*x^4 + 5*a^2*g*h^8*x^4 + 10*c^2*g^6*h^3*x^3 + 20*a*c*g^4*h^5*x \\
& ^3 + 10*a^2*g^2*h^7*x^3 + 10*c^2*g^7*h^2*x^2 + 20*a*c*g^5*h^4*x^2 + 10*a^2* \\
& g^3*h^6*x^2 + 5*c^2*g^8*h*x + 10*a*c*g^6*h^3*x + 5*a^2*g^4*h^5*x + c^2*g^9 \\
& + 2*a*c*g^7*h^2 + a^2*g^5*h^4) + 13/24*(c*x^2 + a)^{(5/2)}*c*f*g^2/(c^2*g^4*h \\
& ^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^ \\
& 6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^ \\
& 2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h \\
& + 2*a*c*g^6*h^3 + a^2*g^4*h^5) - 25/16*sqrt(c*x^2 + a)*c^3*f*g^2/(c^2*g^4* \\
& h^5 + 2*a*c*g^2*h^7 + a^2*h^9) + 3/8*sqrt(c*x^2 + a)*c^3*f*g*x/(c^2*g^4*h^4 \\
& + 2*a*c*g^2*h^6 + a^2*h^8) + 1/48*(c*x^2 + a)^{(5/2)}*c^2*d/(c^3*g^6*h*x^2 + \\
& 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6* \\
& a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g \\
& ^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 1/48*(c*x^2 + a)^{(3/2)}*c^3*d/(c^3*g \\
& ^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 7/30*(c*x^2 + a)^{(5/2)} \\
& )*c*d/g/(c^2*g^4*h^4*x^5 + 2*a*c*g^2*h^6*x^5 + a^2*h^8*x^5 + 5*c^2*g^5*h^3* \\
& x^4 + 10*a*c*g^3*h^5*x^4 + 5*a^2*g*h^7*x^4 + 10*c^2*g^6*h^2*x^3 + 20*a*c*g^ \\
& 4*h^4*x^3 + 10*a^2*g^2*h^6*x^3 + 10*c^2*g^7*h*x^2 + 20*a*c*g^5*h^3*x^2 + 10 \\
& *a^2*g^3*h^5*x^2 + 5*c^2*g^8*x + 10*a*c*g^6*h^2*x + 5*a^2*g^4*h^4*x + c^2*g \\
& ^9/h + 2*a*c*g^7*h + a^2*g^5*h^3) - 7/24*(c*x^2 + a)^{(5/2)}*c*e*g/(c^2*g^4*h \\
& ^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^ \\
& 5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^ \\
& 2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2
\end{aligned}$$

$$\begin{aligned}
& *a*c*g^6*h^2 + a^2*g^4*h^4) + 7/16*\sqrt{c*x^2 + a}*c^3*e*g/(c^2*g^4*h^4 + 2 \\
& *a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^{(5/2)}*c*f*g/(c^2*g^4*h^4*x^3 + 2* \\
& a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a \\
& ^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^ \\
& 7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 3/8*(c*x^2 + a)^{(3/2)}*c^2*f*g/(c^2*g^4 \\
& *h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g* \\
& h^7) - 1/6*(c*x^2 + a)^{(5/2)}*f*g^2/(c*g^2*h^7*x^6 + a*h^9*x^6 + 6*c*g^3*h^6 \\
& *x^5 + 6*a*g*h^8*x^5 + 15*c*g^4*h^5*x^4 + 15*a*g^2*h^7*x^4 + 20*c*g^5*h^4*x \\
& ^3 + 20*a*g^3*h^6*x^3 + 15*c*g^6*h^3*x^2 + 15*a*g^4*h^5*x^2 + 6*c*g^7*h^2*x \\
& + 6*a*g^5*h^4*x + c*g^8*h + a*g^6*h^3) + 1/24*(c*x^2 + a)^{(5/2)}*c*d/(c^2*g \\
& ^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^ \\
& 3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2* \\
& g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h + \\
& 2*a*c*g^6*h + a^2*g^4*h^3) - 1/16*\sqrt{c*x^2 + a}*c^3*d/(c^2*g^4*h^3 + 2*a \\
& *c*g^2*h^5 + a^2*h^7) - 1/8*(c*x^2 + a)^{(5/2)}*c*f/(c^2*g^4*h^3*x^2 + 2*a*c* \\
& g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6 \\
& *x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/8*(c*x^2 + a)^{(3/2)}*c^2*f \\
& /(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/6*(c*x^2 + a)^{(5/2)}*e*g/(c*g^2 \\
& *h^6*x^6 + a*h^8*x^6 + 6*c*g^3*h^5*x^5 + 6*a*g*h^7*x^5 + 15*c*g^4*h^4*x^4 + \\
& 15*a*g^2*h^6*x^4 + 20*c*g^5*h^3*x^3 + 20*a*g^3*h^5*x^3 + 15*c*g^6*h^2*x^2 \\
& + 15*a*g^4*h^4*x^2 + 6*c*g^7*h*x + 6*a*g^5*h^3*x + c*g^8 + a*g^6*h^2) + 2/5 \\
& *(c*x^2 + a)^{(5/2)}*f*g/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g \\
& *h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^ \\
& 3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/6*(c*x \\
& ^2 + a)^{(5/2)}*d/(c*g^2*h^5*x^6 + a*h^7*x^6 + 6*c*g^3*h^4*x^5 + 6*a*g*h^6*x^ \\
& 5 + 15*c*g^4*h^3*x^4 + 15*a*g^2*h^5*x^4 + 20*c*g^5*h^2*x^3 + 20*a*g^3*h^4*x \\
& ^3 + 15*c*g^6*h*x^2 + 15*a*g^4*h^3*x^2 + 6*c*g^7*x + 6*a*g^5*h^2*x + c*g^8/h \\
& + a*g^6*h) - 1/5*(c*x^2 + a)^{(5/2)}*e/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3 \\
& *h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h \\
& ^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2 \\
& ) - 1/4*(c*x^2 + a)^{(5/2)}*f/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + \\
& 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3 \\
& *h^4*x + c*g^6*h + a*g^4*h^3) + 3/8*\sqrt{c*x^2 + a}*c^2*f/(c*g^2*h^5 + a*h^ \\
& 7) + 7/16*c^6*f*g^8*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c} \\
& *\operatorname{abs}(h*x + g)))/((a + c*g^2/h^2)^{(9/2)}*h^{15}) - 7/16*c^6*e*g^7*\operatorname{arcsinh}(c*g*x \\
& /(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/((a + c*g^2/h^2)^ \\
& (9/2)*h^{14}) + 7/16*c^6*d*g^6*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/( \\
& \sqrt{a*c})*\operatorname{abs}(h*x + g))/((a + c*g^2/h^2)^{(9/2)}*h^{13}) - 27/16*c^5*f*g^6*\operatorname{arc} \\
& \operatorname{sinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/((a + c \\
& *g^2/h^2)^{(7/2)}*h^{13}) + 21/16*c^5*e*g^5*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + \\
& g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/((a + c*g^2/h^2)^{(7/2)}*h^{12}) - 15/16*c^ \\
& 5*d*g^4*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g \\
& ))/((a + c*g^2/h^2)^{(7/2)}*h^{11}) + 39/16*c^4*f*g^4*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c} \\
& *\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g))/((a + c*g^2/h^2)^{(5/2)}*h^{11}) \\
& - 21/16*c^4*e*g^3*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*
\end{aligned}$$

```
abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 9/16*c^4*d*g^2*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
5/2)*h^9) - 25/16*c^3*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 7/16*c^3*e*g*arcsinh(
c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/
h^2)^(3/2)*h^8) - 1/16*c^3*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 3/8*c^2*f*arcsinh(c*
g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^
2/h^2)*h^7)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7, x)
```

```
[Out] Timed out
```

$$3.99 \quad \int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8} dx$$

Optimal. Leaf size=532

$$\frac{(a+cx^2)^{5/2} (42a^2fh^4 - ach^2(26fg^2 - h(61eg - 12dh)) - c^2g^2(h(2eg - 51dh) + 5fg^2))}{210h(g+hx)^5 (ah^2 + cg^2)^3} - \frac{ac^2\sqrt{a+cx^2}(ah - cg)}{210h(g+hx)^5 (ah^2 + cg^2)^3}$$

**Rubi [A]** time = 0.89, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {1651, 835, 807, 721, 725, 206}

$$\frac{a^2\sqrt{a+cx^2}(ah-cg)(f^2h^2fg-ah-eg(f^2-4hfg-3ah)+c^2d^2)}{14hg+h^2(ah^2+cg^2)} - \frac{(a+cx^2)^{5/2}(ah-cg)(f^2h^2fg-ah-eg(f^2-4hfg-3ah)+c^2d^2)}{210g+h^2(ah^2+cg^2)} - \frac{(a+cx^2)^{5/2}(42a^2fh^4-ach^2(26fg^2-h(61eg-12dh))-c^2g^2(h(2eg-51dh)+5fg^2))}{210hg+h^2(ah^2+cg^2)} - \frac{a^2\sqrt{a+cx^2}\operatorname{ArcTanh}\left(\frac{ah-cg}{\sqrt{ah^2+cg^2}}\right)}{14(ah^2+cg^2)^{3/2}} - \frac{(f^2h^2fg-ah-eg(f^2-4hfg-3ah)+c^2d^2)}{42hg+h^2(ah^2+cg^2)} - \frac{(a+cx^2)^{5/2}(ah-cg)(f^2-4hfg-3ah)+c^2d^2}{210g+h^2(ah^2+cg^2)}$$

Antiderivative was successfully verified.

[In] Int[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^8, x]

[Out] -(a\*c^2\*(6\*c^2\*d\*g^3 + a^2\*h^2\*(8\*f\*g - e\*h) - a\*c\*g\*(f\*g^2 - h\*(8\*e\*g - 3\*d\*h)))\*(a\*h - c\*g\*x)\*Sqrt[a + c\*x^2])/(16\*(c\*g^2 + a\*h^2)^5\*(g + h\*x)^2) - (c\*(6\*c^2\*d\*g^3 + a^2\*h^2\*(8\*f\*g - e\*h) - a\*c\*g\*(f\*g^2 - h\*(8\*e\*g - 3\*d\*h)))\*(a\*h - c\*g\*x)\*(a + c\*x^2)^(3/2))/(24\*(c\*g^2 + a\*h^2)^4\*(g + h\*x)^4) - ((f\*g^2 - e\*g\*h + d\*h^2)\*(a + c\*x^2)^(5/2))/(7\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^7) + ((5\*c\*f\*g^3 + c\*g\*h\*(2\*e\*g - 9\*d\*h) + 7\*a\*h^2\*(2\*f\*g - e\*h))\*(a + c\*x^2)^(5/2))/(42\*h\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)^6) - ((42\*a^2\*f\*h^4 - c^2\*(5\*f\*g^4 + g^2\*h\*(2\*e\*g - 51\*d\*h)) - a\*c\*h^2\*(26\*f\*g^2 - h\*(61\*e\*g - 12\*d\*h)))\*(a + c\*x^2)^(5/2))/(210\*h\*(c\*g^2 + a\*h^2)^3\*(g + h\*x)^5) - (a^2\*c^3\*(6\*c^2\*d\*g^3 + a^2\*h^2\*(8\*f\*g - e\*h) - a\*c\*g\*(f\*g^2 - h\*(8\*e\*g - 3\*d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(16\*(c\*g^2 + a\*h^2)^(11/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 721

Int[((d\_) + (e\_)\*(x\_)^(m\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(-2\*a\*e + (2\*c\*d)\*x)\*(a + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), x] - Dist[(4\*a\*c\*p)/(2\*(m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 +

$a*e^2, 0]$  && EqQ[m + 2\*p + 2, 0] && GtQ[p, 0]

### Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] :> -Subst[  
Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ  
[{a, c, d, e}, x]

### Rule 807

Int[(((d\_) + (e\_)\*(x\_))^(m\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p  
\_), x\_Symbol] :> -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))  
/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), In  
t[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}  
, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 835

Int[(((d\_) + (e\_)\*(x\_))^(m\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p  
\_), x\_Symbol] :> Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/  
(m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d +  
e\*x)^(m + 1)\*(a + c\*x^2)^p\*Simp[(c\*d\*f + a\*e\*g)\*(m + 1) - c\*(e\*f - d\*g)\*(m  
+ 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 +  
a\*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*  
p])

### Rule 1651

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :>  
With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq,  
d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*  
d^2 + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)  
\*(a + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 + a\*e^2)\*Q + c\*d\*R\*(m + 1) - c\*e\*  
R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]  
&& NeQ[c\*d^2 + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8} dx &= -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{7h(cg^2+ah^2)(g+hx)^7} - \frac{\int \frac{\left(-7(cdg-afg+afh)-\left(7afh+c\left(2eg+\frac{5fg^2}{h}-2dh\right)\right)x\right)}{(g+hx)^7}}{7(cg^2+ah^2)} \\
&= -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{7h(cg^2+ah^2)(g+hx)^7} + \frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-c))}{42h(cg^2+ah^2)^2(g+hx)} \\
&= -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{7h(cg^2+ah^2)(g+hx)^7} + \frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-c))}{42h(cg^2+ah^2)^2(g+hx)} \\
&= -\frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx)(a+cx^2)^{5/2}}{24(cg^2+ah^2)^4(g+hx)^4} \\
&= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx)\sqrt{a+cx^2}}{16(cg^2+ah^2)^5(g+hx)^2} \\
&= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx)\sqrt{a+cx^2}}{16(cg^2+ah^2)^5(g+hx)^2} \\
&= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx)\sqrt{a+cx^2}}{16(cg^2+ah^2)^5(g+hx)^2}
\end{aligned}$$

**Mathematica [A]** time = 2.50, size = 863, normalized size = 1.62

Antiderivative was successfully verified.

[In] Integrate[((a + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^8, x]

[Out] -1/1680\*(Sqrt[a + c\*x^2]\*(240\*(c\*g^2 + a\*h^2)^6\*(f\*g^2 + h\*(-(e\*g) + d\*h)) - 40\*(c\*g^2 + a\*h^2)^5\*(29\*c\*f\*g^3 + c\*g\*h\*(-22\*e\*g + 15\*d\*h) - 7\*a\*h^2\*(-2\*f\*g + e\*h))\*(g + h\*x) + 8\*(c\*g^2 + a\*h^2)^4\*(42\*a^2\*f\*h^4 + a\*c\*h^2\*(314\*f\*g^2 + h\*(-139\*e\*g + 48\*d\*h)) + c^2\*(275\*f\*g^4 + g^2\*h\*(-142\*e\*g + 51\*d\*h)))\*(g + h\*x)^2 - 2\*c\*(c\*g^2 + a\*h^2)^3\*(7\*a^2\*h^4\*(136\*f\*g - 35\*e\*h) + 2\*c^2\*(500\*f\*g^5 + g^3\*h\*(-136\*e\*g + 3\*d\*h)) + a\*c\*g\*h^2\*(1979\*f\*g^2 + h\*(-544\*e

```
*g + 33*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 + c^3*(40
0*f*g^6 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-29*e*g +
8*d*h)) + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h*x)^4 - c
^2*(c*g^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2*(89*f*g^2
+ 44*e*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g - 73*d*h)) +
4*c^3*(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-336*a^4*f*h^
8 + 2*a*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^2*g^2*h^4*(50
5*f*g^2 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4*e*g + 3*d*h))
+ 3*a^3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x)^6)/((c*g^2*h
+ a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) -
a*c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g^2 + a*h^2)^(11/2
)) - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e
*g + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(16*(
c*g^2 + a*h^2)^(11/2))
```

**IntegrateAlgebraic [F]** time = 180.08, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] $Aborted
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 1.12, size = 7936, normalized size = 14.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 - 3*a^3*c^4*d*g*h^2 + 8*a^4*c^3*f*g*h
^2 + 8*a^3*c^4*g^2*h*e - a^4*c^3*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a
^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(-c*g
^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g^3*h^
```



$$\begin{aligned}
& 12 - 105(\sqrt{c}x - \sqrt{c^2x^2 + a})^{13}a^3c^4f^3g^3h^{12} - 315(\sqrt{c}x - \sqrt{c^2x^2 + a})^{13}a^3c^4d^2g^3h^{14} + 840(\sqrt{c}x - \sqrt{c^2x^2 + a})^{13}a^4c^3f^3g^3h^{14} + 840(\sqrt{c}x - \sqrt{c^2x^2 + a})^{13}a^3c^4g^2h^{13}e - 105(\sqrt{c}x - \sqrt{c^2x^2 + a})^{13}a^4c^3h^{15}e - 1680(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}c^{(15/2)}f^3g^{10}h^5 - 8400(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^2c^{(13/2)}f^3g^8h^7 - 16800(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^2c^{(11/2)}f^3g^6h^9 + 8190(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^2c^{(11/2)}d^2g^4h^{11} - 18165(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^3c^{(9/2)}f^3g^4h^{11} - 4095(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^3c^{(9/2)}d^2g^2h^{13} + 2520(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^4c^{(7/2)}f^3g^2h^{13} - 1680(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^5c^{(5/2)}f^3h^{15} + 10920(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^3c^{(9/2)}g^3h^{12}e - 1365(\sqrt{c}x - \sqrt{c^2x^2 + a})^{12}a^4c^{(7/2)}g^3h^{14}e - 5600(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}c^8f^3g^{11}h^4 - 28000(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^2c^6f^3g^7h^8 + 44940(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^2c^6d^2g^5h^{10} - 63490(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^3c^5f^3g^5h^{10} - 26670(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^3c^5d^2g^3h^{12} + 32620(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^4c^4f^3g^3h^{12} + 2100(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^4c^4d^2g^3h^{14} - 11200(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^5c^3f^3g^3h^{14} - 2240(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}c^8g^{10}h^5e - 11200(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^2c^6g^6h^9e + 37520(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^3c^5g^4h^{11}e - 24290(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^4c^4g^2h^{13}e - 1540(\sqrt{c}x - \sqrt{c^2x^2 + a})^{11}a^5c^3h^{15}e - 11200(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}c^{(17/2)}f^3g^{12}h^3 - 3360(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}c^{(17/2)}d^2g^{10}h^5 - 52640(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^2c^{(15/2)}f^3g^{10}h^5 - 16800(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^2c^{(15/2)}d^2g^8h^7 - 95200(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^2c^{(13/2)}f^3g^8h^7 + 100380(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^2c^{(13/2)}d^2g^6h^9 - 100730(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^3c^{(11/2)}f^3g^6h^9 - 146790(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^3c^{(11/2)}d^2g^4h^{11} + 163940(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^4c^{(9/2)}f^3g^4h^{11} + 6300(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^4c^{(9/2)}d^2g^2h^{13} - 56000(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^5c^{(7/2)}f^3g^2h^{13} - 3360(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^5c^{(7/2)}d^2h^{15} + 3360(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^6c^{(5/2)}f^3h^{15} - 4480(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}c^{(17/2)}g^{11}h^4e - 22400(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^2c^{(15/2)}g^9h^6e - 44800(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^2c^{(13/2)}g^7h^8e + 133840(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^3c^{(11/2)}g^5h^{10}e - 106330(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^4c^{(9/2)}g^3h^{12}e + 3220(\sqrt{c}x - \sqrt{c^2x^2 + a})^{10}a^5c^{(7/2)}g^3h^{14}e - 13440(\sqrt{c}x - \sqrt{c^2x^2 + a})^9c^9f^3g^{13}h^2 - 4032(\sqrt{c}x - \sqrt{c^2x^2 + a})^9c^9d^2g^{11}h^4 - 50848(\sqrt{c}x - \sqrt{c^2x^2 + a})^9a^2c^8f^3g^{11}h^4 - 20160(\sqrt{c}x - \sqrt{c^2x^2 + a})^9a^2c^8d^2g^9h^6 - 52640(\sqrt{c}x - \sqrt{c^2x^2 + a})^9a^2c^7f^3g^9h^6 + 191016(\sqrt{c}x - \sqrt{c^2x^2 + a})^9a^2c^7d^2g^7h^8 - 9436(\sqrt{c}x - \sqrt{c^2x^2 + a})^9a^3c^6f^3g^7h^8 - 363
\end{aligned}$$

$$\begin{aligned}
& 216*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^3*c^6*d*g^5*h^{10} + 439306*(\sqrt{c}*x \\
& - \sqrt{c*x^2 + a})^9*a^4*c^5*f*g^5*h^{10} + 95340*(\sqrt{c}*x - \sqrt{c*x^2 + a}) \\
& )^9*a^4*c^5*d*g^3*h^{12} - 209965*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^5*c^4*f* \\
& g^3*h^{12} - 9975*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^5*c^4*d*g*h^{14} + 32200*(\sqrt{c}*x \\
& - \sqrt{c*x^2 + a})^9*a^6*c^3*f*g*h^{14} - 5376*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*c^9*g^{12}*h^3*e \\
& - 25984*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a*c^8*g^{10}*h^5*e - 49280*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^2*c^7*g^8*h^7*e \\
& + 263648*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^3*c^6*g^6*h^9*e - 332780*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^4*c^5*g^4*h^{11}*e \\
& + 49490*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^5*c^4*g^2*h^{13}*e - 1085*(\sqrt{c}*x - \sqrt{c*x^2 + a})^9*a^6*c^3*h^{15}*e \\
& - 8960*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*c^{(19/2)}*f*g^{14}*h - 2688*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*c^{(19/2)}*d*g^{12}*h^3 \\
& - 15232*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a*c^{(17/2)}*f*g^{12}*h^3 - 16800*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a*c^{(17/2)}*d*g^{10}*h^5 \\
& + 53200*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^2*c^{(15/2)}*f*g^{10}*h^5 + 181104*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^2*c^{(15/2)}*d*g^8*h^7 \\
& + 143416*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^3*c^{(13/2)}*f*g^8*h^7 - 651924*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^3*c^{(13/2)}*d*g^6*h^9 \\
& + 580034*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^4*c^{(11/2)}*f*g^6*h^9 + 299460*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^4*c^{(11/2)}*d*g^4*h^{11} \\
& - 568085*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^5*c^{(9/2)}*f*g^4*h^{11} - 72975*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^5*c^{(9/2)}*d*g^2*h^{13} \\
& + 147000*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^6*c^{(7/2)}*f*g^2*h^{13} - 3360*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^6*c^{(7/2)}*d*h^{15} \\
& - 5040*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^7*c^{(5/2)}*f*h^{15} - 3584*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*c^{(19/2)}*g^{13}*h^2*e \\
& - 9856*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a*c^{(17/2)}*g^{11}*h^4*e + 4480*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^2*c^{(15/2)}*g^9*h^6*e \\
& + 344512*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^3*c^{(13/2)}*g^7*h^8*e - 613480*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^4*c^{(11/2)}*g^5*h^{10}*e \\
& + 259210*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^5*c^{(9/2)}*g^3*h^{12}*e - 9765*(\sqrt{c}*x - \sqrt{c*x^2 + a})^8*a^6*c^{(7/2)}*g*h^{14}*e \\
& - 2560*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*c^{10}*f*g^{15} - 768*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*c^{10}*d*g^{13}*h^2 \\
& + 12928*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^9*f*g^{13}*h^2 + 384*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^9*d*g^{11}*h^4 \\
& + 80576*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^8*f*g^{11}*h^4 + 117984*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^8*d*g^9*h^6 \\
& + 101936*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^7*f*g^9*h^6 - 603216*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^7*d*g^7*h^8 \\
& + 256816*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^4*c^6*f*g^7*h^8 + 703752*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^4*c^6*d*g^5*h^{10} \\
& - 941332*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^5*c^5*f*g^5*h^{10} - 184380*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^5*c^5*d*g^3*h^{12} \\
& + 413280*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^6*c^4*f*g^3*h^{12} + 13440*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^6*c^4*d*g*h^{14} \\
& - 47040*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^7*c^3*f*g*h^{14} - 1024*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*c^{10}*g^{14}*h*e \\
& + 4096*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a*c^9*g^{12}*h^3*e + 32768*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^2*c^8*g^{10}*h^5*e \\
& + 205952*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^3*c^7*g^8*h^7*e - 741776*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^4*c^6*g^6*h^9*e \\
& + 608720*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^5*c^5*g^4*h^{11}*e - 92820*(\sqrt{c}*x - \sqrt{c*x^2 + a})^7*a^6*c^4*
\end{aligned}$$

$$\begin{aligned}
&g^2h^{13}e + 8960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a*c^{(19/2)}*f*g^{14}h + 268 \\
&8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a*c^{(19/2)}*d*g^{12}h^3 + 15232*(\text{sqrt}(c)*x \\
&- \text{sqrt}(c*x^2 + a))^6*a^2*c^{(17/2)}*f*g^{12}h^3 + 16800*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 \\
&+ a))^6*a^2*c^{(17/2)}*d*g^{10}h^5 - 53200*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a \\
&^3*c^{(15/2)}*f*g^{10}h^5 - 342384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^3*c^{(15/2)} \\
&)*d*g^8h^7 - 103936*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^4*c^{(13/2)}*f*g^8h^7 \\
&+ 736344*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^4*c^{(13/2)}*d*g^6h^9 - 726404*( \\
&\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^5*c^{(11/2)}*f*g^6h^9 - 488460*(\text{sqrt}(c)*x - \\
&\text{sqrt}(c*x^2 + a))^6*a^5*c^{(11/2)}*d*g^4h^{11} + 764960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 \\
&+ a))^6*a^6*c^{(9/2)}*f*g^4h^{11} + 33600*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^ \\
&6*c^{(9/2)}*d*g^2h^{13} - 168000*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^7*c^{(7/2)}*f \\
&*g^2h^{13} - 6720*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^7*c^{(7/2)}*d*h^{15} + 6720* \\
&(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^8*c^{(5/2)}*f*h^{15} + 3584*(\text{sqrt}(c)*x - \text{sqrt} \\
&(c*x^2 + a))^6*a*c^{(19/2)}*g^{13}h^2e + 9856*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6 \\
&a^2*c^{(17/2)}*g^{11}h^4e + 8960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^3*c^{(15/2)} \\
&)*g^9h^6e - 487312*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^4*c^{(13/2)}*g^7h^8e \\
&+ 807520*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^5*c^{(11/2)}*g^5h^{10}e - 310660* \\
&(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^6*a^6*c^{(9/2)}*g^3h^{12}e + 13440*(\text{sqrt}(c)*x - \\
&\text{sqrt}(c*x^2 + a))^6*a^7*c^{(7/2)}*g*h^{14}e - 13440*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + \\
&a))^5*a^2*c^9*f*g^{13}h^2 - 4032*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^2*c^9*d*g \\
&^{11}h^4 - 50848*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^3*c^8*f*g^{11}h^4 - 47040* \\
&(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^3*c^8*d*g^9h^6 - 50960*(\text{sqrt}(c)*x - \text{sqrt} \\
&(c*x^2 + a))^5*a^4*c^7*f*g^9h^6 + 438816*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a \\
&^4*c^7*d*g^7h^8 - 99736*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^6*f*g^7h^8 \\
&- 556416*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^6*d*g^5h^{10} + 728756*(\text{sqrt} \\
&(c)*x - \text{sqrt}(c*x^2 + a))^5*a^6*c^5*f*g^5h^{10} + 167790*(\text{sqrt}(c)*x - \text{sqrt}(c*x \\
&^2 + a))^5*a^6*c^5*d*g^3h^{12} - 362915*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^7* \\
&c^4*f*g^3h^{12} - 10185*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^7*c^4*d*g*h^{14} + 3 \\
&8360*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^8*c^3*f*g*h^{14} - 5376*(\text{sqrt}(c)*x - s \\
&\text{qrt}(c*x^2 + a))^5*a^2*c^9*g^{12}h^3e - 25984*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^ \\
&5*a^3*c^8*g^{10}h^5e - 86240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^4*c^7*g^8h^ \\
&7e + 574448*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^5*c^6*g^6h^9e - 487480*(sq \\
&\text{rt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^6*c^5*g^4h^{11}e + 89740*(\text{sqrt}(c)*x - \text{sqrt}(c \\
&*x^2 + a))^5*a^7*c^4*g^2h^{13}e + 1085*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^5*a^8* \\
&c^3h^{15}e + 11200*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*c^{(17/2)}*f*g^{12}h^3 \\
&+ 3360*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^3*c^{(17/2)}*d*g^{10}h^5 + 52640*(sq \\
&\text{rt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^4*c^{(15/2)}*f*g^{10}h^5 + 45360*(\text{sqrt}(c)*x - sq \\
&\text{rt}(c*x^2 + a))^4*a^4*c^{(15/2)}*d*g^8h^7 + 96880*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a \\
&))^4*a^5*c^{(13/2)}*f*g^8h^7 - 364728*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^5*c^ \\
&(13/2)*d*g^6h^9 + 215908*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^6*c^{(11/2)}*f*g^ \\
&6h^9 + 220710*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^6*c^{(11/2)}*d*g^4h^{11} - 40 \\
&6735*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^7*c^{(9/2)}*f*g^4h^{11} - 49581*(\text{sqrt}(c \\
&)*x - \text{sqrt}(c*x^2 + a))^4*a^7*c^{(9/2)}*d*g^2h^{13} + 104776*(\text{sqrt}(c)*x - \text{sqrt}( \\
&c*x^2 + a))^4*a^8*c^{(7/2)}*f*g^2h^{13} - 1344*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4 \\
&a^8*c^{(7/2)}*d*h^{15} - 3696*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + a))^4*a^9*c^{(5/2)}*f*h^
\end{aligned}$$

$$\begin{aligned}
& 15 + 4480*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^3*c^{(17/2)}*g^{11}*h^4*e + 29120*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^4*c^{(15/2)}*g^9*h^6*e + 119056*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^5*c^{(13/2)}*g^7*h^8*e - 390656*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^6*c^{(11/2)}*g^5*h^{10}*e + 179900*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^7*c^{(9/2)}*g^3*h^{12}*e - 10703*(\sqrt{c}*x - \sqrt{c*x^2 + a})^4*a^8*c^{(7/2)}*g*h^{14}*e - 5600*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^4*c^8*f*g^{11}*h^4 - 3360*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^4*c^8*d*g^9*h^6 - 29680*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^5*c^7*f*g^9*h^6 - 32592*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^5*c^7*d*g^7*h^8 - 67088*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^6*c^6*f*g^7*h^8 + 172620*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^6*c^6*d*g^5*h^{10} - 156170*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^7*c^5*f*g^5*h^{10} - 62454*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^7*c^5*d*g^3*h^{12} + 140084*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^8*c^4*f*g^3*h^{12} + 5964*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^8*c^4*d*g*h^{14} - 17024*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^9*c^3*f*g*h^{14} - 2240*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^4*c^8*g^{10}*h^5*e - 16576*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^5*c^7*g^8*h^7*e - 72464*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^6*c^6*g^6*h^9*e + 179200*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^7*c^5*g^4*h^{11}*e - 31402*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^8*c^4*g^2*h^{13}*e + 1540*(\sqrt{c}*x - \sqrt{c*x^2 + a})^3*a^9*c^3*h^{15}*e + 1680*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^5*c^{(15/2)}*f*g^{10}*h^5 + 1008*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^5*c^{(15/2)}*d*g^8*h^7 + 9632*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^6*c^{(13/2)}*f*g^8*h^7 + 9996*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^6*c^{(13/2)}*d*g^6*h^9 + 24094*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^7*c^{(11/2)}*f*g^6*h^9 - 54894*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^7*c^{(11/2)}*d*g^4*h^{11} + 56924*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^8*c^{(9/2)}*f*g^4*h^{11} + 9156*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^8*c^{(9/2)}*d*g^2*h^{13} - 32256*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^9*c^{(7/2)}*f*g^2*h^{13} - 672*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^9*c^{(7/2)}*d*h^{15} + 672*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^{10}*c^{(5/2)}*f*h^{15} + 1344*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^5*c^{(15/2)}*g^9*h^6*e + 8624*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^6*c^{(13/2)}*g^7*h^8*e + 30352*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^7*c^{(11/2)}*g^5*h^{10}*e - 47362*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^8*c^{(9/2)}*g^3*h^{12}*e + 3276*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a^9*c^{(7/2)}*g*h^{14}*e - 560*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^6*c^7*f*g^9*h^6 - 168*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^6*c^7*d*g^7*h^8 - 3052*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^7*c^6*f*g^7*h^8 - 1680*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^7*c^6*d*g^5*h^{10} - 7070*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^8*c^5*f*g^5*h^{10} + 9744*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^8*c^5*d*g^3*h^{12} - 12999*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^9*c^4*f*g^3*h^{12} - 1029*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^9*c^4*d*g*h^{14} + 3864*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^{10}*c^3*f*g*h^{14} - 224*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^6*c^7*g^8*h^7*e - 1456*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^7*c^6*g^6*h^9*e - 5180*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^8*c^5*g^4*h^{11}*e + 8442*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^9*c^4*g^2*h^{13}*e + 105*(\sqrt{c}*x - \sqrt{c*x^2 + a})*a^{10}*c^3*h^{15}*e + 40*a^7*c^{(13/2)}*f*g^8*h^7 + 12*a^7*c^{(13/2)}*d*g^6*h^9 + 218*a^8*c^{(11/2)}*f*g^6*h^9 + 120*a^8*c^{(11/2)}*d*g^4*h^{11} + 505*a^9*c^{(9/2)}*f*g^4*h^{11} - 741*a^9*c^{(9/2)}*d*g^2*h^{13} + 936*a^{10}*c^{(7/2)}*f*g^2*h^{13} + 96*a^{10}*c^{(7/2)}*d*h^{15} - 336*a^{11}*c^{(5/2)}
\end{aligned}$$

$2) * f * h^{15} + 16 * a^7 * c^{(13/2)} * g^7 * h^8 * e + 104 * a^8 * c^{(11/2)} * g^5 * h^{10} * e + 370 * a^9 * c^{(9/2)} * g^3 * h^{12} * e - 663 * a^{10} * c^{(7/2)} * g * h^{14} * e) / ((c^5 * g^{10} * h^6 + 5 * a * c^4 * g^8 * h^8 + 10 * a^2 * c^3 * g^6 * h^{10} + 10 * a^3 * c^2 * g^4 * h^{12} + 5 * a^4 * c * g^2 * h^{14} + a^5 * h^{16}) * ((\sqrt{c} * x - \sqrt{c * x^2 + a})^2 * h + 2 * (\sqrt{c} * x - \sqrt{c * x^2 + a})) * \sqrt{c} * g - a * h)^7)$

**maple [B]** time = 0.05, size = 19093, normalized size = 35.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)`

[Out] result too large to display

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")`

[Out] Timed out

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)`

[Out] `int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8, x)`

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)`

[Out] Timed out

$$3.100 \quad \int (a + cx^2)^{5/2} (A + Bx + Cx^2) dx$$

**Optimal.** Leaf size=168

$$\frac{5a^3(8Ac - aC) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}} + \frac{5a^2x\sqrt{a+cx^2}(8Ac - aC)}{128c} + \frac{x(a+cx^2)^{5/2}(8Ac - aC)}{48c} + \frac{5ax(a+cx^2)^{3/2}(8Ac - aC)}{192c}$$

**Rubi [A]** time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1815, 641, 195, 217, 206}

$$\frac{5a^3(8Ac - aC) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}} + \frac{5a^2x\sqrt{a+cx^2}(8Ac - aC)}{128c} + \frac{x(a+cx^2)^{5/2}(8Ac - aC)}{48c} + \frac{5ax(a+cx^2)^{3/2}(8Ac - aC)}{192c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)^(5/2)\*(A + B\*x + C\*x^2), x]

[Out] (5\*a^2\*(8\*A\*c - a\*C)\*x\*sqrt[a + c\*x^2])/(128\*c) + (5\*a\*(8\*A\*c - a\*C)\*x\*(a + c\*x^2)^(3/2))/(192\*c) + ((8\*A\*c - a\*C)\*x\*(a + c\*x^2)^(5/2))/(48\*c) + (B\*(a + c\*x^2)^(7/2))/(7\*c) + (C\*x\*(a + c\*x^2)^(7/2))/(8\*c) + (5\*a^3\*(8\*A\*c - a\*C)\*ArcTanh[(sqrt[c]\*x)/sqrt[a + c\*x^2]])/(128\*c^(3/2))

### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /

; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int (a + cx^2)^{5/2} (A + Bx + Cx^2) dx &= \frac{Cx(a + cx^2)^{7/2}}{8c} + \frac{\int (8Ac - aC + 8Bcx)(a + cx^2)^{5/2} dx}{8c} \\
 &= \frac{B(a + cx^2)^{7/2}}{7c} + \frac{Cx(a + cx^2)^{7/2}}{8c} + \frac{(8Ac - aC) \int (a + cx^2)^{5/2} dx}{8c} \\
 &= \frac{(8Ac - aC)x(a + cx^2)^{5/2}}{48c} + \frac{B(a + cx^2)^{7/2}}{7c} + \frac{Cx(a + cx^2)^{7/2}}{8c} + \frac{(5a(8Ac - aC) \int (a + cx^2)^{3/2} dx)}{48c} \\
 &= \frac{5a(8Ac - aC)x(a + cx^2)^{3/2}}{192c} + \frac{(8Ac - aC)x(a + cx^2)^{5/2}}{48c} + \frac{B(a + cx^2)^{7/2}}{7c} + \frac{(8Ac - aC)x \int (a + cx^2)^{1/2} dx}{48c} \\
 &= \frac{5a^2(8Ac - aC)x\sqrt{a + cx^2}}{128c} + \frac{5a(8Ac - aC)x(a + cx^2)^{3/2}}{192c} + \frac{(8Ac - aC)x \int (a + cx^2)^{-1/2} dx}{48c} \\
 &= \frac{5a^2(8Ac - aC)x\sqrt{a + cx^2}}{128c} + \frac{5a(8Ac - aC)x(a + cx^2)^{3/2}}{192c} + \frac{(8Ac - aC)x \int (a + cx^2)^{-1/2} dx}{48c} \\
 &= \frac{5a^2(8Ac - aC)x\sqrt{a + cx^2}}{128c} + \frac{5a(8Ac - aC)x(a + cx^2)^{3/2}}{192c} + \frac{(8Ac - aC)x \int (a + cx^2)^{-1/2} dx}{48c}
 \end{aligned}$$

**Mathematica** [A] time = 0.31, size = 150, normalized size = 0.89

$$\frac{\sqrt{a + cx^2} \left( \sqrt{c} (3a^3(128B + 35Cx) + 2a^2cx(924A + x(576B + 413Cx)) + 8ac^2x^3(182A + x(144B + 119Cx)) + 16c^3x^5(28A + 3x(8B + 7Cx))) - \frac{105a^{5/2}(aC - 8Ac) \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a} + 1}} \right)}{2688c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)^(5/2)\*(A + B\*x + C\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(Sqrt[c]\*(3\*a^3\*(128\*B + 35\*C\*x) + 16\*c^3\*x^5\*(28\*A + 3\*x\*(8\*B + 7\*C\*x)) + 8\*a\*c^2\*x^3\*(182\*A + x\*(144\*B + 119\*C\*x)) + 2\*a^2\*c\*x\*(924\*A + x\*(576\*B + 413\*C\*x))) - (105\*a^(5/2)\*(-8\*A\*c + a\*C)\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]])/Sqrt[1 + (c\*x^2)/a]))/(2688\*c^(3/2))

**IntegrateAlgebraic [A]** time = 0.55, size = 161, normalized size = 0.96

$$\frac{5(a^4c - 8a^3Ac) \log(\sqrt{a+cx^2} - \sqrt{c}x)}{128c^{3/2}} + \frac{\sqrt{a+cx^2} (384a^3B + 105a^3Cx + 1848a^2Acx + 1152a^2Bcx^2 + 826a^2cCx^3 + 1456aAc^2x^3 + 1152aBc^2x^4 + 952ac^2Cx^5 + 448Ac^3x^5 + 384Bc^3x^6 + 336c^3Cx^7)}{2688c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c\*x^2)^(5/2)\*(A + B\*x + C\*x^2), x]

[Out] (Sqrt[a + c\*x^2]\*(384\*a^3\*B + 1848\*a^2\*A\*c\*x + 105\*a^3\*C\*x + 1152\*a^2\*B\*c\*x^2 + 1456\*a\*A\*c^2\*x^3 + 826\*a^2\*c\*C\*x^3 + 1152\*a\*B\*c^2\*x^4 + 448\*A\*c^3\*x^5 + 952\*a\*c^2\*C\*x^5 + 384\*B\*c^3\*x^6 + 336\*c^3\*C\*x^7))/(2688\*c) + (5\*(-8\*a^3\*A\*c + a^4\*C)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(128\*c^(3/2))

**fricas [A]** time = 1.20, size = 333, normalized size = 1.98

$$\frac{105(Ca^4 - 8Aa^3c) \log(-2cx^2 - 2\sqrt{c}\sqrt{cx^2+a}) - 2(36Ca^4x + 384Ba^4 + 1152Ba^2c^2 + 56(17Ca^3 + 8Aa^2)c^2 + 384Ba^3 + 14(59Ca^2 + 104Aa^2)c^2 + 21(5Ca^2 + 8Aa^2)c) \sqrt{c^2+a}}{5376c^2} + \frac{105(Ca^4 - 8Aa^3c) \arctan\left(\frac{\sqrt{c}x}{\sqrt{cx^2+a}}\right) + (36Ca^4x + 384Ba^4 + 1152Ba^2c^2 + 56(17Ca^3 + 8Aa^2)c^2 + 384Ba^3 + 14(59Ca^2 + 104Aa^2)c^2 + 21(5Ca^2 + 8Aa^2)c) \sqrt{c^2+a}}{2688c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(5/2)\*(C\*x^2+B\*x+A), x, algorithm="fricas")

[Out] [-1/5376\*(105\*(C\*a^4 - 8\*A\*a^3\*c)\*sqrt(c)\*log(-2\*c\*x^2 - 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(336\*C\*c^4\*x^7 + 384\*B\*c^4\*x^6 + 1152\*B\*a\*c^3\*x^4 + 1152\*B\*a^2\*c^2\*x^2 + 56\*(17\*C\*a\*c^3 + 8\*A\*c^4)\*x^5 + 384\*B\*a^3\*c + 14\*(59\*C\*a^2\*c^2 + 104\*A\*a\*c^3)\*x^3 + 21\*(5\*C\*a^3\*c + 88\*A\*a^2\*c^2)\*x)\*sqrt(c\*x^2 + a))/c^2, 1/2688\*(105\*(C\*a^4 - 8\*A\*a^3\*c)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + (336\*C\*c^4\*x^7 + 384\*B\*c^4\*x^6 + 1152\*B\*a\*c^3\*x^4 + 1152\*B\*a^2\*c^2\*x^2 + 56\*(17\*C\*a\*c^3 + 8\*A\*c^4)\*x^5 + 384\*B\*a^3\*c + 14\*(59\*C\*a^2\*c^2 + 104\*A\*a\*c^3)\*x^3 + 21\*(5\*C\*a^3\*c + 88\*A\*a^2\*c^2)\*x)\*sqrt(c\*x^2 + a))/c^2]

**giac [A]** time = 0.21, size = 168, normalized size = 1.00

$$\frac{1}{2688} \left( \frac{384Ba^3}{c} + \left( 2 \left( 576Ba^2 + \left( 4 \left( 144Bac + \left( 6(7C^2x + 8Bc^2)x + \frac{7(17Ca^2 + 8Aa^2)c^2}{c^6} \right) \right) \right) \right) x + \frac{7(59Ca^2c^2 + 104Aa^2c^2)}{c^6} \right) \right) x + \frac{21(5Ca^3c^2 + 88Aa^2c^2)}{c^6} \right) \sqrt{cx^2+a} + \frac{5(Ca^4 - 8Aa^3c) \log\left(\frac{-\sqrt{c}x + \sqrt{cx^2+a}}{128c^3}\right)}{128c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(5/2)\*(C\*x^2+B\*x+A), x, algorithm="giac")

[Out] 1/2688\*(384\*B\*a^3/c + (2\*(576\*B\*a^2 + (4\*(144\*B\*a\*c + (6\*(7\*C\*c^2\*x + 8\*B\*c^2)\*x + 7\*(17\*C\*a\*c^7 + 8\*A\*c^8)/c^6)\*x)\*x + 7\*(59\*C\*a^2\*c^6 + 104\*A\*a\*c^7)/c^6)\*x)\*x + 21\*(5\*C\*a^3\*c^5 + 88\*A\*a^2\*c^6)/c^6)\*x)\*sqrt(c\*x^2 + a) + 5/128\*(C\*a^4 - 8\*A\*a^3\*c)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)



**maple [A]** time = 0.01, size = 181, normalized size = 1.08

$$\frac{5Aa^3 \ln(\sqrt{c}x + \sqrt{cx^2+a})}{16\sqrt{c}} - \frac{5Ca^4 \ln(\sqrt{c}x + \sqrt{cx^2+a})}{128c^{\frac{3}{2}}} + \frac{5\sqrt{cx^2+a}Aa^2x}{16} - \frac{5\sqrt{cx^2+a}Ca^3x}{128c} + \frac{5(cx^2+a)^{\frac{3}{2}}Aax}{24} - \frac{5(cx^2+a)^{\frac{3}{2}}Ca^2x}{192c} + \frac{(cx^2+a)^{\frac{5}{2}}Ax}{6} - \frac{(cx^2+a)^{\frac{5}{2}}Cax}{48c} + \frac{(cx^2+a)^{\frac{7}{2}}Cx}{8c} + \frac{(cx^2+a)^{\frac{7}{2}}B}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(5/2)\*(C\*x^2+B\*x+A), x)

[Out]  $\frac{1}{8}Cx^2(c^2x^2+a)^{7/2}/c - \frac{1}{48}Ca^2(c^2x^2+a)^{5/2}/c - \frac{5}{192}Ca^3(c^2x^2+a)^{3/2}/c - \frac{5}{128}Ca^4(c^2x^2+a)^{1/2}/c - \frac{5}{128}Ca^5/c^3 \ln(c^{1/2}x + (c^2x^2+a)^{1/2}) + \frac{1}{7}B(c^2x^2+a)^{7/2}/c + \frac{1}{6}A^2x^2(c^2x^2+a)^{5/2} + \frac{5}{24}A^2a^2x^2(c^2x^2+a)^{3/2} + \frac{5}{16}A^2a^3x^2(c^2x^2+a)^{1/2} + \frac{5}{16}A^2a^4/c^3 \ln(c^{1/2}x + (c^2x^2+a)^{1/2})$

**maxima [A]** time = 0.45, size = 166, normalized size = 0.99

$$\frac{1}{6}(cx^2+a)^{\frac{5}{2}}Ax + \frac{5}{24}(cx^2+a)^{\frac{3}{2}}Aax + \frac{5}{16}\sqrt{cx^2+a}Aa^2x + \frac{(cx^2+a)^{\frac{7}{2}}Cx}{8c} - \frac{(cx^2+a)^{\frac{5}{2}}Cax}{48c} - \frac{5(cx^2+a)^{\frac{3}{2}}Ca^2x}{192c} - \frac{5\sqrt{cx^2+a}Ca^3x}{128c} - \frac{5Ca^4 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128c^{\frac{3}{2}}} + \frac{5Aa^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{c}} + \frac{(cx^2+a)^{\frac{7}{2}}B}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)^(5/2)\*(C\*x^2+B\*x+A), x, algorithm="maxima")

[Out]  $\frac{1}{6}(c^2x^2+a)^{5/2}A^2x + \frac{5}{24}(c^2x^2+a)^{3/2}A^2a^2x + \frac{5}{16}\sqrt{c^2x^2+a}A^2a^3x + \frac{1}{8}(c^2x^2+a)^{7/2}C^2x/c - \frac{1}{48}(c^2x^2+a)^{5/2}C^2a^2x/c - \frac{5}{192}(c^2x^2+a)^{3/2}C^2a^3x/c - \frac{5}{128}C^2a^4 \operatorname{arcsinh}(cx/\sqrt{ac})/c^3 + \frac{5}{16}A^2a^3 \operatorname{arcsinh}(cx/\sqrt{ac})/\sqrt{c} + \frac{1}{7}(c^2x^2+a)^{7/2}B/c$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (cx^2+a)^{5/2} (Cx^2+Bx+A) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c\*x^2)^(5/2)\*(A + B\*x + C\*x^2), x)

[Out] int((a + c\*x^2)^(5/2)\*(A + B\*x + C\*x^2), x)

**sympy [A]** time = 32.92, size = 510, normalized size = 3.04

$$\frac{Aa^{\frac{5}{2}}\sqrt{1+\frac{c}{a}}}{2} + \frac{3Aa^{\frac{3}{2}}}{16\sqrt{1+\frac{c}{a}}} + \frac{35Aa^{\frac{1}{2}}c^{\frac{3}{2}}}{48\sqrt{1+\frac{c}{a}}} + \frac{17A\sqrt{a}c^{\frac{5}{2}}}{24\sqrt{1+\frac{c}{a}}} + \frac{5Aa^2 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}} + \frac{Aa^2x^2}{6\sqrt{c}\sqrt{1+\frac{c}{a}}} + \operatorname{Re}\left(\frac{(cx^2+a)^{\frac{7}{2}}}{x}\right) + 2\operatorname{Re}\left(\frac{\frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}} + \frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}} + \frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}}}{\frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}}}\right) + \operatorname{Re}\left(\frac{\frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}} - \frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}} + \frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}}}{\frac{5\sqrt{cx^2+a}}{128c^{\frac{3}{2}}}}\right) + \frac{5Ca^{\frac{5}{2}}}{128\sqrt{1+\frac{c}{a}}} + \frac{133Ca^{\frac{3}{2}}}{384\sqrt{1+\frac{c}{a}}} + \frac{127Ca^{\frac{1}{2}}c^{\frac{3}{2}}}{192\sqrt{1+\frac{c}{a}}} + \frac{23C\sqrt{a}c^{\frac{5}{2}}}{48\sqrt{1+\frac{c}{a}}} + \frac{5Ca^4 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{128c^{\frac{3}{2}}} + \frac{C^2x^2}{8\sqrt{c}\sqrt{1+\frac{c}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(5/2)\*(C\*x\*\*2+B\*x+A), x)

```
[Out] A*a**(5/2)*x*sqrt(1 + c*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + c*x**2/a))
+ 35*A*a**(3/2)*c*x**3/(48*sqrt(1 + c*x**2/a)) + 17*A*sqrt(a)*c**2*x**5/(24
*sqrt(1 + c*x**2/a)) + 5*A*a**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + A*c
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 2*B*a*c*Piecewise((-2*a**2
*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a
+ c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + B*c**2*Piecewise((8*a**3*
sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x
**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*
x**6/6, True)) + 5*C*a**(7/2)*x/(128*c*sqrt(1 + c*x**2/a)) + 133*C*a**(5/2)
*x**3/(384*sqrt(1 + c*x**2/a)) + 127*C*a**(3/2)*c*x**5/(192*sqrt(1 + c*x**2
/a)) + 23*C*sqrt(a)*c**2*x**7/(48*sqrt(1 + c*x**2/a)) - 5*C*a**4*asinh(sqrt
(c)*x/sqrt(a))/(128*c**(3/2)) + C*c**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

$$3.101 \quad \int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=325

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg) - 4acg(3h(dh+eg) + fg^2) + 8c^2dg^3\right)}{8c^{5/2}} + \frac{\sqrt{a+cx^2}\left(4(16a^2fh^4 - 4ach^2(5h(dh+eg) + fg^2) + 8c^2dg^3)\right)}{8c^{5/2}}$$

**Rubi [A]** time = 0.66, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1654, 833, 780, 217, 206}

$$\frac{\sqrt{a+cx^2}\left(4(16a^2fh^4 - 4ach^2(5h(dh+eg) + fg^2) + 8c^2dg^3)\right)}{8c^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg) - 4acg(3h(dh+eg) + fg^2) + 8c^2dg^3\right)}{8c^{5/2}} + \frac{\sqrt{a+cx^2}\left(4(16a^2fh^4 - 4ach^2(5h(dh+eg) + fg^2) + 8c^2dg^3)\right)}{8c^{5/2}} + \frac{\sqrt{a+cx^2}\left(4(16a^2fh^4 - 4ach^2(5h(dh+eg) + fg^2) + 8c^2dg^3)\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^3\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] ((4\*(5\*c\*d - 4\*a\*f)\*h^2 - 3\*c\*g\*(f\*g - 5\*e\*h))\*(g + h\*x)^2\*Sqrt[a + c\*x^2]) / (60\*c^2\*h) - ((f\*g - 5\*e\*h)\*(g + h\*x)^3\*Sqrt[a + c\*x^2]) / (20\*c\*h) + (f\*(g + h\*x)^4\*Sqrt[a + c\*x^2]) / (5\*c\*h) + ((4\*(16\*a^2\*f\*h^4 - 4\*a\*c\*h^2\*(13\*f\*g^2 + 5\*h\*(3\*e\*g + d\*h)) - c^2\*g^2\*(3\*f\*g^2 - 5\*h\*(3\*e\*g + 16\*d\*h))) - c\*h\*(6\*c\*f\*g^3 - 10\*c\*g\*h\*(3\*e\*g + 10\*d\*h) + a\*h^2\*(71\*f\*g + 45\*e\*h))\*x)\*Sqrt[a + c\*x^2] / (120\*c^3\*h) + ((8\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) - 4\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]]) / (8\*c^(5/2))

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 217**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

**Rule 780**

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1)) / (2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3)) / (c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^(m*(a + c*x^2)^(p + 1)))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^(m*(a + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && True) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g + hx)^3 (d + ex + fx^2)}{\sqrt{a + cx^2}} dx = \frac{f(g + hx)^4 \sqrt{a + cx^2}}{5ch} + \frac{\int \frac{(g + hx)^3 ((5cd - 4af)h^2 - ch(fg - 5eh)x)}{\sqrt{a + cx^2}} dx}{5ch^2}$$

$$= -\frac{(fg - 5eh)(g + hx)^3 \sqrt{a + cx^2}}{20ch} + \frac{f(g + hx)^4 \sqrt{a + cx^2}}{5ch} + \frac{\int \frac{(g + hx)^2 (ch^2(20cdg - 13af) - ch^2(5cd - 4af)h^2 - ch^2(fg - 5eh)x)}{\sqrt{a + cx^2}} dx}{20ch^2}$$

$$= \frac{(4(5cd - 4af)h^2 - 3cg(fg - 5eh))(g + hx)^2 \sqrt{a + cx^2}}{60c^2h} - \frac{(fg - 5eh)(g + hx)^3 \sqrt{a + cx^2}}{20ch}$$

$$= \frac{(4(5cd - 4af)h^2 - 3cg(fg - 5eh))(g + hx)^2 \sqrt{a + cx^2}}{60c^2h} - \frac{(fg - 5eh)(g + hx)^3 \sqrt{a + cx^2}}{20ch}$$

$$= \frac{(4(5cd - 4af)h^2 - 3cg(fg - 5eh))(g + hx)^2 \sqrt{a + cx^2}}{60c^2h} - \frac{(fg - 5eh)(g + hx)^3 \sqrt{a + cx^2}}{20ch}$$

$$= \frac{(4(5cd - 4af)h^2 - 3cg(fg - 5eh))(g + hx)^2 \sqrt{a + cx^2}}{60c^2h} - \frac{(fg - 5eh)(g + hx)^3 \sqrt{a + cx^2}}{20ch}$$

**Mathematica [A]** time = 0.35, size = 252, normalized size = 0.78

$$\frac{15\sqrt{c} \log\left(\sqrt{c}\sqrt{a+cx^2} + cx\right) \left(3a^2h^2(eh+3fg) - 4acg(3h(dh+eg) + fg^2) + 8c^2dg^2\right) + \sqrt{a+cx^2} \left(8(8a^2fh^2 - 10ach(hdh+3eg) + 3fg^2) + 15c^2g^2(3dh+eg)\right) + 15cx \left(4c(3gh(dh+eg) + fg^2) - 3ah^2(eh+3fg) + 8cha^2(5c(hdh+3eg) + 3fg^2) - 4a/fh^2\right) + 30c^2h^2x^2(eh+3fg) + 24c^2fh^2x^4}{120c^3}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)^3\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] (Sqrt[a + c\*x^2]\*(8\*(8\*a^2\*f\*h^3 + 15\*c^2\*g^2\*(e\*g + 3\*d\*h) - 10\*a\*c\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))) + 15\*c\*(-3\*a\*h^2\*(3\*f\*g + e\*h) + 4\*c\*(f\*g^3 + 3\*g\*h\*(e\*g + d\*h)))\*x + 8\*c\*h\*(-4\*a\*f\*h^2 + 5\*c\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*x^2 + 30\*c^2\*h^2\*(3\*f\*g + e\*h)\*x^3 + 24\*c^2\*f\*h^3\*x^4) + 15\*Sqrt[c]\*(8\*c^2\*d\*g^3 + 3\*a^2\*h^2\*(3\*f\*g + e\*h) - 4\*a\*c\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)))\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]]/(120\*c^3)

**IntegrateAlgebraic [A]** time = 0.87, size = 296, normalized size = 0.91

$$\frac{\log\left(\sqrt{a+cx^2} - \sqrt{c}x\right) \left(-3a^2h^3 - 9a^2fg^2 + 12acgh^2 + 12acgh^2 + 4acfg^2 - 8c^2dg^2\right) + \sqrt{a+cx^2} \left(64a^2fh^3 - 80acdh^2 - 240acgh^2 - 45ach^3x - 240acfg^2h - 135acfg^2h - 32acfh^3x^2 + 360c^2dg^2h + 180c^2dg^2h + 40c^2dh^3x^2 + 120c^2eg^3 + 180c^2eg^3x + 120c^2eg^3x^2 + 30c^2h^3x^3 + 60c^2fg^3x + 120c^2fg^3x^2 + 90c^2fg^3x^2 + 24c^2fh^3x^4\right)}{120c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^3\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] (Sqrt[a + c\*x^2]\*(120\*c^2\*e\*g^3 + 360\*c^2\*d\*g^2\*h - 240\*a\*c\*f\*g^2\*h - 240\*a\*c\*e\*g\*h^2 - 80\*a\*c\*d\*h^3 + 64\*a^2\*f\*h^3 + 60\*c^2\*f\*g^3\*x + 180\*c^2\*e\*g^2\*h\*x + 180\*c^2\*d\*g\*h^2\*x - 135\*a\*c\*f\*g\*h^2\*x - 45\*a\*c\*e\*h^3\*x + 120\*c^2\*f\*g^2\*h\*x^2 + 120\*c^2\*e\*g\*h^2\*x^2 + 40\*c^2\*d\*h^3\*x^2 - 32\*a\*c\*f\*h^3\*x^2 + 90\*c^2\*f\*g\*h^2\*x^3 + 30\*c^2\*e\*h^3\*x^3 + 24\*c^2\*f\*h^3\*x^4))/(120\*c^3) + ((-8\*c^2\*d\*g^3 + 4\*a\*c\*f\*g^3 + 12\*a\*c\*e\*g^2\*h + 12\*a\*c\*d\*g\*h^2 - 9\*a^2\*f\*g\*h^2 - 3\*a^2\*e\*h^3)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(8\*c^(5/2))

**fricas [A]** time = 0.90, size = 559, normalized size = 1.72

$$\frac{\log\left(\sqrt{a+cx^2} - \sqrt{c}x\right) \left(-3a^2h^3 - 9a^2fg^2 + 12acgh^2 + 12acgh^2 + 4acfg^2 - 8c^2dg^2\right) + \sqrt{a+cx^2} \left(64a^2fh^3 - 80acdh^2 - 240acgh^2 - 45ach^3x - 240acfg^2h - 135acfg^2h - 32acfh^3x^2 + 360c^2dg^2h + 180c^2dg^2h + 40c^2dh^3x^2 + 120c^2eg^3 + 180c^2eg^3x + 120c^2eg^3x^2 + 30c^2h^3x^3 + 60c^2fg^3x + 120c^2fg^3x^2 + 90c^2fg^3x^2 + 24c^2fh^3x^4\right)}{120c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [-1/240\*(15\*(12\*a\*c\*e\*g^2\*h - 3\*a^2\*e\*h^3 - 4\*(2\*c^2\*d - a\*c\*f)\*g^3 + 3\*(4\*a\*c\*d - 3\*a^2\*f)\*g\*h^2)\*sqrt(c)\*log(-2\*c\*x^2 - 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(24\*c^2\*f\*h^3\*x^4 + 120\*c^2\*e\*g^3 - 240\*a\*c\*e\*g\*h^2 + 120\*(3\*c^2\*d - 2\*a\*c\*f)\*g^2\*h - 16\*(5\*a\*c\*d - 4\*a^2\*f)\*h^3 + 30\*(3\*c^2\*f\*g\*h^2 + c^2\*e\*h^3)\*x^3 + 8\*(15\*c^2\*f\*g^2\*h + 15\*c^2\*e\*g\*h^2 + (5\*c^2\*d - 4\*a\*c\*f)\*h^3)\*x^2 + 15\*(4\*c^2\*f\*g^3 + 12\*c^2\*e\*g^2\*h - 3\*a\*c\*e\*h^3 + 3\*(4\*c^2\*d - 3\*a\*c\*f)\*g\*h^2)\*x)\*sqrt(c\*x^2 + a))/c^3, 1/120\*(15\*(12\*a\*c\*e\*g^2\*h - 3\*a^2\*e\*h^3 - 4\*(2\*c^2\*d - a\*c\*f)\*g^3 + 3\*(4\*a\*c\*d - 3\*a^2\*f)\*g\*h^2)\*sqrt(-c)\*arctan(sqrt(c\*x^2 + a)/sqrt(-c)) + 1/120\*(15\*(12\*a\*c\*e\*g^2\*h - 3\*a^2\*e\*h^3 - 4\*(2\*c^2\*d - a\*c\*f)\*g^3 + 3\*(4\*a\*c\*d - 3\*a^2\*f)\*g\*h^2)\*sqrt(c)\*log(-2\*c\*x^2 - 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(24\*c^2\*f\*h^3\*x^4 + 120\*c^2\*e\*g^3 - 240\*a\*c\*e\*g\*h^2 + 120\*(3\*c^2\*d - 2\*a\*c\*f)\*g^2\*h - 16\*(5\*a\*c\*d - 4\*a^2\*f)\*h^3 + 30\*(3\*c^2\*f\*g\*h^2 + c^2\*e\*h^3)\*x^3 + 8\*(15\*c^2\*f\*g^2\*h + 15\*c^2\*e\*g\*h^2 + (5\*c^2\*d - 4\*a\*c\*f)\*h^3)\*x^2 + 15\*(4\*c^2\*f\*g^3 + 12\*c^2\*e\*g^2\*h - 3\*a\*c\*e\*h^3 + 3\*(4\*c^2\*d - 3\*a\*c\*f)\*g\*h^2)\*x)\*sqrt(c\*x^2 + a))/c^3]



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{5}\sqrt{c x^2 + a} f h^3 x^4 / c - \frac{4}{15}\sqrt{c x^2 + a} a f h^3 x^2 / c^2 + d g^3 \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c} + \sqrt{c x^2 + a} e g^3 / c + 3 \sqrt{c x^2 + a} d g^2 h / c + \frac{8}{15}\sqrt{c x^2 + a} a^2 f h^3 / c^3 + \frac{1}{4}(3 f g h^2 + e h^3) \sqrt{c x^2 + a} x^3 / c + \frac{1}{3}(3 f g^2 h + 3 e g h^2 + d h^3) \sqrt{c x^2 + a} x^2 / c - \frac{3}{8}(3 f g h^2 + e h^3) \sqrt{c x^2 + a} a x / c^2 + \frac{1}{2}(f g^3 + 3 e g^2 h + 3 d g h^2) \sqrt{c x^2 + a} x / c + \frac{3}{8}(3 f g h^2 + e h^3) a^2 \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{5/2} - \frac{1}{2}(f g^3 + 3 e g^2 h + 3 d g h^2) a \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2} - \frac{2}{3}(3 f g^2 h + 3 e g h^2 + d h^3) \sqrt{c x^2 + a} a / c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^3 (fx^2 + ex + d)}{\sqrt{cx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(1/2),x)

[Out] int(((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(1/2), x)

**sympy** [A] time = 22.20, size = 796, normalized size = 2.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $-3 a^{3/2} e h^3 x / (8 c^2 \sqrt{1 + c x^2 / a}) - 9 a^{3/2} f g h^2 x / (8 c^2 \sqrt{1 + c x^2 / a}) + 3 \sqrt{a} d g h^2 x \sqrt{1 + c x^2 / a} / (2 c) + 3 \sqrt{a} e g^2 h x \sqrt{1 + c x^2 / a} / (2 c) - \sqrt{a} e h^3 x^3 / (8 c \sqrt{1 + c x^2 / a}) + \sqrt{a} f g^3 x \sqrt{1 + c x^2 / a} / (2 c) - 3 \sqrt{a} f g h^2 x^3 / (8 c \sqrt{1 + c x^2 / a}) + 3 a^2 e h^3 \operatorname{asinh}(\sqrt{c} x / \sqrt{a}) / (8 c^{5/2}) + 9 a^2 f g h^2 \operatorname{asinh}(\sqrt{c} x / \sqrt{a}) / (8 c^{5/2}) - 3 a d g h^2 \operatorname{asinh}(\sqrt{c} x / \sqrt{a}) / (2 c^{3/2}) - 3 a e g^2 h \operatorname{asinh}(\sqrt{c} x / \sqrt{a}) / (2 c^{3/2}) - a f g^3 \operatorname{asinh}(\sqrt{c} x / \sqrt{a}) / (2 c^{3/2}) + d g^3 \operatorname{Piecewise}(\left(\sqrt{-a/c} \operatorname{asin}(x \sqrt{-c/a}) / \sqrt{a}, (a > 0) \& (c < 0)\right), \left(\sqrt{a/c} \operatorname{asinh}(x \sqrt{c/a}) / \sqrt{a}, (a > 0) \& (c > 0)\right), \left(\sqrt{-a/c} \operatorname{acosh}(x \sqrt{-c/a}) / \sqrt{-a}, (c > 0) \& (a < 0)\right)) + 3 d g^2 h \operatorname{Piecewise}(\left(x^2 / (2 \sqrt{a}), \operatorname{Eq}(c, 0)\right), \left(\sqrt{a + c x^2} / c, \operatorname{True}\right)) + d h^3 \operatorname{Piecewise}(\left(-2 a \sqrt{a + c x^2} / (3 c^2) + x^2 \sqrt{a + c x^2} / (3 c), \operatorname{Ne}(c, 0)\right)$

```

), (x**4/(4*sqrt(a)), True)) + e*g**3*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)
), (sqrt(a + c*x**2)/c, True)) + 3*e*g*h**2*Piecewise((-2*a*sqrt(a + c*x**2)
)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), Tru
e)) + 3*f*g**2*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a +
c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**3*Piecewise((8*a
**2*sqrt(a + c*x**2)/(15*c**3) - 4*a*x**2*sqrt(a + c*x**2)/(15*c**2) + x**4
*sqrt(a + c*x**2)/(5*c), Ne(c, 0)), (x**6/(6*sqrt(a)), True)) + e*h**3*x**5
/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*f*g*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**
2/a))

```



$$3.102 \quad \int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2 - 4ac(h(dh+2eg) + fg^2) + 8c^2dg^2\right) \sqrt{a+cx^2} \left(4(4ah^2(eh+2fg) + cg(fg^2 - 4h(3a^2fh^2 - 4ac(h(dh+2eg) + fg^2) + 8c^2dg^2)))\right)}{8c^{5/2}}$$

**Rubi [A]** time = 0.37, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1654, 833, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2 - 4ac(h(dh+2eg) + fg^2) + 8c^2dg^2\right) \sqrt{a+cx^2} \left(4(4ah^2(eh+2fg) - 4cgh(3dh+eg) + cf^2g^3) - hx(3h^2(4cd-3af) - 2cg(fg-4eh))\right) - \frac{\sqrt{a+cx^2}(g+hx)^2(fg-4eh)}{12ch} + \frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}}{8c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^2\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] -((f\*g - 4\*e\*h)\*(g + h\*x)^2\*Sqrt[a + c\*x^2])/(12\*c\*h) + (f\*(g + h\*x)^3\*Sqrt[a + c\*x^2])/(4\*c\*h) - ((4\*(c\*f\*g^3 - 4\*c\*g\*h\*(e\*g + 3\*d\*h) + 4\*a\*h^2\*(2\*f\*g + e\*h)) - h\*(3\*(4\*c\*d - 3\*a\*f)\*h^2 - 2\*c\*g\*(f\*g - 4\*e\*h))\*x)\*Sqrt[a + c\*x^2]/(24\*c^2\*h) + ((8\*c^2\*d\*g^2 + 3\*a^2\*f\*h^2 - 4\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*c^(5/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 780

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

### Rule 833

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &&
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1654

```

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}} dx &= \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2((4cd-3af)h^2-ch(fg-4eh)x)}{\sqrt{a+cx^2}} dx}{4ch^2} \\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)(ch^2(12cdg-7afg-4ehx))}{\sqrt{a+cx^2}} dx}{4ch^2} \\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(cf g^3 - 4cgh(eg + fhx) - ah^2))\sqrt{a+cx^2}}{4ch^2} \\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(cf g^3 - 4cgh(eg + fhx) - ah^2))\sqrt{a+cx^2}}{4ch^2} \\
&= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(cf g^3 - 4cgh(eg + fhx) - ah^2))\sqrt{a+cx^2}}{4ch^2}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 164, normalized size = 0.74

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) (3a^2fh^2 - 4ac(h(dh+2eg) + fg^2) + 8c^2dg^2) + \sqrt{c}\sqrt{a+cx^2} (2c(6dh(4g+hx) + 4e(3g^2+3ghx+h^2x^2)) + fx(6g^2+8ghx+3h^2x^2)) - ah(16eh+32fg+9fhx)}{24c^5/2}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)^2\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] (Sqrt[c]\*Sqrt[a + c\*x^2]\*(-(a\*h\*(32\*f\*g + 16\*e\*h + 9\*f\*h\*x)) + 2\*c\*(6\*d\*h\*(4\*g + h\*x) + 4\*e\*(3\*g^2 + 3\*g\*h\*x + h^2\*x^2) + f\*x\*(6\*g^2 + 8\*g\*h\*x + 3\*h^2\*x^2))) + 3\*(8\*c^2\*d\*g^2 + 3\*a^2\*f\*h^2 - 4\*a\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(24\*c^(5/2))

**IntegrateAlgebraic [A]** time = 0.62, size = 178, normalized size = 0.80

$$\frac{\log\left(\sqrt{a+cx^2}-\sqrt{c}x\right)\left(-3a^2fh^2+4acdhl^2+8acegh+4acfg^2-8c^2dg^2\right)}{8c^{5/2}}+\frac{\sqrt{a+cx^2}\left(-16achl^2-32afgh-9afh^2x+48cdgh+12cdlh^2x+24ceg^2+24ceghx+8celh^2x^2+12cfcg^2x+16cfcghx^2+6cflh^2x^3\right)}{24c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^2\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] (Sqrt[a + c\*x^2]\*(24\*c\*e\*g^2 + 48\*c\*d\*g\*h - 32\*a\*f\*g\*h - 16\*a\*e\*h^2 + 12\*c\*f\*g^2\*x + 24\*c\*e\*g\*h\*x + 12\*c\*d\*h^2\*x - 9\*a\*f\*h^2\*x + 16\*c\*f\*g\*h\*x^2 + 8\*c\*e\*h^2\*x^2 + 6\*c\*f\*h^2\*x^3))/(24\*c^2) + ((-8\*c^2\*d\*g^2 + 4\*a\*c\*f\*g^2 + 8\*a\*c\*e\*g\*h + 4\*a\*c\*d\*h^2 - 3\*a^2\*f\*h^2)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(8\*c^(5/2))

**fricas [A]** time = 0.83, size = 381, normalized size = 1.71

$$\frac{3\left(8a^2gh-4\left(2d-a\right)ef\right)^2+\left(4ad-3ef\right)^2\sqrt{c}\log\left(-2c^2-2\sqrt{c^2+a}\sqrt{c}\right)-2\left(6ef\right)^2+24c^2g^2-16acd^2+16\left(3d^2-2af\right)gh+8\left(2d^2gh+c^2d^2\right)+3\left(4d^2g^2+8c^2gh+\left(4d^2-3af\right)h^2\right)\sqrt{c^2+a}}{8c^2}+\frac{3\left(8a^2gh-4\left(2d-a\right)ef\right)^2+\left(4ad-3ef\right)^2\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+\left(6ef\right)^2+24c^2g^2-16acd^2+16\left(3d^2-2af\right)gh+8\left(2d^2gh+c^2d^2\right)+3\left(4d^2g^2+8c^2gh+\left(4d^2-3af\right)h^2\right)\sqrt{c^2+a}}{24c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [-1/48\*(3\*(8\*a\*c\*e\*g\*h - 4\*(2\*c^2\*d - a\*c\*f)\*g^2 + (4\*a\*c\*d - 3\*a^2\*f)\*h^2)\*sqrt(c)\*log(-2\*c\*x^2 - 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(6\*c^2\*f\*h^2\*x^3 + 24\*c^2\*e\*g^2 - 16\*a\*c\*e\*h^2 + 16\*(3\*c^2\*d - 2\*a\*c\*f)\*g\*h + 8\*(2\*c^2\*f\*g\*h + c^2\*e\*h^2)\*x^2 + 3\*(4\*c^2\*f\*g^2 + 8\*c^2\*e\*g\*h + (4\*c^2\*d - 3\*a\*c\*f)\*h^2)\*x)\*sqrt(c\*x^2 + a))/c^3, 1/24\*(3\*(8\*a\*c\*e\*g\*h - 4\*(2\*c^2\*d - a\*c\*f)\*g^2 + (4\*a\*c\*d - 3\*a^2\*f)\*h^2)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + (6\*c^2\*f\*h^2\*x^3 + 24\*c^2\*e\*g^2 - 16\*a\*c\*e\*h^2 + 16\*(3\*c^2\*d - 2\*a\*c\*f)\*g\*h + 8\*(2\*c^2\*f\*g\*h + c^2\*e\*h^2)\*x^2 + 3\*(4\*c^2\*f\*g^2 + 8\*c^2\*e\*g\*h + (4\*c^2\*d - 3\*a\*c\*f)\*h^2)\*x)\*sqrt(c\*x^2 + a))/c^3]

**giac [A]** time = 0.23, size = 206, normalized size = 0.92

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(\frac{3fl^2x}{c}+\frac{4(2c^3fgh+c^3h^2e)}{c^4}\right)\right)x+\frac{3(4c^3fg^2+4c^3dl^2-3ac^2fh^2+8c^3ghe)}{c^4}\right)+\frac{8(6c^3dgh-4ac^2fgh+3c^3g^2e-2ac^2h^2c)}{c^4}-\frac{(8c^2dg^2-4acfg^2-4acdhl^2+3a^2fl^2-8acghe)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{24}\sqrt{c x^2 + a} \left( \frac{2(3 f h^2 x / c + 4(2 c^3 f g h + c^3 h^2 e) / c^4) x + 3(4 c^3 f g^2 + 4 c^3 d h^2 - 3 a c^2 f h^2 + 8 c^3 g h e) / c^4}{c^4} x + 8(6 c^3 d g h - 4 a c^2 f g h + 3 c^3 g^2 e - 2 a c^2 h^2 e) / c^4 \right) - \frac{1}{8} (8 c^2 d g^2 - 4 a c f g^2 - 4 a c d h^2 + 3 a^2 f h^2 - 8 a c g h e) \log(\text{abs}(-\sqrt{c} x + \sqrt{c x^2 + a})) / c^{5/2}$

**maple** [A] time = 0.01, size = 339, normalized size = 1.52

$$\frac{\sqrt{c x^2 + a} f h^2 x^3}{4 c} - \frac{3 \sqrt{c x^2 + a} a f h^2 x}{8 c^2} + \frac{d g^2 \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{\sqrt{c}} + \frac{3 a^2 f h^2 \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{8 c^{\frac{5}{2}}} + \frac{\sqrt{c x^2 + a} e g^2}{c} + \frac{2 \sqrt{c x^2 + a} d g h}{c} + \frac{(2 f g h + e h^2) \sqrt{c x^2 + a} x^2}{3 c} + \frac{(f g^2 + 2 e g h + d h^2) \sqrt{c x^2 + a} x}{2 c} - \frac{(f g^2 + 2 e g h + d h^2) a \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{2 c^{\frac{3}{2}}} - \frac{2(2 f g h + e h^2) \sqrt{c x^2 + a} a}{3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x)

[Out]  $\frac{1}{4} h^2 f x^3 / c (c x^2 + a)^{1/2} - \frac{3}{8} h^2 f a / c^2 x (c x^2 + a)^{1/2} + \frac{3}{8} h^2 f a^2 / c^{5/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) + \frac{1}{3} x^2 / c (c x^2 + a)^{1/2} h^2 e + \frac{2}{3} x^2 / c (c x^2 + a)^{1/2} g h f - \frac{2}{3} a / c^2 (c x^2 + a)^{1/2} h^2 e - \frac{4}{3} a / c^2 (c x^2 + a)^{1/2} g h f + \frac{1}{2} x / c (c x^2 + a)^{1/2} d h^2 + x / c (c x^2 + a)^{1/2} e g h + \frac{1}{2} x / c (c x^2 + a)^{1/2} f g^2 - \frac{1}{2} a / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) * d h^2 - a / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) * e g h - \frac{1}{2} a / c^{3/2} \ln(c^{1/2} x + (c x^2 + a)^{1/2}) * f g^2 + \frac{2}{c} (c x^2 + a)^{1/2} g h d + \frac{1}{c} (c x^2 + a)^{1/2} g^2 e + g^2 d \ln(c^{1/2} x + (c x^2 + a)^{1/2}) / c^{1/2}$

**maxima** [A] time = 0.45, size = 230, normalized size = 1.03

$$\frac{\sqrt{c x^2 + a} f h^2 x^3}{4 c} - \frac{3 \sqrt{c x^2 + a} a f h^2 x}{8 c^2} + \frac{d g^2 \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{\sqrt{c}} + \frac{3 a^2 f h^2 \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{8 c^{\frac{5}{2}}} + \frac{\sqrt{c x^2 + a} e g^2}{c} + \frac{2 \sqrt{c x^2 + a} d g h}{c} + \frac{(2 f g h + e h^2) \sqrt{c x^2 + a} x^2}{3 c} + \frac{(f g^2 + 2 e g h + d h^2) \sqrt{c x^2 + a} x}{2 c} - \frac{(f g^2 + 2 e g h + d h^2) a \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{2 c^{\frac{3}{2}}} - \frac{2(2 f g h + e h^2) \sqrt{c x^2 + a} a}{3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{4} \sqrt{c x^2 + a} f h^2 x^3 / c - \frac{3}{8} \sqrt{c x^2 + a} a f h^2 x / c^2 + d g^2 \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c} + \frac{3}{8} a^2 f h^2 \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{5/2} + \sqrt{c x^2 + a} e g^2 / c + 2 \sqrt{c x^2 + a} d g h / c + \frac{1}{3} (2 f g h + e h^2) \sqrt{c x^2 + a} x^2 / c + \frac{1}{2} (f g^2 + 2 e g h + d h^2) \sqrt{c x^2 + a} x / c - \frac{1}{2} (f g^2 + 2 e g h + d h^2) a \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2} - \frac{2}{3} (2 f g h + e h^2) \sqrt{c x^2 + a} a / c^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + h x)^2 (f x^2 + e x + d)}{\sqrt{c x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(1/2), x)

[Out] int(((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(1/2), x)

**sympy [A]** time = 15.78, size = 518, normalized size = 2.32

$$\frac{\frac{3\sqrt{f}x}{8c\sqrt{1+\frac{c}{a}}} + \frac{\sqrt{d}ab^2\sqrt{1+\frac{c}{a}}}{2c} + \frac{\sqrt{d}egh\sqrt{1+\frac{c}{a}}}{c} + \frac{\sqrt{d}fg^2\sqrt{1+\frac{c}{a}}}{2c} - \frac{\sqrt{d}fh^2x}{8\sqrt{1+\frac{c}{a}}} + \frac{3d^2fh\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{8c^3} + \frac{ab^2\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{2c^2} + \frac{egh\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{c^2} + \frac{dfg^2\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{2c^2} + d\left(\frac{\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{\sqrt{a}} \text{ for } a > 0 \wedge c < 0\right)}{\left(\frac{3\sqrt{f}x}{8c\sqrt{1+\frac{c}{a}}} + \frac{\sqrt{d}ab^2\sqrt{1+\frac{c}{a}}}{2c} + \frac{\sqrt{d}egh\sqrt{1+\frac{c}{a}}}{c} + \frac{\sqrt{d}fg^2\sqrt{1+\frac{c}{a}}}{2c} - \frac{\sqrt{d}fh^2x}{8\sqrt{1+\frac{c}{a}}} + \frac{3d^2fh\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{8c^3} + \frac{ab^2\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{2c^2} + \frac{egh\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{c^2} + \frac{dfg^2\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{2c^2} + d\left(\frac{\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{a}\right)}{\sqrt{a}} \text{ for } a > 0 \wedge c < 0\right)}\right)^2} + 2d\left(\frac{\frac{d}{2\sqrt{c}} \text{ for } c = 0}{\frac{d}{2\sqrt{c}} \text{ otherwise}}\right) + e\left(\frac{\frac{d}{2\sqrt{c}} \text{ for } c = 0}{\frac{d}{2\sqrt{c}} \text{ otherwise}}\right) + ab\left(\frac{\frac{d}{2\sqrt{c}} + \frac{d\sqrt{ac}}{a} \text{ for } c \neq 0}{\frac{d}{2\sqrt{c}} \text{ otherwise}}\right) + 2f\left(\frac{\frac{d}{2\sqrt{c}} + \frac{d\sqrt{ac}}{a} \text{ for } c \neq 0}{\frac{d}{2\sqrt{c}} \text{ otherwise}}\right) + \frac{fh^2x}{4\sqrt{1+\frac{c}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-3*a**(3/2)*f*h**2*x/(8*c**2*\sqrt{1 + c*x**2/a}) + \sqrt{a}*d*h**2*x*\sqrt{1 + c*x**2/a}/(2*c) + \sqrt{a}*e*g*h*x*\sqrt{1 + c*x**2/a}/c + \sqrt{a}*f*g**2*x*\sqrt{1 + c*x**2/a}/(2*c) - \sqrt{a}*f*h**2*x**3/(8*c*\sqrt{1 + c*x**2/a}) + 3*a**2*f*h**2*\operatorname{asinh}(\sqrt{c}*x/\sqrt{a})/(8*c**(5/2)) - a*d*h**2*\operatorname{asinh}(\sqrt{c}*x/\sqrt{a})/(2*c**(3/2)) - a*e*g*h*\operatorname{asinh}(\sqrt{c}*x/\sqrt{a})/c**(3/2) - a*f*g**2*\operatorname{asinh}(\sqrt{c}*x/\sqrt{a})/(2*c**(3/2)) + d*g**2*\operatorname{Piecewise}(\left(\sqrt{-a/c}*\operatorname{asin}(x*\sqrt{-c/a})/\sqrt{a}, (a > 0) \& (c < 0)\right), \left(\sqrt{a/c}*\operatorname{asinh}(x*\sqrt{c/a})/\sqrt{a}, (a > 0) \& (c > 0)\right), \left(\sqrt{-a/c}*\operatorname{acosh}(x*\sqrt{-c/a})/\sqrt{-a}, (c > 0) \& (a < 0)\right)) + 2*d*g*h*\operatorname{Piecewise}(\left(x**2/(2*\sqrt{a}), \operatorname{Eq}(c, 0)\right), \left(\sqrt{a + c*x**2}/c, \operatorname{True}\right)) + e*g**2*\operatorname{Piecewise}(\left(x**2/(2*\sqrt{a}), \operatorname{Eq}(c, 0)\right), \left(\sqrt{a + c*x**2}/c, \operatorname{True}\right)) + e*h**2*\operatorname{Piecewise}(\left(-2*a*\sqrt{a + c*x**2}/(3*c**2) + x**2*\sqrt{a + c*x**2}/(3*c), \operatorname{Ne}(c, 0)\right), \left(x**4/(4*\sqrt{a}), \operatorname{True}\right)) + 2*f*g*h*\operatorname{Piecewise}(\left(-2*a*\sqrt{a + c*x**2}/(3*c**2) + x**2*\sqrt{a + c*x**2}/(3*c), \operatorname{Ne}(c, 0)\right), \left(x**4/(4*\sqrt{a}), \operatorname{True}\right)) + f*h**2*x**5/(4*\sqrt{a}*\sqrt{1 + c*x**2/a})$

$$3.103 \quad \int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=136

$$\frac{\tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)(2cdg - a(eh + fg))}{2c^{3/2}} - \frac{\sqrt{a+cx^2} \left(2(2afh^2 + c(fg^2 - 3h(dh + eg)))\right) + chx(fg - 3eh)}{6c^2h} + \frac{f\sqrt{a+cx^2}}{3ch}$$

**Rubi [A]** time = 0.18, antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1654, 780, 217, 206}

$$-\frac{\sqrt{a+cx^2} \left(2(2afh^2 - 3ch(dh + eg) + cfg^2) + chx(fg - 3eh)\right)}{6c^2h} + \frac{\tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)(2cdg - a(eh + fg))}{2c^{3/2}} + \frac{f\sqrt{a+cx^2}(g + hx)^2}{3ch}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2], x]

[Out] (f\*(g + h\*x)^2\*Sqrt[a + c\*x^2])/(3\*c\*h) - ((2\*(c\*f\*g^2 + 2\*a\*f\*h^2 - 3\*c\*h\*(e\*g + d\*h)) + c\*h\*(f\*g - 3\*e\*h)\*x)\*Sqrt[a + c\*x^2])/(6\*c^2\*h) + ((2\*c\*d\*g - a\*(f\*g + e\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*c^(3/2))

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 780

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

#### Rule 1654

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x

$)^{(m+q-1)*(a+cx^2)^{(p+1)}}/(c*e^{(q-1)*(m+q+2p+1)}), x] + \text{Dist}[1/(c*e^q*(m+q+2p+1)), \text{Int}[(d+e*x)^m*(a+cx^2)^p*\text{ExpandToSum}[c*e^q*(m+q+2p+1)*Pq - c*f*(m+q+2p+1)*(d+e*x)^q - f*(d+e*x)^{(q-2)*(a*e^2*(m+q-1) - c*d^2*(m+q+2p+1) - 2*c*d*e*(m+q+p)*x}], x], x] /; \text{GtQ}[q, 1] \&\& \text{NeQ}[m+q+2p+1, 0] /; \text{FreeQ}[\{a, c, d, e, m, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !(EqQ[d, 0] \&\& \text{True}) \&\& !(IGtQ[m, 0] \&\& \text{RationalQ}[a, c, d, e] \&\& (\text{IntegerQ}[p] || \text{ILtQ}[p + 1/2, 0]))$

### Rubi steps

$$\begin{aligned} \int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}} dx &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} + \frac{\int \frac{(g+hx)((3cd-2af)h^2-ch(fg-3eh)x)}{\sqrt{a+cx^2}} dx}{3ch^2} \\ &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(cf g^2 + 2af h^2 - 3ch(eg+dh)) + ch(fg-3eh)x)\sqrt{a+cx^2}}{6c^2h} \\ &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(cf g^2 + 2af h^2 - 3ch(eg+dh)) + ch(fg-3eh)x)\sqrt{a+cx^2}}{6c^2h} \\ &= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(cf g^2 + 2af h^2 - 3ch(eg+dh)) + ch(fg-3eh)x)\sqrt{a+cx^2}}{6c^2h} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+cx^2} (c(6dh+6eg+3ehx+3fgx+2fhx^2) - 4afh) + 3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right) (2cdg - a(eh+fg))}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((g+h\*x)\*(d+e\*x+f\*x^2))/Sqrt[a+c\*x^2],x]

[Out] (Sqrt[a+c\*x^2]\*(-4\*a\*f\*h+c\*(6\*e\*g+6\*d\*h+3\*f\*g\*x+3\*e\*h\*x+2\*f\*h\*x^2))+3\*Sqrt[c]\*(2\*c\*d\*g-a\*(f\*g+e\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a+c\*x^2]]/(6\*c^2)

**IntegrateAlgebraic [A]** time = 0.44, size = 99, normalized size = 0.73

$$\frac{\log\left(\sqrt{a+cx^2} - \sqrt{c}x\right)(aeh+afg-2cdg)}{2c^{3/2}} + \frac{\sqrt{a+cx^2}(-4afh+6cdh+6ceg+3cehx+3cfgx+2cfhx^2)}{6c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)\*(d + e\*x + f\*x^2))/Sqrt[a + c\*x^2],x]

[Out] (Sqrt[a + c\*x^2]\*(6\*c\*e\*g + 6\*c\*d\*h - 4\*a\*f\*h + 3\*c\*f\*g\*x + 3\*c\*e\*h\*x + 2\*c\*f\*h\*x^2))/(6\*c^2) + ((-2\*c\*d\*g + a\*f\*g + a\*e\*h)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(2\*c^(3/2))

**fricas** [A] time = 1.28, size = 199, normalized size = 1.46

$$\left[ \frac{3(aeh - (2cd - af)g)\sqrt{c} \log(-2cx^2 + 2\sqrt{cx^2 + a}\sqrt{cx - a}) + 2(2cfhx^2 + 6ceg + 2(3cd - 2af)h + 3(cfg + ceh)x)\sqrt{cx^2 + a}}{12c^2}, \frac{3(aeh - (2cd - af)g)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2 + a}}\right) + (2cfhx^2 + 6ceg + 2(3cd - 2af)h + 3(cfg + ceh)x)\sqrt{cx^2 + a}}{6c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/12\*(3\*(a\*e\*h - (2\*c\*d - a\*f)\*g)\*sqrt(c)\*log(-2\*c\*x^2 + 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) + 2\*(2\*c\*f\*h\*x^2 + 6\*c\*e\*g + 2\*(3\*c\*d - 2\*a\*f)\*h + 3\*(c\*f\*g + c\*e\*h)\*x)\*sqrt(c\*x^2 + a))/c^2, 1/6\*(3\*(a\*e\*h - (2\*c\*d - a\*f)\*g)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + (2\*c\*f\*h\*x^2 + 6\*c\*e\*g + 2\*(3\*c\*d - 2\*a\*f)\*h + 3\*(c\*f\*g + c\*e\*h)\*x)\*sqrt(c\*x^2 + a))/c^2]

**giac** [A] time = 0.21, size = 110, normalized size = 0.81

$$\frac{1}{6} \sqrt{cx^2 + a} \left( \left( \frac{2fhx}{c} + \frac{3(c^2fg + c^2he)}{c^3} \right) x + \frac{2(3c^2dh - 2acfh + 3c^2ge)}{c^3} \right) - \frac{(2cdg - afg - ahe) \log\left(\left| -\sqrt{c}x + \sqrt{cx^2 + a} \right|\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="giac")

[Out] 1/6\*sqrt(c\*x^2 + a)\*((2\*f\*h\*x/c + 3\*(c^2\*f\*g + c^2\*h\*e)/c^3)\*x + 2\*(3\*c^2\*d\*h - 2\*a\*c\*f\*h + 3\*c^2\*g\*e)/c^3) - 1/2\*(2\*c\*d\*g - a\*f\*g - a\*h\*e)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

**maple** [A] time = 0.01, size = 172, normalized size = 1.26

$$\frac{\sqrt{cx^2 + a} fhx^2}{3c} - \frac{aeh \ln(\sqrt{c}x + \sqrt{cx^2 + a})}{2c^{\frac{3}{2}}} - \frac{afg \ln(\sqrt{c}x + \sqrt{cx^2 + a})}{2c^{\frac{3}{2}}} + \frac{dgl \ln(\sqrt{c}x + \sqrt{cx^2 + a})}{\sqrt{c}} + \frac{\sqrt{cx^2 + a} ehx}{2c} + \frac{\sqrt{cx^2 + a} fgx}{2c} - \frac{2\sqrt{cx^2 + a} afh}{3c^2} + \frac{\sqrt{cx^2 + a} dh}{c} + \frac{\sqrt{cx^2 + a} eg}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x)

[Out] 1/3\*h\*f\*x^2/c\*(c\*x^2+a)^(1/2)-2/3\*h\*f\*a/c^2\*(c\*x^2+a)^(1/2)+1/2\*x/c\*(c\*x^2+a)^(1/2)\*e\*h+1/2\*x/c\*(c\*x^2+a)^(1/2)\*f\*g-1/2\*a/c^(3/2)\*ln(c^(1/2)\*x+(c\*x^2+a)^(1/2))\*e\*h-1/2\*a/c^(3/2)\*ln(c^(1/2)\*x+(c\*x^2+a)^(1/2))\*f\*g+1/c\*(c\*x^2+a)



$\int (1/2) * d * h + 1/c * (c * x^2 + a)^{1/2} * e * g + d * g * \ln(c^{1/2} * x + (c * x^2 + a)^{1/2}) / c^{1/2} dx$

**maxima [A]** time = 0.44, size = 126, normalized size = 0.93

$$\frac{\sqrt{cx^2+a} f h x^2}{3c} + \frac{d g \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+a} e g}{c} + \frac{\sqrt{cx^2+a} d h}{c} - \frac{2\sqrt{cx^2+a} a f h}{3c^2} + \frac{\sqrt{cx^2+a} (f g + e h) x}{2c} - \frac{(f g + e h) a \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{3} \sqrt{c x^2 + a} f h x^2 / c + d g \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c} + \sqrt{c x^2 + a} e g / c + \sqrt{c x^2 + a} d h / c - 2 / 3 \sqrt{c x^2 + a} a f h / c^2 + 1 / 2 \sqrt{c x^2 + a} (f g + e h) x / c - 1 / 2 (f g + e h) a \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2}$

**mupad [B]** time = 5.17, size = 227, normalized size = 1.67

$$\begin{cases} \frac{2 f g x^3 + 3 e g x^2 + 6 d g x}{6 \sqrt{a}} + \frac{3 f h x^4 + 4 e h x^3 + 6 d h x^2}{12 \sqrt{a}} & \text{if } c = 0 \\ \frac{d g \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{\sqrt{c}} + \frac{d h \sqrt{c x^2 + a}}{c} + \frac{e g \sqrt{c x^2 + a}}{c} + \frac{e h x \sqrt{c x^2 + a}}{2c} + \frac{f g x \sqrt{c x^2 + a}}{2c} - \frac{f h \sqrt{c x^2 + a} (2a - c x^2)}{3c^2} - \frac{a e h \ln(2 \sqrt{c} x + 2 \sqrt{c x^2 + a})}{2c^{3/2}} - \frac{a f g \ln(2 \sqrt{c} x + 2 \sqrt{c x^2 + a})}{2c^{3/2}} & \text{if } c \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(1/2),x)

[Out]  $\operatorname{piecewise}(c == 0, (3 * e * g * x^2 + 2 * f * g * x^3 + 6 * d * g * x) / (6 * a^{1/2}) + (6 * d * h * x^2 + 4 * e * h * x^3 + 3 * f * h * x^4) / (12 * a^{1/2}), c \neq 0, (d * g * \log(c^{1/2} * x + (a + c * x^2)^{1/2})) / c^{1/2} + (d * h * (a + c * x^2)^{1/2}) / c + (e * g * (a + c * x^2)^{1/2}) / c + (e * h * x * (a + c * x^2)^{1/2}) / (2 * c) + (f * g * x * (a + c * x^2)^{1/2}) / (2 * c) - (f * h * (a + c * x^2)^{1/2} * (2 * a - c * x^2)) / (3 * c^2) - (a * e * h * \log(2 * c^{1/2} * x + 2 * (a + c * x^2)^{1/2})) / (2 * c^{3/2}) - (a * f * g * \log(2 * c^{1/2} * x + 2 * (a + c * x^2)^{1/2})) / (2 * c^{3/2}))$

**sympy [A]** time = 9.02, size = 282, normalized size = 2.07

$$\frac{\sqrt{a} e h x \sqrt{1 + \frac{c x^2}{a}}}{2c} + \frac{\sqrt{a} f g x \sqrt{1 + \frac{c x^2}{a}}}{2c} - \frac{a e h \operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}} - \frac{a f g \operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}} + d g \begin{cases} \left(\frac{\sqrt{-\frac{c}{a}} \operatorname{asin}\left(\sqrt{\frac{c}{a}}\right)}{\sqrt{a}}\right) & \text{for } a > 0 \wedge c < 0 \\ \left(\frac{\sqrt{\frac{c}{a}} \operatorname{asinh}\left(\sqrt{\frac{c}{a}}\right)}{\sqrt{a}}\right) & \text{for } a > 0 \wedge c > 0 \\ \left(\frac{\sqrt{-\frac{c}{a}} \operatorname{acosh}\left(\sqrt{\frac{c}{a}}\right)}{\sqrt{-a}}\right) & \text{for } c > 0 \wedge a < 0 \end{cases} + d h \begin{cases} \left(\frac{x^2}{2\sqrt{a}}\right) & \text{for } c = 0 \\ \left(\frac{x^2}{\sqrt{a+c x^2}}\right) & \text{otherwise} \end{cases} + e g \begin{cases} \left(\frac{x^2}{2\sqrt{a}}\right) & \text{for } c = 0 \\ \left(\frac{x^2}{\sqrt{a+c x^2}}\right) & \text{otherwise} \end{cases} + f h \begin{cases} \left(\frac{2 a \sqrt{a+c x^2}}{3c^2} + \frac{x^2 \sqrt{a+c x^2}}{3c}\right) & \text{for } c \neq 0 \\ \left(\frac{x^4}{4\sqrt{a}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $\sqrt{a} * e * h * x * \sqrt{1 + c * x^2 / a} / (2 * c) + \sqrt{a} * f * g * x * \sqrt{1 + c * x^2 / a} / (2 * c) - a * e * h * \operatorname{asinh}(\sqrt{c} * x / \sqrt{a}) / (2 * c^{3/2}) - a * f * g * \operatorname{asinh}(\sqrt{c} * x / \sqrt{a}) / (2 * c^{3/2})$

```

sqrt(a))/(2*c**(3/2)) + d*g*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a)
), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c
> 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + d*h*
Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + e*g*P
iecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + f*h*Pi
iecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c
, 0)), (x**4/(4*sqrt(a)), True))

```

$$3.104 \quad \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}} dx$$

Optimal. Leaf size=74

$$\frac{(2cd - af) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}} + \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c}$$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1815, 641, 217, 206}

$$\frac{(2cd - af) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}} + \frac{e\sqrt{a+cx^2}}{c} + \frac{fx\sqrt{a+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/Sqrt[a + c\*x^2], x]

[Out] (e\*Sqrt[a + c\*x^2])/c + (f\*x\*Sqrt[a + c\*x^2])/(2\*c) + ((2\*c\*d - a\*f)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*c^(3/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x]

], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{\sqrt{a + cx^2}} dx &= \frac{fx\sqrt{a + cx^2}}{2c} + \frac{\int \frac{2cd - af + 2cex}{\sqrt{a + cx^2}} dx}{2c} \\ &= \frac{e\sqrt{a + cx^2}}{c} + \frac{fx\sqrt{a + cx^2}}{2c} + \frac{(2cd - af) \int \frac{1}{\sqrt{a + cx^2}} dx}{2c} \\ &= \frac{e\sqrt{a + cx^2}}{c} + \frac{fx\sqrt{a + cx^2}}{2c} + \frac{(2cd - af) \text{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{2c} \\ &= \frac{e\sqrt{a + cx^2}}{c} + \frac{fx\sqrt{a + cx^2}}{2c} + \frac{(2cd - af) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right)}{2c^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 63, normalized size = 0.85

$$\frac{(2cd - af) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right) + \sqrt{c} \sqrt{a + cx^2} (2e + fx)}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/Sqrt[a + c\*x^2], x]

[Out] (Sqrt[c]\*(2\*e + f\*x)\*Sqrt[a + c\*x^2] + (2\*c\*d - a\*f)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*c^(3/2))

**IntegrateAlgebraic [A]** time = 0.32, size = 64, normalized size = 0.86

$$\frac{(af - 2cd) \log\left(\sqrt{a + cx^2} - \sqrt{c}x\right)}{2c^{3/2}} + \frac{\sqrt{a + cx^2} (2e + fx)}{2c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/Sqrt[a + c\*x^2], x]

[Out] ((2\*e + f\*x)\*Sqrt[a + c\*x^2])/(2\*c) + ((-2\*c\*d + a\*f)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(2\*c^(3/2))

**fricas** [A] time = 0.87, size = 124, normalized size = 1.68

$$\left[ \frac{(2cd - af)\sqrt{c} \log\left(-2cx^2 + 2\sqrt{cx^2 + a}\sqrt{c}x - a\right) - 2(cf x + 2ce)\sqrt{cx^2 + a}}{4c^2}, -\frac{(2cd - af)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2 + a}}\right) - (cf x + 2ce)\sqrt{cx^2 + a}}{2c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [-1/4\*((2\*c\*d - a\*f)\*sqrt(c)\*log(-2\*c\*x^2 + 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(c\*f\*x + 2\*c\*e)\*sqrt(c\*x^2 + a))/c^2, -1/2\*((2\*c\*d - a\*f)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) - (c\*f\*x + 2\*c\*e)\*sqrt(c\*x^2 + a))/c^2]

**giac** [A] time = 0.20, size = 58, normalized size = 0.78

$$\frac{1}{2} \sqrt{cx^2 + a} \left( \frac{fx}{c} + \frac{2e}{c} \right) - \frac{(2cd - af) \log\left(\left| -\sqrt{c}x + \sqrt{cx^2 + a} \right| \right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="giac")

[Out] 1/2\*sqrt(c\*x^2 + a)\*(f\*x/c + 2\*e/c) - 1/2\*(2\*c\*d - a\*f)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

**maple** [A] time = 0.01, size = 76, normalized size = 1.03

$$-\frac{af \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{2c^{\frac{3}{2}}} + \frac{d \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2 + a} fx}{2c} + \frac{\sqrt{cx^2 + a} e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x)

[Out] 1/2\*f\*x\*(c\*x^2+a)^(1/2)/c-1/2\*f\*a/c^(3/2)\*ln(c^(1/2)\*x+(c\*x^2+a)^(1/2))+e\*(c\*x^2+a)^(1/2)/c+d\*ln(c^(1/2)\*x+(c\*x^2+a)^(1/2))/c^(1/2)

**maxima** [A] time = 0.43, size = 61, normalized size = 0.82

$$\frac{\sqrt{cx^2 + a} fx}{2c} + \frac{d \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} - \frac{af \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{3}{2}}} + \frac{\sqrt{cx^2 + a} e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{2}\sqrt{c x^2 + a} f x / c + d \operatorname{arcsinh}(c x / \sqrt{a c}) / \sqrt{c} - \frac{1}{2} a f \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2} + \sqrt{c x^2 + a} e / c$

**mupad [B]** time = 4.56, size = 107, normalized size = 1.45

$$\left\{ \begin{array}{ll} \frac{2 f x^3 + 3 e x^2 + 6 d x}{6 \sqrt{a}} & \text{if } c = 0 \\ \frac{e \sqrt{c x^2 + a}}{c} + \frac{d \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{\sqrt{c}} - \frac{a f \ln(2 \sqrt{c} x + 2 \sqrt{c x^2 + a})}{2 c^{3/2}} + \frac{f x \sqrt{c x^2 + a}}{2 c} & \text{if } c \neq 0 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + c\*x^2)^(1/2),x)

[Out]  $\operatorname{piecewise}(c == 0, (6*d*x + 3*e*x^2 + 2*f*x^3)/(6*a^{1/2}), c \neq 0, (e*(a + c*x^2)^{1/2})/c + (d*\log(c^{1/2}*x + (a + c*x^2)^{1/2}))/c^{1/2} - (a*f*\log(2*c^{1/2}*x + 2*(a + c*x^2)^{1/2}))/ (2*c^{3/2}) + (f*x*(a + c*x^2)^{1/2})/(2*c))$

**sympy [A]** time = 3.50, size = 150, normalized size = 2.03

$$\frac{\sqrt{a} f x \sqrt{1 + \frac{c x^2}{a}}}{2 c} - \frac{a f \operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{2 c^{\frac{3}{2}}} + d \left( \left( \begin{array}{l} \frac{\sqrt{-\frac{a}{c}} \operatorname{asin}\left(x \sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} \quad \text{for } a > 0 \wedge c < 0 \\ \frac{\sqrt{\frac{a}{c}} \operatorname{asinh}\left(x \sqrt{\frac{c}{a}}\right)}{\sqrt{a}} \quad \text{for } a > 0 \wedge c > 0 \\ \frac{\sqrt{-\frac{a}{c}} \operatorname{acosh}\left(x \sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} \quad \text{for } c > 0 \wedge a < 0 \end{array} \right) + e \left( \begin{array}{l} \frac{x^2}{2 \sqrt{a}} \quad \text{for } c = 0 \\ \frac{\sqrt{a + c x^2}}{c} \quad \text{otherwise} \end{array} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $\sqrt{a} f x \sqrt{1 + c x^2 / a} / (2 * c) - a * f * \operatorname{asinh}(\sqrt{c} * x / \sqrt{a}) / (2 * c * (3 / 2)) + d * \operatorname{Piecewise}((\sqrt{-a / c} * \operatorname{asin}(x * \sqrt{-c / a}) / \sqrt{a}, (a > 0) \& (c < 0)), (\sqrt{a / c} * \operatorname{asinh}(x * \sqrt{c / a}) / \sqrt{a}, (a > 0) \& (c > 0)), (\sqrt{-a / c} * \operatorname{acosh}(x * \sqrt{-c / a}) / \sqrt{-a}, (c > 0) \& (a < 0))) + e * \operatorname{Piecewise}((x^2 / (2 * \sqrt{a})), \operatorname{Eq}(c, 0)), (\sqrt{a + c * x^2} / c, \operatorname{True}))$

$$3.105 \quad \int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=130

$$\frac{(dh^2 - egh + fg^2) \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2} \sqrt{ah^2+cg^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg - eh)}{\sqrt{c}h^2} + \frac{f\sqrt{a+cx^2}}{ch}}{h^2\sqrt{ah^2+cg^2}}$$

**Rubi [A]** time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1654, 844, 217, 206, 725}

$$\frac{(dh^2 - egh + fg^2) \tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2} \sqrt{ah^2+cg^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg - eh)}{\sqrt{c}h^2} + \frac{f\sqrt{a+cx^2}}{ch}}{h^2\sqrt{ah^2+cg^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)\*Sqrt[a + c\*x^2]),x]

[Out] (f\*Sqrt[a + c\*x^2])/(c\*h) - ((f\*g - e\*h)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(Sqrt[c]\*h^2) - ((f\*g^2 - e\*g\*h + d\*h^2)\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/(h^2\*Sqrt[c\*g^2 + a\*h^2])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && True) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(g + hx)\sqrt{a + cx^2}} dx &= \frac{f\sqrt{a + cx^2}}{ch} + \frac{\int \frac{cdh^2 - ch(fg - eh)x}{(g + hx)\sqrt{a + cx^2}} dx}{ch^2} \\ &= \frac{f\sqrt{a + cx^2}}{ch} - \frac{(fg - eh) \int \frac{1}{\sqrt{a + cx^2}} dx}{h^2} + \frac{(fg^2 - egh + dh^2) \int \frac{1}{(g + hx)\sqrt{a + cx^2}} dx}{h^2} \\ &= \frac{f\sqrt{a + cx^2}}{ch} - \frac{(fg - eh) \operatorname{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{h^2} - \frac{(fg^2 - egh + dh^2) \operatorname{Subst}\left(\int \frac{1}{cg} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{h^2} \\ &= \frac{f\sqrt{a + cx^2}}{ch} - \frac{(fg - eh) \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right)}{\sqrt{c}h^2} - \frac{(fg^2 - egh + dh^2) \tanh^{-1}\left(\frac{ah - cgx}{\sqrt{cg^2 + ah^2} \sqrt{a + cx^2}}\right)}{h^2 \sqrt{cg^2 + ah^2}} \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 125, normalized size = 0.96

$$\frac{(h(dh - eg) + fg^2) \tanh^{-1}\left(\frac{ah - cgx}{\sqrt{a + cx^2} \sqrt{ah^2 + cg^2}}\right)}{\sqrt{ah^2 + cg^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right)(eh - fg)}{\sqrt{c}} + \frac{fh\sqrt{a + cx^2}}{c}$$


---


$$h^2$$

Antiderivative was successfully verified.



[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)\*Sqrt[a + c\*x^2]),x]

[Out] ((f\*h\*Sqrt[a + c\*x^2])/c + ((-(f\*g) + e\*h)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/Sqrt[c] - ((f\*g^2 + h\*(-(e\*g) + d\*h))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])])/Sqrt[c\*g^2 + a\*h^2])/h^2

**IntegrateAlgebraic [A]** time = 0.47, size = 158, normalized size = 1.22

$$\frac{2\sqrt{-ah^2 - cg^2} (dh^2 - egh + fg^2) \tan^{-1}\left(\frac{-h\sqrt{a+cx^2} + \sqrt{c}g + \sqrt{c}hx}{\sqrt{-ah^2 - cg^2}}\right)}{h^2 (ah^2 + cg^2)} + \frac{\log\left(\sqrt{a+cx^2} - \sqrt{c}x\right)(fg - eh)}{\sqrt{c}h^2} + \frac{f\sqrt{a+cx^2}}{ch}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)\*Sqrt[a + c\*x^2]),x]

[Out] (f\*Sqrt[a + c\*x^2])/(c\*h) + (2\*Sqrt[-(c\*g^2) - a\*h^2]\*(f\*g^2 - e\*g\*h + d\*h^2)\*ArcTan[(Sqrt[c]\*g + Sqrt[c]\*h\*x - h\*Sqrt[a + c\*x^2])/Sqrt[-(c\*g^2) - a\*h^2]])/(h^2\*(c\*g^2 + a\*h^2)) + ((f\*g - e\*h)\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/(Sqrt[c]\*h^2)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [A]** time = 0.22, size = 138, normalized size = 1.06

$$\frac{\sqrt{cx^2 + a} f}{ch} + \frac{2(fg^2 + dh^2 - ghe) \arctan\left(-\frac{(\sqrt{c}x - \sqrt{cx^2 + a})h + \sqrt{c}g}{\sqrt{-cg^2 - ah^2}}\right)}{\sqrt{-cg^2 - ah^2} h^2} + \frac{(\sqrt{c}fg - \sqrt{c}he) \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{ch^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+a)^(1/2),x, algorithm="giac")

[Out] sqrt(c\*x^2 + a)\*f/(c\*h) + 2\*(f\*g^2 + d\*h^2 - g\*h\*e)\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*h + sqrt(c)\*g)/sqrt(-c\*g^2 - a\*h^2))/(sqrt(-c\*g^2 - a\*h^2)\*h^2) + (sqrt(c)\*f\*g - sqrt(c)\*h\*e)\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/(c\*h^2)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\sqrt{a + cx^2} (g + hx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/(sqrt(a + c\*x\*\*2)\*(g + h\*x)), x)

$$3.106 \quad \int \frac{d+ex+fx^2}{(g+hx)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=168

$$-\frac{\sqrt{a+cx^2} (dh^2 - egh + fg^2)}{h(g+hx)(ah^2 + cg^2)} + \frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2} \sqrt{ah^2+cg^2}}\right) (ah^2(2fg - eh) + c(fg^3 - dgh^2))}{h^2 (ah^2 + cg^2)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}$$

**Rubi [A]** time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1651, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2} (dh^2 - egh + fg^2)}{h(g+hx)(ah^2 + cg^2)} + \frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2} \sqrt{ah^2+cg^2}}\right) (ah^2(2fg - eh) + c(fg^3 - dgh^2))}{h^2 (ah^2 + cg^2)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] -(((f\*g^2 - e\*g\*h + d\*h^2)\*Sqrt[a + c\*x^2])/(h\*(c\*g^2 + a\*h^2)\*(g + h\*x))) + (f\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(Sqrt[c]\*h^2) + ((a\*h^2\*(2\*f\*g - e\*h) + c\*(f\*g^3 - d\*g\*h^2))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/(h^2\*(c\*g^2 + a\*h^2)^(3/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(g + hx)^2 \sqrt{a + cx^2}} dx &= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{h(cg^2 + ah^2)(g + hx)} - \frac{\int \frac{-cdg + afg - aeh - f\left(\frac{cg^2}{h} + ah\right)x}{(g + hx)\sqrt{a + cx^2}} dx}{cg^2 + ah^2} \\ &= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{h(cg^2 + ah^2)(g + hx)} + \frac{f \int \frac{1}{\sqrt{a + cx^2}} dx}{h^2} + \frac{\left(cdg - 2afg - \frac{cfg^3}{h^2} + aeh\right) \int \frac{1}{(g + hx)\sqrt{a + cx^2}} dx}{cg^2 + ah^2} \\ &= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{h(cg^2 + ah^2)(g + hx)} + \frac{f \operatorname{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}}\right)}{h^2} - \frac{\left(cdg - 2afg - \frac{cfg^3}{h^2} + aeh\right) \operatorname{arctan}\left(\frac{x}{\sqrt{a + cx^2}}\right)}{(cg^2 + ah^2)\sqrt{a + cx^2}} \\ &= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{h(cg^2 + ah^2)(g + hx)} + \frac{f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right)}{\sqrt{c}h^2} - \frac{\left(cdg - 2afg - \frac{cfg^3}{h^2} + aeh\right) \operatorname{arctan}\left(\frac{x}{\sqrt{a + cx^2}}\right)}{(cg^2 + ah^2)\sqrt{a + cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.36, size = 218, normalized size = 1.30

$$\frac{-\frac{h\sqrt{a+cx^2}(h(dh-eg)+fg^2)}{(g+hx)(ah^2+cg^2)} + \frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)(ah^2(2fg-eh)+c(fg^3-dgh^2))}{(ah^2+cg^2)^{3/2}} + \frac{\log(g+hx)(ah^2(eh-2fg)+c(dgh^2-fg^3))}{(ah^2+cg^2)^{3/2}} + \frac{f \log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*sqrt[a + c*x^2]), x]
```



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^{(1/2)}, x)$

[Out]  $f/h^2*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})/c^{(1/2)}-1/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*d+1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*f*g^2-1/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*d+1/h^2*c*g^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*e-1/h^3*c*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*f-1/h^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*e+2/h^3/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*f*g$

**maxima** [B] time = 0.58, size = 419, normalized size = 2.49

$$\frac{\sqrt{cx^2+ax} \sqrt{cx^2+ax}}{c^2 h^2 x + ah^2 x + cg^2 h + agh^2} + \frac{\sqrt{cx^2+ax}}{c^2 h^2 x + ah^2 x + cg^2 h + agh^2} - \frac{\sqrt{cx^2+ad}}{c^2 h^2 x + ah^2 x + \frac{cg^2}{h} + agh} + \frac{f \operatorname{arcsinh}\left(\frac{cx}{\sqrt{a}}\right)}{\sqrt{c} h^2} + \frac{c f g^3 \operatorname{arcsinh}\left(\frac{cgx}{\sqrt{a} |hx+g|} - \frac{ah}{\sqrt{a} |hx+g|}\right)}{\left(a + \frac{cg^2}{h^2}\right)^{3/2} h^3} - \frac{c g^2 \operatorname{arcsinh}\left(\frac{cgx}{\sqrt{a} |hx+g|} - \frac{ah}{\sqrt{a} |hx+g|}\right)}{\left(a + \frac{cg^2}{h^2}\right)^{3/2} h^4} + \frac{c d g \operatorname{arcsinh}\left(\frac{cgx}{\sqrt{a} |hx+g|} - \frac{ah}{\sqrt{a} |hx+g|}\right)}{\left(a + \frac{cg^2}{h^2}\right)^{3/2} h^3} - \frac{2 f g \operatorname{arcsinh}\left(\frac{cgx}{\sqrt{a} |hx+g|} - \frac{ah}{\sqrt{a} |hx+g|}\right)}{\sqrt{a + \frac{cg^2}{h^2}} h^3} + \frac{e \operatorname{arcsinh}\left(\frac{cgx}{\sqrt{a} |hx+g|} - \frac{ah}{\sqrt{a} |hx+g|}\right)}{\sqrt{a + \frac{cg^2}{h^2}} h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $-\sqrt{c*x^2+a}*f*g^2/(c*g^2*h^2*x+a*h^4*x+c*g^3*h+a*g*h^3)+\sqrt{c*x^2+a}*e*g/(c*g^2*h*x+a*h^3*x+c*g^3+a*g*h^2)-\sqrt{c*x^2+a}*d/(c*g^2*x+a*h^2*x+c*g^3/h+a*g*h)+f*\operatorname{arcsinh}(c*x/\sqrt{a*c})/(\sqrt{c}*h^2)+c*f*g^3*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c}*|h*x+g|))-a*h/(\sqrt{a*c}*|h*x+g|)/((a+c*g^2/h^2)^{(3/2)}*h^5)-c*e*g^2*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c}*|h*x+g|))-a*h/(\sqrt{a*c}*|h*x+g|)/((a+c*g^2/h^2)^{(3/2)}*h^4)+c*d*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c}*|h*x+g|))-a*h/(\sqrt{a*c}*|h*x+g|)/((a+c*g^2/h^2)^{(3/2)}*h^3)-2*f*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c}*|h*x+g|))-a*h/(\sqrt{a*c}*|h*x+g|)/(\sqrt{a+c*g^2/h^2}*h^3)+e*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c}*|h*x+g|))-a*h/(\sqrt{a*c}*|h*x+g|)/(\sqrt{a+c*g^2/h^2}*h^2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f x^2 + e x + d}{(g + h x)^2 \sqrt{c x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\sqrt{a + cx^2} (g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2), x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```



$$3.107 \quad \int \frac{d+ex+fx^2}{(g+hx)^3 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=225

$$\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}} - \frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)} + \frac{\sqrt{a+cx^2}}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

**Rubi [A]** time = 0.29, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1651, 807, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}} - \frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)} + \frac{\sqrt{a+cx^2}\left(2ah^2(2fg-eh)+cgh(eg-3dh)+cfdg^3\right)}{2h(g+hx)\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^3\*Sqrt[a + c\*x^2]), x]

[Out] -((f\*g^2 - e\*g\*h + d\*h^2)\*Sqrt[a + c\*x^2])/(2\*h\*(c\*g^2 + a\*h^2)\*(g + h\*x)^2) + ((c\*f\*g^3 + c\*g\*h\*(e\*g - 3\*d\*h) + 2\*a\*h^2\*(2\*f\*g - e\*h))\*Sqrt[a + c\*x^2])/(2\*h\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)) - ((2\*c^2\*d\*g^2 + 2\*a^2\*f\*h^2 - a\*c\*(f\*g^2 - h\*(3\*e\*g - d\*h)))\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/(2\*(c\*g^2 + a\*h^2)^(5/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

### Rule 807

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}

, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=
  With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
    d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
    d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
    *(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
    R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
  && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\int \frac{d + ex + fx^2}{(g + hx)^3 \sqrt{a + cx^2}} dx = -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{2h (cg^2 + ah^2) (g + hx)^2} - \frac{\int \frac{-2(cdg - afg + aeh) - \left(2afh + c\left(eg + \frac{fg^2}{h} - dh\right)\right)x}{(g + hx)^2 \sqrt{a + cx^2}} dx}{2 (cg^2 + ah^2)}$$

$$= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{2h (cg^2 + ah^2) (g + hx)^2} + \frac{(cfg^3 + cgh(eg - 3dh) + 2ah^2(2fg - eh)) \sqrt{a + cx^2}}{2h (cg^2 + ah^2)^2 (g + hx)}$$

$$= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{2h (cg^2 + ah^2) (g + hx)^2} + \frac{(cfg^3 + cgh(eg - 3dh) + 2ah^2(2fg - eh)) \sqrt{a + cx^2}}{2h (cg^2 + ah^2)^2 (g + hx)}$$

$$= -\frac{(fg^2 - egh + dh^2) \sqrt{a + cx^2}}{2h (cg^2 + ah^2) (g + hx)^2} + \frac{(cfg^3 + cgh(eg - 3dh) + 2ah^2(2fg - eh)) \sqrt{a + cx^2}}{2h (cg^2 + ah^2)^2 (g + hx)}$$

**Mathematica [A]** time = 0.45, size = 254, normalized size = 1.13

$$\frac{(g + hx)^2 \log\left(\sqrt{a + cx^2} \sqrt{ah^2 + cg^2} + ah - cgx\right) \left(-2a^2 fh^2 + ac(hd - 3eg) + fg^2\right) - 2c^2 dg^2 + (g + hx)^2 \log(g + hx) \left(2a^2 fh^2 - ac(hd - 3eg) + fg^2\right) + 2c^2 dg^2 + \sqrt{a + cx^2} \sqrt{ah^2 + cg^2} \left(cg(-dh(4g + 3hx) + eg(2g + hx) + fg^2x) - ah(h(dh + e(g + 2hx)) - fg(3g + 4hx))\right)}{2(g + hx)^2 (ah^2 + cg^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)^3\*Sqrt[a + c\*x^2]), x]

[Out] (Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]\*(c\*g\*(f\*g^2\*x + e\*g\*(2\*g + h\*x) - d\*h\*(4\*g + 3\*h\*x)) - a\*h\*(-(f\*g\*(3\*g + 4\*h\*x)) + h\*(d\*h + e\*(g + 2\*h\*x)))) + (2\*c^2\*d\*g^2 + 2\*a^2\*f\*h^2 - a\*c\*(f\*g^2 + h\*(-3\*e\*g + d\*h)))\*(g + h\*x)^2\*Log[

$g + hx] + (-2*c^2*d*g^2 - 2*a^2*f*h^2 + a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g + hx)^2*\text{Log}[a*h - c*g*x + \text{Sqrt}[c*g^2 + a*h^2]*\text{Sqrt}[a + c*x^2]]/(2*(c*g^2 + a*h^2)^{(5/2)}*(g + hx)^2)$

**IntegrateAlgebraic [A]** time = 1.52, size = 306, normalized size = 1.36

$$\frac{\tan^{-1}\left(\frac{-h\sqrt{a+cx^2} + \sqrt{c}g + \sqrt{c}hx}{\sqrt{a^2-cg^2}}\right) \left(2a^2fh^2\sqrt{-ah^2-cg^2} + 2c^2dg^2\sqrt{-ah^2-cg^2} - acdh^2\sqrt{-ah^2-cg^2} + 3acegh\sqrt{-ah^2-cg^2} - acfg^2\sqrt{-ah^2-cg^2}\right)}{(ah^2 + cg^2)^3} + \frac{\sqrt{a+cx^2}(-adh^2 - aegh^2 - 2neh^2x + 3afg^2h + 4afgh^2x - 4cdgh^2h - 3cdgh^2x + 2ceg^3 + ceg^2hx + cf g^3x)}{2(g+hx)^2(ah^2 + cg^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)^3\*Sqrt[a + c\*x^2]),x]

[Out]  $((2*c*e*g^3 - 4*c*d*g^2*h + 3*a*f*g^2*h - a*e*g*h^2 - a*d*h^3 + c*f*g^3*x + c*e*g^2*h*x - 3*c*d*g*h^2*x + 4*a*f*g*h^2*x - 2*a*e*h^3*x)*\text{Sqrt}[a + c*x^2]) / (2*(c*g^2 + a*h^2)^2*(g + h*x)^2) + ((2*c^2*d*g^2*\text{Sqrt}[-(c*g^2) - a*h^2] - a*c*f*g^2*\text{Sqrt}[-(c*g^2) - a*h^2] + 3*a*c*e*g*h*\text{Sqrt}[-(c*g^2) - a*h^2] - a*c*d*h^2*\text{Sqrt}[-(c*g^2) - a*h^2] + 2*a^2*f*h^2*\text{Sqrt}[-(c*g^2) - a*h^2])*\text{ArcTan}[(\text{Sqrt}[c]*g + \text{Sqrt}[c]*h*x - h*\text{Sqrt}[a + c*x^2])/\text{Sqrt}[-(c*g^2) - a*h^2]]) / (c*g^2 + a*h^2)^3$

**fricas [B]** time = 18.25, size = 1088, normalized size = 4.84

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+a)^(1/2),x, algorithm="fricas")

[Out]  $[1/4*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x)*\text{sqrt}(c*g^2 + a*h^2)*\text{log}((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*\text{sqrt}(c*g^2 + a*h^2)*(c*g*x - a*h)*\text{sqrt}(c*x^2 + a))/((h^2*x^2 + 2*g*h*x + g^2)) + 2*(2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x)*\text{sqrt}(c*x^2 + a))/((c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7)*x), -1/2*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x)*\text{sqrt}(-c*g^2 - a*h^2)*\text{arctan}(\text{sqrt}(-c*g^2 - a*h^2)*(c*g*x - a*h)*\text{sqrt}(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2$

$$\frac{a^2 e h^5 - (3c^2 d - 5a c f) g^3 h^2 - (3a c d - 4a^2 f) g h^4 x \sqrt{c x^2 + a}}{(c^3 g^8 + 3a c^2 g^6 h^2 + 3a^2 c g^4 h^4 + a^3 g^2 h^6 + (c^3 g^6 h^2 + 3a c^2 g^4 h^4 + 3a^2 c g^2 h^6 + a^3 h^8) x^2 + 2(c^3 g^7 h + 3a c^2 g^5 h^3 + 3a^2 c g^3 h^5 + a^3 g h^7) x)}$$

**giac [B]** time = 0.26, size = 848, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+a)^(1/2),x, algorithm="giac")

[Out]  $-(2c^2 d g^2 - a c f g^2 - a c d h^2 + 2a^2 f h^2 + 3a c g h e) \arctan\left(\frac{\sqrt{c} x - \sqrt{c x^2 + a}}{h} + \frac{\sqrt{c} g}{\sqrt{-c g^2 - a h^2}}\right) / \left( (c^2 g^4 + 2a c g^2 h^2 + a^2 h^4) \sqrt{-c g^2 - a h^2} \right) + (2(\sqrt{c} x - \sqrt{c x^2 + a})^3 c^2 f g^4 h - 2(\sqrt{c} x - \sqrt{c x^2 + a})^3 c^2 d g^2 h^3 + 5(\sqrt{c} x - \sqrt{c x^2 + a})^3 a c f g^2 h^3 + (\sqrt{c} x - \sqrt{c x^2 + a})^3 a c d h^5 - 3(\sqrt{c} x - \sqrt{c x^2 + a})^3 a c g h^4 e + 2(\sqrt{c} x - \sqrt{c x^2 + a})^2 c^{5/2} f g^5 - 6(\sqrt{c} x - \sqrt{c x^2 + a})^2 c^{5/2} d g^3 h^2 + 7(\sqrt{c} x - \sqrt{c x^2 + a})^2 a c^{3/2} f g^3 h^2 + 3(\sqrt{c} x - \sqrt{c x^2 + a})^2 a c^{3/2} d g h^4 - 4(\sqrt{c} x - \sqrt{c x^2 + a})^2 a^2 \sqrt{c} f g h^4 + 2(\sqrt{c} x - \sqrt{c x^2 + a})^2 c^{5/2} g^4 h e - 5(\sqrt{c} x - \sqrt{c x^2 + a})^2 a c^{3/2} g^2 h^3 e + 2(\sqrt{c} x - \sqrt{c x^2 + a})^2 a^2 \sqrt{c} h^5 e - 2(\sqrt{c} x - \sqrt{c x^2 + a}) a c^2 f g^4 h + 10(\sqrt{c} x - \sqrt{c x^2 + a}) a c^2 d g^2 h^3 - 11(\sqrt{c} x - \sqrt{c x^2 + a}) a^2 c f g^2 h^3 + (\sqrt{c} x - \sqrt{c x^2 + a}) a^2 c d h^5 - 4(\sqrt{c} x - \sqrt{c x^2 + a}) a c^2 g^3 h^2 e + 5(\sqrt{c} x - \sqrt{c x^2 + a}) a^2 c g h^4 e + a^2 c^{3/2} f g^3 h^2 - 3a^2 c^{3/2} d g h^4 + 4a^3 \sqrt{c} f g h^4 + a^2 c^{3/2} g^2 h^3 e - 2a^3 \sqrt{c} h^5 e) / \left( (c^2 g^4 h^2 + 2a c g^2 h^4 + a^2 h^6) (\sqrt{c} x - \sqrt{c x^2 + a})^2 h + 2(\sqrt{c} x - \sqrt{c x^2 + a}) \sqrt{c} g - a h \right)^2$

**maple [B]** time = 0.02, size = 1574, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+a)^(1/2),x)

[Out]  $-1/h/(a h^2 + c g^2)/(x+g/h) * (-2(x+g/h) c g/h + (x+g/h)^2 c + (a h^2 + c g^2)/h^2)^{1/2} e + 2/h^2/(a h^2 + c g^2)/(x+g/h) * (-2(x+g/h) c g/h + (x+g/h)^2 c + (a h^2 + c g^2)/h^2)^{1/2} f g - 3/2/h^2 c g/(a h^2 + c g^2)/((a h^2 + c g^2)/h^2)^{1/2} \ln\left(\frac{-2(x+g/h) c g/h + 2(a h^2 + c g^2)/h^2 + 2((a h^2 + c g^2)/h^2)^{1/2} (-2(x+g/h) c g/h + (x+g/h)^2 c + (a h^2 + c g^2)/h^2)^{1/2}}{(x+g/h)}\right) e + 5/2/h^3 c g^2/(a h^2 + c g^2)/((a h^2 + c g^2)/h^2)^{1/2} \ln\left(\frac{-2(x+g/h) c g/h + 2(a h^2 + c g^2)/h^2}{(x+g/h)}\right)$

$$\begin{aligned}
& h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} / (x + g / h) * f - f / h^3 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x + g / h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x + g / h) - 1/2 / h / (a * h^2 + c * g^2) / (x + g / h)^2 * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * d + 1/2 / h^2 / (a * h^2 + c * g^2) / (x + g / h)^2 * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * e * g - 1/2 / h^3 / (a * h^2 + c * g^2) / (x + g / h)^2 * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * f * g^2 - 3/2 * c * g / (a * h^2 + c * g^2)^2 / (x + g / h) * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * d + 3/2 / h * c * g^2 / (a * h^2 + c * g^2)^2 / (x + g / h) * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * e - 3/2 / h^2 * c * g^3 / (a * h^2 + c * g^2)^2 / (x + g / h) * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)} * f - 3/2 / h * c^2 * g^2 / (a * h^2 + c * g^2)^2 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x + g / h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x + g / h) * d + 3/2 / h^2 * c^2 * g^3 / (a * h^2 + c * g^2)^2 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x + g / h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x + g / h) * e - 3/2 / h^3 * c^2 * g^4 / (a * h^2 + c * g^2)^2 / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x + g / h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x + g / h) * f + 1/2 / h * c / (a * h^2 + c * g^2) / ((a * h^2 + c * g^2) / h^2)^{(1/2)} * \ln((-2 * (x + g / h) * c * g / h + 2 * (a * h^2 + c * g^2) / h^2 + 2 * ((a * h^2 + c * g^2) / h^2)^{(1/2)} * (-2 * (x + g / h) * c * g / h + (x + g / h)^2 * c + (a * h^2 + c * g^2) / h^2)^{(1/2)}) / (x + g / h) * d
\end{aligned}$$

**maxima [B]** time = 0.67, size = 896, normalized size = 3.98

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+a)^(1/2),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -3/2 * \text{sqrt}(c * x^2 + a) * c * f * g^3 / (c^2 * g^4 * h^2 * x + 2 * a * c * g^2 * h^4 * x + a^2 * h^6 * x + c^2 * g^5 * h + 2 * a * c * g^3 * h^3 + a^2 * g * h^5) + 3/2 * \text{sqrt}(c * x^2 + a) * c * e * g^2 / (c^2 * g^4 * h * x + 2 * a * c * g^2 * h^3 * x + a^2 * h^5 * x + c^2 * g^5 + 2 * a * c * g^3 * h^2 + a^2 * g * h^4) \\
& - 3/2 * \text{sqrt}(c * x^2 + a) * c * d * g / (c^2 * g^4 * x + 2 * a * c * g^2 * h^2 * x + a^2 * h^4 * x + c^2 * g^5 / h + 2 * a * c * g^3 * h + a^2 * g * h^3) - 1/2 * \text{sqrt}(c * x^2 + a) * f * g^2 / (c * g^2 * h^3 * x^2 + a * h^5 * x^2 + 2 * c * g^3 * h^2 * x + 2 * a * g * h^4 * x + c * g^4 * h + a * g^2 * h^3) + 1/2 * \text{sqrt}(c * x^2 + a) * e * g / (c * g^2 * h^2 * x^2 + a * h^4 * x^2 + 2 * c * g^3 * h * x + 2 * a * g * h^3 * x + c * g^4 + a * g^2 * h^2) + 2 * \text{sqrt}(c * x^2 + a) * f * g / (c * g^2 * h^2 * x + a * h^4 * x + c * g^3 * h + a * g * h^3) - 1/2 * \text{sqrt}(c * x^2 + a) * d / (c * g^2 * h * x^2 + a * h^3 * x^2 + 2 * c * g^3 * x + 2 * a * g * h^2 * x + c * g^4 / h + a * g^2 * h) - \text{sqrt}(c * x^2 + a) * e / (c * g^2 * h * x + a * h^3 * x + c * g^3 + a * g * h^2) + 3/2 * c^2 * f * g^4 * \text{arcsinh}(c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g))) - a * h / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) / ((a + c * g^2 / h^2)^{(5/2)} * h^7) - 3/2 * c^2 * e * g^3 * \text{arcsinh}(c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g))) - a * h / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) / ((a + c * g^2 / h^2)^{(5/2)} * h^6) + 3/2 * c^2 * d * g^2 * \text{arcsinh}(c * g * x / (\text{sqrt}(a * c) * \text{abs}(h * x + g))) - a * h / (\text{sqrt}(a * c) * \text{abs}(h * x + g)) / ((a + c * g^2 / h^2)^{(5/2)} * h^5) - 5/2 * c * f * g
\end{aligned}$$

$$\begin{aligned} &^2 \operatorname{arcsinh}(c * g * x / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g)) - a * h / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g))) / \\ &(a + c * g^2 / h^2)^{(3/2)} * h^5) + 3/2 * c * e * g * \operatorname{arcsinh}(c * g * x / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g))) - a * h / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g))) / ((a + c * g^2 / h^2)^{(3/2)} * h^4) - 1/2 * c * d * \operatorname{arcsinh}(c * g * x / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g)) - a * h / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g))) / ((a + c * g^2 / h^2)^{(3/2)} * h^3) + f * \operatorname{arcsinh}(c * g * x / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g)) - a * h / (\operatorname{sqrt}(a * c) * \operatorname{abs}(h * x + g))) / (\operatorname{sqrt}(a + c * g^2 / h^2) * h^3) \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^3 \sqrt{c x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(1/2)), x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{\sqrt{a + c x^2} (g + h x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*3/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral((d + e\*x + f\*x\*\*2)/(sqrt(a + c\*x\*\*2)\*(g + h\*x)\*\*3), x)

$$3.108 \quad \int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}} dx$$

Optimal. Leaf size=229

$$\frac{h\sqrt{a+cx^2} \left( 4(4a^2fh^2 - ac(3h(dh+3eg) + 7fg^2) + 3c^2dg^2) + chx(-9aeh - 11afg + 6cdg) \right) \tanh^{-1} \left( \frac{\sqrt{c}x}{\sqrt{a+cx^2}} \right)}{6ac^3}$$

**Rubi [A]** time = 0.32, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1645, 833, 780, 217, 206}

$$\frac{h\sqrt{a+cx^2} \left( 4(4a^2fh^2 - ac(3h(dh+3eg) + 7fg^2) + 3c^2dg^2) + chx(-9aeh - 11afg + 6cdg) \right) + \frac{\tanh^{-1} \left( \frac{\sqrt{c}x}{\sqrt{a+cx^2}} \right) (-3ah^2(eh+3fg) + 6cgh(dh+eg) + 2c^2fg^3)}{2c^3}}{6ac^3} - \frac{h\sqrt{a+cx^2} (g+hx)^2 (3cd-4af)}{3ac^2} - \frac{(g+hx)^3 (ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2), x]

[Out] -(((a\*e - (c\*d - a\*f)\*x)\*(g + h\*x)^3)/(a\*c\*Sqrt[a + c\*x^2])) - ((3\*c\*d - 4\*a\*f)\*h\*(g + h\*x)^2\*Sqrt[a + c\*x^2])/(3\*a\*c^2) - (h\*(4\*(3\*c^2\*d\*g^2 + 4\*a^2\*f\*h^2 - a\*c\*(7\*f\*g^2 + 3\*h\*(3\*e\*g + d\*h))) + c\*h\*(6\*c\*d\*g - 11\*a\*f\*g - 9\*a\*e\*h)\*x)\*Sqrt[a + c\*x^2])/(6\*a\*c^3) + (((2\*c\*f\*g^3 + 6\*c\*g\*h\*(e\*g + d\*h) - 3\*a\*h^2\*(3\*f\*g + e\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*c^(5/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 780

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

### Rule 833

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &&
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1645

```

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x)/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(g + hx)^3 (d + ex + fx^2)}{(a + cx^2)^{3/2}} dx &= -\frac{(ae - (cd - af)x)(g + hx)^3}{ac\sqrt{a + cx^2}} - \frac{\int \frac{(g+hx)^2(-a(fg+3eh)+(3cd-4af)hx)}{\sqrt{a+cx^2}} dx}{ac} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^3}{ac\sqrt{a + cx^2}} - \frac{(3cd - 4af)h(g + hx)^2\sqrt{a + cx^2}}{3ac^2} - \frac{\int \frac{(g+hx)(-a(2(3d+e)g+3eh+3cd-4af)hx)}{\sqrt{a+cx^2}} dx}{3ac} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^3}{ac\sqrt{a + cx^2}} - \frac{(3cd - 4af)h(g + hx)^2\sqrt{a + cx^2}}{3ac^2} - \frac{h(4(3c^2dg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2))}{3ac} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^3}{ac\sqrt{a + cx^2}} - \frac{(3cd - 4af)h(g + hx)^2\sqrt{a + cx^2}}{3ac^2} - \frac{h(4(3c^2dg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2))}{3ac} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^3}{ac\sqrt{a + cx^2}} - \frac{(3cd - 4af)h(g + hx)^2\sqrt{a + cx^2}}{3ac^2} - \frac{h(4(3c^2dg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2 + 3cdg^2))}{3ac}
\end{aligned}$$

**Mathematica [A]** time = 0.45, size = 246, normalized size = 1.07

$$\frac{-16a^2f^2 + a^2c(3h(4dh+3c(4g+hx)) + f(36g^2+27ghx-8h^2x^2)) + a^2(6dh(-3g^2-3ghx+h^2x^2) - 3c(2g^3+6g^2hx-6gh^2x^2-h^3x^3) + fx(-6g^3+18g^2hx+9gh^2x^2+2h^3x^3)) + 6c^3dg^2x}{a\sqrt{a+cx^2}} + 3\sqrt{c} \log(\sqrt{c}\sqrt{a+cx^2} + cx) (-3ah^2(gh+3fg) + 6cgh(dh+eg) + 2c^2fg^2)$$



Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2),x]

[Out] 
$$\frac{(-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) - 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3)) + f*x*(-6*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) + 3*h*(4*d*h + 3*e*(4*g + h*x)))}{(a*\text{Sqrt}[a + c*x^2]) + 3*\text{Sqrt}[c]*(2*c*f*g^3 + 6*c*g*h*(e*g + d*h) - 3*a*h^2*(3*f*g + e*h))*\text{Log}[c*x + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]]}{(6*c^3)}$$

**IntegrateAlgebraic [A]** time = 1.09, size = 316, normalized size = 1.38

$$\frac{-16a^3fh^3 + 12d^2cgh^3 + 36d^2cgh^3 + 9d^2cgh^3 + 36d^2cgh^3 + 27d^2cgh^3 - 8d^2cgh^3 - 18a^2dgh^2x + 6a^2dgh^2x - 6a^2dgh^2x - 6a^2dgh^2x - 18a^2dgh^2x + 18a^2dgh^2x + 3a^2dgh^2x - 6a^2dgh^2x + 18a^2dgh^2x + 9a^2dgh^2x + 2a^2dgh^2x + 6a^2dgh^2x}{6a^3\sqrt{a+cx^2}} \cdot \frac{\log(\sqrt{a+cx^2} - \sqrt{c}x)(3ah^3 + 9afgh^2 - 6aldh^2 - 6cgh^2 - 2fgh^2)}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2),x]

[Out] 
$$\frac{(-6*a*c^2*e*g^3 - 18*a*c^2*d*g^2*h + 36*a^2*c*f*g^2*h + 36*a^2*c*e*g*h^2 + 12*a^2*c*d*h^3 - 16*a^3*f*h^3 + 6*c^3*d*g^3*x - 6*a*c^2*f*g^3*x - 18*a*c^2*e*g^2*h*x - 18*a*c^2*d*g*h^2*x + 27*a^2*c*f*g*h^2*x + 9*a^2*c*e*h^3*x + 18*a*c^2*f*g^2*h*x^2 + 18*a*c^2*e*g*h^2*x^2 + 6*a*c^2*d*h^3*x^2 - 8*a^2*c*f*h^3*x^2 + 9*a*c^2*f*g*h^2*x^3 + 3*a*c^2*e*h^3*x^3 + 2*a*c^2*f*h^3*x^4)}{(6*a*c^3*\text{Sqrt}[a + c*x^2]) + ((-2*c*f*g^3 - 6*c*e*g^2*h - 6*c*d*g*h^2 + 9*a*f*g*h^2 + 3*a*e*h^3)*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2]])}{(2*c^{5/2})}$$

**fricas [A]** time = 1.00, size = 758, normalized size = 3.31

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/12*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*\text{sqrt}(c)*\log(-2*c*x^2 + 2*\text{sqrt}(c*x^2 + a))*\text{sqrt}(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*\text{sqrt}(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), \\ & -1/6*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*\text{sqrt}(-c)*\text{arctan}(\text{sqrt}(-c)*x/\text{sqrt}(c*x^2 + a)) - (2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*\text{sqrt}(c*x^2 + a) \end{aligned}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="maxima")

[Out]  $\frac{1}{3}f h^3 x^4 / (\sqrt{c x^2 + a} c) - \frac{4}{3} a f h^3 x^2 / (\sqrt{c x^2 + a} c^2) + d g^3 x / (\sqrt{c x^2 + a} a) - e g^3 / (\sqrt{c x^2 + a} c) - 3 d g^2 h / (\sqrt{c x^2 + a} c) - \frac{8}{3} a^2 f h^3 / (\sqrt{c x^2 + a} c^3) + \frac{1}{2} (3 f g h^2 + e h^3) x^3 / (\sqrt{c x^2 + a} c) + (3 f g^2 h + 3 e g h^2 + d h^3) x^2 / (\sqrt{c x^2 + a} c) + \frac{3}{2} (3 f g h^2 + e h^3) a x / (\sqrt{c x^2 + a} c^2) - (f g^3 + 3 e g^2 h + 3 d g h^2) x / (\sqrt{c x^2 + a} c) - \frac{3}{2} (3 f g h^2 + e h^3) a \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{5/2} + (f g^3 + 3 e g^2 h + 3 d g h^2) \operatorname{arcsinh}(c x / \sqrt{a c}) / c^{3/2} + 2 (3 f g^2 h + 3 e g h^2 + d h^3) a / (\sqrt{c x^2 + a} c^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^3 (fx^2 + ex + d)}{(cx^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2),x)

[Out] int(((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)^3 (d + ex + fx^2)}{(a + cx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] Integral((g + h\*x)\*\*3\*(d + e\*x + f\*x\*\*2)/(a + c\*x\*\*2)\*\*(3/2), x)

$$3.109 \quad \int \frac{(g+hx)^2(d+ex+fx^2)}{(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=149

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(h^2(2cd-3af)+2cg(2eh+fg))}{2c^{5/2}} - \frac{h\sqrt{a+cx^2}(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2} - \frac{(g+hx)^2}{a}$$

**Rubi [A]** time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1645, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(h^2(2cd-3af)+2cg(2eh+fg))}{2c^{5/2}} - \frac{h\sqrt{a+cx^2}(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2} - \frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2), x]

[Out] -(((a\*e - (c\*d - a\*f)\*x)\*(g + h\*x)^2)/(a\*c\*Sqrt[a + c\*x^2])) - (h\*(4\*(c\*d\*g - a\*(2\*f\*g + e\*h)) + (2\*c\*d - 3\*a\*f)\*h\*x)\*Sqrt[a + c\*x^2])/(2\*a\*c^2) + (((2\*c\*d - 3\*a\*f)\*h^2 + 2\*c\*g\*(f\*g + 2\*e\*h))\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 780

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

#### Rule 1645

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(g + hx)^2 (d + ex + fx^2)}{(a + cx^2)^{3/2}} dx &= -\frac{(ae - (cd - af)x)(g + hx)^2}{ac\sqrt{a + cx^2}} - \frac{\int \frac{(g+hx)(-a(fg+2eh)+(2cd-3af)hx)}{\sqrt{a+cx^2}} dx}{ac} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^2}{ac\sqrt{a + cx^2}} - \frac{h(4(cdg - a(2fg + eh)) + (2cd - 3af)hx)\sqrt{a + cx^2}}{2ac^2} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^2}{ac\sqrt{a + cx^2}} - \frac{h(4(cdg - a(2fg + eh)) + (2cd - 3af)hx)\sqrt{a + cx^2}}{2ac^2} \\
&= -\frac{(ae - (cd - af)x)(g + hx)^2}{ac\sqrt{a + cx^2}} - \frac{h(4(cdg - a(2fg + eh)) + (2cd - 3af)hx)\sqrt{a + cx^2}}{2ac^2}
\end{aligned}$$

**Mathematica [A]** time = 0.28, size = 177, normalized size = 1.19

$$\frac{\sqrt{c} (a^2 h(4eh + 8fg + 3fhx) + ac(-2dh(2g + hx) - 2e(g^2 + 2ghx - h^2x^2) + fx(-2g^2 + 4ghx + h^2x^2)) + 2c^2 dg^2x) - a^{3/2} \sqrt{\frac{cx^2}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right) (3afh^2 - 2c(h(dh + 2eg) + fg^2))}{2ac^{5/2}\sqrt{a + cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2), x]

[Out] (Sqrt[c]\*(2\*c^2\*d\*g^2\*x + a^2\*h\*(8\*f\*g + 4\*e\*h + 3\*f\*h\*x) + a\*c\*(-2\*d\*h\*(2\*g + h\*x) - 2\*e\*(g^2 + 2\*g\*h\*x - h^2\*x^2) + f\*x\*(-2\*g^2 + 4\*g\*h\*x + h^2\*x^2))) - a^(3/2)\*(3\*a\*f\*h^2 - 2\*c\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*Sqrt[1 + (c\*x^2)/a]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]]/(2\*a\*c^(5/2)\*Sqrt[a + c\*x^2])

**IntegrateAlgebraic [A]** time = 0.73, size = 190, normalized size = 1.28

$$\frac{4a^2eh^2 + 8a^2fgh + 3a^2fh^2x - 4acdgh - 2acd^2x - 2aceg^2 - 4aceghx + 2aceh^2x^2 - 2acfg^2x + 4acfghx^2 + acfh^2x^3 + 2c^2dg^2x}{2ac^2\sqrt{a + cx^2}} + \frac{\log(\sqrt{a + cx^2} - \sqrt{c}x)(3afh^2 - 2cdh^2 - 4cegh - 2c^2fg^2)}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2),x]

[Out]  $(-2*a*c*e*g^2 - 4*a*c*d*g*h + 8*a^2*f*g*h + 4*a^2*e*h^2 + 2*c^2*d*g^2*x - 2*a*c*f*g^2*x - 4*a*c*e*g*h*x - 2*a*c*d*h^2*x + 3*a^2*f*h^2*x + 4*a*c*f*g*h*x^2 + 2*a*c*e*h^2*x^2 + a*c*f*h^2*x^3)/(2*a*c^2*\text{Sqrt}[a + c*x^2]) + ((-2*c*f*g^2 - 4*c*e*g*h - 2*c*d*h^2 + 3*a*f*h^2)*\text{Log}[-(\text{Sqrt}[c]*x) + \text{Sqrt}[a + c*x^2]])/(2*c^(5/2))$

**fricas** [A] time = 0.86, size = 530, normalized size = 3.56

$$\frac{(2af^2e + 4adgh + 2af^2d - 2ad^2g + 2ad^2h + 2ad^2f) \sqrt{a + cx^2} \log\left(\frac{2af^2e + 4adgh + 2af^2d - 2ad^2g + 2ad^2h + 2ad^2f}{2(a^2c^2 + c^2x^2)}\right) - (2af^2e + 4adgh + 2af^2d - 2ad^2g + 2ad^2h + 2ad^2f) \sqrt{a + cx^2} \arctan\left(\frac{\sqrt{cx^2 + a}}{\sqrt{a + cx^2}}\right)}{2(a^2c^2 + c^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="fricas")

[Out]  $[-1/4*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*\text{sqrt}(c)*\log(-2*c*x^2 + 2*\text{sqrt}(c*x^2 + a)*\text{sqrt}(c)*x - a) - 2*(a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*\text{sqrt}(c*x^2 + a)/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^2*c*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*\text{sqrt}(-c)*\arctan(\text{sqrt}(-c)*x/\text{sqrt}(c*x^2 + a)) - (a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*\text{sqrt}(c*x^2 + a)/(a*c^4*x^2 + a^2*c^3)]$

**giac** [A] time = 0.25, size = 219, normalized size = 1.47

$$\left(\frac{fh^2x}{c} + \frac{2(2ac^2fgh+ac^2h^2e)}{ac^4}\right)x + \frac{2c^4dg^2-2ac^3fg^2-2ac^3dh^2+3a^2c^2fh^2-4ac^3ghe}{ac^4}x - \frac{2(2ac^3dgh-4a^2c^2fgh+ac^3g^2e-2a^2c^2h^2e)}{ac^4} \frac{(2cfg^2+2cdh^2-3afh^2+4cgh)e \log\left(\frac{-\sqrt{c}x + \sqrt{cx^2+a}}{2c^2}\right)}{2\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="giac")

[Out]  $1/2*((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*h^2*e)/(a*c^4))*x + (2*c^4*d*g^2 - 2*a*c^3*f*g^2 - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2 - 4*a*c^3*g*h*e)/(a*c^4))*x - 2*(2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + a*c^3*g^2*e - 2*a^2*c^2*h^2*e)/(a*c^4)/\text{sqrt}(c*x^2 + a) - 1/2*(2*c*f*g^2 + 2*c*d*h^2 - 3*a*f*h^2 + 4*c*g*h*e)*\log(\text{abs}(-\text{sqrt}(c)*x + \text{sqrt}(c*x^2 + a)))/c^(5/2)$

**maple** [B] time = 0.01, size = 327, normalized size = 2.19

$$\frac{fh^2x^3}{2\sqrt{cx^2+a}c} + \frac{eh^2x^2}{\sqrt{cx^2+a}c} + \frac{2fghx^2}{\sqrt{cx^2+a}c} + \frac{3afh^2x}{2\sqrt{cx^2+a}c^2} + \frac{dg^2x}{\sqrt{cx^2+a}c} + \frac{dh^2x}{\sqrt{cx^2+a}c} - \frac{2cghx}{\sqrt{cx^2+a}c} - \frac{fg^2x}{\sqrt{cx^2+a}c} - \frac{3afh^2 \ln(\sqrt{c}x + \sqrt{cx^2+a})}{2c^2} + \frac{dh^2 \ln(\sqrt{c}x + \sqrt{cx^2+a})}{c^2} + \frac{2cgh \ln(\sqrt{c}x + \sqrt{cx^2+a})}{c^2} + \frac{fg^2 \ln(\sqrt{c}x + \sqrt{cx^2+a})}{c^2} + \frac{2ach^2}{\sqrt{cx^2+a}c^2} + \frac{4afgh}{\sqrt{cx^2+a}c^2} - \frac{2dgh}{\sqrt{cx^2+a}c^2} - \frac{eg^2}{\sqrt{cx^2+a}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^{(3/2)}, x)$

[Out]  $\frac{1}{2}h^2f*x^3/c/(c*x^2+a)^{(1/2)}+3/2h^2f*a/c^2*x/(c*x^2+a)^{(1/2)}-3/2h^2f*a/c^{(5/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})+x^2/c/(c*x^2+a)^{(1/2)}*h^2e+2*x^2/c/(c*x^2+a)^{(1/2)}*g*h*f+2*a/c^2/(c*x^2+a)^{(1/2)}*h^2e+4*a/c^2/(c*x^2+a)^{(1/2)}*g*h*f-x/c/(c*x^2+a)^{(1/2)}*d*h^2-2*x/c/(c*x^2+a)^{(1/2)}*e*g*h-x/c/(c*x^2+a)^{(1/2)}*f*g^2+1/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*d*h^2+2/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*e*g*h+1/c^{(3/2)}*\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})*f*g^2-2/c/(c*x^2+a)^{(1/2)}*g*h*d-1/c/(c*x^2+a)^{(1/2)}*g^2e+g^2*d*x/a/(c*x^2+a)^{(1/2)}$

**maxima** [A] time = 0.45, size = 227, normalized size = 1.52

$$\frac{fh^2x^3}{2\sqrt{cx^2+ac}} + \frac{dg^2x}{\sqrt{cx^2+aa}} + \frac{3afh^2x}{2\sqrt{cx^2+ac^2}} - \frac{3afh^2 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{5}{2}}} - \frac{eg^2}{\sqrt{cx^2+ac}} - \frac{2dgh}{\sqrt{cx^2+ac}} + \frac{(2fgh+eh^2)x^2}{\sqrt{cx^2+ac}} - \frac{(fg^2+2egh+dh^2)x}{\sqrt{cx^2+ac}} + \frac{(fg^2+2egh+dh^2) \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} + \frac{2(2fgh+eh^2)a}{\sqrt{cx^2+ac^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{2}f*h^2*x^3/(\text{sqrt}(c*x^2+a)*c) + d*g^2*x/(\text{sqrt}(c*x^2+a)*a) + 3/2*a*f*h^2*x/(\text{sqrt}(c*x^2+a)*c^2) - 3/2*a*f*h^2*\operatorname{arcsinh}(c*x/\text{sqrt}(a*c))/c^{(5/2)} - e*g^2/(\text{sqrt}(c*x^2+a)*c) - 2*d*g*h/(\text{sqrt}(c*x^2+a)*c) + (2*f*g*h + e*h^2)*x^2/(\text{sqrt}(c*x^2+a)*c) - (f*g^2 + 2*e*g*h + d*h^2)*x/(\text{sqrt}(c*x^2+a)*c) + (f*g^2 + 2*e*g*h + d*h^2)*\operatorname{arcsinh}(c*x/\text{sqrt}(a*c))/c^{(3/2)} + 2*(2*f*g*h + e*h^2)*a/(\text{sqrt}(c*x^2+a)*c^2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g+hx)^2 (fx^2+ex+d)}{(cx^2+a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((g+h*x)^2*(d+e*x+f*x^2))/(a+c*x^2)^{(3/2)}, x)$

[Out]  $\text{int}(((g+h*x)^2*(d+e*x+f*x^2))/(a+c*x^2)^{(3/2)}, x)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```



$$3.110 \quad \int \frac{(g+hx)(d+ex+fx^2)}{(a+cx^2)^{3/2}} dx$$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}} - \frac{h\sqrt{a+cx^2}(cd-2af)}{ac^2} - \frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

**Rubi** [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {1645, 641, 217, 206}

$$-\frac{h\sqrt{a+cx^2}(cd-2af)}{ac^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}} - \frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2), x]

[Out] -(((a\*e - (c\*d - a\*f)\*x)\*(g + h\*x))/(a\*c\*Sqrt[a + c\*x^2])) - ((c\*d - 2\*a\*f)\*h\*Sqrt[a + c\*x^2])/(a\*c^2) + ((f\*g + e\*h)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/c^(3/2)

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1645

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_))^(m\_.)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + c\*x^2, x], x]}]

```

nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
x], x, 1]], Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
+ 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(g + hx)(d + ex + fx^2)}{(a + cx^2)^{3/2}} dx &= -\frac{(ae - (cd - af)x)(g + hx)}{ac\sqrt{a + cx^2}} - \frac{\int \frac{-a(fg + eh) + (cd - 2af)hx}{\sqrt{a + cx^2}} dx}{ac} \\
&= -\frac{(ae - (cd - af)x)(g + hx)}{ac\sqrt{a + cx^2}} - \frac{(cd - 2af)h\sqrt{a + cx^2}}{ac^2} + \frac{(fg + eh) \int \frac{1}{\sqrt{a + cx^2}} dx}{c} \\
&= -\frac{(ae - (cd - af)x)(g + hx)}{ac\sqrt{a + cx^2}} - \frac{(cd - 2af)h\sqrt{a + cx^2}}{ac^2} + \frac{(fg + eh) \operatorname{Subst}\left(\int \frac{1}{1 - cx^2} dx\right)}{c} \\
&= -\frac{(ae - (cd - af)x)(g + hx)}{ac\sqrt{a + cx^2}} - \frac{(cd - 2af)h\sqrt{a + cx^2}}{ac^2} + \frac{(fg + eh) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a + cx^2}}\right)}{c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 102, normalized size = 1.02

$$\frac{a^{3/2}\sqrt{c}\sqrt{\frac{cx^2}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)(eh + fg) + 2a^2fh - ac(dh + e(g + hx) + fx(g - hx)) + c^2dgx}{ac^2\sqrt{a + cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2), x]

[Out] (2\*a^2\*f\*h + c^2\*d\*g\*x - a\*c\*(d\*h + f\*x\*(g - h\*x) + e\*(g + h\*x)) + a^(3/2)\*Sqrt[c]\*(f\*g + e\*h)\*Sqrt[1 + (c\*x^2)/a]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]]/(a\*c^2\*Sqrt[a + c\*x^2])

**IntegrateAlgebraic [A]** time = 0.55, size = 104, normalized size = 1.04

$$\frac{2a^2fh - acdh - aceg - acehx - acfgx + acfhx^2 + c^2dgx}{ac^2\sqrt{a + cx^2}} + \frac{\log\left(\sqrt{a + cx^2} - \sqrt{cx}\right)(-eh - fg)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2),x]

[Out]  $(-(a*c*e*g) - a*c*d*h + 2*a^2*f*h + c^2*d*g*x - a*c*f*g*x - a*c*e*h*x + a*c*f*h*x^2)/(a*c^2*\sqrt{a + c*x^2}) + ((-(f*g) - e*h)*\text{Log}[-(\sqrt{c}*x) + \sqrt{a + c*x^2}])/c^{3/2}$

**fricas** [A] time = 1.02, size = 278, normalized size = 2.78

$$\frac{\left(\frac{a^2fg + a^2eh + (acfg + acfh)x^2}{2(ac^3x^2 + a^2c^2)}\sqrt{c} \log(-2cx^2 - 2\sqrt{cx^2 + a}\sqrt{cx - a}) + 2(acfhx^2 - aceg - (acd - 2a^2f)h - (aceh - (c^2d - acf)g))\sqrt{cx^2 + a} - \frac{(a^2fg + a^2eh + (acfg + acfh)x^2)\sqrt{-c} \arctan\left(\frac{\sqrt{-c}}{\sqrt{cx^2 + a}}\right) - (acfhx^2 - aceg - (acd - 2a^2f)h - (aceh - (c^2d - acf)g))\sqrt{cx^2 + a}}{ac^3x^2 + a^2c^2}\right)}{2(ac^3x^2 + a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="fricas")

[Out]  $[1/2*((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*\sqrt{c}*\log(-2*c*x^2 - 2*\sqrt{c*x^2 + a}*\sqrt{c}*x - a) + 2*(a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*\sqrt{c*x^2 + a})/(a*c^3*x^2 + a^2*c^2), -((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*\sqrt{-c}*\arctan(\sqrt{-c}*x/\sqrt{c*x^2 + a}) - (a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*\sqrt{c*x^2 + a})/(a*c^3*x^2 + a^2*c^2)]$

**giac** [A] time = 0.25, size = 116, normalized size = 1.16

$$\frac{\left(\frac{f hx}{c} + \frac{c^3 dg - ac^2 fg - ac^2 he}{ac^3}\right)x - \frac{ac^2 dh - 2a^2 c fh + ac^2 ge}{ac^3}}{\sqrt{cx^2 + a}} - \frac{(fg + he) \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="giac")

[Out]  $((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*h*e)/(a*c^3))*x - (a*c^2*d*h - 2*a^2*c*f*h + a*c^2*g*e)/(a*c^3))/\sqrt{c*x^2 + a} - (f*g + h*e)*\log(\text{abs}(-\sqrt{c}*x + \sqrt{c*x^2 + a}))/c^{3/2}$

**maple** [A] time = 0.00, size = 163, normalized size = 1.63

$$\frac{f hx^2}{\sqrt{cx^2 + a}c} + \frac{d gx}{\sqrt{cx^2 + a}a} - \frac{e hx}{\sqrt{cx^2 + a}c} - \frac{f gx}{\sqrt{cx^2 + a}c} + \frac{e h \ln(\sqrt{c}x + \sqrt{cx^2 + a})}{c^{\frac{3}{2}}} + \frac{f g \ln(\sqrt{c}x + \sqrt{cx^2 + a})}{c^{\frac{3}{2}}} + \frac{2afh}{\sqrt{cx^2 + a}c^2} - \frac{dh}{\sqrt{cx^2 + a}c} - \frac{eg}{\sqrt{cx^2 + a}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x)

[Out]  $h*f*x^2/c/(c*x^2+a)^{1/2} + 2*h*f*a/c^2/(c*x^2+a)^{1/2} - x/c/(c*x^2+a)^{1/2} * e*h - x/c/(c*x^2+a)^{1/2} * f*g + 1/c^{3/2} * \ln(c^{1/2}*x + (c*x^2+a)^{1/2}) * e*h + 1/c^{3/2} * \ln(c^{1/2}*x + (c*x^2+a)^{1/2}) * f*g$

$(3/2) \cdot \ln(c^{1/2} \cdot x + (c \cdot x^2 + a)^{1/2}) \cdot f \cdot g - 1/c / (c \cdot x^2 + a)^{1/2} \cdot d \cdot h - 1/c / (c \cdot x^2 + a)^{1/2} \cdot e \cdot g + d \cdot g \cdot x / a / (c \cdot x^2 + a)^{1/2}$

**maxima** [A] time = 0.44, size = 126, normalized size = 1.26

$$\frac{f h x^2}{\sqrt{c x^2 + a c}} + \frac{d g x}{\sqrt{c x^2 + a a}} - \frac{e g}{\sqrt{c x^2 + a c}} - \frac{d h}{\sqrt{c x^2 + a c}} + \frac{2 a f h}{\sqrt{c x^2 + a c^2}} - \frac{(f g + e h) x}{\sqrt{c x^2 + a c}} + \frac{(f g + e h) \operatorname{arsinh}\left(\frac{c x}{\sqrt{a c}}\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="maxima")

[Out]  $f \cdot h \cdot x^2 / (\sqrt{c \cdot x^2 + a} \cdot c) + d \cdot g \cdot x / (\sqrt{c \cdot x^2 + a} \cdot a) - e \cdot g / (\sqrt{c \cdot x^2 + a} \cdot c) - d \cdot h / (\sqrt{c \cdot x^2 + a} \cdot c) + 2 \cdot a \cdot f \cdot h / (\sqrt{c \cdot x^2 + a} \cdot c^2) - (f \cdot g + e \cdot h) \cdot x / (\sqrt{c \cdot x^2 + a} \cdot c) + (f \cdot g + e \cdot h) \cdot \operatorname{arcsinh}(c \cdot x / \sqrt{a \cdot c}) / c^{3/2}$

**mupad** [B] time = 5.28, size = 151, normalized size = 1.51

$$\frac{e h \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{c^{3/2}} + \frac{f g \ln(\sqrt{c} x + \sqrt{c x^2 + a})}{c^{3/2}} - \frac{d h}{c \sqrt{c x^2 + a}} - \frac{e g}{c \sqrt{c x^2 + a}} + \frac{d g x}{a \sqrt{c x^2 + a}} - \frac{e h x}{c \sqrt{c x^2 + a}} - \frac{f g x}{c \sqrt{c x^2 + a}} + \frac{f h (c x^2 + 2 a)}{c^2 \sqrt{c x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)\*(d + e\*x + f\*x^2))/(a + c\*x^2)^(3/2),x)

[Out]  $(e \cdot h \cdot \log(c^{1/2} \cdot x + (a + c \cdot x^2)^{1/2})) / c^{3/2} + (f \cdot g \cdot \log(c^{1/2} \cdot x + (a + c \cdot x^2)^{1/2})) / c^{3/2} - (d \cdot h) / (c \cdot (a + c \cdot x^2)^{1/2}) - (e \cdot g) / (c \cdot (a + c \cdot x^2)^{1/2}) + (d \cdot g \cdot x) / (a \cdot (a + c \cdot x^2)^{1/2}) - (e \cdot h \cdot x) / (c \cdot (a + c \cdot x^2)^{1/2}) - (f \cdot g \cdot x) / (c \cdot (a + c \cdot x^2)^{1/2}) + (f \cdot h \cdot (2 \cdot a + c \cdot x^2)) / (c^2 \cdot (a + c \cdot x^2)^{1/2})$

**sympy** [A] time = 18.84, size = 209, normalized size = 2.09

$$d h \left( \begin{cases} -\frac{1}{c \sqrt{a+c x^2}} & \text{for } c \neq 0 \\ \frac{x^2}{2 a^2} & \text{otherwise} \end{cases} \right) + e g \left( \begin{cases} -\frac{1}{c \sqrt{a+c x^2}} & \text{for } c \neq 0 \\ \frac{x^2}{2 a^2} & \text{otherwise} \end{cases} \right) + e h \left( \frac{\operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a} c \sqrt{1+\frac{c x^2}{a}}} \right) + f g \left( \frac{\operatorname{asinh}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a} c \sqrt{1+\frac{c x^2}{a}}} \right) + f h \left( \begin{cases} \frac{2 a}{c^2 \sqrt{a+c x^2}} + \frac{x^2}{c \sqrt{a+c x^2}} & \text{for } c \neq 0 \\ \frac{x^4}{4 a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{d g x}{a^{\frac{3}{2}} \sqrt{1+\frac{c x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out]  $d \cdot h \cdot \operatorname{Piecewise}((-1 / (c \cdot \sqrt{a + c \cdot x^2})), \operatorname{Ne}(c, 0)), (x^2 / (2 \cdot a^{3/2})), \operatorname{True})) + e \cdot g \cdot \operatorname{Piecewise}((-1 / (c \cdot \sqrt{a + c \cdot x^2})), \operatorname{Ne}(c, 0)), (x^2 / (2 \cdot a^{3/2})), \operatorname{True})) + e \cdot h \cdot (\operatorname{asinh}(\sqrt{c} \cdot x / \sqrt{a}) / c^{3/2} - x / (\sqrt{a} \cdot c \cdot \sqrt{1 + c \cdot x^2 / a})) + f \cdot g \cdot (\operatorname{asinh}(\sqrt{c} \cdot x / \sqrt{a}) / c^{3/2} - x / (\sqrt{a} \cdot c \cdot \sqrt{1 + c \cdot x^2 / a})) + f \cdot h \cdot \operatorname{Piecewise}((2 \cdot a / (c^2 \cdot \sqrt{a + c \cdot x^2}) + x^2 / (c \cdot \sqrt{a + c \cdot x^2})), \operatorname{Ne}(c, 0)), (x^4 / (4 \cdot a^{3/2})), \operatorname{True})) + d \cdot g \cdot x / (a^{3/2} \cdot \sqrt{1 + c \cdot x^2 / a})$

$$3.111 \quad \int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}} dx$$

Optimal. Leaf size=61

$$\frac{f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}} - \frac{ae - x(cd - af)}{ac\sqrt{a + cx^2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1814, 12, 217, 206}

$$\frac{f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}} - \frac{ae - x(cd - af)}{ac\sqrt{a + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/(a + c\*x^2)^(3/2), x]

[Out] -((a\*e - (c\*d - a\*f)\*x)/(a\*c\*Sqrt[a + c\*x^2])) + (f\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/c^(3/2)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int

$[(a + b*x^2)^{(p + 1)}*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /$   
 $; FreeQ[\{a, b\}, x] \&\& PolyQ[Pq, x] \&\& LtQ[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(a + cx^2)^{3/2}} dx &= -\frac{ae - (cd - af)x}{ac\sqrt{a + cx^2}} + \frac{\int \frac{af}{c\sqrt{a+cx^2}} dx}{a} \\ &= -\frac{ae - (cd - af)x}{ac\sqrt{a + cx^2}} + \frac{f \int \frac{1}{\sqrt{a+cx^2}} dx}{c} \\ &= -\frac{ae - (cd - af)x}{ac\sqrt{a + cx^2}} + \frac{f \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{c} \\ &= -\frac{ae - (cd - af)x}{ac\sqrt{a + cx^2}} + \frac{f \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}} \end{aligned}$$

**Mathematica** [A] time = 0.06, size = 74, normalized size = 1.21

$$\frac{a^{3/2} f \sqrt{\frac{cx^2}{a} + 1} \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) + \sqrt{c}(cdx - a(e + fx))}{ac^{3/2}\sqrt{a + cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/(a + c\*x^2)^(3/2), x]

[Out] (Sqrt[c]\*(c\*d\*x - a\*(e + f\*x)) + a^(3/2)\*f\*Sqrt[1 + (c\*x^2)/a]\*ArcSinh[(Sqrt[c]\*x)/Sqrt[a]])/(a\*c^(3/2)\*Sqrt[a + c\*x^2])

**IntegrateAlgebraic** [A] time = 0.37, size = 62, normalized size = 1.02

$$\frac{-ae - afx + cdx}{ac\sqrt{a + cx^2}} - \frac{f \log\left(\sqrt{a + cx^2} - \sqrt{c}x\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/(a + c\*x^2)^(3/2), x]

[Out] (-a\*e) + c\*d\*x - a\*f\*x)/(a\*c\*Sqrt[a + c\*x^2]) - (f\*Log[-(Sqrt[c]\*x) + Sqrt[a + c\*x^2]])/c^(3/2)

**fricas** [A] time = 0.93, size = 181, normalized size = 2.97

$$\left[ \frac{(acf x^2 + a^2 f) \sqrt{c} \log(-2cx^2 - 2\sqrt{cx^2 + a} \sqrt{c} x - a) - 2(ace - (c^2 d - acf)x) \sqrt{cx^2 + a}}{2(ac^3 x^2 + a^2 c^2)}, - \frac{(acf x^2 + a^2 f) \sqrt{-c} \arctan\left(\frac{\sqrt{-c} x}{\sqrt{cx^2 + a}}\right) + (ace - (c^2 d - acf)x) \sqrt{cx^2 + a}}{ac^3 x^2 + a^2 c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/2\*((a\*c\*f\*x^2 + a^2\*f)\*sqrt(c)\*log(-2\*c\*x^2 - 2\*sqrt(c\*x^2 + a)\*sqrt(c)\*x - a) - 2\*(a\*c\*e - (c^2\*d - a\*c\*f)\*x)\*sqrt(c\*x^2 + a))/(a\*c^3\*x^2 + a^2\*c^2), -((a\*c\*f\*x^2 + a^2\*f)\*sqrt(-c)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + (a\*c\*e - (c^2\*d - a\*c\*f)\*x)\*sqrt(c\*x^2 + a))/(a\*c^3\*x^2 + a^2\*c^2)]

**giac** [A] time = 0.20, size = 63, normalized size = 1.03

$$-\frac{\frac{e}{c} - \frac{(c^2 d - acf)x}{ac^2}}{\sqrt{cx^2 + a}} - \frac{f \log\left(\left|-\sqrt{c} x + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="giac")

[Out] -(e/c - (c^2\*d - a\*c\*f)\*x/(a\*c^2))/sqrt(c\*x^2 + a) - f\*log(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

**maple** [A] time = 0.01, size = 69, normalized size = 1.13

$$\frac{dx}{\sqrt{cx^2 + a} a} - \frac{fx}{\sqrt{cx^2 + a} c} + \frac{f \ln\left(\sqrt{c} x + \sqrt{cx^2 + a}\right)}{c^{\frac{3}{2}}} - \frac{e}{\sqrt{cx^2 + a} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x)

[Out] -f\*x/c/(c\*x^2+a)^(1/2)+f/c^(3/2)\*ln(c^(1/2)\*x+(c\*x^2+a)^(1/2))-e/c/(c\*x^2+a)^(1/2)+d\*x/a/(c\*x^2+a)^(1/2)

**maxima** [A] time = 0.43, size = 61, normalized size = 1.00

$$\frac{dx}{\sqrt{cx^2 + a} a} - \frac{fx}{\sqrt{cx^2 + a} c} + \frac{f \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} - \frac{e}{\sqrt{cx^2 + a} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+a)^(3/2),x, algorithm="maxima")

[Out] d\*x/(sqrt(c\*x^2 + a)\*a) - f\*x/(sqrt(c\*x^2 + a)\*c) + f\*arcsinh(c\*x/sqrt(a\*c))/  
c^(3/2) - e/(sqrt(c\*x^2 + a)\*c)

**mupad [B]** time = 4.33, size = 68, normalized size = 1.11

$$\frac{f \ln\left(\sqrt{c} x + \sqrt{c x^2 + a}\right)}{c^{3/2}} - \frac{e}{c \sqrt{c x^2 + a}} + \frac{d x}{a \sqrt{c x^2 + a}} - \frac{f x}{c \sqrt{c x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + c\*x^2)^(3/2),x)

[Out] (f\*log(c^(1/2)\*x + (a + c\*x^2)^(1/2)))/c^(3/2) - e/(c\*(a + c\*x^2)^(1/2)) +  
(d\*x)/(a\*(a + c\*x^2)^(1/2)) - (f\*x)/(c\*(a + c\*x^2)^(1/2))

**sympy [A]** time = 8.86, size = 87, normalized size = 1.43

$$e \left( \begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for } c \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + f \left( \frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}} \right) + \frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] e\*Piecewise((-1/(c\*sqrt(a + c\*x\*\*2)), Ne(c, 0)), (x\*\*2/(2\*a\*\*(3/2)), True))  
+ f\*(asinh(sqrt(c)\*x/sqrt(a))/c\*\*(3/2) - x/(sqrt(a)\*c\*sqrt(1 + c\*x\*\*2/a)))  
+ d\*x/(a\*\*(3/2)\*sqrt(1 + c\*x\*\*2/a))



$$3.112 \quad \int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}} dx$$

Optimal. Leaf size=138

$$-\frac{aafh - cdh + ceg) - cx(aeh - afg + cdg)}{ac\sqrt{a + cx^2} (ah^2 + cg^2)} - \frac{(dh^2 - egh + fg^2) \tanh^{-1}\left(\frac{ah - cgx}{\sqrt{a+cx^2} \sqrt{ah^2+cg^2}}\right)}{(ah^2 + cg^2)^{3/2}}$$

**Rubi [A]** time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1647, 12, 725, 206}

$$-\frac{aafh - cdh + ceg) - cx(aeh - afg + cdg)}{ac\sqrt{a + cx^2} (ah^2 + cg^2)} - \frac{(dh^2 - egh + fg^2) \tanh^{-1}\left(\frac{ah - cgx}{\sqrt{a+cx^2} \sqrt{ah^2+cg^2}}\right)}{(ah^2 + cg^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)\*(a + c\*x^2)^(3/2)), x]

[Out] -((a\*(c\*e\*g - c\*d\*h + a\*f\*h) - c\*(c\*d\*g - a\*f\*g + a\*e\*h)\*x)/(a\*c\*(c\*g^2 + a\*h^2)\*Sqrt[a + c\*x^2])) - ((f\*g^2 - e\*g\*h + d\*h^2)\*ArcTanh[(a\*h - c\*g\*x)/(Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2])))/(c\*g^2 + a\*h^2)^(3/2)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1647

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2}{(g + hx)(a + cx^2)^{3/2}} dx &= -\frac{a(ceg - cdh + afh) - c(cdg - afg + aeh)x}{ac(cg^2 + ah^2)\sqrt{a + cx^2}} + \frac{\int \frac{ac(fg^2 - egh + dh^2)}{(cg^2 + ah^2)(g + hx)\sqrt{a + cx^2}} dx}{ac} \\
&= -\frac{a(ceg - cdh + afh) - c(cdg - afg + aeh)x}{ac(cg^2 + ah^2)\sqrt{a + cx^2}} + \frac{(fg^2 - egh + dh^2) \int \frac{1}{(g + hx)\sqrt{a + cx^2}} dx}{cg^2 + ah^2} \\
&= -\frac{a(ceg - cdh + afh) - c(cdg - afg + aeh)x}{ac(cg^2 + ah^2)\sqrt{a + cx^2}} - \frac{(fg^2 - egh + dh^2) \operatorname{Subst}\left(\int \frac{1}{cg^2 + ah^2 - x}\right)}{cg^2 + ah^2} \\
&= -\frac{a(ceg - cdh + afh) - c(cdg - afg + aeh)x}{ac(cg^2 + ah^2)\sqrt{a + cx^2}} - \frac{(fg^2 - egh + dh^2) \tanh^{-1}\left(\frac{ah - cgx}{\sqrt{cg^2 + ah^2}\sqrt{a + cx^2}}\right)}{(cg^2 + ah^2)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 137, normalized size = 0.99

$$\frac{a^2(-f)h + ac(dh - eg + ehx - fgx) + c^2dgx}{ac\sqrt{a + cx^2}(ah^2 + cg^2)} - \frac{(h(dh - eg) + fg^2) \tanh^{-1}\left(\frac{ah - cgx}{\sqrt{a + cx^2}\sqrt{ah^2 + cg^2}}\right)}{(ah^2 + cg^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)), x]
```

```
[Out] (- (a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2 + a*h^2)*Sqrt[a + c*x^2]) - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```



[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+a)^(3/2),x, algorithm="giac")

[Out] ((c^3\*d\*g^3 - a\*c^2\*f\*g^3 + a\*c^2\*d\*g\*h^2 - a^2\*c\*f\*g\*h^2 + a\*c^2\*g^2\*h\*e + a^2\*c\*h^3\*e)\*x/(a\*c^3\*g^4 + 2\*a^2\*c^2\*g^2\*h^2 + a^3\*c\*h^4) + (a\*c^2\*d\*g^2\*h - a^2\*c\*f\*g^2\*h + a^2\*c\*d\*h^3 - a^3\*f\*h^3 - a\*c^2\*g^3\*e - a^2\*c\*g\*h^2\*e)/(a\*c^3\*g^4 + 2\*a^2\*c^2\*g^2\*h^2 + a^3\*c\*h^4)/sqrt(c\*x^2 + a) - 2\*(f\*g^2 + d\*h^2 - g\*h\*e)\*arctan((sqrt(c)\*x - sqrt(c\*x^2 + a))\*h + sqrt(c)\*g)/sqrt(-c\*g^2 - a\*h^2))/((c\*g^2 + a\*h^2)\*sqrt(-c\*g^2 - a\*h^2))

**maple [B]** time = 0.02, size = 862, normalized size = 6.25

$$\frac{\frac{c^3 d g^3 - a c^2 f g^3 + a c^2 d g h^2 - a^2 c f g h^2 + a c^2 g^2 h e + a^2 c h^3 e}{a^3 c^3 g^4 + 2 a^2 c^2 g^2 h^2 + a^3 c h^4} x + \frac{a c^2 d g^2 h - a^2 c f g^2 h + a^2 c d h^3 - a^3 f h^3 - a c^2 g^3 e - a^2 c g h^2 e}{a^3 c^3 g^4 + 2 a^2 c^2 g^2 h^2 + a^3 c h^4} \sqrt{c x^2 + a} - 2 (f g^2 + d h^2 - g h e) \arctan\left(\frac{\sqrt{c} x - \sqrt{c x^2 + a}}{h} + \frac{g \sqrt{c}}{\sqrt{-c g^2 - a h^2}}\right)}{(c g^2 + a h^2) \sqrt{-c g^2 - a h^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+a)^(3/2),x)

[Out] -1/h\*f/c/(c\*x^2+a)^(1/2)+1/h\*e\*x/a/(c\*x^2+a)^(1/2)-1/h^2\*f\*g\*x/a/(c\*x^2+a)^(1/2)+h/(a\*h^2+c\*g^2)/(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2)\*d-1/(a\*h^2+c\*g^2)/(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2)\*e\*g+1/h/(a\*h^2+c\*g^2)/(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2)\*f\*g^2+g/(a\*h^2+c\*g^2)/a/(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2)\*x\*c\*d-1/h\*g^2/(a\*h^2+c\*g^2)/a/(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2)\*x\*c\*e+1/h^2\*g^3/(a\*h^2+c\*g^2)/a/(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2)\*x\*c\*f-h/(a\*h^2+c\*g^2)/((a\*h^2+c\*g^2)/h^2)^(1/2)\*ln((-2\*(x+g/h)\*c\*g/h+2\*(a\*h^2+c\*g^2)/h^2+2\*((a\*h^2+c\*g^2)/h^2)^(1/2))\*(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2))/(x+g/h)\*d+1/(a\*h^2+c\*g^2)/((a\*h^2+c\*g^2)/h^2)^(1/2)\*ln((-2\*(x+g/h)\*c\*g/h+2\*(a\*h^2+c\*g^2)/h^2+2\*((a\*h^2+c\*g^2)/h^2)^(1/2))\*(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2))/(x+g/h)\*e\*g-1/h/(a\*h^2+c\*g^2)/((a\*h^2+c\*g^2)/h^2)^(1/2)\*ln((-2\*(x+g/h)\*c\*g/h+2\*(a\*h^2+c\*g^2)/h^2+2\*((a\*h^2+c\*g^2)/h^2)^(1/2))\*(-2\*(x+g/h)\*c\*g/h+(x+g/h)^2\*c+(a\*h^2+c\*g^2)/h^2)^(1/2))/(x+g/h)\*f\*g^2

**maxima [B]** time = 0.62, size = 453, normalized size = 3.28

$$\frac{\frac{c f g^3}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{c g^2 e}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{c d g}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{f g^2}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{e g}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{d}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{f g x}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{e x}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} + \frac{f g^2 \operatorname{arsinh}\left(\frac{c x}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} - \frac{a h}{\sqrt{c^2 + a}}\right)}{\left(a + \frac{c^2}{a}\right)^{3/2}} + \frac{e g \operatorname{arsinh}\left(\frac{c x}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} - \frac{a h}{\sqrt{c^2 + a}}\right)}{\left(a + \frac{c^2}{a}\right)^{3/2}} + \frac{d \operatorname{arsinh}\left(\frac{c x}{\sqrt{c^2 + a a g h^2 + \sqrt{c^2 + a} a h^3}} - \frac{a h}{\sqrt{c^2 + a}}\right)}{\left(a + \frac{c^2}{a}\right)^{3/2}} - \frac{f}{\sqrt{c^2 + a}}}{(c g^2 + a h^2) \sqrt{-c g^2 - a h^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+a)^(3/2),x, algorithm="maxima")

[Out] c\*f\*g^3\*x/(sqrt(c\*x^2 + a)\*a\*c\*g^2\*h^2 + sqrt(c\*x^2 + a)\*a^2\*h^4) - c\*e\*g^2\*x/(sqrt(c\*x^2 + a)\*a\*c\*g^2\*h + sqrt(c\*x^2 + a)\*a^2\*h^3) + c\*d\*g\*x/(sqrt(c\*x^2 + a)\*a\*c\*g^2 + sqrt(c\*x^2 + a)\*a^2\*h^2) + f\*g^2/(sqrt(c\*x^2 + a)\*c\*g^2\*h + sqrt(c\*x^2 + a)\*a\*h^3) - e\*g/(sqrt(c\*x^2 + a)\*c\*g^2 + sqrt(c\*x^2 + a)\*a\*h^2) + d/(sqrt(c\*x^2 + a)\*c\*g^2/h + sqrt(c\*x^2 + a)\*a\*h) - f\*g\*x/(sqrt(c\*x^2 + a)\*a\*h^2) + e\*x/(sqrt(c\*x^2 + a)\*a\*h) + f\*g^2\*arcsinh(c\*g\*x/(sqrt(a\*c

```
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3)
- e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(3/2)*h^2) + d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h) - f/(sqrt(c*x^2 + a
)*c*h)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f x^2 + e x + d}{(g + h x) (c x^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)\*(a + c\*x^2)^(3/2)), x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)\*(a + c\*x^2)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{(a + c x^2)^{3/2} (g + h x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] Integral((d + e\*x + f\*x\*\*2)/((a + c\*x\*\*2)\*\*(3/2)\*(g + h\*x)), x)

$$3.113 \quad \int \frac{d+ex+fx^2}{(g+hx)^2(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=239

$$\frac{a(ah(2fg - eh) + cg(eg - 2dh)) - x(a^2fh^2 - ac(fg^2 - h(2eg - dh)) + c^2dg^2)}{a\sqrt{a+cx^2}(ah^2 + cg^2)^2} - \frac{h\sqrt{a+cx^2}(dh^2 - egh + fg^2)}{(g+hx)(ah^2 + cg^2)^2} + \dots$$

**Rubi [A]** time = 0.42, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1647, 807, 725, 206}

$$\frac{a(ah(2fg - eh) + cg(eg - 2dh)) - x(a^2fh^2 - ac(fg^2 - h(2eg - dh)) + c^2dg^2)}{a\sqrt{a+cx^2}(ah^2 + cg^2)^2} - \frac{h\sqrt{a+cx^2}(dh^2 - egh + fg^2)}{(g+hx)(ah^2 + cg^2)^2} - \frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(-ah^2(2fg - eh) - cgh(2eg - 3dh) + cfg^3)}{(ah^2 + cg^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + c\*x^2)^(3/2)), x]

[Out] -((a\*(c\*g\*(e\*g - 2\*d\*h) + a\*h\*(2\*f\*g - e\*h)) - (c^2\*d\*g^2 + a^2\*f\*h^2 - a\*c\*(f\*g^2 - h\*(2\*e\*g - d\*h)))\*x)/(a\*(c\*g^2 + a\*h^2)^2\*sqrt[a + c\*x^2])) - (h\*(f\*g^2 - e\*g\*h + d\*h^2)\*sqrt[a + c\*x^2])/((c\*g^2 + a\*h^2)^2\*(g + h\*x)) - ((c\*f\*g^3 - c\*g\*h\*(2\*e\*g - 3\*d\*h) - a\*h^2\*(2\*f\*g - e\*h))\*ArcTanh[(a\*h - c\*g\*x)/(sqrt[c\*g^2 + a\*h^2]\*sqrt[a + c\*x^2])])/(c\*g^2 + a\*h^2)^(5/2)

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 725**

Int[1/(((d\_) + (e\_.)\*(x\_))\*sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

**Rule 807**

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}

, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1647

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :  
 > With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + c\*x^2, x], x, 1]}, Simp[((a\*g - c\*f\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] + Dist[1/(2\*a\*c\*(p + 1)), Int[(d + e\*x)^m\*(a + c\*x^2)^(p + 1)\*ExpandToSum[(2\*a\*c\*(p + 1)\*Q)/(d + e\*x)^m + (c\*f\*(2\*p + 3))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(g + hx)^2 (a + cx^2)^{3/2}} dx &= -\frac{a(cg(eg - 2dh) + ah(2fg - eh)) - (c^2dg^2 + a^2fh^2 - ac(fg^2 - h(2eg - dh)))x}{a(cg^2 + ah^2)^2 \sqrt{a + cx^2}} \\ &= -\frac{a(cg(eg - 2dh) + ah(2fg - eh)) - (c^2dg^2 + a^2fh^2 - ac(fg^2 - h(2eg - dh)))x}{a(cg^2 + ah^2)^2 \sqrt{a + cx^2}} \\ &= -\frac{a(cg(eg - 2dh) + ah(2fg - eh)) - (c^2dg^2 + a^2fh^2 - ac(fg^2 - h(2eg - dh)))x}{a(cg^2 + ah^2)^2 \sqrt{a + cx^2}} \\ &= -\frac{a(cg(eg - 2dh) + ah(2fg - eh)) - (c^2dg^2 + a^2fh^2 - ac(fg^2 - h(2eg - dh)))x}{a(cg^2 + ah^2)^2 \sqrt{a + cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.61, size = 285, normalized size = 1.19

$$\frac{(ah^2 + cg^2)^{3/2} (-a^2fh^2 + ac(2h(dh - eg + ehx) + fg(g - 2hx)) + 2c^2dghx) + 2h \left( h(a + cx^2) \sqrt{ah^2 + cg^2} (a^2fh^2 + ac(h(3eg - 2dh) - 2fg^2) + c^2dg^2) - ac\sqrt{a + cx^2} (g + hx) \tanh^{-1} \left( \frac{ah - cgx}{\sqrt{a + cx^2} \sqrt{ah^2 + cg^2}} \right) (ah^2(oh - 2fg) + cgh(3dh - 2eg) + cfh^2) \right) - af (ah^2 + cg^2)^{5/2}}{2ach\sqrt{a + cx^2} (g + hx) (ah^2 + cg^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + c\*x^2)^(3/2)),x]

```
[Out] (-(a*f*(c*g^2 + a*h^2)^(5/2)) + (c*g^2 + a*h^2)^(3/2)*(-(a^2*f*h^2) + 2*c^2
*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(e*g) + d*h + e*h*x))) + 2*h*(h*Sqr
t[c*g^2 + a*h^2]*(c^2*d*g^2 + a^2*f*h^2 + a*c*(-2*f*g^2 + h*(3*e*g - 2*d*h)
))* (a + c*x^2) - a*c*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e
h))* (g + h*x)*Sqrt[a + c*x^2]*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])))/(2*a*c*h*(c*g^2 + a*h^2)^(5/2)*(g + h*x)*Sqrt[a + c*x^2])
```

**IntegrateAlgebraic [A]** time = 1.91, size = 383, normalized size = 1.60

$$\frac{-a^2 d h^2 + 2 a^2 e g h^2 + a^2 d h^3 x - 3 a^2 f g^2 h - a^2 f g h^2 x + a^2 f h^2 x^2 + 2 a c d g^2 h + a c d g h^2 x - 2 a c d h^3 x^2 - a c e g^2 + a c e g^2 h x + 3 a c e g h^2 x^2 - a c f g^2 x - 2 a c f g^2 h x^2 + c^2 d g^2 x + c^2 d g^2 h x^2 + 2 \tan^{-1} \left( \frac{4 \sqrt{a} \sqrt{c^2 + a} \sqrt{h}}{\sqrt{a^2 - c g^2}} \right) (3 c d g h^2 \sqrt{-a h^2 - c g^2} - 2 a e g^2 h \sqrt{-a h^2 - c g^2} + a d h^3 \sqrt{-a h^2 - c g^2} - 2 a f g h^2 \sqrt{-a h^2 - c g^2} + c f g^2 \sqrt{-a h^2 - c g^2})}{a \sqrt{a + c x^2} (g + h x) (a h^2 + c g^2)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*c*e*g^3) + 2*a*c*d*g^2*h - 3*a^2*f*g^2*h + 2*a^2*e*g*h^2 - a^2*d*h^3 +
c^2*d*g^3*x - a*c*f*g^3*x + a*c*e*g^2*h*x + a*c*d*g*h^2*x - a^2*f*g*h^2*x
+ a^2*e*h^3*x + c^2*d*g^2*h*x^2 - 2*a*c*f*g^2*h*x^2 + 3*a*c*e*g*h^2*x^2 - 2
*a*c*d*h^3*x^2 + a^2*f*h^3*x^2)/(a*(c*g^2 + a*h^2)^2*(g + h*x)*Sqrt[a + c*x
^2]) + (2*(c*f*g^3*Sqrt[-(c*g^2) - a*h^2] - 2*c*e*g^2*h*Sqrt[-(c*g^2) - a*h
^2] + 3*c*d*g*h^2*Sqrt[-(c*g^2) - a*h^2] - 2*a*f*g*h^2*Sqrt[-(c*g^2) - a*h
^2] + a*e*h^3*Sqrt[-(c*g^2) - a*h^2])*ArcTan[(Sqrt[c]*g + Sqrt[c]*h*x - h*Sq
rt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(c*g^2 + a*h^2)^3
```

**fricas [B]** time = 4.36, size = 1573, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)
*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d -
2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3
*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a
^3*e*h^4 + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g
*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h
^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(a*c^2*e*
g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a*c^2*d - 3*a^2*c*f)
*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^2 + 3*a^2*c*e*g*h^4
+ (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a^2*c*d - a
^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^3*e*h^5 + (c^3*d -
a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d - a^3*f)*g*h^4)*x)*
sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^3*h^4 + a^5*g
*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 + a^4*c*h^7)*x^
3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4*c*g*h^6)*x^2 +
```



$$\begin{aligned} & (a^2c^3g^6h + 3a^3c^2g^4h^3 + 3a^4c^2g^2h^5 + a^5h^7)x, -((a^2 \\ & *c*f*g^4 - 2a^2c^2e*g^3h + a^3e*g^2h^3 + (3a^2c^2d - 2a^3f)*g^2h^2 + \\ & (a^2c^2f*g^3h - 2a^2c^2e*g^2h^2 + a^2c^2e*h^4 + (3a^2c^2d - 2a^2c^2f)* \\ & g^2h^3)*x^3 + (a^2c^2f*g^4 - 2a^2c^2e*g^3h + a^2c^2e*g^2h^3 + (3a^2c^2d - \\ & 2a^2c^2f)*g^2h^2)*x^2 + (a^2c^2f*g^3h - 2a^2c^2e*g^2h^2 + a^3e*h^4 + \\ & (3a^2c^2d - 2a^3f)*g^2h^3)*x)*\sqrt{-c*g^2 - a*h^2}*\arctan(\sqrt{-c*g^2 - a \\ & *h^2}*(c*g*x - a*h)*\sqrt{c*x^2 + a})/(a^2c^2g^2 + a^2h^2 + (c^2g^2 + a^2c^2h^2 \\ & )*x^2)) + (a^2c^2e*g^5 - a^2c^2e*g^3h^2 - 2a^3e*g^2h^4 + a^3d*h^5 - (2a \\ & *c^2d - 3a^2c^2f)*g^4h - (a^2c^2d - 3a^3f)*g^2h^3 - (3a^2c^2e*g^3h^2 \\ & + 3a^2c^2e*g^2h^4 + (c^3d - 2a^2c^2f)*g^4h - (a^2c^2d + a^2c^2f)*g^2h^3 \\ & - (2a^2c^2d - a^3f)*h^5)*x^2 - (a^2c^2e*g^4h + 2a^2c^2e*g^2h^3 + a^3 \\ & e*h^5 + (c^3d - a^2c^2f)*g^5 + 2*(a^2c^2d - a^2c^2f)*g^3h^2 + (a^2c^2d \\ & - a^3f)*g^2h^4)*x)*\sqrt{c*x^2 + a})/(a^2c^3g^7 + 3a^3c^2g^5h^2 + 3a^4 \\ & c^2g^3h^4 + a^5g^2h^6 + (a^2c^4g^6h + 3a^2c^3g^4h^3 + 3a^3c^2g^2h^5 + \\ & a^4c^2h^7)*x^3 + (a^2c^4g^7 + 3a^2c^3g^5h^2 + 3a^3c^2g^3h^4 + \\ & a^4c^2g^2h^6)*x^2 + (a^2c^3g^6h + 3a^3c^2g^4h^3 + 3a^4c^2g^2h^5 + \\ & a^5h^7)*x] \end{aligned}$$

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+a)^(3/2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.02, size = 1663, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+a)^(3/2),x)

[Out] 
$$\begin{aligned} & f/h^2x/a/(c*x^2+a)^{(1/2)}-1/(a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h) \\ & ^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*d+1/h/(a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h \\ & +(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)/(-2*( \\ & x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*f*g^2+3*h*c*g/(a*h^2+c*g^ \\ & 2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*d-3*c*g^2/(a*h^ \\ & 2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e+3/h*c*g \\ & ^3/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*f \\ & +3*c^2*g^2/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^ \\ & 2)^{(1/2)}*x*d-3/h*c^2*g^3/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a \\ & *h^2+c*g^2)/h^2)^{(1/2)}*x*e+3/h^2*c^2*g^4/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/ \\ & h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*f-3*h*c*g/(a*h^2+c*g^2)^2/((a*h^2+ \end{aligned}$$

$$\begin{aligned}
& c*g^2/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h) \\
& )*d+3*c*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2 \\
& *c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*e-3/h*c*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/ \\
& h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h)) \\
& *f-2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)} \\
& *x*c*d+3/h/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*c*e*g-4/h^2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c \\
& *g^2)/h^2)^{(1/2)}*x*c*f*g^2+1/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a \\
& *h^2+c*g^2)/h^2)^{(1/2)}*e-2/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a \\
& *h^2+c*g^2)/h^2)^{(1/2)}*f*g-1/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2 \\
& *(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)* \\
& c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*e+2/h/(a*h^2+c*g^2)/(( \\
& a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+ \\
& c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/( \\
& x+g/h))*f*g
\end{aligned}$$

**maxima** [B] time = 0.77, size = 1085, normalized size = 4.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+a)^(3/2),x, algorithm="maxima")

[Out]  $3*c^2*f*g^4*x/(\sqrt{c*x^2+a})*a*c^2*g^4*h^2+2*\sqrt{c*x^2+a}*a^2*c*g^2*h^4+\sqrt{c*x^2+a}*a^3*h^6)-3*c^2*e*g^3*x/(\sqrt{c*x^2+a})*a*c^2*g^4*h+2*\sqrt{c*x^2+a}*a^2*c*g^2*h^3+\sqrt{c*x^2+a}*a^3*h^5)+3*c^2*d*g^2*x/(\sqrt{c*x^2+a})*a*c^2*g^4+2*\sqrt{c*x^2+a}*a^2*c*g^2*h^2+\sqrt{c*x^2+a}*a^3*h^4)+3*c*f*g^3/(\sqrt{c*x^2+a})*c^2*g^4*h+2*\sqrt{c*x^2+a}*a*c*g^2*h^3+\sqrt{c*x^2+a}*a^2*h^5)-4*c*f*g^2*x/(\sqrt{c*x^2+a})*a*c*g^2*h^2+\sqrt{c*x^2+a}*a^2*h^4)-3*c*e*g^2/(\sqrt{c*x^2+a})*c^2*g^4+2*\sqrt{c*x^2+a}*a*c*g^2*h^2+\sqrt{c*x^2+a}*a^2*h^4)+3*c*e*g*x/(\sqrt{c*x^2+a})*a*c*g^2*h+\sqrt{c*x^2+a}*a^2*h^3)+3*c*d*g/(\sqrt{c*x^2+a})*c^2*g^4/h+2*\sqrt{c*x^2+a}*a*c*g^2*h+\sqrt{c*x^2+a}*a^2*h^3)-f*g^2/(\sqrt{c*x^2+a})*c*g^2*h^2*x+\sqrt{c*x^2+a}*a*h^4*x+\sqrt{c*x^2+a}*c*g^3*h+\sqrt{c*x^2+a}*a*g*h^3)-2*c*d*x/(\sqrt{c*x^2+a})*a*c*g^2+\sqrt{c*x^2+a}*a^2*h^2)+e*g/(\sqrt{c*x^2+a})*c*g^2*h*x+\sqrt{c*x^2+a}*a*h^3*x+\sqrt{c*x^2+a}*c*g^3+\sqrt{c*x^2+a}*a*g*h^2)-2*f*g/(\sqrt{c*x^2+a})*c*g^2*h+\sqrt{c*x^2+a}*a*h^3)-d/(\sqrt{c*x^2+a})*c*g^2*x+\sqrt{c*x^2+a}*a*h^2*x+\sqrt{c*x^2+a}*c*g^3/h+\sqrt{c*x^2+a}*a*g*h)+e/(\sqrt{c*x^2+a})*c*g^2+\sqrt{c*x^2+a}*a*h^2)+f*x/(\sqrt{c*x^2+a})*a*h^2)+3*c*f*g^3*arcsinh(c*g*x/(\sqrt{a*c})*abs(h*x+g))-a*h/(\sqrt{a*c})*abs(h*x+g))/((a+c*g^2/h^2)^(5/2)*h^5)-3*c*e*g^2*arcsinh(c*g*x/(\sqrt{a*c})*abs$

```
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) + 3*
c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(5/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) + e*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^2 (c x^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2), x)
```

```
[Out] Timed out
```

$$3.114 \quad \int \frac{d+ex+fx^2}{(g+hx)^3(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=374

$$\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4 - ach^2(3dh^2 - 9egh + 11fg^2) + 2c^2g^2(6dh^2 - 3egh + fg^2)\right) + a(a^2fh^3 - ach^2)}{2(ah^2 + cg^2)^{7/2}}$$

**Rubi [A]** time = 1.03, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1647, 1651, 807, 725, 206}

$$\frac{cx(a^2h^2(3fg-dh) - acg(fg^2 - 3h(eg-dh)) + c^2dg^2) + a(a^2fh^3 - ach(3fg^2 - h(3eg-dh)) - c^2g^2(eg-3dh))}{a\sqrt{a+cx^2}(ah^2+cg^2)^3} \frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(2a^2fh^4 - ach^2(3dh^2 - 9egh + 11fg^2) + 2c^2g^2(6dh^2 - 3egh + fg^2))}{2(ah^2+cg^2)^{7/2}} - \frac{h\sqrt{a+cx^2}(dh^2 - egh + fg^2)}{2(g+hx)^2(ah^2+cg^2)^2} - \frac{h\sqrt{a+cx^2}(-2ah^2(2fg-dh) - cgh(5eg-7dh) + 3c^2fg^2)}{2(g+hx)(ah^2+cg^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(3/2)), x]

[Out] (a\*(a^2\*f\*h^3 - c^2\*g^2\*(e\*g - 3\*d\*h) - a\*c\*h\*(3\*f\*g^2 - h\*(3\*e\*g - d\*h))) + c\*(c^2\*d\*g^3 + a^2\*h^2\*(3\*f\*g - e\*h) - a\*c\*g\*(f\*g^2 - 3\*h\*(e\*g - d\*h)))\*x)/(a\*(c\*g^2 + a\*h^2)^3\*sqrt[a + c\*x^2]) - (h\*(f\*g^2 - e\*g\*h + d\*h^2)\*sqrt[a + c\*x^2])/(2\*(c\*g^2 + a\*h^2)^2\*(g + h\*x)^2) - (h\*(3\*c\*f\*g^3 - c\*g\*h\*(5\*e\*g - 7\*d\*h) - 2\*a\*h^2\*(2\*f\*g - e\*h))\*sqrt[a + c\*x^2])/(2\*(c\*g^2 + a\*h^2)^3\*(g + h\*x)) - ((2\*a^2\*f\*h^4 - a\*c\*h^2\*(11\*f\*g^2 - 9\*e\*g\*h + 3\*d\*h^2) + 2\*c^2\*g^2\*(f\*g^2 - 3\*e\*g\*h + 6\*d\*h^2))\*ArcTanh[(a\*h - c\*g\*x)/(sqrt[c\*g^2 + a\*h^2]\*sqrt[a + c\*x^2]])]/(2\*(c\*g^2 + a\*h^2)^(7/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 725**

Int[1/(((d\_) + (e\_.)\*(x\_))\*sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

**Rule 807**

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))

$$\frac{1}{(2(p+1)(cd^2 + ae^2))} \int \frac{(cd^2f + ae^2g)}{(cd^2 + ae^2)} \int \frac{(d + ex)^{m+1}(a + cx^2)^p}{x} dx$$

$$/; \text{FreeQ}\{a, c, d, e, f, g, m, p\}, x \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{EqQ}[\text{Simplify}[m + 2p + 3], 0]$$

### Rule 1647

$$\text{Int}[(Pq_)*((d_) + (e_)*(x_))^{(m_)*((a_) + (c_)*(x_)^2)^{(p_)}, x\_Symbol] :$$

$$> \text{With}\{Q = \text{PolynomialQuotient}[(d + ex)^m Pq, a + cx^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + ex)^m Pq, a + cx^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + ex)^m Pq, a + cx^2, x], x, 1]\}, \text{Simp}[\frac{(a*g - c*f*x)*(a + cx^2)^{(p+1)}}{(2*a*c*(p+1))}, x] + \text{Dist}[1/(2*a*c*(p+1)), \text{Int}[(d + ex)^m*(a + cx^2)^{(p+1)}*\text{ExpandToSum}[(2*a*c*(p+1)*Q)/(d + ex)^m + (c*f*(2*p + 3))/(d + ex)^m, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{ILtQ}[m, 0]$$

### Rule 1651

$$\text{Int}[(Pq_)*((d_) + (e_)*(x_))^{(m_)*((a_) + (c_)*(x_)^2)^{(p_)}, x\_Symbol] :>$$

$$\text{With}\{Q = \text{PolynomialQuotient}[Pq, d + ex, x], R = \text{PolynomialRemainder}[Pq, d + ex, x]\}, \text{Simp}[\frac{e*R*(d + ex)^{(m+1)}*(a + cx^2)^{(p+1)}}{(m+1)*(cd^2 + ae^2)}, x] + \text{Dist}[1/((m+1)*(cd^2 + ae^2)), \text{Int}[(d + ex)^{(m+1)}*(a + cx^2)^p*\text{ExpandToSum}[(m+1)*(cd^2 + ae^2)*Q + c*d*R*(m+1) - c*e*R*(m+2*p+3)*x, x], x], x] /; \text{FreeQ}\{a, c, d, e, p\}, x \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{NeQ}[cd^2 + ae^2, 0] \ \&\& \ \text{LtQ}[m, -1]$$

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2}{(g + hx)^3 (a + cx^2)^{3/2}} dx &= \frac{a(a^2fh^3 - c^2g^2(eg - 3dh) - ach(3fg^2 - h(3eg - dh))) + c(c^2dg^3 + a^2h^2(3fg - eh))}{a(cg^2 + ah^2)^3 \sqrt{a + cx^2}} \\
&= \frac{a(a^2fh^3 - c^2g^2(eg - 3dh) - ach(3fg^2 - h(3eg - dh))) + c(c^2dg^3 + a^2h^2(3fg - eh))}{a(cg^2 + ah^2)^3 \sqrt{a + cx^2}} \\
&= \frac{a(a^2fh^3 - c^2g^2(eg - 3dh) - ach(3fg^2 - h(3eg - dh))) + c(c^2dg^3 + a^2h^2(3fg - eh))}{a(cg^2 + ah^2)^3 \sqrt{a + cx^2}} \\
&= \frac{a(a^2fh^3 - c^2g^2(eg - 3dh) - ach(3fg^2 - h(3eg - dh))) + c(c^2dg^3 + a^2h^2(3fg - eh))}{a(cg^2 + ah^2)^3 \sqrt{a + cx^2}} \\
&= \frac{a(a^2fh^3 - c^2g^2(eg - 3dh) - ach(3fg^2 - h(3eg - dh))) + c(c^2dg^3 + a^2h^2(3fg - eh))}{a(cg^2 + ah^2)^3 \sqrt{a + cx^2}}
\end{aligned}$$

**Mathematica [A]** time = 1.18, size = 404, normalized size = 1.08

$$\frac{1}{2} \left( \frac{\log(\sqrt{a + cx^2} \sqrt{ah^2 + cg^2} + ah - cgx) (2a^2fh^4 + ac^2(-3dh^2 + 9egh - 11fg^2) + 2c^2g^2(ahh^2 - 3egh + fg^2))}{(ah^2 + cg^2)^{5/2}} \log(g + hx) (2a^2fh^4 + ac^2(-3dh^2 + 9egh - 11fg^2) + 2c^2g^2(ahh^2 - 3egh + fg^2))}{(ah^2 + cg^2)^{5/2}} \sqrt{a + cx^2} \left( \frac{2(-c^2h^2 + a^2d)(ah - 3eg + hx) + 3f(g - hx) + 2c^2d(ha - g) + c^2g(-3ah + f) + a^2(-2fg + e) + c^2g^2}{d + cx^2} + \frac{4(2a^2h^2 - 2)(g + hx)(ah - 3eg + hx) + 4c^2g^2(ahh^2 - 3egh + fg^2)}{g + hx} + \frac{4(ah^2 + cg^2)(ah - cgx + fg^2)}{(g + hx)^2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(3/2)),x]

[Out] (-((Sqrt[a + c\*x^2]\*((h\*(c\*g^2 + a\*h^2)\*(f\*g^2 + h\*(-e\*g) + d\*h)))/(g + h\*x)^2 + (h\*(3\*c\*f\*g^3 + c\*g\*h\*(-5\*e\*g + 7\*d\*h) + 2\*a\*h^2\*(-2\*f\*g + e\*h)))/(g + h\*x) + (2\*(-a^3\*f\*h^3) - c^3\*d\*g^3\*x + a\*c^2\*g\*(f\*g^2\*x + e\*g\*(g - 3\*h\*x) + 3\*d\*h\*(-g + h\*x)) + a^2\*c\*h\*(3\*f\*g\*(g - h\*x) + h\*(-3\*e\*g + d\*h + e\*h\*x))))/(a\*(a + c\*x^2)))/(c\*g^2 + a\*h^2)^3 + ((2\*a^2\*f\*h^4 + a\*c\*h^2\*(-11\*f\*g^2 + 9\*e\*g\*h - 3\*d\*h^2) + 2\*c^2\*g^2\*(f\*g^2 - 3\*e\*g\*h + 6\*d\*h^2))\*Log[g + h\*x])/(c\*g^2 + a\*h^2)^(7/2) - ((2\*a^2\*f\*h^4 + a\*c\*h^2\*(-11\*f\*g^2 + 9\*e\*g\*h - 3\*d\*h^2) + 2\*c^2\*g^2\*(f\*g^2 - 3\*e\*g\*h + 6\*d\*h^2))\*Log[a\*h - c\*g\*x + Sqrt[c\*g^2 + a\*h^2]\*Sqrt[a + c\*x^2]])/(c\*g^2 + a\*h^2)^(7/2))/2

**IntegrateAlgebraic [B]** time = 17.21, size = 2488, normalized size = 6.65

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(3/2)),x]

[Out] 
$$\begin{aligned} & (-5*a^4*f*g^2*h^3 + a^4*e*g*h^4 + a^4*d*h^5 - 8*a^4*f*g*h^4*x + 2*a^4*e*h^5 \\ & *x - 2*a^4*f*h^5*x^2 + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]*(-10*a^3*f*g^3*h^2 + 4*a^3*e \\ & *g^2*h^3 - 5*a^3*f*g^2*h^3*x + 5*a^3*e*g*h^4*x - 3*a^3*d*h^5*x + 14*a^3*f*g \\ & *h^4*x^2 - 2*a^3*e*h^5*x^2 + 6*a^3*f*h^5*x^3) + c*(10*a^3*f*g^4*h - 12*a^3* \\ & e*g^3*h^2 + 10*a^3*d*g^2*h^3 + 39*a^3*f*g^3*h^2*x - 27*a^3*e*g^2*h^3*x + 11 \\ & *a^3*d*g*h^4*x + 26*a^3*f*g^2*h^3*x^2 - 20*a^3*e*g*h^4*x^2 + 8*a^3*d*h^5*x^ \\ & 2 - 20*a^3*f*g*h^4*x^3 + 2*a^3*e*h^5*x^3 - 10*a^3*f*h^5*x^4) + c^{(7/2)}*\text{Sqrt} \\ & [a + c*x^2]*(-4*d*g^5*x^2 - 8*e*g^5*x^3 + 16*d*g^4*h*x^3 + 12*f*g^5*x^4 - 3 \\ & 6*e*g^4*h*x^4 + 72*d*g^3*h^2*x^4 + 8*f*g^4*h*x^5 - 24*e*g^3*h^2*x^5 + 48*d* \\ & g^2*h^3*x^5) + c^{(5/2)}*\text{Sqrt}[a + c*x^2]*(-2*a*d*g^5 - 6*a*e*g^5*x + 14*a*d*g \\ & ^4*h*x + 19*a*f*g^5*x^2 - 49*a*e*g^4*h*x^2 + 81*a*d*g^3*h^2*x^2 - 14*a*f*g^ \\ & 4*h*x^3 - 14*a*e*g^3*h^2*x^3 + 48*a*d*g^2*h^3*x^3 - 66*a*f*g^3*h^2*x^4 + 54 \\ & *a*e*g^2*h^3*x^4 - 18*a*d*g*h^4*x^4 - 44*a*f*g^2*h^3*x^5 + 36*a*e*g*h^4*x^5 \\ & - 12*a*d*h^5*x^5) + c^{(3/2)}*\text{Sqrt}[a + c*x^2]*(5*a^2*f*g^5 - 11*a^2*e*g^4*h \\ & + 13*a^2*d*g^3*h^2 - 20*a^2*f*g^4*h*x + 14*a^2*e*g^3*h^2*x - 4*a^2*d*g^2*h^ \\ & 3*x - 72*a^2*f*g^3*h^2*x^2 + 54*a^2*e*g^2*h^3*x^2 - 20*a^2*d*g*h^4*x^2 - 53 \\ & *a^2*f*g^2*h^3*x^3 + 39*a^2*e*g*h^4*x^3 - 13*a^2*d*h^5*x^3 + 12*a^2*f*g*h^4 \\ & *x^4 + 8*a^2*f*h^5*x^5) + c^4*(4*d*g^5*x^3 + 8*e*g^5*x^4 - 16*d*g^4*h*x^4 - \\ & 12*f*g^5*x^5 + 36*e*g^4*h*x^5 - 72*d*g^3*h^2*x^5 - 8*f*g^4*h*x^6 + 24*e*g^ \\ & 3*h^2*x^6 - 48*d*g^2*h^3*x^6) + c^3*(4*a*d*g^5*x + 10*a*e*g^5*x^2 - 22*a*d* \\ & g^4*h*x^2 - 25*a*f*g^5*x^3 + 67*a*e*g^4*h*x^3 - 117*a*d*g^3*h^2*x^3 + 10*a* \\ & f*g^4*h*x^4 + 26*a*e*g^3*h^2*x^4 - 72*a*d*g^2*h^3*x^4 + 66*a*f*g^3*h^2*x^5 \\ & - 54*a*e*g^2*h^3*x^5 + 18*a*d*g*h^4*x^5 + 44*a*f*g^2*h^3*x^6 - 36*a*e*g*h^4 \\ & *x^6 + 12*a*d*h^5*x^6) + c^2*(2*a^2*e*g^5 - 6*a^2*d*g^4*h - 13*a^2*f*g^5*x \\ & + 31*a^2*e*g^4*h*x - 45*a^2*d*g^3*h^2*x + 28*a^2*f*g^4*h*x^2 - 10*a^2*e*g^3 \\ & *h^2*x^2 - 14*a^2*d*g^2*h^3*x^2 + 105*a^2*f*g^3*h^2*x^3 - 81*a^2*e*g^2*h^3* \\ & x^3 + 29*a^2*d*g*h^4*x^3 + 75*a^2*f*g^2*h^3*x^4 - 57*a^2*e*g*h^4*x^4 + 19*a \\ & ^2*d*h^5*x^4 - 12*a^2*f*g*h^4*x^5 - 8*a^2*f*h^5*x^6)) / (6*a^5*\text{Sqrt}[c]*h^6*x* \\ & (g + h*x)^2 + 8*c^{(11/2)}*g^6*x^5*(g + h*x)^2 - 2*a^5*h^6*(g + h*x)^2*\text{Sqrt}[a \\ & + c*x^2] - 8*c^5*g^6*x^4*(g + h*x)^2*\text{Sqrt}[a + c*x^2] + 2*c*(g + h*x)^2*\text{Sqr} \\ & t[a + c*x^2]*(-3*a^4*g^2*h^4 - 5*a^4*h^6*x^2) + 2*c^{(3/2)}*(g + h*x)^2*(9*a^ \\ & 4*g^2*h^4*x + 7*a^4*h^6*x^3) + 2*c^4*(g + h*x)^2*\text{Sqrt}[a + c*x^2]*(-5*a*g^6* \\ & x^2 - 12*a*g^4*h^2*x^4) + 2*c^3*(g + h*x)^2*\text{Sqrt}[a + c*x^2]*(-(a^2*g^6) - 1 \\ & 5*a^2*g^4*h^2*x^2 - 12*a^2*g^2*h^4*x^4) + 2*c^2*(g + h*x)^2*\text{Sqrt}[a + c*x^2] \\ & *(-3*a^3*g^4*h^2 - 15*a^3*g^2*h^4*x^2 - 4*a^3*h^6*x^4) + 2*c^{(9/2)}*(g + h*x \\ & )^2*(7*a*g^6*x^3 + 12*a*g^4*h^2*x^5) + 2*c^{(7/2)}*(g + h*x)^2*(3*a^2*g^6*x + \\ & 21*a^2*g^4*h^2*x^3 + 12*a^2*g^2*h^4*x^5) + 2*c^{(5/2)}*(g + h*x)^2*(9*a^3*g^ \\ & 4*h^2*x + 21*a^3*g^2*h^4*x^3 + 4*a^3*h^6*x^5)) - (15*a*f*h^2*\text{ArcTan}[(-\text{Sqrt} \\ & [c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + c*x^2)]/\text{Sqrt}[-(c*g^2) - a*h^2]) / (\text{Sqrt}[-( \\ & c*g^2) - a*h^2]*(c*g^2 + a*h^2)^2) + (21*a*e*h^3*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqr} \\ & t[c]*h*x + h*\text{Sqrt}[a + c*x^2)]/\text{Sqrt}[-(c*g^2) - a*h^2]) / (g*\text{Sqrt}[-(c*g^2) - a \end{aligned}$$

$$\begin{aligned} & *h^2)*(c*g^2 + a*h^2)^2) - (27*a*d*h^4*ArcTan[(-(Sqrt[c]*g) - Sqrt[c]*h*x + \\ & h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g^2*Sqrt[-(c*g^2) - a*h^2]*(c \\ & *g^2 + a*h^2)^2) + (2*f*ArcTan[(-(Sqrt[c]*g) - Sqrt[c]*h*x + h*Sqrt[a + c*x \\ & ^2])/Sqrt[-(c*g^2) - a*h^2]])/(Sqrt[-(c*g^2) - a*h^2]*(c*g^2 + a*h^2)) - (6 \\ & *e*h*ArcTan[(-(Sqrt[c]*g) - Sqrt[c]*h*x + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) \\ & - a*h^2]])/(g*Sqrt[-(c*g^2) - a*h^2]*(c*g^2 + a*h^2)) + (12*d*h^2*ArcTan[(- \\ & (Sqrt[c]*g) - Sqrt[c]*h*x + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g^ \\ & 2*Sqrt[-(c*g^2) - a*h^2]*(c*g^2 + a*h^2)) + ((15*a^2*f*h^4)/(Sqrt[-(c*g^2) \\ & - a*h^2]*(c*g^2 + a*h^2)^3) - (15*a^2*e*h^5)/(g*Sqrt[-(c*g^2) - a*h^2]*(c*g \\ & ^2 + a*h^2)^3) + (15*a^2*d*h^6)/(g^2*Sqrt[-(c*g^2) - a*h^2]*(c*g^2 + a*h^2) \\ & ^3))*ArcTan[(-(Sqrt[c]*g) - Sqrt[c]*h*x + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) \\ & - a*h^2]] \end{aligned}$$

**fricas** [B] time = 21.95, size = 2853, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4\*((2\*a^2\*c^2\*f\*g^6 - 6\*a^2\*c^2\*e\*g^5\*h + 9\*a^3\*c\*e\*g^3\*h^3 + (12\*a^2\*c^2\*d - 11\*a^3\*c\*f)\*g^4\*h^2 - (3\*a^3\*c\*d - 2\*a^4\*f)\*g^2\*h^4 + (2\*a\*c^3\*f\*g^4\*h^2 - 6\*a\*c^3\*e\*g^3\*h^3 + 9\*a^2\*c^2\*e\*g\*h^5 + (12\*a\*c^3\*d - 11\*a^2\*c^2\*f)\*g^2\*h^4 - (3\*a^2\*c^2\*d - 2\*a^3\*c\*f)\*h^6)\*x^4 + 2\*(2\*a\*c^3\*f\*g^5\*h - 6\*a\*c^3\*e\*g^4\*h^2 + 9\*a^2\*c^2\*e\*g^2\*h^4 + (12\*a\*c^3\*d - 11\*a^2\*c^2\*f)\*g^3\*h^3 - (3\*a^2\*c^2\*d - 2\*a^3\*c\*f)\*g\*h^5)\*x^3 + (2\*a\*c^3\*f\*g^6 - 6\*a\*c^3\*e\*g^5\*h + 3\*a^2\*c^2\*e\*g^3\*h^3 + 9\*a^3\*c\*e\*g\*h^5 + 3\*(4\*a\*c^3\*d - 3\*a^2\*c^2\*f)\*g^4\*h^2 + 9\*(a^2\*c^2\*d - a^3\*c\*f)\*g^2\*h^4 - (3\*a^3\*c\*d - 2\*a^4\*f)\*h^6)\*x^2 + 2\*(2\*a^2\*c^2\*f\*g^5\*h - 6\*a^2\*c^2\*e\*g^4\*h^2 + 9\*a^3\*c\*e\*g^2\*h^4 + (12\*a^2\*c^2\*d - 11\*a^3\*c\*f)\*g^3\*h^3 - (3\*a^3\*c\*d - 2\*a^4\*f)\*g\*h^5)\*x)\*sqrt(c\*g^2 + a\*h^2)\*log((2\*a\*c\*g\*h\*x - a\*c\*g^2 - 2\*a^2\*h^2 - (2\*c^2\*g^2 + a\*c\*h^2)\*x^2 - 2\*sqrt(c\*g^2 + a\*h^2)\*(c\*g\*x - a\*h)\*sqrt(c\*x^2 + a))/(h^2\*x^2 + 2\*g\*h\*x + g^2)) - 2\*(2\*a\*c^3\*e\*g^7 - 10\*a^2\*c^2\*e\*g^5\*h^2 - 11\*a^3\*c\*e\*g^3\*h^4 + a^4\*e\*g\*h^6 + a^4\*d\*h^7 - 2\*(3\*a\*c^3\*d - 5\*a^2\*c^2\*f)\*g^6\*h + (4\*a^2\*c^2\*d + 5\*a^3\*c\*f)\*g^4\*h^3 + (11\*a^3\*c\*d - 5\*a^4\*f)\*g^2\*h^5 - (11\*a\*c^3\*e\*g^4\*h^3 + 7\*a^2\*c^2\*e\*g^2\*h^5 - 4\*a^3\*c\*e\*h^7 + (2\*c^4\*d - 5\*a\*c^3\*f)\*g^5\*h^2 - (11\*a\*c^3\*d - 5\*a^2\*c^2\*f)\*g^3\*h^4 - (13\*a^2\*c^2\*d - 10\*a^3\*c\*f)\*g\*h^6)\*x^3 - (16\*a\*c^3\*e\*g^5\*h^2 + 17\*a^2\*c^2\*e\*g^3\*h^4 + a^3\*c\*e\*g\*h^6 + 4\*(c^4\*d - 2\*a\*c^3\*f)\*g^6\*h - (10\*a\*c^3\*d - a^2\*c^2\*f)\*g^4\*h^3 - (17\*a^2\*c^2\*d - 11\*a^3\*c\*f)\*g^2\*h^5 - (3\*a^3\*c\*d - 2\*a^4\*f)\*h^7)\*x^2 - (2\*a\*c^3\*e\*g^6\*h + 17\*a^2\*c^2\*e\*g^4\*h^3 + 13\*a^3\*c\*e\*g^2\*h^5 - 2\*a^4\*e\*h^7 + 2\*(c^4\*d - a\*c^3\*f)\*g^7 + (8\*a\*c^3\*d - 11\*a^2\*c^2\*f)\*g^5\*h^2 - (5\*a^2\*c^2\*d + a^3\*c\*f)\*g^3\*h^4 - (11\*a^3\*c\*d - 8\*a^4\*f)\*g\*h^6)\*x)\*sqrt(c\*x^2 + a))/(a^2\*c^4\*g^10 + 4\*a^3\*c^3\*g^8\*h^2 + 6\*a^4\*c^2\*g^6\*h^4 + 4\*a^5\*c\*g^4\*h^6 + a^6\*g^2\*h^8 + (a\*c^5\*g^8\*h^2 + 4\*a^2\*c^4\*g^6\*h^4 + 6\*a^3\*c^3\*g^4\*h^6 + 4\*a^4\*c^2\*g^2\*h^8 + a^5\*c\*h^10)\*x^4 + 2\*(a\*c^5\*g^9\*h + 4\*a^2\*c^4\*g^7\*h^3 + 6\*a^3\*c^3\*g^5\*h^5 + 4\*a^4\*c^2\*g^3\*h^7 + a^5\*c\*g^





$$\begin{aligned}
& h - 3a^2c^4fg^8h + 8a^2c^4d^2g^6h^3 - 8a^3c^3fg^6h^3 + 6a^3c^3d^2g^4h^5 - 6a^4c^2fg^4h^5 - a^5c^2d^2h^9 + a^6fh^9 - ac^5g^9e \\
& + 6a^3c^3g^5h^4e + 8a^4c^2g^3h^6e + 3a^5c^2g^2h^8e)/(ac^6g^12 + 6a^2c^5g^10h^2 + 15a^3c^4g^8h^4 + 20a^4c^3g^6h^6 + 15a^5c^2g^4h^8 + 6a^6c^2g^2h^10 + a^7h^12))/\sqrt{cx^2 + a} - (2c^2fg^4 + 1 \\
& 2c^2d^2g^2h^2 - 11a^2c^2fg^2h^2 - 3a^2c^2d^2h^4 + 2a^2f^2h^4 - 6c^2g^3h^2e + 9a^2c^2g^3h^2e)\arctan((\sqrt{c}x - \sqrt{cx^2 + a})h + \sqrt{c}g)/\sqrt{-c^2g^2 - ah^2})/((c^3g^6 + 3a^2c^2g^4h^2 + 3a^2c^2g^2h^4 + a^3h^6)\sqrt{-c^2g^2 - ah^2}) - (2(\sqrt{c}x - \sqrt{cx^2 + a})^3c^2fg^4h + 6(\sqrt{c}x - \sqrt{cx^2 + a})^3c^2d^2g^2h^3 - 5(\sqrt{c}x - \sqrt{cx^2 + a})^3a^2c^2fg^2h^3 - (\sqrt{c}x - \sqrt{cx^2 + a})^3a^2c^2d^2h^5 - 4(\sqrt{c}x - \sqrt{cx^2 + a})^3c^2g^3h^2e + 3(\sqrt{c}x - \sqrt{cx^2 + a})^3a^2c^2g^3h^2e + 6(\sqrt{c}x - \sqrt{cx^2 + a})^2c^{5/2}fg^5 + 14(\sqrt{c}x - \sqrt{cx^2 + a})^2c^{5/2}d^2g^3h^2 - 11(\sqrt{c}x - \sqrt{cx^2 + a})^2a^2c^{3/2}fg^3h^2 - 7(\sqrt{c}x - \sqrt{cx^2 + a})^2a^2c^{3/2}d^2g^3h^2 + 4(\sqrt{c}x - \sqrt{cx^2 + a})^2a^2c^{3/2}fg^3h^2 - 10(\sqrt{c}x - \sqrt{cx^2 + a})^2c^{5/2}g^4h^2e + 9(\sqrt{c}x - \sqrt{cx^2 + a})^2a^2c^{3/2}g^2h^3e - 2(\sqrt{c}x - \sqrt{cx^2 + a})^2a^2c^{3/2}g^2h^3e - 10(\sqrt{c}x - \sqrt{cx^2 + a})a^2c^2fg^4h - 22(\sqrt{c}x - \sqrt{cx^2 + a})a^2c^2d^2g^2h^3 + 11(\sqrt{c}x - \sqrt{cx^2 + a})a^2c^2fg^2h^3 - (\sqrt{c}x - \sqrt{cx^2 + a})a^2c^2d^2h^5 + 16(\sqrt{c}x - \sqrt{cx^2 + a})a^2c^2g^3h^2e - 5(\sqrt{c}x - \sqrt{cx^2 + a})a^2c^2g^3h^2e + 3a^2c^{3/2}fg^3h^2 + 7a^2c^{3/2}d^2g^3h^2 - 4a^3\sqrt{c}fg^4h - 5a^2c^{3/2}g^2h^3e + 2a^3\sqrt{c}h^5e)/((c^3g^6 + 3a^2c^2g^4h^2 + 3a^2c^2g^2h^4 + a^3h^6)*((\sqrt{c}x - \sqrt{cx^2 + a})^2h + 2(\sqrt{c}x - \sqrt{cx^2 + a})\sqrt{c}g - ah)^2)
\end{aligned}$$

**maple [B]** time = 0.02, size = 2584, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^{(3/2)}, x)$

[Out]  $f/h/(ah^2+c^2g^2)/(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)}+5/2/h*c^2g^2/(ah^2+c^2g^2)^2/(x+g/h)/(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)}e-5/2/h^2*c^2g^3/(ah^2+c^2g^2)^2/(x+g/h)/(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)}f+15/2*c^3g^3/(ah^2+c^2g^2)^3/a/(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)}*x*d-15/2*h*c^2g^2/(ah^2+c^2g^2)^3/((ah^2+c^2g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c^2g/h+2*(ah^2+c^2g^2)/h^2+2*((ah^2+c^2g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)})/(x+g/h))*d-15/2/h*c^2g^4/(ah^2+c^2g^2)^3/((ah^2+c^2g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c^2g/h+2*(ah^2+c^2g^2)/h^2+2*((ah^2+c^2g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)})/(x+g/h))*f-13/2*c^2g/(ah^2+c^2g^2)^2/a/(-2*(x+g/h)*c^2g/h+(x+g/h)^2*c+(ah^2+c^2g^2)/h^2)^{(1/2)}*x*d-$

$$\begin{aligned} & 2/h/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}* \\ & x*c*e+15/2/h*c*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h) \\ & *c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x \\ & +g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*f+5/h^2/(a*h^2+c*g^2)/a/(-2*(x \\ & +g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*c*f*g-15/2/h*c^3*g^4/(a* \\ & h^2+c*g^2)^3/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*e+1 \\ & 5/2/h^2*c^3*g^5/(a*h^2+c*g^2)^3/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^ \\ & 2)/h^2)^{(1/2)}*x*f+19/2/h*c^2*g^2/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h) \\ & )^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*e-25/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/a/(-2*( \\ & x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*x*f-1/h/(a*h^2+c*g^2)/(x+ \\ & g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e+9/2*c*g/(a*h^ \\ & 2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e+2/h^2/( \\ & a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)} \\ & *f*g-15/2/h*c*g^2/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^ \\ & 2)/h^2)^{(1/2)}*f-9/2*c*g/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x \\ & +g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g \\ & /h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*e+1/2/h^2/(a*h^2+c*g^2)/( \\ & x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e*g-1/2/h^3 \\ & /(a*h^2+c*g^2)/(x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{( \\ & 1/2)}*f*g^2-5/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a \\ & *h^2+c*g^2)/h^2)^{(1/2)}*d+15/2*h*c^2*g^2/(a*h^2+c*g^2)^3/(-2*(x+g/h)*c*g/h+( \\ & x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*d+15/2/h*c^2*g^4/(a*h^2+c*g^2)^3/(-2*(x \\ & +g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*f+15/2*c^2*g^3/(a*h^2+c*g^ \\ & 2)^3/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*( \\ & (a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{( \\ & 1/2)})/(x+g/h))*e+3/2*h*c/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*( \\ & x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c* \\ & g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+g/h))*d-f/h/(a*h^2+c*g^2)/((a* \\ & h^2+c*g^2)/h^2)^{(1/2)}*\ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c* \\ & g^2)/h^2)^{(1/2)}*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)})/(x+ \\ & g/h))-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c* \\ & g^2)/h^2)^{(1/2)}*d-15/2*c^2*g^3/(a*h^2+c*g^2)^3/(-2*(x+g/h)*c*g/h+(x+g/h)^2* \\ & c+(a*h^2+c*g^2)/h^2)^{(1/2)}*e-3/2*h*c/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g \\ & /h)^2*c+(a*h^2+c*g^2)/h^2)^{(1/2)}*d \end{aligned}$$

**maxima [B]** time = 1.02, size = 2254, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+a)^(3/2),x, algorithm="maxima")

[Out]  $15/2*c^3*f*g^5*x/(\text{sqrt}(c*x^2 + a))*a*c^3*g^6*h^2 + 3*\text{sqrt}(c*x^2 + a)*a^2*c^2$   
 $*g^4*h^4 + 3*\text{sqrt}(c*x^2 + a)*a^3*c*g^2*h^6 + \text{sqrt}(c*x^2 + a)*a^4*h^8) - 15/$   
 $2*c^3*e*g^4*x/(\text{sqrt}(c*x^2 + a))*a*c^3*g^6*h + 3*\text{sqrt}(c*x^2 + a)*a^2*c^2*g^4*$   
 $h^3 + 3*\text{sqrt}(c*x^2 + a)*a^3*c*g^2*h^5 + \text{sqrt}(c*x^2 + a)*a^4*h^7) + 15/2*c^3$

$$\begin{aligned}
& *d*g^3*x/(\sqrt{c*x^2 + a})*a*c^3*g^6 + 3*\sqrt{c*x^2 + a}*a^2*c^2*g^4*h^2 + 3 \\
& *\sqrt{c*x^2 + a}*a^3*c*g^2*h^4 + \sqrt{c*x^2 + a}*a^4*h^6) + 15/2*c^2*f*g^4/ \\
& (\sqrt{c*x^2 + a}*c^3*g^6*h + 3*\sqrt{c*x^2 + a}*a*c^2*g^4*h^3 + 3*\sqrt{c*x^2 \\
& + a}*a^2*c*g^2*h^5 + \sqrt{c*x^2 + a}*a^3*h^7) - 25/2*c^2*f*g^3*x/(\sqrt{c*x \\
& ^2 + a}*a*c^2*g^4*h^2 + 2*\sqrt{c*x^2 + a}*a^2*c*g^2*h^4 + \sqrt{c*x^2 + a}*a \\
& ^3*h^6) - 15/2*c^2*e*g^3/(\sqrt{c*x^2 + a}*c^3*g^6 + 3*\sqrt{c*x^2 + a}*a*c^2 \\
& *g^4*h^2 + 3*\sqrt{c*x^2 + a}*a^2*c*g^2*h^4 + \sqrt{c*x^2 + a}*a^3*h^6) + 19/ \\
& 2*c^2*e*g^2*x/(\sqrt{c*x^2 + a}*a*c^2*g^4*h + 2*\sqrt{c*x^2 + a}*a^2*c*g^2*h^ \\
& 3 + \sqrt{c*x^2 + a}*a^3*h^5) + 15/2*c^2*d*g^2/(\sqrt{c*x^2 + a}*c^3*g^6/h + \\
& 3*\sqrt{c*x^2 + a}*a*c^2*g^4*h + 3*\sqrt{c*x^2 + a}*a^2*c*g^2*h^3 + \sqrt{c*x^ \\
& 2 + a}*a^3*h^5) - 5/2*c*f*g^3/(\sqrt{c*x^2 + a}*c^2*g^4*h^2*x + 2*\sqrt{c*x^2 \\
& + a}*a*c*g^2*h^4*x + \sqrt{c*x^2 + a}*a^2*h^6*x + \sqrt{c*x^2 + a}*c^2*g^5*h \\
& + 2*\sqrt{c*x^2 + a}*a*c*g^3*h^3 + \sqrt{c*x^2 + a}*a^2*g*h^5) - 13/2*c^2*d* \\
& g*x/(\sqrt{c*x^2 + a}*a*c^2*g^4 + 2*\sqrt{c*x^2 + a}*a^2*c*g^2*h^2 + \sqrt{c*x \\
& ^2 + a}*a^3*h^4) + 5/2*c*e*g^2/(\sqrt{c*x^2 + a}*c^2*g^4*h*x + 2*\sqrt{c*x^2 \\
& + a}*a*c*g^2*h^3*x + \sqrt{c*x^2 + a}*a^2*h^5*x + \sqrt{c*x^2 + a}*c^2*g^5 + \\
& 2*\sqrt{c*x^2 + a}*a*c*g^3*h^2 + \sqrt{c*x^2 + a}*a^2*g*h^4) - 15/2*c*f*g^2/( \\
& \sqrt{c*x^2 + a}*c^2*g^4*h + 2*\sqrt{c*x^2 + a}*a*c*g^2*h^3 + \sqrt{c*x^2 + a} \\
& *a^2*h^5) + 5*c*f*g*x/(\sqrt{c*x^2 + a}*a*c*g^2*h^2 + \sqrt{c*x^2 + a}*a^2*h^ \\
& 4) - 5/2*c*d*g/(\sqrt{c*x^2 + a}*c^2*g^4*x + 2*\sqrt{c*x^2 + a}*a*c*g^2*h^2*x \\
& + \sqrt{c*x^2 + a}*a^2*h^4*x + \sqrt{c*x^2 + a}*c^2*g^5/h + 2*\sqrt{c*x^2 + a} \\
& )*a*c*g^3*h + \sqrt{c*x^2 + a}*a^2*g*h^3) + 9/2*c*e*g/(\sqrt{c*x^2 + a}*c^2*g \\
& ^4 + 2*\sqrt{c*x^2 + a}*a*c*g^2*h^2 + \sqrt{c*x^2 + a}*a^2*h^4) - 1/2*f*g^2/( \\
& \sqrt{c*x^2 + a}*c*g^2*h^3*x^2 + \sqrt{c*x^2 + a}*a*h^5*x^2 + 2*\sqrt{c*x^2 + \\
& a}*c*g^3*h^2*x + 2*\sqrt{c*x^2 + a}*a*g*h^4*x + \sqrt{c*x^2 + a}*c*g^4*h + sq \\
& rt(c*x^2 + a)*a*g^2*h^3) - 2*c*e*x/(\sqrt{c*x^2 + a}*a*c*g^2*h + \sqrt{c*x^2 \\
& + a}*a^2*h^3) - 3/2*c*d/(\sqrt{c*x^2 + a}*c^2*g^4/h + 2*\sqrt{c*x^2 + a}*a*c* \\
& g^2*h + \sqrt{c*x^2 + a}*a^2*h^3) + 1/2*e*g/(\sqrt{c*x^2 + a}*c*g^2*h^2*x^2 + \\
& \sqrt{c*x^2 + a}*a*h^4*x^2 + 2*\sqrt{c*x^2 + a}*c*g^3*h*x + 2*\sqrt{c*x^2 + a} \\
& )*a*g*h^3*x + \sqrt{c*x^2 + a}*c*g^4 + \sqrt{c*x^2 + a}*a*g^2*h^2) + 2*f*g/(s \\
& qrt(c*x^2 + a)*c*g^2*h^2*x + \sqrt{c*x^2 + a}*a*h^4*x + \sqrt{c*x^2 + a}*c*g^ \\
& 3*h + \sqrt{c*x^2 + a}*a*g*h^3) - 1/2*d/(\sqrt{c*x^2 + a}*c*g^2*h*x^2 + \sqrt{c \\
& x^2 + a}*a*h^3*x^2 + 2*\sqrt{c*x^2 + a}*c*g^3*x + 2*\sqrt{c*x^2 + a}*a*g*h^ \\
& 2*x + \sqrt{c*x^2 + a}*c*g^4/h + \sqrt{c*x^2 + a}*a*g^2*h) - e/(\sqrt{c*x^2 + \\
& a}*c*g^2*h*x + \sqrt{c*x^2 + a}*a*h^3*x + \sqrt{c*x^2 + a}*c*g^3 + \sqrt{c*x^2 \\
& + a}*a*g*h^2) + f/(\sqrt{c*x^2 + a}*c*g^2*h + \sqrt{c*x^2 + a}*a*h^3) + 15/2 \\
& *c^2*f*g^4*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x \\
& + g)))/((a + c*g^2/h^2)^(7/2)*h^7) - 15/2*c^2*e*g^3*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c} \\
& )*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^6) \\
& + 15/2*c^2*d*g^2*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*a \\
& bs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^5) - 15/2*c*f*g^2*\operatorname{arcsinh}(c*g*x/(sq \\
& rt(a*c)*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)))/((a + c*g^2/h^2)^(5/2) \\
& *h^5) + 9/2*c*e*g*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c})*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*a \\
& bs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) - 3/2*c*d*\operatorname{arcsinh}(c*g*x/(\sqrt{a*c} \\
& )*\operatorname{abs}(h*x + g)) - a*h/(\sqrt{a*c})*\operatorname{abs}(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^3)
\end{aligned}$$

+ f\*arcsinh(c\*g\*x/(sqrt(a\*c)\*abs(h\*x + g)) - a\*h/(sqrt(a\*c)\*abs(h\*x + g)))  
/((a + c\*g^2/h^2)^(3/2)\*h^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^3 (c x^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(3/2)), x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + c\*x^2)^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*3/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] Timed out

$$3.115 \quad \int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}} dx$$

Optimal. Leaf size=67

$$\frac{x(aC + 2Ac)}{3a^2c\sqrt{a + cx^2}} - \frac{aB - x(AC - aC)}{3ac(a + cx^2)^{3/2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1814, 12, 191}

$$\frac{x(aC + 2Ac)}{3a^2c\sqrt{a + cx^2}} - \frac{aB - x(AC - aC)}{3ac(a + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2)^(5/2),x]

[Out] -(a\*B - (A\*c - a\*C)\*x)/(3\*a\*c\*(a + c\*x^2)^(3/2)) + ((2\*A\*c + a\*C)\*x)/(3\*a^2\*c\*Sqrt[a + c\*x^2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2}{(a + cx^2)^{5/2}} dx &= -\frac{aB - (Ac - aC)x}{3ac(a + cx^2)^{3/2}} - \frac{\int \frac{-2A - \frac{aC}{c}}{(a+cx^2)^{3/2}} dx}{3a} \\
&= -\frac{aB - (Ac - aC)x}{3ac(a + cx^2)^{3/2}} + \frac{(2Ac + aC) \int \frac{1}{(a+cx^2)^{3/2}} dx}{3ac} \\
&= -\frac{aB - (Ac - aC)x}{3ac(a + cx^2)^{3/2}} + \frac{(2Ac + aC)x}{3a^2c\sqrt{a + cx^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 50, normalized size = 0.75

$$\frac{-a^2B + acx(3A + Cx^2) + 2Ac^2x^3}{3a^2c(a + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2)^(5/2), x]

[Out]  $(-(a^2*B) + 2*A*c^2*x^3 + a*c*x*(3*A + C*x^2))/(3*a^2*c*(a + c*x^2)^(3/2))$

**IntegrateAlgebraic [A]** time = 0.50, size = 50, normalized size = 0.75

$$\frac{-a^2B + 3aAcx + acCx^3 + 2Ac^2x^3}{3a^2c(a + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^(5/2), x]

[Out]  $(-(a^2*B) + 3*a*A*c*x + 2*A*c^2*x^3 + a*c*C*x^3)/(3*a^2*c*(a + c*x^2)^(3/2))$

**fricas [A]** time = 0.75, size = 68, normalized size = 1.01

$$\frac{(3Aacx + (Cac + 2Ac^2)x^3 - Ba^2)\sqrt{cx^2 + a}}{3(a^2c^3x^4 + 2a^3c^2x^2 + a^4c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{3}*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*\text{sqrt}(c*x^2 + a)/(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c)$

**giac** [A] time = 0.21, size = 48, normalized size = 0.72

$$\frac{x\left(\frac{3A}{a} + \frac{(Cac+2Ac^2)x^2}{a^2c}\right) - \frac{B}{c}}{3\left(cx^2 + a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(5/2),x, algorithm="giac")

[Out]  $\frac{1}{3}*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^{(3/2)}$

**maple** [A] time = 0.00, size = 47, normalized size = 0.70

$$\frac{2Ac^2x^3 + Cax^3 + 3Axac - Ba^2}{3\left(cx^2 + a\right)^{\frac{3}{2}}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(c\*x^2+a)^(5/2),x)

[Out]  $\frac{1}{3}*(2*A*c^2*x^3+C*a*c*x^3+3*A*a*c*x-B*a^2)/(c*x^2+a)^{(3/2)}/a^2/c$

**maxima** [A] time = 0.43, size = 83, normalized size = 1.24

$$\frac{2Ax}{3\sqrt{cx^2 + a}a^2} + \frac{Ax}{3\left(cx^2 + a\right)^{\frac{3}{2}}a} - \frac{Cx}{3\left(cx^2 + a\right)^{\frac{3}{2}}c} + \frac{Cx}{3\sqrt{cx^2 + a}ac} - \frac{B}{3\left(cx^2 + a\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(5/2),x, algorithm="maxima")

[Out]  $\frac{2}{3}A*x/(\text{sqrt}(c*x^2 + a)*a^2) + \frac{1}{3}A*x/((c*x^2 + a)^{(3/2)}*a) - \frac{1}{3}C*x/((c*x^2 + a)^{(3/2)}*c) + \frac{1}{3}C*x/(\text{sqrt}(c*x^2 + a)*a*c) - \frac{1}{3}B/((c*x^2 + a)^{(3/2)}*c)$

**mupad** [B] time = 4.22, size = 59, normalized size = 0.88

$$\frac{2Acx\left(cx^2 + a\right) - Ca^2x - Ba^2 + Cax\left(cx^2 + a\right) + Aacx}{3a^2c\left(cx^2 + a\right)^{3/2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*x + C*x^2)/(a + c*x^2)^(5/2), x)`

[Out]  $(2*A*c*x*(a + c*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + c*x^2) + A*a*c*x)/(3*a^2*c*(a + c*x^2)^(3/2))$

**sympy** [A] time = 17.13, size = 194, normalized size = 2.90

$$A \left( \frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}} + 3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}} + \frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}} + 3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}} \right) + B \left( \begin{cases} -\frac{1}{3ac\sqrt{a+cx^2} + 3c^2x^2\sqrt{a+cx^2}} & \text{for } c \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} \right) + \frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{cx^2}{a}} + 3a^{\frac{3}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2), x)`

[Out]  $A*(3*a*x/(3*a**(7/2)*\sqrt{1 + c*x**2/a}) + 3*a**(5/2)*c*x**2*\sqrt{1 + c*x**2/a}) + 2*c*x**3/(3*a**(7/2)*\sqrt{1 + c*x**2/a}) + 3*a**(5/2)*c*x**2*\sqrt{1 + c*x**2/a})) + B*\text{Piecewise}((-1/(3*a*c*\sqrt{a + c*x**2}) + 3*c**2*x**2*\sqrt{a + c*x**2}), \text{Ne}(c, 0)), (x**2/(2*a**(5/2)), \text{True})) + C*x**3/(3*a**(5/2)*\sqrt{1 + c*x**2/a}) + 3*a**(3/2)*c*x**2*\sqrt{1 + c*x**2/a})$

$$3.116 \quad \int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}} dx$$

Optimal. Leaf size=97

$$\frac{2x(aC + 4Ac)}{15a^3c\sqrt{a + cx^2}} + \frac{x(aC + 4Ac)}{15a^2c(a + cx^2)^{3/2}} - \frac{aB - x(AC - aC)}{5ac(a + cx^2)^{5/2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1814, 12, 192, 191}

$$\frac{2x(aC + 4Ac)}{15a^3c\sqrt{a + cx^2}} + \frac{x(aC + 4Ac)}{15a^2c(a + cx^2)^{3/2}} - \frac{aB - x(AC - aC)}{5ac(a + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2)^(7/2), x]

[Out] -(a\*B - (A\*c - a\*C)\*x)/(5\*a\*c\*(a + c\*x^2)^(5/2)) + ((4\*A\*c + a\*C)\*x)/(15\*a^2\*c\*(a + c\*x^2)^(3/2)) + (2\*(4\*A\*c + a\*C)\*x)/(15\*a^3\*c\*sqrt[a + c\*x^2])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[(a\*g -

$b*f*x*(a + b*x^2)^{(p + 1)}/(2*a*b*(p + 1)), x] + \text{Dist}[1/(2*a*(p + 1)), \text{Int}[(a + b*x^2)^{(p + 1)}*\text{ExpandToSum}[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /$   
 $;$  FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{A + Bx + Cx^2}{(a + cx^2)^{7/2}} dx &= -\frac{aB - (Ac - aC)x}{5ac(a + cx^2)^{5/2}} - \frac{\int \frac{-4A - \frac{aC}{c}}{(a+cx^2)^{5/2}} dx}{5a} \\ &= -\frac{aB - (Ac - aC)x}{5ac(a + cx^2)^{5/2}} + \frac{(4Ac + aC) \int \frac{1}{(a+cx^2)^{5/2}} dx}{5ac} \\ &= -\frac{aB - (Ac - aC)x}{5ac(a + cx^2)^{5/2}} + \frac{(4Ac + aC)x}{15a^2c(a + cx^2)^{3/2}} + \frac{(2(4Ac + aC)) \int \frac{1}{(a+cx^2)^{3/2}} dx}{15a^2c} \\ &= -\frac{aB - (Ac - aC)x}{5ac(a + cx^2)^{5/2}} + \frac{(4Ac + aC)x}{15a^2c(a + cx^2)^{3/2}} + \frac{2(4Ac + aC)x}{15a^3c\sqrt{a + cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 71, normalized size = 0.73

$$\frac{-3a^3B + 5a^2cx(3A + Cx^2) + 2ac^2x^3(10A + Cx^2) + 8Ac^3x^5}{15a^3c(a + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2)^(7/2), x]

[Out]  $(-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*x^2))/(15*a^3*c*(a + c*x^2)^{(5/2)})$

**IntegrateAlgebraic [A]** time = 0.60, size = 75, normalized size = 0.77

$$\frac{-3a^3B + 15a^2Acx + 5a^2cCx^3 + 20aAc^2x^3 + 2ac^2Cx^5 + 8Ac^3x^5}{15a^3c(a + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^(7/2), x]

[Out]  $(-3a^3B + 15a^2Acx + 20aAc^2x^3 + 5a^2cCx^3 + 8Aa^3x^5 + 2a^2c^2Cx^5)/(15a^3c(a + cx^2)^{5/2})$

**fricas** [A] time = 0.80, size = 103, normalized size = 1.06

$$\frac{(2(Cac^2 + 4Ac^3)x^5 + 15Aa^2cx - 3Ba^3 + 5(Ca^2c + 4Aac^2)x^3)\sqrt{cx^2 + a}}{15(a^3c^4x^6 + 3a^4c^3x^4 + 3a^5c^2x^2 + a^6c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(7/2),x, algorithm="fricas")

[Out]  $1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*A*a*c^2)*x^3)*\text{sqrt}(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6*c)$

**giac** [A] time = 0.26, size = 80, normalized size = 0.82

$$\frac{\left(x^2\left(\frac{2(Cac^3+4Ac^4)x^2}{a^3c^2} + \frac{5(Ca^2c^2+4Aac^3)}{a^3c^2}\right) + \frac{15A}{a}\right)x - \frac{3B}{c}}{15(cx^2 + a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(7/2),x, algorithm="giac")

[Out]  $1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3)/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^{5/2}$

**maple** [A] time = 0.00, size = 72, normalized size = 0.74

$$\frac{8A c^3 x^5 + 2C a c^2 x^5 + 20A a c^2 x^3 + 5C a^2 c x^3 + 15A x a^2 c - 3B a^3}{15(c x^2 + a)^{\frac{5}{2}} a^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(c\*x^2+a)^(7/2),x)

[Out]  $1/15*(8*A*c^3*x^5+2*C*a*c^2*x^5+20*A*a*c^2*x^3+5*C*a^2*c*x^3+15*A*a^2*c*x-3*B*a^3)/(c*x^2+a)^{5/2}/a^3/c$

**maxima** [A] time = 0.44, size = 118, normalized size = 1.22

$$\frac{8Ax}{15\sqrt{cx^2 + a}a^3} + \frac{4Ax}{15(cx^2 + a)^{\frac{3}{2}}a^2} + \frac{Ax}{5(cx^2 + a)^{\frac{5}{2}}a} - \frac{Cx}{5(cx^2 + a)^{\frac{5}{2}}c} + \frac{2Cx}{15\sqrt{cx^2 + a}a^2c} + \frac{Cx}{15(cx^2 + a)^{\frac{3}{2}}ac} - \frac{B}{5(cx^2 + a)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(7/2),x, algorithm="maxima")

[Out]  $\frac{8}{15}Ax/\sqrt{cx^2+a}a^3 + \frac{4}{15}Ax/((cx^2+a)^{3/2})a^2 + \frac{1}{5}Ax/((cx^2+a)^{5/2})a - \frac{1}{5}Cx/((cx^2+a)^{5/2})c + \frac{2}{15}Cx/\sqrt{cx^2+a}a^2c + \frac{1}{15}Cx/((cx^2+a)^{3/2})a*c - \frac{1}{5}B/((cx^2+a)^{5/2})c$

**mupad [B]** time = 4.28, size = 93, normalized size = 0.96

$$\frac{8Acx(cx^2+a)^2 - 3Ca^3x - 3Ba^3 + 2Cax(cx^2+a)^2 + Ca^2x(cx^2+a) + 3Aa^2cx + 4Aacx(cx^2+a)}{15a^3c(cx^2+a)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/(a + c\*x^2)^(7/2),x)

[Out]  $\frac{(8A*c*x*(a + c*x^2)^2 - 3C*a^3*x - 3B*a^3 + 2C*a*x*(a + c*x^2)^2 + C*a^2*x*(a + c*x^2) + 3*A*a^2*c*x + 4*A*a*c*x*(a + c*x^2))/(15*a^3*c*(a + c*x^2)^{5/2}}$

**sympy [B]** time = 37.22, size = 638, normalized size = 6.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*(7/2),x)

[Out]  $A*(15*a**5*x/(15*a**(17/2)*\sqrt{1 + c*x**2/a} + 45*a**(15/2)*c*x**2*\sqrt{1 + c*x**2/a} + 45*a**(13/2)*c**2*x**4*\sqrt{1 + c*x**2/a} + 15*a**(11/2)*c**3*x**6*\sqrt{1 + c*x**2/a}) + 35*a**4*c*x**3/(15*a**(17/2)*\sqrt{1 + c*x**2/a} + 45*a**(15/2)*c*x**2*\sqrt{1 + c*x**2/a} + 45*a**(13/2)*c**2*x**4*\sqrt{1 + c*x**2/a} + 15*a**(11/2)*c**3*x**6*\sqrt{1 + c*x**2/a}) + 28*a**3*c**2*x**5/(15*a**(17/2)*\sqrt{1 + c*x**2/a} + 45*a**(15/2)*c*x**2*\sqrt{1 + c*x**2/a} + 45*a**(13/2)*c**2*x**4*\sqrt{1 + c*x**2/a} + 15*a**(11/2)*c**3*x**6*\sqrt{1 + c*x**2/a}) + 8*a**2*c**3*x**7/(15*a**(17/2)*\sqrt{1 + c*x**2/a} + 45*a**(15/2)*c*x**2*\sqrt{1 + c*x**2/a} + 45*a**(13/2)*c**2*x**4*\sqrt{1 + c*x**2/a} + 15*a**(11/2)*c**3*x**6*\sqrt{1 + c*x**2/a})) + B*\text{Piecewise}((-1/(5*a**2*c*\sqrt{a + c*x**2}) + 10*a*c**2*x**2*\sqrt{a + c*x**2}) + 5*c**3*x**4*\sqrt{a + c*x**2}), \text{Ne}(c, 0)), (x**2/(2*a**(7/2)), \text{True})) + C*(5*a*x**3/(15*a**(9/2)*\sqrt{1 + c*x**2/a} + 30*a**(7/2)*c*x**2*\sqrt{1 + c*x**2/a} + 15*a**(5/2)*c**2*x**4*\sqrt{1 + c*x**2/a}) + 2*c*x**5/(15*a**(9/2)*\sqrt{1 + c*x**2/a} + 30*a**(7/2)*c*x**2*\sqrt{1 + c*x**2/a} + 15*a**(5/2)*c**2*x**4*\sqrt{1 + c*x**2/a}))$

$$3.117 \quad \int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}} dx$$

**Optimal.** Leaf size=127

$$\frac{8x(aC + 6Ac)}{105a^4c\sqrt{a + cx^2}} + \frac{4x(aC + 6Ac)}{105a^3c(a + cx^2)^{3/2}} + \frac{x(aC + 6Ac)}{35a^2c(a + cx^2)^{5/2}} - \frac{aB - x(AC - aC)}{7ac(a + cx^2)^{7/2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC + 6Ac)}{105a^4c\sqrt{a + cx^2}} + \frac{4x(aC + 6Ac)}{105a^3c(a + cx^2)^{3/2}} + \frac{x(aC + 6Ac)}{35a^2c(a + cx^2)^{5/2}} - \frac{aB - x(AC - aC)}{7ac(a + cx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*x + C\*x^2)/(a + c\*x^2)^(9/2), x]

[Out] -(a\*B - (A\*c - a\*C)\*x)/(7\*a\*c\*(a + c\*x^2)^(7/2)) + ((6\*A\*c + a\*C)\*x)/(35\*a^2\*c\*(a + c\*x^2)^(5/2)) + (4\*(6\*A\*c + a\*C)\*x)/(105\*a^3\*c\*(a + c\*x^2)^(3/2)) + (8\*(6\*A\*c + a\*C)\*x)/(105\*a^4\*c\*Sqrt[a + c\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x,

0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1], Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Bx + Cx^2}{(a + cx^2)^{9/2}} dx &= -\frac{aB - (Ac - aC)x}{7ac(a + cx^2)^{7/2}} - \frac{\int \frac{-6A - \frac{aC}{c}}{(a+cx^2)^{7/2}} dx}{7a} \\
 &= -\frac{aB - (Ac - aC)x}{7ac(a + cx^2)^{7/2}} + \frac{(6Ac + aC) \int \frac{1}{(a+cx^2)^{7/2}} dx}{7ac} \\
 &= -\frac{aB - (Ac - aC)x}{7ac(a + cx^2)^{7/2}} + \frac{(6Ac + aC)x}{35a^2c(a + cx^2)^{5/2}} + \frac{(4(6Ac + aC)) \int \frac{1}{(a+cx^2)^{5/2}} dx}{35a^2c} \\
 &= -\frac{aB - (Ac - aC)x}{7ac(a + cx^2)^{7/2}} + \frac{(6Ac + aC)x}{35a^2c(a + cx^2)^{5/2}} + \frac{4(6Ac + aC)x}{105a^3c(a + cx^2)^{3/2}} + \frac{(8(6Ac + aC)) \int \frac{1}{(a+cx^2)} dx}{105a^3c} \\
 &= -\frac{aB - (Ac - aC)x}{7ac(a + cx^2)^{7/2}} + \frac{(6Ac + aC)x}{35a^2c(a + cx^2)^{5/2}} + \frac{4(6Ac + aC)x}{105a^3c(a + cx^2)^{3/2}} + \frac{8(6Ac + aC)x}{105a^4c\sqrt{a + cx^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 92, normalized size = 0.72

$$\frac{-15a^4B + 35a^3cx(3A + Cx^2) + 14a^2c^2x^3(15A + 2Cx^2) + 8ac^3x^5(21A + Cx^2) + 48Ac^4x^7}{105a^4c(a + cx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*x + C\*x^2)/(a + c\*x^2)^(9/2), x]

[Out] (-15\*a^4\*B + 48\*A\*c^4\*x^7 + 35\*a^3\*c\*x\*(3\*A + C\*x^2) + 8\*a\*c^3\*x^5\*(21\*A + C\*x^2) + 14\*a^2\*c^2\*x^3\*(15\*A + 2\*C\*x^2))/(105\*a^4\*c\*(a + c\*x^2)^(7/2))

**IntegrateAlgebraic [A]** time = 0.74, size = 99, normalized size = 0.78

$$\frac{-15a^4B + 105a^3Acx + 35a^3cCx^3 + 210a^2Ac^2x^3 + 28a^2c^2Cx^5 + 168aAc^3x^5 + 8ac^3Cx^7 + 48Ac^4x^7}{105a^4c(a + cx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + B\*x + C\*x^2)/(a + c\*x^2)^(9/2), x]

[Out]  $(-15*a^4*B + 105*a^3*A*c*x + 210*a^2*A*c^2*x^3 + 35*a^3*c*C*x^3 + 168*a*A*c^3*x^5 + 28*a^2*c^2*C*x^5 + 48*A*c^4*x^7 + 8*a*c^3*C*x^7)/(105*a^4*c*(a + c*x^2)^(7/2))$

**fricas** [A] time = 0.71, size = 137, normalized size = 1.08

$$\frac{(8(Cac^3 + 6Ac^4)x^7 + 105Aa^3cx + 28(Ca^2c^2 + 6Aac^3)x^5 - 15Ba^4 + 35(Ca^3c + 6Aa^2c^2)x^3)\sqrt{cx^2 + a}}{105(a^4c^5x^8 + 4a^5c^4x^6 + 6a^6c^3x^4 + 4a^7c^2x^2 + a^8c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(9/2), x, algorithm="fricas")

[Out]  $1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c^3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*\text{sqrt}(c*x^2 + a)/(a^4*c^5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)$

**giac** [A] time = 0.27, size = 112, normalized size = 0.88

$$\frac{\left(\left(4x^2\left(\frac{2(Cac^5+6Ac^6)x^2}{a^4c^3} + \frac{7(Ca^2c^4+6Aac^5)}{a^4c^3}\right) + \frac{35(Ca^3c^3+6Aa^2c^4)}{a^4c^3}\right)x^2 + \frac{105A}{a}\right)x - \frac{15B}{c}}{105(cx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(9/2), x, algorithm="giac")

[Out]  $1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6))*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a*c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*x - 15*B/c)/(c*x^2 + a)^(7/2)$

**maple** [A] time = 0.01, size = 96, normalized size = 0.76

$$\frac{48A c^4 x^7 + 8C a c^3 x^7 + 168A a c^3 x^5 + 28C a^2 c^2 x^5 + 210A a^2 c^2 x^3 + 35C a^3 c x^3 + 105A x a^3 c - 15B a^4}{105(c x^2 + a)^{\frac{7}{2}} a^4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+B\*x+A)/(c\*x^2+a)^(9/2), x)

[Out]  $1/105*(48*A*c^4*x^7+8*C*a*c^3*x^7+168*A*a*c^3*x^5+28*C*a^2*c^2*x^5+210*A*a^2*c^2*x^3+35*C*a^3*c*x^3+105*A*a^3*c*x-15*B*a^4)/(c*x^2+a)^(7/2)/a^4/c$



**maxima [A]** time = 0.45, size = 153, normalized size = 1.20

$$\frac{16Ax}{35\sqrt{cx^2+aa^4}} + \frac{8Ax}{35(cx^2+a)^{\frac{3}{2}}a^3} + \frac{6Ax}{35(cx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7(cx^2+a)^{\frac{7}{2}}a} - \frac{Cx}{7(cx^2+a)^{\frac{7}{2}}c} + \frac{8Cx}{105\sqrt{cx^2+aa^3c}} + \frac{4Cx}{105(cx^2+a)^{\frac{3}{2}}a^2c} + \frac{Cx}{35(cx^2+a)^{\frac{5}{2}}ac} - \frac{B}{7(cx^2+a)^{\frac{7}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+B\*x+A)/(c\*x^2+a)^(9/2),x, algorithm="maxima")

[Out] 16/35\*A\*x/(sqrt(c\*x^2 + a)\*a^4) + 8/35\*A\*x/((c\*x^2 + a)^(3/2)\*a^3) + 6/35\*A\*x/((c\*x^2 + a)^(5/2)\*a^2) + 1/7\*A\*x/((c\*x^2 + a)^(7/2)\*a) - 1/7\*C\*x/((c\*x^2 + a)^(7/2)\*c) + 8/105\*C\*x/(sqrt(c\*x^2 + a)\*a^3\*c) + 4/105\*C\*x/((c\*x^2 + a)^(3/2)\*a^2\*c) + 1/35\*C\*x/((c\*x^2 + a)^(5/2)\*a\*c) - 1/7\*B/((c\*x^2 + a)^(7/2)\*c)

**mupad [B]** time = 4.37, size = 115, normalized size = 0.91

$$\frac{x(6Ac + Ca)}{35a^2c(c^2x^2 + a)^{5/2}} - \frac{\frac{B}{7c} - x\left(\frac{A}{7a} - \frac{C}{7c}\right)}{(c^2x^2 + a)^{7/2}} + \frac{x(24Ac + 4Ca)}{105a^3c(c^2x^2 + a)^{3/2}} + \frac{x(48Ac + 8Ca)}{105a^4c\sqrt{c^2x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B\*x + C\*x^2)/(a + c\*x^2)^(9/2),x)

[Out] (x\*(6\*A\*c + C\*a))/(35\*a^2\*c\*(a + c\*x^2)^(5/2)) - (B/(7\*c) - x\*(A/(7\*a) - C/(7\*c)))/(a + c\*x^2)^(7/2) + (x\*(24\*A\*c + 4\*C\*a))/(105\*a^3\*c\*(a + c\*x^2)^(3/2)) + (x\*(48\*A\*c + 8\*C\*a))/(105\*a^4\*c\*(a + c\*x^2)^(1/2))

**sympy [B]** time = 78.30, size = 1880, normalized size = 14.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+B\*x+A)/(c\*x\*\*2+a)\*\*(9/2),x)

[Out] A\*(35\*a\*\*14\*x/(35\*a\*\*(37/2)\*sqrt(1 + c\*x\*\*2/a) + 210\*a\*\*(35/2)\*c\*x\*\*2\*sqrt(1 + c\*x\*\*2/a) + 525\*a\*\*(33/2)\*c\*\*2\*x\*\*4\*sqrt(1 + c\*x\*\*2/a) + 700\*a\*\*(31/2)\*c\*\*3\*x\*\*6\*sqrt(1 + c\*x\*\*2/a) + 525\*a\*\*(29/2)\*c\*\*4\*x\*\*8\*sqrt(1 + c\*x\*\*2/a) + 210\*a\*\*(27/2)\*c\*\*5\*x\*\*10\*sqrt(1 + c\*x\*\*2/a) + 35\*a\*\*(25/2)\*c\*\*6\*x\*\*12\*sqrt(1 + c\*x\*\*2/a)) + 175\*a\*\*13\*c\*x\*\*3/(35\*a\*\*(37/2)\*sqrt(1 + c\*x\*\*2/a) + 210\*a\*\*(35/2)\*c\*x\*\*2\*sqrt(1 + c\*x\*\*2/a) + 525\*a\*\*(33/2)\*c\*\*2\*x\*\*4\*sqrt(1 + c\*x\*\*2/a) + 700\*a\*\*(31/2)\*c\*\*3\*x\*\*6\*sqrt(1 + c\*x\*\*2/a) + 525\*a\*\*(29/2)\*c\*\*4\*x\*\*8\*sqrt(1 + c\*x\*\*2/a) + 210\*a\*\*(27/2)\*c\*\*5\*x\*\*10\*sqrt(1 + c\*x\*\*2/a) + 35\*a\*\*(25/2)\*c\*\*6\*x\*\*12\*sqrt(1 + c\*x\*\*2/a)) + 371\*a\*\*12\*c\*\*2\*x\*\*5/(35\*a\*\*(37/2)\*sqrt(1 + c\*x\*\*2/a) + 210\*a\*\*(35/2)\*c\*x\*\*2\*sqrt(1 + c\*x\*\*2/a) + 525\*a\*\*(33/2)\*c\*\*2\*x\*\*4\*sqrt(1 + c\*x\*\*2/a) + 700\*a\*\*(31/2)\*c\*\*3\*x\*\*6\*sqrt(1 + c\*x\*\*2/a) +

$$\begin{aligned}
& 525*a^{(29/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a) + 210*a^{(27/2)}*c^{*5}*x^{*10}*sqrt(1 + c*x^{*2}/a) + 35*a^{(25/2)}*c^{*6}*x^{*12}*sqrt(1 + c*x^{*2}/a) + 429*a^{*11}*c^{*3}*x^{*7}/(35*a^{(37/2)}*sqrt(1 + c*x^{*2}/a) + 210*a^{(35/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 525*a^{(33/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 700*a^{(31/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 525*a^{(29/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a) + 210*a^{(27/2)}*c^{*5}*x^{*10}*sqrt(1 + c*x^{*2}/a) + 35*a^{(25/2)}*c^{*6}*x^{*12}*sqrt(1 + c*x^{*2}/a) + 286*a^{*10}*c^{*4}*x^{*9}/(35*a^{(37/2)}*sqrt(1 + c*x^{*2}/a) + 210*a^{(35/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 525*a^{(33/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 700*a^{(31/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 525*a^{(29/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a) + 210*a^{(27/2)}*c^{*5}*x^{*10}*sqrt(1 + c*x^{*2}/a) + 35*a^{(25/2)}*c^{*6}*x^{*12}*sqrt(1 + c*x^{*2}/a) + 104*a^{*9}*c^{*5}*x^{*11}/(35*a^{(37/2)}*sqrt(1 + c*x^{*2}/a) + 210*a^{(35/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 525*a^{(33/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 700*a^{(31/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 525*a^{(29/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a) + 210*a^{(27/2)}*c^{*5}*x^{*10}*sqrt(1 + c*x^{*2}/a) + 35*a^{(25/2)}*c^{*6}*x^{*12}*sqrt(1 + c*x^{*2}/a) + 16*a^{*8}*c^{*6}*x^{*13}/(35*a^{(37/2)}*sqrt(1 + c*x^{*2}/a) + 210*a^{(35/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 525*a^{(33/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 700*a^{(31/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 525*a^{(29/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a) + 210*a^{(27/2)}*c^{*5}*x^{*10}*sqrt(1 + c*x^{*2}/a) + 35*a^{(25/2)}*c^{*6}*x^{*12}*sqrt(1 + c*x^{*2}/a) + B*Piecewise((-1/(7*a^{*3}*c*sqrt(a + c*x^{*2}) + 21*a^{*2}*c^{*2}*x^{*2}*sqrt(a + c*x^{*2}) + 21*a*c^{*3}*x^{*4}*sqrt(a + c*x^{*2}) + 7*c^{*4}*x^{*6}*sqrt(a + c*x^{*2})), Ne(c, 0)), (x^{*2}/(2*a^{(9/2)}), True)) + C*(35*a^{*5}*x^{*3}/(105*a^{(19/2)}*sqrt(1 + c*x^{*2}/a) + 420*a^{(17/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 630*a^{(15/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 420*a^{(13/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 105*a^{(11/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a)) + 63*a^{*4}*c*x^{*5}/(105*a^{(19/2)}*sqrt(1 + c*x^{*2}/a) + 420*a^{(17/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 630*a^{(15/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 420*a^{(13/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 105*a^{(11/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a)) + 36*a^{*3}*c^{*2}*x^{*7}/(105*a^{(19/2)}*sqrt(1 + c*x^{*2}/a) + 420*a^{(17/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 630*a^{(15/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 420*a^{(13/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 105*a^{(11/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a)) + 8*a^{*2}*c^{*3}*x^{*9}/(105*a^{(19/2)}*sqrt(1 + c*x^{*2}/a) + 420*a^{(17/2)}*c*x^{*2}*sqrt(1 + c*x^{*2}/a) + 630*a^{(15/2)}*c^{*2}*x^{*4}*sqrt(1 + c*x^{*2}/a) + 420*a^{(13/2)}*c^{*3}*x^{*6}*sqrt(1 + c*x^{*2}/a) + 105*a^{(11/2)}*c^{*4}*x^{*8}*sqrt(1 + c*x^{*2}/a))
\end{aligned}$$

$$3.118 \quad \int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}} dx$$

**Optimal.** Leaf size=106

$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4 + \frac{13}{60}\sqrt{3x^2+2}(2x+1)^3 - \frac{19}{540}\sqrt{3x^2+2}(2x+1)^2 - \frac{1}{810}(2073x+3937)\sqrt{3x^2+2} + \frac{5 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1654, 833, 780, 215}

$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4 + \frac{13}{60}\sqrt{3x^2+2}(2x+1)^3 - \frac{19}{540}\sqrt{3x^2+2}(2x+1)^2 - \frac{1}{810}(2073x+3937)\sqrt{3x^2+2} + \frac{5 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/Sqrt[2 + 3\*x^2], x]

[Out] (-19\*(1 + 2\*x)^2\*Sqrt[2 + 3\*x^2])/540 + (13\*(1 + 2\*x)^3\*Sqrt[2 + 3\*x^2])/60 + (2\*(1 + 2\*x)^4\*Sqrt[2 + 3\*x^2])/15 - ((3937 + 2073\*x)\*Sqrt[2 + 3\*x^2])/810 + (5\*ArcSinh[Sqrt[3/2]\*x])/(3\*Sqrt[3])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 780**

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

**Rule 833**

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^p\*Simp[c\*d\*f\*(m + 2\*p + 2) - a\*e\*g\*m + c\*(e\*f\*(m + 2\*p + 2) + d\*g\*m)\*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 1654

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}} dx &= \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{60} \int \frac{(1+2x)^3(-68+156x)}{\sqrt{2+3x^2}} dx \\
 &= \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{720} \int \frac{(-2688-228x)(1+2x)}{\sqrt{2+3x^2}} dx \\
 &= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{810} \int \frac{(-2688-228x)}{\sqrt{2+3x^2}} dx \\
 &= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{810} \int \frac{(-2688-228x)}{\sqrt{2+3x^2}} dx \\
 &= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{810} \int \frac{(-2688-228x)}{\sqrt{2+3x^2}} dx
 \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 54, normalized size = 0.51

$$\frac{1}{405} \left( \sqrt{3x^2+2} (864x^4 + 2430x^3 + 2292x^2 - 135x - 1841) + 225\sqrt{3} \sinh^{-1} \left( \sqrt{\frac{3}{2}} x \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/Sqrt[2 + 3\*x^2], x]

[Out] (Sqrt[2 + 3\*x^2]\*(-1841 - 135\*x + 2292\*x^2 + 2430\*x^3 + 864\*x^4) + 225\*Sqrt[3]\*ArcSinh[Sqrt[3/2]\*x])/405

**IntegrateAlgebraic [A]** time = 0.34, size = 66, normalized size = 0.62

$$\frac{1}{405}\sqrt{3x^2+2}(864x^4+2430x^3+2292x^2-135x-1841)-\frac{5\log(\sqrt{3x^2+2}-\sqrt{3}x)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1+2\*x)^3\*(1+3\*x+4\*x^2))/Sqrt[2+3\*x^2],x]

[Out] (Sqrt[2+3\*x^2]\*(-1841-135\*x+2292\*x^2+2430\*x^3+864\*x^4))/405-(5\*Log[-(Sqrt[3]\*x)+Sqrt[2+3\*x^2]])/(3\*Sqrt[3])

**fricas [A]** time = 0.58, size = 60, normalized size = 0.57

$$\frac{1}{405}(864x^4+2430x^3+2292x^2-135x-1841)\sqrt{3x^2+2}+\frac{5}{18}\sqrt{3}\log(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/405\*(864\*x^4+2430\*x^3+2292\*x^2-135\*x-1841)\*sqrt(3\*x^2+2)+5/18\*sqrt(3)\*log(-sqrt(3)\*sqrt(3\*x^2+2)\*x-3\*x^2-1)

**giac [A]** time = 0.23, size = 54, normalized size = 0.51

$$\frac{1}{405}(3(2(9(16x+45)x+382)x-45)x-1841)\sqrt{3x^2+2}-\frac{5}{9}\sqrt{3}\log(-\sqrt{3}x+\sqrt{3x^2+2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="giac")

[Out] 1/405\*(3\*(2\*(9\*(16\*x+45)\*x+382)\*x-45)\*x-1841)\*sqrt(3\*x^2+2)-5/9\*sqrt(3)\*log(-sqrt(3)\*x+sqrt(3\*x^2+2))

**maple [A]** time = 0.01, size = 79, normalized size = 0.75

$$\frac{32\sqrt{3x^2+2}x^4}{15}+6\sqrt{3x^2+2}x^3+\frac{764\sqrt{3x^2+2}x^2}{135}-\frac{\sqrt{3x^2+2}x}{3}+\frac{5\sqrt{3}\operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}-\frac{1841\sqrt{3x^2+2}}{405}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x)

[Out] 32/15\*x^4\*(3\*x^2+2)^(1/2)+764/135\*x^2\*(3\*x^2+2)^(1/2)-1841/405\*(3\*x^2+2)^(1/2)+6\*x^3\*(3\*x^2+2)^(1/2)-1/3\*x\*(3\*x^2+2)^(1/2)+5/9\*arcsinh(1/2\*x\*6^(1/2))\*3^(1/2)

**maxima [A]** time = 0.95, size = 78, normalized size = 0.74

$$\frac{32}{15} \sqrt{3x^2+2} x^4 + 6 \sqrt{3x^2+2} x^3 + \frac{764}{135} \sqrt{3x^2+2} x^2 - \frac{1}{3} \sqrt{3x^2+2} x + \frac{5}{9} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2} \sqrt{6} x\right) - \frac{1841}{405} \sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="maxima")

[Out] 32/15\*sqrt(3\*x^2 + 2)\*x^4 + 6\*sqrt(3\*x^2 + 2)\*x^3 + 764/135\*sqrt(3\*x^2 + 2)\*x^2 - 1/3\*sqrt(3\*x^2 + 2)\*x + 5/9\*sqrt(3)\*arcsinh(1/2\*sqrt(6)\*x) - 1841/405\*sqrt(3\*x^2 + 2)

**mupad [B]** time = 0.05, size = 45, normalized size = 0.42

$$\frac{5\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9} + \frac{\sqrt{3} \sqrt{x^2 + \frac{2}{3}} \left(\frac{32x^4}{5} + 18x^3 + \frac{764x^2}{45} - x - \frac{1841}{135}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)^3\*(3\*x + 4\*x^2 + 1))/(3\*x^2 + 2)^(1/2),x)

[Out] (5\*3^(1/2)\*asinh((6^(1/2)\*x)/2))/9 + (3^(1/2)\*(x^2 + 2/3)^(1/2)\*((764\*x^2)/45 - x + 18\*x^3 + (32\*x^4)/5 - 1841/135))/3

**sympy [A]** time = 2.20, size = 94, normalized size = 0.89

$$\frac{32x^4\sqrt{3x^2+2}}{15} + 6x^3\sqrt{3x^2+2} + \frac{764x^2\sqrt{3x^2+2}}{135} - \frac{x\sqrt{3x^2+2}}{3} - \frac{1841\sqrt{3x^2+2}}{405} + \frac{5\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*3\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2+2)\*\*(1/2),x)

[Out] 32\*x\*\*4\*sqrt(3\*x\*\*2 + 2)/15 + 6\*x\*\*3\*sqrt(3\*x\*\*2 + 2) + 764\*x\*\*2\*sqrt(3\*x\*\*2 + 2)/135 - x\*sqrt(3\*x\*\*2 + 2)/3 - 1841\*sqrt(3\*x\*\*2 + 2)/405 + 5\*sqrt(3)\*asinh(sqrt(6)\*x/2)/9

$$3.119 \quad \int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}} dx$$

**Optimal.** Leaf size=82

$$\frac{1}{6}\sqrt{3x^2+2}(2x+1)^3 + \frac{5}{18}\sqrt{3x^2+2}(2x+1)^2 - \frac{1}{27}(3x+61)\sqrt{3x^2+2} - \sqrt{3} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

**Rubi [A]** time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1654, 833, 780, 215}

$$\frac{1}{6}\sqrt{3x^2+2}(2x+1)^3 + \frac{5}{18}\sqrt{3x^2+2}(2x+1)^2 - \frac{1}{27}(3x+61)\sqrt{3x^2+2} - \sqrt{3} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/Sqrt[2 + 3\*x^2], x]

[Out] (5\*(1 + 2\*x)^2\*Sqrt[2 + 3\*x^2])/18 + ((1 + 2\*x)^3\*Sqrt[2 + 3\*x^2])/6 - ((61 + 3\*x)\*Sqrt[2 + 3\*x^2])/27 - Sqrt[3]\*ArcSinh[Sqrt[3/2]\*x]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 780

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

#### Rule 833

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^p\*Simp[c\*d\*f\*(m + 2\*p + 2) - a\*e\*g\*m + c\*(e\*f\*(m + 2\*p + 2) + d\*g\*m)\*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

Rule 1654

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}} dx &= \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} + \frac{1}{48} \int \frac{(1+2x)^2(-48+120x)}{\sqrt{2+3x^2}} dx \\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} + \frac{1}{432} \int \frac{(-1392-144x)(1+2x)}{\sqrt{2+3x^2}} dx \\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} - \frac{1}{27}(61+3x)\sqrt{2+3x^2} - 3 \int \frac{\sqrt{2+3x^2}}{\sqrt{2+3x^2}} dx \\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2+3x^2} - \frac{1}{27}(61+3x)\sqrt{2+3x^2} - \sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{3}x}{\sqrt{2+3x^2}}\right) \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 48, normalized size = 0.59

$$\frac{1}{27}\sqrt{3x^2+2}(36x^3+84x^2+54x-49) - \sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{3}x}{\sqrt{2+3x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/Sqrt[2 + 3\*x^2], x]

[Out] (Sqrt[2 + 3\*x^2]\*(-49 + 54\*x + 84\*x^2 + 36\*x^3))/27 - Sqrt[3]\*ArcSinh[Sqrt[3/2]\*x]

**IntegrateAlgebraic** [A] time = 0.30, size = 58, normalized size = 0.71

$$\sqrt{3} \log\left(\sqrt{3x^2+2} - \sqrt{3}x\right) + \frac{1}{27}\sqrt{3x^2+2}(36x^3+84x^2+54x-49)$$



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/Sqrt[2 + 3\*x^2],x]

[Out] (Sqrt[2 + 3\*x^2]\*(-49 + 54\*x + 84\*x^2 + 36\*x^3))/27 + Sqrt[3]\*Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]]

**fricas** [A] time = 0.67, size = 54, normalized size = 0.66

$$\frac{1}{27} (36x^3 + 84x^2 + 54x - 49)\sqrt{3x^2 + 2} + \frac{1}{2} \sqrt{3} \log\left(\sqrt{3}\sqrt{3x^2 + 2}x - 3x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/27\*(36\*x^3 + 84\*x^2 + 54\*x - 49)\*sqrt(3\*x^2 + 2) + 1/2\*sqrt(3)\*log(sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1)

**giac** [A] time = 0.20, size = 48, normalized size = 0.59

$$\frac{1}{27} (6(2(3x + 7)x + 9)x - 49)\sqrt{3x^2 + 2} + \sqrt{3} \log\left(-\sqrt{3}x + \sqrt{3x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="giac")

[Out] 1/27\*(6\*(2\*(3\*x + 7)\*x + 9)\*x - 49)\*sqrt(3\*x^2 + 2) + sqrt(3)\*log(-sqrt(3)\*x + sqrt(3\*x^2 + 2))

**maple** [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{4\sqrt{3x^2 + 2}x^3}{3} + \frac{28\sqrt{3x^2 + 2}x^2}{9} + 2\sqrt{3x^2 + 2}x - \sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right) - \frac{49\sqrt{3x^2 + 2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x)

[Out] 4/3\*(3\*x^2+2)^(1/2)\*x^3+2\*(3\*x^2+2)^(1/2)\*x-arcsinh(1/2\*6^(1/2)\*x)\*3^(1/2)+28/9\*(3\*x^2+2)^(1/2)\*x^2-49/27\*(3\*x^2+2)^(1/2)

**maxima** [A] time = 0.96, size = 64, normalized size = 0.78

$$\frac{4}{3} \sqrt{3x^2 + 2}x^3 + \frac{28}{9} \sqrt{3x^2 + 2}x^2 + 2\sqrt{3x^2 + 2}x - \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{49}{27} \sqrt{3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="maxima")

[Out] 4/3\*sqrt(3\*x^2 + 2)\*x^3 + 28/9\*sqrt(3\*x^2 + 2)\*x^2 + 2\*sqrt(3\*x^2 + 2)\*x - sqrt(3)\*arcsinh(1/2\*sqrt(6)\*x) - 49/27\*sqrt(3\*x^2 + 2)

**mupad [B]** time = 4.10, size = 40, normalized size = 0.49

$$\frac{\sqrt{3} \sqrt{x^2 + \frac{2}{3}} \left(4x^3 + \frac{28x^2}{3} + 6x - \frac{49}{9}\right)}{3} - \sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)^2\*(3\*x + 4\*x^2 + 1))/(3\*x^2 + 2)^(1/2),x)

[Out] (3^(1/2)\*(x^2 + 2/3)^(1/2)\*(6\*x + (28\*x^2)/3 + 4\*x^3 - 49/9))/3 - 3^(1/2)\*a sinh((6^(1/2)\*x)/2)

**sympy [A]** time = 1.19, size = 75, normalized size = 0.91

$$\frac{4x^3\sqrt{3x^2+2}}{3} + \frac{28x^2\sqrt{3x^2+2}}{9} + 2x\sqrt{3x^2+2} - \frac{49\sqrt{3x^2+2}}{27} - \sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*2\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2+2)\*\*(1/2),x)

[Out] 4\*x\*\*3\*sqrt(3\*x\*\*2 + 2)/3 + 28\*x\*\*2\*sqrt(3\*x\*\*2 + 2)/9 + 2\*x\*sqrt(3\*x\*\*2 + 2) - 49\*sqrt(3\*x\*\*2 + 2)/27 - sqrt(3)\*asinh(sqrt(6)\*x/2)

$$3.120 \quad \int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}} dx$$

**Optimal.** Leaf size=62

$$\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2 + \frac{7}{27}(3x+1)\sqrt{3x^2+2} - \frac{7 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1654, 780, 215}

$$\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2 + \frac{7}{27}(3x+1)\sqrt{3x^2+2} - \frac{7 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/Sqrt[2 + 3\*x^2], x]

[Out] (2\*(1 + 2\*x)^2\*Sqrt[2 + 3\*x^2])/9 + (7\*(1 + 3\*x)\*Sqrt[2 + 3\*x^2])/27 - (7\*ArcSinh[Sqrt[3/2]\*x])/(3\*Sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 780

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(((e\*f + d\*g)\*(2\*p + 3) + 2\*e\*g\*(p + 1)\*x)\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)\*(2\*p + 3)), x] - Dist[(a\*e\*g - c\*d\*f\*(2\*p + 3))/(c\*(2\*p + 3)), Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

#### Rule 1654

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_)^(m\_.))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - 2\*c\*d\*e\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, c, d,

```
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && True) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}} dx &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2} + \frac{1}{36} \int \frac{(1+2x)(-28+84x)}{\sqrt{2+3x^2}} dx \\ &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2} + \frac{7}{27}(1+3x)\sqrt{2+3x^2} - \frac{7}{3} \int \frac{1}{\sqrt{2+3x^2}} dx \\ &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2} + \frac{7}{27}(1+3x)\sqrt{2+3x^2} - \frac{7 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 44, normalized size = 0.71

$$\frac{1}{27} \left( \sqrt{3x^2+2} (24x^2+45x+13) - 21\sqrt{3} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1+2*x)*(1+3*x+4*x^2))/Sqrt[2+3*x^2],x]
```

```
[Out] (Sqrt[2+3*x^2]*(13+45*x+24*x^2)-21*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/27
```

**IntegrateAlgebraic [A]** time = 0.22, size = 56, normalized size = 0.90

$$\frac{1}{27} \sqrt{3x^2+2} (24x^2+45x+13) + \frac{7 \log(\sqrt{3x^2+2} - \sqrt{3}x)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1+2*x)*(1+3*x+4*x^2))/Sqrt[2+3*x^2],x]
```

```
[Out] (Sqrt[2+3*x^2]*(13+45*x+24*x^2))/27 + (7*Log[-(Sqrt[3]*x) + Sqrt[2+3*x^2]])/(3*Sqrt[3])
```

**fricas [A]** time = 0.72, size = 49, normalized size = 0.79

$$\frac{1}{27} (24x^2+45x+13)\sqrt{3x^2+2} + \frac{7}{18} \sqrt{3} \log\left(\sqrt{3}\sqrt{3x^2+2}x - 3x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/27\*(24\*x^2 + 45\*x + 13)\*sqrt(3\*x^2 + 2) + 7/18\*sqrt(3)\*log(sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1)

**giac** [A] time = 0.19, size = 44, normalized size = 0.71

$$\frac{1}{27} (3(8x + 15)x + 13)\sqrt{3x^2 + 2} + \frac{7}{9} \sqrt{3} \log\left(-\sqrt{3}x + \sqrt{3x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="giac")

[Out] 1/27\*(3\*(8\*x + 15)\*x + 13)\*sqrt(3\*x^2 + 2) + 7/9\*sqrt(3)\*log(-sqrt(3)\*x + sqrt(3\*x^2 + 2))

**maple** [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{8\sqrt{3x^2 + 2} x^2}{9} + \frac{5\sqrt{3x^2 + 2} x}{3} - \frac{7\sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{6} x}{2}\right)}{9} + \frac{13\sqrt{3x^2 + 2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x)

[Out] 8/9\*(3\*x^2+2)^(1/2)\*x^2+13/27\*(3\*x^2+2)^(1/2)+5/3\*(3\*x^2+2)^(1/2)\*x-7/9\*arcsinh(1/2\*sqrt(6)\*x)\*sqrt(3)

**maxima** [A] time = 0.96, size = 50, normalized size = 0.81

$$\frac{8}{9} \sqrt{3x^2 + 2} x^2 + \frac{5}{3} \sqrt{3x^2 + 2} x - \frac{7}{9} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2} \sqrt{6} x\right) + \frac{13}{27} \sqrt{3x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(1/2),x, algorithm="maxima")

[Out] 8/9\*sqrt(3\*x^2 + 2)\*x^2 + 5/3\*sqrt(3\*x^2 + 2)\*x - 7/9\*sqrt(3)\*arcsinh(1/2\*sqrt(6)\*x) + 13/27\*sqrt(3\*x^2 + 2)

**mupad** [B] time = 0.03, size = 35, normalized size = 0.56

$$\frac{\sqrt{3} \sqrt{x^2 + \frac{2}{3}} \left(\frac{8x^2}{3} + 5x + \frac{13}{9}\right)}{3} - \frac{7\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6} x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2), x)`

[Out]  $(3^{1/2}*(x^2 + 2/3)^{1/2}*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^{1/2}*asinh((6^{1/2}*x)/2))/9$

sympy [A] time = 0.55, size = 63, normalized size = 1.02

$$\frac{8x^2\sqrt{3x^2+2}}{9} + \frac{5x\sqrt{3x^2+2}}{3} + \frac{13\sqrt{3x^2+2}}{27} - \frac{7\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2), x)`

[Out]  $8*x**2*\sqrt{3*x**2 + 2}/9 + 5*x*\sqrt{3*x**2 + 2}/3 + 13*\sqrt{3*x**2 + 2}/27 - 7*\sqrt{3}*asinh(\sqrt{6}*x/2)/9$

$$3.121 \quad \int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}} dx$$

Optimal. Leaf size=67

$$\frac{2}{3}\sqrt{3x^2+2} - \frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

**Rubi [A]** time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1654, 844, 215, 725, 206}

$$\frac{2}{3}\sqrt{3x^2+2} - \frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*Sqrt[2 + 3\*x^2]), x]

[Out] (2\*Sqrt[2 + 3\*x^2])/3 + ArcSinh[Sqrt[3/2]\*x]/(2\*Sqrt[3]) - ArcTanh[(4 - 3\*x)/(Sqrt[11]\*Sqrt[2 + 3\*x^2])]/(2\*Sqrt[11])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d,

e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1654

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1 + 3x + 4x^2}{(1 + 2x)\sqrt{2 + 3x^2}} dx &= \frac{2}{3}\sqrt{2 + 3x^2} + \frac{1}{12} \int \frac{12 + 12x}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
 &= \frac{2}{3}\sqrt{2 + 3x^2} + \frac{1}{2} \int \frac{1}{\sqrt{2 + 3x^2}} dx + \frac{1}{2} \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
 &= \frac{2}{3}\sqrt{2 + 3x^2} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right) \\
 &= \frac{2}{3}\sqrt{2 + 3x^2} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{2\sqrt{11}}
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.90

$$\frac{1}{66} \left( 44\sqrt{3x^2 + 2} - 3\sqrt{11} \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{33x^2 + 22}}\right) + 11\sqrt{3} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*Sqrt[2 + 3\*x^2]), x]

[Out] (44\*Sqrt[2 + 3\*x^2] + 11\*Sqrt[3]\*ArcSinh[Sqrt[3/2]\*x] - 3\*Sqrt[11]\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/66



**IntegrateAlgebraic [A]** time = 0.39, size = 89, normalized size = 1.33

$$\frac{2}{3}\sqrt{3x^2+2} - \frac{\log(\sqrt{3x^2+2} - \sqrt{3}x)}{2\sqrt{3}} + \frac{\tanh^{-1}\left(-\frac{2\sqrt{3x^2+2}}{\sqrt{11}} + 2\sqrt{\frac{3}{11}}x + \sqrt{\frac{3}{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*Sqrt[2 + 3\*x^2]),x]

[Out] (2\*Sqrt[2 + 3\*x^2])/3 + ArcTanh[Sqrt[3/11] + 2\*Sqrt[3/11]\*x - (2\*Sqrt[2 + 3\*x^2])/Sqrt[11]]/Sqrt[11] - Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]]/(2\*Sqrt[3])

**fricas [A]** time = 0.75, size = 88, normalized size = 1.31

$$\frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+2}x - 3x^2 - 1\right) + \frac{1}{44}\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4) + 21x^2 - 12x + 19}{4x^2 + 4x + 1}\right) + \frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/12\*sqrt(3)\*log(-sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1) + 1/44\*sqrt(11)\*log(-(sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) + 21\*x^2 - 12\*x + 19)/(4\*x^2 + 4\*x + 1)) + 2/3\*sqrt(3\*x^2 + 2)

**giac [B]** time = 0.23, size = 99, normalized size = 1.48

$$-\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3x^2+2}\right) + \frac{1}{22}\sqrt{11}\log\left(-\frac{|-2\sqrt{3}x - \sqrt{11} - \sqrt{3} + 2\sqrt{3x^2+2}|}{2\sqrt{3}x - \sqrt{11} + \sqrt{3} - 2\sqrt{3x^2+2}}\right) + \frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(1/2),x, algorithm="giac")

[Out] -1/6\*sqrt(3)\*log(-sqrt(3)\*x + sqrt(3\*x^2 + 2)) + 1/22\*sqrt(11)\*log(-abs(-2\*sqrt(3)\*x - sqrt(11) - sqrt(3) + 2\*sqrt(3\*x^2 + 2))/(2\*sqrt(3)\*x - sqrt(11) + sqrt(3) - 2\*sqrt(3\*x^2 + 2))) + 2/3\*sqrt(3\*x^2 + 2)

**maple [A]** time = 0.01, size = 55, normalized size = 0.82

$$\frac{\sqrt{3}\operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{6} - \frac{\sqrt{11}\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{22} + \frac{2\sqrt{3x^2+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x)`

[Out]  $2/3*(3*x^2+2)^{(1/2)}+1/6*\operatorname{arcsinh}(1/2*6^{(1/2)}*x)*3^{(1/2)}-1/22*11^{(1/2)}*\operatorname{arctanh}(2/11*(4-3*x)*11^{(1/2)})/(12*(1/2+x)^2-12*x+5)^{(1/2)}$

**maxima** [A] time = 0.96, size = 58, normalized size = 0.87

$$\frac{1}{6}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{22}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|}-\frac{2\sqrt{6}}{3|2x+1|}\right)+\frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")`

[Out]  $1/6*\sqrt{3}*\operatorname{arcsinh}(1/2*\sqrt{6}*x)+1/22*\sqrt{11}*\operatorname{arcsinh}(1/2*\sqrt{6}*x/\operatorname{abs}(2*x+1))-2/3*\sqrt{6}/\operatorname{abs}(2*x+1)+2/3*\sqrt{3*x^2+2}$

**mupad** [B] time = 0.19, size = 61, normalized size = 0.91

$$\frac{\sqrt{11}\left(2\ln\left(x+\frac{1}{2}\right)-2\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}-\frac{4}{3}}{3}\right)\right)}{44}+\frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3}+\frac{\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)`

[Out]  $(11^{(1/2)}*(2*\log(x+1/2)-2*\log(x-(3^{(1/2)}*11^{(1/2)}*(x^2+2/3)^{(1/2)})/(3-4/3)))/44+(2*3^{(1/2)}*(x^2+2/3)^{(1/2)})/3+(3^{(1/2)}*\operatorname{asinh}((2^{(1/2)}*3^{(1/2)}*x)/2))/6$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\sqrt{3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)`

[Out] `Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)`

$$3.122 \quad \int \frac{1+3x+4x^2}{(1+2x)^2 \sqrt{2+3x^2}} dx$$

Optimal. Leaf size=71

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)} + \frac{4 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1651, 844, 215, 725, 206}

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)} + \frac{4 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*Sqrt[2 + 3\*x^2]), x]

[Out] -Sqrt[2 + 3\*x^2]/(11\*(1 + 2\*x)) + ArcSinh[Sqrt[3/2]\*x]/Sqrt[3] + (4\*ArcTanh[(4 - 3\*x)/(Sqrt[11]\*Sqrt[2 + 3\*x^2])])/(11\*Sqrt[11])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d,

e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
  With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 \sqrt{2 + 3x^2}} dx &= -\frac{\sqrt{2 + 3x^2}}{11(1 + 2x)} - \frac{1}{11} \int \frac{-7 - 22x}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
 &= -\frac{\sqrt{2 + 3x^2}}{11(1 + 2x)} - \frac{4}{11} \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx + \int \frac{1}{\sqrt{2 + 3x^2}} dx \\
 &= -\frac{\sqrt{2 + 3x^2}}{11(1 + 2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{4}{11} \operatorname{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right) \\
 &= -\frac{\sqrt{2 + 3x^2}}{11(1 + 2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{4 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{11\sqrt{11}}
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 64, normalized size = 0.90

$$-\frac{\sqrt{3x^2 + 2}}{22x + 11} + \frac{4 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{33x^2 + 22}}\right)}{11\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*Sqrt[2 + 3\*x^2]), x]

[Out] -(Sqrt[2 + 3\*x^2]/(11 + 22\*x)) + ArcSinh[Sqrt[3/2]\*x]/Sqrt[3] + (4\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/(11\*Sqrt[11])

**IntegrateAlgebraic** [A] time = 0.50, size = 97, normalized size = 1.37

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)} - \frac{\log(\sqrt{3x^2+2} - \sqrt{3}x)}{\sqrt{3}} - \frac{8 \tanh^{-1}\left(-\frac{2\sqrt{3x^2+2}}{\sqrt{11}} + 2\sqrt{\frac{3}{11}}x + \sqrt{\frac{3}{11}}\right)}{11\sqrt{11}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*sqrt(2 + 3\*x^2)),x]

[Out] -1/11\*sqrt(2 + 3\*x^2)/(1 + 2\*x) - (8\*ArcTanh[Sqrt[3/11] + 2\*Sqrt[3/11]\*x - (2\*Sqrt[2 + 3\*x^2])/Sqrt[11]])/(11\*Sqrt[11]) - Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]]/Sqrt[3]

**fricas** [A] time = 0.75, size = 106, normalized size = 1.49

$$\frac{121\sqrt{3}(2x+1)\log(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1)+12\sqrt{11}(2x+1)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)-66\sqrt{3x^2+2}}{726(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/726\*(121\*sqrt(3)\*(2\*x + 1)\*log(-sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1) + 12\*sqrt(11)\*(2\*x + 1)\*log((sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) - 21\*x^2 + 12\*x - 19)/(4\*x^2 + 4\*x + 1)) - 66\*sqrt(3\*x^2 + 2))/(2\*x + 1)

**giac** [A] time = 0.32, size = 48, normalized size = 0.68

$$\frac{1}{22}\sqrt{3}\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3}}{22\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(1/2),x, algorithm="giac")

[Out] 1/22\*sqrt(3)\*sgn(1/(2\*x + 1)) - 1/22\*sqrt(-6/(2\*x + 1) + 11/(2\*x + 1)^2 + 3)/sgn(1/(2\*x + 1))

**maple** [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{\sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{3} + \frac{4\sqrt{11} \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{121} - \frac{\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{22\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x)`

[Out] `1/3*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+4/121*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)*(3*(x+1/2)^2-3*x+5/4)^(1/2)`

**maxima** [A] time = 0.97, size = 65, normalized size = 0.92

$$\frac{1}{3} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2} \sqrt{6} x\right) - \frac{4}{121} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6} x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) - \frac{\sqrt{3x^2+2}}{11(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")`

[Out] `1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)`

**mupad** [B] time = 0.11, size = 68, normalized size = 0.96

$$\frac{\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{2} \sqrt{3} x}{2}\right)}{3} - \frac{4 \sqrt{11} \ln\left(x + \frac{1}{2}\right)}{121} + \frac{4 \sqrt{11} \ln\left(x - \frac{\sqrt{3} \sqrt{11} \sqrt{x^2 + \frac{2}{3}}}{3} - \frac{4}{3}\right)}{121} - \frac{\sqrt{3} \sqrt{x^2 + \frac{2}{3}}}{22 \left(x + \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)`

[Out] `(3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 + (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 \sqrt{3x^2 + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)`

[Out] `Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)`

$$3.123 \quad \int \frac{1+3x+4x^2}{(1+2x)^3 \sqrt{2+3x^2}} dx$$

Optimal. Leaf size=77

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)} - \frac{\sqrt{3x^2+2}}{22(2x+1)^2} - \frac{103 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

**Rubi** [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1651, 807, 725, 206}

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)} - \frac{\sqrt{3x^2+2}}{22(2x+1)^2} - \frac{103 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*Sqrt[2 + 3\*x^2]),x]

[Out] -Sqrt[2 + 3\*x^2]/(22\*(1 + 2\*x)^2) + (13\*Sqrt[2 + 3\*x^2])/(242\*(1 + 2\*x)) - (103\*ArcTanh[(4 - 3\*x)/(Sqrt[11]\*Sqrt[2 + 3\*x^2])])/(121\*Sqrt[11])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 807

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

#### Rule 1651

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
  With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
    d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
    d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
    *(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
    R*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
  && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
 \int \frac{1 + 3x + 4x^2}{(1 + 2x)^3 \sqrt{2 + 3x^2}} dx &= -\frac{\sqrt{2 + 3x^2}}{22(1 + 2x)^2} - \frac{1}{22} \int \frac{-14 - 41x}{(1 + 2x)^2 \sqrt{2 + 3x^2}} dx \\
 &= -\frac{\sqrt{2 + 3x^2}}{22(1 + 2x)^2} + \frac{13\sqrt{2 + 3x^2}}{242(1 + 2x)} + \frac{103}{121} \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
 &= -\frac{\sqrt{2 + 3x^2}}{22(1 + 2x)^2} + \frac{13\sqrt{2 + 3x^2}}{242(1 + 2x)} - \frac{103}{121} \operatorname{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right) \\
 &= -\frac{\sqrt{2 + 3x^2}}{22(1 + 2x)^2} + \frac{13\sqrt{2 + 3x^2}}{242(1 + 2x)} - \frac{103 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{121\sqrt{11}}
 \end{aligned}$$

**Mathematica** [A] time = 0.06, size = 55, normalized size = 0.71

$$\frac{\frac{11(13x+1)\sqrt{3x^2+2}}{(2x+1)^2} - 103\sqrt{11} \tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{1331}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*Sqrt[2 + 3\*x^2]),x]

[Out] ((11\*(1 + 13\*x)\*Sqrt[2 + 3\*x^2])/((1 + 2\*x)^2 - 103\*Sqrt[11]\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/1331

**IntegrateAlgebraic** [A] time = 0.60, size = 74, normalized size = 0.96

$$\frac{\sqrt{3x^2 + 2}(13x + 1)}{121(2x + 1)^2} + \frac{206 \tanh^{-1}\left(-\frac{2\sqrt{3x^2+2}}{\sqrt{11}} + 2\sqrt{\frac{3}{11}}x + \sqrt{\frac{3}{11}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.



[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*Sqrt[2 + 3\*x^2]),x]

[Out] ((1 + 13\*x)\*Sqrt[2 + 3\*x^2])/(121\*(1 + 2\*x)^2) + (206\*ArcTanh[Sqrt[3/11] + 2\*Sqrt[3/11]\*x - (2\*Sqrt[2 + 3\*x^2])/Sqrt[11]])/(121\*Sqrt[11])

**fricas** [A] time = 0.64, size = 89, normalized size = 1.16

$$\frac{103\sqrt{11}(4x^2 + 4x + 1)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right) + 22\sqrt{3x^2+2}(13x+1)}{2662(4x^2+4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/2662\*(103\*sqrt(11)\*(4\*x^2 + 4\*x + 1)\*log(-(sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) + 21\*x^2 - 12\*x + 19)/(4\*x^2 + 4\*x + 1)) + 22\*sqrt(3\*x^2 + 2)\*(13\*x + 1))/(4\*x^2 + 4\*x + 1)

**giac** [B] time = 0.26, size = 180, normalized size = 2.34

$$\frac{103}{1331}\sqrt{11}\log\left(-\frac{-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right) + \frac{72(\sqrt{3}x-\sqrt{3x^2+2})^3-13\sqrt{3}(\sqrt{3}x-\sqrt{3x^2+2})^2-168\sqrt{3}x+104\sqrt{3}+168\sqrt{3x^2+2}}{484((\sqrt{3}x-\sqrt{3x^2+2})^2+\sqrt{3}(\sqrt{3}x-\sqrt{3x^2+2})-2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(1/2),x, algorithm="giac")

[Out] 103/1331\*sqrt(11)\*log(-abs(-2\*sqrt(3)\*x - sqrt(11) - sqrt(3) + 2\*sqrt(3\*x^2 + 2))/(2\*sqrt(3)\*x - sqrt(11) + sqrt(3) - 2\*sqrt(3\*x^2 + 2))) + 1/484\*(72\*(sqrt(3)\*x - sqrt(3\*x^2 + 2))^3 - 13\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 + 2))^2 - 168\*sqrt(3)\*x + 104\*sqrt(3) + 168\*sqrt(3\*x^2 + 2))/((sqrt(3)\*x - sqrt(3\*x^2 + 2))^2 + sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 + 2)) - 2)^2

**maple** [A] time = 0.01, size = 74, normalized size = 0.96

$$\frac{103\sqrt{11}\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{1331} + \frac{13\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{484\left(x+\frac{1}{2}\right)} - \frac{\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{88\left(x+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(1/2),x)

[Out]  $-103/1331*11^{(1/2)}*\operatorname{arctanh}(2/11*(-3*x+4)*11^{(1/2)})/(-12*x+12*(x+1/2)^2+5)^{(1/2)}+13/484/(x+1/2)*(-3*x+3*(x+1/2)^2+5/4)^{(1/2)}-1/88/(x+1/2)^2*(-3*x+3*(x+1/2)^2+5/4)^{(1/2)}$

**maxima** [A] time = 0.97, size = 76, normalized size = 0.99

$$\frac{103}{1331} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) - \frac{\sqrt{3x^2+2}}{22(4x^2+4x+1)} + \frac{13\sqrt{3x^2+2}}{242(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")`

[Out]  $103/1331*\sqrt{11}*\operatorname{arcsinh}(1/2*\sqrt{6}*x/\operatorname{abs}(2*x+1) - 2/3*\sqrt{6}/\operatorname{abs}(2*x+1)) - 1/22*\sqrt{3*x^2+2}/(4*x^2+4*x+1) + 13/242*\sqrt{3*x^2+2}/(2*x+1)$

**mupad** [B] time = 0.11, size = 77, normalized size = 1.00

$$\frac{103\sqrt{11}\ln\left(x+\frac{1}{2}\right)}{1331} - \frac{103\sqrt{11}\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}-\frac{4}{3}}{3}\right)}{1331} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{88\left(x^2+x+\frac{1}{4}\right)} + \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{484\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)`

[Out]  $(103*11^{(1/2)}*\log(x+1/2))/1331 - (103*11^{(1/2)}*\log(x - (3^{(1/2)}*11^{(1/2)}*(x^2+2/3)^{(1/2)})/3 - 4/3))/1331 - (3^{(1/2)}*(x^2+2/3)^{(1/2)})/(88*(x+x^2+1/4)) + (13*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(484*(x+1/2))$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)`

[Out] Timed out

$$3.124 \quad \int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx$$

Optimal. Leaf size=87

$$\frac{32}{27}\sqrt{3x^2+2}x^2 + 4\sqrt{3x^2+2}x + \frac{292}{81}\sqrt{3x^2+2} + \frac{279x+398}{54\sqrt{3x^2+2}} - \frac{38 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

**Rubi** [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1814, 1815, 641, 215}

$$\frac{32}{27}\sqrt{3x^2+2}x^2 + 4\sqrt{3x^2+2}x + \frac{292}{81}\sqrt{3x^2+2} + \frac{279x+398}{54\sqrt{3x^2+2}} - \frac{38 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (398 + 279\*x)/(54\*sqrt[2 + 3\*x^2]) + (292\*sqrt[2 + 3\*x^2])/81 + 4\*x\*sqrt[2 + 3\*x^2] + (32\*x^2\*sqrt[2 + 3\*x^2])/27 - (38\*ArcSinh[Sqrt[3/2]\*x])/(3\*sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx &= \frac{398+279x}{54\sqrt{2+3x^2}} - \frac{1}{2} \int \frac{\frac{28}{3} - \frac{280x}{9} - 48x^2 - \frac{64x^3}{3}}{\sqrt{2+3x^2}} dx \\ &= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{1}{18} \int \frac{84 - \frac{584x}{3} - 432x^2}{\sqrt{2+3x^2}} dx \\ &= \frac{398+279x}{54\sqrt{2+3x^2}} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{1}{108} \int \frac{1368 - 1168x}{\sqrt{2+3x^2}} dx \\ &= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{38}{3} \int \frac{1}{\sqrt{2+3x^2}} dx \\ &= \frac{398+279x}{54\sqrt{2+3x^2}} + \frac{292}{81}\sqrt{2+3x^2} + 4x\sqrt{2+3x^2} + \frac{32}{27}x^2\sqrt{2+3x^2} - \frac{38 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.67

$$\frac{576x^4 + 1944x^3 + 2136x^2 - 684\sqrt{9x^2 + 6} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) + 2133x + 2362}{162\sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (2362 + 2133\*x + 2136\*x^2 + 1944\*x^3 + 576\*x^4 - 684\*Sqrt[6 + 9\*x^2]\*ArcSinh[Sqrt[3/2]\*x])/(162\*Sqrt[2 + 3\*x^2])

IntegrateAlgebraic [A] time = 0.37, size = 66, normalized size = 0.76

$$\frac{38 \log(\sqrt{3x^2 + 2} - \sqrt{3}x)}{3\sqrt{3}} + \frac{576x^4 + 1944x^3 + 2136x^2 + 2133x + 2362}{162\sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2),x]

[Out] (2362 + 2133\*x + 2136\*x^2 + 1944\*x^3 + 576\*x^4)/(162\*Sqrt[2 + 3\*x^2]) + (38\*Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]])/(3\*Sqrt[3])

**fricas** [A] time = 0.81, size = 76, normalized size = 0.87

$$\frac{342\sqrt{3}(3x^2+2)\log(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1)+(576x^4+1944x^3+2136x^2+2133x+2362)\sqrt{3x^2+2}}{162(3x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(3/2),x, algorithm="fricas")

[Out] 1/162\*(342\*sqrt(3)\*(3\*x^2 + 2)\*log(sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1) + (576\*x^4 + 1944\*x^3 + 2136\*x^2 + 2133\*x + 2362)\*sqrt(3\*x^2 + 2))/(3\*x^2 + 2)

**giac** [A] time = 0.21, size = 54, normalized size = 0.62

$$\frac{38}{9}\sqrt{3}\log(-\sqrt{3}x+\sqrt{3x^2+2})+\frac{3(8(3(8x+27)x+89)x+711)x+2362}{162\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(3/2),x, algorithm="giac")

[Out] 38/9\*sqrt(3)\*log(-sqrt(3)\*x + sqrt(3\*x^2 + 2)) + 1/162\*(3\*(8\*(3\*(8\*x + 27)\*x + 89)\*x + 711)\*x + 2362)/sqrt(3\*x^2 + 2)

**maple** [A] time = 0.01, size = 79, normalized size = 0.91

$$\frac{32x^4}{9\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}+\frac{356x^2}{27\sqrt{3x^2+2}}+\frac{79x}{6\sqrt{3x^2+2}}-\frac{38\sqrt{3}\operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}+\frac{1181}{81\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(3/2),x)

[Out] 32/9\*x^4/(3\*x^2+2)^(1/2)+356/27\*x^2/(3\*x^2+2)^(1/2)+1181/81/(3\*x^2+2)^(1/2)+12\*x^3/(3\*x^2+2)^(1/2)+79/6\*x/(3\*x^2+2)^(1/2)-38/9\*arcsinh(1/2\*sqrt(6)\*x)\*sqrt(3)

**maxima** [A] time = 0.96, size = 78, normalized size = 0.90

$$\frac{32x^4}{9\sqrt{3x^2+2}} + \frac{12x^3}{\sqrt{3x^2+2}} + \frac{356x^2}{27\sqrt{3x^2+2}} - \frac{38}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{79x}{6\sqrt{3x^2+2}} + \frac{1181}{81\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(3/2),x, algorithm="maxima")

[Out] 32/9\*x^4/sqrt(3\*x^2 + 2) + 12\*x^3/sqrt(3\*x^2 + 2) + 356/27\*x^2/sqrt(3\*x^2 + 2) - 38/9\*sqrt(3)\*arcsinh(1/2\*sqrt(6)\*x) + 79/6\*x/sqrt(3\*x^2 + 2) + 1181/81/sqrt(3\*x^2 + 2)

**mupad** [B] time = 0.06, size = 110, normalized size = 1.26

$$\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{32x^2}{9}+12x+\frac{292}{27}\right)}{3} - \frac{38\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{9} - \frac{\sqrt{3}\sqrt{6}\left(-1194+\sqrt{6}279i\right)\sqrt{x^2+\frac{2}{3}}\operatorname{li}}{1944\left(x+\frac{\sqrt{6}i}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(1194+\sqrt{6}279i\right)\sqrt{x^2+\frac{2}{3}}\operatorname{li}}{1944\left(x-\frac{\sqrt{6}i}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)^3\*(3\*x + 4\*x^2 + 1))/(3\*x^2 + 2)^(3/2),x)

[Out] (3^(1/2)\*(x^2 + 2/3)^(1/2)\*(12\*x + (32\*x^2)/9 + 292/27))/3 - (38\*3^(1/2)\*asinh((2^(1/2)\*3^(1/2)\*x)/2))/9 - (3^(1/2)\*6^(1/2)\*(6^(1/2)\*279i - 1194)\*(x^2 + 2/3)^(1/2)\*li)/(1944\*(x + (6^(1/2)\*1i)/3)) - (3^(1/2)\*6^(1/2)\*(6^(1/2)\*279i + 1194)\*(x^2 + 2/3)^(1/2)\*li)/(1944\*(x - (6^(1/2)\*1i)/3))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*3\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2+2)\*\*(3/2),x)

[Out] Integral((2\*x + 1)\*\*3\*(4\*x\*\*2 + 3\*x + 1)/(3\*x\*\*2 + 2)\*\*(3/2), x)

$$3.125 \quad \int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx$$

Optimal. Leaf size=71

$$\frac{70-47x}{18\sqrt{3x^2+2}} + \frac{8}{9}x\sqrt{3x^2+2} + \frac{28}{9}\sqrt{3x^2+2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1814, 1815, 641, 215}

$$\frac{70-47x}{18\sqrt{3x^2+2}} + \frac{8}{9}x\sqrt{3x^2+2} + \frac{28}{9}\sqrt{3x^2+2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (70 - 47\*x)/(18\*sqrt[2 + 3\*x^2]) + (28\*sqrt[2 + 3\*x^2])/9 + (8\*x\*sqrt[2 + 3\*x^2])/9 + (4\*ArcSinh[Sqrt[3/2]\*x])/(3\*sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx &= \frac{70-47x}{18\sqrt{2+3x^2}} - \frac{1}{2} \int \frac{-\frac{56}{9} - \frac{56x}{3} - \frac{32x^2}{3}}{\sqrt{2+3x^2}} dx \\ &= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{8}{9}x\sqrt{2+3x^2} - \frac{1}{12} \int \frac{-16-112x}{\sqrt{2+3x^2}} dx \\ &= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4}{3} \int \frac{1}{\sqrt{2+3x^2}} dx \\ &= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 53, normalized size = 0.75

$$\frac{48x^3 + 168x^2 + 8\sqrt{9x^2 + 6} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) - 15x + 182}{18\sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (182 - 15\*x + 168\*x^2 + 48\*x^3 + 8\*Sqrt[6 + 9\*x^2]\*ArcSinh[Sqrt[3/2]\*x])/(18\*Sqrt[2 + 3\*x^2])

**IntegrateAlgebraic [A]** time = 0.37, size = 61, normalized size = 0.86

$$\frac{48x^3 + 168x^2 - 15x + 182}{18\sqrt{3x^2 + 2}} - \frac{4 \log\left(\sqrt{3x^2 + 2} - \sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]



[Out]  $(182 - 15x + 168x^2 + 48x^3)/(18\sqrt{2 + 3x^2}) - (4\log[-(\sqrt{3}x) + \sqrt{2 + 3x^2}])/(3\sqrt{3})$

**fricas** [A] time = 0.52, size = 72, normalized size = 1.01

$$\frac{4\sqrt{3}(3x^2 + 2)\log(-\sqrt{3}\sqrt{3x^2 + 2}x - 3x^2 - 1) + (48x^3 + 168x^2 - 15x + 182)\sqrt{3x^2 + 2}}{18(3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")`

[Out]  $1/18*(4*\sqrt{3}*(3*x^2 + 2)*\log(-\sqrt{3}*\sqrt{3*x^2 + 2}*x - 3*x^2 - 1) + (48*x^3 + 168*x^2 - 15*x + 182)*\sqrt{3*x^2 + 2})/(3*x^2 + 2)$

**giac** [A] time = 0.20, size = 49, normalized size = 0.69

$$-\frac{4}{9}\sqrt{3}\log(-\sqrt{3}x + \sqrt{3x^2 + 2}) + \frac{3(8(2x + 7)x - 5)x + 182}{18\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")`

[Out]  $-4/9*\sqrt{3}*\log(-\sqrt{3}*x + \sqrt{3*x^2 + 2}) + 1/18*(3*(8*(2*x + 7)*x - 5)*x + 182)/\sqrt{3*x^2 + 2}$

**maple** [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{8x^3}{3\sqrt{3x^2 + 2}} + \frac{28x^2}{3\sqrt{3x^2 + 2}} - \frac{5x}{6\sqrt{3x^2 + 2}} + \frac{4\sqrt{3}\operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{9} + \frac{91}{9\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)`

[Out]  $8/3/(3*x^2+2)^(1/2)*x^3-5/6/(3*x^2+2)^(1/2)*x+4/9*\operatorname{arcsinh}(1/2*6^(1/2)*x)*3^(1/2)+28/3/(3*x^2+2)^(1/2)*x^2+91/9/(3*x^2+2)^(1/2)$

**maxima** [A] time = 0.96, size = 64, normalized size = 0.90

$$\frac{8x^3}{3\sqrt{3x^2 + 2}} + \frac{28x^2}{3\sqrt{3x^2 + 2}} + \frac{4}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{5x}{6\sqrt{3x^2 + 2}} + \frac{91}{9\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(3/2),x, algorithm="maxima")

[Out]  $\frac{8}{3}x^3/\sqrt{3x^2+2} + \frac{28}{3}x^2/\sqrt{3x^2+2} + \frac{4}{9}\sqrt{3}\operatorname{arcsinh}(1/2\sqrt{6}x) - \frac{5}{6}x/\sqrt{3x^2+2} + \frac{91}{9}/\sqrt{3x^2+2}$

**mupad [B]** time = 4.07, size = 105, normalized size = 1.48

$$\frac{4\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{9} + \frac{\sqrt{3}\left(\frac{8x}{3} + \frac{28}{3}\right)\sqrt{x^2 + \frac{2}{3}}}{3} + \frac{\sqrt{3}\sqrt{6}\left(-630 + \sqrt{6}141i\right)\sqrt{x^2 + \frac{2}{3}}1i}{1944\left(x - \frac{\sqrt{6}1i}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(630 + \sqrt{6}141i\right)\sqrt{x^2 + \frac{2}{3}}1i}{1944\left(x + \frac{\sqrt{6}1i}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)^2\*(3\*x + 4\*x^2 + 1))/(3\*x^2 + 2)^(3/2),x)

[Out]  $\frac{4\cdot 3^{1/2}\operatorname{asinh}\left(\frac{2^{1/2}\cdot 3^{1/2}\cdot x}{2}\right)}{9} + \frac{3^{1/2}\left(\frac{8x}{3} + \frac{28}{3}\right)\left(x^2 + \frac{2}{3}\right)^{1/2}}{3} + \frac{3^{1/2}\cdot 6^{1/2}\left(6^{1/2}\cdot 141i - 630\right)\left(x^2 + \frac{2}{3}\right)^{1/2}\cdot 1i}{1944\left(x - \frac{6^{1/2}\cdot 1i}{3}\right)} + \frac{3^{1/2}\cdot 6^{1/2}\left(6^{1/2}\cdot 141i + 630\right)\left(x^2 + \frac{2}{3}\right)^{1/2}\cdot 1i}{1944\left(x + \frac{6^{1/2}\cdot 1i}{3}\right)}$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*2\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2+2)\*\*(3/2),x)

[Out] Integral((2\*x + 1)\*\*2\*(4\*x\*\*2 + 3\*x + 1)/(3\*x\*\*2 + 2)\*\*(3/2), x)

$$3.126 \quad \int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx$$

Optimal. Leaf size=55

$$\frac{2-51x}{18\sqrt{3x^2+2}} + \frac{8}{9}\sqrt{3x^2+2} + \frac{10 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1814, 641, 215}

$$\frac{2-51x}{18\sqrt{3x^2+2}} + \frac{8}{9}\sqrt{3x^2+2} + \frac{10 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (2 - 51\*x)/(18\*sqrt[2 + 3\*x^2]) + (8\*sqrt[2 + 3\*x^2])/9 + (10\*ArcSinh[Sqrt[3/2]\*x])/(3\*sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}} dx &= \frac{2-51x}{18\sqrt{2+3x^2}} - \frac{1}{2} \int \frac{-\frac{20}{3} - \frac{16x}{3}}{\sqrt{2+3x^2}} dx \\
&= \frac{2-51x}{18\sqrt{2+3x^2}} + \frac{8}{9}\sqrt{2+3x^2} + \frac{10}{3} \int \frac{1}{\sqrt{2+3x^2}} dx \\
&= \frac{2-51x}{18\sqrt{2+3x^2}} + \frac{8}{9}\sqrt{2+3x^2} + \frac{10 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 48, normalized size = 0.87

$$\frac{48x^2 + 20\sqrt{9x^2 + 6} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) - 51x + 34}{18\sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (34 - 51\*x + 48\*x^2 + 20\*Sqrt[6 + 9\*x^2]\*ArcSinh[Sqrt[3/2]\*x])/(18\*Sqrt[2 + 3\*x^2])

**IntegrateAlgebraic** [A] time = 0.31, size = 56, normalized size = 1.02

$$\frac{48x^2 - 51x + 34}{18\sqrt{3x^2 + 2}} - \frac{10 \log(\sqrt{3x^2 + 2} - \sqrt{3}x)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(3/2), x]

[Out] (34 - 51\*x + 48\*x^2)/(18\*Sqrt[2 + 3\*x^2]) - (10\*Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]])/(3\*Sqrt[3])

**fricas** [A] time = 0.98, size = 67, normalized size = 1.22

$$\frac{10\sqrt{3}(3x^2 + 2) \log(-\sqrt{3}\sqrt{3x^2 + 2}x - 3x^2 - 1) + (48x^2 - 51x + 34)\sqrt{3x^2 + 2}}{18(3x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(3/2), x, algorithm="fricas")

[Out]  $\frac{1}{18} \cdot (10 \cdot \sqrt{3}) \cdot (3x^2 + 2) \cdot \log(-\sqrt{3} \cdot \sqrt{3x^2 + 2} \cdot x - 3x^2 - 1) + (48x^2 - 51x + 34) \cdot \sqrt{3x^2 + 2} / (3x^2 + 2)$

**giac** [A] time = 0.23, size = 44, normalized size = 0.80

$$-\frac{10}{9} \sqrt{3} \log\left(-\sqrt{3}x + \sqrt{3x^2 + 2}\right) + \frac{3(16x - 17)x + 34}{18\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")`

[Out]  $-10/9 \cdot \sqrt{3} \cdot \log(-\sqrt{3} \cdot x + \sqrt{3x^2 + 2}) + 1/18 \cdot (3 \cdot (16x - 17) \cdot x + 34) / \sqrt{3x^2 + 2}$

**maple** [A] time = 0.00, size = 51, normalized size = 0.93

$$\frac{8x^2}{3\sqrt{3x^2 + 2}} - \frac{17x}{6\sqrt{3x^2 + 2}} + \frac{10\sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{9} + \frac{17}{9\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)`

[Out]  $8/3 / (3x^2 + 2)^{(1/2)} \cdot x^2 + 17/9 / (3x^2 + 2)^{(1/2)} - 17/6 / (3x^2 + 2)^{(1/2)} \cdot x + 10/9 \cdot \operatorname{arcsinh}(1/2 \cdot \sqrt{6} \cdot x) \cdot 3^{(1/2)}$

**maxima** [A] time = 0.96, size = 50, normalized size = 0.91

$$\frac{8x^2}{3\sqrt{3x^2 + 2}} + \frac{10}{9} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2} \sqrt{6}x\right) - \frac{17x}{6\sqrt{3x^2 + 2}} + \frac{17}{9\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")`

[Out]  $8/3 \cdot x^2 / \sqrt{3x^2 + 2} + 10/9 \cdot \sqrt{3} \cdot \operatorname{arcsinh}(1/2 \cdot \sqrt{6} \cdot x) - 17/6 \cdot x / \sqrt{3x^2 + 2} + 17/9 / \sqrt{3x^2 + 2}$

**mupad** [B] time = 0.04, size = 100, normalized size = 1.82

$$\frac{8\sqrt{3}\sqrt{x^2 + \frac{2}{3}}}{9} + \frac{10\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{9} + \frac{\sqrt{3}\sqrt{6}(-6 + \sqrt{6}51i)\sqrt{x^2 + \frac{2}{3}}1i}{648\left(x - \frac{\sqrt{6}1i}{3}\right)} + \frac{\sqrt{3}\sqrt{6}(6 + \sqrt{6}51i)\sqrt{x^2 + \frac{2}{3}}1i}{648\left(x + \frac{\sqrt{6}1i}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2), x)`

[Out]  $(8 \cdot 3^{1/2} \cdot (x^2 + 2/3)^{1/2})/9 + (10 \cdot 3^{1/2} \cdot \operatorname{asinh}((2^{1/2} \cdot 3^{1/2} \cdot x)/2))/9 + (3^{1/2} \cdot 6^{1/2} \cdot (6^{1/2} \cdot 51i - 6) \cdot (x^2 + 2/3)^{1/2} \cdot 1i)/(648 \cdot (x - (6^{1/2} \cdot 1i)/3)) + (3^{1/2} \cdot 6^{1/2} \cdot (6^{1/2} \cdot 51i + 6) \cdot (x^2 + 2/3)^{1/2} \cdot 1i)/(648 \cdot (x + (6^{1/2} \cdot 1i)/3))$

**sympy** [B] time = 15.96, size = 114, normalized size = 2.07

$$\frac{30\sqrt{3}x^2 \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2 + 18} + \frac{8x^2}{3\sqrt{3x^2 + 2}} - \frac{30x\sqrt{3x^2 + 2}}{27x^2 + 18} + \frac{x}{2\sqrt{3x^2 + 2}} + \frac{20\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2 + 18} + \frac{17}{9\sqrt{3x^2 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2), x)`

[Out]  $30 \cdot \sqrt{3} \cdot x^2 \cdot \operatorname{asinh}(\sqrt{6} \cdot x/2)/(27 \cdot x^2 + 18) + 8 \cdot x^2/(3 \cdot \sqrt{3 \cdot x^2 + 2}) - 30 \cdot x \cdot \sqrt{3 \cdot x^2 + 2}/(27 \cdot x^2 + 18) + x/(2 \cdot \sqrt{3 \cdot x^2 + 2}) + 20 \cdot \sqrt{3} \cdot \operatorname{asinh}(\sqrt{6} \cdot x/2)/(27 \cdot x^2 + 18) + 17/(9 \cdot \sqrt{3 \cdot x^2 + 2})$

$$3.127 \quad \int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{3/2}} dx$$

Optimal. Leaf size=53

$$\frac{21x - 38}{66\sqrt{3x^2 + 2}} - \frac{2 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

**Rubi [A]** time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1647, 12, 725, 206}

$$-\frac{38 - 21x}{66\sqrt{3x^2 + 2}} - \frac{2 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(3/2)), x]

[Out] -(38 - 21\*x)/(66\*sqrt[2 + 3\*x^2]) - (2\*ArcTanh[(4 - 3\*x)/(sqrt[11]\*sqrt[2 + 3\*x^2])])/(11\*sqrt[11])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1647

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_)^(m\_.))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + c\*x^2, x], f = Coeff[Pol

```

ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)(2 + 3x^2)^{3/2}} dx &= -\frac{38 - 21x}{66\sqrt{2 + 3x^2}} - \frac{1}{6} \int -\frac{12}{11(1 + 2x)\sqrt{2 + 3x^2}} dx \\
&= -\frac{38 - 21x}{66\sqrt{2 + 3x^2}} + \frac{2}{11} \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
&= -\frac{38 - 21x}{66\sqrt{2 + 3x^2}} - \frac{2}{11} \operatorname{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right) \\
&= -\frac{38 - 21x}{66\sqrt{2 + 3x^2}} - \frac{2 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{11\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 0.96

$$\frac{-12\sqrt{33x^2 + 22} \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{33x^2 + 22}}\right) + 231x - 418}{726\sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(3/2)), x]

[Out] (-418 + 231\*x - 12\*Sqrt[22 + 33\*x^2]\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/(726\*Sqrt[2 + 3\*x^2])

**IntegrateAlgebraic [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1 + 3x + 4x^2}{(1 + 2x)(2 + 3x^2)^{3/2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(3/2)), x]



[Out] Defer[IntegrateAlgebraic] [(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(3/2)), x]

**fricas** [A] time = 0.54, size = 83, normalized size = 1.57

$$\frac{6\sqrt{11}(3x^2 + 2)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right) + 11\sqrt{3x^2+2}(21x-38)}{726(3x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(3/2), x, algorithm="fricas")

[Out] 1/726\*(6\*sqrt(11)\*(3\*x^2 + 2)\*log(-(sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) + 21\*x^2 - 12\*x + 19)/(4\*x^2 + 4\*x + 1)) + 11\*sqrt(3\*x^2 + 2)\*(21\*x - 38))/(3\*x^2 + 2)

**giac** [A] time = 0.21, size = 82, normalized size = 1.55

$$\frac{2}{121}\sqrt{11}\log\left(-\frac{|-2\sqrt{3}x - \sqrt{11} - \sqrt{3} + 2\sqrt{3x^2+2}|}{2\sqrt{3}x - \sqrt{11} + \sqrt{3} - 2\sqrt{3x^2+2}}\right) + \frac{21x-38}{66\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(3/2), x, algorithm="giac")

[Out] 2/121\*sqrt(11)\*log(-abs(-2\*sqrt(3)\*x - sqrt(11) - sqrt(3) + 2\*sqrt(3\*x^2 + 2))/(2\*sqrt(3)\*x - sqrt(11) + sqrt(3) - 2\*sqrt(3\*x^2 + 2))) + 1/66\*(21\*x - 38)/sqrt(3\*x^2 + 2)

**maple** [B] time = 0.01, size = 88, normalized size = 1.66

$$\frac{x}{4\sqrt{3x^2+2}} + \frac{3x}{44\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}} - \frac{2\sqrt{11}\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{121} - \frac{2}{3\sqrt{3x^2+2}} + \frac{1}{11\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(3/2), x)

[Out] -2/3/(3\*x^2+2)^(1/2)+1/4/(3\*x^2+2)^(1/2)\*x+1/11/(-3\*x+3\*(x+1/2)^2+5/4)^(1/2)+3/44\*x/(-3\*x+3\*(x+1/2)^2+5/4)^(1/2)-2/121\*11^(1/2)\*arctanh(2/11\*(-3\*x+4)\*11^(1/2)/(-12\*x+12\*(x+1/2)^2+5)^(1/2))

**maxima** [A] time = 0.96, size = 58, normalized size = 1.09

$$\frac{2}{121} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) + \frac{7x}{22\sqrt{3x^2+2}} - \frac{19}{33\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(3/2),x, algorithm="maxima")

[Out] 2/121\*sqrt(11)\*arcsinh(1/2\*sqrt(6)\*x/abs(2\*x + 1) - 2/3\*sqrt(6)/abs(2\*x + 1)) + 7/22\*x/sqrt(3\*x^2 + 2) - 19/33/sqrt(3\*x^2 + 2)

**mupad** [B] time = 0.14, size = 106, normalized size = 2.00

$$\frac{\sqrt{11} \left( 2 \ln\left(x + \frac{1}{2}\right) - 2 \ln\left(x - \frac{\sqrt{3} \sqrt{11} \sqrt{x^2 + \frac{2}{3}}}{3} - \frac{4}{3}\right) \right)}{121} - \frac{\sqrt{3} \sqrt{6} (-114 + \sqrt{6} 21i) \sqrt{x^2 + \frac{2}{3}} 1i}{2376 \left(x - \frac{\sqrt{6} 1i}{3}\right)} - \frac{\sqrt{3} \sqrt{6} (114 + \sqrt{6} 21i) \sqrt{x^2 + \frac{2}{3}} 1i}{2376 \left(x + \frac{\sqrt{6} 1i}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 4\*x^2 + 1)/((2\*x + 1)\*(3\*x^2 + 2)^(3/2)),x)

[Out] (11^(1/2)\*(2\*log(x + 1/2) - 2\*log(x - (3^(1/2)\*11^(1/2)\*(x^2 + 2/3)^(1/2))/(3 - 4/3))))/121 - (3^(1/2)\*6^(1/2)\*(6^(1/2)\*21i - 114)\*(x^2 + 2/3)^(1/2)\*1i)/(2376\*(x - (6^(1/2)\*1i)/3)) - (3^(1/2)\*6^(1/2)\*(6^(1/2)\*21i + 114)\*(x^2 + 2/3)^(1/2)\*1i)/(2376\*(x + (6^(1/2)\*1i)/3))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)(3x^2 + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)/(1+2\*x)/(3\*x\*\*2+2)\*\*(3/2),x)

[Out] Integral((4\*x\*\*2 + 3\*x + 1)/((2\*x + 1)\*(3\*x\*\*2 + 2)\*\*(3/2)), x)

$$3.128 \quad \int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}} dx$$

Optimal. Leaf size=75

$$\frac{97x-10}{242\sqrt{3x^2+2}} - \frac{4\sqrt{3x^2+2}}{121(2x+1)} + \frac{4 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

**Rubi** [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{242\sqrt{3x^2+2}} - \frac{4\sqrt{3x^2+2}}{121(2x+1)} + \frac{4 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(3/2)), x]

[Out] -(10 - 97\*x)/(242\*sqrt[2 + 3\*x^2]) - (4\*sqrt[2 + 3\*x^2])/(121\*(1 + 2\*x)) + (4\*ArcTanh[(4 - 3\*x)/(sqrt[11]\*sqrt[2 + 3\*x^2])])/(121\*sqrt[11])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 807

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_.), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

Rule 1647

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] & & NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 (2 + 3x^2)^{3/2}} dx &= -\frac{10 - 97x}{242\sqrt{2 + 3x^2}} - \frac{1}{6} \int \frac{-\frac{72}{121} + \frac{120x}{121}}{(1 + 2x)^2 \sqrt{2 + 3x^2}} dx \\
&= -\frac{10 - 97x}{242\sqrt{2 + 3x^2}} - \frac{4\sqrt{2 + 3x^2}}{121(1 + 2x)} - \frac{4}{121} \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
&= -\frac{10 - 97x}{242\sqrt{2 + 3x^2}} - \frac{4\sqrt{2 + 3x^2}}{121(1 + 2x)} + \frac{4}{121} \text{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right) \\
&= -\frac{10 - 97x}{242\sqrt{2 + 3x^2}} - \frac{4\sqrt{2 + 3x^2}}{121(1 + 2x)} + \frac{4 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 71, normalized size = 0.95

$$\frac{11(170x^2 + 77x - 26) + 8(2x + 1)\sqrt{33x^2 + 22} \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{33x^2 + 22}}\right)}{2662(2x + 1)\sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(3/2)), x]

[Out] (11\*(-26 + 77\*x + 170\*x^2) + 8\*(1 + 2\*x)\*Sqrt[22 + 33\*x^2]\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/(2662\*(1 + 2\*x)\*Sqrt[2 + 3\*x^2])

**IntegrateAlgebraic [F]** time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 (2 + 3x^2)^{3/2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(3/2)),x]

[Out] Defer[IntegrateAlgebraic] [(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(3/2)), x]

**fricas** [A] time = 0.84, size = 103, normalized size = 1.37

$$\frac{4\sqrt{11}(6x^3 + 3x^2 + 4x + 2) \log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right) + 11(170x^2 + 77x - 26)\sqrt{3x^2 + 2}}{2662(6x^3 + 3x^2 + 4x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(3/2),x, algorithm="fricas")

[Out] 1/2662\*(4\*sqrt(11)\*(6\*x^3 + 3\*x^2 + 4\*x + 2)\*log((sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) - 21\*x^2 + 12\*x - 19)/(4\*x^2 + 4\*x + 1)) + 11\*(170\*x^2 + 77\*x - 26)\*sqrt(3\*x^2 + 2))/(6\*x^3 + 3\*x^2 + 4\*x + 2)

**giac** [B] time = 0.27, size = 168, normalized size = 2.24

$$-\frac{1}{7986}\sqrt{11}\left(85\sqrt{11}\sqrt{3} + 24\log(\sqrt{11}\sqrt{3} - 3)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{\frac{93}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)} + \frac{44}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)} - \frac{85}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{242\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3}} + \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3} + \frac{\sqrt{11}}{2x+1}\right) - 3\right)}{1331\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(3/2),x, algorithm="giac")

[Out] -1/7986\*sqrt(11)\*(85\*sqrt(11)\*sqrt(3) + 24\*log(sqrt(11)\*sqrt(3) - 3))\*sgn(1/(2\*x + 1)) - 1/242\*((93/sgn(1/(2\*x + 1))) + 44/((2\*x + 1)\*sgn(1/(2\*x + 1))))/(2\*x + 1) - 85/sgn(1/(2\*x + 1))/sqrt(-6/(2\*x + 1) + 11/(2\*x + 1)^2 + 3) + 4/1331\*sqrt(11)\*log(sqrt(11)\*(sqrt(-6/(2\*x + 1) + 11/(2\*x + 1)^2 + 3) + sqrt(11)/(2\*x + 1)) - 3)/sgn(1/(2\*x + 1))

**maple** [A] time = 0.01, size = 98, normalized size = 1.31

$$\frac{x}{2\sqrt{3x^2+2}} - \frac{18x}{121\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}} + \frac{4\sqrt{11}\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{1331} - \frac{2}{121\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}} - \frac{1}{22\left(x+\frac{1}{2}\right)\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(3/2),x)

[Out]  $\frac{1}{2} / (3x^2 + 2)^{(1/2)} * x - 2/121 / (-3x + 3(x+1/2)^2 + 5/4)^{(1/2)} - 18/121 / (-3x + 3(x+1/2)^2 + 5/4)^{(1/2)} * x + 4/1331 * 11^{(1/2)} * \operatorname{arctanh}(2/11 * (-3x + 4) * 11^{(1/2)} / (-12x + 12 * (x+1/2)^2 + 5))^{(1/2)} - 1/22 / (x+1/2) / (-3x + 3(x+1/2)^2 + 5/4)^{(1/2)}$

**maxima [A]** time = 0.97, size = 84, normalized size = 1.12

$$-\frac{4}{1331} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) + \frac{85x}{242\sqrt{3x^2+2}} - \frac{2}{121\sqrt{3x^2+2}} - \frac{1}{11(2\sqrt{3x^2+2}x + \sqrt{3x^2+2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2), x, algorithm="maxima")`

[Out]  $-4/1331 * \sqrt{11} * \operatorname{arcsinh}(1/2 * \sqrt{6} * x / \operatorname{abs}(2x + 1) - 2/3 * \sqrt{6} / \operatorname{abs}(2x + 1)) + 85/242 * x / \sqrt{3x^2 + 2} - 2/121 / \sqrt{3x^2 + 2} - 1/11 / (2 * \sqrt{3x^2 + 2} * x + \sqrt{3x^2 + 2})$

**mupad [B]** time = 4.14, size = 157, normalized size = 2.09

$$\frac{4\sqrt{11} \ln\left(x - \frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3} - \frac{4}{3}\right)}{1331} - \frac{4\sqrt{11} \ln\left(x + \frac{1}{2}\right)}{1331} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x - \frac{\sqrt{6}11}{3}\right)} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x + \frac{\sqrt{6}11}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{121\left(x + \frac{1}{2}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}5i}{1452\left(x - \frac{\sqrt{6}11}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}5i}{1452\left(x + \frac{\sqrt{6}11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)), x)`

[Out]  $(4 * 11^{(1/2)} * \log(x - (3^{(1/2)} * 11^{(1/2)} * (x^2 + 2/3)^{(1/2)}) / 3 - 4/3)) / 1331 - (4 * 11^{(1/2)} * \log(x + 1/2)) / 1331 + (97 * 3^{(1/2)} * (x^2 + 2/3)^{(1/2)}) / (1452 * (x - (6^{(1/2)} * 1i) / 3)) + (97 * 3^{(1/2)} * (x^2 + 2/3)^{(1/2)}) / (1452 * (x + (6^{(1/2)} * 1i) / 3)) - (2 * 3^{(1/2)} * (x^2 + 2/3)^{(1/2)}) / (121 * (x + 1/2)) + (3^{(1/2)} * 6^{(1/2)} * (x^2 + 2/3)^{(1/2)} * 5i) / (1452 * (x - (6^{(1/2)} * 1i) / 3)) - (3^{(1/2)} * 6^{(1/2)} * (x^2 + 2/3)^{(1/2)} * 5i) / (1452 * (x + (6^{(1/2)} * 1i) / 3))$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2), x)`

[Out] Timed out

$$3.129 \quad \int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}} dx$$

Optimal. Leaf size=97

$$\frac{351x + 358}{2662\sqrt{3x^2 + 2}} + \frac{2\sqrt{3x^2 + 2}}{1331(2x + 1)} - \frac{2\sqrt{3x^2 + 2}}{121(2x + 1)^2} - \frac{322 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

**Rubi [A]** time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x + 358}{2662\sqrt{3x^2 + 2}} + \frac{2\sqrt{3x^2 + 2}}{1331(2x + 1)} - \frac{2\sqrt{3x^2 + 2}}{121(2x + 1)^2} - \frac{322 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 + 3\*x^2)^(3/2)), x]

[Out] (358 + 351\*x)/(2662\*sqrt[2 + 3\*x^2]) - (2\*sqrt[2 + 3\*x^2])/(121\*(1 + 2\*x)^2) + (2\*sqrt[2 + 3\*x^2])/(1331\*(1 + 2\*x)) - (322\*ArcTanh[(4 - 3\*x)/(sqrt[11]\*sqrt[2 + 3\*x^2])])/(1331\*sqrt[11])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 807

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] & & NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)^3 (2 + 3x^2)^{3/2}} dx &= \frac{358 + 351x}{2662\sqrt{2 + 3x^2}} - \frac{1}{6} \int \frac{-\frac{2940}{1331} - \frac{7272x}{1331} - \frac{8592x^2}{1331}}{(1 + 2x)^3 \sqrt{2 + 3x^2}} dx \\
&= \frac{358 + 351x}{2662\sqrt{2 + 3x^2}} - \frac{2\sqrt{2 + 3x^2}}{121(1 + 2x)^2} + \frac{1}{132} \int \frac{\frac{3768}{121} + \frac{7800x}{121}}{(1 + 2x)^2 \sqrt{2 + 3x^2}} dx \\
&= \frac{358 + 351x}{2662\sqrt{2 + 3x^2}} - \frac{2\sqrt{2 + 3x^2}}{121(1 + 2x)^2} + \frac{2\sqrt{2 + 3x^2}}{1331(1 + 2x)} + \frac{322 \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx}{1331} \\
&= \frac{358 + 351x}{2662\sqrt{2 + 3x^2}} - \frac{2\sqrt{2 + 3x^2}}{121(1 + 2x)^2} + \frac{2\sqrt{2 + 3x^2}}{1331(1 + 2x)} - \frac{322 \operatorname{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right)}{1331} \\
&= \frac{358 + 351x}{2662\sqrt{2 + 3x^2}} - \frac{2\sqrt{2 + 3x^2}}{121(1 + 2x)^2} + \frac{2\sqrt{2 + 3x^2}}{1331(1 + 2x)} - \frac{322 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{1331\sqrt{11}}
\end{aligned}$$



**Mathematica [A]** time = 0.08, size = 78, normalized size = 0.80

$$\frac{11 \left( 1428x^3 + 2716x^2 + 1799x + 278 \right) - 644(2x + 1)^2 \sqrt{33x^2 + 22} \tanh^{-1} \left( \frac{4-3x}{\sqrt{33x^2+22}} \right)}{29282(2x + 1)^2 \sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 + 3\*x^2)^(3/2)), x]

[Out] (11\*(278 + 1799\*x + 2716\*x^2 + 1428\*x^3) - 644\*(1 + 2\*x)^2\*Sqrt[22 + 33\*x^2]\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/(29282\*(1 + 2\*x)^2\*Sqrt[2 + 3\*x^2])

**IntegrateAlgebraic [A]** time = 0.67, size = 84, normalized size = 0.87

$$\frac{644 \tanh^{-1} \left( -\frac{2\sqrt{3x^2+2}}{\sqrt{11}} + 2\sqrt{\frac{3}{11}}x + \sqrt{\frac{3}{11}} \right)}{1331\sqrt{11}} + \frac{1428x^3 + 2716x^2 + 1799x + 278}{2662(2x + 1)^2 \sqrt{3x^2 + 2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 + 3\*x^2)^(3/2)), x]

[Out] (278 + 1799\*x + 2716\*x^2 + 1428\*x^3)/(2662\*(1 + 2\*x)^2\*Sqrt[2 + 3\*x^2]) + (644\*ArcTanh[Sqrt[3/11] + 2\*Sqrt[3/11]\*x - (2\*Sqrt[2 + 3\*x^2])/Sqrt[11]])/(1331\*Sqrt[11])

**fricas [A]** time = 0.72, size = 119, normalized size = 1.23

$$\frac{322 \sqrt{11} (12x^4 + 12x^3 + 11x^2 + 8x + 2) \log \left( -\frac{\sqrt{11} \sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1} \right) + 11 (1428x^3 + 2716x^2 + 1799x + 278) \sqrt{3x^2 + 2}}{29282 (12x^4 + 12x^3 + 11x^2 + 8x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(3/2), x, algorithm="fricas")

[Out] 1/29282\*(322\*sqrt(11)\*(12\*x^4 + 12\*x^3 + 11\*x^2 + 8\*x + 2)\*log(-(sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) + 21\*x^2 - 12\*x + 19)/(4\*x^2 + 4\*x + 1)) + 11\*(1428\*x^3 + 2716\*x^2 + 1799\*x + 278)\*sqrt(3\*x^2 + 2))/(12\*x^4 + 12\*x^3 + 11\*x^2 + 8\*x + 2)

**giac [B]** time = 0.24, size = 196, normalized size = 2.02

$$\frac{322}{14641} \sqrt{11} \log \left( -\frac{-2\sqrt{3x-\sqrt{11}}-\sqrt{3}+2\sqrt{3x^2+2}}{2\sqrt{3x-\sqrt{11}}+\sqrt{3}-2\sqrt{3x^2+2}} \right) + \frac{351x+358}{2662\sqrt{3x^2+2}} + \frac{36(\sqrt{3x-\sqrt{3x^2+2}})^3 - \sqrt{3}(\sqrt{3x-\sqrt{3x^2+2}})^2 + 48\sqrt{3x} + 8\sqrt{3} - 48\sqrt{3x^2+2}}{1331((\sqrt{3x-\sqrt{3x^2+2}})^2 + \sqrt{3}(\sqrt{3x-\sqrt{3x^2+2}}) - 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(3/2),x, algorithm="giac")

[Out] 322/14641\*sqrt(11)\*log(-abs(-2\*sqrt(3)\*x - sqrt(11) - sqrt(3) + 2\*sqrt(3\*x^2 + 2))/(2\*sqrt(3)\*x - sqrt(11) + sqrt(3) - 2\*sqrt(3\*x^2 + 2))) + 1/2662\*(351\*x + 358)/sqrt(3\*x^2 + 2) + 1/1331\*(36\*(sqrt(3)\*x - sqrt(3\*x^2 + 2))^3 - sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 + 2))^2 + 48\*sqrt(3)\*x + 8\*sqrt(3) - 48\*sqrt(3\*x^2 + 2))/((sqrt(3)\*x - sqrt(3\*x^2 + 2))^2 + sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 + 2)) - 2)^2

maple [A] time = 0.01, size = 107, normalized size = 1.10

$$\frac{357x}{2662\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}} - \frac{322\sqrt{11} \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{14641} + \frac{161}{1331\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}} + \frac{7}{484\left(x+\frac{1}{2}\right)\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}} - \frac{1}{88\left(x+\frac{1}{2}\right)^2\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(3/2),x)

[Out] 161/1331/(-3\*x+3\*(x+1/2)^2+5/4)^(1/2)+357/2662/(-3\*x+3\*(x+1/2)^2+5/4)^(1/2)\*x-322/14641\*11^(1/2)\*arctanh(2/11\*(-3\*x+4)\*11^(1/2)/(-12\*x+12\*(x+1/2)^2+5)^(1/2))+7/484/(x+1/2)/(-3\*x+3\*(x+1/2)^2+5/4)^(1/2)-1/88/(x+1/2)^2/(-3\*x+3\*(x+1/2)^2+5/4)^(1/2)

maxima [A] time = 0.98, size = 124, normalized size = 1.28

$$\frac{322}{14641} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) + \frac{357x}{2662\sqrt{3x^2+2}} + \frac{161}{1331\sqrt{3x^2+2}} - \frac{1}{22(4\sqrt{3x^2+2x^2+4}\sqrt{3x^2+2x+\sqrt{3x^2+2}})} + \frac{7}{242(2\sqrt{3x^2+2x+\sqrt{3x^2+2}})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(3/2),x, algorithm="maxima")

[Out] 322/14641\*sqrt(11)\*arcsinh(1/2\*sqrt(6)\*x/abs(2\*x + 1) - 2/3\*sqrt(6)/abs(2\*x + 1)) + 357/2662\*x/sqrt(3\*x^2 + 2) + 161/1331/sqrt(3\*x^2 + 2) - 1/22/(4\*sqrt(3\*x^2 + 2)\*x^2 + 4\*sqrt(3\*x^2 + 2)\*x + sqrt(3\*x^2 + 2)) + 7/242/(2\*sqrt(3\*x^2 + 2)\*x + sqrt(3\*x^2 + 2))

mupad [B] time = 4.17, size = 180, normalized size = 1.86

$$\frac{322\sqrt{11} \ln\left(x+\frac{1}{2}\right)}{14641} - \frac{322\sqrt{11} \ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}-\frac{4}{3}}{3}\right)}{14641} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}11}{3}\right)} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}11}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{242\left(x^2+x+\frac{1}{4}\right)} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}179i}{15972\left(x-\frac{\sqrt{6}11}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}179i}{15972\left(x+\frac{\sqrt{6}11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 4\*x^2 + 1)/((2\*x + 1)^3\*(3\*x^2 + 2)^(3/2)),x)

```
[Out] (322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324
*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(1/2)
)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2)*(x
^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i
)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i)/(
15972*(x + (6^(1/2)*1i)/3))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

$$3.130 \quad \int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$$

**Optimal.** Leaf size=73

$$\frac{279x + 398}{162(3x^2 + 2)^{3/2}} + \frac{32}{27}\sqrt{3x^2 + 2} - \frac{465x + 152}{54\sqrt{3x^2 + 2}} + \frac{8 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {1814, 641, 215}

$$\frac{279x + 398}{162(3x^2 + 2)^{3/2}} + \frac{32}{27}\sqrt{3x^2 + 2} - \frac{465x + 152}{54\sqrt{3x^2 + 2}} + \frac{8 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (398 + 279\*x)/(162\*(2 + 3\*x^2)^(3/2)) - (152 + 465\*x)/(54\*sqrt[2 + 3\*x^2]) + (32\*sqrt[2 + 3\*x^2])/27 + (8\*ArcSinh[Sqrt[3/2]\*x])/Sqrt[3]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 641

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(e\*(a + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[d, Int[(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx &= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{1}{6} \int \frac{\frac{22}{3} - \frac{280x}{3} - 144x^2 - 64x^3}{(2+3x^2)^{3/2}} dx \\
&= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{1}{12} \int \frac{96 + \frac{128x}{3}}{\sqrt{2+3x^2}} dx \\
&= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + 8 \int \frac{1}{\sqrt{2+3x^2}} dx \\
&= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + \frac{8 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 63, normalized size = 0.86

$$\frac{1728x^4 - 4185x^3 + 936x^2 + 432\sqrt{3}(3x^2 + 2)^{3/2} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) - 2511x + 254}{162(3x^2 + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (254 - 2511\*x + 936\*x^2 - 4185\*x^3 + 1728\*x^4 + 432\*Sqrt[3]\*(2 + 3\*x^2)^(3/2)\*ArcSinh[Sqrt[3/2]\*x])/(162\*(2 + 3\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.45, size = 64, normalized size = 0.88

$$\frac{1728x^4 - 4185x^3 + 936x^2 - 2511x + 254}{162(3x^2 + 2)^{3/2}} - \frac{8 \log(\sqrt{3x^2 + 2} - \sqrt{3}x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (254 - 2511\*x + 936\*x^2 - 4185\*x^3 + 1728\*x^4)/(162\*(2 + 3\*x^2)^(3/2)) - (8\*Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]])/Sqrt[3]

**fricas** [A] time = 0.46, size = 87, normalized size = 1.19

$$\frac{216\sqrt{3}(9x^4 + 12x^2 + 4)\log(-\sqrt{3}\sqrt{3x^2 + 2}x - 3x^2 - 1) + (1728x^4 - 4185x^3 + 936x^2 - 2511x + 254)\sqrt{3x^2 + 2}}{162(9x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x, algorithm="fricas")

[Out] 1/162\*(216\*sqrt(3)\*(9\*x^4 + 12\*x^2 + 4)\*log(-sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1) + (1728\*x^4 - 4185\*x^3 + 936\*x^2 - 2511\*x + 254)\*sqrt(3\*x^2 + 2))/(9\*x^4 + 12\*x^2 + 4)

**giac** [A] time = 0.18, size = 53, normalized size = 0.73

$$-\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3x^2 + 2}\right) + \frac{9((3(64x - 155)x + 104)x - 279)x + 254}{162(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x, algorithm="giac")

[Out] -8/3\*sqrt(3)\*log(-sqrt(3)\*x + sqrt(3\*x^2 + 2)) + 1/162\*(9\*((3\*(64\*x - 155)\*x + 104)\*x - 279)\*x + 254)/(3\*x^2 + 2)^(3/2)

**maple** [A] time = 0.01, size = 91, normalized size = 1.25

$$\frac{32x^4}{3(3x^2 + 2)^{\frac{3}{2}}} - \frac{8x^3}{(3x^2 + 2)^{\frac{3}{2}}} + \frac{52x^2}{9(3x^2 + 2)^{\frac{3}{2}}} - \frac{107x}{18\sqrt{3x^2 + 2}} - \frac{65x}{18(3x^2 + 2)^{\frac{3}{2}}} + \frac{8\sqrt{3}\operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{3} + \frac{127}{81(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x)

[Out] 32/3\*x^4/(3\*x^2+2)^(3/2)+52/9\*x^2/(3\*x^2+2)^(3/2)+127/81/(3\*x^2+2)^(3/2)-8\*x^3/(3\*x^2+2)^(3/2)-107/18/(3\*x^2+2)^(1/2)\*x+8/3\*arcsinh(1/2\*sqrt(6)\*x)\*3^(1/2)-65/18\*x/(3\*x^2+2)^(3/2)

**maxima** [A] time = 0.95, size = 105, normalized size = 1.44

$$\frac{32x^4}{3(3x^2 + 2)^{\frac{3}{2}}} - \frac{8}{3}x\left(\frac{9x^2}{(3x^2 + 2)^{\frac{3}{2}}} + \frac{4}{(3x^2 + 2)^{\frac{3}{2}}}\right) + \frac{8}{3}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{11x}{18\sqrt{3x^2 + 2}} + \frac{52x^2}{9(3x^2 + 2)^{\frac{3}{2}}} - \frac{65x}{18(3x^2 + 2)^{\frac{3}{2}}} + \frac{127}{81(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x, algorithm="maxima")

[Out]  $32/3*x^4/(3*x^2 + 2)^{(3/2)} - 8/3*x*(9*x^2/(3*x^2 + 2)^{(3/2)} + 4/(3*x^2 + 2)^{(3/2)}) + 8/3*\text{sqrt}(3)*\text{arcsinh}(1/2*\text{sqrt}(6)*x) - 11/18*x/\text{sqrt}(3*x^2 + 2) + 52/9*x^2/(3*x^2 + 2)^{(3/2)} - 65/18*x/(3*x^2 + 2)^{(3/2)} + 127/81/(3*x^2 + 2)^{(3/2)}$

**mupad [B]** time = 0.06, size = 212, normalized size = 2.90

$$\frac{32\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{27} + \frac{8\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{3} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}199i}{144}}{x-\frac{\sqrt{6}11}{3}} - \frac{\sqrt{6}\left(\frac{31}{24}+\frac{\sqrt{6}199i}{216}\right)11}{2\left(x-\frac{\sqrt{6}11}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}199i}{144}}{x+\frac{\sqrt{6}11}{3}} + \frac{\sqrt{6}\left(\frac{31}{24}+\frac{\sqrt{6}199i}{216}\right)11}{2\left(x+\frac{\sqrt{6}11}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{6}\left(-1824+\sqrt{6}1953i\right)\sqrt{x^2+\frac{2}{3}}1i}{7776\left(x+\frac{\sqrt{6}11}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(1824+\sqrt{6}1953i\right)\sqrt{x^2+\frac{2}{3}}1i}{7776\left(x-\frac{\sqrt{6}11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)^3\*(3\*x + 4\*x^2 + 1))/(3\*x^2 + 2)^(5/2),x)

[Out]  $(32*3^{(1/2)}*(x^2 + 2/3)^{(1/2)})/27 + (8*3^{(1/2)}*\operatorname{asinh}((2^{(1/2)}*3^{(1/2)}*x)/2))/3 - (3^{(1/2)}*(x^2 + 2/3)^{(1/2)}*((6^{(1/2)}*199i)/144 - 31/16)/(x - (6^{(1/2)}*1i)/3) - (6^{(1/2)}*((6^{(1/2)}*199i)/216 - 31/24)*1i)/(2*(x - (6^{(1/2)}*1i)/3)^2))/27 + (3^{(1/2)}*(x^2 + 2/3)^{(1/2)}*((6^{(1/2)}*199i)/144 + 31/16)/(x + (6^{(1/2)}*1i)/3) + (6^{(1/2)}*((6^{(1/2)}*199i)/216 + 31/24)*1i)/(2*(x + (6^{(1/2)}*1i)/3)^2))/27 + (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)}*1953i - 1824)*(x^2 + 2/3)^{(1/2)}*1i)/(7776*(x + (6^{(1/2)}*1i)/3)) + (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)}*1953i + 1824)*(x^2 + 2/3)^{(1/2)}*1i)/(7776*(x - (6^{(1/2)}*1i)/3))$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3(4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*3\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2+2)\*\*(5/2),x)

[Out] Integral((2\*x + 1)\*\*3\*(4\*x\*\*2 + 3\*x + 1)/(3\*x\*\*2 + 2)\*\*(5/2), x)

$$3.131 \quad \int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$$

Optimal. Leaf size=60

$$\frac{70 - 47x}{54(3x^2 + 2)^{3/2}} - \frac{59x + 168}{54\sqrt{3x^2 + 2}} + \frac{16 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

**Rubi [A]** time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {1814, 12, 215}

$$\frac{70 - 47x}{54(3x^2 + 2)^{3/2}} - \frac{59x + 168}{54\sqrt{3x^2 + 2}} + \frac{16 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (70 - 47\*x)/(54\*(2 + 3\*x^2)^(3/2)) - (168 + 59\*x)/(54\*sqrt[2 + 3\*x^2]) + (16\*ArcSinh[Sqrt[3/2]\*x])/(9\*sqrt[3])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 1814

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

#### Rubi steps



$$\begin{aligned}
\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx &= \frac{70-47x}{54(2+3x^2)^{3/2}} - \frac{1}{6} \int \frac{-\frac{74}{9} - 56x - 32x^2}{(2+3x^2)^{3/2}} dx \\
&= \frac{70-47x}{54(2+3x^2)^{3/2}} - \frac{168+59x}{54\sqrt{2+3x^2}} + \frac{1}{12} \int \frac{64}{3\sqrt{2+3x^2}} dx \\
&= \frac{70-47x}{54(2+3x^2)^{3/2}} - \frac{168+59x}{54\sqrt{2+3x^2}} + \frac{16}{9} \int \frac{1}{\sqrt{2+3x^2}} dx \\
&= \frac{70-47x}{54(2+3x^2)^{3/2}} - \frac{168+59x}{54\sqrt{2+3x^2}} + \frac{16 \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 58, normalized size = 0.97

$$\frac{-177x^3 - 504x^2 + 32\sqrt{3}(3x^2 + 2)^{3/2} \sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right) - 165x - 266}{54(3x^2 + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (-266 - 165\*x - 504\*x^2 - 177\*x^3 + 32\*Sqrt[3]\*(2 + 3\*x^2)^(3/2)\*ArcSinh[Sqrt[3/2]\*x])/(54\*(2 + 3\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.42, size = 61, normalized size = 1.02

$$\frac{-177x^3 - 504x^2 - 165x - 266}{54(3x^2 + 2)^{3/2}} - \frac{16 \log(\sqrt{3x^2 + 2} - \sqrt{3}x)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (-266 - 165\*x - 504\*x^2 - 177\*x^3)/(54\*(2 + 3\*x^2)^(3/2)) - (16\*Log[-(Sqrt[3]\*x) + Sqrt[2 + 3\*x^2]])/(9\*Sqrt[3])

**fricas [A]** time = 0.65, size = 83, normalized size = 1.38

$$\frac{16\sqrt{3}(9x^4 + 12x^2 + 4) \log(-\sqrt{3}\sqrt{3x^2 + 2}x - 3x^2 - 1) - (177x^3 + 504x^2 + 165x + 266)\sqrt{3x^2 + 2}}{54(9x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x, algorithm="fricas")

[Out] 1/54\*(16\*sqrt(3)\*(9\*x^4 + 12\*x^2 + 4)\*log(-sqrt(3)\*sqrt(3\*x^2 + 2)\*x - 3\*x^2 - 1) - (177\*x^3 + 504\*x^2 + 165\*x + 266)\*sqrt(3\*x^2 + 2))/(9\*x^4 + 12\*x^2 + 4)

**giac** [A] time = 0.19, size = 48, normalized size = 0.80

$$-\frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3x^2 + 2}\right) - \frac{3((59x + 168)x + 55)x + 266}{54(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x, algorithm="giac")

[Out] -16/27\*sqrt(3)\*log(-sqrt(3)\*x + sqrt(3\*x^2 + 2)) - 1/54\*(3\*((59\*x + 168)\*x + 55)\*x + 266)/(3\*x^2 + 2)^(3/2)

**maple** [A] time = 0.01, size = 77, normalized size = 1.28

$$-\frac{16x^3}{9(3x^2 + 2)^{\frac{3}{2}}} - \frac{28x^2}{3(3x^2 + 2)^{\frac{3}{2}}} - \frac{x}{2\sqrt{3x^2 + 2}} - \frac{37x}{18(3x^2 + 2)^{\frac{3}{2}}} + \frac{16\sqrt{3}\operatorname{arcsinh}\left(\frac{\sqrt{6}x}{2}\right)}{27} - \frac{133}{27(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x)

[Out] -16/9/(3\*x^2+2)^(3/2)\*x^3-1/2/(3\*x^2+2)^(1/2)\*x+16/27\*arcsinh(1/2\*sqrt(6)\*x)\*x^3-(1/2)-28/3/(3\*x^2+2)^(3/2)\*x^2-133/27/(3\*x^2+2)^(3/2)-37/18/(3\*x^2+2)^(3/2)\*x

**maxima** [B] time = 0.96, size = 91, normalized size = 1.52

$$-\frac{16}{27}x\left(\frac{9x^2}{(3x^2 + 2)^{\frac{3}{2}}} + \frac{4}{(3x^2 + 2)^{\frac{3}{2}}}\right) + \frac{16}{27}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{37x}{54\sqrt{3x^2 + 2}} - \frac{28x^2}{3(3x^2 + 2)^{\frac{3}{2}}} - \frac{37x}{18(3x^2 + 2)^{\frac{3}{2}}} - \frac{133}{27(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2),x, algorithm="maxima")

[Out] -16/27\*x\*(9\*x^2/(3\*x^2 + 2)^(3/2) + 4/(3\*x^2 + 2)^(3/2)) + 16/27\*sqrt(3)\*arcsinh(1/2\*sqrt(6)\*x) + 37/54\*x/sqrt(3\*x^2 + 2) - 28/3\*x^2/(3\*x^2 + 2)^(3/2) - 37/18\*x/(3\*x^2 + 2)^(3/2) - 133/27/(3\*x^2 + 2)^(3/2)

**mupad [B]** time = 0.05, size = 200, normalized size = 3.33

$$\frac{16\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2 + \frac{2}{3}}\left(\frac{-\frac{47}{48} + \frac{\sqrt{6}35i}{48}}{x + \frac{\sqrt{6}11}{3}} + \frac{\sqrt{6}\left(-\frac{47}{72} + \frac{\sqrt{6}35i}{72}\right)11}{2\left(x + \frac{\sqrt{6}11}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2 + \frac{2}{3}}\left(\frac{\frac{47}{48} + \frac{\sqrt{6}35i}{48}}{x - \frac{\sqrt{6}11}{3}} - \frac{\sqrt{6}\left(\frac{47}{72} + \frac{\sqrt{6}35i}{72}\right)11}{2\left(x - \frac{\sqrt{6}11}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{6}\left(-672 + \sqrt{6}63i\right)\sqrt{x^2 + \frac{2}{3}}11}{2592\left(x + \frac{\sqrt{6}11}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(672 + \sqrt{6}63i\right)\sqrt{x^2 + \frac{2}{3}}11}{2592\left(x - \frac{\sqrt{6}11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2), x)`

[Out]  $(16*3^{(1/2)}*\operatorname{asinh}((2^{(1/2)}*3^{(1/2)}*x)/2))/27 + (3^{(1/2)}*(x^2 + 2/3)^{(1/2)}*((6^{(1/2)}*35i)/48 - 47/48)/(x + (6^{(1/2)}*11)/3) + (6^{(1/2)}*((6^{(1/2)}*35i)/72 - 47/72)*11)/(2*(x + (6^{(1/2)}*11)/3)^2))/27 - (3^{(1/2)}*(x^2 + 2/3)^{(1/2)}*((6^{(1/2)}*35i)/48 + 47/48)/(x - (6^{(1/2)}*11)/3) - (6^{(1/2)}*((6^{(1/2)}*35i)/72 + 47/72)*11)/(2*(x - (6^{(1/2)}*11)/3)^2))/27 + (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)}*63i - 672)*(x^2 + 2/3)^{(1/2)}*11)/(2592*(x + (6^{(1/2)}*11)/3)) + (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)}*63i + 672)*(x^2 + 2/3)^{(1/2)}*11)/(2592*(x - (6^{(1/2)}*11)/3))$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x + 1)^2 (4x^2 + 3x + 1)}{(3x^2 + 2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2), x)`

[Out] `Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)`

$$3.132 \quad \int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx$$

Optimal. Leaf size=41

$$\frac{2-51x}{54(3x^2+2)^{3/2}} - \frac{16-13x}{18\sqrt{3x^2+2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {1814, 637}

$$\frac{2-51x}{54(3x^2+2)^{3/2}} - \frac{16-13x}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (2 - 51\*x)/(54\*(2 + 3\*x^2)^(3/2)) - (16 - 13\*x)/(18\*sqrt[2 + 3\*x^2])

Rule 637

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2)^(3/2), x\_Symbol] :> Simp[(-(a\*e) + c\*d\*x)/(a\*c\*Sqrt[a + c\*x^2]), x] /; FreeQ[{a, c, d, e}, x]

Rule 1814

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x^2, x], x, 1]}, Simp[((a\*g - b\*f\*x)\*(a + b\*x^2)^(p + 1))/(2\*a\*b\*(p + 1)), x] + Dist[1/(2\*a\*(p + 1)), Int[(a + b\*x^2)^(p + 1)\*ExpandToSum[2\*a\*(p + 1)\*Q + f\*(2\*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}} dx &= \frac{2-51x}{54(2+3x^2)^{3/2}} - \frac{1}{6} \int \frac{-\frac{26}{3}-16x}{(2+3x^2)^{3/2}} dx \\ &= \frac{2-51x}{54(2+3x^2)^{3/2}} - \frac{16-13x}{18\sqrt{2+3x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 0.73

$$\frac{117x^3 - 144x^2 + 27x - 94}{54(3x^2 + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (-94 + 27\*x - 144\*x^2 + 117\*x^3)/(54\*(2 + 3\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.38, size = 30, normalized size = 0.73

$$\frac{117x^3 - 144x^2 + 27x - 94}{54(3x^2 + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 + 3\*x^2)^(5/2), x]

[Out] (-94 + 27\*x - 144\*x^2 + 117\*x^3)/(54\*(2 + 3\*x^2)^(3/2))

**fricas [A]** time = 0.95, size = 40, normalized size = 0.98

$$\frac{(117x^3 - 144x^2 + 27x - 94)\sqrt{3x^2 + 2}}{54(9x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2), x, algorithm="fricas")

[Out] 1/54\*(117\*x^3 - 144\*x^2 + 27\*x - 94)\*sqrt(3\*x^2 + 2)/(9\*x^4 + 12\*x^2 + 4)

**giac [A]** time = 0.34, size = 25, normalized size = 0.61

$$\frac{9((13x - 16)x + 3)x - 94}{54(3x^2 + 2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2), x, algorithm="giac")

[Out] 1/54\*(9\*((13\*x - 16)\*x + 3)\*x - 94)/(3\*x^2 + 2)^(3/2)

**maple [A]** time = 0.00, size = 27, normalized size = 0.66

$$\frac{117x^3 - 144x^2 + 27x - 94}{54(3x^2 + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2), x)

[Out] 1/54\*(117\*x^3-144\*x^2+27\*x-94)/(3\*x^2+2)^(3/2)

**maxima [A]** time = 0.42, size = 50, normalized size = 1.22

$$\frac{13x}{18\sqrt{3x^2+2}} - \frac{8x^2}{3(3x^2+2)^{\frac{3}{2}}} - \frac{17x}{18(3x^2+2)^{\frac{3}{2}}} - \frac{47}{27(3x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2+2)^(5/2), x, algorithm="maxima")

[Out] 13/18\*x/sqrt(3\*x^2 + 2) - 8/3\*x^2/(3\*x^2 + 2)^(3/2) - 17/18\*x/(3\*x^2 + 2)^(3/2) - 47/27/(3\*x^2 + 2)^(3/2)

**mupad [B]** time = 4.11, size = 185, normalized size = 4.51

$$\frac{\sqrt{3} \sqrt{x^2 + \frac{2}{3}} \left( \frac{\frac{17}{16} + \frac{\sqrt{6} 11}{48}}{x + \frac{\sqrt{6} 11}{3}} + \frac{\sqrt{6} \left( \frac{17}{24} + \frac{\sqrt{6} 11}{72} \right) 11}{2 \left( x + \frac{\sqrt{6} 11}{3} \right)^2} \right)}{27} - \frac{\sqrt{3} \sqrt{x^2 + \frac{2}{3}} \left( \frac{\frac{17}{16} + \frac{\sqrt{6} 11}{48}}{x - \frac{\sqrt{6} 11}{3}} - \frac{\sqrt{6} \left( \frac{17}{24} + \frac{\sqrt{6} 11}{72} \right) 11}{2 \left( x - \frac{\sqrt{6} 11}{3} \right)^2} \right)}{27} - \frac{\sqrt{3} \sqrt{6} (-192 + \sqrt{6} 69i) \sqrt{x^2 + \frac{2}{3}} 11}{2592 \left( x - \frac{\sqrt{6} 11}{3} \right)} - \frac{\sqrt{3} \sqrt{6} (192 + \sqrt{6} 69i) \sqrt{x^2 + \frac{2}{3}} 11}{2592 \left( x + \frac{\sqrt{6} 11}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)\*(3\*x + 4\*x^2 + 1))/(3\*x^2 + 2)^(5/2), x)

[Out] (3^(1/2)\*(x^2 + 2/3)^(1/2)\*(((6^(1/2)\*11)/48 - 17/16)/(x + (6^(1/2)\*11)/3) + (6^(1/2)\*((6^(1/2)\*11)/72 - 17/24)\*11)/(2\*(x + (6^(1/2)\*11)/3)^2))/27 - (3^(1/2)\*(x^2 + 2/3)^(1/2)\*(((6^(1/2)\*11)/48 + 17/16)/(x - (6^(1/2)\*11)/3) - (6^(1/2)\*((6^(1/2)\*11)/72 + 17/24)\*11)/(2\*(x - (6^(1/2)\*11)/3)^2))/27 - (3^(1/2)\*6^(1/2)\*(6^(1/2)\*69i - 192)\*(x^2 + 2/3)^(1/2)\*11)/(2592\*(x - (6^(1/2)\*11)/3)) - (3^(1/2)\*6^(1/2)\*(6^(1/2)\*69i + 192)\*(x^2 + 2/3)^(1/2)\*11)/(2592\*(x + (6^(1/2)\*11)/3))

**sympy [B]** time = 77.50, size = 180, normalized size = 4.39

$$\frac{10x^3}{18x^2\sqrt{3x^2+2} + 12\sqrt{3x^2+2}} + \frac{x^3}{6x^2\sqrt{3x^2+2} + 4\sqrt{3x^2+2}} - \frac{72x^2}{81x^2\sqrt{3x^2+2} + 54\sqrt{3x^2+2}} + \frac{x}{6x^2\sqrt{3x^2+2} + 4\sqrt{3x^2+2}} - \frac{32}{81x^2\sqrt{3x^2+2} + 54\sqrt{3x^2+2}} - \frac{5}{27x^2\sqrt{3x^2+2} + 18\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] 10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 32/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x**2 + 2) + 18*sqrt(3*x**2 + 2))
```

$$3.133 \quad \int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{5/2}} dx$$

Optimal. Leaf size=73

$$\frac{21x - 38}{198(3x^2 + 2)^{3/2}} + \frac{95x + 24}{726\sqrt{3x^2 + 2}} - \frac{8 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

**Rubi [A]** time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1647, 823, 12, 725, 206}

$$-\frac{38 - 21x}{198(3x^2 + 2)^{3/2}} + \frac{95x + 24}{726\sqrt{3x^2 + 2}} - \frac{8 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(5/2)), x]

[Out] -(38 - 21\*x)/(198\*(2 + 3\*x^2)^(3/2)) + (24 + 95\*x)/(726\*sqrt[2 + 3\*x^2]) - (8\*ArcTanh[(4 - 3\*x)/(sqrt[11]\*sqrt[2 + 3\*x^2])])/(121\*sqrt[11])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 823

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(f\*a\*c\*e - a\*g\*c\*d + c\*(c\*d\*f + a



```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

### Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1 + 3x + 4x^2}{(1 + 2x)(2 + 3x^2)^{5/2}} dx &= -\frac{38 - 21x}{198(2 + 3x^2)^{3/2}} - \frac{1}{18} \int \frac{-\frac{78}{11} - \frac{84x}{11}}{(1 + 2x)(2 + 3x^2)^{3/2}} dx \\
 &= -\frac{38 - 21x}{198(2 + 3x^2)^{3/2}} + \frac{24 + 95x}{726\sqrt{2 + 3x^2}} + \frac{\int \frac{864}{11(1+2x)\sqrt{2+3x^2}} dx}{1188} \\
 &= -\frac{38 - 21x}{198(2 + 3x^2)^{3/2}} + \frac{24 + 95x}{726\sqrt{2 + 3x^2}} + \frac{8}{121} \int \frac{1}{(1 + 2x)\sqrt{2 + 3x^2}} dx \\
 &= -\frac{38 - 21x}{198(2 + 3x^2)^{3/2}} + \frac{24 + 95x}{726\sqrt{2 + 3x^2}} - \frac{8}{121} \operatorname{Subst}\left(\int \frac{1}{11 - x^2} dx, x, \frac{4 - 3x}{\sqrt{2 + 3x^2}}\right) \\
 &= -\frac{38 - 21x}{198(2 + 3x^2)^{3/2}} + \frac{24 + 95x}{726\sqrt{2 + 3x^2}} - \frac{8 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{11}\sqrt{2 + 3x^2}}\right)}{121\sqrt{11}}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 58, normalized size = 0.79

$$\frac{855x^3 + 216x^2 + 801x - 274}{2178(3x^2 + 2)^{3/2}} - \frac{8 \tanh^{-1}\left(\frac{4 - 3x}{\sqrt{33x^2 + 22}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(5/2)), x]

[Out] (-274 + 801\*x + 216\*x^2 + 855\*x^3)/(2178\*(2 + 3\*x^2)^(3/2)) - (8\*ArcTanh[(4 - 3\*x)/Sqrt[22 + 33\*x^2]])/(121\*Sqrt[11])

**IntegrateAlgebraic** [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1 + 3x + 4x^2}{(1 + 2x)(2 + 3x^2)^{5/2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(5/2)), x]

[Out] Defer[IntegrateAlgebraic] [(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 + 3\*x^2)^(5/2)), x]

**fricas** [A] time = 0.99, size = 103, normalized size = 1.41

$$\frac{72\sqrt{11}(9x^4 + 12x^2 + 4)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right) + 11(855x^3 + 216x^2 + 801x - 274)\sqrt{3x^2+2}}{23958(9x^4 + 12x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(5/2), x, algorithm="fricas")

[Out] 1/23958\*(72\*sqrt(11)\*(9\*x^4 + 12\*x^2 + 4)\*log(-(sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) + 21\*x^2 - 12\*x + 19)/(4\*x^2 + 4\*x + 1)) + 11\*(855\*x^3 + 216\*x^2 + 801\*x - 274)\*sqrt(3\*x^2 + 2))/(9\*x^4 + 12\*x^2 + 4)

**giac** [A] time = 0.21, size = 91, normalized size = 1.25

$$\frac{8}{1331}\sqrt{11}\log\left(-\frac{|-2\sqrt{3}x - \sqrt{11} - \sqrt{3} + 2\sqrt{3x^2+2}|}{2\sqrt{3}x - \sqrt{11} + \sqrt{3} - 2\sqrt{3x^2+2}}\right) + \frac{9((95x+24)x+89)x-274}{2178(3x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2+2)^(5/2), x, algorithm="giac")

[Out] 8/1331\*sqrt(11)\*log(-abs(-2\*sqrt(3)\*x - sqrt(11) - sqrt(3) + 2\*sqrt(3\*x^2 + 2))/(2\*sqrt(3)\*x - sqrt(11) + sqrt(3) - 2\*sqrt(3\*x^2 + 2))) + 1/2178\*(9\*((95\*x + 24)\*x + 89)\*x - 274)/(3\*x^2 + 2)^(3/2)

**maple [B]** time = 0.01, size = 133, normalized size = 1.82

$$\frac{x}{12(3x^2+2)^{\frac{3}{2}}} + \frac{x}{12\sqrt{3x^2+2}} + \frac{x}{44(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} + \frac{23x}{484\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{8\sqrt{11} \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12(x+\frac{1}{2})^2+5}}\right)}{1331} - \frac{2}{9(3x^2+2)^{\frac{3}{2}}} + \frac{1}{33(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} + \frac{4}{121\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2), x)`

[Out]  $-2/9/(3*x^2+2)^{(3/2)}+1/12/(3*x^2+2)^{(3/2)}*x+1/12/(3*x^2+2)^{(1/2)}*x+1/33/(-3*x+3*(x+1/2)^2+5/4)^{(3/2)}+1/44*x/(-3*x+3*(x+1/2)^2+5/4)^{(3/2)}+23/484/(-3*x+3*(x+1/2)^2+5/4)^{(1/2)}*x+4/121/(-3*x+3*(x+1/2)^2+5/4)^{(1/2)}-8/1331*11^{(1/2)}*\operatorname{arctanh}(2/11*(-3*x+4)*11^{(1/2)}/(-12*x+12*(x+1/2)^2+5)^{(1/2)})$

**maxima [A]** time = 0.97, size = 81, normalized size = 1.11

$$\frac{8}{1331} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) + \frac{95x}{726\sqrt{3x^2+2}} + \frac{4}{121\sqrt{3x^2+2}} + \frac{7x}{66(3x^2+2)^{\frac{3}{2}}} - \frac{19}{99(3x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2), x, algorithm="maxima")`

[Out]  $8/1331*\operatorname{sqrt}(11)*\operatorname{arcsinh}(1/2*\operatorname{sqrt}(6)*x/\operatorname{abs}(2*x+1) - 2/3*\operatorname{sqrt}(6)/\operatorname{abs}(2*x+1)) + 95/726*x/\operatorname{sqrt}(3*x^2+2) + 4/121/\operatorname{sqrt}(3*x^2+2) + 7/66*x/(3*x^2+2)^{(3/2)} - 19/99/(3*x^2+2)^{(3/2)}$

**mupad [B]** time = 0.13, size = 218, normalized size = 2.99

$$\frac{\sqrt{11} \left( 8 \ln\left(x+\frac{1}{2}\right) - 8 \ln\left(x - \frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}} - \frac{4}{3}}{3}\right) \right)}{1331} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}} \left( \frac{\frac{21}{276} + \frac{\sqrt{6}19i}{276}}{x+\frac{\sqrt{6}11}{3}} + \frac{\sqrt{6}\left(\frac{7}{88} + \frac{\sqrt{6}19i}{264}\right)11}{2\left(x+\frac{\sqrt{6}11}{3}\right)^2} \right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}} \left( \frac{\frac{21}{276} + \frac{\sqrt{6}19i}{276}}{x-\frac{\sqrt{6}11}{3}} - \frac{\sqrt{6}\left(\frac{7}{88} + \frac{\sqrt{6}19i}{264}\right)11}{2\left(x-\frac{\sqrt{6}11}{3}\right)^2} \right)}{27} - \frac{\sqrt{3}\sqrt{6}(-288+\sqrt{6}303i)\sqrt{x^2+\frac{2}{3}}11}{104544\left(x+\frac{\sqrt{6}11}{3}\right)} - \frac{\sqrt{3}\sqrt{6}(288+\sqrt{6}303i)\sqrt{x^2+\frac{2}{3}}11}{104544\left(x-\frac{\sqrt{6}11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)), x)`

[Out]  $(11^{(1/2)}*(8*\log(x+1/2) - 8*\log(x - (3^{(1/2)}*11^{(1/2)}*(x^2+2/3)^{(1/2)})/3 - 4/3)))/1331 - (3^{(1/2)}*(x^2+2/3)^{(1/2)}*((6^{(1/2)}*19i)/176 - 21/176)/(x + (6^{(1/2)}*11)/3) + (6^{(1/2)}*((6^{(1/2)}*19i)/264 - 7/88)*11)/(2*(x + (6^{(1/2)}*11)/3)^2))/27 + (3^{(1/2)}*(x^2+2/3)^{(1/2)}*((6^{(1/2)}*19i)/176 + 21/176)/(x - (6^{(1/2)}*11)/3) - (6^{(1/2)}*((6^{(1/2)}*19i)/264 + 7/88)*11)/(2*(x - (6^{(1/2)}*11)/3)^2))/27 - (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)}*303i - 288)*(x^2+2/3)^{(1/2)}*11)/(104544*(x + (6^{(1/2)}*11)/3)) - (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)}*303i + 288)*(x^2+2/3)^{(1/2)}*11)/(104544*(x - (6^{(1/2)}*11)/3))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)/(1+2\*x)/(3\*x\*\*2+2)\*\*(5/2), x)

[Out] Timed out

$$3.134 \quad \int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}} dx$$

Optimal. Leaf size=95

$$\frac{97x-10}{726(3x^2+2)^{3/2}} - \frac{16\sqrt{3x^2+2}}{1331(2x+1)} + \frac{887x+24}{7986\sqrt{3x^2+2}} - \frac{32 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

**Rubi** [A] time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{726(3x^2+2)^{3/2}} - \frac{16\sqrt{3x^2+2}}{1331(2x+1)} + \frac{887x+24}{7986\sqrt{3x^2+2}} - \frac{32 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(5/2)), x]

[Out] -(10 - 97\*x)/(726\*(2 + 3\*x^2)^(3/2)) + (24 + 887\*x)/(7986\*sqrt[2 + 3\*x^2]) - (16\*sqrt[2 + 3\*x^2])/(1331\*(1 + 2\*x)) - (32\*ArcTanh[(4 - 3\*x)/(sqrt[11]\*sqrt[2 + 3\*x^2])])/(1331\*sqrt[11])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 807

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m+1)\*(a + c\*x^2)^(p+1))/(2\*(p+1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m+1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 (2 + 3x^2)^{5/2}} dx &= -\frac{10 - 97x}{726 (2 + 3x^2)^{3/2}} - \frac{1}{18} \int \frac{\frac{798}{121} - \frac{1968x}{121} - \frac{2328x^2}{121}}{(1 + 2x)^2 (2 + 3x^2)^{3/2}} dx \\
&= -\frac{10 - 97x}{726 (2 + 3x^2)^{3/2}} + \frac{24 + 887x}{7986\sqrt{2 + 3x^2}} + \frac{1}{108} \int \frac{\frac{10368}{1331} + \frac{1728x}{1331}}{(1 + 2x)^2 \sqrt{2 + 3x^2}} dx \\
&= -\frac{10 - 97x}{726 (2 + 3x^2)^{3/2}} + \frac{24 + 887x}{7986\sqrt{2 + 3x^2}} - \frac{16\sqrt{2 + 3x^2}}{1331(1 + 2x)} + \frac{32 \int \frac{1}{(1+2x)\sqrt{2+3x^2}} dx}{1331} \\
&= -\frac{10 - 97x}{726 (2 + 3x^2)^{3/2}} + \frac{24 + 887x}{7986\sqrt{2 + 3x^2}} - \frac{16\sqrt{2 + 3x^2}}{1331(1 + 2x)} - \frac{32 \operatorname{Subst}\left(\int \frac{1}{11-x^2} dx, x, \frac{4}{\sqrt{2}}\right)}{1331} \\
&= -\frac{10 - 97x}{726 (2 + 3x^2)^{3/2}} + \frac{24 + 887x}{7986\sqrt{2 + 3x^2}} - \frac{16\sqrt{2 + 3x^2}}{1331(1 + 2x)} - \frac{32 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 91, normalized size = 0.96

$$\frac{11(4458x^4 + 2805x^3 + 4602x^2 + 2717x - 446) - 192\sqrt{33x^2 + 22}(6x^3 + 3x^2 + 4x + 2) \tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{87846(2x+1)(3x^2+2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(5/2)), x]

[Out]  $(11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*\text{Sqrt}[22 + 33*x^2])*(2 + 4*x + 3*x^2 + 6*x^3)*\text{ArcTanh}[(4 - 3*x)/\text{Sqrt}[22 + 33*x^2]]/(87846*(1 + 2*x)*(2 + 3*x^2)^{(3/2)})$

**IntegrateAlgebraic [F]** time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 (2 + 3x^2)^{5/2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(5/2)),x]

[Out] Defer[IntegrateAlgebraic][(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 + 3\*x^2)^(5/2)), x]

**fricas [A]** time = 0.56, size = 134, normalized size = 1.41

$$\frac{96\sqrt{11}(18x^5 + 9x^4 + 24x^3 + 12x^2 + 8x + 4)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right) + 11(4458x^4 + 2805x^3 + 4602x^2 + 2717x - 446)\sqrt{3x^2+2}}{87846(18x^5 + 9x^4 + 24x^3 + 12x^2 + 8x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(5/2),x, algorithm="fricas")

[Out]  $1/87846*(96*\text{sqrt}(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*\log(-(\text{sqrt}(11)*\text{sqrt}(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*\text{sqrt}(3*x^2 + 2))/(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)$

**giac [B]** time = 0.52, size = 233, normalized size = 2.45

$$-\frac{1}{263538}\sqrt{11}(743\sqrt{11}\sqrt{3} - 576\log(\sqrt{11}\sqrt{3} - 3))\text{sgn}\left(\frac{1}{2x+1}\right) - \frac{32\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3} + \frac{\sqrt{11}}{2x+1}\right) - 3\right)}{14641\text{sgn}\left(\frac{1}{2x+1}\right)} + \frac{\frac{11\left(\frac{731}{\text{sgn}\left(\frac{1}{2x+1}\right)} + \frac{528}{(2x+1)\text{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1} - \frac{14163}{\text{sgn}\left(\frac{1}{2x+1}\right)} + \frac{6111}{\text{sgn}\left(\frac{1}{2x+1}\right)} - \frac{2229}{\text{sgn}\left(\frac{1}{2x+1}\right)}}{7986\left(\frac{6}{2x+1} - \frac{11}{(2x+1)^2} - 3\right)\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2+2)^(5/2),x, algorithm="giac")

[Out]  $-1/263538*\text{sqrt}(11)*(743*\text{sqrt}(11)*\text{sqrt}(3) - 576*\log(\text{sqrt}(11)*\text{sqrt}(3) - 3))*\text{sgn}(1/(2*x + 1)) - 32/14641*\text{sqrt}(11)*\log(\text{sqrt}(11)*(\text{sqrt}(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + \text{sqrt}(11)/(2*x + 1)) - 3)/\text{sgn}(1/(2*x + 1)) + 1/7986*((11*(731/\text{sgn}(1/(2*x + 1))) + 528/((2*x + 1)*\text{sgn}(1/(2*x + 1))))/(2*x + 1) - 14163/\text{sgn}(1/(2*x + 1)))/(2*x + 1) + 6111/\text{sgn}(1/(2*x + 1)))/(2*x + 1) - 2229/\text{sgn}(1$

$$\frac{x}{6(3x^2+2)^{\frac{3}{2}}} + \frac{x}{6\sqrt{3x^2+2}} - \frac{10x}{121(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} - \frac{98x}{1331\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{32\sqrt{11} \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12(x+\frac{1}{2})^2+5}}\right)}{14641} + \frac{4}{363(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} + \frac{16}{1331\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{1}{22(x+\frac{1}{2})(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}$$

**maple [A]** time = 0.01, size = 143, normalized size = 1.51

$$\frac{x}{6(3x^2+2)^{\frac{3}{2}}} + \frac{x}{6\sqrt{3x^2+2}} - \frac{10x}{121(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} - \frac{98x}{1331\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{32\sqrt{11} \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12(x+\frac{1}{2})^2+5}}\right)}{14641} + \frac{4}{363(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} + \frac{16}{1331\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{1}{22(x+\frac{1}{2})(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2), x)`

[Out]  $\frac{1}{6(3x^2+2)^{\frac{3}{2}}}x + \frac{1}{6(3x^2+2)^{\frac{1}{2}}}x + \frac{4}{363(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} - \frac{10}{121(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}x - \frac{98}{1331(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{1}{2}}}x + \frac{16}{1331(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{1}{2}}} - \frac{32}{14641}11^{\frac{1}{2}}\operatorname{arctanh}\left(\frac{2(-3x+4)}{11\sqrt{-12x+12(x+\frac{1}{2})^2+5}}\right) - \frac{1}{22(x+\frac{1}{2})}(-3x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}$

**maxima [A]** time = 0.98, size = 107, normalized size = 1.13

$$\frac{32}{14641}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) + \frac{743x}{7986\sqrt{3x^2+2}} + \frac{16}{1331\sqrt{3x^2+2}} + \frac{61x}{726(3x^2+2)^{\frac{3}{2}}} - \frac{1}{11\left(2(3x^2+2)^{\frac{3}{2}}x + (3x^2+2)^{\frac{3}{2}}\right)} + \frac{4}{363(3x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2), x, algorithm="maxima")`

[Out]  $\frac{32}{14641}\sqrt{11}\operatorname{arcsinh}\left(\frac{1}{2}\sqrt{6}x/\operatorname{abs}(2x+1) - \frac{2}{3}\sqrt{6}/\operatorname{abs}(2x+1)\right) + \frac{743}{7986}x/\sqrt{3x^2+2} + \frac{16}{1331}\sqrt{3x^2+2} + \frac{61x}{726(3x^2+2)^{\frac{3}{2}}} - \frac{1}{11(2(3x^2+2)^{\frac{3}{2}}x + (3x^2+2)^{\frac{3}{2}})} + \frac{4}{363(3x^2+2)^{\frac{3}{2}}}$

**mupad [B]** time = 4.31, size = 270, normalized size = 2.84

$$\frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right) - 8\ln\left(x - \frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3} - \frac{4}{3}\right)\right)}{14641} + \frac{\sqrt{11}\left(\frac{48\ln\left(x+\frac{1}{2}\right)}{1331} - \frac{48\ln\left(\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3} - \frac{4}{3}\right)}{1331}\right)}{22} - \frac{8\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{291\sqrt{3}\sqrt{11}}{1936} + \frac{\sqrt{6}\left(\frac{\sqrt{3}\sqrt{11}}{2} + \frac{\sqrt{3}\sqrt{11}}{2}\right)}{2\left(x+\frac{\sqrt{3}\sqrt{11}}{3}\right)}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{291\sqrt{3}\sqrt{11}}{1936} + \frac{\sqrt{6}\left(\frac{\sqrt{3}\sqrt{11}}{2} + \frac{\sqrt{3}\sqrt{11}}{2}\right)}{2\left(x+\frac{\sqrt{3}\sqrt{11}}{3}\right)}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-288+\sqrt{6}2481i\right)\sqrt{x^2+\frac{2}{3}}}{1149984\left(x+\frac{\sqrt{3}\sqrt{11}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}2481i\right)\sqrt{x^2+\frac{2}{3}}}{1149984\left(x-\frac{\sqrt{3}\sqrt{11}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)), x)`

[Out]  $\frac{11^{\frac{1}{2}}(8\log(x+\frac{1}{2}) - 8\log(x - (3^{\frac{1}{2}})11^{\frac{1}{2}}(x^2 + \frac{2}{3})^{\frac{1}{2}}))}{3 - 4/3}}{14641} + \frac{11^{\frac{1}{2}}(48\log(x+\frac{1}{2}) - (48\log(x - (3^{\frac{1}{2}})11^{\frac{1}{2}}(x^2 + \frac{2}{3})^{\frac{1}{2}}))}{3 - 4/3}))}{1331}}{22} - \frac{(8*3^{\frac{1}{2}}(x^2 + \frac{2}{3})^{\frac{1}{2}})/(1331(x + \frac{1}{2})) - (3^{\frac{1}{2}}(x^2 + \frac{2}{3})^{\frac{1}{2}})((6^{\frac{1}{2}}*15i)/1936 - 291/1936)/(x + (6^{\frac{1}{2}}*1i)/3) + (6^{\frac{1}{2}}((6^{\frac{1}{2}}*5i)/968 - 97/968)*1i)}{1149984(x + \frac{\sqrt{3}\sqrt{11}}{3})} - \frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}2481i\right)\sqrt{x^2+\frac{2}{3}}}{1149984\left(x-\frac{\sqrt{3}\sqrt{11}}{3}\right)}$



$$2*(x + (6^{(1/2)*1i}/3)^2))/27 + (3^{(1/2)}*(x^2 + 2/3)^{(1/2)}*((6^{(1/2)*15i})/1936 + 291/1936)/(x - (6^{(1/2)*1i}/3) - (6^{(1/2)}*((6^{(1/2)*5i})/968 + 97/968)*1i)/(2*(x - (6^{(1/2)*1i}/3)^2)))/27 - (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)*2481i} - 288)*(x^2 + 2/3)^{(1/2)*1i)/(1149984*(x + (6^{(1/2)*1i}/3)) - (3^{(1/2)}*6^{(1/2)}*(6^{(1/2)*2481i} + 288)*(x^2 + 2/3)^{(1/2)*1i)/(1149984*(x - (6^{(1/2)*1i}/3))$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)/(1+2\*x)\*\*2/(3\*x\*\*2+2)\*\*(5/2),x)

[Out] Timed out

$$3.135 \quad \int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}} dx$$

Optimal. Leaf size=117

$$\frac{351x + 358}{7986(3x^2 + 2)^{3/2}} - \frac{8\sqrt{3x^2 + 2}}{1331(2x + 1)} - \frac{8\sqrt{3x^2 + 2}}{1331(2x + 1)^2} + \frac{2133x + 1216}{29282\sqrt{3x^2 + 2}} - \frac{1216 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

**Rubi [A]** time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x + 358}{7986(3x^2 + 2)^{3/2}} - \frac{8\sqrt{3x^2 + 2}}{1331(2x + 1)} - \frac{8\sqrt{3x^2 + 2}}{1331(2x + 1)^2} + \frac{2133x + 1216}{29282\sqrt{3x^2 + 2}} - \frac{1216 \tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 + 3\*x^2)^(5/2)), x]

[Out] (358 + 351\*x)/(7986\*(2 + 3\*x^2)^(3/2)) + (1216 + 2133\*x)/(29282\*sqrt[2 + 3\*x^2]) - (8\*sqrt[2 + 3\*x^2])/(1331\*(1 + 2\*x)^2) - (8\*sqrt[2 + 3\*x^2])/(1331\*(1 + 2\*x)) - (1216\*ArcTanh[(4 - 3\*x)/(sqrt[11]\*sqrt[2 + 3\*x^2])])/(14641\*sqrt[11])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 807

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[(c\*d\*f + a\*e\*g)/(c\*d^2 + a\*e^2), Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}

, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :=>
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}} dx &= \frac{358+351x}{7986(2+3x^2)^{3/2}} - \frac{1}{18} \int \frac{-\frac{10926}{1331} - \frac{3132x}{121} - \frac{51048x^2}{1331} - \frac{16848x^3}{1331}}{(1+2x)^3(2+3x^2)^{3/2}} dx \\
&= \frac{358+351x}{7986(2+3x^2)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} + \frac{1}{108} \int \frac{\frac{245376}{14641} + \frac{544320x}{14641} + \frac{525312x^2}{14641}}{(1+2x)^3\sqrt{2+3x^2}} dx \\
&= \frac{358+351x}{7986(2+3x^2)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{\int \frac{\frac{338688}{1331} - \frac{468288x}{1331}}{(1+2x)^2\sqrt{2+3x^2}} dx}{2376} \\
&= \frac{358+351x}{7986(2+3x^2)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} + \frac{1216}{1} \int \frac{1}{(1+2x)^3} dx \\
&= \frac{358+351x}{7986(2+3x^2)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216 \operatorname{Subst}(\int \frac{1}{u^3} du, 1+2x)}{14641} \\
&= \frac{358+351x}{7986(2+3x^2)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216 \operatorname{tanh}^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{14641\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 75, normalized size = 0.64

$$\frac{11(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010)}{(2x+1)^2(3x^2+2)^{3/2}} - 7296\sqrt{11} \operatorname{tanh}^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)$$

966306

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 + 3\*x^2)^(5/2)), x]

[Out] ((11\*(7010 + 57371\*x + 109844\*x^2 + 116937\*x^3 + 111060\*x^4 + 67284\*x^5))/((1 + 2\*x)^2\*(2 + 3\*x^2)^(3/2)) - 7296\*sqrt[11]\*ArcTanh[(4 - 3\*x)/sqrt[22 + 33\*x^2]])/966306

**IntegrateAlgebraic [A]** time = 0.81, size = 94, normalized size = 0.80

$$\frac{2432 \operatorname{tanh}^{-1}\left(-\frac{2\sqrt{3x^2+2}}{\sqrt{11}} + 2\sqrt{\frac{3}{11}}x + \sqrt{\frac{3}{11}}\right)}{14641\sqrt{11}} + \frac{67284x^5 + 111060x^4 + 116937x^3 + 109844x^2 + 57371x + 7010}{87846(2x+1)^2(3x^2+2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 + 3\*x^2)^(5/2)),x]

[Out] (7010 + 57371\*x + 109844\*x^2 + 116937\*x^3 + 111060\*x^4 + 67284\*x^5)/(87846\*(1 + 2\*x)^2\*(2 + 3\*x^2)^(3/2)) + (2432\*ArcTanh[Sqrt[3/11] + 2\*Sqrt[3/11]\*x - (2\*Sqrt[2 + 3\*x^2])/Sqrt[11]])/(14641\*Sqrt[11])

**fricas** [A] time = 0.94, size = 149, normalized size = 1.27

$$\frac{3648\sqrt{11}(36x^6 + 36x^5 + 57x^4 + 48x^3 + 28x^2 + 16x + 4)\log\left(\frac{-\sqrt{11}\sqrt{3x^2+2(3x-4)+21x^2-12x+19}}{4x^2+4x+1}\right) + 11(67284x^5 + 111060x^4 + 116937x^3 + 109844x^2 + 57371x + 7010)\sqrt{3x^2+2}}{966306(36x^6 + 36x^5 + 57x^4 + 48x^3 + 28x^2 + 16x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(5/2),x, algorithm="fricas")

[Out] 1/966306\*(3648\*sqrt(11)\*(36\*x^6 + 36\*x^5 + 57\*x^4 + 48\*x^3 + 28\*x^2 + 16\*x + 4)\*log(-(sqrt(11)\*sqrt(3\*x^2 + 2)\*(3\*x - 4) + 21\*x^2 - 12\*x + 19)/(4\*x^2 + 4\*x + 1)) + 11\*(67284\*x^5 + 111060\*x^4 + 116937\*x^3 + 109844\*x^2 + 57371\*x + 7010)\*sqrt(3\*x^2 + 2))/(36\*x^6 + 36\*x^5 + 57\*x^4 + 48\*x^3 + 28\*x^2 + 16\*x + 4)

**giac** [A] time = 0.29, size = 183, normalized size = 1.56

$$\frac{1216}{161051}\sqrt{11}\log\left(\frac{-2\sqrt{3}x - \sqrt{11} - \sqrt{3} + 2\sqrt{3x^2+2}}{2\sqrt{3}x - \sqrt{11} + \sqrt{3} - 2\sqrt{3x^2+2}}\right) + \frac{9((2133x + 1216)x + 1851)x + 11234}{87846(3x^2+2)^{\frac{3}{2}}} + \frac{4(\sqrt{3}(\sqrt{3}x - \sqrt{3x^2+2})^2 + 24\sqrt{3}x - 8\sqrt{3} - 24\sqrt{3x^2+2})}{1331((\sqrt{3}x - \sqrt{3x^2+2})^2 + \sqrt{3}(\sqrt{3}x - \sqrt{3x^2+2}) - 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(5/2),x, algorithm="giac")

[Out] 1216/161051\*sqrt(11)\*log(-abs(-2\*sqrt(3)\*x - sqrt(11) - sqrt(3) + 2\*sqrt(3\*x^2 + 2))/(2\*sqrt(3)\*x - sqrt(11) + sqrt(3) - 2\*sqrt(3\*x^2 + 2))) + 1/87846\*(9\*((2133\*x + 1216)\*x + 1851)\*x + 11234)/(3\*x^2 + 2)^(3/2) + 4/1331\*(sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 + 2))^2 + 24\*sqrt(3)\*x - 8\*sqrt(3) - 24\*sqrt(3\*x^2 + 2))/((sqrt(3)\*x - sqrt(3\*x^2 + 2))^2 + sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 + 2)) - 2)^2

**maple** [A] time = 0.01, size = 140, normalized size = 1.20

$$\frac{87x}{2662(-3x+3(x+\frac{1}{2})^2+\frac{3}{4})^{\frac{3}{2}}} + \frac{1869x}{29282\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{3}{4}}} - \frac{1216\sqrt{11}\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12(x+\frac{1}{2})^2+5}}\right)}{161051} + \frac{152}{3993(-3x+3(x+\frac{1}{2})^2+\frac{3}{4})^{\frac{3}{2}}} + \frac{608}{14641\sqrt{-3x+3(x+\frac{1}{2})^2+\frac{3}{4}}} + \frac{1}{484(x+\frac{1}{2})(-3x+3(x+\frac{1}{2})^2+\frac{3}{4})^{\frac{3}{2}}} - \frac{1}{88(x+\frac{1}{2})^2(-3x+3(x+\frac{1}{2})^2+\frac{3}{4})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2+2)^(5/2),x)

[Out]  $152/3993/(-3*x+3*(x+1/2)^2+5/4)^{(3/2)}+87/2662/(-3*x+3*(x+1/2)^2+5/4)^{(3/2)}*x+1869/29282/(-3*x+3*(x+1/2)^2+5/4)^{(1/2)}*x+608/14641/(-3*x+3*(x+1/2)^2+5/4)^{(1/2)}-1216/161051*11^{(1/2)}*\operatorname{arctanh}(2/11*(-3*x+4)*11^{(1/2)})/(-12*x+12*(x+1/2)^2+5)^{(1/2)}+1/484/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^{(3/2)}-1/88/(x+1/2)^2/(-3*x+3*(x+1/2)^2+5/4)^{(3/2)}$

**maxima [A]** time = 0.99, size = 147, normalized size = 1.26

$$\frac{1216}{161051} \sqrt{11} \operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|} - \frac{2\sqrt{6}}{3|2x+1|}\right) + \frac{1869x}{29282\sqrt{3x^2+2}} + \frac{608}{14641\sqrt{3x^2+2}} + \frac{87x}{2662(3x^2+2)^{3/2}} - \frac{1}{22\left(4(3x^2+2)^{3/2}x^2+4(3x^2+2)^{3/2}x+(3x^2+2)^{3/2}\right)} + \frac{1}{242\left(2(3x^2+2)^{3/2}x+(3x^2+2)^{3/2}\right)} + \frac{152}{3993(3x^2+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")`

[Out]  $1216/161051*\sqrt{11}*\operatorname{arcsinh}(1/2*\sqrt{6}*x/\operatorname{abs}(2*x+1) - 2/3*\sqrt{6}/\operatorname{abs}(2*x+1)) + 1869/29282*x/\sqrt{3*x^2+2} + 608/14641/\sqrt{3*x^2+2} + 87/2662*x/(3*x^2+2)^{(3/2)} - 1/22/(4*(3*x^2+2)^{(3/2)}*x^2+4*(3*x^2+2)^{(3/2)}*x+(3*x^2+2)^{(3/2)}) + 1/242/(2*(3*x^2+2)^{(3/2)}*x+(3*x^2+2)^{(3/2)}) + 152/3993/(3*x^2+2)^{(3/2)}$

**mupad [B]** time = 4.19, size = 301, normalized size = 2.57

$$\frac{1216\sqrt{11}\ln\left(x+\frac{1}{2}\right)}{161051} - \frac{1216\sqrt{11}\ln\left(x-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3}\right)}{161051} - \frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(x^2+\frac{2\sqrt{6}x}{3}-\frac{2}{3}\right)} + \frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x-\frac{\sqrt{6}i}{3}\right)} + \frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x+\frac{\sqrt{6}i}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x^2+x+\frac{1}{4}\right)} + \frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(-x^2+\frac{2\sqrt{6}x}{3}-\frac{2}{3}\right)} - \frac{4\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}*131}{21296\left(x^2+\frac{2\sqrt{6}x}{3}-\frac{2}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}*9265i}{2108304\left(x-\frac{\sqrt{6}i}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}*9265i}{2108304\left(x+\frac{\sqrt{6}i}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}*131}{21296\left(-x^2+\frac{2\sqrt{6}x}{3}-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x+4*x^2+1)/((2*x+1)^3*(3*x^2+2)^(5/2)),x)`

[Out]  $(1216*11^{(1/2)}*\log(x+1/2))/161051 - (1216*11^{(1/2)}*\log(x-(3^{(1/2)}*11^{(1/2)}*(x^2+2/3)^{(1/2)})/3 - 4/3))/161051 - (179*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(95832*((6^{(1/2)}*x*2i)/3 + x^2 - 2/3)) + (711*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(58564*(x - (6^{(1/2)}*1i)/3)) + (711*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(58564*(x + (6^{(1/2)}*1i)/3)) - (2*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(1331*(x + x^2 + 1/4)) + (179*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(95832*((6^{(1/2)}*x*2i)/3 - x^2 + 2/3)) - (4*3^{(1/2)}*(x^2+2/3)^{(1/2)})/(1331*(x+1/2)) + (3^{(1/2)}*6^{(1/2)}*(x^2+2/3)^{(1/2)}*131)/(21296*((6^{(1/2)}*x*2i)/3 + x^2 - 2/3)) - (3^{(1/2)}*6^{(1/2)}*(x^2+2/3)^{(1/2)}*9265i)/(2108304*(x - (6^{(1/2)}*1i)/3)) + (3^{(1/2)}*6^{(1/2)}*(x^2+2/3)^{(1/2)}*9265i)/(2108304*(x + (6^{(1/2)}*1i)/3)) + (3^{(1/2)}*6^{(1/2)}*(x^2+2/3)^{(1/2)}*131)/(21296*((6^{(1/2)}*x*2i)/3 - x^2 + 2/3))$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)`

[Out] Timed out

$$3.136 \quad \int (a + bx + cx^2)^4 (A + Cx^2) dx$$

Optimal. Leaf size=254

$$a^4 Ax + 2a^3 Abx^2 + abx^4 (a^2 C + A(3ac + b^2)) + \frac{1}{3} a^2 x^3 (a^2 C + 4aAc + 6Ab^2) + \frac{1}{7} x^7 (C(6a^2 c^2 + 12ab^2 c + b^4) + 2A$$

Rubi [A] time = 0.33, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {1657}

$$\frac{1}{5} x^7 (C(6a^2 c^2 + 12ab^2 c + b^4) + 2A^2 (2ac + 3b^2)) + \frac{1}{3} x^5 (A(6a^2 c^2 + 12ab^2 c + b^4) + 2a^2 C(2ac + 3b^2)) + abx^4 (a^2 C + A(3ac + b^2)) + \frac{1}{3} a^2 x^3 (a^2 C + 4aAc + 6Ab^2) + 2a^2 Abx^2 + a^4 Ax + \frac{1}{5} x^7 (4acC + A^2 + 6b^2 C) + \frac{1}{2} b^2 c^2 (3ac + b^2) + A^2 + \frac{2}{3} b^2 c (3ac + b^2) (aC + Ac) + \frac{2}{5} b^2 c^2 x^{10} + \frac{1}{11} c^4 x^{11}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^4\*(A + C\*x^2), x]

[Out] a^4\*A\*x + 2\*a^3\*A\*b\*x^2 + (a^2\*(6\*A\*b^2 + 4\*a\*A\*c + a^2\*C)\*x^3)/3 + a\*b\*(A\*(b^2 + 3\*a\*c) + a^2\*C)\*x^4 + ((A\*(b^4 + 12\*a\*b^2\*c + 6\*a^2\*c^2) + 2\*a^2\*(3\*b^2 + 2\*a\*c)\*C)\*x^5)/5 + (2\*b\*(b^2 + 3\*a\*c)\*(A\*c + a\*C)\*x^6)/3 + ((2\*A\*c^2\*(3\*b^2 + 2\*a\*c) + (b^4 + 12\*a\*b^2\*c + 6\*a^2\*c^2)\*C)\*x^7)/7 + (b\*c\*(A\*c^2 + (b^2 + 3\*a\*c)\*C)\*x^8)/2 + (c^2\*(A\*c^2 + 6\*b^2\*C + 4\*a\*c\*C)\*x^9)/9 + (2\*b\*c^3\*C\*x^10)/5 + (c^4\*C\*x^11)/11

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2)^4 (A + Cx^2) dx &= \int (a^4 A + 4a^3 Abx + a^2 (6Ab^2 + 4aAc + a^2 C) x^2 + 4ab (A(b^2 + 3ac) + a^2 C) x^3 + ab (A(b^2 + 3ac) + a^2 C) x^4 + \frac{1}{3} a^2 (6Ab^2 + 4aAc + a^2 C) x^5 + \frac{1}{7} x^7 (C(6a^2 c^2 + 12ab^2 c + b^4) + 2A) \end{aligned}$$

Mathematica [A] time = 0.09, size = 256, normalized size = 1.01

$$a^4 Ax + 2a^3 Abx^2 + abx^4 (a^2 C + A(3ac + b^2)) + \frac{1}{3} a^2 x^3 (a^2 C + 4aAc + 6Ab^2) + \frac{1}{7} x^7 (C(6a^2 c^2 + 12ab^2 c + b^4) + 2A) + \frac{1}{5} x^5 (A(6a^2 c^2 + 12ab^2 c + b^4) + 2a^2 C(2ac + 3b^2)) + abx^4 (a^2 C + A(3ac + b^2)) + \frac{1}{3} a^2 x^3 (a^2 C + 4aAc + 6Ab^2) + 2a^2 Abx^2 + a^4 Ax + \frac{1}{5} x^7 (4acC + A^2 + 6b^2 C) + \frac{1}{2} b^2 c^2 (3ac + b^2) + A^2 + \frac{2}{3} b^2 c (3ac + b^2) (aC + Ac) + \frac{2}{5} b^2 c^2 x^{10} + \frac{1}{11} c^4 x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^4\*(A + C\*x^2), x]

[Out]  $a^4 A x + 2 a^3 A b x^2 + (a^2 (6 A b^2 + 4 a A c + a^2 C) x^3) / 3 + a b (A b^2 + 3 a A c + a^2 C) x^4 + ((A b^4 + 12 a A b^2 c + 6 a^2 A c^2 + 6 a^2 b^2 C + 4 a^3 c C) x^5) / 5 + (2 b (b^2 + 3 a c) (A c + a C) x^6) / 3 + ((6 A b^2 c^2 + 4 a^3 c^3 + b^4 C + 12 a b^2 c C + 6 a^2 c^2 C) x^7) / 7 + (b c (A c^2 + b^2 C + 3 a c C) x^8) / 2 + (c^2 (A c^2 + 6 b^2 C + 4 a c C) x^9) / 9 + (2 b c^3 C x^{10}) / 5 + (c^4 C x^{11}) / 11$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^4 (A + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^4\*(A + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + b\*x + c\*x^2)^4\*(A + C\*x^2), x]

**fricas** [A] time = 0.54, size = 308, normalized size = 1.21

$\frac{1}{11} c^4 x^{11} + \frac{2}{5} c^3 b x^{10} + \frac{2}{3} c^2 b^2 x^9 + \frac{4}{9} c^3 a x^9 + \frac{1}{9} c^4 a x^9 + \frac{1}{2} c^3 b^3 x^8 + \frac{3}{2} c^2 b^2 a x^8 + \frac{1}{2} c^3 b^3 a x^8 + \frac{1}{7} c^4 b^3 a x^8 + \frac{12}{7} c^2 b^2 c x^7 + \frac{6}{7} c^3 b^2 a x^7 + \frac{6}{7} c^2 b^2 c x^7 + \frac{2}{3} c^3 b^2 a x^7 + 2 c^2 b^2 c x^7 + \frac{2}{3} c^3 b^2 a x^7 + 2 c^2 b^2 c x^7 + \frac{6}{5} c^2 b^2 a x^7 + 2 c^2 b^2 c x^7 + \frac{4}{5} c^3 b^2 a x^7 + \frac{1}{3} c^4 b^2 a x^7 + \frac{12}{5} c^2 b^2 c x^7 + \frac{6}{5} c^3 b^2 a x^7 + c^2 b^2 c x^7 + 3 c^2 b^2 a x^7 + 3 c^2 b^2 c x^7 + \frac{1}{3} c^4 b^2 a x^7 + 2 c^2 b^2 c x^7 + \frac{4}{3} c^3 b^2 a x^7 + x^4 a^4 b^3 a^3 C + x^4 a^4 b^3 a^3 A + 3 x^4 a^4 c^3 b^2 a^2 C + 3 x^4 a^4 c^3 b^2 a^2 A + 1/3 x^3 a^4 c^3 C + 2 x^3 a^3 b^2 a^2 C + 4/3 x^3 a^3 c^3 A + 2 x^2 b^2 a^3 A + x a^4 A$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^4\*(C\*x^2+A), x, algorithm="fricas")

[Out]  $1/11 * x^{11} * c^4 * C + 2/5 * x^{10} * c^3 * b * C + 2/3 * x^9 * c^2 * b^2 * C + 4/9 * x^9 * c^3 * a * C + 1/9 * x^9 * c^4 * A + 1/2 * x^8 * c^3 * b^3 * C + 3/2 * x^8 * c^2 * b^2 * a * C + 1/2 * x^8 * c^3 * b^3 * A + 1/7 * x^7 * b^4 * C + 12/7 * x^7 * c^2 * b^2 * a * C + 6/7 * x^7 * c^2 * a^2 * C + 6/7 * x^7 * c^2 * b^2 * A + 4/7 * x^7 * c^3 * a^2 * A + 2/3 * x^6 * b^3 * a * C + 2 * x^6 * c^3 * b^2 * a^2 * C + 2/3 * x^6 * c^3 * b^3 * A + 2 * x^6 * c^2 * b^2 * a * A + 6/5 * x^5 * b^2 * a^2 * C + 4/5 * x^5 * c^2 * a^3 * C + 1/5 * x^5 * b^4 * A + 12/5 * x^5 * c^2 * b^2 * a * A + 6/5 * x^5 * c^2 * a^2 * A + x^4 * b^3 * a^3 * C + x^4 * b^3 * a^3 * A + 3 * x^4 * c^3 * b^2 * a^2 * A + 1/3 * x^3 * a^4 * C + 2 * x^3 * b^2 * a^2 * A + 4/3 * x^3 * c^3 * a^3 * A + 2 * x^2 * b^2 * a^3 * A + x a^4 A$

**giac** [A] time = 0.15, size = 308, normalized size = 1.21

$\frac{1}{11} C x^{11} + \frac{2}{5} C b c^3 x^{10} + \frac{2}{3} C b^2 c^2 x^9 + \frac{4}{9} C a c^3 x^9 + \frac{1}{9} C^4 x^9 + \frac{1}{2} C b^3 c^3 x^8 + \frac{3}{2} C a b^2 c^2 x^8 + \frac{1}{2} C b^3 c^3 a x^8 + \frac{1}{7} C^4 b^3 a x^8 + \frac{12}{7} C^2 b^2 c x^7 + \frac{6}{7} C^3 b^2 a x^7 + \frac{6}{7} C^2 b^2 c x^7 + \frac{2}{3} C^3 b^2 a x^7 + 2 C^2 b^2 c x^7 + \frac{2}{3} C^3 b^2 a x^7 + 2 C^2 b^2 c x^7 + \frac{6}{5} C^2 b^2 a x^7 + 2 C^2 b^2 c x^7 + \frac{4}{5} C^3 b^2 a x^7 + \frac{1}{3} C^4 b^2 a x^7 + \frac{12}{5} C^2 b^2 c x^7 + \frac{6}{5} C^3 b^2 a x^7 + C^2 b^2 c x^7 + 3 C^2 b^2 a x^7 + 3 C^2 b^2 c x^7 + \frac{1}{3} C^4 b^2 a x^7 + 2 A b^2 c^2 + A b^2 c^3 + \frac{1}{3} C^4 x^4 + 2 A b^2 c^2 + A b^2 c^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^4\*(C\*x^2+A), x, algorithm="giac")

[Out]  $1/11 * C * c^4 * x^{11} + 2/5 * C * b * c^3 * x^{10} + 2/3 * C * b^2 * c^2 * x^9 + 4/9 * C * a * c^3 * x^9 + 1/9 * A * c^4 * x^9 + 1/2 * C * b^3 * c^3 * x^8 + 3/2 * C * a * b^2 * c^2 * x^8 + 1/2 * A * b * c^3 * x^8 + 1/7 * C * b^4 * x^7 + 12/7 * C * a * b^2 * c^2 * x^7 + 6/7 * C * a^2 * c^2 * x^7 + 6/7 * A * b^2 * c^2 * x^7 + 4$



$$\begin{aligned} & /7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 + 2*A* \\ & a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12/5*A* \\ & a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2*b*c*x \\ & ^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x^2 + A* \\ & a^4*x \end{aligned}$$

**maple [A]** time = 0.00, size = 343, normalized size = 1.35

$$\frac{C^4 x^{11}}{11} + \frac{2 C^3 a^2 b^2 x^{10}}{5} + \frac{(A^4 + (4 b^2 c^2 + 2(2 a c + b^2) c) x^2)}{9} + \frac{(4 b^2 c^2 + (4 b^2 c + 4(2 a c + b^2) b) c) x^4}{8} + \frac{(4 b^2 c^2 + 2(2 a c + b^2) c) A + (2 b^2 c^2 + 8 a b^2 c + (2 a c + b^2) c^2) x^2}{7} + A^2 x^4 + \frac{(4 b^2 c^2 + 4(2 a c + b^2) b) A + (4 b^2 c + 4(2 a c + b^2) a b) c x^2}{6} + \frac{(2 b^2 c^2 + 8 a b^2 c + (2 a c + b^2) c^2) A + (4 b^2 c^2 + 2(2 a c + b^2) c) x^2}{5} + \frac{(2 a c^2 b^2 + (4 b^2 c + 4(2 a c + b^2) a b) A) x^4}{4} + \frac{C x^4 + (4 b^2 c^2 + 2(2 a c + b^2) c) A x^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^4\*(C\*x^2+A), x)

$$\begin{aligned} \text{[Out]} & \quad 1/11*c^4*C*x^{11}+2/5*b*c^3*C*x^{10}+1/9*((2*(2*a*c+b^2)*c^2+4*b^2*c^2)*C+c^4*A \\ & )*x^9+1/8*((4*a*b*c^2+4*(2*a*c+b^2)*b*c)*C+4*b*c^3*A)*x^8+1/7*((2*a^2*c^2+8 \\ & *a*b^2*c+(2*a*c+b^2)^2)*C+(2*(2*a*c+b^2)*c^2+4*b^2*c^2)*A)*x^7+1/6*((4*a^2* \\ & b*c+4*a*b*(2*a*c+b^2))*C+(4*a*b*c^2+4*(2*a*c+b^2)*b*c)*A)*x^6+1/5*((2*a^2*( \\ & 2*a*c+b^2)+4*a^2*b^2)*C+(2*a^2*c^2+8*a*b^2*c+(2*a*c+b^2)^2)*A)*x^5+1/4*(4*a \\ & ^3*b*c+(4*a^2*b*c+4*a*b*(2*a*c+b^2))*A)*x^4+1/3*(a^4*C+(2*a^2*(2*a*c+b^2)+4 \\ & *a^2*b^2)*A)*x^3+2*a^3*A*b*x^2+a^4*A*x \end{aligned}$$

**maxima [A]** time = 0.44, size = 263, normalized size = 1.04

$$\frac{1}{11} C^4 x^{11} + \frac{2}{5} C^3 a^2 b^2 x^{10} + \frac{1}{9} (6 C^2 b^2 c^2 + 4 C a c^3 + A c^4) x^9 + \frac{1}{2} (C^3 b^3 c + 3 C^2 a b^2 c + A b^2 c^3) x^8 + \frac{1}{7} (C^3 b^4 + 12 C^2 a b^2 c + 4 A a^2 c^3 + 6 (C^2 a^2 + A b^2) c^2) x^7 + 2 A a^3 b^2 x^2 + \frac{2}{3} (C^2 a^2 b^2 + A b^3 + 3 A a^2 b^2) x^6 + \frac{1}{5} (6 C a^2 b^2 c + A b^4 + 6 A a^2 c^2 + 4 (C a^3 + 3 A a b^2) c) x^5 + \frac{1}{3} (C a^4 + 6 A a^2 b^2 + 4 A a^3 b) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^4\*(C\*x^2+A), x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & \quad 1/11*C*c^4*x^{11} + 2/5*C*b*c^3*x^{10} + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)* \\ & x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c \\ & + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 + \\ & 3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b \\ & ^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A* \\ & a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3 \end{aligned}$$

**mupad [B]** time = 0.13, size = 244, normalized size = 0.96

$$x^4 \left( \frac{4 C a^2 c}{5} + \frac{6 C a^2 b^2}{5} + \frac{6 A a^2 c^2}{5} + \frac{12 A a b^2 c}{5} + \frac{A b^4}{5} \right) + x^7 \left( \frac{6 C a^2 c^2}{7} + \frac{12 C a b^2 c}{7} + \frac{4 A a c^3}{7} + \frac{C b^4}{7} + \frac{6 A b^2 c^2}{7} \right) + x^3 \left( \frac{C a^4}{3} + \frac{4 A c a^2}{3} + 2 A a^2 b^2 \right) + x^6 \left( \frac{2 C b^2 c^2}{3} + \frac{A c^4}{9} + \frac{4 C a c^2}{9} \right) + \frac{C^2 x^{11}}{11} + A a^4 x + \frac{2 b^4 (b^2 + 3 a c) (A c + C a)}{3} + a b^4 (C a^2 + 3 A c a + A b^2) + \frac{b c x^6 (C b^2 + A c^2 + 3 C a c)}{2} + 2 A a^3 b^2 x^2 + \frac{2 C b^2 c^2 x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)\*(a + b\*x + c\*x^2)^4, x)

$$\begin{aligned} \text{[Out]} & \quad x^5*((A*b^4)/5 + (6*A*a^2*c^2)/5 + (6*C*a^2*b^2)/5 + (4*C*a^3*c)/5 + (12*A* \\ & a*b^2*c)/5) + x^7*((C*b^4)/7 + (6*A*b^2*c^2)/7 + (6*C*a^2*c^2)/7 + (4*A*a*c \end{aligned}$$

$$\begin{aligned} & \text{^3)/7 + (12*C*a*b^2*c)/7) + x^3*((C*a^4)/3 + 2*A*a^2*b^2 + (4*A*a^3*c)/3) + \\ & x^9*((A*c^4)/9 + (2*C*b^2*c^2)/3 + (4*C*a*c^3)/9) + (C*c^4*x^11)/11 + A*a^4*x \\ & + (2*b*x^6*(3*a*c + b^2)*(A*c + C*a))/3 + a*b*x^4*(A*b^2 + C*a^2 + 3*A* \\ & a*c) + (b*c*x^8*(A*c^2 + C*b^2 + 3*C*a*c))/2 + 2*A*a^3*b*x^2 + (2*C*b*c^3*x \\ & ^10)/5 \end{aligned}$$

**sympy [A]** time = 0.14, size = 320, normalized size = 1.26

$$Aa^4x + 2Aa^3bx^2 + \frac{2Cb^3c^3x^{10}}{5} + \frac{C^4x^{11}}{11} + x^9\left(\frac{A^4c^4}{9} + \frac{4Ca^3c}{9} + \frac{2Cb^2c^2}{3}\right) + x^8\left(\frac{Abc^3}{2} + \frac{3Cab^2c}{2} + \frac{Cb^3c}{2}\right) + x^7\left(\frac{4Aa^3c}{7} + \frac{6Aa^2b^2c}{7} + \frac{6Ca^2c^2}{7} + \frac{12Cab^2c}{7} + \frac{Cb^4}{7}\right) + x^6\left(2Aab^2c^2 + \frac{2Ab^3c}{3} + 2Ca^2b^2c + \frac{2Cab^3}{3}\right) + x^5\left(\frac{6Aa^2c^2}{5} + \frac{12Aab^2c}{5} + \frac{Ab^4}{5} + \frac{4Ca^3c}{5} + \frac{6Ca^2b^2}{5}\right) + x^4(3Aa^2bc + Ab^3 + Ca^3) + x^3\left(\frac{4Aa^3c}{3} + 2Aa^2b^2 + \frac{Cb^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*4\*(C\*x\*\*2+A), x)

[Out] A\*a\*\*4\*x + 2\*A\*a\*\*3\*b\*x\*\*2 + 2\*C\*b\*c\*\*3\*x\*\*10/5 + C\*c\*\*4\*x\*\*11/11 + x\*\*9\*(A\*c\*\*4/9 + 4\*C\*a\*c\*\*3/9 + 2\*C\*b\*\*2\*c\*\*2/3) + x\*\*8\*(A\*b\*c\*\*3/2 + 3\*C\*a\*b\*c\*\*2/2 + C\*b\*\*3\*c/2) + x\*\*7\*(4\*A\*a\*c\*\*3/7 + 6\*A\*b\*\*2\*c\*\*2/7 + 6\*C\*a\*\*2\*c\*\*2/7 + 12\*C\*a\*b\*\*2\*c/7 + C\*b\*\*4/7) + x\*\*6\*(2\*A\*a\*b\*c\*\*2 + 2\*A\*b\*\*3\*c/3 + 2\*C\*a\*\*2\*b\*c + 2\*C\*a\*b\*\*3/3) + x\*\*5\*(6\*A\*a\*\*2\*c\*\*2/5 + 12\*A\*a\*b\*\*2\*c/5 + A\*b\*\*4/5 + 4\*C\*a\*\*3\*c/5 + 6\*C\*a\*\*2\*b\*\*2/5) + x\*\*4\*(3\*A\*a\*\*2\*b\*c + A\*a\*b\*\*3 + C\*a\*\*3\*b) + x\*\*3\*(4\*A\*a\*\*3\*c/3 + 2\*A\*a\*\*2\*b\*\*2 + C\*a\*\*4/3)

$$3.137 \quad \int (a + bx + cx^2)^3 (A + Cx^2) dx$$

**Optimal.** Leaf size=161

$$a^3 Ax + \frac{1}{4} bx^4 (3a^2 C + A(6ac + b^2)) + \frac{1}{3} ax^3 (a^2 C + 3A(ac + b^2)) + \frac{3}{2} a^2 Abx^2 + \frac{1}{7} cx^7 (3C(ac + b^2) + Ac^2) + \frac{1}{6} bx^6 ($$

**Rubi [A]** time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {1657}

$$\frac{1}{4} bx^4 (3a^2 C + A(6ac + b^2)) + \frac{1}{3} ax^3 (a^2 C + 3A(ac + b^2)) + \frac{3}{2} a^2 Abx^2 + a^3 Ax + \frac{1}{7} cx^7 (3C(ac + b^2) + Ac^2) + \frac{1}{6} bx^6 (C(6ac + b^2) + 3Ac^2) + \frac{3}{5} x^5 (ac + b^2)(ac + Ac) + \frac{3}{8} bc^2 Cx^8 + \frac{1}{9} c^3 Cx^9$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^3\*(A + C\*x^2), x]

[Out] a^3\*A\*x + (3\*a^2\*A\*b\*x^2)/2 + (a\*(3\*A\*(b^2 + a\*c) + a^2\*C)\*x^3)/3 + (b\*(A\*(b^2 + 6\*a\*c) + 3\*a^2\*C)\*x^4)/4 + (3\*(b^2 + a\*c)\*(A\*c + a\*C)\*x^5)/5 + (b\*(3\*A\*c^2 + (b^2 + 6\*a\*c)\*C)\*x^6)/6 + (c\*(A\*c^2 + 3\*(b^2 + a\*c)\*C)\*x^7)/7 + (3\*b\*c^2\*C\*x^8)/8 + (c^3\*C\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2)^3 (A + Cx^2) dx &= \int (a^3 A + 3a^2 Abx + a(3A(b^2 + ac) + a^2 C)x^2 + b(A(b^2 + 6ac) + 3a^2 C)x^3 \\ &\quad + a^3 Ax + \frac{3}{2} a^2 Abx^2 + \frac{1}{3} a(3A(b^2 + ac) + a^2 C)x^3 + \frac{1}{4} b(A(b^2 + 6ac) + 3a^2 C)x^4 \\ &\quad + \frac{3}{5} (b^2 + ac)(A + C)x^5 + \frac{3}{6} b(Ac^2 + C(b^2 + 6ac))x^6 + \frac{3}{7} c(Ac^2 + 3C(b^2 + ac))x^7 \\ &\quad + \frac{3}{8} bc^2 Cx^8 + \frac{1}{9} c^3 Cx^9) dx \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 163, normalized size = 1.01

$$a^3 Ax + \frac{1}{4} bx^4 (3a^2 C + 6aAc + Ab^2) + \frac{1}{3} ax^3 (a^2 C + 3aAc + 3Ab^2) + \frac{3}{2} a^2 Abx^2 + \frac{1}{7} cx^7 (3acC + Ac^2 + 3b^2 C) + \frac{1}{6} bx^6 (6acC + 3Ac^2 + b^2 C) + \frac{3}{5} x^5 (ac + b^2)(ac + Ac) + \frac{3}{8} bc^2 Cx^8 + \frac{1}{9} c^3 Cx^9$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^3\*(A + C\*x^2), x]

[Out]  $a^3Ax + (3a^2Abx^2)/2 + (a(3Ab^2 + 3aAc + a^2C)x^3)/3 + (b(Ab^2 + 6aAc + 3a^2C)x^4)/4 + (3(b^2 + ac)(Ac + aC)x^5)/5 + (b(3Ac^2 + b^2C + 6aC^2)x^6)/6 + (c(Ac^2 + 3b^2C + 3aC^2)x^7)/7 + (3b^2Cx^8)/8 + (c^3Cx^9)/9$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^3 (A + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^3\*(A + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + b\*x + c\*x^2)^3\*(A + C\*x^2), x]

**fricas** [A] time = 0.43, size = 187, normalized size = 1.16

$$\frac{1}{9}c^3x^9 + \frac{3}{8}c^2bx^8 + \frac{3}{7}cb^2cx^7 + \frac{3}{7}c^2ax^7 + \frac{1}{7}c^2x^7 + \frac{1}{6}cb^3x^6 + Cabcx^6 + \frac{1}{2}Abc^2x^6 + \frac{3}{5}Cab^2x^5 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Ab^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ca^2bx^4 + \frac{1}{4}Ab^3x^4 + \frac{3}{2}Aabcx^4 + \frac{1}{3}Ca^3x^3 + Aab^2x^3 + Aa^2cx^3 + \frac{3}{2}Aa^2bx^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^3\*(C\*x^2+A),x, algorithm="fricas")

[Out]  $1/9*x^9*c^3*C + 3/8*x^8*c^2*b*C + 3/7*x^7*c*b^2*C + 3/7*x^7*c^2*a*C + 1/7*x^7*c^3*A + 1/6*x^6*b^3*C + x^6*c*b*a*C + 1/2*x^6*c^2*b*A + 3/5*x^5*b^2*a*C + 3/5*x^5*c*a^2*C + 3/5*x^5*c*b^2*A + 3/5*x^5*c^2*a*A + 3/4*x^4*b*a^2*C + 1/4*x^4*b^3*A + 3/2*x^4*c*b*a*A + 1/3*x^3*a^3*C + x^3*b^2*a*A + x^3*c*a^2*A + 3/2*x^2*b*a^2*A + x*a^3*A$

**giac** [A] time = 0.17, size = 187, normalized size = 1.16

$$\frac{1}{9}C^3x^9 + \frac{3}{8}Cb^2x^8 + \frac{3}{7}Cb^2cx^7 + \frac{3}{7}Ca^2x^7 + \frac{1}{7}Ac^2x^7 + \frac{1}{6}Cb^3x^6 + Cabcx^6 + \frac{1}{2}Abc^2x^6 + \frac{3}{5}Cab^2x^5 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Ab^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ca^2bx^4 + \frac{1}{4}Ab^3x^4 + \frac{3}{2}Aabcx^4 + \frac{1}{3}Ca^3x^3 + Aab^2x^3 + Aa^2cx^3 + \frac{3}{2}Aa^2bx^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^3\*(C\*x^2+A),x, algorithm="giac")

[Out]  $1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^5 + 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4 + 1/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + 3/2*A*a^2*b*x^2 + A*a^3*x$

**maple** [A] time = 0.00, size = 223, normalized size = 1.39

$$\frac{C^3x^9}{9} + \frac{3Cb^2x^8}{8} + \frac{(A^3 + (a^2 + 2b^2 + (2ac + b^2)c)C)x^7}{7} + \frac{3Aa^2bx^2}{2} + \frac{(3Ab^2 + (4abc + (2ac + b^2)b)C)x^6}{6} + \frac{1}{Aa^3x} + \frac{((a^2 + 2b^2 + (2ac + b^2)c)A + (a^2c + 2ab^2 + (2ac + b^2)a)C)x^5}{5} + \frac{(3Ca^2b + (4abc + (2ac + b^2)b)A)x^4}{4} + \frac{(Ca^3 + (a^2c + 2ab^2 + (2ac + b^2)a)A)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^3*(C*x^2+A), x)`

[Out]  $1/9*C*c^3*x^9+3/8*b*c^2*C*x^8+1/7*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*C+c^3*A)*x^7+1/6*((4*a*b*c+b*(2*a*c+b^2))*C+3*b*c^2*A)*x^6+1/5*((a*(2*a*c+b^2)+2*b^2*a+c*a^2)*C+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*A)*x^5+1/4*(3*a^2*b*c+(4*a*b*c+b*(2*a*c+b^2))*A)*x^4+1/3*(a^3*C+(a*(2*a*c+b^2)+2*b^2*a+c*a^2)*A)*x^3+3/2*a^2*A*b*x^2+A*a^3*x$

**maxima** [A] time = 0.43, size = 165, normalized size = 1.02

$$\frac{1}{9}C^3x^9 + \frac{3}{8}Cb^2x^8 + \frac{1}{7}(3Cb^2c + 3Cac^2 + Ac^3)x^7 + \frac{1}{6}(Cb^3 + 6Cabc + 3Abc^2)x^6 + \frac{3}{2}Aa^2bx^5 + \frac{3}{5}(Cab^2 + Aac^2 + (Ca^2 + Ab^2)c)x^5 + Aa^3x + \frac{1}{4}(3Ca^2b + Ab^3 + 6Aabc)x^4 + \frac{1}{3}(Ca^3 + 3Aab^2 + 3Aa^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^3*(C*x^2+A), x, algorithm="maxima")`

[Out]  $1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 + 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^5 + 3/5*(C*a*b^2 + A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3$

**mupad** [B] time = 0.07, size = 149, normalized size = 0.93

$$x^3 \left( \frac{Ca^3}{3} + Aca^2 + Aab^2 \right) + x^7 \left( \frac{3Cb^2c}{7} + \frac{Ac^3}{7} + \frac{3Ca^2c}{7} \right) + \frac{bx^4(3Ca^2 + 6Aca + Ab^2)}{4} + \frac{bx^6(Cb^2 + 3Ac^2 + 6Cac)}{6} + \frac{Cc^3x^9}{9} + Aa^3x + \frac{3x^5(b^2 + ac)(Ac + Ca)}{5} + \frac{3Aa^2bx^2}{2} + \frac{3Cbc^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + C*x^2)*(a + b*x + c*x^2)^3, x)`

[Out]  $x^3*((C*a^3)/3 + A*a*b^2 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7 + (3*C*b^2*c)/7) + (b*x^4*(A*b^2 + 3*C*a^2 + 6*A*a*c))/4 + (b*x^6*(3*A*c^2 + C*b^2 + 6*C*a*c))/6 + (C*c^3*x^9)/9 + A*a^3*x + (3*x^5*(a*c + b^2)*(A*c + C*a))/5 + (3*A*a^2*b*x^2)/2 + (3*C*b*c^2*x^8)/8$

**sympy** [A] time = 0.11, size = 197, normalized size = 1.22

$$Aa^3x + \frac{3Aa^2bx^2}{2} + \frac{3Cb^2x^8}{8} + \frac{Cc^3x^9}{9} + x^7 \left( \frac{Ac^3}{7} + \frac{3Cac^2}{7} + \frac{3Cb^2c}{7} \right) + x^6 \left( \frac{Abc^2}{2} + Cabc + \frac{Cb^3}{6} \right) + x^5 \left( \frac{3Aac^2}{5} + \frac{3Ab^2c}{5} + \frac{3Ca^2c}{5} + \frac{3Cab^2}{5} \right) + x^4 \left( \frac{3Aabc}{2} + \frac{Ab^3}{4} + \frac{3Ca^2b}{4} \right) + x^3 \left( Aa^2c + Aab^2 + \frac{Ca^3}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**3*(C*x**2+A), x)`

[Out]  $A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b**3/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) + x**4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2 + C*a**3/3)$

$$3.138 \quad \int (a + bx + cx^2)^2 (A + Cx^2) dx$$

**Optimal.** Leaf size=96

$$\frac{1}{3}x^3 (a^2C + A(2ac + b^2)) + a^2Ax + \frac{1}{5}x^5 (C(2ac + b^2) + Ac^2) + \frac{1}{2}bx^4(aC + Ac) + aAbx^2 + \frac{1}{3}bcCx^6 + \frac{1}{7}c^2Cx^7$$

**Rubi [A]** time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {1657}

$$\frac{1}{3}x^3 (a^2C + A(2ac + b^2)) + a^2Ax + \frac{1}{5}x^5 (C(2ac + b^2) + Ac^2) + \frac{1}{2}bx^4(aC + Ac) + aAbx^2 + \frac{1}{3}bcCx^6 + \frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^2\*(A + C\*x^2), x]

[Out] a^2\*A\*x + a\*A\*b\*x^2 + ((A\*(b^2 + 2\*a\*c) + a^2\*C)\*x^3)/3 + (b\*(A\*c + a\*C)\*x^4)/2 + ((A\*c^2 + (b^2 + 2\*a\*c)\*C)\*x^5)/5 + (b\*c\*C\*x^6)/3 + (c^2\*C\*x^7)/7

**Rule 1657**

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\begin{aligned} \int (a + bx + cx^2)^2 (A + Cx^2) dx &= \int (a^2A + 2aAbx + (A(b^2 + 2ac) + a^2C)x^2 + 2b(Ac + aC)x^3 + (Ac^2 + (b^2 + 2ac)C)x^4 \\ &\quad + a^2Ax + aAbx^2 + \frac{1}{3}(A(b^2 + 2ac) + a^2C)x^3 + \frac{1}{2}b(Ac + aC)x^4 + \frac{1}{5}(Ac^2 + (b^2 + 2ac)C)x^5) dx \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 96, normalized size = 1.00

$$\frac{1}{3}x^3 (a^2C + 2aAc + Ab^2) + a^2Ax + \frac{1}{5}x^5 (2acC + Ac^2 + b^2C) + \frac{1}{2}bx^4(aC + Ac) + aAbx^2 + \frac{1}{3}bcCx^6 + \frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^2\*(A + C\*x^2), x]

[Out] a^2\*A\*x + a\*A\*b\*x^2 + ((A\*b^2 + 2\*a\*A\*c + a^2\*C)\*x^3)/3 + (b\*(A\*c + a\*C)\*x^4)/2 + ((A\*c^2 + b^2\*C + 2\*a\*c\*C)\*x^5)/5 + (b\*c\*C\*x^6)/3 + (c^2\*C\*x^7)/7

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^2 (A + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)^2\*(A + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + b\*x + c\*x^2)^2\*(A + C\*x^2), x]

**fricas** [A] time = 0.99, size = 99, normalized size = 1.03

$$\frac{1}{7}x^7c^2C + \frac{1}{3}x^6cbC + \frac{1}{5}x^5b^2C + \frac{2}{5}x^5caC + \frac{1}{5}x^5c^2A + \frac{1}{2}x^4baC + \frac{1}{2}x^4cbA + \frac{1}{3}x^3a^2C + \frac{1}{3}x^3b^2A + \frac{2}{3}x^3caA + x^2baA + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^2\*(C\*x^2+A), x, algorithm="fricas")

[Out] 1/7\*x^7\*c^2\*C + 1/3\*x^6\*c\*b\*C + 1/5\*x^5\*b^2\*C + 2/5\*x^5\*c\*a\*C + 1/5\*x^5\*c^2\*A + 1/2\*x^4\*b\*a\*C + 1/2\*x^4\*c\*b\*A + 1/3\*x^3\*a^2\*C + 1/3\*x^3\*b^2\*A + 2/3\*x^3\*c\*a\*A + x^2\*b\*a\*A + x\*a^2\*A

**giac** [A] time = 0.15, size = 99, normalized size = 1.03

$$\frac{1}{7}C^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{5}Cb^2x^5 + \frac{2}{5}Cacx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Cabx^4 + \frac{1}{2}Abcx^4 + \frac{1}{3}Ca^2x^3 + \frac{1}{3}Ab^2x^3 + \frac{2}{3}Aacx^3 + Aabx^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^2\*(C\*x^2+A), x, algorithm="giac")

[Out] 1/7\*C\*c^2\*x^7 + 1/3\*C\*b\*c\*x^6 + 1/5\*C\*b^2\*x^5 + 2/5\*C\*a\*c\*x^5 + 1/5\*A\*c^2\*x^5 + 1/2\*C\*a\*b\*x^4 + 1/2\*A\*b\*c\*x^4 + 1/3\*C\*a^2\*x^3 + 1/3\*A\*b^2\*x^3 + 2/3\*A\*a\*c\*x^3 + A\*a\*b\*x^2 + A\*a^2\*x

**maple** [A] time = 0.00, size = 90, normalized size = 0.94

$$\frac{C^2x^7}{7} + \frac{Cbcx^6}{3} + Aabx^2 + \frac{(Ac^2 + (2ac + b^2)C)x^5}{5} + Aa^2x + \frac{(2bcA + 2abC)x^4}{4} + \frac{(Ca^2 + (2ac + b^2)A)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^2\*(C\*x^2+A), x)

[Out] 1/7\*C\*c^2\*x^7+1/3\*b\*c\*C\*x^6+1/5\*(A\*c^2+(2\*a\*c+b^2)\*C)\*x^5+1/4\*(2\*A\*b\*c+2\*C\*a\*b)\*x^4+1/3\*(A\*(2\*a\*c+b^2)+a^2\*C)\*x^3+a\*A\*b\*x^2+A\*a^2\*x

**maxima [A]** time = 0.43, size = 87, normalized size = 0.91

$$\frac{1}{7}Cc^2x^7 + \frac{1}{3}Cbcx^6 + \frac{1}{5}(Cb^2 + 2Cac + Ac^2)x^5 + Aabx^2 + \frac{1}{2}(Cab + Abc)x^4 + Aa^2x + \frac{1}{3}(Ca^2 + Ab^2 + 2Aac)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^2\*(C\*x^2+A),x, algorithm="maxima")

[Out] 1/7\*C\*c^2\*x^7 + 1/3\*C\*b\*c\*x^6 + 1/5\*(C\*b^2 + 2\*C\*a\*c + A\*c^2)\*x^5 + A\*a\*b\*x^2 + 1/2\*(C\*a\*b + A\*b\*c)\*x^4 + A\*a^2\*x + 1/3\*(C\*a^2 + A\*b^2 + 2\*A\*a\*c)\*x^3

**mupad [B]** time = 4.09, size = 88, normalized size = 0.92

$$x^3 \left( \frac{Ca^2}{3} + \frac{2Aca}{3} + \frac{Ab^2}{3} \right) + x^5 \left( \frac{Cb^2}{5} + \frac{Ac^2}{5} + \frac{2Cac}{5} \right) + \frac{Cc^2x^7}{7} + Aa^2x + \frac{bx^4(Ac + Ca)}{2} + Aabx^2 + \frac{Cbcx^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)\*(a + b\*x + c\*x^2)^2,x)

[Out] x^3\*((A\*b^2)/3 + (C\*a^2)/3 + (2\*A\*a\*c)/3) + x^5\*((A\*c^2)/5 + (C\*b^2)/5 + (2\*C\*a\*c)/5) + (C\*c^2\*x^7)/7 + A\*a^2\*x + (b\*x^4\*(A\*c + C\*a))/2 + A\*a\*b\*x^2 + (C\*b\*c\*x^6)/3

**sympy [A]** time = 0.09, size = 102, normalized size = 1.06

$$Aa^2x + Aabx^2 + \frac{Cbcx^6}{3} + \frac{Cc^2x^7}{7} + x^5 \left( \frac{Ac^2}{5} + \frac{2Cac}{5} + \frac{Cb^2}{5} \right) + x^4 \left( \frac{Abc}{2} + \frac{Cab}{2} \right) + x^3 \left( \frac{2Aac}{3} + \frac{Ab^2}{3} + \frac{Ca^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*2\*(C\*x\*\*2+A),x)

[Out] A\*a\*\*2\*x + A\*a\*b\*x\*\*2 + C\*b\*c\*x\*\*6/3 + C\*c\*\*2\*x\*\*7/7 + x\*\*5\*(A\*c\*\*2/5 + 2\*C\*a\*c/5 + C\*b\*\*2/5) + x\*\*4\*(A\*b\*c/2 + C\*a\*b/2) + x\*\*3\*(2\*A\*a\*c/3 + A\*b\*\*2/3 + C\*a\*\*2/3)



$$3.139 \quad \int (a + bx + cx^2)(A + Cx^2) dx$$

Optimal. Leaf size=46

$$\frac{1}{3}x^3(aC + Ac) + aAx + \frac{1}{2}Abx^2 + \frac{1}{4}bCx^4 + \frac{1}{5}cCx^5$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {1657}

$$\frac{1}{3}x^3(aC + Ac) + aAx + \frac{1}{2}Abx^2 + \frac{1}{4}bCx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)\*(A + C\*x^2), x]

[Out] a\*A\*x + (A\*b\*x^2)/2 + ((A\*c + a\*C)\*x^3)/3 + (b\*C\*x^4)/4 + (c\*C\*x^5)/5

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2)(A + Cx^2) dx &= \int (aA + Abx + (Ac + aC)x^2 + bCx^3 + cCx^4) dx \\ &= aAx + \frac{1}{2}Abx^2 + \frac{1}{3}(Ac + aC)x^3 + \frac{1}{4}bCx^4 + \frac{1}{5}cCx^5 \end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{3}x^3(aC + Ac) + aAx + \frac{1}{2}Abx^2 + \frac{1}{4}bCx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)\*(A + C\*x^2), x]

[Out] a\*A\*x + (A\*b\*x^2)/2 + ((A\*c + a\*C)\*x^3)/3 + (b\*C\*x^4)/4 + (c\*C\*x^5)/5

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)(A + Cx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b\*x + c\*x^2)\*(A + C\*x^2), x]

[Out] IntegrateAlgebraic[(a + b\*x + c\*x^2)\*(A + C\*x^2), x]

**fricas** [A] time = 0.34, size = 40, normalized size = 0.87

$$\frac{1}{5}x^5cC + \frac{1}{4}x^4bC + \frac{1}{3}x^3aC + \frac{1}{3}x^3cA + \frac{1}{2}x^2bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*(C\*x^2+A), x, algorithm="fricas")

[Out] 1/5\*x^5\*c\*C + 1/4\*x^4\*b\*C + 1/3\*x^3\*a\*C + 1/3\*x^3\*c\*A + 1/2\*x^2\*b\*A + x\*a\*A

**giac** [A] time = 0.18, size = 40, normalized size = 0.87

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Abx^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*(C\*x^2+A), x, algorithm="giac")

[Out] 1/5\*C\*c\*x^5 + 1/4\*C\*b\*x^4 + 1/3\*C\*a\*x^3 + 1/3\*A\*c\*x^3 + 1/2\*A\*b\*x^2 + A\*a\*x

**maple** [A] time = 0.00, size = 39, normalized size = 0.85

$$\frac{Ccx^5}{5} + \frac{Cbx^4}{4} + \frac{Abx^2}{2} + Aax + \frac{(Ac + aC)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)\*(C\*x^2+A), x)

[Out] A\*a\*x+1/2\*A\*b\*x^2+1/3\*(A\*c+C\*a)\*x^3+1/4\*b\*C\*x^4+1/5\*C\*c\*x^5

**maxima** [A] time = 0.43, size = 38, normalized size = 0.83

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Cbx^4 + \frac{1}{2}Abx^2 + \frac{1}{3}(Ca + Ac)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*(C\*x^2+A),x, algorithm="maxima")

[Out] 1/5\*C\*c\*x^5 + 1/4\*C\*b\*x^4 + 1/2\*A\*b\*x^2 + 1/3\*(C\*a + A\*c)\*x^3 + A\*a\*x

mupad [B] time = 0.03, size = 39, normalized size = 0.85

$$\frac{Ccx^5}{5} + \frac{Cbx^4}{4} + \left(\frac{Ac}{3} + \frac{Ca}{3}\right)x^3 + \frac{Abx^2}{2} + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)\*(a + b\*x + c\*x^2),x)

[Out] x^3\*((A\*c)/3 + (C\*a)/3) + A\*a\*x + (A\*b\*x^2)/2 + (C\*b\*x^4)/4 + (C\*c\*x^5)/5

sympy [A] time = 0.07, size = 42, normalized size = 0.91

$$Aax + \frac{Abx^2}{2} + \frac{Cbx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*(C\*x\*\*2+A),x)

[Out] A\*a\*x + A\*b\*x\*\*2/2 + C\*b\*x\*\*4/4 + C\*c\*x\*\*5/5 + x\*\*3\*(A\*c/3 + C\*a/3)

$$3.140 \quad \int \frac{A+Cx^2}{a+bx+cx^2} dx$$

Optimal. Leaf size=81

$$\frac{(C(b^2 - 2ac) + 2Ac^2) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{bC \log(a + bx + cx^2)}{2c^2} + \frac{Cx}{c}$$

**Rubi [A]** time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1657, 634, 618, 206, 628}

$$\frac{(C(b^2 - 2ac) + 2Ac^2) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{bC \log(a + bx + cx^2)}{2c^2} + \frac{Cx}{c}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2), x]

[Out] (C\*x)/c - ((2\*A\*c^2 + (b^2 - 2\*a\*c)\*C)\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(c^2\*Sqrt[b^2 - 4\*a\*c]) - (b\*C\*Log[a + b\*x + c\*x^2])/(2\*c^2)

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

### Rule 1657

`Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]`

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Cx^2}{a + bx + cx^2} dx &= \int \left( \frac{C}{c} + \frac{Ac - aC - bCx}{c(a + bx + cx^2)} \right) dx \\
 &= \frac{Cx}{c} + \frac{\int \frac{Ac - aC - bCx}{a + bx + cx^2} dx}{c} \\
 &= \frac{Cx}{c} - \frac{(bC) \int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} + \frac{1}{2} \left( 2A + \frac{(b^2 - 2ac)C}{c^2} \right) \int \frac{1}{a + bx + cx^2} dx \\
 &= \frac{Cx}{c} - \frac{bC \log(a + bx + cx^2)}{2c^2} + \left( -2A - \frac{(b^2 - 2ac)C}{c^2} \right) \text{Subst} \left( \int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx \right) \\
 &= \frac{Cx}{c} - \frac{\left( 2A + \frac{(b^2 - 2ac)C}{c^2} \right) \tanh^{-1} \left( \frac{b+2cx}{\sqrt{b^2 - 4ac}} \right) - \frac{bC \log(a + bx + cx^2)}{2c^2}}{\sqrt{b^2 - 4ac}}
 \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 84, normalized size = 1.04

$$\frac{(-2acC + 2Ac^2 + b^2C) \tan^{-1} \left( \frac{b+2cx}{\sqrt{4ac-b^2}} \right) - \frac{bC \log(a + bx + cx^2)}{2c^2} + \frac{Cx}{c}}{c^2 \sqrt{4ac - b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2), x]

[Out] (C\*x)/c + ((2\*A\*c^2 + b^2\*C - 2\*a\*c\*C)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(c^2\*Sqrt[-b^2 + 4\*a\*c]) - (b\*C\*Log[a + b\*x + c\*x^2])/(2\*c^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{a + bx + cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2), x]

[Out] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2), x]

**fricas** [A] time = 0.77, size = 265, normalized size = 3.27

$$\left[ \frac{(Cb^2 - 2Cac + 2Ac^2)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2(Cb^2c - 4Cabc)x - (Cb^3 - 4Cabc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)}, -\frac{2(Cb^2 - 2Cac + 2Ac^2)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - 2(Cb^2c - 4Cac^2)x + (Cb^3 - 4Cabc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a), x, algorithm="fricas")

[Out] [1/2\*((C\*b^2 - 2\*C\*a\*c + 2\*A\*c^2)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c - sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) + 2\*(C\*b^2\*c - 4\*C\*a\*c^2)\*x - (C\*b^3 - 4\*C\*a\*b\*c)\*log(c\*x^2 + b\*x + a))/(b^2\*c^2 - 4\*a\*c^3), -1/2\*(2\*(C\*b^2 - 2\*C\*a\*c + 2\*A\*c^2)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) - 2\*(C\*b^2\*c - 4\*C\*a\*c^2)\*x + (C\*b^3 - 4\*C\*a\*b\*c)\*log(c\*x^2 + b\*x + a))/(b^2\*c^2 - 4\*a\*c^3)]

**giac** [A] time = 0.15, size = 78, normalized size = 0.96

$$\frac{Cx}{c} - \frac{Cb \log(cx^2 + bx + a)}{2c^2} + \frac{(Cb^2 - 2Cac + 2Ac^2) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a), x, algorithm="giac")

[Out] C\*x/c - 1/2\*C\*b\*log(c\*x^2 + b\*x + a)/c^2 + (C\*b^2 - 2\*C\*a\*c + 2\*A\*c^2)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/(sqrt(-b^2 + 4\*a\*c)\*c^2)

**maple** [A] time = 0.01, size = 140, normalized size = 1.73

$$\frac{2A \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} - \frac{2Ca \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} c} + \frac{C b^2 \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2} c^2} - \frac{Cb \ln(cx^2 + bx + a)}{2c^2} + \frac{Cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+A)/(c\*x^2+b\*x+a), x)

[Out] C/c\*x - 1/2\*b\*C\*ln(c\*x^2 + b\*x + a)/c^2 + 2/(4\*a\*c - b^2)^(1/2)\*arctan((2\*c\*x + b)/(4\*a\*c - b^2)^(1/2))\*A - 2/c/(4\*a\*c - b^2)^(1/2)\*arctan((2\*c\*x + b)/(4\*a\*c - b^2)^(1/2))\*a\*C + 1/c^2/(4\*a\*c - b^2)^(1/2)\*arctan((2\*c\*x + b)/(4\*a\*c - b^2)^(1/2))\*b^2\*C

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 0.19, size = 224, normalized size = 2.77

$$\frac{2A \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{Cx}{c} + \frac{Cb^3 \ln(cx^2 + bx + a)}{2(4ac^3 - b^2c^2)} - \frac{2Ca \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}} + \frac{Cb^2 \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}} - \frac{2Cab \ln(cx^2 + bx + a)}{4ac^3 - b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)/(a + b\*x + c\*x^2),x)

[Out]  $(2A \operatorname{atan}(b/(4ac - b^2)^{1/2} + (2cx)/(4ac - b^2)^{1/2})) / (4ac - b^2)^{1/2} + (Cx)/c + (Cb^3 \log(a + bx + cx^2)) / (2(4ac^3 - b^2c^2)) - (2Ca \operatorname{atan}(b/(4ac - b^2)^{1/2} + (2cx)/(4ac - b^2)^{1/2})) / (c(4ac - b^2)^{1/2}) + (Cb^2 \operatorname{atan}(b/(4ac - b^2)^{1/2} + (2cx)/(4ac - b^2)^{1/2})) / (c^2(4ac - b^2)^{1/2}) - (2Cab \log(a + bx + cx^2)) / (4ac^3 - b^2c^2)$

**sympy** [B] time = 1.21, size = 413, normalized size = 5.10

$$\frac{Cx}{c} + \left( \frac{Cb}{2c^2} \frac{\sqrt{4ac + b^2} (-2Aa^2 + 2Cac - Cb^2)}{2c^2(4ac - b^2)} \right) \log \left( x + \frac{-Abc - Ccb - 4ac^2 \left( \frac{Cb}{2c^2} + \frac{\sqrt{4ac + b^2} (-2Aa^2 + 2Cac - Cb^2)}{2c^2(4ac - b^2)} \right) + b^2c \left( -\frac{Cb}{2c^2} - \frac{\sqrt{4ac + b^2} (-2Aa^2 + 2Cac - Cb^2)}{2c^2(4ac - b^2)} \right)}{-2Aa^2 + 2Cac - Cb^2} \right) + \left( \frac{Cb}{2c^2} + \frac{\sqrt{4ac + b^2} (-2Aa^2 + 2Cac - Cb^2)}{2c^2(4ac - b^2)} \right) \log \left( x + \frac{-Abc - Ccb - 4ac^2 \left( \frac{Cb}{2c^2} + \frac{\sqrt{4ac + b^2} (-2Aa^2 + 2Cac - Cb^2)}{2c^2(4ac - b^2)} \right) + b^2c \left( -\frac{Cb}{2c^2} - \frac{\sqrt{4ac + b^2} (-2Aa^2 + 2Cac - Cb^2)}{2c^2(4ac - b^2)} \right)}{-2Aa^2 + 2Cac - Cb^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+A)/(c\*x\*\*2+b\*x+a),x)

[Out]  $Cx/c + (-Cb/(2c**2) - \sqrt{-4ac + b**2} * (-2A*c**2 + 2C*a*c - C*b**2) / (2*c**2*(4ac - b**2))) * \log(x + (-A*b*c - C*a*b - 4*a*c**2*(-Cb/(2c**2) - \sqrt{-4ac + b**2} * (-2A*c**2 + 2C*a*c - C*b**2) / (2*c**2*(4ac - b**2)))) + b**2*c*(-Cb/(2c**2) - \sqrt{-4ac + b**2} * (-2A*c**2 + 2C*a*c - C*b**2) / (2*c**2*(4ac - b**2)))) / (-2A*c**2 + 2C*a*c - C*b**2) + (-Cb/(2c**2) + \sqrt{-4ac + b**2} * (-2A*c**2 + 2C*a*c - C*b**2) / (2*c**2*(4ac - b**2))) * \log(x + (-A*b*c - C*a*b - 4*a*c**2*(-Cb/(2c**2) + \sqrt{-4ac + b**2} * (-2A*c**2 + 2C*a*c - C*b**2) / (2*c**2*(4ac - b**2)))) + b**2*c*(-Cb/(2c**2) + \sqrt{-4ac + b**2} * (-2A*c**2 + 2C*a*c - C*b**2) / (2*c**2*(4ac - b**2)))) / (-2A*c**2 + 2C*a*c - C*b**2)$

$$3.141 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=100

$$\frac{4(aC + Ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}} - \frac{x(C(b^2 - 2ac) + 2Ac^2) + bc\left(\frac{aC}{c} + A\right)}{c(b^2 - 4ac)(a + bx + cx^2)}$$

**Rubi [A]** time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1660, 12, 618, 206}

$$\frac{4(aC + Ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}} - \frac{x(C(b^2 - 2ac) + 2Ac^2) + bc\left(\frac{aC}{c} + A\right)}{c(b^2 - 4ac)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^2, x]

[Out] -((b\*c\*(A + (a\*C)/c) + (2\*A\*c^2 + (b^2 - 2\*a\*c)\*C)\*x)/(c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)) + (4\*(A\*c + a\*C)\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(b^2 - 4\*a\*c)^(3/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660



```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Cx^2}{(a + bx + cx^2)^2} dx &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{2(Ac + aC)}{a + bx + cx^2} dx}{-b^2 + 4ac} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{c(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2(Ac + aC)) \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{(4(Ac + aC)) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{4(Ac + aC) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 98, normalized size = 0.98

$$\frac{aC(b - 2cx) + Ac(b + 2cx) + b^2Cx}{c(4ac - b^2)(a + x(b + cx))} + \frac{4(aC + Ac) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^2, x]

[Out] (b^2\*C\*x + a\*C\*(b - 2\*c\*x) + A\*c\*(b + 2\*c\*x))/(c\*(-b^2 + 4\*a\*c)\*(a + x\*(b + c\*x))) + (4\*(A\*c + a\*C)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(-b^2 + 4\*a\*c)^(3/2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^2, x]

[Out] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^2, x]

**fricas** [B] time = 0.67, size = 511, normalized size = 5.11

$$\frac{Cb^2 - 4Abc^2 + 2(Ca^2 + Aac^2 + (Ca^2 + Ac^2)^2 + (Cac + Abc^2))\sqrt{-b^2 + 4ac} \log\left(\frac{2c^2x + b - \sqrt{-b^2 + 4ac}}{2c^2x + b + \sqrt{-b^2 + 4ac}}\right) - (4Cb^2 - Ab^3) + (C^2 - 6Cab^2 - 8Aac^2 + 2(4C^2 + Ab^2)^2)c}{ab^2c - 8a^2b^2c^2 + 16a^2c^3} + \frac{Cb^2 - 4Abc^2 - 4(Ca^2 + Aac^2 + (Ca^2 + Ac^2)^2 + (Cac + Abc^2))\sqrt{-b^2 + 4ac} \arctan\left(\frac{\sqrt{-b^2 + 4ac}}{2c^2x + b}\right) - (4Cb^2 - Ab^3)c + (C^2 - 6Cab^2 - 8Aac^2 + 2(4C^2 + Ab^2)^2)c}{ab^2c - 8a^2b^2c^2 + 16a^2c^3} + \frac{(b^2c - 8ab^2c^2 + 16a^2c^3)^2 + (b^2c - 8ab^2c^2 + 16a^2c^3)c}{ab^2c - 8a^2b^2c^2 + 16a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out]  $[-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3))*x^2 + (C*a*b*c + A*b*c^2)*x]*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a) - (4*C*a^2*b - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 - 4*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3))*x^2 + (C*a*b*c + A*b*c^2)*x]*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c})*(2*c*x + b)/(b^2 - 4*a*c)) - (4*C*a^2*b - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]$

**giac** [A] time = 0.16, size = 108, normalized size = 1.08

$$-\frac{4(Ca + Ac) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{Cb^2x - 2Cacx + 2Ac^2x + Cab + Abc}{(b^2c - 4ac^2)(cx^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out]  $-4*(C*a + A*c)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2 - 4*a*c)*\sqrt{-b^2 + 4*a*c}) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))$

**maple [A]** time = 0.01, size = 146, normalized size = 1.46

$$\frac{4Ac \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{4Ca \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{\frac{(Ac+aC)b}{(4ac-b^2)c} + \frac{(2Ac^2-2Cac+Cb^2)x}{(4ac-b^2)c}}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+A)/(c\*x^2+b\*x+a)^2,x)

[Out] ((2\*A\*c^2-2\*C\*a\*c+C\*b^2)/c/(4\*a\*c-b^2)\*x+b/c\*(A\*c+C\*a)/(4\*a\*c-b^2))/(c\*x^2+b\*x+a)+4/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*A\*c+4/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*C

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad [B]** time = 4.53, size = 172, normalized size = 1.72

$$\frac{\frac{Abc+Cab}{c(4ac-b^2)} + \frac{x(Cb^2+2Ac^2-2Cac)}{c(4ac-b^2)}}{cx^2+bx+a} - \frac{4 \operatorname{atan}\left(\frac{\left(\frac{2(Ac+C a)(b^3-4abc)}{(4ac-b^2)^{5/2}} - \frac{4cx(Ac+C a)}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2Ac+2Ca}\right)}{(4ac-b^2)^{3/2}}}{(Ac+Ca)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)/(a + b\*x + c\*x^2)^2,x)

[Out] ((A\*b\*c + C\*a\*b)/(c\*(4\*a\*c - b^2)) + (x\*(2\*A\*c^2 + C\*b^2 - 2\*C\*a\*c))/(c\*(4\*a\*c - b^2)))/(a + b\*x + c\*x^2) - (4\*atan((((2\*(A\*c + C\*a))\*(b^3 - 4\*a\*b\*c))/(4\*a\*c - b^2)^(5/2) - (4\*c\*x\*(A\*c + C\*a))/(4\*a\*c - b^2)^(3/2))\*((4\*a\*c - b^2)^(3/2)))/(2\*A\*c + 2\*C\*a))\*(A\*c + C\*a))/(4\*a\*c - b^2)^(3/2)

sympy [B] time = 1.21, size = 376, normalized size = 3.76

$$-2 \sqrt{\frac{1}{(4ac - b^2)^3}} (Ac + Ca) \log\left(x + \frac{2Abc + 2Cab - 32a^2 \sqrt{\frac{1}{(4ac - b^2)}} (Ac + Ca) + 16ab^2 \sqrt{\frac{1}{(4ac - b^2)}} (Ac + Ca) - 2b^3 \sqrt{\frac{1}{(4ac - b^2)}} (Ac + Ca)}{4Ac^2 + 4Cac}\right) + 2 \sqrt{\frac{1}{(4ac - b^2)^3}} (Ac + Ca) \log\left(x + \frac{2Abc + 2Cab + 32a^2 \sqrt{\frac{1}{(4ac - b^2)}} (Ac + Ca) - 16ab^2 \sqrt{\frac{1}{(4ac - b^2)}} (Ac + Ca) + 2b^3 \sqrt{\frac{1}{(4ac - b^2)}} (Ac + Ca)}{4Ac^2 + 4Cac}\right) + \frac{Abc + Cab + x(2Ac^2 - 2Cac + Cb^2)}{4Ac^2 - ab^2c + x^2(4ac^3 - b^2c^2) + x(4abc^2 - b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+A)/(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out]  $-2\sqrt{-1/(4ac - b^2)^3}(Ac + Ca)\log(x + (2Abc + 2Cab - 32a^2c^2\sqrt{-1/(4ac - b^2)^3}(Ac + Ca) + 16ab^2\sqrt{-1/(4ac - b^2)^3}(Ac + Ca) - 2b^3\sqrt{-1/(4ac - b^2)^3}(Ac + Ca)) / (4A^2c + 4Cac)) + 2\sqrt{-1/(4ac - b^2)^3}(Ac + Ca)\log(x + (2Abc + 2Cab + 32a^2c^2\sqrt{-1/(4ac - b^2)^3}(Ac + Ca) - 16ab^2\sqrt{-1/(4ac - b^2)^3}(Ac + Ca) + 2b^3\sqrt{-1/(4ac - b^2)^3}(Ac + Ca)) / (4A^2c + 4Cac)) + (Abc + Cab + x(2A^2c - 2Cac + Cb^2)) / (4a^2c^2 - ab^2c + x^2(4ac^3 - b^2c^2) + x(4abc^2 - b^3c))$

$$3.142 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^3} dx$$

**Optimal.** Leaf size=161

$$\frac{x(C(b^2-2ac)+2Ac^2)+bc\left(\frac{aC}{c}+A\right)}{2c(b^2-4ac)(a+bx+cx^2)^2} - \frac{2(C(2ac+b^2)+6Ac^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{(b+2cx)(2aC+6Ac+\frac{b^2C}{c})}{2(b^2-4ac)^2(a+bx+cx^2)}$$

**Rubi [A]** time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1660, 12, 614, 618, 206}

$$\frac{x(C(b^2-2ac)+2Ac^2)+bc\left(\frac{aC}{c}+A\right)}{2c(b^2-4ac)(a+bx+cx^2)^2} - \frac{2(C(2ac+b^2)+6Ac^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}} + \frac{(b+2cx)(2aC+6Ac+\frac{b^2C}{c})}{2(b^2-4ac)^2(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^3, x]

[Out]  $-(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(2*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/(b^2 - 4*a*c)^{(5/2)}$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{A + Cx^2}{(a + bx + cx^2)^3} dx &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{2c(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{\int \frac{6Ac + 2aC + \frac{b^2C}{c}}{(a + bx + cx^2)^2} dx}{2(b^2 - 4ac)} \\
 &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{2c(b^2 - 4ac)(a + bx + cx^2)^2} - \frac{\left(6Ac + 2aC + \frac{b^2C}{c}\right) \int \frac{1}{(a + bx + cx^2)^2} dx}{2(b^2 - 4ac)} \\
 &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{2c(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{\left(6Ac + 2aC + \frac{b^2C}{c}\right)(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{(6Ac^2 + (b^2 - 2ac)C)}{2(b^2 - 4ac)^2} \\
 &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{2c(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{\left(6Ac + 2aC + \frac{b^2C}{c}\right)(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{2(6Ac^2 + (b^2 - 2ac)C)}{2(b^2 - 4ac)^2} \\
 &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{2c(b^2 - 4ac)(a + bx + cx^2)^2} + \frac{\left(6Ac + 2aC + \frac{b^2C}{c}\right)(b + 2cx)}{2(b^2 - 4ac)^2(a + bx + cx^2)} - \frac{2(6Ac^2 + (b^2 - 2ac)C)}{2(b^2 - 4ac)^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 160, normalized size = 0.99

$$\frac{1}{2} \left( \frac{(b+2cx)(C(2ac+b^2)+6Ac^2)}{c(b^2-4ac)^2(a+x(b+cx))} + \frac{4(C(2ac+b^2)+6Ac^2) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}} + \frac{aC(b-2cx)+Ac(b+2cx)+b^2Cx}{c(4ac-b^2)(a+x(b+cx))^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^3,x]

[Out] (((6\*A\*c^2 + (b^2 + 2\*a\*c)\*C)\*(b + 2\*c\*x))/(c\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))) + (b^2\*C\*x + a\*C\*(b - 2\*c\*x) + A\*c\*(b + 2\*c\*x))/(c\*(-b^2 + 4\*a\*c)\*(a + x\*(b + c\*x))^2) + (4\*(6\*A\*c^2 + (b^2 + 2\*a\*c)\*C)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(-b^2 + 4\*a\*c)^(5/2))/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^3,x]

[Out] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^3, x]

**fricas [B]** time = 0.60, size = 1199, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^3,x, algorithm="fricas")

[Out] [1/2\*(6\*C\*a^2\*b^3 - A\*b^5 - 40\*A\*a^2\*b\*c^2 + 2\*(C\*b^4\*c - 2\*C\*a\*b^2\*c^2 - 2\*4\*A\*a\*c^4 - 2\*(4\*C\*a^2 - 3\*A\*b^2)\*c^3)\*x^3 + 3\*(C\*b^5 - 2\*C\*a\*b^3\*c - 24\*A\*a\*b\*c^3 - 2\*(4\*C\*a^2\*b - 3\*A\*b^3)\*c^2)\*x^2 + 2\*(C\*a^2\*b^2 + 2\*C\*a^3\*c + 6\*A\*a^2\*c^2 + (C\*b^2\*c^2 + 2\*C\*a\*c^3 + 6\*A\*c^4)\*x^4 + 2\*(C\*b^3\*c + 2\*C\*a\*b\*c^2 + 6\*A\*b\*c^3)\*x^3 + (C\*b^4 + 4\*C\*a\*b^2\*c + 12\*A\*a\*c^3 + 2\*(2\*C\*a^2 + 3\*A\*b^2)\*c^2)\*x^2 + 2\*(C\*a\*b^3 + 2\*C\*a^2\*b\*c + 6\*A\*a\*b\*c^2)\*x)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c - sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) - 2\*(12\*C\*a^3\*b - 7\*A\*a\*b^3)\*c + 2\*(5\*C\*a\*b^4 - 40\*A\*a^2\*c^3 + 2\*(4\*C\*a^3 + A\*a\*b^2)\*c^2 - 2\*(11\*C\*a^2\*b^2 - A\*b^4)\*c)\*x)/(a^2\*b^6 - 1\*2\*a^3\*b^4\*c + 48\*a^4\*b^2\*c^2 - 64\*a^5\*c^3 + (b^6\*c^2 - 12\*a\*b^4\*c^3 + 48\*a^2\*b^2\*c^4 - 64\*a^3\*c^5)\*x^4 + 2\*(b^7\*c - 12\*a\*b^5\*c^2 + 48\*a^2\*b^3\*c^3 - 64\*a^3\*b\*c^4)\*x^3 + (b^8 - 10\*a\*b^6\*c + 24\*a^2\*b^4\*c^2 + 32\*a^3\*b^2\*c^3 - 128\*a^4\*c^4)\*x^2 + 2\*(a\*b^7 - 12\*a^2\*b^5\*c + 48\*a^3\*b^3\*c^2 - 64\*a^4\*b\*c^3)\*x)

,  $1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 - 24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2 + 2*C*a^3*c + 6*A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*\sqrt{-b^2 + 4*a*c})*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]$

**giac [A]** time = 0.18, size = 217, normalized size = 1.35

$$\frac{2(Cb^2 + 2Cac + 6Ac^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + 2Cb^2cx^3 + 4Cac^2x^3 + 12Ac^3x^3 + 3Cb^3x^2 + 6Cabcx^2 + 18Abc^2x^2 + 10Cab^2x - 4Ca^2cx + 4Ab^2cx + 20Aac^2x + 6Ca^2b - Ab^3 + 10Aabc}{(b^4 - 8ab^2c + 16a^2c^2)\sqrt{-b^2 + 4ac}} + \frac{2(Cb^2 + 2Cac + 6Ac^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + 2Cb^2cx^3 + 4Cac^2x^3 + 12Ac^3x^3 + 3Cb^3x^2 + 6Cabcx^2 + 18Abc^2x^2 + 10Cab^2x - 4Ca^2cx + 4Ab^2cx + 20Aac^2x + 6Ca^2b - Ab^3 + 10Aabc}{2(b^4 - 8ab^2c + 16a^2c^2)(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^3,x, algorithm="giac")

[Out]  $2*(C*b^2 + 2*C*a*c + 6*A*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*\sqrt{-b^2 + 4*a*c}) + 1/2*(2*C*b^2*c*x^3 + 4*C*a*c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 + 10*C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^3 + 10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)$

**maple [B]** time = 0.01, size = 373, normalized size = 2.32

$$\frac{12A c^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{4Cac \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{2Cb^2 \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2 - 8ab^2c + b^4)\sqrt{4ac-b^2}} + \frac{\frac{(6A c^2 + 2Cac + C b^2) c x^3}{16a^2c^2 - 8ab^2c + b^4} + \frac{3(6A c^2 + 2Cac + C b^2) b x^2}{2(16a^2c^2 - 8ab^2c + b^4)} + \frac{(10Aac - Ab^2 + 6C a^2) b}{32a^2c^2 - 16ab^2c + 2b^4} + \frac{(10Aa^2 + 2A b^2c - 2C a^2c + 5Ca b^2) x}{16a^2c^2 - 8ab^2c + b^4}}{(cx^2 + bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+A)/(c\*x^2+b\*x+a)^3,x)

[Out]  $(c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2*c+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2+4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*a*c+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*b^2$





$$\begin{aligned}
& )*(6*A*c**2 + 2*C*a*c + C*b**2)*\log(x + (6*A*b*c**2 + 2*C*a*b*c + C*b**3 + \\
& 64*a**3*c**3*\sqrt{-1/(4*a*c - b**2)**5}*(6*A*c**2 + 2*C*a*c + C*b**2) - 48* \\
& a**2*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**5}*(6*A*c**2 + 2*C*a*c + C*b**2) + 1 \\
& 2*a*b**4*c*\sqrt{-1/(4*a*c - b**2)**5}*(6*A*c**2 + 2*C*a*c + C*b**2) - b**6* \\
& \sqrt{-1/(4*a*c - b**2)**5}*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*c**3 + 4*C* \\
& a*c**2 + 2*C*b**2*c) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**3*(12*A*c**3 \\
& + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c + 3*C*b**3) + x \\
& *(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a**4*c**2 - 16* \\
& a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c* \\
& *2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c** \\
& 3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5)
\end{aligned}$$

$$3.143 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^4} dx$$

**Optimal.** Leaf size=206

$$\frac{2(b+2cx)(C(ac+b^2)+5Ac^2)}{(b^2-4ac)^3(a+bx+cx^2)} - \frac{x(C(b^2-2ac)+2Ac^2)+bc\left(\frac{aC}{c}+A\right)}{3c(b^2-4ac)(a+bx+cx^2)^3} + \frac{8c(C(ac+b^2)+5Ac^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

**Rubi [A]** time = 0.19, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1660, 12, 614, 618, 206}

$$\frac{2(b+2cx)(C(ac+b^2)+5Ac^2)}{(b^2-4ac)^3(a+bx+cx^2)} - \frac{x(C(b^2-2ac)+2Ac^2)+bc\left(\frac{aC}{c}+A\right)}{3c(b^2-4ac)(a+bx+cx^2)^3} + \frac{8c(C(ac+b^2)+5Ac^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}} + \frac{(b+2cx)\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)}{3(b^2-4ac)^2(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^4, x]

[Out]  $-(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^{(7/2)}$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 614**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Cx^2}{(a + bx + cx^2)^4} dx &= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{3c(b^2 - 4ac)(a + bx + cx^2)^3} - \frac{\int \frac{2\left(5Ac + \left(a + \frac{b^2}{c}\right)C\right)}{(a + bx + cx^2)^3} dx}{3(b^2 - 4ac)} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{3c(b^2 - 4ac)(a + bx + cx^2)^3} - \frac{\left(2\left(5Ac + \left(a + \frac{b^2}{c}\right)C\right)\right) \int \frac{1}{(a + bx + cx^2)^3} dx}{3(b^2 - 4ac)} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{3c(b^2 - 4ac)(a + bx + cx^2)^3} + \frac{\left(5Ac + \left(a + \frac{b^2}{c}\right)C\right)(b + 2cx)}{3(b^2 - 4ac)^2(a + bx + cx^2)^2} + \frac{2(5Ac^2 + (b^2 - 2ac)C)}{(b^2 - 4ac)} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{3c(b^2 - 4ac)(a + bx + cx^2)^3} + \frac{\left(5Ac + \left(a + \frac{b^2}{c}\right)C\right)(b + 2cx)}{3(b^2 - 4ac)^2(a + bx + cx^2)^2} - \frac{2(5Ac^2 + (b^2 - 2ac)C)}{(b^2 - 4ac)} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{3c(b^2 - 4ac)(a + bx + cx^2)^3} + \frac{\left(5Ac + \left(a + \frac{b^2}{c}\right)C\right)(b + 2cx)}{3(b^2 - 4ac)^2(a + bx + cx^2)^2} - \frac{2(5Ac^2 + (b^2 - 2ac)C)}{(b^2 - 4ac)} \\
&= -\frac{bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x}{3c(b^2 - 4ac)(a + bx + cx^2)^3} + \frac{\left(5Ac + \left(a + \frac{b^2}{c}\right)C\right)(b + 2cx)}{3(b^2 - 4ac)^2(a + bx + cx^2)^2} - \frac{2(5Ac^2 + (b^2 - 2ac)C)}{(b^2 - 4ac)}
\end{aligned}$$

**Mathematica [A]** time = 0.41, size = 204, normalized size = 0.99

$$\frac{1}{3} \left( \frac{6(b+2cx)(C(ac+b^2)+5Ac^2)}{(b^2-4ac)^3(a+x(b+cx))} + \frac{(b+2cx)(C(ac+b^2)+5Ac^2)}{c(b^2-4ac)^2(a+x(b+cx))^2} + \frac{24c(C(ac+b^2)+5Ac^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{7/2}} + \frac{aC(b-2cx)+Ac(b+2cx)+b^2Cx}{c(4ac-b^2)(a+x(b+cx))^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^4, x]

[Out] (((5\*A\*c^2 + (b^2 + a\*c)\*C)\*(b + 2\*c\*x))/(c\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))^2) - (6\*(5\*A\*c^2 + (b^2 + a\*c)\*C)\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)^3\*(a + x\*(b + c\*x))) + (b^2\*C\*x + a\*C\*(b - 2\*c\*x) + A\*c\*(b + 2\*c\*x))/(c\*(-b^2 + 4\*a\*c)\*(a + x\*(b + c\*x))^3) + (24\*c\*(5\*A\*c^2 + (b^2 + a\*c)\*C)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(-b^2 + 4\*a\*c)^(7/2))/3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^4, x]

[Out] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^4, x]

**fricas [B]** time = 0.87, size = 2103, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^4,x, algorithm="fricas")

[Out] [-1/3\*(C\*a^2\*b^5 + A\*b^7 - 264\*A\*a^3\*b\*c^3 + 12\*(C\*b^4\*c^3 - 3\*C\*a\*b^2\*c^4 - 20\*A\*a\*c^6 - (4\*C\*a^2 - 5\*A\*b^2)\*c^5)\*x^5 + 30\*(C\*b^5\*c^2 - 3\*C\*a\*b^3\*c^3 - 20\*A\*a\*b\*c^5 - (4\*C\*a^2\*b - 5\*A\*b^3)\*c^4)\*x^4 + 2\*(11\*C\*b^6\*c - 17\*C\*a\*b^4\*c^2 - 320\*A\*a^2\*c^5 - 4\*(16\*C\*a^3 + 35\*A\*a\*b^2)\*c^4 - (92\*C\*a^2\*b^2 - 55\*A\*b^4)\*c^3)\*x^3 - 2\*(52\*C\*a^4\*b - 59\*A\*a^2\*b^3)\*c^2 + 3\*(C\*b^7 + 13\*C\*a\*b^5\*c - 320\*A\*a^2\*b\*c^4 - 4\*(16\*C\*a^3\*b - 15\*A\*a\*b^3)\*c^3 - (52\*C\*a^2\*b^3 - 5\*A\*b^5)\*c^2)\*x^2 + 12\*(C\*a^3\*b^2\*c + C\*a^4\*c^2 + 5\*A\*a^3\*c^3 + (C\*b^2\*c^4 + C\*a\*c^5 + 5\*A\*c^6)\*x^6 + 3\*(C\*b^3\*c^3 + C\*a\*b\*c^4 + 5\*A\*b\*c^5)\*x^5 + 3\*(C\*b^4\*c^2 + 2\*C\*a\*b^2\*c^3 + 5\*A\*a\*c^5 + (C\*a^2 + 5\*A\*b^2)\*c^4)\*x^4 + (C\*b^5\*c + 7\*C\*a\*b^3\*c^2 + 30\*A\*a\*b\*c^4 + (6\*C\*a^2\*b + 5\*A\*b^3)\*c^3)\*x^3 + 3\*(C\*a\*b^4\*c + 2\*C\*a^2\*b^2\*c^2 + 5\*A\*a^2\*c^4 + (C\*a^3 + 5\*A\*a\*b^2)\*c^3)\*x^2 + 3\*(C\*a^2\*b^3\*c + C\*a^3\*b\*c^2 + 5\*A\*a^2\*b\*c^3)\*x)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c - sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a

) + (22\*C\*a^3\*b^3 - 17\*A\*a\*b^5)\*c + 3\*(C\*a\*b^6 - 176\*A\*a^3\*c^4 + 4\*(4\*C\*a^4 - 7\*A\*a^2\*b^2)\*c^3 - 2\*(46\*C\*a^3\*b^2 - 11\*A\*a\*b^4)\*c^2 + (18\*C\*a^2\*b^4 - A\*b^6)\*c)\*x)/(a^3\*b^8 - 16\*a^4\*b^6\*c + 96\*a^5\*b^4\*c^2 - 256\*a^6\*b^2\*c^3 + 256\*a^7\*c^4 + (b^8\*c^3 - 16\*a\*b^6\*c^4 + 96\*a^2\*b^4\*c^5 - 256\*a^3\*b^2\*c^6 + 256\*a^4\*c^7)\*x^6 + 3\*(b^9\*c^2 - 16\*a\*b^7\*c^3 + 96\*a^2\*b^5\*c^4 - 256\*a^3\*b^3\*c^5 + 256\*a^4\*b\*c^6)\*x^5 + 3\*(b^10\*c - 15\*a\*b^8\*c^2 + 80\*a^2\*b^6\*c^3 - 160\*a^3\*b^4\*c^4 + 256\*a^5\*c^6)\*x^4 + (b^11 - 10\*a\*b^9\*c + 320\*a^3\*b^5\*c^3 - 1280\*a^4\*b^3\*c^4 + 1536\*a^5\*b\*c^5)\*x^3 + 3\*(a\*b^10 - 15\*a^2\*b^8\*c + 80\*a^3\*b^6\*c^2 - 160\*a^4\*b^4\*c^3 + 256\*a^6\*c^5)\*x^2 + 3\*(a^2\*b^9 - 16\*a^3\*b^7\*c + 96\*a^4\*b^5\*c^2 - 256\*a^5\*b^3\*c^3 + 256\*a^6\*b\*c^4)\*x), -1/3\*(C\*a^2\*b^5 + A\*b^7 - 264\*A\*a^3\*b\*c^3 + 12\*(C\*b^4\*c^3 - 3\*C\*a\*b^2\*c^4 - 20\*A\*a\*c^6 - (4\*C\*a^2 - 5\*A\*b^2)\*c^5)\*x^5 + 30\*(C\*b^5\*c^2 - 3\*C\*a\*b^3\*c^3 - 20\*A\*a\*b\*c^5 - (4\*C\*a^2\*b - 5\*A\*b^3)\*c^4)\*x^4 + 2\*(11\*C\*b^6\*c - 17\*C\*a\*b^4\*c^2 - 320\*A\*a^2\*c^5 - 4\*(16\*C\*a^3 + 35\*A\*a\*b^2)\*c^4 - (92\*C\*a^2\*b^2 - 55\*A\*b^4)\*c^3)\*x^3 - 2\*(52\*C\*a^4\*b - 59\*A\*a^2\*b^3)\*c^2 + 3\*(C\*b^7 + 13\*C\*a\*b^5\*c - 320\*A\*a^2\*b\*c^4 - 4\*(16\*C\*a^3\*b - 15\*A\*a\*b^3)\*c^3 - (52\*C\*a^2\*b^3 - 5\*A\*b^5)\*c^2)\*x^2 - 24\*(C\*a^3\*b^2\*c + C\*a^4\*c^2 + 5\*A\*a^3\*c^3 + (C\*b^2\*c^4 + C\*a\*c^5 + 5\*A\*c^6)\*x^6 + 3\*(C\*b^3\*c^3 + C\*a\*b\*c^4 + 5\*A\*b\*c^5)\*x^5 + 3\*(C\*b^4\*c^2 + 2\*C\*a\*b^2\*c^3 + 5\*A\*a\*c^5 + (C\*a^2 + 5\*A\*b^2)\*c^4)\*x^4 + (C\*b^5\*c + 7\*C\*a\*b^3\*c^2 + 30\*A\*a\*b\*c^4 + (6\*C\*a^2\*b + 5\*A\*b^3)\*c^3)\*x^3 + 3\*(C\*a\*b^4\*c + 2\*C\*a^2\*b^2\*c^2 + 5\*A\*a^2\*c^4 + (C\*a^3 + 5\*A\*a\*b^2)\*c^3)\*x^2 + 3\*(C\*a^2\*b^3\*c + C\*a^3\*b\*c^2 + 5\*A\*a^2\*b\*c^3)\*x)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) + (22\*C\*a^3\*b^3 - 17\*A\*a\*b^5)\*c + 3\*(C\*a\*b^6 - 176\*A\*a^3\*c^4 + 4\*(4\*C\*a^4 - 7\*A\*a^2\*b^2)\*c^3 - 2\*(46\*C\*a^3\*b^2 - 11\*A\*a\*b^4)\*c^2 + (18\*C\*a^2\*b^4 - A\*b^6)\*c)\*x)/(a^3\*b^8 - 16\*a^4\*b^6\*c + 96\*a^5\*b^4\*c^2 - 256\*a^6\*b^2\*c^3 + 256\*a^7\*c^4 + (b^8\*c^3 - 16\*a\*b^6\*c^4 + 96\*a^2\*b^4\*c^5 - 256\*a^3\*b^2\*c^6 + 256\*a^4\*c^7)\*x^6 + 3\*(b^9\*c^2 - 16\*a\*b^7\*c^3 + 96\*a^2\*b^5\*c^4 - 256\*a^3\*b^3\*c^5 + 256\*a^4\*b\*c^6)\*x^5 + 3\*(b^10\*c - 15\*a\*b^8\*c^2 + 80\*a^2\*b^6\*c^3 - 160\*a^3\*b^4\*c^4 + 256\*a^5\*c^6)\*x^4 + (b^11 - 10\*a\*b^9\*c + 320\*a^3\*b^5\*c^3 - 1280\*a^4\*b^3\*c^4 + 1536\*a^5\*b\*c^5)\*x^3 + 3\*(a\*b^10 - 15\*a^2\*b^8\*c + 80\*a^3\*b^6\*c^2 - 160\*a^4\*b^4\*c^3 + 256\*a^6\*c^5)\*x^2 + 3\*(a^2\*b^9 - 16\*a^3\*b^7\*c + 96\*a^4\*b^5\*c^2 - 256\*a^5\*b^3\*c^3 + 256\*a^6\*b\*c^4)\*x]

**giac [B]** time = 0.17, size = 407, normalized size = 1.98

$$\frac{8(C^2c + Ca^2 + 5Aa^2)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - 12C^2b^2c^2 + 12Ca^4c^2 + 60A^2b^2c + 30C^2b^2c^4 + 30Ca^2b^2c^4 + 150Ab^4c^4 + 22C^2c^3 + 54Ca^2b^2c^3 + 32C^2b^2c^3 + 110Ab^2c^3 + 160Aa^4c^3 + 3C^2c^2 + 54Ca^2b^2c^2 + 88Ca^2b^2c^2 + 15A^2b^2c^2 + 240Ab^2c^2 + 3Ca^2c + 66Ca^2c^2 - 3Ab^2c - 12C^2b^2c + 54Aa^2b^2c + 132A^2b^2c + C^2b^2 + Ab^2 + 26Ca^2c - 13Aa^2c + 66A^2b^2c}{(b^2 - 12Ab^2c + 48A^2b^2c^2 - 64A^2c^3)\sqrt{-b^2 + 4ac}} \cdot \frac{1}{3(b^2 - 12Ab^2c + 48A^2b^2c^2 - 64A^2c^3)(c^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^4,x, algorithm="giac")

[Out] -8\*(C\*b^2\*c + C\*a\*c^2 + 5\*A\*c^3)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/((b^6 - 12\*a\*b^4\*c + 48\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*sqrt(-b^2 + 4\*a\*c)) - 1/3\*(12\*C\*b^2\*c^3\*x^5 + 12\*C\*a\*c^4\*x^5 + 60\*A\*c^5\*x^5 + 30\*C\*b^3\*c^2\*x^4 + 30\*C\*a\*b\*c^3\*x^4 + 150\*A\*b\*c^4\*x^4 + 22\*C\*b^4\*c\*x^3 + 54\*C\*a\*b^2\*c^2\*x^3 + 32\*C\*a^2\*c^3\*x^3 + 110\*A\*b^2\*c^3\*x^3 + 160\*A\*a\*c^4\*x^3 + 3\*C\*b^5\*x^2 + 51\*C\*a\*b^3



```
[Out] - ((A*b^5 + C*a^2*b^3 - 13*A*a*b^3*c + 26*C*a^3*b*c + 66*A*a^2*b*c^2)/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(44*A*a^2*c^3 - 4*C*a^3*c^2 - A*b^4*c + C*a*b^4 + 18*A*a*b^2*c^2 + 22*C*a^2*b^2*c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (2*x^3*(16*a*c^2 + 11*b^2*c)*(5*A*c^2 + C*b^2 + C*a*c))/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x^2*(b^3 + 16*a*b*c)*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^4*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a*b^2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*c - b^2)^(7/2) + (4*c*(5*A*c^2 + C*b^2 + C*a*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(20*A*c^3 + 4*C*a*c^2 + 4*C*b^2*c)*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*c - b^2)^(7/2)
```

**sympy [B]** time = 4.22, size = 1224, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66*A*a**2*b*c**2 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x**5*(60*A*c**5 + 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*a*b*c**3 + 30*C*b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a**2*c**3 + 54*C*a*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b**3*c**2 + 48*C*a**2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c**3 + 54*A*a*b**2*c**2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c + 3*C*a*b**4))/(192*a**6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3
```



$$\begin{aligned} & *c^{**6} - 144*a^{**2}*b^{**2}*c^{**5} + 36*a*b^{**4}*c^{**4} - 3*b^{**6}*c^{**3}) + x^{**5}*(576*a^{**3} \\ & *b*c^{**5} - 432*a^{**2}*b^{**3}*c^{**4} + 108*a*b^{**5}*c^{**3} - 9*b^{**7}*c^{**2}) + x^{**4}*(576*a \\ & **4*c^{**5} + 144*a^{**3}*b^{**2}*c^{**4} - 324*a^{**2}*b^{**4}*c^{**3} + 99*a*b^{**6}*c^{**2} - 9*b^{** \\ & 8*c) + x^{**3}*(1152*a^{**4}*b*c^{**4} - 672*a^{**3}*b^{**3}*c^{**3} + 72*a^{**2}*b^{**5}*c^{**2} + 18 \\ & *a*b^{**7}*c - 3*b^{**9}) + x^{**2}*(576*a^{**5}*c^{**4} + 144*a^{**4}*b^{**2}*c^{**3} - 324*a^{**3}*b \\ & **4*c^{**2} + 99*a^{**2}*b^{**6}*c - 9*a*b^{**8}) + x*(576*a^{**5}*b*c^{**3} - 432*a^{**4}*b^{**3}* \\ & c^{**2} + 108*a^{**3}*b^{**5}*c - 9*a^{**2}*b^{**7})) \end{aligned}$$

$$3.144 \quad \int \frac{(d+ex)^3(f+gx+hx^2)}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=591

$$\frac{\log(a+bx+cx^2)(c^2e(a^2e^2h+2abe(3dh+eg)+b^2(3d^2h+3deg+e^2f))-b^2ce^2(3aeh+3bdh+beg)-c^3(ae(3h+2a+bx+cx^2)+b^2(3d^2h+3deg+e^2f)))}{2c^5}$$

**Rubi [A]** time = 1.43, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1628, 634, 618, 206, 628}

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

[Out] -(((b^3\*e^3\*h - c^3\*d\*(3\*e^2\*f + 3\*d\*e\*g + d^2\*h) - b\*c\*e^2\*(b\*e\*g + 3\*b\*d\*h + 2\*a\*e\*h) + c^2\*e\*(a\*e\*(e\*g + 3\*d\*h) + b\*(e^2\*f + 3\*d\*e\*g + 3\*d^2\*h)))\*x)/c^4 + (e\*(b^2\*e^2\*h + c^2\*(e^2\*f + 3\*d\*e\*g + 3\*d^2\*h) - c\*e\*(b\*e\*g + 3\*b\*d\*h + a\*e\*h))\*x^2)/(2\*c^3) + (e^2\*(c\*e\*g + 3\*c\*d\*h - b\*e\*h)\*x^3)/(3\*c^2) + (e^3\*h\*x^4)/(4\*c) - ((2\*c^5\*d^3\*f - b^5\*e^3\*h + b^3\*c\*e^2\*(b\*e\*g + 3\*b\*d\*h + 5\*a\*e\*h) - c^4\*d\*(b\*d\*(3\*e\*f + d\*g) + 2\*a\*(3\*e^2\*f + 3\*d\*e\*g + d^2\*h)) - b\*c^2\*e\*(5\*a^2\*e^2\*h + 4\*a\*b\*e\*(e\*g + 3\*d\*h) + b^2\*(e^2\*f + 3\*d\*e\*g + 3\*d^2\*h)) + c^3\*(2\*a^2\*e^2\*(e\*g + 3\*d\*h) + b^2\*d\*(3\*e^2\*f + 3\*d\*e\*g + d^2\*h) + 3\*a\*b\*e\*(e^2\*f + 3\*d\*e\*g + 3\*d^2\*h)))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/(c^5\*Sqrt[b^2 - 4\*a\*c]) + ((c^4\*d^2\*(3\*e\*f + d\*g) + b^4\*e^3\*h - b^2\*c\*e^2\*(b\*e\*g + 3\*b\*d\*h + 3\*a\*e\*h) + c^2\*e\*(a^2\*e^2\*h + 2\*a\*b\*e\*(e\*g + 3\*d\*h) + b^2\*(e^2\*f + 3\*d\*e\*g + 3\*d^2\*h)) - c^3\*(b\*d\*(3\*e^2\*f + 3\*d\*e\*g + d^2\*h) + a\*e\*(e^2\*f + 3\*d\*e\*g + 3\*d^2\*h)))\*Log[a + b\*x + c\*x^2])/(2\*c^5)

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628



```
[Out] (12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*
h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g
+ 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c^4*e
^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h + 5*a*
e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) - b*c^2*
e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) +
c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) + 3*a*b*e
*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]/Sqrt
[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2*(b*e*g +
3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f
+ 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a*e*(e^2*f
+ 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)]/(12*c^5)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3 (f + gx + hx^2)}{a + bx + cx^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] IntegrateAlgebraic[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2), x]
```

**fricas** [A] time = 2.52, size = 2150, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*
(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4
- 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^
3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c^5*d^
3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3
)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*
e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 -
3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2
- (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*((3*(b^2*c^4
- 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*c^5)*d
^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*
```

$$\begin{aligned}
& c^2 - 5ab^2c^3 + 4a^2c^4) * d * e^2 - (b^5c - 6ab^3c^2 + 8a^2b^2c^3) * \\
& e^3) * h) * x + 6 * ((3 * (b^2c^4 - 4a^2c^5) * d^2 * e - 3 * (b^3c^3 - 4ab^2c^4) * d * e^2 \\
& + (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * e^3) * f + ((b^2c^4 - 4a^2c^5) * d^3 - \\
& 3 * (b^3c^3 - 4ab^2c^4) * d^2 * e + 3 * (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * d * e^2 \\
& - (b^5c - 6ab^3c^2 + 8a^2b^2c^3) * e^3) * g - ((b^3c^3 - 4ab^2c^4) * d^3 \\
& - 3 * (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * d^2 * e + 3 * (b^5c - 6ab^3c^2 + 8a^2b^2c^3) * d * e^2 - \\
& (b^6 - 7ab^4c + 13a^2b^2c^2 - 4a^3c^3) * e^3) * h) * \\
& \log(cx^2 + bx + a) / (b^2c^5 - 4a^2c^6), 1/12 * (3 * (b^2c^4 - 4a^2c^5) * e^3 * h \\
& * x^4 + 4 * ((b^2c^4 - 4a^2c^5) * e^3 * g + (3 * (b^2c^4 - 4a^2c^5) * d * e^2 - (b^3c^3 - \\
& 4ab^2c^4) * e^3) * h) * x^3 + 6 * ((b^2c^4 - 4a^2c^5) * e^3 * f + (3 * (b^2c^4 - \\
& 4a^2c^5) * d * e^2 - (b^3c^3 - 4ab^2c^4) * e^3) * g + (3 * (b^2c^4 - 4a^2c^5) * d^2 * \\
& e - 3 * (b^3c^3 - 4ab^2c^4) * d * e^2 + (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * e^3) \\
& ) * h) * x^2 - 12 * \sqrt{-b^2 + 4ac} * ((2c^5 * d^3 - 3b^2c^4 * d^2 * e + 3 * (b^2c^3 - \\
& 2a^2c^4) * d * e^2 - (b^3c^2 - 3ab^2c^3) * e^3) * f - (b^2c^4 * d^3 - 3 * (b^2c^3 - \\
& 2a^2c^4) * d^2 * e + 3 * (b^3c^2 - 3ab^2c^3) * d * e^2 - (b^4c - 4ab^2c^2 + 2a^2c^3) * e^3) * g + \\
& ((b^2c^3 - 2a^2c^4) * d^3 - 3 * (b^3c^2 - 3ab^2c^3) * d^2 * e + \\
& 3 * (b^4c - 4ab^2c^2 + 2a^2c^3) * d * e^2 - (b^5 - 5ab^3c + 5a^2b^2c^2) \\
& ) * e^3) * h) * \arctan(-\sqrt{-b^2 + 4ac} * (2cx + b) / (b^2 - 4ac)) + 12 * ((3 * (b^2c^4 - \\
& 4a^2c^5) * d * e^2 - (b^3c^3 - 4ab^2c^4) * e^3) * f + (3 * (b^2c^4 - 4a^2c^5) * d^2 * e - \\
& 3 * (b^3c^3 - 4ab^2c^4) * d * e^2 + (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * e^3) * g + \\
& ((b^2c^4 - 4a^2c^5) * d^3 - 3 * (b^3c^3 - 4ab^2c^4) * d^2 * e + 3 * (b^4c^2 - 5ab^2c^3 + \\
& 4a^2c^4) * d * e^2 - (b^5c - 6ab^3c^2 + 8a^2b^2c^3) * e^3) * h) * x + 6 * ((3 * (b^2c^4 - \\
& 4a^2c^5) * d^2 * e - 3 * (b^3c^3 - 4ab^2c^4) * d * e^2 + (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * e^3) * f + \\
& ((b^2c^4 - 4a^2c^5) * d^3 - 3 * (b^3c^3 - 4ab^2c^4) * d^2 * e + 3 * (b^4c^2 - 5ab^2c^3 + \\
& 4a^2c^4) * d * e^2 - (b^5c - 6ab^3c^2 + 8a^2b^2c^3) * e^3) * g - ((b^3c^3 - 4ab^2c^4) * d^3 - \\
& 3 * (b^4c^2 - 5ab^2c^3 + 4a^2c^4) * d^2 * e + 3 * (b^5c - 6ab^3c^2 + 8a^2b^2c^3) * d * e^2 - \\
& (b^6 - 7ab^4c + 13a^2b^2c^2 - 4a^3c^3) * e^3) * h) * \log(cx^2 + bx + a) / (b^2c^5 - 4a^2c^6)]
\end{aligned}$$

**giac** [A] time = 0.17, size = 771, normalized size = 1.30

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out]  $1/12 * (3 * c^3 * h * x^4 * e^3 + 12 * c^3 * d * h * x^3 * e^2 + 18 * c^3 * d^2 * h * x^2 * e + 12 * c^3 * d^3 * h * x * e + 4 * c^3 * g * x^3 * e^3 - 4 * b * c^2 * h * x^3 * e^3 + 18 * c^3 * d * g * x^2 * e^2 - 18 * b * c^2 * d * h * x^2 * e^2 + 36 * c^3 * d^2 * g * x * e - 36 * b * c^2 * d^2 * h * x * e + 6 * c^3 * f * x^2 * e^3 - 6 * b * c^2 * g * x^2 * e^3 + 6 * b^2 * c * h * x^2 * e^3 - 6 * a * c^2 * h * x^2 * e^3 + 36 * c^3 * d * f * x * e^2 - 36 * b * c^2 * d * g * x * e^2 + 36 * b^2 * c * d * h * x * e^2 - 36 * a * c^2 * d * h * x * e^2 - 12 * b * c^2 * f * x * e^3 + 12 * b^2 * c * g * x * e^3 - 12 * a * c^2 * g * x * e^3 - 12 * b^3 * h * x * e^3 + 24 * a * b * c * h * x * e^3) / c^4 + 1/2 * (c^4 * d^3 * g - b * c^3 * d^3 * h + 3 * c^4 * d^2 * f * e - 3 * b * c^3 * d^2 * g * e + 3 * b^2 * c^2 * d^2 * h * e - 3 * a * c^3 * d^2 * h * e - 3 * b * c^3 * d * f * e^2 + 3 * b^2 * c^2 * d * g * e^2$

$$\begin{aligned}
& 2 - 3*a*c^3*d*g*e^2 - 3*b^3*c*d*h*e^2 + 6*a*b*c^2*d*h*e^2 + b^2*c^2*f*e^3 - \\
& a*c^3*f*e^3 - b^3*c*g*e^3 + 2*a*b*c^2*g*e^3 + b^4*h*e^3 - 3*a*b^2*c*h*e^3 \\
& + a^2*c^2*h*e^3)*\log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - b*c^4*d^3*g + b^ \\
& 2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b*c^4*d^2*f*e + 3*b^2*c^3*d^2*g*e - 6*a*c^4 \\
& *d^2*g*e - 3*b^3*c^2*d^2*h*e + 9*a*b*c^3*d^2*h*e + 3*b^2*c^3*d*f*e^2 - 6*a* \\
& c^4*d*f*e^2 - 3*b^3*c^2*d*g*e^2 + 9*a*b*c^3*d*g*e^2 + 3*b^4*c*d*h*e^2 - 12* \\
& a*b^2*c^2*d*h*e^2 + 6*a^2*c^3*d*h*e^2 - b^3*c^2*f*e^3 + 3*a*b*c^3*f*e^3 + b \\
& ^4*c*g*e^3 - 4*a*b^2*c^2*g*e^3 + 2*a^2*c^3*g*e^3 - b^5*h*e^3 + 5*a*b^3*c*h* \\
& e^3 - 5*a^2*b*c^2*h*e^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 \\
& + 4*a*c})*c^5)
\end{aligned}$$

**maple [B]** time = 0.01, size = 1738, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a), x)`

[Out]  $1/4*e^3*h*x^4/c+1/2/c*\ln(c*x^2+b*x+a)*d^3*g+2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*d^3*f+1/3/c*x^3*e^3*g+1/2/c*x^2*e^3*f+1/c*d^3*h*x+9/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*d^2*g+9/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*d^2*e*h-12/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*d*e^2*h-1/2/c^4*\ln(c*x^2+b*x+a)*b^3*e^3*g+1/2/c^3*\ln(c*x^2+b*x+a)*b^2*e^3*f-1/2/c^2*\ln(c*x^2+b*x+a)*b*d^3*h+3/2/c*\ln(c*x^2+b*x+a)*d^2*e*f-1/3/c^2*x^3*b*e^3*h+1/c*x^3*d*e^2*h-1/2/c^2*x^2*a*e^3*h+1/2/c^3*x^2*b^2*e^3*h-1/2/c^2*x^2*b*e^3*g+3/2/c*x^2*d^2*e*h+3/2/c*x^2*d*e^2*g+1/c^3*\ln(c*x^2+b*x+a)*a*b*e^3*g-3/2/c^2*\ln(c*x^2+b*x+a)*a*d^2*e*h-3/2/c^2*\ln(c*x^2+b*x+a)*a*d*e^2*g-3/2/c^4*\ln(c*x^2+b*x+a)*b^3*d*e^2*h+3/2/c^3*\ln(c*x^2+b*x+a)*b^2*d^2*e*h+3/2/c^3*\ln(c*x^2+b*x+a)*b^2*d*e^2*g-3/2/c^2*\ln(c*x^2+b*x+a)*b*d^2*e*g-3/2/c^2*\ln(c*x^2+b*x+a)*b*d*e^2*f+1/c^4/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^4*e^3*g+5/c^4/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^3*e^3*h+3/c^4/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^4*d*e^2*h-3/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*d^2*e*f+3/c^3*\ln(c*x^2+b*x+a)*a*b*d*e^2*h+6/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a^2*d*e^2*h-6/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*d^2*e*g-6/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*d*e^2*f-4/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*e^3*g+3/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*e^3*f-5/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a^2*b*e^3*h-3/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*d^2*e*h-3/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*d^2*g+3/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*d^2*e*g+3/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*d^2*f-1/c^2*a*e^3*g*x-1/c^4*b^3*e^3*h*x+1/c^3*b^2*e^3*g*x-1/c^2*b*e^3*f*x+3/c*d^2*e*g*x+3/c*d*e^$

$$2fx+1/2/c^3*\ln(cx^2+bx+a)*a^2*e^3h-1/2/c^2*\ln(cx^2+bx+a)*a*e^3f+1/2/c^5*\ln(cx^2+bx+a)*b^4*e^3h-3/2/c^2*x^2*b*d*e^2*h+2/c^3*a*b*e^3h*x-3/c^2*a*d*e^2*h*x+3/c^3*b^2*d*e^2*h*x-3/c^2*b*d^2*e*h*x-3/c^2*b*d*e^2*g*x-1/c^3/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3f+1/c^2/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^3*h-1/c/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^3*g-1/c^5/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^3h+2/c^2/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^3g-2/c/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^3*h-3/2/c^4*\ln(cx^2+bx+a)*a*b^2*e^3h$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 5.46, size = 967, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2),x)

[Out]  $x^3*((e^3g + 3d^2e^2h)/(3c) - (b^3e^3h)/(3c^2)) + x*((d^3h + 3d^2e^2f + 3d^2e^2g)/c + (b*((e^3g + 3d^2e^2h)/c - (b^3e^3h)/c^2))/c - (e^3f + 3d^2e^2g + 3d^2e^2h)/c + (a^3e^3h)/c^2)/c - (a*((e^3g + 3d^2e^2h)/c - (b^3e^3h)/c^2))/c - x^2*((b*((e^3g + 3d^2e^2h)/c - (b^3e^3h)/c^2))/(2c) - (e^3f + 3d^2e^2g + 3d^2e^2h)/(2c) + (a^3e^3h)/(2c^2)) - (\log(a + b*x + c*x^2)*(b^6e^3h + 4a^2c^4e^3f + b^2c^4d^3g + b^4c^2e^3f - 4a^3c^3e^3h - b^3c^3d^3h - 4a^2c^5d^3g - b^5c^3e^3g + 4a^2b^2c^4d^3h - 7a^2b^4c^2e^3h - 12a^2c^5d^2e^2f - 3b^5c^3d^2e^2h - 5a^2b^2c^3e^3f + 6a^2b^3c^2e^3g - 8a^2b^2c^3e^3g + 12a^2c^4d^2e^2g + 3b^2c^4d^2e^2f - 3b^3c^3d^2e^2f + 12a^2c^4d^2e^2h - 3b^3c^3d^2e^2g + 3b^4c^2d^2e^2g + 3b^4c^2d^2e^2h + 13a^2b^2c^2e^3h + 12a^2b^2c^4d^2e^2f + 12a^2b^2c^4d^2e^2g - 15a^2b^2c^3d^2e^2g - 15a^2b^2c^3d^2e^2h + 18a^2b^3c^2d^2e^2h - 24a^2b^2c^3d^2e^2h))/(2*(4a^2c^6 - b^2c^5)) + (e^3h*x^4)/(4c) + (atan(b/(4a^2c - b^2)^(1/2) + (2c*x)/(4a^2c - b^2)^(1/2))*(2c^5d^3f - b^5e^3h + 2a^2c^3e^3g - b^3c^2e^3f + b^2c^3d^3h - 2a^2c^4d^3h - b^3c^4d^3g + b^4c^2e^3g + 3a^2b^2c^3e^3f + 5a^2b^3$

$$\frac{c^3 e^{3h} - 6a^2 c^4 d^2 e^2 f - 6a^2 c^4 d^2 e^2 g - 3b^2 c^4 d^2 e^2 f + 3b^4 c^2 d^2 e^2 h - 4a^2 b^2 c^2 e^3 g - 5a^2 b^2 c^2 e^3 h + 3b^2 c^3 d^2 e^2 f + 6a^2 c^3 d^2 e^2 h + 3b^2 c^3 d^2 e^2 g - 3b^3 c^2 d^2 e^2 h + 9a^2 b^2 c^3 d^2 e^2 g + 9a^2 b^2 c^3 d^2 e^2 h - 12a^2 b^2 c^2 d^2 e^2 h)}{(c^5(4a^2 c - b^2))^{1/2}}$$

**sympy [B]** time = 118.42, size = 4972, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(h\*x\*\*2+g\*x+f)/(c\*x\*\*2+b\*x+a),x)

[Out]  $x^3 \left( \frac{-b e^{3h}}{3c^2} + \frac{d e^{2h}}{c} + \frac{e^3 g}{3c} \right) + x^2 \left( \frac{-a e^{3h}}{2c^2} + \frac{b^2 e^{3h}}{2c^3} - \frac{3b d e^{2h}}{2c^2} - \frac{b e^3 g}{2c^2} + \frac{3d^2 e^2 h}{2c} + \frac{3d e^2 g}{2c} + \frac{e^3 f}{2c} \right) + x \left( \frac{2a b e^{3h}}{c^3} - \frac{3a d e^{2h}}{c^2} - \frac{a e^3 g}{c^2} - \frac{b^3 e^{3h}}{c^4} + \frac{3b^2 d e^{2h}}{c^3} + \frac{b^2 e^3 g}{c^3} - \frac{3b d^2 e^2 h}{c^2} - \frac{3b d e^2 g}{c^2} - \frac{b e^3 f}{c^2} + \frac{d^3 h}{c} + \frac{3d^2 e^2 g}{c} + \frac{3d e^2 f}{c} \right) + \frac{(-\sqrt{-4ac + b^2})(5a^2 b^2 c^2 e^{3h} - 6a^2 c^3 d e^{2h} - 2a^2 c^3 e^3 g - 5a^2 b^3 c e^3 h + 12a^2 b^2 c^2 d e^{2h} + 4a^2 b^2 c^2 e^3 g - 9a^2 b^3 c^2 d e^2 h - 9a^2 b^3 c^2 d e^2 g - 3a^2 b^3 c^2 e^3 f + 2a^2 c^4 d^3 h + 6a^2 c^4 d^2 e^2 g + 6a^2 c^4 d^2 e^2 f + b^5 e^3 h - 3b^4 c^2 d e^{2h} - b^4 c^2 e^3 g + 3b^3 c^2 d^2 e^2 h + 3b^3 c^2 d e^2 g + b^3 c^2 e^3 f - b^2 c^3 d^3 h - 3b^2 c^3 d^2 e^2 g - 3b^2 c^3 d e^2 f + b^2 c^4 d^3 g + 3b^2 c^4 d^2 e^2 f - 2c^5 d^3 f)}{(2c^5(4ac - b^2))} + \frac{(a^2 c^2 e^{3h} - 3a^2 b^2 c e^{3h} + 6a^2 b^2 c^2 d e^{2h} + 2a^2 b^2 c^2 e^3 g - 3a^2 c^3 d^2 e^2 h - 3a^2 c^3 d e^2 g - a^2 c^3 e^3 f + b^4 e^3 h - 3b^3 c^2 d e^2 h - b^3 c^2 e^3 g + 3b^2 c^2 d^2 e^2 h + 3b^2 c^2 d e^2 g + b^2 c^2 e^3 f - b^2 c^3 d^3 h - 3b^2 c^3 d^2 e^2 g - 3b^2 c^3 d e^2 f + c^4 d^3 g + 3c^4 d^2 e^2 f)}{(2c^5)} \log(x + \frac{2a^3 c^2 e^{3h} - 4a^2 b^2 c e^{3h} + 9a^2 b^2 c^2 d e^{2h} + 3a^2 b^2 c^2 e^3 g - 6a^2 c^3 d^2 e^2 h - 6a^2 c^3 d e^2 g - 2a^2 c^3 e^3 f + a^2 b^4 e^3 h - 3a^2 b^3 c^2 d e^2 h - a^2 b^3 c^2 e^3 g + 3a^2 b^2 c^2 d^2 e^2 h + 3a^2 b^2 c^2 d e^2 g + a^2 b^2 c^2 e^3 f - a^2 b^2 c^3 d^3 h - 3a^2 b^2 c^3 d^2 e^2 g - 3a^2 b^2 c^3 d e^2 f - 4a^2 c^5 (-\sqrt{-4ac + b^2})(5a^2 b^2 c^2 e^{3h} - 6a^2 c^3 d e^{2h} - 2a^2 c^3 e^3 g - 5a^2 b^3 c e^3 h + 12a^2 b^2 c^2 d e^{2h} + 4a^2 b^2 c^2 e^3 g - 9a^2 b^3 c^2 d e^2 h - 9a^2 b^3 c^2 d e^2 g - 3a^2 b^3 c^2 e^3 f + 2a^2 c^4 d^3 h + 6a^2 c^4 d^2 e^2 g + 6a^2 c^4 d^2 e^2 f + b^5 e^3 h - 3b^4 c^2 d e^{2h} - b^4 c^2 e^3 g + 3b^3 c^2 d^2 e^2 h + 3b^3 c^2 d e^2 g + b^3 c^2 e^3 f - b^2 c^3 d^3 h - 3b^2 c^3 d^2 e^2 g - 3b^2 c^3 d e^2 f + b^2 c^4 d^3 g + 3b^2 c^4 d^2 e^2 f - 2c^5 d^3 f)}{(2c^5(4ac - b^2))} + \frac{(a^2 c^2 e^{3h} - 3a^2 b^2 c e^{3h} + 6a^2 b^2 c^2 d e^{2h} + 2a^2 b^2 c^2 e^3 g - 3a^2 c^3 d^2 e^2 h - 3a^2 c^3 d e^2 g - a^2 c^3 e^3 f + b^4 e^3 h - 3b^3 c^2 d e^2 h - b^3 c^2 e^3 g + 3b^2 c^2 d^2 e^2 h + 3b^2 c^2 d e^2 g + b^2 c^2 e^3 f - b^2 c^3 d^3 h - 3b^2 c^3 d^2 e^2 g - 3b^2 c^3 d e^2 f + c^4 d^3 g + 3c^4 d^2 e^2 f)}{(2c^5)} \log(x + \frac{2a^3 c^2 e^{3h} - 4a^2 b^2 c e^{3h} + 9a^2 b^2 c^2 d e^{2h} + 3a^2 b^2 c^2 e^3 g - 6a^2 c^3 d^2 e^2 h - 6a^2 c^3 d e^2 g - 2a^2 c^3 e^3 f + a^2 b^4 e^3 h - 3a^2 b^3 c^2 d e^2 h - a^2 b^3 c^2 e^3 g + 3a^2 b^2 c^2 d^2 e^2 h + 3a^2 b^2 c^2 d e^2 g + a^2 b^2 c^2 e^3 f - a^2 b^2 c^3 d^3 h - 3a^2 b^2 c^3 d^2 e^2 g - 3a^2 b^2 c^3 d e^2 f - 4a^2 c^5 (-\sqrt{-4ac + b^2})(5a^2 b^2 c^2 e^{3h} - 6a^2 c^3 d e^{2h} - 2a^2 c^3 e^3 g - 5a^2 b^3 c e^3 h + 12a^2 b^2 c^2 d e^{2h} + 4a^2 b^2 c^2 e^3 g - 9a^2 b^3 c^2 d e^2 h - 9a^2 b^3 c^2 d e^2 g - 3a^2 b^3 c^2 e^3 f + 2a^2 c^4 d^3 h + 6a^2 c^4 d^2 e^2 g + 6a^2 c^4 d^2 e^2 f + b^5 e^3 h - 3b^4 c^2 d e^{2h} - b^4 c^2 e^3 g + 3b^3 c^2 d^2 e^2 h + 3b^3 c^2 d e^2 g + b^3 c^2 e^3 f - b^2 c^3 d^3 h - 3b^2 c^3 d^2 e^2 g - 3b^2 c^3 d e^2 f + b^2 c^4 d^3 g + 3b^2 c^4 d^2 e^2 f - 2c^5 d^3 f)}{(2c^5(4ac - b^2))} + \frac{(a^2 c^2 e^{3h} - 3a^2 b^2 c e^{3h} + 6a^2 b^2 c^2 d e^{2h} + 2a^2 b^2 c^2 e^3 g - 3a^2 c^3 d^2 e^2 h - 3a^2 c^3 d e^2 g - a^2 c^3 e^3 f + b^4 e^3 h - 3b^3 c^2 d e^2 h - b^3 c^2 e^3 g + 3b^2 c^2 d^2 e^2 h + 3b^2 c^2 d e^2 g + b^2 c^2 e^3 f - b^2 c^3 d^3 h - 3b^2 c^3 d^2 e^2 g - 3b^2 c^3 d e^2 f + c^4 d^3 g + 3c^4 d^2 e^2 f)}{(2c^5)}$



$$\begin{aligned}
& + 3b^{**2}c^{**2}d^{**2}e^*h + 3b^{**2}c^{**2}d^*e^{**2}g + b^{**2}c^{**2}e^{**3}f - b^{**3}c^{**3}d^{**3}h - 3b^*c^{**3}d^{**2}e^*g - 3b^*c^{**3}d^*e^{**2}f + c^{**4}d^{**3}g + 3c^{**4}d^{**2}e^*f)/(2c^{**5}) + 2a^*c^{**4}d^{**3}g + 6a^*c^{**4}d^{**2}e^*f + b^{**2}c^{**4}(-\sqrt{-4a^*c + b^{**2}})(5a^{**2}b^*c^{**2}e^{**3}h - 6a^{**2}c^{**3}d^*e^{**2}h - 2a^{**2}c^{**3}e^{**3}g - 5a^*b^{**3}c^*e^{**3}h + 12a^*b^{**2}c^{**2}d^*e^{**2}h + 4a^*b^{**2}c^{**2}e^{**3}g - 9a^*b^*c^{**3}d^{**2}e^*h - 9a^*b^*c^{**3}d^*e^{**2}g - 3a^*b^*c^{**3}e^{**3}f + 2a^*c^{**4}d^{**3}h + 6a^*c^{**4}d^{**2}e^*g + 6a^*c^{**4}d^*e^{**2}f + b^{**5}e^{**3}h - 3b^{**4}c^*d^*e^{**2}h - b^{**4}c^*e^{**3}g + 3b^{**3}c^{**2}d^{**2}e^*h + 3b^{**3}c^{**2}d^*e^{**2}g + b^{**3}c^{**2}e^{**3}f - b^{**2}c^{**3}d^{**3}h - 3b^{**2}c^{**3}d^*e^{**2}g - 3b^{**2}c^{**3}d^*e^{**2}f + b^*c^{**4}d^{**3}g + 3b^*c^{**4}d^{**2}e^*f - 2c^{**5}d^{**3}f)/(2c^{**5}(4a^*c - b^{**2})) + (a^{**2}c^{**2}e^{**3}h - 3a^*b^{**2}c^*e^{**3}h + 6a^*b^*c^{**2}d^*e^{**2}h + 2a^*b^*c^{**2}e^{**3}g - 3a^*c^{**3}d^{**2}e^*h - 3a^*c^{**3}d^*e^{**2}g - a^*c^{**3}e^{**3}f + b^{**4}e^{**3}h - 3b^{**3}c^*d^*e^{**2}h - b^{**3}c^*e^{**3}g + 3b^{**2}c^{**2}d^{**2}e^*h + 3b^{**2}c^{**2}d^*e^{**2}g + b^{**2}c^{**2}e^{**3}f - b^*c^{**3}d^{**3}h - 3b^*c^{**3}d^*e^{**2}g - 3b^*c^{**3}d^*e^{**2}f + c^{**4}d^{**3}g + 3c^{**4}d^{**2}e^*f)/(2c^{**5}) - b^*c^{**4}d^{**3}f)/(5a^{**2}b^*c^{**2}e^{**3}h - 6a^{**2}c^{**3}d^*e^{**2}h - 2a^{**2}c^{**3}e^{**3}g - 5a^*b^{**3}c^*e^{**3}h + 12a^*b^{**2}c^{**2}d^*e^{**2}h + 4a^*b^{**2}c^{**2}e^{**3}g - 9a^*b^*c^{**3}d^{**2}e^*h - 9a^*b^*c^{**3}d^*e^{**2}g - 3a^*b^*c^{**3}e^{**3}f + 2a^*c^{**4}d^{**3}h + 6a^*c^{**4}d^{**2}e^*g + 6a^*c^{**4}d^*e^{**2}f + b^{**5}e^{**3}h - 3b^{**4}c^*d^*e^{**2}h - b^{**4}c^*e^{**3}g + 3b^{**3}c^{**2}d^{**2}e^*h + 3b^{**3}c^{**2}d^*e^{**2}g + b^{**3}c^{**2}e^{**3}f - b^{**2}c^{**3}d^{**3}h - 3b^{**2}c^{**3}d^*e^{**2}g - 3b^{**2}c^{**3}d^*e^{**2}f + b^*c^{**4}d^{**3}g + 3b^*c^{**4}d^{**2}e^*f - 2c^{**5}d^{**3}f)/(2c^{**5}(4a^*c - b^{**2})) + (\sqrt{-4a^*c + b^{**2}})(5a^{**2}b^*c^{**2}e^{**3}h - 6a^{**2}c^{**3}d^*e^{**2}h - 2a^{**2}c^{**3}e^{**3}g - 5a^*b^{**3}c^*e^{**3}h + 12a^*b^{**2}c^{**2}d^*e^{**2}h + 4a^*b^{**2}c^{**2}e^{**3}g - 9a^*b^*c^{**3}d^{**2}e^*h - 9a^*b^*c^{**3}d^*e^{**2}g - 3a^*b^*c^{**3}e^{**3}f + 2a^*c^{**4}d^{**3}h + 6a^*c^{**4}d^{**2}e^*g + 6a^*c^{**4}d^*e^{**2}f + b^{**5}e^{**3}h - 3b^{**4}c^*d^*e^{**2}h - b^{**4}c^*e^{**3}g + 3b^{**3}c^{**2}d^{**2}e^*h + 3b^{**3}c^{**2}d^*e^{**2}g + b^{**3}c^{**2}e^{**3}f - b^{**2}c^{**3}d^{**3}h - 3b^{**2}c^{**3}d^*e^{**2}g - 3b^{**2}c^{**3}d^*e^{**2}f + b^*c^{**4}d^{**3}g + 3b^*c^{**4}d^{**2}e^*f - 2c^{**5}d^{**3}f)/(2c^{**5}(4a^*c - b^{**2})) + (a^{**2}c^{**2}e^{**3}h - 3a^*b^{**2}c^*e^{**3}h + 6a^*b^*c^{**2}d^*e^{**2}h + 2a^*b^*c^{**2}e^{**3}g - 3a^*c^{**3}d^{**2}e^*h - 3a^*c^{**3}d^*e^{**2}g - a^*c^{**3}e^{**3}f + b^{**4}e^{**3}h - 3b^{**3}c^*d^*e^{**2}h - b^{**3}c^*e^{**3}g + 3b^{**2}c^{**2}d^{**2}e^*h + 3b^{**2}c^{**2}d^*e^{**2}g + b^{**2}c^{**2}e^{**3}f - b^*c^{**3}d^{**3}h - 3b^*c^{**3}d^*e^{**2}g - 3b^*c^{**3}d^*e^{**2}f + c^{**4}d^{**3}g + 3c^{**4}d^{**2}e^*f)/(2c^{**5})*\log(x + (2a^{**3}c^{**2}e^{**3}h - 4a^{**2}b^{**2}c^*e^{**3}h + 9a^{**2}b^*c^{**2}d^*e^{**2}h + 3a^{**2}b^*c^{**2}e^{**3}g - 6a^{**2}c^{**3}d^{**2}e^*h - 6a^{**2}c^{**3}d^*e^{**2}g - 2a^{**2}c^{**3}e^{**3}f + a^*b^{**4}e^{**3}h - 3a^*b^{**3}c^*d^*e^{**2}h - a^*b^{**3}c^*e^{**3}g + 3a^*b^{**2}c^{**2}d^{**2}e^*h + 3a^*b^{**2}c^{**2}d^*e^{**2}g + a^*b^{**2}c^{**2}e^{**3}f - a^*b^*c^{**3}d^{**3}h - 3a^*b^*c^{**3}d^*e^{**2}g - 3a^*b^*c^{**3}d^*e^{**2}f - 4a^*c^{**5}(\sqrt{-4a^*c + b^{**2}})(5a^{**2}b^*c^{**2}e^{**3}h - 6a^{**2}c^{**3}d^*e^{**2}h - 2a^{**2}c^{**3}e^{**3}g - 5a^*b^{**3}c^*e^{**3}h + 12a^*b^{**2}c^{**2}d^*e^{**2}h + 4a^*b^{**2}c^{**2}e^{**3}g - 9a^*b^*c^{**3}d^{**2}e^*h - 9a^*b^*c^{**3}d^*e^{**2}g - 3a^*b^*c^{**3}e^{**3}f + 2a^*c^{**4}d^{**3}h + 6a^*c^{**4}d^{**2}e^*g + 6a^*c^{**4}d^*e^{**2}f + b^{**5}e^{**3}h - 3b^{**4}c^*d^*e^{**2}h - b^{**4}c^*e^{**3}g + 3b^{**3}c^{**2}d^{**2}e^*h + 3b^{**3}c^{**2}d^*e^{**2}g + b^{**3}c^{**2}e^{**3}f - b^{**2}c^{**3}d^{**3}h - 3b^{**2}c^{**3}d^*e^{**2}g - 3b^{**2}c^{**3}d^*e^{**2}f + b^*c^{**4}d^{**3}g + 3b^*c^{**4}d^{**2}e^*f - 2c^{**5}d^{**3}f))
\end{aligned}$$

$$\begin{aligned}
& / (2c^{*5}(4a*c - b^{*2})) + (a^{*2}c^{*2}e^{*3}h - 3a*b^{*2}c*e^{*3}h + 6a*b*c^{*2}d*e^{*2}h + 2a*b*c^{*2}e^{*3}g - 3a*c^{*3}d^{*2}e*h - 3a*c^{*3}d*e^{*2}g - a \\
& *c^{*3}e^{*3}f + b^{*4}e^{*3}h - 3b^{*3}c*d*e^{*2}h - b^{*3}c*e^{*3}g + 3b^{*2}c^{*2}d^{*2}e*h + 3b^{*2}c^{*2}d*e^{*2}g + b^{*2}c^{*2}e^{*3}f - b*c^{*3}d^{*3}h - 3b \\
& c^{*3}d^{*2}e*g - 3b*c^{*3}d*e^{*2}f + c^{*4}d^{*3}g + 3c^{*4}d^{*2}e*f) / (2c^{*5}) \\
& ) + 2a*c^{*4}d^{*3}g + 6a*c^{*4}d^{*2}e*f + b^{*2}c^{*4}(\sqrt{-4a*c + b^{*2}})(5 \\
& a^{*2}b*c^{*2}e^{*3}h - 6a^{*2}c^{*3}d*e^{*2}h - 2a^{*2}c^{*3}e^{*3}g - 5a*b^{*3}c \\
& e^{*3}h + 12a*b^{*2}c^{*2}d*e^{*2}h + 4a*b^{*2}c^{*2}e^{*3}g - 9a*b*c^{*3}d^{*2} \\
& e*h - 9a*b*c^{*3}d*e^{*2}g - 3a*b*c^{*3}e^{*3}f + 2a*c^{*4}d^{*3}h + 6a*c^{*4} \\
& d^{*2}e*g + 6a*c^{*4}d*e^{*2}f + b^{*5}e^{*3}h - 3b^{*4}c*d*e^{*2}h - b^{*4}c*e \\
& ^{*3}g + 3b^{*3}c^{*2}d^{*2}e*h + 3b^{*3}c^{*2}d*e^{*2}g + b^{*3}c^{*2}e^{*3}f - b^{*2} \\
& c^{*3}d^{*3}h - 3b^{*2}c^{*3}d^{*2}e*g - 3b^{*2}c^{*3}d*e^{*2}f + b*c^{*4}d^{*3}g \\
& + 3b*c^{*4}d^{*2}e*f - 2c^{*5}d^{*3}f) / (2c^{*5}(4a*c - b^{*2})) + (a^{*2}c^{*2}e \\
& ^{*3}h - 3a*b^{*2}c*e^{*3}h + 6a*b*c^{*2}d*e^{*2}h + 2a*b*c^{*2}e^{*3}g - 3a \\
& c^{*3}d^{*2}e*h - 3a*c^{*3}d*e^{*2}g - a*c^{*3}e^{*3}f + b^{*4}e^{*3}h - 3b^{*3}c \\
& d*e^{*2}h - b^{*3}c*e^{*3}g + 3b^{*2}c^{*2}d^{*2}e*h + 3b^{*2}c^{*2}d*e^{*2}g + b \\
& ^{*2}c^{*2}e^{*3}f - b*c^{*3}d^{*3}h - 3b*c^{*3}d^{*2}e*g - 3b*c^{*3}d*e^{*2}f + c \\
& ^{*4}d^{*3}g + 3c^{*4}d^{*2}e*f) / (2c^{*5}) - b*c^{*4}d^{*3}f) / (5a^{*2}b*c^{*2}e^{*3} \\
& h - 6a^{*2}c^{*3}d*e^{*2}h - 2a^{*2}c^{*3}e^{*3}g - 5a*b^{*3}c*e^{*3}h + 12a*b \\
& ^{*2}c^{*2}d*e^{*2}h + 4a*b^{*2}c^{*2}e^{*3}g - 9a*b*c^{*3}d^{*2}e*h - 9a*b*c^{*3} \\
& d*e^{*2}g - 3a*b*c^{*3}e^{*3}f + 2a*c^{*4}d^{*3}h + 6a*c^{*4}d^{*2}e*g + 6a*c \\
& ^{*4}d*e^{*2}f + b^{*5}e^{*3}h - 3b^{*4}c*d*e^{*2}h - b^{*4}c*e^{*3}g + 3b^{*3}c^{*2} \\
& d^{*2}e*h + 3b^{*3}c^{*2}d*e^{*2}g + b^{*3}c^{*2}e^{*3}f - b^{*2}c^{*3}d^{*3}h - 3 \\
& b^{*2}c^{*3}d^{*2}e*g - 3b^{*2}c^{*3}d*e^{*2}f + b*c^{*4}d^{*3}g + 3b*c^{*4}d^{*2}e \\
& ^{*2}f - 2c^{*5}d^{*3}f)) + e^{*3}h*x^{*4}/(4*c)
\end{aligned}$$

$$3.145 \quad \int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=348

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\right)\right)}{c^4\sqrt{b^2-4ac}}$$

**Rubi [A]** time = 0.68, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\right)\right)}{c^4\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

[Out] ((b^2\*e^2\*h + c^2\*(e^2\*f + 2\*d\*e\*g + d^2\*h) - c\*e\*(b\*e\*g + 2\*b\*d\*h + a\*e\*h))\*x)/c^3 + (e\*(c\*e\*g + 2\*c\*d\*h - b\*e\*h)\*x^2)/(2\*c^2) + (e^2\*h\*x^3)/(3\*c) - ((2\*c^4\*d^2\*f + b^4\*e^2\*h - b^2\*c\*e\*(b\*e\*g + 2\*b\*d\*h + 4\*a\*e\*h) - c^3\*(b\*d\*(2\*e\*f + d\*g) + 2\*a\*(e^2\*f + 2\*d\*e\*g + d^2\*h)) + c^2\*(2\*a^2\*e^2\*h + 3\*a\*b\*e\*(e\*g + 2\*d\*h) + b^2\*(e^2\*f + 2\*d\*e\*g + d^2\*h)))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/(c^4\*Sqrt[b^2 - 4\*a\*c]) + ((c^3\*d\*(2\*e\*f + d\*g) - b^3\*e^2\*h + b\*c\*e\*(b\*e\*g + 2\*b\*d\*h + 2\*a\*e\*h) - c^2\*(a\*e\*(e\*g + 2\*d\*h) + b\*(e^2\*f + 2\*d\*e\*g + d^2\*h)))\*Log[a + b\*x + c\*x^2])/(2\*c^4)

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d + ex)^2 (f + gx + hx^2)}{a + bx + cx^2} dx = \int \left( \frac{b^2 e^2 h + c^2 (e^2 f + 2deg + d^2 h) - ce(beg + 2bdh + aeh)}{c^3} + \frac{e(ceg + 2cdh - beh)}{c^2} \right) dx$$

$$= \frac{(b^2 e^2 h + c^2 (e^2 f + 2deg + d^2 h) - ce(beg + 2bdh + aeh)) x}{c^3} + \frac{e(ceg + 2cdh - beh)}{2c^2}$$

$$= \frac{(b^2 e^2 h + c^2 (e^2 f + 2deg + d^2 h) - ce(beg + 2bdh + aeh)) x}{c^3} + \frac{e(ceg + 2cdh - beh)}{2c^2}$$

$$= \frac{(b^2 e^2 h + c^2 (e^2 f + 2deg + d^2 h) - ce(beg + 2bdh + aeh)) x}{c^3} + \frac{e(ceg + 2cdh - beh)}{2c^2}$$

$$= \frac{(b^2 e^2 h + c^2 (e^2 f + 2deg + d^2 h) - ce(beg + 2bdh + aeh)) x}{c^3} + \frac{e(ceg + 2cdh - beh)}{2c^2}$$

**Mathematica [A]** time = 0.38, size = 345, normalized size = 0.99

$$\frac{6 \operatorname{atan}\left(\frac{\sqrt{a+bx+cx^2}}{\sqrt{a-cx}}\right) \left( c^2 (2a^2 b^2 + 3ab(2bh+cg) + 2f^2(b+2dg+e^2 f)) + 2c^2(a^2 b + 2bh + cg) - 2(d^2(b+2dg+e^2 f) + 2d(dg+2ef) + 4f^2 b - 2d^2 f) \right) + 3 \log(a + x(b + cx)) \left( -c^2 (a(2bh + cg) + b(f^2 h + 2deg + e^2 f)) + bc(2ah + 2bh + bg) + b^2(-c^2)h + c^2(d(dg + 2ef)) + 6c(-c(a^2 h + 2bh + bg) + b^2 e^2 h + c^2(f^2 h + 2deg + e^2 f)) + 3c^2 c^2(-beh + ceg) + 2c^2 d h^2 \right)}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2), x]
```

```
[Out] (6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*(e*g
```

+ 2\*d\*h) + b^2\*(e^2\*f + 2\*d\*e\*g + d^2\*h))\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]]/Sqrt[-b^2 + 4\*a\*c] + 3\*(c^3\*d\*(2\*e\*f + d\*g) - b^3\*e^2\*h + b\*c\*e\*(b\*e\*g + 2\*b\*d\*h + 2\*a\*e\*h) - c^2\*(a\*e\*(e\*g + 2\*d\*h) + b\*(e^2\*f + 2\*d\*e\*g + d^2\*h)))\*Log[a + x\*(b + c\*x)]/(6\*c^4)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2 (f + gx + hx^2)}{a + bx + cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

**fricas** [A] time = 1.20, size = 1273, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a), x, algorithm="fricas")

[Out] [1/6\*(2\*(b^2\*c^3 - 4\*a\*c^4)\*e^2\*h\*x^3 + 3\*((b^2\*c^3 - 4\*a\*c^4)\*e^2\*g + (2\*(b^2\*c^3 - 4\*a\*c^4)\*d\*e - (b^3\*c^2 - 4\*a\*b\*c^3)\*e^2)\*h)\*x^2 + 3\*sqrt(b^2 - 4\*a\*c)\*((2\*c^4\*d^2 - 2\*b\*c^3\*d\*e + (b^2\*c^2 - 2\*a\*c^3)\*e^2)\*f - (b\*c^3\*d^2 - 2\*(b^2\*c^2 - 2\*a\*c^3)\*d\*e + (b^3\*c - 3\*a\*b\*c^2)\*e^2)\*g + ((b^2\*c^2 - 2\*a\*c^3)\*d^2 - 2\*(b^3\*c - 3\*a\*b\*c^2)\*d\*e + (b^4 - 4\*a\*b^2\*c + 2\*a^2\*c^2)\*e^2)\*h)\*log(((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c - sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) + 6\*((b^2\*c^3 - 4\*a\*c^4)\*e^2\*f + (2\*(b^2\*c^3 - 4\*a\*c^4)\*d\*e - (b^3\*c^2 - 4\*a\*b\*c^3)\*e^2)\*g + ((b^2\*c^3 - 4\*a\*c^4)\*d^2 - 2\*(b^3\*c^2 - 4\*a\*b\*c^3)\*d\*e + (b^4\*c - 5\*a\*b^2\*c^2 + 4\*a^2\*c^3)\*e^2)\*h)\*x + 3\*((2\*(b^2\*c^3 - 4\*a\*c^4)\*d\*e - (b^3\*c^2 - 4\*a\*b\*c^3)\*e^2)\*f + ((b^2\*c^3 - 4\*a\*c^4)\*d^2 - 2\*(b^3\*c^2 - 4\*a\*b\*c^3)\*d\*e + (b^4\*c - 5\*a\*b^2\*c^2 + 4\*a^2\*c^3)\*e^2)\*g - ((b^3\*c^2 - 4\*a\*b\*c^3)\*d^2 - 2\*(b^4\*c - 5\*a\*b^2\*c^2 + 4\*a^2\*c^3)\*d\*e + (b^5 - 6\*a\*b^3\*c + 8\*a^2\*b\*c^2)\*e^2)\*h)\*log(c\*x^2 + b\*x + a)]/(b^2\*c^4 - 4\*a\*c^5), 1/6\*(2\*(b^2\*c^3 - 4\*a\*c^4)\*e^2\*h\*x^3 + 3\*((b^2\*c^3 - 4\*a\*c^4)\*e^2\*g + (2\*(b^2\*c^3 - 4\*a\*c^4)\*d\*e - (b^3\*c^2 - 4\*a\*b\*c^3)\*e^2)\*h)\*x^2 - 6\*sqrt(-b^2 + 4\*a\*c)\*((2\*c^4\*d^2 - 2\*b\*c^3\*d\*e + (b^2\*c^2 - 2\*a\*c^3)\*e^2)\*f - (b\*c^3\*d^2 - 2\*(b^2\*c^2 - 2\*a\*c^3)\*d\*e + (b^3\*c - 3\*a\*b\*c^2)\*e^2)\*g + ((b^2\*c^2 - 2\*a\*c^3)\*d^2 - 2\*(b^3\*c - 3\*a\*b\*c^2)\*d\*e + (b^4 - 4\*a\*b^2\*c + 2\*a^2\*c^2)\*e^2)\*h)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) + 6\*((b^2\*c^3 - 4\*a\*c^4)\*e^2\*f + (2\*(b^2\*c^3 - 4\*a\*c^4)\*d\*e - (b^3\*c^2 - 4\*a\*b\*c^3)\*e^2)\*g + ((b^2\*c^3 - 4\*a\*c^4)\*d^2 - 2\*(b^3\*c^2 - 4\*a\*b\*c^3)\*d\*e + (b^4\*c - 5\*a\*b^2\*c^2 + 4\*a^2\*c^3)\*e^2)\*h)\*x + 3\*((2\*(b^2\*c^3 - 4\*a\*c^4)\*d\*e - (b^3\*c^2 - 4\*a\*b\*c^3)\*e^2)\*f + ((b^2\*c^3 - 4\*a\*c^4)\*d^2 - 2\*(b^3\*c^2 - 4\*a\*b\*c^3)\*d\*e +

$$(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*\log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]$$

**giac [A]** time = 0.16, size = 426, normalized size = 1.22

$$\frac{2*b^4*c^2 + 6*b^3*c^2 + 6*b^2*c^2 + 3*c^2*d^2 - 3*b^3*c^2 + 12*b^2*c^2 - 12*b^2*c^2 + 6*c^2*d^2 - 6*b^3*c^2 + 6*b^2*c^2 - 6*b^2*c^2}{2*c^2} \cdot \frac{(2*c^2*b^2*d^2 + 2*b^2*c^2*d^2 - 2*b^2*c^2*d^2 - 2*b^2*c^2*d^2 + 2*b^2*c^2*d^2)\log(c^2*x + b*x + a)}{2*c^2} - \frac{(2*c^2*d^2*b^2*c^2 + 2*b^2*c^2*d^2*b^2*c^2 - 2*b^2*c^2*d^2*b^2*c^2 - 2*b^2*c^2*d^2*b^2*c^2 + 2*b^2*c^2*d^2*b^2*c^2)\arctan\left(\frac{2*c*x + b}{\sqrt{-b^2 + 4*a*c}}\right)}{2*c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a),x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & \frac{1}{6}*(2*c^2*h*x^3*e^2 + 6*c^2*d*h*x^2*e + 6*c^2*d^2*h*x + 3*c^2*g*x^2*e^2 - 3*b*c*h*x^2*e^2 + 12*c^2*d*g*x*e - 12*b*c*d*h*x*e + 6*c^2*f*x*e^2 - 6*b*c*g*x*e^2 + 6*b^2*h*x*e^2 - 6*a*c*h*x*e^2)/c^3 + \frac{1}{2}*(c^3*d^2*g - b*c^2*d^2*h + 2*c^3*d*f*e - 2*b*c^2*d*g*e + 2*b^2*c*d*h*e - 2*a*c^2*d*h*e - b*c^2*f*e^2 + b^2*c*g*e^2 - a*c^2*g*e^2 - b^3*h*e^2 + 2*a*b*c*h*e^2)*\log(c*x^2 + b*x + a)/c^4 + (2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*c^3*d*f*e + 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h*e + b^2*c^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*e^2 - 4*a*b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c})*c^4 \end{aligned}$$

**maple [B]** time = 0.01, size = 1028, normalized size = 2.95

$$\frac{2*b^4*c^2 + 6*b^3*c^2 + 6*b^2*c^2 + 3*c^2*d^2 - 3*b^3*c^2 + 12*b^2*c^2 - 12*b^2*c^2 + 6*c^2*d^2 - 6*b^3*c^2 + 6*b^2*c^2 - 6*b^2*c^2}{2*c^2} \cdot \frac{(2*c^2*b^2*d^2 + 2*b^2*c^2*d^2 - 2*b^2*c^2*d^2 - 2*b^2*c^2*d^2 + 2*b^2*c^2*d^2)\log(c^2*x + b*x + a)}{2*c^2} - \frac{(2*c^2*d^2*b^2*c^2 + 2*b^2*c^2*d^2*b^2*c^2 - 2*b^2*c^2*d^2*b^2*c^2 - 2*b^2*c^2*d^2*b^2*c^2 + 2*b^2*c^2*d^2*b^2*c^2)\arctan\left(\frac{2*c*x + b}{\sqrt{-b^2 + 4*a*c}}\right)}{2*c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a),x)

$$\begin{aligned} \text{[Out]} & \frac{1}{3}e^2*h*x^3/c + \frac{1}{2}c*\ln(c*x^2+b*x+a)*d^2*g + \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*d^2*f + \frac{1}{2}c*x^2*e^2*g + \frac{1}{c}d^2*h*x + \frac{1}{c}e^2*f*x + \frac{6}{c^2} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*a*b*d*e*h - \frac{2}{c^2}b*d*e*h*x - \frac{1}{c^2}*\ln(c*x^2+b*x+a)*b*d*e*g + \frac{2}{c^2} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*a^2*e^2*h - \frac{2}{c} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*a*d^2*h - \frac{2}{c} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*a*e^2*f + \frac{1}{c^4} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*b^4*e^2*h + \frac{1}{c^2} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*b^2*d^2*h + \frac{1}{c^2} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*b^2*e^2*f - \frac{1}{c} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*b*d^2*g - \frac{1}{c^3} \cdot \frac{2}{(4*a*c-b^2)^{1/2}}*\arctan\left(\frac{2*c*x+b}{(4*a*c-b^2)^{1/2}}\right)*b^3*e^2*g + \frac{1}{c^3}*\ln(c*x^2+b*x+a)*a*b*e^2*h - \frac{1}{c^2}*\ln(c*x^2+b*x+a)*a*d*e*h + \frac{1}{c^3}*\ln(c*x^2+b*x+a)*b^2*d*e*h - \frac{1}{2} \cdot \frac{1}{c^4}*\ln(c*x^2+b*x+a)*b^3*e^2*h + \frac{1}{2} \cdot \frac{1}{c^3}*\ln(c*x^2+b*x+a)*b^2*e^2*g - \frac{1}{2} \cdot \frac{1}{c^2}*\ln(c*x^2+b*x+a)*b*d^2*h - \frac{1}{2} \cdot \frac{1}{c^2}*\ln(c*x^2+b*x+a)*a*e^2*g - \frac{1}{2} \cdot \frac{1}{c^2}x^2*b*e^2*h + \frac{1}{c*x^2}d*e*h - \frac{1}{c^2}a*e^2*h*x + \frac{1}{c^3}b^2*e^2*h*x - \frac{1}{c^2}b*e^2*g*x + \frac{2}{c*d} \end{aligned}$$







$$\begin{aligned}
& f - b^{**3}d^{**2}g - 2b^{**3}d^{**2}ef + 2c^{**4}d^{**2}f)/(2c^{**4}(4a^{**c} - b^{**2})) \\
& + (2a^{**b}c^{**e}e^{**2}h - 2a^{**c}e^{**2}d^{**e}h - a^{**c}e^{**2}e^{**2}g - b^{**3}e^{**2}h + 2b^{**2}c^{**d}e^{**h} + b^{**2}c^{**e}e^{**2}g - b^{**c}e^{**2}d^{**2}h - 2b^{**c}e^{**2}d^{**e}g - b^{**c}e^{**2}e^{**2}f + \\
& c^{**3}d^{**2}g + 2c^{**3}d^{**e}f)/(2c^{**4})) - 2a^{**c}e^{**3}d^{**2}g - 4a^{**c}e^{**3}d^{**e}f - \\
& b^{**2}c^{**3}(sqrt(-4a^{**c} + b^{**2})*(2a^{**2}c^{**2}e^{**2}h - 4a^{**b}e^{**2}c^{**e}e^{**2}h + 6a^{**b}c^{**2}d^{**e}h + 3a^{**b}c^{**2}e^{**2}g - 2a^{**c}e^{**3}d^{**2}h - 4a^{**c}e^{**3}d^{**e}g - 2a^{**c}e^{**3}e^{**2}f + b^{**4}e^{**2}h - 2b^{**3}c^{**d}e^{**h} - b^{**3}c^{**e}e^{**2}g + b^{**2}c^{**2}d^{**2}h + 2b^{**2}c^{**2}d^{**e}g + b^{**2}c^{**2}e^{**2}f - b^{**c}e^{**3}d^{**2}g - 2b^{**c}e^{**3}d^{**e}f + 2c^{**4}d^{**2}f)/(2c^{**4}(4a^{**c} - b^{**2})) + (2a^{**b}c^{**e}e^{**2}h - 2a^{**c}e^{**2}d^{**e}h - a^{**c}e^{**2}e^{**2}g - b^{**3}e^{**2}h + 2b^{**2}c^{**d}e^{**h} + b^{**2}c^{**e}e^{**2}g - b^{**c}e^{**2}d^{**2}h - 2b^{**c}e^{**2}d^{**e}g - b^{**c}e^{**2}e^{**2}f + c^{**3}d^{**2}g + 2c^{**3}d^{**e}f)/(2c^{**4})) + b^{**c}e^{**3}d^{**2}f)/(2a^{**2}c^{**2}e^{**2}h - 4a^{**b}e^{**2}c^{**e}e^{**2}h + 6a^{**b}c^{**2}d^{**e}h + 3a^{**b}c^{**2}e^{**2}g - 2a^{**c}e^{**3}d^{**2}h - 4a^{**c}e^{**3}d^{**e}g - 2a^{**c}e^{**3}e^{**2}f + b^{**4}e^{**2}h - 2b^{**3}c^{**d}e^{**h} - b^{**3}c^{**e}e^{**2}g + b^{**2}c^{**2}d^{**2}h + 2b^{**2}c^{**2}d^{**e}g + b^{**2}c^{**2}e^{**2}f - b^{**c}e^{**3}d^{**2}g - 2b^{**c}e^{**3}d^{**e}f + 2c^{**4}d^{**2}f)) + e^{**2}h^{**x}e^{**3}/(3c)
\end{aligned}$$

$$3.146 \quad \int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=177

$$\frac{\log(a+bx+cx^2)(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef))}{2c^3} - \frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-c^2(2adh+2aeg+bdg+bef))}{c^3\sqrt{b^2-4ac}}$$

**Rubi [A]** time = 0.35, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$ , Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log(a+bx+cx^2)(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef))}{2c^3} - \frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-c^2(2adh+2aeg+bdg+bef)+bc(3aeh+bdh+beg)+b^3(-e)+2c^3df)}{c^3\sqrt{b^2-4ac}} + \frac{x(-beh+cdh+ceg)}{c^2} + \frac{ehx^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

[Out] ((c\*e\*g + c\*d\*h - b\*e\*h)\*x)/c^2 + (e\*h\*x^2)/(2\*c) - ((2\*c^3\*d\*f - b^3\*e\*h - c^2\*(b\*e\*f + b\*d\*g + 2\*a\*e\*g + 2\*a\*d\*h) + b\*c\*(b\*e\*g + b\*d\*h + 3\*a\*e\*h))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/(c^3\*Sqrt[b^2 - 4\*a\*c]) + ((c^2\*(e\*f + d\*g) + b^2\*e\*h - c\*(b\*e\*g + b\*d\*h + a\*e\*h))\*Log[a + b\*x + c\*x^2])/(2\*c^3)

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2} dx &= \int \left( \frac{ceg+cdh-beh}{c^2} + \frac{ehx}{c} + \frac{c^2df+abeh-ac(eg+dh)+(c^2(ef+dg)+b^2eh)}{c^2(a+bx+cx^2)} \right) dx \\ &= \frac{(ceg+cdh-beh)x}{c^2} + \frac{ehx^2}{2c} + \frac{\int \frac{c^2df+abeh-ac(eg+dh)+(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh))x}{a+bx+cx^2} dx}{c^2} \\ &= \frac{(ceg+cdh-beh)x}{c^2} + \frac{ehx^2}{2c} + \frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)) \int \frac{b+2c}{a+bx+cx^2} dx}{2c^3} \\ &= \frac{(ceg+cdh-beh)x}{c^2} + \frac{ehx^2}{2c} + \frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)) \log(a+bx+cx^2)}{2c^3} \\ &= \frac{(ceg+cdh-beh)x}{c^2} + \frac{ehx^2}{2c} - \frac{(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh))+b^3c}{c^3\sqrt{b^2-4ac}} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 173, normalized size = 0.98

$$\frac{\log(a+x(b+cx))(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)) - \frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(c^2(2adh+2aeg+bdg+bef)-bc(3aeh+bdh+beg)+b^3eh-2c^3df)}{\sqrt{4ac-b^2}} + 2cx(-beh+cdh+ceg)+c^2ehx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2), x]
```

```
[Out] (2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h + c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)]/(2*c^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)(f + gx + hx^2)}{a + bx + cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2), x]

**fricas [A]** time = 0.75, size = 654, normalized size = 3.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a), x, algorithm="fricas")

[Out] [1/2\*((b^2\*c^2 - 4\*a\*c^3)\*e\*h\*x^2 + sqrt(b^2 - 4\*a\*c)\*((2\*c^3\*d - b\*c^2\*e)\*f - (b\*c^2\*d - (b^2\*c - 2\*a\*c^2)\*e)\*g + ((b^2\*c - 2\*a\*c^2)\*d - (b^3 - 3\*a\*b\*c)\*e)\*h)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c - sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) + 2\*((b^2\*c^2 - 4\*a\*c^3)\*e\*g + ((b^2\*c^2 - 4\*a\*c^3)\*d - (b^3\*c - 4\*a\*b\*c^2)\*e)\*h)\*x + ((b^2\*c^2 - 4\*a\*c^3)\*e\*f + ((b^2\*c^2 - 4\*a\*c^3)\*d - (b^3\*c - 4\*a\*b\*c^2)\*e)\*g - ((b^3\*c - 4\*a\*b\*c^2)\*d - (b^4 - 5\*a\*b^2\*c + 4\*a^2\*c^2)\*e)\*h)\*log(c\*x^2 + b\*x + a))/(b^2\*c^3 - 4\*a\*c^4), 1/2\*((b^2\*c^2 - 4\*a\*c^3)\*e\*h\*x^2 - 2\*sqrt(-b^2 + 4\*a\*c)\*((2\*c^3\*d - b\*c^2\*e)\*f - (b\*c^2\*d - (b^2\*c - 2\*a\*c^2)\*e)\*g + ((b^2\*c - 2\*a\*c^2)\*d - (b^3 - 3\*a\*b\*c)\*e)\*h)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) + 2\*((b^2\*c^2 - 4\*a\*c^3)\*e\*g + ((b^2\*c^2 - 4\*a\*c^3)\*d - (b^3\*c - 4\*a\*b\*c^2)\*e)\*h)\*x + ((b^2\*c^2 - 4\*a\*c^3)\*e\*f + ((b^2\*c^2 - 4\*a\*c^3)\*d - (b^3\*c - 4\*a\*b\*c^2)\*e)\*g - ((b^3\*c - 4\*a\*b\*c^2)\*d - (b^4 - 5\*a\*b^2\*c + 4\*a^2\*c^2)\*e)\*h)\*log(c\*x^2 + b\*x + a))/(b^2\*c^3 - 4\*a\*c^4)]

**giac [A]** time = 0.19, size = 201, normalized size = 1.14

$$\frac{chx^2e + 2cdhx + 2cge - 2bhxe}{2c^2} + \frac{(c^2dg - bcdh + c^2fe - bce + b^2he - ache)\log(cx^2 + bx + a)}{2c^3} + \frac{(2c^3df - bc^2dg + b^2cdh - 2ac^2dh - bc^2fe + b^2cge - 2ac^2ge - b^3he + 3abche)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a), x, algorithm="giac")

[Out] 1/2\*(c\*h\*x^2\*e + 2\*c\*d\*h\*x + 2\*c\*g\*x\*e - 2\*b\*h\*x\*e)/c^2 + 1/2\*(c^2\*d\*g - b\*c\*d\*h + c^2\*f\*e - b\*c\*g\*e + b^2\*h\*e - a\*c\*h\*e)\*log(c\*x^2 + b\*x + a)/c^3 + (2\*c^3\*d\*f - b\*c^2\*d\*g + b^2\*c\*d\*h - 2\*a\*c^2\*d\*h - b\*c^2\*f\*e + b^2\*c\*g\*e - 2\*a\*c^2\*g\*e - b^3\*h\*e + 3\*a\*b\*c\*h\*e)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/(sqrt(-b^2 + 4\*a\*c)\*c^3)

maple [B] time = 0.01, size = 510, normalized size = 2.88

$$\frac{3ab\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{2afh\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{2ag\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{f^2bh\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{f^2bg\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{bf\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{ghx^2}{2c} + \frac{2f\operatorname{arctan}\left(\frac{2xh}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{abh\ln(x^2+bx+a)}{2c^2} + \frac{f^2bh\ln(x^2+bx+a)}{2c^2} + \frac{bh\ln(x^2+bx+a)}{2c^2} + \frac{hg\ln(x^2+bx+a)}{2c^2} + \frac{bhx}{c^2} + \frac{d\ln(x^2+bx+a)}{2c} + \frac{ah}{c} + \frac{ef\ln(x^2+bx+a)}{2c} + \frac{ghx^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a), x)`

[Out]  $\frac{1}{2}e*h*x^2/c - 1/c^2*b*e*h*x + 1/c*d*h*x + 1/c*e*g*x - 1/2/c^2*\ln(c*x^2+b*x+a)*a*e*h + 1/2/c^3*\ln(c*x^2+b*x+a)*b^2*e*h - 1/2/c^2*\ln(c*x^2+b*x+a)*b*d*h - 1/2/c^2*\ln(c*x^2+b*x+a)*b*e*g + 1/2/c*\ln(c*x^2+b*x+a)*d*g + 1/2/c*\ln(c*x^2+b*x+a)*e*f + 3/c^2/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*e*h - 2/c/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*d*h - 2/c/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*e*g + 2/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*d*f - 1/c^3/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*e*h + 1/c^2/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*d*h + 1/c^2/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*e*g - 1/c/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*d*g - 1/c/(4*a*c-b^2)^{(1/2)}*\operatorname{arctan}((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*e*f$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a), x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

mupad [B] time = 0.53, size = 273, normalized size = 1.54

$$x \left( \frac{dh+eg}{c} - \frac{beh}{c^2} \right) \frac{\ln(x^2+bx+a) (b^2eh-4a^2dg-4a^2ef-b^2cdh-b^2ceg+b^2^2dg+b^2^2ef+4a^2^2eh+4ab^2dh+4ab^2cg-5ab^2ceh)}{2(4a^4-b^2c^2)} - \frac{\operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx}{\sqrt{4ac-b^2}}\right) (b^2eh-2c^2df+2a^2dh+2a^2eg+b^2^2dg+b^2^2ef-b^2cdh-b^2ceg-3abceh)}{c^3\sqrt{4ac-b^2}} + \frac{ehx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2), x)`

[Out]  $x*((d*h + e*g)/c - (b*e*h)/c^2) - (\log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3*d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4*a^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4 - b^2*c^3)) - (\operatorname{atan}(b/(4*a*c - b^2)^{(1/2)} + (2*c*x)/(4*a*c - b^2)^{(1/2)})*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*$

$c*d*h - b^2*c*e*g - 3*a*b*c*e*h)/(c^3*(4*a*c - b^2)^{(1/2)}) + (e*h*x^2)/(2*c)$

**sympy [B]** time = 14.46, size = 1265, normalized size = 7.15

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x\*\*2+g\*x+f)/(c\*x\*\*2+b\*x+a),x)

[Out]  $x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-\sqrt{-4*a*c + b**2}*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*\log(x + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-\sqrt{-4*a*c + b**2}*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f))/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*a*c**2*e*f - b**2*c**2*(-\sqrt{-4*a*c + b**2}*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f))/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)) + (\sqrt{-4*a*c + b**2}*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*\log(x + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(\sqrt{-4*a*c + b**2}*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f))/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*a*c**2*e*f - b**2*c**2*(\sqrt{-4*a*c + b**2}*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f))/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)) + e*h*x**2/(2*c)$

$$3.147 \quad \int \frac{f+gx+hx^2}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=92

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-2ach + b^2h - bcg + 2c^2f)}{c^2\sqrt{b^2-4ac}} + \frac{(cg - bh)\log(a + bx + cx^2)}{2c^2} + \frac{hx}{c}$$

**Rubi [A]** time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-2ach + b^2h - bcg + 2c^2f)}{c^2\sqrt{b^2-4ac}} + \frac{(cg - bh)\log(a + bx + cx^2)}{2c^2} + \frac{hx}{c}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2), x]

[Out] (h\*x)/c - ((2\*c^2\*f - b\*c\*g + b^2\*h - 2\*a\*c\*h)\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(c^2\*Sqrt[b^2 - 4\*a\*c]) + ((c\*g - b\*h)\*Log[a + b\*x + c\*x^2])/(2\*c^2)

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 1657

$\text{Int}[(\text{Pq}_*)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x\_Symbol] \ :> \ \text{Int}[\text{Expand} \ \text{Integrand}[\text{Pq}*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{PolyQ}[\text{Pq}, x] \ \&\& \ \text{IGtQ}[p, -2]$

### Rubi steps

$$\begin{aligned} \int \frac{f + gx + hx^2}{a + bx + cx^2} dx &= \int \left( \frac{h}{c} + \frac{cf - ah + (cg - bh)x}{c(a + bx + cx^2)} \right) dx \\ &= \frac{hx}{c} + \frac{\int \frac{cf - ah + (cg - bh)x}{a + bx + cx^2} dx}{c} \\ &= \frac{hx}{c} + \frac{(cg - bh) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^2} + \frac{(2c^2f - bcg + b^2h - 2ach) \int \frac{1}{a + bx + cx^2} dx}{2c^2} \\ &= \frac{hx}{c} + \frac{(cg - bh) \log(a + bx + cx^2)}{2c^2} - \frac{(2c^2f - bcg + b^2h - 2ach) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^2} \\ &= \frac{hx}{c} - \frac{(2c^2f - bcg + b^2h - 2ach) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} + \frac{(cg - bh) \log(a + bx + cx^2)}{2c^2} \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 95, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)(-2ach + b^2h - bcg + 2c^2f)}{c^2 \sqrt{4ac - b^2}} + \frac{(cg - bh) \log(a + bx + cx^2)}{2c^2} + \frac{hx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2), x]

[Out] (h\*x)/c + ((2\*c^2\*f - b\*c\*g + b^2\*h - 2\*a\*c\*h)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(c^2\*Sqrt[-b^2 + 4\*a\*c]) + ((c\*g - b\*h)\*Log[a + b\*x + c\*x^2])/(2\*c^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{a + bx + cx^2} dx$$



Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2), x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2), x]

**fricas** [A] time = 0.74, size = 302, normalized size = 3.28

$$\frac{2(b^2c - 4ac^2)hx - (2c^2f - bcg + (b^2 - 2ac)h)\sqrt{b^2 - 4ac} \log\left(\frac{2x^2 + 2bx + b^2 - 2ac\sqrt{b^2 - 4ac}(2cx + b)}{x^2 + bx + a}\right) + ((b^2c - 4ac^2)g - (b^3 - 4abc)h) \log(cx^2 + bx + a) - 2(b^2c - 4ac^2)hx - 2(2c^2f - bcg + (b^2 - 2ac)h)\sqrt{-b^2 + 4ac} \arctan\left(\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{x^2 + bx + a}\right) + ((b^2c - 4ac^2)g - (b^3 - 4abc)h) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a), x, algorithm="fricas")

[Out] [1/2\*(2\*(b^2\*c - 4\*a\*c^2)\*h\*x - (2\*c^2\*f - b\*c\*g + (b^2 - 2\*a\*c)\*h)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c + sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) + ((b^2\*c - 4\*a\*c^2)\*g - (b^3 - 4\*a\*b\*c)\*h)\*log(c\*x^2 + b\*x + a)/(b^2\*c^2 - 4\*a\*c^3), 1/2\*(2\*(b^2\*c - 4\*a\*c^2)\*h\*x - 2\*(2\*c^2\*f - b\*c\*g + (b^2 - 2\*a\*c)\*h)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) + ((b^2\*c - 4\*a\*c^2)\*g - (b^3 - 4\*a\*b\*c)\*h)\*log(c\*x^2 + b\*x + a)/(b^2\*c^2 - 4\*a\*c^3)]

**giac** [A] time = 0.16, size = 89, normalized size = 0.97

$$\frac{hx}{c} + \frac{(cg - bh) \log(cx^2 + bx + a)}{2c^2} + \frac{(2c^2f - bcg + b^2h - 2ach) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a), x, algorithm="giac")

[Out] h\*x/c + 1/2\*(c\*g - b\*h)\*log(c\*x^2 + b\*x + a)/c^2 + (2\*c^2\*f - b\*c\*g + b^2\*h - 2\*a\*c\*h)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/(sqrt(-b^2 + 4\*a\*c)\*c^2)

**maple** [B] time = 0.00, size = 196, normalized size = 2.13

$$-\frac{2ah \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{b^2h \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c^2} - \frac{bg \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}c} + \frac{2f \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{bh \ln(cx^2 + bx + a)}{2c^2} + \frac{g \ln(cx^2 + bx + a)}{2c} + \frac{hx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a), x)

[Out] h\*x/c - 1/2/c^2\*ln(c\*x^2+b\*x+a)\*h\*b + 1/2/c\*ln(c\*x^2+b\*x+a)\*g - 2/c/(4\*a\*c - b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c - b^2)^(1/2))\*a\*h + 2/(4\*a\*c - b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c - b^2)^(1/2))\*f + 1/c^2/(4\*a\*c - b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c

$-b^2)^{(1/2)} * b^2 * h - 1/c / (4 * a * c - b^2)^{(1/2)} * \arctan((2 * c * x + b) / (4 * a * c - b^2)^{(1/2)}) * b * g$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 0.25, size = 132, normalized size = 1.43

$$\frac{hx}{c} + \frac{\ln(cx^2 + bx + a)(hb^3 - gb^2c - 4ahbc + 4agc^2)}{2(4ac^3 - b^2c^2)} + \frac{\operatorname{atan}\left(\frac{b}{\sqrt{4ac - b^2}} + \frac{2cx}{\sqrt{4ac - b^2}}\right)(hb^2 - gbc + 2fc^2 - 2ahc)}{c^2\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g\*x + h\*x^2)/(a + b\*x + c\*x^2),x)

[Out] (h\*x)/c + (log(a + b\*x + c\*x^2)\*(b^3\*h + 4\*a\*c^2\*g - b^2\*c\*g - 4\*a\*b\*c\*h))/(2\*(4\*a\*c^3 - b^2\*c^2)) + (atan(b/(4\*a\*c - b^2)^(1/2) + (2\*c\*x)/(4\*a\*c - b^2)^(1/2))\*(2\*c^2\*f + b^2\*h - 2\*a\*c\*h - b\*c\*g))/(c^2\*(4\*a\*c - b^2)^(1/2))

**sympy** [B] time = 2.14, size = 488, normalized size = 5.30

$$\left(\frac{\sqrt{-4ac + b^2}(2ach - b^2h + hcg - 2c^2f)}{2c^2(4ac - b^2)} - \frac{bh - cg}{2c^2}\right) \log\left(1 + \frac{-abh - 4ac^2\left(\frac{\sqrt{4ac^3 - b^2h - hcg - 2c^2f}}{2c(4ac - b^2)} - \frac{bh - cg}{2c^2}\right) + 2ahg + b^2c\left(\frac{\sqrt{4ac^3 - b^2h - hcg - 2c^2f}}{2c(4ac - b^2)} - \frac{bh - cg}{2c^2}\right) - bcf}{2ach - b^2h + hcg - 2c^2f}\right) + \left(\frac{\sqrt{-4ac + b^2}(2ach - b^2h + hcg - 2c^2f)}{2c^2(4ac - b^2)} - \frac{bh - cg}{2c^2}\right) \log\left(1 + \frac{-abh - 4ac^2\left(\frac{\sqrt{4ac^3 - b^2h - hcg - 2c^2f}}{2c(4ac - b^2)} - \frac{bh - cg}{2c^2}\right) + 2ahg + b^2c\left(\frac{\sqrt{4ac^3 - b^2h - hcg - 2c^2f}}{2c(4ac - b^2)} - \frac{bh - cg}{2c^2}\right) - bcf}{2ach - b^2h + hcg - 2c^2f}\right) + \frac{hx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x\*\*2+g\*x+f)/(c\*x\*\*2+b\*x+a),x)

[Out] (-sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*h - c\*g)/(2\*c\*\*2))\*log(x + (-a\*b\*h - 4\*a\*c\*\*2\*(-sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*h - c\*g)/(2\*c\*\*2)) + 2\*a\*c\*g + b\*\*2\*c\*(-sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*h - c\*g)/(2\*c\*\*2)) - b\*c\*f)/(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)) + (sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*h - c\*g)/(2\*c\*\*2))\*log(x + (-a\*b\*h - 4\*a\*c\*\*2\*(sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*h - c\*g)/(2\*c\*\*2)) + 2\*a\*c\*g + b\*\*2\*c\*(sqrt(-4\*a\*c + b\*\*2)\*(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)/(2\*c\*\*2\*(4\*a\*c - b\*\*2)) - (b\*h - c\*g)/(2\*c\*\*2)) - b\*c\*f)/(2\*a\*c\*h - b\*\*2\*h + b\*c\*g - 2\*c\*\*2\*f)) + h\*x/c

$$3.148 \quad \int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=196

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)\log(a+bx+cx^2)(-aeh+bdh-cd)}{c\sqrt{b^2-4ac}(ae^2-bde+cd^2) \cdot 2c(ae^2-bde+cd^2)}$$

**Rubi [A]** time = 0.35, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}(ae^2-bde+cd^2)} - \frac{\log(a+bx+cx^2)(-aeh+bdh-cd+cef)}{2c(ae^2-bde+cd^2)} + \frac{\log(d+ex)(d^2h-deg+e^2f)}{e(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)), x]

[Out] -(((2\*c^2\*d\*f + b\*(b\*d - a\*e)\*h - c\*(b\*e\*f + b\*d\*g - 2\*a\*e\*g + 2\*a\*d\*h))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(c\*Sqrt[b^2 - 4\*a\*c]\*(c\*d^2 - b\*d\*e + a\*e^2)) + ((e^2\*f - d\*e\*g + d^2\*h)\*Log[d + e\*x])/(e\*(c\*d^2 - b\*d\*e + a\*e^2)) - ((c\*e\*f - c\*d\*g + b\*d\*h - a\*e\*h)\*Log[a + b\*x + c\*x^2])/(2\*c\*(c\*d^2 - b\*d\*e + a\*e^2))

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 634**

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)} dx &= \int \left( \frac{e^2 f - deg + d^2 h}{(cd^2 - bde + ae^2)(d + ex)} + \frac{cdf - bef + aeg - adh - (cef - cdg + bdh - aeh)}{(cd^2 - bde + ae^2)(a + bx + cx^2)} \right) dx \\ &= \frac{(e^2 f - deg + d^2 h) \log(d + ex)}{e(cd^2 - bde + ae^2)} + \frac{\int \frac{cdf - bef + aeg - adh - (cef - cdg + bdh - aeh)x}{a + bx + cx^2} dx}{cd^2 - bde + ae^2} \\ &= \frac{(e^2 f - deg + d^2 h) \log(d + ex)}{e(cd^2 - bde + ae^2)} - \frac{(cef - cdg + bdh - aeh) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c(cd^2 - bde + ae^2)} + \frac{(2c^2 df - b^2 h)}{2c(cd^2 - bde + ae^2)} \\ &= \frac{(e^2 f - deg + d^2 h) \log(d + ex)}{e(cd^2 - bde + ae^2)} - \frac{(cef - cdg + bdh - aeh) \log(a + bx + cx^2)}{2c(cd^2 - bde + ae^2)} - \frac{(2c^2 df - b^2 h)}{2c(cd^2 - bde + ae^2)} \\ &= -\frac{(2c^2 df + b(bd - ae)h - c(bef + bdg - 2aeg + 2adh)) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c\sqrt{b^2 - 4ac}(cd^2 - bde + ae^2)} + \frac{(e^2 f - deg + d^2 h) \log(d + ex)}{e(cd^2 - bde + ae^2)} \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 193, normalized size = 0.98

$$\frac{-2e \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) (c(2adh - 2aeg + bdg + bef) + bh(ae - bd) - 2c^2 df) + 2c\sqrt{4ac - b^2} \log(d + ex) (d^2 h - deg + e^2 f) - e\sqrt{4ac - b^2} \log(a + x(b + cx)) (-aeh + bdh - cdg + cef)}{2ce\sqrt{4ac - b^2} (e(ac - bd) + cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)), x]
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```
[Out] (-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2*f - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b*d*h)
```

$h - a*e*h)*\text{Log}[a + x*(b + c*x)]/(2*c*\text{Sqrt}[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)),x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)), x]

**fricas** [A] time = 89.67, size = 625, normalized size = 3.19

$$\frac{\sqrt{-b^2 + 4ac} \left( (2c^2d^2e - b^2c^2e^2) f - (b^2c^2d^2e - 2a^2c^2e^2) g - (a^2b^2e^2 - (b^2 - 2ac)d^2e) h \right) \log\left(\frac{2c^2x^2 + 2b^2cx + b^2 - 2ac}{c^2x^2 + bx + a}\right) + ((b^2c - 4a^2c^2)e^2 f - (b^2c - 4a^2c^2)d^2e g + ((b^3 - 4a^2b^2c)d^2e - (a^2b^2 - 4a^2c^2)e^2) h) \log(cx^2 + bx + a) - 2((b^2c - 4a^2c^2)e^2 f - (b^2c - 4a^2c^2)d^2e g + (b^2c - 4a^2c^2)d^2h) \log(ex + d)}{(b^2c^2 - 4a^2c^3)d^2e - (b^3c - 4a^2b^2c^2)d^2e + (a^2b^2c - 4a^2c^2)e^3} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{(b^2 - 4ac)}\right) + ((b^2c - 4a^2c^2)e^2 f - (b^2c - 4a^2c^2)d^2e g + ((b^3 - 4a^2b^2c)d^2e - (a^2b^2 - 4a^2c^2)e^2) h) \log(cx^2 + bx + a) - 2((b^2c - 4a^2c^2)e^2 f - (b^2c - 4a^2c^2)d^2e g + (b^2c - 4a^2c^2)d^2h) \log(ex + d)}{(b^2c^2 - 4a^2c^3)d^2e - (b^3c - 4a^2b^2c^2)d^2e + (a^2b^2c - 4a^2c^2)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out]  $[-1/2*(\text{sqrt}(b^2 - 4*a*c))*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \text{sqrt}(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d^2*e - (a*b^2 - 4*a^2*c)*e^2)*h)*\log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*\log(e*x + d)]/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d^2e + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*\text{sqrt}(-b^2 + 4*a*c))*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d^2e - (a*b^2 - 4*a^2*c)*e^2)*h)*\log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*\log(e*x + d)]/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d^2e + (a*b^2*c - 4*a^2*c^2)*e^3)]$

**giac** [A] time = 0.16, size = 204, normalized size = 1.04

$$\frac{(cdg - bdh - cfe + ahe) \log(cx^2 + bx + a)}{2(c^2d^2 - bcde + ace^2)} + \frac{(d^2h - dge + fe^2) \log(ex + d)}{cd^2e - bde^2 + ae^3} + \frac{(2c^2df - bcdg + b^2dh - 2acd h - bcfe + 2aceg - abhe) \arctan\left(\frac{2cx + b}{\sqrt{-b^2 + 4ac}}\right)}{(c^2d^2 - bcde + ace^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out]  $1/2*(c*d*g - b*d*h - c*f*e + a*h*e)*\log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + (d^2*h - d*g*e + f*e^2)*\log(\text{abs}(x*e + d))/(c*d^2*e - b*d*e^2$

$$+ a * e^3) + (2 * c^2 * d * f - b * c * d * g + b^2 * d * h - 2 * a * c * d * h - b * c * f * e + 2 * a * c * g * e - a * b * h * e) * \arctan((2 * c * x + b) / \sqrt{-b^2 + 4 * a * c}) / ((c^2 * d^2 - b * c * d * e + a * c * e^2) * \sqrt{-b^2 + 4 * a * c})$$

**maple [B]** time = 0.01, size = 622, normalized size = 3.17

$$\frac{\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} - \frac{2ab\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} + \frac{2ac\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} - \frac{b^2d\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} + \frac{bdg\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} - \frac{bef\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} + \frac{2cdf\operatorname{arctan}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2-bde+e^2)\sqrt{4ac-b^2}} - \frac{ab\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{bd\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c} - \frac{d^2\ln(x^2+bx+a)}{2(a^2-bde+e^2)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a), x)

[Out] 1/2/(a\*e^2-b\*d\*e+c\*d^2)/c\*ln(c\*x^2+b\*x+a)\*a\*e\*h-1/2/(a\*e^2-b\*d\*e+c\*d^2)/c\*ln(c\*x^2+b\*x+a)\*b\*d\*h+1/2/(a\*e^2-b\*d\*e+c\*d^2)\*ln(c\*x^2+b\*x+a)\*d\*g-1/2/(a\*e^2-b\*d\*e+c\*d^2)\*ln(c\*x^2+b\*x+a)\*e\*f-2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*d\*h+2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*e\*g-1/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*b\*e\*f+2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*c\*d\*f-1/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))/c\*b\*a\*e\*h+1/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))/c\*b^2\*d\*h-1/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)^(1/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*b\*d\*g+1/(a\*e^2-b\*d\*e+c\*d^2)/e\*ln(e\*x+d)\*d^2\*h-1/(a\*e^2-b\*d\*e+c\*d^2)\*ln(e\*x+d)\*d\*g+1/(a\*e^2-b\*d\*e+c\*d^2)\*e\*ln(e\*x+d)\*f

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad [B]** time = 10.45, size = 2467, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)), x)

[Out] (log(a^2\*b\*e^4\*g - 2\*a\*b^2\*e^4\*f - 2\*a^3\*e^4\*h + 6\*a^2\*c\*e^4\*f - 4\*a\*c^2\*d^4\*h + b^2\*c\*d^4\*h + b^3\*d^3\*e\*h - 2\*b^3\*e^4\*f\*x + a^2\*e^4\*g\*(b^2 - 4\*a\*c))^(

$$\begin{aligned}
& 1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - \\
& 10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c \\
& *e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a*c)^{(1/2)} - \\
& c^2*d^3*e*f*(b^2 - 4*a*c)^{(1/2)} - b^2*d^3*e*h*(b^2 - 4*a*c)^{(1/2)} - \\
& 2*b^2*e^4*f*x*(b^2 - 4*a*c)^{(1/2)} - a^2*e^4*h*x*(b^2 - 4*a*c)^{(1/2)} - 2*c^2 \\
& *d^4*h*x*(b^2 - 4*a*c)^{(1/2)} - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2 \\
& *c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*(b^2 - 4* \\
& a*c)^{(1/2)} - b*c*d^4*h*(b^2 - 4*a*c)^{(1/2)} + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3* \\
& e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^{(1/2)} - b^2*d^2*e^2 \\
& *h*x*(b^2 - 4*a*c)^{(1/2)} + a*b*d*e^3*g*(b^2 - 4*a*c)^{(1/2)} + 7*a*c*d*e^3*f* \\
& (b^2 - 4*a*c)^{(1/2)} + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^{(1/2)} + 2*b*c*d^3*e*g*(b^ \\
& 2 - 4*a*c)^{(1/2)} + a*b*e^4*g*x*(b^2 - 4*a*c)^{(1/2)} + 3*a*c*e^4*f*x*(b^2 - 4 \\
& *a*c)^{(1/2)} + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - \\
& 10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h* \\
& x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} - 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^{(1/ \\
& 2)} - b*c*d^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} + b^2*d*e^3*g*x*(b^2 - 4*a*c)^{(1/2)} \\
& + 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^{(1/2)} + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e \\
& ^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^{(1/2)} - 2*b \\
& *c*d^2*e^2*g*x*(b^2 - 4*a*c)^{(1/2)} - 7*a*b*c*d*e^3*g*x - 5*a*c*d*e^3*g*x*(b \\
& ^2 - 4*a*c)^{(1/2)} + 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^{(1/2)} + b*c*d^3*e*h*x*(b^ \\
& 2 - 4*a*c)^{(1/2)} + a*b*c*d^2*e^2*h*x*(b^3*d*h + 4*a*c^2*d*g - 4*a*c^2*e*f \\
& - a*b^2*e*h - b^2*c*d*g + b^2*c*e*f + 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c) \\
& ^{(1/2)} - b^2*d*h*(b^2 - 4*a*c)^{(1/2)} - 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c) \\
& ^{(1/2)} + 2*a*c*d*h*(b^2 - 4*a*c)^{(1/2)} - 2*a*c*e*g*(b^2 - 4*a*c)^{(1/2)} + b*c \\
& *d*g*(b^2 - 4*a*c)^{(1/2)} + b*c*e*f*(b^2 - 4*a*c)^{(1/2)})) / (2*(4*a*c^3*d^2 + \\
& 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) - ( \\
& \log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^4 \\
& *h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x - a^2*e^4*g*(b^2 - 4*a*c)^{(1 \\
& /2)} + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - 1 \\
& 0*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c \\
& *e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x + 3*a^2*d*e^3*h*(b^2 - 4*a*c)^{(1 \\
& /2)} + c^2*d^3*e*f*(b^2 - 4*a*c)^{(1/2)} + b^2*d^3*e*h*(b^2 - 4*a*c)^{(1/2)} + 2 \\
& *b^2*e^4*f*x*(b^2 - 4*a*c)^{(1/2)} + a^2*e^4*h*x*(b^2 - 4*a*c)^{(1/2)} + 2*c^2* \\
& d^4*h*x*(b^2 - 4*a*c)^{(1/2)} - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2*c \\
& *d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x + 2*a*b*e^4*f*(b^2 - 4*a \\
& *c)^{(1/2)} + b*c*d^4*h*(b^2 - 4*a*c)^{(1/2)} + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*e \\
& *h + 7*a*b*c*e^4*f*x + 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^{(1/2)} + b^2*d^2*e^2* \\
& h*x*(b^2 - 4*a*c)^{(1/2)} - a*b*d*e^3*g*(b^2 - 4*a*c)^{(1/2)} - 7*a*c*d*e^3*f*( \\
& b^2 - 4*a*c)^{(1/2)} - 5*a*c*d^3*e*h*(b^2 - 4*a*c)^{(1/2)} - 2*b*c*d^3*e*g*(b^2 \\
& - 4*a*c)^{(1/2)} - a*b*e^4*g*x*(b^2 - 4*a*c)^{(1/2)} - 3*a*c*e^4*f*x*(b^2 - 4* \\
& a*c)^{(1/2)} + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - 1 \\
& 0*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*x \\
& - 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} + 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^{(1/2} \\
& ) + b*c*d^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} - b^2*d*e^3*g*x*(b^2 - 4*a*c)^{(1/2)} - \\
& 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^{(1/2)} + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e
\end{aligned}$$

$$\begin{aligned} &^2*f*x - 2*b^2*c*d^2*e^2*g*x - 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^{(1/2)} + 2*b* \\ &c*d^2*e^2*g*x*(b^2 - 4*a*c)^{(1/2)} - 7*a*b*c*d*e^3*g*x + 5*a*c*d*e^3*g*x*(b^ \\ &2 - 4*a*c)^{(1/2)} - 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^{(1/2)} - b*c*d^3*e*h*x*(b^2 \\ &- 4*a*c)^{(1/2)} + a*b*c*d^2*e^2*h*x*(4*a*c^2*e*f - 4*a*c^2*d*g - b^3*d*h + \\ &a*b^2*e*h + b^2*c*d*g - b^2*c*e*f - 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)^{(1/2)} \\ &(1/2) - b^2*d*h*(b^2 - 4*a*c)^{(1/2)} + 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^{(1/2)} \\ &+ 2*a*c*d*h*(b^2 - 4*a*c)^{(1/2)} - 2*a*c*e*g*(b^2 - 4*a*c)^{(1/2)} + b*c* \\ &d*g*(b^2 - 4*a*c)^{(1/2)} + b*c*e*f*(b^2 - 4*a*c)^{(1/2)))/(2*(4*a*c^3*d^2 + 4 \\ &*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) + (1 \\ &og(d + e*x)*(e^2*f + d^2*h - d*e*g))/(a*e^3 - b*d*e^2 + c*d^2*e) \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x\*\*2+g\*x+f)/(e\*x+d)/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out



$$3.149 \quad \int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h - c\left(2a\left(d^2h - 2deg + e^2f\right) + bd(dg + 2ef)\right) - abe(2dh + eg) + b^2\left(d^2h + e^2f\right) + 2c^2d\right)}{\sqrt{b^2 - 4ac}\left(ae^2 - bde + cd^2\right)^2}$$

**Rubi [A]** time = 0.76, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h - c\left(2a\left(d^2h - 2deg + e^2f\right) + bd(dg + 2ef)\right) - abe(2dh + eg) + b^2\left(d^2h + e^2f\right) + 2c^2d\right)}{\sqrt{b^2 - 4ac}\left(ae^2 - bde + cd^2\right)^2} - \frac{\log(a+bx+cx^2)\left(ae(eg-2dh) - b\left(e^2f - d^2h\right) + cd(2ef - dg)\right)}{2\left(ae^2 - bde + cd^2\right)^2} - \frac{d^2h - deg + e^2f}{e(d+ex)\left(ae^2 - bde + cd^2\right)} + \frac{\log(d+ex)\left(ae(eg-2dh) - b\left(e^2f - d^2h\right) + cd(2ef - dg)\right)}{\left(ae^2 - bde + cd^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)),x]

[Out] -((e^2\*f - d\*e\*g + d^2\*h)/(e\*(c\*d^2 - b\*d\*e + a\*e^2)\*(d + e\*x))) - ((2\*c^2\*d^2\*f + 2\*a^2\*e^2\*h - a\*b\*e\*(e\*g + 2\*d\*h) + b^2\*(e^2\*f + d^2\*h) - c\*(b\*d\*(2\*e\*f + d\*g) + 2\*a\*(e^2\*f - 2\*d\*e\*g + d^2\*h)))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(Sqrt[b^2 - 4\*a\*c]\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + ((c\*d\*(2\*e\*f - d\*g) + a\*e\*(e\*g - 2\*d\*h) - b\*(e^2\*f - d^2\*h))\*Log[d + e\*x])/(c\*d^2 - b\*d\*e + a\*e^2)^2 - ((c\*d\*(2\*e\*f - d\*g) + a\*e\*(e\*g - 2\*d\*h) - b\*(e^2\*f - d^2\*h))\*Log[a + b\*x + c\*x^2])/(2\*(c\*d^2 - b\*d\*e + a\*e^2)^2)

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)} dx &= \int \left( \frac{e^2 f - deg + d^2 h}{(cd^2 - bde + ae^2)(d + ex)^2} + \frac{e(cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h))}{(cd^2 - bde + ae^2)^2 (d + ex)} \right) dx \\ &= -\frac{e^2 f - deg + d^2 h}{e(cd^2 - bde + ae^2)(d + ex)} + \frac{(cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)) \log(d + ex)}{(cd^2 - bde + ae^2)^2} \\ &= -\frac{e^2 f - deg + d^2 h}{e(cd^2 - bde + ae^2)(d + ex)} + \frac{(cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)) \log(d + ex)}{(cd^2 - bde + ae^2)^2} \\ &= -\frac{e^2 f - deg + d^2 h}{e(cd^2 - bde + ae^2)(d + ex)} + \frac{(cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)) \log(d + ex)}{(cd^2 - bde + ae^2)^2} \\ &= -\frac{e^2 f - deg + d^2 h}{e(cd^2 - bde + ae^2)(d + ex)} - \frac{(2c^2 d^2 f + 2a^2 e^2 h - abe(eg + 2dh) + b^2(e^2 f + d^2 h)) \sqrt{b^2 - 4ac}}{2(e(ac - bd) + cd^2)^2} \end{aligned}$$

**Mathematica [A]** time = 0.54, size = 281, normalized size = 0.89

$$\frac{2 \tan^{-1}\left(\frac{bx+2c}{\sqrt{4ac-3d^2}}\right) (2a^2 e^2 h - (2a(d^2 h - 2d dg + c^2 f) + bd(dg + 2ef)) - abe(2dh + eg) + e^2(d^2 h + c^2 f) + 2c^2 d^2 f) - \frac{2(ac - bd + cd^2)(d^2 h - dg + c^2 f)}{e(d + ex)} + 2 \log(d + ex) (ae(eg - 2dh) + b(d^2 h - c^2 f) + cd(2ef - dg)) + \log(a + x(b + cx)) (ae(2dh - eg) + b(e^2 f - d^2 h) + cd(dg - 2ef))}{2(e(ac - bd) + cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)), x]
```

[Out]  $((-2*(c*d^2 + e*(-(b*d) + a*e)))*(e^{2*f} - d*e*g + d^2*h))/(e*(d + e*x)) + (2*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^{2*f} + d^2*h) - c*(b*d*(2*e*f + d*g) + 2*a*(e^{2*f} - 2*d*e*g + d^2*h)))*\text{ArcTan}[(b + 2*c*x)/\text{Sqrt}[-b^2 + 4*a*c]])/\text{Sqrt}[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) + b*(-(e^{2*f}) + d^2*h))*\text{Log}[d + e*x] + (c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^{2*f} - d^2*h))*\text{Log}[a + x*(b + c*x)]/(2*(c*d^2 + e*(-(b*d) + a*e))^2)$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)),x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)), x]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^2/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] Timed out

**giac** [A] time = 0.20, size = 449, normalized size = 1.42

$$\frac{(2c^2d^2f^2 - bcd^2g^2 + b^2d^2h^2 - 2acd^2he^2 - 2bcdf^2 + 4acdge^3 - 2abdh^3 + b^2fe^4 - 2acfe^4 - abge^4 + 2a^2he^4) \arctan\left(\frac{(2cd - \frac{2ad}{bd} - \frac{4e}{bd} - \frac{2a^2}{bd})\sqrt{-b^2 + 4ac}}{\sqrt{-b^2 + 4ac}}\right) e^{-2d}}{(c^2d^4 - 2bcd^2e + b^2d^2e^2 + 2acd^2e^2 - 2abd^3 + a^2e^4)\sqrt{-b^2 + 4ac}} + \frac{(cdfg - bd^2h - 2cdf e + 2adhe + bfe^2 - agc^2) \log\left(c - \frac{2ad}{bd} + \frac{a^2}{bd} - \frac{bc}{bd} + \frac{bc}{bd} + \frac{a^2}{bd}\right) - \frac{d^2he - dg^2}{bd} + \frac{f^2}{bd}}{2(c^2d^4 - 2bcd^2e + b^2d^2e^2 + 2acd^2e^2 - 2abd^3 + a^2e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^2/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out]  $(2*c^2*d^2*f*e^2 - b*c*d^2*g*e^2 + b^2*d^2*h*e^2 - 2*a*c*d^2*h*e^2 - 2*b*c*d*f*e^3 + 4*a*c*d*g*e^3 - 2*a*b*d*h*e^3 + b^2*f*e^4 - 2*a*c*f*e^4 - a*b*g*e^4 + 2*a^2*h*e^4)*\text{arctan}((2*c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^{-1}/\text{sqrt}(-b^2 + 4*a*c))*e^{-2}/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*\text{sqrt}(-b^2 + 4*a*c)) + 1/2*(c*d^2*g - b*d^2*h - 2*c*d*f*e + 2*a*d*h*e + b*f*e^2 - a*g*e^2)*\text{log}(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - (d^2*h*e/(x*e + d) - d*g*e^2/(x*e + d) + f*e^3/(x*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4)$

maple [B] time = 0.01, size = 1125, normalized size = 3.56

---

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a), x)$

[Out] 
$$\begin{aligned} & -1/(a*e^2-b*d*e+c*d^2)*e/(e*x+d)*f+1/(a*e^2-b*d*e+c*d^2)/(e*x+d)*d*g-2/(a*e \\ & ^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*c \\ & *e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *b*c*d^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *a*b*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *a*c*d^2*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *b*c*d*e*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *a*b*d*e*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *a*c*d*e*g-1/2/(a*e^2-b*d*e+c*d^2)^2*\ln(c*x^2+b*x+a)*g*e^2*a-1/2/(a*e^2-b*d*e+c*d^2)^2 \\ & *\ln(c*x^2+b*x+a)*b*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *b^2*e^2*f+2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *c^2*d^2*f-2/(a*e^2-b*d*e+c*d^2)^2*\ln(e*x+d)*a*d*e*h+2/(a*e^2-b*d*e+c*d^2)^2*\ln(e*x+d)*c*d*f*e+1/(a \\ & e^2-b*d*e+c*d^2)^2*\ln(c*x^2+b*x+a)*a*d*e*h-1/(a*e^2-b*d*e+c*d^2)^2*c*\ln(c*x^2+b*x+a) \\ & *d*f*e+2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *a^2*e^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *b^2*d^2*h+1/2/(a*e^2-b*d*e+c*d^2)^2*\ln(c*x^2+b*x+a)*f*e^2*b+1/2/(a*e^2-b*d*e+c*d^2)^2*c*\ln(c*x^2+b*x+a) \\ & *g*d^2-1/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2*\ln(e*x+d)*a*e^2*g+1/( \\ & a*e^2-b*d*e+c*d^2)^2*\ln(e*x+d)*b*d^2*h-1/(a*e^2-b*d*e+c*d^2)^2*\ln(e*x+d)*b \\ & e^2*f-1/(a*e^2-b*d*e+c*d^2)^2*\ln(e*x+d)*c*d^2*g \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive or negative?

mupad [B] time = 14.71, size = 3991, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)$

[Out]  $(\log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2) + (\log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g + b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x + 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^{(1/2)} + a^3*e^4*h*(b^2 - 4*a*c)^{(1/2)} - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d^3*e^3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x + 2*a*b^2*e^4*f*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^{(1/2)} - a^2*c*e^4*f*(b^2 - 4*a*c)^{(1/2)} + a*c^2*d^4*h*(b^2 - 4*a*c)^{(1/2)} + 2*b*c^2*d^4*g*(b^2 - 4*a*c)^{(1/2)} - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^{(1/2)} + 2*b^3*e^4*f*x*(b^2 - 4*a*c)^{(1/2)} + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^{(1/2)} + 16*a^2*c^2*d^3*e^3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d^3*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} + b^2*c*d^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} - b^3*d^2*e^2*h*x*(b^2 - 4*a*c)^{(1/2)} + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*b^2*c*d^3*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d^3*e^3*g - 6*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d^3*e^3*h*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d^3*e^3*f*x + 2*b^3*c*d^3*e*h*x - 8*a*c^2*d^3*e*g*(b^2 - 4*a*c)^{(1/2)} - 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*b*d^3*e^3*h*(b^2 - 4*a*c)^{(1/2)} + 8*a^2*c*d^3*e^3*g*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^2*e^4*g*x*(b^2 - 4*a*c)^{(1/2)} + a^2*b*e^4*h*x*(b^2 - 4*a*c)^{(1/2)} + 3*a^2*c*e^4*g*x*(b^2 - 4*a*c)^{(1/2)} - 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^{(1/2)} - 8*c^3*d^3*e*f*x*(b^2 - 4*a*c)^{(1/2)} + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g + 10*a^2*b*c*d^2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d^3*e^3*g*x - 2*b^2*c^2*d^3*e*g*x + b^3*c*d^2*e^2*g*x + 8*a*c^2*d^3*e^3*f*x*(b^2 - 4*a*c)^{(1/2)} + 2*a*b^2*d^3*e^3*h*x*(b^2 - 4*a*c)^{(1/2)} - 8*b^2*c*d^3*e^3*f*x*(b^2 - 4*a*c)^{(1/2)} + 8*a*c^2*d^3*e*h*x*(b^2 - 4*a*c)^{(1/2)} - 2*b*c^2*d^3*e*g*x*(b^2 - 4*a*c)^{(1/2)} - 8*a^2*c*d^3*e^3*h*x*(b^2 - 4*a*c)^{(1/2)} + 2*b^2*c*d^3*e*h*x*(b^2 - 4*a*c)^{(1/2)} - 10*a*b*c^2*d^2*e^2*g*x - 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*c)^{(1/2)} + 12*b*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^{(1/2)} + b^2*c*d^2*e^2*g*x*(b^2 - 4*a*c)^{(1/2)} - 10*a*b*c*d^2*e^3*f*(b^2 - 4*a*c)^{(1/2)} + 10*a*b*c*d^3*e^3*h*(b^2 - 4*a*c)^{(1/2)} - 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^{(1/2)} + 28*a*b*c^2*d^3*e^3*f*x + 6*a*b^2*c*d^3*e^3*g*x - 12*a*b*c^2*d^3*e^3*h*x - 12*a^2*b*c*d^3*e^3*h*x + 6*a*b*c*d^3*e^3*g*x*(b^2 - 4*a*c)^{(1/2)} - 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^{(1/2)}*(b^3*d^2*h - b^3*e^2*f + a*b^2*e^2*g + 4*a*c^2*d^2*g - 4*a^2*c*e^2*g - b^2*c*d^2*g - b^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*c^2*d^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} - b^2*d^2*h*(b^2 - 4*a*c)^{(1/2)} + 4*a*b*c*e^2*f - 4*a*b*c*d^2*h - 8*a*c^2*d^2*e*f - 2*a*b^2*d^2*e*h + 2*b^2*c*d^2*e*f + 8*a^2*c*d^2*e*h + a*b*e^2*g*(b^2 - 4*a*c)^{(1/2)} + 2*a*c*e^2*f*(b^2 - 4*a*c)^{(1/2)})$

$$\begin{aligned}
& c)^{(1/2)} + 2*a*c*d^2*h*(b^2 - 4*a*c)^{(1/2)} + b*c*d^2*g*(b^2 - 4*a*c)^{(1/2)} \\
& + 2*a*b*d*e*h*(b^2 - 4*a*c)^{(1/2)} - 4*a*c*d*e*g*(b^2 - 4*a*c)^{(1/2)} + 2*b*c \\
& *d*e*f*(b^2 - 4*a*c)^{(1/2)})/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - \\
& b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3 \\
& *e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (\log(2*a*b^3 \\
& *e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g + b*c^3*d^4*f + \\
& a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x + 2*c^4*d^4*f*x \\
& + c^3*d^4*f*(b^2 - 4*a*c)^{(1/2)} - a^3*e^4*h*(b^2 - 4*a*c)^{(1/2)} - 7*a^2*b* \\
& c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^3*h - 2*a*b^3*e \\
& ^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x - 2*a*b^2*e^4*f* \\
& (b^2 - 4*a*c)^{(1/2)} + 2*a^2*b*e^4*g*(b^2 - 4*a*c)^{(1/2)} + a^2*c*e^4*f*(b^2 \\
& - 4*a*c)^{(1/2)} - a*c^2*d^4*h*(b^2 - 4*a*c)^{(1/2)} - 2*b*c^2*d^4*g*(b^2 - 4*a \\
& *c)^{(1/2)} + 2*b^2*c*d^4*h*(b^2 - 4*a*c)^{(1/2)} - 2*b^3*e^4*f*x*(b^2 - 4*a*c) \\
& ^{(1/2)} - 3*c^3*d^4*g*x*(b^2 - 4*a*c)^{(1/2)} + 16*a^2*c^2*d*e^3*f - a*b^3*d^2 \\
& *e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e^2*f + 16*a^2*c \\
& ^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d^4*h*x - b^4*d^ \\
& 2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g - 14*a*c^2*d^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} + \\
& a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} - b^2*c*d^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} \\
& + 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^{(1/2)} + b^3*d^2*e^2*h*x*(b^2 - 4*a*c)^{(1 \\
& /2)} + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*b^2*c*d*e^3*f + \\
& 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4* \\
& f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3* \\
& d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x + 8*a*c^2*d^3*e*g*(b^2 - \\
& 4*a*c)^{(1/2)} + 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*b*d*e^3*h*(b^2 - \\
& 4*a*c)^{(1/2)} - 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^{(1/2)} + 2*a*b^2*e^4*g*x*(b^2 \\
& - 4*a*c)^{(1/2)} - a^2*b*e^4*h*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*c*e^4*g*x*(b^2 - \\
& 4*a*c)^{(1/2)} + 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^{(1/2)} + 8*c^3*d^3*e*f*x*(b^2 \\
& - 4*a*c)^{(1/2)} + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g + 10*a^2*b*c*d^ \\
& 2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^2*c^2*d^3*e*g*x \\
& + b^3*c*d^2*e^2*g*x - 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^2*d*e^ \\
& 3*h*x*(b^2 - 4*a*c)^{(1/2)} + 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^{(1/2)} - 8*a*c^2 \\
& *d^3*e*h*x*(b^2 - 4*a*c)^{(1/2)} + 2*b*c^2*d^3*e*g*x*(b^2 - 4*a*c)^{(1/2)} + 8* \\
& a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^{(1/2)} - 2*b^2*c*d^3*e*h*x*(b^2 - 4*a*c)^{(1/2)} \\
& - 10*a*b*c^2*d^2*e^2*g*x + 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*c)^{(1/2)} - 12*b \\
& *c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^{(1/2)} - b^2*c*d^2*e^2*g*x*(b^2 - 4*a*c)^{(1/2)} \\
& ) + 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^{(1/2)} - 10*a*b*c*d^3*e*h*(b^2 - 4*a*c)^{( \\
& 1/2)} + 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^{(1/2)} + 28*a*b*c^2*d*e^3*f*x + 6*a*b^2 \\
& *c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x - 6*a*b*c*d*e^3* \\
& g*x*(b^2 - 4*a*c)^{(1/2)} + 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^{(1/2)})*(b^3*e^2 \\
& *f - b^3*d^2*h - a*b^2*e^2*g - 4*a*c^2*d^2*g + 4*a^2*c*e^2*g + b^2*c*d^2*g \\
& - b^2*e^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*c^2*d^2*f*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*e \\
& ^2*h*(b^2 - 4*a*c)^{(1/2)} - b^2*d^2*h*(b^2 - 4*a*c)^{(1/2)} - 4*a*b*c*e^2*f + \\
& 4*a*b*c*d^2*h + 8*a*c^2*d*e*f + 2*a*b^2*d*e*h - 2*b^2*c*d*e*f - 8*a^2*c*d*e \\
& *h + a*b*e^2*g*(b^2 - 4*a*c)^{(1/2)} + 2*a*c*e^2*f*(b^2 - 4*a*c)^{(1/2)} + 2*a* \\
& c*d^2*h*(b^2 - 4*a*c)^{(1/2)} + b*c*d^2*g*(b^2 - 4*a*c)^{(1/2)} + 2*a*b*d*e*h*(
\end{aligned}$$

$$\frac{(b^2 - 4ac)^{1/2} - 4acd*eg*(b^2 - 4ac)^{1/2} + 2bcd*ef*(b^2 - 4ac)^{1/2}}{(2*(4a^3c^3d^4 + 4a^3c^3e^4 - a^2b^2e^4 - b^2c^2d^4 - b^4d^2e^2 + 8a^2c^2d^2e^2 + 2ab^3d^3e + 2b^3c^3d^3e - 8ab^2c^2d^3e - 8a^2b^2c^2d^3e + 2ab^2c^2d^2e^2)) - (e^2f + d^2h - d*eg)/(e*(d + ex)*(ae^2 + cd^2 - bde))}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x\*\*2+g\*x+f)/(e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

$$3.150 \quad \int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=509

$$\frac{\log(a+bx+cx^2) \left( e^3(a^2h-abg+b^2f) - c(ae(3d^2h-3deg+e^2f) + b(3de^2f-d^3h)) + c^2d^2(3ef-dg) \right)}{2(ae^2-bde+cd^2)^3} + \frac{\log\left(\frac{d+ex}{a+bx+cx^2}\right)}{2(ae^2-bde+cd^2)^3}$$

**Rubi [A]** time = 1.25, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log(a+bx+cx^2) \left( e^3(a^2h-abg+b^2f) - c(ae(3d^2h-3deg+e^2f) + b(3de^2f-d^3h)) + c^2d^2(3ef-dg) \right)}{2(ae^2-bde+cd^2)^3} + \frac{\log\left(\frac{d+ex}{a+bx+cx^2}\right)}{2(ae^2-bde+cd^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/((d + e\*x)^3\*(a + b\*x + c\*x^2)),x]

[Out]  $-\frac{(e^2f - d*eg + d^2h)}{(2e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2)} - \frac{(c*d*(2e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2f - d^2h))}{((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))} - \frac{((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*(b*d*(3e*f + d*g) + 2*a*(3e^2*f - 3*d*eg + d^2h)) - c*(2*a^2*e^2*(e*g - 3*d*h) - 3*a*b*e*(e^2f - d*eg - d^2h) - b^2*(3*d*e^2*f + d^3h)))}{(c*d^2 - b*d*e + a*e^2)^3} * \text{ArcTanh}\left[\frac{(b + 2*c*x)}{\sqrt{b^2 - 4*a*c}}\right] / (\sqrt{b^2 - 4*a*c} * (c*d^2 - b*d*e + a*e^2)^3) + \frac{((c^2*d^2*(3e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2f - 3*d*eg + 3*d^2h) - b*c*(3*d*e^2*f - d^3h))}{(c*d^2 - b*d*e + a*e^2)^3} * \text{Log}[d + e*x] + \frac{((c^2*d^2*(3e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2f - 3*d*eg + 3*d^2h) - b*c*(3*d*e^2*f - d^3h))}{(c*d^2 - b*d*e + a*e^2)^3} * \text{Log}[a + b*x + c*x^2]$

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628



```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{f + gx + hx^2}{(d + ex)^3 (a + bx + cx^2)} dx &= \int \left( \frac{e^2 f - deg + d^2 h}{(cd^2 - bde + ae^2) (d + ex)^3} + \frac{e (cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h))}{(cd^2 - bde + ae^2)^2 (d + ex)^2} \right) dx \\ &= -\frac{e^2 f - deg + d^2 h}{2e (cd^2 - bde + ae^2) (d + ex)^2} - \frac{cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\ &= -\frac{e^2 f - deg + d^2 h}{2e (cd^2 - bde + ae^2) (d + ex)^2} - \frac{cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\ &= -\frac{e^2 f - deg + d^2 h}{2e (cd^2 - bde + ae^2) (d + ex)^2} - \frac{cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\ &= -\frac{e^2 f - deg + d^2 h}{2e (cd^2 - bde + ae^2) (d + ex)^2} - \frac{cd(2ef - dg) + ae(eg - 2dh) - b(e^2 f - d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \end{aligned}$$

**Mathematica [A]** time = 0.77, size = 504, normalized size = 0.99

$$\frac{\log(d + ex) \left( -\frac{e^2 (e^2 h - abg + d^2 f)}{(ae^2 - bde + cd^2)} + \frac{ae(2ef - abg + d^2 f) + b(2de^2 - abg + d^2 f) + c(2de^2 - abg + d^2 f)}{2(ae^2 - bde + cd^2)} \right) + \frac{\log(a + bx + cx^2) \left( -\frac{e^2 (e^2 h - abg + d^2 f)}{(ae^2 - bde + cd^2)} + \frac{ae(2ef - abg + d^2 f) + b(2de^2 - abg + d^2 f) + c(2de^2 - abg + d^2 f)}{2(ae^2 - bde + cd^2)} \right)}{2(ae^2 - bde + cd^2)} - \frac{\arcsin\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) \left( -\frac{e^2 (e^2 h - abg + d^2 f)}{(ae^2 - bde + cd^2)} + \frac{ae(2ef - abg + d^2 f) + b(2de^2 - abg + d^2 f) + c(2de^2 - abg + d^2 f)}{2(ae^2 - bde + cd^2)} \right) + \frac{e^2 d(2e(ef - abg + d^2 f) + ae(eg - 2dh) + b(2de^2 - abg + d^2 f) - 2e^2 h)}{\sqrt{4ac - b^2} (ae^2 - bde + cd^2)}}{2cd + e^2 (ae^2 - bde + cd^2)} - \frac{e^2 h - abg + d^2 f}{2cd + e^2 (ae^2 - bde + cd^2)} + \frac{ae(2ef - abg + d^2 f) + b(2de^2 - abg + d^2 f) + c(2de^2 - abg + d^2 f)}{d + e(x + \frac{b}{2c})}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2)/((d + e\*x)^3\*(a + b\*x + c\*x^2)),x]

[Out] 
$$-1/2*(e^{2f} - d*eg + d^2h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (c*d*(-2*ef + dg) + a*e*(-(eg) + 2*dh) + b*(e^{2f} - d^2h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a^2*h) + c^2*d*(b*d*(3*ef + dg) + 2*a*(3*e^{2f} - 3*d*eg + d^2h)) - c*(-2*a^2*e^2*(eg - 3*dh) + 3*a*b*e*(e^{2f} - d*eg - d^2h) + b^2*(3*d*e^{2f} + d^3h)))*ArcTan[(b + 2*c*x)/\sqrt{-b^2 + 4*a*c}]/(\sqrt{-b^2 + 4*a*c}*(-(c*d^2) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*ef + dg) - e^3*(b^2*f - a*b*g + a^2*h) + a*c*e*(e^{2f} - 3*d*eg + 3*d^2h) + b*c*(3*d*e^{2f} - d^3h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*ef + dg) - e^3*(b^2*f - a*b*g + a^2*h) + a*c*e*(e^{2f} - 3*d*eg + 3*d^2h) + b*c*(3*d*e^{2f} - d^3h))*Log[a + x*(b + c*x)]/(2*(c*d^2 + e*(-(b*d) + a*e))^3)$$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{(d + ex)^3 (a + bx + cx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)^3\*(a + b\*x + c\*x^2)),x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)^3\*(a + b\*x + c\*x^2)), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^3/(c\*x^2+b\*x+a),x, algorithm="fricas")

[Out] Timed out

**giac [A]** time = 0.21, size = 1002, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^3/(c\*x^2+b\*x+a),x, algorithm="giac")

[Out] 
$$1/2*(c^2*d^3*g - b*c*d^3*h - 3*c^2*d^2*f*e + 3*a*c*d^2*h*e + 3*b*c*d*f*e^2 - 3*a*c*d*g*e^2 - b^2*f*e^3 + a*c*f*e^3 + a*b*g*e^3 - a^2*h*e^3)*\log(c*x^2 + b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*$$

$$d^5e^5 + a^3e^6) - (c^2d^3g^e - b^2cd^3h^e - 3c^2d^2f^e^2 + 3a^2cd^2h^e^2 + 3b^2cd^2f^e^3 - 3a^2cd^2g^e^3 - b^2f^e^4 + a^2cf^e^4 + ab^2g^e^4 - a^2h^e^4) \log(\text{abs}(xe + d)) / (c^3d^6e - 3b^2c^2d^5e^2 + 3b^2c^2d^4e^3 + 3a^2c^2d^4e^3 - b^3d^3e^4 - 6a^2b^2c^2d^3e^4 + 3a^2b^2d^2e^5 + 3a^2c^2d^2e^5 - 3a^2b^2d^2e^6 + a^3e^7) + (2c^3d^3f - b^2c^2d^3g + b^2c^2d^3h - 2a^2c^2d^3h - 3b^2c^2d^2f^e + 6a^2c^2d^2g^e - 3a^2b^2c^2d^2h^e + 3b^2c^2d^2f^e^2 - 6a^2c^2d^2f^e^2 - 3a^2b^2c^2d^2g^e^2 + 6a^2c^2d^2h^e^2 - b^3f^e^3 + 3a^2b^2c^2f^e^3 + a^2b^2g^e^3 - 2a^2c^2g^e^3 - a^2b^2h^e^3) \arctan((2cx + b) / \sqrt{-b^2 + 4ac}) / ((c^3d^6 - 3b^2c^2d^5e + 3b^2c^2d^4e^2 + 3a^2c^2d^4e^2 - b^3d^3e^3 - 6a^2b^2c^2d^3e^3 + 3a^2b^2d^2e^4 + 3a^2c^2d^2e^4 - 3a^2b^2d^2e^5 + a^3e^6) \sqrt{-b^2 + 4ac}) - 1/2(c^2d^6h - 3c^2d^5g^e + 5c^2d^4f^e^2 + 4b^2c^2d^4g^e^2 - b^2d^4h^e^2 - 2a^2c^2d^4h^e^2 - 8b^2c^2d^3f^e^3 - b^2d^3g^e^3 - 2a^2c^2d^3g^e^3 + 4a^2b^2d^3h^e^3 + 3b^2d^2f^e^4 + 6a^2c^2d^2f^e^4 - 3a^2d^2h^e^4 - 4a^2b^2d^2f^e^5 + a^2d^2g^e^5 + a^2f^e^6 - 2(c^2d^4g^e^2 - b^2c^2d^4h^e^2 - 2c^2d^3f^e^3 - b^2c^2d^3g^e^3 + b^2d^3h^e^3 + 2a^2c^2d^3h^e^3 + 3b^2c^2d^2f^e^4 - 3a^2b^2d^2h^e^4 - b^2d^2f^e^5 - 2a^2c^2d^2f^e^5 + a^2b^2d^2g^e^5 + 2a^2d^2d^2h^e^5 + a^2b^2f^e^6 - a^2g^e^6) * x) * e^{-1} / ((c^2d^2 - b^2d^2e + a^2e^2)^3 * (xe + d)^2)$$

**maple [B]** time = 0.02, size = 1945, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a), x)$

[Out] 
$$-3/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * b^2c^2d^2e^2f+6/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * a^2c^2d^2e^2h+3/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * a^2b^2c^2e^3f+6/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * a^2c^2d^2e^2g-6/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * a^2c^2d^2e^2f+3/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * b^2c^2d^2e^2f-1/2/(a^2e^2-b^2d^2e+c^2d^2) * e/(e*x+d)^2f+1/2/(a^2e^2-b^2d^2e+c^2d^2)/(e*x+d)^2d^2g+1/2/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(c*x^2+b*x+a) * g^e^3 * b^2a+1/2/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(c*x^2+b*x+a) * f^e^3 * a-1/2/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(c*x^2+b*x+a) * b^2d^3h-3/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * a^2b^2c^2d^2e^2g-3/(a^2e^2-b^2d^2e+c^2d^2)^3/(4ac-b^2)^{1/2} \arctan((2cx+b)/(4ac-b^2)^{1/2}) * a^2b^2c^2d^2e^2h+1/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(e*x+d) * a^2e^3h+1/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(e*x+d) * b^2e^3f-1/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(e*x+d) * c^2d^3g-1/(a^2e^2-b^2d^2e+c^2d^2)^2/(e*x+d) * a^2e^2g-1/(a^2e^2-b^2d^2e+c^2d^2)^2/(e*x+d) * b^2d^2h+1/(a^2e^2-b^2d^2e+c^2d^2)^2/(e*x+d) * b^2e^2f+1/(a^2e^2-b^2d^2e+c^2d^2)^2/(e*x+d) * c^2g^2d^2-3/2/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(c*x^2+b*x+a) * d^2g^e^2 * a+3/2/(a^2e^2-b^2d^2e+c^2d^2)^3 * \ln(c*x^2+$$

$$\begin{aligned}
& b*x+a)*d*f*e^2*b-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b) \\
& / (4*a*c-b^2)^{(1/2)})*a^2*b*e^3*h-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^{(1/2)}*a \\
& rctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a^2*c*e^3*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a \\
& *c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*e^3*g-2/(a*e^2-b*d* \\
& e+c*d^2)^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*c^2*d^3* \\
& h+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1 \\
& /2)})*b^2*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b) \\
& / (4*a*c-b^2)^{(1/2)})*b*c^2*d^3*g+3/2/(a*e^2-b*d*e+c*d^2)^3*c*\ln(c*x^2+b*x+a) \\
& *a*d^2*e*h-3/(a*e^2-b*d*e+c*d^2)^3*\ln(e*x+d)*a*c*d^2*e*h+3/(a*e^2-b*d*e+c*d \\
& ^2)^3*\ln(e*x+d)*a*c*d*e^2*g-3/(a*e^2-b*d*e+c*d^2)^3*\ln(e*x+d)*b*c*d*e^2*f-3 \\
& /2/(a*e^2-b*d*e+c*d^2)^3*c^2*\ln(c*x^2+b*x+a)*d^2*f*e-1/(a*e^2-b*d*e+c*d^2)^ \\
& 3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*e^3*f+2/(a*e^2- \\
& b*d*e+c*d^2)^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*c^3*d^ \\
& 3*f-1/(a*e^2-b*d*e+c*d^2)^3*\ln(e*x+d)*a*b*e^3*g-1/(a*e^2-b*d*e+c*d^2)^3*\ln( \\
& e*x+d)*a*c*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3*\ln(e*x+d)*b*c*d^3*h+3/(a*e^2-b*d*e \\
& +c*d^2)^3*\ln(e*x+d)*c^2*d^2*f*e+2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*a*d*e*h-2/( \\
& a*e^2-b*d*e+c*d^2)^2/(e*x+d)*c*d*e*f+1/2/(a*e^2-b*d*e+c*d^2)^3*c^2*\ln(c*x^2 \\
& +b*x+a)*g*d^3-1/2/(a*e^2-b*d*e+c*d^2)^3*\ln(c*x^2+b*x+a)*a^2*e^3*h-1/2/(a*e^ \\
& 2-b*d*e+c*d^2)^3*\ln(c*x^2+b*x+a)*f*e^3*b^2-1/2/(a*e^2-b*d*e+c*d^2)/e/(e*x+d) \\
& )^2*d^2*h
\end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^3/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 6.82, size = 12784, normalized size = 25.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g\*x + h\*x^2)/((d + e\*x)^3\*(a + b\*x + c\*x^2)),x)

[Out] symsum(log(root(24\*a^6\*b\*c\*d\*e^11\*z^3 + 24\*a\*b\*c^6\*d^11\*e\*z^3 + 240\*a^4\*b\*c^3\*d^5\*e^7\*z^3 + 240\*a^3\*b\*c^4\*d^7\*e^5\*z^3 + 120\*a^5\*b\*c^2\*d^3\*e^9\*z^3 + 120\*a^2\*b\*c^5\*d^9\*e^3\*z^3 - 54\*a^5\*b^2\*c\*d^2\*e^10\*z^3 - 54\*a\*b^2\*c^5\*d^10\*e^2\*z^3 + 50\*a^4\*b^3\*c\*d^3\*e^9\*z^3 + 50\*a\*b^3\*c^4\*d^9\*e^3\*z^3 - 36\*a^2\*b^5\*c\*d^5\*e^7\*z^3 - 36\*a\*b^5\*c^2\*d^7\*e^5\*z^3 + 26\*a\*b^6\*c\*d^6\*e^6\*z^3 - 340\*a^3\*b^2\*c^3\*d^6\*e^6\*z^3 - 225\*a^4\*b^2\*c^2\*d^4\*e^8\*z^3 - 225\*a^2\*b^2\*c^4\*d^8\*e^4\*z^3

$$\begin{aligned}
&^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4 \\
&*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d \\
&*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e \\
&^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4* \\
&e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^1 \\
&0*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d \\
&^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^ \\
&6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e \\
&*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2 \\
&*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^ \\
&2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*d*e^5*f*g*z + \\
&27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2* \\
&e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^ \\
&3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3*d^2*e^4*f*g* \\
&z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z + 27*a*b^3*c^2* \\
&d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6* \\
&g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^ \\
&2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d \\
&^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^ \\
&3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e^5*g^2*z - 66 \\
&*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c^3*d*e^5*f^2* \\
&z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^ \\
&2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z - b^5*c*d^3* \\
&e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^ \\
&6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5* \\
&h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^ \\
&2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^ \\
&2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d^2*e^4*f^2*z \\
&+ 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*h*z - b^2*c^4 \\
&*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2* \\
&z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d^4*e^2*h^2*z \\
&- 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2*c^4*d^4*e^2* \\
&g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^ \\
&4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5* \\
&d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^ \\
&2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*c \\
&^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z \\
&- a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2 \\
&*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^ \\
&3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d \\
&*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2* \\
&e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f \\
&*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3* \\
&a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d \\
&^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*
\end{aligned}$$

$$\begin{aligned}
& h + 3a^3c^2d^2e^2g^2h^2 - 3a^2c^3d^2e^2g^2h + 9a^2c^3d^2e^2f^2h^2 + \\
& 3b^2c^3d^2e^2f^2g - 3ab^2c^2e^3f^2h + 2a^2b^2c^2e^3f^2h^2 - a^2b^2c^2d^3g^2h^2 + 2ab^2c^2e^3f^2g^2 - 3a^3c^2e^3f^2h^2 + 3a^2c^3 \\
& e^3f^2h - b^3c^2e^3f^2g - a^2c^3d^3g^2h^2 - a^2c^3e^3f^2g^2 - 3a^3c^2d^2e^2h^3 + 3a^2c^3d^2e^2g^3 - a^2b^2c^2e^3g^3 - 3b^2c^4d^2e^2 \\
& f^3 + a^2b^2c^2e^3g^2h + a^3c^2e^3g^2h + b^3c^2d^3f^2h^2 + a^2b^2c^2d^3h^3 + b^4c^2e^3f^2h + b^2c^4d^3f^2h + b^2c^4d^3f^2g^2 - c^5d^3 \\
& f^2g + 3c^5d^2e^2f^3 - a^4c^4e^3f^3 - a^4c^4d^3g^3 + b^2c^3e^3f^3 + a^4c^4e^3h^3, z, k) * (\text{root}(24a^6b^2c^2d^2e^11z^3 + 24a^5b^3c^2d^11e^2z^3 \\
& + 240a^4b^3c^3d^5e^7z^3 + 240a^3b^4c^4d^7e^5z^3 + 120a^5b^3c^2d^3e^9z^3 + 120a^2b^5c^5d^9e^3z^3 - 54a^5b^2c^2d^2e^10z^3 - 54a^4b^2 \\
& c^5d^10e^2z^3 + 50a^4b^3c^2d^3e^9z^3 + 50a^3b^3c^4d^9e^3z^3 - 36a^2b^5c^2d^7e^5z^3 + 26a^3b^6c^2d^6e^6z^3 - 340a^3b^2c^3d^6e^6z^3 - 225a^4b^2c^2d^4e^8z^3 - 225a^2b^2c^4d^8e^4z^3 + 180a^3b^3c^2d^5e^7z^3 + 180a^2b^3c^3d^7e^5z^3 \\
& - 30a^2b^4c^2d^6e^6z^3 - 6b^7c^2d^7e^5z^3 - 6b^3c^5d^11e^2z^3 - 6a^5b^3d^11e^2z^3 - 6a^3b^7d^5e^7z^3 - 20b^5c^3d^9e^3z^3 + 15b^6c^2d^8e^4z^3 + 15b^4c^4d^10e^2z^3 - 80a^4c^4d^6e^6z^3 - 60a^5c^3d^4e^8z^3 - 60a^3c^5d^8e^4z^3 - 24a^6c^2d^2e^10z^3 - 24a^2c^6d^10e^2z^3 - 20a^3b^5d^3e^9z^3 + 15a^4b^4d^2e^10z^3 + 15a^2b^6d^4e^8z^3 - 4a^7c^2e^12z^3 - 4a^5c^7d^12z^3 + b^8d^6e^6z^3 + b^2c^6d^12z^3 + a^6b^2e^12z^3 - 9a^3b^2c^2d^2e^5g^2h^2z - 9a^2b^2c^3d^5e^2g^2h^2z - 30a^3b^2c^2d^2e^5f^2h^2z + 9a^2b^3c^2d^2e^5f^2h^2z + 3a^2b^4c^2d^2e^4f^2h^2z + 27a^2b^3c^4d^4e^2f^2g^2h^2z + 6a^2b^2c^2d^3e^3g^2h^2z - 33a^2b^2c^2d^2e^4f^2g^2h^2z + 18a^2b^3c^4d^5e^2f^2g^2h^2z - 12a^2b^4c^2d^2e^5f^2g^2h^2z + 27a^3b^2c^2d^2e^4g^2h^2z + 27a^2b^2c^3d^4e^2g^2h^2z - 3a^2b^3c^2d^2e^4g^2h^2z - 3a^2b^3c^2d^4e^2g^2h^2z + 52a^2b^2c^3d^3e^3f^2h^2z - 4a^2b^3c^2d^3e^3f^2h^2z - 3a^2b^2c^3d^4e^2f^2h^2z - 93a^2b^2c^3d^2e^4f^2g^2h^2z + 51a^2b^2c^2d^2e^5f^2g^2h^2z - 34a^2b^2c^3d^3e^3f^2g^2h^2z + 27a^2b^3c^2d^2e^4f^2g^2h^2z - 24a^2c^5d^5e^2f^2g^2h^2z - 7a^4b^2c^2e^6g^2h^2z - 7a^2b^2c^4d^6g^2h^2z + a^2b^4c^2d^3e^3g^2h^2z - 80a^3c^3d^3e^3g^2h^2z + 3b^4c^2d^4e^2f^2g^2h^2z - 66a^2c^4d^4e^2f^2g^2h^2z + 54a^3c^3d^2e^4f^2g^2h^2z - 3b^3c^3d^4e^2f^2g^2h^2z + 80a^2c^4d^3e^3f^2g^2h^2z - 21a^2b^2c^3d^5e^2h^2z + 6a^2b^3c^2d^5e^2h^2z - 21a^3b^2c^2d^2e^5g^2h^2z + 6a^2b^3c^2d^2e^5g^2h^2z - 66a^2b^2c^4d^3e^3f^2g^2h^2z - 30a^2b^3c^2d^2e^5f^2g^2h^2z + 27a^2b^2c^3d^2e^5f^2g^2h^2z - 12a^2b^2c^2d^4e^2h^2z - 12a^2b^2c^2d^2e^4g^2h^2z + 24a^4c^2d^2e^5g^2h^2z + 24a^2c^4d^5e^2g^2h^2z - 3b^3c^3d^5e^2f^2g^2h^2z - b^5c^2d^3e^3f^2g^2h^2z + 3b^2c^4d^5e^2f^2g^2h^2z - 24a^3c^3d^2e^5f^2g^2h^2z + 9a^3b^2c^2e^6f^2g^2h^2z - 10a^2b^3c^2e^6f^2g^2h^2z + 9a^3b^2c^2e^6f^2g^2h^2z + 3a^4b^2c^2d^2e^5h^2z + 3a^2b^2c^4d^5e^2g^2h^2z + 14a^3b^2c^2d^3e^3h^2z + 3a^3b^2c^2d^2e^4h^2z - a^2b^3c^2d^3e^3h^2z + 14a^2b^2c^3d^3e^3g^2h^2z + 3a^2b^2c^3d^4e^2g^2h^2z - a^2b^3c^2d^3e^3g^2h^2z + 63a^2b^2c^3d^2e^4f^2g^2h^2z + 2b^3c^3d^6g^2h^2z - 6a^4c^2e^6f^2g^2h^2z + 2a^3b^3e^6f^2g^2h^2z - b^2c^4d^6f^2g^2h^2z - 2a^2b^4e^6f^2g^2h^2z + 6b^5c^2d^2e^5f^2g^2h^2z + 3b^2c^5d^5e^2f^2g^2h^2z + 6a^2b^4c^2e^6f^2g^2h^2z + b^4c^2d^3e^3f^2g^2h^2z + 33a^3c^3d^3
\end{aligned}$$

$$\begin{aligned}
&^4e^2h^2z - 27a^4c^2d^2e^4h^2z + 33a^3c^3d^2e^4g^2z - 27a^2 \\
&*c^4d^4e^2g^2z + 19b^3c^3d^3e^3f^2z - 15b^4c^2d^2e^4f^2z - \\
&12b^2c^4d^4e^2f^2z - 27a^2c^4d^2e^4f^2z - 9a^2b^2c^2e^6f^2 \\
&*z + 2a^2c^5d^6f^2h^2z + 2a^2b^5e^6f^2g^2z + 33a^2c^5d^4e^2f^2z + 4a^3 \\
&*b^2c^2e^6g^2z + 4a^2b^2c^3d^6h^2z - b^4c^2d^6h^2z - b^2c^4d^6 \\
&*g^2z - a^4c^2e^6g^2z - a^4b^2e^6h^2z - a^2c^4d^6h^2z + 3a^3c \\
&^3e^6f^2z - a^2b^4e^6g^2z + b^2c^5d^6f^2g^2z + 3a^5c^2e^6h^2z + 3a \\
&*a^2c^5d^6g^2z - c^6d^6f^2z - b^6e^6f^2z + 6a^2b^2c^2d^2e^2f^2g^2h^2 - \\
&2a^2b^3c^2e^3f^2g^2h^2 + 3a^2b^2c^2d^2e^2g^2h^2 - 3a^2b^2c^2d^2e^2f^2g^2h^2 - \\
&3a^2b^2c^2d^2e^2f^2g^2h^2 - 6a^2c^3d^2e^2f^2g^2h^2 + \\
&2a^2b^2c^2e^3f^2g^2h^2 + 6a^2b^2c^3d^2e^2f^2g^2h^2 - 6a^2b^2c^3d^2e^2f^2g^2h^2 - 2 \\
&b^2c^3d^3f^2g^2h^2 - 9a^2c^4d^2e^2f^2g^2h^2 - 3b^2c^4d^2e^2f^2g^2h^2 + 3a^2c^4d^2 \\
&*e^2f^2g^2h^2 + 3a^2c^4d^2e^2f^2g^2h^2 - 2a^3b^2c^2e^3f^2g^2h^2 + 2a^2b^2c^3d^3g^2h^2 \\
&- 2a^2b^2c^3d^3f^2g^2h^2 + 2a^2c^4d^3f^2g^2h^2 - 3b^3c^2d^2e^2f^2g^2h^2 + 3b^2c^2 \\
&^3d^2e^2f^2g^2h^2 + 3a^3c^2d^2e^2g^2h^2 - 3a^2c^3d^2e^2g^2h^2 + 9a^2c^3d^2 \\
&d^2e^2f^2g^2h^2 + 3b^2c^3d^2e^2f^2g^2h^2 - 3a^2b^2c^2e^3f^2g^2h^2 + 2a^2b^2c^2e \\
&^3f^2g^2h^2 - a^2b^2c^2d^3g^2h^2 + 2a^2b^2c^2e^3f^2g^2h^2 - 3a^3c^2e^3f^2g^2h^2 \\
&2 + 3a^2c^3e^3f^2g^2h^2 - b^3c^2e^3f^2g^2h^2 - a^2c^3d^3g^2h^2 - a^2c^3e \\
&^3f^2g^2h^2 - 3a^3c^2d^2e^2h^3 + 3a^2c^3d^2e^2g^3 - a^2b^2c^2e^3g^3 - \\
&3b^2c^4d^2e^2f^3 + a^2b^2c^2e^3g^2h^2 + a^3c^2e^3g^2h^2 + b^3c^2d^3f \\
&*h^2 + a^2b^2c^2d^3h^3 + b^4c^2e^3f^2g^2h^2 + b^2c^4d^3f^2g^2h^2 + b^2c^4d^3f \\
&*g^2 - c^5d^3f^2g^2h^2 + 3c^5d^2e^2f^3 - a^2c^4e^3f^3 - a^2c^4d^3g^3 + b^2 \\
&*c^3e^3f^3 + a^4c^2e^3h^3, z, k) * ((8a^2c^6d^9e^2 + 8a^5c^2d^2e^10 - \\
&b^6c^2d^5e^6 + 32a^2c^5d^7e^4 + 48a^3c^4d^5e^6 + 32a^4c^3d^3e^8 \\
&+ 3b^2c^5d^9e^2 - 2b^3c^4d^8e^3 - 2b^4c^3d^7e^4 + 3b^5c^2d^6 \\
&^6e^5 - a^5b^2c^2e^11 - b^2c^6d^10e^6 + 114a^2b^2c^3d^5e^6 - 38a^2b^3 \\
&*c^2d^4e^7 + 60a^3b^2c^2d^3e^8 - 37a^2b^2c^5d^8e^3 + 3a^2b^5c^2d^4 \\
&e^7 + 3a^4b^2c^2d^2e^10 + 60a^2b^2c^4d^7e^4 - 38a^2b^3c^3d^6e^5 + 4a \\
&a^2b^4c^2d^5e^6 - 106a^2b^2c^4d^6e^5 - 2a^2b^4c^2d^3e^8 - 106a^3b \\
&*c^3d^4e^7 - 2a^3b^3c^2d^2e^9 - 37a^4b^2c^2d^2e^9) / (a^4e^8 + c^4d^8 \\
&+ b^4d^4e^4 - 4a^2b^3d^3e^5 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 - 4a \\
&b^3c^2d^5e^3 + 6a^2b^2d^2e^6 + 6a^2c^2d^4e^4 + 6b^2c^2d^6e^2 - \\
&4a^3b^2d^7e^7 - 4b^2c^3d^7e^7 - 12a^2b^2c^2d^5e^3 + 12a^2b^2c^2d^4e^4 - \\
&12a^2b^2c^2d^3e^5) + (x * (6a^5c^2e^11 - 2c^7d^10e^6 - 2a^4b^2c^2e^11 \\
&- 2a^2c^6d^8e^3 + 10b^2c^6d^9e^2 - 2b^6c^2d^4e^7 + 12a^2c^5d^6e^5 \\
&+ 28a^3c^4d^4e^7 + 22a^4c^3d^2e^9 - 22b^2c^5d^8e^3 + 28b^3c^4 \\
&d^7e^4 - 22b^4c^3d^6e^5 + 10b^5c^2d^5e^6 + 24a^2b^2c^3d^4e^7 \\
&+ 12a^2b^3c^2d^3e^8 + 20a^3b^2c^2d^2e^9 + 8a^2b^2c^5d^7e^4 + 8 \\
&a^2b^5c^2d^3e^8 + 8a^3b^3c^2d^2e^10 - 22a^4b^2c^2d^2e^10 - 20a^2b^2c^4 \\
&d^6e^5 + 32a^2b^3c^3d^5e^6 - 26a^2b^4c^2d^4e^7 - 36a^2b^2c^4d^5e^6 \\
&- 12a^2b^4c^2d^2e^9 - 56a^3b^2c^3d^3e^8)) / (a^4e^8 + c^4d^8 + b^4d^4 \\
&e^4 - 4a^2b^3d^3e^5 + 4a^2c^3d^6e^2 + 4a^3c^2d^2e^6 - 4b^3c^2d^5 \\
&*e^3 + 6a^2b^2d^2e^6 + 6a^2c^2d^4e^4 + 6b^2c^2d^6e^2 - 4a^3b^2 \\
&d^7e^7 - 4b^2c^3d^7e^7 - 12a^2b^2c^2d^5e^3 + 12a^2b^2c^2d^4e^4 - 12a^2b^2 \\
&c^2d^3e^5)) + (a^4c^2e^8g^2 + c^6d^7e^2f^2 + a^4b^2c^2e^8h^2 - a^2c^5d^7e^2h^2
\end{aligned}$$

$$\begin{aligned}
& - b^5 c^5 d^7 e^g + a^2 b^3 c^3 e^8 f - 2 a^3 b^2 c^2 e^8 f - a^3 b^2 c^2 e^8 g + 3 \\
& * a^5 c^5 d^5 e^3 f + a^3 c^3 d^3 e^7 f + a^5 c^5 d^6 e^2 g - b^5 c^5 d^6 e^2 f + b^5 \\
& * c^5 d^2 e^6 f - a^4 c^2 d^2 e^7 h + b^2 c^4 d^7 e^h + 3 a^2 c^4 d^3 e^5 f + 3 \\
& * a^2 c^4 d^4 e^4 g + 3 a^3 c^3 d^2 e^6 g - 3 b^2 c^4 d^5 e^3 f + 6 b^3 c^3 d^4 \\
& * e^4 f - 4 b^4 c^2 d^3 e^5 f - 3 a^2 c^4 d^5 e^3 h - 3 a^3 c^3 d^3 e^5 h \\
& + 2 b^2 c^4 d^6 e^2 g - b^3 c^3 d^5 e^3 g - 2 b^3 c^3 d^6 e^2 h + b^4 c^2 d^5 \\
& * e^3 h - a b^2 c^3 d^3 e^5 f + 4 a^2 b^3 c^2 d^2 e^6 f - 5 a^2 b^3 c^3 d^2 e^6 \\
& * f + 2 a^2 b^2 c^2 d^2 e^7 f - 2 a^2 b^2 c^3 d^4 e^4 g + 4 a^2 b^3 c^2 d^3 e^5 g \\
& - a^2 b^3 c^3 d^3 e^5 g + 5 a^2 b^2 c^3 d^5 e^3 h - 4 a^2 b^3 c^2 d^4 e^4 h + a^2 \\
& * b^3 c^3 d^4 e^4 h + a^2 b^3 c^3 d^2 e^6 h + 2 a^3 b^2 c^2 d^2 e^6 h - 2 a^2 b^4 c \\
& * d^2 e^7 f - 5 a^2 b^2 c^2 d^2 e^6 g + 2 a^2 b^2 c^2 d^3 e^5 h - 4 a^2 b^3 c^4 d \\
& * e^4 f - 2 a^2 b^3 c^4 d^5 e^3 g - a^2 b^4 c^4 d^2 e^6 g + 2 a^2 b^3 c^3 d^2 e^7 g - \\
& 2 a^3 b^2 c^3 d^2 e^7 h) / (a^4 e^8 + c^4 d^8 + b^4 d^4 e^4 - 4 a^2 b^3 d^3 e^5 + 4 \\
& * a^3 c^3 d^6 e^2 + 4 a^3 c^3 d^2 e^6 - 4 b^3 c^3 d^5 e^3 + 6 a^2 b^2 d^2 e^6 + 6 a^2 \\
& * c^2 d^4 e^4 + 6 b^2 c^2 d^6 e^2 - 4 a^3 b^2 d^2 e^7 - 4 b^3 c^3 d^7 e - 12 a^2 b \\
& * c^2 d^5 e^3 + 12 a^2 b^2 c^2 d^4 e^4 - 12 a^2 b^2 c^3 d^3 e^5) + (x * (3 a^4 c^2 e^8 \\
& * h - 3 a^3 c^3 e^8 f + 5 c^6 d^6 e^2 f - c^6 d^7 e^g + b^5 c^5 d^7 e^h - 2 a^3 \\
& * b^2 c^2 e^8 g + 7 a^5 c^5 d^4 e^4 f + 5 a^5 c^5 d^5 e^3 g - 15 b^5 c^5 d^5 e^3 f \\
& + 7 a^3 c^3 d^2 e^7 g - 5 a^5 c^5 d^6 e^2 h + b^5 c^5 d^6 e^2 g + 2 a^2 b^2 c^2 e^8 \\
& * f - a^2 c^4 d^2 e^6 f + 13 a^2 c^4 d^3 e^5 g + 17 b^2 c^4 d^4 e^4 f - 9 b^3 c^3 d^3 \\
& * e^5 f + 2 b^4 c^2 d^2 e^6 f - 7 a^2 c^4 d^4 e^4 h + a^3 c^3 d^2 e^6 h + b^2 c^4 d^5 \\
& * e^3 g - b^3 c^3 d^4 e^4 g - b^2 c^4 d^6 e^2 h - b^3 c^3 d^5 e^3 h + b^4 c^2 d^4 e^4 \\
& * h + 11 a^2 b^2 c^3 d^2 e^6 f + 13 a^2 b^2 c^3 d^3 e^5 g - 2 a^2 b^3 c^2 d^2 e^6 g - \\
& 19 a^2 b^2 c^3 d^2 e^6 g + 4 a^2 b^2 c^2 d^2 e^7 g - a^2 b^2 c^3 d^4 e^4 h - 4 a^2 b^3 \\
& * c^2 d^3 e^5 h + a^2 b^3 c^3 d^3 e^5 h + 8 a^2 b^2 c^2 d^2 e^6 h - 14 a^2 b^2 c^4 d^3 e^5 \\
& * f - 4 a^2 b^3 c^2 d^2 e^7 f + a^2 b^3 c^3 d^2 e^7 f - 16 a^2 b^3 c^4 d^4 e^4 g + 10 a^2 \\
& * b^3 c^4 d^5 e^3 h - 8 a^3 b^2 c^2 d^2 e^7 h) / (a^4 e^8 + c^4 d^8 + b^4 d^4 e^4 - 4 a^2 b^3 \\
& * d^3 e^5 + 4 a^3 c^3 d^6 e^2 + 4 a^3 c^3 d^2 e^6 - 4 b^3 c^3 d^5 e^3 + 6 a^2 b^2 d^2 e^6 + \\
& 6 a^2 c^2 d^4 e^4 + 6 b^2 c^2 d^6 e^2 - 4 a^3 b^2 d^2 e^7 - 4 b^3 c^3 d^7 e - 12 a^2 b^2 \\
& * c^2 d^5 e^3 + 12 a^2 b^2 c^2 d^4 e^4 - 12 a^2 b^2 c^3 d^3 e^5) - (2 c^5 d^3 e^2 f^2 - b^3 c^2 \\
& * e^5 f^2 - c^5 d^4 e^f g + 2 a^2 c^3 d^3 e^2 h^2 + a b^3 c^3 e^5 f^2 - 2 a^2 c^4 d^2 e^4 \\
& * f^2 - a^2 c^3 e^5 f^2 g + a^3 c^2 e^5 g^2 h - a^2 b^3 c^2 e^5 g^2 - 2 a^2 c^4 d^3 e^2 \\
& * g^2 - 5 b^3 c^4 d^2 e^3 f^2 + 2 a^2 c^3 d^2 e^4 g^2 + 4 b^2 c^3 d^2 e^4 f^2 - 2 a^3 c^2 \\
& * d^2 e^4 h^2 + a b^3 c^3 d^2 e^3 g^2 - b^2 c^3 d^2 e^3 f^2 g - 6 a^2 c^3 d^2 e^3 g^2 h - \\
& 2 b^2 c^3 d^3 e^2 f^2 h + b^3 c^2 d^2 e^3 f^2 h + a^2 c^4 d^4 e^4 g^2 h + b^3 c^4 d^4 e^4 f^2 \\
& * h + a^2 b^3 c^2 d^2 e^3 h^2 - a b^3 c^3 d^4 e^4 h^2 + 2 a^2 b^2 c^2 e^5 f^2 g - a^2 b^2 c^2 \\
& * e^5 f^2 h + 6 a^2 c^4 d^2 e^3 f^2 g - 4 a^2 c^4 d^3 e^2 f^2 h + 2 b^3 c^4 d^3 e^2 f^2 g + \\
& 4 a^2 c^3 d^2 e^4 f^2 h + 4 a^2 b^3 c^3 d^2 e^3 f^2 h - 2 a^2 b^2 c^2 d^2 e^4 f^2 h + 2 a^2 b^2 \\
& * c^3 d^3 e^2 g^2 h + 2 a^2 b^2 c^2 d^2 e^4 g^2 h - a b^2 c^2 d^2 e^3 g^2 h - 6 a^2 b^2 c^3 \\
& * d^2 e^4 f^2 g) / (a^4 e^8 + c^4 d^8 + b^4 d^4 e^4 - 4 a^2 b^3 d^3 e^5 + 4 a^3 c^3 d^6 e^2 \\
& + 4 a^3 c^3 d^2 e^6 - 4 b^3 c^3 d^5 e^3 + 6 a^2 b^2 d^2 e^6 + 6 a^2 c^2 d^4 e^4 + 6 b^2 c^2 \\
& * d^6 e^2 - 4 a^3 b^2 d^2 e^7 - 4 b^3 c^3 d^7 e - 12 a^2 b^2 c^2 d^5 e^3 + 12 a^2 b^2 c^2 \\
& * d^4 e^4 - 12 a^2 b^2 c^3 d^3 e^5) + (x * (c^5 d^4 e^g^2 + a^2 c^3 e^5 g^2 + b^2 c^3 e^5 f^2 + \\
& 4 c^5 d^2
\end{aligned}$$



$$\begin{aligned}
& e^3 f^2 + 4a^2 c^3 d^2 e^3 h^2 - 4b^4 c^4 d e^4 f^2 - 4c^5 d^3 e^2 f g - \\
& 2a^4 c^4 d^2 e^3 g^2 + b^2 c^3 d^4 e^4 h^2 - 4a^3 b^2 c^3 d^3 e^2 h^2 - 2b^2 c^3 \\
& d^2 e^3 f h - 2a^3 b^2 c^3 e^5 f g + 4a^4 c^4 d e^4 f g - 2b^4 c^4 d^4 e^4 g h - \\
& 8a^4 c^4 d^2 e^3 f h + 2b^4 c^4 d^2 e^3 f g + 4a^4 c^4 d^3 e^2 g h + 4b^4 c^4 d \\
& ^3 e^2 f h - 4a^2 c^3 d e^4 g h + 2a^3 b^2 c^3 d^2 e^3 g h + 4a^3 b^2 c^3 d e^4 \\
& f h) / (a^4 e^8 + c^4 d^8 + b^4 d^4 e^4 - 4a^3 b^3 d^3 e^5 + 4a^2 c^3 d^6 e^2 \\
& + 4a^3 c^3 d^2 e^6 - 4b^3 c^3 d^5 e^3 + 6a^2 b^2 d^2 e^6 + 6a^2 c^2 d^4 e^4 \\
& + 6b^2 c^2 d^6 e^2 - 4a^3 b^3 d^7 e - 4b^3 c^3 d^7 e - 12a^3 b^2 c^2 d^5 e^3 + \\
& 12a^3 b^2 c^2 d^4 e^4 - 12a^2 b^3 c^3 d^3 e^5) \cdot \text{root}(24a^6 b^3 c^3 d^3 e^11 z^3 + 24a \\
& a^3 b^3 c^6 d^11 e^z^3 + 240a^4 b^3 c^3 d^5 e^7 z^3 + 240a^3 b^3 c^4 d^7 e^5 z^3 \\
& + 120a^5 b^3 c^2 d^3 e^9 z^3 + 120a^2 b^3 c^5 d^9 e^3 z^3 - 54a^5 b^2 c^3 d^2 e \\
& ^10 z^3 - 54a^4 b^2 c^5 d^10 e^2 z^3 + 50a^4 b^3 c^3 d^3 e^9 z^3 + 50a^4 b^3 c^3 \\
& c^4 d^9 e^3 z^3 - 36a^2 b^5 c^3 d^5 e^7 z^3 - 36a^4 b^5 c^2 d^7 e^5 z^3 + 26a \\
& a^3 b^6 c^3 d^6 e^6 z^3 - 340a^3 b^2 c^3 d^6 e^6 z^3 - 225a^4 b^2 c^2 d^4 e^8 \\
& z^3 - 225a^2 b^2 c^4 d^8 e^4 z^3 + 180a^3 b^3 c^2 d^5 e^7 z^3 + 180a^2 b^3 \\
& b^3 c^3 d^7 e^5 z^3 - 30a^2 b^4 c^2 d^6 e^6 z^3 - 6b^7 c^3 d^7 e^5 z^3 - 6b \\
& b^3 c^5 d^11 e^z^3 - 6a^5 b^3 d^3 e^11 z^3 - 6a^3 b^7 d^5 e^7 z^3 - 20b^5 c^3 \\
& 3 d^9 e^3 z^3 + 15b^6 c^2 d^8 e^4 z^3 + 15b^4 c^4 d^10 e^2 z^3 - 80a^4 c^4 \\
& ^4 d^6 e^6 z^3 - 60a^5 c^3 d^4 e^8 z^3 - 60a^3 c^5 d^8 e^4 z^3 - 24a^6 c^2 \\
& ^2 d^2 e^10 z^3 - 24a^2 c^6 d^10 e^2 z^3 - 20a^3 b^5 d^3 e^9 z^3 + 15a^4 \\
& b^4 d^2 e^10 z^3 + 15a^2 b^6 d^4 e^8 z^3 - 4a^7 c^3 e^12 z^3 - 4a^3 c^7 d^1 \\
& 2 z^3 + b^8 d^6 e^6 z^3 + b^2 c^6 d^12 z^3 + a^6 b^2 e^12 z^3 - 9a^3 b^2 c^3 \\
& d^5 e^5 g^2 h^2 z - 9a^3 b^2 c^3 d^5 e^5 g^2 h^2 z - 30a^3 b^2 c^2 d^5 e^5 f^2 h^2 z + 9a^2 b^3 \\
& ^3 c^3 d^5 e^5 f^2 h^2 z + 3a^3 b^4 c^3 d^2 e^4 f^2 h^2 z + 27a^3 b^2 c^4 d^4 e^2 f^2 g^2 z + 6a^2 \\
& ^2 b^2 c^2 d^3 e^3 g^2 h^2 z - 33a^2 b^2 c^2 d^2 e^4 f^2 h^2 z + 18a^3 b^2 c^4 d^5 e^5 \\
& f^2 h^2 z - 12a^3 b^4 c^3 d^2 e^5 f^2 g^2 z + 27a^3 b^3 c^2 d^2 e^4 g^2 h^2 z + 27a^2 b^3 c^3 \\
& d^4 e^2 g^2 h^2 z - 3a^2 b^3 c^3 d^2 e^4 g^2 h^2 z - 3a^3 b^3 c^2 d^4 e^2 g^2 h^2 z + 52a^2 \\
& a^2 b^3 c^3 d^3 e^3 f^2 h^2 z - 4a^3 b^3 c^2 d^3 e^3 f^2 h^2 z - 3a^3 b^2 c^3 d^4 e^2 f^2 \\
& h^2 z - 93a^2 b^3 c^3 d^2 e^4 f^2 g^2 z + 51a^2 b^2 c^2 d^2 e^5 f^2 g^2 z - 34a^3 b^2 c^3 \\
& ^3 d^3 e^3 f^2 g^2 z + 27a^3 b^3 c^2 d^2 e^4 f^2 g^2 z - 24a^3 c^5 d^5 e^5 f^2 g^2 z - 7a^4 \\
& 4 b^3 c^3 e^6 g^2 h^2 z - 7a^3 b^3 c^4 d^6 g^2 h^2 z + a^3 b^4 c^3 d^3 e^3 g^2 h^2 z - 80a^3 c^3 \\
& d^3 e^3 g^2 h^2 z + 3b^4 c^2 d^4 e^2 f^2 h^2 z - 66a^2 c^4 d^4 e^2 f^2 h^2 z + 54a^3 \\
& c^3 d^2 e^4 f^2 h^2 z - 3b^3 c^3 d^4 e^2 f^2 g^2 z + 80a^2 c^4 d^3 e^3 f^2 g^2 z - 2 \\
& 1a^2 b^3 c^3 d^5 e^4 h^2 z + 6a^3 b^3 c^2 d^5 e^4 h^2 z - 21a^3 b^3 c^2 d^5 e^5 g^2 z \\
& z + 6a^2 b^3 c^3 d^5 e^5 g^2 z - 66a^3 b^3 c^4 d^3 e^3 f^2 z - 30a^3 b^3 c^2 d^5 e^5 \\
& f^2 z + 27a^2 b^3 c^3 d^5 e^5 f^2 z - 12a^2 b^2 c^2 d^4 e^2 h^2 z - 12a^2 b^2 \\
& ^2 c^2 d^2 e^4 g^2 z + 24a^4 c^2 d^2 e^5 g^2 h^2 z + 24a^2 c^4 d^5 e^5 g^2 h^2 z - 3b^3 \\
& b^3 c^3 d^5 e^5 f^2 h^2 z - b^5 c^3 d^3 e^3 f^2 h^2 z + 3b^2 c^4 d^5 e^5 f^2 g^2 z - 24a^3 \\
& c^3 d^5 e^5 f^2 g^2 z + 9a^3 b^2 c^3 e^6 f^2 h^2 z - 10a^2 b^3 c^3 e^6 f^2 g^2 z + 9a^3 b^3 \\
& c^2 e^6 f^2 g^2 z + 3a^4 b^3 c^3 d^5 e^5 h^2 z + 3a^3 b^3 c^4 d^5 e^5 g^2 z + 14a^3 b^3 c^ \\
& 2 d^3 e^3 h^2 z + 3a^3 b^2 c^3 d^2 e^4 h^2 z - a^2 b^3 c^3 d^3 e^3 h^2 z + 14a^2 \\
& a^2 b^3 c^3 d^3 e^3 g^2 z + 3a^3 b^2 c^3 d^4 e^2 g^2 z - a^3 b^3 c^2 d^3 e^3 g^2 \\
& z + 63a^3 b^2 c^3 d^2 e^4 f^2 z + 2b^3 c^3 d^6 g^2 h^2 z - 6a^4 c^2 e^6 f^2 h^2 z \\
& + 2a^3 b^3 e^6 g^2 h^2 z - b^2 c^4 d^6 f^2 h^2 z - 2a^2 b^4 e^6 f^2 h^2 z + 6b^5 c^3 \\
& d^5 e^5 f^2 z + 3b^3 c^5 d^5 e^5 f^2 z + 6a^3 b^4 c^3 e^6 f^2 z + b^4 c^2 d^3 e^3 f^2
\end{aligned}$$

```

*g*z + 33*a^3*c^3*d^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2
*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c
^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*
a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^
4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h
^2*z - b^2*c^4*d^6*g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*
d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a
^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^
2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b
*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^
2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c
^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*
f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2
*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*
e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e
*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^
2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 -
3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^
3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a
^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^
2*h + b^3*c^2*d^3*f*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f
^2*h + b*c^4*d^3*f*g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 -
a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k), k, 1, 3) - ((a*e^4*
f + c*d^4*h + a*d*e^3*g - 3*b*d*e^3*f + b*d^3*e*h - 3*c*d^3*e*g - 3*a*d^2*e
^2*h + b*d^2*e^2*g + 5*c*d^2*e^2*f)/(2*e*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 -
2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)) + (x*(a*e^3*g - b*e^3*f - 2*a*
d*e^2*h + 2*c*d*e^2*f + b*d^2*e*h - c*d^2*e*g))/(a^2*e^4 + c^2*d^4 + b^2*d^
2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*
x)

```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x\*\*2+g\*x+f)/(e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

$$3.151 \quad \int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=288

$$\frac{(d+ex)^2 \left( c \left( 2ag - b \left( \frac{ah}{c} + f \right) \right) - x(-2ach + b^2h - bcg + 2c^2f) \right)}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{e^2x(-6ach + 2b^2h - bcg + 2c^2f)}{c^2(b^2 - 4ac)} + \frac{\tanh^{-1} \left( \frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{c}$$

**Rubi [A]** time = 0.70, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1644, 773, 634, 618, 206, 628}

$$\frac{\tanh^{-1} \left( \frac{b+2cx}{\sqrt{b^2-4ac}} \right) (b^2cx(2ach + 2b^2h + bcg) - c^3(2bd(dg + 2ef) - 4a(f^2h + 2d^2g + e^2f)) - 6ac^2e(2ach + 2b^2h + bcg) - 2b^4e^2h + 4c^4d^2f) + (d+ex)^2 \left( c \left( 2ag - b \left( \frac{ah}{c} + f \right) \right) - x(-2ach + b^2h - bcg + 2c^2f) \right)}{c^3(b^2 - 4ac)^{3/2}} + \frac{e^2x(-6ach + 2b^2h - bcg + 2c^2f)}{c^2(b^2 - 4ac)} + \frac{c \log(a + bx + cx^2)(-2b^2h + 2cdh + ceg)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x]

[Out] (e^2\*(2\*c^2\*f - b\*c\*g + 2\*b^2\*h - 6\*a\*c\*h)\*x)/(c^2\*(b^2 - 4\*a\*c)) + ((d + e\*x)^2\*(c\*(2\*a\*g - b\*(f + (a\*h)/c)) - (2\*c^2\*f - b\*c\*g + b^2\*h - 2\*a\*c\*h)\*x))/(c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)) + ((4\*c^4\*d^2\*f - 2\*b^4\*e^2\*h - 6\*a\*c^2\*e\*(b\*e\*g + 2\*b\*d\*h + 2\*a\*e\*h) + b^2\*c\*e\*(b\*e\*g + 2\*b\*d\*h + 12\*a\*e\*h) - c^3\*(2\*b\*d\*(2\*e\*f + d\*g) - 4\*a\*(e^2\*f + 2\*d\*e\*g + d^2\*h)))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/(c^3\*(b^2 - 4\*a\*c)^(3/2)) + (e\*(c\*e\*g + 2\*c\*d\*h - 2\*b\*e\*h)\*Log[a + b\*x + c\*x^2])/(2\*c^3)

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2} dx = \frac{(d+ex)^2 \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach) x \right)}{c(b^2 - 4ac)(a + bx + cx^2)} + \int \frac{(d+ex)(2cdf - \dots)}{(a + bx + cx^2)^2}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)x}{c^2(b^2 - 4ac)} + \frac{(d+ex)^2 \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x \right)}{c(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)x}{c^2(b^2 - 4ac)} + \frac{(d+ex)^2 \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x \right)}{c(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)x}{c^2(b^2 - 4ac)} + \frac{(d+ex)^2 \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x \right)}{c(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{e^2(2c^2f - bcg + 2b^2h - 6ach)x}{c^2(b^2 - 4ac)} + \frac{(d+ex)^2 \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x \right)}{c(b^2 - 4ac)(a + bx + cx^2)}$$

**Mathematica [A]** time = 0.81, size = 398, normalized size = 1.38

$$\frac{2b(-3a^2b^2 + e^2(b^2 + 2ag + 3ah)^2 + f^2 - 2fg) - c^2(2b^2f - bcg + b^2h - 2ach) + 2e^2(2c^2f - bcg + 2b^2h - 6ach)x}{(b^2 - 4ac)^2(a + bx + cx^2)} + \frac{2 \operatorname{atan}\left(\frac{2cdx}{a + bx + cx^2}\right) \left( (d+ex)^2 \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x \right) \right)}{(a + bx + cx^2)^2} + c \log(a + x(b + cx))(-2beh + 2cdh + ceg) + 2c^2hx$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x]

[Out] (2\*c\*e^2\*h\*x - (2\*(b^4\*e^2\*h\*x + b^3\*e\*(a\*e\*h - c\*(e\*g + 2\*d\*h)\*x) + b^2\*c\*(c\*(e^2\*f + 2\*d\*e\*g + d^2\*h)\*x - a\*e\*(e\*g + 2\*d\*h + 4\*e\*h\*x)) + 2\*c^2\*(c^2\*d^2\*f\*x - a\*c\*(e^2\*f\*x + 2\*d\*e\*(f + g\*x) + d^2\*(g + h\*x)) + a^2\*e\*(2\*d\*h + e\*(g + h\*x))) + b\*c\*(-3\*a^2\*e^2\*h + c^2\*d\*(-2\*e\*f\*x + d\*(f - g\*x)) + a\*c\*(d^2\*h + e^2\*(f + 3\*g\*x) + 2\*d\*e\*(g + 3\*h\*x))))/((b^2 - 4\*a\*c)\*(a + x\*(b + c\*x))) + (2\*(4\*c^4\*d^2\*f - 2\*b^4\*e^2\*h - 6\*a\*c^2\*e\*(b\*e\*g + 2\*b\*d\*h + 2\*a\*e\*h) + b^2\*c\*e\*(b\*e\*g + 2\*b\*d\*h + 12\*a\*e\*h) + c^3\*(-2\*b\*d\*(2\*e\*f + d\*g) + 4\*a\*(e^2\*f + 2\*d\*e\*g + d^2\*h)))\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]]/(-b^2 + 4\*a\*c)^(3/2) + e\*(c\*e\*g + 2\*c\*d\*h - 2\*b\*e\*h)\*Log[a + x\*(b + c\*x)]/(2\*c^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2, x]

**fricas** [B] time = 1.75, size = 2771, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f - \\ & (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)*x^2 \\ & + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c - 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x]*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*e^2)*h)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e^2*g + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 + 2*((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^2)*h + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)*h)*x]*\log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), 1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + 2*((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f - (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8$$

$$\begin{aligned}
& a^2c^3de - (ab^3c - 6a^2b^2c^2)e^2 * g + 2(2a^2c^3d^2 + (ab^3c \\
& - 6a^2b^2c^2)de - (ab^4 - 6a^2b^2c + 6a^3c^2)e^2) * h + (4(b^4c^4 \\
& d^2 - b^2c^3de + ab^2c^3e^2) * f - (2b^2c^3d^2 - 8ab^2c^3de - (b^4c \\
& c - 6ab^2c^2)e^2) * g + 2(2ab^2c^3d^2 + (b^4c - 6ab^2c^2)de - (b \\
& ^5 - 6ab^3c + 6a^2b^2c^2)e^2) * h) * x) * \sqrt{-b^2 + 4ac} * \arctan(-\sqrt{-b \\
& ^2 + 4ac}) * (2cx + b) / (b^2 - 4ac) - 2((b^3c^3 - 4ab^2c^4)d^2 - 4( \\
& ab^2c^3 - 4a^2c^4)de + (ab^3c^2 - 4a^2b^2c^3)e^2) * f + 2(2(ab^2 \\
& c^3 - 4a^2c^4)d^2 - 2(ab^3c^2 - 4a^2b^2c^3)de + (ab^4c - 6a^2b^2 \\
& b^2c^2 + 8a^3c^3)e^2) * g - 2((ab^3c^2 - 4a^2b^2c^3)d^2 - 2(ab^4c \\
& - 6a^2b^2c^2 + 8a^3c^3)de + (ab^5 - 7a^2b^3c + 12a^3b^2c^2)e^2) * h \\
& - 2((2(b^2c^4 - 4ac^5)d^2 - 2(b^3c^3 - 4ab^2c^4)de + (b^4c \\
& ^2 - 6ab^2c^3 + 8a^2c^4)e^2) * f - ((b^3c^3 - 4ab^2c^4)d^2 - 2(b^4c \\
& c^2 - 6ab^2c^3 + 8a^2c^4)de + (b^5c - 7ab^3c^2 + 12a^2b^2c^3)e \\
& ^2) * g + ((b^4c^2 - 6ab^2c^3 + 8a^2c^4)d^2 - 2(b^5c - 7ab^3c^2 + \\
& 12a^2b^2c^3)de + (b^6 - 9ab^4c + 26a^2b^2c^2 - 24a^3c^3)e^2) * h) \\
& ) * x + ((ab^4c - 8a^2b^2c^2 + 16a^3c^3)e^2 * g + ((b^4c^2 - 8ab^2c \\
& ^3 + 16a^2c^4)e^2 * g + 2((b^4c^2 - 8ab^2c^3 + 16a^2c^4)de - (b^5 \\
& c - 8ab^3c^2 + 16a^2b^2c^3)e^2) * h) * x^2 + 2((ab^4c - 8a^2b^2c^2 \\
& + 16a^3c^3)de - (ab^5 - 8a^2b^3c + 16a^3b^2c^2)e^2) * h + ((b^5c - \\
& 8ab^3c^2 + 16a^2b^2c^3)e^2 * g + 2((b^5c - 8ab^3c^2 + 16a^2b^2c^3) \\
& )de - (b^6 - 8ab^4c + 16a^2b^2c^2)e^2) * h) * x) * \log(cx^2 + bx + a) \\
& / (ab^4c^3 - 8a^2b^2c^4 + 16a^3c^5 + (b^4c^4 - 8ab^2c^5 + 16a^2c^6) \\
& x^2 + (b^5c^3 - 8ab^3c^4 + 16a^2b^2c^5) * x) ]
\end{aligned}$$

**giac [A]** time = 0.18, size = 540, normalized size = 1.88

$$\frac{\sqrt{4ac^3 - 2b^2c^2 + 4a^2c^2 - 4b^2c^2 + 8a^2c^2 + 2b^2c^2 - 12ab^2c^2 + 4a^2c^2 + 8c^2} \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) + \frac{1}{2} \log\left(\frac{cx^2 + bx + a}{(cx^2 + b)^2 - 4ac^2}\right) + \frac{1}{2} \log\left(\frac{cx^2 + bx + a}{(cx^2 + b)^2 - 4ac^2}\right)}{(cx^2 + b)^2 - 4ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out]  $\begin{aligned}
& h*x*e^2/c^2 - (4c^4d^2f - 2b^2c^3d^2g + 4a^2c^3d^2h - 4b^2c^3d^2f * e \\
& + 8a^2c^3d^2g * e + 2b^3c^3d^2h * e - 12a^2b^2c^2d^2h * e + 4a^2c^3f * e^2 + b^3c^3 \\
& g * e^2 - 6a^2b^2c^2g * e^2 - 2b^4h * e^2 + 12a^2b^2c^2h * e^2 - 12a^2c^2h * e^2) \\
& * \arctan((2cx + b) / \sqrt{-b^2 + 4ac}) / ((b^2c^3 - 4a^2c^4) * \sqrt{-b^2 + 4 \\
& ac}) + 1/2 * (2c^2d^2h * e + c * g * e^2 - 2b^2h * e^2) * \log(cx^2 + bx + a) / c^3 - ( \\
& (2c^4d^2f - b^2c^3d^2g + b^2c^2d^2h - 2a^2c^3d^2h - 2b^2c^3d^2f * e \\
& + 2b^2c^2d^2g * e - 4a^2c^3d^2g * e - 2b^3c^3d^2h * e + 6a^2b^2c^2d^2h * e + b^2c^2 \\
& ^2f * e^2 - 2a^2c^3f * e^2 - b^3c^3g * e^2 + 3a^2b^2c^2g * e^2 + b^4h * e^2 - 4a^2 \\
& b^2c^2h * e^2 + 2a^2c^2h * e^2) * x / c + (b^2c^3d^2f - 2a^2c^3d^2g + a^2b^2c^2 \\
& d^2h - 4a^2c^3d^2f * e + 2a^2b^2c^2d^2g * e - 2a^2b^2c^2d^2h * e + 4a^2c^2d^2h * \\
& e + a^2b^2c^2f * e^2 - a^2b^2c^2g * e^2 + 2a^2c^2g * e^2 + a^2b^3h * e^2 - 3a^2b^2 \\
& c^2h * e^2) / c / ((cx^2 + bx + a) * (b^2 - 4ac) * c^2)
\end{aligned}$

**maple [B]** time = 0.02, size = 1712, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] e^2*h/c^2*x+6/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*d*e*h-4/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*d*e*g+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*d^2*h+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*e^2*f+4/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*d*e*h+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*e^2*f-1/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*e^2*g-3/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^2*h-1/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*e^2*g-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^2*g+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e^2*f-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*g-2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d^2*g+1/(c*x^2+b*x+a)/(4*a*c-b^2)*b*d^2*f+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2*f+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^2*h-4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^2*h-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^2*g+1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e^2*g-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*d^2*h-1/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d^2*g-4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d*e*f+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3*e^2*h+2*c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*d^2*f+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*e^2*g+8/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e*g+3/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*e^2*g+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*d*e*g+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d*e*g-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^2*e^2*h-2/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d*e*h-2/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*d*e*h-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h+4/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*d*e*h+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2*h+2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e*h-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*b*e^2*h-1/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*d*e*h-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d*e*f
```

```
maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```



[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is  $4ac - b^2$  positive or negative?

**mupad [B]** time = 5.78, size = 742, normalized size = 2.58

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((d + ex)^2 * (f + gx + hx^2)) / (a + bx + cx^2)^2, x)$

[Out] 
$$\begin{aligned} & ((2a^2c^2e^2g - 2a^2c^3d^2g + bc^3d^2f + ab^3e^2h + abc^2e^2 \\ & *f + abc^2d^2h - ab^2c^2e^2g - 3a^2b^2c^2d^2h + 4a^2c^2d^2e^2h - 4a^2c^3d^2e^2f + 2ab^2c^2d^2e^2g - 2ab^2c^2d^2e^2h) / (c(4ac - b^2)) + (x(2c^4d^2f + b^4e^2h + b^2c^2e^2f + 2a^2c^2e^2h + b^2c^2d^2h - 2a^2c^3e^2f - 2a^2c^3d^2h - bc^3d^2g - b^3c^2e^2g + 3ab^2c^2e^2g - 4ab^2c^2e^2h + 2b^2c^2d^2e^2g - 4a^2c^3d^2e^2g - 2bc^3d^2e^2f - 2b^3c^2d^2e^2h + 6abc^2d^2e^2h)) / (c(4ac - b^2))) / (a^2c^2 + c^3x^2 + b^2cx) \\ & + (\log(a + bx + cx^2) * (2b^7e^2h + 64a^3c^4e^2g - b^6c^2e^2g - 24a^2b^5c^2e^2h + 128a^3c^4d^2e^2h + 12a^2b^4c^2e^2g - 128a^3b^2c^3e^2h - 2b^6c^2d^2e^2h - 48a^2b^2c^3e^2g + 96a^2b^3c^2e^2h + 24a^2b^4c^2d^2e^2h - 96a^2b^2c^3d^2e^2h)) / (2(64a^3c^6 - b^6c^3 + 12a^2b^4c^4 - 48a^2b^2c^5)) + (\text{atan}((2cx) / (4ac - b^2)^{1/2}) - (b^3c^2 - 4ab^2c^3) / (c^2(4ac - b^2)^{3/2})) * (4c^4d^2f - 2b^4e^2h - 12a^2c^2e^2h + 4a^2c^3e^2f + 4a^2c^3d^2h - 2bc^3d^2g + b^3c^2e^2g - 6ab^2c^2e^2g + 12ab^2c^2e^2h + 8a^2c^3d^2e^2g - 4bc^3d^2e^2f + 2b^3c^2d^2e^2h - 12ab^2c^2d^2e^2h)) / (c^3(4ac - b^2)^{3/2}) + (e^2hx) / c^2 \end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((ex+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)$

[Out] Timed out

$$3.152 \quad \int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=178

$$\frac{(d+ex) \left( c \left( 2ag - b \left( \frac{ah}{c} + f \right) \right) - x(-2ach + b^2h - bcg + 2c^2f) \right)}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{\tanh^{-1} \left( \frac{b+2cx}{\sqrt{b^2-4ac}} \right) (-2c^2(b(dg + ef) - 2a(dh + eg)) - 6abcch + b^3eh + 4c^3df)}{c^2(b^2 - 4ac)^{3/2}}$$

**Rubi [A]** time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$ , Rules used = {1644, 634, 618, 206, 628}

$$\frac{(d+ex) \left( c \left( 2ag - b \left( \frac{ah}{c} + f \right) \right) - x(-2ach + b^2h - bcg + 2c^2f) \right)}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{\tanh^{-1} \left( \frac{b+2cx}{\sqrt{b^2-4ac}} \right) (-2c^2(b(dg + ef) - 2a(dh + eg)) - 6abcch + b^3eh + 4c^3df)}{c^2(b^2 - 4ac)^{3/2}} + \frac{eh \log(a + bx + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x]

[Out] ((d + e\*x)\*(c\*(2\*a\*g - b\*(f + (a\*h)/c)) - (2\*c^2\*f - b\*c\*g + b^2\*h - 2\*a\*c\*h)\*x))/(c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)) + ((4\*c^3\*d\*f + b^3\*e\*h - 6\*a\*b\*c\*e\*h - 2\*c^2\*(b\*(e\*f + d\*g) - 2\*a\*(e\*g + d\*h)))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(c^2\*(b^2 - 4\*a\*c)^(3/2)) + (e\*h\*Log[a + b\*x + c\*x^2])/(2\*c^2)

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 634**

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(
p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2} dx &= \frac{(d+ex) \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach) x \right)}{c(b^2 - 4ac)(a+bx+cx^2)} + \int \frac{2cdf - b(ef+dg)}{a+bx+cx^2} dx \\ &= \frac{(d+ex) \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach) x \right)}{c(b^2 - 4ac)(a+bx+cx^2)} + \frac{(eh) \int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} \\ &= \frac{(d+ex) \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach) x \right)}{c(b^2 - 4ac)(a+bx+cx^2)} + \frac{eh \log(a+bx+cx^2)}{2c^2} \\ &= \frac{(d+ex) \left( c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach) x \right)}{c(b^2 - 4ac)(a+bx+cx^2)} + \frac{(4c^3df + b^3eh)}{2c^2} \end{aligned}$$

**Mathematica [A]** time = 0.46, size = 225, normalized size = 1.26

$$\frac{-2(2(a^2eh - ac(d(g+hx) + e(f+gx)) + c^2dfx) + b^2(cx(dh+eg) - aeh) + bc(adh + ae(g+3hx) + cd(f-gx) - cefx) + b^3(-e)hx)}{(b^2 - 4ac)(a + x(b + cx))} + \frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) \left(-2c^2(b(dg+ef) - 2a(dh+eg)) - 6abcch + b^3eh + 4c^3df\right)}{(4ac-b^2)^{3/2}} + eh \log(a + x(b + cx))}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x]

[Out] ((-2\*(-(b^3\*e\*h\*x) + b^2\*(-(a\*e\*h) + c\*(e\*g + d\*h)\*x) + b\*c\*(a\*d\*h - c\*e\*f\*x + c\*d\*(f - g\*x) + a\*e\*(g + 3\*h\*x)) + 2\*c\*(a^2\*e\*h + c^2\*d\*f\*x - a\*c\*(e\*(f + g\*x) + d\*(g + h\*x)))))/((b^2 - 4\*a\*c)\*(a + x\*(b + c\*x))) + (2\*(4\*c^3\*d\*f + b^3\*e\*h - 6\*a\*b\*c\*e\*h - 2\*c^2\*(b\*(e\*f + d\*g) - 2\*a\*(e\*g + d\*h)))\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(-b^2 + 4\*a\*c)^(3/2) + e\*h\*Log[a + x\*(b + c\*x)]/(2\*c^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)(f + gx + hx^2)}{(a + bx + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2, x]

fricas [B] time = 1.57, size = 1413, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] [1/2\*(((2\*(2\*c^4\*d - b\*c^3\*e)\*f - 2\*(b\*c^3\*d - 2\*a\*c^3\*e)\*g + (4\*a\*c^3\*d + (b^3\*c - 6\*a\*b\*c^2)\*e)\*h)\*x^2 + 2\*(2\*a\*c^3\*d - a\*b\*c^2\*e)\*f - 2\*(a\*b\*c^2\*d - 2\*a^2\*c^2\*e)\*g + (4\*a^2\*c^2\*d + (a\*b^3 - 6\*a^2\*b\*c)\*e)\*h + (2\*(2\*b\*c^3\*d - b^2\*c^2\*e)\*f - 2\*(b^2\*c^2\*d - 2\*a\*b\*c^2\*e)\*g + (4\*a\*b\*c^2\*d + (b^4 - 6\*a\*b^2\*c)\*e)\*h)\*x)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c + sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) - 2\*((b^3\*c^2 - 4\*a\*b\*c^3)\*d - 2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*e)\*f + 2\*(2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*d - (a\*b^3\*c - 4\*a^2\*b\*c^2)\*e)\*g - 2\*((a\*b^3\*c - 4\*a^2\*b\*c^2)\*d - (a\*b^4 - 6\*a^2\*b^2\*c + 8\*a^3\*c^2)\*e)\*h - 2\*((2\*(b^2\*c^3 - 4\*a\*c^4)\*d - (b^3\*c^2 - 4\*a\*b\*c^3)\*e)\*f - ((b^3\*c^2 - 4\*a\*b\*c^3)\*d - (b^4\*c - 6\*a\*b^2\*c^2 + 8\*a^2\*c^3)\*e)\*g + ((b^4\*c - 6\*a\*b^2\*c^2 + 8\*a^2\*c^3)\*d - (b^5 - 7\*a\*b^3\*c + 12\*a^2\*b\*c^2)\*e)\*h)\*x + ((b^4\*c - 8\*a\*b^2\*c^2 + 16\*a^2\*c^3)\*e\*h\*x^2 + (b^5 - 8\*a\*b^3\*c + 16\*a^2\*b\*c^2)\*e\*h\*x + (a\*b^4 - 8\*a^2\*b^2\*c + 16\*a^3\*c^2)\*e\*h)\*log(c\*x^2 + b\*x + a)]/(a\*b^4\*c^2 - 8\*a^2\*b^2\*c^3 + 16\*a^3\*c^4 + (b^4\*c^3 - 8\*a\*b^2\*c^4 + 16\*a^2\*c^5)\*x^2 + (b^5\*c^2 - 8\*a\*b^3\*c^3 + 16\*a^2\*b\*c^4)\*x), 1/2\*(2\*(2\*c^4\*d - b\*c^3\*e)\*f - 2\*(b\*c^3\*d - 2\*a\*c^3\*e)\*g + (4\*a\*c^3\*d + (b^3\*c - 6\*a\*b\*c^2)\*e)\*h)\*x^2 + 2\*(2\*a\*c^3\*d - a\*b\*c^2\*e)\*f - 2\*(a\*b\*c^2\*d - 2\*a^2\*c^2\*e

)\*g + (4\*a^2\*c^2\*d + (a\*b^3 - 6\*a^2\*b\*c)\*e)\*h + (2\*(2\*b\*c^3\*d - b^2\*c^2\*e)\*f - 2\*(b^2\*c^2\*d - 2\*a\*b\*c^2\*e)\*g + (4\*a\*b\*c^2\*d + (b^4 - 6\*a\*b^2\*c)\*e)\*h)\*x)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) - 2\*((b^3\*c^2 - 4\*a\*b\*c^3)\*d - 2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*e)\*f + 2\*(2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*d - (a\*b^3\*c - 4\*a^2\*b\*c^2)\*e)\*g - 2\*((a\*b^3\*c - 4\*a^2\*b\*c^2)\*d - (a\*b^4 - 6\*a^2\*b^2\*c + 8\*a^3\*c^2)\*e)\*h - 2\*((2\*(b^2\*c^3 - 4\*a\*c^4)\*d - (b^3\*c^2 - 4\*a\*b\*c^3)\*e)\*f - ((b^3\*c^2 - 4\*a\*b\*c^3)\*d - (b^4\*c - 6\*a\*b^2\*c^2 + 8\*a^2\*c^3)\*e)\*g + ((b^4\*c - 6\*a\*b^2\*c^2 + 8\*a^2\*c^3)\*d - (b^5 - 7\*a\*b^3\*c + 12\*a^2\*b\*c^2)\*e)\*h)\*x + ((b^4\*c - 8\*a\*b^2\*c^2 + 16\*a^2\*c^3)\*e)\*h\*x^2 + (b^5 - 8\*a\*b^3\*c + 16\*a^2\*b\*c^2)\*e\*h\*x + (a\*b^4 - 8\*a^2\*b^2\*c + 16\*a^3\*c^2)\*e\*h\*log(c\*x^2 + b\*x + a)/(a\*b^4\*c^2 - 8\*a^2\*b^2\*c^3 + 16\*a^3\*c^4 + (b^4\*c^3 - 8\*a\*b^2\*c^4 + 16\*a^2\*c^5)\*x^2 + (b^5\*c^2 - 8\*a\*b^3\*c^3 + 16\*a^2\*b\*c^4)\*x)]

**giac** [A] time = 0.17, size = 285, normalized size = 1.60

$$\frac{he \log(cx^2 + bx + a)}{2c^2} - \frac{(4c^3df - 2b^2dg + 4a^2dh - 2b^2fe + 4ac^2ge + b^3he - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^2 - 4ac^3)\sqrt{-b^2+4ac}} - \frac{bc^2df - 2ac^2dg + abcdh - 2ac^2fe + abce - ab^2he + 2a^2che + (2c^3df - b^2dg + b^2cdh - 2ac^2dh - b^2fe + b^2cge - 2ac^2ge - b^3he + 3abc)h}{(cx^2 + bx + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out] 1/2\*h\*e\*log(c\*x^2 + b\*x + a)/c^2 - (4\*c^3\*d\*f - 2\*b\*c^2\*d\*g + 4\*a\*c^2\*d\*h - 2\*b\*c^2\*f\*e + 4\*a\*c^2\*g\*e + b^3\*h\*e - 6\*a\*b\*c\*h\*e)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/((b^2\*c^2 - 4\*a\*c^3)\*sqrt(-b^2 + 4\*a\*c)) - (b\*c^2\*d\*f - 2\*a\*c^2\*d\*g + a\*b\*c\*d\*h - 2\*a\*c^2\*f\*e + a\*b\*c\*g\*e - a\*b^2\*h\*e + 2\*a^2\*c\*h\*e + (2\*c^3\*d\*f - b\*c^2\*d\*g + b^2\*c\*d\*h - 2\*a\*c^2\*d\*h - b\*c^2\*f\*e + b^2\*c\*g\*e - 2\*a\*c^2\*g\*e - b^3\*h\*e + 3\*a\*b\*c\*h\*e)\*x)/((c\*x^2 + b\*x + a)\*(b^2 - 4\*a\*c)\*c^2)

**maple** [B] time = 0.01, size = 500, normalized size = 2.81

$$\frac{6abeh \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c} + \frac{4abfh \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{4aeg \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{b^3dh \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2c^2} - \frac{2bdg \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} - \frac{2bef \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{4cdf \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^2} + \frac{2aeh \ln(cx^2 + bx + a)}{(4ac-b^2)c} + \frac{b^2dh \ln(cx^2 + bx + a)}{2(4ac-b^2)c^2} + \frac{(3abeh-2a^2b-2a^2c^2ge-b^3he+2b^2dg+4a^2dh-2b^2fe+4ac^2ge-b^3he+3abc)h}{(4ac-b^2)^2} + \frac{2c^3df-b^2dg+b^2cdh-2ac^2dh-b^2fe+b^2cge-2ac^2ge-b^3he+3abc}{(4ac-b^2)^2} \frac{x}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x)

[Out] ((3\*a\*b\*c\*e\*h-2\*a\*c^2\*d\*h-2\*a\*c^2\*e\*g-b^3\*e\*h+b^2\*c\*d\*h+b^2\*c\*e\*g-b\*c^2\*d\*g -b\*c^2\*e\*f+2\*c^3\*d\*f)/c^2/(4\*a\*c-b^2)\*x+(2\*a^2\*c\*e\*h-a\*b^2\*e\*h+a\*b\*c\*d\*h+a\*b\*c\*e\*g-2\*a\*c^2\*d\*g-2\*a\*c^2\*e\*f+b\*c^2\*d\*f)/(4\*a\*c-b^2)/c^2)/(c\*x^2+b\*x+a)+2/c/(4\*a\*c-b^2)\*ln(c\*x^2+b\*x+a)\*a\*e\*h-1/2/c^2/(4\*a\*c-b^2)\*ln(c\*x^2+b\*x+a)\*b^2\*e\*h-6/c/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*b\*e\*h+4/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*d\*h+4/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*a\*e\*g-2/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*b\*d\*g-2/(4\*a\*c-b^2)^(3/2)\*arctan((2\*c\*x+b)/(4\*a\*c-b^2)^(1/2))\*b^3\*h

$(c-b^2)^{1/2}) * b * e * f + 4 * c / (4 * a * c - b^2)^{3/2} * \arctan((2 * c * x + b) / (4 * a * c - b^2)^{1/2}) * d * f + 1 / c^2 / (4 * a * c - b^2)^{3/2} * \arctan((2 * c * x + b) / (4 * a * c - b^2)^{1/2}) * b^3 * e * h$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* h elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for mo re details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 5.04, size = 376, normalized size = 2.11

$$\frac{\frac{b^2 d f - 2 a^2 f - 2 a^2 d g - a^2 e h + 2 a^2 c b + a b c d h + b c e g}{c^2 (4 a c - b^2)} - \frac{(b^2 h - 2 c^2 d f + 2 a^2 d h + 2 a^2 e g + b^2 d g + b^2 e f - b^2 c d h + b^2 c e g - 3 a b c e h)}{c^2 (4 a c - b^2)}}{c^2 + b x + a} - \frac{\ln(c x^2 + b x + a) (-64 c h a^3 c^3 + 48 c h a^2 b^2 c^2 - 12 c h a b^4 c + c h b^6)}{2 (64 a^2 c^5 - 48 a^2 b^2 c^4 + 12 a b^4 c^3 - b^6 c^2)} + \frac{\operatorname{atan}\left(\frac{2 c x}{\sqrt{4 a c - b^2}} - \frac{b^2 - 4 a b c}{c (4 a c - b^2)}\right) (4 c^3 d f + b^3 e h + 4 a^2 d h + 4 a^2 e g - 2 b^2 d g - 2 b^2 e f - 6 a b c e h)}{c^2 (4 a c - b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(f + g\*x + h\*x^2))/(a + b\*x + c\*x^2)^2,x)

[Out] ((b\*c^2\*d\*f - 2\*a\*c^2\*e\*f - 2\*a\*c^2\*d\*g - a\*b^2\*e\*h + 2\*a^2\*c\*e\*h + a\*b\*c\*d\*h + a\*b\*c\*e\*g)/(c^2\*(4\*a\*c - b^2)) - (x\*(b^3\*e\*h - 2\*c^3\*d\*f + 2\*a\*c^2\*d\*h + 2\*a\*c^2\*e\*g + b\*c^2\*d\*g + b\*c^2\*e\*f - b^2\*c\*d\*h - b^2\*c\*e\*g - 3\*a\*b\*c\*e\*h))/(c^2\*(4\*a\*c - b^2)))/(a + b\*x + c\*x^2) - (log(a + b\*x + c\*x^2)\*(b^6\*e\*h - 64\*a^3\*c^3\*e\*h + 48\*a^2\*b^2\*c^2\*e\*h - 12\*a\*b^4\*c\*e\*h))/(2\*(64\*a^3\*c^5 - b^6\*c^2 + 12\*a\*b^4\*c^3 - 48\*a^2\*b^2\*c^4)) + (atan((2\*c\*x)/(4\*a\*c - b^2)^(1/2) - (b^3\*c - 4\*a\*b\*c^2)/(c\*(4\*a\*c - b^2)^(3/2)))\*(4\*c^3\*d\*f + b^3\*e\*h + 4\*a\*c^2\*d\*h + 4\*a\*c^2\*e\*g - 2\*b\*c^2\*d\*g - 2\*b\*c^2\*e\*f - 6\*a\*b\*c\*e\*h))/(c^2\*(4\*a\*c - b^2)^(3/2))

**sympy** [B] time = 60.26, size = 1535, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(h\*x\*\*2+g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] (e\*h/(2\*c\*\*2) - sqrt(-(4\*a\*c - b\*\*2)\*\*3)\*(6\*a\*b\*c\*e\*h - 4\*a\*c\*\*2\*d\*h - 4\*a\*c\*\*2\*e\*g - b\*\*3\*e\*h + 2\*b\*c\*\*2\*d\*g + 2\*b\*c\*\*2\*e\*f - 4\*c\*\*3\*d\*f)/(2\*c\*\*2\*(64\*a\*\*3\*c\*\*3 - 48\*a\*\*2\*b\*\*2\*c\*\*2 + 12\*a\*b\*\*4\*c - b\*\*6)))\*log(x + (-16\*a\*\*2\*c\*\*3\*(e\*h/(2\*c\*\*2) - sqrt(-(4\*a\*c - b\*\*2)\*\*3)\*(6\*a\*b\*c\*e\*h - 4\*a\*c\*\*2\*d\*h - 4\*a\*c\*\*2\*e\*g - b\*\*3\*e\*h + 2\*b\*c\*\*2\*d\*g + 2\*b\*c\*\*2\*e\*f - 4\*c\*\*3\*d\*f)/(2\*c\*\*2\*(64\*a\*\*3\*c\*\*3 - 48\*a\*\*2\*b\*\*2\*c\*\*2 + 12\*a\*b\*\*4\*c - b\*\*6))) + 8\*a\*\*2\*c\*e\*h +

$$\begin{aligned}
& 8*a*b**2*c**2*(e*h/(2*c**2) - \text{sqrt}(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - \text{sqrt}(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)) + (e*h/(2*c**2) + \text{sqrt}(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) * \log(x + (-16*a**2*c**3*(e*h/(2*c**2) + \text{sqrt}(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))) + 8*a**2*c*e*h + 8*a*b**2*c**2*(e*h/(2*c**2) + \text{sqrt}(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) + \text{sqrt}(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)) + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g - 2*a*c**2*d*g - 2*a*c**2*e*f + b*c**2*d*f + x*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
\end{aligned}$$

$$3.153 \quad \int \frac{f+gx+hx^2}{(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=118

$$\frac{c\left(2ag - b\left(\frac{ah}{c} + f\right)\right) - x(-2ach + b^2h - bcg + 2c^2f)}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah - bg + 2cf)}{(b^2 - 4ac)^{3/2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {1660, 12, 618, 206}

$$\frac{c\left(2ag - b\left(\frac{ah}{c} + f\right)\right) - x(-2ach + b^2h - bcg + 2c^2f)}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah - bg + 2cf)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2)^2,x]

[Out] (c\*(2\*a\*g - b\*(f + (a\*h)/c)) - (2\*c^2\*f - b\*c\*g + b^2\*h - 2\*a\*c\*h)\*x)/(c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)) + (2\*(2\*c\*f - b\*g + 2\*a\*h)\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(b^2 - 4\*a\*c)^(3/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660



```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{f + gx + hx^2}{(a + bx + cx^2)^2} dx &= \frac{c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{2cf - bg + 2ah}{a + bx + cx^2} dx}{-b^2 + 4ac} \\ &= \frac{c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x}{c(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2cf - bg + 2ah) \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\ &= \frac{c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{(2(2cf - bg + 2ah)) \operatorname{Subst} \left( \int \frac{1}{b^2 - 4ac} dx \right)}{b^2 - 4ac} \\ &= \frac{c \left( 2ag - b \left( f + \frac{ah}{c} \right) \right) - (2c^2f - bcg + b^2h - 2ach)x}{c(b^2 - 4ac)(a + bx + cx^2)} + \frac{2(2cf - bg + 2ah) \tanh^{-1} \left( \frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 114, normalized size = 0.97

$$\frac{abh - 2ac(g + hx) + b^2hx + bc(f - gx) + 2c^2fx}{c(4ac - b^2)(a + x(b + cx))} - \frac{2 \tan^{-1} \left( \frac{b + 2cx}{\sqrt{4ac - b^2}} \right) (-2ah + bg - 2cf)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2)^2, x]

[Out] (a\*b\*h + 2\*c^2\*f\*x + b^2\*h\*x + b\*c\*(f - g\*x) - 2\*a\*c\*(g + h\*x))/(c\*(-b^2 + 4\*a\*c)\*(a + x\*(b + c\*x))) - (2\*(-2\*c\*f + b\*g - 2\*a\*h)\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]])/(-b^2 + 4\*a\*c)^(3/2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{(a + bx + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2)^2,x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/(a + b\*x + c\*x^2)^2, x]

**fricas [B]** time = 1.06, size = 632, normalized size = 5.36

$$\frac{(2c^2f - b^2c^2g + 2a^2c^2h + (2c^3f - b^2c^2g + 2a^2c^2h)x^2 + (2b^2c^2f - b^2c^2g + 2a^2b^2c^2h)x)\sqrt{b^2 - 4ac} \log\left(\frac{(2c^2x^2 + 2b^2cx + b^2 - 2ac - \sqrt{b^2 - 4ac})(2cx + b)}{(cx^2 + bx + a)}\right) + (b^3c - 4a^2b^2c^2)f - 2(a^2b^2c - 4a^2c^2)g + (a^2b^3 - 4a^2b^2c)h + (2(b^2c^2 - 4a^2c^3)f - (b^3c - 4a^2b^2c^2)g + (b^4 - 6a^2b^2c + 8a^2c^2)h)x}{(a^2b^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8a^2b^2c^3 + 16a^2c^4)x^2 + (b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x), (2(2a^2c^2f - a^2b^2c^2g + 2a^2c^2h + (2c^3f - b^2c^2g + 2a^2c^2h)x^2 + (2b^2c^2f - b^2c^2g + 2a^2b^2c^2h)x)\sqrt{-b^2 + 4ac} \arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) - (b^3c - 4a^2b^2c^2)f + 2(a^2b^2c - 4a^2c^2)g - (a^2b^3 - 4a^2b^2c)h - (2(b^2c^2 - 4a^2c^3)f - (b^3c - 4a^2b^2c^2)g + (b^4 - 6a^2b^2c + 8a^2c^2)h)x}{(a^2b^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8a^2b^2c^3 + 16a^2c^4)x^2 + (b^5c - 8a^2b^3c^2 + 16a^2b^2c^3)x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] [-(2\*a\*c^2\*f - a\*b\*c\*g + 2\*a^2\*c\*h + (2\*c^3\*f - b\*c^2\*g + 2\*a\*c^2\*h)\*x^2 + (2\*b\*c^2\*f - b^2\*c\*g + 2\*a\*b\*c\*h)\*x)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c - sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) + (b^3\*c - 4\*a\*b\*c^2)\*f - 2\*(a\*b^2\*c - 4\*a^2\*c^2)\*g + (a\*b^3 - 4\*a^2\*b\*c)\*h + (2\*(b^2\*c^2 - 4\*a\*c^3)\*f - (b^3\*c - 4\*a\*b\*c^2)\*g + (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*h)\*x]/(a\*b^4\*c - 8\*a^2\*b^2\*c^2 + 16\*a^3\*c^3 + (b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*x^2 + (b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b^2\*c^3)\*x), (2\*(2\*a\*c^2\*f - a\*b\*c\*g + 2\*a^2\*c\*h + (2\*c^3\*f - b\*c^2\*g + 2\*a\*c^2\*h)\*x^2 + (2\*b\*c^2\*f - b^2\*c\*g + 2\*a\*b\*c\*h)\*x)\*sqrt(-b^2 + 4\*a\*c)\*arctan(-sqrt(-b^2 + 4\*a\*c)\*(2\*c\*x + b)/(b^2 - 4\*a\*c)) - (b^3\*c - 4\*a\*b\*c^2)\*f + 2\*(a\*b^2\*c - 4\*a^2\*c^2)\*g - (a\*b^3 - 4\*a^2\*b\*c)\*h - (2\*(b^2\*c^2 - 4\*a\*c^3)\*f - (b^3\*c - 4\*a\*b\*c^2)\*g + (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*h)\*x]/(a\*b^4\*c - 8\*a^2\*b^2\*c^2 + 16\*a^3\*c^3 + (b^4\*c^2 - 8\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*x^2 + (b^5\*c - 8\*a\*b^3\*c^2 + 16\*a^2\*b^2\*c^3)\*x)]

**giac [A]** time = 0.16, size = 125, normalized size = 1.06

$$\frac{2(2cf - bg + 2ah) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2c^2fx - b^2cx + b^2hx - 2achx + bcf - 2acg + abh}{(b^2c - 4ac^2)(cx^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out] -2\*(2\*c\*f - b\*g + 2\*a\*h)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/((b^2 - 4\*a\*c)\*sqrt(-b^2 + 4\*a\*c)) - (2\*c^2\*f\*x - b\*c\*g\*x + b^2\*h\*x - 2\*a\*c\*h\*x + b\*c\*f - 2\*a\*c\*g + a\*b\*h)/((b^2\*c - 4\*a\*c^2)\*(c\*x^2 + b\*x + a))

**maple [A]** time = 0.01, size = 194, normalized size = 1.64

$$\frac{4ah \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} - \frac{2bg \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{4cf \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}} + \frac{-\frac{(2ach-b^2h+bcg-2c^2f)x}{(4ac-b^2)c} + \frac{abh-2acg+bcf}{(4ac-b^2)c}}{cx^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x)

[Out]  $(-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/(4*a*c-b^2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*h-2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*g+4/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*c*f$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad [B]** time = 3.90, size = 203, normalized size = 1.72

$$\frac{\frac{abh-2acg+bcf}{c(4ac-b^2)} + \frac{x(hb^2-gbc+2fc^2-2ahc)}{c(4ac-b^2)}}{cx^2+bx+a} - \frac{2 \operatorname{atan}\left(\frac{\left(\frac{(b^3-4abc)(2ah-bg+2cf)}{(4ac-b^2)^{5/2}} - \frac{2cx(2ah-bg+2cf)}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2ah-bg+2cf}\right)}{(4ac-b^2)^{3/2}}(2ah-bg+2cf)}{(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g\*x + h\*x^2)/(a + b\*x + c\*x^2)^2,x)

[Out]  $((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*\operatorname{atan}((((b^3 - 4*a*b*c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^{(5/2)} - (2*c*x*(2*a*h - b*g + 2*c*f)))/(4*a*c - b^2)^{(3/2)}*(4*a*c - b^2)))/(2*a*h - b*g + 2*c*f))*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^{(3/2)}$

sympy [B] time = 2.24, size = 459, normalized size = 3.89

$$-\frac{1}{\sqrt{4ac-b^2}}(2ah-bg+2f)\log\left(1+\frac{-8a^2\sqrt{\frac{4ac-b^2}{4ac-b^2}}(2ah-bg+2f)+8a^2\sqrt{\frac{4ac-b^2}{4ac-b^2}}(2ah-bg+2f)+2ab\sqrt{\frac{4ac-b^2}{4ac-b^2}}(2ah-bg+2f)-b^2g+2bcf}{4ac-2bg+4c^2}\right)+\frac{1}{\sqrt{4ac-b^2}}(2ah-bg+2f)\log\left(1+\frac{8a^2\sqrt{\frac{4ac-b^2}{4ac-b^2}}(2ah-bg+2f)-8a^2\sqrt{\frac{4ac-b^2}{4ac-b^2}}(2ah-bg+2f)+2ab\sqrt{\frac{4ac-b^2}{4ac-b^2}}(2ah-bg+2f)-b^2g+2bcf}{4ac-2bg+4c^2}\right)+\frac{ab^2-2a^2c+bcf+a(-2ab+bg+2f)(4ac-b^2)}{4a^2c^2-ab^2c+a^2(4ac-b^2)+a(4ab^2-b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x\*\*2+g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out]  $-\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)\log(x+(-16a^2c^2\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)+8ab^2c\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)+2ab^2h-b^4\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)-b^2g+2bcf)/(4ac^2h-2bcg+4c^2f))} + \sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)\log(x+(16a^2c^2\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)-8ab^2c\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)+2ab^2h+b^4\sqrt{-1/(4ac-b^2)^3}(2ah-bg+2cf)-b^2g+2bcf)/(4ac^2h-2bcg+4c^2f))} + (abh-2acg+bcf+x(-2ac^2h+b^2h-bcg+2c^2f))/(4a^2c^2-ab^2c+x^2(4ac^3-b^2c^2)+x(4ab^2c^2-b^3c))$

$$3.154 \quad \int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=407

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-\right)\right)\right)}{(b^2-4ac)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

**Rubi [A]** time = 1.09, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-\right)\right)\right)}{(b^2-4ac)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)^2), x]

[Out] (b^2\*e\*f - b\*(c\*d\*f + a\*e\*g + a\*d\*h) - 2\*a\*(c\*e\*f - c\*d\*g - a\*e\*h) - (2\*c^2\*d\*f + b\*(b\*d - a\*e)\*h - c\*(b\*e\*f + b\*d\*g - 2\*a\*e\*g + 2\*a\*d\*h))\*x)/((b^2 - 4\*a\*c)\*(c\*d^2 - b\*d\*e + a\*e^2)\*(a + b\*x + c\*x^2)) + ((4\*c^3\*d^3\*f + b\*e\*(4\*a\*b\*d\*e\*h - 2\*a^2\*e^2\*h + b^2\*(e^2\*f - d\*e\*g - d^2\*h)) - 2\*c^2\*d\*(b\*d\*(3\*e\*f + d\*g) - 2\*a\*(3\*e^2\*f - d\*e\*g + d^2\*h)) + 2\*c\*e\*(2\*b^2\*d^2\*g + 2\*a^2\*e\*(e\*g - d\*h) - a\*b\*(3\*e^2\*f + d\*e\*g + d^2\*h)))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/((b^2 - 4\*a\*c)^(3/2)\*(c\*d^2 - b\*d\*e + a\*e^2)^2) + (e\*(e^2\*f - d\*e\*g + d^2\*h)\*Log[d + e\*x])/(c\*d^2 - b\*d\*e + a\*e^2)^2 - (e\*(e^2\*f - d\*e\*g + d^2\*h)\*Log[a + b\*x + c\*x^2])/(2\*(c\*d^2 - b\*d\*e + a\*e^2)^2)

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 800

Int[(((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_)^m)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)^2} dx &= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bd^2 - bde + ae^2))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)} \\
&= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bd^2 - bde + ae^2))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)} \\
&= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bd^2 - bde + ae^2))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)} \\
&= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bd^2 - bde + ae^2))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)} \\
&= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bd^2 - bde + ae^2))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)} \\
&= \frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bd^2 - bde + ae^2))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.90, size = 405, normalized size = 1.00

$$\frac{-2d^2eh + ab^2dh + c(dg - hx) + 2ac(e(f + gx) - d(g + hx)) + b^2(dhx - ef) + bcd(f - gx) - cf(a) + 2c^2dfx}{(b^2 - 4ac)(a + x(b + cx))(bd - ae - cd^2)} \frac{\tan^{-1}\left(\frac{bx+cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} \left[ 2cx(2d^2e(dh - cg) + ab(d^2h + dfg + 3c^2f) - 2d^2d^2g) + b(2d^2d^2h - 4abdh + b^2(d^2h + dfg - c^2f)) + 2c^2d(bhdg + 3cf) - 2c(d^2h - dfg + 3c^2f) - 4c^2d^2f \right] + \frac{e \log(d + cx)(d^2h - dfg + c^2f)}{e(ax - bd) + cd^2} - \frac{e \log(a + x(b + cx))(d^2h - dfg + c^2f)}{2(e(ax - bd) + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)^2), x]

[Out]  $(-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f - g*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2 - 4*a*c)*(-c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x)) - ((-4*c^3*d^3*f + 2*c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b*d*e*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2*g + 2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2 + (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^2)$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)^2), x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)^2), x]

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [B] time = 0.19, size = 860, normalized size = 2.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out] 
$$-1/2*(d^2*h*e - d*g*e^2 + f*e^3)*\log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (d^2*h*e^2 - d*g*e^3 + f*e^4)*\log(\text{abs}(x*e + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 2*b*c^2*d^3*g + 4*a*c^2*d^3*h - 6*b*c^2*d^2*f*e + 4*b^2*c*d^2*g*e - 4*a*c^2*d^2*g*e - b^3*d^2*h*e - 2*a*b*c*d^2*h*e + 12*a*c^2*d*f*e^2 - b^3*d*g*e^2 - 2*a*b*c*d*g*e^2 + 4*a*b^2*d*h*e^2 - 4*a^2*c*d*h*e^2 + b^3*f*e^3 - 6*a*b*c*f*e^3 + 4*a^2*c*g*e^3 - 2*a^2*b*h*e^3)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4)*\sqrt{-b^2 + 4*a*c}) - (b*c^2*d^3*f - 2*a*c^2*d^3*g + a*b*c*d^3*h - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e - a*b^2*d^2*h*e - 2*a^2*c*d^2*h*e + b^3*d*f*e^2 - a*b*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g*e^2 + 3*a^2*b*d*h*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3 + a^2*b*g*e^3 - 2*a^3*h*e^3 + (2*c^3*d^3*f - b*c^2*d^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*d^2*g*e - b^3*d^2*h*e + a*b*c*d^2*h*e + b^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 2*a*b^2*d*h*e^2 - 2*a^2*$$





$$\begin{aligned} & \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} x a b c d e^{2g+1/2} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)} \ln(c^2x^2+bx+a) b^2 d^2 e^{h-1/2} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)} \\ & \ln(c^2x^2+bx+a) b^2 d^2 e^{2g-2} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) a^2 b^3 e^{3h+4} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \\ & \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) a^2 c^3 e^{3g+2} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} x c^3 d^3 f+1 \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \\ & \frac{1}{(4ac-b^2)} a^2 b^3 e^{3g+2} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} a^2 c^3 e^{3f-1} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} a b^2 \\ & e^{3f-2} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} a c^2 d^3 g+4 \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) a \\ & c^2 d^3 h-1 \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) b^3 d^2 e^{h-1} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) \\ & b^3 d^2 e^{2g+e} \frac{1}{(a^2-bd+cd^2)^2} \ln(ex+d) d^2 h-e^2 \frac{1}{(a^2-bd+cd^2)^2} \ln(ex+d) d^2 g-2 \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} a^3 e^{3h+4} \\ & \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) c^3 d^3 f+1 \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) b^3 e^{3f+1/2} \\ & \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)} \ln(c^2x^2+bx+a) b^2 e^{3f+1} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} b^3 d^2 e^{2f+1} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} \\ & b^3 d^2 e^{2f+1} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(c^2x^2+bx+a)} \frac{1}{(4ac-b^2)} b^3 d^2 e^{2f+1} \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) b^3 c^2 d^3 f-2 \\ & \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx+b}{(4ac-b^2)^{1/2}}\right) b^3 c^2 d^3 g-2 \frac{1}{(a^2-bd+cd^2)^2} \frac{1}{(4ac-b^2)} c \ln(c^2x^2+bx+a) a e^{3f} \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 6.70, size = 13698, normalized size = 33.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g\*x + h\*x^2)/((d + e\*x)\*(a + b\*x + c\*x^2)^2),x)

[Out] symsum(log(root(768\*a^5\*b\*c^4\*d^3\*e^5\*z^3 + 768\*a^4\*b\*c^5\*d^5\*e^3\*z^3 - 192\*a^5\*b^3\*c^2\*d\*e^7\*z^3 - 192\*a^2\*b^3\*c^5\*d^7\*e\*z^3 - 68\*a^3\*b^6\*c\*d^2\*e^6\*z^3 - 68\*a\*b^6\*c^3\*d^6\*e^2\*z^3 + 36\*a^2\*b^7\*c\*d^3\*e^5\*z^3 + 36\*a\*b^7\*c^2\*d^5\*e^3\*z^3 + 256\*a^6\*b\*c^3\*d\*e^7\*z^3 + 256\*a^3\*b\*c^6\*d^7\*e\*z^3 + 48\*a^4\*b^5\*c

$$\begin{aligned}
& *d^7e^7z^3 + 48*a*b^5c^4d^7e^7z^3 - 480*a^4b^2c^4d^4e^4z^3 + 440*a^3 \\
& *b^4c^3d^4e^4z^3 - 320*a^4b^3c^3d^3e^5z^3 - 320*a^3b^3c^4d^5e^ \\
& 3z^3 + 240*a^4b^4c^2d^2e^6z^3 + 240*a^2b^4c^4d^6e^2z^3 - 192*a^5 \\
& *b^2c^3d^2e^6z^3 - 192*a^3b^2c^5d^6e^2z^3 - 90*a^2b^6c^2d^4e^4 \\
& *z^3 - 48*a^3b^5c^2d^3e^5z^3 - 48*a^2b^5c^3d^5e^3z^3 - 4*b^9c*d^ \\
& 5e^3z^3 - 4*b^7c^3d^7e^7z^3 - 4*a^3b^7d^7e^7z^3 - 4*a*b^9d^3e^5z^3 \\
& - 12*a^5b^4c^8z^3 - 12*a*b^4c^5d^8z^3 + 6*b^8c^2d^6e^2z^3 - 38 \\
& 4*a^5c^5d^4e^4z^3 - 256*a^6c^4d^2e^6z^3 - 256*a^4c^6d^6e^2z^3 + \\
& 6*a^2b^8d^2e^6z^3 + 48*a^6b^2c^2e^8z^3 + 48*a^2b^2c^6d^8z^3 - \\
& 64*a^7c^3e^8z^3 - 64*a^3c^7d^8z^3 + b^10d^4e^4z^3 + b^6c^4d^8z^ \\
& 3 + a^4b^6e^8z^3 - 28*a*b^4c*d^3e^3g*h*z - 10*a^3b^2c*d^5e^5g*h*z - \\
& 10*a*b^2c^3d^5e^5g*h*z + 16*a*b^4c*d^2e^4f*h*z + 14*a^2b^3c*d^5e^5f \\
& *h*z + 4*a*b*c^4d^4e^2f*g*z + 84*a^2b^2c^2d^3e^3g*h*z - 108*a^2b^2 \\
& *c^2d^2e^4f*h*z + 16*a*b*c^4d^5e^5f*h*z - 20*a*b^4c*d^5e^5f*g*z + 8*a^ \\
& 2b^3c*d^2e^4g*h*z + 8*a*b^3c^2d^4e^2g*h*z - 4*a^3b*c^2d^2e^4g*h \\
& *z - 4*a^2b*c^3d^4e^2g*h*z + 16*a^2b*c^3d^3e^3f*h*z + 16*a*b^3c^2* \\
& d^3e^3f*h*z - 14*a*b^2c^3d^4e^2f*h*z + 66*a^2b^2c^2d^5e^5f*g*z - 3 \\
& 6*a*b^2c^3d^3e^3f*g*z + 20*a*b^3c^2d^2e^4f*g*z + 12*a^2b*c^3d^2e^ \\
& ^4f*g*z + 8*a*c^5d^5e^5f*g*z + 4*a^4b*c^6g*h*z - 2*a*b^5d^5e^5f*h*z \\
& + 4*a*b*c^4d^6g*h*z - 112*a^3c^3d^3e^3g*h*z - 3*b^4c^2d^4e^2f*h*z \\
& + 120*a^3c^3d^2e^4f*h*z - 16*a^2c^4d^4e^2f*h*z + 14*b^3c^3d^4e^ \\
& 2f*g*z - 2*b^4c^2d^3e^3f*g*z + 16*a^2c^4d^3e^3f*g*z + 8*a*b^4c*d^ \\
& 4e^2h^2z + 4*a^2b*c^3d^5e^5h^2z + 2*a*b^3c^2d^5e^5h^2z + 8*a*b^4c \\
& *d^2e^4g^2z + 4*a^3b*c^2d^5e^5g^2z + 2*a^2b^3c*d^5e^5g^2z + 48*a*b \\
& *c^4d^3e^3f^2z + 36*a^2b*c^3d^5e^5f^2z - 6*a*b^3c^2d^5e^5f^2z - 4 \\
& 5*a^2b^2c^2d^4e^2h^2z - 45*a^2b^2c^2d^2e^4g^2z + 2*b^5c*d^4e^ \\
& 2g*h*z - b^4c^2d^5e^5g*h*z + 8*a^4c^2d^5e^5g*h*z + 8*a^2c^4d^5e^5g*h \\
& *z + 2*b^3c^3d^5e^5f*h*z - 14*b^2c^4d^5e^5f*g*z - 2*b^5c*d^2e^4f*g*z \\
& + 2*a*b^5d^2e^4g*h*z - a^2b^4d^5e^5g*h*z - 120*a^3c^3d^5e^5f*g*z - \\
& 6*a^3b^2c^5e^5f*h*z + 12*a^3b*c^2e^6f*g*z - 2*a^2b^3c^5e^6f*g*z - 4* \\
& a^4b*c*d^5e^5h^2z - 4*a*b*c^4d^5e^5g^2z + 6*a^3b^2c*d^2e^4h^2z + 2 \\
& *a^2b^3c*d^3e^3h^2z + 6*a*b^2c^3d^4e^2g^2z + 2*a*b^3c^2d^3e^3g^ \\
& 2z - 18*a*b^2c^3d^2e^4f^2z - b^6d^2e^4f*h*z + 12*b*c^5d^5e^5f^2 \\
& *z + 12*a*b^4c^5e^6f^2z + 56*a^3c^3d^4e^2h^2z - 5*b^4c^2d^4e^2g^ \\
& 2z - 4*a^4c^2d^2e^4h^2z + 56*a^3c^3d^2e^4g^2z - 9*b^2c^4d^4e^ \\
& 2f^2z - 5*a^2b^4d^2e^4h^2z - 4*a^2c^4d^4e^2g^2z + 3*b^4c^2d^2 \\
& *e^4f^2z - 2*b^3c^3d^3e^3f^2z - 36*a^2c^4d^2e^4f^2z - 45*a^2b^ \\
& 2c^2e^6f^2z + 2*b^6d^5e^5f*g*z - 8*a*c^5d^6f*h*z + 4*b*c^5d^6f*g*z \\
& + 4*b^3c^3d^5e^5g^2z + 2*b^5c*d^3e^3g^2z + 4*a^3b^3d^5e^5h^2z + \\
& 2*a*b^5d^3e^3h^2z - 24*a*c^5d^4e^2f^2z + b^6d^3e^3g*h*z + a^2b^ \\
& 4e^6f*h*z - b^6d^4e^2h^2z - b^6d^2e^4g^2z - 4*a^4c^2e^6g^2z - \\
& 4*a^2c^4d^6h^2z - b^2c^4d^6g^2z - a^4b^2e^6h^2z + 48*a^3c^3e^ \\
& ^6f^2z - 4*c^6d^6f^2z - b^6e^6f^2z - 16*a*b*c^2d^2e^3f*g*h - 4*a \\
& *b^2c*d^4e^4f*g*h - 4*b*c^3d^4e^4f*g*h - 4*a^2b*c^5e^5f*g*h + 6*b^2c^2* \\
& d^3e^2f*g*h - 8*a^2b*c*d^2e^3g^2h + 8*a*b*c^2d^3e^2g^2h + 2*a*b^2
\end{aligned}$$

$$\begin{aligned}
& *c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k)*((a*b^5*c*e^6*f - 8*a^4*c^3*e^6*g + 8*a*c^6*d^5*e*f - b^6*c*d*e^5*f + 20*a^3*b*c^3*e^6*f - a^3*b^3*c*e^6*h + 8*a^3*c^4*d*e^5*f + 4*a^4*b*c^2*e^6*h - 2*b^2*c^5*d^5*e*f + 8*a^2*c^5*d^5*e*h + 8*a^4*c^3*d*e^5*h + b^3*c^4*d^5*e*g + b^6*c*d^2*e^4*g - b^6*c*d^3*e^3*h - 9*a^2*b^3*c^2*e^6*f + 2*a^3*b^2*c^2*e^6*g + 16*a^2*c^5*d^3*e^3*f - 8*a^2*c^5*d^4*e^2*g - 16*a^3*c^4*d^2*e^4*g + 3*b^3*c^4*d^4*e^2*f + 16*a^3*c^4*d^3*e^3*h - 2*b^4*c^3*d^4*e^2*g + b^5*c^2*d^4*e^2*h - 4*a*b^2*c^4*d^3*e^3*f - 2*a*b^3*c^3*d^2*e^4*f + 8*a^2*b*c^4*d^2*e^4*f - 26*a^2*b^2*c^3*d*e^5*f + 10*a*b^2*c^4*d^4*e^2*g + 2*a*b^3*c^3*d^3*e^3*g - 8*a*b^4*c^2*d^2*e^4*g - 8*a^2*b*c^4*d^3*e^3*g + 5*a^2*b^3*c^2*d*e^5*g - 5*a*b^3*c^3*d^4*e^2*h + 8*a*b^4*c^2*d^3*e^3*h + 4*a^2*b*c^4*d^4*e^2*h + 8*a^3*b*c^3*d^2*e^4*h - 10*a^3*b^2*c^2*d*e^5*h - 4*a*b*c^5*d^5*e*g - a*b^5*c*d*e^5*g + 20*a^2*b^2*c^3*d^2*e^4*g - 20*a^2*b^2*c^3*d^3*e^3*h - 2*a^2*b^3*c^2*d^2*e^4*h - 12*a*b*c^5*d^4*e^2*f + 10*a*b^4*c^2*d*e^5*f - 4*a^3*b*c^3*d*e^5*g - 2*a*b^2*c^4*d^5*e*h + 2*a^2*b^4*c*d*e^5*h)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^
\end{aligned}$$

$$\begin{aligned}
&6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*
\end{aligned}$$

$$\begin{aligned}
& c^3d^3e^2g^2h - 4b^3c^3d^3e^2f^2h - 2b^3c^3d^3e^2f^2h^2 + 24a^3c^3d^2e^3f^2h + 9b^2c^2d^2e^4f^2g + 4b^3c^3d^3e^2f^2g^2 + 2a^3b^3d^2e^3g^2h^2 - a^2b^2d^2e^4g^2h^2 + 8a^3c^3d^2e^3f^2g^2 + 4a^2b^3c^3d^2e^2h^3 - 4a^3b^3c^2d^2e^3g^3 - b^4d^2e^3g^2h - 4c^4d^3e^2f^2g - b^4d^2e^3f^2h^2 + 4a^2c^2e^5f^2g^2 + 4a^2c^2d^4e^2h^3 + 2b^3c^3d^2e^3g^3 - 4a^2c^2d^2e^4g^3 - 2a^3b^3d^3e^2h^3 + 4c^4d^4e^2f^2h + 2b^3c^3e^5f^2g - 4b^3c^3d^2e^4f^3 + b^2c^2d^4e^2g^2h - b^2c^2d^3e^2g^3 + b^4d^3e^2g^2h^2 + a^2b^2e^5f^2h^2 + 4c^4d^2e^3f^3 - 3b^2c^2e^5f^3 + a^2b^2d^2e^3h^3 - b^4e^5f^2h + 16a^3c^3e^5f^3, z, k) * \\
& ((128a^5c^4d^2e^6 - 16a^5b^3c^3e^7 - a^3b^5c^3e^7 - b^5c^4d^6e - b^8c^3d^3e^4 + 8a^4b^3c^2e^7 + 128a^3c^6d^5e^2 + 256a^4c^5d^3e^4 + b^6c^3d^5e^2 + b^7c^2d^4e^3 - 48a^2b^2c^5d^5e^2 + 168a^2b^3c^4d^4e^3 - 80a^2b^4c^3d^3e^4 - 27a^2b^5c^2d^2e^5 + 32a^3b^2c^4d^3e^4 + 168a^3b^3c^3d^2e^5 + 8a^3b^3c^5d^6e + a^3b^7c^3d^2e^5 - 16a^2b^3c^6d^6e + a^2b^6c^3d^4e^3 + 18a^3b^6c^2d^3e^4 - 304a^3b^3c^5d^4e^3 - 304a^4b^3c^4d^2e^5 - 48a^4b^2c^3d^3e^6)/(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8a^3b^2c^3d^4 - 8a^3b^2c^3e^4 + 32a^3c^3d^2e^2 - 2a^3b^5d^2e^3 - 2b^5c^3d^3e + 16a^3b^3c^2d^3e - 6a^3b^4c^3d^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3c^3d^3e^3 - 32a^3b^3c^2d^3e^3) - (x*(2a^2b^6c^3e^7 - 96a^5c^4e^7 + 32a^2c^7d^6e + 2b^4c^5d^6e + 2b^8c^3d^2e^5 - 2a^3b^4c^2e^7 + 80a^4b^2c^3e^7 - 32a^3c^6d^4e^3 - 160a^4c^5d^2e^5 - 6b^5c^4d^5e^2 + 8b^6c^3d^4e^3 - 6b^7c^2d^3e^4 - 4a^3b^7c^3d^2e^6 + 144a^2b^2c^5d^4e^3 - 128a^2b^3c^4d^3e^4 + 6a^2b^4c^3d^2e^5 + 112a^3b^2c^4d^2e^5 - 16a^3b^2c^6d^6e + 160a^4b^3c^4d^2e^6 + 48a^3b^3c^5d^5e^2 - 66a^3b^4c^4d^4e^3 + 52a^3b^5c^3d^3e^4 - 14a^3b^6c^2d^2e^5 - 96a^2b^3c^6d^5e^2 + 42a^2b^5c^2d^2e^6 + 64a^3b^3c^5d^3e^4 - 144a^3b^3c^3d^2e^6))/(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8a^3b^2c^3d^4 - 8a^3b^2c^3e^4 + 32a^3c^3d^2e^2 - 2a^3b^5d^2e^3 - 2b^5c^3d^3e + 16a^3b^3c^2d^3e - 6a^3b^4c^3d^2e^2 - 32a^2b^3c^3d^3e + 16a^2b^3c^3d^3e^3 - 32a^3b^3c^2d^3e^3) - (x*(8a^3b^3c^3e^6g - 2a^3b^4c^2e^6f - 48a^3c^4e^6f - 16a^3c^6d^4e^2f + a^2b^4c^3e^6h + 32a^3c^4d^2e^5g + 2b^5c^2d^2e^5f + b^6c^3d^2e^4h + 20a^2b^2c^3e^6f - 2a^2b^3c^2e^6g - 64a^2c^5d^2e^4f - 4a^3b^2c^2e^6h + 32a^2c^5d^3e^3g + 4b^2c^5d^4e^2f - 8b^3c^4d^3e^3f + 2b^4c^3d^2e^4f - 32a^2c^5d^4e^2h - 32a^3c^4d^2e^4h - 2b^3c^4d^4e^2g + 6b^4c^3d^3e^3g - 4b^5c^2d^2e^4g - b^4c^3d^4e^2h + 8a^3b^2c^4d^2e^4f - 32a^3b^2c^4d^3e^3g + 20a^3b^3c^3d^2e^4g - 16a^2b^3c^4d^2e^4g - 32a^2b^2c^3d^2e^5g + 12a^3b^2c^4d^4e^2h - 8a^3b^3c^3d^3e^3h - 4a^3b^4c^2d^2e^4h + 32a^2b^3c^4d^3e^3h + 8a^2b^3c^2d^2e^5h - 2a^3b^5c^3d^2e^5h + 8a^2b^2c^3d^2e^4h + 32a^3b^3c^5d^3e^3f - 24a^3b^3c^3d^2e^5f + 64a^2b^3c^4d^2e^5f + 8a^3b^3c^5d^4e^2g + 6a^3b^4c^2d^2e^5g))/(a^2b^4e^4 + 16a^2c^4d^4 + 16a^4c^2e^4 + b^4c^2d^4 + b^6d^2e^2 - 8a^3b^2c^3d^4 - 8a^3b^2c^3e^4 + 32a^3c^3d^2e^2 - 2a^3b^5d^2e^3 - 2b^5c^3d^3e
\end{aligned}$$

$$\begin{aligned}
&^3e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2 \\
&*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)) - (4*a^2*c^3*d^3*e^2*h^2 - 4*c^5*d^3*e^ \\
&2*f^2 - b^3*c^2*e^5*f^2 - b^2*c^3*d^3*e^2*g^2 + b^3*c^2*d^2*e^3*g^2 + 4*a*b \\
&*c^3*e^5*f^2 - 8*a*c^4*d*e^4*f^2 - 8*a^2*c^3*e^5*f*g + 4*b*c^4*d^2*e^3*f^2 \\
&+ 4*a^2*c^3*d*e^4*g^2 + b^2*c^3*d*e^4*f^2 - 2*a*b^2*c^2*d*e^4*g^2 + a*b^3*c \\
&*d^2*e^3*h^2 - a^2*b^2*c*d*e^4*h^2 - 4*b^2*c^3*d^2*e^3*f*g - 8*a^2*c^3*d^2* \\
&e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + b^3*c^2*d^3*e^2*g*h \\
&- a*b^3*c*e^5*f*h + b^4*c*d*e^4*f*h - 2*a*b^2*c^2*d^3*e^2*h^2 + 2*a*b^2*c^ \\
&2*e^5*f*g + 4*a^2*b*c^2*e^5*f*h + 4*b*c^4*d^3*e^2*f*g + 8*a^2*c^3*d*e^4*f*h \\
&- b^4*c*d^2*e^3*g*h + 4*a*b*c^3*d^2*e^3*f*h - 8*a*b^2*c^2*d*e^4*f*h + 2*a* \\
&b^2*c^2*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*f*g + a*b^3*c*d*e^4*g*h)/(a^2*b^4*e^4 \\
&+ 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^ \\
&3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3* \\
&e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^ \\
&3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + (x*(4*a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 + \\
&4*c^5*d^2*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 + b^2*c^3*d^2*e^3*g^2 - 4*b*c^4* \\
&d*e^4*f^2 + a^2*b^2*c*e^5*h^2 + b^4*c*d^2*e^3*h^2 + 4*a^2*b*c^2*d*e^4*h^2 + \\
&4*b^2*c^3*d^2*e^3*f*h - 2*b^3*c^2*d^2*e^3*g*h - 4*a*b*c^3*e^5*f*g + 8*a*c^ \\
&4*d*e^4*f*g - 4*a*b^2*c^2*d^2*e^3*h^2 - 4*a*b*c^3*d*e^4*g^2 - 2*a*b^3*c*d*e \\
&^4*h^2 + 2*a*b^2*c^2*e^5*f*h - 4*a^2*b*c^2*e^5*g*h - 8*a*c^4*d^2*e^3*f*h - \\
&4*b*c^4*d^2*e^3*f*g + 2*b^2*c^3*d*e^4*f*g - 8*a^2*c^3*d*e^4*g*h - 2*b^3*c^2 \\
&*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*g*h + 6*a*b^2*c^2*d*e^4*g*h))/(a^2*b^4*e^4 + \\
&16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3* \\
&d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e \\
&+ 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3* \\
&c*d*e^3 - 32*a^3*b*c^2*d*e^3))*root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c \\
&^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68 \\
&*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^ \\
&3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7* \\
&e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d \\
&^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 32 \\
&0*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d \\
&^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90 \\
&*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5* \\
&e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - \\
&4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8* \\
&c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a \\
&^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^ \\
&2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4* \\
&z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3 \\
&*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + \\
&14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3 \\
&*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4* \\
&c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a \\
&^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f
\end{aligned}$$

$$\begin{aligned}
& *h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2 \\
& *c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z \\
& + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - \\
& 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b \\
& ^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z \\
& + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3* \\
& f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5 \\
& *e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c* \\
& d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3 \\
& *c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^ \\
& 2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + \\
& 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2 \\
& *b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^ \\
& 3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2* \\
& b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2 \\
& *c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + \\
& 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z \\
& + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - \\
& 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2* \\
& z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g \\
& ^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e \\
& ^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z \\
& + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^ \\
& 3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^ \\
& 3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4 \\
& *a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6* \\
& h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2 \\
& *d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^ \\
& 5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3 \\
& *e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c* \\
& d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2* \\
& d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e \\
& ^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h \\
& ^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a \\
& *b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2 \\
& *c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4* \\
& a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3 \\
& *d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2 \\
& *d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^ \\
& 4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3 \\
& *g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2* \\
& c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4 \\
& *g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^ \\
& 3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 \\
& + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*
\end{aligned}$$



$$e^3 h^3 - b^4 e^5 f^2 h + 16 a^3 c^3 e^5 f^3, z, k), k, 1, 3) - ((a b d h - 2 a^2 e h - b^2 e f + a b e g - 2 a c d g + 2 a c e f + b c d f) / (a b^2 e^2 - 4 a c^2 d^2 - 4 a^2 c e^2 + b^2 c d^2 - b^3 d e + 4 a b c d e) - (x (a b e h - b^2 d h - 2 c^2 d f + 2 a c d h - 2 a c e g + b c d g + b c e f)) / (a b^2 e^2 - 4 a c^2 d^2 - 4 a^2 c e^2 + b^2 c d^2 - b^3 d e + 4 a b c d e)) / (a + b x + c x^2)$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x\*\*2+g\*x+f)/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] Timed out

$$3.155 \quad \int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2} dx$$

Optimal. Leaf size=673

$$\frac{cx(2a^2e^2h - c(2a(d^2h - 2deg + e^2f) + bd(dg + 2ef)) - abe(2dh + eg) + b^2(d^2h + e^2f) + 2c^2d^2f) + b(a^2e^2h - (b^2 - 4ac)(a + bx + cx^2)(ae$$

**Rubi [A]** time = 2.56, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1646, 1628, 634, 618, 206, 628}

[[{"rule": "1646", "order": 1}, {"rule": "1628", "order": 2}, {"rule": "634", "order": 3}, {"rule": "618", "order": 4}, {"rule": "206", "order": 5}, {"rule": "628", "order": 6}]]

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)^2), x]

[Out]  $-\frac{((e^2f - d^2h) + (c^2d^2f + a^2e^2h - a^2c(3e^2f - 2d^2h) + c(2c^2d^2f + 2a^2e^2h - a^2b(e^2f + d^2h) + b^2(e^2f + d^2h) - c(bd(2e^2f + d^2g) + 2a(e^2f - 2d^2h)))x))}{(b^2 - 4ac)(c^2d^2 - b^2d^2e + a^2e^2)(a + b^2x + c^2x^2)} + \frac{((4c^4d^4f - b^3e^3(2b^2e^2f - b^2d^2g - a^2e^2h) - 2c^3d^2(b^2d^2(4e^2f + d^2g) - 2a(6e^2f - 2d^2e^2g + d^2h)) - 6c^2e(4a^2b^2d^2e^2f - b^2d^3g + 2a^2e(e^2f - 2d^2e^2g + 2d^2h)) - c(6a^2b^2e^3g - 4a^3e^3h - b^3d(4e^2f - 3d^2e^2g - 2d^2h) - 6a^2b^2e(2e^2f - d^2e^2g + 2d^2h)))}{(b^2 - 4ac)^{3/2}(c^2d^2 - b^2d^2e + a^2e^2)^3} - \frac{(e^2(2b^2e^2f - b^2d^2g - a^2e^2h) - c^2d^2(4e^2f - 3d^2e^2g + 2d^2h)) \text{Log}[d + e^2x]}{(c^2d^2 - b^2d^2e + a^2e^2)^3} + \frac{(e^2(2b^2e^2f - b^2d^2g - a^2e^2h) - c^2d^2(4e^2f - 3d^2e^2g + 2d^2h)) \text{Log}[a + b^2x + c^2x^2]}{2(c^2d^2 - b^2d^2e + a^2e^2)^3} \text{ArcTanh}\left[\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right]$

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.)^m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^p_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

### Rule 1646

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.)^m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^p_.), x\_Symbol] \rightarrow \text{With}\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]\}, \text{Simp}[(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^{p+1})/((p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p+1)*(b^2 - 4*a*c)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^{p+1}*\text{ExpandToSum}[(p+1)*(b^2 - 4*a*c)*Q]/(d + e*x)^m - ((2*p+3)*(2*c*f - b*g))/(d + e*x)^m, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{LtQ}[p, -1] \&\& \text{ILtQ}[m, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)^2} dx &= -\frac{b^3 e^2 f - b^2 e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2 d^2 f + a^2 e^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\
&= -\frac{e(e^2 f - deg + d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} - \frac{b^3 e^2 f - b^2 e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2 d^2 f + a^2 e^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\
&= -\frac{e(e^2 f - deg + d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} - \frac{b^3 e^2 f - b^2 e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2 d^2 f + a^2 e^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\
&= -\frac{e(e^2 f - deg + d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} - \frac{b^3 e^2 f - b^2 e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2 d^2 f + a^2 e^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} \\
&= -\frac{e(e^2 f - deg + d^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)} - \frac{b^3 e^2 f - b^2 e(2cdf + aeg) + 2ac(cd(2ef - dg) + ae(eg - 2dh)) + b(c^2 d^2 f + a^2 e^2 h)}{(cd^2 - bde + ae^2)^2 (d + ex)}
\end{aligned}$$

**Mathematica [A]** time = 2.21, size = 650, normalized size = 0.97

(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)^2), x

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)^2), x]

[Out]  $-\frac{e(e^2 f - d e g + d^2 h)}{(c d^2 + e(-b d) + a e)^2 (d + e x)} + (-b^3 e^2 f + b^2(a e^2 g - c(-2 d e f + e^2 f x + d^2 h x)) + b(-a^2 e^2 h + c^2 d(-d f) + 2 e f x + d g x) + a c(-d^2 h + e^2(3 f + g x) - 2 d e(g - h x))) + 2 c(-c^2 d^2 f x + a c(e^2 f x - 2 d e(f + g x) + d^2(g + h x)) - a^2 e(-2 d h + e(g + h x)))}{(b^2 - 4 a c)(c d^2 + e(-b d) + a e)^2 (a + x(b + c x))} - ((4 c^4 d^4 f + b^3 e^3(-2 b e f + b d g + a e g - 2 a d h) - 2 c^3 d^2(b d(4 e f + d g) - 2 a(6 e^2 f - 2 d e g + d^2 h)) - 6 c^2 e(4 a b d e^2 f - b^2 d^3 g + 2 a^2 e(e^2 f - 2 d e g + 2 d^2 h)) + c e(-6 a^2 b e^3 g + 4 a^3 e^3 h + b^3 d(4 e^2 f - 3 d e g - 2 d^2 h) + 6 a b^2 e(2 e^2 f - d e g + 2 d^2 h))) \operatorname{ArcTan}[(b + 2 c$

$x)/\text{Sqrt}[-b^2 + 4*a*c]]/((-b^2 + 4*a*c)^{(3/2)}*(-(c*d^2) + e*(b*d - a*e))^3 + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*\text{Log}[d + e*x])/((c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*\text{Log}[a + x*(b + c*x)]))/(2*(c*d^2 + e*(-(b*d) + a*e))^3)$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)^2), x]

[Out] IntegrateAlgebraic[(f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)^2), x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^2/(c\*x^2+b\*x+a)^2,x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 0.33, size = 1437, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^2/(c\*x^2+b\*x+a)^2,x, algorithm="giac")

[Out]  $-(4*c^4*d^4*f*e^2 - 2*b*c^3*d^4*g*e^2 + 4*a*c^3*d^4*h*e^2 - 8*b*c^3*d^3*f*e^3 + 6*b^2*c^2*d^3*g*e^3 - 8*a*c^3*d^3*g*e^3 - 2*b^3*c*d^3*h*e^3 + 24*a*c^3*d^2*f*e^4 - 3*b^3*c*d^2*g*e^4 + 12*a*b^2*c*d^2*h*e^4 - 24*a^2*c^2*d^2*h*e^4 + 4*b^3*c*d*f*e^5 - 24*a*b*c^2*d*f*e^5 + b^4*d*g*e^5 - 6*a*b^2*c*d*g*e^5 + 24*a^2*c^2*d*g*e^5 - 2*a*b^3*d*h*e^5 - 2*b^4*f*e^6 + 12*a*b^2*c*f*e^6 - 12*a^2*c^2*f*e^6 + a*b^3*g*e^6 - 6*a^2*b*c*g*e^6 + 4*a^3*c*h*e^6)*\arctan((2*c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^{-1}/\text{sqrt}(-b^2 + 4*a*c))*e^{-2}/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*\text{sqrt}(-b^2 + 4*a*c)) - 1/2*(2*c*d^3*h*e - 3*c*d^2*g*e^2 + 4*c*d*f*e^3 + b*d*g*e^3 - 2*a*d*h*e^3 - 2*b*f*e^4 + a$

```

*g*e^4)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e
/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (d^2*h*e^5/(x*e + d) - d*g*e^6/(x
*e + d) + f*e^7/(x*e + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6 + 2*a
*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - ((2*c^4*d^3*f*e - b*c^3*d^3*g*e + b^2
*c^2*d^3*h*e - 2*a*c^3*d^3*h*e - 3*b*c^3*d^2*f*e^2 + 6*a*c^3*d^2*g*e^2 - 3*
a*b*c^2*d^2*h*e^2 + 3*b^2*c^2*d*f*e^3 - 6*a*c^3*d*f*e^3 - 3*a*b*c^2*d*g*e^3
+ 6*a^2*c^2*d*h*e^3 - b^3*c*f*e^4 + 3*a*b*c^2*f*e^4 + a*b^2*c*g*e^4 - 2*a^
2*c^2*g*e^4 - a^2*b*c*h*e^4)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*f*e^2 - b
*c^3*d^4*g*e^2 + b^2*c^2*d^4*h*e^2 - 2*a*c^3*d^4*h*e^2 - 4*b*c^3*d^3*f*e^3
+ 8*a*c^3*d^3*g*e^3 - 4*a*b*c^2*d^3*h*e^3 + 6*b^2*c^2*d^2*f*e^4 - 12*a*c^3*
d^2*f*e^4 - 6*a*b*c^2*d^2*g*e^4 + 12*a^2*c^2*d^2*h*e^4 - 4*b^3*c*d*f*e^5 +
12*a*b*c^2*d*f*e^5 + 4*a*b^2*c*d*g*e^5 - 8*a^2*c^2*d*g*e^5 - 4*a^2*b*c*d*h*
e^5 + b^4*f*e^6 - 4*a*b^2*c*f*e^6 + 2*a^2*c^2*f*e^6 - a*b^3*g*e^6 + 3*a^2*b
*c*g*e^6 + a^2*b^2*h*e^6 - 2*a^3*c*h*e^6)*e^(-1)/((c*d^2 - b*d*e + a*e^2)*(
x*e + d)))/((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(x*e + d) +
c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2))

```

**maple [B]** time = 0.04, size = 4716, normalized size = 7.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2, x)$

[Out] 
$$\begin{aligned}
& -e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*f-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/ \\
& (4*a*c-b^2)*a*b*c^2*d^2*e^2*f-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*\ln(c*x^ \\
& 2+b*x+a)*a*b*d*e^3*g+12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c \\
& *x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*c*d^2*e^2*h-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b \\
& ^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*c*d*e^3*g-24/(a*e^2-b*d \\
& *e+c*d^2)^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*c^2*d \\
& *e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a^2*b*e^4*g+4/ \\
& (a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a^2*d*e^3*g+1/(a*e^2- \\
& b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b^2*e^4*f+4/(a*e^2-b*d*e+c*d \\
& ^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*a*d^3*e*g-1/(a*e^2-b*d*e+c*d^2)^3/(c* \\
& x^2+b*x+a)*c/(4*a*c-b^2)*x*b^3*d^3*e*h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a \\
& )*c/(4*a*c-b^2)*x*b^3*d^3*e*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4* \\
& a*c-b^2)*x*b^2*d^3*e*g+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2 \\
& )*x*b^2*d^2*e^2*f-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b \\
& *d^3*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*d^2*e^2* \\
& h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^3*e*h-3/(a*e^ \\
& 2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^2*e^2*g+1/(a*e^2-b*d*e \\
& +c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^2*e^2*f+4/(a*e^2-b*d*e+c*d^2)^3 \\
& /((c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^3*e*g-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b
\end{aligned}$$

$$\begin{aligned}
& *x+a) *c^2 / (4*a*c-b^2) *x*a*b*d^2 *e^2 *g-4 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) \\
& *c / (4*a*c-b^2) *x*a^2*b*d*e^3 *h+3 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) *c / (4*a \\
& *c-b^2) *x*a*b^2*d^2 *e^2 *h+1 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) *c / (4*a*c-b^ \\
& 2) *x*a*b^2*d*e^3 *g-2 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+ \\
& b) / (4*a*c-b^2)^{(1/2)}) *b^4 *e^4 *f-1 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *ln(c*x^ \\
& 2+b*x+a) *b^3 *e^4 *f+4 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+ \\
& b) / (4*a*c-b^2)^{(1/2)}) *c^4 *d^4 *f-2 *e^3 / (a*e^2-b*d*e+c*d^2)^3 *ln(e*x+d) *a*d*h \\
& +e^3 / (a*e^2-b*d*e+c*d^2)^3 *ln(e*x+d) *b*d*g+2 *e / (a*e^2-b*d*e+c*d^2)^3 *ln(e*x \\
& +d) *c*d^3 *h-3 *e^2 / (a*e^2-b*d*e+c*d^2)^3 *ln(e*x+d) *c*d^2 *g+4 *e^3 / (a*e^2-b*d* \\
& e+c*d^2)^3 *ln(e*x+d) *c*d*f+4 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *c*ln(c*x^2+b \\
& *x+a) *a^2 *d*e^3 *h+4 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *c*ln(c*x^2+b*x+a) *a*b \\
& *e^4 *f+6 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *c^2 *ln(c*x^2+b*x+a) *a*d^2 *e^2 *g+ \\
& 1 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a*b*c^2 *d^4 *h+4 / (a*e^2-b* \\
& d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a*c^3 *d^3 *e*f+3 / (a*e^2-b*d*e+c*d^2)^ \\
& 3 / (c*x^2+b*x+a) / (4*a*c-b^2) *b^3 *c*d^2 *e^2 *f-8 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c- \\
& b^2) *c^2 *ln(c*x^2+b*x+a) *a*d*e^3 *f+1 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *c*ln \\
& (c*x^2+b*x+a) *b^2 *d^3 *e*h-3 / 2 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *c*ln(c*x^2+ \\
& b*x+a) *b^2 *d^2 *e^2 *g+2 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *c*ln(c*x^2+b*x+a) * \\
& b^2 *d *e^3 *f-1 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2) *ln(c*x^2+b*x+a) *a*b^2 *d *e^3 \\
& *h-8 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/ \\
& 2)}) *b*c^3 *d^3 *e*f-6 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x \\
& +b) / (4*a*c-b^2)^{(1/2)}) *a^2 *b*c *e^4 *g-24 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{( \\
& 3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/2)}) *a^2 *c^2 *d^2 *e^2 *h+24 / (a*e^2-b*d*e+ \\
& c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/2)}) *a^2 *c^2 *d *e^ \\
& 3 *g-2 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{( \\
& 1/2)}) *a*b^3 *d *e^3 *h+12 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c \\
& *x+b) / (4*a*c-b^2)^{(1/2)}) *a*b^2 *c *e^4 *f-8 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{( \\
& 3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/2)}) *a*c^3 *d^3 *e *g+24 / (a*e^2-b*d*e+c*d \\
& ^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/2)}) *a*c^3 *d^2 *e^2 *f \\
& -2 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/ \\
& 2)}) *b^3 *c *d^3 *e *h-3 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/2)} *arctan((2*c*x+b \\
& ) / (4*a*c-b^2)^{(1/2)}) *b^3 *c *d^2 *e^2 *g+4 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3 \\
& /2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/2)}) *b^3 *c *d *e^3 *f-3 / (a*e^2-b*d*e+c*d^2) \\
& ^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *b^2 *c^2 *d^3 *e *f-4 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c \\
& -b^2) *c^2 *ln(c*x^2+b*x+a) *a*d^3 *e *h+6 / (a*e^2-b*d*e+c*d^2)^3 / (4*a*c-b^2)^{(3/ \\
& 2)} *arctan((2*c*x+b) / (4*a*c-b^2)^{(1/2)}) *b^2 *c^2 *d^3 *e *g-4 / (a*e^2-b*d*e+c*d^2 \\
& )^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a^2 *c^2 *d^3 *e *h+4 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^ \\
& 2+b*x+a) / (4*a*c-b^2) *a^2 *c^2 *d *e^3 *f+1 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / \\
& (4*a*c-b^2) *a*b^3 *d *e^3 *g-4 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) \\
& *a^3 *c *d *e^3 *h-1 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a^2 *b^2 *d * \\
& e^3 *h-3 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a^2 *b *c *e^4 *f+1 / (a* \\
& e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a*b^3 *e^4 *f-2 / (a*e^2-b*d*e+c*d \\
& ^2)^3 / (c*x^2+b*x+a) / (4*a*c-b^2) *a*c^3 *d^4 *g-1 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+ \\
& b*x+a) / (4*a*c-b^2) *b^4 *d *e^3 *f+1 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) / (4*a*c \\
& -b^2) *b*c^3 *d^4 *f+2 / (a*e^2-b*d*e+c*d^2)^3 / (c*x^2+b*x+a) *c^4 / (4*a*c-b^2) *x*d
\end{aligned}$$

$$\begin{aligned} & ^4*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*b*e^4*h+2/(a*e^2 \\ & -b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*c*e^4*g-1/(a*e^2-b*d*e+c*d^2) \\ & ^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*e^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c \\ & -b^2)*\ln(c*x^2+b*x+a)*a*b^2*e^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*\ln( \\ & c*x^2+b*x+a)*b^3*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*\ln(c*x^2+b*x \\ & +a)*a^2*e^4*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4 \\ & *a*c-b^2)^(1/2))*a*b^3*e^4*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*\arct \\ & \arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*c*e^4*h-12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c \\ & -b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*e^4*f-2/(a*e^2-b*d* \\ & e+c*d^2)^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^4* \\ & g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1 \\ & /2))*b^4*d*e^3*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b) \\ & /(4*a*c-b^2)^(1/2))*a*c^3*d^4*h-e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d^2*h*e^2/( \\ & a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d*g*e^4/(a*e^2-b*d*e+c*d^2)^3*\ln(e*x+d)*a*g-2* \\ & e^4/(a*e^2-b*d*e+c*d^2)^3*\ln(e*x+d)*b*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+ \\ & a)*c/(4*a*c-b^2)*x*a^3*e^4*h-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a \\ & *c-b^2)*x*a^2*e^4*f-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x \\ & *a*d^4*h+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*b^2*d^4*h- \\ & 1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b*d^4*g \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x^2+g\*x+f)/(e\*x+d)^2/(c\*x^2+b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive or negative?

**mupad** [B] time = 8.93, size = 26278, normalized size = 39.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g\*x + h\*x^2)/((d + e\*x)^2\*(a + b\*x + c\*x^2)^2),x)

[Out] 
$$\frac{(a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f - 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^2*c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*e^2*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f + 2*c^3*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2*a^2*c$$



$$\begin{aligned}
& e^3g + b^2cd^3h - b^3d^2e^2g + b^3d^2e^2h + 2a^2c^2d^2e^2f + 2a^2c^2d^2e^2g - b^2c^2d^2e^2f - b^2c^2d^2e^2g - 2a^2c^2d^2e^2h - 7a^2b^2c^2e^3f \\
& + 5a^2b^2c^2d^2e^2g - 5a^2b^2c^2d^2e^2h)/(4a^2c^3d^4 + 4a^3c^2e^4 - a^2b^2e^4 - b^2c^2d^4 - b^4d^2e^2 + 8a^2c^2d^2e^2 + 2a^2b^3d^2e^3 + 2b^3c^2d^3e - 8a^2b^2c^2d^3e - 8a^2b^2c^2d^3e + 2a^2b^2c^2d^2e^2) - (x^2(6a^2c^2e^3f - 2b^2c^2e^3f - 2a^2c^2e^3h - 2c^3d^2e^2f - 8a^2c^2d^2e^2g + 2b^2c^2d^2e^2f + 6a^2c^2d^2e^2h + b^2c^2d^2e^2g + b^2c^2d^2e^2g - 2b^2c^2d^2e^2h + a^2b^2c^2e^3g + 2a^2b^2c^2d^2e^2h))/(4a^2c^3d^4 + 4a^3c^2e^4 - a^2b^2e^4 - b^2c^2d^4 - b^4d^2e^2 + 8a^2c^2d^2e^2 + 2a^2b^3d^2e^3 + 2b^3c^2d^3e - 8a^2b^2c^2d^3e - 8a^2b^2c^2d^3e + 2a^2b^2c^2d^2e^2) \\
& ))/(a(d + x(ae + bd)) + x^2(b(e + cd) + ce*x^3) + \text{symsum}(\log((x*(36a^2c^5e^7f^2 + 4b^4c^3e^7f^2 + 4a^4c^3e^7h^2 + 4c^7d^4e^3f^2 + a^2b^2c^3e^7g^2 + 64a^2c^5d^2e^5g^2 + 12b^2c^5d^2e^5f^2 + 36a^2c^5d^4e^3h^2 - 24a^3c^4d^2e^5h^2 + b^2c^5d^4e^3g^2 + 2b^3c^4d^3e^4g^2 + b^4c^3d^2e^5g^2 + 4b^4c^3d^4e^3h^2 - 24a^3c^4e^7f*h - 24a^2b^2c^4e^7f^2 - 24a^2c^6d^2e^5f^2 - 8b^2c^6d^3e^4f^2 - 8b^3c^4d^2e^6f^2 - 16a^2b^2c^5d^3e^4g^2 + 2a^2b^3c^3d^2e^6g^2 - 16a^2b^2c^4d^2e^6g^2 - 8a^3b^2c^3d^2e^6h^2 + 8a^2b^2c^3e^7f*h + 80a^2c^5d^2e^5f*h - 96a^2c^5d^3e^4g*h + 8b^2c^5d^4e^3f*h - 8b^3c^4d^3e^4f*h + 8b^4c^3d^2e^5f*h - 4b^3c^4d^4e^3g*h - 4b^4c^3d^3e^4g*h - 14a^2b^2c^4d^2e^5g^2 - 24a^2b^2c^4d^4e^3h^2 - 8a^2b^3c^3d^3e^4h^2 + 24a^2b^2c^4d^3e^4h^2 + 24a^2b^2c^5d^2e^6f^2 - 4a^2b^3c^3e^7f*g + 12a^2b^2c^4e^7f*g + 32a^2c^6d^3e^4f*g - 96a^2c^5d^2e^6f*g - 4a^3b^2c^3e^7g*h - 24a^2c^6d^4e^3f*h - 4b^2c^6d^4e^3f*g - 4b^4c^3d^2e^6f*g + 32a^3c^4d^2e^6g*h + 12a^2b^2c^3d^2e^5h^2 - 24a^2b^2c^5d^2e^5f*g + 48a^2b^2c^4d^2e^6f*g + 16a^2b^2c^5d^3e^4f*h - 8a^2b^3c^3d^2e^6f*h + 16a^2b^2c^4d^2e^6f*h + 12a^2b^2c^5d^4e^3g*h - 40a^2b^2c^4d^2e^5f*h + 48a^2b^2c^4d^3e^4g*h - 24a^2b^2c^4d^2e^5g*h)))/(16a^2c^6d^8 + a^4b^4e^8 + 16a^6c^2e^8 + b^4c^4d^8 + b^8d^4e^4 - 8a^2b^2c^5d^8 - 8a^5b^2c^2e^8 - 4a^2b^7d^3e^5 - 4a^3b^5d^2e^7 - 4b^5c^3d^7e - 4b^7c^2d^5e^3 + 6a^2b^6d^2e^6 + 64a^3c^5d^6e^2 + 96a^4c^4d^4e^4 + 64a^5c^3d^2e^6 + 6b^6c^2d^6e^2 + 64a^2b^2c^4d^6e^2 + 32a^2b^3c^3d^5e^3 - 74a^2b^4c^2d^4e^4 + 144a^3b^2c^3d^4e^4 + 32a^3b^3c^2d^3e^5 + 64a^4b^2c^2d^2e^6 + 32a^2b^3c^4d^7e + 4a^2b^6c^4d^4e^4 - 64a^2b^2c^5d^7e + 32a^4b^3c^2d^5e^7 - 64a^5b^2c^2d^2e^7 - 44a^2b^4c^3d^6e^2 + 20a^2b^5c^2d^5e^3 + 20a^2b^5c^2d^3e^5 - 192a^3b^2c^4d^5e^3 - 44a^3b^4c^2d^2e^6 - 192a^4b^2c^3d^3e^5) - \text{root}(3840a^6b^2c^5d^5e^7z^3 + 3840a^5b^2c^6d^7e^5z^3 + 1920a^7b^2c^4d^3e^9z^3 + 1920a^4b^2c^7d^9e^3z^3 - 288a^7b^3c^2d^2e^11z^3 - 288a^2b^3c^7d^11e^z^3 + 210a^4b^7c^2d^3e^9z^3 + 210a^2b^7c^4d^9e^3z^3 - 174a^5b^6c^2d^2e^10z^3 - 174a^2b^6c^5d^10e^2z^3 - 120a^3b^8c^3d^8e^4z^3 + 12a^2b^9c^3d^5e^7z^3 + 12a^2b^9c^2d^7e^5z^3 + 384a^8b^2c^3d^2e^11z^3 + 384a^3b^2c^8d^11e^z^3 + 72a^6b^5c^2d^2e^11z^3 + 72a^2b^5c^6d^11e^z^3 + 18a^2b^10c^2d^6e^6z^3 - 4800a^5b^2c^5d^6e^6z^3 - 3120a^6b^2c^
\end{aligned}$$

$$\begin{aligned}
& 4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 \\
& - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7 \\
& *z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3 \\
& *b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10 \\
& *e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372* \\
& a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6 \\
& *e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160* \\
& a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4 \\
& *z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 \\
& - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 2 \\
& 0*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - \\
& 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 \\
& - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9 \\
& *z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2 \\
& *e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12 \\
& *z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4* \\
& c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a \\
& ^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z \\
& - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4 \\
& *d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g* \\
& h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a \\
& ^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2* \\
& d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g \\
& *z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^ \\
& 6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a \\
& *b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6* \\
& g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b \\
& *c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f* \\
& h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^ \\
& 5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z \\
& - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^ \\
& 3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z \\
& - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z \\
& + 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + \\
& 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4 \\
& *f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4 \\
& *d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b \\
& ^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g* \\
& z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e \\
& ^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4* \\
& d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b \\
& ^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z \\
& + 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c \\
& ^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*
\end{aligned}$$

$$\begin{aligned}
& h^2 * z - 20 * a^3 * b^3 * c^2 * d^3 * e^5 * h^2 * z - 20 * a^2 * b^3 * c^3 * d^5 * e^3 * h^2 * z - 426 * a \\
& ^2 * b^2 * c^4 * d^4 * e^4 * g^2 * z + 336 * a^3 * b^2 * c^3 * d^2 * e^6 * g^2 * z + 274 * a^2 * b^3 * c^3 * \\
& d^3 * e^5 * g^2 * z - 120 * a^2 * b^4 * c^2 * d^2 * e^6 * g^2 * z - 864 * a^2 * b^2 * c^4 * d^2 * e^6 * f^2 \\
& * z - 2 * b^7 * c * d^4 * e^4 * g * h * z - 2 * b^4 * c^4 * d^7 * e * g * h * z - 240 * a^5 * c^3 * d * e^7 * g * h * \\
& z + 16 * a^2 * c^6 * d^7 * e * g * h * z + 4 * b^7 * c * d^3 * e^5 * f * h * z + 4 * b^3 * c^5 * d^7 * e * f * h * z \\
& - 20 * b^7 * c * d^2 * e^6 * f * g * z - 20 * b^2 * c^6 * d^7 * e * f * g * z + 4 * a^2 * b^6 * d * e^7 * g * h * z + \\
& 4 * a * b^7 * d^2 * e^6 * g * h * z + 528 * a^4 * c^4 * d * e^7 * f * g * z + 12 * a^5 * b * c^2 * e^8 * g * h * z - \\
& 2 * a^4 * b^3 * c * e^8 * g * h * z + 4 * a^3 * b^4 * c * e^8 * f * h * z - 228 * a^4 * b * c^3 * e^8 * f * g * z - \\
& 48 * a^2 * b^5 * c * e^8 * f * g * z - 8 * a * b * c^6 * d^7 * e * g^2 * z + 36 * a^3 * b^4 * c * d^2 * e^6 * h^2 * z \\
& + 36 * a * b^4 * c^3 * d^6 * e^2 * h^2 * z + 12 * a^2 * b^5 * c * d^3 * e^5 * h^2 * z + 12 * a * b^5 * c^2 * d \\
& ^5 * e^3 * h^2 * z - 312 * a^3 * b * c^4 * d^3 * e^5 * g^2 * z + 104 * a * b^4 * c^3 * d^4 * e^4 * g^2 * z - \\
& 102 * a^3 * b^3 * c^2 * d * e^7 * g^2 * z - 66 * a * b^5 * c^2 * d^3 * e^5 * g^2 * z + 24 * a^2 * b * c^5 * d^5 \\
& * e^3 * g^2 * z + 24 * a * b^2 * c^5 * d^6 * e^2 * g^2 * z - 18 * a * b^3 * c^4 * d^5 * e^3 * g^2 * z + 744 * \\
& a^2 * b^3 * c^3 * d * e^7 * f^2 * z + 240 * a^2 * b * c^5 * d^3 * e^5 * f^2 * z + 216 * a * b^4 * c^3 * d^2 * e \\
& ^6 * f^2 * z - 120 * a * b^2 * c^5 * d^4 * e^4 * f^2 * z + 24 * a^5 * c^3 * e^8 * f * h * z + 16 * b^7 * c * d * \\
& e^7 * f^2 * z + 16 * b * c^7 * d^7 * e * f^2 * z - 2 * a * b^7 * d * e^7 * g^2 * z + 48 * a * b^6 * c * e^8 * f^2 \\
& * z - 4 * b^6 * c^2 * d^6 * e^2 * h^2 * z - 536 * a^4 * c^4 * d^4 * e^4 * h^2 * z + 240 * a^5 * c^3 * d^2 * \\
& e^6 * h^2 * z + 240 * a^3 * c^5 * d^6 * e^2 * h^2 * z - 12 * b^6 * c^2 * d^4 * e^4 * g^2 * z - 12 * b^4 * c \\
& ^4 * d^6 * e^2 * g^2 * z + 10 * b^5 * c^3 * d^5 * e^3 * g^2 * z + 528 * a^3 * c^5 * d^4 * e^4 * g^2 * z - 4 \\
& 32 * a^4 * c^4 * d^2 * e^6 * g^2 * z + 20 * b^4 * c^4 * d^4 * e^4 * f^2 * z - 16 * b^6 * c^2 * d^2 * e^6 * f^ \\
& 2 * z - 16 * b^2 * c^6 * d^6 * e^2 * f^2 * z - 16 * a^2 * c^6 * d^6 * e^2 * g^2 * z - 8 * b^5 * c^3 * d^3 * e \\
& ^5 * f^2 * z - 8 * b^3 * c^5 * d^5 * e^3 * f^2 * z - 4 * a^2 * b^6 * d^2 * e^6 * h^2 * z + 912 * a^3 * c^5 * \\
& d^2 * e^6 * f^2 * z - 120 * a^2 * c^6 * d^4 * e^4 * f^2 * z - 45 * a^4 * b^2 * c^2 * e^8 * g^2 * z + 264 * \\
& a^3 * b^2 * c^3 * e^8 * f^2 * z - 192 * a^2 * b^4 * c^2 * e^8 * f^2 * z + 4 * b^8 * d * e^7 * f * g * z - 8 * a \\
& * c^7 * d^8 * f * h * z + 4 * b * c^7 * d^8 * f * g * z + 4 * a * b^7 * e^8 * f * g * z + 6 * b^7 * c * d^3 * e^5 * g^ \\
& 2 * z + 6 * b^3 * c^5 * d^7 * e * g^2 * z - 48 * a * c^7 * d^6 * e^2 * f^2 * z + 12 * a^3 * b^4 * c * e^8 * g^2 \\
& * z - b^8 * d^2 * e^6 * g^2 * z - 4 * a^6 * c^2 * e^8 * h^2 * z + 48 * a^5 * c^3 * e^8 * g^2 * z - 4 * a^2 \\
& * c^6 * d^8 * h^2 * z - b^2 * c^6 * d^8 * g^2 * z - 36 * a^4 * c^4 * e^8 * f^2 * z - a^2 * b^6 * e^8 * g^2 \\
& * z - 4 * c^8 * d^8 * f^2 * z - 4 * b^8 * e^8 * f^2 * z - 80 * a * b * c^4 * d^3 * e^3 * f * g * h + 24 * a^2 * \\
& b * c^3 * d * e^5 * f * g * h + 16 * a * b^3 * c^2 * d * e^5 * f * g * h - 72 * a * b^2 * c^3 * d^2 * e^4 * f * g * h - \\
& 48 * a^2 * b * c^3 * d^3 * e^3 * g * h^2 + 16 * a * b^3 * c^2 * d^3 * e^3 * g * h^2 - 12 * a * b^2 * c^3 * d^3 \\
& * e^3 * g^2 * h - 6 * a^2 * b^2 * c^2 * d * e^5 * g^2 * h - 72 * a^2 * b^2 * c^2 * d * e^5 * f * h^2 + 48 * a * \\
& b^2 * c^3 * d^3 * e^3 * f * h^2 + 24 * a^2 * b * c^3 * d^2 * e^4 * f * h^2 - 8 * a * b^3 * c^2 * d^2 * e^4 * f * \\
& h^2 - 8 * b^5 * c * d * e^5 * f * g * h - 8 * b * c^5 * d^5 * e * f * g * h - 8 * a * b^4 * c * e^6 * f * g * h + 24 * \\
& b^3 * c^3 * d^3 * e^3 * f * g * h + 16 * b^4 * c^2 * d^2 * e^4 * f * g * h + 16 * b^2 * c^4 * d^4 * e^2 * f * g * h \\
& + 48 * a^2 * c^4 * d^2 * e^4 * f * g * h + 48 * a^2 * b^2 * c^2 * e^6 * f * g * h + 40 * a^3 * b * c^2 * d * e^5 \\
& * g * h^2 + 28 * a * b * c^4 * d^4 * e^2 * g^2 * h - 8 * a^2 * b^3 * c * d * e^5 * g * h^2 - 8 * a * b^4 * c * d^2 \\
& * e^4 * g * h^2 + 96 * a * b^2 * c^3 * d * e^5 * f^2 * h + 24 * a * b * c^4 * d^2 * e^4 * f^2 * h + 16 * a * b * c \\
& ^4 * d^4 * e^2 * f * h^2 + 96 * a * b * c^4 * d^2 * e^4 * f * g^2 - 48 * a * b^2 * c^3 * d * e^5 * f * g^2 + 12 \\
& * a^2 * b^2 * c^2 * d^2 * e^4 * g * h^2 - 56 * a * c^5 * d^4 * e^2 * f * g * h - 8 * a * b * c^4 * d^5 * e * g * h^2 \\
& + 4 * a * b^4 * c * d * e^5 * g^2 * h + 16 * a * b^4 * c * d * e^5 * f * h^2 - 48 * a * b * c^4 * d * e^5 * f^2 * g \\
& - 24 * a^3 * c^3 * e^6 * f * g * h + 16 * a * c^5 * d^5 * e * f * h^2 - 6 * b^4 * c^2 * d^3 * e^3 * g^2 * h - 6 \\
& * b^3 * c^3 * d^4 * e^2 * g^2 * h + 4 * b^4 * c^2 * d^4 * e^2 * g * h^2 + 80 * a^2 * c^4 * d^3 * e^3 * g^2 * h \\
& - 44 * a^2 * c^4 * d^4 * e^2 * g * h^2 + 24 * a^3 * c^3 * d^2 * e^4 * g * h^2 - 16 * b^3 * c^3 * d^2 * e^4 \\
& * f^2 * h - 16 * b^2 * c^4 * d^3 * e^3 * f^2 * h - 8 * b^4 * c^2 * d^3 * e^3 * f * h^2 - 8 * b^3 * c^3 * d^4
\end{aligned}$$

$$\begin{aligned}
& *e^{2*f*h^2} + 60*b^2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24 \\
& *a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 \\
& + 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h \\
& - 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - \\
& 168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + \\
& 12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g \\
& + 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h \\
& + 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - \\
& 8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + \\
& 12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - \\
& 6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4 \\
& *c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5 \\
& *e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + \\
& 8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d \\
& ^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2 \\
& *e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 \\
& + 16*a^3*c^3*e^6*g^3, z, k) * ((8*a^6*c^3*e^9*h - 24*a^5*c^4*e^9*f - 8*a*c^8* \\
& d^8*e*f + 2*a^2*b^6*c*e^9*f - a^3*b^5*c*e^9*g - 20*a^5*b*c^3*e^9*g + 16*a^5 \\
& *c^4*d*e^8*g + 2*b^2*c^7*d^8*e*f + 2*b^8*c*d^2*e^7*f - 8*a^2*c^7*d^8*e*h - \\
& b^3*c^6*d^8*e*g - b^8*c*d^3*e^6*g - 18*a^3*b^4*c^2*e^9*f + 46*a^4*b^2*c^3*e \\
& ^9*f + 9*a^4*b^3*c^2*e^9*g - 48*a^2*c^7*d^6*e^3*f - 96*a^3*c^6*d^4*e^5*f - \\
& 80*a^4*c^5*d^2*e^7*f - 2*a^5*b^2*c^2*e^9*h + 16*a^2*c^7*d^7*e^2*g + 48*a^3* \\
& c^6*d^5*e^4*g + 48*a^4*c^5*d^3*e^6*g - 6*b^3*c^6*d^7*e^2*f + 4*b^4*c^5*d^6* \\
& e^3*f + 4*b^6*c^3*d^4*e^5*f - 6*b^7*c^2*d^3*e^6*f - 16*a^3*c^6*d^6*e^3*h + \\
& 16*a^5*c^4*d^2*e^7*h + 4*b^4*c^5*d^7*e^2*g - 3*b^5*c^4*d^6*e^3*g - 3*b^6*c^ \\
& 3*d^5*e^4*g + 4*b^7*c^2*d^4*e^5*g - 2*b^5*c^4*d^7*e^2*h + 4*b^6*c^3*d^6*e^3 \\
& *h - 2*b^7*c^2*d^5*e^4*h - 4*a*b^2*c^6*d^6*e^3*f - 14*a*b^3*c^5*d^5*e^4*f - \\
& 38*a*b^4*c^4*d^4*e^5*f + 54*a*b^5*c^3*d^3*e^6*f - 10*a*b^6*c^2*d^2*e^7*f + \\
& 56*a^2*b*c^6*d^5*e^4*f + 34*a^2*b^5*c^2*d*e^8*f + 40*a^3*b*c^5*d^3*e^6*f - \\
& 74*a^3*b^3*c^3*d*e^8*f - 20*a*b^2*c^6*d^7*e^2*g + 10*a*b^3*c^5*d^6*e^3*g + \\
& 34*a*b^4*c^4*d^5*e^4*g - 33*a*b^5*c^3*d^4*e^5*g + 4*a*b^6*c^2*d^3*e^6*g + \\
& 8*a^2*b*c^6*d^6*e^3*g - 16*a^3*b*c^5*d^4*e^5*g - 10*a^3*b^4*c^2*d*e^8*g - 4 \\
& 0*a^4*b*c^4*d^2*e^7*g + 20*a^4*b^2*c^3*d*e^8*g + 10*a*b^3*c^5*d^7*e^2*h - 2 \\
& 6*a*b^4*c^4*d^6*e^3*h + 12*a*b^5*c^3*d^5*e^4*h - 8*a^2*b*c^6*d^7*e^2*h - 4* \\
& a^2*b^6*c*d^2*e^7*h - 8*a^3*b*c^5*d^5*e^4*h + 8*a^4*b*c^4*d^3*e^6*h - 10*a^ \\
& 4*b^3*c^2*d*e^8*h - 4*a*b^7*c*d*e^8*f + 4*a*b*c^7*d^8*e*g + 112*a^2*b^2*c^5 \\
& *d^4*e^5*f - 130*a^2*b^3*c^4*d^3*e^6*f - 28*a^2*b^4*c^3*d^2*e^7*f + 164*a^3 \\
& *b^2*c^4*d^2*e^7*f - 100*a^2*b^2*c^5*d^5*e^4*g + 72*a^2*b^3*c^4*d^4*e^5*g + \\
& 12*a^2*b^4*c^3*d^3*e^6*g - 7*a^2*b^5*c^2*d^2*e^7*g - 60*a^3*b^2*c^4*d^3*e^ \\
& 6*g + 22*a^3*b^3*c^3*d^2*e^7*g + 44*a^2*b^2*c^5*d^6*e^3*h - 14*a^2*b^3*c^4* \\
& d^5*e^4*h - 12*a^2*b^5*c^2*d^3*e^6*h + 14*a^3*b^3*c^3*d^3*e^6*h + 26*a^3*b^ \\
& 4*c^2*d^2*e^7*h - 44*a^4*b^2*c^3*d^2*e^7*h + 24*a*b*c^7*d^7*e^2*f + 8*a^4*b \\
& *c^4*d*e^8*f + a*b^7*c*d^2*e^7*g + a^2*b^6*c*d*e^8*g + 2*a*b^2*c^6*d^8*e*h \\
& + 2*a*b^7*c*d^3*e^6*h + 2*a^3*b^5*c*d*e^8*h + 8*a^5*b*c^3*d*e^8*h)/(16*a^2*
\end{aligned}$$

$$\begin{aligned}
& c^6d^8 + a^4b^4e^8 + 16a^6c^2e^8 + b^4c^4d^8 + b^8d^4e^4 - 8a^*b^2c^5d^8 - 8a^5b^2c^*e^8 - 4a^*b^7d^3e^5 - 4a^3b^5d^*e^7 - 4b^5c^3d^7e - 4b^7c^*d^5e^3 + 6a^2b^6d^2e^6 + 64a^3c^5d^6e^2 + 96a^4c^4d^4e^4 + 64a^5c^3d^2e^6 + 6b^6c^2d^6e^2 + 64a^2b^2c^4d^6e^2 + 32a^2b^3c^3d^5e^3 - 74a^2b^4c^2d^4e^4 + 144a^3b^2c^3d^4e^4 + 32a^3b^3c^2d^3e^5 + 64a^4b^2c^2d^2e^6 + 32a^*b^3c^4d^7e + 4a^*b^6c^d^4e^4 - 64a^2b^*c^5d^7e + 32a^4b^3c^*d^e^7 - 64a^5b^*c^2d^e^7 - 44a^*b^4c^3d^6e^2 + 20a^*b^5c^2d^5e^3 + 20a^2b^5c^*d^3e^5 - 192a^3b^*c^4d^5e^3 - 44a^3b^4c^*d^2e^6 - 192a^4b^*c^3d^3e^5) + \\
& \text{root}(3840a^6b^*c^5d^5e^7z^3 + 3840a^5b^*c^6d^7e^5z^3 + 1920a^7b^*c^4d^3e^9z^3 + 1920a^4b^*c^7d^9e^3z^3 - 288a^7b^3c^2d^*e^11z^3 - 288a^2b^3c^7d^11e^*z^3 + 210a^4b^7c^*d^3e^9z^3 + 210a^*b^7c^4d^9e^3z^3 - 174a^5b^6c^*d^2e^10z^3 - 174a^*b^6c^5d^10e^2z^3 - 120a^3b^8c^*d^4e^8z^3 - 120a^*b^8c^3d^8e^4z^3 + 12a^2b^9c^*d^5e^7z^3 + 12a^*b^9c^2d^7e^5z^3 + 384a^8b^*c^3d^e^11z^3 + 384a^3b^*c^8d^11e^*z^3 + 72a^6b^5c^*d^e^11z^3 + 72a^*b^5c^6d^11e^*z^3 + 18a^*b^10c^*d^6e^6z^3 - 4800a^5b^2c^5d^6e^6z^3 - 3120a^6b^2c^4d^4e^8z^3 - 3120a^4b^2c^6d^8e^4z^3 + 2160a^4b^4c^4d^6e^6z^3 - 1776a^4b^5c^3d^5e^7z^3 - 1776a^3b^5c^4d^7e^5z^3 + 1740a^5b^4c^3d^4e^8z^3 + 1740a^3b^4c^5d^8e^4z^3 + 960a^5b^3c^4d^5e^7z^3 + 960a^4b^3c^5d^7e^5z^3 - 672a^7b^2c^3d^2e^10z^3 - 672a^3b^2c^7d^10e^2z^3 + 648a^6b^4c^2d^2e^10z^3 + 648a^2b^4c^6d^10e^2z^3 - 600a^5b^5c^2d^3e^9z^3 - 600a^2b^5c^5d^9e^3z^3 + 372a^3b^7c^2d^5e^7z^3 + 372a^2b^7c^3d^7e^5z^3 + 316a^3b^6c^3d^6e^6z^3 - 222a^2b^8c^2d^6e^6z^3 - 160a^6b^3c^3d^3e^9z^3 - 160a^3b^3c^6d^9e^3z^3 + 15a^4b^6c^2d^4e^8z^3 + 15a^2b^6c^4d^8e^4z^3 - 6b^11c^*d^7e^5z^3 - 6b^7c^5d^11e^*z^3 - 6a^5b^7d^*e^11z^3 - 6a^*b^11d^5e^7z^3 - 12a^7b^4c^*e^12z^3 - 12a^*b^4c^7d^12z^3 - 20b^9c^3d^9e^3z^3 + 15b^10c^2d^8e^4z^3 + 15b^8c^4d^10e^2z^3 - 1280a^6c^6d^6e^6z^3 - 960a^7c^5d^4e^8z^3 - 960a^5c^7d^8e^4z^3 - 384a^8c^4d^2e^10z^3 - 384a^4c^8d^10e^2z^3 - 20a^3b^9d^3e^9z^3 + 15a^4b^8d^2e^10z^3 + 15a^2b^10d^4e^8z^3 + 48a^8b^2c^2e^12z^3 + 48a^2b^2c^8d^12z^3 - 64a^9c^3e^12z^3 - 64a^3c^9d^12z^3 + b^12d^6e^6z^3 + b^6c^6d^12z^3 + a^6b^6e^12z^3 - 44a^3b^4c^*d^e^7g^*h^*z - 20a^*b^6c^*d^3e^5g^*h^*z - 12a^*b^2c^5d^7e^*g^*h^*z + 432a^4b^*c^3d^*e^7f^*h^*z + 84a^2b^5c^*d^e^7f^*h^*z + 28a^*b^6c^*d^2e^6f^*h^*z - 8a^*b^*c^6d^6e^2f^*g^*z - 804a^3b^2c^3d^3e^5g^*h^*z + 564a^2b^2c^4d^5e^3g^*h^*z + 222a^3b^3c^2d^2e^6g^*h^*z + 186a^2b^4c^2d^3e^5g^*h^*z - 166a^2b^3c^3d^4e^4g^*h^*z + 792a^3b^2c^3d^2e^6f^*h^*z - 744a^2b^2c^4d^4e^4f^*h^*z + 492a^2b^3c^3d^3e^5f^*h^*z - 264a^2b^4c^2d^2e^6f^*h^*z + 996a^2b^2c^4d^3e^5f^*g^*z - 870a^2b^3c^3d^2e^6f^*g^*z + 16a^*b^*c^6d^7e^*f^*h^*z - 56a^*b^6c^*d^e^7f^*g^*z - 264a^4b^*c^3d^2e^6g^*h^*z + 208a^3b^*c^4d^4e^4g^*h^*z + 156a^4b^2c^2d^*e^7g^*h^*z - 148a^*b^4c^3d^5e^3g^*h^*z + 54a^*b^5c^2d^4e^4g^*h^*z - 48a^2b^5c^*d^2e^6g^*h^*z - 24a^2b^*c^5d^6e^2g^*h^*z + 10a^*b^3c^4d^6e^2g^*h^*z - 656a^3b^*c^4d^3e^5f^*h^*z
\end{aligned}$$

$$\begin{aligned}
& - 308a^3b^3c^2d^2e^7f^2h^2z + 116a^4b^4c^3d^4e^4f^2h^2z - 84a^5b^5c^4d^3e^5f^2h^2z + 68a^6b^6c^5d^4e^3f^2h^2z - 48a^7b^7c^6d^5e^2f^2h^2z - \\
& 24a^8b^8c^7d^6e^2f^2h^2z + 1320a^3b^3c^4d^2e^6f^2g^2z - 732a^3b^3c^4d^2e^6f^2g^2z + 306a^2b^4c^2d^2e^7f^2g^2z - 304a^2b^4c^2d^2e^7f^2g^2z + \\
& 222a^2b^5c^2d^2e^6f^2g^2z + 110a^2b^5c^2d^2e^6f^2g^2z - 84a^2b^5c^2d^2e^6f^2g^2z + 16a^2b^6c^2d^2e^6f^2g^2z - 8a^2b^6c^2d^2e^6f^2g^2z + 4a^2b^6c^2d^2e^6f^2g^2z \\
& h^2z + 6b^6c^2d^2e^6f^2g^2h^2z + 6b^5c^3d^2e^6f^2g^2h^2z + 1072a^4c^4d^3e^5g^2h^2z - 720a^3c^5d^5e^3g^2h^2z - 8b^6c^2d^4e^4f^2h^2z - 8b^4c^4d^6e^2f^2h^2z \\
& + 1072a^3c^5d^4e^4f^2h^2z - 960a^4c^4d^2e^6f^2h^2z + 30b^6c^2d^3e^5f^2g^2z + 30b^3c^5d^6e^2f^2g^2z - 10b^5c^3d^4e^4f^2g^2z - 10b^4c^4d^5e^3f^2g^2z - 1488a^3c^5d^3e^5f^2g^2z \\
& + 48a^2c^6d^5e^3f^2g^2z - 24a^4b^2c^2e^8f^2h^2z + 186a^3b^3c^2e^8f^2g^2z + 4a^4b^3c^2d^2e^7h^2z + 4a^4b^3c^2d^2e^7h^2z + 4a^4b^3c^2d^2e^7h^2z + 168a^4b^3c^2d^2e^7h^2z \\
& + 24a^2b^5c^2d^2e^7g^2z + 18a^2b^6c^2d^2e^6g^2z - 912a^3b^3c^4d^2e^7f^2z - 192a^2b^5c^2d^2e^7f^2z + 144a^2b^5c^2d^2e^7f^2z + 432a^3b^2c^3d^4e^4h^2z - 168a^4b^2c^2d^2e^6h^2z - \\
& 168a^2b^2c^4d^6e^2h^2z - 108a^2b^4c^2d^4e^4h^2z - 20a^3b^3c^2d^3e^5h^2z - 20a^2b^3c^3d^5e^3h^2z - 426a^2b^2c^4d^4e^4g^2z + 336a^3b^2c^3d^2e^6g^2z + 274a^2b^3c^3d^3e^5g^2z - 12 \\
& 0a^2b^4c^2d^2e^6g^2z - 864a^2b^2c^4d^2e^6f^2z - 2b^7c^2d^4e^4g^2h^2z - 2b^4c^4d^7e^2g^2h^2z - 240a^5c^3d^2e^7g^2h^2z + 16a^2c^6d^7e^2g^2h^2z + 4b^7c^2d^3e^5f^2h^2z + 4b^3c^5d^7e^2f^2h^2z - 20b^7c^2d^2e^6f^2g^2z - 20b^2c^6d^7e^2f^2g^2z + 4a^2b^6d^2e^7g^2h^2z + 4a^2b^7d^2e^6g^2h^2z + 528a^4c^4d^2e^7f^2g^2z + 12a^5b^2c^2e^8g^2h^2z - 2a^4b^3c^2e^8g^2h^2z + 4a^3b^4c^2e^8f^2h^2z - 228a^4b^3c^3e^8f^2g^2z - 48a^2b^5c^2e^8f^2g^2z - 8a^2b^6c^2d^7e^2g^2z + 36a^3b^4c^2d^2e^6h^2z + 36a^2b^4c^3d^6e^2h^2z + 12a^2b^5c^2d^3e^5h^2z + 12a^2b^5c^2d^5e^3h^2z - 312a^3b^3c^4d^3e^5g^2z + 104a^2b^4c^3d^4e^4g^2z - 102a^3b^3c^2d^2e^7g^2z - 66a^2b^5c^2d^3e^5g^2z + 24a^2b^3c^5d^5e^3g^2z + 24a^2b^2c^5d^6e^2g^2z - 18a^2b^3c^4d^5e^3g^2z + 744a^2b^3c^3d^2e^7f^2z + 240a^2b^3c^5d^3e^5f^2z + 216a^2b^4c^3d^2e^6f^2z - 120a^2b^2c^5d^4e^4f^2z + 24a^5c^3e^8f^2h^2z + 16b^7c^2d^2e^7f^2z + 16b^7c^2d^2e^7f^2z - 2a^2b^7d^2e^7g^2z + 48a^2b^6c^2e^8f^2z - 4b^6c^2d^6e^2h^2z - 536a^4c^4d^4e^4h^2z + 240a^5c^3d^2e^6h^2z + 240a^3c^5d^6e^2h^2z - 12b^6c^2d^4e^4g^2z - 12b^4c^4d^6e^2g^2z + 10b^5c^3d^5e^3g^2z + 528a^3c^5d^4e^4g^2z - 432a^4c^4d^2e^6g^2z + 20b^4c^4d^4e^4f^2z - 16b^6c^2d^2e^6f^2z - 16b^2c^6d^6e^2f^2z - 16a^2c^6d^6e^2g^2z - 8b^5c^3d^3e^5f^2z - 8b^3c^5d^5e^3f^2z - 4a^2b^6d^2e^6h^2z + 912a^3c^5d^2e^6f^2z - 120a^2c^6d^4e^4f^2z - 45a^4b^2c^2e^8g^2z + 264a^3b^2c^3e^8f^2z - 192a^2b^4c^2e^8f^2z + 4b^8d^2e^7f^2g^2z - 8a^2c^7d^8f^2h^2z + 4b^2c^7d^8f^2g^2z + 4a^2b^7e^8f^2g^2z + 6b^7c^2d^3e^5g^2z + 6b^3c^5d^7e^2g^2z - 48a^2c^7d^6e^2f^2z + 12a^3b^4c^2e^8g^2z - b^8d^2e^6g^2z - 4a^6c^2e^8h^2z + 48a^5c^3e^8g^2z - 4a^2c^6d^8h^2z - b^2c^6d^8g^2z - 36a^4c^4e^8f^2z - a^2b^6e^8g^2z - 4c^8d^8f^2z
\end{aligned}$$

$$\begin{aligned}
& *z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h \\
& + 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3* \\
& e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2 \\
& *b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f* \\
& h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^ \\
& 5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f* \\
& g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2* \\
& e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^ \\
& 4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a* \\
& b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 + \\
& 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e \\
& ^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5 \\
& *g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f \\
& *g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g \\
& ^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4* \\
& e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^ \\
& 4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^ \\
& 2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 - \\
& 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2 \\
& *h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2 \\
& *e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5 \\
& *g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5* \\
& f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5* \\
& f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5 \\
& *f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6* \\
& g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f \\
& *h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f \\
& *g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5* \\
& g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 3 \\
& 6*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^ \\
& 4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h \\
& + 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4 \\
& *c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b \\
& ^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g \\
& ^3, z, k)*((a^5*b^5*c*e^11 + 16*a^7*b*c^3*e^11 - 128*a^7*c^4*d*e^10 + b^5*c \\
& ^6*d^10*e + b^10*c*d^5*e^6 - 8*a^6*b^3*c^2*e^11 - 128*a^3*c^8*d^9*e^2 - 512 \\
& *a^4*c^7*d^7*e^4 - 768*a^5*c^6*d^5*e^6 - 512*a^6*c^5*d^3*e^8 - 3*b^6*c^5*d^ \\
& 9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + 16*a^2* \\
& b^2*c^7*d^9*e^2 - 264*a^2*b^3*c^6*d^8*e^3 + 480*a^2*b^4*c^5*d^7*e^4 - 246*a \\
& ^2*b^5*c^4*d^6*e^5 - 66*a^2*b^6*c^3*d^5*e^6 + 62*a^2*b^7*c^2*d^4*e^7 - 704* \\
& a^3*b^2*c^6*d^7*e^4 - 240*a^3*b^3*c^5*d^6*e^5 + 800*a^3*b^4*c^4*d^5*e^6 - 2 \\
& 46*a^3*b^5*c^3*d^4*e^7 - 76*a^3*b^6*c^2*d^3*e^8 - 1440*a^4*b^2*c^5*d^5*e^6 \\
& - 240*a^4*b^3*c^4*d^4*e^7 + 480*a^4*b^4*c^3*d^3*e^8 + 21*a^4*b^5*c^2*d^2*e^ \\
& 9 - 704*a^5*b^2*c^4*d^3*e^8 - 264*a^5*b^3*c^3*d^2*e^9 - 8*a*b^3*c^7*d^10*e \\
& - 3*a*b^9*c*d^4*e^7 + 16*a^2*b*c^8*d^10*e - 3*a^4*b^6*c*d*e^10 + 16*a*b^4*c
\end{aligned}$$

$$\begin{aligned}
& ^6*d^9*e^2 + 21*a*b^5*c^5*d^8*e^3 - 76*a*b^6*c^4*d^7*e^4 + 62*a*b^7*c^3*d^6 \\
& *e^5 - 12*a*b^8*c^2*d^5*e^6 + 2*a^2*b^8*c*d^3*e^8 + 592*a^3*b*c^7*d^8*e^3 + \\
& 2*a^3*b^7*c*d^2*e^9 + 1696*a^4*b*c^6*d^6*e^5 + 1696*a^5*b*c^5*d^4*e^7 + 16 \\
& *a^5*b^4*c^2*d*e^10 + 592*a^6*b*c^4*d^2*e^9 + 16*a^6*b^2*c^3*d*e^10)/(16*a^ \\
& 2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a* \\
& b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c \\
& ^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^ \\
& 4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6 \\
& *e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^ \\
& 4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7* \\
& e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b* \\
& c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3* \\
& e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5) \\
& + (x*(2*a^4*b^6*c*e^11 - 96*a^7*c^4*e^11 + 32*a^2*c^9*d^10*e + 2*b^4*c^7*d \\
& ^10*e + 2*b^10*c*d^4*e^7 - 22*a^5*b^4*c^2*e^11 + 80*a^6*b^2*c^3*e^11 + 32*a \\
& ^3*c^8*d^8*e^3 - 192*a^4*c^7*d^6*e^5 - 448*a^5*c^6*d^4*e^7 - 352*a^6*c^5*d^ \\
& 2*e^9 - 10*b^5*c^6*d^9*e^2 + 22*b^6*c^5*d^8*e^3 - 28*b^7*c^4*d^7*e^4 + 22*b \\
& ^8*c^3*d^6*e^5 - 10*b^9*c^2*d^5*e^6 + 336*a^2*b^2*c^7*d^8*e^3 - 384*a^2*b^3 \\
& *c^6*d^7*e^4 + 180*a^2*b^4*c^5*d^6*e^5 + 132*a^2*b^5*c^4*d^5*e^6 - 200*a^2* \\
& b^6*c^3*d^4*e^7 + 52*a^2*b^7*c^2*d^3*e^8 + 416*a^3*b^2*c^6*d^6*e^5 - 800*a^ \\
& 3*b^3*c^5*d^5*e^6 + 580*a^3*b^4*c^4*d^4*e^7 + 24*a^3*b^5*c^3*d^3*e^8 - 116* \\
& a^3*b^6*c^2*d^2*e^9 - 160*a^4*b^2*c^5*d^4*e^7 - 640*a^4*b^3*c^4*d^3*e^8 + 3 \\
& 30*a^4*b^4*c^3*d^2*e^9 - 144*a^5*b^2*c^4*d^2*e^9 - 16*a*b^2*c^8*d^10*e - 8* \\
& a*b^9*c*d^3*e^8 - 8*a^3*b^7*c*d*e^10 + 352*a^6*b*c^4*d*e^10 + 80*a*b^3*c^7* \\
& d^9*e^2 - 174*a*b^4*c^6*d^8*e^3 + 216*a*b^5*c^5*d^7*e^4 - 156*a*b^6*c^4*d^6 \\
& *e^5 + 48*a*b^7*c^3*d^5*e^6 + 10*a*b^8*c^2*d^4*e^7 - 160*a^2*b*c^8*d^9*e^2 \\
& + 12*a^2*b^8*c*d^2*e^9 - 128*a^3*b*c^7*d^7*e^4 + 576*a^4*b*c^6*d^5*e^6 + 86 \\
& *a^4*b^5*c^2*d*e^10 + 896*a^5*b*c^5*d^3*e^8 - 304*a^5*b^3*c^3*d*e^10))/(16* \\
& a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8* \\
& a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5 \\
& *c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96* \\
& a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^ \\
& ^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3* \\
& d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^ \\
& 7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5* \\
& b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^ \\
& 3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^ \\
& 5)) - (x*(48*a^5*c^4*e^9*g - 72*a^4*b*c^4*e^9*f + 16*a*c^8*d^7*e^2*f + 144* \\
& a^4*c^5*d*e^8*f - 8*a^5*b*c^3*e^9*h - 80*a^5*c^4*d*e^8*h - 4*a^2*b^5*c^2*e^ \\
& 9*f + 34*a^3*b^3*c^3*e^9*f + 2*a^3*b^4*c^2*e^9*g - 20*a^4*b^2*c^3*e^9*g + 1 \\
& 76*a^2*c^7*d^5*e^4*f + 304*a^3*c^6*d^3*e^6*f + 2*a^4*b^3*c^2*e^9*h - 80*a^2 \\
& *c^7*d^6*e^3*g - 112*a^3*c^6*d^4*e^5*g + 16*a^4*c^5*d^2*e^7*g - 4*b^2*c^7*d \\
& ^7*e^2*f + 14*b^3*c^6*d^6*e^3*f - 10*b^4*c^5*d^5*e^4*f - 10*b^5*c^4*d^4*e^5 \\
& *f + 14*b^6*c^3*d^3*e^6*f - 4*b^7*c^2*d^2*e^7*f + 48*a^2*c^7*d^7*e^2*h + 16 \\
& *a^3*c^6*d^5*e^4*h - 112*a^4*c^5*d^3*e^6*h + 2*b^3*c^6*d^7*e^2*g - 12*b^4*c
\end{aligned}$$



$$\begin{aligned}
&^5d^6e^3g + 20b^5c^4d^5e^4g - 12b^6c^3d^4e^5g + 2b^7c^2d^3e^6g + 2b^4c^5d^7e^2h - 2b^5c^4d^6e^3h - 2b^6c^3d^5e^4h + 2 \\
&*b^7c^2d^4e^5h - 4a*b^2c^6d^5e^4f + 150a*b^3c^5d^4e^5f - 128a \\
&*b^4c^4d^3e^6f + 14a*b^5c^3d^2e^7f - 440a^2b*c^6d^4e^5f - 62 \\
&*a^2b^4c^3d*e^8f - 456a^3b*c^5d^2e^7f + 84a^3b^2c^4d*e^8f + 6 \\
&8*a*b^2c^6d^6e^3g - 118a*b^3c^5d^5e^4g + 54a*b^4c^4d^4e^5g + \\
&6a*b^5c^3d^3e^6g - 2a*b^6c^2d^2e^7g + 152a^2b*c^6d^5e^4g - 2 \\
&*a^2b^5c^2d*e^8g + 72a^3b*c^5d^3e^6g + 30a^3b^3c^3d*e^8g - 20 \\
&*a*b^2c^6d^7e^2h + 30a*b^3c^5d^6e^3h - 4a*b^4c^4d^5e^4h + 6a \\
&*b^5c^3d^4e^5h - 12a*b^6c^2d^3e^6h - 88a^2b*c^6d^6e^3h + 72a \\
&^3b*c^5d^4e^5h - 12a^3b^4c^2d*e^8h + 152a^4b*c^4d^2e^7h + 68 \\
&a^4b^2c^3d*e^8h + 212a^2b^2c^5d^3e^6f + 122a^2b^3c^4d^2e^7f \\
&+ 4a^2b^2c^5d^4e^5g - 74a^2b^3c^4d^3e^6g - 4a^2b^4c^3d^2e \\
&^7g + 44a^3b^2c^4d^2e^7g + 44a^2b^2c^5d^5e^4h - 74a^2b^3c^4 \\
&d^4e^5h + 54a^2b^4c^3d^3e^6h + 20a^2b^5c^2d^2e^7h + 4a^3b^ \\
&2c^4d^3e^6h - 118a^3b^3c^3d^2e^7h - 56a*b*c^7d^6e^3f + 8a*b^ \\
&6c^2d*e^8f - 8a*b*c^7d^7e^2g - 88a^4b*c^4d*e^8g)/(16a^2c^6d^ \\
&8 + a^4b^4e^8 + 16a^6c^2e^8 + b^4c^4d^8 + b^8d^4e^4 - 8a*b^2c^5 \\
&d^8 - 8a^5b^2c*e^8 - 4a*b^7d^3e^5 - 4a^3b^5d*e^7 - 4b^5c^3d^7e \\
&- 4b^7c*d^5e^3 + 6a^2b^6d^2e^6 + 64a^3c^5d^6e^2 + 96a^4c^4d^ \\
&4e^4 + 64a^5c^3d^2e^6 + 6b^6c^2d^6e^2 + 64a^2b^2c^4d^6e^2 + 3 \\
&2a^2b^3c^3d^5e^3 - 74a^2b^4c^2d^4e^4 + 144a^3b^2c^3d^4e^4 + \\
&32a^3b^3c^2d^3e^5 + 64a^4b^2c^2d^2e^6 + 32a*b^3c^4d^7e + 4a*a \\
&b^6c*d^4e^4 - 64a^2b*c^5d^7e + 32a^4b^3c*d*e^7 - 64a^5b*c^2d*e^ \\
&7 - 44a*b^4c^3d^6e^2 + 20a*b^5c^2d^5e^3 + 20a^2b^5c*d^3e^5 - 19 \\
&2a^3b*c^4d^5e^3 - 44a^3b^4c*d^2e^6 - 192a^4b*c^3d^3e^5)) - (32 \\
&a^2c^5d^3e^4g^2 - 4c^7d^5e^2f^2 - a^2b^3c^2e^7g^2 - 4b^5c^2e \\
&^7f^2 - 4b^2c^5d^3e^4f^2 - 4b^3c^4d^2e^5f^2 + 12a^2c^5d^5e^2 \\
&*h^2 - 40a^3c^4d^3e^4h^2 - b^2c^5d^5e^2g^2 + b^3c^4d^4e^3g^2 + \\
&b^4c^3d^3e^4g^2 - b^5c^2d^2e^5g^2 + 24a^3c^4e^7f*g - 8a^4c^3 \\
&*e^7g*h + 28a*b^3c^3e^7f^2 - 48a^2b*c^4e^7f^2 + 4a^3b*c^3e^7g^ \\
&2 - 8a*c^6d^3e^4f^2 + 60a^2c^5d*e^6f^2 + 8b*c^6d^4e^3f^2 - 32a \\
&^3c^4d*e^6g^2 + 8b^4c^3d*e^6f^2 + 12a^4c^3d*e^6h^2 + 24a*b*c^5 \\
&d^2e^5f^2 - 48a*b^2c^4d*e^6f^2 + 4a*b*c^5d^4e^3g^2 - 2a*b^4c^2 \\
&d*e^6g^2 - 22a^2b^2c^3e^7f*g - 4a^2b^3c^2e^7f*h - 112a^2c^5d^ \\
&2e^5f*g + 2a^3b^2c^2e^7g*h + 80a^2c^5d^3e^4f*h - 6b^2c^5d^4 \\
&e^3f*g + 4b^3c^4d^3e^4f*g - 6b^4c^3d^2e^5f*g - 40a^2c^5d^4e^ \\
&3g*h + 80a^3c^4d^2e^5g*h - 4b^2c^5d^5e^2f*h + 4b^3c^4d^4e^3 \\
&f*h + 4b^4c^3d^3e^4f*h - 4b^5c^2d^2e^5f*h + 2b^3c^4d^5e^2g*h \\
&- 4b^4c^3d^4e^3g*h + 2b^5c^2d^3e^4g*h - 18a*b^2c^4d^3e^4g^2 \\
&+ 12a*b^3c^3d^2e^5g^2 - 24a^2b*c^4d^2e^5g^2 + 15a^2b^2c^3d*e \\
&^6g^2 - 4a*b^2c^4d^5e^2h^2 + 4a*b^3c^3d^4e^3h^2 - 4a*b^4c^2d^ \\
&3e^4h^2 - 8a^2b*c^4d^4e^3h^2 - 8a^3b*c^3d^2e^5h^2 - 4a^3b^2c \\
&^2d*e^6h^2 + 4a*b^4c^2e^7f*g + 16a^3b*c^3e^7f*h - 8a*c^6d^4e^3 \\
&*f*g + 8a*c^6d^5e^2f*h + 4b*c^6d^5e^2f*g - 56a^3c^4d*e^6f*h + 4
\end{aligned}$$

$$\begin{aligned}
& *b^5*c^2*d*e^6*f*g + 20*a^2*b^2*c^3*d^3*e^4*h^2 + 4*a^2*b^3*c^2*d^2*e^5*h^2 \\
& + 8*a*b*c^5*d^3*e^4*f*g - 40*a*b^3*c^3*d*e^6*f*g + 100*a^2*b*c^4*d*e^6*f*g \\
& - 4*a*b*c^5*d^5*e^2*g*h - 4*a^3*b*c^3*d*e^6*g*h + 44*a*b^2*c^4*d^2*e^5*f*g \\
& - 48*a*b^2*c^4*d^3*e^4*f*h + 32*a*b^3*c^3*d^2*e^5*f*h - 48*a^2*b*c^4*d^2*e^5*f*h \\
& + 12*a^2*b^2*c^3*d*e^6*f*h + 18*a*b^2*c^4*d^4*e^3*g*h - 8*a*b^3*c^3*d^3*e^4*g*h \\
& + 2*a*b^4*c^2*d^2*e^5*g*h + 24*a^2*b*c^4*d^3*e^4*g*h + 2*a^2*b^3*c^2*d*e^6*g*h \\
& - 36*a^2*b^2*c^3*d^2*e^5*g*h)/(16*a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 \\
& + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 \\
& - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 \\
& + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 \\
& + 64*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 \\
& + 144*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 \\
& + 32*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 \\
& - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 \\
& - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5))*root(3840*a^6*b*c^5*d^5 \\
& *e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920 \\
& *a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11 \\
& *e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6 \\
& *c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 - \\
& 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5 \\
& *z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d \\
& *e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^2 \\
& *c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4 \\
& *z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3 \\
& *b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8 \\
& *e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672 \\
& *a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2 \\
& *e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - \\
& 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3 \\
& *d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - \\
& 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2 \\
& *d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5 \\
& *d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12 \\
& *z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4 \\
& *z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4 \\
& *e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8 \\
& *d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2 \\
& *b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64 \\
& *a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 \\
& + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z \\
& - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7 \\
& *f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3 \\
& *d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6 \\
& *g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 79
\end{aligned}$$

$$\begin{aligned}
& 2a^3b^2c^3d^2e^6f^2h^2z - 744a^2b^2c^4d^4e^4f^2h^2z + 492a^2b^3c^3d^3e^5f^2h^2z - 264a^2b^4c^2d^2e^6f^2h^2z + 996a^2b^2c^4d^3e^5f^2g^2z - 870a^2b^3c^3d^2e^6f^2g^2z + 16a^2b^3c^6d^7e^2f^2h^2z - 56a^2b^6c^2d^2e^7f^2g^2z - 264a^4b^2c^3d^2e^6g^2h^2z + 208a^3b^2c^4d^4e^4g^2h^2z + 156a^4b^2c^2d^2e^7g^2h^2z - 148a^2b^4c^3d^5e^3g^2h^2z + 54a^2b^5c^2d^4e^4g^2h^2z - 48a^2b^5c^2d^2e^6g^2h^2z - 24a^2b^2c^5d^6e^2g^2h^2z + 10a^2b^3c^4d^6e^2g^2h^2z - 656a^3b^2c^4d^3e^5f^2h^2z - 308a^3b^3c^2d^2e^7f^2h^2z + 116a^2b^4c^3d^4e^4f^2h^2z - 84a^2b^5c^2d^3e^5f^2h^2z + 68a^2b^3c^4d^5e^3f^2h^2z - 48a^2b^2c^5d^5e^3f^2h^2z - 24a^2b^2c^5d^6e^2f^2h^2z + 1320a^3b^2c^4d^2e^6f^2g^2z - 732a^3b^2c^3d^2e^7f^2g^2z + 306a^2b^4c^2d^2e^7f^2g^2z - 304a^2b^4c^3d^3e^5f^2g^2z + 222a^2b^5c^2d^2e^6f^2g^2z + 110a^2b^3c^4d^4e^4f^2g^2z - 84a^2b^2c^5d^5e^3f^2g^2z + 16a^2c^7d^7e^2f^2g^2z - 8a^2b^7d^2e^7f^2h^2z + 4a^2b^2c^6d^8g^2h^2z + 6b^6c^2d^5e^3g^2h^2z + 6b^5c^3d^6e^2g^2h^2z + 1072a^4c^4d^3e^5g^2h^2z - 720a^3c^5d^5e^3g^2h^2z - 8b^6c^2d^4e^4f^2h^2z - 8b^4c^4d^6e^2f^2h^2z + 1072a^3c^5d^4e^4f^2h^2z - 960a^4c^4d^2e^6f^2h^2z + 30b^6c^2d^3e^5f^2g^2z + 30b^3c^5d^6e^2f^2g^2z - 10b^5c^3d^4e^4f^2g^2z - 10b^4c^4d^5e^3f^2g^2z - 1488a^3c^5d^3e^5f^2g^2z + 48a^2c^6d^5e^3f^2g^2z - 24a^4b^2c^2e^8f^2h^2z + 186a^3b^3c^2e^8f^2g^2z + 4a^4b^3c^2d^2e^7h^2z + 4a^2b^6c^2d^4e^4h^2z + 4a^2b^3c^4d^7e^2h^2z + 168a^4b^2c^3d^2e^7g^2z + 24a^2b^5c^2d^2e^7g^2z + 18a^2b^6c^2d^2e^6g^2z - 912a^3b^2c^4d^2e^7f^2z - 192a^2b^5c^2d^2e^7f^2z + 144a^2b^2c^6d^5e^3f^2z + 432a^3b^2c^3d^4e^4h^2z - 168a^4b^2c^2d^2e^6h^2z - 168a^2b^2c^4d^6e^2h^2z - 108a^2b^4c^2d^4e^4h^2z - 20a^3b^3c^2d^3e^5h^2z - 20a^2b^3c^3d^5e^3h^2z - 426a^2b^2c^4d^4e^4g^2z + 336a^3b^2c^3d^2e^6g^2z + 274a^2b^3c^3d^3e^5g^2z - 120a^2b^4c^2d^2e^6g^2z - 864a^2b^2c^4d^2e^6f^2z - 2b^7c^2d^4e^4g^2h^2z - 2b^4c^4d^7e^2g^2h^2z - 240a^5c^3d^2e^7g^2h^2z + 16a^2c^6d^7e^2g^2h^2z + 4b^7c^2d^3e^5f^2h^2z + 4b^3c^5d^7e^2f^2h^2z - 20b^7c^2d^2e^6f^2g^2z - 20b^2c^6d^7e^2f^2g^2z + 4a^2b^6d^2e^7g^2h^2z + 4a^2b^7d^2e^6g^2h^2z + 528a^4c^4d^2e^7f^2g^2z + 12a^5b^2c^2e^8g^2h^2z - 2a^4b^3c^2e^8g^2h^2z + 4a^3b^4c^2e^8f^2h^2z - 228a^4b^2c^3e^8f^2g^2z - 48a^2b^5c^2e^8f^2g^2z - 8a^2b^2c^6d^7e^2g^2z + 36a^3b^4c^2d^2e^6h^2z + 36a^2b^4c^3d^6e^2h^2z + 12a^2b^5c^2d^3e^5h^2z + 12a^2b^5c^2d^5e^3h^2z - 312a^3b^2c^4d^3e^5g^2z + 104a^2b^4c^3d^4e^4g^2z - 102a^3b^3c^2d^2e^7g^2z - 66a^2b^5c^2d^3e^5g^2z + 24a^2b^2c^5d^6e^2g^2z - 18a^2b^3c^4d^5e^3g^2z + 744a^2b^3c^3d^2e^7f^2z + 240a^2b^2c^5d^3e^5f^2z + 216a^2b^4c^3d^2e^6f^2z - 120a^2b^2c^5d^4e^4f^2z + 24a^5c^3e^8f^2h^2z + 16b^7c^2d^2e^7f^2z + 16b^2c^7d^7e^2f^2z - 2a^2b^7d^2e^7g^2z + 48a^2b^6c^2e^8f^2z - 4b^6c^2d^6e^2h^2z - 536a^4c^4d^4e^4h^2z + 240a^5c^3d^2e^6h^2z + 240a^3c^5d^6e^2h^2z - 12b^6c^2d^4e^4g^2z - 12b^4c^4d^6e^2g^2z + 10b^5c^3d^5e^3g^2z + 528a^3c^5d^4e^4g^2z - 432a^4c^4d^2e^6g^2z + 20b^4c^4d^4e^4f^2z - 16b^6c^2d^2e^6f^2z - 16b^2c^6d^6e^2f^2z - 16a^2c^6d^6e^2g^2z - 8b^5c^3d^3e^5f^2z - 8b^3c^5d^5e^3f^2z - 4a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2* \\
& z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2* \\
& e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a \\
& *b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d \\
& ^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h \\
& ^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36* \\
& a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z - \\
& 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f \\
& *g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c \\
& ^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h - \\
& 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2 \\
& *e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5* \\
& e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e \\
& ^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2 \\
& *c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^ \\
& 2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h + \\
& 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f* \\
& g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^ \\
& 4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d* \\
& e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e* \\
& f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e \\
& ^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3 \\
& *d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^ \\
& 4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g - \\
& 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f* \\
& g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2 \\
& *e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2 \\
& *e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e \\
& ^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e \\
& ^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e \\
& ^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3* \\
& e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e \\
& ^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^ \\
& 6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d* \\
& e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f \\
& ^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g - \\
& 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5 \\
& *d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3 \\
& - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b \\
& ^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^ \\
& 6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g^3, z, k), k, 1, 3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

$$3.156 \quad \int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx$$

**Optimal.** Leaf size=62

$$\frac{x^2}{2} + \frac{2(2-x)}{3(x^2-x+1)} + 2 \log(x^2-x+1) + 3x + \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{x^2}{2} + \frac{2(2-x)}{3(x^2-x+1)} + 2 \log(x^2-x+1) + 3x + \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(1 + x + x^2))/(1 - x + x^2)^2,x]

[Out] 3\*x + x^2/2 + (2\*(2 - x))/(3\*(1 - x + x^2)) + (10\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + 2\*Log[1 - x + x^2]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 1657

$\text{Int}[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow \text{Int}[\text{Expand}[\text{Integrand}[Pq*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

### Rule 1660

$\text{Int}[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow \text{With}[\{Q = \text{PolynomialQuotient}[Pq, a + b*x + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x + c*x^2, x], x, 1]\}, \text{Simp}[\frac{(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1)}{(p + 1)*(b^2 - 4*a*c)}, x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx &= \frac{2(2-x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{-2+6x+6x^2+3x^3}{1-x+x^2} dx \\ &= \frac{2(2-x)}{3(1-x+x^2)} + \frac{1}{3} \int \left( 9+3x - \frac{11-12x}{1-x+x^2} \right) dx \\ &= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3(1-x+x^2)} - \frac{1}{3} \int \frac{11-12x}{1-x+x^2} dx \\ &= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3(1-x+x^2)} - \frac{5}{3} \int \frac{1}{1-x+x^2} dx + 2 \int \frac{-1+2x}{1-x+x^2} dx \\ &= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3(1-x+x^2)} + 2 \log(1-x+x^2) + \frac{10}{3} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\ &= 3x + \frac{x^2}{2} + \frac{2(2-x)}{3(1-x+x^2)} + \frac{10 \tan^{-1} \left( \frac{1-2x}{\sqrt{3}} \right)}{3\sqrt{3}} + 2 \log(1-x+x^2) \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 60, normalized size = 0.97

$$\frac{x^2}{2} - \frac{2(x-2)}{3(x^2-x+1)} + 2 \log(x^2-x+1) + 3x - \frac{10 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(1 + x + x^2))/(1 - x + x^2)^2,x]

[Out] 3\*x + x^2/2 - (2\*(-2 + x))/(3\*(1 - x + x^2)) - (10\*ArcTan[(-1 + 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + 2\*Log[1 - x + x^2]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3\*(1 + x + x^2))/(1 - x + x^2)^2,x]

[Out] IntegrateAlgebraic[(x^3\*(1 + x + x^2))/(1 - x + x^2)^2, x]

**fricas** [A] time = 1.93, size = 75, normalized size = 1.21

$$\frac{9x^4 + 45x^3 - 20\sqrt{3}(x^2 - x + 1) \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) - 45x^2 + 36(x^2 - x + 1) \log(x^2 - x + 1) + 42x + 24}{18(x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")

[Out] 1/18\*(9\*x^4 + 45\*x^3 - 20\*sqrt(3)\*(x^2 - x + 1)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) - 45\*x^2 + 36\*(x^2 - x + 1)\*log(x^2 - x + 1) + 42\*x + 24)/(x^2 - x + 1)

**giac** [A] time = 0.16, size = 51, normalized size = 0.82

$$\frac{1}{2}x^2 - \frac{10}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x - 1)\right) + 3x - \frac{2(x-2)}{3(x^2-x+1)} + 2 \log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")



[Out]  $\frac{1}{2}x^2 - \frac{10\sqrt{3}}{9}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3x - \frac{2}{3}\frac{(x-2)}{(x^2-x+1)} + 2\log(x^2-x+1)$

**maple** [A] time = 0.01, size = 53, normalized size = 0.85

$$\frac{x^2}{2} + 3x - \frac{10\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + 2\ln(x^2-x+1) + \frac{-\frac{2x}{3} + \frac{4}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(x^2+x+1)/(x^2-x+1)^2,x)`

[Out]  $\frac{1}{2}x^2 + 3x + \frac{-2/3x + 4/3}{(x^2-x+1)} + 2\ln(x^2-x+1) - \frac{10}{9}3^{(1/2)}\arctan\left(\frac{1}{3}(2x-1)\right)3^{(1/2)}$

**maxima** [A] time = 0.96, size = 51, normalized size = 0.82

$$\frac{1}{2}x^2 - \frac{10}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3x - \frac{2(x-2)}{3(x^2-x+1)} + 2\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}x^2 - \frac{10\sqrt{3}}{9}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3x - \frac{2}{3}\frac{(x-2)}{(x^2-x+1)} + 2\log(x^2-x+1)$

**mupad** [B] time = 0.04, size = 55, normalized size = 0.89

$$3x + 2\ln(x^2-x+1) - \frac{\frac{2x}{3} - \frac{4}{3}}{x^2-x+1} - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(x+x^2+1))/(x^2-x+1)^2,x)`

[Out]  $3x + 2\log(x^2-x+1) - \frac{((2x)/3 - 4/3)}{(x^2-x+1)} - \frac{(10*3^{(1/2)}*\operatorname{atan}((2*3^{(1/2)}*x)/3 - 3^{(1/2)}/3))}{9} + x^2/2$

**sympy** [A] time = 0.16, size = 60, normalized size = 0.97

$$\frac{x^2}{2} + 3x + \frac{4-2x}{3x^2-3x+3} + 2\log(x^2-x+1) - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqrt  
(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

$$3.157 \quad \int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx$$

Optimal. Leaf size=55

$$\frac{2(1-2x)}{3(x^2-x+1)} + \frac{3}{2} \log(x^2-x+1) + x - \frac{7 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{2(1-2x)}{3(x^2-x+1)} + \frac{3}{2} \log(x^2-x+1) + x - \frac{7 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(1 + x + x^2))/(1 - x + x^2)^2,x]

[Out] x + (2\*(1 - 2\*x))/(3\*(1 - x + x^2)) - (7\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + (3\*Log[1 - x + x^2])/2

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ  
 [2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand  
 Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq  
 , x] && IGtQ[p, -2]

### Rule 1660

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q =  
 PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq,  
 a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x +  
 c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(  
 p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[  
 (a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(  
 2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 -  
 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx &= \frac{2(1-2x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{2+6x+3x^2}{1-x+x^2} dx \\
 &= \frac{2(1-2x)}{3(1-x+x^2)} + \frac{1}{3} \int \left( 3 - \frac{1-9x}{1-x+x^2} \right) dx \\
 &= x + \frac{2(1-2x)}{3(1-x+x^2)} - \frac{1}{3} \int \frac{1-9x}{1-x+x^2} dx \\
 &= x + \frac{2(1-2x)}{3(1-x+x^2)} + \frac{7}{6} \int \frac{1}{1-x+x^2} dx + \frac{3}{2} \int \frac{-1+2x}{1-x+x^2} dx \\
 &= x + \frac{2(1-2x)}{3(1-x+x^2)} + \frac{3}{2} \log(1-x+x^2) - \frac{7}{3} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
 &= x + \frac{2(1-2x)}{3(1-x+x^2)} - \frac{7 \tan^{-1} \left( \frac{1-2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{3}{2} \log(1-x+x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 55, normalized size = 1.00

$$-\frac{2(2x-1)}{3(x^2-x+1)} + \frac{3}{2} \log(x^2-x+1) + x + \frac{7 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(1 + x + x^2))/(1 - x + x^2)^2,x]

[Out] x - (2\*(-1 + 2\*x))/(3\*(1 - x + x^2)) + (7\*ArcTan[(-1 + 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + (3\*Log[1 - x + x^2])/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2\*(1 + x + x^2))/(1 - x + x^2)^2,x]

[Out] IntegrateAlgebraic[(x^2\*(1 + x + x^2))/(1 - x + x^2)^2, x]

**fricas [A]** time = 0.69, size = 70, normalized size = 1.27

$$\frac{18x^3 + 14\sqrt{3}(x^2 - x + 1) \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 18x^2 + 27(x^2 - x + 1) \log(x^2 - x + 1) - 6x + 12}{18(x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")

[Out] 1/18\*(18\*x^3 + 14\*sqrt(3)\*(x^2 - x + 1)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) - 18\*x^2 + 27\*(x^2 - x + 1)\*log(x^2 - x + 1) - 6\*x + 12)/(x^2 - x + 1)

**giac [A]** time = 0.16, size = 46, normalized size = 0.84

$$\frac{7}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + x - \frac{2(2x-1)}{3(x^2-x+1)} + \frac{3}{2} \log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")

[Out]  $7/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1) + 3/2*\log(x^2 - x + 1)$

maple [A] time = 0.01, size = 46, normalized size = 0.84

$$x + \frac{7\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + \frac{3 \ln(x^2 - x + 1)}{2} + \frac{-\frac{4x}{3} + \frac{2}{3}}{x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x^2+x+1)/(x^2-x+1)^2,x)`

[Out]  $x + (-4/3*x + 2/3)/(x^2 - x + 1) + 3/2*\ln(x^2 - x + 1) + 7/9*3^{(1/2)}*\arctan(1/3*(2*x - 1)*3^{(1/2)})$

maxima [A] time = 0.95, size = 46, normalized size = 0.84

$$\frac{7}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + x - \frac{2(2x - 1)}{3(x^2 - x + 1)} + \frac{3}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")`

[Out]  $7/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1) + 3/2*\log(x^2 - x + 1)$

mupad [B] time = 0.04, size = 48, normalized size = 0.87

$$x + \frac{3 \ln(x^2 - x + 1)}{2} - \frac{\frac{4x}{3} - \frac{2}{3}}{x^2 - x + 1} + \frac{7\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)`

[Out]  $x + (3*\log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^{(1/2)}*\operatorname{atan}((2*3^{(1/2)}*x)/3 - 3^{(1/2)}/3))/9$

sympy [A] time = 0.15, size = 54, normalized size = 0.98

$$x + \frac{2 - 4x}{3x^2 - 3x + 3} + \frac{3 \log(x^2 - x + 1)}{2} + \frac{7\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(2  
*sqrt(3)*x/3 - sqrt(3)/3)/9
```

$$3.158 \quad \int \frac{x(1+x+x^2)}{(1-x+x^2)^2} dx$$

Optimal. Leaf size=52

$$-\frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2} \log(x^2-x+1) - \frac{11 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {1660, 634, 618, 204, 628}

$$-\frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2} \log(x^2-x+1) - \frac{11 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
+ Log[1 - x + x^2]/2
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

#### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```



$t[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

### Rule 1660

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x\_Symbol] \rightarrow \text{With}\{Q = \text{PolynomialQuotient}[Pq, a + bx + cx^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + bx + cx^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + bx + cx^2, x], x, 1]\}, \text{Simp}[(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + bx + cx^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + bx + cx^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{x(1+x+x^2)}{(1-x+x^2)^2} dx &= -\frac{2(1+x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{4+3x}{1-x+x^2} dx \\ &= -\frac{2(1+x)}{3(1-x+x^2)} + \frac{1}{2} \int \frac{-1+2x}{1-x+x^2} dx + \frac{11}{6} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{2(1+x)}{3(1-x+x^2)} + \frac{1}{2} \log(1-x+x^2) - \frac{11}{3} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\ &= -\frac{2(1+x)}{3(1-x+x^2)} - \frac{11 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2} \log(1-x+x^2) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 1.00

$$-\frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2} \log(x^2-x+1) + \frac{11 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(1 + x + x^2))/(1 - x + x^2)^2, x]

[Out] (-2\*(1 + x))/(3\*(1 - x + x^2)) + (11\*ArcTan[(-1 + 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + Log[1 - x + x^2]/2

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x\*(1+x+x^2))/(1-x+x^2)^2,x]

[Out] IntegrateAlgebraic[(x\*(1+x+x^2))/(1-x+x^2)^2, x]

**fricas** [A] time = 1.04, size = 60, normalized size = 1.15

$$\frac{22\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+9(x^2-x+1)\log(x^2-x+1)-12x-12}{18(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")

[Out] 1/18\*(22\*sqrt(3)\*(x^2-x+1)\*arctan(1/3\*sqrt(3)\*(2\*x-1))+9\*(x^2-x+1)\*log(x^2-x+1)-12\*x-12)/(x^2-x+1)

**giac** [A] time = 0.15, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{2(x+1)}{3(x^2-x+1)}+\frac{1}{2}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")

[Out] 11/9\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x-1))-2/3\*(x+1)/(x^2-x+1)+1/2\*log(x^2-x+1)

**maple** [A] time = 0.00, size = 45, normalized size = 0.87

$$\frac{11\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{\ln(x^2-x+1)}{2}+\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(x^2+x+1)/(x^2-x+1)^2,x)

[Out]  $(-2/3*x-2/3)/(x^2-x+1)+1/2*\ln(x^2-x+1)+11/9*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})$

**maxima [A]** time = 0.96, size = 43, normalized size = 0.83

$$\frac{11}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - \frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2} \log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")`

[Out]  $11/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/2*\log(x^2 - x + 1)$

**mupad [B]** time = 3.84, size = 59, normalized size = 1.13

$$\frac{\ln(x^2-x+1)}{2} - \frac{2x}{3(x^2-x+1)} - \frac{2}{3(x^2-x+1)} + \frac{11\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)`

[Out]  $\log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3^{(1/2)}*\operatorname{atan}((2*3^{(1/2)}*x)/3 - 3^{(1/2)}/3))/9$

**sympy [A]** time = 0.15, size = 53, normalized size = 1.02

$$\frac{-2x-2}{3x^2-3x+3} + \frac{\log(x^2-x+1)}{2} + \frac{11\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)`

[Out]  $(-2*x - 2)/(3*x**2 - 3*x + 3) + \log(x**2 - x + 1)/2 + 11*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/9$

$$3.159 \quad \int \frac{1+x+x^2}{(1-x+x^2)^2} dx$$

Optimal. Leaf size=41

$$-\frac{2(2-x)}{3(x^2-x+1)} - \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {1660, 12, 618, 204}

$$-\frac{2(2-x)}{3(x^2-x+1)} - \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(1 - x + x^2)^2, x]

[Out] (-2\*(2 - x))/(3\*(1 - x + x^2)) - (10\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(3\*Sqrt[3])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^

$(p + 1)/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /;$  FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{(1-x+x^2)^2} dx &= -\frac{2(2-x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{5}{1-x+x^2} dx \\ &= -\frac{2(2-x)}{3(1-x+x^2)} + \frac{5}{3} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{2(2-x)}{3(1-x+x^2)} - \frac{10}{3} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\ &= -\frac{2(2-x)}{3(1-x+x^2)} - \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.95

$$\frac{2(x-2)}{3(x^2-x+1)} + \frac{10 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(1 - x + x^2)^2, x]

[Out] (2\*(-2 + x))/(3\*(1 - x + x^2)) + (10\*ArcTan[(-1 + 2\*x)/Sqrt[3]])/(3\*Sqrt[3])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(1 - x + x^2)^2, x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(1 - x + x^2)^2, x]

**fricas** [A] time = 0.61, size = 41, normalized size = 1.00

$$\frac{2 \left( 5 \sqrt{3} (x^2 - x + 1) \arctan \left( \frac{1}{3} \sqrt{3} (2x - 1) \right) + 3x - 6 \right)}{9 (x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")

[Out] 2/9\*(5\*sqrt(3)\*(x^2 - x + 1)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 3\*x - 6)/(x^2 - x + 1)

**giac** [A] time = 0.16, size = 32, normalized size = 0.78

$$\frac{10}{9} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} (2x - 1) \right) + \frac{2(x - 2)}{3(x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")

[Out] 10/9\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 2/3\*(x - 2)/(x^2 - x + 1)

**maple** [A] time = 0.00, size = 34, normalized size = 0.83

$$\frac{10\sqrt{3} \arctan \left( \frac{(2x-1)\sqrt{3}}{3} \right)}{9} + \frac{\frac{2x}{3} - \frac{4}{3}}{x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/(x^2-x+1)^2,x)

[Out] (2/3\*x-4/3)/(x^2-x+1)+10/9\*3^(1/2)\*arctan(1/3\*(2\*x-1)\*3^(1/2))

**maxima** [A] time = 0.95, size = 32, normalized size = 0.78

$$\frac{10}{9} \sqrt{3} \arctan \left( \frac{1}{3} \sqrt{3} (2x - 1) \right) + \frac{2(x - 2)}{3(x^2 - x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")

[Out] 10/9\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) + 2/3\*(x - 2)/(x^2 - x + 1)

mupad [B] time = 3.83, size = 35, normalized size = 0.85

$$\frac{\frac{2x}{3} - \frac{4}{3}}{x^2 - x + 1} + \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 + 1)/(x^2 - x + 1)^2,x)`

[Out] `((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9`

sympy [A] time = 0.14, size = 41, normalized size = 1.00

$$\frac{2x - 4}{3x^2 - 3x + 3} + \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x+1)/(x**2-x+1)**2,x)`

[Out] `(2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9`

$$3.160 \quad \int \frac{1+x+x^2}{x(1-x+x^2)^2} dx$$

Optimal. Leaf size=56

$$-\frac{2(1-2x)}{3(x^2-x+1)} - \frac{1}{2} \log(x^2-x+1) + \log(x) - \frac{11 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{2(1-2x)}{3(x^2-x+1)} - \frac{1}{2} \log(x^2-x+1) + \log(x) - \frac{11 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(x\*(1 - x + x^2)^2),x]

[Out] (-2\*(1 - 2\*x))/(3\*(1 - x + x^2)) - (11\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + Log[x] - Log[1 - x + x^2]/2

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In



$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 800

$\text{Int}[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[m]$

### Rule 1646

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x\_Symbol] \text{:>} \text{With}\{\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]\}, \text{Simp}[(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q]/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]\} /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{LtQ}[p, -1] \&\& \text{ILtQ}[m, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{x(1-x+x^2)^2} dx &= -\frac{2(1-2x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{3+4x}{x(1-x+x^2)} dx \\ &= -\frac{2(1-2x)}{3(1-x+x^2)} + \frac{1}{3} \int \left( \frac{3}{x} + \frac{7-3x}{1-x+x^2} \right) dx \\ &= -\frac{2(1-2x)}{3(1-x+x^2)} + \log(x) + \frac{1}{3} \int \frac{7-3x}{1-x+x^2} dx \\ &= -\frac{2(1-2x)}{3(1-x+x^2)} + \log(x) - \frac{1}{2} \int \frac{-1+2x}{1-x+x^2} dx + \frac{11}{6} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{2(1-2x)}{3(1-x+x^2)} + \log(x) - \frac{1}{2} \log(1-x+x^2) - \frac{11}{3} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\ &= -\frac{2(1-2x)}{3(1-x+x^2)} - \frac{11 \tan^{-1} \left( \frac{1-2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \log(x) - \frac{1}{2} \log(1-x+x^2) \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 56, normalized size = 1.00

$$\frac{2(2x-1)}{3(x^2-x+1)} - \frac{1}{2} \log(x^2-x+1) + \log(x) + \frac{11 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(x\*(1 - x + x^2)^2), x]

[Out] (2\*(-1 + 2\*x))/(3\*(1 - x + x^2)) + (11\*ArcTan[(-1 + 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + Log[x] - Log[1 - x + x^2]/2

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{x(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(x\*(1 - x + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(x\*(1 - x + x^2)^2), x]

**fricas** [A] time = 1.09, size = 72, normalized size = 1.29

$$\frac{22\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 9(x^2-x+1)\log(x^2-x+1) + 18(x^2-x+1)\log(x) + 24x - 12}{18(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")

[Out] 1/18\*(22\*sqrt(3)\*(x^2 - x + 1)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) - 9\*(x^2 - x + 1)\*log(x^2 - x + 1) + 18\*(x^2 - x + 1)\*log(x) + 24\*x - 12)/(x^2 - x + 1)

**giac** [A] time = 0.15, size = 48, normalized size = 0.86

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2(2x-1)}{3(x^2-x+1)} - \frac{1}{2}\log(x^2-x+1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")

[Out]  $11/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) - 1/2*\log(x^2 - x + 1) + \log(\text{abs}(x))$

**maple** [A] time = 0.01, size = 48, normalized size = 0.86

$$\frac{11\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + \ln(x) - \frac{\ln(x^2 - x + 1)}{2} - \frac{-\frac{4x}{3} + \frac{2}{3}}{x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x+1)/x/(x^2-x+1)^2,x)`

[Out]  $-(-4/3*x+2/3)/(x^2-x+1)-1/2*\ln(x^2-x+1)+11/9*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})+\ln(x)$

**maxima** [A] time = 0.95, size = 47, normalized size = 0.84

$$\frac{11}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2(2x-1)}{3(x^2-x+1)} - \frac{1}{2}\log(x^2-x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")`

[Out]  $11/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) - 1/2*\log(x^2 - x + 1) + \log(x)$

**mupad** [B] time = 0.10, size = 58, normalized size = 1.04

$$\ln(x) + \frac{\frac{4x}{3} - \frac{2}{3}}{x^2 - x + 1} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} 11i}{18}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(-\frac{1}{2} + \frac{\sqrt{3} 11i}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)`

[Out]  $\log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - \log(x - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*11i)/18 + 1/2) + \log(x + (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*11i)/18 - 1/2)$

**sympy** [A] time = 0.18, size = 54, normalized size = 0.96

$$\frac{4x - 2}{3x^2 - 3x + 3} + \log(x) - \frac{\log(x^2 - x + 1)}{2} + \frac{11\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

```
[Out] (4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

$$3.161 \quad \int \frac{1+x+x^2}{x^2(1-x+x^2)^2} dx$$

Optimal. Leaf size=61

$$\frac{2(x+1)}{3(x^2-x+1)} - \frac{3}{2} \log(x^2-x+1) - \frac{1}{x} + 3 \log(x) - \frac{7 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(x+1)}{3(x^2-x+1)} - \frac{3}{2} \log(x^2-x+1) - \frac{1}{x} + 3 \log(x) - \frac{7 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(x^2\*(1 - x + x^2)^2), x]

[Out] -x^(-1) + (2\*(1 + x))/(3\*(1 - x + x^2)) - (7\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + 3\*Log[x] - (3\*Log[1 - x + x^2])/2

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1628

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{1+x+x^2}{x^2(1-x+x^2)^2} dx &= \frac{2(1+x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{3+6x+2x^2}{x^2(1-x+x^2)} dx \\
 &= \frac{2(1+x)}{3(1-x+x^2)} + \frac{1}{3} \int \left( \frac{3}{x^2} + \frac{9}{x} + \frac{8-9x}{1-x+x^2} \right) dx \\
 &= -\frac{1}{x} + \frac{2(1+x)}{3(1-x+x^2)} + 3 \log(x) + \frac{1}{3} \int \frac{8-9x}{1-x+x^2} dx \\
 &= -\frac{1}{x} + \frac{2(1+x)}{3(1-x+x^2)} + 3 \log(x) + \frac{7}{6} \int \frac{1}{1-x+x^2} dx - \frac{3}{2} \int \frac{-1+2x}{1-x+x^2} dx \\
 &= -\frac{1}{x} + \frac{2(1+x)}{3(1-x+x^2)} + 3 \log(x) - \frac{3}{2} \log(1-x+x^2) - \frac{7}{3} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, -1+2x \right) \\
 &= -\frac{1}{x} + \frac{2(1+x)}{3(1-x+x^2)} - \frac{7 \tan^{-1} \left( \frac{1-2x}{\sqrt{3}} \right)}{3\sqrt{3}} + 3 \log(x) - \frac{3}{2} \log(1-x+x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 61, normalized size = 1.00

$$\frac{2(x+1)}{3(x^2-x+1)} - \frac{3}{2} \log(x^2-x+1) - \frac{1}{x} + 3 \log(x) + \frac{7 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(x^2\*(1 - x + x^2)^2), x]

[Out]  $-x^{-1} + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{x^2(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(x^2\*(1 - x + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(x^2\*(1 - x + x^2)^2), x]

**fricas [A]** time = 1.25, size = 85, normalized size = 1.39

$$\frac{14\sqrt{3}(x^3-x^2+x)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 6x^2 - 27(x^3-x^2+x)\log(x^2-x+1) + 54(x^3-x^2+x)\log(x) + 30x - 18}{18(x^3-x^2+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")

[Out]  $1/18*(14*\sqrt{3}*(x^3 - x^2 + x)*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 6*x^2 - 27*(x^3 - x^2 + x)*\log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*\log(x) + 30*x - 18)/(x^3 - x^2 + x)$

**giac [A]** time = 0.15, size = 55, normalized size = 0.90

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x^2-5x+3}{3(x^3-x^2+x)} - \frac{3}{2}\log(x^2-x+1) + 3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")

[Out]  $7/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2 + x) - 3/2*\log(x^2 - x + 1) + 3*\log(\text{abs}(x))$

**maple** [A] time = 0.01, size = 55, normalized size = 0.90

$$\frac{7\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + 3 \ln(x) - \frac{3 \ln(x^2 - x + 1)}{2} - \frac{1}{x} - \frac{-\frac{2x}{3} - \frac{2}{3}}{x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x+1)/x^2/(x^2-x+1)^2,x)`

[Out]  $-(-2/3*x-2/3)/(x^2-x+1)-3/2*\ln(x^2-x+1)+7/9*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})-1/x+3*\ln(x)$

**maxima** [A] time = 0.96, size = 54, normalized size = 0.89

$$\frac{7}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) - \frac{x^2 - 5x + 3}{3(x^3 - x^2 + x)} - \frac{3}{2} \log(x^2 - x + 1) + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")`

[Out]  $7/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2 + x) - 3/2*\log(x^2 - x + 1) + 3*\log(x)$

**mupad** [B] time = 4.13, size = 68, normalized size = 1.11

$$3 \ln(x) - \frac{\frac{x^2}{3} - \frac{5x}{3} + 1}{x^3 - x^2 + x} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(\frac{3}{2} + \frac{\sqrt{3} 7i}{18}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(-\frac{3}{2} + \frac{\sqrt{3} 7i}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)`

[Out]  $3*\log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - \log(x - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*7i)/18 + 3/2) + \log(x + (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*7i)/18 - 3/2)$

**sympy** [A] time = 0.20, size = 65, normalized size = 1.07

$$\frac{-x^2 + 5x - 3}{3x^3 - 3x^2 + 3x} + 3 \log(x) - \frac{3 \log(x^2 - x + 1)}{2} + \frac{7\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

```
[Out] (-x**2 + 5*x - 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)/  
2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

$$3.162 \quad \int \frac{1+x+x^2}{x^3(1-x+x^2)^2} dx$$

Optimal. Leaf size=68

$$\frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{2x^2} - 2 \log(x^2-x+1) - \frac{3}{x} + 4 \log(x) + \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{2x^2} - 2 \log(x^2-x+1) - \frac{3}{x} + 4 \log(x) + \frac{10 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(x^3\*(1 - x + x^2)^2), x]

[Out] -1/(2\*x^2) - 3/x + (2\*(2 - x))/(3\*(1 - x + x^2)) + (10\*ArcTan[(1 - 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + 4\*Log[x] - 2\*Log[1 - x + x^2]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

### Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

### Rule 1646

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x\_Symbol] :> \text{With}\{Q = \text{PolynomialQuotient}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]\}, \text{Simp}[(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^{(p + 1)} / ((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1 / ((p + 1)*(b^2 - 4*a*c)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^{(p + 1)}*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q] / (d + e*x)^m - ((2*p + 3)*(2*c*f - b*g)) / (d + e*x)^m, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{LtQ}[p, -1] \&\& \text{ILtQ}[m, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{x^3(1-x+x^2)^2} dx &= \frac{2(2-x)}{3(1-x+x^2)} + \frac{1}{3} \int \frac{3+6x+6x^2-2x^3}{x^3(1-x+x^2)} dx \\ &= \frac{2(2-x)}{3(1-x+x^2)} + \frac{1}{3} \int \left( \frac{3}{x^3} + \frac{9}{x^2} + \frac{12}{x} + \frac{1-12x}{1-x+x^2} \right) dx \\ &= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3(1-x+x^2)} + 4 \log(x) + \frac{1}{3} \int \frac{1-12x}{1-x+x^2} dx \\ &= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3(1-x+x^2)} + 4 \log(x) - \frac{5}{3} \int \frac{1}{1-x+x^2} dx - 2 \int \frac{-1+2x}{1-x+x^2} dx \\ &= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3(1-x+x^2)} + 4 \log(x) - 2 \log(1-x+x^2) + \frac{10}{3} \text{Subst} \left( \int \frac{1}{-3-x^2} dx, \right. \\ &= -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3(1-x+x^2)} + \frac{10 \tan^{-1} \left( \frac{1-2x}{\sqrt{3}} \right)}{3\sqrt{3}} + 4 \log(x) - 2 \log(1-x+x^2) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.97

$$-\frac{2(x-2)}{3(x^2-x+1)} - \frac{1}{2x^2} - 2 \log(x^2-x+1) - \frac{3}{x} + 4 \log(x) - \frac{10 \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(x^3\*(1 - x + x^2)^2), x]

[Out] -1/2\*1/x^2 - 3/x - (2\*(-2 + x))/(3\*(1 - x + x^2)) - (10\*ArcTan[(-1 + 2\*x)/Sqrt[3]])/(3\*Sqrt[3]) + 4\*Log[x] - 2\*Log[1 - x + x^2]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(x^3\*(1 - x + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(x^3\*(1 - x + x^2)^2), x]

**fricas [A]** time = 1.69, size = 98, normalized size = 1.44

$$\frac{66x^3 + 20\sqrt{3}(x^4 - x^3 + x^2) \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 69x^2 + 36(x^4 - x^3 + x^2) \log(x^2 - x + 1) - 72(x^4 - x^3 + x^2) \log(x) + 45x + 9}{18(x^4 - x^3 + x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")

[Out] -1/18\*(66\*x^3 + 20\*sqrt(3)\*(x^4 - x^3 + x^2)\*arctan(1/3\*sqrt(3)\*(2\*x - 1)) - 69\*x^2 + 36\*(x^4 - x^3 + x^2)\*log(x^2 - x + 1) - 72\*(x^4 - x^3 + x^2)\*log(x) + 45\*x + 9)/(x^4 - x^3 + x^2)

**giac [A]** time = 0.16, size = 63, normalized size = 0.93

$$-\frac{10}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{22x^3 - 23x^2 + 15x + 3}{6(x^2 - x + 1)x^2} - 2 \log(x^2 - x + 1) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")

[Out]  $-10/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x + 3)/((x^2 - x + 1)*x^2) - 2*\log(x^2 - x + 1) + 4*\log(\text{abs}(x))$

**maple** [A] time = 0.01, size = 60, normalized size = 0.88

$$-\frac{10\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + 4\ln(x) - 2\ln(x^2 - x + 1) - \frac{3}{x} - \frac{1}{2x^2} - \frac{\frac{2x}{3} - \frac{4}{3}}{x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^2+x+1)/x^3/(x^2-x+1)^2, x)$

[Out]  $-(2/3*x-4/3)/(x^2-x+1)-2*\ln(x^2-x+1)-10/9*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})-1/2/x^2-3/x+4*\ln(x)$

**maxima** [A] time = 0.95, size = 63, normalized size = 0.93

$$-\frac{10}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{22x^3 - 23x^2 + 15x + 3}{6(x^4 - x^3 + x^2)} - 2\log(x^2 - x + 1) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((x^2+x+1)/x^3/(x^2-x+1)^2, x, \text{algorithm}=\text{"maxima"})$

[Out]  $-10/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x + 3)/(x^4 - x^3 + x^2) - 2*\log(x^2 - x + 1) + 4*\log(x)$

**mupad** [B] time = 0.10, size = 75, normalized size = 1.10

$$4\ln(x) + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(-2 + \frac{\sqrt{3}5i}{9}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(2 + \frac{\sqrt{3}5i}{9}\right) - \frac{\frac{11x^3}{3} - \frac{23x^2}{6} + \frac{5x}{2} + \frac{1}{2}}{x^4 - x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2), x)$

[Out]  $4*\log(x) + \log(x - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*5i)/9 - 2) - \log(x + (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3)/3 + 1/2)/(x^2 - x^3 + x^4)$

**sympy** [A] time = 0.22, size = 71, normalized size = 1.04

$$4\log(x) - 2\log(x^2 - x + 1) - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9} + \frac{-22x^3 + 23x^2 - 15x - 3}{6x^4 - 6x^3 + 6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

```
[Out] 4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)
/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

$$3.163 \quad \int \frac{1-x^2}{(1+x+x^2)^2} dx$$

Optimal. Leaf size=10

$$\frac{x}{x^2 + x + 1}$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {1588}

$$\frac{x}{x^2 + x + 1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1-x^2}{(1+x+x^2)^2} dx = \frac{x}{1+x+x^2}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{x}{x^2 + x + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - x^2}{(1 + x + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] IntegrateAlgebraic[(1 - x^2)/(1 + x + x^2)^2, x]

**fricas** [A] time = 1.11, size = 10, normalized size = 1.00

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")

[Out] x/(x^2 + x + 1)

**giac** [A] time = 0.15, size = 8, normalized size = 0.80

$$\frac{1}{x + \frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")

[Out] 1/(x + 1/x + 1)

**maple** [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^2+x+1)^2,x)

[Out] x/(x^2+x+1)

**maxima** [A] time = 0.43, size = 10, normalized size = 1.00

$$\frac{x}{x^2 + x + 1}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")

[Out] x/(x^2 + x + 1)

**mupad** [B] time = 0.05, size = 10, normalized size = 1.00

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(x + x^2 + 1)^2,x)

[Out] x/(x + x^2 + 1)

**sympy** [A] time = 0.10, size = 7, normalized size = 0.70

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x\*\*2+1)/(x\*\*2+x+1)\*\*2,x)

[Out] x/(x\*\*2 + x + 1)

$$3.164 \quad \int \frac{1+x^2}{1+x+x^2} dx$$

**Optimal.** Leaf size=31

$$-\frac{1}{2} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {1657, 634, 618, 204, 628}

$$-\frac{1}{2} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x + x^2), x]

[Out] x + ArcTan[(1 + 2\*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

### Rule 1657

`Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand  
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,  
x] && IGtQ[p, -2]`

### Rubi steps

$$\begin{aligned}
 \int \frac{1+x^2}{1+x+x^2} dx &= \int \left(1 - \frac{x}{1+x+x^2}\right) dx \\
 &= x - \int \frac{x}{1+x+x^2} dx \\
 &= x + \frac{1}{2} \int \frac{1}{1+x+x^2} dx - \frac{1}{2} \int \frac{1+2x}{1+x+x^2} dx \\
 &= x - \frac{1}{2} \log(1+x+x^2) - \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\
 &= x + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2} \log(1+x+x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x + x^2), x]

[Out] x + ArcTan[(1 + 2\*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+x+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + x + x^2), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + x + x^2), x]

**fricas** [A] time = 1.95, size = 27, normalized size = 0.87

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + x - \frac{1}{2} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + x - 1/2\*log(x^2 + x + 1)

**giac** [A] time = 0.17, size = 27, normalized size = 0.87

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + x - \frac{1}{2} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + x - 1/2\*log(x^2 + x + 1)

**maple** [A] time = 0.00, size = 28, normalized size = 0.90

$$x + \frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} - \frac{\ln(x^2 + x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^2+x+1),x)

[Out] x-1/2\*ln(x^2+x+1)+1/3\*arctan(1/3\*(1+2\*x)\*3^(1/2))\*3^(1/2)

**maxima** [A] time = 0.95, size = 27, normalized size = 0.87

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) + x - \frac{1}{2} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + 1)) + x - 1/2\*log(x^2 + x + 1)

mupad [B] time = 0.03, size = 29, normalized size = 0.94

$$x - \frac{\ln(x^2 + x + 1)}{2} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x + x^2 + 1), x)

[Out] x - log(x + x^2 + 1)/2 + (3^(1/2)\*atan((2\*3^(1/2)\*x)/3 + 3^(1/2)/3))/3

sympy [A] time = 0.12, size = 36, normalized size = 1.16

$$x - \frac{\log(x^2 + x + 1)}{2} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+1)/(x\*\*2+x+1), x)

[Out] x - log(x\*\*2 + x + 1)/2 + sqrt(3)\*atan(2\*sqrt(3)\*x/3 + sqrt(3)/3)/3

$$3.165 \quad \int \frac{-1+x^2}{25-6x+x^2} dx$$

Optimal. Leaf size=23

$$3 \log(x^2 - 6x + 25) + x - 2 \tan^{-1}\left(\frac{x-3}{4}\right)$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {1657, 634, 618, 204, 628}

$$3 \log(x^2 - 6x + 25) + x - 2 \tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(25 - 6\*x + x^2), x]

[Out] x - 2\*ArcTan[(-3 + x)/4] + 3\*Log[25 - 6\*x + x^2]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand  
Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,  
x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{-1 + x^2}{25 - 6x + x^2} dx &= \int \left(1 - \frac{2(13 - 3x)}{25 - 6x + x^2}\right) dx \\
 &= x - 2 \int \frac{13 - 3x}{25 - 6x + x^2} dx \\
 &= x + 3 \int \frac{-6 + 2x}{25 - 6x + x^2} dx - 8 \int \frac{1}{25 - 6x + x^2} dx \\
 &= x + 3 \log(25 - 6x + x^2) + 16 \operatorname{Subst}\left(\int \frac{1}{-64 - x^2} dx, x, -6 + 2x\right) \\
 &= x - 2 \tan^{-1}\left(\frac{1}{4}(-3 + x)\right) + 3 \log(25 - 6x + x^2)
 \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 1.00

$$3 \log(x^2 - 6x + 25) + x - 2 \tan^{-1}\left(\frac{x - 3}{4}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(25 - 6\*x + x^2), x]

[Out] x - 2\*ArcTan[(-3 + x)/4] + 3\*Log[25 - 6\*x + x^2]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + x^2}{25 - 6x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^2)/(25 - 6\*x + x^2), x]

[Out] IntegrateAlgebraic[(-1 + x^2)/(25 - 6\*x + x^2), x]

**fricas** [A] time = 1.58, size = 21, normalized size = 0.91

$$x - 2 \arctan\left(\frac{1}{4}x - \frac{3}{4}\right) + 3 \log(x^2 - 6x + 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2-6\*x+25),x, algorithm="fricas")

[Out] x - 2\*arctan(1/4\*x - 3/4) + 3\*log(x^2 - 6\*x + 25)

**giac** [A] time = 0.15, size = 21, normalized size = 0.91

$$x - 2 \arctan\left(\frac{1}{4}x - \frac{3}{4}\right) + 3 \log(x^2 - 6x + 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2-6\*x+25),x, algorithm="giac")

[Out] x - 2\*arctan(1/4\*x - 3/4) + 3\*log(x^2 - 6\*x + 25)

**maple** [A] time = 0.00, size = 22, normalized size = 0.96

$$x - 2 \arctan\left(\frac{x}{4} - \frac{3}{4}\right) + 3 \ln(x^2 - 6x + 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^2-6\*x+25),x)

[Out] x-2\*arctan(-3/4+1/4\*x)+3\*ln(x^2-6\*x+25)

**maxima** [A] time = 0.95, size = 21, normalized size = 0.91

$$x - 2 \arctan\left(\frac{1}{4}x - \frac{3}{4}\right) + 3 \log(x^2 - 6x + 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2-6\*x+25),x, algorithm="maxima")

[Out] x - 2\*arctan(1/4\*x - 3/4) + 3\*log(x^2 - 6\*x + 25)

**mupad** [B] time = 0.04, size = 21, normalized size = 0.91

$$x + 3 \ln(x^2 - 6x + 25) - 2 \operatorname{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)/(x^2 - 6\*x + 25),x)



[Out]  $x + 3 \log(x^2 - 6x + 25) - 2 \operatorname{atan}(x/4 - 3/4)$

sympy [A] time = 0.11, size = 22, normalized size = 0.96

$$x + 3 \log(x^2 - 6x + 25) - 2 \operatorname{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-1)/(x**2-6*x+25),x)`

[Out]  $x + 3 \log(x^2 - 6x + 25) - 2 \operatorname{atan}(x/4 - 3/4)$

$$3.166 \quad \int \frac{-10+3x^2}{4-4x+x^2} dx$$

Optimal. Leaf size=21

$$3x + \frac{2}{2-x} + 12 \log(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {27, 697}

$$3x + \frac{2}{2-x} + 12 \log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(-10 + 3\*x^2)/(4 - 4\*x + x^2), x]

[Out] 2/(2 - x) + 3\*x + 12\*Log[2 - x]

Rule 27

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[u\*Cancel[(b/2 + c\*x)^(2\*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4\*a\*c, 0] && IntegerQ[p]

Rule 697

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{-10+3x^2}{4-4x+x^2} dx &= \int \frac{-10+3x^2}{(-2+x)^2} dx \\ &= \int \left( 3 + \frac{2}{(-2+x)^2} + \frac{12}{-2+x} \right) dx \\ &= \frac{2}{2-x} + 3x + 12 \log(2-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$3(x-2) - \frac{2}{x-2} + 12 \log(x-2)$$

Antiderivative was successfully verified.

[In] Integrate[(-10 + 3\*x^2)/(4 - 4\*x + x^2), x]

[Out] -2/(-2 + x) + 3\*(-2 + x) + 12\*Log[-2 + x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-10 + 3x^2}{4 - 4x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-10 + 3\*x^2)/(4 - 4\*x + x^2), x]

[Out] IntegrateAlgebraic[(-10 + 3\*x^2)/(4 - 4\*x + x^2), x]

**fricas** [A] time = 3.10, size = 25, normalized size = 1.19

$$\frac{3x^2 + 12(x - 2)\log(x - 2) - 6x - 2}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-10)/(x^2-4\*x+4), x, algorithm="fricas")

[Out] (3\*x^2 + 12\*(x - 2)\*log(x - 2) - 6\*x - 2)/(x - 2)

**giac** [A] time = 0.17, size = 18, normalized size = 0.86

$$3x - \frac{2}{x - 2} + 12 \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-10)/(x^2-4\*x+4), x, algorithm="giac")

[Out] 3\*x - 2/(x - 2) + 12\*log(abs(x - 2))

**maple** [A] time = 0.01, size = 18, normalized size = 0.86

$$3x + 12 \ln(x - 2) - \frac{2}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2-10)/(x^2-4\*x+4), x)

[Out] 3\*x+12\*ln(x-2)-2/(x-2)

**maxima [A]** time = 0.42, size = 17, normalized size = 0.81

$$3x - \frac{2}{x-2} + 12 \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-10)/(x^2-4\*x+4),x, algorithm="maxima")

[Out] 3\*x - 2/(x - 2) + 12\*log(x - 2)

**mupad [B]** time = 0.04, size = 17, normalized size = 0.81

$$3x + 12 \ln(x-2) - \frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2 - 10)/(x^2 - 4\*x + 4),x)

[Out] 3\*x + 12\*log(x - 2) - 2/(x - 2)

**sympy [A]** time = 0.09, size = 14, normalized size = 0.67

$$3x + 12 \log(x-2) - \frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x\*\*2-10)/(x\*\*2-4\*x+4),x)

[Out] 3\*x + 12\*log(x - 2) - 2/(x - 2)

$$3.167 \quad \int \frac{8+x^2}{6-5x+x^2} dx$$

Optimal. Leaf size=18

$$x - 12 \log(2 - x) + 17 \log(3 - x)$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1657, 632, 31}

$$x - 12 \log(2 - x) + 17 \log(3 - x)$$

Antiderivative was successfully verified.

[In] Int[(8 + x^2)/(6 - 5\*x + x^2), x]

[Out] x - 12\*Log[2 - x] + 17\*Log[3 - x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 632

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(c\*d - e\*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c\*x), x], x] - Dist[(c\*d - e\*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rubi steps

$$\begin{aligned}
\int \frac{8+x^2}{6-5x+x^2} dx &= \int \left(1 + \frac{2+5x}{6-5x+x^2}\right) dx \\
&= x + \int \frac{2+5x}{6-5x+x^2} dx \\
&= x - 12 \int \frac{1}{-2+x} dx + 17 \int \frac{1}{-3+x} dx \\
&= x - 12 \log(2-x) + 17 \log(3-x)
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$x - 12 \log(2 - x) + 17 \log(3 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(8 + x^2)/(6 - 5\*x + x^2), x]

[Out] x - 12\*Log[2 - x] + 17\*Log[3 - x]

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8+x^2}{6-5x+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + x^2)/(6 - 5\*x + x^2), x]

[Out] IntegrateAlgebraic[(8 + x^2)/(6 - 5\*x + x^2), x]

**fricas** [A] time = 1.07, size = 14, normalized size = 0.78

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+8)/(x^2-5\*x+6), x, algorithm="fricas")

[Out] x - 12\*log(x - 2) + 17\*log(x - 3)

**giac** [A] time = 0.15, size = 16, normalized size = 0.89

$$x - 12 \log(|x - 2|) + 17 \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+8)/(x^2-5\*x+6),x, algorithm="giac")

[Out]  $x - 12 \cdot \log(\text{abs}(x - 2)) + 17 \cdot \log(\text{abs}(x - 3))$

**maple** [A] time = 0.01, size = 15, normalized size = 0.83

$$x + 17 \ln(x - 3) - 12 \ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+8)/(x^2-5\*x+6),x)

[Out]  $x - 12 \cdot \ln(x - 2) + 17 \cdot \ln(x - 3)$

**maxima** [A] time = 0.43, size = 14, normalized size = 0.78

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+8)/(x^2-5\*x+6),x, algorithm="maxima")

[Out]  $x - 12 \cdot \log(x - 2) + 17 \cdot \log(x - 3)$

**mupad** [B] time = 3.92, size = 14, normalized size = 0.78

$$x - 12 \ln(x - 2) + 17 \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 8)/(x^2 - 5\*x + 6),x)

[Out]  $x - 12 \cdot \log(x - 2) + 17 \cdot \log(x - 3)$

**sympy** [A] time = 0.11, size = 14, normalized size = 0.78

$$x + 17 \log(x - 3) - 12 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+8)/(x\*\*2-5\*x+6),x)

[Out]  $x + 17 \cdot \log(x - 3) - 12 \cdot \log(x - 2)$

$$3.168 \quad \int \frac{-4+3x+x^2}{-8-2x+x^2} dx$$

Optimal. Leaf size=14

$$x + 4 \log(4 - x) + \log(x + 2)$$

**Rubi [A]** time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1657, 632, 31}

$$x + 4 \log(4 - x) + \log(x + 2)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 3\*x + x^2)/(-8 - 2\*x + x^2), x]

[Out] x + 4\*Log[4 - x] + Log[2 + x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 632

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(c\*d - e\*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c\*x), x], x] - Dist[(c\*d - e\*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c\*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps



$$\begin{aligned}
\int \frac{-4 + 3x + x^2}{-8 - 2x + x^2} dx &= \int \left( 1 + \frac{4 + 5x}{-8 - 2x + x^2} \right) dx \\
&= x + \int \frac{4 + 5x}{-8 - 2x + x^2} dx \\
&= x + 4 \int \frac{1}{-4 + x} dx + \int \frac{1}{2 + x} dx \\
&= x + 4 \log(4 - x) + \log(2 + x)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 1.00

$$x + 4 \log(4 - x) + \log(x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 3\*x + x^2)/(-8 - 2\*x + x^2), x]

[Out] x + 4\*Log[4 - x] + Log[2 + x]

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4 + 3x + x^2}{-8 - 2x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 3\*x + x^2)/(-8 - 2\*x + x^2), x]

[Out] IntegrateAlgebraic[(-4 + 3\*x + x^2)/(-8 - 2\*x + x^2), x]

**fricas [A]** time = 0.74, size = 12, normalized size = 0.86

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3\*x-4)/(x^2-2\*x-8), x, algorithm="fricas")

[Out] x + log(x + 2) + 4\*log(x - 4)

**giac [A]** time = 0.16, size = 14, normalized size = 1.00

$$x + \log(|x + 2|) + 4 \log(|x - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3\*x-4)/(x^2-2\*x-8),x, algorithm="giac")

[Out] x + log(abs(x + 2)) + 4\*log(abs(x - 4))

**maple [A]** time = 0.01, size = 13, normalized size = 0.93

$$x + \ln(x + 2) + 4 \ln(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3\*x-4)/(x^2-2\*x-8),x)

[Out] x+ln(2+x)+4\*ln(x-4)

**maxima [A]** time = 0.43, size = 12, normalized size = 0.86

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3\*x-4)/(x^2-2\*x-8),x, algorithm="maxima")

[Out] x + log(x + 2) + 4\*log(x - 4)

**mupad [B]** time = 3.85, size = 12, normalized size = 0.86

$$x + \ln(x + 2) + 4 \ln(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3\*x + x^2 - 4)/(2\*x - x^2 + 8),x)

[Out] x + log(x + 2) + 4\*log(x - 4)

**sympy [A]** time = 0.11, size = 12, normalized size = 0.86

$$x + 4 \log(x - 4) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+3\*x-4)/(x\*\*2-2\*x-8),x)

[Out] x + 4\*log(x - 4) + log(x + 2)

$$3.169 \quad \int \frac{7+5x+4x^2}{5+4x+4x^2} dx$$

Optimal. Leaf size=27

$$\frac{1}{8} \log(4x^2 + 4x + 5) + x + \frac{3}{8} \tan^{-1}\left(x + \frac{1}{2}\right)$$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8} \log(4x^2 + 4x + 5) + x + \frac{3}{8} \tan^{-1}\left(x + \frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(7 + 5\*x + 4\*x^2)/(5 + 4\*x + 4\*x^2), x]

[Out] x + (3\*ArcTan[1/2 + x])/8 + Log[5 + 4\*x + 4\*x^2]/8

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

`Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[Expand  
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,  
x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned}
 \int \frac{7 + 5x + 4x^2}{5 + 4x + 4x^2} dx &= \int \left( 1 + \frac{2 + x}{5 + 4x + 4x^2} \right) dx \\
 &= x + \int \frac{2 + x}{5 + 4x + 4x^2} dx \\
 &= x + \frac{1}{8} \int \frac{4 + 8x}{5 + 4x + 4x^2} dx + \frac{3}{2} \int \frac{1}{5 + 4x + 4x^2} dx \\
 &= x + \frac{1}{8} \log(5 + 4x + 4x^2) - 3 \operatorname{Subst} \left( \int \frac{1}{-64 - x^2} dx, x, 4 + 8x \right) \\
 &= x + \frac{3}{8} \tan^{-1} \left( \frac{1}{2} + x \right) + \frac{1}{8} \log(5 + 4x + 4x^2)
 \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{8} \log(4x^2 + 4x + 5) + x + \frac{3}{8} \tan^{-1} \left( \frac{1}{2}(2x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5\*x + 4\*x^2)/(5 + 4\*x + 4\*x^2), x]

[Out] x + (3\*ArcTan[(1 + 2\*x)/2])/8 + Log[5 + 4\*x + 4\*x^2]/8

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7 + 5x + 4x^2}{5 + 4x + 4x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(7 + 5\*x + 4\*x^2)/(5 + 4\*x + 4\*x^2), x]

[Out] IntegrateAlgebraic[(7 + 5\*x + 4\*x^2)/(5 + 4\*x + 4\*x^2), x]

**fricas** [A] time = 0.80, size = 21, normalized size = 0.78

$$x + \frac{3}{8} \arctan \left( x + \frac{1}{2} \right) + \frac{1}{8} \log(4x^2 + 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+5\*x+7)/(4\*x^2+4\*x+5),x, algorithm="fricas")

[Out] x + 3/8\*arctan(x + 1/2) + 1/8\*log(4\*x^2 + 4\*x + 5)

**giac** [A] time = 0.15, size = 21, normalized size = 0.78

$$x + \frac{3}{8} \arctan\left(x + \frac{1}{2}\right) + \frac{1}{8} \log(4x^2 + 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+5\*x+7)/(4\*x^2+4\*x+5),x, algorithm="giac")

[Out] x + 3/8\*arctan(x + 1/2) + 1/8\*log(4\*x^2 + 4\*x + 5)

**maple** [A] time = 0.00, size = 22, normalized size = 0.81

$$x + \frac{3 \arctan\left(x + \frac{1}{2}\right) + \ln(4x^2 + 4x + 5)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+5\*x+7)/(4\*x^2+4\*x+5),x)

[Out] x+3/8\*arctan(x+1/2)+1/8\*ln(4\*x^2+4\*x+5)

**maxima** [A] time = 0.94, size = 21, normalized size = 0.78

$$x + \frac{3}{8} \arctan\left(x + \frac{1}{2}\right) + \frac{1}{8} \log(4x^2 + 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+5\*x+7)/(4\*x^2+4\*x+5),x, algorithm="maxima")

[Out] x + 3/8\*arctan(x + 1/2) + 1/8\*log(4\*x^2 + 4\*x + 5)

**mupad** [B] time = 3.80, size = 17, normalized size = 0.63

$$x + \frac{\ln\left(x^2 + x + \frac{5}{4}\right) + 3 \operatorname{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x + 4\*x^2 + 7)/(4\*x + 4\*x^2 + 5),x)

[Out]  $x + \log(x + x^2 + 5/4)/8 + (3*\operatorname{atan}(x + 1/2))/8$

sympy [A] time = 0.12, size = 22, normalized size = 0.81

$$x + \frac{\log\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3 \operatorname{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)`

[Out]  $x + \log(x^2 + x + 5/4)/8 + 3*\operatorname{atan}(x + 1/2)/8$

$$3.170 \quad \int \frac{2-x+x^2}{-5+2x+x^2} dx$$

Optimal. Leaf size=48

$$x - \frac{1}{6}(9 - 5\sqrt{6}) \log(x - \sqrt{6} + 1) - \frac{1}{6}(9 + 5\sqrt{6}) \log(x + \sqrt{6} + 1)$$

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1657, 632, 31}

$$x - \frac{1}{6}(9 - 5\sqrt{6}) \log(x - \sqrt{6} + 1) - \frac{1}{6}(9 + 5\sqrt{6}) \log(x + \sqrt{6} + 1)$$

Antiderivative was successfully verified.

[In] Int[(2 - x + x^2)/(-5 + 2\*x + x^2), x]

[Out] x - ((9 - 5\*Sqrt[6])\*Log[1 - Sqrt[6] + x])/6 - ((9 + 5\*Sqrt[6])\*Log[1 + Sqrt[6] + x])/6

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(c\*d - e\*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c\*x), x], x] - Dist[(c\*d - e\*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Int[Expand[Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{2-x+x^2}{-5+2x+x^2} dx &= \int \left( 1 + \frac{7-3x}{-5+2x+x^2} \right) dx \\
&= x + \int \frac{7-3x}{-5+2x+x^2} dx \\
&= x + \frac{1}{6}(-9+5\sqrt{6}) \int \frac{1}{1-\sqrt{6}+x} dx - \frac{1}{6}(9+5\sqrt{6}) \int \frac{1}{1+\sqrt{6}+x} dx \\
&= x - \frac{1}{6}(9-5\sqrt{6}) \log(1-\sqrt{6}+x) - \frac{1}{6}(9+5\sqrt{6}) \log(1+\sqrt{6}+x)
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 48, normalized size = 1.00

$$x + \frac{1}{6}(5\sqrt{6}-9) \log(-x + \sqrt{6} - 1) + \frac{1}{6}(-9 - 5\sqrt{6}) \log(x + \sqrt{6} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x + x^2)/(-5 + 2\*x + x^2), x]

[Out] x + ((-9 + 5\*Sqrt[6])\*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5\*Sqrt[6])\*Log[1 + Sqrt[6] + x])/6

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-x+x^2}{-5+2x+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - x + x^2)/(-5 + 2\*x + x^2), x]

[Out] IntegrateAlgebraic[(2 - x + x^2)/(-5 + 2\*x + x^2), x]

**fricas [A]** time = 0.74, size = 55, normalized size = 1.15

$$\frac{5}{6} \sqrt{3} \sqrt{2} \log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1) - x^2 - 2x - 7}{x^2 + 2x - 5}\right) + x - \frac{3}{2} \log(x^2 + 2x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x+2)/(x^2+2\*x-5), x, algorithm="fricas")

[Out] 5/6\*sqrt(3)\*sqrt(2)\*log(-(2\*sqrt(3)\*sqrt(2)\*(x + 1) - x^2 - 2\*x - 7)/(x^2 + 2\*x - 5)) + x - 3/2\*log(x^2 + 2\*x - 5)



**giac [A]** time = 0.19, size = 45, normalized size = 0.94

$$\frac{5}{6} \sqrt{6} \log \left( \frac{|2x - 2\sqrt{6} + 2|}{|2x + 2\sqrt{6} + 2|} \right) + x - \frac{3}{2} \log(|x^2 + 2x - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x+2)/(x^2+2\*x-5),x, algorithm="giac")

[Out] 5/6\*sqrt(6)\*log(abs(2\*x - 2\*sqrt(6) + 2)/abs(2\*x + 2\*sqrt(6) + 2)) + x - 3/2\*log(abs(x^2 + 2\*x - 5))

**maple [A]** time = 0.00, size = 30, normalized size = 0.62

$$x - \frac{5\sqrt{6} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)}{3} - \frac{3 \ln(x^2 + 2x - 5)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-x+2)/(x^2+2\*x-5),x)

[Out] x-3/2\*ln(x^2+2\*x-5)-5/3\*6^(1/2)\*arctanh(1/12\*(2\*x+2)\*6^(1/2))

**maxima [A]** time = 0.96, size = 36, normalized size = 0.75

$$\frac{5}{6} \sqrt{6} \log \left( \frac{x - \sqrt{6} + 1}{x + \sqrt{6} + 1} \right) + x - \frac{3}{2} \log(x^2 + 2x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x+2)/(x^2+2\*x-5),x, algorithm="maxima")

[Out] 5/6\*sqrt(6)\*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2\*log(x^2 + 2\*x - 5)

**mupad [B]** time = 0.11, size = 35, normalized size = 0.73

$$x - \ln(x + \sqrt{6} + 1) \left( \frac{5\sqrt{6}}{6} + \frac{3}{2} \right) + \ln(x - \sqrt{6} + 1) \left( \frac{5\sqrt{6}}{6} - \frac{3}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - x + 2)/(2\*x + x^2 - 5),x)

[Out] x - log(x + 6^(1/2) + 1)\*((5\*6^(1/2))/6 + 3/2) + log(x - 6^(1/2) + 1)\*((5\*6^(1/2))/6 - 3/2)

sympy [A] time = 0.12, size = 46, normalized size = 0.96

$$x + \left(-\frac{5\sqrt{6}}{6} - \frac{3}{2}\right) \log(x + 1 + \sqrt{6}) + \left(-\frac{3}{2} + \frac{5\sqrt{6}}{6}\right) \log(x - \sqrt{6} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2-x+2)/(x\*\*2+2\*x-5),x)

[Out] x + (-5\*sqrt(6)/6 - 3/2)\*log(x + 1 + sqrt(6)) + (-3/2 + 5\*sqrt(6)/6)\*log(x - sqrt(6) + 1)

$$3.171 \quad \int \frac{1+4x+3x^2}{(4+7x+2x^2)^2} dx$$

Optimal. Leaf size=21

$$-\frac{3x+2}{2(2x^2+7x+4)}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {1660, 8}

$$-\frac{3x+2}{2(2x^2+7x+4)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x + 3\*x^2)/(4 + 7\*x + 2\*x^2)^2, x]

[Out] -(2 + 3\*x)/(2\*(4 + 7\*x + 2\*x^2))

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1+4x+3x^2}{(4+7x+2x^2)^2} dx &= -\frac{2+3x}{2(4+7x+2x^2)} - \frac{\int 0 dx}{17} \\ &= -\frac{2+3x}{2(4+7x+2x^2)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{-3x - 2}{2(2x^2 + 7x + 4)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x + 3\*x^2)/(4 + 7\*x + 2\*x^2)^2,x]

[Out] (-2 - 3\*x)/(2\*(4 + 7\*x + 2\*x^2))

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 4x + 3x^2}{(4 + 7x + 2x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4\*x + 3\*x^2)/(4 + 7\*x + 2\*x^2)^2,x]

[Out] IntegrateAlgebraic[(1 + 4\*x + 3\*x^2)/(4 + 7\*x + 2\*x^2)^2, x]

**fricas** [A] time = 0.78, size = 19, normalized size = 0.90

$$-\frac{3x + 2}{2(2x^2 + 7x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+4\*x+1)/(2\*x^2+7\*x+4)^2,x, algorithm="fricas")

[Out] -1/2\*(3\*x + 2)/(2\*x^2 + 7\*x + 4)

**giac** [A] time = 0.16, size = 19, normalized size = 0.90

$$-\frac{3x + 2}{2(2x^2 + 7x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2+4\*x+1)/(2\*x^2+7\*x+4)^2,x, algorithm="giac")

[Out] -1/2\*(3\*x + 2)/(2\*x^2 + 7\*x + 4)

**maple** [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{-\frac{3x}{4} - \frac{1}{2}}{x^2 + \frac{7}{2}x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x)`

[Out]  $(-3/4*x-1/2)/(x^2+7/2*x+2)$

**maxima** [A] time = 0.43, size = 19, normalized size = 0.90

$$-\frac{3x+2}{2(2x^2+7x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")`

[Out]  $-1/2*(3*x+2)/(2*x^2+7*x+4)$

**mupad** [B] time = 3.84, size = 17, normalized size = 0.81

$$-\frac{\frac{3x}{4} + \frac{1}{2}}{x^2 + \frac{7x}{2} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x+3*x^2+1)/(7*x+2*x^2+4)^2,x)`

[Out]  $-((3*x)/4+1/2)/((7*x)/2+x^2+2)$

**sympy** [A] time = 0.12, size = 15, normalized size = 0.71

$$\frac{-3x-2}{4x^2+14x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)`

[Out]  $(-3*x-2)/(4*x**2+14*x+8)$

$$3.172 \quad \int \frac{1+x+x^2}{(3+2x+x^2)^2} dx$$

Optimal. Leaf size=39

$$\frac{1-x}{4(x^2+2x+3)} + \frac{3 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {1660, 12, 618, 204}

$$\frac{1-x}{4(x^2+2x+3)} + \frac{3 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2)/(3 + 2\*x + x^2)^2,x]

[Out] (1 - x)/(4\*(3 + 2\*x + x^2)) + (3\*ArcTan[(1 + x)/Sqrt[2]])/(4\*Sqrt[2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^

$(p + 1)/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /;$  FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^2}{(3+2x+x^2)^2} dx &= \frac{1-x}{4(3+2x+x^2)} + \frac{1}{8} \int \frac{6}{3+2x+x^2} dx \\ &= \frac{1-x}{4(3+2x+x^2)} + \frac{3}{4} \int \frac{1}{3+2x+x^2} dx \\ &= \frac{1-x}{4(3+2x+x^2)} - \frac{3}{2} \text{Subst}\left(\int \frac{1}{-8-x^2} dx, x, 2+2x\right) \\ &= \frac{1-x}{4(3+2x+x^2)} + \frac{3 \tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 39, normalized size = 1.00

$$\frac{1-x}{4(x^2+2x+3)} + \frac{3 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2)/(3 + 2\*x + x^2)^2, x]

[Out] (1 - x)/(4\*(3 + 2\*x + x^2)) + (3\*ArcTan[(1 + x)/Sqrt[2]])/(4\*Sqrt[2])

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2)/(3 + 2\*x + x^2)^2, x]

[Out] IntegrateAlgebraic[(1 + x + x^2)/(3 + 2\*x + x^2)^2, x]

**fricas** [A] time = 0.86, size = 39, normalized size = 1.00

$$\frac{3\sqrt{2}(x^2 + 2x + 3)\arctan\left(\frac{1}{2}\sqrt{2}(x + 1)\right) - 2x + 2}{8(x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+2\*x+3)^2,x, algorithm="fricas")

[Out] 1/8\*(3\*sqrt(2)\*(x^2 + 2\*x + 3)\*arctan(1/2\*sqrt(2)\*(x + 1)) - 2\*x + 2)/(x^2 + 2\*x + 3)

**giac** [A] time = 0.16, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x + 1)\right) - \frac{x - 1}{4(x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+2\*x+3)^2,x, algorithm="giac")

[Out] 3/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(x + 1)) - 1/4\*(x - 1)/(x^2 + 2\*x + 3)

**maple** [A] time = 0.01, size = 34, normalized size = 0.87

$$\frac{3\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{8} + \frac{-\frac{x}{4} + \frac{1}{4}}{x^2 + 2x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/(x^2+2\*x+3)^2,x)

[Out] (-1/4\*x+1/4)/(x^2+2\*x+3)+3/8\*2^(1/2)\*arctan(1/4\*(2\*x+2)\*2^(1/2))

**maxima** [A] time = 0.96, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x + 1)\right) - \frac{x - 1}{4(x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+2\*x+3)^2,x, algorithm="maxima")

[Out] 3/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(x + 1)) - 1/4\*(x - 1)/(x^2 + 2\*x + 3)



mupad [B] time = 3.84, size = 36, normalized size = 0.92

$$\frac{3\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2} + \frac{\sqrt{2}}{2}\right)}{8} - \frac{\frac{x}{4} - \frac{1}{4}}{x^2 + 2x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 + 1)/(2*x + x^2 + 3)^2, x)`

[Out] `(3*2^(1/2)*atan((2^(1/2)*x)/2 + 2^(1/2)/2))/8 - (x/4 - 1/4)/(2*x + x^2 + 3)`

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$\frac{1 - x}{4x^2 + 8x + 12} + \frac{3\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2} + \frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x+1)/(x**2+2*x+3)**2, x)`

[Out] `(1 - x)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8`

$$3.173 \quad \int \frac{-1+2x+5x^2}{(1+x+x^2)^4} dx$$

Optimal. Leaf size=11

$$-\frac{x}{(x^2+x+1)^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {1588}

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2\*x + 5\*x^2)/(1 + x + x^2)^4,x]

[Out] -(x/(1 + x + x^2)^3)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4} dx = -\frac{x}{(1+x+x^2)^3}$$

**Mathematica [A]** time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2\*x + 5\*x^2)/(1 + x + x^2)^4,x]

[Out]  $-(x/(1 + x + x^2)^3)$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 2x + 5x^2}{(1 + x + x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 2\*x + 5\*x^2)/(1 + x + x^2)^4,x]

[Out] IntegrateAlgebraic[(-1 + 2\*x + 5\*x^2)/(1 + x + x^2)^4, x]

**fricas** [B] time = 0.82, size = 33, normalized size = 3.00

$$-\frac{x}{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x-1)/(x^2+x+1)^4,x, algorithm="fricas")

[Out]  $-x/(x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1)$

**giac** [A] time = 0.15, size = 11, normalized size = 1.00

$$-\frac{x}{(x^2 + x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x-1)/(x^2+x+1)^4,x, algorithm="giac")

[Out]  $-x/(x^2 + x + 1)^3$

**maple** [A] time = 0.00, size = 12, normalized size = 1.09

$$-\frac{x}{(x^2 + x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x-1)/(x^2+x+1)^4,x)

[Out]  $-x/(x^2+x+1)^3$

**maxima** [B] time = 0.44, size = 33, normalized size = 3.00

$$-\frac{x}{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x-1)/(x^2+x+1)^4,x, algorithm="maxima")

[Out] -x/(x^6 + 3\*x^5 + 6\*x^4 + 7\*x^3 + 6\*x^2 + 3\*x + 1)

**mupad** [B] time = 3.80, size = 11, normalized size = 1.00

$$-\frac{x}{(x^2 + x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 5\*x^2 - 1)/(x + x^2 + 1)^4,x)

[Out] -x/(x + x^2 + 1)^3

**sympy** [B] time = 0.13, size = 31, normalized size = 2.82

$$-\frac{x}{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x-1)/(x\*\*2+x+1)\*\*4,x)

[Out] -x/(x\*\*6 + 3\*x\*\*5 + 6\*x\*\*4 + 7\*x\*\*3 + 6\*x\*\*2 + 3\*x + 1)

$$3.174 \quad \int (a + bx + cx^2)^{5/2} (A + Cx^2) dx$$

**Optimal.** Leaf size=267

$$\frac{5(b^2 - 4ac)^3 (-4acC + 32Ac^2 + 9b^2C) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}} + \frac{5(b^2 - 4ac)^2 (b + 2cx)\sqrt{a + bx + cx^2} (-4acC + 32Ac^2 + 9b^2C)}{16384c^5}$$

**Rubi [A]** time = 0.24, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)(a+bx+cx^2)^{5/2}(-4acC+32Ac^2+9b^2C)}{384c^3} - \frac{5(b^2-4ac)(b+2cx)(a+bx+cx^2)^{3/2}(-4acC+32Ac^2+9b^2C)}{6144c^4} + \frac{5(b^2-4ac)^2(b+2cx)\sqrt{a+bx+cx^2}(-4acC+32Ac^2+9b^2C)}{16384c^5} - \frac{5(b^2-4ac)^3(-4acC+32Ac^2+9b^2C)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}} - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(5/2)\*(A + C\*x^2), x]

[Out] (5\*(b^2 - 4\*a\*c)^2\*(32\*A\*c^2 + 9\*b^2\*C - 4\*a\*c\*C)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(16384\*c^5) - (5\*(b^2 - 4\*a\*c)\*(32\*A\*c^2 + 9\*b^2\*C - 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(6144\*c^4) + ((32\*A\*c^2 + 9\*b^2\*C - 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(5/2))/(384\*c^3) - (9\*b\*C\*(a + b\*x + c\*x^2)^(7/2))/(112\*c^2) + (C\*x\*(a + b\*x + c\*x^2)^(7/2))/(8\*c) - (5\*(b^2 - 4\*a\*c)^3\*(32\*A\*c^2 + 9\*b^2\*C - 4\*a\*c\*C)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(32768\*c^(11/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:= Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{5/2} (A + Cx^2) dx &= \frac{Cx(a + bx + cx^2)^{7/2}}{8c} + \frac{\int \left(8Ac - aC - \frac{9bCx}{2}\right) (a + bx + cx^2)^{5/2} dx}{8c} \\
&= -\frac{9bC(a + bx + cx^2)^{7/2}}{112c^2} + \frac{Cx(a + bx + cx^2)^{7/2}}{8c} + \frac{\left(\frac{9b^2C}{2} + 2c(8Ac - aC)\right) \int (a + bx + cx^2)^{3/2} dx}{16c^2} \\
&= \frac{(32Ac^2 + 9b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{5/2}}{384c^3} - \frac{9bC(a + bx + cx^2)^{7/2}}{112c^2} \\
&= -\frac{5(b^2 - 4ac)(32Ac^2 + 9b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{3/2}}{6144c^4} + \frac{(32Ac^2 + 9b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{5/2}}{16384c^5} \\
&= \frac{5(b^2 - 4ac)^2(32Ac^2 + 9b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{16384c^5} - \frac{5(b^2 - 4ac)(32Ac^2 + 9b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{3/2}}{16384c^5} \\
&= \frac{5(b^2 - 4ac)^2(32Ac^2 + 9b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{16384c^5} - \frac{5(b^2 - 4ac)(32Ac^2 + 9b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{3/2}}{16384c^5} \\
&= \frac{5(b^2 - 4ac)^2(32Ac^2 + 9b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{16384c^5} - \frac{5(b^2 - 4ac)(32Ac^2 + 9b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{3/2}}{16384c^5}
\end{aligned}$$

**Mathematica [A]** time = 0.86, size = 344, normalized size = 1.29

$$\frac{1120A(b^2 - 4ac)\sqrt{16c^2(b + 2cx)(a + bx + cx^2)^{3/2} - 3(b^2 - 4ac)\sqrt{2}\sqrt{b + 2cx}\sqrt{a + bx + cx^2}}{(b^2 - 4ac)\sqrt{16c^2(b + 2cx)(a + bx + cx^2)^{3/2} - 3(b^2 - 4ac)\sqrt{2}\sqrt{b + 2cx}\sqrt{a + bx + cx^2}}} + 57344A(b + 2cx)(a + bx + cx^2)^{3/2} + \frac{7C(b^2 - 4ac)\sqrt{256c^2(b + 2cx)(a + bx + cx^2)^{3/2} - 5(b^2 - 4ac)\sqrt{16c^2(b + 2cx)(a + bx + cx^2)^{3/2} - 3(b^2 - 4ac)\sqrt{2}\sqrt{b + 2cx}\sqrt{a + bx + cx^2}}}{\sqrt{16c^2(b + 2cx)(a + bx + cx^2)^{3/2} - 3(b^2 - 4ac)\sqrt{2}\sqrt{b + 2cx}\sqrt{a + bx + cx^2}}} - \frac{35296C(a + bx + cx^2)^{3/2}}{\sqrt{16c^2(b + 2cx)(a + bx + cx^2)^{3/2} - 3(b^2 - 4ac)\sqrt{2}\sqrt{b + 2cx}\sqrt{a + bx + cx^2}}} + 86016Cx(a + bx + cx^2)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (57344*A*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - (55296*b*C*(a + x*(b + c*x))^(7/2))/c + 86016*C*x*(a + x*(b + c*x))^(7/2) - (1120*A*(b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (7*(9*b^2 - 4*a*c)*C*(256*c^(5/2)*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(9/2))/(688128*c)
```

**IntegrateAlgebraic [A]** time = 2.15, size = 489, normalized size = 1.83

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(3360*A*b^5*c^2 - 35840*a*A*b^3*c^3 + 118272*a^2*A*b*c^4 + 945*b^7*C - 10500*a*b^5*c*C + 37744*a^2*b^3*c^2*C - 42432*a^3*b*c^3*C - 2240*A*b^4*c^3*x + 21504*a*A*b^2*c^4*x + 236544*a^2*A*c^5*x - 630*b^6*c*C*x + 6328*a*b^4*c^2*C*x - 19104*a^2*b^2*c^3*C*x + 13440*a^3*c^4*C*x + 1792*A*b^3*c^4*x^2 + 279552*a*A*b*c^5*x^2 + 504*b^5*c^2*C*x^2 - 4544*a*b^3*c^3*C*x^2 + 11136*a^2*b*c^4*C*x^2 + 96768*A*b^2*c^5*x^3 + 186368*a*A*c^6*x^3 - 432*b^4*c^3*C*x^3 + 3456*a*b^2*c^4*C*x^3 + 105728*a^2*c^5*C*x^3 + 143360*A*b*c^6*x^4 + 384*b^3*c^4*C*x^4 + 157184*a*b*c^5*C*x^4 + 57344*A*c^7*x^5 + 62208*b^2*c^5*C*x^5 + 121856*a*c^6*C*x^5 + 101376*b*c^6*C*x^6 + 43008*c^7*C*x^7))/(344064*c^5) + (5*(32*A*b^6*c^2 - 384*a*A*b^4*c^3 + 1536*a^2*A*b^2*c^4 - 2048*a^3*A*c^5 + 9*b^8*C - 112*a*b^6*c*C + 480*a^2*b^4*c^2*C - 768*a^3*b^2*c^3*C + 256*a^4*c^4*C)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(32768*c^(11/2))
```

**fricas [B]** time = 1.66, size = 953, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a))*(2*
```

$$\begin{aligned}
& c*x + b)*\text{sqrt}(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C \\
& *b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476* \\
& C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^ \\
& 3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5) \\
& *c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + \\
& 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + \\
& 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 \\
& - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 7 \\
& 0*A*b^4)*c^4)*x)*\text{sqrt}(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C*b^8 - 112*C \\
& *a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^ \\
& 2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*\text{sqrt}(-c)*\text{arctan}(1/2*\text{sqrt}( \\
& c*x^2 + b*x + a)*(2*c*x + b)*\text{sqrt}(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*C \\
& *c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A \\
& a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a \\
& ^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7) \\
& *x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2* \\
& c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 \\
& - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - \\
& 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8 \\
& *A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 70*A*b^4)*c^4)*x)*\text{sqrt}(c*x^2 + b*x + a) \\
& )/c^6]
\end{aligned}$$

**giac [B]** time = 0.27, size = 482, normalized size = 1.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(5/2)\*(C\*x^2+A),x, algorithm="giac")

[Out]  $1/344064*\text{sqrt}(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c))*x + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228*C*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608*C*a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*C*a*b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x - (315*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 - 6720*C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^7*c^2 - 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C*a^3*b*c^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b^8 - 112*C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3 - 384*A*a*b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*\text{log}(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^(11/2))$

**maple [B]** time = 0.01, size = 997, normalized size = 3.73

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x)`

[Out] 
$$-9/112*b*C*(c*x^2+b*x+a)^{(7/2)}/c^2+1/8*C*x*(c*x^2+b*x+a)^{(7/2)}/c-95/2048*C/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*x*a-5/32*A/c*(c*x^2+b*x+a)^{(1/2)}*x*a*b^2+55/512*C/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x*a^2+25/384*C/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}*x*a+5/16*A/c^{(1/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^3-5/1024*A/c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^6-5/128*C*a^4/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-45/32768*C/c^{(11/2)}*b^8*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+3/128*C/c^3*b^3*(c*x^2+b*x+a)^{(5/2)}-15/2048*C/c^4*b^5*(c*x^2+b*x+a)^{(3/2)}+45/16384*C/c^5*b^7*(c*x^2+b*x+a)^{(1/2)}+1/12*A/c*(c*x^2+b*x+a)^{(5/2)}*b+5/24*A*(c*x^2+b*x+a)^{(3/2)}*x*a-5/192*A/c^2*(c*x^2+b*x+a)^{(3/2)}*b^3+5/16*A*(c*x^2+b*x+a)^{(1/2)}*x*a^2+5/512*A/c^3*(c*x^2+b*x+a)^{(1/2)}*b^5+1/6*A*(c*x^2+b*x+a)^{(5/2)}*x-15/1024*C/c^3*b^4*(c*x^2+b*x+a)^{(3/2)}*x+25/768*C/c^3*b^3*(c*x^2+b*x+a)^{(3/2)}*a-5/96*A/c*(c*x^2+b*x+a)^{(3/2)}*x*b^2+5/48*A/c*(c*x^2+b*x+a)^{(3/2)}*b*a+5/256*A/c^2*(c*x^2+b*x+a)^{(1/2)}*x*b^4+35/2048*C/c^{(9/2)}*b^6*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a-1/48*C*a/c*(c*x^2+b*x+a)^{(5/2)}*x-1/96*C*a/c^2*(c*x^2+b*x+a)^{(5/2)}*b-5/192*C*a^2/c*(c*x^2+b*x+a)^{(3/2)}*x+45/8192*C/c^4*b^6*(c*x^2+b*x+a)^{(1/2)}*x+55/1024*C/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*a^2-95/4096*C/c^4*b^5*(c*x^2+b*x+a)^{(1/2)}*a-5/384*C*a^2/c^2*(c*x^2+b*x+a)^{(3/2)}*b-5/128*C*a^3/c*(c*x^2+b*x+a)^{(1/2)}*x-5/256*C*a^3/c^2*(c*x^2+b*x+a)^{(1/2)}*b+3/64*C/c^2*b^2*(c*x^2+b*x+a)^{(5/2)}*x+15/128*C/c^{(5/2)}*b^2*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^3-75/1024*C/c^{(7/2)}*b^4*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2+5/32*A/c*(c*x^2+b*x+a)^{(1/2)}*b*a^2-5/64*A/c^2*(c*x^2+b*x+a)^{(1/2)}*b^3*a-15/64*A/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^2*a^2+15/256*A/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^4*a$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (Cx^2 + A) (cx^2 + bx + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)`

[Out] `int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Cx^2)(a + bx + cx^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A), x)`

[Out] `Integral((A + C*x**2)*(a + b*x + c*x**2)**(5/2), x)`

$$3.175 \quad \int (a + bx + cx^2)^{3/2} (A + Cx^2) dx$$

Optimal. Leaf size=212

$$\frac{(b^2 - 4ac)^2 (-4acC + 24Ac^2 + 7b^2C) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}} - \frac{(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2} (-4acC + 24Ac^2 + 7b^2C)}{512c^4}$$

Rubi [A] time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)(a+bx+cx^2)^{3/2}(-4acC+24Ac^2+7b^2C)}{192c^3} - \frac{(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}(-4acC+24Ac^2+7b^2C)}{512c^4} + \frac{(b^2-4ac)^2(-4acC+24Ac^2+7b^2C)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}} - \frac{7bC(a+bx+cx^2)^{5/2}}{60c^2} + \frac{Cx(a+bx+cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)\*(A + C\*x^2), x]

[Out] -((b^2 - 4\*a\*c)\*(24\*A\*c^2 + 7\*b^2\*C - 4\*a\*c\*C)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(512\*c^4) + ((24\*A\*c^2 + 7\*b^2\*C - 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(192\*c^3) - (7\*b\*C\*(a + b\*x + c\*x^2)^(5/2))/(60\*c^2) + (C\*x\*(a + b\*x + c\*x^2)^(5/2))/(6\*c) + ((b^2 - 4\*a\*c)^2\*(24\*A\*c^2 + 7\*b^2\*C - 4\*a\*c\*C)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(1024\*c^(9/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 640

```
Int[((d._) + (e._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{3/2} (A + Cx^2) dx &= \frac{Cx(a + bx + cx^2)^{5/2}}{6c} + \frac{\int \left(6Ac - aC - \frac{7bCx}{2}\right) (a + bx + cx^2)^{3/2} dx}{6c} \\
&= -\frac{7bC(a + bx + cx^2)^{5/2}}{60c^2} + \frac{Cx(a + bx + cx^2)^{5/2}}{6c} + \frac{\left(\frac{7b^2C}{2} + 2c(6Ac - aC)\right) \int (a + bx + cx^2)^{1/2} dx}{12c^2} \\
&= \frac{(24Ac^2 + 7b^2C - 4acC)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} - \frac{7bC(a + bx + cx^2)^{5/2}}{60c^2} \\
&= -\frac{(b^2 - 4ac)(24Ac^2 + 7b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24Ac^2 + 7b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{192c^3} \\
&= -\frac{(b^2 - 4ac)(24Ac^2 + 7b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24Ac^2 + 7b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{192c^3} \\
&= -\frac{(b^2 - 4ac)(24Ac^2 + 7b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(24Ac^2 + 7b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{192c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.58, size = 267, normalized size = 1.26

$$\frac{360A(b^2 - 4ac) \left( (b^2 - 4ac) \operatorname{tanh}^{-1} \left( \frac{b + 2cx}{2\sqrt{a + bx + cx^2}} \right) - 2\sqrt{c} (b + 2cx) \sqrt{a + bx + cx^2} \right) + 1920A(b + 2cx)(a + x(b + cx))^{3/2} + \frac{\left( 5(7b^2 - 4ac) \left( \frac{3(b^2 - 4ac) \left( (b^2 - 4ac) \operatorname{tanh}^{-1} \left( \frac{b + 2cx}{2\sqrt{a + bx + cx^2}} \right) - 2\sqrt{c} (b + 2cx) \sqrt{a + bx + cx^2} \right) + 16(b + 2cx)(a + x(b + cx))^{3/2}}{c^2} \right) - 1792b(a + x(b + cx))^{5/2} \right)}{c}}{15360c} + 2560Cx(a + x(b + cx))^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)\*(A + C\*x^2), x]

```
[Out] (1920*A*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 2560*C*x*(a + x*(b + c*x))^(5/2) + (360*A*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/c^(3/2) + (C*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x))*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/c^(5/2)))/c/(15360*c)
```

**IntegrateAlgebraic [A]** time = 1.12, size = 295, normalized size = 1.39

$\sqrt{b+cx^2} \frac{(-1296b^2c^2 + 480a^2c^2 + 2400abc^2 + 4800a^2c^2 + 7680a^3c^2 - 4320a^2c^2c^2 + 2880abc^2c^2 + 2240a^2c^2c^2 - 360a^3c^2c^2 + 240a^4c^2c^2 + 2880a^2c^2c^2 + 1920a^3c^2c^2 - 105b^2c^2 + 70b^2c^2c^2 + 56b^2c^2c^2 + 48b^2c^2c^2 + 1664b^2c^2c^2 + 1280b^2c^2c^2)}{7680c^4} \frac{(64b^2c^2 - 384b^2c^2 - 144b^2c^2 + 192a^2c^2 + 60ab^2c^2 - 24a^2c^2 - 7b^2c^2) \log(-2\sqrt{b+cx^2} + b + 2a)}{1024c^4}$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)*(A + C*x^2), x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-360*A*b^3*c^2 + 2400*a*A*b*c^3 - 105*b^5*C + 760*a*b^3*c*C - 1296*a^2*b*c^2*C + 240*A*b^2*c^3*x + 4800*a*A*c^4*x + 70*b^4*c*C*x - 432*a*b^2*c^2*C*x + 480*a^2*c^3*C*x + 2880*A*b*c^4*x^2 - 56*b^3*c^2*C*x^2 + 288*a*b*c^3*C*x^2 + 1920*A*c^5*x^3 + 48*b^2*c^3*C*x^3 + 2240*a*c^4*C*x^3 + 1664*b*c^4*C*x^4 + 1280*c^5*C*x^5))/(7680*c^4) + ((-24*A*b^4*c^2 + 192*a*A*b^2*c^3 - 384*a^2*A*c^4 - 7*b^6*C + 60*a*b^4*c*C - 144*a^2*b^2*c^2*C + 64*a^3*c^3*C)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(1024*c^(9/2))
```

**fricas [A]** time = 0.93, size = 605, normalized size = 2.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A), x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2
```

$$c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*\sqrt{(c*x^2 + b*x + a)}/c^5]$$

**giac** [A] time = 0.28, size = 297, normalized size = 1.40

$$\frac{1}{7680} \sqrt{c^2 + bx + a} \left( \frac{1}{2} \left( \frac{1}{2} \left( 8100 Ccx + 13 C^2 b^2 + \frac{3 C^2 a^2 + 140 C a^2 + 120 A a^2}{c^2} \right) - \frac{7 C^2 b^2 - 36 C a b^2 - 360 A b^2}{c^2} \right) + \frac{35 C^2 a^2 - 216 C a b^2 + 240 C a^2 a + 120 A b^2 a + 2400 A a^2}{c^2} - \frac{105 C^2 b^2 - 760 C a b^2 + 1296 C^2 a^2 + 360 A b^2 a - 2400 A a^2}{c^2} \right) \frac{(7 C^2 - 60 C a b^2 + 144 C a^2 b^2 + 24 A b^2 - 64 C^2 a^2 - 192 A a b^2 + 384 A b^2 a) \log \left( \frac{-2(\sqrt{c^2 + bx + a})\sqrt{c} - b}{1024 c^2} \right)}{1024 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(C\*x^2+A),x, algorithm="giac")

[Out]  $\frac{1}{7680} \sqrt{c*x^2 + b*x + a} * (2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*b^2*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a^2*b*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5 - \frac{1}{1024} * (7*C*b^6 - 60*C*a*b^4*c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3 + 384*A*a^2*c^4) * \log(\text{abs}(-2*(\sqrt{c})x - \sqrt{c*x^2 + b*x + a})*\sqrt{c} - b))/c^{9/2}$

**maple** [B] time = 0.01, size = 613, normalized size = 2.89

$$\frac{1}{8} C \sqrt{c^2 + bx + a} \left( \frac{1}{2} \left( \frac{1}{2} \left( 8100 Ccx + 13 C^2 b^2 + \frac{3 C^2 a^2 + 140 C a^2 + 120 A a^2}{c^2} \right) - \frac{7 C^2 b^2 - 36 C a b^2 - 360 A b^2}{c^2} \right) + \frac{35 C^2 a^2 - 216 C a b^2 + 240 C a^2 a + 120 A b^2 a + 2400 A a^2}{c^2} - \frac{105 C^2 b^2 - 760 C a b^2 + 1296 C^2 a^2 + 360 A b^2 a - 2400 A a^2}{c^2} \right) \frac{(7 C^2 - 60 C a b^2 + 144 C a^2 b^2 + 24 A b^2 - 64 C^2 a^2 - 192 A a b^2 + 384 A b^2 a) \log \left( \frac{-2(\sqrt{c^2 + bx + a})\sqrt{c} - b}{1024 c^2} \right)}{1024 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(C\*x^2+A),x)

[Out]  $\frac{1}{8} C \sqrt{c^2 + bx + a} \left( \frac{1}{2} \left( \frac{1}{2} \left( 8100 Ccx + 13 C^2 b^2 + \frac{3 C^2 a^2 + 140 C a^2 + 120 A a^2}{c^2} \right) - \frac{7 C^2 b^2 - 36 C a b^2 - 360 A b^2}{c^2} \right) + \frac{35 C^2 a^2 - 216 C a b^2 + 240 C a^2 a + 120 A b^2 a + 2400 A a^2}{c^2} - \frac{105 C^2 b^2 - 760 C a b^2 + 1296 C^2 a^2 + 360 A b^2 a - 2400 A a^2}{c^2} \right) \frac{(7 C^2 - 60 C a b^2 + 144 C a^2 b^2 + 24 A b^2 - 64 C^2 a^2 - 192 A a b^2 + 384 A b^2 a) \log \left( \frac{-2(\sqrt{c^2 + bx + a})\sqrt{c} - b}{1024 c^2} \right)}{1024 c^2}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (Cx^2 + A) (cx^2 + bx + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + C*x^2)*(a + b*x + c*x^2)^(3/2),x)`

[Out] `int((A + C*x^2)*(a + b*x + c*x^2)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Cx^2) (a + bx + cx^2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)`

[Out] `Integral((A + C*x**2)*(a + b*x + c*x**2)**(3/2), x)`

### 3.176 $\int \sqrt{a + bx + cx^2} (A + Cx^2) dx$

**Optimal.** Leaf size=157

$$\frac{(b^2 - 4ac)(-4acC + 16Ac^2 + 5b^2C) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}} + \frac{(b + 2cx)\sqrt{a + bx + cx^2} (-4acC + 16Ac^2 + 5b^2C)}{64c^3}$$

**Rubi [A]** time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} (-4acC + 16Ac^2 + 5b^2C)}{64c^3} - \frac{(b^2 - 4ac)(-4acC + 16Ac^2 + 5b^2C) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}} - \frac{5bC(a + bx + cx^2)^{3/2}}{24c^2} + \frac{Cx(a + bx + cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]\*(A + C\*x^2), x]

[Out] ((16\*A\*c^2 + 5\*b^2\*C - 4\*a\*c\*C)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(64\*c^3) - (5\*b\*C\*(a + b\*x + c\*x^2)^(3/2))/(24\*c^2) + (C\*x\*(a + b\*x + c\*x^2)^(3/2))/(4\*c) - ((b^2 - 4\*a\*c)\*(16\*A\*c^2 + 5\*b^2\*C - 4\*a\*c\*C)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(7/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640



```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  ] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + bx + cx^2} (A + Cx^2) dx &= \frac{Cx(a + bx + cx^2)^{3/2}}{4c} + \frac{\int \left(4Ac - aC - \frac{5bCx}{2}\right) \sqrt{a + bx + cx^2} dx}{4c} \\ &= -\frac{5bC(a + bx + cx^2)^{3/2}}{24c^2} + \frac{Cx(a + bx + cx^2)^{3/2}}{4c} + \frac{\left(\frac{5b^2C}{2} + 2c(4Ac - aC)\right) \int \sqrt{a + bx + cx^2} dx}{8c^2} \\ &= \frac{(16Ac^2 + 5b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} - \frac{5bC(a + bx + cx^2)^{3/2}}{24c^2} + \\ &= \frac{(16Ac^2 + 5b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} - \frac{5bC(a + bx + cx^2)^{3/2}}{24c^2} + \\ &= \frac{(16Ac^2 + 5b^2C - 4acC)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} - \frac{5bC(a + bx + cx^2)^{3/2}}{24c^2} + \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 144, normalized size = 0.92

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(C\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)+15b^3-10b^2cx\right)+48Ac^2\left(b+2cx\right)-3\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2), x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2
*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) - 3*(b^2 - 4*a*c)
```

$(16A^2c^2 + 5b^2C - 4a^2c^2) \operatorname{ArcTanh}\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right] / (384c^{7/2})$

**IntegrateAlgebraic [A]** time = 0.55, size = 161, normalized size = 1.03

$$\frac{(16a^2c^2C - 64aAc^3 - 24ab^2cC + 16Ab^2c^2 + 5b^4C) \log\left(-2\sqrt{c}\sqrt{a + bx + cx^2} + b + 2cx\right)}{128c^{7/2}} + \frac{\sqrt{a + bx + cx^2}(-52abcC + 24ac^2Cx + 48Abc^2 + 96Ac^3x + 15b^3C - 10b^2cCx + 8bc^2Cx^2 + 48c^3Cx^3)}{192c^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b\*x + c\*x^2]\*(A + C\*x^2), x]

[Out]  $(\sqrt{a + bx + cx^2} * (48Ab^2c^2 + 15b^3C - 52a^2b^2c^2 + 96A^2c^3x - 10b^2c^2Cx + 24a^2c^2Cx + 8b^2c^2Cx^2 + 48c^3Cx^3)) / (192c^3) + ((16A^2b^2c^2 - 64A^2Ac^3 + 5b^4C - 24a^2b^2c^2 + 16a^2c^2C) * \operatorname{Log}[b + 2cx - 2\sqrt{c}\sqrt{a + bx + cx^2}]) / (128c^{7/2})$

**fricas [A]** time = 0.79, size = 355, normalized size = 2.26

$$\frac{3(5C^3 - 24Ca^2b - 64Aa^3 + 16(C^2 + Ab^2)\sqrt{c}) \log\left(-2\sqrt{c}\sqrt{a + bx + cx^2} + b + 2cx\right) - 4(48Ca^2 + 8Cb^2 + 15C^2 - 52Ca^2 + 48Aa^2 - 2(5C^2 - 12Ca^2 - 48Aa^2)\sqrt{c^2 + bx + a}) \sqrt{a + bx + cx^2}}{384c^3} + \frac{3(5C^3 - 24Ca^2b - 64Aa^3 + 16(C^2 + Ab^2)\sqrt{c}) \operatorname{arctan}\left(\frac{\sqrt{a + bx + cx^2}}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right) + 2(48Ca^2 + 8Cb^2 + 15C^2 - 52Ca^2 + 48Aa^2 - 2(5C^2 - 12Ca^2 - 48Aa^2)\sqrt{c^2 + bx + a}) \sqrt{a + bx + a}}{384c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(C\*x^2+A), x, algorithm="fricas")

[Out]  $[-1/768 * (3 * (5 * C * b^4 - 24 * C * a * b^2 * c - 64 * A * a * c^3 + 16 * (C * a^2 + A * b^2) * c^2) * \operatorname{sqrt}(c) * \log(-8 * c^2 * x^2 - 8 * b * c * x - b^2 - 4 * \operatorname{sqrt}(c * x^2 + b * x + a) * (2 * c * x + b) * \operatorname{sqrt}(c) - 4 * a * c) - 4 * (48 * C * c^4 * x^3 + 8 * C * b * c^3 * x^2 + 15 * C * b^3 * c - 52 * C * a * b * c^2 + 48 * A * b * c^3 - 2 * (5 * C * b^2 * c^2 - 12 * C * a * c^3 - 48 * A * c^4) * x) * \operatorname{sqrt}(c * x^2 + b * x + a)) / c^4, 1/384 * (3 * (5 * C * b^4 - 24 * C * a * b^2 * c - 64 * A * a * c^3 + 16 * (C * a^2 + A * b^2) * c^2) * \operatorname{sqrt}(-c) * \operatorname{arctan}(1/2 * \operatorname{sqrt}(c * x^2 + b * x + a) * (2 * c * x + b) * \operatorname{sqrt}(-c) / (c^2 * x^2 + b * c * x + a * c)) + 2 * (48 * C * c^4 * x^3 + 8 * C * b * c^3 * x^2 + 15 * C * b^3 * c - 52 * C * a * b * c^2 + 48 * A * b * c^3 - 2 * (5 * C * b^2 * c^2 - 12 * C * a * c^3 - 48 * A * c^4) * x) * \operatorname{sqrt}(c * x^2 + b * x + a)) / c^4]$

**giac [A]** time = 0.22, size = 160, normalized size = 1.02

$$\frac{1}{192} \sqrt{cx^2 + bx + a} \left( 2 \left( 4 \left( 6Cx + \frac{Cb}{c} \right) x - \frac{5Cb^2c - 12Cac^2 - 48Ac^3}{c^3} \right) x + \frac{15Cb^3 - 52Cabc + 48Abc^2}{c^3} \right) + \frac{(5Cb^4 - 24Cab^2c + 16Ca^2c^2 + 16Ab^2c^2 - 64Aac^3) \log\left(-2\left(\sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right)}{128c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(C\*x^2+A), x, algorithm="giac")

[Out]  $1/192 * \operatorname{sqrt}(c * x^2 + b * x + a) * (2 * (4 * (6 * C * x + C * b / c) * x - (5 * C * b^2 * c - 12 * C * a * c^2 - 48 * A * c^3) / c^3) * x + (15 * C * b^3 - 52 * C * a * b * c + 48 * A * b * c^2) / c^3) + 1/128 * (5 * C * b^4 - 24 * C * a * b^2 * c + 16 * C * a^2 * c^2 + 16 * A * b^2 * c^2 - 64 * A * a * c^3) * \log(\operatorname{abs}(-2 * (\operatorname{sqrt}(c) * x - \operatorname{sqrt}(c * x^2 + b * x + a)) * \operatorname{sqrt}(c) - b)) / c^{7/2})$

**maple [B]** time = 0.01, size = 327, normalized size = 2.08

$$\frac{A \ln\left(\frac{cx^2 + \sqrt{cx^2 + bx + a}}{2\sqrt{c}}\right)}{2\sqrt{c}} + \frac{A^2 \ln\left(\frac{cx^2 + \sqrt{cx^2 + bx + a}}{8c^2}\right)}{8c^2} + \frac{C^2 \ln\left(\frac{cx^2 + \sqrt{cx^2 + bx + a}}{8c^2}\right)}{8c^2} + \frac{3C^2 \ln\left(\frac{cx^2 + \sqrt{cx^2 + bx + a}}{16c^2}\right)}{16c^2} + \frac{5C^2 \ln\left(\frac{cx^2 + \sqrt{cx^2 + bx + a}}{128c^2}\right)}{128c^2} + \frac{\sqrt{cx^2 + bx + a} A x}{2} + \frac{\sqrt{cx^2 + bx + a} C a x}{8c} + \frac{5\sqrt{cx^2 + bx + a} C^2 x}{32c^2} + \frac{\sqrt{cx^2 + bx + a} A b}{4c} + \frac{\sqrt{cx^2 + bx + a} C a b}{16c^2} + \frac{5\sqrt{cx^2 + bx + a} C^2 b}{64c^3} + \frac{(cx^2 + bx + a)^{3/2} C x}{4c} + \frac{5(cx^2 + bx + a)^{3/2} C^2}{24c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)\*(C\*x^2+A), x)

[Out]  $\frac{1}{4} C x (c x^2 + b x + a)^{3/2} / c - 5/24 b C (c x^2 + b x + a)^{3/2} / c^2 + 5/32 C^2 / c^2 * b^2 * (c x^2 + b x + a)^{1/2} * x + 5/64 C / c^3 * b^3 * (c x^2 + b x + a)^{1/2} + 3/16 C / c^{5/2} * b^2 * \ln((c x + 1/2 * b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a - 5/128 C / c^{7/2} * b^4 * \ln((c x + 1/2 * b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - 1/8 C * a / c * (c x^2 + b x + a)^{1/2} * x - 1/16 C * a / c^2 * (c x^2 + b x + a)^{1/2} * b - 1/8 C * a^2 / c^{3/2} * \ln((c x + 1/2 * b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + 1/2 * A * (c x^2 + b x + a)^{1/2} * x + 1/4 * A / c * (c x^2 + b x + a)^{1/2} * b + 1/2 * A / c^{1/2} * \ln((c x + 1/2 * b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * a - 1/8 * A / c^{3/2} * \ln((c x + 1/2 * b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) * b^2$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(C\*x^2+A), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad [B]** time = 4.26, size = 240, normalized size = 1.53

$$A \left( \frac{x}{2} + \frac{b}{4c} \right) \sqrt{cx^2 + bx + a} - \frac{C a \left( \frac{x}{2} + \frac{b}{4c} \right) \sqrt{cx^2 + bx + a} + \frac{\ln\left(\frac{\frac{x}{2} + \frac{b}{4c} + \sqrt{cx^2 + bx + a}}{2c^{3/2}}\right) (cx - \frac{b^2}{4})}{2c^{3/2}}}{4c} + \frac{A \ln\left(\frac{\frac{x}{2} + \frac{b}{4c} + \sqrt{cx^2 + bx + a}}{2c^{3/2}}\right) (cx - \frac{b^2}{4})}{2c^{3/2}} - \frac{5 C b \left( \frac{\ln\left(\frac{b^2 c x + 2 \sqrt{cx^2 + bx + a}}{16 c^{3/2}}\right) (b^2 - 4 a b c) + (-3 b^2 + 2 c x b + 8 c (c x^2 + a)) \sqrt{cx^2 + bx + a}}{24 c^2} \right)}{8c} + \frac{C x (c x^2 + b x + a)^{3/2}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)\*(a + b\*x + c\*x^2)^(1/2), x)

[Out]  $A * (x/2 + b/(4*c)) * (a + b*x + c*x^2)^{1/2} - (C*a*((x/2 + b/(4*c)) * (a + b*x + c*x^2)^{1/2} + (\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{1/2})) * (a*c - b^2/4)) / (2*c^{3/2})) / (4*c) + (A*\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{1/2})) * (a*c - b^2/4) / (2*c^{3/2}) - (5*C*b*((\log((b + 2*c*x)/c^{1/2} + 2*(a + b*x + c*x^2)^{1/2})) * (b^3 - 4*a*b*c)) / (16*c^{5/2}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x) * (a + b*x + c*x^2)^{1/2}) / (24*c^2)) / (8*c) + (C*x*(a + b*x + c*x^2)^{3/2}) / (4*c)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + Cx^2) \sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)\*(C\*x\*\*2+A), x)

[Out] Integral((A + C\*x\*\*2)\*sqrt(a + b\*x + c\*x\*\*2), x)

$$3.177 \quad \int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=104

$$\frac{(-4acC + 8Ac^2 + 3b^2C) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}} - \frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

**Rubi [A]** time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1661, 640, 621, 206}

$$\frac{(-4acC + 8Ac^2 + 3b^2C) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}} - \frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] (-3\*b\*C\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + (C\*x\*Sqrt[a + b\*x + c\*x^2])/(2\*c) + ((8\*A\*c^2 + 3\*b^2\*C - 4\*a\*c\*C)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 640

Int[((d\_) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + Cx^2}{\sqrt{a + bx + cx^2}} dx &= \frac{Cx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2Ac - aC - \frac{3bCx}{2}}{\sqrt{a + bx + cx^2}} dx}{2c} \\ &= -\frac{3bC\sqrt{a + bx + cx^2}}{4c^2} + \frac{Cx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(\frac{3b^2C}{2} + 2c(2Ac - aC)\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{4c^2} \\ &= -\frac{3bC\sqrt{a + bx + cx^2}}{4c^2} + \frac{Cx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(\frac{3b^2C}{2} + 2c(2Ac - aC)\right) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \right)}{2c^2} \\ &= -\frac{3bC\sqrt{a + bx + cx^2}}{4c^2} + \frac{Cx\sqrt{a + bx + cx^2}}{2c} + \frac{(8Ac^2 + 3b^2C - 4acC) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 86, normalized size = 0.83

$$\frac{(-4acC + 8Ac^2 + 3b^2C) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)}{8c^{5/2}} + \frac{C(2cx - 3b)\sqrt{a + x(b + cx)}}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] (C\*(-3\*b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)]/(4\*c^2) + ((8\*A\*c^2 + 3\*b^2\*C - 4\*a\*c\*C)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(8\*c^(5/2))

**IntegrateAlgebraic [A]** time = 0.45, size = 92, normalized size = 0.88

$$\frac{(4acC - 8Ac^2 - 3b^2C) \log\left(-2c^{5/2}\sqrt{a + bx + cx^2} + bc^2 + 2c^3x\right)}{8c^{5/2}} - \frac{C(3b - 2cx)\sqrt{a + bx + cx^2}}{4c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + C\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out]  $-1/4*(C*(3*b - 2*c*x)*\text{Sqrt}[a + b*x + c*x^2])/c^2 + ((-8*A*c^2 - 3*b^2*C + 4*a*c*C)*\text{Log}[b*c^2 + 2*c^3*x - 2*c^{(5/2)}*\text{Sqrt}[a + b*x + c*x^2]])/(8*c^{(5/2)})$

**fricas** [A] time = 0.94, size = 203, normalized size = 1.95

$$\left[ \frac{(3Cb^2 - 4Cac + 8Ac^2)\sqrt{c} \log\left(\frac{-8c^2x^2 - 8bcx - b^2 - 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac}{16c^3}\right) + 4(2C^2x - 3Cbc)\sqrt{cx^2 + bx + a}}{16c^3}, \frac{(3Cb^2 - 4Cac + 8Ac^2)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{-c}}{2(c^2x^2 + bcx + ac)}\right) - 2(2C^2x - 3Cbc)\sqrt{cx^2 + bx + a}}{8c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out]  $[1/16*((3C*b^2 - 4C*a*c + 8A*c^2)*\text{sqrt}(c)*\log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*\text{sqrt}(c*x^2 + b*x + a)*(2*c*x + b)*\text{sqrt}(c) - 4*a*c) + 4*(2*C*c^2*x - 3*C*b*c)*\text{sqrt}(c*x^2 + b*x + a))/c^3, -1/8*((3C*b^2 - 4C*a*c + 8A*c^2)*\text{sqrt}(-c)*\arctan(1/2*\text{sqrt}(c*x^2 + b*x + a)*(2*c*x + b)*\text{sqrt}(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*\text{sqrt}(c*x^2 + b*x + a))/c^3]$

**giac** [A] time = 0.25, size = 84, normalized size = 0.81

$$\frac{1}{4} \sqrt{cx^2 + bx + a} \left( \frac{2Cx}{c} - \frac{3Cb}{c^2} \right) - \frac{(3Cb^2 - 4Cac + 8Ac^2) \log\left(\left| -2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b \right|\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="giac")

[Out]  $1/4*\text{sqrt}(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3C*b^2 - 4C*a*c + 8A*c^2)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(5/2)}$

**maple** [A] time = 0.01, size = 136, normalized size = 1.31

$$\frac{A \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{\sqrt{c}} - \frac{Ca \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}} + \frac{3Cb^2 \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{8c^{\frac{5}{2}}} + \frac{\sqrt{cx^2 + bx + a} Cx}{2c} - \frac{3\sqrt{cx^2 + bx + a} Cb}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+A)/(c\*x^2+b\*x+a)^(1/2), x)

[Out]  $1/2*C*x*(c*x^2+b*x+a)^{(1/2)}/c - 3/4*b*C*(c*x^2+b*x+a)^{(1/2)}/c^2 + 3/8*C/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) - 1/2*C*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) + A*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C x^2 + A}{\sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((A + C\*x^2)/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+A)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((A + C\*x\*\*2)/sqrt(a + b\*x + c\*x\*\*2), x)



$$3.178 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=98

$$\frac{C \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

**Rubi** [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1660, 12, 621, 206}

$$\frac{C \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (-2\*(b\*c\*(A + (a\*C)/c) + (2\*A\*c^2 + (b^2 - 2\*a\*c)\*C)\*x)/(c\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + (C\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/c^(3/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + Cx^2}{(a + bx + cx^2)^{3/2}} dx &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} - \frac{2\int -\frac{(b^2 - 4ac)C}{2c\sqrt{a + bx + cx^2}} dx}{b^2 - 4ac} \\
&= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{C\int \frac{1}{\sqrt{a + bx + cx^2}} dx}{c} \\
&= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{(2C)\text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{c} \\
&= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{C \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 104, normalized size = 1.06

$$\frac{\frac{2\sqrt{c}(aC(b-2cx) + Ac(b+2cx) + b^2Cx)}{\sqrt{a+x(b+cx)}} - C(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] ((2\*Sqrt[c]\*(b^2\*C\*x + a\*C\*(b - 2\*c\*x) + A\*c\*(b + 2\*c\*x)))/Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*C\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(c^(3/2)\*(-b^2 + 4\*a\*c))

**IntegrateAlgebraic [A]** time = 0.48, size = 100, normalized size = 1.02

$$\frac{2(abC - 2acCx + Abc + 2Ac^2x + b^2Cx)}{c(4ac - b^2)\sqrt{a + bx + cx^2}} - \frac{C \log\left(-2c^{3/2}\sqrt{a + bx + cx^2} + bc + 2c^2x\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(A\*b\*c + a\*b\*C + 2\*A\*c^2\*x + b^2\*C\*x - 2\*a\*c\*C\*x))/(c\*(-b^2 + 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) - (C\*Log[b\*c + 2\*c^2\*x - 2\*c^(3/2)\*Sqrt[a + b\*x + c\*x^2]])/c^(3/2)

**fricas [B]** time = 1.73, size = 403, normalized size = 4.11

$$\frac{(Cab^2 - 4Ca^2c + (C^2c - 4Ca^2)c^2 + (C^3 - 4Cab)c)\sqrt{c} \log\left(\frac{-8c^2d^2 - 8bcx - b^2 - 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^2)c^2 + (b^2c^3 - 4abc^2)c)}\right) - 4(Cabc + Abc^2 + (C^2c - 2Ca^2 + 2Ac^2))\sqrt{cx^2 + bx + a}}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^2)c^2 + (b^2c^3 - 4abc^2)c)} - \frac{(Cab^2 - 4Ca^2c + (C^2c - 4Ca^2)c^2 + (C^3 - 4Cab)c)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^2 + bx + a}}{2(b^2c^2 - 4ac^2)}\right) + 2(Cabc + Abc^2 + (C^2c - 2Ca^2 + 2Ac^2))\sqrt{cx^2 + bx + a}}{ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^2)c^2 + (b^2c^3 - 4abc^2)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="fricas")

[Out] [1/2\*((C\*a\*b^2 - 4\*C\*a^2\*c + (C\*b^2\*c - 4\*C\*a\*c^2)\*x^2 + (C\*b^3 - 4\*C\*a\*b\*c)\*x)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(C\*a\*b\*c + A\*b\*c^2 + (C\*b^2\*c - 2\*C\*a\*c^2 + 2\*A\*c^3)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x), -((C\*a\*b^2 - 4\*C\*a^2\*c + (C\*b^2\*c - 4\*C\*a\*c^2)\*x^2 + (C\*b^3 - 4\*C\*a\*b\*c)\*x)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(C\*a\*b\*c + A\*b\*c^2 + (C\*b^2\*c - 2\*C\*a\*c^2 + 2\*A\*c^3)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x)]

**giac [A]** time = 0.27, size = 110, normalized size = 1.12

$$-\frac{2\left(\frac{(Cb^2 - 2Cac + 2Ac^2)x}{b^2c - 4ac^2} + \frac{Cab + Abc}{b^2c - 4ac^2}\right)}{\sqrt{cx^2 + bx + a}} - \frac{C \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="giac")

[Out] -2\*((C\*b^2 - 2\*C\*a\*c + 2\*A\*c^2)\*x/(b^2\*c - 4\*a\*c^2) + (C\*a\*b + A\*b\*c)/(b^2\*c - 4\*a\*c^2))/sqrt(c\*x^2 + b\*x + a) - C\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(3/2)

**maple [A]** time = 0.01, size = 169, normalized size = 1.72

$$\frac{Cb^2x}{(4ac-b^2)\sqrt{cx^2+bx+a}} + \frac{Cb^3}{2(4ac-b^2)\sqrt{cx^2+bx+a}c^2} + \frac{2(2cx+b)A}{(4ac-b^2)\sqrt{cx^2+bx+a}} - \frac{Cx}{\sqrt{cx^2+bx+a}c} + \frac{C \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)}{c^{\frac{3}{2}}} + \frac{Cb}{2\sqrt{cx^2+bx+a}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+A)/(c\*x^2+b\*x+a)^(3/2),x)

[Out]  $-Cx/c/(cx^2+bx+a)^{1/2} + 1/2 * C/c^2 * b/(cx^2+bx+a)^{1/2} + C/c * b^2/(4*a*c - b^2)/(cx^2+bx+a)^{1/2} * x + 1/2 * C/c^2 * b^3/(4*a*c - b^2)/(cx^2+bx+a)^{1/2} + C/c^{3/2} * \ln((cx + 1/2 * b)/c^{1/2} + (cx^2+bx+a)^{1/2}) + 2 * A * (2 * cx + b)/(4 * a * c - b^2)/(cx^2+bx+a)^{1/2}$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details) Is 4\*a\*c-b^2 zero or nonzero?

**mupad [B]** time = 4.21, size = 108, normalized size = 1.10

$$\frac{C \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)}{c^{3/2}} + \frac{A\left(\frac{b}{2}+cx\right)}{\left(ac-\frac{b^2}{4}\right)\sqrt{cx^2+bx+a}} + \frac{C\left(\frac{ab}{2}-x\left(ac-\frac{b^2}{2}\right)\right)}{c\left(ac-\frac{b^2}{4}\right)\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)/(a + b\*x + c\*x^2)^(3/2),x)

[Out]  $(C * \log((b/2 + cx)/c^{1/2} + (a + bx + cx^2)^{1/2}))/c^{3/2} + (A * (b/2 + cx))/((ac - b^2/4) * (a + bx + cx^2)^{1/2}) + (C * ((a*b)/2 - x * (ac - b^2/2)))/(c * (ac - b^2/4) * (a + bx + cx^2)^{1/2})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

$$3.179 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}} dx$$

**Optimal.** Leaf size=114

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {1660, 12, 613}

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*(b\*c\*(A + (a\*C)/c) + (2\*A\*c^2 + (b^2 - 2\*a\*c)\*C)\*x)/(3\*c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(3/2)) + (2\*(8\*A\*c + 4\*a\*C + (b^2\*C)/c)\*(b + 2\*c\*x))/(3\*(b^2 - 4\*a\*c)^2\*sqrt[a + b\*x + c\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2

- 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{A + Cx^2}{(a + bx + cx^2)^{5/2}} dx &= -\frac{2 \left( bc \left( A + \frac{aC}{c} \right) + (2Ac^2 + (b^2 - 2ac)C)x \right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} - \frac{2 \int \frac{8Ac + 4aC + \frac{b^2C}{c}}{2(a + bx + cx^2)^{3/2}} dx}{3(b^2 - 4ac)} \\ &= -\frac{2 \left( bc \left( A + \frac{aC}{c} \right) + (2Ac^2 + (b^2 - 2ac)C)x \right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} - \frac{\left( 8Ac + 4aC + \frac{b^2C}{c} \right) \int \frac{1}{(a + bx + cx^2)^{3/2}} dx}{3(b^2 - 4ac)} \\ &= -\frac{2 \left( bc \left( A + \frac{aC}{c} \right) + (2Ac^2 + (b^2 - 2ac)C)x \right)}{3c(b^2 - 4ac)(a + bx + cx^2)^{3/2}} + \frac{2 \left( 8Ac + 4aC + \frac{b^2C}{c} \right) (b + 2cx)}{3(b^2 - 4ac)^2 \sqrt{a + bx + cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.89, size = 107, normalized size = 0.94

$$\frac{2C(8a^2b + 4ax(3b^2 + 3bcx + 2c^2x^2) + b^2x^2(3b + 2cx)) - 2A(b + 2cx)(-4c(3a + 2cx^2) + b^2 - 8bcx)}{3(b^2 - 4ac)^2(a + x(b + cx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*A\*(b + 2\*c\*x)\*(b^2 - 8\*b\*c\*x - 4\*c\*(3\*a + 2\*c\*x^2)) + 2\*C\*(8\*a^2\*b + b^2\*x^2\*(3\*b + 2\*c\*x) + 4\*a\*x\*(3\*b^2 + 3\*b\*c\*x + 2\*c^2\*x^2)))/(3\*(b^2 - 4\*a\*c)^2\*(a + x\*(b + c\*x))^(3/2))

**IntegrateAlgebraic [A]** time = 0.97, size = 128, normalized size = 1.12

$$\frac{2(-8a^2bC - 12aAbc - 24aAc^2x - 12ab^2Cx - 12abcCx^2 - 8ac^2Cx^3 + Ab^3 - 6Ab^2cx - 24Abc^2x^2 - 16Ac^3x^3 - 3b^3Cx^2 - 2b^2cCx^3)}{3(b^2 - 4ac)^2(a + bx + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (-2\*(A\*b^3 - 12\*a\*A\*b\*c - 8\*a^2\*b\*C - 6\*A\*b^2\*c\*x - 24\*a\*A\*c^2\*x - 12\*a\*b^2\*C\*x - 24\*A\*b\*c^2\*x^2 - 3\*b^3\*C\*x^2 - 12\*a\*b\*c\*C\*x^2 - 16\*A\*c^3\*x^3 - 2\*b^2\*c\*C\*x^3 - 8\*a\*c^2\*C\*x^3))/(3\*(b^2 - 4\*a\*c)^2\*(a + b\*x + c\*x^2)^(3/2))

**fricas** [B] time = 2.07, size = 242, normalized size = 2.12

$$\frac{2(8Ca^2b - Ab^3 + 12Aabc + 2(Cb^2c + 4Cac^2 + 8Ac^3)x^3 + 3(Cb^3 + 4Cabc + 8Abc^2)x^2 + 6(2Cab^2 + Ab^2c + 4Aac^2)x)\sqrt{cx^2 + bx + a}}{3(a^2b^4 - 8a^3b^2c + 16a^4c^2 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + 2(b^5c - 8ab^3c^2 + 16a^2bc^3)x^3 + (b^6 - 6ab^4c + 32a^3c^3)x^2 + 2(ab^5 - 8a^2b^3c + 16a^3bc^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="fricas")

[Out]  $\frac{2}{3} * (8 * C * a^2 * b - A * b^3 + 12 * A * a * b * c + 2 * (C * b^2 * c + 4 * C * a * c^2 + 8 * A * c^3) * x^3 + 3 * (C * b^3 + 4 * C * a * b * c + 8 * A * b * c^2) * x^2 + 6 * (2 * C * a * b^2 + A * b^2 * c + 4 * A * a * c^2) * x) * \text{sqrt}(c * x^2 + b * x + a) / (a^2 * b^4 - 8 * a^3 * b^2 * c + 16 * a^4 * c^2 + (b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * x^4 + 2 * (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * x^3 + (b^6 - 6 * a * b^4 * c + 32 * a^3 * c^3) * x^2 + 2 * (a * b^5 - 8 * a^2 * b^3 * c + 16 * a^3 * b * c^2) * x)$

**giac** [A] time = 0.26, size = 193, normalized size = 1.69

$$\frac{2 \left( \left( \frac{2(Cb^2c + 4Cac^2 + 8Ac^3)x}{b^4 - 8ab^2c + 16a^2c^2} + \frac{3(Cb^3 + 4Cabc + 8Abc^2)}{b^4 - 8ab^2c + 16a^2c^2} \right) x + \frac{6(2Cab^2 + Ab^2c + 4Aac^2)}{b^4 - 8ab^2c + 16a^2c^2} \right) x + \frac{8Ca^2b - Ab^3 + 12Aabc}{b^4 - 8ab^2c + 16a^2c^2}}{3(cx^2 + bx + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out]  $\frac{2}{3} * ((2 * (C * b^2 * c + 4 * C * a * c^2 + 8 * A * c^3) * x / (b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2) + 3 * (C * b^3 + 4 * C * a * b * c + 8 * A * b * c^2) / (b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2)) * x + 6 * (2 * C * a * b^2 + A * b^2 * c + 4 * A * a * c^2) / (b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2)) * x + (8 * C * a^2 * b - A * b^3 + 12 * A * a * b * c) / (b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2)) / (c * x^2 + b * x + a)^{(3/2)}$

**maple** [A] time = 0.01, size = 137, normalized size = 1.20

$$\frac{\frac{32}{3}Ac^3x^3 + \frac{16}{3}Ca^2c^2x^3 + \frac{4}{3}Cb^2cx^3 + 16Ab^2c^2x^2 + 8Cabcx^2 + 2Cb^3x^2 + 16Aa^2c^2x + 4Ab^2cx + 8Ca^2b^2x + 8Aabc - \frac{2}{3}Ab^3 + \frac{16}{3}Ca^2b}{(cx^2 + bx + a)^{\frac{3}{2}}(16a^2c^2 - 8ab^2c + b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C\*x^2+A)/(c\*x^2+b\*x+a)^(5/2),x)

[Out]  $\frac{2}{3} / (c * x^2 + b * x + a)^{(3/2)} * (16 * A * c^3 * x^3 + 8 * C * a * c^2 * x^3 + 2 * C * b^2 * c * x^3 + 24 * A * b * c^2 * x^2 + 12 * C * a * b * c * x^2 + 3 * C * b^3 * x^2 + 24 * A * a * c^2 * x + 6 * A * b^2 * c * x + 12 * C * a * b^2 * x + 12 * A * a * b * c - A * b^3 + 8 * C * a^2 * b) / (16 * a^2 * c^2 - 8 * a * b^2 * c + b^4)$



**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [B] time = 4.14, size = 127, normalized size = 1.11

$$\frac{2(8Ca^2b + 12Cab^2x + 12Cabcx^2 + 12Aabc + 8Cac^2x^3 + 24Aac^2x + 3Cb^3x^2 - Ab^3 + 2Cb^2cx^3 + 6Ab^2cx + 24Abc^2x^2 + 16Ac^3x^3)}{3(4ac - b^2)^2(cx^2 + bx + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C\*x^2)/(a + b\*x + c\*x^2)^(5/2),x)

[Out] (2\*(16\*A\*c^3\*x^3 - A\*b^3 + 3\*C\*b^3\*x^2 + 8\*C\*a^2\*b + 24\*A\*a\*c^2\*x + 6\*A\*b^2\*c\*x + 12\*C\*a\*b^2\*x + 24\*A\*b\*c^2\*x^2 + 8\*C\*a\*c^2\*x^3 + 2\*C\*b^2\*c\*x^3 + 12\*A\*a\*b\*c + 12\*C\*a\*b\*c\*x^2))/(3\*(4\*a\*c - b^2)^2\*(a + b\*x + c\*x^2)^(3/2))

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+A)/(c\*x\*\*2+b\*x+a)\*\*(5/2),x)

[Out] Timed out

$$3.180 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}} dx$$

**Optimal.** Leaf size=167

$$\frac{16(b+2cx)(4acC+16Ac^2+3b^2C)}{15(b^2-4ac)^3\sqrt{a+bx+cx^2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c(b^2-4ac)(a+bx+cx^2)^{5/2}} + \frac{2(b+2cx)(4aC+16Ac+\frac{3b^2C}{c})}{15(b^2-4ac)^2(a+bx+cx^2)}$$

**Rubi [A]** time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1660, 12, 614, 613}

$$\frac{16(b+2cx)(4acC+16Ac^2+3b^2C)}{15(b^2-4ac)^3\sqrt{a+bx+cx^2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c(b^2-4ac)(a+bx+cx^2)^{5/2}} + \frac{2(b+2cx)\left(4aC+16Ac+\frac{3b^2C}{c}\right)}{15(b^2-4ac)^2(a+bx+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^(7/2), x]

[Out] (-2\*(b\*c\*(A + (a\*C)/c) + (2\*A\*c^2 + (b^2 - 2\*a\*c)\*C)\*x)/(5\*c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(5/2)) + (2\*(16\*A\*c + 4\*a\*C + (3\*b^2\*C)/c)\*(b + 2\*c\*x))/(15\*(b^2 - 4\*a\*c)^2\*(a + b\*x + c\*x^2)^(3/2)) - (16\*(16\*A\*c^2 + 3\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x))/(15\*(b^2 - 4\*a\*c)^3\*Sqrt[a + b\*x + c\*x^2])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4\*p]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{A + Cx^2}{(a + bx + cx^2)^{7/2}} dx &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{5c(b^2 - 4ac)(a + bx + cx^2)^{5/2}} - \frac{2\int \frac{16Ac + 4aC + \frac{3b^2C}{c}}{2(a + bx + cx^2)^{5/2}} dx}{5(b^2 - 4ac)} \\ &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{5c(b^2 - 4ac)(a + bx + cx^2)^{5/2}} - \frac{\left(16Ac + 4aC + \frac{3b^2C}{c}\right)\int \frac{1}{(a + bx + cx^2)^{5/2}} dx}{5(b^2 - 4ac)} \\ &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{5c(b^2 - 4ac)(a + bx + cx^2)^{5/2}} + \frac{2\left(16Ac + 4aC + \frac{3b^2C}{c}\right)(b + 2cx)}{15(b^2 - 4ac)^2(a + bx + cx^2)^{3/2}} + \dots \\ &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{5c(b^2 - 4ac)(a + bx + cx^2)^{5/2}} + \frac{2\left(16Ac + 4aC + \frac{3b^2C}{c}\right)(b + 2cx)}{15(b^2 - 4ac)^2(a + bx + cx^2)^{3/2}} - \dots \end{aligned}$$

**Mathematica [A]** time = 1.93, size = 148, normalized size = 0.89

$$\frac{2\left((b^2 - 4ac)(b + 2cx)(a + x(b + cx))(4acC + 16Ac^2 + 3b^2C) - 8c(b + 2cx)(a + x(b + cx))^2(4acC + 16Ac^2 + 3b^2C) - 3(b^2 - 4ac)^2(ac(b - 2cx) + Ac(b + 2cx) + b^2Cx)\right)}{15c(b^2 - 4ac)^3(a + x(b + cx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^(7/2), x]

[Out] (2\*((b^2 - 4\*a\*c)\*(16\*A\*c^2 + 3\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + x\*(b + c\*x)) - 8\*c\*(16\*A\*c^2 + 3\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^2 - 3\*(b^2 - 4\*a\*c)^2\*(b^2\*C\*x + a\*C\*(b - 2\*c\*x) + A\*c\*(b + 2\*c\*x)))/(15\*c\*(b^2 - 4\*a\*c)^3\*(a + x\*(b + c\*x))^(5/2))

**IntegrateAlgebraic [A]** time = 2.45, size = 295, normalized size = 1.77

$$\frac{2(96a^3bc + 240a^2b^2c + 480a^2b^2c^2 + 8a^2b^3c + 240a^2b^2c^2 + 240a^2b^2c^2 + 160a^2b^2c^2 + 40a^2b^3c + 240a^2b^2c^2 + 960a^2b^2c^2 + 640a^2b^2c^2 + 20a^2b^3c + 200a^2b^2c^2 + 240a^2b^2c^2 + 160a^2b^2c^2 + 64a^2b^2c^2 + 3a^2b^3 - 10a^2b^3c + 80a^2b^2c^2 + 480a^2b^2c^2 + 640a^2b^2c^2 + 256a^2b^2c^2 + 15a^2b^2c^2 + 90a^2b^2c^2 + 120a^2b^2c^2 + 48a^2b^2c^2)}{15(b^2 - 4ac)(a + bx + cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^(7/2),x]

[Out] (-2\*(3\*A\*b^5 - 40\*a\*A\*b^3\*c + 240\*a^2\*A\*b\*c^2 + 8\*a^2\*b^3\*C + 96\*a^3\*b\*c\*C - 10\*A\*b^4\*c\*x + 240\*a\*A\*b^2\*c^2\*x + 480\*a^2\*A\*c^3\*x + 20\*a\*b^4\*C\*x + 240\*a^2\*b^2\*c\*C\*x + 80\*A\*b^3\*c^2\*x^2 + 960\*a\*A\*b\*c^3\*x^2 + 15\*b^5\*C\*x^2 + 200\*a\*b^3\*c\*C\*x^2 + 240\*a^2\*b\*c^2\*C\*x^2 + 480\*A\*b^2\*c^3\*x^3 + 640\*a\*A\*c^4\*x^3 + 90\*b^4\*c\*C\*x^3 + 240\*a\*b^2\*c^2\*C\*x^3 + 160\*a^2\*c^3\*C\*x^3 + 640\*A\*b\*c^4\*x^4 + 120\*b^3\*c^2\*C\*x^4 + 160\*a\*b\*c^3\*C\*x^4 + 256\*A\*c^5\*x^5 + 48\*b^2\*c^3\*C\*x^5 + 64\*a\*c^4\*C\*x^5))/(15\*(b^2 - 4\*a\*c)^3\*(a + b\*x + c\*x^2)^(5/2))

**fricas [B]** time = 17.20, size = 563, normalized size = 3.37

$$\frac{2(8C^2b^3 + 3Ab^5 + 240A^2b^2c^2 + 16(3C^2b^2 + 4Ca^2 + 16A^2)c^2 + 40(3C^2b^2 + 4Cab^2 + 16Ab^2)c^2 + 10(9C^2c + 24Cab^2 + 64Aac^2 + 16(C^2 + 3Ab^2)c^2) + 5(3C^2 + 40Cab^2 + 192Aab^2 + 16(3C^2b^2 + Ab^2)c^2) + 8(12Ca^2b - 5Aab^2)c + 10(2Cab^4 + 24Aab^2c^2 + 48Aa^2c^2 + (24C^2b^2 - Ab^4)c)}{15(b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2 + (b^2 - 12ab^2c + 48a^2b^2c^2 - 64a^2b^3c^2) + 3(b^2 - 12ab^2c + 48a^2b^2c^2 - 64a^2b^3c^2) + 3(b^2 - 11ab^2c + 36a^2b^2c^2 - 16a^2b^3c^2 - 64a^2b^4c^2) + (b^5 - 6ab^4c - 24a^2b^3c^2 + 224a^2b^2c^3 - 384a^2b^3c^3) + 3(ab^4 - 11a^2b^3c + 36a^2b^2c^2 - 16a^2b^3c^2 - 64a^2b^4c^2) + 3(a^2b^4 - 12a^2b^3c + 48a^2b^2c^2 - 64a^2b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(7/2),x, algorithm="fricas")

[Out] -2/15\*(8\*C\*a^2\*b^3 + 3\*A\*b^5 + 240\*A\*a^2\*b\*c^2 + 16\*(3\*C\*b^2\*c^3 + 4\*C\*a\*c^4 + 16\*A\*c^5)\*x^5 + 40\*(3\*C\*b^3\*c^2 + 4\*C\*a\*b\*c^3 + 16\*A\*b\*c^4)\*x^4 + 10\*(9\*C\*b^4\*c + 24\*C\*a\*b^2\*c^2 + 64\*A\*a\*c^4 + 16\*(C\*a^2 + 3\*A\*b^2)\*c^3)\*x^3 + 5\*(3\*C\*b^5 + 40\*C\*a\*b^3\*c + 192\*A\*a\*b\*c^3 + 16\*(3\*C\*a^2\*b + A\*b^3)\*c^2)\*x^2 + 8\*(12\*C\*a^3\*b - 5\*A\*a\*b^3)\*c + 10\*(2\*C\*a\*b^4 + 24\*A\*a\*b^2\*c^2 + 48\*A\*a^2\*c^3 + (24\*C\*a^2\*b^2 - A\*b^4)\*c)\*x)\*sqrt(c\*x^2 + b\*x + a)/(a^3\*b^6 - 12\*a^4\*b^4\*c + 48\*a^5\*b^2\*c^2 - 64\*a^6\*c^3 + (b^6\*c^3 - 12\*a\*b^4\*c^4 + 48\*a^2\*b^2\*c^5 - 64\*a^3\*c^6)\*x^6 + 3\*(b^7\*c^2 - 12\*a\*b^5\*c^3 + 48\*a^2\*b^3\*c^4 - 64\*a^3\*b\*c^5)\*x^5 + 3\*(b^8\*c - 11\*a\*b^6\*c^2 + 36\*a^2\*b^4\*c^3 - 16\*a^3\*b^2\*c^4 - 64\*a^4\*c^5)\*x^4 + (b^9 - 6\*a\*b^7\*c - 24\*a^2\*b^5\*c^2 + 224\*a^3\*b^3\*c^3 - 384\*a^4\*b\*c^4)\*x^3 + 3\*(a\*b^8 - 11\*a^2\*b^6\*c + 36\*a^3\*b^4\*c^2 - 16\*a^4\*b^2\*c^3 - 64\*a^5\*c^4)\*x^2 + 3\*(a^2\*b^7 - 12\*a^3\*b^5\*c + 48\*a^4\*b^3\*c^2 - 64\*a^5\*b\*c^3)\*x)

**giac [B]** time = 0.28, size = 452, normalized size = 2.71

$$\frac{2\left(\left(4\left(\frac{2(3C^2b^2 + 4Ca^2 + 16A^2)c^2}{b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2} + \frac{5(3C^2b^2 + 4Cab^2 + 16Ab^2)c^2}{b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2}\right)x + \frac{5(9C^2b^2 + 24Cab^2 + 16C^2b^2 + 48Ab^2c^2 + 64Aac^2)}{b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2}\right)x + \frac{5(3C^2 + 40Cab^2 + 192Aab^2 + 16(3C^2b^2 + Ab^2)c^2)}{b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2}\right)x + \frac{10(2Cab^4 + 24Aab^2c^2 + 48Aa^2c^2 + (24C^2b^2 - Ab^4)c)}{b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2}\right)x + \frac{8C^2b^3 + 3Ab^5 + 240A^2b^2c^2 + 16(3C^2b^2 + 4Ca^2 + 16A^2)c^2 + 40(3C^2b^2 + 4Cab^2 + 16Ab^2)c^2 + 10(9C^2c + 24Cab^2 + 64Aac^2 + 16(C^2 + 3Ab^2)c^2) + 5(3C^2 + 40Cab^2 + 192Aab^2 + 16(3C^2b^2 + Ab^2)c^2) + 8(12Ca^2b - 5Aab^2)c + 10(2Cab^4 + 24Aab^2c^2 + 48Aa^2c^2 + (24C^2b^2 - Ab^4)c)}{15(b^5 - 12a^2b^4c + 48a^2b^2c^2 - 64a^2b^3c^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(7/2),x, algorithm="giac")

[Out] 
$$-2/15 * (((2 * (4 * (2 * (3 * C * b^2 * c^3 + 4 * C * a * c^4 + 16 * A * c^5)) * x / (b^6 - 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 - 64 * a^3 * c^3)) + 5 * (3 * C * b^3 * c^2 + 4 * C * a * b * c^3 + 16 * A * b * c^4) / (b^6 - 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 - 64 * a^3 * c^3)) * x + 5 * (9 * C * b^4 * c + 24 * C * a * b^2 * c^2 + 16 * C * a^2 * c^3 + 48 * A * b^2 * c^3 + 64 * A * a * c^4) / (b^6 - 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 - 64 * a^3 * c^3)) * x + 5 * (3 * C * b^5 + 40 * C * a * b^3 * c + 48 * C * a^2 * b * c^2 + 16 * A * b^3 * c^2 + 192 * A * a * b * c^3) / (b^6 - 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 - 64 * a^3 * c^3)) * x + 10 * (2 * C * a * b^4 + 24 * C * a^2 * b^2 * c - A * b^4 * c + 24 * A * a * b^2 * c^2 + 48 * A * a^2 * c^3) / (b^6 - 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 - 64 * a^3 * c^3)) * x + (8 * C * a^2 * b^3 + 3 * A * b^5 + 96 * C * a^3 * b * c - 40 * A * a * b^3 * c + 240 * A * a^2 * b * c^2) / (b^6 - 12 * a * b^4 * c + 48 * a^2 * b^2 * c^2 - 64 * a^3 * c^3)) / (c * x^2 + b * x + a)^{(5/2)}$$

**maple [B]** time = 0.01, size = 316, normalized size = 1.89

$$\frac{\frac{22}{15} C a^4 b^3 + \frac{22}{15} C^2 b^3 c^2 + \frac{22}{15} A b^4 c^3 + 16 C^2 b^2 c^4 + \frac{22}{15} A a^2 c^5 + 64 A^2 b^2 c^3 + \frac{22}{15} C^2 b^2 c^3 + 12 C^2 b^2 c^3 + \frac{22}{15} A^2 b^2 c^3 + 64 A^2 b^2 c^3 - \frac{2}{3} A^2 b^2 c^3 + \frac{2}{3} C a^2 b^2 c^3 + 32 A^2 b^2 c^3 - \frac{16}{3} A b^2 c^3 + \frac{22}{15} C^2 b^2 c^3 + \frac{2}{3} A b^2 c^3 + 128 A a b^2 c^3 + 32 C^2 b^2 c^3 + \frac{22}{15} C a^2 b^2 c^3 + 32 A a b^2 c^3 + 32 C^2 b^2 c^3 + \frac{22}{15} A^2 c^5 + 2 C^2 b^2 c^3 + \frac{22}{15} C^2 b^2 c^3 + \frac{22}{15} C a b^2 c^3 + 32 C a b^2 c^3}{(c^2 + b x + a)^2 (4 a^3 b^3 - 48 a^2 b^2 c + 12 a b^4 c - b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x)`

[Out] 
$$2/15 / (c * x^2 + b * x + a)^{(5/2)} * (256 * A * c^5 * x^5 + 64 * C * a * c^4 * x^5 + 48 * C * b^2 * c^3 * x^5 + 640 * A * b * c^4 * x^4 + 160 * C * a * b * c^3 * x^4 + 120 * C * b^3 * c^2 * x^4 + 640 * A * a * c^4 * x^3 + 480 * A * b^2 * c^3 * x^3 + 160 * C * a^2 * c^3 * x^3 + 240 * C * a * b^2 * c^2 * x^3 + 90 * C * b^4 * c * x^3 + 960 * A * a * b * c^3 * x^2 + 80 * A * b^3 * c^2 * x^2 + 240 * C * a^2 * b * c^2 * x^2 + 200 * C * a * b^3 * c * x^2 + 15 * C * b^5 * x^2 + 480 * A * a^2 * c^3 * x + 240 * A * a * b^2 * c^2 * x - 10 * A * b^4 * c * x + 240 * C * a^2 * b^2 * c * x + 20 * C * a * b^4 * x + 240 * A * a^2 * b * c^2 - 40 * A * a * b^3 * c + 3 * A * b^5 + 96 * C * a^3 * b * c + 8 * C * a^2 * b^3) / (64 * a^3 * c^3 - 48 * a^2 * b^2 * c^2 + 12 * a * b^4 * c - b^6)$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad [B]** time = 4.53, size = 578, normalized size = 3.46

$$\frac{\frac{4}{15} \frac{(8 C^2 b^2 + 256 A^2 + 32 C a)}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{2}{15} \frac{(8 C^2 b^2 + 256 A^2 + 32 C a)}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{8 C b}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{16 C^2 a}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{4 C x}{15 (4 a c - b^2)} - \frac{2 C b}{15 (4 a c - b^2)} + \frac{4 A a^2}{5 (4 a^2 - b^2)} + \frac{2 C b^2}{5 (4 a^2 - b^2)} - \frac{4 C a c}{5 (4 a^2 - b^2)} + \frac{2 A a b}{5 (4 a^2 - b^2)} + \frac{2 C a b}{5 (4 a^2 - b^2)} + \frac{2 (8 C^2 b^2 + 256 A^2 + 8 C a)}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{16 C a^2}{15 (4 a^2 - b^2) (4 a c - b^2)} - \frac{8 C b^2}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{8 (8 C^2 b^2 + 256 A^2 + 8 C a)}{15 (4 a^2 - b^2) (4 a c - b^2)} + \frac{8 C a b}{15 (4 a^2 - b^2) (4 a c - b^2)}}{\sqrt{c x^2 + b x + a} (c x^2 + b x + a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + C*x^2)/(a + b*x + c*x^2)^(7/2),x)`

```
[Out] ((b*c*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (2*c^2*x*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((8*C*b*c)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c^2*x)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(1/2) - ((4*C*x)/(15*(4*a*c - b^2)) - (2*C*b)/(15*c*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) + (x*((4*A*c^2)/(5*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(5*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(5*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(5*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^(5/2) + (x*((2*c*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

$$3.181 \quad \int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}} dx$$

Optimal. Leaf size=220

$$\frac{256c(b+2cx)(4acC+24Ac^2+5b^2C)}{105(b^2-4ac)^4\sqrt{a+bx+cx^2}} - \frac{32(b+2cx)(4acC+24Ac^2+5b^2C)}{105(b^2-4ac)^3(a+bx+cx^2)^{3/2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\right)}{7c(b^2-4ac)(a+bx+cx^2)}$$

Rubi [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1660, 12, 614, 613}

$$\frac{256c(b+2cx)(4acC+24Ac^2+5b^2C)}{105(b^2-4ac)^4\sqrt{a+bx+cx^2}} - \frac{32(b+2cx)(4acC+24Ac^2+5b^2C)}{105(b^2-4ac)^3(a+bx+cx^2)^{3/2}} - \frac{2\left(x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{ac}{c}+A\right)\right)}{7c(b^2-4ac)(a+bx+cx^2)^{7/2}} + \frac{2(b+2cx)\left(4ac+24Ac+\frac{5b^2c}{c}\right)}{35(b^2-4ac)^2(a+bx+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C\*x^2)/(a + b\*x + c\*x^2)^(9/2), x]

[Out] (-2\*(b\*c\*(A + (a\*C)/c) + (2\*A\*c^2 + (b^2 - 2\*a\*c)\*C)\*x)/(7\*c\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(7/2)) + (2\*(24\*A\*c + 4\*a\*C + (5\*b^2\*C)/c)\*(b + 2\*c\*x))/(35\*(b^2 - 4\*a\*c)^2\*(a + b\*x + c\*x^2)^(5/2)) - (32\*(24\*A\*c^2 + 5\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x))/(105\*(b^2 - 4\*a\*c)^3\*(a + b\*x + c\*x^2)^(3/2)) + (256\*c\*(24\*A\*c^2 + 5\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x))/(105\*(b^2 - 4\*a\*c)^4\*sqrt[a + b\*x + c\*x^2])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 614

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] - Dist[(2\*c\*(2\*p + 3))/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int

egerQ[4\*p]

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq,
a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{A + Cx^2}{(a + bx + cx^2)^{9/2}} dx &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{7c(b^2 - 4ac)(a + bx + cx^2)^{7/2}} - \frac{2\int \frac{24Ac + 4aC + \frac{5b^2C}{c}}{2(a + bx + cx^2)^{7/2}} dx}{7(b^2 - 4ac)} \\
 &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{7c(b^2 - 4ac)(a + bx + cx^2)^{7/2}} - \frac{\left(24Ac + 4aC + \frac{5b^2C}{c}\right)\int \frac{1}{(a + bx + cx^2)^{7/2}} dx}{7(b^2 - 4ac)} \\
 &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{7c(b^2 - 4ac)(a + bx + cx^2)^{7/2}} + \frac{2\left(24Ac + 4aC + \frac{5b^2C}{c}\right)(b + 2cx)}{35(b^2 - 4ac)^2(a + bx + cx^2)^{5/2}} + \frac{16}{1} \\
 &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{7c(b^2 - 4ac)(a + bx + cx^2)^{7/2}} + \frac{2\left(24Ac + 4aC + \frac{5b^2C}{c}\right)(b + 2cx)}{35(b^2 - 4ac)^2(a + bx + cx^2)^{5/2}} - \frac{32}{1} \\
 &= -\frac{2\left(bc\left(A + \frac{aC}{c}\right) + (2Ac^2 + (b^2 - 2ac)C)x\right)}{7c(b^2 - 4ac)(a + bx + cx^2)^{7/2}} + \frac{2\left(24Ac + 4aC + \frac{5b^2C}{c}\right)(b + 2cx)}{35(b^2 - 4ac)^2(a + bx + cx^2)^{5/2}} - \frac{32}{1}
 \end{aligned}$$

**Mathematica [A]** time = 1.74, size = 199, normalized size = 0.90

$$\frac{2\left(3(b^2 - 4ac)^2(b + 2cx)(a + x(b + cx))(4acC + 24Ac^2 + 5b^2C) - 16c(b^2 - 4ac)(b + 2cx)(a + x(b + cx))^2(4acC + 24Ac^2 + 5b^2C) + 128c^2(b + 2cx)(a + x(b + cx))^3(4acC + 24Ac^2 + 5b^2C) - 15(b^2 - 4ac)^3(aC(b - 2cx) + Ac(b + 2cx) + b^2Cx)\right)}{105c(b^2 - 4ac)^3(a + x(b + cx))^{7/2}}$$

Antiderivative was successfully verified.



[In] Integrate[(A + C\*x^2)/(a + b\*x + c\*x^2)^(9/2),x]

[Out] (2\*(3\*(b^2 - 4\*a\*c)^2\*(24\*A\*c^2 + 5\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + x\*(b + c\*x)) - 16\*c\*(b^2 - 4\*a\*c)\*(24\*A\*c^2 + 5\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^2 + 128\*c^2\*(24\*A\*c^2 + 5\*b^2\*C + 4\*a\*c\*C)\*(b + 2\*c\*x)\*(a + x\*(b + c\*x))^3 - 15\*(b^2 - 4\*a\*c)^3\*(b^2\*C\*x + a\*C\*(b - 2\*c\*x) + A\*c\*(b + 2\*c\*x))))/(105\*c\*(b^2 - 4\*a\*c)^4\*(a + x\*(b + c\*x))^(7/2))

**IntegrateAlgebraic [B]** time = 5.09, size = 525, normalized size = 2.39

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(A + C\*x^2)/(a + b\*x + c\*x^2)^(9/2),x]

[Out] (-2\*(15\*A\*b^7 - 252\*a\*A\*b^5\*c + 1680\*a^2\*A\*b^3\*c^2 - 6720\*a^3\*A\*b\*c^3 + 8\*a^2\*b^5\*C - 320\*a^3\*b^3\*c\*C - 1920\*a^4\*b\*c^2\*C - 42\*A\*b^6\*c\*x + 840\*a\*A\*b^4\*c^2\*x - 10080\*a^2\*A\*b^2\*c^3\*x - 13440\*a^3\*A\*c^4\*x + 28\*a\*b^6\*C\*x - 1120\*a^2\*b^4\*c\*C\*x - 6720\*a^3\*b^2\*c^2\*C\*x + 168\*A\*b^5\*c^2\*x^2 - 6720\*a\*A\*b^3\*c^3\*x^2 - 40320\*a^2\*A\*b\*c^4\*x^2 + 35\*b^7\*C\*x^2 - 1372\*a\*b^5\*c\*C\*x^2 - 9520\*a^2\*b^3\*c^2\*C\*x^2 - 6720\*a^3\*b\*c^3\*C\*x^2 - 1680\*A\*b^4\*c^3\*x^3 - 40320\*a\*A\*b^2\*c^4\*x^3 - 26880\*a^2\*A\*c^5\*x^3 - 350\*b^6\*c\*C\*x^3 - 8680\*a\*b^4\*c^2\*C\*x^3 - 12320\*a^2\*b^2\*c^3\*C\*x^3 - 4480\*a^3\*c^4\*C\*x^3 - 13440\*A\*b^3\*c^4\*x^4 - 53760\*a\*A\*b\*c^5\*x^4 - 2800\*b^5\*c^2\*C\*x^4 - 13440\*a\*b^3\*c^3\*C\*x^4 - 8960\*a^2\*b\*c^4\*C\*x^4 - 26880\*A\*b^2\*c^5\*x^5 - 21504\*a\*A\*c^6\*x^5 - 5600\*b^4\*c^3\*C\*x^5 - 8960\*a\*b^2\*c^4\*C\*x^5 - 3584\*a^2\*c^5\*C\*x^5 - 21504\*A\*b\*c^6\*x^6 - 4480\*b^3\*c^4\*C\*x^6 - 3584\*a\*b\*c^5\*C\*x^6 - 6144\*A\*c^7\*x^7 - 1280\*b^2\*c^5\*C\*x^7 - 1024\*a\*c^6\*C\*x^7))/(105\*(b^2 - 4\*a\*c)^4\*(a + b\*x + c\*x^2)^(7/2))

**fricas [B]** time = 49.88, size = 978, normalized size = 4.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(9/2),x, algorithm="fricas")

[Out] -2/105\*(8\*C\*a^2\*b^5 + 15\*A\*b^7 - 6720\*A\*a^3\*b\*c^3 - 256\*(5\*C\*b^2\*c^5 + 4\*C\*a\*c^6 + 24\*A\*c^7)\*x^7 - 896\*(5\*C\*b^3\*c^4 + 4\*C\*a\*b\*c^5 + 24\*A\*b\*c^6)\*x^6 - 224\*(25\*C\*b^4\*c^3 + 40\*C\*a\*b^2\*c^4 + 96\*A\*a\*c^6 + 8\*(2\*C\*a^2 + 15\*A\*b^2)\*c^5)\*x^5 - 560\*(5\*C\*b^5\*c^2 + 24\*C\*a\*b^3\*c^3 + 96\*A\*a\*b\*c^5 + 8\*(2\*C\*a^2\*b + 3\*A\*b^3)\*c^4)\*x^4 - 70\*(5\*C\*b^6\*c + 124\*C\*a\*b^4\*c^2 + 384\*A\*a^2\*c^5 + 64\*(C\*a^3 + 9\*A\*a\*b^2)\*c^4 + 8\*(22\*C\*a^2\*b^2 + 3\*A\*b^4)\*c^3)\*x^3 - 240\*(8\*C\*a^4\*b - 7\*A\*a^2\*b^3)\*c^2 + 7\*(5\*C\*b^7 - 196\*C\*a\*b^5\*c - 5760\*A\*a^2\*b\*c^4 - 960\*(C\*a^3\*b + A\*a\*b^3)\*c^3 - 8\*(170\*C\*a^2\*b^3 - 3\*A\*b^5)\*c^2)\*x^2 - 4\*(80\*C\*a^3\*b^3 + 63\*A\*a\*b^5)\*c + 14\*(2\*C\*a\*b^6 - 720\*A\*a^2\*b^2\*c^3 - 960\*A\*a^3\*c^4 -

$$60*(8*C*a^3*b^2 - A*a*b^4)*c^2 - (80*C*a^2*b^4 + 3*A*b^6)*c)*x)*\text{sqrt}(c*x^2 + b*x + a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 + 256*a^8*c^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 256*a^4*c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*a^2*b^6*c^4 - 576*a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^11*c - 13*a*b^9*c^2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 + 768*a^5*b*c^6)*x^5 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3 - 2240*a^4*b^4*c^4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 13*a^2*b^9*c + 48*a^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*b*c^5)*x^3 + 2*(3*a^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b^4*c^3 + 256*a^6*b^2*c^4 + 512*a^7*c^5)*x^2 + 4*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6*b^3*c^3 + 256*a^7*b*c^4)*x)$$

**giac [B]** time = 0.31, size = 805, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x^2+A)/(c\*x^2+b\*x+a)^(9/2),x, algorithm="giac")

[Out]  $\frac{2}{105} * (((2 * (8 * (2 * (4 * (2 * (5 * C * b^2 * c^5 + 4 * C * a * c^6 + 24 * A * c^7)) * x / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) + 7 * (5 * C * b^3 * c^4 + 4 * C * a * b * c^5 + 24 * A * b * c^6) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) * x + 7 * (25 * C * b^4 * c^3 + 40 * C * a * b^2 * c^4 + 16 * C * a^2 * c^5 + 120 * A * b^2 * c^5 + 96 * A * a * c^6) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) * x + 35 * (5 * C * b^5 * c^2 + 24 * C * a * b^3 * c^3 + 16 * C * a^2 * b * c^4 + 24 * A * b^3 * c^4 + 96 * A * a * b * c^5) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) * x + 35 * (5 * C * b^6 * c + 124 * C * a * b^4 * c^2 + 176 * C * a^2 * b^2 * c^3 + 24 * A * b^4 * c^3 + 64 * C * a^3 * c^4 + 576 * A * a * b^2 * c^4 + 384 * A * a^2 * c^5) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) * x - 7 * (5 * C * b^7 - 196 * C * a * b^5 * c - 1360 * C * a^2 * b^3 * c^2 + 24 * A * b^5 * c^2 - 960 * C * a^3 * b * c^3 - 960 * A * a * b^3 * c^3 - 5760 * A * a^2 * b * c^4) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) * x - 14 * (2 * C * a * b^6 - 80 * C * a^2 * b^4 * c - 3 * A * b^6 * c - 480 * C * a^3 * b^2 * c^2 + 60 * A * a * b^4 * c^2 - 720 * A * a^2 * b^2 * c^3 - 960 * A * a^3 * c^4) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) * x - (8 * C * a^2 * b^5 + 15 * A * b^7 - 320 * C * a^3 * b^3 * c - 252 * A * a * b^5 * c - 1920 * C * a^4 * b * c^2 + 1680 * A * a^2 * b^3 * c^2 - 6720 * A * a^3 * b * c^3) / (b^8 - 16 * a * b^6 * c + 96 * a^2 * b^4 * c^2 - 256 * a^3 * b^2 * c^3 + 256 * a^4 * c^4)) / (c * x^2 + b * x + a)^(7/2)$

**maple [B]** time = 0.01, size = 555, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((C*x^2+A)/(c*x^2+b*x+a)^{(9/2)},x)$

[Out]  $\frac{2}{105} \frac{(c*x^2+b*x+a)^{(7/2)} * (6144*A*c^7*x^7 + 1024*C*a*c^6*x^7 + 1280*C*b^2*c^5*x^7 + 21504*A*b*c^6*x^6 + 3584*C*a*b*c^5*x^6 + 4480*C*b^3*c^4*x^6 + 21504*A*a*c^6*x^5 + 26880*A*b^2*c^5*x^5 + 3584*C*a^2*c^5*x^5 + 8960*C*a*b^2*c^4*x^5 + 5600*C*b^4*c^3*x^5 + 53760*A*a*b*c^5*x^4 + 13440*A*b^3*c^4*x^4 + 8960*C*a^2*b*c^4*x^4 + 13440*C*a*b^3*c^3*x^4 + 2800*C*b^5*c^2*x^4 + 26880*A*a^2*c^5*x^3 + 40320*A*a*b^2*c^4*x^3 + 1680*A*b^4*c^3*x^3 + 4480*C*a^3*c^4*x^3 + 12320*C*a^2*b^2*c^3*x^3 + 8680*C*a*b^4*c^2*x^3 + 350*C*b^6*c*x^3 + 40320*A*a^2*b*c^4*x^2 + 6720*A*a*b^3*c^3*x^2 - 168*A*b^5*c^2*x^2 + 6720*C*a^3*b*c^3*x^2 + 9520*C*a^2*b^3*c^2*x^2 + 1372*C*a*b^5*c*x^2 - 35*C*b^7*x^2 + 13440*A*a^3*c^4*x + 10080*A*a^2*b^2*c^3*x - 840*A*a*b^4*c^2*x + 42*A*b^6*c*x + 6720*C*a^3*b^2*c^2*x + 1120*C*a^2*b^4*c*x - 28*C*a*b^6*x + 6720*A*a^3*b*c^3 - 1680*A*a^2*b^3*c^2 + 252*A*a*b^5*c - 15*A*b^7 + 1920*C*a^4*b*c^2 + 320*C*a^3*b^3*c - 8*C*a^2*b^5)}{(256*a^4*c^4 - 256*a^3*b^2*c^3 + 96*a^2*b^4*c^2 - 16*a*b^6*c + b^8)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((C*x^2+A)/(c*x^2+b*x+a)^{(9/2)},x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [B] time = 5.06, size = 1018, normalized size = 4.63

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A + C*x^2)/(a + b*x + c*x^2)^{(9/2)},x)$

[Out]  $(x*((2*c^2*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c))*(4*a*c - b^2)^2) - (64*C*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*b^2*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (b*c*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*a*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^{(3/2)} - ((8*C*b)/(105*(4*a*c - b^2)^2) - (16*C*c*x)/(105*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^{(3/2)} + ((8*C*b*c)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c^2*x)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^{(3/2)} - ((4*C*x)/(35*(4*a*c - b^2)) - (2*C*b)/(35*c*(4*a*c - b^2)))/(a + b*x + c*x^2)^{(5/2)} + ((b*c*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3) + (2*c^2*x*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4$

$$\begin{aligned} & *a*c^2 - b^2*c)*(4*a*c - b^2)^3)/(a + b*x + c*x^2)^{1/2} + (x*((4*A*c^2)/(7*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(7*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(7*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(7*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(7*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^{7/2} + (x*((2*c*(48*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))) + (b*(4*8*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^{5/2} - ((32*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*C*c^3*x)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^{1/2} + ((64*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (128*C*c^3*x)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^{1/2} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C\*x\*\*2+A)/(c\*x\*\*2+b\*x+a)\*\*(9/2),x)

[Out] Timed out

$$3.182 \quad \int (g + hx)^3 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Optimal. Leaf size=930

$$\frac{f(cx^2 + bx + a)^{3/2} (g + hx)^4 (6cfg - 14ceh + 11bfh)(cx^2 + bx + a)^{3/2} (g + hx)^3}{7ch} - \frac{(6cfg - 14ceh + 11bfh)(cx^2 + bx + a)^{3/2} (g + hx)^3}{84c^2h} + \frac{(-4(3fg^2 - 7h(eg + 2dh))c}{7ch}$$

**Rubi [A]** time = 3.01, antiderivative size = 927, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 621, 206}

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^3\*Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((256\*c^5\*d\*g^3 - 33\*b^5\*f\*h^3 - 64\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + 2\*b\*g\*(e\*g + 3\*d\*h)) + 6\*b^3\*c\*h^2\*(20\*a\*f\*h + 7\*b\*(3\*f\*g + e\*h)) - 8\*b\*c^2\*h\*(10\*a^2\*f\*h^2 + 14\*a\*b\*h\*(3\*f\*g + e\*h) + 7\*b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 16\*c^3\*(2\*a^2\*h^2\*(3\*f\*g + e\*h) + 5\*b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 6\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(10\*24\*c^6) + (((33\*b^2\*f\*h^2 - 2\*c\*h\*(8\*b\*f\*g + 21\*b\*e\*h + 16\*a\*f\*h) - 4\*c^2\*(3\*f\*g^2 - 7\*h\*(e\*g + 2\*d\*h)))\*(g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2))/(280\*c^3\*h) - ((6\*c\*f\*g - 14\*c\*e\*h + 11\*b\*f\*h)\*(g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2))/(84\*c^2\*h) + (f\*(g + h\*x)^4\*(a + b\*x + c\*x^2)^(3/2))/(7\*c\*h) + ((1155\*b^4\*f\*h^4 - 128\*c^4\*(3\*f\*g^4 - 7\*g^2\*h\*(e\*g + 12\*d\*h)) - 42\*b^2\*c\*h^3\*(78\*a\*f\*h + 35\*b\*(3\*f\*g + e\*h)) + 8\*c^2\*h^2\*(128\*a^2\*f\*h^2 + 343\*a\*b\*h\*(3\*f\*g + e\*h) + b^2\*(537\*f\*g^2 + 245\*h\*(3\*e\*g + d\*h))) - 16\*c^3\*h\*(16\*a\*h\*(15\*f\*g^2 + 7\*h\*(3\*e\*g + d\*h)) + b\*g\*(17\*f\*g^2 + 21\*h\*(19\*e\*g + 25\*d\*h))) - 6\*c\*h\*(231\*b^3\*f\*h^3 - 6\*b\*c\*h^2\*(59\*b\*f\*g + 49\*b\*e\*h + 74\*a\*f\*h) + 16\*c^3\*(3\*f\*g^3 - 7\*g\*h\*(e\*g + 7\*d\*h)) + 8\*c^2\*h\*(5\*b\*f\*g^2 + 7\*b\*h\*(9\*e\*g + 7\*d\*h) + a\*h\*(41\*f\*g + 35\*e\*h))))\*x\*(a + b\*x + c\*x^2)^(3/2))/(13440\*c^5\*h) - ((b^2 - 4\*a\*c)\*(256\*c^5\*d\*g^3 - 33\*b^5\*f\*h^3 - 64\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + 2\*b\*g\*(e\*g + 3\*d\*h)) + 6\*b^3\*c\*h^2\*(20\*a\*f\*h + 7\*b\*(3\*f\*g + e\*h)) - 8\*b\*c^2\*h\*(10\*a^2\*f\*h^2 + 14\*a\*b\*h\*(3\*f\*g + e\*h) + 7\*b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 16\*c^3\*(2\*a^2\*h^2\*(3\*f\*g + e\*h) + 5\*b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 6\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2048\*c^(13/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 612

$Int[(a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_)}, x\_Symbol] := Simp[(b + 2*c*x)*(a + b*x + c*x^2)^p / (2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c)) / (2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^{(p - 1)}, x], x] /; FreeQ[{a, b, c}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& GtQ[p, 0] \&\& IntegerQ[4*p]$

### Rule 621

$Int[1/Sqrt[(a_) + (b_.)(x_) + (c_.)(x_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] \&\& NeQ[b^2 - 4*a*c, 0]$

### Rule 779

$Int[((d_.) + (e_.)(x_))*((f_.) + (g_.)(x_))*((a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_)}, x\_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^{(p + 1)} / (2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3)) / (2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& !LeQ[p, -1]$

### Rule 832

$Int[((d_.) + (e_.)(x_))^{(m_)}*((f_.) + (g_.)(x_))*((a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_)}, x\_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^{(p + 1)}) / (c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^{(m - 1)}*(a + b*x + c*x^2)^p * Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& NeQ[c*d^2 - b*d*e + a*e^2, 0] \&\& GtQ[m, 0] \&\& NeQ[m + 2*p + 2, 0] \&\& (IntegerQ[m] \parallel IntegerQ[p] \parallel IntegersQ[2*m, 2*p]) \&\& !(IGtQ[m, 0] \&\& EqQ[f, 0])$

### Rule 1653

$Int[(Pq_)*((d_.) + (e_.)(x_))^{(m_)}*((a_.) + (b_.)(x_) + (c_.)(x_)^2]^{(p_)}, x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^{(m + q - 1)}*(a + b*x + c*x^2)^{(p + 1)}) / (c*e^{(q - 1)}*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p * ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^{(q - 2)}*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q, 1] \&\& NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] \&\& Poly$

$Q[Pq, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ !(IGtQ[m, 0] \ \&\& \ \text{RationalQ}[a, b, c, d, e] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{ILtQ}[p + 1/2, 0]))$

Rubi steps

$$\begin{aligned}
 \int (g + hx)^3 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx &= \frac{f(g + hx)^4 (a + bx + cx^2)^{3/2}}{7ch} + \frac{\int (g + hx)^3 \left(-\frac{1}{2}h(3bfg - 14cdh + \dots)\right)}{\dots} \\
 &= -\frac{(6cfg - 14ceh + 11bfh)(g + hx)^3 (a + bx + cx^2)^{3/2}}{84c^2h} + \frac{f(g + hx)^4 (a + bx + cx^2)^{3/2}}{\dots} \\
 &= \frac{(33b^2fh^2 - 2ch(8bfg + 21beh + 16afh) - 4c^2(3fg^2 - 7h(eg + dh) + 2bh^2)) (g + hx)^3 (a + bx + cx^2)^{3/2}}{280c^3h} \\
 &= \frac{(33b^2fh^2 - 2ch(8bfg + 21beh + 16afh) - 4c^2(3fg^2 - 7h(eg + dh) + 2bh^2)) (g + hx)^3 (a + bx + cx^2)^{3/2}}{280c^3h} \\
 &= \frac{(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + dh) + 2bh^2)) (g + hx)^3 (a + bx + cx^2)^{3/2}}{280c^3h} \\
 &= \frac{(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + dh) + 2bh^2)) (g + hx)^3 (a + bx + cx^2)^{3/2}}{280c^3h} \\
 &= \frac{(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg + dh) + 2bg(eg + dh) + 2bh^2)) (g + hx)^3 (a + bx + cx^2)^{3/2}}{280c^3h}
 \end{aligned}$$

**Mathematica [A]** time = 2.42, size = 1093, normalized size = 1.18

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] (2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]\*(-3465\*b^6\*f\*h^3 + 210\*b^5\*c\*h^2\*(63\*f\*g + 21\*e\*h + 11\*f\*h\*x) - 84\*b^4\*c\*h\*(-260\*a\*f\*h^2 + 35\*c\*h\*(6\*e\*g + 2\*d\*h + e\*h\*x) + c\*f\*(210\*g^2 + 105\*g\*h\*x + 22\*h^2\*x^2)) - 16\*b^2\*c^2\*(2163\*a^2\*f\*h^3 - 2\*a\*c\*h\*(7\*h\*(345\*e\*g + 115\*d\*h + 56\*e\*h\*x) + 3\*f\*(805\*g^2 + 392\*g\*h\*x + 81\*h^2\*x^2)) + 2\*c^2\*(7\*d\*h\*(180\*g^2 + 75\*g\*h\*x + 14\*h^2\*x^2) + 21\*e\*(20\*g^3 + 25\*g^2\*h\*x + 14\*g\*h^2\*x^2 + 3\*h^3\*x^3) + f\*x\*(175\*g^3 + 294\*g^2\*h\*x + 189\*g\*h^2\*x^2 + 44\*h^3\*x^3))) + 16\*b^3\*c^2\*(-42\*a\*h^2\*(35\*e\*h + 3\*f\*(35\*g + 6\*h\*x)) + c\*(f\*(525\*g^3 + 735\*g^2\*h\*x + 441\*g\*h^2\*x^2 + 99\*h^3\*x^3) + 7\*h\*

$$\begin{aligned}
& (5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2))) + 32*b*c^3*( \\
& a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 609*g^2*h*x \\
& + 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(195*g^2 + 87 \\
& *g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g*h^2*x^2 + h \\
& ^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + f*x*(35* \\
& g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3))) + 64*c^3*(128*a^3*f*h^3 - \\
& a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315*g*h*x + 64*h^2 \\
& *x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45 \\
& *g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^ \\
& 2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4 \\
& *h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f* \\
& x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3)))) + 105*(b^2 - 4*a*c) \\
& *(-256*c^5*d*g^3 + 33*b^5*f*h^3 + 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2 \\
& *b*g*(e*g + 3*d*h)) - 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) + 8*b*c^2* \\
& h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2 \\
& )) - 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + \\
& 6*a*b*h*(3*f*g^2 + h*(3*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a \\
& + x*(b + c*x)])]/(215040*c^(13/2))
\end{aligned}$$

**IntegrateAlgebraic [A]** time = 13.41, size = 1767, normalized size = 1.90

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)^3\*sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] (sqrt[a + b\*x + c\*x^2]\*(26880\*b\*c^5\*d\*g^3 - 13440\*b^2\*c^4\*e\*g^3 + 35840\*a\*c^5\*e\*g^3 + 8400\*b^3\*c^3\*f\*g^3 - 29120\*a\*b\*c^4\*f\*g^3 - 40320\*b^2\*c^4\*d\*g^2\*h + 107520\*a\*c^5\*d\*g^2\*h + 25200\*b^3\*c^3\*e\*g^2\*h - 87360\*a\*b\*c^4\*e\*g^2\*h - 17640\*b^4\*c^2\*f\*g^2\*h + 77280\*a\*b^2\*c^3\*f\*g^2\*h - 43008\*a^2\*c^4\*f\*g^2\*h + 25200\*b^3\*c^3\*d\*g\*h^2 - 87360\*a\*b\*c^4\*d\*g\*h^2 - 17640\*b^4\*c^2\*e\*g\*h^2 + 77280\*a\*b^2\*c^3\*e\*g\*h^2 - 43008\*a^2\*c^4\*e\*g\*h^2 + 13230\*b^5\*c\*f\*g\*h^2 - 70560\*a\*b^3\*c^2\*f\*g\*h^2 + 75936\*a^2\*b\*c^3\*f\*g\*h^2 - 5880\*b^4\*c^2\*d\*h^3 + 25760\*a\*b^2\*c^3\*d\*h^3 - 14336\*a^2\*c^4\*d\*h^3 + 4410\*b^5\*c\*e\*h^3 - 23520\*a\*b^3\*c^2\*e\*h^3 + 25312\*a^2\*b\*c^3\*e\*h^3 - 3465\*b^6\*f\*h^3 + 21840\*a\*b^4\*c\*f\*h^3 - 34608\*a^2\*b^2\*c^2\*f\*h^3 + 8192\*a^3\*c^3\*f\*h^3 + 53760\*c^6\*d\*g^3\*x + 8960\*b\*c^5\*e\*g^3\*x - 5600\*b^2\*c^4\*f\*g^3\*x + 13440\*a\*c^5\*f\*g^3\*x + 26880\*b\*c^5\*d\*g^2\*h\*x - 16800\*b^2\*c^4\*e\*g^2\*h\*x + 40320\*a\*c^5\*e\*g^2\*h\*x + 11760\*b^3\*c^3\*f\*g^2\*h\*x - 38976\*a\*b\*c^4\*f\*g^2\*h\*x - 16800\*b^2\*c^4\*d\*g\*h^2\*x + 40320\*a\*c^5\*d\*g\*h^2\*x + 11760\*b^3\*c^3\*e\*g\*h^2\*x - 38976\*a\*b\*c^4\*e\*g\*h^2\*x - 8820\*b^4\*c^2\*f\*g\*h^2\*x + 37632\*a\*b^2\*c^3\*f\*g\*h^2\*x - 20160\*a^2\*c^4\*f\*g\*h^2\*x + 3920\*b^3\*c^3\*d\*h^3\*x - 12992\*a\*b\*c^4\*d\*h^3\*x - 2940\*b^4\*c^2\*e\*h^3\*x + 12544\*a\*b^2\*c^3\*e\*h^3\*x - 6720\*a^2\*c^4\*e\*h^3\*x + 2310\*b^5\*c\*f\*h^3\*x - 12096\*a\*b^3\*c^2\*f\*h^3\*x + 12704\*a^2\*b\*c^3\*f\*h^3\*x + 35840\*c^6\*e\*g^3\*x^2 + 4480\*b\*c^5\*f\*g^3\*x^2 + 107520\*c^6\*d\*g^2\*h\*x^2 + 13440\*b\*c^5\*e\*g^2\*h\*x^2 - 9408\*b^2\*c^4\*f\*g^2\*h\*x^2 + 215



$$\begin{aligned}
& 04*a*c^5*f*g^2*h*x^2 + 13440*b*c^5*d*g*h^2*x^2 - 9408*b^2*c^4*e*g*h^2*x^2 + \\
& 21504*a*c^5*e*g*h^2*x^2 + 7056*b^3*c^3*f*g*h^2*x^2 - 22848*a*b*c^4*f*g*h^2 \\
& *x^2 - 3136*b^2*c^4*d*h^3*x^2 + 7168*a*c^5*d*h^3*x^2 + 2352*b^3*c^3*e*h^3*x \\
& ^2 - 7616*a*b*c^4*e*h^3*x^2 - 1848*b^4*c^2*f*h^3*x^2 + 7776*a*b^2*c^3*f*h^3 \\
& *x^2 - 4096*a^2*c^4*f*h^3*x^2 + 26880*c^6*f*g^3*x^3 + 80640*c^6*e*g^2*h*x^3 \\
& + 8064*b*c^5*f*g^2*h*x^3 + 80640*c^6*d*g*h^2*x^3 + 8064*b*c^5*e*g*h^2*x^3 \\
& - 6048*b^2*c^4*f*g*h^2*x^3 + 13440*a*c^5*f*g*h^2*x^3 + 2688*b*c^5*d*h^3*x^3 \\
& - 2016*b^2*c^4*e*h^3*x^3 + 4480*a*c^5*e*h^3*x^3 + 1584*b^3*c^3*f*h^3*x^3 - \\
& 5056*a*b*c^4*f*h^3*x^3 + 64512*c^6*f*g^2*h*x^4 + 64512*c^6*e*g*h^2*x^4 + 5 \\
& 376*b*c^5*f*g*h^2*x^4 + 21504*c^6*d*h^3*x^4 + 1792*b*c^5*e*h^3*x^4 - 1408*b \\
& ^2*c^4*f*h^3*x^4 + 3072*a*c^5*f*h^3*x^4 + 53760*c^6*f*g*h^2*x^5 + 17920*c^6 \\
& *e*h^3*x^5 + 1280*b*c^5*f*h^3*x^5 + 15360*c^6*f*h^3*x^6)/(107520*c^6) + (( \\
& 256*b^2*c^5*d*g^3 - 1024*a*c^6*d*g^3 - 128*b^3*c^4*e*g^3 + 512*a*b*c^5*e*g^ \\
& 3 + 80*b^4*c^3*f*g^3 - 384*a*b^2*c^4*f*g^3 + 256*a^2*c^5*f*g^3 - 384*b^3*c^ \\
& 4*d*g^2*h + 1536*a*b*c^5*d*g^2*h + 240*b^4*c^3*e*g^2*h - 1152*a*b^2*c^4*e*g \\
& ^2*h + 768*a^2*c^5*e*g^2*h - 168*b^5*c^2*f*g^2*h + 960*a*b^3*c^3*f*g^2*h - \\
& 1152*a^2*b*c^4*f*g^2*h + 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*g*h^2 + 768 \\
& *a^2*c^5*d*g*h^2 - 168*b^5*c^2*e*g*h^2 + 960*a*b^3*c^3*e*g*h^2 - 1152*a^2*b \\
& *c^4*e*g*h^2 + 126*b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*a^2*b^2*c^3 \\
& *f*g*h^2 - 384*a^3*c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^3*d*h^3 - 3 \\
& 84*a^2*b*c^4*d*h^3 + 42*b^6*c*e*h^3 - 280*a*b^4*c^2*e*h^3 + 480*a^2*b^2*c^3 \\
& *e*h^3 - 128*a^3*c^4*e*h^3 - 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 - 560*a^2*b^3 \\
& *c^2*f*h^3 + 320*a^3*b*c^3*f*h^3)*Log[b + 2*c*x - 2*sqrt[c]*sqrt[a + b*x + \\
& c*x^2]]/(2048*c^(13/2))
\end{aligned}$$

**fricas** [A] time = 3.62, size = 2817, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/430080\*(105\*(16\*(16\*(b^2\*c^5 - 4\*a\*c^6)\*d - 8\*(b^3\*c^4 - 4\*a\*b\*c^5)\*e + (5\*b^4\*c^3 - 24\*a\*b^2\*c^4 + 16\*a^2\*c^5)\*f)\*g^3 - 24\*(16\*(b^3\*c^4 - 4\*a\*b\*c^5)\*d - 2\*(5\*b^4\*c^3 - 24\*a\*b^2\*c^4 + 16\*a^2\*c^5)\*e + (7\*b^5\*c^2 - 40\*a\*b^3\*c^3 + 48\*a^2\*b\*c^4)\*f)\*g^2\*h + 6\*(8\*(5\*b^4\*c^3 - 24\*a\*b^2\*c^4 + 16\*a^2\*c^5)\*d - 4\*(7\*b^5\*c^2 - 40\*a\*b^3\*c^3 + 48\*a^2\*b\*c^4)\*e + (21\*b^6\*c - 140\*a\*b^4\*c^2 + 240\*a^2\*b^2\*c^3 - 64\*a^3\*c^4)\*f)\*g\*h^2 - (8\*(7\*b^5\*c^2 - 40\*a\*b^3\*c^3 + 48\*a^2\*b\*c^4)\*d - 2\*(21\*b^6\*c - 140\*a\*b^4\*c^2 + 240\*a^2\*b^2\*c^3 - 64\*a^3\*c^4)\*e + (33\*b^7 - 252\*a\*b^5\*c + 560\*a^2\*b^3\*c^2 - 320\*a^3\*b\*c^3)\*f)\*h^3)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(15360\*c^7\*f\*h^3\*x^6 + 1280\*(42\*c^7\*f\*g\*h^2 + (14\*c^7\*e + b\*c^6\*f)\*h^3)\*x^5 + 128\*(504\*c^7\*f\*g^2\*h + 42\*(12\*c^7\*e + b\*c^6\*f)\*g\*h^2 + (168\*c^7\*d + 14\*b\*c^6\*e - (11\*b^2\*c^5 - 24\*a\*c^6)\*f)\*h^3)\*x^4 + 560\*(48\*b\*c^6\*d - 8\*(3\*b^2\*c^5 - 8\*a\*c^6)\*e + (15\*b^3\*c^4 - 52\*a\*b\*c^5)\*f)\*g^3 -

$$\begin{aligned}
& 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b*c^6*e - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 20*a*c^6)*e + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c^6*f)*g^3 + 168*(80*c^7*d + 10*b*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d - 4*(7*b^2*c^5 - 16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8*b*c^6*e - (5*b^2*c^5 - 12*a*c^6)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5 - 12*a*c^6)*e + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d - 4*(35*b^3*c^4 - 116*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a)/c^7, 1/215040*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e + (5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b*c^6*e - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 20*a*c^6)*e + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c^6*f)*g^3 + 168*(80*c^7*d + 10*b*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d - 4*(7*b^2*c^5 - 16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8*b*c^6*e - (5*b^2*c^5 - 12*a*c^6)
\end{aligned}$$

$$\begin{aligned} & ) * f) * g^3 + 168 * (80 * b * c^6 * d - 10 * (5 * b^2 * c^5 - 12 * a * c^6) * e + (35 * b^3 * c^4 - 11 \\ & 6 * a * b * c^5) * f) * g^2 * h - 42 * (40 * (5 * b^2 * c^5 - 12 * a * c^6) * d - 4 * (35 * b^3 * c^4 - 116 \\ & * a * b * c^5) * e + (105 * b^4 * c^3 - 448 * a * b^2 * c^4 + 240 * a^2 * c^5) * f) * g * h^2 + (56 * (3 \\ & 5 * b^3 * c^4 - 116 * a * b * c^5) * d - 14 * (105 * b^4 * c^3 - 448 * a * b^2 * c^4 + 240 * a^2 * c^5) \\ & * e + (1155 * b^5 * c^2 - 6048 * a * b^3 * c^3 + 6352 * a^2 * b * c^4) * f) * h^3) * x) * \text{sqrt}(c * x^2 \\ & + b * x + a) / c^7 \end{aligned}$$

**giac** [A] time = 0.31, size = 1702, normalized size = 1.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] 1/107520\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(2\*(8\*(10\*(12\*f\*h^3\*x + (42\*c^6\*f\*g\*h^2 + b\*c^5\*f\*h^3 + 14\*c^6\*h^3\*e)/c^6)\*x + (504\*c^6\*f\*g^2\*h + 42\*b\*c^5\*f\*g\*h^2 + 168\*c^6\*d\*h^3 - 11\*b^2\*c^4\*f\*h^3 + 24\*a\*c^5\*f\*h^3 + 504\*c^6\*g\*h^2\*e + 14\*b\*c^5\*h^3\*e)/c^6)\*x + (1680\*c^6\*f\*g^3 + 504\*b\*c^5\*f\*g^2\*h + 5040\*c^6\*d\*g\*h^2 - 378\*b^2\*c^4\*f\*g\*h^2 + 840\*a\*c^5\*f\*g\*h^2 + 168\*b\*c^5\*d\*h^3 + 99\*b^3\*c^3\*f\*h^3 - 316\*a\*b\*c^4\*f\*h^3 + 5040\*c^6\*g^2\*h\*e + 504\*b\*c^5\*g\*h^2\*e - 126\*b^2\*c^4\*h^3\*e + 280\*a\*c^5\*h^3\*e)/c^6)\*x + (560\*b\*c^5\*f\*g^3 + 13440\*c^6\*d\*g^2\*h - 1176\*b^2\*c^4\*f\*g^2\*h + 2688\*a\*c^5\*f\*g^2\*h + 1680\*b\*c^5\*d\*g\*h^2 + 882\*b^3\*c^3\*f\*g\*h^2 - 2856\*a\*b\*c^4\*f\*g\*h^2 - 392\*b^2\*c^4\*d\*h^3 + 896\*a\*c^5\*d\*h^3 - 231\*b^4\*c^2\*f\*h^3 + 972\*a\*b^2\*c^3\*f\*h^3 - 512\*a^2\*c^4\*f\*h^3 + 4480\*c^6\*g^3\*e + 1680\*b\*c^5\*g^2\*h\*e - 1176\*b^2\*c^4\*g\*h^2\*e + 2688\*a\*c^5\*g\*h^2\*e + 294\*b^3\*c^3\*h^3\*e - 952\*a\*b\*c^4\*h^3\*e)/c^6)\*x + (26880\*c^6\*d\*g^3 - 2800\*b^2\*c^4\*f\*g^3 + 6720\*a\*c^5\*f\*g^3 + 13440\*b\*c^5\*d\*g^2\*h + 5880\*b^3\*c^3\*f\*g^2\*h - 19488\*a\*b\*c^4\*f\*g^2\*h - 8400\*b^2\*c^4\*d\*g\*h^2 + 20160\*a\*c^5\*d\*g\*h^2 - 4410\*b^4\*c^2\*f\*g\*h^2 + 18816\*a\*b^2\*c^3\*f\*g\*h^2 - 10080\*a^2\*c^4\*f\*g\*h^2 + 1960\*b^3\*c^3\*d\*h^3 - 6496\*a\*b\*c^4\*d\*h^3 + 1155\*b^5\*c\*f\*h^3 - 6048\*a\*b^3\*c^2\*f\*h^3 + 6352\*a^2\*b\*c^3\*f\*h^3 + 4480\*b\*c^5\*g^3\*e - 8400\*b^2\*c^4\*g^2\*h\*e + 20160\*a\*c^5\*g^2\*h\*e + 5880\*b^3\*c^3\*g\*h^2\*e - 19488\*a\*b\*c^4\*g\*h^2\*e - 1470\*b^4\*c^2\*h^3\*e + 6272\*a\*b^2\*c^3\*h^3\*e - 3360\*a^2\*c^4\*h^3\*e)/c^6)\*x + (26880\*b\*c^5\*d\*g^3 + 8400\*b^3\*c^3\*f\*g^3 - 29120\*a\*b\*c^4\*f\*g^3 - 40320\*b^2\*c^4\*d\*g^2\*h + 107520\*a\*c^5\*d\*g^2\*h - 17640\*b^4\*c^2\*f\*g^2\*h + 77280\*a\*b^2\*c^3\*f\*g^2\*h - 43008\*a^2\*c^4\*f\*g^2\*h + 25200\*b^3\*c^3\*d\*g\*h^2 - 87360\*a\*b\*c^4\*d\*g\*h^2 + 13230\*b^5\*c\*f\*g\*h^2 - 70560\*a\*b^3\*c^2\*f\*g\*h^2 + 75936\*a^2\*b\*c^3\*f\*g\*h^2 - 5880\*b^4\*c^2\*d\*h^3 + 25760\*a\*b^2\*c^3\*d\*h^3 - 14336\*a^2\*c^4\*d\*h^3 - 3465\*b^6\*f\*h^3 + 21840\*a\*b^4\*c\*f\*h^3 - 34608\*a^2\*b^2\*c^2\*f\*h^3 + 8192\*a^3\*c^3\*f\*h^3 - 13440\*b^2\*c^4\*g^3\*e + 35840\*a\*c^5\*g^3\*e + 25200\*b^3\*c^3\*g^2\*h\*e - 87360\*a\*b\*c^4\*g^2\*h\*e - 17640\*b^4\*c^2\*g\*h^2\*e + 77280\*a\*b^2\*c^3\*g\*h^2\*e - 43008\*a^2\*c^4\*g\*h^2\*e + 4410\*b^5\*c\*h^3\*e - 23520\*a\*b^3\*c^2\*h^3\*e + 25312\*a^2\*b\*c^3\*h^3\*e)/c^6) + 1/2048\*(256\*b^2\*c^5\*d\*g^3 - 1024\*a\*c^6\*d\*g^3 + 80\*b^4\*c^3\*f\*g^3 - 384\*a\*b^2\*c^4\*f\*g^3 + 256\*a^2\*c^5\*f\*g^3 - 384\*b^3\*c^4\*d\*g^2\*h + 1536\*a\*b\*c^5\*d\*g^2\*h - 168\*b^5\*c^2\*f\*g^2\*h + 960\*a\*b^3\*c^3\*f\*g^2\*h - 1152\*a^2\*b\*c^4\*f\*g^2\*h

$$\begin{aligned}
& + 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*g*h^2 + 768*a^2*c^5*d*g*h^2 + 126* \\
& b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*a^2*b^2*c^3*f*g*h^2 - 384*a^3* \\
& c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^3*d*h^3 - 384*a^2*b*c^4*d*h^3 \\
& - 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 - 560*a^2*b^3*c^2*f*h^3 + 320*a^3*b*c^3* \\
& f*h^3 - 128*b^3*c^4*g^3*e + 512*a*b*c^5*g^3*e + 240*b^4*c^3*g^2*h*e - 1152* \\
& a*b^2*c^4*g^2*h*e + 768*a^2*c^5*g^2*h*e - 168*b^5*c^2*g*h^2*e + 960*a*b^3*c^3* \\
& g*h^2*e - 1152*a^2*b*c^4*g*h^2*e + 42*b^6*c*h^3*e - 280*a*b^4*c^2*h^3*e \\
& + 480*a^2*b^2*c^3*h^3*e - 128*a^3*c^4*h^3*e) * \log(\text{abs}(-2 * (\text{sqrt}(c) * x - \text{sqrt}(c) \\
& * x^2 + b * x + a)) * \text{sqrt}(c) - b) / c^{(13/2)})
\end{aligned}$$

**maple [B]** time = 0.02, size = 3543, normalized size = 3.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^{(1/2)}, x)$

[Out] 
$$\begin{aligned}
& 3/5*x^2*(c*x^2+b*x+a)^{(3/2)}/c*f*g^2*h-7/40/c^2*b*x*(c*x^2+b*x+a)^{(3/2)}*d*h^3 \\
& -3/8*a^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e*g^2*h+3/4*x \\
& *(c*x^2+b*x+a)^{(3/2)}/c*d*g*h^2+3/4*x*(c*x^2+b*x+a)^{(3/2)}/c*e*g^2*h-5/8/c^2* \\
& b*(c*x^2+b*x+a)^{(3/2)}*d*g*h^2+15/64/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*d*g*h^2+15/ \\
& 64/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*e*g^2*h+3/16/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+ \\
& (c*x^2+b*x+a)^{(1/2)})*a*f*g^3-15/128/c^{(7/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+ \\
& (c*x^2+b*x+a)^{(1/2)})*d*g*h^2-15/128/c^{(7/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x \\
& ^2+b*x+a)^{(1/2)})*e*g^2*h-1/8*a/c*x*(c*x^2+b*x+a)^{(1/2)}*f*g^3-1/16*a/c^2*(c* \\
& x^2+b*x+a)^{(1/2)}*b*f*g^3-3/8*a^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+ \\
& a)^{(1/2)})*d*g*h^2-33/320*f*h^3/c^4*b^3*x*(c*x^2+b*x+a)^{(3/2)}-33/512*f*h^3/c \\
& ^5*b^5*x*(c*x^2+b*x+a)^{(1/2)}+15/128*f*h^3/c^5*b^4*a*(c*x^2+b*x+a)^{(1/2)}-39/ \\
& 160*f*h^3/c^4*b^2*a*(c*x^2+b*x+a)^{(3/2)}-5/64*f*h^3/c^4*b^2*a^2*(c*x^2+b*x+a \\
& )^{(1/2)}-63/512*f*h^3/c^{(11/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2} \\
& ))*a+35/128*f*h^3/c^{(9/2)}*b^3*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2} \\
& ))-5/32*f*h^3/c^{(7/2)}*b*a^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-4/3 \\
& 5*f*h^3*a/c^2*x^2*(c*x^2+b*x+a)^{(3/2)}-11/84*f*h^3/c^2*b*x^3*(c*x^2+b*x+a)^{(3/2)}+33/280*f*h^3/c^3*b^2*x^2*(c*x^2+b*x+a)^{(3/2)}+7/16/c^3*b^2*(c*x^2+b*x+a)^{(3/2)}*f*g^2*h-7/64/c^3*b^3*x*(c*x^2+b*x+a)^{(1/2)}*d*h^3-21/128/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*e*g*h^2-21/128/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*f*g^2*h-2/5*a/c^2*(c*x^2+b*x+a)^{(3/2)}*e*g*h^2-2/5*a/c^2*(c*x^2+b*x+a)^{(3/2)}*f*g^2*h-5/32/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*d*h^3+21/256/c^{(9/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e*g*h^2-3/8*a/c*x*(c*x^2+b*x+a)^{(1/2)}*e*g^2*h-3/16*a/c^2*(c*x^2+b*x+a)^{(1/2)}*b*d*g*h^2-3/16*a/c^2*(c*x^2+b*x+a)^{(1/2)}*b*e*g^2*h-3/4/c*b*x*(c*x^2+b*x+a)^{(1/2)}*d*g^2*h-3/4/c^{(3/2)}*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*d*g^2*h-3/8*a/c*x*(c*x^2+b*x+a)^{(1/2)}*d*g*h^2+1/3*(c*x^2+b*x+a)^{(3/2)}/c*e*g^3+1/2*d*g^3*x*(c*x^2+b*x+a)^{(1/2)}+21/256/c^{(9/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*g^2*h+3/32/c^3*b^2*a*(c*x^2+b*x+a)^{(1/2)}*d*h^3+3/16/c^{(5/2)}*b*a^2*\ln((c*x
\end{aligned}$$

$$\begin{aligned}
& +1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*h^3+7/16/c^3*b^2*(c*x^2+b*x+a)^{(3/2)} \\
& *e*g*h^2-5/8/c^2*b*(c*x^2+b*x+a)^{(3/2)}*e*g^2*h+5/32/c^2*b^2*x*(c*x^2+b*x+a) \\
& ^{(1/2)}*f*g^3-1/4/c*b*x*(c*x^2+b*x+a)^{(1/2)}*e*g^3-3/8/c^2*b^2*(c*x^2+b*x+a) \\
& ^{(1/2)}*d*g^2*h-1/4/c^{(3/2)}*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e \\
& *g^3+3/16/c^{(5/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*g^2*h+2 \\
& 1/512/c^5*b^5*(c*x^2+b*x+a)^{(1/2)}*e*h^3+15/64*f*h^3/c^4*b^3*a*x*(c*x^2+b*x+ \\
& a)^{(1/2)}+111/560*f*h^3/c^3*b*a*x*(c*x^2+b*x+a)^{(3/2)}-15/32/c^{(7/2)}*b^3*\ln(( \\
& c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*f*g^2*h+3/16/c^2*b*a*x*(c*x^2+b*x \\
& +a)^{(1/2)}*d*h^3+9/32/c^3*b^2*a*(c*x^2+b*x+a)^{(1/2)}*e*g*h^2+9/32/c^3*b^2*a*( \\
& c*x^2+b*x+a)^{(1/2)}*f*g^2*h+9/16/c^{(5/2)}*b*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2 \\
& +b*x+a)^{(1/2)})*e*g*h^2+9/16/c^{(5/2)}*b*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x \\
& +a)^{(1/2)})*f*g^2*h-21/40/c^2*b*x*(c*x^2+b*x+a)^{(3/2)}*e*g*h^2-21/40/c^2*b*x* \\
& (c*x^2+b*x+a)^{(3/2)}*f*g^2*h-21/64/c^3*b^3*x*(c*x^2+b*x+a)^{(1/2)}*e*g*h^2-21/ \\
& 64/c^3*b^3*x*(c*x^2+b*x+a)^{(1/2)}*f*g^2*h+7/48/c^3*b^2*(c*x^2+b*x+a)^{(3/2)}*d \\
& *h^3-7/128/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*d*h^3+7/256/c^{(9/2)}*b^5*\ln((c*x+1/2* \\
& b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*h^3-2/15*a/c^2*(c*x^2+b*x+a)^{(3/2)}*d*h^3+ \\
& 1/4*x*(c*x^2+b*x+a)^{(3/2)}/c*f*g^3-5/24/c^2*b*(c*x^2+b*x+a)^{(3/2)}*f*g^3+5/64 \\
& /c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*f*g^3-5/128/c^{(7/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)} \\
& +(c*x^2+b*x+a)^{(1/2)})*f*g^3-1/8*a^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b \\
& *x+a)^{(1/2)})*f*g^3+(c*x^2+b*x+a)^{(3/2)}/c*d*g^2*h-1/8/c^2*b^2*(c*x^2+b*x+a) \\
& ^{(1/2)}*e*g^3+1/16/c^{(5/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e \\
& g^3+1/4*d*g^3/c*(c*x^2+b*x+a)^{(1/2)}*b+1/2*d*g^3/c^{(1/2)}*\ln((c*x+1/2*b)/c^{(1 \\
& /2)}+(c*x^2+b*x+a)^{(1/2)})*a-1/8*d*g^3/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+ \\
& b*x+a)^{(1/2)})*b^2+3/32*a^2/c^3*(c*x^2+b*x+a)^{(1/2)}*b*f*g*h^2-3/8*a/c^2*x*(c \\
& *x^2+b*x+a)^{(3/2)}*f*g*h^2+3/16*a^2/c^2*x*(c*x^2+b*x+a)^{(1/2)}*f*g*h^2+105/25 \\
& 6/c^{(9/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*f*g*h^2-7/32/c^ \\
& 3*b^2*a*x*(c*x^2+b*x+a)^{(1/2)}*e*h^3-21/64/c^4*b^3*a*(c*x^2+b*x+a)^{(1/2)}*f*g \\
& *h^2-45/64/c^{(7/2)}*b^2*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*g* \\
& h^2+49/80/c^3*b*a*(c*x^2+b*x+a)^{(3/2)}*f*g*h^2+3/5*x^2*(c*x^2+b*x+a)^{(3/2)}/c \\
& *e*g*h^2+63/160/c^3*b^2*x*(c*x^2+b*x+a)^{(3/2)}*f*g*h^2+63/256/c^4*b^4*x*(c*x \\
& ^2+b*x+a)^{(1/2)}*f*g*h^2-5/32*f*h^3/c^3*b*a^2*x*(c*x^2+b*x+a)^{(1/2)}-15/32/c^ \\
& (7/2)*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*g*h^2+15/32/c^2*b \\
& ^2*x*(c*x^2+b*x+a)^{(1/2)}*d*g*h^2+15/32/c^2*b^2*x*(c*x^2+b*x+a)^{(1/2)}*e*g^2* \\
& h+9/16/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*d*g*h^2+9/ \\
& 16/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*g^2*h-21/32/ \\
& c^3*b^2*a*x*(c*x^2+b*x+a)^{(1/2)}*f*g*h^2+9/16/c^2*b*a*x*(c*x^2+b*x+a)^{(1/2)}* \\
& e*g*h^2+9/16/c^2*b*a*x*(c*x^2+b*x+a)^{(1/2)}*f*g^2*h-7/64/c^4*b^3*(c*x^2+b*x+ \\
& a)^{(3/2)}*e*h^3-9/20/c^2*b*x^2*(c*x^2+b*x+a)^{(3/2)}*f*g*h^2-3/20/c^2*b*x^2*(c \\
& *x^2+b*x+a)^{(3/2)}*e*h^3+21/160/c^3*b^2*x*(c*x^2+b*x+a)^{(3/2)}*e*h^3-21/64/c^ \\
& 4*b^3*(c*x^2+b*x+a)^{(3/2)}*f*g*h^2+21/256/c^4*b^4*x*(c*x^2+b*x+a)^{(1/2)}*e*h^ \\
& 3+63/512/c^5*b^5*(c*x^2+b*x+a)^{(1/2)}*f*g*h^2+35/256/c^{(9/2)}*b^4*\ln((c*x+1/2 \\
& *b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*h^3-63/1024/c^{(11/2)}*b^6*\ln((c*x+1/2*b \\
& )/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*g*h^2-7/64/c^4*b^3*a*(c*x^2+b*x+a)^{(1/2)}*e \\
& *h^3-15/64/c^{(7/2)}*b^2*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e*h^ \\
& 3+49/240/c^3*b*a*(c*x^2+b*x+a)^{(3/2)}*e*h^3-1/8*a/c^2*x*(c*x^2+b*x+a)^{(3/2)}*
\end{aligned}$$

$$e^{h^3+1/16*a^2/c^2*x*(c*x^2+b*x+a)^{1/2}}*e^{h^3+1/32*a^2/c^3*(c*x^2+b*x+a)^{1/2}}*b*e^{h^3+3/16*a^3/c^{5/2}}*\ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2})*f*g*h^2+1/2*x^3*(c*x^2+b*x+a)^{3/2}/c*f*g*h^2-21/1024/c^{11/2}*b^6*\ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2})*e^{h^3+1/16*a^3/c^{5/2}}*\ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2})*e^{h^3+1/6*x^3*(c*x^2+b*x+a)^{3/2}/c}*e^{h^3+33/2048*f*h^3/c^{13/2}}*b^7*\ln((c*x+1/2*b)/c^{1/2}+(c*x^2+b*x+a)^{1/2}))+8/105*f*h^3*a^2/c^3*(c*x^2+b*x+a)^{3/2}+11/128*f*h^3/c^5*b^4*(c*x^2+b*x+a)^{3/2}-33/1024*f*h^3/c^6*b^6*(c*x^2+b*x+a)^{1/2}+1/7*f*h^3*x^4*(c*x^2+b*x+a)^{3/2}/c+1/5*x^2*(c*x^2+b*x+a)^{3/2}/c*d*h^3$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [B] time = 14.70, size = 3262, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h\*x)^3\*(a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2),x)

[Out]  $d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^{1/2} + (8*a^3*f*h^3*(a + b*x + c*x^2)^{1/2})/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^{1/2})/(1024*c^6) + (d*h^3*x^2*(a + b*x + c*x^2)^{3/2})/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)^{3/2})/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^{3/2})/(7*c) - (a*f*g^3*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{1/2} + (\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{1/2}))*((a*c - b^2/4))/(2*c^{3/2}))))/(4*c) + (d*g^3*\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{1/2}))*((a*c - b^2/4))/(2*c^{3/2}) + (e*g^3*\log((b + 2*c*x)/c^{1/2} + 2*(a + b*x + c*x^2)^{1/2}))*((b^3 - 4*a*b*c))/(16*c^{5/2}) - (2*a*d*h^3*((\log((b + 2*c*x)/c^{1/2} + 2*(a + b*x + c*x^2)^{1/2}))*((b^3 - 4*a*b*c))/(16*c^{5/2}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{1/2})/(24*c^2)))/(5*c) - (5*b*f*g^3*((\log((b + 2*c*x)/c^{1/2} + 2*(a + b*x + c*x^2)^{1/2}))*((b^3 - 4*a*b*c))/(16*c^{5/2}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{1/2})/(24*c^2)))/(8*c) + (e*g^3*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{1/2})/(24*c^2) + (33*b^7*f*h^3*\log(b + 2*c^{1/2}*(a + b*x + c*x^2)^{1/2} + 2*c*x))/(2048*c^{13/2}) + (f*g^3*x*(a + b*x + c*x^2)^{3/2})/(4*c) + (a*e*h^3*((5*b*((\log((b + 2*c*x)/c^{1/2} + 2*(a + b*x + c*x^2)^{1/2}))*((a*c - b^2/4))/(2*c^{3/2}))))/(4*c)$

$$\begin{aligned}
& (1/2) + 2*(a + b*x + c*x^2)^{(1/2)}*(b^3 - 4*a*b*c)/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c))/(2*c) + (7*b*d*h^3*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c)))/(10*c) - (3*b*e*h^3*((7*b*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c)))/(10*c) - (2*a*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^{(3/2)})/(5*c)))/(4*c) + (3*d*g*h^2*x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (3*e*g^2*h*x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (3*a*f*g*h^2*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c)))/(2*c) + (21*b*e*g*h^2*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c)))/(10*c) + (21*b*f*g^2*h*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c)))/(10*c) - (9*b*f*g*h^2*((7*b*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4)))/(2*c^{(3/2)})))/(4*c)))/(10*c) - (2*a*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^{(3/2)})/(5*c)))/(4*c) + (35*a^2*b^3*f*h^3*\log(b + 2*c^{(1/2)}*(a + b*x + c*x^2)^{(1/2)} + 2*c*x))/(128*c^{(9/2)}) + (13*a*b^4*f*h^3*(a + b*x + c*x^2)^{(1/2)})/(64*c^5) - (4*a*f*h^3*x^2*(a + b*x + c*x^2)^{(3/2)})/(35*c^2) - (11*b*f*h^3*x^3*(a + b*x + c*x^2)^{(3/2)})/(8*4*c^2) - (33*b^3*f*h^3*x*(a + b*x + c*x^2)^{(3/2)})/(320*c^4) + (11*b^5*f*h^3
\end{aligned}$$

```

*x*(a + b*x + c*x^2)^(1/2))/(512*c^5) + (3*e*g*h^2*x^2*(a + b*x + c*x^2)^(3
/2))/(5*c) + (3*f*g^2*h*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (f*g*h^2*x^3*(
a + b*x + c*x^2)^(3/2))/(2*c) - (3*a*d*g*h^2*((x/2 + b/(4*c))*(a + b*x + c*
x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2
/4)))/(2*c^(3/2)))/(4*c) - (3*a*e*g^2*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4)))/
(2*c^(3/2)))/(4*c) + (3*d*g^2*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (103*a^2*b^2*f*h^3*(a + b*x + c*
x^2)^(1/2))/(320*c^4) - (6*a*e*g*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
+ c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*d*g*h^2*((log(
(b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/
2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2
)))/(8*c) - (6*a*f*g^2*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(
a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*e*g^2*h*((log((b + 2*c*x)/
c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*
(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) +
(8*a^2*f*h^3*x^2*(a + b*x + c*x^2)^(1/2))/(105*c^2) + (33*b^2*f*h^3*x^2*(a
+ b*x + c*x^2)^(3/2))/(280*c^3) + (11*b^4*f*h^3*x^2*(a + b*x + c*x^2)^(1/2)
)/(128*c^4) + (d*g^2*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2
)^(1/2))/(8*c^2) - (5*a^3*b*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2)
+ 2*c*x))/(32*c^(7/2)) - (63*a*b^5*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^
2)^(1/2) + 2*c*x))/(512*c^(11/2)) - (39*a*b^2*f*h^3*x^2*(a + b*x + c*x^2)^(
1/2))/(160*c^3) + (111*a*b*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(560*c^3) - (26
9*a^2*b*f*h^3*x*(a + b*x + c*x^2)^(1/2))/(3360*c^3) - (3*a*b^3*f*h^3*x*(a +
b*x + c*x^2)^(1/2))/(320*c^4)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^3 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*(f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((g + h\*x)\*\*3\*sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2), x)



$$3.183 \quad \int (g + hx)^2 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Optimal. Leaf size=584

$$(b^2 - 4ac) \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c} \sqrt{a+bx+cx^2}} \right) (8c^2 (2a^2 fh^2 + 6abh(eh + 2fg) + 5b^2 (dh^2 + 2egh + fg^2)) - 28b^2 ch(2afh +$$

1024c<sup>11/2</sup>)

**Rubi** [A] time = 1.44, antiderivative size = 581, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^2\*sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((128\*c^4\*d\*g^2 + 21\*b^4\*f\*h^2 - 28\*b^2\*c\*h\*(2\*b\*f\*g + b\*e\*h + 2\*a\*f\*h) - 3\*2\*c^3\*(a\*f\*g^2 + a\*h\*(2\*e\*g + d\*h) + 2\*b\*g\*(e\*g + 2\*d\*h)) + 8\*c^2\*(2\*a^2\*f\*h^2 + 6\*a\*b\*h\*(2\*f\*g + e\*h) + 5\*b^2\*(f\*g^2 + h\*(2\*e\*g + d\*h))))\*(b + 2\*c\*x)\*sqrt[a + b\*x + c\*x^2]/(512\*c^5) - ((2\*c\*f\*g - 4\*c\*e\*h + 3\*b\*f\*h)\*(g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2))/(20\*c^2\*h) + (f\*(g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2))/(6\*c\*h) - ((105\*b^3\*f\*h^3 + 64\*c^3\*(f\*g^3 - 2\*g\*h\*(e\*g + 5\*d\*h)) - 2\*8\*b\*c\*h^2\*(7\*a\*f\*h + 5\*b\*(2\*f\*g + e\*h)) + 8\*c^2\*h\*(7\*b\*f\*g^2 + 25\*b\*h\*(2\*e\*g + d\*h) + 16\*a\*h\*(2\*f\*g + e\*h)) - 6\*c\*h\*(21\*b^2\*f\*h^2 - 4\*c\*h\*(2\*b\*f\*g + 7\*b\*e\*h + 5\*a\*f\*h) - 8\*c^2\*(f\*g^2 - h\*(2\*e\*g + 5\*d\*h)))\*x\*(a + b\*x + c\*x^2)^(3/2))/(960\*c^4\*h) - ((b^2 - 4\*a\*c)\*(128\*c^4\*d\*g^2 + 21\*b^4\*f\*h^2 - 28\*b^2\*c\*h\*(2\*b\*f\*g + b\*e\*h + 2\*a\*f\*h) - 32\*c^3\*(a\*f\*g^2 + a\*h\*(2\*e\*g + d\*h) + 2\*b\*g\*(e\*g + 2\*d\*h)) + 8\*c^2\*(2\*a^2\*f\*h^2 + 6\*a\*b\*h\*(2\*f\*g + e\*h) + 5\*b^2\*(f\*g^2 + h\*(2\*e\*g + d\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2])]/(1024\*c^(11/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int (g + hx)^2 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx &= \frac{f(g + hx)^3 (a + bx + cx^2)^{3/2}}{6ch} + \frac{\int (g + hx)^2 \left(-\frac{3}{2}h(bfg - 4cdh + 2\right)}{20c^2h} \\
&= -\frac{(2cfg - 4ceh + 3bfh)(g + hx)^2 (a + bx + cx^2)^{3/2}}{20c^2h} + \frac{f(g + hx)^3}{20c^2h} \\
&= -\frac{(2cfg - 4ceh + 3bfh)(g + hx)^2 (a + bx + cx^2)^{3/2}}{20c^2h} + \frac{f(g + hx)^3}{20c^2h} \\
&= \frac{(128c^4dg^2 + 21b^4fh^2 - 28b^2ch(2bfg + beh + 2afh) - 32c^3(afg)}{20c^2h} \\
&= \frac{(128c^4dg^2 + 21b^4fh^2 - 28b^2ch(2bfg + beh + 2afh) - 32c^3(afg)}{20c^2h} \\
&= \frac{(128c^4dg^2 + 21b^4fh^2 - 28b^2ch(2bfg + beh + 2afh) - 32c^3(afg)}{20c^2h}
\end{aligned}$$

**Mathematica [A]** time = 0.97, size = 436, normalized size = 0.75

$$\frac{\int (g + hx)^2 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx}{20c^2h} = \frac{(128c^4dg^2 + 21b^4fh^2 - 28b^2ch(2bfg + beh + 2afh) - 32c^3(afg)}{20c^2h}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] ((-3\*(2\*c\*f\*g - 4\*c\*e\*h + 3\*b\*f\*h)\*(g + h\*x)^2\*(a + x\*(b + c\*x))^(3/2))/(10\*c) + f\*(g + h\*x)^3\*(a + x\*(b + c\*x))^(3/2) - ((a + x\*(b + c\*x))^(3/2)\*(105\*b^3\*f\*h^3 - 14\*b\*c\*h^2\*(14\*a\*f\*h + b\*(20\*f\*g + 10\*e\*h + 9\*f\*h\*x)) + 8\*c^2\*h\*(b\*f\*g\*(7\*g + 6\*h\*x) + b\*h\*(50\*e\*g + 25\*d\*h + 21\*e\*h\*x) + a\*h\*(32\*f\*g + 16\*e\*h + 15\*f\*h\*x)) + 16\*c^3\*(f\*g^2\*(4\*g + 3\*h\*x) - h\*(2\*e\*g\*(4\*g + 3\*h\*x) + 5\*d\*h\*(8\*g + 3\*h\*x)))))/(160\*c^3) + (3\*h\*(128\*c^4\*d\*g^2 + 21\*b^4\*f\*h^2 - 28\*b^2\*c\*h\*(2\*b\*f\*g + b\*e\*h + 2\*a\*f\*h) - 32\*c^3\*(a\*f\*g^2 + a\*h\*(2\*e\*g + d\*h) + 2\*b\*g\*(e\*g + 2\*d\*h)) + 8\*c^2\*(2\*a^2\*f\*h^2 + 6\*a\*b\*h\*(2\*f\*g + e\*h) + 5\*b^2\*(f\*g^2 + h\*(2\*e\*g + d\*h))))\*(2\*Sqrt[c]\*(b + 2\*c\*x)\*Sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]))/(512\*c^(9/2))/(6\*c\*h)

**IntegrateAlgebraic [A]** time = 5.52, size = 1037, normalized size = 1.78

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(1920\*b\*c^4\*d\*g^2 - 960\*b^2\*c^3\*e\*g^2 + 2560\*a\*c^4\*e\*g^2 + 600\*b^3\*c^2\*f\*g^2 - 2080\*a\*b\*c^3\*f\*g^2 - 1920\*b^2\*c^3\*d\*g\*h + 5120\*a\*c^4\*d\*g\*h + 1200\*b^3\*c^2\*e\*g\*h - 4160\*a\*b\*c^3\*e\*g\*h - 840\*b^4\*c\*f\*g\*h + 3680\*a\*b^2\*c^2\*f\*g\*h - 2048\*a^2\*c^3\*f\*g\*h + 600\*b^3\*c^2\*d\*h^2 - 2080\*a\*b\*c^3\*d\*h^2 - 420\*b^4\*c\*e\*h^2 + 1840\*a\*b^2\*c^2\*e\*h^2 - 1024\*a^2\*c^3\*e\*h^2 + 315\*b^5\*f\*h^2 - 1680\*a\*b^3\*c\*f\*h^2 + 1808\*a^2\*b\*c^2\*f\*h^2 + 3840\*c^5\*d\*g^2\*x + 640\*b\*c^4\*e\*g^2\*x - 400\*b^2\*c^3\*f\*g^2\*x + 960\*a\*c^4\*f\*g^2\*x + 1280\*b\*c^4\*d\*g\*h\*x - 800\*b^2\*c^3\*e\*g\*h\*x + 1920\*a\*c^4\*e\*g\*h\*x + 560\*b^3\*c^2\*f\*g\*h\*x - 1856\*a\*b\*c^3\*f\*g\*h\*x - 400\*b^2\*c^3\*d\*h^2\*x + 960\*a\*c^4\*d\*h^2\*x + 280\*b^3\*c^2\*e\*h^2\*x - 928\*a\*b\*c^3\*e\*h^2\*x - 210\*b^4\*c\*f\*h^2\*x + 896\*a\*b^2\*c^2\*f\*h^2\*x - 480\*a^2\*c^3\*f\*h^2\*x + 2560\*c^5\*e\*g^2\*x^2 + 320\*b\*c^4\*f\*g^2\*x^2 + 5120\*c^5\*d\*g\*h\*x^2 + 640\*b\*c^4\*e\*g\*h\*x^2 - 448\*b^2\*c^3\*f\*g\*h\*x^2 + 1024\*a\*c^4\*f\*g\*h\*x^2 + 320\*b\*c^4\*d\*h^2\*x^2 - 224\*b^2\*c^3\*e\*h^2\*x^2 + 512\*a\*c^4\*e\*h^2\*x^2 + 168\*b^3\*c^2\*f\*h^2\*x^2 - 544\*a\*b\*c^3\*f\*h^2\*x^2 + 1920\*c^5\*f\*g^2\*x^3 + 3840\*c^5\*e\*g\*h\*x^3 + 384\*b\*c^4\*f\*g\*h\*x^3 + 1920\*c^5\*d\*h^2\*x^3 + 192\*b\*c^4\*e\*h^2\*x^3 - 144\*b^2\*c^3\*f\*h^2\*x^3 + 320\*a\*c^4\*f\*h^2\*x^3 + 3072\*c^5\*f\*g\*h\*x^4 + 1536\*c^5\*e\*h^2\*x^4 + 128\*b\*c^4\*f\*h^2\*x^4 + 1280\*c^5\*f\*h^2\*x^5))/(7680\*c^5) + ((128\*b^2\*c^4\*d\*g^2 - 512\*a\*c^5\*d\*g^2 - 64\*b^3\*c^3\*e\*g^2 + 256\*a\*b\*c^4\*e\*g^2 + 40\*b^4\*c^2\*f\*g^2 - 192\*a\*b^2\*c^3\*f\*g^2 + 128\*a^2\*c^4\*f\*g^2 - 128\*b^3\*c^3\*d\*g\*h + 512\*a\*b\*c^4\*d\*g\*h + 80\*b^4\*c^2\*e\*g\*h - 384\*a\*b^2\*c^3\*e\*g\*h + 256\*a^2\*c^4\*e\*g\*h - 56\*b^5\*c\*f\*g\*h + 320\*a\*b^3\*c^2\*f\*g\*h - 384\*a^2\*b\*c^3\*f\*g\*h + 40\*b^4\*c^2\*d\*h^2 - 192\*a\*b^2\*c^3\*d\*h^2 + 128\*a^2\*c^4\*d\*h^2 - 28\*b^5\*c\*e\*h^2 + 160\*a\*b^3\*c^2\*e\*h^2 - 192\*a^2\*b\*c^3\*e\*h^2 + 21\*b^6\*f\*h^2 - 140\*a\*b^4\*c\*f\*h^2 + 240\*a^2\*b^2\*c^2\*f\*h^2 - 64\*a^3\*c^3\*f\*h^2)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]]/(1024\*c^(11/2))

**fricas** [A] time = 2.06, size = 1791, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/30720\*(15\*(8\*(16\*(b^2\*c^4 - 4\*a\*c^5)\*d - 8\*(b^3\*c^3 - 4\*a\*b\*c^4)\*e + (5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*f)\*g^2 - 8\*(16\*(b^3\*c^3 - 4\*a\*b\*c^4)\*d - 2\*(5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*e + (7\*b^5\*c - 40\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3)\*f)\*g\*h + (8\*(5\*b^4\*c^2 - 24\*a\*b^2\*c^3 + 16\*a^2\*c^4)\*d - 4\*(7\*b^5\*c - 40\*a\*b^3\*c^2 + 48\*a^2\*b\*c^3)\*e + (21\*b^6 - 140\*a\*b^4\*c + 240\*a^2\*b^2\*c^2 - 64\*a^3\*c^3)\*f)\*h^2)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1280\*c^6\*f\*h^2\*x^5 + 128\*(24\*c^6\*f\*g\*h + (12\*c^6\*e + b\*c^5\*f)\*h^2)\*x^4 + 16\*(120\*c^6\*f\*g^2 + 24\*(10\*c^6\*e + b\*c^5\*f)\*g\*h + (120\*c^6\*d + 12\*b\*c^5\*e - (9\*b^2\*c^4 - 20\*a\*c^5)\*f)\*h^2)\*x^3 + 40\*(48\*b\*c^5\*d - 8\*(3\*b^2\*c^4 - 8\*a\*c^5)\*e + (15\*b^3\*c^3 - 52

```

*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*
c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c
^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f
)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^
5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 +
2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d
- 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5
*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 44
8*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(
15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5*b^4*c^2 -
24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*d - 2*(5*b^
4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7*b^5*c - 40
*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64
*a^3*c^3)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f*h^2*x^5 + 128*(24*c^6*f*g*h
+ (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(10*c^6*e + b*c^5*
f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*f)*h^2)*x^3 + 40*
(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2
- 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*c^4)*e + (105*b^4
*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^4)*
d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315*b^5*c - 1680*a*b
^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f)*g^2 + 8*(80*c^6
*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^5*d - 4*(7*b^2*c^
4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(40*(48*c^6*d +
8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4
- 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c
^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 448*a*b^2*c^3 + 240
*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6]

```

**giac** [A] time = 0.31, size = 1012, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + b*c^
4*f*h^2 + 12*c^5*h^2*e)/c^5)*x + (120*c^5*f*g^2 + 24*b*c^4*f*g*h + 120*c^5*
d*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^2 + 240*c^5*g*h*e + 12*b*c^4*h^2*e)/
c^5)*x + (40*b*c^4*f*g^2 + 640*c^5*d*g*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g
*h + 40*b*c^4*d*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2 + 320*c^5*g^2*e +
80*b*c^4*g*h*e - 28*b^2*c^3*h^2*e + 64*a*c^4*h^2*e)/c^5)*x + (1920*c^5*d*g
^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c^4*d*g*h + 280*b^3*c^2*f*

```

$$\begin{aligned}
& g*h - 928*a*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 - 105*b^4*c*f \\
& *h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h^2 + 320*b*c^4*g^2*e - 400*b^2*c \\
& ^3*g*h*e + 960*a*c^4*g*h*e + 140*b^3*c^2*h^2*e - 464*a*b*c^3*h^2*e)/c^5)*x \\
& + (1920*b*c^4*d*g^2 + 600*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^ \\
& ^3*d*g*h + 5120*a*c^4*d*g*h - 840*b^4*c*f*g*h + 3680*a*b^2*c^2*f*g*h - 2048* \\
& a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2 + 315*b^5*f*h^2 - 16 \\
& 80*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2 - 960*b^2*c^3*g^2*e + 2560*a*c^4*g^ \\
& ^2*e + 1200*b^3*c^2*g*h*e - 4160*a*b*c^3*g*h*e - 420*b^4*c*h^2*e + 1840*a*b^ \\
& ^2*c^2*h^2*e - 1024*a^2*c^3*h^2*e)/c^5) + 1/1024*(128*b^2*c^4*d*g^2 - 512*a* \\
& c^5*d*g^2 + 40*b^4*c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 12 \\
& 8*b^3*c^3*d*g*h + 512*a*b*c^4*d*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h \\
& - 384*a^2*b*c^3*f*g*h + 40*b^4*c^2*d*h^2 - 192*a*b^2*c^3*d*h^2 + 128*a^2*c^ \\
& ^4*d*h^2 + 21*b^6*f*h^2 - 140*a*b^4*c*f*h^2 + 240*a^2*b^2*c^2*f*h^2 - 64*a^3 \\
& *c^3*f*h^2 - 64*b^3*c^3*g^2*e + 256*a*b*c^4*g^2*e + 80*b^4*c^2*g*h*e - 384* \\
& a*b^2*c^3*g*h*e + 256*a^2*c^4*g*h*e - 28*b^5*c*h^2*e + 160*a*b^3*c^2*h^2*e \\
& - 192*a^2*b*c^3*h^2*e)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}( \\
& c) - b))/c^{(11/2)}
\end{aligned}$$

**maple [B]** time = 0.02, size = 2179, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^{(1/2)}, x)$

[Out] 
$$\begin{aligned}
& -1/2/c^{(3/2)}*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*d*g*h-1/4*a/c* \\
& (c*x^2+b*x+a)^{(1/2)}*x*e*g*h-1/8*a/c^2*(c*x^2+b*x+a)^{(1/2)}*b*e*g*h-7/32/c^3* \\
& b^3*(c*x^2+b*x+a)^{(1/2)}*x*f*g*h-5/16/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c* \\
& x^2+b*x+a)^{(1/2)})*a*f*g*h+1/3*(c*x^2+b*x+a)^{(3/2)}/c*e*g^2+1/2*d*g^2*(c*x^2+ \\
& b*x+a)^{(1/2)}*x-1/4*a^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})* \\
& e*g*h+1/16*f*h^2*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*x+1/32*f*h^2*a^2/c^3*(c*x^2+b* \\
& x+a)^{(1/2)}*b+21/256*f*h^2/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*x+35/256*f*h^2/c^{(9/2)} \\
& )*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a-7/64*f*h^2/c^4*b^3*a*(c \\
& *x^2+b*x+a)^{(1/2)}-15/64*f*h^2/c^{(7/2)}*b^2*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2 \\
& +b*x+a)^{(1/2)})+49/240*f*h^2/c^3*b*a*(c*x^2+b*x+a)^{(3/2)}-1/8*f*h^2*a/c^2*x*( \\
& c*x^2+b*x+a)^{(3/2)}+5/32/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x*f*g^2+5/32/c^3*b^3*(c \\
& *x^2+b*x+a)^{(1/2)}*e*g*h+3/16/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+ \\
& a)^{(1/2)})*a*d*h^2+2/5*x^2*(c*x^2+b*x+a)^{(3/2)}/c*f*g*h-7/40/c^2*b*x*(c*x^2+b \\
& *x+a)^{(3/2)}*e*h^2+7/24/c^3*b^2*(c*x^2+b*x+a)^{(3/2)}*f*g*h-7/64/c^3*b^3*(c*x^ \\
& ^2+b*x+a)^{(1/2)}*x*e*h^2-7/64/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*f*g*h-5/32/c^{(7/2)}* \\
& b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*h^2+7/128/c^{(9/2)}*b^5*1 \\
& n((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*g*h+3/32/c^3*b^2*a*(c*x^2+b*x+ \\
& a)^{(1/2)}*e*h^2+3/16/c^{(5/2)}*b*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)} \\
& ))*e*h^2-4/15*a/c^2*(c*x^2+b*x+a)^{(3/2)}*f*g*h-5/12/c^2*b*(c*x^2+b*x+a)^{(3/2)} \\
& )*e*g*h+5/32/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x*d*h^2-3/20*f*h^2/c^2*b*x^2*(c*x^
\end{aligned}$$

$$\begin{aligned}
& 2+bx+a)^{(3/2)}+21/160*f*h^2/c^3*b^2*x*(c*x^2+b*x+a)^{(3/2)}-1/8*a/c*(c*x^2+b*x+a)^{(1/2)}*x*d*h^2-1/8*a/c*(c*x^2+b*x+a)^{(1/2)}*x*f*g^2-1/16*a/c^2*(c*x^2+b*x+a)^{(1/2)}*b*d*h^2-1/16*a/c^2*(c*x^2+b*x+a)^{(1/2)}*b*f*g^2-1/8*d*g^2/c^{(3/2)} \\
& *ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^2+1/5*x^2*(c*x^2+b*x+a)^{(3/2)}/c*e*h^2+7/48/c^3*b^2*(c*x^2+b*x+a)^{(3/2)}*e*h^2-7/128/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*e*h^2+7/256/c^{(9/2)}*b^5*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& *e*h^2-2/15*a/c^2*(c*x^2+b*x+a)^{(3/2)}*e*h^2+1/6*f*h^2*x^3*(c*x^2+b*x+a)^{(3/2)}/c-7/64*f*h^2/c^4*b^3*(c*x^2+b*x+a)^{(3/2)}+21/512*f*h^2/c^5*b^5*(c*x^2+b*x+a)^{(1/2)}-21/1024*f*h^2/c^{(11/2)}*b^6*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& +1/16*f*h^2*a^3/c^{(5/2)}*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-5/24/c^2*b*(c*x^2+b*x+a)^{(3/2)}*f*g^2+3/16/c^2*b*a*(c*x^2+b*x+a)^{(1/2)}*x*e*h^2+3/16/c^3*b^2*a*(c*x^2+b*x+a)^{(1/2)}*f*g*h+3/8/c^{(5/2)}*b*a^2*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& *f*g*h+5/16/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x*e*g*h+3/8/c^{(5/2)}*b^2*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*g*h-1/2/c*b*(c*x^2+b*x+a)^{(1/2)}*x*d*g*h-7/32*f*h^2/c^3*b^2*a*(c*x^2+b*x+a)^{(1/2)}*x-7/20/c^2*b*x*(c*x^2+b*x+a)^{(3/2)} \\
& *f*g*h+1/2*d*g^2/c^{(1/2)}*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a+1/4*d*g^2/c*(c*x^2+b*x+a)^{(1/2)}*b+5/64/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*d*h^2+5/64/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*f*g^2-5/128/c^{(7/2)}*b^4*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& *d*h^2-5/128/c^{(7/2)}*b^4*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*g^2-1/8*a^2/c^{(3/2)}*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*h^2-1/8*a^2/c^{(3/2)}*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& *f*g^2+2/3*(c*x^2+b*x+a)^{(3/2)}/c*d*g*h-1/8/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*e*g^2+1/16/c^{(5/2)}*b^3*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e*g^2+1/4*x*(c*x^2+b*x+a)^{(3/2)}/c*d*h^2+1/4*x*(c*x^2+b*x+a)^{(3/2)}/c*f*g^2-5/24/c^2*b*(c*x^2+b*x+a)^{(3/2)}*d*h^2+3/8/c^2*b*a*(c*x^2+b*x+a)^{(1/2)}*x*f*g*h+3/16/c^{(5/2)}*b^2*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*f*g^2-5/64/c^{(7/2)}*b^4*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e*g*h+1/2*x*(c*x^2+b*x+a)^{(3/2)}/c*e*g*h-1/4/c*b*(c*x^2+b*x+a)^{(1/2)}*x*e*g^2-1/4/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*d*g*h-1/4/c^{(3/2)}*b*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*g^2+1/8/c^{(5/2)}*b^3*ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*g*h
\end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [B] time = 7.91, size = 1881, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g + h*x)^2*(a + b*x + c*x^2)^{(1/2)}*(d + e*x + f*x^2), x)$

[Out]  $d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (e*h^2*x^2*(a + b*x + c*x^2)^{(3/2)})/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^{(3/2)})/(6*c) - (a*d*h^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c) - (a*f*g^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c) + (d*g^2*\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4))/(2*c^{(3/2)}) + (e*g^2*\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) - (2*a*e*h^2*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2))/(5*c) - (5*b*d*h^2*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2))/(8*c) - (5*b*f*g^2*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2))/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (f*g^2*x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*f*h^2*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c)))/(2*c) + (7*b*e*h^2*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c)))/(10*c) - (3*b*f*h^2*((7*b*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c)))/(10*c) - (2*a*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^{(3/2)})/(5*c)))/(4*c) + (2*f*g*h*x^2*(a + b*x + c*x^2)^{(3/2)})/(5*c) - (a*e*g*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(2*c) + (d*g*h*\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(8*c^{(5/2)}) - (4*a*f*g*h*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a +$



```

c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b
*e*g*h*(log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(4*c) + (d*g*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*
x + c*x^2)^(1/2))/(12*c^2) + (e*g*h*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7*b
*f*g*h*((5*b*(log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
+ b/(4*c))* (a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^2 \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((g + h\*x)\*\*2\*sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2), x)

$$3.184 \quad \int (g + hx) \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=322

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} \left( -8c^2(aeh + afg + 2bdh + 2beg) + 2bc(6afh + 5b(eh + fg)) - 7b^3fh + 32c^3dg \right) + (a + b^2)}{128c^4}$$

**Rubi [A]** time = 0.50, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1653, 779, 612, 621, 206}

$$\frac{(a + bx + cx^2)^{3/2} (-2b(16afh + 25b(eh + fg)) + 35b^2fh^2 - 6c(7b^2h - 10c(ah + fg)) + c^2(-48fg^2 - 80b(dh + eg))) + (b + 2cx)\sqrt{a + bx + cx^2} (-8c^2(aeh + afg + 2bdh + 2beg) + 2bc(6afh + 5b(eh + fg)) - 7b^3fh + 32c^3dg)}{128c^4} - \frac{(c^2 - 4a) \operatorname{arctanh}\left(\frac{b + 2cx}{2\sqrt{a + bx + cx^2}}\right) (-8c^2(aeh + afg + 2bdh + 2beg) + 2bc(6afh + 5b(eh + fg)) - 7b^3fh + 32c^3dg)}{256c^{9/2}} + \frac{f(g + hx)^2 (a + bx + cx^2)^{3/2}}{5ch}$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)\*Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2),x]

[Out] ((32\*c^3\*d\*g - 7\*b^3\*f\*h - 8\*c^2\*(2\*b\*e\*g + a\*f\*g + 2\*b\*d\*h + a\*e\*h) + 2\*b\*c\*(6\*a\*f\*h + 5\*b\*(f\*g + e\*h)))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(128\*c^4) + (f\*(g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2))/(5\*c\*h) + ((35\*b^2\*f\*h^2 - c^2\*(48\*f\*g^2 - 80\*h\*(e\*g + d\*h)) - 2\*c\*h\*(16\*a\*f\*h + 25\*b\*(f\*g + e\*h)) - 6\*c\*h\*(6\*c\*f\*g - 10\*c\*e\*h + 7\*b\*f\*h)\*x)\*(a + b\*x + c\*x^2)^(3/2))/(240\*c^3\*h) - ((b^2 - 4\*a\*c)\*(32\*c^3\*d\*g - 7\*b^3\*f\*h - 8\*c^2\*(2\*b\*e\*g + a\*f\*g + 2\*b\*d\*h + a\*e\*h) + 2\*b\*c\*(6\*a\*f\*h + 5\*b\*(f\*g + e\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(256\*c^(9/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rubi steps

$$\begin{aligned}
 \int (g + hx) \sqrt{a + bx + cx^2} (d + ex + fx^2) dx &= \frac{f(g + hx)^2 (a + bx + cx^2)^{3/2}}{5ch} + \frac{\int (g + hx) \left(-\frac{1}{2}h(3bfg - 10cdh + \dots)\right)}{5ch} \\
 &= \frac{f(g + hx)^2 (a + bx + cx^2)^{3/2}}{5ch} + \frac{(35b^2fh^2 - c^2(48fg^2 - 80h(eg + \dots)))}{128c^4} \\
 &= \frac{(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + \dots))}{128c^4} \\
 &= \frac{(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + \dots))}{128c^4} \\
 &= \frac{(32c^3dg - 7b^3fh - 8c^2(2beg + afg + 2bdh + aeh) + 2bc(6afh + \dots))}{128c^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.48, size = 258, normalized size = 0.80

$$\frac{(a+x(b+cx))^2(-2ch(16afh+b(25ch+25fg+21f/hx))+35f^2j^2+c^2(20h(4dh+4cg+3chx)-12fg(4g+3hx)))}{48c^2} - \frac{\operatorname{Si}\left(2\sqrt{c(b+2cx)}\sqrt{a+x(b+cx)} - (b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c(b+cx)}}\right)\right)(8c^2(ah+afg+2bdh+2bcg)-2bc(6afh+5b(ch+fg))+7b^3fh-32^2dg)}{256c^{7/2}} + f(g+hx)^2(a+x(b+cx))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(3/2) + ((a + x*(b + c*x))^(3/2)*(35*b^2*f
*h^2 + c^2*(-12*f*g*(4*g + 3*h*x) + 20*h*(4*e*g + 4*d*h + 3*e*h*x)) - 2*c*h
*(16*a*f*h + b*(25*f*g + 25*e*h + 21*f*h*x))))/(48*c^2) - (5*h*(-32*c^3*d*g
+ 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
5*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/(256*c^(7/2))
)/(5*c*h)
```

**IntegrateAlgebraic [A]** time = 2.07, size = 498, normalized size = 1.55

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(480*b*c^3*d*g - 240*b^2*c^2*e*g + 640*a*c^3*e*g + 1
50*b^3*c*f*g - 520*a*b*c^2*f*g - 240*b^2*c^2*d*h + 640*a*c^3*d*h + 150*b^3*
c*e*h - 520*a*b*c^2*e*h - 105*b^4*f*h + 460*a*b^2*c*f*h - 256*a^2*c^2*f*h +
960*c^4*d*g*x + 160*b*c^3*e*g*x - 100*b^2*c^2*f*g*x + 240*a*c^3*f*g*x + 16
0*b*c^3*d*h*x - 100*b^2*c^2*e*h*x + 240*a*c^3*e*h*x + 70*b^3*c*f*h*x - 232*
a*b*c^2*f*h*x + 640*c^4*e*g*x^2 + 80*b*c^3*f*g*x^2 + 640*c^4*d*h*x^2 + 80*b
*c^3*e*h*x^2 - 56*b^2*c^2*f*h*x^2 + 128*a*c^3*f*h*x^2 + 480*c^4*f*g*x^3 + 4
80*c^4*e*h*x^3 + 48*b*c^3*f*h*x^3 + 384*c^4*f*h*x^4))/(1920*c^4) + ((32*b^2
*c^3*d*g - 128*a*c^4*d*g - 16*b^3*c^2*e*g + 64*a*b*c^3*e*g + 10*b^4*c*f*g -
48*a*b^2*c^2*f*g + 32*a^2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h + 10*b
^4*c*e*h - 48*a*b^2*c^2*e*h + 32*a^2*c^3*e*h - 7*b^5*f*h + 40*a*b^3*c*f*h -
48*a^2*b*c^2*f*h)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(256*c
^(9/2))
```

**fricas [A]** time = 2.76, size = 1009, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5*
b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2*(
5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2
)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10*c^5*
e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*c^4*e
```

$$\begin{aligned}
& - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h \\
& + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*x)*\sqrt{c*x^2 + b*x + a})/c^5, \\
& 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*f)*h)*\sqrt{-c})*\arctan(1/2*\sqrt{c*x^2 + b*x + a}*(2*c*x + b)*\sqrt{-c}/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*x)*\sqrt{c*x^2 + b*x + a})/c^5]
\end{aligned}$$

**giac [A]** time = 0.24, size = 495, normalized size = 1.54

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] 1/1920\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(6\*(8\*f\*h\*x + (10\*c^4\*f\*g + b\*c^3\*f\*h + 10\*c^4\*h\*e)/c^4)\*x + (10\*b\*c^3\*f\*g + 80\*c^4\*d\*h - 7\*b^2\*c^2\*f\*h + 16\*a\*c^3\*f\*h + 80\*c^4\*g\*e + 10\*b\*c^3\*h\*e)/c^4)\*x + (480\*c^4\*d\*g - 50\*b^2\*c^2\*f\*g + 120\*a\*c^3\*f\*g + 80\*b\*c^3\*d\*h + 35\*b^3\*c\*f\*h - 116\*a\*b\*c^2\*f\*h + 80\*b\*c^3\*g\*e - 50\*b^2\*c^2\*h\*e + 120\*a\*c^3\*h\*e)/c^4)\*x + (480\*b\*c^3\*d\*g + 150\*b^3\*c\*f\*g - 520\*a\*b\*c^2\*f\*g - 240\*b^2\*c^2\*d\*h + 640\*a\*c^3\*d\*h - 105\*b^4\*f\*h + 460\*a\*b^2\*c\*f\*h - 256\*a^2\*c^2\*f\*h - 240\*b^2\*c^2\*g\*e + 640\*a\*c^3\*g\*e + 150\*b^3\*c\*h\*e - 520\*a\*b\*c^2\*h\*e)/c^4) + 1/256\*(32\*b^2\*c^3\*d\*g - 128\*a\*c^4\*d\*g + 10\*b^4\*c\*f\*g - 48\*a\*b^2\*c^2\*f\*g + 32\*a^2\*c^3\*f\*g - 16\*b^3\*c^2\*d\*h + 64\*a\*b\*c^3\*d\*h - 7\*b^5\*f\*h + 40\*a\*b^3\*c\*f\*h - 48\*a^2\*b\*c^2\*f\*h - 16\*b^3\*c^2\*g\*e + 64\*a\*b\*c^3\*g\*e + 10\*b^4\*c\*h\*e - 48\*a\*b^2\*c^2\*h\*e + 32\*a^2\*c^3\*h\*e)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(9/2)

**maple [B]** time = 0.01, size = 1117, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x)

```
[Out] 1/3*(c*x^2+b*x+a)^(3/2)/c*d*h+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g+1/2*d*g*(c*x^2+
b*x+a)^(1/2)*x+3/16*h*f/c^2*b*a*(c*x^2+b*x+a)^(1/2)*x-7/40*h*f/c^2*b*x*(c*x
^2+b*x+a)^(3/2)+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*e*h+3/16*h*f/c^(5/2)*b*a
^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/32*h*f/c^3*b^2*a*(c*x^2+b*
x+a)^(1/2)+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e*h+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g-
5/32*h*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/64*h*f
/c^3*b^3*(c*x^2+b*x+a)^(1/2)*x+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*f*g+3/16/
c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h+3/16/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g-1/8*a/c*(c*x^2+b*x+a)
^(1/2)*x*e*h-1/8*a/c*(c*x^2+b*x+a)^(1/2)*x*f*g-1/16*a/c^2*(c*x^2+b*x+a)^(1/
2)*b*e*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h-5/
24/c^2*b*(c*x^2+b*x+a)^(3/2)*e*h-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g+5/64/c^
3*b^3*(c*x^2+b*x+a)^(1/2)*e*h+1/2*d*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*a-1/8*d*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)*b^2-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*d*h-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*e
*g+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/4*d*g/c*(c*x^2+
b*x+a)^(1/2)*b+1/5*h*f*x^2*(c*x^2+b*x+a)^(3/2)/c+7/48*h*f/c^3*b^2*(c*x^2+b*
x+a)^(3/2)-7/128*h*f/c^4*b^4*(c*x^2+b*x+a)^(1/2)+7/256*h*f/c^(9/2)*b^5*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/15*h*f*a/c^2*(c*x^2+b*x+a)^(3/2)+
5/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*e*h-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*f*g-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e*h-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/1
6*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*a*e*g-1/4/c*b*(c*x^2+b*x+a)^(1/2)*x*d*h-1/4/c*b*(c*x^2+b*x
+a)^(1/2)*x*e*g
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [B] time = 5.62, size = 877, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)`

[Out]  $d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} - (2*a*f*h*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)}))*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2))/(5*c) - (5*b*e*h*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)}))*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2))/(8*c) - (5*b*f*g*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)}))*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2))/(8*c) + (d*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2) + (e*g*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2) + (e*h*x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (f*g*x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (7*b*f*h*((5*b*((\log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)}))*(b^3 - 4*a*b*c)))/(16*c^{(5/2)}) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^{(1/2)})/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^{(3/2)})/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c)))/(10*c) + (f*h*x^2*(a + b*x + c*x^2)^{(3/2)})/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{(1/2)} + (\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))*(a*c - b^2/4))/(2*c^{(3/2)})))/(4*c) + (d*g*log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)})*(a*c - b^2/4))/(2*c^{(3/2)}) + (d*h*log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)}) + (e*g*log((b + 2*c*x)/c^{(1/2)} + 2*(a + b*x + c*x^2)^{(1/2)})*(b^3 - 4*a*b*c))/(16*c^{(5/2)})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx) \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((g + h*x)*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)`

$$3.185 \quad \int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=175

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 2be) + 5b^2f + 16c^2d)}{128c^{7/2}} + \frac{(b + 2cx)\sqrt{a + bx + cx^2} (-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3}$$

**Rubi [A]** time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b + 2cx)\sqrt{a + bx + cx^2} (-4acf + 5b^2f - 8bce + 16c^2d)}{64c^3} - \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 2be) + 5b^2f + 16c^2d)}{128c^{7/2}} + \frac{(a + bx + cx^2)^{3/2} (8ce - 5bf)}{24c^2} + \frac{fx(a + bx + cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2), x]

[Out] ((16\*c^2\*d - 8\*b\*c\*e + 5\*b^2\*f - 4\*a\*c\*f)\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(64\*c^3) + ((8\*c\*e - 5\*b\*f)\*(a + b\*x + c\*x^2)^(3/2))/(24\*c^2) + (f\*x\*(a + b\*x + c\*x^2)^(3/2))/(4\*c) - ((b^2 - 4\*a\*c)\*(16\*c^2\*d + 5\*b^2\*f - 4\*c\*(2\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(7/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 640



```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{3/2}}{4c} + \frac{\int \left(4cd - af + \frac{1}{2}(8ce - 5bf)x\right) \sqrt{a + bx + cx^2} dx}{4c} \\ &= \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c^2} + \frac{fx(a + bx + cx^2)^{3/2}}{4c} + \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c} \\ &= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c} \\ &= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c} \\ &= \frac{(16c^2d - 8bce + 5b^2f - 4acf)(b + 2cx)\sqrt{a + bx + cx^2}}{64c^3} + \frac{(8ce - 5bf)(a + bx + cx^2)^{3/2}}{24c} \end{aligned}$$

**Mathematica [A]** time = 0.30, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+bx+cx^2}(4bc(2c(6d+2ex+fx^2)-13af)+8c^2(a(8e+3fx)+2cx(6d+4ex+3fx^2))+15b^3f-2b^2c(12e+5fx))-3(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+2be)+5b^2f+16c^2d)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
```



$$f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{7/2})$$

**maple [B]** time = 0.01, size = 453, normalized size = 2.59

$$\frac{f \sqrt{b \sqrt{c^2 + \sqrt{c^2 + 4bx + a}}}}{4c^3} - \frac{52bf \sqrt{b \sqrt{c^2 + \sqrt{c^2 + 4bx + a}}}}{128c^3} - \frac{24e \sqrt{b \sqrt{c^2 + \sqrt{c^2 + 4bx + a}}}}{4c^3} + \frac{64e \sqrt{b \sqrt{c^2 + \sqrt{c^2 + 4bx + a}}}}{128c^3} + \frac{1}{128} \left( \frac{16b^2c^2d}{c^3} - \frac{64ac^3d}{c^3} + \frac{5b^4f}{c^3} - \frac{24ab^2cf}{c^3} + \frac{16a^2c^2f}{c^3} - \frac{8b^3ce}{c^3} + \frac{32ab^2ce}{c^3} \right) \log\left(\frac{\text{abs}(-2(\sqrt{c}x - \sqrt{c^2x^2 + bx + a})\sqrt{c} - b)}{c^{7/2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $\frac{1}{4}f*x*(c*x^2+b*x+a)^{3/2}/c - 5/24*f/c^2*b*(c*x^2+b*x+a)^{3/2} + 5/32*f/c^2*b^2*(c*x^2+b*x+a)^{3/2} + 5/64*f/c^3*b^3*(c*x^2+b*x+a)^{3/2} + 3/16*f/c^{5/2}*b^2*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2})*a - 5/128*f/c^{7/2}*b^4*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2}) - 1/8*f*a/c*(c*x^2+b*x+a)^{1/2}*x - 1/16*f*a/c^2*(c*x^2+b*x+a)^{1/2}*b - 1/8*f*a^2/c^{3/2}*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2}) + 1/3*e*(c*x^2+b*x+a)^{3/2}/c - 1/4*e/c*b*(c*x^2+b*x+a)^{1/2}*x - 1/8*e/c^2*b^2*(c*x^2+b*x+a)^{1/2} - 1/4*e/c^{3/2}*b*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2})*a + 1/16*e/c^{5/2}*b^3*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2}) + 1/2*d*(c*x^2+b*x+a)^{1/2}*x + 1/4*d/c*(c*x^2+b*x+a)^{1/2}*b + 1/2*d/c^{1/2}*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2})*a - 1/8*d/c^{3/2}*\ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{1/2})*b^2$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(1/2)\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad [B]** time = 4.24, size = 320, normalized size = 1.83

$$d \left( \frac{a}{2} + \frac{b}{4c} \right) \sqrt{c^2 + bx + a} - \frac{af \left( \frac{a}{2} + \frac{b}{4c} \right) \sqrt{c^2 + bx + a} + \frac{5 \left( \frac{2a^2c^2 + \sqrt{c^2 + 4bx + a}}{2c^3} \right) \left( c - \frac{a}{2} \right)}{4c} + \frac{d \ln \left( \frac{\frac{a}{2} + \sqrt{c^2 + bx + a}}{2c^3} \right) \left( c - \frac{a}{2} \right)}{2c^3} + \frac{e \ln \left( \frac{2a^2c^2 + 2\sqrt{c^2 + bx + a}}{16c^3} \right) (b^2 - 4abc)}{16c^3} - \frac{5bf \left( \frac{5 \left( \frac{2a^2c^2 + \sqrt{c^2 + 4bx + a}}{2c^3} \right) (b^2 - 4abc)}{8c} + \frac{(-3b^2 + 2cxb + 8c(c^2 + a)) \sqrt{c^2 + bx + a}}{24c^3} \right)}{8c} + \frac{e \left( -3b^2 + 2cxb + 8c(c^2 + a) \right) \sqrt{c^2 + bx + a}}{24c^3} + \frac{fx(c^2 + bx + a)^{3/2}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2),x)

[Out]  $d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^{1/2} - (a*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^{1/2} + (\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{1/2}))*a*c -$

```

b^2/4))/(2*c^(3/2)))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*(log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)\*(f\*x\*\*2+e\*x+d), x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2), x)

$$3.186 \quad \int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{g+hx} dx$$

**Optimal.** Leaf size=321

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh) - (-4ch(bg-ah) - b^2h^2 + 8c^2g^2)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4} \sqrt{\dots}$$

**Rubi [A]** time = 0.78, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh) - (-4ch(bg-ah) - b^2h^2 + 8c^2g^2)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4} - \frac{\sqrt{a+bx+cx^2}\left(4ch(bfg-2cdh) + 2chx(bfh-2ceh+2cfg) - (4cg-bh)(bfh-2ceh+2cfg)\right)}{8c^2h^3} + \frac{\sqrt{ah^2-bgh+cg^2}\left(ah^2-cgh+fg^2\right)\tanh^{-1}\left(\frac{-2ah+2cg-bh+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right) + f(a+bx+cx^2)^{3/2}}{h^4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] -((4\*c\*h\*(b\*f\*g - 2\*c\*d\*h) - (4\*c\*g - b\*h)\*(2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h) + 2\*c\*h\*(2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*x)\*Sqrt[a + b\*x + c\*x^2])/(8\*c^2\*h^3) + (f\*(a + b\*x + c\*x^2)^(3/2))/(3\*c\*h) + ((4\*c\*h\*(2\*c\*g - b\*h)\*(b\*f\*g - 2\*c\*d\*h) - (2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*(8\*c^2\*g^2 - b^2\*h^2 - 4\*c\*h\*(b\*g - a\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(16\*c^(5/2)\*h^4) + (Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*(f\*g^2 - e\*g\*h + d\*h^2)\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/h^4

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 724**

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 814

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{g+hx} dx &= \frac{f(a+bx+cx^2)^{3/2}}{3ch} + \frac{\int \frac{\left(-\frac{3}{2}h(bfg-2cdh)-\frac{3}{2}h(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+cx^2}}{g+hx} dx}{3ch^2} \\
&= -\frac{(4ch(bfg-2cdh) - (4cg-bh)(2cfg-2ceh+bfh) + 2ch(2cfg-2ceh+bfh))\sqrt{a+bx+cx^2}}{8c^2h^3} \\
&= -\frac{(4ch(bfg-2cdh) - (4cg-bh)(2cfg-2ceh+bfh) + 2ch(2cfg-2ceh+bfh))\sqrt{a+bx+cx^2}}{8c^2h^3} \\
&= -\frac{(4ch(bfg-2cdh) - (4cg-bh)(2cfg-2ceh+bfh) + 2ch(2cfg-2ceh+bfh))\sqrt{a+bx+cx^2}}{8c^2h^3} \\
&= -\frac{(4ch(bfg-2cdh) - (4cg-bh)(2cfg-2ceh+bfh) + 2ch(2cfg-2ceh+bfh))\sqrt{a+bx+cx^2}}{8c^2h^3}
\end{aligned}$$

**Mathematica [A]** time = 0.79, size = 331, normalized size = 1.03

$$\frac{2\sqrt{c}\left(h\sqrt{a+bx+cx^2}\left(2ch(4afh+b(3ah-3fg+fx)) - 3b^2fh^2 + 4c^2(3h(2ah-2cg+ahx) + f(4g^2-3ghx+2f^2x^2))\right) - 24c^2\sqrt{h(ad-bh)} + c\sqrt{g}\left(h(ad-cg) + fg^2\right)\operatorname{tanh}^{-1}\left(\frac{2ah-3gh-2fg}{2\sqrt{a+bx+cx^2}\sqrt{h(ad-bh)}}\right) - 3\operatorname{tanh}^{-1}\left(\frac{b-2cx}{2\sqrt{a+bx+cx^2}}\right)\left(-8c^2h(adh-fg) + h(ad-cg) + bf\right) + 2ch^2(2afh+hah-bfg) - b^2fh^2 + 16c^2g(h(ad-cg) + fg^2)\right)}{48c^2h^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] (-3\*(-(b^3\*f\*h^3) + 2\*b\*c\*h^2\*(-(b\*f\*g) + b\*e\*h + 2\*a\*f\*h) + 16\*c^3\*g\*(f\*g^2 + h\*(-(e\*g) + d\*h)) - 8\*c^2\*h\*(b\*f\*g^2 + b\*h\*(-(e\*g) + d\*h) + a\*h\*(-(f\*g) + e\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])] + 2\*Sqrt[c]\*(h\*Sqrt[a + x\*(b + c\*x)]\*(-3\*b^2\*f\*h^2 + 2\*c\*h\*(4\*a\*f\*h + b\*(-3\*f\*g + 3\*e\*h + f\*h\*x)) + 4\*c^2\*(3\*h\*(-2\*e\*g + 2\*d\*h + e\*h\*x) + f\*(6\*g^2 - 3\*g\*h\*x + 2\*h^2\*x^2))) - 24\*c^2\*Sqrt[c\*g^2 + h\*(-(b\*g) + a\*h)]\*(f\*g^2 + h\*(-(e\*g) + d\*h))\*ArcTanh[(-(b\*g) + 2\*a\*h - 2\*c\*g\*x + b\*h\*x)/(2\*Sqrt[c\*g^2 + h\*(-(b\*g) + a\*h)]\*Sqrt[a + x\*(b + c\*x)])])/(48\*c^(5/2)\*h^4)

**IntegrateAlgebraic [A]** time = 2.06, size = 384, normalized size = 1.20

$$\frac{\sqrt{a+bx+cx^2}\left(8acfh^2-3b^2fh^2+6bcfh-6c^2fgh+2bcfh^2+24c^2dh^2-24c^2gh+12c^2dh^2+24c^2fg^2-12c^2fgh+8c^2fh^2\right)}{24c^3h^4} \cdot \log\left(-2\sqrt{a+bx+cx^2} + b+2cx\right)\left(4abcfh-8ac^2dh+8ac^2fgh-b^2fh^2+2b^2ah^2-2b^2fgh^2-8ac^2dh+8ac^2fgh-8ac^2fgh+16c^2dgh-16c^2gh+16c^2fg^2\right) \cdot \frac{2\sqrt{-a^2+bh-cg^2}\left(dh-cg+h+fg\right)\operatorname{tanh}^{-1}\left(\frac{2ah-3gh-2fg}{2\sqrt{a+bx+cx^2}\sqrt{-a^2+bh-cg^2}}\right)}{h^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x), x]

```
[Out] (Sqrt[a + b*x + c*x^2]*(24*c^2*f*g^2 - 24*c^2*e*g*h - 6*b*c*f*g*h + 24*c^2*d*h^2 + 6*b*c*e*h^2 - 3*b^2*f*h^2 + 8*a*c*f*h^2 - 12*c^2*f*g*h*x + 12*c^2*e*h^2*x + 2*b*c*f*h^2*x + 8*c^2*f*h^2*x^2))/(24*c^2*h^3) + (2*Sqrt[-(c*g^2) + b*g*h - a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTan[(Sqrt[c]*g + Sqrt[c]*h*x - h*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*g^2) + b*g*h - a*h^2]])/h^4 + ((16*c^3*f*g^3 - 16*c^3*e*g^2*h - 8*b*c^2*f*g^2*h + 16*c^3*d*g*h^2 + 8*b*c^2*e*g*h^2 - 2*b^2*c*f*g*h^2 + 8*a*c^2*f*g*h^2 - 8*b*c^2*d*h^3 + 2*b^2*c*e*h^3 - 8*a*c^2*e*h^3 - b^3*f*h^3 + 4*a*b*c*f*h^3)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^(5/2)*h^4)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

**maple** [B] time = 0.02, size = 2549, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x)
```

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/2/h*e*(c*x^2+b*x+a)^(1/2)*x-1/h^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g+1/h^3*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/h*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d+1/h^3*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g^2*e-1/h^4*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g^3*f-1/h/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*((x+g/h)^2*
```



$$\begin{aligned}
& c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} / (x + g / h) * a^d - 1/8 / h * \\
& f / c^2 * b^2 * (c^2 x^2 + b^2 x + a)^{1/2} + 1/16 / h * f / c^{5/2} * b^3 * \ln((c^2 x + 1/2 * b) / c^{1/2} + ( \\
& c^2 x^2 + b^2 x + a)^{1/2}) + 1/4 / h * e / c * (c^2 x^2 + b^2 x + a)^{1/2} * b + 1/2 / h * e / c^{1/2} * \ln((c^2 x \\
& + 1/2 * b) / c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}) * a - 1/8 / h * e / c^{3/2} * \ln((c^2 x + 1/2 * b) / c^{1/2} \\
& + (c^2 x^2 + b^2 x + a)^{1/2}) * b^2 - 1/2 / h^2 * f * g * (c^2 x^2 + b^2 x + a)^{1/2} * x - 1/2 / h^2 * \ln((1 \\
& / 2 * (b^2 h - 2 c^2 g) / h + (x + g / h) * c) / c^{1/2} + ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - \\
& b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / c^{1/2} * b * e * g + 1/2 / h^3 * \ln((1/2 * (b^2 h - 2 c^2 g) / h + (x + \\
& g / h) * c) / c^{1/2} + ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2) \\
& ^{1/2}) / c^{1/2} * b * f * g^2 + 1 / h^2 / ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * \ln((2 * (a^2 h^2 - \\
& b^2 g^2 + c^2 g^2) / h^2 + (b^2 h - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * (( \\
& x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / (x + g / h)) * a \\
& * e * g - 1 / h^3 / ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * \ln((2 * (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2 + (b \\
& ^2 h - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * ((x + g / h)^2 * c + (b^2 h - 2 c^2 \\
& * g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / (x + g / h)) * a * f * g^2 + 1 / h^2 / ((a^2 h^2 - \\
& b^2 g^2 + c^2 g^2) / h^2)^{1/2} * \ln((2 * (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2 + (b^2 h - 2 c^2 g) / h * (x + g / \\
& h) + 2 * ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 \\
& h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / (x + g / h)) * b * g * d - 1 / h^3 / ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2) \\
& ^{1/2} * \ln((2 * (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2 + (b^2 h - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 h^2 - b^2 g^2 + \\
& c^2 g^2) / h^2)^{1/2} * ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2) \\
& ^{1/2}) / (x + g / h)) * b * g^2 * e + 1/2 / h * \ln((1/2 * (b^2 h - 2 c^2 g) / h + (x + g / h) * c) / c^{1/2} + ( \\
& (x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / c^{1/2} * b \\
& * d - 1 / h^2 * \ln((1/2 * (b^2 h - 2 c^2 g) / h + (x + g / h) * c) / c^{1/2} + ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / \\
& h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) * c^{1/2} * g * d + 1 / h^4 / ((a^2 h^2 - b^2 g^2 + c^2 \\
& * g^2) / h^2)^{1/2} * \ln((2 * (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2 + (b^2 h - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 \\
& h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + \\
& c^2 g^2) / h^2)^{1/2}) / (x + g / h)) * b * g^3 * f - 1 / h^3 / ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * \\
& \ln((2 * (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2 + (b^2 h - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 h^2 - b^2 g^2 + c^2 g^2) / \\
& h^2)^{1/2} * ((x + g / h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} \\
& )) / (x + g / h)) * c * g^2 * d + 1 / h^4 / ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * \ln((2 * (a^2 h^2 - b^2 g^2 + \\
& h + c^2 g^2) / h^2 + (b^2 h - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * ((x + g / \\
& h)^2 * c + (b^2 h - 2 c^2 g) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / (x + g / h)) * c * g^3 \\
& * e - 1 / h^5 / ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * \ln((2 * (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2 + (b^2 h - \\
& - 2 c^2 g) / h * (x + g / h) + 2 * ((a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2} * ((x + g / h)^2 * c + (b^2 h - 2 c^2 g \\
& ) / h * (x + g / h) + (a^2 h^2 - b^2 g^2 + c^2 g^2) / h^2)^{1/2}) / (x + g / h)) * c * g^4 * f - 1/4 / h * f / c * b * (c \\
& ^2 x^2 + b^2 x + a)^{1/2} * x - 1/4 / h * f / c^{3/2} * b * \ln((c^2 x + 1/2 * b) / c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}) \\
& * a - 1/4 / h^2 * f * g / c * (c^2 x^2 + b^2 x + a)^{1/2} * b - 1/2 / h^2 * f * g / c^{1/2} * \ln((c^2 x + 1/2 * b) / c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}) \\
& * a + 1/8 / h^2 * f * g / c^{3/2} * \ln((c^2 x + 1/2 * b) / c^{1/2} + (c^2 x^2 + b^2 x + a)^{1/2}) * b^2
\end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*h-2\*c\*g>0)', see `assume?` for more details) Is b\*h-2\*c\*g zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)}{g + hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x)

[Out] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{g + hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(h\*x+g), x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x), x)

$$3.187 \quad \int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^2} dx$$

**Optimal.** Leaf size=459

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(4ch(-afh - beh + 2bfg) + b^2fh^2 - 8c^2(3fg^2 - h(2eg - dh))\right) \sqrt{a+bx+cx^2} \left(2ch^2x - 2h\right)}{8c^{3/2}h^4}$$

**Rubi [A]** time = 1.10, antiderivative size = 453, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(4ch(-afh - beh + 2bfg) + b^2fh^2 - 8c^2(3fg^2 - h(2eg - dh))\right) \sqrt{a+bx+cx^2} \left(2ch^2x - 2h\right)}{8c^{3/2}h^4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] -((b\*f\*h\*(b\*g - a\*h) - 4\*c^2\*g\*(2\*e\*g - (3\*f\*g^2)/h - d\*h) + 4\*a\*c\*h\*(2\*f\*g - e\*h) - b\*c\*(13\*f\*g^2 - 8\*e\*g\*h + 4\*d\*h^2) + 2\*c\*h\*(2\*c\*e\*g + b\*f\*g - (3\*c\*f\*g^2)/h - 2\*c\*d\*h - a\*f\*h)\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*c\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(3/2))/(h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)) - ((b^2\*f\*h^2 + 4\*c\*h\*(2\*b\*f\*g - b\*e\*h - a\*f\*h) - 8\*c^2\*(3\*f\*g^2 - h\*(2\*e\*g - d\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(3/2)\*h^4) - (((2\*c\*(3\*f\*g^3 - g\*h\*(2\*e\*g - d\*h)) - h\*(5\*b\*f\*g^2 - b\*h\*(3\*e\*g - d\*h) - 2\*a\*h\*(2\*f\*g - e\*h)))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*h^4\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^2} dx &= \frac{(fg^2 - h(eg - dh))(a+bx+cx^2)^{3/2}}{h(cg^2 - bgh + ah^2)(g+hx)} - \frac{\int \frac{1}{2} \left( -2cdg + 3beg + 2afg - \frac{3bf g^2}{h} - bdh - 2a \right)}{cg^2} \\
&= \frac{\left( bfh(bg - ah) - 4c^2g \left( 2eg - \frac{3fg^2}{h} - dh \right) + 4ach(2fg - eh) - bc(13fg^2 - \dots) \right)}{4ch^2 (cg^2 - \dots)} \\
&= \frac{\left( bfh(bg - ah) - 4c^2g \left( 2eg - \frac{3fg^2}{h} - dh \right) + 4ach(2fg - eh) - bc(13fg^2 - \dots) \right)}{4ch^2 (cg^2 - \dots)} \\
&= \frac{\left( bfh(bg - ah) - 4c^2g \left( 2eg - \frac{3fg^2}{h} - dh \right) + 4ach(2fg - eh) - bc(13fg^2 - \dots) \right)}{4ch^2 (cg^2 - \dots)} \\
&= \frac{\left( bfh(bg - ah) - 4c^2g \left( 2eg - \frac{3fg^2}{h} - dh \right) + 4ach(2fg - eh) - bc(13fg^2 - \dots) \right)}{4ch^2 (cg^2 - \dots)}
\end{aligned}$$

**Mathematica [A]** time = 1.56, size = 486, normalized size = 1.06

$$\frac{\frac{(16ah^2g^2 + 2ah^2g) \sqrt{a+bx+cx^2}}{2\sqrt{a+bx+cx^2}} + 2h\sqrt{a+bx+cx^2} (102ah^2g - 4fg^2 + 4h^2d) + 4h^2d(2ah - 4fg^2 + 4h^2d) + 4h^2d(2ah - 4fg^2 + 4h^2d) + 4h^2d(2ah - 4fg^2 + 4h^2d)}{4h^2(bg - ah)^2} - \frac{2h \int \frac{1}{2} \left( -2cdg + 3beg + 2afg - \frac{3bf g^2}{h} - bdh - 2a \right)}{(g+hx)^2} + \frac{f(x+cx)^2}{g+hx}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] ((f\*(a + x\*(b + c\*x))^(3/2))/(g + h\*x) - ((3\*c\*f\*g^2 + f\*h\*(-(b\*g) + a\*h) + 2\*c\*h\*(-(e\*g) + d\*h))\*(a + x\*(b + c\*x))^(3/2))/((c\*g^2 + h\*(-(b\*g) + a\*h)) \* (g + h\*x)) - (2\*h\*Sqrt[a + x\*(b + c\*x)]\*(b\*f\*h^2\*(-(b\*g) + a\*h) + c\*h\*(4\*b\*h\*(-2\*e\*g + d\*h) + b\*f\*g\*(13\*g - 2\*h\*x) + 2\*a\*h\*(-4\*f\*g + 2\*e\*h + f\*h\*x)) + c^2\*(6\*f\*g^2\*(-2\*g + h\*x) + 4\*h\*(e\*g\*(2\*g - h\*x) + d\*h\*(-g + h\*x)))) + ((c\*g^2 + h\*(-(b\*g) + a\*h))\*(-(b^2\*f\*h^2) + 4\*c\*h\*(-2\*b\*f\*g + b\*e\*h + a\*f\*h) + 8\*c^2\*(3\*f\*g^2 + h\*(-2\*e\*g + d\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/Sqrt[c] + 4\*c\*Sqrt[c\*g^2 + h\*(-(b\*g) + a\*h)]\*(2\*c\*(3\*f\*g^3 + g\*h\*(-2\*e\*g + d\*h)) - h\*(5\*b\*f\*g^2 + b\*h\*(-3\*e\*g + d\*h) + 2\*a\*h\*(-2\*f\*g + e\*h)))\*ArcTanh[(-(b\*g) + 2\*a\*h - 2\*c\*g\*x + b\*h\*x)/(2\*Sqrt[c\*g^2 + h\*(-(b\*g) + a\*h)]\*Sqrt[a + x\*(b + c\*x)])])/(4\*h^3\*(-(c\*g^2) + h\*(b\*g - a\*h)))/(2\*c\*h)

**IntegrateAlgebraic [B]** time = 8.32, size = 5655, normalized size = 12.32

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] Result too large to show
```

```
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

```
maple [B] time = 0.02, size = 6218, normalized size = 13.55
```

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

```
maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(b\*h-2\*c\*g>0)', see `assume?` for more details) Is b\*h-2\*c\*g zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)}{(g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2, x)

[Out] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{(g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(h\*x+g)\*\*2, x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*2, x)

$$3.188 \quad \int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^3} dx$$

**Optimal.** Leaf size=448

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2(8a^2fh^2-4abh(6fg-eh))+b^2(15fg^2-h(dh+3eg))\right)-4ch(bg^2(10fg-3eh))}{8h^4(ah^2-bgh+cg^2)^{3/2}}$$

**Rubi [A]** time = 0.87, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 812, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2(8a^2fh^2-4abh(6fg-eh))+b^2(15fg^2-h(dh+3eg))\right)-4ch(bg^2(10fg-3eh))}{8h^4(ah^2-bgh+cg^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] -((11\*b\*f\*g^2 - b\*h\*(3\*e\*g + d\*h) - (4\*c\*g^2\*(3\*f\*g - e\*h))/h - 4\*a\*h\*(3\*f\*g - e\*h) + 2\*h\*(c\*e\*g + 2\*b\*f\*g - (3\*c\*f\*g^2)/h - c\*d\*h - 2\*a\*f\*h)\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(3/2))/(2\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^2) - ((6\*c\*f\*g - 2\*c\*e\*h - b\*f\*h)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*h^4) + ((8\*c^2\*g^3\*(3\*f\*g - e\*h) - 4\*c\*h\*(b\*g^2\*(10\*f\*g - 3\*e\*h) - a\*h\*(9\*f\*g^2 - 3\*e\*g\*h + d\*h^2)) + h^2\*(8\*a^2\*f\*h^2 - 4\*a\*b\*h\*(6\*f\*g - e\*h) + b^2\*(15\*f\*g^2 - h\*(3\*e\*g + d\*h))))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(8\*h^4\*(c\*g^2 - b\*g\*h + a\*h^2)^(3/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 724



```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x]
+ Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^3} dx &= \frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{3/2}}{2h(cg^2 - bgh + ah^2)(g + hx)^2} - \frac{\int \frac{\left(\frac{1}{2}(-4cdg+3beg+4afg-\frac{3bf^2g^2}{h}+bdh-4ach)\right)}{2(cg^2 - bgh + ah^2)} dx}{2(cg^2 - bgh + ah^2)} \\
&= \frac{\left(11bfg^2 - bh(3eg + dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg - eh) + 2h(ceg + 2bfg)\right)}{4h^2(cg^2 - bgh + ah^2)(g + hx)} \\
&= \frac{\left(11bfg^2 - bh(3eg + dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg - eh) + 2h(ceg + 2bfg)\right)}{4h^2(cg^2 - bgh + ah^2)(g + hx)} \\
&= \frac{\left(11bfg^2 - bh(3eg + dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg - eh) + 2h(ceg + 2bfg)\right)}{4h^2(cg^2 - bgh + ah^2)(g + hx)} \\
&= \frac{\left(11bfg^2 - bh(3eg + dh) - \frac{4cg^2(3fg-eh)}{h} - 4ah(3fg - eh) + 2h(ceg + 2bfg)\right)}{4h^2(cg^2 - bgh + ah^2)(g + hx)}
\end{aligned}$$

**Mathematica [A]** time = 3.58, size = 645, normalized size = 1.44

---

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] ((f\*(a + x\*(b + c\*x))^(3/2))/(g + h\*x)^2 - ((3\*c\*f\*g^2 + 2\*f\*h\*(-(b\*g) + a\*h) + c\*h\*(-(e\*g) + d\*h))\*(a + x\*(b + c\*x))^(3/2))/(2\*(c\*g^2 + h\*(-(b\*g) + a\*h))\*(g + h\*x)^2) - ((-2\*c\*(6\*c\*f\*g^3 - 2\*c\*g\*h\*(e\*g + d\*h) - 4\*a\*h^2\*(-2\*f\*g + e\*h) + b\*h\*(-7\*f\*g^2 + h\*(3\*e\*g + d\*h)))\*(a + x\*(b + c\*x))^(3/2))/(g + h\*x) + (2\*c\*Sqrt[a + x\*(b + c\*x)]\*(h^2\*(-4\*a^2\*f\*h^2 - 4\*a\*b\*h\*(-4\*f\*g + e\*h) + b^2\*(-11\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) - 2\*c^2\*(3\*f\*g^3\*(2\*g - h\*x) + g\*h\*(d\*h^2\*x + e\*g\*(-2\*g + h\*x))) + c\*h\*(-2\*a\*h\*(f\*g\*(9\*g - 4\*h\*x) + h\*(-3\*e\*g + d\*h + 2\*e\*h\*x)) + b\*(f\*g^2\*(23\*g - 7\*h\*x) + h\*(d\*h\*(-g + h\*x) + e\*g\*(-7\*g + 3\*h\*x)))))/h^2 + (4\*Sqrt[c]\*(6\*c\*f\*g - 2\*c\*e\*h - b\*f\*h)\*(c\*g^2 + h\*(-(b\*g) + a\*h))^2\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])] + c\*Sqrt[c\*g^2 + h\*(-(b\*g) + a\*h)]\*(8\*c^2\*g^3\*(3\*f\*g - e\*h) + 4\*c\*h\*(b\*g^2\*(-10\*f\*g + 3\*e\*h) + a\*h\*(9\*f\*g^2 - 3\*e\*g\*h + d\*h^2)) + h^2\*(8\*a^2\*f\*h^2 + 4\*a\*b\*h\*(-6\*f\*g + e\*h) + b^2\*(15\*f\*g^2 - h\*(3\*e\*g + d\*h))))\*ArcTanh[(-(b\*g) + 2\*a

$$\frac{h - 2cgx + bhx}{(2\sqrt{c^2g^2 + h(-bg + ah)}\sqrt{a + x(b + cx)})^3} / (8(c^2g^2 + h(-bg + ah))^2) / (ch)$$

**IntegrateAlgebraic** [F] time = 180.11, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x);;OUTPUT:Evaluation time: 0.81Unable to divide, perhaps due to rounding error%%{1, [6,0,0,7,0,0,0,0]%%}+%%{%%{[-6,0]: [1,0,%%{-1, [1]%%}]}%%}, [5,0,0,6,1,0,0,0]%%}+%%{3, [4,1,0,6,1,0,0,0]%%}+%%{-3, [4,0,0,7,0,0,1,0]%%}+%%{%%{12, [1]%%}, [4,0,0,5,2,0,0,0]%%}+%%{%%{[-12,0]: [1,0,%%{-1, [1]%%}]}%%}, [3,1,0,5,2,0,0,0]%%}+%%{%%{12,0]: [1,0,%%{-1, [1]%%}]}%%}, [3,0,0,6,1,0,1,0]%%}+%%{%%{[-8, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%}, [3,0,0,4,3,0,0,0]%%}+%%{3, [2,2,0,5,2,0,0,0]%%}+%%{-6, [2,1,0,6,1,0,1,0]%%}+%%{%%{12, [1]%%}, [2,1,0,4,3,0,0,0]%%}+%%{3, [2,0,0,7,0,0,2,0]%%}+%%{%%{[-12, [1]%%}, [2,0,0,5,2,0,1,0]%%}+%%{%%{[-6,0]: [1,0,%%{-1, [1]%%}]}%%}, [1,2,0,4,3,0,0,0]%%}+%%{%%{12,0]: [1,0,%%{-1, [1]%%}]}%%}, [1,1,0,5,2,0,1,0]%%}+%%{%%{[-6,0]: [1,0,%%{-1, [1]%%}]}%%}, [1,0,0,6,1,0,2,0]%%}+%%{1, [0,3,0,4,3,0,0,0]%%}+%%{-3, [0,2,0,5,2,0,1,0]%%}+%%{3, [0,1,0,6,1,0,2,0]%%}+%%{-1, [0,0,0,7,0,0,3,0]%%} / %%{%%{poly1[%%{1, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%}, [6,0,0,3,0,0,0,0]%%}+%%{%%{[-6, [2]%%}, [5,0,0,2,1,0,

```

0,0]%%}+%%{%%{[%%{3, [1]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [4,1,0,2,1,0,0,0]
%%}+%%{%%{poly1[%%{-3, [1]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [4,0,0,3,0,0,1,
0]%%}+%%{%%{poly1[%%{12, [2]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [4,0,0,1,2,0,
0,0]%%}+%%{%%{-12, [2]%%}, [3,1,0,1,2,0,0,0]%%}+%%{%%{12, [2]%%}, [3,0,
0,2,1,0,1,0]%%}+%%{%%{-8, [3]%%}, [3,0,0,0,3,0,0,0]%%}+%%{%%{3, [1]
%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [2,2,0,1,2,0,0,0]%%}+%%{%%{-6, [1]%%
}, 0] : [1,0,%%{-1, [1]%%}]%%}, [2,1,0,2,1,0,1,0]%%}+%%{%%{12, [2]%%}, 0
] : [1,0,%%{-1, [1]%%}]%%}, [2,1,0,0,3,0,0,0]%%}+%%{%%{poly1[%%{3, [1]%%},
0] : [1,0,%%{-1, [1]%%}]%%}, [2,0,0,3,0,0,2,0]%%}+%%{%%{poly1[%%{-12, [2]%%
}, 0] : [1,0,%%{-1, [1]%%}]%%}, [2,0,0,1,2,0,1,0]%%}+%%{%%{-6, [2]%%}, [1,2
,0,0,3,0,0,0]%%}+%%{%%{12, [2]%%}, [1,1,0,1,2,0,1,0]%%}+%%{%%{-6, [2]%%
}, [1,0,0,2,1,0,2,0]%%}+%%{%%{[%%{1, [1]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [
0,3,0,0,3,0,0,0]%%}+%%{%%{[%%{-3, [1]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [0,2
,0,1,2,0,1,0]%%}+%%{%%{[%%{3, [1]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [0,1,0,2
,1,0,2,0]%%}+%%{%%{poly1[%%{-1, [1]%%}, 0] : [1,0,%%{-1, [1]%%}]%%}, [0,0,0
,3,0,0,3,0]%%} Error: Bad Argument Value

```

**maple [B]** time = 0.02, size = 12139, normalized size = 27.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x)`

[Out] result too large to display

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)}{(g + hx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)`

[Out] `int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{(g + hx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3, x)`

[Out] `Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)`

$$3.189 \quad \int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^4} dx$$

**Optimal.** Leaf size=603

$$\frac{\sqrt{a+bx+cx^2} \left( hx \left( h^2 \left( 8a^2fh^2 - 2abh(10fg - eh) + b^2 \left( 11fg^2 - h(dh + eg) \right) \right) \right) + 2cgh \left( 2ah(6fg - eh) - b \left( 12fg^2 \right) \right) \right)}{\dots}$$

**Rubi [A]** time = 1.45, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 810, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2} \left( hx \left( h^2 \left( 8a^2fh^2 - 2abh(10fg - eh) + b^2 \left( 11fg^2 - h(dh + eg) \right) \right) \right) + 2cgh \left( 2ah(6fg - eh) - b \left( 12fg^2 \right) \right) \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x]

[Out] -(((8\*c^2\*f\*g^5)/h + 4\*a^2\*e\*h^4 + 4\*a\*c\*g\*h\*(3\*f\*g^2 + d\*h^2) + b^2\*g\*h\*(5\*f\*g^2 + h\*(e\*g + d\*h)) - 2\*b\*(a\*h^2\*(3\*f\*g^2 + 2\*e\*g\*h + d\*h^2) + c\*(7\*f\*g^4 + d\*g^2\*h^2)) + h\*(8\*a^2\*f\*h^3 + 4\*a\*c\*g\*h\*(6\*f\*g - e\*h) + c^2\*((12\*f\*g^4)/h - 4\*d\*g^2\*h) + b^2\*h\*(11\*f\*g^2 - h\*(e\*g + d\*h)) - 2\*b\*(12\*c\*f\*g^3 - c\*g\*h\*(e\*g + 2\*d\*h) + a\*h^2\*(10\*f\*g - e\*h)))\*x)\*Sqrt[a + b\*x + c\*x^2])/(8\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^2) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(3/2))/(3\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^3) + (Sqrt[c]\*f\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/h^4 - ((16\*c^3\*f\*g^5 - 8\*c^2\*g\*h\*(5\*b\*f\*g^3 - 5\*a\*f\*g^2\*h + a\*d\*h^3) - b\*h^3\*(8\*a^2\*f\*h^2 - 2\*a\*b\*h\*(6\*f\*g + e\*h) + b^2\*(5\*f\*g^2 + e\*g\*h + d\*h^2)) + 2\*c\*h^2\*(4\*a^2\*h^2\*(4\*f\*g - e\*h) - 2\*a\*b\*h\*(15\*f\*g^2 - e\*g\*h - d\*h^2) + b^2\*(15\*f\*g^3 + d\*g\*h^2)))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(16\*h^4\*(c\*g^2 - b\*g\*h + a\*h^2)^(5/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 810

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*((d\*g - e\*f\*(m + 2))\*(c\*d^2 - b\*d\*e + a\*e^2) - d\*p\*(2\*c\*d - b\*e)\*(e\*f - d\*g) - e\*(g\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2) + p\*(2\*c\*d - b\*e)\*(e\*f - d\*g))\*x)/(e^2\*(m + 1)\*(m + 2)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[p/(e^2\*(m + 1)\*(m + 2)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[2\*a\*c\*e\*(e\*f - d\*g)\*(m + 2) + b^2\*e\*(d\*g\*(p + 1) - e\*f\*(m + p + 2)) + b\*(a\*e^2\*g\*(m + 1) - c\*d\*(d\*g\*(2\*p + 1) - e\*f\*(m + 2\*p + 2))) - c\*(2\*c\*d\*(d\*g\*(2\*p + 1) - e\*f\*(m + 2\*p + 2)) - e\*(2\*a\*e\*g\*(m + 1) - b\*(d\*g\*(m - 2\*p) + e\*f\*(m + 2\*p + 2)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2\*p, 0] && !ILtQ[m + 2\*p + 3, 0]

Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^4} dx &= \frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{3/2}}{3h(cg^2 - bgh + ah^2)(g+hx)^3} - \frac{\int \frac{\left(-\frac{3}{2}\left(2cdg - beg - 2afg + \frac{bf g^2}{h} - bdh + 2aeh\right)\right)}{(g+hx)^3}}{3(cg^2 - bgh)} \\
&= \frac{\left(\frac{8c^2fg^5}{h} + 4a^2eh^4 + 4acgh(3fg^2 + dh^2) + b^2gh(5fg^2 + h(eg + dh)) - 2b\right)}{3(cg^2 - bgh)} \\
&= \frac{\left(\frac{8c^2fg^5}{h} + 4a^2eh^4 + 4acgh(3fg^2 + dh^2) + b^2gh(5fg^2 + h(eg + dh)) - 2b\right)}{3(cg^2 - bgh)} \\
&= \frac{\left(\frac{8c^2fg^5}{h} + 4a^2eh^4 + 4acgh(3fg^2 + dh^2) + b^2gh(5fg^2 + h(eg + dh)) - 2b\right)}{3(cg^2 - bgh)} \\
&= \frac{\left(\frac{8c^2fg^5}{h} + 4a^2eh^4 + 4acgh(3fg^2 + dh^2) + b^2gh(5fg^2 + h(eg + dh)) - 2b\right)}{3(cg^2 - bgh)}
\end{aligned}$$

**Mathematica [A]** time = 1.93, size = 439, normalized size = 0.73

$$\frac{\left(\frac{(b^2-4ac) \operatorname{tanh}^{-1}\left(\frac{2ah-bx+bt+2cqx}{2\sqrt{(a+bx+cx^2)}\sqrt{(ah-bx)+cg^2}}\right)}{8(h(ab-bx)+cg^2)^{3/2}} + \frac{\sqrt{a+x(b+cx)}(-2ah+b(x-b)+2cqx)}{4(g+hx)^2(h(ab-bx)+cg^2)}\right) \left(2ah^2(eh-2fg)-bh(h(d+eg)-3fg^2)+c(2dgh^2-2fg^3)\right)}{2(h(ab-bx)+cg^2)} - \frac{h(a+x(b+cx))^2(h(d+eg)+fg^2)}{3(g+hx)^3(h(ab-bx)+cg^2)} + \frac{\left(\frac{(2cg-bt) \operatorname{tanh}^{-1}\left(\frac{2ah-bx+bt+2cqx}{2\sqrt{(a+bx+cx)}\sqrt{(ah-bx)+cg^2}}\right)}{2\sqrt{(ah-bx)+cg^2}} - \frac{h\sqrt{a+x(b+cx)}}{g+hx} + \sqrt{c} \operatorname{tanh}^{-1}\left(\frac{b+2cx}{2\sqrt{(a+bx+cx)}}\right)\right)}{h^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x]

[Out]  $(-1/3*(h*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^3) + ((2*a*h^2*(-2*f*g + e*h) + c*(-2*f*g^3 + 2*d*g*h^2) - b*h*(-3*f*g^2 + h*(e*g + d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])*Sqrt[a + x*(b + c*x)]])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(2*(c*g^2 + h*(-(b*g) + a*h))) + (f*(-((h*Sqrt[a + x*(b + c*x)])/(g + h*x)) + Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) + ((2*c*g - b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])*Sqrt[a + x*(b + c*x)]])/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])))/h^2$



IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x]

[Out] \$Aborted

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^4,x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^4,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 42.01Unable to divide, perhaps due to rounding error%%{[-1,0]:[1,0,%%{-1,[1]%%}]%%},[8,0,0,0,0,0,8,0]%%}+%%{%%{8,[1]%%},[7,0,0,0,0,0,7,1]%%}+%%{%%{-4,0}:[1,0,%%{-1,[1]%%}]%%},[6,0,1,0,0,0,7,1]%%}+%%{%%{4,0}:[1,0,%%{-1,[1]%%}]%%},[6,0,0,0,1,0,8,0]%%}+%%{%%{%%{-24,[1]%%},0}:[1,0,%%{-1,[1]%%}]%%},[6,0,0,0,0,6,2]%%}+%%{%%{24,[1]%%},[5,0,1,0,0,0,6,2]%%}+%%{%%{-24,[1]%%},[5,0,0,0,1,0,7,1]%%}+%%{%%{32,[2]%%},[5,0,0,0,0,5,3]%%}+%%{%%{-6,0}:[1,0,%%{-1,[1]%%}]%%},[4,0,2,0,0,0,6,2]%%}+%%{%%{12,0}:[1,0,%%{-1,[1]%%}]%%},[4,0,1,0,1,0,7,1]%%}+%%{%%{%%{-48,[1]%%},0}:[1,0,%%{-1,[1]%%}]%%},[4,0,1,0,0,0,5,3]%%}+%%{%%{-6,0}:[1,0,%%{-1,[1]%%}]%%},[4,0,0,0,2,0,8,0]%%}+%%{%%{%%{48,[1]%%},0}:[1,0,%%{-1,[1]%%}]%%},[4,0,0,0,1,0,6,2]%%}+%%{%%{%%{-16,[2]%%},0}:[1,0,%%{-1,[1]%%}]%%},[4,0,0,0,0,4,4]%%}+%%{%%{24,[1]%%},[3,0,2,0,0,0,5,3]%%}+%%{%%{-48,[1]%%},[3,0,1,0,1,0,6,2]%%}+%%{%%{32,[2]%%},[3,0,1,0,0,0,4,4]%%}+%%{%%{24,[1]%%},[3,0,0,0,2,0,7,1]%%}+%%{%%{-32,[2]%%},[3,0,0,0,1,0,5,3]%%}+%%{%%{-4,0}:[1,0,%%{-1,[1]%%}]%%},[2,0,3,0,0,0,5,3]%%}+%%{%%{12,0}:[1,0,%%{-1,[1]%%}]%%},[2,0,2,0,1,0,6,2]%%}+%%{%%{%%{-24,[1]%%},0}:[1,0,%%

```

%{-1, [1]%%}]]%}, [2,0,2,0,0,0,4,4]%%}+%%%{%%{-12,0}:[1,0,%%{-1, [1]%%}]]%
%}, [2,0,1,0,2,0,7,1]%%}+%%%{%%{[%%{-48, [1]%%}, 0]:[1,0,%%{-1, [1]%%}]]%},
[2,0,1,0,1,0,5,3]%%}+%%%{%%{-4,0}:[1,0,%%{-1, [1]%%}]]%}, [2,0,0,0,3,0,8,0
]%%}+%%%{%%{-24, [1]%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [2,0,0,0,2,0,6,2]%%
%}+%%%{%%{-8, [1]%%}, [1,0,3,0,0,0,4,4]%%}+%%%{%%{-24, [1]%%}, [1,0,2,0,1,
0,5,3]%%}+%%%{%%{-24, [1]%%}, [1,0,1,0,2,0,6,2]%%}+%%%{%%{-8, [1]%%}, [1,0
,0,0,3,0,7,1]%%}+%%%{%%{-1,0}:[1,0,%%{-1, [1]%%}]]%}, [0,0,4,0,0,0,4,4]%%
%}+%%%{%%{-4,0}:[1,0,%%{-1, [1]%%}]]%}, [0,0,3,0,1,0,5,3]%%}+%%%{%%{-6,0]
:[1,0,%%{-1, [1]%%}]]%}, [0,0,2,0,2,0,6,2]%%}+%%%{%%{-4,0}:[1,0,%%{-1, [1]
%%}]]%}, [0,0,1,0,3,0,7,1]%%}+%%%{%%{-1,0}:[1,0,%%{-1, [1]%%}]]%}, [0,0,0
,0,4,0,8,0]%%} / %%%{%%{-1, [2]%%}, [8,0,0,0,0,0,4,0]%%}+%%%{%%{poly1[%%{-
8, [2]%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [7,0,0,0,0,0,3,1]%%}+%%%{%%{-4, [2]%%
%}, [6,0,1,0,0,0,3,1]%%}+%%%{%%{-4, [2]%%}, [6,0,0,0,1,0,4,0]%%}+%%%{%%{-2
4, [3]%%}, [6,0,0,0,0,0,2,2]%%}+%%%{%%{-24, [2]%%}, 0]:[1,0,%%{-1, [1]%%
%}]]%}, [5,0,1,0,0,0,2,2]%%}+%%%{%%{poly1[%%{-24, [2]%%}, 0]:[1,0,%%{-1, [1]
%%}]]%}, [5,0,0,0,1,0,3,1]%%}+%%%{%%{poly1[%%{-32, [3]%%}, 0]:[1,0,%%{-1
, [1]%%}]]%}, [5,0,0,0,0,0,1,3]%%}+%%%{%%{-6, [2]%%}, [4,0,2,0,0,0,2,2]%%}+
%%%{%%{-12, [2]%%}, [4,0,1,0,1,0,3,1]%%}+%%%{%%{-48, [3]%%}, [4,0,1,0,0,0,1
,3]%%}+%%%{%%{-6, [2]%%}, [4,0,0,0,2,0,4,0]%%}+%%%{%%{-48, [3]%%}, [4,0,0,
0,1,0,2,2]%%}+%%%{%%{-16, [4]%%}, [4,0,0,0,0,0,0,4]%%}+%%%{%%{poly1[%%{-2
4, [2]%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [3,0,2,0,0,0,1,3]%%}+%%%{%%{-48, [
2]%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [3,0,1,0,1,0,2,2]%%}+%%%{%%{-32, [3]
%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [3,0,1,0,0,0,0,4]%%}+%%%{%%{poly1[%%{-24,
[2]%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [3,0,0,0,2,0,3,1]%%}+%%%{%%{poly1[%%{-3
2, [3]%%}, 0]:[1,0,%%{-1, [1]%%}]]%}, [3,0,0,0,1,0,1,3]%%}+%%%{%%{-4, [2]%%
%}, [2,0,3,0,0,0,1,3]%%}+%%%{%%{-12, [2]%%}, [2,0,2,0,1,0,2,2]%%}+%%%{%%{-2
4, [3]%%}, [2,0,2,0,0,0,0,4]%%}+%%%{%%{-12, [2]%%}, [2,0,1,0,2,0,3,1]%%}+%%
%{%%{-48, [3]%%}, [2,0,1,0,1,0,1,3]%%}+%%%{%%{-4, [2]%%}, [2,0,0,0,3,0,4,0
]%%}+%%%{%%{-24, [3]%%}, [2,0,0,0,2,0,2,2]%%}+%%%{%%{-8, [2]%%}, 0]:[1
,0,%%{-1, [1]%%}]]%}, [1,0,3,0,0,0,0,4]%%}+%%%{%%{poly1[%%{-24, [2]%%}, 0]
:[1,0,%%{-1, [1]%%}]]%}, [1,0,2,0,1,0,1,3]%%}+%%%{%%{-24, [2]%%}, 0]:[1
,0,%%{-1, [1]%%}]]%}, [1,0,1,0,2,0,2,2]%%}+%%%{%%{poly1[%%{-8, [2]%%}, 0]
:[1,0,%%{-1, [1]%%}]]%}, [1,0,0,0,3,0,3,1]%%}+%%%{%%{-1, [2]%%}, [0,0,4,0,0,0
,0,4]%%}+%%%{%%{-4, [2]%%}, [0,0,3,0,1,0,1,3]%%}+%%%{%%{-6, [2]%%}, [0,0,2
,0,2,0,2,2]%%}+%%%{%%{-4, [2]%%}, [0,0,1,0,3,0,3,1]%%}+%%%{%%{-1, [2]%%},
[0,0,0,0,4,0,4,0]%%} Error: Bad Argument Value

```

**maple [B]** time = 0.02, size = 19321, normalized size = 32.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (f*x^2+e*x+d)*(c*x^2+b*x+a)^{(1/2)}/(h*x+g)^4, x$

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)}{(g + hx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x)

[Out] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{(g + hx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(h\*x+g)\*\*4,x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*4, x)

$$3.190 \quad \int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^5} dx$$

**Optimal.** Leaf size=497

$$\frac{\sqrt{a+bx+cx^2} (-2ah + x(2cg - bh) + bg) (16a^2fh^2 - 4c(a(dh^2 - 5egh + fg^2) + 2bg(2dh + eg)) - 8abh(eh + 2fg))}{64(g+hx)^2 (ah^2 - bgh + cg^2)^3}$$

**Rubi [A]** time = 0.86, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1650, 806, 720, 724, 206}

$$\frac{\sqrt{a+bx+cx^2} (-2ah + x(2cg - bh) + bg) (16a^2fh^2 - 4c(a(dh^2 - 5egh + fg^2) + 2bg(2dh + eg)) - 8abh(eh + 2fg))}{64(g+hx)^2 (ah^2 - bgh + cg^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] ((16\*c^2\*d\*g^2 + 16\*a^2\*f\*h^2 - 8\*a\*b\*h\*(2\*f\*g + e\*h) - 4\*c\*(a\*f\*g^2 - a\*h\*(5\*e\*g - d\*h) + 2\*b\*g\*(e\*g + 2\*d\*h)) + b^2\*(5\*f\*g^2 + h\*(3\*e\*g + 5\*d\*h)))\*(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)\*Sqrt[a + b\*x + c\*x^2])/(64\*(c\*g^2 - b\*g\*h + a\*h^2)^3\*(g + h\*x)^2) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(3/2))/(4\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^4) + ((6\*c\*f\*g^3 + 2\*c\*g\*h\*(e\*g - 5\*d\*h) + 8\*a\*h^2\*(2\*f\*g - e\*h) - b\*h\*(11\*f\*g^2 - h\*(3\*e\*g + 5\*d\*h)))\*(a + b\*x + c\*x^2)^(3/2))/(24\*h\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^3) - ((b^2 - 4\*a\*c)\*(16\*c^2\*d\*g^2 + 16\*a^2\*f\*h^2 - 8\*a\*b\*h\*(2\*f\*g + e\*h) - 4\*c\*(a\*f\*g^2 - a\*h\*(5\*e\*g - d\*h) + 2\*b\*g\*(e\*g + 2\*d\*h)) + b^2\*(5\*f\*g^2 + h\*(3\*e\*g + 5\*d\*h)))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])]/(128\*(c\*g^2 - b\*g\*h + a\*h^2)^(7/2))

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 720

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(d\*b - 2\*a\*e + (2\*c\*d - b\*e)\*x)\*(a + b\*x + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[(p\*(b^2 - 4\*a\*c))/(2\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && EqQ[m + 2\*p + 2, 0]

] && GtQ[p, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :=> Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :=> -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :=> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^5} dx &= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{3/2}}{4h(CG^2-bgh+ah^2)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}(-8cdg+3beg+8afg-\frac{3bf^2g^2}{h}+5bdh-8ac)\right)}{4(CG^2-bgh+ah^2)(g+hx)^4} dx}{4(CG^2-bgh+ah^2)(g+hx)^4} \\
&= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{3/2}}{4h(CG^2-bgh+ah^2)(g+hx)^4} + \frac{(6cf^2g^3+2cgh(eg-5dh)+8ah^2)}{24h} \\
&= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh))+2bg(eg-dh))}{64(CG^2-bgh+ah^2)(g+hx)^4} \\
&= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh))+2bg(eg-dh))}{64(CG^2-bgh+ah^2)(g+hx)^4} \\
&= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh))+2bg(eg-dh))}{64(CG^2-bgh+ah^2)(g+hx)^4}
\end{aligned}$$

**Mathematica [A]** time = 3.90, size = 447, normalized size = 0.90

$$\frac{\left(\frac{(d^2-4ac)\operatorname{tanh}^{-1}\left(\frac{2ah-by+bx-2cy}{2\sqrt{a+bx+cx^2}}\right)}{8(h(ab-by)+c^2)^{3/2}} + \frac{\sqrt{a+bx+cx^2}(c-2ah+(g-bx)+2gx)}{4(g+hx)^2(h(ab-by)+c^2)}\right)\frac{(16c^2fg^2-4c(ah(dh-5eg)+f^2g^2+2hg(2dh+eg))-8abh(dh+2fg)+h^2(5dh+3eg)+5fg^2)+16c^2dg^2}{24(h(ab-by)+c^2)^2} + \frac{(a+bx+cx)^{3/2}(4f^2h(ab-bg)+ch(eg-dh)+3cf^2)}{4(g+hx)^4(h(ab-by)+c^2)} + \frac{f(a+bx+cx)^{3/2}}{(g+hx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x]

[Out] 
$$\begin{aligned}
& -\left(\frac{f(a+bx+cx)^{3/2}}{(g+hx)^4} + \frac{(3c^2fg^2+4f^2h(-b*g)+a*h)+c*h*(e*g-d*h)}{4*(c*g^2+h*(-b*g)+a*h)}\right)*(g+hx)^4 \\
& + \frac{(c*(6c^2fg^3+2c*gh*(e*g-5*d*h))-8*a*h^2*(-2*f*g+e*h)+b*h*(-11*f*g^2+h*(3*e*g+5*d*h)))*(a+bx+cx)^{3/2}}{(g+hx)^3} \\
& + \frac{(3c^2h*(16c^2d*g^2+16a^2f*h^2-8a*b*h*(2f*g+e*h))-4c*(a*f*g^2+a*h*(-5*e*g+d*h))+2*b*g*(e*g+2*d*h)+b^2*(5*f*g^2+h*(3*e*g+5*d*h))}{24*(c*g^2+h*(-b*g)+a*h)} \\
& * \frac{(\operatorname{Sqrt}[a+bx+cx]*(-2*a*h+2*c*g*x+b*(g-h*x)))}{(g+hx)^2} \\
& + \frac{(b^2-4*a*c)*\operatorname{ArcTanh}\left(\frac{-(b*g)+2*a*h-2*c*g*x+b*h*x}{2*\operatorname{Sqrt}[c*g^2+h*(-b*g)+a*h]}\right)*\operatorname{Sqrt}[a+bx+cx]}{(8*(c*g^2+h*(-b*g)+a*h))^{3/2}} \\
& \left. \right) / (24*(c*g^2+h*(-b*g)+a*h)^2) / (c*h)
\end{aligned}$$

IntegrateAlgebraic [F] time = 180.08, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] \$Aborted

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^5,x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^5,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.03, size = 29161, normalized size = 58.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^5,x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^5,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

[Out] int(((a + b\*x + c\*x^2)^(1/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(h\*x+g)\*\*5, x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*5, x)



$$3.191 \quad \int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^6} dx$$

**Optimal.** Leaf size=824

$$\frac{(4c^2(3fg^2 + h(2eg - 27dh))g^2 - 5h^2((3fg^2 + 3ehg + 7dh^2)b^2 - 2ah(6fg + 5eh)b + 16a^2fh^2) - 2ch(bg(16fg^2 + 5e^2h) + 3d^2h^2))g^2 + 240h(cg^2 - bhg + ah^2)^3(g + hx)^3}{240h(cg^2 - bhg + ah^2)^3(g + hx)^3}$$

**Rubi [A]** time = 2.33, antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 834, 806, 720, 724, 206}

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x]

[Out] ((32\*c^3\*d\*g^3 - 8\*c^2\*g\*(a\*f\*g^2 - 3\*a\*h\*(2\*e\*g - d\*h) + 2\*b\*g\*(e\*g + 3\*d\*h)) + 2\*c\*(4\*a^2\*h^2\*(6\*f\*g - e\*h) - 6\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g - d\*h)) + b^2\*(5\*f\*g^3 + 3\*g\*h\*(2\*e\*g + 5\*d\*h))) - b\*h\*(16\*a^2\*f\*h^2 - 2\*a\*b\*h\*(6\*f\*g + 5\*e\*h) + b^2\*(3\*f\*g^2 + h\*(3\*e\*g + 7\*d\*h))))\*(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)\*Sqrt[a + b\*x + c\*x^2]/(128\*(c\*g^2 - b\*g\*h + a\*h^2)^4\*(g + h\*x)^2) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(3/2))/(5\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^5) + ((6\*c\*f\*g^3 + 2\*c\*g\*h\*(2\*e\*g - 7\*d\*h) + 10\*a\*h^2\*(2\*f\*g - e\*h) - b\*h\*(13\*f\*g^2 - h\*(3\*e\*g + 7\*d\*h)))\*(a + b\*x + c\*x^2)^(3/2))/(40\*h\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^4) + (((4\*c^2\*(3\*f\*g^4 + g^2\*h\*(2\*e\*g - 27\*d\*h)) - 5\*h^2\*(16\*a^2\*f\*h^2 - 2\*a\*b\*h\*(6\*f\*g + 5\*e\*h) + b^2\*(3\*f\*g^2 + 3\*e\*g\*h + 7\*d\*h^2)) - 2\*c\*h\*(b\*g\*(16\*f\*g^2 - 21\*e\*g\*h - 54\*d\*h^2) - 2\*a\*h\*(18\*f\*g^2 - 33\*e\*g\*h + 8\*d\*h^2)))\*(a + b\*x + c\*x^2)^(3/2))/(240\*h\*(c\*g^2 - b\*g\*h + a\*h^2)^3\*(g + h\*x)^3) - ((b^2 - 4\*a\*c)\*(32\*c^3\*d\*g^3 - 8\*c^2\*g\*(a\*f\*g^2 - 3\*a\*h\*(2\*e\*g - d\*h) + 2\*b\*g\*(e\*g + 3\*d\*h)) + 2\*c\*(4\*a^2\*h^2\*(6\*f\*g - e\*h) - 6\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g - d\*h)) + b^2\*(5\*f\*g^3 + 3\*g\*h\*(2\*e\*g + 5\*d\*h))) - b\*h\*(16\*a^2\*f\*h^2 - 2\*a\*b\*h\*(6\*f\*g + 5\*e\*h) + b^2\*(3\*f\*g^2 + h\*(3\*e\*g + 7\*d\*h))))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])]/(256\*(c\*g^2 - b\*g\*h + a\*h^2)^(9/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 720**

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

### Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

&& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{(g + hx)^6} dx &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{3/2}}{5h(cg^2 - bgh + ah^2)(g + hx)^5} - \int \frac{\left(\frac{1}{2}\left(-10cdg + 3beg + 10afg - \frac{3bf^2}{h} + 7bdh\right)\right)}{5(cg^2 - bgh + ah^2)(g + hx)^6} dx \\
 &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{3/2}}{5h(cg^2 - bgh + ah^2)(g + hx)^5} + \frac{(6cfg^3 + 2cgh(2eg - 7dh) + 10afg^2 - 7bdfg + 3bdh^2)}{4h^2(cg^2 - bgh + ah^2)(g + hx)^4} \\
 &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{3/2}}{5h(cg^2 - bgh + ah^2)(g + hx)^5} + \frac{(6cfg^3 + 2cgh(2eg - 7dh) + 10afg^2 - 7bdfg + 3bdh^2)}{4h^2(cg^2 - bgh + ah^2)(g + hx)^4} \\
 &= \frac{(32c^3dg^3 - 8c^2g(afg^2 - 3ah(2eg - dh) + 2bg(eg + 3dh)) + 2c(4a^2h^2(6fg^2 - 7bdfg + 3bdh^2)))}{4h^2(cg^2 - bgh + ah^2)(g + hx)^4} \\
 &= \frac{(32c^3dg^3 - 8c^2g(afg^2 - 3ah(2eg - dh) + 2bg(eg + 3dh)) + 2c(4a^2h^2(6fg^2 - 7bdfg + 3bdh^2)))}{4h^2(cg^2 - bgh + ah^2)(g + hx)^4} \\
 &= \frac{(32c^3dg^3 - 8c^2g(afg^2 - 3ah(2eg - dh) + 2bg(eg + 3dh)) + 2c(4a^2h^2(6fg^2 - 7bdfg + 3bdh^2)))}{4h^2(cg^2 - bgh + ah^2)(g + hx)^4}
 \end{aligned}$$

**Mathematica [A]** time = 6.33, size = 1128, normalized size = 1.37



Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x]

[Out]  $-\frac{1}{2} \frac{(f(a + bx + cx^2) \sqrt{a + x(b + cx)})}{(c h (g + hx)^5)} + \frac{(f(a + bx + cx^2) \sqrt{a + x(b + cx)} (-1/5 ((h(3b^2fg + 4c^2d^2h - 10a^2f^2h))/2 - (g(6c^2fg + 4c^2e^2h - 7b^2f^2h))/2) (a + bx + cx^2)^{3/2})}{((c^2g^2 - b^2gh + a^2h^2)(g + hx)^5)} - \frac{(-1/4 ((2c^2g(3c^2fg^2 - 5f^2h(bg - ah)) + 2c^2h(e^2g - 2efg + fh^2) - 2c^2h^2(eg + 3dh)) + 2c(4a^2h^2(6fg^2 - 7bdfg + 3bdh^2)))}{4h^2(cg^2 - bgh + ah^2)(g + hx)^4}$

$$\begin{aligned}
& d*h)) - c*h*(3*b*f*g^2 - b*h*(3*e*g + 7*d*h) + 10*h*(c*d*g - a*f*g + a*e*h \\
& ))*(a + b*x + c*x^2)^{(3/2)}/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (((c^2 \\
& *g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13* \\
& f*g^2 - h*(3*e*g + 7*d*h))) - (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) + \\
& 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h \\
& *(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2)*(a + \\
& b*x + c*x^2)^{(3/2)}/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-2*(a*c^2*h \\
& *(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*f* \\
& g^2 - h*(3*e*g + 7*d*h))) + (c^2*g*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) + \\
& 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h \\
& *(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2) + b*( \\
& c^2*g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*( \\
& 13*f*g^2 - h*(3*e*g + 7*d*h))) + (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h) \\
& ) + 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 1 \\
& 6*h*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2))* \\
& ((b*g - 2*a*h + (2*c*g - b*h)*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*(c*g^2 - b*g*h + \\
& a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*\text{ArcTanh}[(-(b*g) + 2*a*h - (2*c*g - b* \\
& h)*x)/(2*\text{Sqrt}[c*g^2 - b*g*h + a*h^2]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[c*g^2 \\
& - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(2*(c*g^2 - b*g*h + a*h^ \\
& 2)))/(4*(c*g^2 - b*g*h + a*h^2)))/(5*(c*g^2 - b*g*h + a*h^2)))/(2*c*h*\text{Sqrt} \\
& [a + b*x + c*x^2])
\end{aligned}$$

**IntegrateAlgebraic** [F] time = 180.05, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[a + b\*x + c\*x^2]\*(d + e\*x + f\*x^2))/(g + h\*x)^6,x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)\*(c\*x^2+b\*x+a)^(1/2)/(h\*x+g)^6,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")`

[Out] Timed out

**maple** [B] time = 0.04, size = 40336, normalized size = 48.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x)`

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{cx^2 + bx + a} (fx^2 + ex + d)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)`

[Out] `int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} (d + ex + fx^2)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

$$3.192 \quad \int (g + hx)^3 (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

Optimal. Leaf size=1169

$$\frac{f(cx^2 + bx + a)^{5/2} (g + hx)^4}{9ch} - \frac{(10cfg - 18ceh + 13bfh)(cx^2 + bx + a)^{5/2} (g + hx)^3}{144c^2h} + \frac{(-12(5fg^2 - 3h(3eg + 8dh))}{144c^2h}$$

**Rubi [A]** time = 3.70, antiderivative size = 1166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 621, 206}

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] -((b^2 - 4\*a\*c)\*(1536\*c^5\*d\*g^3 - 143\*b^5\*f\*h^3 - 256\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + 3\*b\*g\*(e\*g + 3\*d\*h)) + 22\*b^3\*c\*h^2\*(20\*a\*f\*h + 9\*b\*(3\*f\*g + e\*h)) - 48\*b\*c^2\*h\*(5\*a^2\*f\*h^2 + 9\*a\*b\*h\*(3\*f\*g + e\*h) + 6\*b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 32\*c^3\*(3\*a^2\*h^2\*(3\*f\*g + e\*h) + 14\*b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 12\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2]/(32768\*c^7) + ((1536\*c^5\*d\*g^3 - 143\*b^5\*f\*h^3 - 256\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + 3\*b\*g\*(e\*g + 3\*d\*h)) + 22\*b^3\*c\*h^2\*(20\*a\*f\*h + 9\*b\*(3\*f\*g + e\*h)) - 48\*b\*c^2\*h\*(5\*a^2\*f\*h^2 + 9\*a\*b\*h\*(3\*f\*g + e\*h) + 6\*b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 32\*c^3\*(3\*a^2\*h^2\*(3\*f\*g + e\*h) + 14\*b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 12\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))))\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(12288\*c^6) + ((143\*b^2\*f\*h^2 - 2\*c\*h\*(2\*4\*b\*f\*g + 99\*b\*e\*h + 64\*a\*f\*h) - 12\*c^2\*(5\*f\*g^2 - 3\*h\*(3\*e\*g + 8\*d\*h)))\*(g + h\*x)^2\*(a + b\*x + c\*x^2)^(5/2))/(2016\*c^3\*h) - ((10\*c\*f\*g - 18\*c\*e\*h + 1\*3\*b\*f\*h)\*(g + h\*x)^3\*(a + b\*x + c\*x^2)^(5/2))/(144\*c^2\*h) + (f\*(g + h\*x)^4\*(a + b\*x + c\*x^2)^(5/2))/(9\*c\*h) + ((3003\*b^4\*f\*h^4 - 192\*c^4\*(5\*f\*g^4 - 3\*g^2\*h\*(3\*e\*g + 64\*d\*h)) - 198\*b^2\*c\*h^3\*(38\*a\*f\*h + 21\*b\*(3\*f\*g + e\*h)) + 8\*c^2\*h^2\*(256\*a^2\*f\*h^2 + 837\*a\*b\*h\*(3\*f\*g + e\*h) + b^2\*(1553\*f\*g^2 + 756\*h\*(3\*e\*g + d\*h))) - 16\*c^3\*h\*(32\*a\*h\*(17\*f\*g^2 + 9\*h\*(3\*e\*g + d\*h)) + b\*g\*(1\*3\*f\*g^2 + 9\*h\*(141\*e\*g + 196\*d\*h))) - 10\*c\*h\*(429\*b^3\*f\*h^3 - 22\*b\*c\*h^2\*(2\*9\*b\*f\*g + 27\*b\*e\*h + 34\*a\*f\*h) + 16\*c^3\*(5\*f\*g^3 - 9\*g\*h\*(e\*g + 12\*d\*h)) + 8\*c^2\*h\*(a\*h\*(61\*f\*g + 63\*e\*h) + 3\*b\*(f\*g^2 + 6\*h\*(7\*e\*g + 6\*d\*h))))\*x\*(a + b\*x + c\*x^2)^(5/2))/(80640\*c^5\*h) + ((b^2 - 4\*a\*c)^2\*(1536\*c^5\*d\*g^3 - 143\*b^5\*f\*h^3 - 256\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + 3\*b\*g\*(e\*g + 3\*d\*h)) + 22\*b^3\*c\*h^2\*(20\*a\*f\*h + 9\*b\*(3\*f\*g + e\*h)) - 48\*b\*c^2\*h\*(5\*a^2\*f\*h^2 + 9\*a\*b\*h\*(3\*f\*g + e\*h) + 6\*b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 32\*c^3\*(3\*a^2\*h^2\*(3\*f\*g + e\*h) + 14\*b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 12\*a\*b\*h\*(3\*f\*g^2

+ h\*(3\*e\*g + d\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])]  
)/(65536\*c^(15/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/  
Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt  
Q[a, 0] || LtQ[b, 0])

### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x  
)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2  
\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N  
eQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int  
t[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a,  
b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 779

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(  
x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -  
2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x  
] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p +  
3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d  
, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 832

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c  
\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p +  
1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)  
\*(a + b\*x + c\*x^2)^p\*Simp[m\*(c\*d\*f - a\*e\*g) + d\*(2\*c\*f - b\*g)\*(p + 1) + (m\*  
(c\*e\*f + c\*d\*g - b\*e\*g) + e\*(p + 1)\*(2\*c\*f - b\*g))\*x, x], x] /; FreeQ[{  
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a  
\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]  
|| IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 1653

Int[(Pq)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p  
\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S



```

imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (g + hx)^3 (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{f(g + hx)^4 (a + bx + cx^2)^{5/2}}{9ch} + \frac{\int (g + hx)^3 \left(-\frac{1}{2}h(5bfg - 18cd)\right)}{9ch} \\
&= -\frac{(10cfg - 18ceh + 13bfh)(g + hx)^3 (a + bx + cx^2)^{5/2}}{144c^2h} + \frac{f(g + hx)^4 (a + bx + cx^2)^{5/2}}{9ch} \\
&= \frac{(143b^2fh^2 - 2ch(24bfg + 99beh + 64afh) - 12c^2(5fg^2 - 3hd^2)) (g + hx)^3 (a + bx + cx^2)^{5/2}}{2016c^3h} \\
&= \frac{(143b^2fh^2 - 2ch(24bfg + 99beh + 64afh) - 12c^2(5fg^2 - 3hd^2)) (g + hx)^3 (a + bx + cx^2)^{5/2}}{2016c^3h} \\
&= \frac{(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3b^2fg)) (g + hx)^3 (a + bx + cx^2)^{5/2}}{2016c^3h} \\
&= -\frac{(b^2 - 4ac)(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3b^2fg)) (g + hx)^3 (a + bx + cx^2)^{5/2}}{2016c^3h} \\
&= -\frac{(b^2 - 4ac)(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3b^2fg)) (g + hx)^3 (a + bx + cx^2)^{5/2}}{2016c^3h} \\
&= -\frac{(b^2 - 4ac)(1536c^5dg^3 - 143b^5fh^3 - 256c^4g(afg^2 + 3ah(eg + dh) + 3b^2fg)) (g + hx)^3 (a + bx + cx^2)^{5/2}}{2016c^3h}
\end{aligned}$$

**Mathematica [A]** time = 2.71, size = 721, normalized size = 0.62

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

```
[Out] (((143*b^2*f*h^2 - 2*c*h*(24*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2
- 3*h*(3*e*g + 8*d*h)))*(g + h*x)^2*(a + x*(b + c*x))^(5/2))/(224*c^2) - (
(13*b*f*h + 2*c*(5*f*g - 9*e*h))*(g + h*x)^3*(a + x*(b + c*x))^(5/2))/(16*c
) + f*(g + h*x)^4*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(3003*
b^4*f*h^4 - 66*b^2*c*h^3*(114*a*f*h + b*(189*f*g + 63*e*h + 65*f*h*x)) - 32
*c^4*(5*f*g^3*(6*g + 5*h*x) - 9*g*h*(e*g*(6*g + 5*h*x) + 4*d*h*(32*g + 15*h
*x))) + 4*c^2*h^2*(512*a^2*f*h^2 + 2*a*b*h*(2511*f*g + 837*e*h + 935*f*h*x)
+ b^2*(f*g*(3106*g + 1595*h*x) + 27*h*(168*e*g + 56*d*h + 55*e*h*x))) - 16
*c^3*h*(a*h*(f*g*(544*g + 305*h*x) + 9*h*(96*e*g + 32*d*h + 35*e*h*x)) + b*
(f*g^2*(13*g + 15*h*x) + 9*h*(4*d*h*(49*g + 15*h*x) + e*g*(141*g + 70*h*x))
))))/(8960*c^4) + (3*h*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2
+ 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*
(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3
*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*
g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(2*sqrt[c]*
(b + 2*c*x)*sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2))
+ 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])])
)/(65536*c^(13/2))/(9*c*h)
```

**IntegrateAlgebraic [B]** time = 9.59, size = 3156, normalized size = 2.70

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (sqrt[a + b*x + c*x^2]*(-483840*b^3*c^5*d*g^3 + 3225600*a*b*c^6*d*g^3 + 241
920*b^4*c^4*e*g^3 - 1612800*a*b^2*c^5*e*g^3 + 2064384*a^2*c^6*e*g^3 - 14112
0*b^5*c^3*f*g^3 + 1021440*a*b^3*c^4*f*g^3 - 1741824*a^2*b*c^5*f*g^3 + 72576
0*b^4*c^4*d*g^2*h - 4838400*a*b^2*c^5*d*g^2*h + 6193152*a^2*c^6*d*g^2*h - 4
23360*b^5*c^3*e*g^2*h + 3064320*a*b^3*c^4*e*g^2*h - 5225472*a^2*b*c^5*e*g^2
*h + 272160*b^6*c^2*f*g^2*h - 2177280*a*b^4*c^3*f*g^2*h + 4741632*a^2*b^2*c
^4*f*g^2*h - 1769472*a^3*c^5*f*g^2*h - 423360*b^5*c^3*d*g*h^2 + 3064320*a*b
^3*c^4*d*g*h^2 - 5225472*a^2*b*c^5*d*g*h^2 + 272160*b^6*c^2*e*g*h^2 - 21772
80*a*b^4*c^3*e*g*h^2 + 4741632*a^2*b^2*c^4*e*g*h^2 - 1769472*a^3*c^5*e*g*h^
2 - 187110*b^7*c*f*g*h^2 + 1655640*a*b^5*c^2*f*g*h^2 - 4408992*a^2*b^3*c^3*
f*g*h^2 + 3176064*a^3*b*c^4*f*g*h^2 + 90720*b^6*c^2*d*h^3 - 725760*a*b^4*c^
3*d*h^3 + 1580544*a^2*b^2*c^4*d*h^3 - 589824*a^3*c^5*d*h^3 - 62370*b^7*c*e*
h^3 + 551880*a*b^5*c^2*e*h^3 - 1469664*a^2*b^3*c^3*e*h^3 + 1058688*a^3*b*c^
4*e*h^3 + 45045*b^8*f*h^3 - 438900*a*b^6*c*f*h^3 + 1383984*a^2*b^4*c^2*f*h^
3 - 1467072*a^3*b^2*c^3*f*h^3 + 262144*a^4*c^4*f*h^3 + 322560*b^2*c^6*d*g^3
*x + 6451200*a*c^7*d*g^3*x - 161280*b^3*c^5*e*g^3*x + 903168*a*b*c^6*e*g^3*
x + 94080*b^4*c^4*f*g^3*x - 580608*a*b^2*c^5*f*g^3*x + 645120*a^2*c^6*f*g^3
*x - 483840*b^3*c^5*d*g^2*h*x + 2709504*a*b*c^6*d*g^2*h*x + 282240*b^4*c^4*
e*g^2*h*x - 1741824*a*b^2*c^5*e*g^2*h*x + 1935360*a^2*c^6*e*g^2*h*x - 18144
```

$$\begin{aligned}
& 0*b^5*c^3*f*g^2*h*x + 1257984*a*b^3*c^4*f*g^2*h*x - 2018304*a^2*b*c^5*f*g^2 \\
& *h*x + 282240*b^4*c^4*d*g*h^2*x - 1741824*a*b^2*c^5*d*g*h^2*x + 1935360*a^2 \\
& *c^6*d*g*h^2*x - 181440*b^5*c^3*e*g*h^2*x + 1257984*a*b^3*c^4*e*g*h^2*x - 2 \\
& 018304*a^2*b*c^5*e*g*h^2*x + 124740*b^6*c^2*f*g*h^2*x - 970704*a*b^4*c^3*f* \\
& g*h^2*x + 2040768*a^2*b^2*c^4*f*g*h^2*x - 725760*a^3*c^5*f*g*h^2*x - 60480* \\
& b^5*c^3*d*h^3*x + 419328*a*b^3*c^4*d*h^3*x - 672768*a^2*b*c^5*d*h^3*x + 415 \\
& 80*b^6*c^2*e*h^3*x - 323568*a*b^4*c^3*e*h^3*x + 680256*a^2*b^2*c^4*e*h^3*x \\
& - 241920*a^3*c^5*e*h^3*x - 30030*b^7*c*f*h^3*x + 260568*a*b^5*c^2*f*h^3*x - \\
& 677664*a^2*b^3*c^3*f*h^3*x + 473728*a^3*b*c^4*f*h^3*x + 3870720*b*c^7*d*g^ \\
& 3*x^2 + 129024*b^2*c^6*e*g^3*x^2 + 4128768*a*c^7*e*g^3*x^2 - 75264*b^3*c^5* \\
& f*g^3*x^2 + 387072*a*b*c^6*f*g^3*x^2 + 387072*b^2*c^6*d*g^2*h*x^2 + 1238630 \\
& 4*a*c^7*d*g^2*h*x^2 - 225792*b^3*c^5*e*g^2*h*x^2 + 1161216*a*b*c^6*e*g^2*h* \\
& x^2 + 145152*b^4*c^4*f*g^2*h*x^2 - 857088*a*b^2*c^5*f*g^2*h*x^2 + 884736*a^ \\
& 2*c^6*f*g^2*h*x^2 - 225792*b^3*c^5*d*g*h^2*x^2 + 1161216*a*b*c^6*d*g*h^2*x^ \\
& 2 + 145152*b^4*c^4*e*g*h^2*x^2 - 857088*a*b^2*c^5*e*g*h^2*x^2 + 884736*a^2* \\
& c^6*e*g*h^2*x^2 - 99792*b^5*c^3*f*g*h^2*x^2 + 673920*a*b^3*c^4*f*g*h^2*x^2 \\
& - 1043712*a^2*b*c^5*f*g*h^2*x^2 + 48384*b^4*c^4*d*h^3*x^2 - 285696*a*b^2*c^ \\
& 5*d*h^3*x^2 + 294912*a^2*c^6*d*h^3*x^2 - 33264*b^5*c^3*e*h^3*x^2 + 224640*a \\
& *b^3*c^4*e*h^3*x^2 - 347904*a^2*b*c^5*e*h^3*x^2 + 24024*b^6*c^2*f*h^3*x^2 - \\
& 183744*a*b^4*c^3*f*h^3*x^2 + 378240*a^2*b^2*c^4*f*h^3*x^2 - 131072*a^3*c^5 \\
& *f*h^3*x^2 + 2580480*c^8*d*g^3*x^3 + 2838528*b*c^7*e*g^3*x^3 + 64512*b^2*c^ \\
& 6*f*g^3*x^3 + 3010560*a*c^7*f*g^3*x^3 + 8515584*b*c^7*d*g^2*h*x^3 + 193536* \\
& b^2*c^6*e*g^2*h*x^3 + 9031680*a*c^7*e*g^2*h*x^3 - 124416*b^3*c^5*f*g^2*h*x^ \\
& 3 + 608256*a*b*c^6*f*g^2*h*x^3 + 193536*b^2*c^6*d*g*h^2*x^3 + 9031680*a*c^7 \\
& *d*g*h^2*x^3 - 124416*b^3*c^5*e*g*h^2*x^3 + 608256*a*b*c^6*e*g*h^2*x^3 + 85 \\
& 536*b^4*c^4*f*g*h^2*x^3 - 490752*a*b^2*c^5*f*g*h^2*x^3 + 483840*a^2*c^6*f*g \\
& *h^2*x^3 - 41472*b^3*c^5*d*h^3*x^3 + 202752*a*b*c^6*d*h^3*x^3 + 28512*b^4*c \\
& ^4*e*h^3*x^3 - 163584*a*b^2*c^5*e*h^3*x^3 + 161280*a^2*c^6*e*h^3*x^3 - 2059 \\
& 2*b^5*c^3*f*h^3*x^3 + 136576*a*b^3*c^4*f*h^3*x^3 - 206592*a^2*b*c^5*f*h^3*x \\
& ^3 + 2064384*c^8*e*g^3*x^4 + 2236416*b*c^7*f*g^3*x^4 + 6193152*c^8*d*g^2*h* \\
& x^4 + 6709248*b*c^7*e*g^2*h*x^4 + 110592*b^2*c^6*f*g^2*h*x^4 + 7077888*a*c^ \\
& 7*f*g^2*h*x^4 + 6709248*b*c^7*d*g*h^2*x^4 + 110592*b^2*c^6*e*g*h^2*x^4 + 70 \\
& 77888*a*c^7*e*g*h^2*x^4 - 76032*b^3*c^5*f*g*h^2*x^4 + 359424*a*b*c^6*f*g*h^ \\
& 2*x^4 + 36864*b^2*c^6*d*h^3*x^4 + 2359296*a*c^7*d*h^3*x^4 - 25344*b^3*c^5*e \\
& *h^3*x^4 + 119808*a*b*c^6*e*h^3*x^4 + 18304*b^4*c^4*f*h^3*x^4 - 102912*a*b^ \\
& 2*c^5*f*h^3*x^4 + 98304*a^2*c^6*f*h^3*x^4 + 1720320*c^8*f*g^3*x^5 + 5160960 \\
& *c^8*e*g^2*h*x^5 + 5529600*b*c^7*f*g^2*h*x^5 + 5160960*c^8*d*g*h^2*x^5 + 55 \\
& 29600*b*c^7*e*g*h^2*x^5 + 69120*b^2*c^6*f*g*h^2*x^5 + 5806080*a*c^7*f*g*h^2 \\
& *x^5 + 1843200*b*c^7*d*h^3*x^5 + 23040*b^2*c^6*e*h^3*x^5 + 1935360*a*c^7*e* \\
& h^3*x^5 - 16640*b^3*c^5*f*h^3*x^5 + 76800*a*b*c^6*f*h^3*x^5 + 4423680*c^8*f \\
& *g^2*h*x^6 + 4423680*c^8*e*g*h^2*x^6 + 4700160*b*c^7*f*g*h^2*x^6 + 1474560* \\
& c^8*d*h^3*x^6 + 1566720*b*c^7*e*h^3*x^6 + 15360*b^2*c^6*f*h^3*x^6 + 1638400 \\
& *a*c^7*f*h^3*x^6 + 3870720*c^8*f*g*h^2*x^7 + 1290240*c^8*e*h^3*x^7 + 136192 \\
& 0*b*c^7*f*h^3*x^7 + 1146880*c^8*f*h^3*x^8)/(10321920*c^7) + ((-1536*b^4*c^ \\
& 5*d*g^3 + 12288*a*b^2*c^6*d*g^3 - 24576*a^2*c^7*d*g^3 + 768*b^5*c^4*e*g^3 -
\end{aligned}$$

$$\begin{aligned}
& 6144*a*b^3*c^5*e*g^3 + 12288*a^2*b*c^6*e*g^3 - 448*b^6*c^3*f*g^3 + 3840*a* \\
& b^4*c^4*f*g^3 - 9216*a^2*b^2*c^5*f*g^3 + 4096*a^3*c^6*f*g^3 + 2304*b^5*c^4* \\
& d*g^2*h - 18432*a*b^3*c^5*d*g^2*h + 36864*a^2*b*c^6*d*g^2*h - 1344*b^6*c^3* \\
& e*g^2*h + 11520*a*b^4*c^4*e*g^2*h - 27648*a^2*b^2*c^5*e*g^2*h + 12288*a^3*c \\
& ^6*e*g^2*h + 864*b^7*c^2*f*g^2*h - 8064*a*b^5*c^3*f*g^2*h + 23040*a^2*b^3*c \\
& ^4*f*g^2*h - 18432*a^3*b*c^5*f*g^2*h - 1344*b^6*c^3*d*g*h^2 + 11520*a*b^4*c \\
& ^4*d*g*h^2 - 27648*a^2*b^2*c^5*d*g*h^2 + 12288*a^3*c^6*d*g*h^2 + 864*b^7*c^ \\
& 2*e*g*h^2 - 8064*a*b^5*c^3*e*g*h^2 + 23040*a^2*b^3*c^4*e*g*h^2 - 18432*a^3* \\
& b*c^5*e*g*h^2 - 594*b^8*c*f*g*h^2 + 6048*a*b^6*c^2*f*g*h^2 - 20160*a^2*b^4* \\
& c^3*f*g*h^2 + 23040*a^3*b^2*c^4*f*g*h^2 - 4608*a^4*c^5*f*g*h^2 + 288*b^7*c^ \\
& 2*d*h^3 - 2688*a*b^5*c^3*d*h^3 + 7680*a^2*b^3*c^4*d*h^3 - 6144*a^3*b*c^5*d* \\
& h^3 - 198*b^8*c*e*h^3 + 2016*a*b^6*c^2*e*h^3 - 6720*a^2*b^4*c^3*e*h^3 + 768 \\
& 0*a^3*b^2*c^4*e*h^3 - 1536*a^4*c^5*e*h^3 + 143*b^9*f*h^3 - 1584*a*b^7*c*f*h \\
& ^3 + 6048*a^2*b^5*c^2*f*h^3 - 8960*a^3*b^3*c^3*f*h^3 + 3840*a^4*b*c^4*f*h^3 \\
& ) * \text{Log}[b + 2*c*x - 2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]] / (65536*c^(15/2))
\end{aligned}$$

**fricas [B]** time = 14.76, size = 4751, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] [-1/41287680\*(315\*(64\*(24\*(b^4\*c^5 - 8\*a\*b^2\*c^6 + 16\*a^2\*c^7)\*d - 12\*(b^5\*c^4 - 8\*a\*b^3\*c^5 + 16\*a^2\*b\*c^6)\*e + (7\*b^6\*c^3 - 60\*a\*b^4\*c^4 + 144\*a^2\*b^2\*c^5 - 64\*a^3\*c^6)\*f)\*g^3 - 96\*(24\*(b^5\*c^4 - 8\*a\*b^3\*c^5 + 16\*a^2\*b\*c^6)\*d - 2\*(7\*b^6\*c^3 - 60\*a\*b^4\*c^4 + 144\*a^2\*b^2\*c^5 - 64\*a^3\*c^6)\*e + 3\*(3\*b^7\*c^2 - 28\*a\*b^5\*c^3 + 80\*a^2\*b^3\*c^4 - 64\*a^3\*b\*c^5)\*f)\*g^2\*h + 6\*(32\*(7\*b^6\*c^3 - 60\*a\*b^4\*c^4 + 144\*a^2\*b^2\*c^5 - 64\*a^3\*c^6)\*d - 48\*(3\*b^7\*c^2 - 28\*a\*b^5\*c^3 + 80\*a^2\*b^3\*c^4 - 64\*a^3\*b\*c^5)\*e + 3\*(33\*b^8\*c - 336\*a\*b^6\*c^2 + 1120\*a^2\*b^4\*c^3 - 1280\*a^3\*b^2\*c^4 + 256\*a^4\*c^5)\*f)\*g\*h^2 - (96\*(3\*b^7\*c^2 - 28\*a\*b^5\*c^3 + 80\*a^2\*b^3\*c^4 - 64\*a^3\*b\*c^5)\*d - 6\*(33\*b^8\*c - 336\*a\*b^6\*c^2 + 1120\*a^2\*b^4\*c^3 - 1280\*a^3\*b^2\*c^4 + 256\*a^4\*c^5)\*e + (143\*b^9 - 1584\*a\*b^7\*c + 6048\*a^2\*b^5\*c^2 - 8960\*a^3\*b^3\*c^3 + 3840\*a^4\*b\*c^4)\*f)\*h^3)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(1146880\*c^9\*f\*h^3\*x^8 + 71680\*(54\*c^9\*f\*g\*h^2 + (18\*c^9\*e + 19\*b\*c^8\*f)\*h^3)\*x^7 + 5120\*(864\*c^9\*f\*g^2\*h + 54\*(16\*c^9\*e + 17\*b\*c^8\*f)\*g\*h^2 + (288\*c^9\*d + 306\*b\*c^8\*e + (3\*b^2\*c^7 + 320\*a\*c^8)\*f)\*h^3)\*x^6 + 1280\*(1344\*c^9\*f\*g^3 + 288\*(14\*c^9\*e + 15\*b\*c^8\*f)\*g^2\*h + 18\*(224\*c^9\*d + 240\*b\*c^8\*e + 3\*(b^2\*c^7 + 84\*a\*c^8)\*f)\*g\*h^2 + (1440\*b\*c^8\*d + 18\*(b^2\*c^7 + 84\*a\*c^8)\*e - (13\*b^3\*c^6 - 60\*a\*b\*c^7)\*f)\*h^3)\*x^5 + 128\*(1344\*(12\*c^9\*e + 13\*b\*c^8\*f)\*g^3 + 288\*(168\*c^9\*d + 182\*b\*c^8\*e + 3\*(b^2\*c^7 + 64\*a\*c^8)\*f)\*g^2\*h + 18\*(2912\*b\*c^8\*d + 48\*(b^2\*c^7 + 64\*a\*c^8)\*e - 3\*(11\*b^3\*c^6 - 52\*a\*b\*c^7)\*f)\*g\*h^2 + (288\*(b^2\*c^7 + 64\*a\*c^8)\*d - 18\*(11\*b^3\*c^6 - 52\*a\*b\*c^7)\*e + (143\*b^4\*c^5 - 804\*a\*b^2\*c^6 + 768\*a^2\*c^7)\*f)\*h^3)

$$\begin{aligned}
& *x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*b^2*c^6 \\
& + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3 \\
& + 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5*c^4 - \\
& 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488 \\
& *a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^ \\
& 5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 \\
& - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4 \\
& - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2 \\
& *b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2 \\
& *b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 1383984*a \\
& ^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*(120*c \\
& ^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8*d + 1 \\
& 4*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 18*(224 \\
& *(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^4*c^5 \\
& - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^7)*d - \\
& 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 8536*a*b \\
& ^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^2*c^7 \\
& + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 + 32*a \\
& c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^6 + 12 \\
& 8*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4*c^5 - \\
& 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^ \\
& 2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d - 18* \\
& (231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 22968*a \\
& *b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(120*(b \\
& ^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 - 216*a \\
& b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d - 14*(3 \\
& 5*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b^3*c^5 \\
& + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2 \\
& *c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(1155*b^6 \\
& *c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - (288*( \\
& 105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 - 8988*a \\
& *b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130284*a \\
& b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^2 + b \\
& x + a))/c^8, -1/20643840*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)* \\
& d - 12*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 \\
& + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 1 \\
& 6*a^2*b*c^6)*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6 \\
& )*e + 3*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h \\
& + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*( \\
& 3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - \\
& 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^ \\
& 2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(3 \\
& 3*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5 \\
& )*e + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840 \\
& a^4*b*c^4)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a))*(2*c*x + b)*sq
\end{aligned}$$

$$\begin{aligned} & \text{rt}(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9* \\ & f*g*h^2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16 \\ & *c^9*e + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a* \\ & c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h \\ & + 18*(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b* \\ & c^8*d + 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + \\ & 128*(1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*( \\ & b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e \\ & - 3*(11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18* \\ & (11*b^3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f \\ & )*h^3)*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a* \\ & b^2*c^6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f \\ & )*g^3 + 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5* \\ & c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 \\ & + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a* \\ & b^3*c^5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b \\ & ^2*c^5 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^ \\ & 3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 54 \\ & 88*a^2*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 816 \\ & 48*a^2*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 138 \\ & 3984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344* \\ & (120*c^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8 \\ & *d + 14*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 1 \\ & 8*(224*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^ \\ & 4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^ \\ & 7)*d - 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 85 \\ & 36*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^ \\ & 2*c^7 + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 + \\ & 32*a*c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^ \\ & 6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4 \\ & *c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2 \\ & 416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d \\ & - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 2 \\ & 2968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*( \\ & 120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 - \\ & 216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d - \\ & 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b^ \\ & 3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 2 \\ & 40*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(11 \\ & 55*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - \\ & (288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 - \\ & 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130 \\ & 284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^ \\ & 2 + b*x + a))/c^8] \end{aligned}$$

**giac [B]** time = 0.44, size = 2977, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out]  $1/10321920*\sqrt{c*x^2 + b*x + a}*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (54*c^9*f*g*h^2 + 19*b*c^8*f*h^3 + 18*c^9*h^3*e)/c^8)*x + (864*c^9*f*g^2*h + 9*18*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 3*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3 + 864*c^9*g*h^2*e + 306*b*c^8*h^3*e)/c^8)*x + (1344*c^9*f*g^3 + 4320*b*c^8*f*g^2*h + 4032*c^9*d*g*h^2 + 54*b^2*c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8*d*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3 + 4032*c^9*g^2*h*e + 4320*b*c^8*g*h^2*e + 18*b^2*c^7*h^3*e + 1512*a*c^8*h^3*e)/c^8)*x + (17472*b*c^8*f*g^3 + 48384*c^9*d*g^2*h + 864*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b*c^8*d*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^2 + 288*b^2*c^7*d*h^3 + 18432*a*c^8*d*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*c^7*f*h^3 + 16128*c^9*g^3*e + 52416*b*c^8*g^2*h*e + 864*b^2*c^7*g*h^2*e + 55296*a*c^8*g*h^2*e - 198*b^3*c^6*h^3*e + 936*a*b*c^7*h^3*e)/c^8)*x + (161280*c^9*d*g^3 + 4032*b^2*c^7*f*g^3 + 188160*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*g*h^2 + 564480*a*c^8*d*g*h^2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 30240*a^2*c^7*f*g*h^2 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 - 1287*b^5*c^4*f*h^3 + 8536*a*b^3*c^5*f*h^3 - 12912*a^2*b*c^6*f*h^3 + 177408*b*c^8*g^3*e + 12096*b^2*c^7*g^2*h*e + 564480*a*c^8*g^2*h*e - 7776*b^3*c^6*g*h^2*e + 38016*a*b*c^7*g*h^2*e + 1782*b^4*c^5*h^3*e - 10224*a*b^2*c^6*h^3*e + 10080*a^2*c^7*h^3*e)/c^8)*x + (483840*b*c^8*d*g^3 - 9408*b^3*c^6*f*g^3 + 48384*a*b*c^7*f*g^3 + 48384*b^2*c^7*d*g^2*h + 1548288*a*c^8*d*g^2*h + 18144*b^4*c^5*f*g^2*h - 107136*a*b^2*c^6*f*g^2*h + 110592*a^2*c^7*f*g^2*h - 28224*b^3*c^6*d*g*h^2 + 145152*a*b*c^7*d*g*h^2 - 12474*b^5*c^4*f*g*h^2 + 84240*a*b^3*c^5*f*g*h^2 - 130464*a^2*b*c^6*f*g*h^2 + 6048*b^4*c^5*d*h^3 - 35712*a*b^2*c^6*d*h^3 + 36864*a^2*c^7*d*h^3 + 3003*b^6*c^3*f*h^3 - 22968*a*b^4*c^4*f*h^3 + 47280*a^2*b^2*c^5*f*h^3 - 16384*a^3*c^6*f*h^3 + 16128*b^2*c^7*g^3*e + 516096*a*c^8*g^3*e - 28224*b^3*c^6*g^2*h*e + 145152*a*b*c^7*g^2*h*e + 18144*b^4*c^5*g*h^2*e - 107136*a*b^2*c^6*g*h^2*e + 110592*a^2*c^7*g*h^2*e - 4158*b^5*c^4*h^3*e + 28080*a*b^3*c^5*h^3*e - 43488*a^2*b*c^6*h^3*e)/c^8)*x + (161280*b^2*c^7*d*g^3 + 3225600*a*c^8*d*g^3 + 47040*b^4*c^5*f*g^3 - 290304*a*b^2*c^6*f*g^3 + 322560*a^2*c^7*f*g^3 - 241920*b^3*c^6*d*g^2*h + 1354752*a*b*c^7*d*g^2*h - 90720*b^5*c^4*f*g^2*h + 628992*a*b^3*c^5*f*g^2*h - 1009152*a^2*b*c^6*f*g^2*h + 141120*b^4*c^5*d*g*h^2 - 870912*a*b^2*c^6*d*g*h^2 + 967680*a^2*c^7*d*g*h^2 + 62370*b^6*c^3*f*g*h^2 - 485352*a*b^4*c^4*f*g*h^2 + 1020384*a^2*b^2*c^5*f*g*h^2 - 362880*a^3*c^6*f*g*h^2 - 30240*b^5*c^4*d*h^3 + 209664*a*b^3*c^5*d*h^3 - 336384*a^2*b*c^6*d*h^3 - 15015*b^7*c^2*f*h^3 + 130284*a*b^5*c^3*f*h^3 - 338832*a^2*b^3*c^4*f*h^3 + 236864*a^3*b*c^5*f*h^3 - 80640*b^3*c^6*g^3*e + 451584*a*b*c^7*g^3*e + 141120*b^4*c^5*g^2*h*e - 870912*a*b^2*$

$$\begin{aligned}
& c^6 g^2 h^2 e + 967680 a^2 c^7 g^2 h^2 e - 90720 b^5 c^4 g^2 h^2 e + 628992 a b^3 \\
& c^5 g^2 h^2 e - 1009152 a^2 b c^6 g^2 h^2 e + 20790 b^6 c^3 h^3 e - 161784 a b \\
& ^4 c^4 h^3 e + 340128 a^2 b^2 c^5 h^3 e - 120960 a^3 c^6 h^3 e) / c^8) x - (4 \\
& 83840 b^3 c^6 d g^3 - 3225600 a b c^7 d g^3 + 141120 b^5 c^4 f g^3 - 102144 \\
& 0 a b^3 c^5 f g^3 + 1741824 a^2 b c^6 f g^3 - 725760 b^4 c^5 d g^2 h + 4838 \\
& 400 a b^2 c^6 d g^2 h - 6193152 a^2 c^7 d g^2 h - 272160 b^6 c^3 f g^2 h + \\
& 2177280 a b^4 c^4 f g^2 h - 4741632 a^2 b^2 c^5 f g^2 h + 1769472 a^3 c^6 f \\
& g^2 h + 423360 b^5 c^4 d g^2 h - 3064320 a b^3 c^5 d g^2 h + 5225472 a^2 b \\
& c^6 d g^2 h + 187110 b^7 c^2 f g^2 h - 1655640 a b^5 c^3 f g^2 h + 4408992 \\
& a^2 b^3 c^4 f g^2 h - 3176064 a^3 b c^5 f g^2 h - 90720 b^6 c^3 d h^3 + 72 \\
& 5760 a b^4 c^4 d h^3 - 1580544 a^2 b^2 c^5 d h^3 + 589824 a^3 c^6 d h^3 - 4 \\
& 5045 b^8 c f h^3 + 438900 a b^6 c^2 f h^3 - 1383984 a^2 b^4 c^3 f h^3 + 146 \\
& 7072 a^3 b^2 c^4 f h^3 - 262144 a^4 c^5 f h^3 - 241920 b^4 c^5 g^3 e + 1612 \\
& 800 a b^2 c^6 g^3 e - 2064384 a^2 c^7 g^3 e + 423360 b^5 c^4 g^2 h e - 3064 \\
& 320 a b^3 c^5 g^2 h e + 5225472 a^2 b c^6 g^2 h e - 272160 b^6 c^3 g^2 h e \\
& + 2177280 a b^4 c^4 g^2 h e - 4741632 a^2 b^2 c^5 g^2 h e + 1769472 a^3 c^6 \\
& g^2 h e + 62370 b^7 c^2 h^3 e - 551880 a b^5 c^3 h^3 e + 1469664 a^2 b^3 c \\
& ^4 h^3 e - 1058688 a^3 b c^5 h^3 e) / c^8) - 1/65536 (1536 b^4 c^5 d g^3 - 12 \\
& 288 a b^2 c^6 d g^3 + 24576 a^2 c^7 d g^3 + 448 b^6 c^3 f g^3 - 3840 a b^4 c \\
& ^4 f g^3 + 9216 a^2 b^2 c^5 f g^3 - 4096 a^3 c^6 f g^3 - 2304 b^5 c^4 d g^2 \\
& 2 h + 18432 a b^3 c^5 d g^2 h - 36864 a^2 b c^6 d g^2 h - 864 b^7 c^2 f g^2 \\
& h + 8064 a b^5 c^3 f g^2 h - 23040 a^2 b^3 c^4 f g^2 h + 18432 a^3 b c^5 f \\
& g^2 h + 1344 b^6 c^3 d g^2 h - 11520 a b^4 c^4 d g^2 h + 27648 a^2 b^2 c^5 \\
& d g^2 h - 12288 a^3 c^6 d g^2 h + 594 b^8 c f g^2 h - 6048 a b^6 c^2 f g^2 h \\
& ^2 + 20160 a^2 b^4 c^3 f g^2 h - 23040 a^3 b^2 c^4 f g^2 h + 4608 a^4 c^5 f \\
& g^2 h - 288 b^7 c^2 d h^3 + 2688 a b^5 c^3 d h^3 - 7680 a^2 b^3 c^4 d h^3 \\
& + 6144 a^3 b c^5 d h^3 - 143 b^9 f h^3 + 1584 a b^7 c f h^3 - 6048 a^2 b^5 c \\
& ^2 f h^3 + 8960 a^3 b^3 c^3 f h^3 - 3840 a^4 b c^4 f h^3 - 768 b^5 c^4 g^3 \\
& e + 6144 a b^3 c^5 g^3 e - 12288 a^2 b c^6 g^3 e + 1344 b^6 c^3 g^2 h e - \\
& 11520 a b^4 c^4 g^2 h e + 27648 a^2 b^2 c^5 g^2 h e - 12288 a^3 c^6 g^2 h e \\
& - 864 b^7 c^2 g^2 h e + 8064 a b^5 c^3 g^2 h e - 23040 a^2 b^3 c^4 g^2 h e \\
& + 18432 a^3 b c^5 g^2 h e + 198 b^8 c h^3 e - 2016 a b^6 c^2 h^3 e + 6720 a \\
& ^2 b^4 c^3 h^3 e - 7680 a^3 b^2 c^4 h^3 e + 1536 a^4 c^5 h^3 e) * \log(\text{abs}(- \\
& * (\text{sqrt}(c) * x - \text{sqrt}(c * x^2 + b * x + a)) * \text{sqrt}(c) - b)) / c^{(15/2)}
\end{aligned}$$

**maple [B]** time = 0.03, size = 5881, normalized size = 5.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^3*(c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d), x)$

[Out] result too large to display



**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + hx)^3 (cx^2 + bx + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x)

[Out] int((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^3 (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*(c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d),x)

[Out] Integral((g + h\*x)\*\*3\*(a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2), x)

$$3.193 \quad \int (g + hx)^2 (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=753

$$(b^2 - 4ac)^2 \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right) (16c^2 (3a^2fh^2 + 12abh(eh + 2fg) + 14b^2 (dh^2 + 2egh + fg^2)) - 72b^2ch(3afh) - 72b^2ch(3afh)) - 72b^2ch(3afh)$$


---


$$32768c^{13/2}$$

**Rubi [A]** time = 2.10, antiderivative size = 749, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 621, 206}

Antiderivative was successfully verified.

[In] Int[(g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out]  $-(b^2 - 4ac) \cdot (768c^4d^2g^2 + 99b^4f^2h^2 - 72b^2c^2h(4b^2fg + 2b^2eh + 3a^2fh) - 128c^3(a^2fg^2 + ah(2eh + d) + 3b^2g(eh + 2d))) \cdot (b + 2cx) \cdot \sqrt{a + bx + cx^2} / (16384c^6) + ((768c^4d^2g^2 + 99b^4f^2h^2 - 72b^2c^2h(4b^2fg + 2b^2eh + 3a^2fh) - 128c^3(a^2fg^2 + ah(2eh + d) + 3b^2g(eh + 2d))) \cdot (b + 2cx) \cdot (a + bx + cx^2)^{3/2} / (6144c^5) - ((10c^2fg - 16ceh + 11bfh) \cdot (g + hx)^2 \cdot (a + bx + cx^2)^{5/2} / (112c^2h) + (f \cdot (g + hx)^3 \cdot (a + bx + cx^2)^{5/2} / (8ch) - ((693b^3f^2h^3 + 96c^3(5f^2g^3 - 8gh(eh + 7d))) - 36b^2c^2h(31afh + 28b(2fg + eh)) + 8c^2h(31b^2fg^2 + 196b^2h(2eh + d) + 96ah(2fg + eh)) - 10ch(99b^2f^2h^2 - 8c^2(5f^2g^2 - 4h(2eh + 7d))) - 12ch(7afh + 2b(fg + 6eh))) \cdot x) \cdot (a + bx + cx^2)^{5/2} / (13440c^4h) + ((b^2 - 4ac)^2 \cdot (768c^4d^2g^2 + 99b^4f^2h^2 - 72b^2c^2h(4b^2fg + 2b^2eh + 3a^2fh) - 128c^3(a^2fg^2 + ah(2eh + d) + 3b^2g(eh + 2d))) + 16c^2 \cdot (3a^2f^2h^2 + 12ab^2h(2fg + eh) + 14b^2(fg^2 + h(2eh + d)))) \cdot \text{ArcTanh}[(b + 2cx) / (2\sqrt{c} \cdot \sqrt{a + bx + cx^2})]) / (32768c^{13/2})$

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

### Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int (g + hx)^2 (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{f(g + hx)^3 (a + bx + cx^2)^{5/2}}{8ch} + \frac{\int (g + hx)^2 \left(-\frac{1}{2}h(5bfg - 16cdh)\right)}{8ch} \\
&= -\frac{(10cfg - 16ceh + 11bfh)(g + hx)^2 (a + bx + cx^2)^{5/2}}{112c^2h} + \frac{f(g + hx)^3 (a + bx + cx^2)^{5/2}}{8ch} \\
&= -\frac{(10cfg - 16ceh + 11bfh)(g + hx)^2 (a + bx + cx^2)^{5/2}}{112c^2h} + \frac{f(g + hx)^3 (a + bx + cx^2)^{5/2}}{8ch} \\
&= \frac{(768c^4dg^2 + 99b^4fh^2 - 72b^2ch(4bfg + 2beh + 3afh) - 128c^3(afg + beh + 3afh))}{112c^2h} \\
&= -\frac{(b^2 - 4ac)(768c^4dg^2 + 99b^4fh^2 - 72b^2ch(4bfg + 2beh + 3afh) - 128c^3(afg + beh + 3afh))}{112c^2h} \\
&= -\frac{(b^2 - 4ac)(768c^4dg^2 + 99b^4fh^2 - 72b^2ch(4bfg + 2beh + 3afh) - 128c^3(afg + beh + 3afh))}{112c^2h} \\
&= -\frac{(b^2 - 4ac)(768c^4dg^2 + 99b^4fh^2 - 72b^2ch(4bfg + 2beh + 3afh) - 128c^3(afg + beh + 3afh))}{112c^2h}
\end{aligned}$$

**Mathematica [A]** time = 1.60, size = 468, normalized size = 0.62

$$\frac{(-1/14*((11*b*f*h + 2*c*(5*f*g - 8*e*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + f*(g + h*x)^3*(a + x*(b + c*x))^(5/2) - ((a + x*(b + c*x))^(5/2)*(69*3*b^3*f*h^3 + 8*c^2*h*(b*f*g*(31*g + 30*h*x) + 4*b*h*(98*e*g + 49*d*h + 45*e*h*x) + 3*a*h*(64*f*g + 32*e*h + 35*f*h*x)) - 18*b*c*h^2*(62*a*f*h + b*(11*2*f*g + 56*e*h + 55*f*h*x)) + 16*c^3*(5*f*g^2*(6*g + 5*h*x) - 4*h*(2*e*g*(6*g + 5*h*x) + 7*d*h*(12*g + 5*h*x)))))/(1680*c^3) + (h*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])])/(12288*c^(11/2)))/(8*c*h)$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] (-1/14\*((11\*b\*f\*h + 2\*c\*(5\*f\*g - 8\*e\*h))\*(g + h\*x)^2\*(a + x\*(b + c\*x))^(5/2) + f\*(g + h\*x)^3\*(a + x\*(b + c\*x))^(5/2) - ((a + x\*(b + c\*x))^(5/2)\*(69\*3\*b^3\*f\*h^3 + 8\*c^2\*h\*(b\*f\*g\*(31\*g + 30\*h\*x) + 4\*b\*h\*(98\*e\*g + 49\*d\*h + 45\*e\*h\*x) + 3\*a\*h\*(64\*f\*g + 32\*e\*h + 35\*f\*h\*x)) - 18\*b\*c\*h^2\*(62\*a\*f\*h + b\*(11\*2\*f\*g + 56\*e\*h + 55\*f\*h\*x)) + 16\*c^3\*(5\*f\*g^2\*(6\*g + 5\*h\*x) - 4\*h\*(2\*e\*g\*(6\*g + 5\*h\*x) + 7\*d\*h\*(12\*g + 5\*h\*x)))))/(1680\*c^3) + (h\*(768\*c^4\*d\*g^2 + 99\*b^4\*f\*h^2 - 72\*b^2\*c\*h\*(4\*b\*f\*g + 2\*b\*e\*h + 3\*a\*f\*h) - 128\*c^3\*(a\*f\*g^2 + a\*h\*(2\*e\*g + d\*h) + 3\*b\*g\*(e\*g + 2\*d\*h)) + 16\*c^2\*(3\*a^2\*f\*h^2 + 12\*a\*b\*h\*(2\*f\*g + e\*h) + 14\*b^2\*(f\*g^2 + h\*(2\*e\*g + d\*h))))\*(2\*sqrt[c]\*(b + 2\*c\*x)\*sqrt[a + x\*(b + c\*x)]\*(-3\*b^2 + 8\*b\*c\*x + 4\*c\*(5\*a + 2\*c\*x^2)) + 3\*(b^2 - 4\*a\*c)^2\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)])])/(12288\*c^(11/2)))/(8\*c\*h)

IntegrateAlgebraic [B] time = 17.70, size = 1943, normalized size = 2.58

result too large to display

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(-80640\*b^3\*c^4\*d\*g^2 + 537600\*a\*b\*c^5\*d\*g^2 + 40320\*b^4\*c^3\*e\*g^2 - 268800\*a\*b^2\*c^4\*e\*g^2 + 344064\*a^2\*c^5\*e\*g^2 - 23520\*b^5\*c^2\*f\*g^2 + 170240\*a\*b^3\*c^3\*f\*g^2 - 290304\*a^2\*b\*c^4\*f\*g^2 + 80640\*b^4\*c^3\*d\*g\*h - 537600\*a\*b^2\*c^4\*d\*g\*h + 688128\*a^2\*c^5\*d\*g\*h - 47040\*b^5\*c^2\*e\*g\*h + 340480\*a\*b^3\*c^3\*e\*g\*h - 580608\*a^2\*b\*c^4\*e\*g\*h + 30240\*b^6\*c\*f\*g\*h - 241920\*a\*b^4\*c^2\*f\*g\*h + 526848\*a^2\*b^2\*c^3\*f\*g\*h - 196608\*a^3\*c^4\*f\*g\*h - 23520\*b^5\*c^2\*d\*h^2 + 170240\*a\*b^3\*c^3\*d\*h^2 - 290304\*a^2\*b\*c^4\*d\*h^2 + 15120\*b^6\*c\*e\*h^2 - 120960\*a\*b^4\*c^2\*e\*h^2 + 263424\*a^2\*b^2\*c^3\*e\*h^2 - 98304\*a^3\*c^4\*e\*h^2 - 10395\*b^7\*f\*h^2 + 91980\*a\*b^5\*c\*f\*h^2 - 244944\*a^2\*b^3\*c^2\*f\*h^2 + 176448\*a^3\*b\*c^3\*f\*h^2 + 53760\*b^2\*c^5\*d\*g^2\*x + 1075200\*a\*c^6\*d\*g^2\*x - 26880\*b^3\*c^4\*e\*g^2\*x + 150528\*a\*b\*c^5\*e\*g^2\*x + 15680\*b^4\*c^3\*f\*g^2\*x - 96768\*a\*b^2\*c^4\*f\*g^2\*x + 107520\*a^2\*c^5\*f\*g^2\*x - 53760\*b^3\*c^4\*d\*g\*h\*x + 301056\*a\*b\*c^5\*d\*g\*h\*x + 31360\*b^4\*c^3\*e\*g\*h\*x - 193536\*a\*b^2\*c^4\*e\*g\*h\*x + 215040\*a^2\*c^5\*e\*g\*h\*x - 20160\*b^5\*c^2\*f\*g\*h\*x + 139776\*a\*b^3\*c^3\*f\*g\*h\*x - 224256\*a^2\*b\*c^4\*f\*g\*h\*x + 15680\*b^4\*c^3\*d\*h^2\*x - 96768\*a\*b^2\*c^4\*d\*h^2\*x + 107520\*a^2\*c^5\*d\*h^2\*x - 10080\*b^5\*c^2\*e\*h^2\*x + 69888\*a\*b^3\*c^3\*e\*h^2\*x - 112128\*a^2\*b\*c^4\*e\*h^2\*x + 6930\*b^6\*c\*f\*h^2\*x - 53928\*a\*b^4\*c^2\*f\*h^2\*x + 113376\*a^2\*b^2\*c^3\*f\*h^2\*x - 40320\*a^3\*c^4\*f\*h^2\*x + 645120\*b\*c^6\*d\*g^2\*x^2 + 21504\*b^2\*c^5\*e\*g^2\*x^2 + 688128\*a\*c^6\*e\*g^2\*x^2 - 12544\*b^3\*c^4\*f\*g^2\*x^2 + 64512\*a\*b\*c^5\*f\*g^2\*x^2 + 43008\*b^2\*c^5\*d\*g\*h\*x^2 + 1376256\*a\*c^6\*d\*g\*h\*x^2 - 25088\*b^3\*c^4\*e\*g\*h\*x^2 + 129024\*a\*b\*c^5\*e\*g\*h\*x^2 + 16128\*b^4\*c^3\*f\*g\*h\*x^2 - 95232\*a\*b^2\*c^4\*f\*g\*h\*x^2 + 98304\*a^2\*c^5\*f\*g\*h\*x^2 - 12544\*b^3\*c^4\*d\*h^2\*x^2 + 64512\*a\*b\*c^5\*d\*h^2\*x^2 + 8064\*b^4\*c^3\*e\*h^2\*x^2 - 47616\*a\*b^2\*c^4\*e\*h^2\*x^2 + 49152\*a^2\*c^5\*e\*h^2\*x^2 - 5544\*b^5\*c^2\*f\*h^2\*x^2 + 37440\*a\*b^3\*c^3\*f\*h^2\*x^2 - 57984\*a^2\*b\*c^4\*f\*h^2\*x^2 + 430080\*c^7\*d\*g^2\*x^3 + 473088\*b\*c^6\*e\*g^2\*x^3 + 10752\*b^2\*c^5\*f\*g^2\*x^3 + 501760\*a\*c^6\*f\*g^2\*x^3 + 946176\*b\*c^6\*d\*g\*h\*x^3 + 21504\*b^2\*c^5\*e\*g\*h\*x^3 + 1003520\*a\*c^6\*e\*g\*h\*x^3 - 13824\*b^3\*c^4\*f\*g\*h\*x^3 + 67584\*a\*b\*c^5\*f\*g\*h\*x^3 + 10752\*b^2\*c^5\*d\*h^2\*x^3 + 501760\*a\*c^6\*d\*h^2\*x^3 - 6912\*b^3\*c^4\*e\*h^2\*x^3 + 33792\*a\*b\*c^5\*e\*h^2\*x^3 + 4752\*b^4\*c^3\*f\*h^2\*x^3 - 27264\*a\*b^2\*c^4\*f\*h^2\*x^3 + 26880\*a^2\*c^5\*f\*h^2\*x^3 + 344064\*c^7\*e\*g^2\*x^4 + 372736\*b\*c^6\*f\*g^2\*x^4 + 688128\*c^7\*d\*g\*h\*x^4 + 745472\*b\*c^6\*e\*g\*h\*x^4 + 12288\*b^2\*c^5\*f\*g\*h\*x^4 + 786432\*a\*c^6\*f\*g\*h\*x^4 + 372736\*b\*c^6\*d\*h^2\*x^4 + 6144\*b^2\*c^5\*e\*h^2\*x^4 + 393216\*a\*c^6\*e\*h^2\*x^4 - 4224\*b^3\*c^4\*f\*h^2\*x^4 + 19968\*a\*b\*c^5\*f\*h^2\*x^4 + 286720\*c^7\*f\*g^2\*x^5 + 573440\*c^7\*e\*g\*h\*x^5 + 614400\*b\*c^6\*f\*g\*h\*x^5 + 286720\*c^7\*d\*h^2\*x^5 + 307200\*b\*c^6\*e\*h^2\*x^5 + 3840\*b^2\*c^5\*f\*h^2\*x^5 + 322560\*a\*c^6\*f\*h^2\*x^5 + 491520\*c^7\*f\*g\*h\*x^6 + 245760\*c^7\*e\*h^2\*x^6 + 261120\*b\*c^6\*f\*h^2\*x^6 + 215040\*c^7\*f\*h^2\*x^7))/(1720320\*c^6) + ((-768\*b^4\*c^4\*d\*g^2 + 6144\*a\*

$$b^2c^5dg^2 - 12288a^2c^6d^2g^2 + 384b^5c^3eg^2 - 3072ab^3c^4eg^2 + 6144a^2b^2c^5efg^2 - 224b^6c^2f^2g^2 + 1920ab^4c^3f^2g^2 - 4608a^2b^2c^4f^2g^2 + 2048a^3c^5f^2g^2 + 768b^5c^3d^2gh - 6144ab^3c^4d^2gh + 12288a^2b^2c^5d^2gh - 448b^6c^2e^2gh + 3840ab^4c^3e^2gh - 9216a^2b^2c^4e^2gh + 4096a^3c^5e^2gh + 288b^7c^2f^2gh - 2688ab^5c^2f^2gh + 7680a^2b^3c^3f^2gh - 6144a^3b^2c^4f^2gh - 224b^6c^2d^2h^2 + 1920ab^4c^3d^2h^2 - 4608a^2b^2c^4d^2h^2 + 2048a^3c^5d^2h^2 + 144b^7c^2e^2h^2 - 1344ab^5c^2e^2h^2 + 3840a^2b^3c^3e^2h^2 - 3072a^3b^2c^4e^2h^2 - 99b^8f^2h^2 + 1008ab^6c^2f^2h^2 - 3360a^2b^4c^2f^2h^2 + 3840a^3b^2c^3f^2h^2 - 768a^4c^4f^2h^2) \cdot \text{Log}[b + 2cx - 2\sqrt{c}]\sqrt{a + bx + cx^2} / (32768c^{13/2})$$

**fricas [B]** time = 7.90, size = 3145, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="fricas")

[Out] [1/6881280\*(105\*(32\*(24\*(b^4c^4 - 8\*a\*b^2c^5 + 16\*a^2c^6)\*d - 12\*(b^5c^3 - 8\*a\*b^3c^4 + 16\*a^2b^2c^5)\*e + (7\*b^6c^2 - 60\*a\*b^4c^3 + 144\*a^2b^2c^4 - 64\*a^3c^5)\*f)\*g^2 - 32\*(24\*(b^5c^3 - 8\*a\*b^3c^4 + 16\*a^2b^2c^5)\*d - 2\*(7\*b^6c^2 - 60\*a\*b^4c^3 + 144\*a^2b^2c^4 - 64\*a^3c^5)\*e + 3\*(3\*b^7c - 28\*a\*b^5c^2 + 80\*a^2b^3c^3 - 64\*a^3b^2c^4)\*f)\*g\*h + (32\*(7\*b^6c^2 - 60\*a\*b^4c^3 + 144\*a^2b^2c^4 - 64\*a^3c^5)\*d - 48\*(3\*b^7c - 28\*a\*b^5c^2 + 80\*a^2b^3c^3 - 64\*a^3b^2c^4)\*e + 3\*(33\*b^8 - 336\*a\*b^6c + 1120\*a^2b^4c^2 - 1280\*a^3b^2c^3 + 256\*a^4c^4)\*f)\*h^2)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(215040\*c^8\*f\*h^2\*x^7 + 15360\*(32\*c^8\*f\*g\*h + (16\*c^8\*e + 17\*b\*c^7\*f)\*h^2)\*x^6 + 1280\*(224\*c^8\*f^2g^2 + 32\*(14\*c^8\*e + 15\*b\*c^7\*f)\*g\*h + (224\*c^8\*d + 240\*b\*c^7\*e + 3\*(b^2\*c^6 + 84\*a\*c^7)\*f)\*h^2)\*x^5 + 128\*(224\*(12\*c^8\*e + 13\*b\*c^7\*f)\*g^2 + 32\*(168\*c^8\*d + 182\*b\*c^7\*e + 3\*(b^2\*c^6 + 64\*a\*c^7)\*f)\*g\*h + (2912\*b\*c^7\*d + 48\*(b^2\*c^6 + 64\*a\*c^7)\*e - 3\*(11\*b^3c^5 - 52\*a\*b^2c^6)\*f)\*h^2)\*x^4 + 16\*(224\*(120\*c^8\*d + 132\*b\*c^7\*e + (3\*b^2c^6 + 140\*a\*c^7)\*f)\*g^2 + 32\*(1848\*b\*c^7\*d + 14\*(3\*b^2c^6 + 140\*a\*c^7)\*e - 3\*(9\*b^3c^5 - 44\*a\*b^2c^6)\*f)\*g\*h + (224\*(3\*b^2c^6 + 140\*a\*c^7)\*d - 48\*(9\*b^3c^5 - 44\*a\*b^2c^6)\*e + 3\*(99\*b^4c^4 - 568\*a\*b^2c^5 + 560\*a^2c^6)\*f)\*h^2)\*x^3 - 224\*(120\*(3\*b^3c^5 - 20\*a\*b^2c^6)\*d - 12\*(15\*b^4c^4 - 100\*a\*b^2c^5 + 128\*a^2c^6)\*e + (105\*b^5c^3 - 760\*a\*b^3c^4 + 1296\*a^2b^2c^5)\*f)\*g^2 + 32\*(168\*(15\*b^4c^4 - 100\*a\*b^2c^5 + 128\*a^2c^6)\*d - 14\*(105\*b^5c^3 - 760\*a\*b^3c^4 + 1296\*a^2b^2c^5)\*e + 3\*(315\*b^6c^2 - 2520\*a\*b^4c^3 + 5488\*a^2b^2c^4 - 2048\*a^3c^5)\*f)\*g\*h - (224\*(105\*b^5c^3 - 760\*a\*b^3c^4 + 1296\*a^2b^2c^5)\*d - 48\*(315\*b^6c^2 - 2520\*a\*b^4c^3 + 5488\*a^2b^2c^4 - 2048\*a^3c^5)\*e + 3\*(3465\*b^7c - 30660\*a\*b^5c^2 + 81648\*a^2b^3c^3 - 58816\*a^3b^2c^4)\*f)\*h^2 + 8\*(224\*(360\*b\*c^7\*d + 12\*(b^2c^6 + 32\*a\*c^7)\*e - (7\*b^3c^5 - 36\*a\*b^2c^6)\*

$$\begin{aligned}
& f) * g^2 + 32 * (168 * (b^2 * c^6 + 32 * a * c^7) * d - 14 * (7 * b^3 * c^5 - 36 * a * b * c^6) * e + 3 \\
& * (21 * b^4 * c^4 - 124 * a * b^2 * c^5 + 128 * a^2 * c^6) * f) * g * h - (224 * (7 * b^3 * c^5 - 36 * a \\
& * b * c^6) * d - 48 * (21 * b^4 * c^4 - 124 * a * b^2 * c^5 + 128 * a^2 * c^6) * e + 3 * (231 * b^5 * c^3 \\
& - 1560 * a * b^3 * c^4 + 2416 * a^2 * b * c^5) * f) * h^2) * x^2 + 2 * (224 * (120 * (b^2 * c^6 + 2 \\
& 0 * a * c^7) * d - 12 * (5 * b^3 * c^5 - 28 * a * b * c^6) * e + (35 * b^4 * c^4 - 216 * a * b^2 * c^5 + \\
& 240 * a^2 * c^6) * f) * g^2 - 32 * (168 * (5 * b^3 * c^5 - 28 * a * b * c^6) * d - 14 * (35 * b^4 * c^4 - \\
& 216 * a * b^2 * c^5 + 240 * a^2 * c^6) * e + 3 * (105 * b^5 * c^3 - 728 * a * b^3 * c^4 + 1168 * a^2 \\
& * b * c^5) * f) * g * h + (224 * (35 * b^4 * c^4 - 216 * a * b^2 * c^5 + 240 * a^2 * c^6) * d - 48 * (10 \\
& 5 * b^5 * c^3 - 728 * a * b^3 * c^4 + 1168 * a^2 * b * c^5) * e + 3 * (1155 * b^6 * c^2 - 8988 * a * b^4 \\
& * c^3 + 18896 * a^2 * b^2 * c^4 - 6720 * a^3 * c^5) * f) * h^2) * x) * \text{sqrt}(c * x^2 + b * x + a) \\
& / c^7, -1/3440640 * (105 * (32 * (24 * (b^4 * c^4 - 8 * a * b^2 * c^5 + 16 * a^2 * c^6) * d - 12 * ( \\
& b^5 * c^3 - 8 * a * b^3 * c^4 + 16 * a^2 * b * c^5) * e + (7 * b^6 * c^2 - 60 * a * b^4 * c^3 + 144 * a \\
& ^2 * b^2 * c^4 - 64 * a^3 * c^5) * f) * g^2 - 32 * (24 * (b^5 * c^3 - 8 * a * b^3 * c^4 + 16 * a^2 * b * \\
& c^5) * d - 2 * (7 * b^6 * c^2 - 60 * a * b^4 * c^3 + 144 * a^2 * b^2 * c^4 - 64 * a^3 * c^5) * e + 3 * \\
& (3 * b^7 * c - 28 * a * b^5 * c^2 + 80 * a^2 * b^3 * c^3 - 64 * a^3 * b * c^4) * f) * g * h + (32 * (7 * b^ \\
& 6 * c^2 - 60 * a * b^4 * c^3 + 144 * a^2 * b^2 * c^4 - 64 * a^3 * c^5) * d - 48 * (3 * b^7 * c - 28 * a \\
& * b^5 * c^2 + 80 * a^2 * b^3 * c^3 - 64 * a^3 * b * c^4) * e + 3 * (33 * b^8 - 336 * a * b^6 * c + 112 \\
& 0 * a^2 * b^4 * c^2 - 1280 * a^3 * b^2 * c^3 + 256 * a^4 * c^4) * f) * h^2) * \text{sqrt}(-c) * \arctan(1/2 \\
& * \text{sqrt}(c * x^2 + b * x + a) * (2 * c * x + b) * \text{sqrt}(-c) / (c^2 * x^2 + b * c * x + a * c)) - 2 * (2 \\
& 15040 * c^8 * f * h^2 * x^7 + 15360 * (32 * c^8 * f * g * h + (16 * c^8 * e + 17 * b * c^7 * f) * h^2) * x^ \\
& 6 + 1280 * (224 * c^8 * f * g^2 + 32 * (14 * c^8 * e + 15 * b * c^7 * f) * g * h + (224 * c^8 * d + 240 \\
& * b * c^7 * e + 3 * (b^2 * c^6 + 84 * a * c^7) * f) * h^2) * x^5 + 128 * (224 * (12 * c^8 * e + 13 * b * c \\
& ^7 * f) * g^2 + 32 * (168 * c^8 * d + 182 * b * c^7 * e + 3 * (b^2 * c^6 + 64 * a * c^7) * f) * g * h + ( \\
& 2912 * b * c^7 * d + 48 * (b^2 * c^6 + 64 * a * c^7) * e - 3 * (11 * b^3 * c^5 - 52 * a * b * c^6) * f) * h \\
& ^2) * x^4 + 16 * (224 * (120 * c^8 * d + 132 * b * c^7 * e + (3 * b^2 * c^6 + 140 * a * c^7) * f) * g^2 \\
& + 32 * (1848 * b * c^7 * d + 14 * (3 * b^2 * c^6 + 140 * a * c^7) * e - 3 * (9 * b^3 * c^5 - 44 * a * b * \\
& c^6) * f) * g * h + (224 * (3 * b^2 * c^6 + 140 * a * c^7) * d - 48 * (9 * b^3 * c^5 - 44 * a * b * c^6) * \\
& e + 3 * (99 * b^4 * c^4 - 568 * a * b^2 * c^5 + 560 * a^2 * c^6) * f) * h^2) * x^3 - 224 * (120 * (3 * \\
& b^3 * c^5 - 20 * a * b * c^6) * d - 12 * (15 * b^4 * c^4 - 100 * a * b^2 * c^5 + 128 * a^2 * c^6) * e + \\
& (105 * b^5 * c^3 - 760 * a * b^3 * c^4 + 1296 * a^2 * b * c^5) * f) * g^2 + 32 * (168 * (15 * b^4 * c^ \\
& 4 - 100 * a * b^2 * c^5 + 128 * a^2 * c^6) * d - 14 * (105 * b^5 * c^3 - 760 * a * b^3 * c^4 + 1296 \\
& * a^2 * b * c^5) * e + 3 * (315 * b^6 * c^2 - 2520 * a * b^4 * c^3 + 5488 * a^2 * b^2 * c^4 - 2048 * a \\
& ^3 * c^5) * f) * g * h - (224 * (105 * b^5 * c^3 - 760 * a * b^3 * c^4 + 1296 * a^2 * b * c^5) * d - 48 \\
& * (315 * b^6 * c^2 - 2520 * a * b^4 * c^3 + 5488 * a^2 * b^2 * c^4 - 2048 * a^3 * c^5) * e + 3 * (34 \\
& 65 * b^7 * c - 30660 * a * b^5 * c^2 + 81648 * a^2 * b^3 * c^3 - 58816 * a^3 * b * c^4) * f) * h^2 + \\
& 8 * (224 * (360 * b * c^7 * d + 12 * (b^2 * c^6 + 32 * a * c^7) * e - (7 * b^3 * c^5 - 36 * a * b * c^6) * \\
& f) * g^2 + 32 * (168 * (b^2 * c^6 + 32 * a * c^7) * d - 14 * (7 * b^3 * c^5 - 36 * a * b * c^6) * e + 3 \\
& * (21 * b^4 * c^4 - 124 * a * b^2 * c^5 + 128 * a^2 * c^6) * f) * g * h - (224 * (7 * b^3 * c^5 - 36 * a \\
& * b * c^6) * d - 48 * (21 * b^4 * c^4 - 124 * a * b^2 * c^5 + 128 * a^2 * c^6) * e + 3 * (231 * b^5 * c^3 \\
& - 1560 * a * b^3 * c^4 + 2416 * a^2 * b * c^5) * f) * h^2) * x^2 + 2 * (224 * (120 * (b^2 * c^6 + 2 \\
& 0 * a * c^7) * d - 12 * (5 * b^3 * c^5 - 28 * a * b * c^6) * e + (35 * b^4 * c^4 - 216 * a * b^2 * c^5 + \\
& 240 * a^2 * c^6) * f) * g^2 - 32 * (168 * (5 * b^3 * c^5 - 28 * a * b * c^6) * d - 14 * (35 * b^4 * c^4 - \\
& 216 * a * b^2 * c^5 + 240 * a^2 * c^6) * e + 3 * (105 * b^5 * c^3 - 728 * a * b^3 * c^4 + 1168 * a^2 \\
& * b * c^5) * f) * g * h + (224 * (35 * b^4 * c^4 - 216 * a * b^2 * c^5 + 240 * a^2 * c^6) * d - 48 * (10 \\
& 5 * b^5 * c^3 - 728 * a * b^3 * c^4 + 1168 * a^2 * b * c^5) * e + 3 * (1155 * b^6 * c^2 - 8988 * a * b^
\end{aligned}$$

$$\frac{4c^3 + 18896a^2b^2c^4 - 6720a^3c^5)f)h^2)x) \sqrt{cx^2 + bx + a}}{c^7}$$

**giac [B]** time = 0.37, size = 1852, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out]  $\frac{1}{1720320} \sqrt{cx^2 + bx + a} \left( \frac{2(4(2(8(10(12(14cfh^2x + (32c^8fgh + 17b^7f^2h^2 + 16c^8h^2e)/c^7)x + (224c^8f^2g^2 + 480b^7c^8fgh + 224c^8d^2h^2 + 3b^2c^6f^2h^2 + 252a^7c^8f^2h^2 + 448c^8g^2h^2e + 240b^7c^8h^2e)/c^7)x + (2912b^7c^8fg^2 + 5376c^8d^2gh + 96b^2c^6f^2gh + 6144a^7c^8fgh + 2912b^7c^8d^2h^2 - 33b^3c^5f^2h^2 + 156a^6b^7c^6f^2h^2 + 2688c^8g^2e + 5824b^7c^8g^2h^2e + 48b^2c^6h^2e + 3072a^7c^8h^2e)/c^7)x + (26880c^8d^2g^2 + 672b^2c^6f^2g^2 + 31360a^7c^8f^2g^2 + 59136b^7c^8d^2gh - 864b^3c^5f^2gh + 4224a^6b^7c^6f^2gh + 672b^2c^6d^2h^2 + 31360a^7c^8d^2h^2 + 297b^4c^4f^2h^2 - 1704a^6b^2c^5f^2h^2 + 1680a^2c^6f^2h^2 + 29568b^7c^8g^2e + 1344b^2c^6g^2h^2e + 62720a^7c^8g^2h^2e - 432b^3c^5h^2e + 2112a^6b^7c^6h^2e)/c^7)x + (80640b^7c^8d^2g^2 - 1568b^3c^5f^2g^2 + 8064a^6b^7c^6f^2g^2 + 5376b^2c^6d^2gh + 172032a^7c^8d^2gh + 2016b^4c^4f^2gh - 11904a^6b^2c^5f^2gh + 12288a^2c^6f^2gh - 1568b^3c^5d^2h^2 + 8064a^6b^7c^6d^2h^2 - 693b^5c^3f^2h^2 + 4680a^6b^3c^4f^2h^2 - 7248a^2b^7c^5f^2h^2 + 2688b^2c^6g^2e + 86016a^7c^8g^2e - 3136b^3c^5g^2h^2e + 16128a^6b^7c^6g^2h^2e + 1008b^4c^4h^2e - 5952a^6b^2c^5h^2e + 6144a^2c^6h^2e)/c^7)x + (26880b^2c^6d^2g^2 + 537600a^7c^8d^2g^2 + 7840b^4c^4f^2g^2 - 48384a^6b^2c^5f^2g^2 + 53760a^2c^6f^2g^2 - 26880b^3c^5d^2gh + 150528a^6b^7c^6d^2gh - 10080b^5c^3f^2gh + 69888a^6b^3c^4f^2gh - 112128a^2b^7c^5f^2gh + 7840b^4c^4d^2h^2 - 48384a^6b^2c^5d^2h^2 + 53760a^2c^6d^2h^2 + 3465b^6c^2f^2h^2 - 26964a^6b^4c^3f^2h^2 + 56688a^2b^2c^4f^2h^2 - 20160a^3c^5f^2h^2 - 13440b^3c^5g^2e + 75264a^6b^7c^6g^2e + 15680b^4c^4g^2h^2e - 96768a^6b^2c^5g^2h^2e + 107520a^2c^6g^2h^2e - 5040b^5c^3h^2e + 34944a^6b^3c^4h^2e - 56064a^2b^7c^5h^2e)/c^7)x - (80640b^3c^5d^2g^2 - 537600a^6b^7c^6d^2g^2 + 23520b^5c^3f^2g^2 - 170240a^6b^3c^4f^2g^2 + 290304a^2b^7c^5f^2g^2 - 80640b^4c^4d^2gh + 537600a^6b^2c^5d^2gh - 688128a^2c^6d^2gh - 30240b^6c^2f^2gh + 241920a^6b^4c^3f^2gh - 526848a^2b^2c^4f^2gh + 196608a^3c^5f^2gh + 23520b^5c^3d^2h^2 - 170240a^6b^3c^4d^2h^2 + 290304a^2b^7c^5d^2h^2 + 10395b^7c^3f^2h^2 - 91980a^6b^5c^2f^2h^2 + 244944a^2b^3c^3f^2h^2 - 176448a^3b^7c^4f^2h^2 - 40320b^4c^4g^2e + 268800a^6b^2c^5g^2e - 344064a^2c^6g^2e + 47040b^5c^3g^2h^2e - 340480a^6b^3c^4g^2h^2e + 580608a^2b^7c^5g^2h^2e - 15120b^6c^2h^2e + 120960a^6b^4c^3h^2e - 263424a^2b^2c^4h^2e + 98304a^3c^5h^2e)/c^7) - \frac{1}{32768} (768b^4c^4d^2g^2 - 6144a^6b^2c^5d^2g^2 + 12288a^2c^6d^2g^2 + 224b^6c^2f^2g^2 - 1920a^6b^4c^3f^2g^2$



$$\begin{aligned} &^2 + 4608a^2b^2c^4f^2g^2 - 2048a^3c^5f^2g^2 - 768b^5c^3d^2g^2h + 6144 \\ & *ab^3c^4d^2g^2h - 12288a^2b^3c^5d^2g^2h - 288b^7c^3f^2g^2h + 2688a^5b^2c^2 \\ & *f^2g^2h - 7680a^2b^3c^3f^2g^2h + 6144a^3b^2c^4f^2g^2h + 224b^6c^2d^2h^2 \\ & - 1920a^4b^3c^3d^2h^2 + 4608a^2b^2c^4d^2h^2 - 2048a^3c^5d^2h^2 + 99b^8 \\ & *f^2h^2 - 1008a^6b^6c^3f^2h^2 + 3360a^2b^4c^2f^2h^2 - 3840a^3b^2c^3f^2 \\ & *h^2 + 768a^4c^4f^2h^2 - 384b^5c^3g^2e + 3072a^3b^3c^4g^2e - 6144a^2 \\ & *b^2c^5g^2e + 448b^6c^2g^2h^2e - 3840a^4b^4c^3g^2h^2e + 9216a^2b^2c^4 \\ & *g^2h^2e - 4096a^3c^5g^2h^2e - 144b^7c^3h^2e + 1344a^5b^5c^2h^2e - 38 \\ & 40a^2b^3c^3h^2e + 3072a^3b^3c^4h^2e) * \log(\text{abs}(-2 * (\text{sqrt}(c) * x - \text{sqrt}(c \\ & * x^2 + b * x + a)) * \text{sqrt}(c) - b)) / c^{13/2} \end{aligned}$$

**maple [B]** time = 0.02, size = 3769, normalized size = 5.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^2*(c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d), x)$

[Out]  $\frac{1}{5}(c*x^2+b*x+a)^{(5/2)}/c*e*g^2+1/4*d*g^2*(c*x^2+b*x+a)^{(3/2)}*x+3/16/c^{(5/2)}$   
 $*b*a^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*f*g*h-15/128/c^{(7/2)}*b^4*$   
 $\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*g*h-1/12*a/c*(c*x^2+b*x+a)^{(3/2)}$   
 $*x*e*g*h-1/24*a/c^2*(c*x^2+b*x+a)^{(3/2)}*b*e*g*h-1/8*a^2/c*(c*x^2+b*x+a)^{(1/2)}$   
 $*x*e*g*h-7/30/c^2*b*(c*x^2+b*x+a)^{(5/2)}*e*g*h+3/20/c^3*b^2*(c*x^2+b*x+a)^{(5/2)}$   
 $*f*g*h-3/64/c^4*b^4*(c*x^2+b*x+a)^{(3/2)}*f*g*h+9/512/c^5*b^6*(c*x^2+b*x+a)^{(1/2)}$   
 $*f*g*h-15/128/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a^2*e*h^2+21/512/c^{(9/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a*e*h^2+3/32/c^{(5/2)}*b*a^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*e*h^2-3/28/c^2*b*x*(c*x^2+b*x+a)^{(5/2)}*e*h^2-3/64/c^3*b^3*(c*x^2+b*x+a)^{(3/2)}$   
 $*x*e*h^2-9/256*f*h^2/c^4*b^3*a*(c*x^2+b*x+a)^{(3/2)}-57/1024*f*h^2/c^4*b^3*a^2*$   
 $(c*x^2+b*x+a)^{(1/2)}-11/112*f*h^2/c^2*b*x^2*(c*x^2+b*x+a)^{(5/2)}+93/1120*f*h^2/c^3*$   
 $b*a*(c*x^2+b*x+a)^{(5/2)}+105/1024*f*h^2/c^{(9/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a^2-63/2048*f*h^2/c^{(11/2)}*b^6*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a+3/128/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*e*g^2-3/256/c^{(7/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*e*g^2+1/8*d*g^2/c*(c*x^2+b*x+a)^{(3/2)}*b+3/8*d*g^2*(c*x^2+b*x+a)^{(1/2)}*x*a-3/64*d*g^2/c^2*$   
 $(c*x^2+b*x+a)^{(1/2)}*b^3+3/8*d*g^2/c^{(1/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a^2+7/96/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}*x*d*h^2+7/96/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}$   
 $*x*f*g^2+7/96/c^3*b^3*(c*x^2+b*x+a)^{(3/2)}*e*g*h-7/256/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}$   
 $*x*d*h^2-9/128*f*h^2/c^3*b^2*a*(c*x^2+b*x+a)^{(3/2)}*x-57/512*f*h^2/c^3*b^2*a^2*$   
 $(c*x^2+b*x+a)^{(1/2)}*x-1/16/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}*e*g^2+2/5*(c*x^2+b*x+a)^{(5/2)}/c*d*g^2-3/32/c^3*b^3*$   
 $(c*x^2+b*x+a)^{(1/2)}*x*a*e*h^2-15/64/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a^2*f*g*h+21/256/c^{(9/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})$   
 $*a*f*g*h+1/16/c^2*b*a*(c*x^2+b*x+a)^{(3/2)}*x*e*h^2+1/16/c^3*b^2*a*(c*x^2+b*x+a)^{(3/2)}$   
 $*f*g*h+3/32/c^3*b^2*a^2*(c*x^2+b*x+a)^{(1/2)}*f*g*h+3/32/c^2*b*a^2*(c*x^2$

$$\begin{aligned}
& +b*x+a)^{(1/2)}*x*e*h^2+9/32/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a) \\
& ^{(1/2)})*a^2*e*g*h+1/8/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*a*e*g*h-1/4/c*b*(c*x^2+b* \\
& x+a)^{(3/2)}*x*d*g*h-3/16/c*b*(c*x^2+b*x+a)^{(1/2)}*x*a*e*g^2+3/32/c^2*b^3*(c*x \\
& ^2+b*x+a)^{(1/2)}*x*d*g*h-3/16/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*a*d*g*h-3/8/c^{(3/2)} \\
& )*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*d*g*h+3/16/c^{(5/2)}*b^3* \\
& \ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*d*g*h+9/256/c^4*b^5*(c*x^2+b* \\
& x+a)^{(1/2)}*x*f*g*h-3/14/c^2*b*x*(c*x^2+b*x+a)^{(5/2)}*f*g*h-3/32/c^3*b^3*(c*x \\
& ^2+b*x+a)^{(3/2)}*x*f*g*h-3/32/c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*a*f*g*h-3/32/c^2*b \\
& ^2*(c*x^2+b*x+a)^{(1/2)}*a*e*g^2+3/64/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*d*g*h-3/16/ \\
& c^{(3/2)}*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*e*g^2+3/32/c^{(5/2)} \\
& )*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e*g^2-3/128/c^{(7/2)}*b^5 \\
& *\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*g*h-3/32*d*g^2/c*(c*x^2+b*x+ \\
& a)^{(1/2)}*x*b^2+2/7*x^2*(c*x^2+b*x+a)^{(5/2)}/c*f*g*h-4/35*a/c^2*(c*x^2+b*x+a) \\
& ^{(5/2)}*f*g*h-15/128*f*h^2/c^{(7/2)}*b^2*a^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x \\
& +a)^{(1/2)})+1/128*f*h^2*a^2/c^3*(c*x^2+b*x+a)^{(3/2)}*b+3/128*f*h^2*a^3/c^2*(c \\
& *x^2+b*x+a)^{(1/2)}*x+3/256*f*h^2*a^3/c^3*(c*x^2+b*x+a)^{(1/2)}*b-1/16*f*h^2*a/ \\
& c^2*x*(c*x^2+b*x+a)^{(5/2)}+1/64*f*h^2*a^2/c^2*(c*x^2+b*x+a)^{(3/2)}*x+33/448*f \\
& *h^2/c^3*b^2*x*(c*x^2+b*x+a)^{(5/2)}+33/1024*f*h^2/c^4*b^4*(c*x^2+b*x+a)^{(3/2)} \\
& )*x-99/8192*f*h^2/c^5*b^6*(c*x^2+b*x+a)^{(1/2)}*x-1/8/c*b*(c*x^2+b*x+a)^{(3/2)} \\
& *x*e*g^2-1/8/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}*d*g*h+3/64/c^2*b^3*(c*x^2+b*x+a)^{( \\
& 1/2)}*x*e*g^2+9/512/c^4*b^5*(c*x^2+b*x+a)^{(1/2)}*x*e*h^2-3/64/c^4*b^4*(c*x^2+ \\
& b*x+a)^{(1/2)}*a*e*h^2-9/1024/c^{(11/2)}*b^7*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+ \\
& a)^{(1/2)})*f*g*h+1/32/c^3*b^2*a*(c*x^2+b*x+a)^{(3/2)}*e*h^2+3/64/c^3*b^2*a^2*( \\
& c*x^2+b*x+a)^{(1/2)}*e*h^2-15/256/c^{(7/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b \\
& *x+a)^{(1/2)})*a*d*h^2-15/256/c^{(7/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a) \\
& )^{(1/2)})*a*f*g^2+7/512/c^{(9/2)}*b^6*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/ \\
& 2)})*e*g*h-1/24*a/c*(c*x^2+b*x+a)^{(3/2)}*x*d*h^2-1/24*a/c*(c*x^2+b*x+a)^{(3/2)} \\
& *x*f*g^2-1/48*a/c^2*(c*x^2+b*x+a)^{(3/2)}*b*d*h^2-1/48*a/c^2*(c*x^2+b*x+a)^{(3 \\
& /2)}*b*f*g^2-1/16*a^2/c*(c*x^2+b*x+a)^{(1/2)}*x*d*h^2-1/16*a^2/c*(c*x^2+b*x+a) \\
& ^{(1/2)}*x*f*g^2-1/32*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b*d*h^2-1/32*a^2/c^2*(c*x^2 \\
& +b*x+a)^{(1/2)}*b*f*g^2-1/8*a^3/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{( \\
& 1/2)})*e*g*h+1/3*x*(c*x^2+b*x+a)^{(5/2)}/c*e*g*h+3/16*d*g^2/c*(c*x^2+b*x+a)^{( \\
& 1/2)}*b*a-3/16*d*g^2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*b^2 \\
& *a-1/16*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b*e*g*h+7/48/c^2*b^2*(c*x^2+b*x+a)^{(3/2)} \\
& )*x*e*g*h+1/8/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x*a*d*h^2+1/8/c^2*b^2*(c*x^2+b*x+ \\
& a)^{(1/2)}*x*a*f*g^2-7/128/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*x*e*g*h+153/2048*f*h^2 \\
& /c^4*b^4*(c*x^2+b*x+a)^{(1/2)}*x*a-7/256/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*x*f*g^2+ \\
& 1/16/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*a*d*h^2+1/16/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*a \\
& *f*g^2-7/256/c^4*b^5*(c*x^2+b*x+a)^{(1/2)}*e*g*h+9/64/c^{(5/2)}*b^2*\ln((c*x+1/2 \\
& *b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*d*h^2+9/64/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/ \\
& c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*f*g^2+153/4096*f*h^2/c^5*b^5*(c*x^2+b*x+a) \\
& ^{(1/2)}*a+33/2048*f*h^2/c^5*b^5*(c*x^2+b*x+a)^{(3/2)}-99/16384*f*h^2/c^6*b^7*( \\
& c*x^2+b*x+a)^{(1/2)}+3/128*d*g^2/c^{(5/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a) \\
& ^{(1/2)})*b^4-9/2048/c^{(11/2)}*b^7*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\
& *e*h^2+1/7*x^2*(c*x^2+b*x+a)^{(5/2)}/c*e*h^2-2/35*a/c^2*(c*x^2+b*x+a)^{(5/2)}*e
\end{aligned}$$

```

*h^2+3/40/c^3*b^2*(c*x^2+b*x+a)^(5/2)*e*h^2-3/128/c^4*b^4*(c*x^2+b*x+a)^(3/
2)*e*h^2+9/1024/c^5*b^6*(c*x^2+b*x+a)^(1/2)*e*h^2+1/6*x*(c*x^2+b*x+a)^(5/2)
/c*d*h^2+1/6*x*(c*x^2+b*x+a)^(5/2)/c*f*g^2-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*d
*h^2-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*f*g^2+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)
*d*h^2+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)*f*g^2-7/512/c^4*b^5*(c*x^2+b*x+a)^(
1/2)*d*h^2-7/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*f*g^2+7/1024/c^(9/2)*b^6*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+7/1024/c^(9/2)*b^6*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*f*g^2+3/128*f*h^2*a^4/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+99/32768*f*h^2/c^(13/2)*b^8*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+1/8*f*h^2*x^3*(c*x^2+b*x+a)^(5/2)/c-33/640*f*h^2/c^4*b^3*(c*x
^2+b*x+a)^(5/2)-3/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*x*a*f*g*h+1/8/c^2*b*a*(c*x
^2+b*x+a)^(3/2)*x*f*g*h+3/16/c^2*b*a^2*(c*x^2+b*x+a)^(1/2)*x*f*g*h+1/4/c^2*
b^2*(c*x^2+b*x+a)^(1/2)*x*a*e*g*h-3/8/c*b*(c*x^2+b*x+a)^(1/2)*x*a*d*g*h

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + hx)^2 (cx^2 + bx + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^2 (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**2*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

$$3.194 \quad \int (g + hx) (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

Optimal. Leaf size=418

$$\frac{(b + 2cx) (a + bx + cx^2)^{3/2} (-8c^2(aeh + afg + 3bdh + 3beg) + 2bc(6afh + 7b(eh + fg)) - 9b^3fh + 48c^3dg)}{384c^4} + \dots$$

**Rubi [A]** time = 0.65, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1653, 779, 612, 621, 206}

$$\frac{(a + bx + cx^2)^{3/2} (-2b(2bx + a) + 48c^2) + \dots}{384c^4} + \dots$$

Antiderivative was successfully verified.

[In] Int[(g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] -((b^2 - 4\*a\*c)\*(48\*c^3\*d\*g - 9\*b^3\*f\*h - 8\*c^2\*(3\*b\*e\*g + a\*f\*g + 3\*b\*d\*h + a\*e\*h) + 2\*b\*c\*(6\*a\*f\*h + 7\*b\*(f\*g + e\*h)))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(1024\*c^5) + ((48\*c^3\*d\*g - 9\*b^3\*f\*h - 8\*c^2\*(3\*b\*e\*g + a\*f\*g + 3\*b\*d\*h + a\*e\*h) + 2\*b\*c\*(6\*a\*f\*h + 7\*b\*(f\*g + e\*h)))\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(384\*c^4) + (f\*(g + h\*x)^2\*(a + b\*x + c\*x^2)^(5/2))/(7\*c\*h) + ((63\*b^2\*f\*h^2 - 24\*c^2\*(5\*f\*g^2 - 7\*h\*(e\*g + d\*h)) - 2\*c\*h\*(24\*a\*f\*h + 49\*b\*(f\*g + e\*h)) - 10\*c\*h\*(10\*c\*f\*g - 14\*c\*e\*h + 9\*b\*f\*h)\*x)\*(a + b\*x + c\*x^2)^(5/2))/(840\*c^3\*h) + ((b^2 - 4\*a\*c)^2\*(48\*c^3\*d\*g - 9\*b^3\*f\*h - 8\*c^2\*(3\*b\*e\*g + a\*f\*g + 3\*b\*d\*h + a\*e\*h) + 2\*b\*c\*(6\*a\*f\*h + 7\*b\*(f\*g + e\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2048\*c^(11/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a,

b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int (g + hx) (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{f(g + hx)^2 (a + bx + cx^2)^{5/2}}{7ch} + \frac{\int (g + hx) \left(-\frac{1}{2}h(5bfg - 14cdh\right.}{7ch} \\
&= \frac{f(g + hx)^2 (a + bx + cx^2)^{5/2}}{7ch} + \frac{(63b^2fh^2 - 24c^2(5fg^2 - 7h(eg}{7ch} \\
&= \frac{(48c^3dg - 9b^3fh - 8c^2(3beg + afg + 3bdh + aeh) + 2bc(6afh}{384c^4} \\
&= -\frac{(b^2 - 4ac)(48c^3dg - 9b^3fh - 8c^2(3beg + afg + 3bdh + aeh)}{1024c^5} \\
&= -\frac{(b^2 - 4ac)(48c^3dg - 9b^3fh - 8c^2(3beg + afg + 3bdh + aeh)}{1024c^5} \\
&= -\frac{(b^2 - 4ac)(48c^3dg - 9b^3fh - 8c^2(3beg + afg + 3bdh + aeh)}{1024c^5}
\end{aligned}$$

**Mathematica [A]** time = 0.78, size = 285, normalized size = 0.68

$$\frac{(a+x(b+cx))^{5/2}(-2c(24af/b+349c^2/49fg+45f/hx)+63b^2f/h^2-4c^2(5fg(6g+5hx)-7h(6dh+6eg+5ahx))}{120c^2} - \frac{7(2\sqrt{c(b+2cx)}\sqrt{a+x(b+cx)}(4c(5a+2cx^2)-3b^2+8bcx)+3(b^2-4ac)^2 \operatorname{tanh}^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right))(8c^2(ah^2+afg+3bdh+3bcg)-2c(6af/h+7b(gh+f)))+9b^2fh-48c^2dg}{6144c^{9/2}} + f(g+hx)^2(a+x(b+cx))^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

[Out] (f\*(g + h\*x)^2\*(a + x\*(b + c\*x))^(5/2) + ((a + x\*(b + c\*x))^(5/2)\*(63\*b^2\*f\*h^2 - 4\*c^2\*(5\*f\*g\*(6\*g + 5\*h\*x) - 7\*h\*(6\*e\*g + 6\*d\*h + 5\*e\*h\*x)) - 2\*c\*h\*(24\*a\*f\*h + b\*(49\*f\*g + 49\*e\*h + 45\*f\*h\*x)))/(120\*c^2) - (7\*h\*(-48\*c^3\*d\*g + 9\*b^3\*f\*h + 8\*c^2\*(3\*b\*e\*g + a\*f\*g + 3\*b\*d\*h + a\*e\*h) - 2\*b\*c\*(6\*a\*f\*h + 7\*b\*(f\*g + e\*h)))\*(2\*sqrt[c]\*(b + 2\*c\*x)\*sqrt[a + x\*(b + c\*x)]\*(-3\*b^2 + 8\*b\*c\*x + 4\*c\*(5\*a + 2\*c\*x^2)) + 3\*(b^2 - 4\*a\*c)^2\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)])]))/(6144\*c^(9/2)))/(7\*c\*h)

**IntegrateAlgebraic [B]** time = 6.20, size = 984, normalized size = 2.35

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x]

```
[Out] (Sqrt[a + b*x + c*x^2]*(-5040*b^3*c^3*d*g + 33600*a*b*c^4*d*g + 2520*b^4*c^2*e*g - 16800*a*b^2*c^3*e*g + 21504*a^2*c^4*e*g - 1470*b^5*c*f*g + 10640*a*b^3*c^2*f*g - 18144*a^2*b*c^3*f*g + 2520*b^4*c^2*d*h - 16800*a*b^2*c^3*d*h + 21504*a^2*c^4*d*h - 1470*b^5*c*e*h + 10640*a*b^3*c^2*e*h - 18144*a^2*b*c^3*e*h + 945*b^6*f*h - 7560*a*b^4*c*f*h + 16464*a^2*b^2*c^2*f*h - 6144*a^3*c^3*f*h + 3360*b^2*c^4*d*g*x + 67200*a*c^5*d*g*x - 1680*b^3*c^3*e*g*x + 9408*a*b*c^4*e*g*x + 980*b^4*c^2*f*g*x - 6048*a*b^2*c^3*f*g*x + 6720*a^2*c^4*f*g*x - 1680*b^3*c^3*d*h*x + 9408*a*b*c^4*d*h*x + 980*b^4*c^2*e*h*x - 6048*a*b^2*c^3*e*h*x + 6720*a^2*c^4*e*h*x - 630*b^5*c*f*h*x + 4368*a*b^3*c^2*f*h*x - 7008*a^2*b*c^3*f*h*x + 40320*b*c^5*d*g*x^2 + 1344*b^2*c^4*e*g*x^2 + 43008*a*c^5*e*g*x^2 - 784*b^3*c^3*f*g*x^2 + 4032*a*b*c^4*f*g*x^2 + 1344*b^2*c^4*d*h*x^2 + 43008*a*c^5*d*h*x^2 - 784*b^3*c^3*e*h*x^2 + 4032*a*b*c^4*e*h*x^2 + 504*b^4*c^2*f*h*x^2 - 2976*a*b^2*c^3*f*h*x^2 + 3072*a^2*c^4*f*h*x^2 + 26880*c^6*d*g*x^3 + 29568*b*c^5*e*g*x^3 + 672*b^2*c^4*f*g*x^3 + 31360*a*c^5*f*g*x^3 + 29568*b*c^5*d*h*x^3 + 672*b^2*c^4*e*h*x^3 + 31360*a*c^5*e*h*x^3 - 432*b^3*c^3*f*h*x^3 + 2112*a*b*c^4*f*h*x^3 + 21504*c^6*e*g*x^4 + 23296*b*c^5*f*g*x^4 + 21504*c^6*d*h*x^4 + 23296*b*c^5*e*h*x^4 + 384*b^2*c^4*f*h*x^4 + 24576*a*c^5*f*h*x^4 + 17920*c^6*f*g*x^5 + 17920*c^6*e*h*x^5 + 19200*b*c^5*f*h*x^5 + 15360*c^6*f*h*x^6))/(107520*c^5) + ((-48*b^4*c^3*d*g + 384*a*b^2*c^4*d*g - 768*a^2*c^5*d*g + 24*b^5*c^2*e*g - 192*a*b^3*c^3*e*g + 384*a^2*b*c^4*e*g - 14*b^6*c*f*g + 120*a*b^4*c^2*f*g - 288*a^2*b^2*c^3*f*g + 128*a^3*c^4*f*g + 24*b^5*c^2*d*h - 192*a*b^3*c^3*d*h + 384*a^2*b*c^4*d*h - 14*b^6*c*e*h + 120*a*b^4*c^2*e*h - 288*a^2*b^2*c^3*e*h + 128*a^3*c^4*e*h + 9*b^7*f*h - 84*a*b^5*c*f*h + 240*a^2*b^3*c^2*f*h - 192*a^3*b*c^3*f*h)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(2048*c^(11/2))
```

**fricas [B]** time = 1.91, size = 1833, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g + (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 36*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^
```



$$\begin{aligned}
& 3c^4 - 20ab^2c^5)d - 12(15b^4c^3 - 100ab^2c^4 + 128a^2c^5)e + (105b^5c^2 - 760ab^3c^3 + 1296a^2b^2c^4)fg + (168(15b^4c^3 - 100ab^2c^4 + 128a^2c^5)d - 14(105b^5c^2 - 760ab^3c^3 + 1296a^2b^2c^4)fg + 3(315b^6c - 2520ab^4c^2 + 5488a^2b^2c^3 - 2048a^3c^4)fh) * h + 2(14(120(b^2c^5 + 20a^2c^6)d - 12(5b^3c^4 - 28ab^2c^5)e + (35b^4c^3 - 216ab^2c^4 + 240a^2c^5)fg) * g - (168(5b^3c^4 - 28ab^2c^5)d - 14(35b^4c^3 - 216ab^2c^4 + 240a^2c^5)fg) * h + 3(105b^5c^2 - 728ab^3c^3 + 1168a^2b^2c^4)fh) * x) * \sqrt{cx^2 + bx + a} / c^6, -1/215 \\
& 040(105(2(24(b^4c^3 - 8ab^2c^4 + 16a^2c^5)d - 12(b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4)fg) * g + (7b^6c - 60ab^4c^2 + 144a^2b^2c^3 - 64a^3c^4)fh) * h - (24(b^5c^2 - 8ab^3c^3 + 16a^2b^2c^4)d - 2(7b^6c - 60ab^4c^2 + 144a^2b^2c^3 - 64a^3c^4)fg) * h + 3(3b^7 - 28ab^5c + 80a^2b^3c^2 - 64a^3b^2c^3)fh) * x) * \sqrt{-c} * \arctan(1/2\sqrt{cx^2 + bx + a}) * (2cx + b) * \sqrt{-c} / (c^2x^2 + bcx + ac) - 2(15360c^7fghx^6 + 1280(14c^7fg + (14c^7e + 15b^2c^6f)h) * x^5 + 128(14(12c^7e + 13b^2c^6f) * g + (168c^7d + 182b^2c^6e + 3(b^2c^5 + 64a^2c^6)fh) * h) * x^4 + 16(14(120c^7d + 132b^2c^6e + (3b^2c^5 + 140a^2c^6)fg) * g + (1848b^2c^6d + 14(3b^2c^5 + 140a^2c^6)e - 3(9b^3c^4 - 44ab^2c^5)fh) * h) * x^3 + 8(14(360b^2c^6d + 12(b^2c^5 + 32a^2c^6)e - (7b^3c^4 - 36ab^2c^5)fg) * g + (168(b^2c^5 + 32a^2c^6)d - 14(7b^3c^4 - 36ab^2c^5)e + 3(21b^4c^3 - 124ab^2c^4 + 128a^2c^5)fh) * h) * x^2 - 14(120(3b^3c^4 - 20ab^2c^5)d - 12(15b^4c^3 - 100ab^2c^4 + 128a^2c^5)fg) * g + (105b^5c^2 - 760ab^3c^3 + 1296a^2b^2c^4)fh) * h) * x) * \sqrt{cx^2 + bx + a} / c^6]
\end{aligned}$$

**giac [B]** time = 0.29, size = 955, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="giac")

[Out] 1/107520\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*(2\*(8\*(10\*(12\*c\*f\*h\*x + (14\*c^7\*f\*g + 15\*b\*c^6\*f\*h + 14\*c^7\*h\*e)/c^6)\*x + (182\*b\*c^6\*f\*g + 168\*c^7\*d\*h + 3\*b^2\*c^5\*f\*h + 192\*a\*c^6\*f\*h + 168\*c^7\*g\*e + 182\*b\*c^6\*h\*e)/c^6)\*x + (1680\*c^7\*d\*g + 42\*b^2\*c^5\*f\*g + 1960\*a\*c^6\*f\*g + 1848\*b\*c^6\*d\*h - 27\*b^3\*c^4\*f\*h + 132\*a\*b\*c^5\*f\*h + 1848\*b\*c^6\*g\*e + 42\*b^2\*c^5\*h\*e + 1960\*a\*c^6\*h\*e)/c^6)\*x + (5040\*b\*c^6\*d\*g - 98\*b^3\*c^4\*f\*g + 504\*a\*b\*c^5\*f\*g + 168\*b^2\*c^5\*d\*h + 5376\*a\*c^6\*d\*h + 63\*b^4\*c^3\*f\*h - 372\*a\*b^2\*c^4\*f\*h + 384\*a^2\*c^5\*f\*h + 168\*b^2\*c^5\*g\*e + 5376\*a\*c^6\*g\*e - 98\*b^3\*c^4\*h\*e + 504\*a\*b\*c^5\*h\*e)/c^6)\*x + (1680\*

$$\begin{aligned}
& b^2c^5d^2g + 33600a^2c^6d^2g + 490b^4c^3f^2g - 3024a^2b^2c^4f^2g + 3360 \\
& a^2c^5f^2g - 840b^3c^4d^2h + 4704a^2b^2c^5d^2h - 315b^5c^2f^2h + 2184a \\
& a^2b^3c^3f^2h - 3504a^2b^2c^4f^2h - 840b^3c^4g^2e + 4704a^2b^2c^5g^2e + 4 \\
& 90b^4c^3h^2e - 3024a^2b^2c^4h^2e + 3360a^2c^5h^2e)/c^6) * x - (5040b^3c \\
& c^4d^2g - 33600a^2b^2c^5d^2g + 1470b^5c^2f^2g - 10640a^2b^3c^3f^2g + 1814 \\
& 4a^2b^2c^4f^2g - 2520b^4c^3d^2h + 16800a^2b^2c^4d^2h - 21504a^2c^5d^2h \\
& h - 945b^6c^2f^2h + 7560a^2b^4c^2f^2h - 16464a^2b^2c^3f^2h + 6144a^3c \\
& ^4f^2h - 2520b^4c^3g^2e + 16800a^2b^2c^4g^2e - 21504a^2c^5g^2e + 1470 \\
& b^5c^2h^2e - 10640a^2b^3c^3h^2e + 18144a^2b^2c^4h^2e)/c^6) - 1/2048*(48 \\
& b^4c^3d^2g - 384a^2b^2c^4d^2g + 768a^2c^5d^2g + 14b^6c^2f^2g - 120a^2b^4 \\
& c^2f^2g + 288a^2b^2c^3f^2g - 128a^3c^4f^2g - 24b^5c^2d^2h + 192a^2 \\
& b^3c^3d^2h - 384a^2b^2c^4d^2h - 9b^7f^2h + 84a^2b^5c^2f^2h - 240a^2b^3c \\
& ^2f^2h + 192a^3b^2c^3f^2h - 24b^5c^2g^2e + 192a^2b^3c^3g^2e - 384a^2b \\
& b^2c^4g^2e + 14b^6c^2h^2e - 120a^2b^4c^2h^2e + 288a^2b^2c^3h^2e - 128a^3 \\
& c^4h^2e) * \log(\text{abs}(-2 * (\text{sqrt}(c) * x - \text{sqrt}(c * x^2 + b * x + a)) * \text{sqrt}(c) - b)) / c^{( \\
& 11/2)})
\end{aligned}$$

**maple [B]** time = 0.01, size = 2026, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)*(c*x^2+b*x+a)^{(3/2)}*(f*x^2+e*x+d), x)$

[Out]  $\begin{aligned}
& 1/4*d^2g*(c*x^2+b*x+a)^{(3/2)}*x+1/5*(c*x^2+b*x+a)^{(5/2)}/c*d^2h+1/5*(c*x^2+b*x+ \\
& a)^{(5/2)}/c*e^2g+9/1024*h^2f/c^5*b^6*(c*x^2+b*x+a)^{(1/2)}-3/32*h^2f/c^3*b^3*(c*x \\
& ^2+b*x+a)^{(1/2)}*x+a+3/32*h^2f/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x+1/16*h^2f/c^2*b \\
& *a*(c*x^2+b*x+a)^{(3/2)}*x-3/16/c*b*(c*x^2+b*x+a)^{(1/2)}*x*a*d^2h-3/16/c*b*(c*x \\
& ^2+b*x+a)^{(1/2)}*x*a*e^2g+1/8/c^2*b^2*(c*x^2+b*x+a)^{(1/2)}*x*a*f^2g+1/8/c^2*b^2 \\
& *(c*x^2+b*x+a)^{(1/2)}*x*a*e^2h-3/128*h^2f/c^4*b^4*(c*x^2+b*x+a)^{(3/2)}-15/128*h \\
& ^2f/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2-15/256/c^{(7/ \\
& 2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*f^2g-1/24*a/c*(c*x^2+b* \\
& x+a)^{(3/2)}*x*e^2h-1/48*a/c^2*(c*x^2+b*x+a)^{(3/2)}*b*f^2g-1/48*a/c^2*(c*x^2+b*x \\
& +a)^{(3/2)}*b*e^2h+1/6*x*(c*x^2+b*x+a)^{(5/2)}/c*e^2h+1/6*x*(c*x^2+b*x+a)^{(5/2)}/c \\
& *f^2g-1/16/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}*d^2h-1/16/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}* \\
& e^2g+3/128/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*d^2h+3/128/c^3*b^4*(c*x^2+b*x+a)^{(1/2)} \\
& *e^2g-3/256/c^{(7/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d^2h-7/25 \\
& 6/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*x*e^2h-7/256/c^3*b^4*(c*x^2+b*x+a)^{(1/2)}*x*f^2g \\
& +1/16/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*a*e^2h+1/16/c^3*b^3*(c*x^2+b*x+a)^{(1/2)}*a* \\
& f^2g+9/64/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*e^2h+9/ \\
& 64/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a^2*f^2g-15/256/c \\
& ^{(7/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*a*e^2h-1/16*a^2/c*(c* \\
& x^2+b*x+a)^{(1/2)}*x*e^2h-1/16*a^2/c*(c*x^2+b*x+a)^{(1/2)}*x*f^2g-1/32*a^2/c^2*(c \\
& *x^2+b*x+a)^{(1/2)}*b*e^2h-1/32*a^2/c^2*(c*x^2+b*x+a)^{(1/2)}*b*f^2g+7/96/c^2*b^2 \\
& *(c*x^2+b*x+a)^{(3/2)}*x*e^2h+7/96/c^2*b^2*(c*x^2+b*x+a)^{(3/2)}*x*f^2g-1/8/c*b*(
\end{aligned}$

```

c*x^2+b*x+a)^(3/2)*x*d*h-1/8/c*b*(c*x^2+b*x+a)^(3/2)*x*e*g+3/64/c^2*b^3*(c*
x^2+b*x+a)^(1/2)*x*d*h+3/64/c^2*b^3*(c*x^2+b*x+a)^(1/2)*x*e*g-3/32/c^2*b^2*
(c*x^2+b*x+a)^(1/2)*a*d*h-3/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*a*e*g-3/16/c^(3/
2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h-3/16/c^(3/2)*b*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*g+3/32/c^(5/2)*b^3*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h+3/32/c^(5/2)*b^3*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g-3/32*d*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*
d*g/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16*d*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*b^2*a-1/24*a/c*(c*x^2+b*x+a)^(3/2)*x*f*g+1/32*h*f/c^3*b^2*a
*(c*x^2+b*x+a)^(3/2)+3/64*h*f/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)-3/28*h*f/c^2*
b*x*(c*x^2+b*x+a)^(5/2)+21/512*h*f/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a+3/32*h*f/c^(5/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))-3/64*h*f/c^3*b^3*(c*x^2+b*x+a)^(3/2)*x+9/512*h*f/c^4*b^5*(c*x^2+b
*x+a)^(1/2)*x-3/64*h*f/c^4*b^4*(c*x^2+b*x+a)^(1/2)*a-3/256/c^(7/2)*b^5*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/8*d*g/c*(c*x^2+b*x+a)^(3/2)*b
+3/8*d*g*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d*g/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d
*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d*g/c^(5/2
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-7/60/c^2*b*(c*x^2+b*x+a)^(
5/2)*e*h-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*f*g+7/192/c^3*b^3*(c*x^2+b*x+a)^(3
/2)*e*h+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)*f*g-7/512/c^4*b^5*(c*x^2+b*x+a)^(
1/2)*e*h-7/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*f*g+7/1024/c^(9/2)*b^6*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h+7/1024/c^(9/2)*b^6*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e*h-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))*f*g-9/2048*h*f/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))+1/7*h*f*x^2*(c*x^2+b*x+a)^(5/2)/c-2/35*h*f*a/c^2*(c*x^2+b*x+a)^(5/2)+3/
40*h*f/c^3*b^2*(c*x^2+b*x+a)^(5/2)

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (g + hx) (cx^2 + bx + a)^{3/2} (fx^2 + ex + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)`

[Out] `int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx) (a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d), x)`

[Out] `Integral((g + h*x)*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)`

$$3.195 \quad \int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx$$

**Optimal.** Leaf size=236

$$\frac{(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4c(af + 3be) + 7b^2f + 24c^2d)}{1024c^{9/2}} - \frac{(b^2 - 4ac)(b + 2cx)\sqrt{a + bx + cx^2} (-4c(a + bx + cx^2)^{5/2} (12ce - 7bf) + fx(a + bx + cx^2)^{5/2})}{512c^4}$$

**Rubi [A]** time = 0.24, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)(a+bx+cx^2)^{3/2}(-4acf+7b^2f-12bc+24c^2d)}{192c^3} - \frac{(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}(-4c(af+3be)+7b^2f+24c^2d)}{512c^4} + \frac{(b^2-4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+3be)+7b^2f+24c^2d)}{1024c^{9/2}} + \frac{(a+bx+cx^2)^{5/2}(12ce-7bf)}{60c^2} + \frac{fx(a+bx+cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

[Out] -((b^2 - 4\*a\*c)\*(24\*c^2\*d + 7\*b^2\*f - 4\*c\*(3\*b\*e + a\*f))\*(b + 2\*c\*x)\*Sqrt[a + b\*x + c\*x^2])/(512\*c^4) + ((24\*c^2\*d - 12\*b\*c\*e + 7\*b^2\*f - 4\*a\*c\*f)\*(b + 2\*c\*x)\*(a + b\*x + c\*x^2)^(3/2))/(192\*c^3) + ((12\*c\*e - 7\*b\*f)\*(a + b\*x + c\*x^2)^(5/2))/(60\*c^2) + (f\*x\*(a + b\*x + c\*x^2)^(5/2))/(6\*c) + ((b^2 - 4\*a\*c)^2\*(24\*c^2\*d + 7\*b^2\*f - 4\*c\*(3\*b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(1024\*c^(9/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 640

```
Int[((d._) + (e._)*(x_))*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol
] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2)^{3/2} (d + ex + fx^2) dx &= \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{\int (6cd - af + \frac{1}{2}(12ce - 7bf)x)(a + bx + cx^2)^{3/2}}{6c} \\
&= \frac{(12ce - 7bf)(a + bx + cx^2)^{5/2}}{60c^2} + \frac{fx(a + bx + cx^2)^{5/2}}{6c} + \frac{(2c(6cd - af) - (12ce - 7bf)x)(a + bx + cx^2)^{3/2}}{192c^3} \\
&= \frac{(24c^2d - 12bce + 7b^2f - 4acf)(b + 2cx)(a + bx + cx^2)^{3/2}}{192c^3} + \frac{(12ce - 7bf)x(a + bx + cx^2)^{3/2}}{192c^3} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(12ce - 7bf)x(a + bx + cx^2)^{3/2}}{192c^3} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(12ce - 7bf)x(a + bx + cx^2)^{3/2}}{192c^3} \\
&= -\frac{(b^2 - 4ac)(24c^2d + 7b^2f - 4c(3be + af))(b + 2cx)\sqrt{a + bx + cx^2}}{512c^4} + \frac{(12ce - 7bf)x(a + bx + cx^2)^{3/2}}{192c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.69, size = 392, normalized size = 1.66

$$\frac{300c^2(d - 4a)\sqrt{a + bx + cx^2} \operatorname{atanh}\left(\frac{bx + 2cx}{2\sqrt{a + bx + cx^2}}\right) + 2\sqrt{c}(b + 2cx)\sqrt{a + bx + cx^2}}{15360c} - \frac{30c^2(d - 4a)\sqrt{a + bx + cx^2} \operatorname{atanh}\left(\frac{bx + 2cx}{2\sqrt{a + bx + cx^2}}\right) + 2\sqrt{c}(b + 2cx)\sqrt{a + bx + cx^2}}{15360c} + \frac{5(37c^2d - 4a)\sqrt{a + bx + cx^2} \operatorname{atanh}\left(\frac{bx + 2cx}{2\sqrt{a + bx + cx^2}}\right) + 2\sqrt{c}(b + 2cx)\sqrt{a + bx + cx^2}}{15360c} + \frac{168b^2c^2(a + b^2 + c^2)\sqrt{a + bx + cx^2}}{15360c} + \frac{1920c^2(b + 2cx)(a + x(b + cx))^2 + 3072c^2(a + x(b + cx))^2 + 2560c^2(a + x(b + cx))^2}{15360c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x]

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2)
+ 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c/(15360*c)
```

**IntegrateAlgebraic [A]** time = 0.00, size = 408, normalized size = 1.73

---

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-360*b^3*c^2*d + 2400*a*b*c^3*d + 180*b^4*c*c*e - 120
0*a*b^2*c^2*e + 1536*a^2*c^3*e - 105*b^5*f + 760*a*b^3*c*f - 1296*a^2*b*c^2
*f + 240*b^2*c^3*d*x + 4800*a*c^4*d*x - 120*b^3*c^2*e*x + 672*a*b*c^3*e*x +
70*b^4*c*f*x - 432*a*b^2*c^2*f*x + 480*a^2*c^3*f*x + 2880*b*c^4*d*x^2 + 96
*b^2*c^3*e*x^2 + 3072*a*c^4*e*x^2 - 56*b^3*c^2*f*x^2 + 288*a*b*c^3*f*x^2 +
1920*c^5*d*x^3 + 2112*b*c^4*e*x^3 + 48*b^2*c^3*f*x^3 + 2240*a*c^4*f*x^3 + 1
536*c^5*e*x^4 + 1664*b*c^4*f*x^4 + 1280*c^5*f*x^5))/(7680*c^4) + ((-24*b^4*
c^2*d + 192*a*b^2*c^3*d - 384*a^2*c^4*d + 12*b^5*c*e - 96*a*b^3*c^2*e + 192
*a^2*b*c^3*e - 7*b^6*f + 60*a*b^4*c*f - 144*a^2*b^2*c^2*f + 64*a^3*c^3*f)*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]]/(1024*c^(9/2))
```

**fricas [A]** time = 1.02, size = 839, normalized size = 3.56

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
```





$$2) - 7/512 * f / c^4 * b^5 * (c * x^2 + b * x + a)^{(1/2)} + 7/1024 * f / c^{(9/2)} * b^6 * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) - 1/16 * f * a^3 / c^{(3/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) + 3/16 * d / c * (c * x^2 + b * x + a)^{(1/2)} * b * a - 3/16 * d / c^{(3/2)} * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * b^2 * a + 1/4 * d * (c * x^2 + b * x + a)^{(3/2)} * x + 1/5 * e * (c * x^2 + b * x + a)^{(5/2)} / c - 1/24 * f * a / c * (c * x^2 + b * x + a)^{(3/2)} * x - 1/8 * e / c * b * (c * x^2 + b * x + a)^{(3/2)} * x - 1/32 * f * a^2 / c^2 * (c * x^2 + b * x + a)^{(1/2)} * b - 1/48 * f * a / c^2 * (c * x^2 + b * x + a)^{(3/2)} * b - 1/16 * f * a^2 / c * (c * x^2 + b * x + a)^{(1/2)} * x - 3/32 * d / c * (c * x^2 + b * x + a)^{(1/2)} * x * b^2 + 3/64 * e / c^2 * b^3 * (c * x^2 + b * x + a)^{(1/2)} * x - 3/32 * e / c^2 * b^2 * (c * x^2 + b * x + a)^{(1/2)} * a - 3/16 * e / c^{(3/2)} * b * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * a^2 + 3/32 * e / c^{(5/2)} * b^3 * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * a + 7/96 * f / c^2 * b^2 * (c * x^2 + b * x + a)^{(3/2)} * x - 7/256 * f / c^3 * b^4 * (c * x^2 + b * x + a)^{(1/2)} * x + 1/16 * f / c^3 * b^3 * (c * x^2 + b * x + a)^{(1/2)} * a + 9/64 * f / c^{(5/2)} * b^2 * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * a^2 - 15/256 * f / c^{(7/2)} * b^4 * \ln((c * x + 1/2 * b) / c^{(1/2)} + (c * x^2 + b * x + a)^{(1/2)}) * a$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c x^2 + b x + a)^{3/2} (f x^2 + e x + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2),x)

[Out] int((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b x + c x^2)^{\frac{3}{2}} (d + e x + f x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d),x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2), x)

$$3.196 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx} dx$$

**Optimal.** Leaf size=660

$$\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)\left(8ch(bg-2ah)(bfg-2cdh)-g(-4ach-3b^2h+8bcg)(bfh-2ceh+2c\right)\right)$$

**Rubi [A]** time = 1.83, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 814, 843, 621, 206, 724}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] ((3\*b^4\*f\*h^4 + 6\*b^2\*c\*h^3\*(b\*f\*g - b\*e\*h - 2\*a\*f\*h) - 32\*c^3\*h\*(5\*b\*g - 4\*a\*h)\*(f\*g^2 - h\*(e\*g - d\*h)) + 128\*c^4\*(f\*g^4 - g^2\*h\*(e\*g - d\*h)) - 8\*b\*c^2\*h^2\*(3\*a\*h\*(f\*g - e\*h) - 2\*b\*(f\*g^2 - e\*g\*h + d\*h^2)) + 2\*c\*h\*(8\*c\*h\*(2\*c\*g - b\*h)\*(b\*f\*g - 2\*c\*d\*h) - (2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*(16\*c^2\*g^2 - 3\*b^2\*h^2 - 4\*c\*h\*(2\*b\*g - 3\*a\*h))))\*x)\*Sqrt[a + b\*x + c\*x^2]/(128\*c^3\*h^5) - ((8\*c\*h\*(b\*f\*g - 2\*c\*d\*h) - (8\*c\*g - 3\*b\*h)\*(2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h) + 6\*c\*h\*(2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*x)\*(a + b\*x + c\*x^2)^(3/2))/(48\*c^2\*h^3) + (f\*(a + b\*x + c\*x^2)^(5/2))/(5\*c\*h) - ((4\*c\*h\*(2\*c\*g - b\*h)\*(8\*c\*h\*(b\*g - 2\*a\*h)\*(b\*f\*g - 2\*c\*d\*h) - g\*(8\*b\*c\*g - 3\*b^2\*h - 4\*a\*c\*h)\*(2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)) - 2\*(4\*c^2\*g^2 - (b^2\*h^2)/2 - 2\*c\*h\*(b\*g - a\*h))\*(8\*c\*h\*(2\*c\*g - b\*h)\*(b\*f\*g - 2\*c\*d\*h) - (2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*(16\*c^2\*g^2 - 3\*b^2\*h^2 - 4\*c\*h\*(2\*b\*g - 3\*a\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])]/(256\*c^(7/2)\*h^6) + ((c\*g^2 - b\*g\*h + a\*h^2)^(3/2)\*(f\*g^2 - h\*(e\*g - d\*h))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/h^6

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a,

b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{g + hx} dx &= \frac{f(a + bx + cx^2)^{5/2}}{5ch} + \frac{\int \frac{\left(-\frac{5}{2}h(bfg - 2cdh) - \frac{5}{2}h(2cfg - 2ceh + bfh)x\right)(a + bx + cx^2)^{3/2}}{g + hx} dx}{5ch^2} \\
&= -\frac{(8ch(bfg - 2cdh) - (8cg - 3bh)(2cfg - 2ceh + bfh) + 6ch(2cfg - 2ceh + bfh))}{48c^2h^3} \\
&= \frac{(3b^4fh^4 + 6b^2ch^3(bfg - beh - 2afh) - 32c^3h(5bg - 4ah)(fg^2 - h(eg - h^2)))}{48c^2h^3} \\
&= \frac{(3b^4fh^4 + 6b^2ch^3(bfg - beh - 2afh) - 32c^3h(5bg - 4ah)(fg^2 - h(eg - h^2)))}{48c^2h^3} \\
&= \frac{(3b^4fh^4 + 6b^2ch^3(bfg - beh - 2afh) - 32c^3h(5bg - 4ah)(fg^2 - h(eg - h^2)))}{48c^2h^3} \\
&= \frac{(3b^4fh^4 + 6b^2ch^3(bfg - beh - 2afh) - 32c^3h(5bg - 4ah)(fg^2 - h(eg - h^2)))}{48c^2h^3}
\end{aligned}$$

**Mathematica [A]** time = 2.21, size = 635, normalized size = 0.96

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x]

[Out] (f\*(a + x\*(b + c\*x))^(5/2))/(5\*c\*h) - ((a + x\*(b + c\*x))^(3/2)\*(3\*b^2\*f\*h^2 + 6\*b\*c\*h\*(-(e\*h) + f\*(g + h\*x)) - 4\*c^2\*(f\*g\*(4\*g - 3\*h\*x) + h\*(-4\*e\*g + 4\*d\*h + 3\*e\*h\*x)))/(48\*c^2\*h^3) + (Sqrt[c]\*h\*Sqrt[a + x\*(b + c\*x)]\*(3\*b^4\*f\*h^4 + 64\*c^4\*g\*(f\*g^2 + h\*(-(e\*g) + d\*h))\*(2\*g - h\*x) + 6\*b^2\*c\*h^3\*(-(b\*e\*h) - 2\*a\*f\*h + b\*f\*(g + h\*x)) + 4\*b\*c^2\*h^2\*(6\*a\*e\*h^2 - 6\*a\*f\*h\*(g + h\*x) + b\*f\*g\*(4\*g + 3\*h\*x) + b\*h\*(-4\*e\*g + 4\*d\*h - 3\*e\*h\*x)) - 16\*c^3\*h\*(2\*b\*(f\*g^2 + h\*(-(e\*g) + d\*h))\*(5\*g - h\*x) + a\*h\*(f\*g\*(-8\*g + 3\*h\*x) + h\*(8\*e\*g - 8\*d\*h - 3\*e\*h\*x))) - (2\*c\*h\*(2\*c\*g - b\*h)\*(8\*c\*h\*(b\*g - 2\*a\*h)\*(b\*f\*g - 2\*c\*d\*h) - g\*(8\*b\*c\*g - 3\*b^2\*h - 4\*a\*c\*h)\*(2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)) + (-4\*c^2\*g^2 + (b^2\*h^2)/2 + 2\*c\*h\*(b\*g - a\*h))\*(8\*c\*h\*(2\*c\*g - b\*h)\*(b\*f\*g - 2\*c\*d\*h) - (2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*(16\*c^2\*g^2 - 3\*b^2\*h^2 + 4\*c\*h\*(-2\*b\*g + 3\*a\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])] - 1

$$28*c^{(7/2)}*(c*g^2 + h*(-(b*g) + a*h))^{(3/2)}*(f*g^2 + h*(-(e*g) + d*h))*\text{ArcTanh}\left[\frac{(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)}{(2*\sqrt{c*g^2 + h*(-(b*g) + a*h)})*\sqrt{a + x*(b + c*x)}}\right]\bigg]/(128*c^{(7/2)}*h^6)$$

**IntegrateAlgebraic** [B] time = 89.95, size = 26959, normalized size = 40.85

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x),x]

[Out] Result too large to show

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x, algorithm="fricas")

[Out] Timed out

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Error: Bad Argument Type

**maple** [B] time = 0.02, size = 6715, normalized size = 10.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x)

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*h-2\*c\*g>0)', see `assume?` for more details)Is b\*h-2\*c\*g zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{g + hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x),x)

[Out] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{g + hx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g),x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x), x)

$$3.197 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2} dx$$

**Optimal.** Leaf size=754

$$\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2(a^2fh^2 - 2abh(2fg - eh) + b^2(dh^2 - 2egh + 3fg^2)) + 8b^2ch^3(-3afh - beh + 2bh^2)\right)$$

128c

**Rubi [A]** time = 2.50, antiderivative size = 750, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 814, 843, 621, 206, 724}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2,x]

[Out] -((3\*b^3\*f\*h^3 + 4\*b\*c\*h^2\*(4\*b\*f\*g - 2\*b\*e\*h - 3\*a\*f\*h) + 64\*c^3\*(5\*f\*g^3 - g\*h\*(4\*e\*g - 3\*d\*h)) - 16\*c^2\*h\*(19\*b\*f\*g^2 - b\*h\*(14\*e\*g - 9\*d\*h) - 4\*a\*h\*(2\*f\*g - e\*h)) + 2\*c\*h\*(3\*b^2\*f\*h^2 + 4\*c\*h\*(4\*b\*f\*g - 2\*b\*e\*h - 3\*a\*f\*h) - 16\*c^2\*(5\*f\*g^2 - h\*(4\*e\*g - 3\*d\*h)))\*x)\*Sqrt[a + b\*x + c\*x^2])/((64\*c^2\*h^5) - ((3\*b\*f\*h\*(b\*g - a\*h) + (8\*c^2\*(5\*f\*g^3 - g\*h\*(4\*e\*g - 3\*d\*h)))/h - c\*(43\*b\*f\*g^2 - 8\*b\*h\*(4\*e\*g - 3\*d\*h) - 8\*a\*h\*(2\*f\*g - e\*h)) + 6\*c\*h\*(4\*c\*e\*g + b\*f\*g - (5\*c\*f\*g^2)/h - 4\*c\*d\*h - a\*f\*h)\*x)\*(a + b\*x + c\*x^2)^(3/2))/((24\*c\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(5/2))/(h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)) + ((3\*b^4\*f\*h^4 + 8\*b^2\*c\*h^3\*(2\*b\*f\*g - b\*e\*h - 3\*a\*f\*h) + 128\*c^4\*(5\*f\*g^4 - g^2\*h\*(4\*e\*g - 3\*d\*h)) + 48\*c^2\*h^2\*(a^2\*f\*h^2 - 2\*a\*b\*h\*(2\*f\*g - e\*h) + b^2\*(3\*f\*g^2 - 2\*e\*g\*h + d\*h^2)) + 192\*c^3\*h\*(a\*h\*(3\*f\*g^2 - 2\*e\*g\*h + d\*h^2) - b\*g\*(4\*f\*g^2 - 3\*e\*g\*h + 2\*d\*h^2)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(5/2)\*h^6) - (Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*(2\*c\*(5\*f\*g^3 - g\*h\*(4\*e\*g - 3\*d\*h)) - h\*(7\*b\*f\*g^2 - b\*h\*(5\*e\*g - 3\*d\*h) - 2\*a\*h\*(2\*f\*g - e\*h)))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*h^6)

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 814

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```





$$\begin{aligned}
& x) + b*c*h^2*(-6*a*f*h + b*(8*f*g - 4*e*h + 3*f*h*x)) - 4*c^2*h*(a*h*(-16*f \\
& *g + 8*e*h + 3*f*h*x) + 2*b*(f*g*(19*g - 2*h*x) + h*(-14*e*g + 9*d*h + e*h* \\
& x)))) + \text{Sqrt}[c]*(c*g^2 + h*(-(b*g) + a*h))*(2*c*h*(2*c*g - b*h)*(3*b^2*f*g* \\
& h + 4*a*c*h*(5*f*g - 4*e*h) - 8*b*c*(5*f*g^2 + h*(-4*e*g + 3*d*h))) + ((8*c \\
& ^2*g^2 - b^2*h^2 + 4*c*h*(-(b*g) + a*h))*(-3*b^2*f*h^2 + 4*c*h*(-4*b*f*g + \\
& 2*b*e*h + 3*a*f*h) + 16*c^2*(5*f*g^2 + h*(-4*e*g + 3*d*h))))/2)*\text{ArcTanh}[(b \\
& + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])] + 32*c^3*(c*g^2 + h*(-(b*g) + a \\
& *h))^(3/2)*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*g^2 + b*h*(5*e \\
& *g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*\text{ArcTanh}[(-(b*g) + 2*a*h - 2*c*g*x + b* \\
& h*x)/(2*\text{Sqrt}[c*g^2 + h*(-(b*g) + a*h)]*\text{Sqrt}[a + x*(b + c*x)])]/(16*c^2*h^5 \\
& ))/(-(c*g^2) + h*(b*g - a*h))/(c*h)
\end{aligned}$$

**IntegrateAlgebraic** [B] time = 47.04, size = 17196, normalized size = 22.81

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^2, x]

[Out] Result too large to show

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^2,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.02, size = 14734, normalized size = 19.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)`

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*h-2\*c\*g>0)', see `assume?` for more details) Is b\*h-2\*c\*g zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)`

[Out] `int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)`

[Out] `Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)`

$$3.198 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3} dx$$

Optimal. Leaf size=824

$$\frac{(fg^2 - h(eg - dh))(cx^2 + bx + a)^{5/2} \left(4cg \left(-\frac{10fg^2}{h} + 6eg - 3dh\right) - 4ah(7fg - 3eh) + b(31fg^2 - 3h(5eg - dh))\right)}{2h(cg^2 - bhg + ah^2)(g + hx)^2} - \frac{12h^2(cg^2 - bhg + ah^2)}{2h(cg^2 - bhg + ah^2)(g + hx)^2}$$

**Rubi [A]** time = 2.14, antiderivative size = 819, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1650, 812, 814, 843, 621, 206, 724}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out]  $-\left(\frac{b^2 f h^2 (b g - a h) + 8 c^3 g^2 (6 e g - (10 f g^2)/h - 3 d h) - 2 c^2 (2 a h (19 f g^2 - 9 e g h + 3 d h^2) - 3 b g (22 f g^2 - 12 e g h + 5 d h^2)) - c h (8 a^2 f h^2 - 18 a b h (3 f g - e h) + b^2 (53 f g^2 - 6 h (4 e g - d h))) + 2 c (b f h^2 (b g - a h) + 2 c^2 (10 f g^3 - 3 g h (2 e g - d h)) + c h (2 a h (7 f g - 3 e h) - 3 b (6 f g^2 - 3 e g h + d h^2))) x \sqrt{a + b x + c x^2}}{8 c h^4 (c g^2 - b g h + a h^2)} - \frac{((31 b f g^2 + 4 c g (6 e g - (10 f g^2)/h - 3 d h) - 3 b h (5 e g - d h) - 4 a h (7 f g - 3 e h) + 2 h (3 c e g + 2 b f g - (5 c f g^2)/h - 3 c d h - 2 a f h)) x (a + b x + c x^2)^{3/2}}{12 h^2 (c g^2 - b g h + a h^2) (g + h x)} - \frac{(f g^2 - h (e g - d h)) (a + b x + c x^2)^{5/2}}{(2 h (c g^2 - b g h + a h^2) (g + h x)^2} - \frac{((b^3 f h^3 + 6 b c h^2 (3 b f g - b e h - 2 a f h) + 16 c^3 (10 f g^3 - 3 g h (2 e g - d h)) - 24 c^2 h (6 b f g^2 - b h (3 e g - d h) - a h (3 f g - e h))) \operatorname{ArcTanh}\left[\frac{b + 2 c x}{2 \sqrt{c} \sqrt{a + b x + c x^2}}\right]}{16 c^{3/2} h^6} + \frac{((8 c^2 (10 f g^4 - 3 g^2 h (2 e g - d h)) - 4 c h (28 b f g^3 - 3 b g h (5 e g - 2 d h) - a h (19 f g^2 - 9 e g h + 3 d h^2)) + h^2 (8 a^2 f h^2 - 4 a b h (10 f g - 3 e h) + b^2 (35 f g^2 - 3 h (5 e g - d h))) \operatorname{ArcTanh}\left[\frac{b g - 2 a h + (2 c g - b h) x}{2 \sqrt{c g^2 - b g h + a h^2}}\right] \sqrt{a + b x + c x^2}}{8 h^6 \sqrt{c g^2 - b g h + a h^2}}$

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 812

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 814

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^3} dx = -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{2h(cg^2 - bgh + ah^2)(g + hx)^2} - \frac{\int \frac{\frac{1}{2}(-4cdg + 5beg + 4afg - \frac{5bf g^2}{h} - bdh - 4a^2)}{2(cg^2 - bgh + ah^2)} dx}{2(cg^2 - bgh + ah^2)}$$

$$= -\frac{(31bfg^2 + 4cg(6eg - \frac{10fg^2}{h} - 3dh) - 3bh(5eg - dh) - 4ah(7fg - 3eh))}{12h^2(cg^2 - bgh + ah^2)}$$

$$= -\frac{(b^2fh^2(bg - ah) + 8c^3g^2(6eg - \frac{10fg^2}{h} - 3dh) - 2c^2(2ah(19fg^2 - 9egh)))}{12h^2(cg^2 - bgh + ah^2)}$$

$$= -\frac{(b^2fh^2(bg - ah) + 8c^3g^2(6eg - \frac{10fg^2}{h} - 3dh) - 2c^2(2ah(19fg^2 - 9egh)))}{12h^2(cg^2 - bgh + ah^2)}$$

$$= -\frac{(b^2fh^2(bg - ah) + 8c^3g^2(6eg - \frac{10fg^2}{h} - 3dh) - 2c^2(2ah(19fg^2 - 9egh)))}{12h^2(cg^2 - bgh + ah^2)}$$

$$= -\frac{(b^2fh^2(bg - ah) + 8c^3g^2(6eg - \frac{10fg^2}{h} - 3dh) - 2c^2(2ah(19fg^2 - 9egh)))}{12h^2(cg^2 - bgh + ah^2)}$$

**Mathematica** [B] time = 6.27, size = 4162, normalized size = 5.05

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3,x]

[Out] 
$$\begin{aligned} & (f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(3*c*h*(g + h*x)^2) - ((a + x \\ & *(b + c*x))^(3/2)*(-1/2*((h*(5*b*f*g - 6*c*d*h - 4*a*f*h))/2 - (g*(10*c*f* \\ & g - 6*c*e*h + b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)* \\ & (g + h*x)^2) - (((-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h) \\ & ) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 \\ & )*(a + b*x + c*x^2)^(5/2))/((-c*g^2) + b*g*h - a*h^2)*(g + h*x) + (((4*c* \\ & (4*c*g - (3*b*h)/2)*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d \\ & *h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)) \\ & )/2) + 4*c*h*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) \\ & - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 \\ & + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/ \\ & 2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c* \\ & h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2) - 1 \\ & 2*c^2*h*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c* \\ & h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2)*x*(a + \\ & b*x + c*x^2)^(3/2))/(12*c*h^2) - (((2*c*h*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + \\ & (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + \\ & (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + \\ & 4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - \\ & d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e \\ & *h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a* \\ & e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) \\ & - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2)) \\ & /2)) - (2*c*g - (b*h)/2)*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + \\ & 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c* \\ & h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h \\ & *(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h) \\ & ) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)) \\ & )/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)) \\ & )/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3* \\ & c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) \\ & + c*h*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5* \\ & c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h* \\ & (5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3* \\ & a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f* \\ & *g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b* \\ & f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*( \\ & 5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b* \\ & h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))*x)*Sqrt[a + b*x + c \\ & *x^2])/(2*c*h^2) - (((2*c*h*(2*c*g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + \\ & (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + ($$

$$\begin{aligned}
& 3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))/2) + 4 \\
& *c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - \\
& d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e* \\
& h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e \\
& *h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - \\
& (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/ \\
& 2)) + (-4*c^2*g^2 + (b^2*h^2)/2 - c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 + \\
& (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a* \\
& h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d* \\
& g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h* \\
& (b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) \\
& + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) \\
& + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - \\
& a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c* \\
& d*g - a*f*g + a*e*h)))/2))/2)))*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + b*x \\
& + c*x^2]])/(sqrt[c]*h) - (4*sqrt[c*g^2 - b*g*h + a*h^2]*(-(g*(2*c*h*(2*c* \\
& g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - \\
& 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d \\
& *h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c \\
& *f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h \\
& *(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b* \\
& h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 \\
& - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - \\
& d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (-4*c^2*g^2 + (b^2*h^2)/2 - \\
& c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6* \\
& a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h* \\
& (5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*( \\
& 2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) \\
& - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 \\
& + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/ \\
& 2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c* \\
& h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))) \\
& + h*(2*c*h*(b*g - 2*a*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c \\
& *g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 \\
& - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h \\
& )*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g* \\
& (5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h \\
& *(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3 \\
& *c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^ \\
& 2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (2*a*c*g*h \\
& + b*g*(-2*c*g + (b*h)/2))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + \\
& 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c \\
& *h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c* \\
& h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h \\
& )) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
\end{aligned}$$



$$\begin{aligned} & )/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h) \\ & ))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3 \\ & *c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) \\ & ))*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*sqrt[c*g^2 - b*g*h + a*h^2 \\ & ]*sqrt[a + b*x + c*x^2])]/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2))/(4*c*h^2)/(8 \\ & *c*h^2)/(-(c*g^2) + b*g*h - a*h^2)/(2*(c*g^2 - b*g*h + a*h^2))/(3*c*h*( \\ & a + b*x + c*x^2)^(3/2)) \end{aligned}$$

**IntegrateAlgebraic [F]** time = 180.17, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x]

[Out] \$Aborted

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 1.47Unable to divide, perhaps  
 due to rounding error%%{1, [6,0,0,0,9,0,0,0]%%}+%%{%%{[-6,0]: [1,0,%%{-1, [1]%%}]}%%}, [5,0,0,0,8,1,0,0]%%}+%%{-3, [4,1,0,0,9,0,0,0]%%}+%%{3, [4,0,1,0,8,1,0,0]%%}+%%{%%{12, [1]%%}, [4,0,0,0,7,2,0,0]%%}+%%{%%{[12,0]: [1,0,%%{-1, [1]%%}]}%%}, [3,1,0,0,8,1,0,0]%%}+%%{%%{[-12,0]: [1,0,%%{-1, [1]%%}]}%%}, [3,0,1,0,7,2,0,0]%%}+%%{%%{[-8, [1]%%}, 0]: [1,0,%%{-1, [1]%%}]}%%}, [3,0,0,0,6,3,0,0]%%}+%%{3, [2,2,0,0,9,0,0,0]%%}+%%{-6, [2,1,1,0,8,1,0,0]%%}+%%{%%{-12, [1]%%}, [2,1,0,0,7,2,0,0]%%}+%%{3, [2,0,2,0,7,2,0,0]

```

%%}{+%%{%%{12, [1]%%}, [2, 0, 1, 0, 6, 3, 0, 0]%%}+%%{%%{[-6, 0] : [1, 0, %%{-1, [1]
%%}]}%%}, [1, 2, 0, 0, 8, 1, 0, 0]%%}+%%{%%{[12, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [1, 1, 1
, 0, 7, 2, 0, 0]%%}+%%{%%{[-6, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [1, 0, 2, 0, 6, 3, 0, 0]%%}
+%%{-1, [0, 3, 0, 0, 9, 0, 0, 0]%%}+%%{3, [0, 2, 1, 0, 8, 1, 0, 0]%%}+%%{-3, [0, 1, 2, 0, 7
, 2, 0, 0]%%}+%%{1, [0, 0, 3, 0, 6, 3, 0, 0]%%} / %%{%%{poly1[%%{1, [1]%%}, 0] : [1,
0, %%{-1, [1]%%}]}%%}, [6, 0, 0, 0, 3, 0, 0, 0]%%}+%%{%%{-6, [2]%%}, [5, 0, 0, 0, 2, 1,
0, 0]%%}+%%{%%{-3, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [4, 1, 0, 0, 3, 0, 0, 0
]%%}+%%{%%{poly1[%%{3, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [4, 0, 1, 0, 2, 1, 0,
0]%%}+%%{%%{poly1[%%{12, [2]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [4, 0, 0, 0, 1, 2,
0, 0]%%}+%%{%%{12, [2]%%}, [3, 1, 0, 0, 2, 1, 0, 0]%%}+%%{%%{-12, [2]%%}, [3, 0,
1, 0, 1, 2, 0, 0]%%}+%%{%%{-8, [3]%%}, [3, 0, 0, 0, 0, 3, 0, 0]%%}+%%{%%{3, [1]
%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [2, 2, 0, 0, 3, 0, 0, 0]%%}+%%{%%{-6, [1]%%
}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [2, 1, 1, 0, 2, 1, 0, 0]%%}+%%{%%{-12, [2]%%},
0] : [1, 0, %%{-1, [1]%%}]}%%}, [2, 1, 0, 0, 1, 2, 0, 0]%%}+%%{%%{poly1[%%{3, [1]%%}
, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [2, 0, 2, 0, 1, 2, 0, 0]%%}+%%{%%{poly1[%%{12, [2]%%
}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [2, 0, 1, 0, 0, 3, 0, 0]%%}+%%{%%{-6, [2]%%}, [1, 2
, 0, 0, 2, 1, 0, 0]%%}+%%{%%{12, [2]%%}, [1, 1, 1, 0, 1, 2, 0, 0]%%}+%%{%%{-6, [2]%%
}, [1, 0, 2, 0, 0, 3, 0, 0]%%}+%%{%%{-1, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%},
[0, 3, 0, 0, 3, 0, 0, 0]%%}+%%{%%{3, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [0, 2
, 1, 0, 2, 1, 0, 0]%%}+%%{%%{-3, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [0, 1, 2,
0, 1, 2, 0, 0]%%}+%%{%%{poly1[%%{1, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]}%%}, [0, 0, 3
, 0, 0, 3, 0, 0]%%} Error: Bad Argument Value

```

**maple [B]** time = 0.02, size = 26596, normalized size = 32.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h +c*g^2 zero or nonze
ro?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x)

[Out] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*3, x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*3, x)

$$3.199 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4} dx$$

**Optimal.** Leaf size=833

$$\frac{(fg^2 - h(eg - dh))(cx^2 + bx + a)^{5/2} \left( 2cg \left( -\frac{10fg^2}{h} + 4eg - dh \right) - 6ah(3fg - eh) + b(17fg^2 - h(5eg + dh)) + 2h^2 \right)}{3h(cg^2 - bhg + ah^2)(g + hx)^3} \quad \frac{12h^2(cg^2 - bhg + ah^2)}{12h^2(cg^2 - bhg + ah^2)}$$

**Rubi [A]** time = 2.26, antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 812, 843, 621, 206, 724}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x]

[Out] -((12\*a^2\*f\*h^3 - 8\*c^2\*g^2\*(4\*e\*g - (10\*f\*g^2)/h - d\*h) - 6\*a\*b\*h^2\*(7\*f\*g - e\*h) + 4\*a\*c\*h\*(23\*f\*g^2 - 2\*h\*(4\*e\*g - d\*h)) - 6\*b\*c\*g\*(18\*f\*g^2 - h\*(6\*e\*g - d\*h)) + b^2\*h\*(29\*f\*g^2 - h\*(5\*e\*g + d\*h)) + 2\*(3\*b\*f\*h^2\*(b\*g - a\*h) + 2\*c^2\*(10\*f\*g^3 - g\*h\*(4\*e\*g - d\*h)) - c\*h\*(22\*b\*f\*g^2 - b\*h\*(7\*e\*g - d\*h) - 6\*a\*h\*(3\*f\*g - e\*h))) \* Sqrt[a + b\*x + c\*x^2]) / (8\*h^4\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)) - ((17\*b\*f\*g^2 + 2\*c\*g\*(4\*e\*g - (10\*f\*g^2)/h - d\*h) - b\*h\*(5\*e\*g + d\*h) - 6\*a\*h\*(3\*f\*g - e\*h) + 2\*h\*(2\*c\*e\*g + 3\*b\*f\*g - (5\*c\*f\*g^2)/h - 2\*c\*d\*h - 3\*a\*f\*h)) \* x) \* (a + b\*x + c\*x^2)^(3/2) / (12\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^2) - ((f\*g^2 - h\*(e\*g - d\*h)) \* (a + b\*x + c\*x^2)^(5/2)) / (3\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^3) + ((3\*b^2\*f\*h^2 - 12\*c\*h\*(4\*b\*f\*g - b\*e\*h - a\*f\*h) + 8\*c^2\*(10\*f\*g^2 - h\*(4\*e\*g - d\*h))) \* ArcTanh[(b + 2\*c\*x) / (2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])]) / (8\*Sqrt[c]\*h^6) - ((16\*c^3\*(10\*f\*g^5 - g^3\*h\*(4\*e\*g - d\*h)) - b\*h^3\*(24\*a^2\*f\*h^2 - 6\*a\*b\*h\*(10\*f\*g - e\*h) + b^2\*(35\*f\*g^2 - 5\*e\*g\*h - d\*h^2)) + 6\*c\*h^2\*(4\*a^2\*h^2\*(4\*f\*g - e\*h) + b^2\*g\*(35\*f\*g^2 - 10\*e\*g\*h + d\*h^2) - 2\*a\*b\*h\*(25\*f\*g^2 - 7\*e\*g\*h + d\*h^2)) - 24\*c^2\*g\*h\*(b\*g\*(14\*f\*g^2 - 5\*e\*g\*h + d\*h^2) - a\*h\*(11\*f\*g^2 - 4\*e\*g\*h + d\*h^2))) \* ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x) / (2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])]) / (16\*h^6\*(c\*g^2 - b\*g\*h + a\*h^2)^(3/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^4} dx &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{3h(cg^2 - bgh + ah^2)(g + hx)^3} - \int \frac{\left(\frac{1}{2}(-6cdg + 5beg + 6afg - \frac{5bf g^2}{h} + bdh - 6\right)}{3(cg^2 - bgh + ah^2)(g + hx)^3} dx \\
&= -\frac{\left(17bfg^2 + 2cg\left(4eg - \frac{10fg^2}{h} - dh\right) - bh(5eg + dh) - 6ah(3fg - eh) + 2\right)}{12h^2(cg^2 - bgh + ah^2)} \\
&= -\frac{\left(12a^2fh^3 - 8c^2g^2\left(4eg - \frac{10fg^2}{h} - dh\right) - 6abh^2(7fg - eh) + 4ach(23fg^2 - 2ehg)\right)}{12h^2(cg^2 - bgh + ah^2)} \\
&= -\frac{\left(12a^2fh^3 - 8c^2g^2\left(4eg - \frac{10fg^2}{h} - dh\right) - 6abh^2(7fg - eh) + 4ach(23fg^2 - 2ehg)\right)}{12h^2(cg^2 - bgh + ah^2)} \\
&= -\frac{\left(12a^2fh^3 - 8c^2g^2\left(4eg - \frac{10fg^2}{h} - dh\right) - 6abh^2(7fg - eh) + 4ach(23fg^2 - 2ehg)\right)}{12h^2(cg^2 - bgh + ah^2)} \\
&= -\frac{\left(12a^2fh^3 - 8c^2g^2\left(4eg - \frac{10fg^2}{h} - dh\right) - 6abh^2(7fg - eh) + 4ach(23fg^2 - 2ehg)\right)}{12h^2(cg^2 - bgh + ah^2)}
\end{aligned}$$

**Mathematica [B]** time = 6.49, size = 7806, normalized size = 9.37

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x]

[Out] Result too large to show

**IntegrateAlgebraic [F]** time = 180.06, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^4,x, algorithm="fricas")

[Out] Timed out

**giac** [B] time = 26.17, size = 7319, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^4,x, algorithm="giac")

[Out] 
$$\frac{1}{4}\sqrt{c x^2 + b x + a} \left( \frac{2 c f x}{h^4} - \frac{16 c^2 f g h^{10} - 5 b c f h^{11} - 4 c^2 h^{11} e}{c h^{15}} \right) - \frac{1}{8} \frac{160 c^3 f g^5 - 336 b c^2 f g^4 h + 16 c^3 d g^3 h^2 + 210 b^2 c f g^3 h^2 + 264 a c^2 f g^3 h^2 - 24 b c^2 d g^2 h^3 - 35 b^3 f g^2 h^3 - 300 a b c f g^2 h^3 + 6 b^2 c d g h^4 + 24 a c^2 d g h^4 + 60 a b^2 f g h^4 + 96 a^2 c f g h^4 + b^3 d h^5 - 12 a b c d h^5 - 24 a^2 b f h^5 - 64 c^3 g^4 h e + 120 b c^2 g^3 h^2 e - 60 b^2 c g^2 h^3 e - 96 a c^2 g^2 h^3 e + 5 b^3 g h^4 e + 84 a b c g h^4 e - 6 a b^2 h^5 e - 24 a^2 c h^5 e}{(c g^2 h^6 - b g h^7 + a h^8) \sqrt{-c g^2 + b g h - a h^2}} - \frac{1}{24} \frac{480 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 c^{7/2} f g^5 h^2 - 912 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b c^{5/2} f g^4 h^3 + 144 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 c^{7/2} d g^3 h^4 + 522 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b^2 c^{3/2} f g^3 h^4 + 552 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a c^{5/2} f g^3 h^4 - 216 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b c^{5/2} d g^2 h^5 - 87 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b^3 \sqrt{c} f g^2 h^5 - 540 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a b c^{3/2} f g^2 h^5 + 78 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b^2 c^{3/2} d g h^6 + 120 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a c^{5/2} d g h^6 + 108 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a b^2 \sqrt{c} f g h^6 + 96 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a^2 c^{3/2} f g h^6 - 3 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b^3 \sqrt{c} d h^7 - 60 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a b c^{3/2} d h^7 - 24 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a^2 b \sqrt{c} f h^7 - 288 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 c^{7/2} g^4 h^3 e + 504 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b c^{5/2} g^3 h^4 e - 252 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b^2 c^{3/2} g^2 h^5 e - 288 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a c^{5/2} g^2 h^5 e + 33 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 b^3 \sqrt{c} g h^6 e + 228 (\sqrt{c} x - \sqrt{c x^2 + b x + a})^5 a b c^{3/2} g h^6 e$$

$$\begin{aligned}
& e - 30*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a*b^2*\text{sqrt}(c)*h^7*e - 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^5*a^2*c^{(3/2)}*h^7*e + 1680*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*c^4*f*g^6*h - 2880*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b*c^3*f*g^5*h^2 + 432*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*c^4*d*g^4*h^3 + 1362*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^2*c^2*f*g^4*h^3 + 1464*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*c^3*f*g^4*h^3 - 504*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b*c^3*d*g^3*h^4 - 147*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^3*c*f*g^3*h^4 - 876*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b*c^2*f*g^3*h^4 + 54*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^2*c^2*d*g^2*h^5 + 216*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*c^3*d*g^2*h^5 - 36*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b^2*c*f*g^2*h^5 - 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*c^2*f*g^2*h^5 + 33*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^3*c*d*g*h^6 + 84*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b*c^2*d*g*h^6 + 216*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*b*c*f*g*h^6 - 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b^2*c*d*h^7 - 96*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*c^2*d*h^7 - 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^3*c*f*h^7 - 960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*c^4*g^5*h^2*e + 1464*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b*c^3*g^4*h^3*e - 540*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^2*c^2*g^3*h^4*e - 672*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*c^3*g^3*h^4*e + 21*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*b^3*c*g^2*h^5*e + 180*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b*c^2*g^2*h^5*e + 90*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a*b^2*c*g*h^6*e + 168*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*c^2*g*h^6*e - 96*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^4*a^2*b*c*h^7*e + 1504*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*c^{(9/2)}*f*g^7 - 1072*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b*c^{(7/2)}*f*g^6*h + 352*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*c^{(9/2)}*d*g^5*h^2 - 1308*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^2*c^{(5/2)}*f*g^5*h^2 - 656*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*c^{(7/2)}*f*g^5*h^2 - 16*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b*c^{(7/2)}*d*g^4*h^3 + 1042*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^3*c^{(3/2)}*f*g^4*h^3 + 4056*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b*c^{(5/2)}*f*g^4*h^3 - 420*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^2*c^{(5/2)}*d*g^3*h^4 - 272*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*c^{(7/2)}*d*g^3*h^4 - 136*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^4*\text{sqrt}(c)*f*g^3*h^4 - 2712*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^2*c^{(3/2)}*f*g^3*h^4 - 2208*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^2*c^{(5/2)}*f*g^3*h^4 + 106*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^3*c^{(3/2)}*d*g^2*h^5 + 840*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b*c^{(5/2)}*d*g^2*h^5 + 328*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^3*\text{sqrt}(c)*f*g^2*h^5 + 1920*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^2*b*c^{(3/2)}*f*g^2*h^5 + 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*b^4*\text{sqrt}(c)*d*g*h^6 - 144*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^2*c^{(3/2)}*d*g*h^6 - 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^2*c^{(5/2)}*d*g*h^6 - 240*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^2*b^2*\text{sqrt}(c)*f*g*h^6 - 288*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^3*c^{(3/2)}*f*g*h^6 - 8*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a*b^3*\text{sqrt}(c)*d*h^7 + 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*a^3*b*\text{sqrt}(c)*f*h^7 - 832*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))^3*c^{(9/2)}*g^6*h*e + 400*(\text{sqrt}(c)*x -
\end{aligned}$$



$$\begin{aligned}
& \sqrt{c*x^2 + b*x + a})^3*b*c^{(7/2)}*g^5*h^2*e + 840*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*b^2*c^{(5/2)}*g^4*h^3*e + 512*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*c^{(7/2)}*g^4*h^3*e - 478*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*b^3*c^{(3/2)}*g^3*h^4*e - 2232*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*b*c^{(5/2)}*g^3*h^4*e + 40*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*b^4*\sqrt{c}*g^2*h^5*e + 1092*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*b^2*c^{(3/2)}*g^2*h^5*e + 1104*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a^2*c^{(5/2)}*g^2*h^5*e - 88*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*b^3*\sqrt{c}*g*h^6*e - 576*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a^2*b*c^{(3/2)}*g*h^6*e + 48*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a^2*b^2*\sqrt{c}*h^7*e + 2256*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b*c^4*f*g^7 - 3420*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^2*c^3*f*g^6*h - 2832*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*c^4*f*g^6*h + 528*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b*c^4*d*g^5*h^2 + 1218*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^3*c^2*f*g^5*h^2 + 5976*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b*c^3*f*g^5*h^2 - 516*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^2*c^3*d*g^4*h^3 - 624*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*c^4*d*g^4*h^3 - 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^4*c*f*g^4*h^3 - 1944*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^2*c^2*f*g^4*h^3 - 2208*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*c^3*f*g^4*h^3 - 6*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^3*c^2*d*g^3*h^4 + 840*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b*c^3*d*g^3*h^4 - 264*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^3*c*f*g^3*h^4 + 192*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*b*c^2*f*g^3*h^4 + 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^4*c*d*g^2*h^5 + 144*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^2*c^2*d*g^2*h^5 - 288*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*c^3*d*g^2*h^5 + 720*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*b^2*c*f*g^2*h^5 + 480*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*c^2*f*g^2*h^5 - 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^3*c*d*g*h^6 - 288*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*b*c^2*d*g*h^6 - 528*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*b*c*f*g*h^6 + 96*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*c^2*d*h^7 + 96*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^4*c*f*h^7 - 1248*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b*c^4*g^6*h*e + 1656*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^2*c^3*g^5*h^2*e + 1536*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*c^4*g^5*h^2*e - 414*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^3*c^2*g^4*h^3*e - 2760*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b*c^3*g^4*h^3*e - 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b^4*c*g^3*h^4*e + 420*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^2*c^2*g^3*h^4*e + 912*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*c^3*g^3*h^4*e + 168*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^3*c*g^2*h^5*e + 432*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*b*c^2*g^2*h^5*e - 288*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*b^2*c*g*h^6*e - 384*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*c^2*g*h^6*e + 144*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*b*c*h^7*e + 1128*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*b^2*c^{(7/2)}*f*g^7 - 1776*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*b^3*c^{(5/2)}*f*g^6*h - 2832*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*a*b*c^{(7/2)}*f*g^6*h + 264*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*b^2*c^{(7/2)}*d*g^5*h^2 + 720*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*b^4*c^{(3/2)}*f*g^5*h^2 + 5580*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})
\end{aligned}$$

$$\begin{aligned}
& + b*x + a)) * a*b^2*c^{(5/2)} * f*g^5*h^2 + 1776*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*c^{(7/2)} * f*g^5*h^2 - 288*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^3*c^{(5/2)} * d*g^4*h^3 - 624*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b*c^{(7/2)} * d*g^4*h^3 - 57*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^5*\text{sqrt}(c)*f*g^4*h^3 - 2514*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^3*c^{(3/2)} * f*g^4*h^3 - 5688*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b*c^{(5/2)} * f*g^4*h^3 + 36*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^4*c^{(3/2)} * d*g^3*h^4 + 852*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^2*c^{(5/2)} * d*g^3*h^4 + 384*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*c^{(7/2)} * d*g^3*h^4 + 198*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^4*\text{sqrt}(c)*f*g^3*h^4 + 3078*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b^2*c^{(3/2)} * f*g^3*h^4 + 1848*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*c^{(5/2)} * f*g^3*h^4 + 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^5*\text{sqrt}(c)*d*g^2*h^5 - 90*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^3*c^{(3/2)} * d*g^2*h^5 - 864*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b*c^{(5/2)} * d*g^2*h^5 - 249*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b^3*\text{sqrt}(c)*f*g^2*h^5 - 1476*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*b*c^{(3/2)} * f*g^2*h^5 - 6*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^4*\text{sqrt}(c)*d*g*h^6 + 90*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b^2*c^{(3/2)} * d*g*h^6 + 264*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*c^{(5/2)} * d*g*h^6 + 132*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*b^2*\text{sqrt}(c)*f*g*h^6 + 192*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^4*c^{(3/2)} * f*g*h^6 + 3*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b^3*\text{sqrt}(c)*d*h^7 - 36*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*b*c^{(3/2)} * d*h^7 - 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^4*b*\text{sqrt}(c)*f*h^7 - 624*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^2*c^{(7/2)} * g^6*h*e + 876*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^3*c^{(5/2)} * g^5*h^2*e + 1536*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b*c^{(7/2)} * g^5*h^2*e - 282*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^4*c^{(3/2)} * g^4*h^3*e - 2664*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^2*c^{(5/2)} * g^4*h^3*e - 960*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*c^{(7/2)} * g^4*h^3*e + 15*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * b^5*\text{sqrt}(c)*g^3*h^4*e + 894*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^3*c^{(3/2)} * g^3*h^4*e + 2640*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b*c^{(5/2)} * g^3*h^4*e - 48*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a*b^4*\text{sqrt}(c)*g^2*h^5*e - 936*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b^2*c^{(3/2)} * g^2*h^5*e - 816*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*c^{(5/2)} * g^2*h^5*e + 51*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^2*b^3*\text{sqrt}(c)*g*h^6*e + 300*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*b*c^{(3/2)} * g*h^6*e - 18*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^3*b^2*\text{sqrt}(c)*h^7*e + 24*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a)) * a^4*c^{(3/2)} * h^7*e + 188*b^3*c^3*f*g^7 - 272*b^4*c^2*f*g^6*h - 708*a*b^2*c^3*f*g^6*h + 44*b^3*c^3*d*g^5*h^2 + 87*b^5*c*f*g^5*h^2 + 1214*a*b^3*c^2*f*g^5*h^2 + 888*a^2*b*c^3*f*g^5*h^2 - 44*b^4*c^2*d*g^4*h^3 - 156*a*b^2*c^3*d*g^4*h^3 - 426*a*b^4*c*f*g^4*h^3 - 2010*a^2*b^2*c^2*f*g^4*h^3 - 376*a^3*c^3*f*g^4*h^3 + 3*b^5*c*d*g^3*h^4 + 182*a*b^3*c^2*d*g^3*h^4 + 192*a^2*b*c^3*d*g^3*h^4 + 807*a^2*b^3*c*f*g^3*h^4 + 1468*a^3*b*c^2*f*g^3*h^4 - 6*a*b^4*c*d*g^2*h^5 - 294*a^2*b^2*c^2*d*g^2*h^5 - 88*a^3*c^3*d*g^2*h^5 - 732*a^3*b^2*c*f*g^2*h^5 - 400*a^4*c^2*f*g^2*h^5 + 3*a^2*b^3*c*d*g*h^6 + 220*a^3*b*c^2*d*g*h^6 + 312*a^4*b*c*f*g*h^6 - 64*a^4*c^2*d*h^7 - 48*a^5*c*f*h^7 - 104*b^3*c^3*g^6*h*e + 134*b^4*c^2*g^5*h^2*e + 384*a*b^
\end{aligned}$$

$$2*c^3*g^5*h^2*e - 33*b^5*c*g^4*h^3*e - 578*a*b^3*c^2*g^4*h^3*e - 480*a^2*b*c^3*g^4*h^3*e + 144*a*b^4*c*g^3*h^4*e + 936*a^2*b^2*c^2*g^3*h^4*e + 208*a^3*c^3*g^3*h^4*e - 237*a^2*b^3*c*g^2*h^5*e - 676*a^3*b*c^2*g^2*h^5*e + 174*a^3*b^2*c*g*h^6*e + 184*a^4*c^2*g*h^6*e - 48*a^4*b*c*h^7*e)/((c^(3/2)*g^2*h^6 - b*sqrt(c)*g*h^7 + a*sqrt(c)*h^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^3) - 1/8*(80*c^2*f*g^2 - 48*b*c*f*g*h + 8*c^2*d*h^2 + 3*b^2*f*h^2 + 12*a*c*f*h^2 - 32*c^2*g*h*e + 12*b*c*h^2*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(sqrt(c)*h^6)$$

**maple [B]** time = 0.03, size = 40092, normalized size = 48.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^4,x)

[Out] result too large to display

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4,x)

[Out] int((((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*4,x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*4, x)

$$3.200 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5} dx$$

Optimal. Leaf size=1097

$$\frac{(fg^2 - h(eg - dh))(cx^2 + bx + a)^{5/2} (16c^2(5fg - eh)g^4 - 4ch(bg(31fg^2 - 5ehg + 3dh^2) - ah(25fg^2 - 5ehg)) - 4h(CG^2 - bhg + ah^2)(g + hx)^4}{4h(CG^2 - bhg + ah^2)(g + hx)^4}$$

**Rubi [A]** time = 3.12, antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1650, 810, 812, 843, 621, 206, 724}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x]

[Out] (((64\*c^3\*g^4\*(5\*f\*g - e\*h))/h - 16\*c^2\*g^2\*(b\*g\*(41\*f\*g - 7\*e\*h) - 8\*a\*h\*(5\*f\*g - e\*h) + 4\*c\*h\*(2\*b^2\*g^2\*(46\*f\*g - 5\*e\*h) + 16\*a^2\*h^2\*(5\*f\*g - e\*h) - a\*b\*h\*(173\*f\*g^2 - 25\*e\*g\*h - 3\*d\*h^2)) - b\*h^2\*(48\*a^2\*f\*h^2 - 8\*a\*b\*h\*(10\*f\*g + e\*h) + b^2\*(35\*f\*g^2 + 5\*e\*g\*h + 3\*d\*h^2)) + 2\*c\*(16\*c^2\*g^3\*(5\*f\*g - e\*h) - 4\*c\*h\*(6\*b\*g^2\*(6\*f\*g - e\*h) - a\*h\*(35\*f\*g^2 - h\*(7\*e\*g - 3\*d\*h))) + h^2\*(48\*a^2\*f\*h^2 - 8\*a\*b\*h\*(14\*f\*g - e\*h) + b^2\*(61\*f\*g^2 - h\*(5\*e\*g + 3\*d\*h))))\*x)\*Sqrt[a + b\*x + c\*x^2])/((64\*h^4\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)) - (((16\*c^2\*g^4\*(5\*f\*g - e\*h))/h - h\*(16\*a^2\*h^2\*(f\*g - 2\*e\*h) - b^2\*g\*(35\*f\*g^2 + 5\*e\*g\*h + 3\*d\*h^2) + 4\*a\*b\*h\*(7\*f\*g^2 + 7\*e\*g\*h + 3\*d\*h^2)) - 4\*c\*g\*(b\*g\*(31\*f\*g^2 - 5\*e\*g\*h + 3\*d\*h^2) - a\*h\*(25\*f\*g^2 - 5\*e\*g\*h + 9\*d\*h^2)) + 3\*h\*((40\*c^2\*f\*g^4)/h + 16\*a^2\*f\*h^3 - 8\*c^2\*g^2\*(e\*g + d\*h) - 8\*a\*b\*h^2\*(6\*f\*g - e\*h) + 4\*a\*c\*h\*(17\*f\*g^2 - h\*(5\*e\*g - d\*h)) - 8\*b\*c\*g\*(9\*f\*g^2 - h\*(2\*e\*g + d\*h)) + b^2\*h\*(29\*f\*g^2 - h\*(5\*e\*g + 3\*d\*h))))\*x)\*(a + b\*x + c\*x^2)^(3/2))/(96\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^3 - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(5/2))/(4\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^4) - (Sqrt[c]\*(10\*c\*f\*g - 2\*c\*e\*h - 3\*b\*f\*h)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*h^6) + ((128\*c^4\*g^5\*(5\*f\*g - e\*h) - 64\*c^3\*g^3\*h\*(b\*g\*(28\*f\*g - 5\*e\*h) - 5\*a\*h\*(5\*f\*g - e\*h)) + 8\*c\*h^3\*(24\*a^3\*f\*h^3 - 12\*a^2\*b\*h^2\*(10\*f\*g - e\*h) - 5\*b^3\*g^2\*(14\*f\*g - e\*h) + 3\*a\*b^2\*h\*(55\*f\*g^2 - 5\*e\*g\*h - d\*h^2)) - 48\*c^2\*h^2\*(10\*a\*b\*g^2\*h\*(6\*f\*g - e\*h) - 5\*b^2\*g^3\*(7\*f\*g - e\*h) - a^2\*h^2\*(25\*f\*g^2 - 5\*e\*g\*h + d\*h^2)) + b^2\*h^4\*(48\*a^2\*f\*h^2 - 8\*a\*b\*h\*(10\*f\*g + e\*h) + b^2\*(35\*f\*g^2 + 5\*e\*g\*h + 3\*d\*h^2))))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])]/(128\*h^6\*(c\*g^2 - b\*g\*h + a\*h^2)^(5/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 810

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*((d\*g - e\*f\*(m + 2))\*(c\*d^2 - b\*d\*e + a\*e^2) - d\*p\*(2\*c\*d - b\*e)\*(e\*f - d\*g) - e\*(g\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2) + p\*(2\*c\*d - b\*e)\*(e\*f - d\*g))\*x)/(e^2\*(m + 1)\*(m + 2)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[p/(e^2\*(m + 1)\*(m + 2)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[2\*a\*c\*e\*(e\*f - d\*g)\*(m + 2) + b^2\*e\*(d\*g\*(p + 1) - e\*f\*(m + p + 2)) + b\*(a\*e^2\*g\*(m + 1) - c\*d\*(d\*g\*(2\*p + 1) - e\*f\*(m + 2\*p + 2))) - c\*(2\*c\*d\*(d\*g\*(2\*p + 1) - e\*f\*(m + 2\*p + 2)) - e\*(2\*a\*e\*g\*(m + 1) - b\*(d\*g\*(m - 2\*p) + e\*f\*(m + 2\*p + 2)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2\*p, 0] && !ILtQ[m + 2\*p + 3, 0]

Rule 812

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(e\*f\*(m + 2\*p + 2) - d\*g\*(2\*p + 1) + e\*g\*(m + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(e^2\*(m + 1)\*(m + 2\*p + 2)), x] + Dist[p/(e^2\*(m + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[g\*(b\*d + 2\*a\*e + 2\*a\*e\*m + 2\*b\*d\*p) - f\*b\*e\*(m + 2\*p + 2) + (g\*(2\*c\*d + b\*e + b\*e\*m + 4\*c\*d\*p) - 2\*c\*e\*f\*(m + 2\*p + 2))\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2\*p

+ 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^5} dx &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{4h(cg^2 - bgh + ah^2)(g + hx)^4} - \int \frac{\left(\frac{1}{2}(-8cdg + 5beg + 8afg - \frac{5bf^2g^2}{h} + 3bdh - \right.}{4(d} \\
&= -\frac{\left(\frac{16c^2g^4(5fg - eh)}{h} - h(16a^2h^2(fg - 2eh) - b^2g(35fg^2 + 5egh + 3dh^2) + 4}{4} \right.}{4} \\
&= \frac{\left(\frac{64c^3g^4(5fg - eh)}{h} - 16c^2g^2(bg(41fg - 7eh) - 8ah(5fg - eh)) + 4ch(2b^2g^2(4}{4} \right.}{4} \\
&= \frac{\left(\frac{64c^3g^4(5fg - eh)}{h} - 16c^2g^2(bg(41fg - 7eh) - 8ah(5fg - eh)) + 4ch(2b^2g^2(4}{4} \right.}{4} \\
&= \frac{\left(\frac{64c^3g^4(5fg - eh)}{h} - 16c^2g^2(bg(41fg - 7eh) - 8ah(5fg - eh)) + 4ch(2b^2g^2(4}{4} \right.}{4} \\
&= \frac{\left(\frac{64c^3g^4(5fg - eh)}{h} - 16c^2g^2(bg(41fg - 7eh) - 8ah(5fg - eh)) + 4ch(2b^2g^2(4}{4} \right.}{4}
\end{aligned}$$

**Mathematica [B]** time = 6.62, size = 46895, normalized size = 42.75

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5,x]

[Out] Result too large to show

**IntegrateAlgebraic [F]** time = 180.10, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x]



[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.06, size = 57957, normalized size = 52.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x)

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^5,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

[Out] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*5, x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*5, x)

$$3.201 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6} dx$$

Optimal. Leaf size=1226

$$\frac{(fg^2 - h(eg - dh))(cx^2 + bx + a)^{5/2} (16c^2fg^5 - 2ch(13bfg^3 - 10afhg^2 + 3bdh^2g - 6adh^3)g - h^2(-g(7fg^2 - 5h(cg^2 - bhg + ah^2))(g + hx)^5))}{5h(cg^2 - bhg + ah^2)(g + hx)^5}$$

Rubi [A] time = 4.00, antiderivative size = 1223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 810, 843, 621, 206, 724}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6,x]

[Out] -((((128\*c^4\*f\*g^7)/h - 32\*c^3\*f\*g^5\*(11\*b\*g - 10\*a\*h) + 8\*c^2\*g\*h\*(38\*b^2\*f\*g^4 + 2\*a^2\*h^2\*(13\*f\*g^2 + 3\*d\*h^2) - a\*b\*g\*h\*(65\*f\*g^2 + 3\*d\*h^2)) - b\*h^3\*(b\*g - 2\*a\*h)\*(16\*a^2\*f\*h^2 - 2\*a\*b\*h\*(10\*f\*g + 3\*e\*h) + b^2\*(7\*f\*g^2 + 3\*h\*(e\*g + d\*h))) - 2\*c\*h^2\*(8\*a^3\*h^3\*(2\*f\*g - 3\*e\*h) - 2\*a\*b^2\*g^2\*h\*(34\*f\*g + 3\*e\*h) + b^3\*(35\*f\*g^4 - 3\*d\*g^2\*h^2) + 4\*a^2\*b\*h^2\*(5\*f\*g^2 + 3\*h\*(2\*e\*g + d\*h))) + (128\*c\*f\*(c\*g^2 - h\*(b\*g - a\*h))^3 + (2\*c\*g - b\*h)\*(32\*c^3\*f\*g^5 - 8\*c^2\*g\*h\*(10\*b\*f\*g^3 - 11\*a\*f\*g^2\*h + 3\*a\*d\*h^3) + 2\*c\*h^2\*(4\*a^2\*h^2\*(10\*f\*g - 3\*e\*h) - 6\*a\*b\*h\*(11\*f\*g^2 - e\*g\*h - d\*h^2) + b^2\*(29\*f\*g^3 + 3\*d\*g\*h^2)) - b\*h^3\*(16\*a^2\*f\*h^2 - 2\*a\*b\*h\*(10\*f\*g + 3\*e\*h) + b^2\*(7\*f\*g^2 + 3\*h\*(e\*g + d\*h)))))\*x)\*Sqrt[a + b\*x + c\*x^2])/((128\*h^4\*(c\*g^2 - b\*g\*h + a\*h^2)^3\*(g + h\*x)^2) - ((16\*c^2\*f\*g^5 - 2\*c\*g\*h\*(13\*b\*f\*g^3 - 10\*a\*f\*g^2\*h + 3\*b\*d\*g\*h^2 - 6\*a\*d\*h^3) - h^2\*(4\*a^2\*h^2\*(2\*f\*g - 3\*e\*h) - b^2\*g\*(7\*f\*g^2 + 3\*h\*(e\*g + d\*h)) + 2\*a\*b\*h\*(f\*g^2 + 3\*h\*(2\*e\*g + d\*h))) + h^2\*(16\*a^2\*f\*h^3 + 4\*a\*c\*g\*h\*(14\*f\*g - 3\*e\*h) + c^2\*((28\*f\*g^4)/h - 12\*d\*g^2\*h) + b^2\*h\*(25\*f\*g^2 - 3\*h\*(e\*g + d\*h)) - b\*(56\*c\*f\*g^3 - 6\*c\*g\*h\*(e\*g + 2\*d\*h) + 2\*a\*h^2\*(22\*f\*g - 3\*e\*h)))\*x)\*(a + b\*x + c\*x^2)^(3/2))/(48\*h^3\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^4) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(5/2))/(5\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^5) + (c^(3/2)\*f\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/h^6 - ((256\*c^5\*f\*g^7 - 896\*c^4\*f\*g^5\*h\*(b\*g - a\*h) + 32\*c^3\*g\*h^2\*(35\*b^2\*f\*g^4 - 70\*a\*b\*f\*g^3\*h + a^2\*h^2\*(35\*f\*g^2 - 3\*d\*h^2)) - 16\*c^2\*h^3\*(35\*b^3\*f\*g^4 - 6\*a^3\*h^3\*(6\*f\*g - e\*h) + 3\*a^2\*b\*h^2\*(35\*f\*g^2 - e\*g\*h - d\*h^2) - 3\*a\*b^2\*g\*h\*(35\*f\*g^2 + d\*h^2)) + b^3\*h^5\*(16\*a^2\*f\*h^2 - 2\*a\*b\*h\*(10\*f\*g + 3\*e\*h) + b^2\*(7\*f\*g^2 + 3\*h\*(e\*g + d\*h))) - 2\*b\*c\*h^4\*(96\*a^3\*f\*h^3 - 24\*a^2\*b\*h^2\*(8\*f\*g + e\*h) - b^3\*(35\*f\*g^3 - 3\*d\*g\*h^2) + 4\*a\*b^2\*h\*(35\*f\*g^2 + 3\*h\*(e\*g + d\*h))))\*ArcTanh[(b\*g -

$$\frac{2*a*h + (2*c*g - b*h)*x}{(2*\sqrt{c*g^2 - b*g*h + a*h^2}*\sqrt{a + b*x + c*x^2})} / (256*h^6*(c*g^2 - b*g*h + a*h^2)^{(7/2)})$$

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 810

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/ (e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

### Rule 843

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1650

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^6} dx &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{5h(cg^2 - bgh + ah^2)(g + hx)^5} - \frac{\int \frac{\left(-\frac{5}{2}\left(2cdg - beg - 2afg + \frac{bfg^2}{h} - bdh + 2\right)\right)}{(g + hx)^6} dx}{5(cg^2 - bgh + ah^2)} \\
&= -\frac{(16c^2fg^5 - 2cgh(13bfg^3 - 10afg^2h + 3bdgh^2 - 6adh^3) - h^2(4a^2h^2 - 2ahg + g^2))}{5h(cg^2 - bgh + ah^2)(g + hx)^5} \\
&= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg - 10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13fg^2 - 2ahg + g^2)) - h^2(4a^2h^2 - 2ahg + g^2)\right)}{5h(cg^2 - bgh + ah^2)(g + hx)^5} \\
&= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg - 10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13fg^2 - 2ahg + g^2)) - h^2(4a^2h^2 - 2ahg + g^2)\right)}{5h(cg^2 - bgh + ah^2)(g + hx)^5} \\
&= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg - 10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13fg^2 - 2ahg + g^2)) - h^2(4a^2h^2 - 2ahg + g^2)\right)}{5h(cg^2 - bgh + ah^2)(g + hx)^5} \\
&= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg - 10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13fg^2 - 2ahg + g^2)) - h^2(4a^2h^2 - 2ahg + g^2)\right)}{5h(cg^2 - bgh + ah^2)(g + hx)^5}
\end{aligned}$$

**Mathematica [A]** time = 6.30, size = 1111, normalized size = 0.91

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x]

[Out] 
$$-\left(\frac{(a + x(b + cx))^{\frac{3}{2}}((gh(2fg - eh) - h(fg^2 - dh^2))(a + bx + cx^2)^{\frac{5}{2}})}{5(cg^2 - bgh + ah^2)(g + hx)^5} - \frac{((-2ah^2(2fg - eh) + cg(fg^2 - dh^2) + b(gh(2fg - eh) + h(fg^2 - dh^2))))((bg - 2ah + (2cg - bh)x)(a + bx + cx^2)^{\frac{3}{2}})}{8(cg^2 - bgh + ah^2)(g + hx)^4} - \frac{(3(b^2 - 4ac)((bg - 2ah + (2cg - bh)x) \sqrt{a + bx + cx^2})}{4(cg^2 - bgh + ah^2)(g + hx)^2} + \frac{(b^2 - 4ac) \operatorname{ArcTanh}\left(\frac{-(bg) + 2ah - (2cg - bh)x}{2\sqrt{cg^2 - bgh + ah^2}}\right) \sqrt{a + bx + cx^2}}{2\sqrt{cg^2 - bgh + ah^2}(4cg^2 - 4bgh + 4ah^2)}\right) / (16(cg^2 - bgh + ah^2)) / (2(cg^2 - bgh + ah^2)) / (h^2(a + bx + cx^2)^{\frac{3}{2}}) + \frac{f(a + x(b + cx))^{\frac{3}{2}}(-1/3(a + bx + cx^2)^{\frac{3}{2}}/h(g + hx)^3 + (-1/2((-2cg + bh)(a + bx + cx^2)^{\frac{3}{2}})/((cg^2 - bgh + ah^2)(g + hx)^2 - (((-cg(2cg - bh)) + (h(2b*cg + b^2h - 8ac*ah))/2)(a + bx + cx^2)^{\frac{3}{2}})/((-cg^2 + bgh - ah^2)(g + hx)) + (((-c(2cg - bh)/2)(4c^2g^2 - b^2h^2 - 4c*ah*(bg - 2ah))) + (c*ah(-10b^2*cg*ah + 8ac^2g*ah - b^3h^2 + 4b*c(2cg^2 + 3ah^2)))/2 + c^2h(4c^2g^2 - b^2h^2 - 4c*ah*(bg - 2ah))x) \sqrt{a + bx + cx^2}}{2c*ah^2} - \frac{((-16c^{5/2})(cg^2 - h(bg - ah))^2 \operatorname{ArcTanh}\left(\frac{b + 2cx}{2\sqrt{c} \sqrt{a + bx + cx^2}}\right))}{h} - \frac{4\sqrt{cg^2 - bgh + ah^2}(-c*ah(cg^2 - bgh + ah^2)(8b*c^2g^2 - 6b^2*cg*ah - 8ac^2g*ah - b^3h^2 + 12a*b*c*ah^2) + 16c^3g*(cg^2 - h(bg - ah))^2) \operatorname{ArcTanh}\left(\frac{-(bg) + 2ah - (2cg - bh)x}{2\sqrt{cg^2 - bgh + ah^2}}\right) \sqrt{a + bx + cx^2}}{h(4cg^2 - 4bgh + 4ah^2)} / (4c*ah^2) / (-cg^2 + bgh - ah^2) / (2(cg^2 - bgh + ah^2)) / (2*ah) / (h^2(a + bx + cx^2)^{\frac{3}{2}})$$

**IntegrateAlgebraic [F]** time = 180.06, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
```

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

[Out] Timed out

**maple** [B] time = 0.07, size = 76693, normalized size = 62.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x)

[Out] int(((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^6, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)\*(f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*6, x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)\*(d + e\*x + f\*x\*\*2)/(g + h\*x)\*\*6, x)



$$3.202 \quad \int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7} dx$$

Optimal. Leaf size=657

$$\frac{(a+bx+cx^2)^{3/2}(-2ah+x(2cg-bh)+bg)(24a^2fh^2-4c(a(dh^2-7egh+fg^2)+3bg(2dh+eg))-12abh(eh-192(g+hx)^4(ah^2-bgh+cg^2)^3))}{192(g+hx)^4(ah^2-bgh+cg^2)^3}$$

**Rubi [A]** time = 1.22, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1650, 806, 720, 724, 206}

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^7,x]

[Out] -((b^2 - 4\*a\*c)\*(24\*c^2\*d\*g^2 + 24\*a^2\*f\*h^2 - 12\*a\*b\*h\*(2\*f\*g + e\*h) - 4\*c\*(a\*f\*g^2 - a\*h\*(7\*e\*g - d\*h) + 3\*b\*g\*(e\*g + 2\*d\*h)) + b^2\*(7\*f\*g^2 + h\*(5\*e\*g + 7\*d\*h)))\*(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)\*Sqrt[a + b\*x + c\*x^2])/(512\*(c\*g^2 - b\*g\*h + a\*h^2)^4\*(g + h\*x)^2) + ((24\*c^2\*d\*g^2 + 24\*a^2\*f\*h^2 - 12\*a\*b\*h\*(2\*f\*g + e\*h) - 4\*c\*(a\*f\*g^2 - a\*h\*(7\*e\*g - d\*h) + 3\*b\*g\*(e\*g + 2\*d\*h)) + b^2\*(7\*f\*g^2 + h\*(5\*e\*g + 7\*d\*h)))\*(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)\*(a + b\*x + c\*x^2)^(3/2))/(192\*(c\*g^2 - b\*g\*h + a\*h^2)^3\*(g + h\*x)^4) - ((f\*g^2 - h\*(e\*g - d\*h))\*(a + b\*x + c\*x^2)^(5/2))/(6\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^6) + ((2\*c\*(5\*f\*g^3 + g\*h\*(e\*g - 7\*d\*h)) - h\*(17\*b\*f\*g^2 - b\*h\*(5\*e\*g + 7\*d\*h) - 12\*a\*h\*(2\*f\*g - e\*h)))\*(a + b\*x + c\*x^2)^(5/2))/(60\*h\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^5) + ((b^2 - 4\*a\*c)^2\*(24\*c^2\*d\*g^2 + 24\*a^2\*f\*h^2 - 12\*a\*b\*h\*(2\*f\*g + e\*h) - 4\*c\*(a\*f\*g^2 - a\*h\*(7\*e\*g - d\*h) + 3\*b\*g\*(e\*g + 2\*d\*h)) + b^2\*(7\*f\*g^2 + h\*(5\*e\*g + 7\*d\*h)))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])]/(1024\*(c\*g^2 - b\*g\*h + a\*h^2)^(9/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 720

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(d\*b - 2\*a\*e + (2\*c\*d - b\*e)\*x)\*(a + b\*x

```

+ c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]

```

### Rule 724

```

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

```

### Rule 806

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]

```

### Rule 1650

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^7} dx &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{6h(cg^2 - bgh + ah^2)(g + hx)^6} - \frac{\int \left( \frac{1}{2} \left( -12cdg + 5beg + 12afg - \frac{5bf_8^2}{h} + 7b \right) \right)}{6h(cg^2 - bgh + ah^2)(g + hx)^6} \\
&= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{6h(cg^2 - bgh + ah^2)(g + hx)^6} + \frac{(2c(5fg^3 + gh(eg - 7dh)) - h)}{6h(cg^2 - bgh + ah^2)(g + hx)^6} \\
&= \frac{(24c^2dg^2 + 24a^2fh^2 - 12abh(2fg + eh) - 4c(afg^2 - ah(7eg - dh)) + 3)}{192(cg^2 - bgh + ah^2)(g + hx)^6} \\
&= -\frac{(b^2 - 4ac)(24c^2dg^2 + 24a^2fh^2 - 12abh(2fg + eh) - 4c(afg^2 - ah(7eg - dh)) + 3)}{192(cg^2 - bgh + ah^2)(g + hx)^6} \\
&= -\frac{(b^2 - 4ac)(24c^2dg^2 + 24a^2fh^2 - 12abh(2fg + eh) - 4c(afg^2 - ah(7eg - dh)) + 3)}{192(cg^2 - bgh + ah^2)(g + hx)^6} \\
&= -\frac{(b^2 - 4ac)(24c^2dg^2 + 24a^2fh^2 - 12abh(2fg + eh) - 4c(afg^2 - ah(7eg - dh)) + 3)}{192(cg^2 - bgh + ah^2)(g + hx)^6}
\end{aligned}$$

**Mathematica [A]** time = 6.24, size = 766, normalized size = 1.17

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^7,x]

[Out] -((f\*(a + b\*x + c\*x^2)\*(a + x\*(b + c\*x))^(3/2))/(c\*h\*(g + h\*x)^6)) + ((a + x\*(b + c\*x))^(3/2)\*(-1/6\*((h\*(5\*b\*f\*g + 2\*c\*d\*h - 12\*a\*f\*h))/2 - (g\*(-7\*b\*f\*h + 2\*c\*(5\*f\*g + e\*h)))/2)\*(a + b\*x + c\*x^2)^(5/2))/((c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^6) - (((-c\*g\*(5\*c\*f\*g^2 - 6\*f\*h\*(b\*g - a\*h) + c\*h\*(e\*g - d\*h))) + (c\*h\*(5\*b\*f\*g^2 - b\*h\*(5\*e\*g + 7\*d\*h) + 12\*h\*(c\*d\*g - a\*f\*g + a\*e\*h)))/2)\*(a + b\*x + c\*x^2)^(5/2))/(5\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^5) - ((-

$$2*(-(a*c*h*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h)) - (c^2*g*(5*b*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h))/2) + b*(-(c*g*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h)) - (c*h*(5*b*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4 - (3*(b^2 - 4*a*c)*((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2]))/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])))/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(16*(c*g^2 - b*g*h + a*h^2)))/(2*(c*g^2 - b*g*h + a*h^2))/(6*(c*g^2 - b*g*h + a*h^2)))/(c*h*(a + b*x + c*x^2)^(3/2))$$

**IntegrateAlgebraic** [F] time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^7, x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^7,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^7,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.13, size = 100754, normalized size = 153.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)`

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details) Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(cx^2 + bx + a)^{3/2} (fx^2 + ex + d)}{(g + hx)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)`

[Out] `int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)`

[Out] `Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)`



Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Sym
bol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^8} dx &= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{7h(cg^2 - bgh + ah^2)(g + hx)^7} - \int \frac{\left(\frac{1}{2}(-14cdg + 5beg + 14afg - \frac{5bfg^2}{h} + 9bd^2)\right)}{7h(cg^2 - bgh + ah^2)(g + hx)^7} dx \\
&= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{7h(cg^2 - bgh + ah^2)(g + hx)^7} + \frac{(2c(5fg^3 + gh(2eg - 9dh)) - h^2d^2)}{7h(cg^2 - bgh + ah^2)(g + hx)^7} \\
&= -\frac{(fg^2 - h(eg - dh))(a + bx + cx^2)^{5/2}}{7h(cg^2 - bgh + ah^2)(g + hx)^7} + \frac{(2c(5fg^3 + gh(2eg - 9dh)) - h^2d^2)}{7h(cg^2 - bgh + ah^2)(g + hx)^7} \\
&= \frac{(48c^3dg^3 - 8c^2g(afg^2 - ah(8eg - 3dh)) + 3bg(eg + 3dh)) - bh(24a^2fh^2)}{7h(cg^2 - bgh + ah^2)(g + hx)^7} \\
&= -\frac{(b^2 - 4ac)(48c^3dg^3 - 8c^2g(afg^2 - ah(8eg - 3dh)) + 3bg(eg + 3dh)) - bh(24a^2fh^2)}{7h(cg^2 - bgh + ah^2)(g + hx)^7} \\
&= -\frac{(b^2 - 4ac)(48c^3dg^3 - 8c^2g(afg^2 - ah(8eg - 3dh)) + 3bg(eg + 3dh)) - bh(24a^2fh^2)}{7h(cg^2 - bgh + ah^2)(g + hx)^7} \\
&= -\frac{(b^2 - 4ac)(48c^3dg^3 - 8c^2g(afg^2 - ah(8eg - 3dh)) + 3bg(eg + 3dh)) - bh(24a^2fh^2)}{7h(cg^2 - bgh + ah^2)(g + hx)^7}
\end{aligned}$$



**Mathematica [A]** time = 6.42, size = 1221, normalized size = 1.15



Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^8, x]

[Out] 
$$-1/2*(f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(c*h*(g + h*x)^7) + ((a + x*(b + c*x))^(3/2)*(-1/7*((h*(5*b*f*g + 4*c*d*h - 14*a*f*h))/2 - (g*(10*c*f*g + 4*c*e*h - 9*b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) - (-1/6*((2*c*g*(5*c*f*g^2 - 7*f*h*(b*g - a*h) + 2*c*h*(e*g - d*h)) - c*h*(5*b*f*g^2 - b*h*(5*e*g + 9*d*h) + 14*h*(c*d*g - a*f*g + a*e*h)))*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^6) - ((c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h))) - (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h)) + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h))))/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*c^2*h*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h))) + (c^2*g*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h)) + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h))))/2) + b*(c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h))) + (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h)) + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h))))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2]))/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(16*(c*g^2 - b*g*h + a*h^2)))/(2*(c*g^2 - b*g*h + a*h^2))/(6*(c*g^2 - b*g*h + a*h^2))/(7*(c*g^2 - b*g*h + a*h^2))/(2*c*h*(a + b*x + c*x^2)^(3/2))$$

**IntegrateAlgebraic [F]** time = 180.12, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b\*x + c\*x^2)^(3/2)\*(d + e\*x + f\*x^2))/(g + h\*x)^8, x]

[Out] \$Aborted

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^8,x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^8,x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.18, size = 126612, normalized size = 119.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^8,x)

[Out] result too large to display

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)^(3/2)\*(f\*x^2+e\*x+d)/(h\*x+g)^8,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 zero or nonzero?

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

$$3.204 \quad \int (1 + 2x)^3 \sqrt{2 - x + 3x^2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=143

$$\frac{2}{21} (3x^2 - x + 2)^{3/2} (2x+1)^4 + \frac{67}{378} (3x^2 - x + 2)^{3/2} (2x+1)^3 + \frac{17}{105} (3x^2 - x + 2)^{3/2} (2x+1)^2 - \frac{(26982x + 75295)(3x^2 - x + 2)^{3/2}}{68040}$$

**Rubi [A]** time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{21} (3x^2 - x + 2)^{3/2} (2x+1)^4 + \frac{67}{378} (3x^2 - x + 2)^{3/2} (2x+1)^3 + \frac{17}{105} (3x^2 - x + 2)^{3/2} (2x+1)^2 - \frac{(26982x + 75295)(3x^2 - x + 2)^{3/2}}{68040} + \frac{5393(1 - 6x)\sqrt{3x^2 - x + 2}}{15552} + \frac{124039 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (5393\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/15552 + (17\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2))/105 + (67\*(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2))/378 + (2\*(1 + 2\*x)^4\*(2 - x + 3\*x^2)^(3/2))/21 - ((75295 + 26982\*x)\*(2 - x + 3\*x^2)^(3/2))/68040 + (124039\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(31104\*Sqrt[3])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (1+2x)^3 \sqrt{2-x+3x^2} (1+3x+4x^2) dx &= \frac{2}{21} (1+2x)^4 (2-x+3x^2)^{3/2} + \frac{1}{84} \int (1+2x)^3 (-32+268x) \sqrt{2-x+3x^2} dx \\
&= \frac{67}{378} (1+2x)^3 (2-x+3x^2)^{3/2} + \frac{2}{21} (1+2x)^4 (2-x+3x^2)^{3/2} + \frac{1}{84} \int (1+2x)^3 (-32+268x) \sqrt{2-x+3x^2} dx \\
&= \frac{17}{105} (1+2x)^2 (2-x+3x^2)^{3/2} + \frac{67}{378} (1+2x)^3 (2-x+3x^2)^{3/2} + \frac{2}{21} (1+2x)^4 (2-x+3x^2)^{3/2} \\
&= \frac{17}{105} (1+2x)^2 (2-x+3x^2)^{3/2} + \frac{67}{378} (1+2x)^3 (2-x+3x^2)^{3/2} + \frac{2}{21} (1+2x)^4 (2-x+3x^2)^{3/2} \\
&= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105} (1+2x)^2 (2-x+3x^2)^{3/2} + \frac{67}{378} (1+2x)^3 (2-x+3x^2)^{3/2} \\
&= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105} (1+2x)^2 (2-x+3x^2)^{3/2} + \frac{67}{378} (1+2x)^3 (2-x+3x^2)^{3/2} \\
&= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105} (1+2x)^2 (2-x+3x^2)^{3/2} + \frac{67}{378} (1+2x)^3 (2-x+3x^2)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 70, normalized size = 0.49

$$\frac{6\sqrt{3x^2-x+2} (2488320x^6 + 6462720x^5 + 7491456x^4 + 5497776x^3 + 3280872x^2 + 1493894x - 543069) - 4341365\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{3265920}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(-543069 + 1493894\*x + 3280872\*x^2 + 5497776\*x^3 + 7491456\*x^4 + 6462720\*x^5 + 2488320\*x^6) - 4341365\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/3265920

**IntegrateAlgebraic [A]** time = 0.68, size = 85, normalized size = 0.59

$$\frac{124039 \log(2\sqrt{3}\sqrt{3x^2-x+2} - 6x+1)}{31104\sqrt{3}} + \frac{\sqrt{3x^2-x+2} (2488320x^6 + 6462720x^5 + 7491456x^4 + 5497776x^3 + 3280872x^2 + 1493894x - 543069)}{544320}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out]  $(\text{Sqrt}[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 + 7491456*x^4 + 6462720*x^5 + 2488320*x^6))/544320 + (124039*\text{Log}[1 - 6*x + 2*\text{Sqrt}[3]*\text{Sqrt}[2 - x + 3*x^2]])/(31104*\text{Sqrt}[3])$

**fricas** [A] time = 1.81, size = 83, normalized size = 0.58

$$\frac{1}{544320} (2488320x^6 + 6462720x^5 + 7491456x^4 + 5497776x^3 + 3280872x^2 + 1493894x - 543069)\sqrt{3x^2 - x + 2} + \frac{124039}{186624} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")`

[Out]  $1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*x^2 + 1493894*x - 543069)*\text{sqrt}(3*x^2 - x + 2) + 124039/186624*\text{sqrt}(3)*\log(4*\text{sqrt}(3)*\text{sqrt}(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)$

**giac** [A] time = 0.26, size = 78, normalized size = 0.55

$$\frac{1}{544320} (2(12(6(8(30(72x + 187)x + 6503)x + 38179)x + 136703)x + 746947)x - 543069)\sqrt{3x^2 - x + 2} + \frac{124039}{93312} \sqrt{3} \log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")`

[Out]  $1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x + 746947)*x - 543069)*\text{sqrt}(3*x^2 - x + 2) + 124039/93312*\text{sqrt}(3)*\log(-2*\text{sqrt}(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)$

**maple** [A] time = 0.02, size = 115, normalized size = 0.80

$$\frac{32(3x^2 - x + 2)^{\frac{3}{2}}x^4}{21} + \frac{844(3x^2 - x + 2)^{\frac{3}{2}}x^3}{189} + \frac{1594(3x^2 - x + 2)^{\frac{3}{2}}x^2}{315} + \frac{7849(3x^2 - x + 2)^{\frac{3}{2}}x}{3780} - \frac{124039\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x - \frac{1}{6}\right)}{23}\right)}{93312} - \frac{5393(6x - 1)\sqrt{3x^2 - x + 2}}{15552} - \frac{45739(3x^2 - x + 2)^{\frac{3}{2}}}{68040}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)`

[Out]  $32/21*x^4*(3*x^2-x+2)^(3/2)+844/189*x^3*(3*x^2-x+2)^(3/2)+7849/3780*x*(3*x^2-x+2)^(3/2)+1594/315*x^2*(3*x^2-x+2)^(3/2)-124039/93312*3^(1/2)*\operatorname{arcsinh}(6/23*23^(1/2)*(x-1/6))-5393/15552*(6*x-1)*(3*x^2-x+2)^(1/2)-45739/68040*(3*x^2-x+2)^(3/2)$

**maxima** [A] time = 0.95, size = 126, normalized size = 0.88

$$\frac{32}{21} (3x^2 - x + 2)^{\frac{3}{2}}x^4 + \frac{844}{189} (3x^2 - x + 2)^{\frac{3}{2}}x^3 + \frac{1594}{315} (3x^2 - x + 2)^{\frac{3}{2}}x^2 + \frac{7849}{3780} (3x^2 - x + 2)^{\frac{3}{2}}x - \frac{45739}{68040} (3x^2 - x + 2)^{\frac{3}{2}} - \frac{5393}{2592} \sqrt{3x^2 - x + 2}x - \frac{124039}{93312} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(6x - 1)\right) + \frac{5393}{15552} \sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
[Out] 32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594/
315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739/6
8040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93312
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x + 2)
mupad [B] time = 5.54, size = 170, normalized size = 1.19
```

$$\frac{1594x^2(3x^2-x+2)^{3/2}}{315} + \frac{844x^3(3x^2-x+2)^{3/2}}{189} + \frac{32x^4(3x^2-x+2)^{3/2}}{21} - \frac{137057\sqrt{3}\ln\left(\sqrt{3x^2-x+2} + \frac{\sqrt{3}(x-1)}{3}\right)}{136080} - \frac{5959\left(\frac{x}{2} - \frac{1}{12}\right)\sqrt{3x^2-x+2}}{1890} - \frac{45739\sqrt{3x^2-x+2}(72x^2-6x+45)}{1632960} + \frac{7849x(3x^2-x+2)^{3/2}}{3780} - \frac{1051997\sqrt{3}\ln\left(2\sqrt{3x^2-x+2} + \frac{\sqrt{3}(6x-1)}{3}\right)}{3265920}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
[Out] (1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
+ (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)^(
1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x + 2
)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/1632960 +
(7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 - x +
2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (2x + 1)^3 \sqrt{3x^2 - x + 2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
[Out] Integral((2*x + 1)**3*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```



$$3.205 \quad \int (1 + 2x)^2 \sqrt{2 - x + 3x^2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=118

$$\frac{1}{9} (3x^2 - x + 2)^{3/2} (2x+1)^3 + \frac{1}{5} (3x^2 - x + 2)^{3/2} (2x+1)^2 + \frac{1}{810} (306x+25) (3x^2 - x + 2)^{3/2} + \frac{235(1 - 6x)\sqrt{3x^2 - x + 2}}{1296}$$

**Rubi [A]** time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{9} (3x^2 - x + 2)^{3/2} (2x+1)^3 + \frac{1}{5} (3x^2 - x + 2)^{3/2} (2x+1)^2 + \frac{1}{810} (306x+25) (3x^2 - x + 2)^{3/2} + \frac{235(1 - 6x)\sqrt{3x^2 - x + 2}}{1296} + \frac{5405 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (235\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/1296 + ((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2))/5 + ((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2))/9 + ((25 + 306\*x)\*(2 - x + 3\*x^2)^(3/2))/810 + (5405\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(2592\*Sqrt[3])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x]

] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 832

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p\*Simp[m\*(c\*d\*f - a\*e\*g) + d\*(2\*c\*f - b\*g)\*(p + 1) + (m\*(c\*e\*f + c\*d\*g - b\*e\*g) + e\*(p + 1)\*(2\*c\*f - b\*g))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int (1+2x)^2 \sqrt{2-x+3x^2} (1+3x+4x^2) dx &= \frac{1}{9}(1+2x)^3 (2-x+3x^2)^{3/2} + \frac{1}{72} \int (1+2x)^2 (-12+216x) \sqrt{2-x+3x^2} dx \\
&= \frac{1}{5}(1+2x)^2 (2-x+3x^2)^{3/2} + \frac{1}{9}(1+2x)^3 (2-x+3x^2)^{3/2} + \frac{\int (1+2x)^2 (-12+216x) \sqrt{2-x+3x^2} dx}{72} \\
&= \frac{1}{5}(1+2x)^2 (2-x+3x^2)^{3/2} + \frac{1}{9}(1+2x)^3 (2-x+3x^2)^{3/2} + \frac{1}{810} \int (1+2x)^2 (-12+216x) \sqrt{2-x+3x^2} dx \\
&= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296} + \frac{1}{5}(1+2x)^2 (2-x+3x^2)^{3/2} + \frac{1}{9}(1+2x)^3 (2-x+3x^2)^{3/2} \\
&= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296} + \frac{1}{5}(1+2x)^2 (2-x+3x^2)^{3/2} + \frac{1}{9}(1+2x)^3 (2-x+3x^2)^{3/2} \\
&= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296} + \frac{1}{5}(1+2x)^2 (2-x+3x^2)^{3/2} + \frac{1}{9}(1+2x)^3 (2-x+3x^2)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 65, normalized size = 0.55

$$\frac{6\sqrt{3x^2-x+2} (17280x^5 + 35712x^4 + 33552x^3 + 22344x^2 + 14638x + 5607) - 27025\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{38880}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(5607 + 14638\*x + 22344\*x^2 + 33552\*x^3 + 35712\*x^4 + 17280\*x^5) - 27025\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/38880

**IntegrateAlgebraic [A]** time = 0.58, size = 80, normalized size = 0.68

$$\frac{5405 \log(2\sqrt{3}\sqrt{3x^2-x+2} - 6x + 1)}{2592\sqrt{3}} + \frac{\sqrt{3x^2-x+2} (17280x^5 + 35712x^4 + 33552x^3 + 22344x^2 + 14638x + 5607)}{6480}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(5607 + 14638\*x + 22344\*x^2 + 33552\*x^3 + 35712\*x^4 + 17280\*x^5))/6480 + (5405\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(2592\*Sqrt[3])

**fricas** [A] time = 0.75, size = 78, normalized size = 0.66

$$\frac{1}{6480} (17280x^5 + 35712x^4 + 33552x^3 + 22344x^2 + 14638x + 5607)\sqrt{3x^2 - x + 2} + \frac{5405}{15552} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2),x, algorithm="fricas")

[Out] 1/6480\*(17280\*x^5 + 35712\*x^4 + 33552\*x^3 + 22344\*x^2 + 14638\*x + 5607)\*sqrt(3\*x^2 - x + 2) + 5405/15552\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac** [A] time = 0.30, size = 73, normalized size = 0.62

$$\frac{1}{6480} (2(12(6(8(15x + 31)x + 233)x + 931)x + 7319)x + 5607)\sqrt{3x^2 - x + 2} + \frac{5405}{7776} \sqrt{3} \log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2),x, algorithm="giac")

[Out] 1/6480\*(2\*(12\*(6\*(8\*(15\*x + 31)\*x + 233)\*x + 931)\*x + 7319)\*x + 5607)\*sqrt(3\*x^2 - x + 2) + 5405/7776\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple** [A] time = 0.01, size = 98, normalized size = 0.83

$$\frac{8(3x^2 - x + 2)^{\frac{3}{2}}x^3}{9} + \frac{32(3x^2 - x + 2)^{\frac{3}{2}}x^2}{15} + \frac{83(3x^2 - x + 2)^{\frac{3}{2}}x}{45} - \frac{5405\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776} + \frac{277(3x^2 - x + 2)^{\frac{3}{2}}}{810} - \frac{235(6x - 1)\sqrt{3x^2 - x + 2}}{1296}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)^2\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2),x)

[Out] 8/9\*(3\*x^2-x+2)^(3/2)\*x^3+32/15\*(3\*x^2-x+2)^(3/2)\*x^2+83/45\*(3\*x^2-x+2)^(3/2)\*x+277/810\*(3\*x^2-x+2)^(3/2)-235/1296\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)-5405/7776\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))

**maxima** [A] time = 0.96, size = 109, normalized size = 0.92

$$\frac{8}{9}(3x^2 - x + 2)^{\frac{3}{2}}x^3 + \frac{32}{15}(3x^2 - x + 2)^{\frac{3}{2}}x^2 + \frac{83}{45}(3x^2 - x + 2)^{\frac{3}{2}}x + \frac{277}{810}(3x^2 - x + 2)^{\frac{3}{2}} - \frac{235}{216}\sqrt{3x^2 - x + 2}x - \frac{5405}{7776}\sqrt{3} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x - 1)\right) + \frac{235}{1296}\sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2),x, algorithm="maxima")

[Out] 8/9\*(3\*x^2 - x + 2)^(3/2)\*x^3 + 32/15\*(3\*x^2 - x + 2)^(3/2)\*x^2 + 83/45\*(3\*x^2 - x + 2)^(3/2)\*x + 277/810\*(3\*x^2 - x + 2)^(3/2) - 235/216\*sqrt(3\*x^2 -

$x + 2) * x - 5405/7776 * \sqrt{3} * \operatorname{arcsinh}(1/23 * \sqrt{23} * (6 * x - 1)) + 235/1296 * \sqrt{3 * x^2 - x + 2}$

**mupad [B]** time = 5.15, size = 153, normalized size = 1.30

$$\frac{32x^2(3x^2-x+2)^{3/2}}{15} + \frac{8x^3(3x^2-x+2)^{3/2}}{9} - \frac{2783\sqrt{3}\ln\left(\sqrt{3x^2-x+2} + \frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{3240} - \frac{121\left(\frac{x}{2} - \frac{1}{12}\right)\sqrt{3x^2-x+2}}{45} + \frac{277\sqrt{3x^2-x+2}(72x^2-6x+45)}{19440} + \frac{83x(3x^2-x+2)^{3/2}}{45} + \frac{6371\sqrt{3}\ln\left(2\sqrt{3x^2-x+2} + \frac{\sqrt{3}(6x-1)}{3}\right)}{38880}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1), x)`

[Out]  $(32 * x^2 * (3 * x^2 - x + 2)^{(3/2)}) / 15 + (8 * x^3 * (3 * x^2 - x + 2)^{(3/2)}) / 9 - (2783 * 3^{(1/2)} * \log((3 * x^2 - x + 2)^{(1/2)} + (3^{(1/2)} * (3 * x - 1/2)) / 3)) / 3240 - (121 * (x/2 - 1/12) * (3 * x^2 - x + 2)^{(1/2)}) / 45 + (277 * (3 * x^2 - x + 2)^{(1/2)} * (72 * x^2 - 6 * x + 45)) / 19440 + (83 * x * (3 * x^2 - x + 2)^{(3/2)}) / 45 + (6371 * 3^{(1/2)} * \log(2 * (3 * x^2 - x + 2)^{(1/2)} + (3^{(1/2)} * (6 * x - 1)) / 3)) / 38880$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 1)^2 \sqrt{3x^2 - x + 2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2), x)`

[Out] `Integral((2*x + 1)**2*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)`

$$3.206 \quad \int (1 + 2x)\sqrt{2 - x + 3x^2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=93

$$\frac{2}{15} (3x^2 - x + 2)^{3/2} (2x+1)^2 + \frac{(738x + 745)(3x^2 - x + 2)^{3/2}}{1620} + \frac{19(1 - 6x)\sqrt{3x^2 - x + 2}}{2592} + \frac{437 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{15} (3x^2 - x + 2)^{3/2} (2x+1)^2 + \frac{(738x + 745)(3x^2 - x + 2)^{3/2}}{1620} + \frac{19(1 - 6x)\sqrt{3x^2 - x + 2}}{2592} + \frac{437 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (19\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/2592 + (2\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2))/15 + ((745 + 738\*x)\*(2 - x + 3\*x^2)^(3/2))/1620 + (437\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(5184\*Sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 779

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (1 + 2x)\sqrt{2 - x + 3x^2} (1 + 3x + 4x^2) dx &= \frac{2}{15}(1 + 2x)^2 (2 - x + 3x^2)^{3/2} + \frac{1}{60} \int (1 + 2x)(8 + 164x)\sqrt{2 - x + 3x^2} dx \\
&= \frac{2}{15}(1 + 2x)^2 (2 - x + 3x^2)^{3/2} + \frac{(745 + 738x)(2 - x + 3x^2)^{3/2}}{1620} - \frac{1}{21} \int (1 + 2x)\sqrt{2 - x + 3x^2} dx \\
&= \frac{19(1 - 6x)\sqrt{2 - x + 3x^2}}{2592} + \frac{2}{15}(1 + 2x)^2 (2 - x + 3x^2)^{3/2} + \frac{(745 + 738x)(2 - x + 3x^2)^{3/2}}{1620} \\
&= \frac{19(1 - 6x)\sqrt{2 - x + 3x^2}}{2592} + \frac{2}{15}(1 + 2x)^2 (2 - x + 3x^2)^{3/2} + \frac{(745 + 738x)(2 - x + 3x^2)^{3/2}}{1620} \\
&= \frac{19(1 - 6x)\sqrt{2 - x + 3x^2}}{2592} + \frac{2}{15}(1 + 2x)^2 (2 - x + 3x^2)^{3/2} + \frac{(745 + 738x)(2 - x + 3x^2)^{3/2}}{1620}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.65

$$\frac{6\sqrt{3x^2 - x + 2} (20736x^4 + 31536x^3 + 24072x^2 + 17374x + 15471) - 2185\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{77760}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(15471 + 17374\*x + 24072\*x^2 + 31536\*x^3 + 20736\*x^4) - 2185\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/77760

**IntegrateAlgebraic [A]** time = 0.45, size = 75, normalized size = 0.81

$$\frac{437 \log\left(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1\right)}{5184\sqrt{3}} + \frac{\sqrt{3x^2-x+2}\left(20736x^4+31536x^3+24072x^2+17374x+15471\right)}{12960}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2), x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(15471 + 17374\*x + 24072\*x^2 + 31536\*x^3 + 20736\*x^4))/12960 + (437\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(5184\*Sqrt[3])

**fricas [A]** time = 0.86, size = 73, normalized size = 0.78

$$\frac{1}{12960}\left(20736x^4+31536x^3+24072x^2+17374x+15471\right)\sqrt{3x^2-x+2} + \frac{437}{31104}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2), x, algorithm="fricas")

[Out] 1/12960\*(20736\*x^4 + 31536\*x^3 + 24072\*x^2 + 17374\*x + 15471)\*sqrt(3\*x^2 - x + 2) + 437/31104\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac [A]** time = 0.26, size = 68, normalized size = 0.73

$$\frac{1}{12960}\left(2(12(18(48x+73)x+1003)x+8687)x+15471\right)\sqrt{3x^2-x+2} + \frac{437}{15552}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2), x, algorithm="giac")

[Out] 1/12960\*(2\*(12\*(18\*(48\*x + 73)\*x + 1003)\*x + 8687)\*x + 15471)\*sqrt(3\*x^2 - x + 2) + 437/15552\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple [A]** time = 0.01, size = 81, normalized size = 0.87

$$\frac{8(3x^2-x+2)^{\frac{3}{2}}x^2}{15} + \frac{89(3x^2-x+2)^{\frac{3}{2}}x}{90} - \frac{437\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552} + \frac{961(3x^2-x+2)^{\frac{3}{2}}}{1620} - \frac{19(6x-1)\sqrt{3x^2-x+2}}{2592}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((2\*x+1)\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2), x)

[Out]  $8/15*(3*x^2-x+2)^{(3/2)}*x^2+89/90*(3*x^2-x+2)^{(3/2)}*x+961/1620*(3*x^2-x+2)^{(3/2)}-19/2592*(6*x-1)*(3*x^2-x+2)^{(1/2)}-437/15552*3^{(1/2)}*\operatorname{arcsinh}(6/23*23^{(1/2)}*(x-1/6))$

**maxima** [A] time = 0.96, size = 92, normalized size = 0.99

$\frac{8}{15}(3x^2-x+2)^{\frac{3}{2}}x^2 + \frac{89}{90}(3x^2-x+2)^{\frac{3}{2}}x + \frac{961}{1620}(3x^2-x+2)^{\frac{3}{2}} - \frac{19}{432}\sqrt{3x^2-x+2}x - \frac{437}{15552}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{19}{2592}\sqrt{3x^2-x+2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2), x, algorithm="maxima")

[Out]  $8/15*(3*x^2-x+2)^{(3/2)}*x^2+89/90*(3*x^2-x+2)^{(3/2)}*x+961/1620*(3*x^2-x+2)^{(3/2)}-19/432*\operatorname{sqrt}(3*x^2-x+2)*x-437/15552*\operatorname{sqrt}(3)*\operatorname{arc}\operatorname{sinh}(1/23*\operatorname{sqrt}(23)*(6*x-1))+19/2592*\operatorname{sqrt}(3*x^2-x+2)$

**mupad** [B] time = 4.88, size = 136, normalized size = 1.46

$\frac{8x^2(3x^2-x+2)^{3/2}}{15} - \frac{253\sqrt{3}\ln\left(\sqrt{3x^2-x+2} + \frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{810} - \frac{44\left(\frac{x}{2} - \frac{1}{12}\right)\sqrt{3x^2-x+2}}{45} + \frac{961\sqrt{3x^2-x+2}(72x^2-6x+45)}{38880} + \frac{89x(3x^2-x+2)^{3/2}}{90} + \frac{22103\sqrt{3}\ln\left(2\sqrt{3x^2-x+2} + \frac{\sqrt{3}(6x-1)}{3}\right)}{77760}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 1)\*(3\*x^2 - x + 2)^(1/2)\*(3\*x + 4\*x^2 + 1), x)

[Out]  $(8*x^2*(3*x^2-x+2)^{(3/2)})/15 - (253*3^{(1/2)}*\log((3*x^2-x+2)^{(1/2)} + (3^{(1/2)}*(3*x-1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2-x+2)^{(1/2)})/45 + (961*(3*x^2-x+2)^{(1/2)}*(72*x^2-6*x+45))/38880 + (89*x*(3*x^2-x+2)^{(3/2)})/90 + (22103*3^{(1/2)}*\log(2*(3*x^2-x+2)^{(1/2)} + (3^{(1/2)}*(6*x-1))/3))/77760$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 1)\sqrt{3x^2 - x + 2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x\*\*2+3\*x+1)\*(3\*x\*\*2-x+2)\*\*(1/2), x)

[Out] Integral((2\*x + 1)\*sqrt(3\*x\*\*2 - x + 2)\*(4\*x\*\*2 + 3\*x + 1), x)

$$3.207 \quad \int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x} dx$$

**Optimal.** Leaf size=101

$$\frac{2}{9}(3x^2-x+2)^{3/2} + \frac{1}{72}(30x+13)\sqrt{3x^2-x+2} - \frac{1}{8}\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - \frac{43 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{9}(3x^2-x+2)^{3/2} + \frac{1}{72}(30x+13)\sqrt{3x^2-x+2} - \frac{1}{8}\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - \frac{43 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] ((13 + 30\*x)\*Sqrt[2 - x + 3\*x^2])/72 + (2\*(2 - x + 3\*x^2)^(3/2))/9 - (43\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(144\*Sqrt[3]) - (Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/8

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x} dx &= \frac{2}{9}(2-x+3x^2)^{3/2} + \frac{1}{36} \int \frac{(48+60x)\sqrt{2-x+3x^2}}{1+2x} dx \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}(2-x+3x^2)^{3/2} - \frac{\int \frac{-3324-1032x}{(1+2x)\sqrt{2-x+3x^2}} dx}{1728} \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}(2-x+3x^2)^{3/2} + \frac{43}{144} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}(2-x+3x^2)^{3/2} - \frac{13}{4} \text{Subst} \left( \int \frac{1}{52-x^2} dx \right) \\
&= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}(2-x+3x^2)^{3/2} - \frac{43 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{144\sqrt{3}} - \frac{1}{8} \sqrt{13}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 0.85

$$\frac{1}{432} \left( 6\sqrt{3x^2-x+2}(48x^2+14x+45) - 54\sqrt{13} \tanh^{-1} \left( \frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}} \right) + 43\sqrt{3} \sinh^{-1} \left( \frac{6x-1}{\sqrt{23}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(45 + 14\*x + 48\*x^2) + 43\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]] - 54\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/432

**IntegrateAlgebraic [A]** time = 0.44, size = 114, normalized size = 1.13

$$\frac{1}{72}\sqrt{3x^2-x+2}(48x^2+14x+45) - \frac{43 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{144\sqrt{3}} + \frac{1}{4}\sqrt{13} \tanh^{-1} \left( -\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(45 + 14\*x + 48\*x^2))/72 + (Sqrt[13]\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/4 - (43\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(144\*Sqrt[3])

**fricas** [A] time = 1.33, size = 115, normalized size = 1.14

$$\frac{1}{72}(48x^2 + 14x + 45)\sqrt{3x^2 - x + 2} + \frac{43}{864}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right) + \frac{1}{16}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2 - x + 2}(8x - 9) + 220x^2 - 196x + 185}{4x^2 + 4x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(1+2\*x),x, algorithm="fricas")

[Out] 1/72\*(48\*x^2 + 14\*x + 45)\*sqrt(3\*x^2 - x + 2) + 43/864\*sqrt(3)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 1/16\*sqrt(13)\*log(-4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)

**giac** [A] time = 0.39, size = 126, normalized size = 1.25

$$\frac{1}{72}(2(24x + 7)x + 45)\sqrt{3x^2 - x + 2} - \frac{43}{432}\sqrt{3}\log\left(-6\sqrt{3}x + \sqrt{3} + 6\sqrt{3x^2 - x + 2}\right) + \frac{1}{8}\sqrt{13}\log\left(-\frac{-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2 - x + 2}}{2(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2 - x + 2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(1+2\*x),x, algorithm="giac")

[Out] 1/72\*(2\*(24\*x + 7)\*x + 45)\*sqrt(3\*x^2 - x + 2) - 43/432\*sqrt(3)\*log(-6\*sqrt(3)\*x + sqrt(3) + 6\*sqrt(3\*x^2 - x + 2)) + 1/8\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2)))

**maple** [A] time = 0.01, size = 95, normalized size = 0.94

$$\frac{43\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{432} - \frac{\sqrt{13}\operatorname{arctanh}\left(\frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{8} + \frac{2(3x^2-x+2)^{\frac{3}{2}}}{9} + \frac{5(6x-1)\sqrt{3x^2-x+2}}{72} + \frac{\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(2\*x+1),x)

[Out] 2/9\*(3\*x^2-x+2)^(3/2)+5/72\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)+43/432\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))+1/8\*(12\*(x+1/2)^2-16\*x+5)^(1/2)-1/8\*13^(1/2)\*arctanh(2/13\*(9/2-4\*x)\*13^(1/2)/(12\*(x+1/2)^2-16\*x+5)^(1/2))

**maxima** [A] time = 0.96, size = 96, normalized size = 0.95

$$\frac{2}{9}(3x^2 - x + 2)^{\frac{3}{2}} + \frac{5}{12}\sqrt{3x^2 - x + 2}x + \frac{43}{432}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{1}{8}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{13}{72}\sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(1+2\*x),x, algorithm="maxima")

[Out] 2/9\*(3\*x^2 - x + 2)^(3/2) + 5/12\*sqrt(3\*x^2 - x + 2)\*x + 43/432\*sqrt(3)\*arc  
sinh(6/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 1/8\*sqrt(13)\*arcsinh(8/23\*sqrt(23))\*  
x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1) + 13/72\*sqrt(3\*x^2 - x + 2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3x^2 - x + 2} (4x^2 + 3x + 1)}{2x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3\*x^2 - x + 2)^(1/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1),x)

[Out] int(((3\*x^2 - x + 2)^(1/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2 - x + 2} (4x^2 + 3x + 1)}{2x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)\*(3\*x\*\*2-x+2)\*\*(1/2)/(1+2\*x),x)

[Out] Integral(sqrt(3\*x\*\*2 - x + 2)\*(4\*x\*\*2 + 3\*x + 1)/(2\*x + 1), x)

$$3.208 \quad \int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2} dx$$

**Optimal.** Leaf size=108

$$-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)} - \frac{1}{156}(67-96x)\sqrt{3x^2-x+2} + \frac{17 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}} - \frac{11 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)} - \frac{1}{156}(67-96x)\sqrt{3x^2-x+2} + \frac{17 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}} - \frac{11 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2, x]

[Out] -((67 - 96\*x)\*Sqrt[2 - x + 3\*x^2])/156 - (2 - x + 3\*x^2)^(3/2)/(13\*(1 + 2\*x)) - (11\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(6\*Sqrt[3]) + (17\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(8\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /;

FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /;

FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{2-x+3x^2} (1+3x+4x^2)}{(1+2x)^2} dx &= -\frac{(2-x+3x^2)^{3/2}}{13(1+2x)} - \frac{1}{13} \int \frac{\left(-\frac{15}{2} - 32x\right) \sqrt{2-x+3x^2}}{1+2x} dx \\
&= -\frac{1}{156} (67-96x) \sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} + \frac{1}{624} \int \frac{-182+22x}{(1+2x)\sqrt{2-x+3x^2}} dx \\
&= -\frac{1}{156} (67-96x) \sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} + \frac{11}{6} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\
&= -\frac{1}{156} (67-96x) \sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} + \frac{17}{4} \text{Subst} \left( \int \frac{1}{52-x^2} dx \right) \\
&= -\frac{1}{156} (67-96x) \sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} - \frac{11 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{6\sqrt{3}} + \frac{17}{4} \text{Subst} \left( \int \frac{1}{52-x^2} dx \right)
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 92, normalized size = 0.85

$$\frac{\sqrt{3x^2-x+2} (12x^2-2x-7)}{24x+12} + \frac{17 \tanh^{-1} \left( \frac{9-8x}{2\sqrt{13} \sqrt{3x^2-x+2}} \right)}{8\sqrt{13}} + \frac{11 \sinh^{-1} \left( \frac{6x-1}{\sqrt{23}} \right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2,x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(-7 - 2\*x + 12\*x^2))/(12 + 24\*x) + (11\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/(6\*Sqrt[3]) + (17\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(8\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.63, size = 121, normalized size = 1.12

$$\frac{\sqrt{3x^2-x+2} (12x^2-2x-7)}{12(2x+1)} - \frac{11 \log(2\sqrt{3} \sqrt{3x^2-x+2} - 6x+1)}{6\sqrt{3}} - \frac{17 \tanh^{-1} \left( -\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}} \right)}{4\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2,x]

[Out]  $(\text{Sqrt}[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12*(1 + 2*x)) - (17*\text{ArcTanh}[\text{Sqrt}[3/13] + 2*\text{Sqrt}[3/13]*x - (2*\text{Sqrt}[2 - x + 3*x^2])/ \text{Sqrt}[13]])/(4*\text{Sqrt}[13]) - (11*\text{Log}[1 - 6*x + 2*\text{Sqrt}[3]*\text{Sqrt}[2 - x + 3*x^2]])/(6*\text{Sqrt}[3])$

**fricas** [A] time = 0.87, size = 133, normalized size = 1.23

$$\frac{572\sqrt{3}(2x+1)\log(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25)+153\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+156(12x^2-2x-7)\sqrt{3x^2-x+2}}{1872(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")`

[Out]  $1/1872*(572*\text{sqrt}(3)*(2*x + 1)*\log(-4*\text{sqrt}(3)*\text{sqrt}(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 153*\text{sqrt}(13)*(2*x + 1)*\log((4*\text{sqrt}(13)*\text{sqrt}(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*x^2 - 2*x - 7)*\text{sqrt}(3*x^2 - x + 2))/(2*x + 1)$

**giac** [B] time = 0.72, size = 380, normalized size = 3.52

$$\frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\text{sgn}\left(\frac{1}{2x+1}\right)+\frac{11}{18}\sqrt{3}\log\left(\frac{-2\sqrt{3}+2\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}}{2\left(\sqrt{3}+\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\text{sgn}\left(\frac{1}{2x+1}\right)+\frac{1}{8}\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}\text{sgn}\left(\frac{1}{2x+1}\right)+\frac{57\left(\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}\right)^2\text{sgn}\left(\frac{1}{2x+1}\right)-57\sqrt{13}\left(\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}\right)\text{sgn}\left(\frac{1}{2x+1}\right)+129\left(\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}\right)\text{sgn}\left(\frac{1}{2x+1}\right)+27\sqrt{13}\text{sgn}\left(\frac{1}{2x+1}\right)}{12\left(\left(\sqrt{\frac{8}{2x+1}+\frac{13}{2x+17}+3}+\frac{\sqrt{13}}{2x+1}\right)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")`

[Out]  $17/104*\text{sqrt}(13)*\log(\text{sqrt}(13)*(\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + \text{sqrt}(13)/(2*x + 1)) - 4)*\text{sgn}(1/(2*x + 1)) - 11/18*\text{sqrt}(3)*\log(1/2*\text{abs}(-2*\text{sqrt}(3) + 2*\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*\text{sqrt}(13)/(2*x + 1)))/(\text{sqrt}(3) + \text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + \text{sqrt}(13)/(2*x + 1)))*\text{sgn}(1/(2*x + 1)) - 1/8*\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*\text{sgn}(1/(2*x + 1)) + 1/12*(67*(\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + \text{sqrt}(13)/(2*x + 1))^3*\text{sgn}(1/(2*x + 1)) - 57*\text{sqrt}(13)*(\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + \text{sqrt}(13)/(2*x + 1))^2*\text{sgn}(1/(2*x + 1)) + 129*(\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + \text{sqrt}(13)/(2*x + 1))*\text{sgn}(1/(2*x + 1)) + 27*\text{sqrt}(13)*\text{sgn}(1/(2*x + 1))))/((\text{sqrt}(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + \text{sqrt}(13)/(2*x + 1))^2 - 3)^2$

**maple** [A] time = 0.01, size = 123, normalized size = 1.14

$$\frac{11\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{2}\right)}{23}\right)}{18} + \frac{17\sqrt{13}\operatorname{arctanh}\left(\frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{104} + \frac{(6x-1)\sqrt{3x^2-x+2}}{12} - \frac{17\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}{104} - \frac{\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{9}{4}\right)^{\frac{3}{2}}}{26\left(x+\frac{1}{2}\right)} + \frac{(6x-1)\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^2,x)`

[Out]  $\frac{1}{12}(6x-1)(3x^2-x+2)^{1/2} + \frac{11}{18}3^{1/2}\operatorname{arcsinh}\left(\frac{6}{23}\sqrt{23}\right)(x-1/6) - \frac{17}{104}(-16x+12(x+1/2)^2+5)^{1/2} + \frac{17}{104}13^{1/2}\operatorname{arctanh}\left(\frac{2}{13}(-4x+9/2)\right) \cdot 13^{1/2} / (-16x+12(x+1/2)^2+5)^{1/2} - \frac{1}{26}(x+1/2)(3(x+1/2)^2-4x+5/4)^{3/2} + \frac{1}{52}(6x-1)(3(x+1/2)^2-4x+5/4)^{1/2}$

**maxima** [A] time = 0.97, size = 103, normalized size = 0.95

$$\frac{1}{2}\sqrt{3x^2-x+2}x + \frac{11}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{17}{104}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) - \frac{1}{3}\sqrt{3x^2-x+2} - \frac{\sqrt{3x^2-x+2}}{4(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}\sqrt{3x^2-x+2}x + \frac{11}{18}\sqrt{3}\operatorname{arcsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{17}{104}\sqrt{13}\operatorname{arcsinh}\left(\frac{8}{23}\sqrt{23}x/\operatorname{abs}(2x+1) - \frac{9}{23}\sqrt{23}/\operatorname{abs}(2x+1)\right) - \frac{1}{3}\sqrt{3x^2-x+2} - \frac{1}{4}\sqrt{3x^2-x+2}/(2x+1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{(2x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((3*x^2-x+2)^(1/2)*(3*x+4*x^2+1))/(2*x+1)^2,x)`

[Out] `int(((3*x^2-x+2)^(1/2)*(3*x+4*x^2+1))/(2*x+1)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{(2x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)`

[Out] `Integral(sqrt(3*x**2-x+2)*(4*x**2+3*x+1)/(2*x+1)**2, x)`

$$3.209 \quad \int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} dx$$

**Optimal.** Leaf size=115

$$-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2} + \frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)} - \frac{803 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}} + \frac{11 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2} + \frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)} - \frac{803 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}} + \frac{11 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] (11\*(7 + 10\*x)\*Sqrt[2 - x + 3\*x^2])/(104\*(1 + 2\*x)) - (2 - x + 3\*x^2)^(3/2)/(26\*(1 + 2\*x)^2) + (11\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(8\*Sqrt[3]) - (803\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(208\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 812

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(e\*f\*(m + 2\*p + 2) - d\*g\*(2\*p + 1) + e\*g\*(m + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(e^2\*(m + 1)\*(m + 2\*p + 2)), x] + Dist[p/(e^2\*(m + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[g\*(b\*d + 2\*a\*e + 2\*a\*e\*m + 2\*b\*d\*p) - f\*b\*e\*(m + 2\*p + 2) + (g\*(2\*c\*d + b\*e + b\*e\*m + 4\*c\*d\*p) - 2\*c\*e\*f\*(m + 2\*p + 2))\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2\*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-x+3x^2} (1+3x+4x^2)}{(1+2x)^3} dx &= -\frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} - \frac{1}{26} \int \frac{\left(-\frac{33}{2} - 55x\right) \sqrt{2-x+3x^2}}{(1+2x)^2} dx \\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{1}{208} \int \frac{517-572x}{(1+2x)\sqrt{2-x+3x^2}} dx \\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} - \frac{11}{8} \int \frac{1}{\sqrt{2-x+3x^2}} dx + \frac{8}{2} \\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} - \frac{803}{104} \text{Subst} \left( \int \frac{1}{52-x^2} dx, \frac{1+2x}{\sqrt{2-x+3x^2}} \right) \\
&= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{11 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{8\sqrt{3}} - \frac{803 \tanh^{-1} \left( \frac{1+2x}{\sqrt{23}} \right)}{104}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 93, normalized size = 0.81

$$\frac{78\sqrt{3x^2-x+2}(208x^2+268x+69)}{(2x+1)^2} - 2409\sqrt{13} \tanh^{-1} \left( \frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}} \right) - 3718\sqrt{3} \sinh^{-1} \left( \frac{6x-1}{\sqrt{23}} \right)$$

8112

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] ((78\*Sqrt[2 - x + 3\*x^2]\*(69 + 268\*x + 208\*x^2))/(1 + 2\*x)^2 - 3718\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]] - 2409\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/8112

**IntegrateAlgebraic [A]** time = 0.66, size = 121, normalized size = 1.05

$$\frac{\sqrt{3x^2-x+2}(208x^2+268x+69)}{104(2x+1)^2} + \frac{11 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{8\sqrt{3}} + \frac{803 \tanh^{-1} \left( -\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}} \right)}{104\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[2 - x + 3\*x^2]\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(69 + 268\*x + 208\*x^2))/((104\*(1 + 2\*x)^2) + (803\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(104\*Sqrt[13])) + (11\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(8\*Sqrt[3])

**fricas** [A] time = 1.39, size = 149, normalized size = 1.30

$$\frac{3718\sqrt{3}(4x^2+4x+1)\log(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25)+2409\sqrt{13}(4x^2+4x+1)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+156(208x^2+268x+69)\sqrt{3x^2-x+2}}{16224(4x^2+4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(1+2\*x)^3,x, algorithm="fricas")

[Out] 1/16224\*(3718\*sqrt(3)\*(4\*x^2 + 4\*x + 1)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 2409\*sqrt(13)\*(4\*x^2 + 4\*x + 1)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 156\*(208\*x^2 + 268\*x + 69)\*sqrt(3\*x^2 - x + 2))/(4\*x^2 + 4\*x + 1)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(1+2\*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error%%{-389344, [6]%%}+%%{%%{ [1168032,0] : [1,0,-3]%%}, [5]%%}+%%{-584016, [4]%%}+%%{%%{ [-4672128,0] : [1,0,-3]%%}, [3]%%}+%%{1460040, [2]%%}+%%{%%{ [7300200,0] : [1,0,-3]%%}, [1]%%}+%%{6083500, [0]%%} / %%{%%{ [24,0] : [1,0,-3]%%}, [6]%%}+%%{-216, [5]%%}+%%{%%{ [36,0] : [1,0,-3]%%}, [4]%%}+%%{864, [3]%%}+%%{%%{ [-90,0] : [1,0,-3]%%}, [2]%%}+%%{-1350, [1]%%}+%%{%%{ [-375,0] : [1,0,-3]%%}, [0]%%} Error: Bad Argument Value

**maple** [A] time = 0.01, size = 125, normalized size = 1.09

$$\frac{11\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{2704}+\frac{803\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}{2704}+\frac{11\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}{338\left(x+\frac{1}{2}\right)}-\frac{11(6x-1)\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{676}-\frac{\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}{104\left(x+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(2\*x+1)^3,x)

[Out] 803/2704\*(-16\*x+12\*(x+1/2)^2+5)^(1/2)-11/24\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))-803/2704\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))+11/338/(x+1/2)\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)-11/676\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)

**maxima** [A] time = 0.99, size = 114, normalized size = 0.99

$$-\frac{11}{24}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{803}{2704}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{55}{104}\sqrt{3x^2-x+2}-\frac{(3x^2-x+2)^{\frac{3}{2}}}{26(4x^2+4x+1)}+\frac{11\sqrt{3x^2-x+2}}{52(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)\*(3\*x^2-x+2)^(1/2)/(1+2\*x)^3,x, algorithm="maxima")

[Out] -11/24\*sqrt(3)\*arcsinh(6/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 803/2704\*sqrt(13)\*arcsinh(8/23\*sqrt(23)\*x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1)) + 55/104\*sqrt(3\*x^2 - x + 2) - 1/26\*(3\*x^2 - x + 2)^(3/2)/(4\*x^2 + 4\*x + 1) + 11/52\*sqrt(3\*x^2 - x + 2)/(2\*x + 1)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3x^2 - x + 2} (4x^2 + 3x + 1)}{(2x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3\*x^2 - x + 2)^(1/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1)^3,x)

[Out] int(((3\*x^2 - x + 2)^(1/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2 - x + 2} (4x^2 + 3x + 1)}{(2x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)\*(3\*x\*\*2-x+2)\*\*(1/2)/(1+2\*x)\*\*3,x)

[Out] Integral(sqrt(3\*x\*\*2 - x + 2)\*(4\*x\*\*2 + 3\*x + 1)/(2\*x + 1)\*\*3, x)



$$3.210 \quad \int (1 + 2x)^3 (2 - x + 3x^2)^{3/2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=158

$$\frac{2}{27} (3x^2 - x + 2)^{5/2} (2x+1)^4 + \frac{913}{486} x^2 (3x^2 - x + 2)^{5/2} - \frac{11(283 - 5850x) (3x^2 - x + 2)^{5/2}}{58320} + \frac{54593(1 - 6x) (3x^2 - x + 2)^{3/2}}{559872}$$

**Rubi [A]** time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 12, 779, 612, 619, 215}

$$\frac{2}{27} (3x^2 - x + 2)^{5/2} (2x+1)^4 + \frac{77}{81} x^3 (3x^2 - x + 2)^{5/2} + \frac{913}{486} x^2 (3x^2 - x + 2)^{5/2} - \frac{11(283 - 5850x) (3x^2 - x + 2)^{5/2}}{58320} + \frac{54593(1 - 6x) (3x^2 - x + 2)^{3/2}}{559872} + \frac{1255639(1 - 6x) \sqrt{3x^2 - x + 2}}{4478976} + \frac{28879697 \sinh^{-1} \left( \frac{1-6x}{\sqrt{3}} \right)}{8957952\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (1255639\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/4478976 + (54593\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(3/2))/559872 - (11\*(283 - 5850\*x)\*(2 - x + 3\*x^2)^(5/2))/58320 + (913\*x^2\*(2 - x + 3\*x^2)^(5/2))/486 + (77\*x^3\*(2 - x + 3\*x^2)^(5/2))/81 + (2\*(1 + 2\*x)^4\*(2 - x + 3\*x^2)^(5/2))/27 + (28879697\*ArcSinh[(1 - 6\*x)/Sqrt[2 - x + 3\*x^2]])/(8957952\*Sqrt[3])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b

+ 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int (1+2x)^3 (2-x+3x^2)^{3/2} (1+3x+4x^2) dx &= \frac{2}{27}(1+2x)^4 (2-x+3x^2)^{5/2} + \frac{1}{108} \int 308x(1+2x)^3 (2-x+3x^2)^{3/2} dx \\
&= \frac{2}{27}(1+2x)^4 (2-x+3x^2)^{5/2} + \frac{77}{27} \int x(1+2x)^3 (2-x+3x^2)^{3/2} dx \\
&= \frac{77}{81}x^3 (2-x+3x^2)^{5/2} + \frac{2}{27}(1+2x)^4 (2-x+3x^2)^{5/2} + \frac{77}{648} \int (1+2x)^3 (2-x+3x^2)^{3/2} dx \\
&= \frac{913}{486}x^2 (2-x+3x^2)^{5/2} + \frac{77}{81}x^3 (2-x+3x^2)^{5/2} + \frac{2}{27}(1+2x)^4 (2-x+3x^2)^{5/2} \\
&= -\frac{11(283-5850x)(2-x+3x^2)^{5/2}}{58320} + \frac{913}{486}x^2 (2-x+3x^2)^{5/2} + \frac{2}{27}(1+2x)^4 (2-x+3x^2)^{5/2} \\
&= \frac{54593(1-6x)(2-x+3x^2)^{3/2}}{559872} - \frac{11(283-5850x)(2-x+3x^2)^{5/2}}{58320} + \frac{2}{27}(1+2x)^4 (2-x+3x^2)^{5/2} \\
&= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)(2-x+3x^2)^{3/2}}{559872} - \frac{11(283-5850x)(2-x+3x^2)^{5/2}}{58320} \\
&= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)(2-x+3x^2)^{3/2}}{559872} - \frac{11(283-5850x)(2-x+3x^2)^{5/2}}{58320} \\
&= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)(2-x+3x^2)^{3/2}}{559872} - \frac{11(283-5850x)(2-x+3x^2)^{5/2}}{58320}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 80, normalized size = 0.51

$$\frac{6\sqrt{3x^2-x+2} (238878720x^8 + 510105600x^7 + 635765760x^6 + 711210240x^5 + 649452672x^4 + 421626672x^3 + 201289704x^2 + 84014278x + 12499587) - 144398485\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{134369280}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*sqrt[2 - x + 3\*x^2]\*(12499587 + 84014278\*x + 201289704\*x^2 + 421626672\*x^3 + 649452672\*x^4 + 711210240\*x^5 + 635765760\*x^6 + 510105600\*x^7 + 238878720\*x^8) - 144398485\*sqrt[3]\*ArcSinh[(-1 + 6\*x)/sqrt[23]])/134369280

**IntegrateAlgebraic [A]** time = 0.92, size = 95, normalized size = 0.60

$$\frac{288796971 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{8957952\sqrt{3}} + \frac{\sqrt{3x^2-x+2} (238878720x^8 + 510105600x^7 + 635765760x^6 + 711210240x^5 + 649452672x^4 + 421626672x^3 + 201289704x^2 + 84014278x + 12499587)}{22394880}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(12499587 + 84014278\*x + 201289704\*x^2 + 421626672\*x^3 + 649452672\*x^4 + 711210240\*x^5 + 635765760\*x^6 + 510105600\*x^7 + 238878720\*x^8))/22394880 + (28879697\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(8957952\*Sqrt[3])

**fricas** [A] time = 1.50, size = 93, normalized size = 0.59

$$\frac{1}{22394880} (238878720x^8 + 510105600x^7 + 635765760x^6 + 711210240x^5 + 649452672x^4 + 421626672x^3 + 201289704x^2 + 84014278x + 12499587)\sqrt{3x^2 - x + 2} + \frac{28879697}{53747712} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1), x, algorithm="fricas")

[Out] 1/22394880\*(238878720\*x^8 + 510105600\*x^7 + 635765760\*x^6 + 711210240\*x^5 + 649452672\*x^4 + 421626672\*x^3 + 201289704\*x^2 + 84014278\*x + 12499587)\*sqrt(3\*x^2 - x + 2) + 28879697/53747712\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac** [A] time = 0.26, size = 88, normalized size = 0.56

$$\frac{1}{22394880} (2(12(6(8(30(36(2(96x + 205)x + 511)x + 20579)x + 563761)x + 2927963)x + 8387071)x + 42007139)x + 12499587)\sqrt{3x^2 - x + 2} + \frac{28879697}{26873856} \sqrt{3} \log(-2\sqrt{3}(\sqrt{3x - \sqrt{3x^2 - x + 2}}) + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1), x, algorithm="giac")

[Out] 1/22394880\*(2\*(12\*(6\*(8\*(30\*(36\*(2\*(96\*x + 205)\*x + 511)\*x + 20579)\*x + 563761)\*x + 2927963)\*x + 8387071)\*x + 42007139)\*x + 12499587)\*sqrt(3\*x^2 - x + 2) + 28879697/26873856\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple** [A] time = 0.02, size = 134, normalized size = 0.85

$$\frac{32(3x^2 - x + 2)^{\frac{5}{2}}x^4}{27} + \frac{269(3x^2 - x + 2)^{\frac{5}{2}}x^3}{81} + \frac{1777(3x^2 - x + 2)^{\frac{5}{2}}x^2}{486} + \frac{1099(3x^2 - x + 2)^{\frac{5}{2}}x}{648} - \frac{28879697\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}(x - \frac{1}{6})}{23}\right)}{26873856} - \frac{54593(6x - 1)(3x^2 - x + 2)^{\frac{3}{2}}}{559872} - \frac{1255639(6x - 1)\sqrt{3x^2 - x + 2}}{4478976} + \frac{1207(3x^2 - x + 2)^{\frac{5}{2}}}{58320}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)^3\*(3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1), x)

[Out] 32/27\*x^4\*(3\*x^2-x+2)^(5/2)+269/81\*x^3\*(3\*x^2-x+2)^(5/2)+1777/486\*x^2\*(3\*x^2-x+2)^(5/2)+1099/648\*x\*(3\*x^2-x+2)^(5/2)-54593/559872\*(6\*x-1)\*(3\*x^2-x+2)^(3/2)-28879697/26873856\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))-1255639/4478976\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)+1207/58320\*(3\*x^2-x+2)^(5/2)

**maxima** [A] time = 0.98, size = 155, normalized size = 0.98

$$\frac{32}{27}(3x^2 - x + 2)^{\frac{5}{2}}x^4 + \frac{269}{81}(3x^2 - x + 2)^{\frac{5}{2}}x^3 + \frac{1777}{486}(3x^2 - x + 2)^{\frac{5}{2}}x^2 + \frac{1099}{648}(3x^2 - x + 2)^{\frac{5}{2}}x + \frac{1207}{58320}(3x^2 - x + 2)^{\frac{5}{2}} - \frac{54593}{93312}(3x^2 - x + 2)^{\frac{3}{2}} + \frac{54593}{559872}(3x^2 - x + 2)^{\frac{3}{2}} - \frac{1255639}{746496}\sqrt{3x^2 - x + 2} - \frac{28879697}{26873856}\sqrt{3} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(6x - 1)\right) + \frac{1255639}{4478976}\sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
[Out] 32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/4
86*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/5832
0*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55987
2*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 28879697/2
6873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqrt(3*x
^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x + 1)^3 (3x^2 - x + 2)^{3/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 1)^3 (3x^2 - x + 2)^{3/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

$$3.211 \quad \int (1 + 2x)^2 (2 - x + 3x^2)^{3/2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=141

$$\frac{1}{12} (3x^2 - x + 2)^{5/2} (2x+1)^3 + \frac{8}{63} (3x^2 - x + 2)^{5/2} (2x+1)^2 + \frac{13(50x + 29) (3x^2 - x + 2)^{5/2}}{2520} + \frac{91(1 - 6x) (3x^2 - x + 2)}{3456}$$

**Rubi [A]** time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{12} (3x^2 - x + 2)^{5/2} (2x+1)^3 + \frac{8}{63} (3x^2 - x + 2)^{5/2} (2x+1)^2 + \frac{13(50x + 29) (3x^2 - x + 2)^{5/2}}{2520} + \frac{91(1 - 6x) (3x^2 - x + 2)^{3/2}}{3456} + \frac{2093(1 - 6x) \sqrt{3x^2 - x + 2}}{27648} + \frac{48139 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (2093\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/27648 + (91\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(3/2))/3456 + (8\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2))/63 + ((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2))/12 + (13\*(29 + 50\*x)\*(2 - x + 3\*x^2)^(5/2))/2520 + (48139 \*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(55296\*Sqrt[3])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (1+2x)^2 (2-x+3x^2)^{3/2} (1+3x+4x^2) dx &= \frac{1}{12} (1+2x)^3 (2-x+3x^2)^{5/2} + \frac{1}{96} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx \\
&= \frac{8}{63} (1+2x)^2 (2-x+3x^2)^{5/2} + \frac{1}{12} (1+2x)^3 (2-x+3x^2)^{5/2} + \frac{1}{96} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx \\
&= \frac{8}{63} (1+2x)^2 (2-x+3x^2)^{5/2} + \frac{1}{12} (1+2x)^3 (2-x+3x^2)^{5/2} + \frac{1}{96} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx \\
&= \frac{91(1-6x)(2-x+3x^2)^{3/2}}{3456} + \frac{8}{63} (1+2x)^2 (2-x+3x^2)^{5/2} + \frac{1}{12} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx \\
&= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)(2-x+3x^2)^{3/2}}{3456} + \frac{8}{63} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx \\
&= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)(2-x+3x^2)^{3/2}}{3456} + \frac{8}{63} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx \\
&= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)(2-x+3x^2)^{3/2}}{3456} + \frac{8}{63} \int (1+2x)^2 (20+256x) (2-x+3x^2)^{3/2} dx
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 75, normalized size = 0.53

$$\frac{6\sqrt{3x^2-x+2} (5806080x^7 + 9262080x^6 + 10656000x^5 + 12173952x^4 + 10119792x^3 + 5694024x^2 + 2735918x + 1517367) - 1684865\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{5806080}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(1517367 + 2735918\*x + 5694024\*x^2 + 10119792\*x^3 + 12173952\*x^4 + 10656000\*x^5 + 9262080\*x^6 + 5806080\*x^7) - 1684865\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/5806080

**IntegrateAlgebraic [A]** time = 0.83, size = 90, normalized size = 0.64

$$\frac{48139 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{55296\sqrt{3}} + \frac{\sqrt{3x^2-x+2} (5806080x^7 + 9262080x^6 + 10656000x^5 + 12173952x^4 + 10119792x^3 + 5694024x^2 + 2735918x + 1517367)}{967680}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]



[Out]  $(\sqrt{2-x+3x^2} \cdot (1517367 + 2735918x + 5694024x^2 + 10119792x^3 + 12173952x^4 + 10656000x^5 + 9262080x^6 + 5806080x^7)) / 967680 + (48139 \cdot \log[1 - 6x + 2\sqrt{3} \cdot \sqrt{2-x+3x^2}]) / (55296 \cdot \sqrt{3})$

**fricas** [A] time = 1.26, size = 88, normalized size = 0.62

$$\frac{1}{967680} (5806080x^7 + 9262080x^6 + 10656000x^5 + 12173952x^4 + 10119792x^3 + 5694024x^2 + 2735918x + 1517367) \sqrt{3x^2 - x + 2} + \frac{48139}{331776} \sqrt{3} \log(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")`

[Out]  $1/967680 \cdot (5806080x^7 + 9262080x^6 + 10656000x^5 + 12173952x^4 + 10119792x^3 + 5694024x^2 + 2735918x + 1517367) \cdot \sqrt{3x^2 - x + 2} + 48139/331776 \cdot \sqrt{3} \cdot \log(4 \cdot \sqrt{3} \cdot \sqrt{3x^2 - x + 2} \cdot (6x - 1) - 72x^2 + 24x - 25)$

**giac** [A] time = 0.27, size = 83, normalized size = 0.59

$$\frac{1}{967680} (2(12(2(8(30(12(42x + 67)x + 925)x + 31703)x + 210829)x + 237251)x + 1367959)x + 1517367) \sqrt{3x^2 - x + 2} + \frac{48139}{165888} \sqrt{3} \log(-2\sqrt{3}(\sqrt{3x - \sqrt{3x^2 - x + 2}}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")`

[Out]  $1/967680 \cdot (2 \cdot (12 \cdot (2 \cdot (8 \cdot (30 \cdot (12 \cdot (42x + 67)x + 925)x + 31703)x + 210829)x + 237251)x + 1367959)x + 1517367) \cdot \sqrt{3x^2 - x + 2} + 48139/165888 \cdot \sqrt{3} \cdot \log(-2 \cdot \sqrt{3} \cdot (\sqrt{3x - \sqrt{3x^2 - x + 2}}) + 1)$

**maple** [A] time = 0.01, size = 117, normalized size = 0.83

$$\frac{2(3x^2 - x + 2)^{\frac{5}{2}} x^3}{3} + \frac{95(3x^2 - x + 2)^{\frac{5}{2}} x^2}{63} + \frac{319(3x^2 - x + 2)^{\frac{5}{2}} x}{252} - \frac{48139\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}(x-1/6)}{23}\right)}{165888} - \frac{91(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{3456} - \frac{2093(6x-1)\sqrt{3x^2-x+2}}{27648} + \frac{907(3x^2-x+2)^{\frac{5}{2}}}{2520}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)`

[Out]  $2/3 \cdot (3x^2 - x + 2)^{\frac{5}{2}} x^3 + 95/63 \cdot (3x^2 - x + 2)^{\frac{5}{2}} x^2 + 319/252 \cdot (3x^2 - x + 2)^{\frac{5}{2}} x - 91/3456 \cdot (6x - 1) \cdot (3x^2 - x + 2)^{\frac{3}{2}} - 48139/165888 \cdot 3^{\frac{1}{2}} \cdot \operatorname{arcsinh}(6/23 \cdot 23^{\frac{1}{2}} \cdot (x - 1/6)) - 2093/27648 \cdot (6x - 1) \cdot (3x^2 - x + 2)^{\frac{1}{2}} + 907/2520 \cdot (3x^2 - x + 2)^{\frac{5}{2}}$

**maxima** [A] time = 0.93, size = 138, normalized size = 0.98

$$\frac{2}{3} (3x^2 - x + 2)^{\frac{5}{2}} x^3 + \frac{95}{63} (3x^2 - x + 2)^{\frac{5}{2}} x^2 + \frac{319}{252} (3x^2 - x + 2)^{\frac{5}{2}} x + \frac{907}{2520} (3x^2 - x + 2)^{\frac{5}{2}} - \frac{91}{576} (3x^2 - x + 2)^{\frac{3}{2}} - \frac{91}{3456} (3x^2 - x + 2)^{\frac{3}{2}} - \frac{2093}{4608} \sqrt{3x^2 - x + 2} - \frac{48139}{165888} \sqrt{3} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23}(6x - 1)\right) + \frac{2093}{27648} \sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
[Out] 2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*(
3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 - x
+ 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x +
2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/27648*s
qrt(3*x^2 - x + 2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x + 1)^2 (3x^2 - x + 2)^{3/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 1)^2 (3x^2 - x + 2)^{\frac{3}{2}} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

$$3.212 \quad \int (1 + 2x) (2 - x + 3x^2)^{3/2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=116

$$\frac{2}{21}(2x+1)^2(3x^2-x+2)^{5/2} + \frac{1}{378}(102x+109)(3x^2-x+2)^{5/2} - \frac{71(1-6x)(3x^2-x+2)^{3/2}}{2592} - \frac{1633(1-6x)\sqrt{3x^2-x+2}}{20736}$$

**Rubi** [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{21}(2x+1)^2(3x^2-x+2)^{5/2} + \frac{1}{378}(102x+109)(3x^2-x+2)^{5/2} - \frac{71(1-6x)(3x^2-x+2)^{3/2}}{2592} - \frac{1633(1-6x)\sqrt{3x^2-x+2}}{20736} - \frac{37559 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (-1633\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/20736 - (71\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(3/2))/2592 + (2\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2))/21 + ((109 + 102\*x)\*(2 - x + 3\*x^2)^(5/2))/378 - (37559\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(41472\*Sqrt[3])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (1 + 2x)(2 - x + 3x^2)^{3/2}(1 + 3x + 4x^2) dx &= \frac{2}{21}(1 + 2x)^2(2 - x + 3x^2)^{5/2} + \frac{1}{84} \int (1 + 2x)(40 + 204x)(2 - x + 3x^2)^{3/2} dx \\
&= \frac{2}{21}(1 + 2x)^2(2 - x + 3x^2)^{5/2} + \frac{1}{378}(109 + 102x)(2 - x + 3x^2)^{5/2} \\
&= -\frac{71(1 - 6x)(2 - x + 3x^2)^{3/2}}{2592} + \frac{2}{21}(1 + 2x)^2(2 - x + 3x^2)^{5/2} + \frac{1}{378}(109 + 102x)(2 - x + 3x^2)^{5/2} \\
&= -\frac{1633(1 - 6x)\sqrt{2 - x + 3x^2}}{20736} - \frac{71(1 - 6x)(2 - x + 3x^2)^{3/2}}{2592} + \frac{2}{21}(1 + 2x)^2(2 - x + 3x^2)^{5/2} \\
&= -\frac{1633(1 - 6x)\sqrt{2 - x + 3x^2}}{20736} - \frac{71(1 - 6x)(2 - x + 3x^2)^{3/2}}{2592} + \frac{2}{21}(1 + 2x)^2(2 - x + 3x^2)^{5/2} \\
&= -\frac{1633(1 - 6x)\sqrt{2 - x + 3x^2}}{20736} - \frac{71(1 - 6x)(2 - x + 3x^2)^{3/2}}{2592} + \frac{2}{21}(1 + 2x)^2(2 - x + 3x^2)^{5/2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 70, normalized size = 0.60

$$\frac{6\sqrt{3x^2 - x + 2}(497664x^6 + 518400x^5 + 653184x^4 + 744336x^3 + 531384x^2 + 275410x + 203337) + 262913\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{870912}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*sqrt[2 - x + 3\*x^2]\*(203337 + 275410\*x + 531384\*x^2 + 744336\*x^3 + 653184\*x^4 + 518400\*x^5 + 497664\*x^6) + 262913\*sqrt[3]\*ArcSinh[(-1 + 6\*x)/sqrt[23]])/870912

**IntegrateAlgebraic [A]** time = 0.65, size = 85, normalized size = 0.73

$$\frac{\sqrt{3x^2 - x + 2} (497664x^6 + 518400x^5 + 653184x^4 + 744336x^3 + 531384x^2 + 275410x + 203337)}{145152} - \frac{37559 \log(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1)}{41472\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (sqrt[2 - x + 3\*x^2]\*(203337 + 275410\*x + 531384\*x^2 + 744336\*x^3 + 653184\*x^4 + 518400\*x^5 + 497664\*x^6))/145152 - (37559\*Log[1 - 6\*x + 2\*sqrt[3]\*sqrt[2 - x + 3\*x^2]])/(41472\*sqrt[3])

**fricas [A]** time = 2.01, size = 83, normalized size = 0.72

$$\frac{1}{145152} (497664x^6 + 518400x^5 + 653184x^4 + 744336x^3 + 531384x^2 + 275410x + 203337)\sqrt{3x^2 - x + 2} + \frac{37559}{248832} \sqrt{3} \log(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1), x, algorithm="fricas")

[Out] 1/145152\*(497664\*x^6 + 518400\*x^5 + 653184\*x^4 + 744336\*x^3 + 531384\*x^2 + 275410\*x + 203337)\*sqrt(3\*x^2 - x + 2) + 37559/248832\*sqrt(3)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac [A]** time = 0.21, size = 78, normalized size = 0.67

$$\frac{1}{145152} (2(12(18(24(2(24x + 25)x + 63)x + 1723)x + 22141)x + 137705)x + 203337)\sqrt{3x^2 - x + 2} - \frac{37559}{124416} \sqrt{3} \log(-2\sqrt{3}(\sqrt{3x - \sqrt{3x^2 - x + 2}}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1), x, algorithm="giac")

[Out] 1/145152\*(2\*(12\*(18\*(24\*(2\*(24\*x + 25)\*x + 63)\*x + 1723)\*x + 22141)\*x + 137705)\*x + 203337)\*sqrt(3\*x^2 - x + 2) - 37559/124416\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple [A]** time = 0.00, size = 100, normalized size = 0.86

$$\frac{8(3x^2 - x + 2)^{\frac{5}{2}} x^2}{21} + \frac{41(3x^2 - x + 2)^{\frac{5}{2}} x}{63} + \frac{37559\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x - \frac{1}{6}\right)}{23}\right)}{124416} + \frac{145(3x^2 - x + 2)^{\frac{5}{2}}}{378} + \frac{71(6x - 1)(3x^2 - x + 2)^{\frac{3}{2}}}{2592} + \frac{1633(6x - 1)\sqrt{3x^2 - x + 2}}{20736}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)`

[Out]  $8/21*(3*x^2-x+2)^{(5/2)}*x^2+41/63*(3*x^2-x+2)^{(5/2)}*x+145/378*(3*x^2-x+2)^{(5/2)}+71/2592*(6*x-1)*(3*x^2-x+2)^{(3/2)}+1633/20736*(6*x-1)*(3*x^2-x+2)^{(1/2)}+37559/124416*3^{(1/2)}*\operatorname{arcsinh}(6/23*23^{(1/2)}*(x-1/6))$

**maxima** [A] time = 0.97, size = 121, normalized size = 1.04

$$\frac{8}{21}(3x^2-x+2)^{\frac{5}{2}}x^2 + \frac{41}{63}(3x^2-x+2)^{\frac{5}{2}}x + \frac{145}{378}(3x^2-x+2)^{\frac{5}{2}} + \frac{71}{432}(3x^2-x+2)^{\frac{3}{2}}x - \frac{71}{2592}(3x^2-x+2)^{\frac{3}{2}} + \frac{1633}{3456}\sqrt{3x^2-x+2}x + \frac{37559}{124416}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{1633}{20736}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")`

[Out]  $8/21*(3*x^2-x+2)^{(5/2)}*x^2+41/63*(3*x^2-x+2)^{(5/2)}*x+145/378*(3*x^2-x+2)^{(5/2)}+71/432*(3*x^2-x+2)^{(3/2)}*x-71/2592*(3*x^2-x+2)^{(3/2)}+1633/3456*\operatorname{sqrt}(3*x^2-x+2)*x+37559/124416*\operatorname{sqrt}(3)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(6*x-1))-1633/20736*\operatorname{sqrt}(3*x^2-x+2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)(3x^2-x+2)^{3/2}(4x^2+3x+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)*(3*x^2-x+2)^(3/2)*(3*x+4*x^2+1),x)`

[Out] `int((2*x+1)*(3*x^2-x+2)^(3/2)*(3*x+4*x^2+1),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)(3x^2-x+2)^{\frac{3}{2}}(4x^2+3x+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)`

[Out] `Integral((2*x+1)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)`

$$3.213 \quad \int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x} dx$$

**Optimal.** Leaf size=124

$$\frac{2}{15} (3x^2 - x + 2)^{5/2} + \frac{1}{144} (30x+7) (3x^2 - x + 2)^{3/2} + \frac{(402x + 869)\sqrt{3x^2 - x + 2}}{1152} - \frac{13}{32} \sqrt{13} \tanh^{-1} \left( \frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}} \right)$$

**Rubi [A]** time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{15} (3x^2 - x + 2)^{5/2} + \frac{1}{144} (30x + 7) (3x^2 - x + 2)^{3/2} + \frac{(402x + 869)\sqrt{3x^2 - x + 2}}{1152} - \frac{13}{32} \sqrt{13} \tanh^{-1} \left( \frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}} \right) + \frac{2203 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{2304\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] ((869 + 402\*x)\*Sqrt[2 - x + 3\*x^2])/1152 + ((7 + 30\*x)\*(2 - x + 3\*x^2)^(3/2))/144 + (2\*(2 - x + 3\*x^2)^(5/2))/15 + (2203\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(2304\*Sqrt[3]) - (13\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/32

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps



$$\begin{aligned}
\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x} dx &= \frac{2}{15}(2-x+3x^2)^{5/2} + \frac{1}{60} \int \frac{(80+100x)(2-x+3x^2)^{3/2}}{1+2x} dx \\
&= \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{15}(2-x+3x^2)^{5/2} - \frac{\int \frac{(-13380-8040x)\sqrt{2-x+3x^2}}{1+2x} dx}{5760} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{15}(2-x+3x^2)^{5/2} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{15}(2-x+3x^2)^{5/2} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{15}(2-x+3x^2)^{5/2} \\
&= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{15}(2-x+3x^2)^{5/2}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 96, normalized size = 0.77

$$\frac{-14040\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + 6\sqrt{3x^2-x+2} (6912x^4 - 1008x^3 + 9624x^2 + 1058x + 7977) - 11015\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{34560}$$

Antiderivative was successfully verified.

[In] Integrate[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(7977 + 1058\*x + 9624\*x^2 - 1008\*x^3 + 6912\*x^4) - 11015\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]] - 14040\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/34560

**IntegrateAlgebraic [A]** time = 0.56, size = 124, normalized size = 1.00

$$\frac{2203 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{2304\sqrt{3}} + \frac{13}{16}\sqrt{13} \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right) + \frac{\sqrt{3x^2-x+2} (6912x^4 - 1008x^3 + 9624x^2 + 1058x + 7977)}{5760}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(7977 + 1058\*x + 9624\*x^2 - 1008\*x^3 + 6912\*x^4))/5760 + (13\*Sqrt[13]\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])])

)]/Sqrt[13]])/16 + (2203\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(230  
4\*Sqrt[3])

**fricas** [A] time = 0.96, size = 125, normalized size = 1.01

$$\frac{1}{5760} (6912x^4 - 1008x^3 + 9624x^2 + 1058x + 7977)\sqrt{3x^2 - x + 2} + \frac{2203}{13824} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right) + \frac{13}{64} \sqrt{13} \log\left(-\frac{4\sqrt{13}\sqrt{3x^2 - x + 2}(8x - 9) + 220x^2 - 196x + 185}{4x^2 + 4x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(1+2\*x),x, algorithm="fricas")

[Out] 1/5760\*(6912\*x^4 - 1008\*x^3 + 9624\*x^2 + 1058\*x + 7977)\*sqrt(3\*x^2 - x + 2)  
+ 2203/13824\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2  
+ 24\*x - 25) + 13/64\*sqrt(13)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9  
) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1))

**giac** [A] time = 0.28, size = 136, normalized size = 1.10

$$\frac{1}{5760} (2(12(6(48x - 7)x + 401)x + 529)x + 7977)\sqrt{3x^2 - x + 2} + \frac{2203}{6912} \sqrt{3} \log\left(-6\sqrt{3}x + \sqrt{3} + 6\sqrt{3x^2 - x + 2}\right) + \frac{13}{32} \sqrt{13} \log\left(-\frac{-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2 - x + 2}}{2(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2 - x + 2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(1+2\*x),x, algorithm="giac")

[Out] 1/5760\*(2\*(12\*(6\*(48\*x - 7)\*x + 401)\*x + 529)\*x + 7977)\*sqrt(3\*x^2 - x + 2)  
+ 2203/6912\*sqrt(3)\*log(-6\*sqrt(3)\*x + sqrt(3) + 6\*sqrt(3\*x^2 - x + 2)) +  
13/32\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(  
3\*x^2 - x + 2)))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2)))

**maple** [A] time = 0.01, size = 151, normalized size = 1.22

$$\frac{2203\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{2}\right)}{23}\right)}{6912} - \frac{13\sqrt{13} \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{1}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{32} + \frac{2\left(3x^2-x+2\right)^{\frac{5}{2}}}{15} + \frac{5(6x-1)\left(3x^2-x+2\right)^{\frac{3}{2}}}{144} + \frac{115(6x-1)\sqrt{3x^2-x+2}}{1152} + \frac{\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}{12} - \frac{(6x-1)\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{24} + \frac{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(2\*x+1),x)

[Out] 2/15\*(3\*x^2-x+2)^(5/2)+5/144\*(6\*x-1)\*(3\*x^2-x+2)^(3/2)+115/1152\*(6\*x-1)\*(3\*  
x^2-x+2)^(1/2)-2203/6912\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))+1/12\*(-4\*x+  
3\*(x+1/2)^2+5/4)^(3/2)-1/24\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)+13/32\*(-16  
\*x+12\*(x+1/2)^2+5)^(1/2)-13/32\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-  
16\*x+12\*(x+1/2)^2+5)^(1/2))

**maxima** [A] time = 0.99, size = 125, normalized size = 1.01

$$\frac{2}{15} (3x^2 - x + 2)^{\frac{5}{2}} + \frac{5}{24} (3x^2 - x + 2)^{\frac{3}{2}} x + \frac{7}{144} (3x^2 - x + 2)^{\frac{3}{2}} + \frac{67}{192} \sqrt{3x^2 - x + 2} x - \frac{2203}{6912} \sqrt{3} \operatorname{arcsinh}\left(\frac{6}{23} \sqrt{23} x - \frac{1}{23} \sqrt{23}\right) + \frac{13}{32} \sqrt{13} \operatorname{arcsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{869}{1152} \sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
[Out] 2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6/2
3*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(
2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
mupad [F]   time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{(3x^2 - x + 2)^{3/2} (4x^2 + 3x + 1)}{2x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
sympy [F]   time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(3x^2 - x + 2)^{\frac{3}{2}} (4x^2 + 3x + 1)}{2x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

$$3.214 \quad \int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} dx$$

**Optimal.** Leaf size=131

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)} - \frac{1}{104}(23-38x)(3x^2-x+2)^{3/2} - \frac{1}{192}(349-294x)\sqrt{3x^2-x+2} + \frac{25}{32}\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)$$

**Rubi [A]** time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)} - \frac{1}{104}(23-38x)(3x^2-x+2)^{3/2} - \frac{1}{192}(349-294x)\sqrt{3x^2-x+2} + \frac{25}{32}\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - \frac{2327 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2, x]

[Out] -((349 - 294\*x)\*Sqrt[2 - x + 3\*x^2])/192 - ((23 - 38\*x)\*(2 - x + 3\*x^2)^(3/2))/104 - (2 - x + 3\*x^2)^(5/2)/(13\*(1 + 2\*x)) - (2327\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(384\*Sqrt[3]) + (25\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/32

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(2-x+3x^2)^{3/2} (1+3x+4x^2)}{(1+2x)^2} dx &= -\frac{(2-x+3x^2)^{5/2}}{13(1+2x)} - \frac{1}{13} \int \frac{\left(-\frac{13}{2} - 38x\right) (2-x+3x^2)^{3/2}}{1+2x} dx \\
&= -\frac{1}{104} (23-38x) (2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)} + \int \frac{(-78+7644x)\sqrt{2-x+3x^2}}{1+2x} dx \\
&= -\frac{1}{192} (349-294x)\sqrt{2-x+3x^2} - \frac{1}{104} (23-38x) (2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)} \\
&= -\frac{1}{192} (349-294x)\sqrt{2-x+3x^2} - \frac{1}{104} (23-38x) (2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)} \\
&= -\frac{1}{192} (349-294x)\sqrt{2-x+3x^2} - \frac{1}{104} (23-38x) (2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)} \\
&= -\frac{1}{192} (349-294x)\sqrt{2-x+3x^2} - \frac{1}{104} (23-38x) (2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 103, normalized size = 0.79

$$\frac{900\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{6\sqrt{3x^2-x+2}(288x^4-96x^3+564x^2-332x-493)}{2x+1} + 2327\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1152}$$

Antiderivative was successfully verified.

[In] Integrate[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2,x]

[Out] ((6\*Sqrt[2 - x + 3\*x^2]\*(-493 - 332\*x + 564\*x^2 - 96\*x^3 + 288\*x^4))/(1 + 2\*x) + 2327\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]] + 900\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/1152

**IntegrateAlgebraic [A]** time = 0.63, size = 131, normalized size = 1.00

$$-\frac{2327 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{384\sqrt{3}} - \frac{25}{16}\sqrt{13} \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right) + \frac{\sqrt{3x^2-x+2}(288x^4-96x^3+564x^2-332x-493)}{192(2x+1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2,x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(-493 - 332\*x + 564\*x^2 - 96\*x^3 + 288\*x^4))/(192\*(1 + 2\*x)) - (25\*Sqrt[13]\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/16 - (2327\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(384\*Sqrt[3])

**fricas** [A] time = 1.40, size = 143, normalized size = 1.09

$$\frac{2327\sqrt{3}(2x+1)\log(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25)+900\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+12(288x^4-96x^3+564x^2-332x-493)\sqrt{3x^2-x+2}}{2304(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^2,x, algorithm="fricas")

[Out] 1/2304\*(2327\*sqrt(3)\*(2\*x + 1)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 900\*sqrt(13)\*(2\*x + 1)\*log((4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) - 220\*x^2 + 196\*x - 185)/(4\*x^2 + 4\*x + 1)) + 12\*(288\*x^4 - 96\*x^3 + 564\*x^2 - 332\*x - 493)\*sqrt(3\*x^2 - x + 2))/(2\*x + 1)

**giac** [B] time = 0.84, size = 570, normalized size = 4.35

$$\frac{25\sqrt{13}\sqrt{3x^2-x+2}\operatorname{arcsinh}\left(\frac{6\sqrt{3}\sqrt{3x^2-x+2}}{23}\right)+25\sqrt{13}\sqrt{3x^2-x+2}\operatorname{arctanh}\left(\frac{x(4x+2)\sqrt{13}}{13\sqrt{36x+12}\sqrt{3x^2-x+2}}\right)+\frac{(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{24}+\frac{23(6x-1)\sqrt{3x^2-x+2}}{192}-\frac{25(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4})^{\frac{3}{2}}}{156}+\frac{13(6x-1)\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{96}-\frac{25\sqrt{-16x+12}\left(x+\frac{1}{2}\right)^2+5}{32}-\frac{(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4})^{\frac{3}{2}}}{26\left(x+\frac{1}{2}\right)}+\frac{(6x-1)\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}{52}}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^2,x, algorithm="giac")

[Out] 25/32\*sqrt(13)\*log(sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1)) - 4)\*sgn(1/(2\*x + 1)) - 2327/1152\*sqrt(3)\*log(1/2\*abs(-2\*sqrt(3) + 2\*sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + 2\*sqrt(13)/(2\*x + 1)))/(sqrt(3) + sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))\*sgn(1/(2\*x + 1)) - 13/32\*sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3)\*sgn(1/(2\*x + 1)) + 1/192\*(5929\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^7\*sgn(1/(2\*x + 1)) - 7272\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^6\*sgn(1/(2\*x + 1)) + 25101\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^5\*sgn(1/(2\*x + 1)) - 48\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^4\*sgn(1/(2\*x + 1)) + 112359\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^3\*sgn(1/(2\*x + 1)) - 69336\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^2\*sgn(1/(2\*x + 1)) + 71955\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))\*sgn(1/(2\*x + 1)) + 24624\*sqrt(13)\*sgn(1/(2\*x + 1)))/(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^2 - 3)^4

**maple** [A] time = 0.01, size = 179, normalized size = 1.37

$$\frac{2327\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{3}\sqrt{3x^2-x+2}}{23}\right)+25\sqrt{13}\operatorname{arctanh}\left(\frac{x(4x+2)\sqrt{13}}{13\sqrt{36x+12}\sqrt{3x^2-x+2}}\right)+\frac{(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{24}+\frac{23(6x-1)\sqrt{3x^2-x+2}}{192}-\frac{25(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4})^{\frac{3}{2}}}{156}+\frac{13(6x-1)\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{96}-\frac{25\sqrt{-16x+12}\left(x+\frac{1}{2}\right)^2+5}{32}-\frac{(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4})^{\frac{3}{2}}}{26\left(x+\frac{1}{2}\right)}+\frac{(6x-1)\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}{52}}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^2,x)`

[Out]  $\frac{1}{24}(6x-1)(3x^2-x+2)^{3/2} + \frac{23}{192}(6x-1)(3x^2-x+2)^{1/2} + \frac{2327}{1152}3^{1/2} \operatorname{arcsinh}\left(\frac{6}{23}\sqrt{23}(x-1/6)\right) - \frac{25}{156}(-4x+3(x+1/2)^{2+5/4})^{3/2} + \frac{3}{96}(6x-1)(-4x+3(x+1/2)^{2+5/4})^{1/2} - \frac{25}{32}(-16x+12(x+1/2)^{2+5/4})^{1/2} + \frac{25}{32}13^{1/2} \operatorname{arctanh}\left(\frac{2}{13}(-4x+9/2)13^{1/2}/(-16x+12(x+1/2)^{2+5/4})^{1/2}\right) - \frac{1}{26}(x+1/2)(-4x+3(x+1/2)^{2+5/4})^{5/2} + \frac{1}{52}(6x-1)(-4x+3(x+1/2)^{2+5/4})^{3/2}$

**maxima** [A] time = 0.98, size = 132, normalized size = 1.01

$$\frac{1}{4}(3x^2-x+2)^{3/2}x - \frac{1}{8}(3x^2-x+2)^{3/2} + \frac{49}{32}\sqrt{3x^2-x+2}x + \frac{2327}{1152}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{25}{32}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) - \frac{349}{192}\sqrt{3x^2-x+2} - \frac{(3x^2-x+2)^{3/2}}{4(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{4}(3x^2-x+2)^{3/2}x - \frac{1}{8}(3x^2-x+2)^{3/2} + \frac{49}{32}\sqrt{3x^2-x+2}x + \frac{2327}{1152}\sqrt{3}\operatorname{arcsinh}\left(\frac{6}{23}\sqrt{3x^2-x+2}x - \frac{1}{23}\sqrt{3x^2-x+2}\right) - \frac{5}{32}\sqrt{13}\operatorname{arcsinh}\left(\frac{8\sqrt{23}\sqrt{3x^2-x+2}x}{23|2x+1|} - \frac{9\sqrt{23}\sqrt{3x^2-x+2}}{23|2x+1|}\right) - \frac{349}{192}\sqrt{3x^2-x+2} - \frac{1}{4}(3x^2-x+2)^{3/2}/(2x+1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{(2x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((3*x^2-x+2)^(3/2)*(3*x+4*x^2+1))/(2*x+1)^2,x)`

[Out] `int(((3*x^2-x+2)^(3/2)*(3*x+4*x^2+1))/(2*x+1)^2,x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{(2x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)`

[Out] `Integral((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(2*x+1)**2,x)`



$$3.215 \quad \int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3} dx$$

**Optimal.** Leaf size=138

$$-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2} + \frac{(122x+151)(3x^2-x+2)^{3/2}}{312(2x+1)} + \frac{1}{624}(1858-771x)\sqrt{3x^2-x+2} - \frac{1153 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}$$

**Rubi [A]** time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2} + \frac{(122x+151)(3x^2-x+2)^{3/2}}{312(2x+1)} + \frac{1}{624}(1858-771x)\sqrt{3x^2-x+2} - \frac{1153 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}} + \frac{1519 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] ((1858 - 771\*x)\*Sqrt[2 - x + 3\*x^2])/624 + ((151 + 122\*x)\*(2 - x + 3\*x^2)^(3/2))/(312\*(1 + 2\*x)) - (2 - x + 3\*x^2)^(5/2)/(26\*(1 + 2\*x)^2) + (1519\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(192\*Sqrt[3]) - (1153\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(64\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$

### Rule 812

$\text{Int}[\{(d\_.) + (e\_.)*(x\_)\}^{(m\_)}*\{(f\_.) + (g\_.)*(x\_)\}*\{(a\_.) + (b\_.)*(x\_.) + (c\_.)*(x\_.)^2\}^{(p\_.)}, x\_Symbol] \rightarrow \text{Simp}[\{(d + e*x)^{(m + 1)}*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p\}/(e^2*(m + 1)*(m + 2*p + 2)), x] + \text{Dist}[p/(e^2*(m + 1)*(m + 2*p + 2)), \text{Int}[(d + e*x)^{(m + 1)}*(a + b*x + c*x^2)^{(p - 1)}*\text{Simp}[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{RationalQ}[p] \ \&\& \ p > 0 \ \&\& \ (\text{LtQ}[m, -1] \ || \ \text{EqQ}[p, 1] \ || \ (\text{IntegerQ}[p] \ \&\& \ !\text{RationalQ}[m])) \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ !\text{ILtQ}[m + 2*p + 1, 0] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{IntegersQ}[2*m, 2*p])]$

### Rule 814

$\text{Int}[\{(d\_.) + (e\_.)*(x\_)\}^{(m\_)}*\{(f\_.) + (g\_.)*(x\_)\}*\{(a\_.) + (b\_.)*(x\_.) + (c\_.)*(x\_.)^2\}^{(p\_.)}, x\_Symbol] \rightarrow \text{Simp}[\{(d + e*x)^{(m + 1)}*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p\}/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - \text{Dist}[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^{(p - 1)}*\text{Simp}[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ !\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 0])) \ \&\& \ !\text{ILtQ}[m + 2*p, 0] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{IntegersQ}[2*m, 2*p])]$

### Rule 843

$\text{Int}[\{(d\_.) + (e\_.)*(x\_)\}^{(m\_)}*\{(f\_.) + (g\_.)*(x\_)\}*\{(a\_.) + (b\_.)*(x\_.) + (c\_.)*(x\_.)^2\}^{(p\_.)}, x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^{(m + 1)}*(a + b*x + c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, p\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ !\text{IGtQ}[m, 0]$

### Rule 1650

$\text{Int}[(Pq\_)*\{(d\_.) + (e\_.)*(x\_)\}^{(m\_)}*\{(a\_.) + (b\_.)*(x\_.) + (c\_.)*(x\_.)^2\}^{(p\_.)}, x\_Symbol] \rightarrow \text{With}[\{Q = \text{PolynomialQuotient}[Pq, d + e*x, x], R = \text{PolynomialRemainder}[Pq, d + e*x, x]\}, \text{Simp}[(e*R*(d + e*x)^{(m + 1)}*(a + b*x + c*x^2)^{(p + 1)})/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + \text{Dist}[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), \text{Int}[(d + e*x)^{(m + 1)}*(a + b*x + c*x^2)^p, x], x]$

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3} dx &= -\frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} - \frac{1}{26} \int \frac{\left(-\frac{31}{2} - 61x\right)(2-x+3x^2)^{3/2}}{(1+2x)^2} dx \\
&= \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} + \frac{1}{208} \int \frac{(639-1028x)}{1+2x} dx \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} \\
&= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 103, normalized size = 0.75

$$\frac{-10377\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{156\sqrt{3x^2-x+2}(96x^4-68x^3+390x^2+627x+182)}{(2x+1)^2} - 19747\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{7488}$$

Antiderivative was successfully verified.

[In] Integrate[((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] ((156\*sqrt[2 - x + 3\*x^2]\*(182 + 627\*x + 390\*x^2 - 68\*x^3 + 96\*x^4))/(1 + 2\*x)^2 - 19747\*sqrt[3]\*ArcSinh[(-1 + 6\*x)/sqrt[23]] - 10377\*sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*sqrt[13]\*sqrt[2 - x + 3\*x^2])])/7488

**IntegrateAlgebraic [A]** time = 0.79, size = 131, normalized size = 0.95

$$\frac{1519 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{192\sqrt{3}} + \frac{1153 \tanh^{-1}\left(\frac{-2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{32\sqrt{13}} + \frac{\sqrt{3x^2-x+2}(96x^4-68x^3+390x^2+627x+182)}{48(2x+1)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((2 - x + 3\*x^2)^(3/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3,x)

[Out] (Sqrt[2 - x + 3\*x^2]\*(182 + 627\*x + 390\*x^2 - 68\*x^3 + 96\*x^4))/(48\*(1 + 2\*x)^2) + (1153\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(32\*Sqrt[13]) + (1519\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(192\*Sqrt[3])

**fricas [A]** time = 0.81, size = 159, normalized size = 1.15

$$\frac{19747\sqrt{3}(4x^2+4x+1)\log(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25)+10377\sqrt{13}(4x^2+4x+1)\log\left(\frac{-4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+312(96x^4-68x^3+390x^2+627x+182)\sqrt{3x^2-x+2}}{14976(4x^2+4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^3,x, algorithm="fricas")

[Out] 1/14976\*(19747\*sqrt(3)\*(4\*x^2 + 4\*x + 1)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 10377\*sqrt(13)\*(4\*x^2 + 4\*x + 1)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 312\*(96\*x^4 - 68\*x^3 + 390\*x^2 + 627\*x + 182)\*sqrt(3\*x^2 - x + 2))/(4\*x^2 + 4\*x + 1)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(3/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{-18688512, [6]%%}%+%%{%{[56065536,0]: [1,0,-3]%%}, [5]%%}%+%%{-28032768, [4]%%}%+%%{%{[-224262144,0]: [1,0,-3]%%}, [3]%%}%+%%{70081920, [2]%%}%+%%{%{[350409600,0]: [1,0,-3]%%}, [1]%%}%+%%{292008000, [0]%%}% / %%{%{[24,0]: [1,0,-3]%%}, [6]%%}%+%%{-216, [5]%%}%+%%{%{[36,0]: [1,0,-3]%%}, [4]%%}%+%%{864, [3]%%}%+%%{%{[-90,0]: [1,0,-3]%%}, [2]%%}%+%%{-1350, [1]%%}%+%%{%{[-375,0]: [1,0,-3]%%}, [0]%%}% Error: Bad Argument Value

**maple [A]** time = 0.01, size = 162, normalized size = 1.17

$$\frac{1519\sqrt{3} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-\frac{1}{2})}{23}\right) - 1153\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{1}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right) + 1153(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}} - 257(6x-1)\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}} + 1153\sqrt{-16x+12(x+\frac{1}{2})^2+5} - 15(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{5}{2}} - 15(6x-1)(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}} - \frac{(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{5}{2}}}{104(x+\frac{1}{2})^2}}{576} - \frac{1153\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{1}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{832} + \frac{1153(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}{4056} - \frac{257(6x-1)\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}{1248} + \frac{1153\sqrt{-16x+12(x+\frac{1}{2})^2+5}}{832} + \frac{15(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{5}{2}}}{338(x+\frac{1}{2})} - \frac{15(6x-1)(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}{676} - \frac{(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{5}{2}}}{104(x+\frac{1}{2})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^3,x)`

[Out]  $1153/4056*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-257/1248*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-1519/576*3^(1/2)*\operatorname{arcsinh}(6/23*23^(1/2)*(x-1/6))+1153/832*(-16*x+12*(x+1/2)^2+5)^(1/2)-1153/832*13^(1/2)*\operatorname{arctanh}(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))+15/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-15/676*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+5/4)^(5/2)$

**maxima [A]** time = 0.98, size = 143, normalized size = 1.04

$$\frac{61}{312}(3x^2-x+2)^{\frac{3}{2}} - \frac{(3x^2-x+2)^{\frac{5}{2}}}{26(4x^2+4x+1)} - \frac{257}{208}\sqrt{3x^2-x+2} - \frac{1519}{576}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{1153}{832}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{929}{312}\sqrt{3x^2-x+2} + \frac{15(3x^2-x+2)^{\frac{3}{2}}}{52(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")`

[Out]  $61/312*(3*x^2-x+2)^(3/2) - 1/26*(3*x^2-x+2)^(5/2)/(4*x^2+4*x+1) - 257/208*\operatorname{sqrt}(3*x^2-x+2)*x - 1519/576*\operatorname{sqrt}(3)*\operatorname{arcsinh}(6/23*\operatorname{sqrt}(23)*x - 1/23*\operatorname{sqrt}(23)) + 1153/832*\operatorname{sqrt}(13)*\operatorname{arcsinh}(8/23*\operatorname{sqrt}(23)*x/\operatorname{abs}(2*x+1) - 9/23*\operatorname{sqrt}(23)/\operatorname{abs}(2*x+1)) + 929/312*\operatorname{sqrt}(3*x^2-x+2) + 15/52*(3*x^2-x+2)^(3/2)/(2*x+1)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{(2x+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((3*x^2-x+2)^(3/2)*(3*x+4*x^2+1))/(2*x+1)^3,x)`

[Out] `int(((3*x^2-x+2)^(3/2)*(3*x+4*x^2+1))/(2*x+1)^3,x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2-x+2)^{\frac{3}{2}}(4x^2+3x+1)}{(2x+1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

$$3.216 \quad \int (1 + 2x)^3 (2 - x + 3x^2)^{5/2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=189

$$\frac{2}{33} (3x^2 - x + 2)^{7/2} (2x+1)^4 + \frac{29}{330} (3x^2 - x + 2)^{7/2} (2x+1)^3 + \frac{133 (3x^2 - x + 2)^{7/2} (2x+1)^2 (26353 - 21350x)}{1485} - \frac{(26353 - 21350x) (3x^2 - x + 2)^{7/2}}{498960}$$

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{33} (3x^2 - x + 2)^{7/2} (2x+1)^4 + \frac{29}{330} (3x^2 - x + 2)^{7/2} (2x+1)^3 + \frac{133 (3x^2 - x + 2)^{7/2} (2x+1)^2}{1485} - \frac{(26353 - 21350x) (3x^2 - x + 2)^{7/2}}{498960} + \frac{5089(1 - 6x) (3x^2 - x + 2)^{5/2}}{155520} + \frac{117047(1 - 6x) (3x^2 - x + 2)^{3/2}}{1492992} + \frac{2692081(1 - 6x) \sqrt{3x^2 - x + 2}}{11943936} + \frac{61917863 \sinh^{-1} \left( \frac{1-6x}{\sqrt{3}} \right)}{23887872\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (2692081\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/11943936 + (117047\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(3/2))/1492992 + (5089\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(5/2))/155520 - ((26353 - 21350\*x)\*(2 - x + 3\*x^2)^(7/2))/498960 + (133\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(7/2))/1485 + (29\*(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(7/2))/330 + (2\*(1 + 2\*x)^4\*(2 - x + 3\*x^2)^(7/2))/33 + (61917863\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(23887872\*Sqrt[3])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

### Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps



$$\begin{aligned}
\int (1+2x)^3 (2-x+3x^2)^{5/2} (1+3x+4x^2) dx &= \frac{2}{33} (1+2x)^4 (2-x+3x^2)^{7/2} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{29}{330} (1+2x)^3 (2-x+3x^2)^{7/2} + \frac{2}{33} (1+2x)^4 (2-x+3x^2)^{7/2} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{133(1+2x)^2 (2-x+3x^2)^{7/2}}{1485} + \frac{29}{330} (1+2x)^3 (2-x+3x^2)^{7/2} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= -\frac{(26353-21350x)(2-x+3x^2)^{7/2}}{498960} + \frac{133(1+2x)^2 (2-x+3x^2)^{7/2}}{1485} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{5089(1-6x)(2-x+3x^2)^{5/2}}{155520} - \frac{(26353-21350x)(2-x+3x^2)^{7/2}}{498960} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{117047(1-6x)(2-x+3x^2)^{3/2}}{1492992} + \frac{5089(1-6x)(2-x+3x^2)^{5/2}}{155520} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x+3x^2)^{3/2}}{1492992} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x+3x^2)^{3/2}}{1492992} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx \\
&= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x+3x^2)^{3/2}}{1492992} + \frac{1}{132} \int (1+2x)^3 (32+348x) (2-x+3x^2)^{5/2} dx
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 90, normalized size = 0.48

$$\frac{6\sqrt{3x^2-x+2} (120394874880x^{10} + 207681159168x^9 + 308846297088x^8 + 419978151936x^7 + 415908006912x^6 + 347247744768x^5 + 263636134272x^4 + 161269204752x^3 + 72088585464x^2 + 26646633218x + 9173509857) - 23838377255\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{3}}\right)}{27590492160}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(9173509857 + 26646633218\*x + 72088585464\*x^2 + 161269204752\*x^3 + 263636134272\*x^4 + 347247744768\*x^5 + 415908006912\*x^6 + 419978151936\*x^7 + 308846297088\*x^8 + 207681159168\*x^9 + 120394874880\*x^10) - 23838377255\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/27590492160

**IntegrateAlgebraic [A]** time = 1.11, size = 105, normalized size = 0.56

$$\frac{61917863 \log\left(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1\right) + \sqrt{3x^2-x+2} (120394874880x^{10} + 207681159168x^9 + 308846297088x^8 + 419978151936x^7 + 415908006912x^6 + 347247744768x^5 + 263636134272x^4 + 161269204752x^3 + 72088585464x^2 + 26646633218x + 9173509857)}{23887872\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2),x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(9173509857 + 26646633218\*x + 72088585464\*x^2 + 161269204752\*x^3 + 263636134272\*x^4 + 347247744768\*x^5 + 415908006912\*x^6 + 419978151936\*x^7 + 308846297088\*x^8 + 207681159168\*x^9 + 120394874880\*x^10))/4598415360 + (61917863\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(23887872\*Sqrt[3])

**fricas** [A] time = 1.62, size = 103, normalized size = 0.54

$$\frac{1}{4598415360} (120394874880 x^{10} + 207681159168 x^9 + 308846297088 x^8 + 419978151936 x^7 + 415908006912 x^6 + 347247744768 x^5 + 263636134272 x^4 + 161269204752 x^3 + 26646633218 x^2 + 9173509857) \sqrt{3x^2 - x + 2} + \frac{61917863}{143327232} \sqrt{3} \log(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="fricas")

[Out] 1/4598415360\*(120394874880\*x^10 + 207681159168\*x^9 + 308846297088\*x^8 + 419978151936\*x^7 + 415908006912\*x^6 + 347247744768\*x^5 + 263636134272\*x^4 + 161269204752\*x^3 + 72088585464\*x^2 + 26646633218\*x + 9173509857)\*sqrt(3\*x^2 - x + 2) + 61917863/143327232\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac** [A] time = 0.20, size = 98, normalized size = 0.52

$$\frac{1}{4598415360} (2(12(6(8(6(36(14(48(18(40x + 69)x + 1847)x + 120557)x + 1671441)x + 50238389)x + 228850811)x + 1119925033)x + 3003691061)x + 13323316609)x + 9173509857) \sqrt{3x^2 - x + 2} + \frac{61917863}{71663616} \sqrt{3} \log(-2\sqrt{3}(\sqrt{3}x - \sqrt{3x^2 - x + 2}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="giac")

[Out] 1/4598415360\*(2\*(12\*(6\*(8\*(6\*(36\*(14\*(48\*(18\*(40\*x + 69)\*x + 1847)\*x + 120557)\*x + 1671441)\*x + 50238389)\*x + 228850811)\*x + 1119925033)\*x + 3003691061)\*x + 13323316609)\*x + 9173509857)\*sqrt(3\*x^2 - x + 2) + 61917863/71663616\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple** [A] time = 0.02, size = 153, normalized size = 0.81

$$\frac{32(3x^2 - x + 2)^{\frac{7}{2}} x^4}{33} + \frac{436(3x^2 - x + 2)^{\frac{7}{2}} x^3}{165} + \frac{4258(3x^2 - x + 2)^{\frac{7}{2}} x^2}{1485} + \frac{10073(3x^2 - x + 2)^{\frac{7}{2}} x}{7128} - \frac{61917863\sqrt{3} \operatorname{arcsinh}\left(\frac{4\sqrt{3}(x-1)}{23}\right)}{71663616} - \frac{5089(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{155520} - \frac{117047(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{1492992} - \frac{2692081(6x-1)\sqrt{3x^2-x+2}}{11943936} + \frac{92423(3x^2-x+2)^{\frac{7}{2}}}{498960}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)^3\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x)

[Out] 32/33\*x^4\*(3\*x^2-x+2)^(7/2)+436/165\*x^3\*(3\*x^2-x+2)^(7/2)+4258/1485\*x^2\*(3\*x^2-x+2)^(7/2)+10073/7128\*x\*(3\*x^2-x+2)^(7/2)-5089/155520\*(6\*x-1)\*(3\*x^2-x+

$(2)^{(5/2)} - 117047/1492992 * (6*x-1) * (3*x^2-x+2)^{(3/2)} - 61917863/71663616 * 3^{(1/2)}$   
 $* \operatorname{arcsinh}(6/23 * 23^{(1/2)} * (x-1/6)) - 2692081/11943936 * (6*x-1) * (3*x^2-x+2)^{(1/2)} +$   
 $92423/498960 * (3*x^2-x+2)^{(7/2)}$

**maxima** [A] time = 0.98, size = 184, normalized size = 0.97

$\frac{32}{33} (3x^2-x+2)^{7/2} + \frac{436}{165} (3x^2-x+2)^{7/2} x + \frac{4258}{1485} (3x^2-x+2)^{7/2} x^2 + \frac{10073}{7128} (3x^2-x+2)^{7/2} x^3 + \frac{92423}{498960} (3x^2-x+2)^{7/2} x^4 + \frac{5089}{25920} (3x^2-x+2)^{5/2} x + \frac{5089}{15520} (3x^2-x+2)^{5/2} x^2 - \frac{117047}{248832} (3x^2-x+2)^{3/2} x + \frac{117047}{1492992} (3x^2-x+2)^{3/2} x^2 - \frac{2692081}{1990656} \sqrt{3x^2-x+2} x - \frac{61917863}{71663616} \sqrt{3} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23} (6x-1)\right) + \frac{2692081}{11943936} \sqrt{3x^2-x+2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="maxima")

[Out] 32/33\*(3\*x^2 - x + 2)^(7/2)\*x^4 + 436/165\*(3\*x^2 - x + 2)^(7/2)\*x^3 + 4258/1485\*(3\*x^2 - x + 2)^(7/2)\*x^2 + 10073/7128\*(3\*x^2 - x + 2)^(7/2)\*x + 92423/498960\*(3\*x^2 - x + 2)^(7/2) - 5089/25920\*(3\*x^2 - x + 2)^(5/2)\*x + 5089/15520\*(3\*x^2 - x + 2)^(5/2) - 117047/248832\*(3\*x^2 - x + 2)^(3/2)\*x + 117047/1492992\*(3\*x^2 - x + 2)^(3/2) - 2692081/1990656\*sqrt(3\*x^2 - x + 2)\*x - 61917863/71663616\*sqrt(3)\*arcsinh(1/23\*sqrt(23)\*(6\*x - 1)) + 2692081/11943936\*sqrt(3\*x^2 - x + 2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)^3 (3x^2-x+2)^{5/2} (4x^2+3x+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 1)^3\*(3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1),x)

[Out] int((2\*x + 1)^3\*(3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3 (3x^2-x+2)^{5/2} (4x^2+3x+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*3\*(3\*x\*\*2-x+2)\*\*(5/2)\*(4\*x\*\*2+3\*x+1),x)

[Out] Integral((2\*x + 1)\*\*3\*(3\*x\*\*2 - x + 2)\*\*(5/2)\*(4\*x\*\*2 + 3\*x + 1), x)

$$3.217 \quad \int (1 + 2x)^2 (2 - x + 3x^2)^{5/2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=164

$$\frac{1}{15}(2x+1)^3 (3x^2 - x + 2)^{7/2} + \frac{37}{405}(2x+1)^2 (3x^2 - x + 2)^{7/2} + \frac{(3430x + 2731)(3x^2 - x + 2)^{7/2}}{17010} - \frac{293(1 - 6x)(3x^2 - x + 2)^{7/2}}{58320}$$

**Rubi [A]** time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{15}(2x+1)^3 (3x^2 - x + 2)^{7/2} + \frac{37}{405}(2x+1)^2 (3x^2 - x + 2)^{7/2} + \frac{(3430x + 2731)(3x^2 - x + 2)^{7/2}}{17010} - \frac{293(1 - 6x)(3x^2 - x + 2)^{7/2}}{58320} - \frac{6739(1 - 6x)(3x^2 - x + 2)^{3/2}}{559872} - \frac{154997(1 - 6x)\sqrt{3x^2 - x + 2}}{4478976} - \frac{3564931 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (-154997\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/4478976 - (6739\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(3/2))/559872 - (293\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(5/2))/58320 + (37\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(7/2))/405 + ((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(7/2))/15 + ((2731 + 3430\*x)\*(2 - x + 3\*x^2)^(7/2))/17010 - (3564931\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(8957952\*Sqrt[3])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (1+2x)^2 (2-x+3x^2)^{5/2} (1+3x+4x^2) dx &= \frac{1}{15}(1+2x)^3 (2-x+3x^2)^{7/2} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= \frac{37}{405}(1+2x)^2 (2-x+3x^2)^{7/2} + \frac{1}{15}(1+2x)^3 (2-x+3x^2)^{7/2} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= \frac{37}{405}(1+2x)^2 (2-x+3x^2)^{7/2} + \frac{1}{15}(1+2x)^3 (2-x+3x^2)^{7/2} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= -\frac{293(1-6x)(2-x+3x^2)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2 (2-x+3x^2)^{7/2} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= -\frac{6739(1-6x)(2-x+3x^2)^{3/2}}{559872} - \frac{293(1-6x)(2-x+3x^2)^{5/2}}{58320} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)(2-x+3x^2)^{3/2}}{559872} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)(2-x+3x^2)^{3/2}}{559872} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx \\
&= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)(2-x+3x^2)^{3/2}}{559872} + \frac{1}{120} \int (1+2x)^2 (52+296x) (2-x+3x^2)^{5/2} dx
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 85, normalized size = 0.52

$$\frac{6\sqrt{3x^2-x+2} (2257403904x^9 + 2675441664x^8 + 4427716608x^7 + 5671627776x^6 + 4996802304x^5 + 4171579776x^4 + 3096104976x^3 + 1693765752x^2 + 692659234x + 387182961) + 124772585\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{2x}}\right)}{940584960}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(387182961 + 692659234\*x + 1693765752\*x^2 + 3096104976\*x^3 + 4171579776\*x^4 + 4996802304\*x^5 + 5671627776\*x^6 + 4427716608\*x^7 + 2675441664\*x^8 + 2257403904\*x^9) + 124772585\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/940584960

**IntegrateAlgebraic [A]** time = 1.05, size = 100, normalized size = 0.61

$$\frac{\sqrt{3x^2-x+2} (2257403904x^9 + 2675441664x^8 + 4427716608x^7 + 5671627776x^6 + 4996802304x^5 + 4171579776x^4 + 3096104976x^3 + 1693765752x^2 + 692659234x + 387182961)}{156764160} - \frac{3564931 \log(2\sqrt{3}\sqrt{3x^2-x+2} - 6x + 1)}{8957952\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2),x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(387182961 + 692659234\*x + 1693765752\*x^2 + 3096104976\*x^3 + 4171579776\*x^4 + 4996802304\*x^5 + 5671627776\*x^6 + 4427716608\*x^7 + 2675441664\*x^8 + 2257403904\*x^9))/156764160 - (3564931\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(8957952\*Sqrt[3])

**fricas** [A] time = 1.08, size = 98, normalized size = 0.60

$$\frac{1}{156764160} (2257403904x^9 + 2675441664x^8 + 4427716608x^7 + 5671627776x^6 + 4996802304x^5 + 4171579776x^4 + 3096104976x^3 + 1693765752x^2 + 692659234x + 387182961) \sqrt{3x^2 - x + 2} + \frac{3564931}{53747712} \sqrt{3} \log(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="fricas")

[Out] 1/156764160\*(2257403904\*x^9 + 2675441664\*x^8 + 4427716608\*x^7 + 5671627776\*x^6 + 4996802304\*x^5 + 4171579776\*x^4 + 3096104976\*x^3 + 1693765752\*x^2 + 692659234\*x + 387182961)\*sqrt(3\*x^2 - x + 2) + 3564931/53747712\*sqrt(3)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac** [A] time = 0.21, size = 93, normalized size = 0.57

$$\frac{1}{156764160} (2(12(6(8(6(36(14(24(27x + 32)x + 1271)x + 22793)x + 722917)x + 3621163)x + 21500729)x + 70573573)x + 346329617)x + 387182961) \sqrt{3x^2 - x + 2} - \frac{3564931}{26873856} \sqrt{3} \log(-2\sqrt{3}(\sqrt{3x^2 - x + 2}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="giac")

[Out] 1/156764160\*(2\*(12\*(6\*(8\*(6\*(36\*(14\*(24\*(27\*x + 32)\*x + 1271)\*x + 22793)\*x + 722917)\*x + 3621163)\*x + 21500729)\*x + 70573573)\*x + 346329617)\*x + 387182961)\*sqrt(3\*x^2 - x + 2) - 3564931/26873856\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple** [A] time = 0.01, size = 136, normalized size = 0.83

$$\frac{8(3x^2 - x + 2)^{\frac{7}{2}}x^3}{15} + \frac{472(3x^2 - x + 2)^{\frac{7}{2}}x^2}{405} + \frac{235(3x^2 - x + 2)^{\frac{7}{2}}x}{243} + \frac{3564931\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}(x-\frac{1}{6})}{23}\right)}{26873856} + \frac{293(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{58320} + \frac{6739(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{559872} + \frac{154997(6x-1)\sqrt{3x^2-x+2}}{4478976} + \frac{5419(3x^2-x+2)^{\frac{7}{2}}}{17010}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)^2\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x)

[Out] 8/15\*(3\*x^2-x+2)^(7/2)\*x^3+472/405\*(3\*x^2-x+2)^(7/2)\*x^2+235/243\*(3\*x^2-x+2)^(7/2)\*x+293/58320\*(6\*x-1)\*(3\*x^2-x+2)^(5/2)+6739/559872\*(6\*x-1)\*(3\*x^2-x+2)^(3/2)+3564931/26873856\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))+154997/4478976\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)+5419/17010\*(3\*x^2-x+2)^(7/2)

**maxima** [A] time = 0.97, size = 167, normalized size = 1.02

$$\frac{8}{15} (3x^2 - x + 2)^{\frac{7}{2}}x^3 + \frac{472}{405} (3x^2 - x + 2)^{\frac{7}{2}}x^2 + \frac{235}{243} (3x^2 - x + 2)^{\frac{7}{2}}x + \frac{5419}{17010} (3x^2 - x + 2)^{\frac{7}{2}} + \frac{293}{9720} (3x^2 - x + 2)^{\frac{5}{2}} - \frac{293}{58320} (3x^2 - x + 2)^{\frac{3}{2}} + \frac{6739}{93312} (3x^2 - x + 2)^{\frac{1}{2}}x - \frac{6739}{559872} (3x^2 - x + 2)^{\frac{1}{2}} + \frac{154997}{746496} \sqrt{3x^2 - x + 2}x + \frac{3564931}{26873856} \sqrt{3} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23}(6x - 1)\right) - \frac{154997}{4478976} \sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
[Out] 8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/24
3*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(3*
x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x^2
- x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sqrt(3
*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
- 154997/4478976*sqrt(3*x^2 - x + 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x + 1)^2 (3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 1)^2 (3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```



$$3.218 \quad \int (1 + 2x) (2 - x + 3x^2)^{5/2} (1 + 3x + 4x^2) dx$$

Optimal. Leaf size=139

$$\frac{2}{27}(2x+1)^2(3x^2-x+2)^{7/2} + \frac{1}{648}(122x+137)(3x^2-x+2)^{7/2} - \frac{445(1-6x)(3x^2-x+2)^{5/2}}{15552} - \frac{51175(1-6x)(3x^2-x+2)^{3/2}}{746496}$$

**Rubi [A]** time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{27}(2x+1)^2(3x^2-x+2)^{7/2} + \frac{1}{648}(122x+137)(3x^2-x+2)^{7/2} - \frac{445(1-6x)(3x^2-x+2)^{5/2}}{15552} - \frac{51175(1-6x)(3x^2-x+2)^{3/2}}{746496} - \frac{1177025(1-6x)\sqrt{3x^2-x+2}}{5971968} - \frac{27071575 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2\*x)\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2), x]

[Out] (-1177025\*(1 - 6\*x)\*Sqrt[2 - x + 3\*x^2])/5971968 - (51175\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(3/2))/746496 - (445\*(1 - 6\*x)\*(2 - x + 3\*x^2)^(5/2))/15552 + (2\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(7/2))/27 + ((137 + 122\*x)\*(2 - x + 3\*x^2)^(7/2))/648 - (27071575\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(11943936\*Sqrt[3])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

$2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x$   
 $] + \text{Dist}[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), \text{Int}[(a + b*x + c*x^2)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 1653

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x\_Symbol] := \text{With}[\{q = \text{Expon}[Pq, x], f = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + \text{Dist}[1/(c*e^q*(m + q + 2*p + 1)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p*\text{ExpandToSum}[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /;$  GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned} \int (1 + 2x)(2 - x + 3x^2)^{5/2}(1 + 3x + 4x^2) dx &= \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{108} \int (1 + 2x)(72 + 244x)(2 - x + 3x^2)^{7/2} dx \\ &= \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{648}(137 + 122x)(2 - x + 3x^2)^{7/2} \\ &= -\frac{445(1 - 6x)(2 - x + 3x^2)^{5/2}}{15552} + \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{648}(137 + 122x)(2 - x + 3x^2)^{7/2} \\ &= -\frac{51175(1 - 6x)(2 - x + 3x^2)^{3/2}}{746496} - \frac{445(1 - 6x)(2 - x + 3x^2)^{5/2}}{15552} + \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{648}(137 + 122x)(2 - x + 3x^2)^{7/2} \\ &= -\frac{1177025(1 - 6x)\sqrt{2 - x + 3x^2}}{5971968} - \frac{51175(1 - 6x)(2 - x + 3x^2)^{3/2}}{746496} + \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{648}(137 + 122x)(2 - x + 3x^2)^{7/2} \\ &= -\frac{1177025(1 - 6x)\sqrt{2 - x + 3x^2}}{5971968} - \frac{51175(1 - 6x)(2 - x + 3x^2)^{3/2}}{746496} + \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{648}(137 + 122x)(2 - x + 3x^2)^{7/2} \\ &= -\frac{1177025(1 - 6x)\sqrt{2 - x + 3x^2}}{5971968} - \frac{51175(1 - 6x)(2 - x + 3x^2)^{3/2}}{746496} + \frac{2}{27}(1 + 2x)^2(2 - x + 3x^2)^{7/2} + \frac{1}{648}(137 + 122x)(2 - x + 3x^2)^{7/2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 80, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2} (47775744x^8 + 30357504x^7 + 79377408x^6 + 80034048x^5 + 66969216x^4 + 58946544x^3 + 41031048x^2 + 19860062x + 10960335) + 27071575\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{35831808}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2\*x)\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2),x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(10960335 + 19860062\*x + 41031048\*x^2 + 58946544\*x^3 + 66969216\*x^4 + 80034048\*x^5 + 79377408\*x^6 + 30357504\*x^7 + 47775744\*x^8) + 27071575\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/35831808

**IntegrateAlgebraic [A]** time = 0.91, size = 95, normalized size = 0.68

$$\frac{\sqrt{3x^2-x+2} (47775744x^8 + 30357504x^7 + 79377408x^6 + 80034048x^5 + 66969216x^4 + 58946544x^3 + 41031048x^2 + 19860062x + 10960335)}{5971968} - \frac{27071575 \log(2\sqrt{3}\sqrt{3x^2-x+2} - 6x + 1)}{11943936\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2\*x)\*(2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2),x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(10960335 + 19860062\*x + 41031048\*x^2 + 58946544\*x^3 + 66969216\*x^4 + 80034048\*x^5 + 79377408\*x^6 + 30357504\*x^7 + 47775744\*x^8))/5971968 - (27071575\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(11943936\*Sqrt[3])

**fricas [A]** time = 1.09, size = 93, normalized size = 0.67

$$\frac{1}{5971968} (47775744x^8 + 30357504x^7 + 79377408x^6 + 80034048x^5 + 66969216x^4 + 58946544x^3 + 41031048x^2 + 19860062x + 10960335)\sqrt{3x^2-x+2} + \frac{27071575}{71663616} \sqrt{3} \log(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1) - 72x^2 + 24x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="fricas")

[Out] 1/5971968\*(47775744\*x^8 + 30357504\*x^7 + 79377408\*x^6 + 80034048\*x^5 + 66969216\*x^4 + 58946544\*x^3 + 41031048\*x^2 + 19860062\*x + 10960335)\*sqrt(3\*x^2 - x + 2) + 27071575/71663616\*sqrt(3)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac [A]** time = 0.19, size = 88, normalized size = 0.63

$$\frac{1}{5971968} (2(12(6(8(6(36(2(96x+61)x+319)x+11579)x+58133)x+409351)x+1709627)x+9930031)x+10960335)\sqrt{3x^2-x+2} - \frac{27071575}{35831808} \sqrt{3} \log(-2\sqrt{3}(\sqrt{3x-\sqrt{3x^2-x+2}})+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1),x, algorithm="giac")

[Out] 1/5971968\*(2\*(12\*(6\*(8\*(6\*(36\*(2\*(96\*x + 61)\*x + 319)\*x + 11579)\*x + 58133)\*x + 409351)\*x + 1709627)\*x + 9930031)\*x + 10960335)\*sqrt(3\*x^2 - x + 2) -

27071575/35831808\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

maple [A] time = 0.01, size = 119, normalized size = 0.86

$$\frac{8(3x^2-x+2)^{\frac{7}{2}}x^2}{27} + \frac{157(3x^2-x+2)^{\frac{7}{2}}x}{324} + \frac{27071575\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}(x-\frac{1}{6})}{23}\right)}{35831808} + \frac{185(3x^2-x+2)^{\frac{7}{2}}}{648} + \frac{445(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{15552} + \frac{51175(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{746496} + \frac{1177025(6x-1)\sqrt{3x^2-x+2}}{5971968}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1), x)

[Out] 8/27\*(3\*x^2-x+2)^(7/2)\*x^2+157/324\*(3\*x^2-x+2)^(7/2)\*x+185/648\*(3\*x^2-x+2)^(7/2)+445/15552\*(6\*x-1)\*(3\*x^2-x+2)^(5/2)+51175/746496\*(6\*x-1)\*(3\*x^2-x+2)^(3/2)+1177025/5971968\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)+27071575/35831808\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))

maxima [A] time = 0.97, size = 150, normalized size = 1.08

$$\frac{8}{27}(3x^2-x+2)^{\frac{7}{2}}x^2 + \frac{157}{324}(3x^2-x+2)^{\frac{7}{2}}x + \frac{185}{648}(3x^2-x+2)^{\frac{7}{2}} + \frac{445}{2592}(3x^2-x+2)^{\frac{5}{2}}x - \frac{445}{15552}(3x^2-x+2)^{\frac{5}{2}} + \frac{51175}{124416}(3x^2-x+2)^{\frac{3}{2}}x - \frac{51175}{746496}(3x^2-x+2)^{\frac{3}{2}} + \frac{1177025}{995328}\sqrt{3x^2-x+2}x + \frac{27071575}{35831808}\sqrt{3}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{1177025}{5971968}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1), x, algorithm="maxima")

[Out] 8/27\*(3\*x^2 - x + 2)^(7/2)\*x^2 + 157/324\*(3\*x^2 - x + 2)^(7/2)\*x + 185/648\*(3\*x^2 - x + 2)^(7/2) + 445/2592\*(3\*x^2 - x + 2)^(5/2)\*x - 445/15552\*(3\*x^2 - x + 2)^(5/2) + 51175/124416\*(3\*x^2 - x + 2)^(3/2)\*x - 51175/746496\*(3\*x^2 - x + 2)^(3/2) + 1177025/995328\*sqrt(3\*x^2 - x + 2)\*x + 27071575/35831808\*sqrt(3)\*arcsinh(1/23\*sqrt(23)\*(6\*x - 1)) - 1177025/5971968\*sqrt(3\*x^2 - x + 2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x + 1) (3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 1)\*(3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1), x)

[Out] int((2\*x + 1)\*(3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 1) (3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

$$3.219 \quad \int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x} dx$$

**Optimal.** Leaf size=147

$$\frac{2}{21} (3x^2 - x + 2)^{7/2} + \frac{(150x + 29)(3x^2 - x + 2)^{5/2}}{1080} + \frac{(2154x + 2449)(3x^2 - x + 2)^{3/2}}{10368} + \frac{(221999 - 17850x)\sqrt{3x^2 - x + 2}}{82944}$$

**Rubi [A]** time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{21} (3x^2 - x + 2)^{7/2} + \frac{(150x + 29)(3x^2 - x + 2)^{5/2}}{1080} + \frac{(2154x + 2449)(3x^2 - x + 2)^{3/2}}{10368} + \frac{(221999 - 17850x)\sqrt{3x^2 - x + 2}}{82944} - \frac{169}{128} \sqrt{13} \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}}\right) + \frac{944521 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] ((221999 - 17850\*x)\*Sqrt[2 - x + 3\*x^2])/82944 + ((2449 + 2154\*x)\*(2 - x + 3\*x^2)^(3/2))/10368 + ((29 + 150\*x)\*(2 - x + 3\*x^2)^(5/2))/1080 + (2\*(2 - x + 3\*x^2)^(7/2))/21 + (944521\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(165888\*Sqrt[3]) - (169\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/128

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{(2-x+3x^2)^{5/2} (1+3x+4x^2)}{1+2x} dx &= \frac{2}{21} (2-x+3x^2)^{7/2} + \frac{1}{84} \int \frac{(112+140x)(2-x+3x^2)^{5/2}}{1+2x} dx \\
&= \frac{(29+150x)(2-x+3x^2)^{5/2}}{1080} + \frac{2}{21} (2-x+3x^2)^{7/2} - \int \frac{(-29708-20104x)(2-x+3x^2)^{3/2}}{1+2x} dx \\
&= \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{(29+150x)(2-x+3x^2)^{5/2}}{1080} + \frac{2}{21} (2-x+3x^2)^{7/2} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{2}{21} (2-x+3x^2)^{7/2} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{2}{21} (2-x+3x^2)^{7/2} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{2}{21} (2-x+3x^2)^{7/2} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{2}{21} (2-x+3x^2)^{7/2} \\
&= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{2}{21} (2-x+3x^2)^{7/2}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 106, normalized size = 0.72

$$\frac{-22997520\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + 6\sqrt{3x^2-x+2} (7464960x^6 - 3836160x^5 + 15700608x^4 - 3646512x^3 + 12466776x^2 - 2120998x + 11665053) - 33058235\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{17418240}$$

Antiderivative was successfully verified.

[In] Integrate[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x), x]

[Out] (6\*sqrt[2 - x + 3\*x^2]\*(11665053 - 2120998\*x + 12466776\*x^2 - 3646512\*x^3 + 15700608\*x^4 - 3836160\*x^5 + 7464960\*x^6) - 33058235\*sqrt[3]\*ArcSinh[(-1 + 6\*x)/sqrt[23]] - 22997520\*sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*sqrt[13]\*sqrt[2 - x + 3\*x^2])])/17418240

**IntegrateAlgebraic [A]** time = 0.78, size = 134, normalized size = 0.91

$$\frac{944521 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{165888\sqrt{3}} + \frac{169}{64}\sqrt{13} \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right) + \frac{\sqrt{3x^2-x+2} (7464960x^6 - 3836160x^5 + 15700608x^4 - 3646512x^3 + 12466776x^2 - 2120998x + 11665053)}{2903040}$$

Antiderivative was successfully verified.



[In] IntegrateAlgebraic[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x),x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(11665053 - 2120998\*x + 12466776\*x^2 - 3646512\*x^3 + 15700608\*x^4 - 3836160\*x^5 + 7464960\*x^6))/2903040 + (169\*Sqrt[13]\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/64 + (944521\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(165888\*Sqrt[3])

**fricas** [A] time = 1.14, size = 135, normalized size = 0.92

$$\frac{1}{2903040} (7464960x^6 - 3836160x^5 + 15700608x^4 - 3646512x^3 + 12466776x^2 - 2120998x + 11665053) \sqrt{3x^2 - x + 2} + \frac{944521}{995328} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right) + \frac{169}{256} \sqrt{13} \log\left(\frac{4\sqrt{13}\sqrt{3x^2 - x + 2}(8x - 9) + 220x^2 - 196x + 185}{4x^2 + 4x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x),x, algorithm="fricas")

[Out] 1/2903040\*(7464960\*x^6 - 3836160\*x^5 + 15700608\*x^4 - 3646512\*x^3 + 12466776\*x^2 - 2120998\*x + 11665053)\*sqrt(3\*x^2 - x + 2) + 944521/995328\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 169/256\*sqrt(13)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1))

**giac** [A] time = 0.31, size = 146, normalized size = 0.99

$$\frac{1}{2903040} (2(12(18(8(30(72x - 37)x + 4543)x - 8441)x + 519449)x - 1060499)x + 11665053) \sqrt{3x^2 - x + 2} + \frac{944521}{497664} \sqrt{3} \log(-6\sqrt{3}x + \sqrt{3} + 6\sqrt{3x^2 - x + 2}) + \frac{169}{128} \sqrt{13} \log\left(\frac{-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2 - x + 2}}{2(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2 - x + 2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x),x, algorithm="giac")

[Out] 1/2903040\*(2\*(12\*(18\*(8\*(30\*(72\*x - 37)\*x + 4543)\*x - 8441)\*x + 519449)\*x - 1060499)\*x + 11665053)\*sqrt(3\*x^2 - x + 2) + 944521/497664\*sqrt(3)\*log(-6\*sqrt(3)\*x + sqrt(3) + 6\*sqrt(3\*x^2 - x + 2)) + 169/128\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2)))

**maple** [A] time = 0.01, size = 207, normalized size = 1.41

$$\frac{944521\sqrt{3} \operatorname{arcsinh}\left(\frac{\sqrt{3}(x-\frac{1}{2})}{2}\right)}{497664} - \frac{169\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{1}{2})\sqrt{3}}{13\sqrt{-3x+2}(\frac{1}{2}+x)}\right)}{128} + \frac{2(3x^2-x+2)^{\frac{3}{2}}}{21} + \frac{5(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{216} + \frac{575(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{10368} + \frac{13225(6x-1)\sqrt{3x^2-x+2}}{82944} + \frac{(-4x+3(\frac{1}{2}+x)^2+\frac{1}{2})^{\frac{3}{2}}}{20} - \frac{(6x-1)(-4x+3(\frac{1}{2}+x)^2+\frac{1}{2})^{\frac{3}{2}}}{48} - \frac{25(6x-1)\sqrt{-4x+3(\frac{1}{2}+x)^2+\frac{1}{2}}}{128} - \frac{13(-4x+3(\frac{1}{2}+x)^2+\frac{1}{2})^{\frac{3}{2}}}{48} - \frac{169\sqrt{-16x+12(\frac{1}{2}+x)^2+5}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(2\*x+1),x)

[Out] 2/21\*(3\*x^2-x+2)^(7/2)+5/216\*(6\*x-1)\*(3\*x^2-x+2)^(5/2)+575/10368\*(6\*x-1)\*(3\*x^2-x+2)^(3/2)+13225/82944\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)-944521/497664\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))+1/20\*(-4\*x+3\*(x+1/2)^2+5/4)^(5/2)-1/48\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)-25/128\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)

)+13/48\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)+169/128\*(-16\*x+12\*(x+1/2)^2+5)^(1/2)-169/128\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))

**maxima** [A] time = 0.97, size = 154, normalized size = 1.05

$$\frac{2}{21}(3x^2-x+2)^{\frac{7}{2}} + \frac{5}{36}(3x^2-x+2)^{\frac{5}{2}}x + \frac{29}{1080}(3x^2-x+2)^{\frac{3}{2}} + \frac{359}{1728}(3x^2-x+2)^{\frac{1}{2}}x + \frac{2449}{10368}(3x^2-x+2)^{\frac{1}{2}} - \frac{2975}{13824}\sqrt{3x^2-x+2}x - \frac{944521}{497664}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{169}{128}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{221999}{82944}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x),x, algorithm="maxima")

[Out] 2/21\*(3\*x^2 - x + 2)^(7/2) + 5/36\*(3\*x^2 - x + 2)^(5/2)\*x + 29/1080\*(3\*x^2 - x + 2)^(5/2) + 359/1728\*(3\*x^2 - x + 2)^(3/2)\*x + 2449/10368\*(3\*x^2 - x + 2)^(3/2) - 2975/13824\*sqrt(3\*x^2 - x + 2)\*x - 944521/497664\*sqrt(3)\*arcsinh(6/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 169/128\*sqrt(13)\*arcsinh(8/23\*sqrt(23)\*x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1)) + 221999/82944\*sqrt(3\*x^2 - x + 2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1)}{2x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1),x)

[Out] int(((3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 - x + 2)^{\frac{5}{2}} (4x^2 + 3x + 1)}{2x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x\*\*2-x+2)\*\*(5/2)\*(4\*x\*\*2+3\*x+1)/(1+2\*x),x)

[Out] Integral((3\*x\*\*2 - x + 2)\*\*(5/2)\*(4\*x\*\*2 + 3\*x + 1)/(2\*x + 1), x)

$$3.220 \quad \int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2} dx$$

**Optimal.** Leaf size=154

$$\frac{(3x^2 - x + 2)^{7/2}}{13(2x + 1)} - \frac{11(37 - 60x)(3x^2 - x + 2)^{5/2}}{2340} - \frac{11}{864}(67 - 78x)(3x^2 - x + 2)^{3/2} - \frac{11(4727 - 3090x)\sqrt{3x^2 - x + 2}}{6912}$$

**Rubi [A]** time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$ , Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2 - x + 2)^{7/2}}{13(2x + 1)} - \frac{11(37 - 60x)(3x^2 - x + 2)^{5/2}}{2340} - \frac{11}{864}(67 - 78x)(3x^2 - x + 2)^{3/2} - \frac{11(4727 - 3090x)\sqrt{3x^2 - x + 2}}{6912} + \frac{429}{128}\sqrt{13} \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}}\right) - \frac{315623 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2, x]

[Out] (-11\*(4727 - 3090\*x)\*Sqrt[2 - x + 3\*x^2])/6912 - (11\*(67 - 78\*x)\*(2 - x + 3\*x^2)^(3/2))/864 - (11\*(37 - 60\*x)\*(2 - x + 3\*x^2)^(5/2))/2340 - (2 - x + 3\*x^2)^(7/2)/(13\*(1 + 2\*x)) - (315623\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(13824\*Sqrt[3]) + (429\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/128

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(2-x+3x^2)^{5/2} (1+3x+4x^2)}{(1+2x)^2} dx &= \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} - \frac{1}{13} \int \frac{\left(-\frac{11}{2} - 44x\right) (2-x+3x^2)^{5/2}}{1+2x} dx \\
&= -\frac{11(37-60x)(2-x+3x^2)^{5/2}}{2340} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} + \int \frac{(-286+14872x)(2-x+3x^2)^{3/2}}{1+2x} dx \\
&= -\frac{11}{864}(67-78x)(2-x+3x^2)^{3/2} - \frac{11(37-60x)(2-x+3x^2)^{5/2}}{2340} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} \\
&= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)(2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} \\
&= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)(2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} \\
&= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)(2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} \\
&= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)(2-x+3x^2)^{3/2} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 113, normalized size = 0.73

$$\frac{694980\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{6\sqrt{3x^2-x+2}(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)}{2x+1} + 1578115\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{207360}$$

Antiderivative was successfully verified.

[In] Integrate[((2-x+3\*x^2)^(5/2)\*(1+3\*x+4\*x^2))/(1+2\*x)^2,x]

[Out] ((6\*Sqrt[2-x+3\*x^2]\*(-364257-322972\*x+310660\*x^2-115680\*x^3+251424\*x^4-65664\*x^5+103680\*x^6))/(1+2\*x)+1578115\*Sqrt[3]\*ArcSinh[(-1+6\*x)/Sqrt[23]]+694980\*Sqrt[13]\*ArcTanh[(9-8\*x)/(2\*Sqrt[13]\*Sqrt[2-x+3\*x^2])])/207360

**IntegrateAlgebraic [A]** time = 0.87, size = 141, normalized size = 0.92

$$-\frac{315623 \log(2\sqrt{3}\sqrt{3x^2-x+2}-6x+1)}{13824\sqrt{3}} - \frac{429}{64}\sqrt{13} \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right) + \frac{\sqrt{3x^2-x+2}(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)}{34560(2x+1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^2,x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(-364257 - 322972\*x + 310660\*x^2 - 115680\*x^3 + 251424\*x^4 - 65664\*x^5 + 103680\*x^6))/(34560\*(1 + 2\*x)) - (429\*Sqrt[13]\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/64 - (315623\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(13824\*Sqrt[3])

**fricas** [A] time = 0.62, size = 153, normalized size = 0.99

$$\frac{1578115\sqrt{3}(2x+1)\log(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25)+694980\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+12(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)\sqrt{3x^2-x+2}}{414720(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^2,x, algorithm="fricas")

[Out] 1/414720\*(1578115\*sqrt(3)\*(2\*x + 1)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 694980\*sqrt(13)\*(2\*x + 1)\*log((4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) - 220\*x^2 + 196\*x - 185)/(4\*x^2 + 4\*x + 1)) + 12\*(103680\*x^6 - 65664\*x^5 + 251424\*x^4 - 115680\*x^3 + 310660\*x^2 - 322972\*x - 364257)\*sqrt(3\*x^2 - x + 2))/(2\*x + 1)

**giac** [B] time = 1.10, size = 760, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^2,x, algorithm="giac")

[Out] 429/128\*sqrt(13)\*log(sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1)) - 4)\*sgn(1/(2\*x + 1)) - 315623/41472\*sqrt(3)\*log(1/2\*abs(-2\*sqrt(3) + 2\*sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + 2\*sqrt(13)/(2\*x + 1))/(sqrt(3) + sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1)))\*sgn(1/(2\*x + 1)) - 169/128\*sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3)\*sgn(1/(2\*x + 1)) + 1/34560\*(5154065\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^11\*sgn(1/(2\*x + 1)) - 7837020\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^10\*sgn(1/(2\*x + 1)) + 39468815\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^9\*sgn(1/(2\*x + 1)) - 14445540\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^8\*sgn(1/(2\*x + 1)) + 460893402\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^7\*sgn(1/(2\*x + 1)) - 343084680\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^6\*sgn(1/(2\*x + 1)) + 944150094\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^5\*sgn(1/(2\*x + 1)) - 22871160\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^4\*sgn(1/(2\*x + 1)) + 1397032245\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^3\*sgn(1/(2\*x + 1)) -

683367516\*sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^2\*sgn(1/(2\*x + 1)) + 392684355\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))\*sgn(1/(2\*x + 1)) + 197538588\*sqrt(13)\*sgn(1/(2\*x + 1)))/((sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1))^2 - 3)^6

**maple [A]** time = 0.01, size = 235, normalized size = 1.53

$\frac{315623\sqrt{3} \operatorname{arsinh}\left(\frac{\sqrt{23}}{23}\right)}{41472} + \frac{429\sqrt{13} \operatorname{arctanh}\left(\frac{\sqrt{-16x+12}}{2\sqrt{2x+1}}\right)}{128} + \frac{(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{36} + \frac{115(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{1728} + \frac{2645(6x-1)\sqrt{3x^2-x+2}}{13824} + \frac{31(-4x+3\sqrt{x+\frac{1}{2}})^{\frac{5}{2}}}{260} + \frac{19(6x-1)(-4x+3\sqrt{x+\frac{1}{2}})^{\frac{3}{2}}}{192} + \frac{965(6x-1)\sqrt{-4x+3\sqrt{x+\frac{1}{2}}}}{1536} + \frac{429\sqrt{-16x+12}\sqrt{x+\frac{1}{2}}}{128} + \frac{11(-4x+3\sqrt{x+\frac{1}{2}})^{\frac{3}{2}}}{16} + \frac{429\sqrt{-16x+12}\sqrt{x+\frac{1}{2}}}{128} + \frac{(6x-1)(-4x+3\sqrt{x+\frac{1}{2}})^{\frac{5}{2}}}{28\sqrt{x+\frac{1}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(2\*x+1)^2, x)

[Out] 1/36\*(6\*x-1)\*(3\*x^2-x+2)^(5/2)+115/1728\*(6\*x-1)\*(3\*x^2-x+2)^(3/2)+2645/13824\*(6\*x-1)\*(3\*x^2-x+2)^(1/2)+315623/41472\*3^(1/2)\*arsinh(6/23\*23^(1/2)\*(x-1/6))-33/260\*(-4\*x+3\*(x+1/2)^2+5/4)^(5/2)+19/192\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)+965/1536\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)-11/16\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)-429/128\*(-16\*x+12\*(x+1/2)^2+5)^(1/2)+429/128\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)\*(-4\*x+3\*(x+1/2)^2+5/4)^(7/2)+1/52\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(5/2)

**maxima [A]** time = 1.00, size = 161, normalized size = 1.05

$\frac{1}{6}(3x^2-x+2)^{\frac{5}{2}}x - \frac{7}{90}(3x^2-x+2)^{\frac{5}{2}} + \frac{143}{144}(3x^2-x+2)^{\frac{3}{2}}x - \frac{737}{864}(3x^2-x+2)^{\frac{3}{2}} - \frac{(3x^2-x+2)^{\frac{5}{2}}}{4(2x+1)} + \frac{5665}{1152}\sqrt{3x^2-x+2}x + \frac{315623}{41472}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{429}{128}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) - \frac{51997}{6912}\sqrt{3x^2-x+2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^2, x, algorithm="maxima")

[Out] 1/6\*(3\*x^2 - x + 2)^(5/2)\*x - 7/90\*(3\*x^2 - x + 2)^(5/2) + 143/144\*(3\*x^2 - x + 2)^(3/2)\*x - 737/864\*(3\*x^2 - x + 2)^(3/2) - 1/4\*(3\*x^2 - x + 2)^(5/2)/(2\*x + 1) + 5665/1152\*sqrt(3\*x^2 - x + 2)\*x + 315623/41472\*sqrt(3)\*arsinh(6/23\*sqrt(23)\*x - 1/23\*sqrt(23)) - 429/128\*sqrt(13)\*arsinh(8/23\*sqrt(23)\*x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1)) - 51997/6912\*sqrt(3\*x^2 - x + 2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1)}{(2x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1)^2, x)

[Out] `int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 - x + 2)^{\frac{5}{2}} (4x^2 + 3x + 1)}{(2x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)`

[Out] `Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)`



$$3.221 \quad \int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3} dx$$

**Optimal.** Leaf size=161

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2} + \frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)} + \frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2} + \frac{(21317-10470x)\sqrt{3x^2-x+2}}{1536}$$

**Rubi [A]** time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2} + \frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)} + \frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2} + \frac{(21317-10470x)\sqrt{3x^2-x+2}}{1536} - \frac{1631}{256}\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{118423 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] ((21317 - 10470\*x)\*Sqrt[2 - x + 3\*x^2])/1536 + ((1227 - 838\*x)\*(2 - x + 3\*x^2)^(3/2))/832 + ((257 + 134\*x)\*(2 - x + 3\*x^2)^(5/2))/(520\*(1 + 2\*x)) - (2 - x + 3\*x^2)^(7/2)/(26\*(1 + 2\*x)^2) + (118423\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(3072\*Sqrt[3]) - (1631\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/256

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$$\frac{a^2e - b^2d - (2cd - b^2e)x}{\sqrt{ax^2 + bx + c}}$$
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && NeQ[2cd - b^2e, 0]

### Rule 812

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(e\*f\*(m + 2\*p + 2) - d\*g\*(2\*p + 1) + e\*g\*(m + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(e^2\*(m + 1)\*(m + 2\*p + 2)), x] + Dist[p/(e^2\*(m + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[g\*(b\*d + 2\*a\*e + 2\*a\*e\*m + 2\*b\*d\*p) - f\*b\*e\*(m + 2\*p + 2) + (g\*(2\*c\*d + b\*e + b\*e\*m + 4\*c\*d\*p) - 2\*c\*e\*f\*(m + 2\*p + 2))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2\*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1), x], x]

\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(2-x+3x^2)^{5/2} (1+3x+4x^2)}{(1+2x)^3} dx &= -\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} - \frac{1}{26} \int \frac{\left(-\frac{29}{2} - 67x\right) (2-x+3x^2)^{5/2}}{(1+2x)^2} dx \\
 &= \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} + \frac{1}{208} \int \frac{(793-1676x+1024x^2)(2-x+3x^2)^{3/2}}{(1+2x)^2} dx \\
 &= \frac{1}{832} (1227-838x) (2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} \\
 &= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832} (1227-838x) (2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} \\
 &= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832} (1227-838x) (2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} \\
 &= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832} (1227-838x) (2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} \\
 &= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832} (1227-838x) (2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} \\
 &= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832} (1227-838x) (2-x+3x^2)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 113, normalized size = 0.70

$$\frac{-293580\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{6\sqrt{3x^2-x+2}(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)}{(2x+1)^2} - 592115\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{46080}$$

Antiderivative was successfully verified.

[In] Integrate[((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3, x]

[Out] ((6\*Sqrt[2 - x + 3\*x^2]\*(142057 + 464446\*x + 256564\*x^2 - 76200\*x^3 + 83616\*x^4 - 22464\*x^5 + 27648\*x^6))/(1 + 2\*x)^2 - 592115\*Sqrt[3]\*ArcSinh[(-1 + 6

$\frac{x}{\sqrt{23}} - 293580\sqrt{13}\operatorname{ArcTanh}\left(\frac{9 - 8x}{(2\sqrt{13}\sqrt{2 - x + 3x^2})}\right)/46080$

**IntegrateAlgebraic [A]** time = 1.03, size = 141, normalized size = 0.88

$$\frac{118423\log\left(\frac{2\sqrt{3}\sqrt{3x^2-x+2}-6x+1}{3072\sqrt{3}}\right) + \frac{1631\sqrt{13}\tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{128} + \frac{\sqrt{3x^2-x+2}(27648x^6 - 22464x^5 + 83616x^4 - 76200x^3 + 256564x^2 + 464446x + 142057)}{7680(2x+1)^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((2 - x + 3\*x^2)^(5/2)\*(1 + 3\*x + 4\*x^2))/(1 + 2\*x)^3,x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(142057 + 464446\*x + 256564\*x^2 - 76200\*x^3 + 83616\*x^4 - 22464\*x^5 + 27648\*x^6))/(7680\*(1 + 2\*x)^2) + (1631\*Sqrt[13]\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/128 + (118423\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(3072\*Sqrt[3])

**fricas [A]** time = 1.11, size = 169, normalized size = 1.05

$$\frac{592115\sqrt{3}(4x^2+4x+1)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right) + 293580\sqrt{13}(4x^2+4x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right) + 12(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)\sqrt{3x^2-x+2}}{92160(4x^2+4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^3,x, algorithm="fricas")

[Out] 1/92160\*(592115\*sqrt(3)\*(4\*x^2 + 4\*x + 1)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 293580\*sqrt(13)\*(4\*x^2 + 4\*x + 1)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 12\*(27648\*x^6 - 22464\*x^5 + 83616\*x^4 - 76200\*x^3 + 256564\*x^2 + 464446\*x + 142057)\*sqrt(3\*x^2 - x + 2))/(4\*x^2 + 4\*x + 1)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{-299016192, [6]%%}%+%%{%{[897048576,0]: [1,0,-3]%%}, [5]%%}%+%%{-448524288, [4]%%}%+%%{%{[-3588194304,0]: [1,0,-3]%%}, [3]%%}%+%%{1121310720, [2]%%}%+%%{%{[5606553600,0]: [1,0,-3]%%}, [1]%%}%+%%{4672128000, [0]%%}% / %%{%{[24,0]: [1,0,-3]%%}, [6]%%}%+%%{-216, [5]%%}%+%%{%{[36,0]: [1,0,-3]%%}, [4]%%}%+%%{864, [3]%%}%+%%{%{[-90,0]: [1,0,-3]%%}, [2]%%}%+%%{-1350, [1]%%}%+%%{%{[-375,0]: [1,0,-3]%%}, [0]%%}% Error: Bad Argument Value

**maple [A]** time = 0.02, size = 199, normalized size = 1.24

$$\frac{118423\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}(x-1)}{23}\right)}{9216} - \frac{1631\sqrt{13} \operatorname{arctanh}\left(\frac{2(4x+3)\sqrt{13}}{(13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{256} - \frac{1631(-4x+3(x+\frac{1}{2})^2+\frac{1}{2})^{\frac{5}{2}}}{6760} - \frac{1631(-4x+3(x+\frac{1}{2})^2+\frac{1}{2})^{\frac{3}{2}}}{1248} - \frac{1631\sqrt{-16x+12(x+\frac{1}{2})^2+5}}{256} - \frac{(-4x+3(x+\frac{1}{2})^2+\frac{1}{2})^{\frac{7}{2}}}{104(x+\frac{1}{2})^2} - \frac{1745(6x-1)\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{1}{2}}}{1536} - \frac{19(-4x+3(x+\frac{1}{2})^2+\frac{1}{2})^{\frac{7}{2}}}{338(x+\frac{1}{2})} - \frac{19(6x-1)(-4x+3(x+\frac{1}{2})^2+\frac{1}{2})^{\frac{5}{2}}}{676} - \frac{419(6x-1)(-4x+3(x+\frac{1}{2})^2+\frac{1}{2})^{\frac{3}{2}}}{2496}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(2\*x+1)^3,x)

[Out] 1631/6760\*(-4\*x+3\*(x+1/2)^2+5/4)^(5/2)+1631/1248\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)+1631/256\*(-16\*x+12\*(x+1/2)^2+5)^(1/2)-1/104/(x+1/2)^2\*(-4\*x+3\*(x+1/2)^2+5/4)^(7/2)-1745/1536\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)+19/338/(x+1/2)\*(-4\*x+3\*(x+1/2)^2+5/4)^(7/2)-19/676\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(5/2)-419/2496\*(6\*x-1)\*(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)-1631/256\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))-118423/9216\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))

**maxima [A]** time = 0.99, size = 172, normalized size = 1.07

$$\frac{67}{520}(3x^2-x+2)^{\frac{5}{2}} - \frac{(3x^2-x+2)^{\frac{7}{2}}}{26(4x^2+4x+1)} - \frac{419}{416}(3x^2-x+2)^{\frac{3}{2}}x + \frac{1227}{832}(3x^2-x+2)^{\frac{3}{2}} + \frac{19(3x^2-x+2)^{\frac{5}{2}}}{52(2x+1)} - \frac{1745}{256}\sqrt{3x^2-x+2}x - \frac{118423}{9216}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{1631}{256}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23(2x+1)} - \frac{9\sqrt{23}}{23(2x+1)}\right) + \frac{21317}{1536}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2-x+2)^(5/2)\*(4\*x^2+3\*x+1)/(1+2\*x)^3,x, algorithm="maxima")

[Out] 67/520\*(3\*x^2 - x + 2)^(5/2) - 1/26\*(3\*x^2 - x + 2)^(7/2)/(4\*x^2 + 4\*x + 1) - 419/416\*(3\*x^2 - x + 2)^(3/2)\*x + 1227/832\*(3\*x^2 - x + 2)^(3/2) + 19/52\*(3\*x^2 - x + 2)^(5/2)/(2\*x + 1) - 1745/256\*sqrt(3\*x^2 - x + 2)\*x - 118423/9216\*sqrt(3)\*arcsinh(6/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 1631/256\*sqrt(13)\*arcsinh(8/23\*sqrt(23)\*x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1)) + 21317/1536\*sqrt(3\*x^2 - x + 2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2 - x + 2)^{5/2} (4x^2 + 3x + 1)}{(2x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1)^3,x)

[Out] int(((3\*x^2 - x + 2)^(5/2)\*(3\*x + 4\*x^2 + 1))/(2\*x + 1)^3, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 - x + 2)^{\frac{5}{2}} (4x^2 + 3x + 1)}{(2x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

$$3.222 \quad \int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=693

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3\left(a^2h^2(eh+3fg)+2abh\left(h(dh+3eg)+3fg^2\right)+b^2g\left(3h(dh+eg)+fg^2\right)\right)-80bc^2\right)}{\sqrt{a+bx+cx^2}}$$

**Rubi [A]** time = 2.10, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1653, 832, 779, 621, 206}

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^3\*(d + e\*x + f\*x^2))/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((63\*b^2\*f\*h^2 - 2\*c\*h\*(24\*b\*f\*g + 35\*b\*e\*h + 32\*a\*f\*h) - c^2\*(12\*f\*g^2 - 20\*h\*(3\*e\*g + 4\*d\*h)))\*(g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2])/(240\*c^3\*h) - ((9\*b\*f\*h + 2\*c\*(f\*g - 5\*e\*h))\*(g + h\*x)^3\*Sqrt[a + b\*x + c\*x^2])/(40\*c^2\*h) + (f\*(g + h\*x)^4\*Sqrt[a + b\*x + c\*x^2])/(5\*c\*h) + ((945\*b^4\*f\*h^4 - 64\*c^4\*(3\*f\*g^4 - 5\*g^2\*h\*(3\*e\*g + 16\*d\*h)) - 210\*b^2\*c\*h^3\*(14\*a\*f\*h + 5\*b\*(3\*f\*g + e\*h)) + 8\*c^2\*h^2\*(128\*a^2\*f\*h^2 + 275\*a\*b\*h\*(3\*f\*g + e\*h) + 3\*b^2\*(129\*f\*g^2 + 50\*h\*(3\*e\*g + d\*h))) - 16\*c^3\*h\*(16\*a\*h\*(13\*f\*g^2 + 5\*h\*(3\*e\*g + d\*h)) + b\*g\*(39\*f\*g^2 + 5\*h\*(47\*e\*g + 54\*d\*h))) - 2\*c\*h\*(315\*b^3\*f\*h^3 - 14\*b\*c\*h^2\*(39\*b\*f\*g + 25\*b\*e\*h + 46\*a\*f\*h) + 16\*c^3\*(3\*f\*g^3 - 5\*g\*h\*(3\*e\*g + 10\*d\*h)) + 8\*c^2\*h\*(21\*b\*f\*g^2 + 10\*b\*h\*(8\*e\*g + 5\*d\*h) + a\*h\*(71\*f\*g + 45\*e\*h)))\*x)\*Sqrt[a + b\*x + c\*x^2])/(1920\*c^5\*h) + ((256\*c^5\*d\*g^3 - 63\*b^5\*f\*h^3 + 70\*b^3\*c\*h^2\*(3\*b\*f\*g + b\*e\*h + 4\*a\*f\*h) - 128\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + b\*g\*(e\*g + 3\*d\*h)) - 80\*b\*c^2\*h\*(3\*a^2\*f\*h^2 + 3\*a\*b\*h\*(3\*f\*g + e\*h) + b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 96\*c^3\*(a^2\*h^2\*(3\*f\*g + e\*h) + b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 2\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])]/(256\*c^(11/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a,

b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 832

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p\*Simp[m\*(c\*d\*f - a\*e\*g) + d\*(2\*c\*f - b\*g)\*(p + 1) + (m\*(c\*e\*f + c\*d\*g - b\*e\*g) + e\*(p + 1)\*(2\*c\*f - b\*g))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps



$$\begin{aligned}
\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx &= \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left(-\frac{1}{2}h(bfg-10cdh+8afh)-\frac{1}{2}h(2cfg-10ceh+9bfh)x\right)}{\sqrt{a+bx+cx^2}} dx}{5ch^2} \\
&= -\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h} + \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} \\
&= \frac{(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12fg^2-20h(3eg+4dh)))(g+hx)^3\sqrt{a+bx+cx^2}}{240c^3h} \\
&= \frac{(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12fg^2-20h(3eg+4dh)))(g+hx)^2\sqrt{a+bx+cx^2}}{240c^3h} \\
&= \frac{(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12fg^2-20h(3eg+4dh)))(g+hx)\sqrt{a+bx+cx^2}}{240c^3h} \\
&= \frac{(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12fg^2-20h(3eg+4dh)))\sqrt{a+bx+cx^2}}{240c^3h}
\end{aligned}$$

**Mathematica [A]** time = 1.25, size = 588, normalized size = 0.85

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)^3\*(d + e\*x + f\*x^2))/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(945\*b^4\*f\*h^3 - 210\*b^2\*c\*h^2\*(5\*b\*e\*h + 14\*a\*f\*h + 3\*b\*f\*(5\*g + h\*x)) + 32\*c^4\*(10\*d\*h\*(18\*g^2 + 9\*g\*h\*x + 2\*h^2\*x^2) + 15\*e\*(4\*g^3 + 6\*g^2\*h\*x + 4\*g\*h^2\*x^2 + h^3\*x^3) + 3\*f\*x\*(10\*g^3 + 20\*g^2\*h\*x + 15\*g\*h^2\*x^2 + 4\*h^3\*x^3)) + 4\*c^2\*h\*(256\*a^2\*f\*h^2 + 2\*a\*b\*h\*(825\*f\*g + 275\*e\*h + 161\*f\*h\*x) + b^2\*(25\*h\*(36\*e\*g + 12\*d\*h + 7\*e\*h\*x) + 3\*f\*(300\*g^2 + 175\*g\*h\*x + 42\*h^2\*x^2))) - 16\*c^3\*(a\*h\*(5\*h\*(48\*e\*g + 16\*d\*h + 9\*e\*h\*x) + f\*(240\*g^2 + 135\*g\*h\*x + 32\*h^2\*x^2)) + b\*(3\*f\*(30\*g^3 + 50\*g^2\*h\*x + 35\*g\*h^2\*x^2 + 9\*h^3\*x^3) + 5\*h\*(2\*d\*h\*(27\*g + 5\*h\*x) + e\*(54\*g^2 + 30\*g\*h\*x + 7\*h^2\*x^2)))))/(1920\*c^5) + (((256\*c^5\*d\*g^3 - 63\*b^5\*f\*h^3 + 70\*b^3\*c\*h^2\*(3\*b\*f\*g + b\*e\*h + 4\*a\*f\*h) - 128\*c^4\*g\*(a\*f\*g^2 + 3\*a\*h\*(e\*g + d\*h) + b\*g\*(e\*g + 3\*d\*h)) - 80\*b\*c^2\*h\*(3\*a^2\*f\*h^2 + 3\*a\*b\*h\*(3\*f\*g + e\*h) + b^2\*(3\*f\*g^2 + 3\*e\*g\*h + d\*h^2)) + 96\*c^3\*(a^2\*h^2\*(3\*f\*g + e\*h) + b^2\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 2\*a\*b\*h\*(3\*f\*g^2 + h\*(3\*e\*g + d\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])]/(256\*c^(11/2))

IntegrateAlgebraic [A] time = 3.58, size = 820, normalized size = 1.18

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^3\*(d + e\*x + f\*x^2))/Sqrt[a + b\*x + c\*x^2],x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(1920\*c^4\*e\*g^3 - 1440\*b\*c^3\*f\*g^3 + 5760\*c^4\*d\*g^2\*h - 4320\*b\*c^3\*e\*g^2\*h + 3600\*b^2\*c^2\*f\*g^2\*h - 3840\*a\*c^3\*f\*g^2\*h - 4320\*b\*c^3\*d\*g\*h^2 + 3600\*b^2\*c^2\*e\*g\*h^2 - 3840\*a\*c^3\*e\*g\*h^2 - 3150\*b^3\*c\*f\*g\*h^2 + 6600\*a\*b\*c^2\*f\*g\*h^2 + 1200\*b^2\*c^2\*d\*h^3 - 1280\*a\*c^3\*d\*h^3 - 1050\*b^3\*c\*e\*h^3 + 2200\*a\*b\*c^2\*e\*h^3 + 945\*b^4\*f\*h^3 - 2940\*a\*b^2\*c\*f\*h^3 + 1024\*a^2\*c^2\*f\*h^3 + 960\*c^4\*f\*g^3\*x + 2880\*c^4\*e\*g^2\*h\*x - 2400\*b\*c^3\*f\*g^2\*h\*x + 2880\*c^4\*d\*g\*h^2\*x - 2400\*b\*c^3\*e\*g\*h^2\*x + 2100\*b^2\*c^2\*f\*g\*h^2\*x - 2160\*a\*c^3\*f\*g\*h^2\*x - 800\*b\*c^3\*d\*h^3\*x + 700\*b^2\*c^2\*e\*h^3\*x - 720\*a\*c^3\*e\*h^3\*x - 630\*b^3\*c\*f\*h^3\*x + 1288\*a\*b\*c^2\*f\*h^3\*x + 1920\*c^4\*f\*g^2\*h\*x^2 + 1920\*c^4\*e\*g\*h^2\*x^2 - 1680\*b\*c^3\*f\*g\*h^2\*x^2 + 640\*c^4\*d\*h^3\*x^2 - 560\*b\*c^3\*e\*h^3\*x^2 + 504\*b^2\*c^2\*f\*h^3\*x^2 - 512\*a\*c^3\*f\*h^3\*x^2 + 1440\*c^4\*f\*g\*h^2\*x^3 + 480\*c^4\*e\*h^3\*x^3 - 432\*b\*c^3\*f\*h^3\*x^3 + 384\*c^4\*f\*h^3\*x^4))/(1920\*c^5) + ((-256\*c^5\*d\*g^3 + 128\*b\*c^4\*e\*g^3 - 96\*b^2\*c^3\*f\*g^3 + 128\*a\*c^4\*f\*g^3 + 384\*b\*c^4\*d\*g^2\*h - 288\*b^2\*c^3\*e\*g^2\*h + 384\*a\*c^4\*e\*g^2\*h + 240\*b^3\*c^2\*f\*g^2\*h - 576\*a\*b\*c^3\*f\*g^2\*h - 288\*b^2\*c^3\*d\*g\*h^2 + 384\*a\*c^4\*d\*g\*h^2 + 240\*b^3\*c^2\*e\*g\*h^2 - 576\*a\*b\*c^3\*e\*g\*h^2 - 210\*b^4\*c\*f\*g\*h^2 + 720\*a\*b^2\*c^2\*f\*g\*h^2 - 288\*a^2\*c^3\*f\*g\*h^2 + 80\*b^3\*c^2\*d\*h^3 - 192\*a\*b\*c^3\*d\*h^3 - 70\*b^4\*c\*e\*h^3 + 240\*a\*b^2\*c^2\*e\*h^3 - 96\*a^2\*c^3\*e\*h^3 + 63\*b^5\*f\*h^3 - 280\*a\*b^3\*c\*f\*h^3 + 240\*a^2\*b\*c^2\*f\*h^3)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(256\*c^(11/2))

fricas [A] time = 2.30, size = 1435, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/7680\*(15\*(32\*(8\*c^5\*d - 4\*b\*c^4\*e + (3\*b^2\*c^3 - 4\*a\*c^4)\*f)\*g^3 - 48\*(8\*b\*c^4\*d - 2\*(3\*b^2\*c^3 - 4\*a\*c^4)\*e + (5\*b^3\*c^2 - 12\*a\*b\*c^3)\*f)\*g^2\*h + 6\*(16\*(3\*b^2\*c^3 - 4\*a\*c^4)\*d - 8\*(5\*b^3\*c^2 - 12\*a\*b\*c^3)\*e + (35\*b^4\*c - 120\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*f)\*g\*h^2 - (16\*(5\*b^3\*c^2 - 12\*a\*b\*c^3)\*d - 2\*(35\*b^4\*c - 120\*a\*b^2\*c^2 + 48\*a^2\*c^3)\*e + (63\*b^5 - 280\*a\*b^3\*c + 240\*a^2\*b\*c^2)\*f)\*h^3)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(384\*c^5\*f\*h^3\*x^4 + 480\*(4\*c^5\*e - 3\*b\*c^4\*f)\*g^3 + 240\*(24\*c^5\*d - 18\*b\*c^4\*e + (15\*b^2\*c^3 - 16\*a\*c^4)\*f)\*g^2\*h - 30\*(144\*b\*c^4\*d - 8\*(15\*b^2\*c^3 - 16\*a\*c^4)\*e + 5\*(21\*b^3\*c^2 - 44\*a

$$\begin{aligned}
& b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d - 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a))*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*f*h^3*x^4 + 480*(4*c^5*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 - 44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^6]
\end{aligned}$$

**giac** [A] time = 0.32, size = 822, normalized size = 1.19

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 - 9*b*c^3*f*h^3 + 10*c^4*h^3*e)/c^5)*x + (240*c^4*f*g^2*h - 210*b*c^3*f*g*h^2 + 80*c^4*d*h^3 + 63*b^2*c^2*f*h^3 - 64*a*c^3*f*h^3 + 240*c^4*g*h^2*e - 70*b*c^3*h^3*e)/c^5)*x + (480*c^4*f*g^3 - 1200*b*c^3*f*g^2*h + 1440*c^4*d*g*h^2 + 1050*b^2*c^2*f*g*h^2 - 1080*a*c^3*f*g*h^2 - 400*b*c^3*d*h^3 - 315*b^3*c*f*h^3 + 644*a*b*c^2*f*h^3 + 1440*c^4*g^2*h*e - 1200*b*c^3*g*h^2*e + 350*b^2*c^2*h^3*e - 360*a*c^3*h^3*e)/c^5)*x - (1440*b*c^3*f*g^3 - 5760*c^4*d*g^2*h - 3600*b^2*c^2*f*g^2*h + 3840*a*c^3*f*g^2*h + 4320*b*c^3*d*g*h^2 + 3150*b^3*c*f*g*h^2 - 6600*a*b*c^2*f*g*h^2 - 1200*b^2*c^2*d*h^3 + 1280*a*c^3*d*h^3 - 945*b^4*f*h^3 + 2940*a*b^2*c*f*h^3 - 1024*a^2*c^2*f*h^3 - 1920*c^4*g^3*e + 4320*b*c^3*g^2*h*e - 3600*b^2*c^2*g*h^2*e + 3840*a*c^3*g*h^2*e + 1050*b^3*c*h^3*e - 2200*a*b*c^2*h^3*e)/c^5) - 1/256*(256*c^5*d*g^3 + 96*b^2*c^3*f*g^3 - 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f$

$$*g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 + 210*b^4*c*f*g*h^2 - 720*a*b^2*c^2*f*g*h^2 + 288*a^2*c^3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*a^2*b*c^2*f*h^3 - 128*b*c^4*g^3*e + 288*b^2*c^3*g^2*h*e - 384*a*c^4*g^2*h*e - 240*b^3*c^2*g*h^2*e + 576*a*b*c^3*g*h^2*e + 70*b^4*c*h^3*e - 240*a*b^2*c^2*h^3*e + 96*a^2*c^3*h^3*e)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(11/2)}$$

**maple [B]** time = 0.02, size = 1869, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out]  $g^3*d*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}+1/c*(c*x^2+b*x+a)^{(1/2)}*g^3*e-3/4/c^2*b*(c*x^2+b*x+a)^{(1/2)}*g^3*f+3/8/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^3*f-1/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^3*f+161/240*h^3*f/c^3*b*a*x*(c*x^2+b*x+a)^{(1/2)}-7/8/c^2*b*x^2*(c*x^2+b*x+a)^{(1/2)}*g*h^2*f+35/32/c^3*b^2*x*(c*x^2+b*x+a)^{(1/2)}*g*h^2*f-45/16/c^{(7/2)}*b^2*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g*h^2*f+55/16/c^3*b*a*(c*x^2+b*x+a)^{(1/2)}*g*h^2*f-9/8*a/c^2*x*(c*x^2+b*x+a)^{(1/2)}*g*h^2*f-5/4/c^2*b*x*(c*x^2+b*x+a)^{(1/2)}*g*h^2*e-5/4/c^2*b*x*(c*x^2+b*x+a)^{(1/2)}*g^2*h*f+9/4/c^{(5/2)}*b*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g*h^2*e+9/4/c^{(5/2)}*b*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^2*h*f+3/4*x^3/c*(c*x^2+b*x+a)^{(1/2)}*g*h^2*f-7/24/c^2*b*x^2*(c*x^2+b*x+a)^{(1/2)}*h^3*e-21/64*h^3*f/c^4*b^3*x*(c*x^2+b*x+a)^{(1/2)}+35/32*h^3*f/c^{(9/2)}*b^3*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-49/32*h^3*f/c^4*b^2*a*(c*x^2+b*x+a)^{(1/2)}-15/16*h^3*f/c^{(7/2)}*b*a^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-9/40*h^3*f/c^2*b*x^3*(c*x^2+b*x+a)^{(1/2)}+21/80*h^3*f/c^3*b^2*x^2*(c*x^2+b*x+a)^{(1/2)}+15/8/c^3*b^2*(c*x^2+b*x+a)^{(1/2)}*g^2*h*f-15/16/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^2*h*e-15/16/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^2*h*f+3/4/c^{(5/2)}*b*a*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*h^3*d-2*a/c^2*(c*x^2+b*x+a)^{(1/2)}*g^2*h*f+3/2*x/c*(c*x^2+b*x+a)^{(1/2)}*g*h^2*d+3/2*x/c*(c*x^2+b*x+a)^{(1/2)}*g^2*h*e-9/4/c^2*b*(c*x^2+b*x+a)^{(1/2)}*g*h^2*d-9/4/c^2*b*(c*x^2+b*x+a)^{(1/2)}*g^2*h*e+9/8/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g*h^2*d+9/8/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^2*h*e-3/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g*h^2*d-3/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^2*h*e-3/2*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g^2*h*d+1/5*h^3*f*x^4/c*(c*x^2+b*x+a)^{(1/2)}+63/128*h^3*f/c^5*b^4*(c*x^2+b*x+a)^{(1/2)}-63/256*h^3*f/c^{(11/2)}*b^5*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+35/96/c^3*b^2*x*(c*x^2+b*x+a)^{(1/2)}*h^3*e+8/15*h^3*f*a^2/c^3*(c*x^2+b*x+a)^{(1/2)}+1/4*x^3/c*(c*x^2+b*x+a)^{(1/2)}*h^3*e-35/64/c^4*b^3*(c*x^2+b*x+a)^{(1/2)}*h^3*e+35/128/c^{(9/2)}*b^4*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*h^3*e+3/8*a^2/c$

$$\begin{aligned} & \left( \frac{5}{2} \right) \ln \left( \frac{c*x+1/2*b}{c^{1/2}} + (c*x^2+b*x+a)^{1/2} \right) * h^3 * e + 1/3 * x^2 / c * (c*x^2+b*x+a)^{1/2} * h^3 * d + 5/8 / c^3 * b^2 * (c*x^2+b*x+a)^{1/2} * h^3 * d - 5/16 / c^{7/2} * b^3 * \ln \\ & \left( \frac{c*x+1/2*b}{c^{1/2}} + (c*x^2+b*x+a)^{1/2} \right) * h^3 * d - 2/3 * a / c^2 * (c*x^2+b*x+a)^{1/2} * h^3 * d + 1/2 * x / c * (c*x^2+b*x+a)^{1/2} * g^3 * f + 3/c * (c*x^2+b*x+a)^{1/2} * g^2 * h * d - \\ & 1/2 * b / c^{3/2} * \ln \left( \frac{c*x+1/2*b}{c^{1/2}} + (c*x^2+b*x+a)^{1/2} \right) * g^3 * e - 105/64 / c^4 * b^3 * (c*x^2+b*x+a)^{1/2} * g * h^2 * f + 105/128 / c^{9/2} * b^4 * \ln \left( \frac{c*x+1/2*b}{c^{1/2}} + \right. \\ & \left. (c*x^2+b*x+a)^{1/2} \right) * g * h^2 * f - 15/16 / c^{7/2} * b^2 * a * \ln \left( \frac{c*x+1/2*b}{c^{1/2}} + (c*x^2+b*x+a)^{1/2} \right) * h^3 * e + 55/48 / c^3 * b * a * (c*x^2+b*x+a)^{1/2} * h^3 * e - 3/8 * a / c^2 * x * \\ & (c*x^2+b*x+a)^{1/2} * h^3 * e + 9/8 * a^2 / c^{5/2} * \ln \left( \frac{c*x+1/2*b}{c^{1/2}} + (c*x^2+b*x+a)^{1/2} \right) * g * h^2 * f + x^2 / c * (c*x^2+b*x+a)^{1/2} * g * h^2 * e + x^2 / c * (c*x^2+b*x+a)^{1/2} * g^2 * h * f - \\ & 5/12 / c^2 * b * x * (c*x^2+b*x+a)^{1/2} * h^3 * d + 15/8 / c^3 * b^2 * (c*x^2+b*x+a)^{1/2} * g * h^2 * e - 4/15 * h^3 * f * a / c^2 * x^2 * (c*x^2+b*x+a)^{1/2} \end{aligned}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^3 (fx^2 + ex + d)}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int(((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)^3 (d + ex + fx^2)}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*3\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((g + h\*x)\*\*3\*(d + e\*x + f\*x\*\*2)/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.223 \quad \int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=420

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2(dh^2+2egh+fg^2)\right)-40b^2ch(3afh+beh+2bfg)\right)}{128c^{9/2}}$$

**Rubi [A]** time = 1.01, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1653, 832, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2(dh^2+2egh+fg^2)\right)-40b^2ch(3afh+beh+2bfg)\right)}{128c^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^2\*(d + e\*x + f\*x^2))/Sqrt[a + b\*x + c\*x^2], x]

[Out] -((2\*c\*f\*g - 8\*c\*e\*h + 7\*b\*f\*h)\*(g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2])/(24\*c^2\*h) + (f\*(g + h\*x)^3\*Sqrt[a + b\*x + c\*x^2])/(4\*c\*h) - ((105\*b^3\*f\*h^3 + 32\*c^3\*(f\*g^3 - 4\*g\*h\*(e\*g + 3\*d\*h)) - 20\*b\*c\*h^2\*(11\*a\*f\*h + 6\*b\*(2\*f\*g + e\*h)) + 8\*c^2\*h\*(11\*b\*f\*g^2 + 18\*b\*h\*(2\*e\*g + d\*h) + 16\*a\*h\*(2\*f\*g + e\*h)) - 2\*c\*h\*(35\*b^2\*f\*h^2 - 4\*c\*h\*(6\*b\*f\*g + 10\*b\*e\*h + 9\*a\*f\*h) - 8\*c^2\*(f\*g^2 - 2\*h\*(2\*e\*g + 3\*d\*h)))\*x)\*Sqrt[a + b\*x + c\*x^2])/(192\*c^4\*h) + (((128\*c^4\*d\*g^2 + 35\*b^4\*f\*h^2 - 40\*b^2\*c\*h\*(2\*b\*f\*g + b\*e\*h + 3\*a\*f\*h) - 64\*c^3\*(a\*f\*g^2 + a\*h\*(2\*e\*g + d\*h) + b\*g\*(e\*g + 2\*d\*h)) + 48\*c^2\*(a^2\*f\*h^2 + 2\*a\*b\*h\*(2\*f\*g + e\*h) + b^2\*(f\*g^2 + h\*(2\*e\*g + d\*h))))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(9/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

$2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^{(p + 1)}/(2*c^2*(p + 1)*(2*p + 3)), x$   
 $] + \text{Dist}[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), \text{Int}[(a + b*x + c*x^2)^p, x], x] /;$ 
 $\text{FreeQ}\{a, b, c, d, e, f, g, p\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rule 832

$\text{Int}[(d_.) + (e_.)*(x_.))^{(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(g*(d + e*x)^m*(a + b*x + c*x^2)^{(p + 1)})/(c*(m + 2*p + 2)), x] + \text{Dist}[1/(c*(m + 2*p + 2)), \text{Int}[(d + e*x)^{(m - 1)}*(a + b*x + c*x^2)^p*\text{Simp}[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /;$ 
 $\text{FreeQ}\{a, b, c, d, e, f, g, p\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[m + 2*p + 2, 0] \&\& (\text{IntegerQ}[m] \|\| \text{IntegerQ}[p] \|\| \text{IntegersQ}[2*m, 2*p]) \&\& !( \text{IGtQ}[m, 0] \&\& \text{EqQ}[f, 0])$

### Rule 1653

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.))^{(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{With}\{q = \text{Expon}[Pq, x], f = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(f*(d + e*x)^{(m + q - 1)}*(a + b*x + c*x^2)^{(p + 1)})/(c*e^{(q - 1)}*(m + q + 2*p + 1)), x] + \text{Dist}[1/(c*e^q*(m + q + 2*p + 1)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p*\text{ExpandToSum}[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^{(q - 2)}*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /;$ 
 $\text{GtQ}[q, 1] \&\& \text{NeQ}[m + q + 2*p + 1, 0] /;$ 
 $\text{FreeQ}\{a, b, c, d, e, m, p\}, x\} \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !( \text{IGtQ}[m, 0] \&\& \text{RationalQ}[a, b, c, d, e] \&\& (\text{IntegerQ}[p] \|\| \text{ILtQ}[p + 1/2, 0]))$

### Rubi steps





$$\begin{aligned} & b^2*c*e*h^2 - 128*a*c^2*e*h^2 - 105*b^3*f*h^2 + 220*a*b*c*f*h^2 + 96*c^3*f* \\ & g^2*x + 192*c^3*e*g*h*x - 160*b*c^2*f*g*h*x + 96*c^3*d*h^2*x - 80*b*c^2*e*h \\ & ^2*x + 70*b^2*c*f*h^2*x - 72*a*c^2*f*h^2*x + 128*c^3*f*g*h*x^2 + 64*c^3*e*h \\ & ^2*x^2 - 56*b*c^2*f*h^2*x^2 + 48*c^3*f*h^2*x^3)/(192*c^4) + ((-128*c^4*d*g \\ & ^2 + 64*b*c^3*e*g^2 - 48*b^2*c^2*f*g^2 + 64*a*c^3*f*g^2 + 128*b*c^3*d*g*h - \\ & 96*b^2*c^2*e*g*h + 128*a*c^3*e*g*h + 80*b^3*c*f*g*h - 192*a*b*c^2*f*g*h - \\ & 48*b^2*c^2*d*h^2 + 64*a*c^3*d*h^2 + 40*b^3*c*e*h^2 - 96*a*b*c^2*e*h^2 - 35* \\ & b^4*f*h^2 + 120*a*b^2*c*f*h^2 - 48*a^2*c^2*f*h^2)*\text{Log}[b + 2*c*x - 2*\text{Sqrt}[c] \\ & *\text{Sqrt}[a + b*x + c*x^2]]/(128*c^{(9/2)}) \end{aligned}$$

**fricas** [A] time = 1.98, size = 861, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/768\*(3\*(16\*(8\*c^4\*d - 4\*b\*c^3\*e + (3\*b^2\*c^2 - 4\*a\*c^3)\*f)\*g^2 - 16\*(8\*b\*c^3\*d - 2\*(3\*b^2\*c^2 - 4\*a\*c^3)\*e + (5\*b^3\*c - 12\*a\*b\*c^2)\*f)\*g\*h + (16\*(3\*b^2\*c^2 - 4\*a\*c^3)\*d - 8\*(5\*b^3\*c - 12\*a\*b\*c^2)\*e + (35\*b^4 - 120\*a\*b^2\*c + 48\*a^2\*c^2)\*f)\*h^2)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(48\*c^4\*f\*h^2\*x^3 + 48\*(4\*c^4\*e - 3\*b\*c^3\*f)\*g^2 + 16\*(24\*c^4\*d - 18\*b\*c^3\*e + (15\*b^2\*c^2 - 16\*a\*c^3)\*f)\*g\*h - (144\*b\*c^3\*d - 8\*(15\*b^2\*c^2 - 16\*a\*c^3)\*e + 5\*(21\*b^3\*c - 44\*a\*b\*c^2)\*f)\*h^2 + 8\*(16\*c^4\*f\*g\*h + (8\*c^4\*e - 7\*b\*c^3\*f)\*h^2)\*x^2 + 2\*(48\*c^4\*f\*g^2 + 16\*(6\*c^4\*e - 5\*b\*c^3\*f)\*g\*h + (48\*c^4\*d - 40\*b\*c^3\*e + (35\*b^2\*c^2 - 36\*a\*c^3)\*f)\*h^2)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5, -1/384\*(3\*(16\*(8\*c^4\*d - 4\*b\*c^3\*e + (3\*b^2\*c^2 - 4\*a\*c^3)\*f)\*g^2 - 16\*(8\*b\*c^3\*d - 2\*(3\*b^2\*c^2 - 4\*a\*c^3)\*e + (5\*b^3\*c - 12\*a\*b\*c^2)\*f)\*g\*h + (16\*(3\*b^2\*c^2 - 4\*a\*c^3)\*d - 8\*(5\*b^3\*c - 12\*a\*b\*c^2)\*e + (35\*b^4 - 120\*a\*b^2\*c + 48\*a^2\*c^2)\*f)\*h^2)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(48\*c^4\*f\*h^2\*x^3 + 48\*(4\*c^4\*e - 3\*b\*c^3\*f)\*g^2 + 16\*(24\*c^4\*d - 18\*b\*c^3\*e + (15\*b^2\*c^2 - 16\*a\*c^3)\*f)\*g\*h - (144\*b\*c^3\*d - 8\*(15\*b^2\*c^2 - 16\*a\*c^3)\*e + 5\*(21\*b^3\*c - 44\*a\*b\*c^2)\*f)\*h^2 + 8\*(16\*c^4\*f\*g\*h + (8\*c^4\*e - 7\*b\*c^3\*f)\*h^2)\*x^2 + 2\*(48\*c^4\*f\*g^2 + 16\*(6\*c^4\*e - 5\*b\*c^3\*f)\*g\*h + (48\*c^4\*d - 40\*b\*c^3\*e + (35\*b^2\*c^2 - 36\*a\*c^3)\*f)\*h^2)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^5]

**giac** [A] time = 0.30, size = 457, normalized size = 1.09

$\frac{1}{128} \sqrt{c} \sqrt{a + b x + c x^2} \left( \left( \frac{48 c^4 d - 4 b c^3 e + (3 b^2 c^2 - 4 a c^3) f}{c^5} \log \left( \frac{-8 c^2 x^2 - 8 b c x - b^2 - 4 \sqrt{c} \sqrt{a + b x + c x^2}}{c^2 x^2 + b c x + a c} \right) + 4 \left( \frac{48 c^4 f h^2 x^3 + 48 (4 c^4 e - 3 b c^3 f) g^2 + 16 (24 c^4 d - 18 b c^3 e + (15 b^2 c^2 - 16 a c^3) f) g h - (144 b c^3 d - 8 (15 b^2 c^2 - 16 a c^3) e + 5 (21 b^3 c - 44 a b c^2) f) h^2 + 8 (16 c^4 f g h + (8 c^4 e - 7 b c^3 f) h^2) x^2 + 2 (48 c^4 f g^2 + 16 (6 c^4 e - 5 b c^3 f) g h + (48 c^4 d - 40 b c^3 e + (35 b^2 c^2 - 36 a c^3) f) h^2) x \right) \sqrt{c x^2 + b x + a}}{c^5} \right) - \frac{1}{384} \sqrt{-c} \arctan \left( \frac{1}{2} \sqrt{c x^2 + b x + a} \frac{(2 c x + b) \sqrt{-c}}{c^2 x^2 + b c x + a c} \right) - 2 \left( \frac{48 c^4 f h^2 x^3 + 48 (4 c^4 e - 3 b c^3 f) g^2 + 16 (24 c^4 d - 18 b c^3 e + (15 b^2 c^2 - 16 a c^3) f) g h - (144 b c^3 d - 8 (15 b^2 c^2 - 16 a c^3) e + 5 (21 b^3 c - 44 a b c^2) f) h^2 + 8 (16 c^4 f g h + (8 c^4 e - 7 b c^3 f) h^2) x^2 + 2 (48 c^4 f g^2 + 16 (6 c^4 e - 5 b c^3 f) g h + (48 c^4 d - 40 b c^3 e + (35 b^2 c^2 - 36 a c^3) f) h^2) x \right) \sqrt{c x^2 + b x + a}}{c^5} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h - 7*b*c^2*f*
h^2 + 8*c^3*h^2*e)/c^4)*x + (48*c^3*f*g^2 - 80*b*c^2*f*g*h + 48*c^3*d*h^2 +
35*b^2*c*f*h^2 - 36*a*c^2*f*h^2 + 96*c^3*g*h*e - 40*b*c^2*h^2*e)/c^4)*x -
(144*b*c^2*f*g^2 - 384*c^3*d*g*h - 240*b^2*c*f*g*h + 256*a*c^2*f*g*h + 144*
b*c^2*d*h^2 + 105*b^3*f*h^2 - 220*a*b*c*f*h^2 - 192*c^3*g^2*e + 288*b*c^2*g
*h*e - 120*b^2*c*h^2*e + 128*a*c^2*h^2*e)/c^4) - 1/128*(128*c^4*d*g^2 + 48*
b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h - 80*b^3*c*f*g*h + 192*a*b
*c^2*f*g*h + 48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 + 35*b^4*f*h^2 - 120*a*b^2*c
*f*h^2 + 48*a^2*c^2*f*h^2 - 64*b*c^3*g^2*e + 96*b^2*c^2*g*h*e - 128*a*c^3*g
*h*e - 40*b^3*c*h^2*e + 96*a*b*c^2*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.01, size = 1069, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2), x)
```

```
[Out] 1/c*(c*x^2+b*x+a)^(1/2)*g^2*e+g^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)-5/6/c^2*b*x*(c*x^2+b*x+a)^(1/2)*g*h*f+3/2/c^(5/2)*b*a*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h*f+5/4/c^3*b^2*(c*x^2+b*x+a)^(1/2)*g
*h*f-4/3*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h*f+x/c*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4/
c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e+2/3*x^2/c*(c*
x^2+b*x+a)^(1/2)*g*h*f-5/12/c^2*b*x*(c*x^2+b*x+a)^(1/2)*h^2*e+2/c*(c*x^2+b*
x+a)^(1/2)*g*h*d-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
g^2*e+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+1/3
*x^2/c*(c*x^2+b*x+a)^(1/2)*h^2*e-5/8/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*g*h*f+35/96*h^2*f/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)-15/16*h^2
*f/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*h^2*f/c^
3*b*a*(c*x^2+b*x+a)^(1/2)-3/8*h^2*f*a/c^2*x*(c*x^2+b*x+a)^(1/2)-7/24*h^2*f/
c^2*b*x^2*(c*x^2+b*x+a)^(1/2)-3/2/c^2*b*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4/c^(5/
2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-a/c^(3/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*g*h*d-2/3*a/c^2*(c*x^2+b*x+a)^(1/2)*h^2*e+1/2*x/c*(c*x
^2+b*x+a)^(1/2)*d*h^2+1/4*h^2*f*x^3/c*(c*x^2+b*x+a)^(1/2)-35/64*h^2*f/c^4*b
^3*(c*x^2+b*x+a)^(1/2)+35/128*h^2*f/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+3/8*h^2*f*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g^2-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*d*h
^2-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/2*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-3/4/c^2*b*(c*x^2+b*
x+a)^(1/2)*f*g^2+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)*f*g^2+5/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*h^2*e-5/16/c^(7/2)*b^3*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^2 (fx^2 + ex + d)}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int(((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)^2 (d + ex + fx^2)}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((g + h\*x)\*\*2\*(d + e\*x + f\*x\*\*2)/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.224 \quad \int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)}{16c^{7/2}} + \frac{\sqrt{a+bx+cx^2}}{3ch}$$

**Rubi [A]** time = 0.30, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}(-2ch(8afh+9b(dh+fg))+15b^2fh^2-2cht(5bflh-6ceh+2cfg)-8c^2(fg^2-3l(dh+eg)))}{24c^3h} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg)}{16c^{7/2}} + \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)\*(d + e\*x + f\*x^2))/Sqrt[a + b\*x + c\*x^2], x]

[Out] (f\*(g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2])/(3\*c\*h) + ((15\*b^2\*f\*h^2 - 8\*c^2\*(f\*g^2 - 3\*h\*(e\*g + d\*h)) - 2\*c\*h\*(8\*a\*f\*h + 9\*b\*(f\*g + e\*h)) - 2\*c\*h\*(2\*c\*f\*g - 6\*c\*e\*h + 5\*b\*f\*h)\*x)\*Sqrt[a + b\*x + c\*x^2])/(24\*c^3\*h) + ((16\*c^3\*d\*g - 5\*b^3\*f\*h - 8\*c^2\*(b\*e\*g + a\*f\*g + b\*d\*h + a\*e\*h) + 6\*b\*c\*(b\*f\*g + b\*e\*h + 2\*a\*f\*h))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(16\*c^(7/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 779**

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g)))\*(2\*p + 3)/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d

, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(g + hx)(d + ex + fx^2)}{\sqrt{a + bx + cx^2}} dx &= \frac{f(g + hx)^2 \sqrt{a + bx + cx^2}}{3ch} + \frac{\int \frac{(g + hx) \left( -\frac{1}{2}h(bfg - 6cdh + 4afh) - \frac{1}{2}h(2cfg - 6ceh + 5bfh)x \right)}{\sqrt{a + bx + cx^2}} dx}{3ch^2} \\ &= \frac{f(g + hx)^2 \sqrt{a + bx + cx^2}}{3ch} + \frac{(15b^2fh^2 - 8c^2(fg^2 - 3h(eg + dh)) - 2ch(8afh + 2d^2fg - 6cdh + 4afh)) \sqrt{a + bx + cx^2}}{3ch^2} \\ &= \frac{f(g + hx)^2 \sqrt{a + bx + cx^2}}{3ch} + \frac{(15b^2fh^2 - 8c^2(fg^2 - 3h(eg + dh)) - 2ch(8afh + 2d^2fg - 6cdh + 4afh)) \sqrt{a + bx + cx^2}}{3ch^2} \\ &= \frac{f(g + hx)^2 \sqrt{a + bx + cx^2}}{3ch} + \frac{(15b^2fh^2 - 8c^2(fg^2 - 3h(eg + dh)) - 2ch(8afh + 2d^2fg - 6cdh + 4afh)) \sqrt{a + bx + cx^2}}{3ch^2} \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 215, normalized size = 0.96

$$\frac{3h \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right) (8c^2(ach+afg+bdh+beg) - 6bc(2afh+beh+bfh) + 5b^2fh - 16c^3dg) + \frac{\sqrt{a+x(b+cx)} (-2ch(8afh+b(9eh+9fg+5fhx)) + 15b^2fh^2 - 4c^2(fg(2g+hx) - 3h(2dh+2eg+ehx)))}{8c^2} + f(g+hx)^2 \sqrt{a+x(b+cx)}}{16c^5/2} + \frac{\dots}{3ch}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)\*(d + e\*x + f\*x^2))/Sqrt[a + b\*x + c\*x^2], x]

[Out] (f\*(g + h\*x)^2\*Sqrt[a + x\*(b + c\*x)] + (Sqrt[a + x\*(b + c\*x)]\*(15\*b^2\*f\*h^2 - 4\*c^2\*(f\*g\*(2\*g + h\*x) - 3\*h\*(2\*e\*g + 2\*d\*h + e\*h\*x)) - 2\*c\*h\*(8\*a\*f\*h + b\*(9\*f\*g + 9\*e\*h + 5\*f\*h\*x))))/(8\*c^2) - (3\*h\*(-16\*c^3\*d\*g + 5\*b^3\*f\*h + 8



$$e + 18*b*c*h*e)/c^3) - 1/16*(16*c^3*d*g + 6*b^2*c*f*g - 8*a*c^2*f*g - 8*b*c^2*d*h - 5*b^3*f*h + 12*a*b*c*f*h - 8*b*c^2*g*e + 6*b^2*c*h*e - 8*a*c^2*h*e) * \log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(7/2)}$$

**maple [B]** time = 0.01, size = 505, normalized size = 2.26

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Verification of antiderivative is not currently implemented for this CAS.

[In] `int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)`

[Out]  $1/3*h*f*x^2/c*(c*x^2+b*x+a)^{(1/2)} - 5/12*h*f/c^2*b*x*(c*x^2+b*x+a)^{(1/2)} + 5/8*h*f/c^3*b^2*(c*x^2+b*x+a)^{(1/2)} - 5/16*h*f/c^{(7/2)}*b^3*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) + 3/4*h*f/c^{(5/2)}*b*a*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)}) - 2/3*h*f*a/c^2*(c*x^2+b*x+a)^{(1/2)} + 1/2*x/c*(c*x^2+b*x+a)^{(1/2)}*e*h + 1/2*x/c*(c*x^2+b*x+a)^{(1/2)}*f*g - 3/4/c^2*b*(c*x^2+b*x+a)^{(1/2)}*e*h - 3/4/c^2*b*(c*x^2+b*x+a)^{(1/2)}*f*g + 3/8/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})*e*h + 3/8/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})*f*g - 1/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})*e*h - 1/2*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})*f*g + 1/c*(c*x^2+b*x+a)^{(1/2)}*d*h + 1/c*(c*x^2+b*x+a)^{(1/2)}*e*g - 1/2*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})*d*h - 1/2*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})*e*g + d*g*\ln((c*x+1/2*b)/c^{(1/2)} + (c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* h elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for mo re details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)(fx^2 + ex + d)}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)`

[Out] `int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)(d + ex + fx^2)}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral((g + h*x)*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)`



$$3.225 \quad \int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

**Rubi [A]** time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.160, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f)\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + (f\*x\*Sqrt[a + b\*x + c\*x^2])/(2\*c) + ((8\*c^2\*d + 3\*b^2\*f - 4\*c\*(b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(5/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}} dx &= \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2cd - af + \frac{1}{2}(4ce - 3bf)x}{\sqrt{a + bx + cx^2}} dx}{2c} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \int \frac{dx}{\sqrt{a + bx + cx^2}}}{4c^2} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{\left(2c(2cd - af) - \frac{1}{2}b(4ce - 3bf)\right) \operatorname{Subst}\left(\int \frac{dx}{\sqrt{a + bx + cx^2}}\right)}{2c^2} \\ &= \frac{(4ce - 3bf)\sqrt{a + bx + cx^2}}{4c^2} + \frac{fx\sqrt{a + bx + cx^2}}{2c} + \frac{(8c^2d + 3b^2f - 4c(be + af)) \tanh^{-1}\left(\frac{2cx + b}{\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((4\*c\*e - 3\*b\*f + 2\*c\*f\*x)\*Sqrt[a + x\*(b + c\*x)]/(4\*c^2) + ((8\*c^2\*d + 3\*b^2\*f - 4\*c\*(b\*e + a\*f))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(8\*c^(5/2))

**IntegrateAlgebraic [A]** time = 0.00, size = 102, normalized size = 0.88

$$\frac{\log\left(-2c^{5/2}\sqrt{a + bx + cx^2} + bc^2 + 2c^3x\right)\left(4acf - 3b^2f + 4bce - 8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a + bx + cx^2}(-3bf + 4ce + 2cfx)}{4c^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out]  $\frac{((4*c*e - 3*b*f + 2*c*f*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2) + ((-8*c^2*d + 4*b*c*e - 3*b^2*f + 4*a*c*f)*\text{Log}[b*c^2 + 2*c^3*x - 2*c^{(5/2)}*\text{Sqrt}[a + b*x + c*x^2]])/(8*c^{(5/2)})}$

**fricas** [A] time = 2.73, size = 227, normalized size = 1.96

$$\frac{(8c^2d - 4bce + (3b^2 - 4ac)f)\sqrt{c} \log\left(\frac{-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c} - 4ac}{16c^3}\right) - 4(2c^2fx + 4c^2e - 3bcf)\sqrt{cx^2 + bx + a} - (8c^2d - 4bce + (3b^2 - 4ac)f)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^2 + bx + a}(2cx + b)\sqrt{c}}{2(\sqrt{c^2 + bcx + ac})}\right) - 2(2c^2fx + 4c^2e - 3bcf)\sqrt{cx^2 + bx + a}}{8c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out]  $[-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*\text{sqrt}(c)*\log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*\text{sqrt}(c*x^2 + b*x + a)*(2*c*x + b)*\text{sqrt}(c) - 4*a*c) - 4*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)*\text{sqrt}(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*\text{sqrt}(-c)*\arctan(1/2*\text{sqrt}(c*x^2 + b*x + a)*(2*c*x + b)*\text{sqrt}(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)*\text{sqrt}(c*x^2 + b*x + a))/c^3]$

**giac** [A] time = 0.25, size = 98, normalized size = 0.84

$$\frac{1}{4} \sqrt{cx^2 + bx + a} \left( \frac{2fx}{c} - \frac{3bf - 4ce}{c^2} \right) - \frac{(8c^2d + 3b^2f - 4acf - 4bce) \log\left(\left| -2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b \right|\right)}{8c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="giac")

[Out]  $1/4*\text{sqrt}(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d + 3*b^2*f - 4*a*c*f - 4*b*c*e)*\log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(5/2)}$

**maple** [A] time = 0.01, size = 185, normalized size = 1.59

$$\frac{af \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}} + \frac{3b^2f \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{8c^{\frac{5}{2}}} - \frac{bef \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}} + \frac{d \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2 + bx + a} fx}{2c} - \frac{3\sqrt{cx^2 + bx + a} bf}{4c^2} + \frac{\sqrt{cx^2 + bx + a} e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x)

[Out]  $1/2*f*x*(c*x^2+b*x+a)^{(1/2)}/c - 3/4*f/c^2*b*(c*x^2+b*x+a)^{(1/2)} + 3/8*f/c^{(5/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) - 1/2*f*a/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) + e/c*(c*x^2+b*x+a)^{(1/2)} - 1/2*e*b/c^{(3/2)}*1$

$n((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+d*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f x^2 + e x + d}{\sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(1/2),x)

[Out] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{\sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.226 \quad \int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=179

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg) + (fg^2-h(eg-dh))\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{2c^{3/2}h^2} + \frac{f\sqrt{a+bx}}{ch}$$

**Rubi [A]** time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1653, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg) + (fg^2-h(eg-dh))\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{2c^{3/2}h^2} + \frac{f\sqrt{a+bx+cx^2}}{ch}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (f\*Sqrt[a + b\*x + c\*x^2])/(c\*h) - ((2\*c\*f\*g - 2\*c\*e\*h + b\*f\*h)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*c^(3/2)\*h^2) + ((f\*g^2 - h\*(e\*g - d\*h))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(h^2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(g + hx)\sqrt{a + bx + cx^2}} dx &= \frac{f\sqrt{a + bx + cx^2}}{ch} + \frac{\int \frac{-\frac{1}{2}h(bfg - 2cdh) - \frac{1}{2}h(2cfg - 2ceh + bfh)x}{(g + hx)\sqrt{a + bx + cx^2}} dx}{ch^2} \\ &= \frac{f\sqrt{a + bx + cx^2}}{ch} - \frac{(2cfg - 2ceh + bfh) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{2ch^2} + \frac{(fg^2 - egh + dh^2) \int \frac{1}{g + hx} dx}{h^2} \\ &= \frac{f\sqrt{a + bx + cx^2}}{ch} - \frac{(2cfg - 2ceh + bfh) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{ch^2} - \frac{(2fg^2 - egh + dh^2) \int \frac{1}{g + hx} dx}{h^2} \\ &= \frac{f\sqrt{a + bx + cx^2}}{ch} - \frac{(2cfg - 2ceh + bfh) \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{2c^{3/2}h^2} + \frac{(fg^2 - h(eg - dh)) \int \frac{1}{g + hx} dx}{h^2} \end{aligned}$$

**Mathematica** [A] time = 0.28, size = 172, normalized size = 0.96

$$\frac{\tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)(bfh - 2ceh + 2cfg)}{c^{3/2}} + \frac{2(h(dh - eg) + fg^2) \tanh^{-1}\left(\frac{2ah - bg + bhx - 2cgx}{2\sqrt{a + x(b + cx)}\sqrt{h(ah - bg) + cg^2}}\right)}{\sqrt{h(ah - bg) + cg^2}} - \frac{2fh\sqrt{a + x(b + cx)}}{c}$$


---


$$2h^2$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$-1/2*((-2*f*h*Sqrt[a + x*(b + c*x)])/c + ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^{3/2} + (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])*Sqrt[a + x*(b + c*x)])]/Sqrt[c*g^2 + h*(-(b*g) + a*h)]/h^2$$

**IntegrateAlgebraic [A]** time = 0.78, size = 198, normalized size = 1.11

$$\frac{\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2} + \frac{2\sqrt{-ah^2+bgh-cg^2}(dh^2-egh+fg^2)\tan^{-1}\left(\frac{-h\sqrt{a+bx+cx^2}+\sqrt{c}g+\sqrt{c}hx}{\sqrt{-ah^2+bgh-cg^2}}\right)}{h^2(ah^2-bgh+cg^2)} + \frac{f\sqrt{a+bx+cx^2}}{ch}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$(f*Sqrt[a + b*x + c*x^2])/(c*h) + (2*Sqrt[-(c*g^2) + b*g*h - a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTan[(Sqrt[c]*g + Sqrt[c]*h*x - h*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*g^2) + b*g*h - a*h^2]])/(h^2*(c*g^2 - b*g*h + a*h^2)) + ((2*c*f*g - 2*c*e*h + b*f*h)*Log[b*c + 2*c^2*x - 2*c^{3/2}*Sqrt[a + b*x + c*x^2]])/(2*c^{3/2}*h^2)$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT>Error: Bad Argument Type

**maple [B]** time = 0.02, size = 599, normalized size = 3.35

$$d \ln \left( \frac{(b-2c)(-f)}{\sqrt{a+bx+cx^2}} \sqrt{\frac{a+bx+cx^2}{(b-2c)(-f)}} \right) + c \ln \left( \frac{(b-2c)(-f)}{\sqrt{a+bx+cx^2}} \sqrt{\frac{a+bx+cx^2}{(b-2c)(-f)}} \right) + fg^2 \ln \left( \frac{(b-2c)(-f)}{\sqrt{a+bx+cx^2}} \sqrt{\frac{a+bx+cx^2}{(b-2c)(-f)}} \right) + \frac{bf \ln \left( \frac{a+bx+cx^2}{\sqrt{a+bx+cx^2}} \right)}{2ch} + \frac{c \ln \left( \frac{a+bx+cx^2}{\sqrt{a+bx+cx^2}} \right)}{\sqrt{c}h} + \frac{fg \ln \left( \frac{a+bx+cx^2}{\sqrt{a+bx+cx^2}} \right)}{\sqrt{c}h^2} + \frac{\sqrt{a+bx+cx^2}}{ch}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out]  $f*(c*x^2+b*x+a)^{(1/2)}/c/h-1/2/h*f*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+1/h*e*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-1/h^2*f*g*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-1/h/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+g/h)*d+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+g/h)*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+g/h))*f*g^2$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume((b/h-(2\*c\*g)/h^2)^2>0)', see `assume?` for more details)Is (b/h-(2\*c\*g)/h^2)^2 - (4\*c \* ((- (b\*g)/h) + (c\*g^2)/h^2+a)) /h^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f x^2 + e x + d}{(g + h x) \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^{(1/2)}), x)$

[Out]  $\text{int}((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^{(1/2)}), x)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{(g + h x) \sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

$$3.227 \quad \int \frac{d+ex+fx^2}{(g+hx)^2 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=241

$$\frac{\sqrt{a+bx+cx^2} (fg^2 - h(eg - dh)) \tanh^{-1} \left( \frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2} \sqrt{ah^2-bgh+cg^2}} \right) (h(2ah(2fg - eh) - b(-dh^2 - egh + 3fg^2))}{h(g+hx)(ah^2 - bgh + cg^2)} - \frac{2h^2 (ah^2 - bgh + cg^2)^{3/2}}{2h^2 (ah^2 - bgh + cg^2)^{3/2}}}{2h^2 (ah^2 - bgh + cg^2)^{3/2}}$$

**Rubi [A]** time = 0.37, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1650, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2} (fg^2 - h(eg - dh))}{h(g+hx)(ah^2 - bgh + cg^2)} - \frac{\tanh^{-1} \left( \frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2} \sqrt{ah^2-bgh+cg^2}} \right) (2c(fg^3 - dgh^2) - h(-2ah(2fg - eh) - bh(dh + eg) + 3bfg^2))}{2h^2 (ah^2 - bgh + cg^2)^{3/2}} + \frac{f \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c} \sqrt{a+bx+cx^2}} \right)}{\sqrt{c} h^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] -(((f\*g^2 - h\*(e\*g - d\*h))\*Sqrt[a + b\*x + c\*x^2])/(h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x))) + (f\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(Sqrt[c]\*h^2) - (((2\*c\*(f\*g^3 - d\*g\*h^2) - h\*(3\*b\*f\*g^2 - b\*h\*(e\*g + d\*h) - 2\*a\*h\*(2\*f\*g - e\*h))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*h^2\*(c\*g^2 - b\*g\*h + a\*h^2)^(3/2)))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c,

$d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[2*c*d - b*e, 0]$

### Rule 843

$\text{Int}[(d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

### Rule 1650

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x\_Symbol] \rightarrow \text{With}\{Q = \text{PolynomialQuotient}[Pq, d + e*x, x], R = \text{PolynomialRemainder}[Pq, d + e*x, x]\}, \text{Simp}[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + \text{Dist}[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), \text{Int}[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*\text{ExpandToSum}[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x]] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{LtQ}[m, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(g + hx)^2 \sqrt{a + bx + cx^2}} dx &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{h(cg^2 - bgh + ah^2)(g + hx)} - \frac{\int \frac{\frac{1}{2}(-2cdg + beg + 2afg - \frac{bfg^2}{h} + bdh - 2aeh) + f(bg - \frac{cg^2}{h})}{(g + hx)\sqrt{a + bx + cx^2}} dx}{cg^2 - bgh + ah^2} \\ &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{h(cg^2 - bgh + ah^2)(g + hx)} + \frac{f \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{h^2} - \frac{(2c(fg^3 - dgh^2) - \dots)}{h^2} \\ &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{h(cg^2 - bgh + ah^2)(g + hx)} + \frac{(2f) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{h^2} + \dots \\ &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{h(cg^2 - bgh + ah^2)(g + hx)} + \frac{f \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{\sqrt{c}h^2} - \frac{(2c(fg^3 - dgh^2) - \dots)}{h^2} \end{aligned}$$

**Mathematica [A]** time = 0.35, size = 227, normalized size = 0.94

$$\frac{\frac{h\sqrt{a+x(b+cx)}(h(dh-eg)+fg^2)}{(g+hx)(h(ah-bg)+cg^2)} + \frac{\tanh^{-1}\left(\frac{2ah-bg+bx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)(h(-2ah(eh-2fg)+bh(dh+eg)-3bfg^2)+2c(fg^3-dgh^2))}{2(h(ah-bg)+cg^2)^{3/2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$\frac{(-(h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + ((2*c*(f*g^3 - d*g*h^2) + h*(-3*b*f*g^2 + b*h*(e*g + d*h) - 2*a*h*(-2*f*g + e*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/h^2$$

**IntegrateAlgebraic [A]** time = 1.27, size = 379, normalized size = 1.57

$$\frac{\sqrt{a+bx+cx^2}(-dh^2+egh-fg^2)}{h(g+hx)(ah^2-bgh+cg^2)} + \frac{\tan^{-1}\left(\frac{h\sqrt{a+bx+cx^2}+fg\sqrt{c}h}{\sqrt{ah^2-bgh+cg^2}}\right)(2adgh^2\sqrt{ah^2-bgh+cg^2}-bdh^3\sqrt{ah^2-bgh+cg^2}-bcgh^2\sqrt{ah^2-bgh+cg^2}+2adh^3\sqrt{ah^2-bgh+cg^2}+3bfg^2h\sqrt{ah^2-bgh+cg^2}-4afgh^2\sqrt{ah^2-bgh+cg^2}-2cfg^3\sqrt{ah^2-bgh+cg^2})}{h^2(ah^2-bgh+cg^2)^2} - \frac{f \log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{\sqrt{c}h^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)^2\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$\frac{((-f*g^2) + e*g*h - d*h^2)*Sqrt[a + b*x + c*x^2]/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((-2*c*f*g^3*Sqrt[-(c*g^2) + b*g*h - a*h^2] + 3*b*f*g^2*h*Sqrt[-(c*g^2) + b*g*h - a*h^2] + 2*c*d*g*h^2*Sqrt[-(c*g^2) + b*g*h - a*h^2] - b*e*g*h^2*Sqrt[-(c*g^2) + b*g*h - a*h^2] - 4*a*f*g*h^2*Sqrt[-(c*g^2) + b*g*h - a*h^2] - b*d*h^3*Sqrt[-(c*g^2) + b*g*h - a*h^2] + 2*a*e*h^3*Sqrt[-(c*g^2) + b*g*h - a*h^2])*ArcTan[(Sqrt[c]*g + Sqrt[c]*h*x - h*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*g^2) + b*g*h - a*h^2])/(h^2*(c*g^2 - b*g*h + a*h^2)^2) - (f*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2)])/(Sqrt[c]*h^2)$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.71Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 1671, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f/h^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/(a*h^2-b*g*h+c*
g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/2/(a*h^2-
b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*d-1/2/h/(a*h^2-b
*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e*g+1/2/h^2/(a*h^
2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*
(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h
-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*f*g^2-1/h/(a*h
^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2
*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*
h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g*d+1/h^2/(a*
h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+
2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b
*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c*g^2*e-1/h^3/(a*h^2-b*
g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*
h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*f-1/h^
2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h
+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g
/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2))/(x+g/h))*f*g
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume((b/h-(2\*c\*g)/h^2)^2>0)', see `assume?` for more details)Is (b/h-(2\*c\*g)/h^2)^2 - (4\*c\*(b/h-(2\*c\*g)/h^2) + (c\*g^2)/h^2+a) /h^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^2 \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + b\*x + c\*x^2)^(1/2)),x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + b\*x + c\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{(g + h x)^2 \sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/((g + h\*x)\*\*2\*sqrt(a + b\*x + c\*x\*\*2)), x)

$$3.228 \quad \int \frac{d+ex+fx^2}{(g+hx)^3 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=336

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(a\left(dh^2-3egh+fg^2\right)+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2(3a+2gh)\right)}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

**Rubi [A]** time = 0.66, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(a\left(dh^2-3egh+fg^2\right)+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2(3a+2gh)\right)}{8\left(ah^2-bgh+cg^2\right)^{5/2}} - \frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)\left(ah^2-bgh+cg^2\right)} + \frac{\sqrt{a+bx+cx^2}\left(2c\left(gh(eg-3dh)+fg^2\right)-h\left(-4ah(2fg-eh)-bh(3dh+eg)+5bfg^2\right)\right)}{4h(g+hx)\left(ah^2-bgh+cg^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^3\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] -((f\*g^2 - h\*(e\*g - d\*h))\*Sqrt[a + b\*x + c\*x^2])/(2\*h\*(c\*g^2 - b\*g\*h + a\*h^2)\*(g + h\*x)^2) + ((2\*c\*(f\*g^3 + g\*h\*(e\*g - 3\*d\*h)) - h\*(5\*b\*f\*g^2 - b\*h\*(e\*g + 3\*d\*h) - 4\*a\*h\*(2\*f\*g - e\*h)))\*Sqrt[a + b\*x + c\*x^2])/(4\*h\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)) + ((8\*c^2\*d\*g^2 + 8\*a^2\*f\*h^2 - 4\*a\*b\*h\*(2\*f\*g + e\*h) - 4\*c\*(a\*f\*g^2 - a\*h\*(3\*e\*g - d\*h) + b\*g\*(e\*g + 2\*d\*h)) + b^2\*(3\*f\*g^2 + h\*(e\*g + 3\*d\*h)))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])]/(8\*(c\*g^2 - b\*g\*h + a\*h^2)^(5/2))

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f

+ d\*g) - 2\*(c\*d\*f + a\*e\*g)/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] & & NeQ[b^2 - 4\*a\*c, 0] & & NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] & & EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1650

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] & & PolyQ[Pq, x] & & NeQ[b^2 - 4\*a\*c, 0] & & NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] & & LtQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(g + hx)^3 \sqrt{a + bx + cx^2}} dx &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{2h (cg^2 - bgh + ah^2) (g + hx)^2} - \int \frac{\frac{1}{2} \left( -4cdg + beg + 4afg - \frac{bf g^2}{h} + 3bdh - 4aeh \right) - (ceg - 2bf g)}{(g + hx)^2 \sqrt{a + bx + cx^2}} \\ &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{2h (cg^2 - bgh + ah^2) (g + hx)^2} + \frac{(2c (fg^3 + gh(eg - 3dh)) - h (5bfg^2 - bh^2))}{4h (cg^2 - bgh + ah^2)} \\ &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{2h (cg^2 - bgh + ah^2) (g + hx)^2} + \frac{(2c (fg^3 + gh(eg - 3dh)) - h (5bfg^2 - bh^2))}{4h (cg^2 - bgh + ah^2)} \\ &= -\frac{(fg^2 - h(eg - dh)) \sqrt{a + bx + cx^2}}{2h (cg^2 - bgh + ah^2) (g + hx)^2} + \frac{(2c (fg^3 + gh(eg - 3dh)) - h (5bfg^2 - bh^2))}{4h (cg^2 - bgh + ah^2)} \end{aligned}$$

**Mathematica [A]** time = 1.10, size = 367, normalized size = 1.09

$$\frac{ch \tanh^{-1} \left( \frac{2ah - bg + hbx - 2cx}{2\sqrt{a+bx+cx^2}} \right) \left( 8a^2 f h^2 - 4c(ah(dh - 3eg) + af g^2 + bg(2dh + eg)) - 4abh(eh + 2fg) + h^2(3dh + eg) + 3fg^2 + 8c^2 dg^2 \right)}{8(h(ah - bg) + cg^2)^{5/2}} + \frac{\sqrt{a+bx+cx^2} (2f h(ah - bg) + ch(eg - dh) + cf g^2)}{2(g + hx)^2 (h(ah - bg) + cg^2)} + \frac{c \sqrt{a+bx+cx^2} (h(-4ah(eh - 2fg) + bh(3dh + eg) - 5bf g^2) + 2c(g h(eg - 3dh) + fg^2))}{4(g + hx)(h(ah - bg) + cg^2)} - \frac{f \sqrt{a+bx+cx^2}}{(g + hx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)^3\*Sqrt[a + b\*x + c\*x^2]), x]



```
[Out] (-((f*Sqrt[a + x*(b + c*x)])/(g + h*x)^2) + ((c*f*g^2 + 2*f*h*(-(b*g) + a*h) + c*h*(e*g - d*h))*Sqrt[a + x*(b + c*x)]/(2*(c*g^2 + h*(-(b*g) + a*h)))*(g + h*x)^2) + (c*(2*c*(f*g^3 + g*h*(e*g - 3*d*h)) + h*(-5*b*f*g^2 + b*h*(e*g + 3*d*h) - 4*a*h*(-2*f*g + e*h)))*Sqrt[a + x*(b + c*x)]/(4*(c*g^2 + h*(-(b*g) + a*h))^2*(g + h*x)) - (c*h*(8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 + a*h*(-3*e*g + d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g^2 + h*(e*g + 3*d*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]/(8*(c*g^2 + h*(-(b*g) + a*h))^(5/2)))/(c*h)
```

**IntegrateAlgebraic [F]** time = 180.26, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] $Aborted
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.54, size = 2307, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h - 8*a*b*f*g*h + 3*b^2*d*h^2 - 4*a*c*d*h^2 + 8*a^2*f*h^2 - 4*b*c*g^2*e + b^2*g*h*e + 12*a*c*g*h*e - 4*a*b*h^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h^2 + 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*g*h^2 - 8*a*b*f*g*h^2 + 3*b^2*d*h^2 - 4*a*c*d*h^2 + 8*a^2*f*h^2 - 4*b*c*g^2*e + b^2*g*h*e + 12*a*c*g*h*e - 4*a*b*h^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))
```

$$\begin{aligned}
& t(cx^2 + bx + a)^3 * b * c * d * g * h^4 - 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^3 * b^2 * d * h^5 + 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^3 * a * c * d * h^5 + 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^3 * b * c * g^2 * h^3 * e - (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^3 * b^2 * g * h^4 * e - 12 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^3 * a * c * g * h^4 * e + 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^3 * a * b * h^5 * e + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * c^{(5/2)} * f * g^5 - 16 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * b * c^{(3/2)} * f * g^4 * h - 24 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * c^{(5/2)} * d * g^3 * h^2 - (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * b^2 * \sqrt{c} * f * g^3 * h^2 + 28 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a * c^{(3/2)} * f * g^3 * h^2 + 24 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * b * c^{(3/2)} * d * g^2 * h^3 + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a * b * \sqrt{c} * f * g^2 * h^3 - 9 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * b^2 * \sqrt{c} * d * g * h^4 + 12 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a * c^{(3/2)} * d * g * h^4 - 16 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a^2 * \sqrt{c} * f * g * h^4 + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * c^{(5/2)} * g^4 * h * e - 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * b * c^{(3/2)} * g^3 * h^2 * e + 5 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * b^2 * \sqrt{c} * g^2 * h^3 * e - 20 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a * c^{(3/2)} * g^2 * h^3 * e - 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a * b * \sqrt{c} * g * h^4 * e + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * a^2 * \sqrt{c} * h^5 * e + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b * c^2 * f * g^5 - 20 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b^2 * c * f * g^4 * h - 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * c^2 * f * g^4 * h - 24 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b * c^2 * d * g^3 * h^2 + 3 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b^3 * f * g^3 * h^2 + 60 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * b * c * f * g^3 * h^2 + 20 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b^2 * c * d * g^2 * h^3 + 40 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * c^2 * d * g^2 * h^3 - 11 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * b^2 * f * g^2 * h^3 - 44 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a^2 * c * f * g^2 * h^3 - 5 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b^3 * d * g * h^4 - 28 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * b * c * d * g * h^4 + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a^2 * b * f * g * h^4 + 5 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * b^2 * d * h^5 + 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a^2 * c * d * h^5 + 8 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b * c^2 * g^4 * h * e - 16 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * c^2 * g^3 * h^2 * e + (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * b^3 * g^2 * h^3 * e - 16 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * b * c * g^2 * h^3 * e + 3 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a * b^2 * g * h^4 * e + 20 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a^2 * c * g * h^4 * e - 4 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * a^2 * b * h^5 * e + 2 * b^2 * c^{(3/2)} * f * g^5 - 5 * b^3 * \sqrt{c} * f * g^4 * h - 4 * a * b * c^{(3/2)} * f * g^4 * h - 6 * b^2 * c^{(3/2)} * d * g^3 * h^2 + 21 * a * b^2 * \sqrt{c} * f * g^3 * h^2 + 4 * a^2 * c^{(3/2)} * f * g^3 * h^2 + 3 * b^3 * \sqrt{c} * d * g^2 * h^3 + 20 * a * b * c^{(3/2)} * d * g^2 * h^3 - 32 * a^2 * b * \sqrt{c} * f * g^2 * h^3 - 11 * a * b^2 * \sqrt{c} * d * g * h^4 - 12 * a^2 * c^{(3/2)} * d * g * h^4 + 16 * a^3 * \sqrt{c} * f * g * h^4 + 8 * a^2 * b * \sqrt{c} * d * h^5 + 2 * b^2 * c^{(3/2)} * g^4 * h * e + b^3 * \sqrt{c} * g^3 * h^2 * e - 8 * a * b * c^{(3/2)} * g^3 * h^2 * e - 5 * a * b^2 * \sqrt{c} * g^2 * h^3 * e + 4 * a^2 * c^{(3/2)} * g^2 * h^3 * e + 12 * a^2 * b * \sqrt{c} * g * h^4 * e - 8 * a^3 * \sqrt{c} * h^5 * e) / ((c^2 * g^4 * h^2 - 2 * b * c * g^3 * h^3 + b^2 * g^2 * h^4 + 2 * a * c * g^2 * h^4 - 2 * a * b * g * h^5 + a^2 * h^6) * ((\sqrt{c} * x - \sqrt{cx^2 + bx + a})^2 * h + 2 * (\sqrt{c} * x - \sqrt{cx^2 + bx + a}) * \sqrt{c} * g + b * g - a * h)^2)
\end{aligned}$$

maple [B] time = 0.02, size = 3615, normalized size = 10.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out] 
$$-f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+g/h)^{-3/2}/h/(a*h^2-b*g*h+c*g^2)^2/(((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*b*c*g^2*e+3/2/h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*b*c*g^3*f-3/2/h^2/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*c*g^3*f-3/8/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*b^2*f*g^2+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*c^2*g^2*d+3/2/h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*c^2*g^3*e-3/2/h^3/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*c^2*g^4*f-1/2/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*d-1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*f*g+1/2/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})))/(x+g/h)*b*e+1/2/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*e*g-1/2/h^3/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*f*g^2+3/4*h/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c$$

$$g^2/h^2)^{1/2} * b*d - 3/4 / (a*h^2 - b*g*h + c*g^2)^2 / (x+g/h) * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * b*e*g - 3/2 / (a*h^2 - b*g*h + c*g^2)^2 / (x+g/h) * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * c*g*d - 3/8 * h / (a*h^2 - b*g*h + c*g^2)^2 / ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * \ln(((b*h - 2*c*g) * (x+g/h) / h + 2 * (a*h^2 - b*g*h + c*g^2) / h^2 + 2 * ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2})) / (x+g/h) * b^2*d + 3/8 / (a*h^2 - b*g*h + c*g^2)^2 / ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * \ln(((b*h - 2*c*g) * (x+g/h) / h + 2 * (a*h^2 - b*g*h + c*g^2) / h^2 + 2 * ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2})) / (x+g/h) * b^2*e*g + 1/2 / h * c / (a*h^2 - b*g*h + c*g^2) / ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * \ln(((b*h - 2*c*g) * (x+g/h) / h + 2 * (a*h^2 - b*g*h + c*g^2) / h^2 + 2 * ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2})) / (x+g/h) * d - 1/h^2 / (a*h^2 - b*g*h + c*g^2) / ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * \ln(((b*h - 2*c*g) * (x+g/h) / h + 2 * (a*h^2 - b*g*h + c*g^2) / h^2 + 2 * ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2})) / (x+g/h) * b*f*g - 3/2 / h^2 / (a*h^2 - b*g*h + c*g^2) / ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * \ln(((b*h - 2*c*g) * (x+g/h) / h + 2 * (a*h^2 - b*g*h + c*g^2) / h^2 + 2 * ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2})) / (x+g/h) * c*g*e + 5/2 / h^3 / (a*h^2 - b*g*h + c*g^2) / ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * \ln(((b*h - 2*c*g) * (x+g/h) / h + 2 * (a*h^2 - b*g*h + c*g^2) / h^2 + 2 * ((a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2})) / (x+g/h) * c*g^2*f + 3/4 / h / (a*h^2 - b*g*h + c*g^2)^2 / (x+g/h) * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * b*f*g^2 + 3/2 / h / (a*h^2 - b*g*h + c*g^2)^2 / (x+g/h) * ((x+g/h)^2*c + (b*h - 2*c*g) * (x+g/h) / h + (a*h^2 - b*g*h + c*g^2) / h^2)^{1/2} * c*g^2*e$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details) Is a\*h^2-b\*g\*h +c\*g^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^3 \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)`

[Out] `int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(g + hx)^3 \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)`



Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((b*e*g*(p + 2) - c*(e*f + d*g))*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx &= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right) - (2c^2d - bce + b^2f - 2acf)x\right)(g+hx)^3}{c(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+hx)^2(-}{ \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right) - (2c^2d - bce + b^2f - 2acf)x\right)(g+hx)^3}{c(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{(12c^2d+7b^2}{ \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right) - (2c^2d - bce + b^2f - 2acf)x\right)(g+hx)^3}{c(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{(12c^2d+7b^2}{ \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right) - (2c^2d - bce + b^2f - 2acf)x\right)(g+hx)^3}{c(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{(12c^2d+7b^2}{ \\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right) - (2c^2d - bce + b^2f - 2acf)x\right)(g+hx)^3}{c(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{(12c^2d+7b^2}{
\end{aligned}$$

**Mathematica [A]** time = 1.59, size = 715, normalized size = 1.42

Antiderivative was successfully verified.

[In] Integrate[((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (-2\*Sqrt[c]\*(105\*b^5\*f\*h^3\*x + 5\*b^4\*h^2\*(21\*a\*f\*h + c\*x\*(-54\*f\*g - 18\*e\*h + 7\*f\*h\*x)) - 2\*b^3\*c\*h\*(5\*a\*h\*(27\*f\*g + 9\*e\*h + 53\*f\*h\*x) + c\*x\*(3\*h\*(-36\*e\*g - 12\*d\*h + 5\*e\*h\*x) + f\*(-108\*g^2 + 45\*g\*h\*x + 7\*h^2\*x^2))) - 16\*c^2\*(-16\*a^3\*f\*h^3 + 6\*c^3\*d\*g^3\*x + a\*c^2\*(6\*d\*h\*(-3\*g^2 - 3\*g\*h\*x + h^2\*x^2) - 3\*e\*(2\*g^3 + 6\*g^2\*h\*x - 6\*g\*h^2\*x^2 - h^3\*x^3) + f\*x\*(-6\*g^3 + 18\*g^2\*h\*x + 9\*g\*h^2\*x^2 + 2\*h^3\*x^3)) + a^2\*c\*h\*(f\*(36\*g^2 + 27\*g\*h\*x - 8\*h^2\*x^2) + 3\*h\*(4\*d\*h + 3\*e\*(4\*g + h\*x)))) - 8\*b\*c^2\*(-6\*c^2\*g^2\*(-(d\*g) + e\*g\*x + 3\*d\*h\*x) - a^2\*h^2\*(117\*f\*g + 39\*e\*h + 61\*f\*h\*x) + a\*c\*(f\*(6\*g^3 + 90\*g^2\*h\*x - 45\*g\*h^2\*x^2 - 7\*h^3\*x^3) + 3\*h\*(2\*d\*h\*(3\*g + 5\*h\*x) + e\*(6\*g^2 + 30\*g\*h\*x - 5\*h^2\*x^2)))) + 4\*b^2\*c\*(-115\*a^2\*f\*h^3 + a\*c\*h\*(3\*h\*(18\*e\*g + 6\*d\*h + 31\*e\*h\*x) + f\*(54\*g^2 + 279\*g\*h\*x - 43\*h^2\*x^2)) + c^2\*x\*(f\*(-12\*g^3 + 18\*g^2\*h\*x + 9\*g\*h^2\*x^2 + 2\*h^3\*x^3) + 3\*h\*(2\*d\*h\*(-6\*g + h\*x) + e\*(-12\*g^2 + 6\*g\*h\*x + h^2\*x^2)))) + 3\*(b^2 - 4\*a\*c)\*(35\*b^3\*f\*h^3 - 30\*b\*c\*h^2\*(3\*b\*f\*g + b\*e\*h + 2\*a\*f\*h) - 16\*c^3\*g\*(f\*g^2 + 3\*h\*(e\*g + d\*h)) + 24\*c^2\*h\*(3\*b\*f



$*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)) * \text{Sqrt}[a + x*(b + c*x)] * \text{ArcTan} \frac{h[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])]}{(48*c^{(9/2)}*(-b^2 + 4*a*c)) * \text{Sqrt}[a + x*(b + c*x)]}$

**IntegrateAlgebraic [B]** time = 4.93, size = 1034, normalized size = 2.05

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^3\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(3/2), x]

[Out] 
$$-1/24*(-48*b*c^4*d*g^3 + 96*a*c^4*e*g^3 - 48*a*b*c^3*f*g^3 + 288*a*c^4*d*g^2*h - 144*a*b*c^3*e*g^2*h + 216*a*b^2*c^2*f*g^2*h - 576*a^2*c^3*f*g^2*h - 144*a*b*c^3*d*g*h^2 + 216*a*b^2*c^2*e*g*h^2 - 576*a^2*c^3*e*g*h^2 - 270*a*b^3*c*f*g*h^2 + 936*a^2*b*c^2*f*g*h^2 + 72*a*b^2*c^2*d*h^3 - 192*a^2*c^3*d*h^3 - 90*a*b^3*c*e*h^3 + 312*a^2*b*c^2*e*h^3 + 105*a*b^4*f*h^3 - 460*a^2*b^2*c*f*h^3 + 256*a^3*c^2*f*h^3 - 96*c^5*d*g^3*x + 48*b*c^4*e*g^3*x - 48*b^2*c^3*f*g^3*x + 96*a*c^4*f*g^3*x + 144*b*c^4*d*g^2*h*x - 144*b^2*c^3*e*g^2*h*x + 288*a*c^4*e*g^2*h*x + 216*b^3*c^2*f*g^2*h*x - 720*a*b*c^3*f*g^2*h*x - 144*b^2*c^3*d*g*h^2*x + 288*a*c^4*d*g*h^2*x + 216*b^3*c^2*e*g*h^2*x - 720*a*b*c^3*e*g*h^2*x - 270*b^4*c*f*g*h^2*x + 1116*a*b^2*c^2*f*g*h^2*x - 432*a^2*c^3*f*g*h^2*x + 72*b^3*c^2*d*h^3*x - 240*a*b*c^3*d*h^3*x - 90*b^4*c*e*h^3*x + 372*a*b^2*c^2*e*h^3*x - 144*a^2*c^3*e*h^3*x + 105*b^5*f*h^3*x - 530*a*b^3*c*f*h^3*x + 488*a^2*b*c^2*f*h^3*x + 72*b^2*c^3*f*g^2*h*x^2 - 288*a*c^4*f*g^2*h*x^2 + 72*b^2*c^3*e*g*h^2*x^2 - 288*a*c^4*e*g*h^2*x^2 - 90*b^3*c^2*f*g*h^2*x^2 + 360*a*b*c^3*f*g*h^2*x^2 + 24*b^2*c^3*d*h^3*x^2 - 96*a*c^4*d*h^3*x^2 - 30*b^3*c^2*e*h^3*x^2 + 120*a*b*c^3*e*h^3*x^2 + 35*b^4*c*f*h^3*x^2 - 172*a*b^2*c^2*f*h^3*x^2 + 128*a^2*c^3*f*h^3*x^2 + 36*b^2*c^3*f*g*h^2*x^3 - 144*a*c^4*f*g*h^2*x^3 + 12*b^2*c^3*e*h^3*x^3 - 48*a*c^4*e*h^3*x^3 - 14*b^3*c^2*f*h^3*x^3 + 56*a*b*c^3*f*h^3*x^3 + 8*b^2*c^3*f*h^3*x^4 - 32*a*c^4*f*h^3*x^4)/(c^4*(-b^2 + 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((-16*c^3*f*g^3 - 48*c^3*e*g^2*h + 72*b*c^2*f*g^2*h - 48*c^3*d*g*h^2 + 72*b*c^2*e*g*h^2 - 90*b^2*c*f*g*h^2 + 72*a*c^2*f*g*h^2 + 24*b*c^2*d*h^3 - 30*b^2*c*e*h^3 + 24*a*c^2*e*h^3 + 35*b^3*f*h^3 - 60*a*b*c*f*h^3)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(16*c^{(9/2)})$$

**fricas [B]** time = 45.68, size = 2937, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^3\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out]  $\left[ \frac{1}{96} \left( 3 \left( 16 \left( a^2 b^2 c^3 - 4 a^2 c^4 \right) f g^3 + 24 \left( 2 \left( a b^2 c^3 - 4 a^2 c^4 \right) e - 3 \left( a b^3 c^2 - 4 a^2 b c^3 \right) f \right) g^2 h + 6 \left( 8 \left( a b^2 c^3 - 4 a^2 c^4 \right) d - 12 \left( a b^3 c^2 - 4 a^2 b c^3 \right) e + 3 \left( 5 a b^4 c - 24 a^2 b^2 c^2 + 16 a^3 c^3 \right) f \right) g h^2 - \left( 24 \left( a b^3 c^2 - 4 a^2 b c^3 \right) d - 6 \left( 5 a b^4 c - 24 a^2 b^2 c^2 + 16 a^3 c^3 \right) e + 5 \left( 7 a b^5 - 40 a^2 b^3 c + 48 a^3 b c^2 \right) f \right) h^3 + \left( 16 \left( b^2 c^4 - 4 a c^5 \right) f g^3 + 24 \left( 2 \left( b^2 c^4 - 4 a c^5 \right) e - 3 \left( b^3 c^3 - 4 a b c^4 \right) f \right) g^2 h + 6 \left( 8 \left( b^2 c^4 - 4 a c^5 \right) d - 12 \left( b^3 c^3 - 4 a b c^4 \right) e + 3 \left( 5 b^4 c^2 - 24 a b^2 c^3 + 16 a^2 c^4 \right) f \right) g h^2 - \left( 24 \left( b^3 c^3 - 4 a b c^4 \right) d - 6 \left( 5 b^4 c^2 - 24 a b^2 c^3 + 16 a^2 c^4 \right) e + 5 \left( 7 b^5 c - 40 a b^3 c^2 + 48 a^2 b c^3 \right) f \right) h^3 \right) x^2 + \left( 16 \left( b^3 c^3 - 4 a b c^4 \right) f g^3 + 24 \left( 2 \left( b^3 c^3 - 4 a b c^4 \right) e - 3 \left( b^4 c^2 - 4 a b^2 c^3 \right) f \right) g^2 h + 6 \left( 8 \left( b^3 c^3 - 4 a b c^4 \right) d - 12 \left( b^4 c^2 - 4 a b^2 c^3 \right) e + 3 \left( 5 b^5 c - 24 a b^3 c^2 + 16 a^2 b c^3 \right) f \right) g h^2 - \left( 24 \left( b^4 c^2 - 4 a b^2 c^3 \right) d - 6 \left( 5 b^5 c - 24 a b^3 c^2 + 16 a^2 b c^3 \right) e + 5 \left( 7 b^6 - 40 a b^4 c + 48 a^2 b^2 c^2 \right) f \right) h^3 \right) x \right) \sqrt{c} \log(-8 c^2 x^2 - 8 b c x - b^2 - 4 \sqrt{c x^2 + b x + a}) \left( 2 c x + b \right) \sqrt{c} - 4 a c) + 4 \left( 8 \left( b^2 c^4 - 4 a c^5 \right) f h^3 x^4 - 48 \left( b c^5 d - 2 a c^5 e + a b c^4 f \right) g^3 + 72 \left( 4 a c^5 d - 2 a b c^4 e + \left( 3 a b^2 c^3 - 8 a^2 c^4 \right) f \right) g^2 h - 18 \left( 8 a b c^4 d - 4 \left( 3 a b^2 c^3 - 8 a^2 c^4 \right) e + \left( 15 a b^3 c^2 - 52 a^2 b c^3 \right) f \right) g h^2 + \left( 24 \left( 3 a b^2 c^3 - 8 a^2 c^4 \right) d - 6 \left( 15 a b^3 c^2 - 52 a^2 b c^3 \right) e + \left( 105 a b^4 c - 460 a^2 b^2 c^2 + 256 a^3 c^3 \right) f \right) h^3 + 2 \left( 18 \left( b^2 c^4 - 4 a c^5 \right) f g h^2 + \left( 6 \left( b^2 c^4 - 4 a c^5 \right) e - 7 \left( b^3 c^3 - 4 a b c^4 \right) f \right) h^3 \right) x^3 + \left( 72 \left( b^2 c^4 - 4 a c^5 \right) f g^2 h + 18 \left( 4 \left( b^2 c^4 - 4 a c^5 \right) e - 5 \left( b^3 c^3 - 4 a b c^4 \right) f \right) g h^2 + \left( 24 \left( b^2 c^4 - 4 a c^5 \right) d - 30 \left( b^3 c^3 - 4 a b c^4 \right) e + \left( 35 b^4 c^2 - 172 a b^2 c^3 + 128 a^2 c^4 \right) f \right) h^3 \right) x^2 - \left( 48 \left( 2 c^6 d - b c^5 e + \left( b^2 c^4 - 2 a c^5 \right) f \right) g^3 - 72 \left( 2 b c^5 d - 2 \left( b^2 c^4 - 2 a c^5 \right) e + \left( 3 b^3 c^3 - 10 a b c^4 \right) f \right) g^2 h + 18 \left( 8 \left( b^2 c^4 - 2 a c^5 \right) d - 4 \left( 3 b^3 c^3 - 10 a b c^4 \right) e + \left( 15 b^4 c^2 - 62 a b^2 c^3 + 24 a^2 c^4 \right) f \right) g h^2 - \left( 24 \left( 3 b^3 c^3 - 10 a b c^4 \right) d - 6 \left( 15 b^4 c^2 - 62 a b^2 c^3 + 24 a^2 c^4 \right) e + \left( 105 b^5 c - 530 a b^3 c^2 + 488 a^2 b c^3 \right) f \right) h^3 \right) x \right) \sqrt{c x^2 + b x + a} \right) / \left( a b^2 c^5 - 4 a^2 c^6 + \left( b^2 c^6 - 4 a c^7 \right) x^2 + \left( b^3 c^5 - 4 a b c^6 \right) x \right), -1/48 \left( 3 \left( 16 \left( a b^2 c^3 - 4 a^2 c^4 \right) f g^3 + 24 \left( 2 \left( a b^2 c^3 - 4 a^2 c^4 \right) e - 3 \left( a b^3 c^2 - 4 a^2 b c^3 \right) f \right) g^2 h + 6 \left( 8 \left( a b^2 c^3 - 4 a^2 c^4 \right) d - 12 \left( a b^3 c^2 - 4 a^2 b c^3 \right) e + 3 \left( 5 a b^4 c - 24 a^2 b^2 c^2 + 16 a^3 c^3 \right) f \right) g h^2 - \left( 24 \left( a b^3 c^2 - 4 a^2 b c^3 \right) d - 6 \left( 5 a b^4 c - 24 a^2 b^2 c^2 + 16 a^3 c^3 \right) e + 5 \left( 7 a b^5 - 40 a^2 b^3 c + 48 a^3 b c^2 \right) f \right) h^3 + \left( 16 \left( b^2 c^4 - 4 a c^5 \right) f g^3 + 24 \left( 2 \left( b^2 c^4 - 4 a c^5 \right) e - 3 \left( b^3 c^3 - 4 a b c^4 \right) f \right) g^2 h + 6 \left( 8 \left( b^2 c^4 - 4 a c^5 \right) d - 12 \left( b^3 c^3 - 4 a b c^4 \right) e + 3 \left( 5 b^4 c^2 - 24 a b^2 c^3 + 16 a^2 c^4 \right) f \right) g h^2 - \left( 24 \left( b^3 c^3 - 4 a b c^4 \right) d - 6 \left( 5 b^4 c^2 - 24 a b^2 c^3 + 16 a^2 c^4 \right) e + 5 \left( 7 b^5 c - 40 a b^3 c^2 + 48 a^2 b c^3 \right) f \right) h^3 \right) x^2 + \left( 16 \left( b^3 c^3 - 4 a b c^4 \right) f g^3 + 24 \left( 2 \left( b^3 c^3 - 4 a b c^4 \right) e - 3 \left( b^4 c^2 - 4 a b^2 c^3 \right) f \right) g^2 h + 6 \left( 8 \left( b^3 c^3 - 4 a b c^4 \right) d - 12 \left( b^4 c^2 - 4 a b^2 c^3 \right) e + 3 \left( 5 b^5 c - 24 a b^3 c^2 + 16 a^2 b c^3 \right) f \right) g h^2 - \left( 24 \left( b^4 c^2 - 4 a b^2 c^3 \right) d - 6 \left( 5 b^5 c - 24 a b^3 c^2 + 16 a^2 b c^3 \right) e + 5 \left( 7 b^6 - 40 a b^4 c + 48 a^2 b^2 c^2 \right) f \right) h^3 \right) x \right) \sqrt{c}$

$$(-c) \arctan(1/2 \sqrt{c x^2 + b x + a}) (2 c x + b) \sqrt{-c} / (c^2 x^2 + b c x + a c) - 2 (8 (b^2 c^4 - 4 a c^5) f h^3 x^4 - 48 (b c^5 d - 2 a c^5 e + a b c^4 f) g^3 + 72 (4 a c^5 d - 2 a b c^4 e + (3 a b^2 c^3 - 8 a^2 c^4) f) g^2 h - 18 (8 a b c^4 d - 4 (3 a b^2 c^3 - 8 a^2 c^4) e + (15 a b^3 c^2 - 5 2 a^2 b c^3) f) g h^2 + (24 (3 a b^2 c^3 - 8 a^2 c^4) d - 6 (15 a b^3 c^2 - 5 2 a^2 b c^3) e + (105 a b^4 c - 460 a^2 b^2 c^2 + 256 a^3 c^3) f) h^3 + 2 (18 (b^2 c^4 - 4 a c^5) f g h^2 + (6 (b^2 c^4 - 4 a c^5) e - 7 (b^3 c^3 - 4 a b c^4) f) h^3) x^3 + (72 (b^2 c^4 - 4 a c^5) f g^2 h + 18 (4 (b^2 c^4 - 4 a c^5) e - 5 (b^3 c^3 - 4 a b c^4) f) g h^2 + (24 (b^2 c^4 - 4 a c^5) d - 30 (b^3 c^3 - 4 a b c^4) e + (35 b^4 c^2 - 172 a b^2 c^3 + 128 a^2 c^4) f) h^3) x^2 - (48 (2 c^6 d - b c^5 e + (b^2 c^4 - 2 a c^5) f) g^3 - 72 (2 b c^5 d - 2 (b^2 c^4 - 2 a c^5) e + (3 b^3 c^3 - 10 a b c^4) f) g^2 h + 18 (8 (b^2 c^4 - 2 a c^5) d - 4 (3 b^3 c^3 - 10 a b c^4) e + (15 b^4 c^2 - 62 a b^2 c^3 + 24 a^2 c^4) f) g h^2 - (24 (3 b^3 c^3 - 10 a b c^4) d - 6 (15 b^4 c^2 - 62 a b^2 c^3 + 24 a^2 c^4) e + (105 b^5 c - 530 a b^3 c^2 + 488 a^2 b c^3) f) h^3) x) \sqrt{c x^2 + b x + a} / (a b^2 c^5 - 4 a^2 c^6 + (b^2 c^6 - 4 a c^7) x^2 + (b^3 c^5 - 4 a b c^6) x)]$$

**giac [B]** time = 0.35, size = 1054, normalized size = 2.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] 1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b^2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 - 7*b^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3 + 6*b^2*c^3*h^3*e - 24*a*c^4*h^3*e)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^2*h - 288*a*c^4*f*g^2*h - 90*b^3*c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^3*d*h^3 - 96*a*c^4*d*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3 + 128*a^2*c^3*f*h^3 + 72*b^2*c^3*g*h^2*e - 288*a*c^4*g*h^2*e - 30*b^3*c^2*h^3*e + 120*a*b*c^3*h^3*e)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 + 48*b^2*c^3*f*g^3 - 96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^2*h + 144*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*c^2*f*h^3 - 48*b*c^4*g^3*e + 144*b^2*c^3*g^2*h*e - 288*a*c^4*g^2*h*e - 216*b^3*c^2*g*h^2*e + 720*a*b*c^3*g*h^2*e + 90*b^4*c*h^3*e - 372*a*b^2*c^2*h^3*e + 144*a^2*c^3*h^3*e)/(b^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 + 48*a*b*c^3*f*g^3 - 288*a*c^4*d*g^2*h - 216*a*b^2*c^2*f*g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 + 270*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a*b^2*c^2*d*h^3 + 192*a^2*c^3*d*h^3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3 - 96*a*c^4*g^3*e + 144*a*b*c^3*g^2*h*e - 216*a*b^2*c^2*g*h^2*e + 576*a^2*c^3*g*h^2*e + 90*a*b^3*c*h^3*e - 312*a^2*b*c^2*h^3*e)/(b^2*c^4 - 4*a*c^5))/sqrt(c*x^2 + b*x + a) - 1/16*(16*c^3*f*g^3 - 72*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 90*
```

$$b^2*c*f*g*h^2 - 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 35*b^3*f*h^3 + 60*a*b*c*f*h^3 + 48*c^3*g^2*h*e - 72*b*c^2*g*h^2*e + 30*b^2*c*h^3*e - 24*a*c^2*h^3*e) * \log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{(9/2)})$$

**maple [B]** time = 0.02, size = 2780, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out]  $\frac{1}{c^{(3/2)}} * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g^3*f - \frac{1}{c} / (c*x^2+b*x+a)^{(1/2)} * g^3*e - \frac{3/4}{c^3*b^2} / (c*x^2+b*x+a)^{(1/2)} * h^3*d - \frac{3/2}{c^{(5/2)}} * b * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * h^3*d + \frac{2*a}{c^2} / (c*x^2+b*x+a)^{(1/2)} * h^3*d - \frac{35/32}{c^5*b^4} / (c*x^2+b*x+a)^{(1/2)} - \frac{35/16}{c^9/2} * b^3 * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) - \frac{8/3}{c^3} * h^3*f * a^2 / (c*x^2+b*x+a)^{(1/2)} + \frac{1/2}{c} * x^3 / (c*x^2+b*x+a)^{(1/2)} * h^3*e + \frac{15/16}{c^4*b^3} / (c*x^2+b*x+a)^{(1/2)} * h^3*e + \frac{15/8}{c^{(7/2)}} * b^2 * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * h^3*e - \frac{3/2}{c^5} * a / (c*x^2+b*x+a)^{(1/2)} * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * h^3*e + \frac{1/3}{c} * h^3*f * x^4 / (c*x^2+b*x+a)^{(1/2)} + 2 * g^3*d * (2*c*x+b) / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} + \frac{3}{c^{(3/2)}} * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g*h^2*d + \frac{3}{c^{(3/2)}} * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g^2*h*e - \frac{3}{c} / (c*x^2+b*x+a)^{(1/2)} * g^2*h*d + \frac{x^2}{c} / (c*x^2+b*x+a)^{(1/2)} * h^3*d + \frac{45/16}{c^4*b^5} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * g*h^2*f - \frac{39/4}{c^3*b*a} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*f - \frac{13/4}{c^3*b^3*a} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * h^3*e - \frac{9/4}{c^3*b^4} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * g*h^2*e - \frac{9/4}{c^3*b^4} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * g^2*h*f + \frac{2*a}{c^2*b^2} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * h^3*d + \frac{9/2}{c^2*b*x} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*e + \frac{12*a}{c*b} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * x * g^2*h*f + \frac{3/2}{c^2*b^3} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * g*h^2*d + \frac{1}{c*b^2} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * x * g^3*f - \frac{3/2}{c^2*b^3} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * x * h^3*d - \frac{3*b^2}{c} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * g^2*h*d + \frac{115/24}{c^4*b^4*a} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} - \frac{15/4}{c^3*b*a*x} / (c*x^2+b*x+a)^{(1/2)} - \frac{8/3}{c^3*b^2} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} - \frac{35/16}{c^4*b^5} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * x + \frac{9/2}{c^2*b*x} / (c*x^2+b*x+a)^{(1/2)} * g^2*h*f - \frac{15/4}{c^2*b*x^2} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*f - \frac{45/8}{c^3*b^2*x} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*f + \frac{15/8}{c^3*b^4} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * x * h^3*e - \frac{9/4}{c^3*b^2} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*e - \frac{9/4}{c^3*b^2} / (c*x^2+b*x+a)^{(1/2)} * g^2*h*f - \frac{3/4}{c^3*b^4} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * h^3*d - \frac{9/2}{c^{(5/2)}} * b * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g*h^2*e + \frac{3/2}{c} * x^3 / (c*x^2+b*x+a)^{(1/2)} * g*h^2*f - \frac{5/4}{c^2*b*x^2} / (c*x^2+b*x+a)^{(1/2)} * h^3*e - \frac{15/8}{c^3*b^2*x} / (c*x^2+b*x+a)^{(1/2)} * h^3*e + \frac{45/16}{c^4*b^3} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*f + \frac{15/16}{c^4*b^5} / (4*a*c-b^2) / (c*x^2+b*x+a)^{(1/2)} * h^3*e + \frac{45/8}{c^{(7/2)}} * b^2 * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g*h^2*f - \frac{13/4}{c^3*b*a} / (c*x^2+b*x+a)^{(1/2)} * h^3*e + \frac{3/2}{c^2*x} / (c*x^2+b*x+a)^{(1/2)} * h^3*e - \frac{9/2}{c^{(5/2)}} * b * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g*h^2*f - \frac{9/2}{c^{(5/2)}} * b * \ln\left(\frac{c*x+1/2*b}{c^{(1/2)}} + (c*x^2+b*x+a)^{(1/2)}\right) * g^2*h*f + \frac{6*a}{c^2} / (c*x^2+b*x+a)^{(1/2)} * g*h^2*$

```

e+6*a/c^2/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g*h^2*e+9
/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*g*h^2*f+3/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*g^2*h*e-6*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*d+45/8/c^3*b^4/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-13/2/c^2*b^2*a/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*h^3*e-39/4/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2*
f+115/12*h^3*f/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-16/3*h^3*f*a^2/c
^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+6*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*g*h^2*e+6*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*f+4*a/c*b
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*d+3/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*x*g*h^2*d+3/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*e-9/2/c^2*b
^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*e-9/2/c^2*b^3/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*g^2*h*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3/2/c^2*b*x/(c*
x^2+b*x+a)^(1/2)*h^3*d-3*x/c/(c*x^2+b*x+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+b*x+a
)^(1/2)*g^2*h*e+3/2/c^2*b/(c*x^2+b*x+a)^(1/2)*g*h^2*d+3/2/c^2*b/(c*x^2+b*x+
a)^(1/2)*g^2*h*e+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^3*f-2*b/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*e-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g
^3*e-7/12*h^3*f/c^2*b*x^3/(c*x^2+b*x+a)^(1/2)+35/24*h^3*f/c^3*b^2*x^2/(c*x^
2+b*x+a)^(1/2)+35/16*h^3*f/c^4*b^3*x/(c*x^2+b*x+a)^(1/2)-35/32*h^3*f/c^5*b^
6/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+115/24*h^3*f/c^4*b^2*a/(c*x^2+b*x+a)^(1/2
)+15/4*h^3*f/c^(7/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3*h^
3*f*a/c^2*x^2/(c*x^2+b*x+a)^(1/2)+12*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x*g*h^2*e-39/2/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-x/c/(c*x
^2+b*x+a)^(1/2)*g^3*f+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*g^3*f

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^3 (fx^2 + ex + d)}{(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)`

[Out] `int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)^3 (d + ex + fx^2)}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2), x)`

[Out] `Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)`

$$3.230 \quad \int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=289

$$\frac{2(g+hx)^2 \left( c \left( 2ae - b \left( \frac{af}{c} + d \right) \right) - x(-2acf + b^2f - bce + 2c^2d) \right) \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right) (-12ch(afh + beh + \dots)}{c(b^2 - 4ac)\sqrt{a+bx+cx^2}} + \dots$$

**Rubi [A]** time = 0.39, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1644, 779, 621, 206}

$$\frac{\tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right) (-12ch(afh + beh + 2bf)g + 15b^2f^2 + 8c^2(h(dh + 2eg) + fg^2))}{8c^{7/2}} + \frac{2(g+hx)^2 \left( c \left( 2ae - b \left( \frac{af}{c} + d \right) \right) - x(-2acf + b^2f - bce + 2c^2d) \right)}{c(b^2 - 4ac)\sqrt{a+bx+cx^2}} + \frac{h\sqrt{a+bx+cx^2} (2hx(-12acf + 5b^2f - 4bce + 8c^2d) - 8c(4ach + 8afg + bdh + 2bcg) + 4h(13afh + 3beh + 6bfg) - \frac{15b^2h}{c} + 32c^2dg)}{4c^2(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(c\*(2\*a\*e - b\*(d + (a\*f)/c)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x)\*(g + h\*x)^2)/(c\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + (h\*(32\*c^2\*d\*g - (15\*b^3\*f\*h)/c - 8\*c\*(2\*b\*e\*g + 8\*a\*f\*g + b\*d\*h + 4\*a\*e\*h) + 4\*b\*(6\*b\*f\*g + 3\*b\*e\*h + 13\*a\*f\*h) + 2\*(8\*c^2\*d - 4\*b\*c\*e + 5\*b^2\*f - 12\*a\*c\*f)\*h\*x)\*Sqrt[a + b\*x + c\*x^2]/(4\*c^2\*(b^2 - 4\*a\*c)) + ((15\*b^2\*f\*h^2 - 12\*c\*h\*(2\*b\*f\*g + b\*e\*h + a\*f\*h) + 8\*c^2\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(7/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 779

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x





[In] Integrate[((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(3/2),x]

[Out] (2\*sqrt[c]\*(15\*b^4\*f\*h^2\*x + b^3\*h\*(15\*a\*f\*h + c\*x\*(-24\*f\*g - 12\*e\*h + 5\*f\*h\*x)) + 4\*b\*c\*(-13\*a^2\*f\*h^2 + 2\*c^2\*g\*(-e\*g\*x) + d\*(g - 2\*h\*x)) + a\*c\*(2\*h\*(2\*e\*g + d\*h + 5\*e\*h\*x) + f\*(2\*g^2 + 20\*g\*h\*x - 5\*h^2\*x^2))) - 2\*b^2\*c\*(a\*h\*(12\*f\*g + 6\*e\*h + 31\*f\*h\*x) + c\*x\*(2\*h\*(-4\*e\*g - 2\*d\*h + e\*h\*x) + f\*(-4\*g^2 + 4\*g\*h\*x + h^2\*x^2))) + 8\*c^2\*(2\*c^2\*d\*g^2\*x + a^2\*h\*(8\*f\*g + 4\*e\*h + 3\*f\*h\*x) + a\*c\*(-2\*d\*h\*(2\*g + h\*x) - 2\*e\*(g^2 + 2\*g\*h\*x - h^2\*x^2) + f\*x\*(-2\*g^2 + 4\*g\*h\*x + h^2\*x^2))) - (b^2 - 4\*a\*c)\*(15\*b^2\*f\*h^2 - 12\*c\*h\*(2\*b\*f\*g + b\*e\*h + a\*f\*h) + 8\*c^2\*(f\*g^2 + h\*(2\*e\*g + d\*h)))\*sqrt[a + x\*(b + c\*x)]\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)])]/(8\*c^(7/2)\*(-b^2 + 4\*a\*c)\*sqrt[a + x\*(b + c\*x)])

**IntegrateAlgebraic [A]** time = 3.10, size = 552, normalized size = 1.91

http://www.sagemath.org/doc/development/patching/

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((g + h\*x)^2\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(3/2),x]

[Out] -1/4\*(-8\*b\*c^3\*d\*g^2 + 16\*a\*c^3\*e\*g^2 - 8\*a\*b\*c^2\*f\*g^2 + 32\*a\*c^3\*d\*g\*h - 16\*a\*b\*c^2\*e\*g\*h + 24\*a\*b^2\*c\*f\*g\*h - 64\*a^2\*c^2\*f\*g\*h - 8\*a\*b\*c^2\*d\*h^2 + 12\*a\*b^2\*c\*e\*h^2 - 32\*a^2\*c^2\*e\*h^2 - 15\*a\*b^3\*f\*h^2 + 52\*a^2\*b\*c\*f\*h^2 - 16\*c^4\*d\*g^2\*x + 8\*b\*c^3\*e\*g^2\*x - 8\*b^2\*c^2\*f\*g^2\*x + 16\*a\*c^3\*f\*g^2\*x + 16\*b\*c^3\*d\*g\*h\*x - 16\*b^2\*c^2\*e\*g\*h\*x + 32\*a\*c^3\*e\*g\*h\*x + 24\*b^3\*c\*f\*g\*h\*x - 80\*a\*b\*c^2\*f\*g\*h\*x - 8\*b^2\*c^2\*d\*h^2\*x + 16\*a\*c^3\*d\*h^2\*x + 12\*b^3\*c\*e\*h^2\*x - 40\*a\*b\*c^2\*e\*h^2\*x - 15\*b^4\*f\*h^2\*x + 62\*a\*b^2\*c\*f\*h^2\*x - 24\*a^2\*c^2\*f\*h^2\*x + 8\*b^2\*c^2\*f\*g\*h\*x^2 - 32\*a\*c^3\*f\*g\*h\*x^2 + 4\*b^2\*c^2\*e\*h^2\*x^2 - 16\*a\*c^3\*e\*h^2\*x^2 - 5\*b^3\*c\*f\*h^2\*x^2 + 20\*a\*b\*c^2\*f\*h^2\*x^2 + 2\*b^2\*c^2\*f\*h^2\*x^3 - 8\*a\*c^3\*f\*h^2\*x^3)/(c^3\*(-b^2 + 4\*a\*c)\*sqrt[a + b\*x + c\*x^2]) + ((-8\*c^2\*f\*g^2 - 16\*c^2\*e\*g\*h + 24\*b\*c\*f\*g\*h - 8\*c^2\*d\*h^2 + 12\*b\*c\*e\*h^2 - 15\*b^2\*f\*h^2 + 12\*a\*c\*f\*h^2)\*Log[b\*c^3 + 2\*c^4\*x - 2\*c^(7/2)\*sqrt[a + b\*x + c\*x^2]])/(8\*c^(7/2))

**fricas [B]** time = 28.87, size = 1769, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] [-1/16\*((8\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*f\*g^2 + 8\*(2\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*e - 3\*(a\*b^3\*c - 4\*a^2\*b\*c^2)\*f)\*g\*h + (8\*(a\*b^2\*c^2 - 4\*a^2\*c^3)\*d - 12\*(a\*b^3\*c - 4\*a^2\*b\*c^2)\*e + 3\*(5\*a\*b^4 - 24\*a^2\*b^2\*c + 16\*a^3\*c^2)\*f)\*h^2 + (8\*

$$\begin{aligned}
& (b^2c^3 - 4ac^4)fg^2 + 8*(2*(b^2c^3 - 4ac^4)e - 3*(b^3c^2 - 4ab*c^3)*f)*g*h + (8*(b^2c^3 - 4ac^4)d - 12*(b^3c^2 - 4ab*c^3)*e + 3*(5*b^4c - 24*ab^2c^2 + 16*a^2c^3)*f)*h^2)*x^2 + (8*(b^3c^2 - 4ab*c^3)*f*g^2 + 8*(2*(b^3c^2 - 4ab*c^3)*e - 3*(b^4c - 4ab^2c^2)*f)*g*h + (8*(b^3c^2 - 4ab*c^3)*d - 12*(b^4c - 4ab^2c^2)*e + 3*(5*b^5 - 24*ab^3c + 16*a^2b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a))*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2c^3 - 4ac^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d - 2*a*b*c^3*e + (3*a*b^2c^2 - 8*a^2c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*a*b^2c^2 - 8*a^2c^3)*e + (15*a*b^3c - 52*a^2b*c^2)*f)*h^2 + (8*(b^2c^3 - 4ac^4)*f*g*h + (4*(b^2c^3 - 4ac^4)*e - 5*(b^3c^2 - 4ab*c^3)*f)*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d - 2*(b^2c^3 - 2*a*c^4)*e + (3*b^3c^2 - 10*ab*c^3)*f)*g*h + (8*(b^2c^3 - 2*a*c^4)*d - 4*(3*b^3c^2 - 10*ab*c^3)*e + (15*b^4c - 62*ab^2c^2 + 24*a^2c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2c^4 - 4*a^2c^5 + (b^2c^5 - 4*a*c^6)*x^2 + (b^3c^4 - 4*ab*c^5)*x), -1/8*((8*(a*b^2c^2 - 4*a^2c^3)*f*g^2 + 8*(2*(a*b^2c^2 - 4*a^2c^3)*e - 3*(a*b^3c - 4*a^2b*c^2)*f)*g*h + (8*(a*b^2c^2 - 4*a^2c^3)*d - 12*(a*b^3c - 4*a^2b*c^2)*e + 3*(5*a*b^4 - 24*a^2b^2c + 16*a^3c^2)*f)*h^2 + (8*(b^2c^3 - 4ac^4)*f*g^2 + 8*(2*(b^2c^3 - 4ac^4)*e - 3*(b^3c^2 - 4ab*c^3)*f)*g*h + (8*(b^2c^3 - 4ac^4)*d - 12*(b^3c^2 - 4ab*c^3)*e + 3*(5*b^4c - 24*ab^2c^2 + 16*a^2c^3)*f)*h^2)*x^2 + (8*(b^3c^2 - 4ab*c^3)*f*g^2 + 8*(2*(b^3c^2 - 4ab*c^3)*e - 3*(b^4c - 4ab^2c^2)*f)*g*h + (8*(b^3c^2 - 4ab*c^3)*d - 12*(b^4c - 4ab^2c^2)*e + 3*(5*b^5 - 24*ab^3c + 16*a^2b*c^2)*f)*h^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*(b^2c^3 - 4ac^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d - 2*a*b*c^3*e + (3*a*b^2c^2 - 8*a^2c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*a*b^2c^2 - 8*a^2c^3)*e + (15*a*b^3c - 52*a^2b*c^2)*f)*h^2 + (8*(b^2c^3 - 4ac^4)*f*g*h + (4*(b^2c^3 - 4ac^4)*e - 5*(b^3c^2 - 4ab*c^3)*f)*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d - 2*(b^2c^3 - 2*a*c^4)*e + (3*b^3c^2 - 10*ab*c^3)*f)*g*h + (8*(b^2c^3 - 2*a*c^4)*d - 4*(3*b^3c^2 - 10*ab*c^3)*e + (15*b^4c - 62*ab^2c^2 + 24*a^2c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2c^4 - 4*a^2c^5 + (b^2c^5 - 4*a*c^6)*x^2 + (b^3c^4 - 4*ab*c^5)*x)]
\end{aligned}$$

**giac [B]** time = 0.32, size = 580, normalized size = 2.01

$$\frac{\left(\frac{11c^2d^2 - 24bd^2g + 8c^2d^2 + 15b^2d^2 - 12ad^2f + 16c^2gh - 12bd^2g \log\left|\frac{-2(\sqrt{c}x - \sqrt{c^2 + bx + a})d - \|}{\sqrt{c}}\right.\right)}{4\sqrt{c^2 + bx + a}}\right)}{a^2b^2c^4 - 4a^2c^5 + (b^2c^5 - 4ac^6)x^2 + (b^3c^4 - 4ab^2c^5)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] 1/4\*(((2\*(b^2c^2\*f\*h^2 - 4ac^3\*f\*h^2)\*x/(b^2c^3 - 4ac^4) + (8\*b^2c^2\*f\*g\*h - 32\*a\*c^3\*f\*g\*h - 5\*b^3c\*f\*h^2 + 20\*ab\*c^2\*f\*h^2 + 4\*b^2c^2\*h^2\*e - 16\*a\*c^3\*h^2\*e)/(b^2c^3 - 4ac^4))\*x - (16\*c^4\*d\*g^2 + 8\*b^2c^2\*f\*g^2

$$\begin{aligned}
& 2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8 \\
& *b^2*c^2*d*h^2 - 16*a*c^3*d*h^2 + 15*b^4*f*h^2 - 62*a*b^2*c*f*h^2 + 24*a^2* \\
& c^2*f*h^2 - 8*b*c^3*g^2*e + 16*b^2*c^2*g*h*e - 32*a*c^3*g*h*e - 12*b^3*c*h^ \\
& 2*e + 40*a*b*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 + 8*a*b*c^2 \\
& *f*g^2 - 32*a*c^3*d*g*h - 24*a*b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d \\
& *h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2 - 16*a*c^3*g^2*e + 16*a*b*c^2*g*h* \\
& e - 12*a*b^2*c*h^2*e + 32*a^2*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + \\
& b*x + a) - 1/8*(8*c^2*f*g^2 - 24*b*c*f*g*h + 8*c^2*d*h^2 + 15*b^2*f*h^2 - 1 \\
& 2*a*c*f*h^2 + 16*c^2*g*h*e - 12*b*c*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x \\
& ^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
\end{aligned}$$

**maple [B]** time = 0.01, size = 1557, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out] 
$$\begin{aligned}
& -13/2*h^2*f/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+1/c^{(3/2)}*\ln((c*x+1 \\
& /2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*d*h^2+1/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+( \\
& c*x^2+b*x+a)^{(1/2)})*f*g^2-1/c/(c*x^2+b*x+a)^{(1/2)}*g^2*e+4*a/c*b/(4*a*c-b^2) \\
& / (c*x^2+b*x+a)^{(1/2)}*x*h^2*e+4*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*g* \\
& h*f+2/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*e*g*h-3/c^2*b^3/(4*a*c-b^2)/( \\
& c*x^2+b*x+a)^{(1/2)}*x*g*h*f+8*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*g*h*f+ \\
& 2*g^2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+15/16*h^2*f/c^4*b^3/(c*x^ \\
& 2+b*x+a)^{(1/2)}+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*f*g^2-5/4*h^2*f/c^ \\
& 2*b*x^2/(c*x^2+b*x+a)^{(1/2)}-15/8*h^2*f/c^3*b^2*x/(c*x^2+b*x+a)^{(1/2)}+15/16* \\
& h^2*f/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}-13/4*h^2*f/c^3*b*a/(c*x^2+b*x \\
& +a)^{(1/2)}+3/2*h^2*f*a/c^2*x/(c*x^2+b*x+a)^{(1/2)}+2*x^2/c/(c*x^2+b*x+a)^{(1/2)} \\
& *g*h*f+3/2/c^2*b*x/(c*x^2+b*x+a)^{(1/2)}*h^2*e-3/2/c^3*b^2/(c*x^2+b*x+a)^{(1/2)} \\
& )*g*h*f-3/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*h^2*e-3/c^{(5/2)}*b*\ln((c \\
& *x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*g*h*f+4*a/c^2/(c*x^2+b*x+a)^{(1/2)}*g* \\
& h*f-2*x/c/(c*x^2+b*x+a)^{(1/2)}*e*g*h+1/c^2*b/(c*x^2+b*x+a)^{(1/2)}*e*g*h+1/2/c \\
& ^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d*h^2+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2 \\
& +b*x+a)^{(1/2)}*f*g^2-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*g^2*e-b^2/c/(4*a* \\
& c-b^2)/(c*x^2+b*x+a)^{(1/2)}*g^2*e+1/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}* \\
& e*g*h-3/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*h^2*e-3/2/c^3*b^4/(4*a* \\
& c-b^2)/(c*x^2+b*x+a)^{(1/2)}*g*h*f+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} \\
& )*h^2*e+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*d*h^2+3/c^2*b*x/(c*x^2+b* \\
& x+a)^{(1/2)}*g*h*f-4*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*g*h*d-2*b^2/c/(4*a*c \\
& -b^2)/(c*x^2+b*x+a)^{(1/2)}*g*h*d+15/8*h^2*f/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a \\
& )^{(1/2)}*x-13/4*h^2*f/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+15/8*h^2*f/c \\
& ^{(7/2)}*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-3/2*h^2*f*a/c^{(5/2)}* \\
& \ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-x/c/(c*x^2+b*x+a)^{(1/2)}*d*h^2-x \\
& /c/(c*x^2+b*x+a)^{(1/2)}*f*g^2+1/2/c^2*b/(c*x^2+b*x+a)^{(1/2)}*d*h^2+1/2/c^2*b/
\end{aligned}$$

$$(c*x^2+b*x+a)^{(1/2)}*f*g^2+2/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*e*g*h-2/c/(c*x^2+b*x+a)^{(1/2)}*g*h*d+1/2*h^2*f*x^3/c/(c*x^2+b*x+a)^{(1/2)}+x^2/c/(c*x^2+b*x+a)^{(1/2)}*h^2*e-3/4/c^3*b^2/(c*x^2+b*x+a)^{(1/2)}*h^2*e-3/2/c^{(5/2)}*b*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})*h^2*e+2*a/c^2/(c*x^2+b*x+a)^{(1/2)}*h^2*e$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+hx)^2 (fx^2+ex+d)}{(cx^2+bx+a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g+h\*x)^2\*(d+e\*x+f\*x^2))/(a+b\*x+c\*x^2)^(3/2),x)

[Out] int(((g+h\*x)^2\*(d+e\*x+f\*x^2))/(a+b\*x+c\*x^2)^(3/2),x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Integral((g+h\*x)\*\*2\*(d+e\*x+f\*x\*\*2)/(a+b\*x+c\*x\*\*2)\*\*(3/2),x)

$$3.231 \quad \int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=186

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c^2(b^2-4ac)}$$

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {1644, 640, 621, 206}

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c^2(b^2-4ac)} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(3bfh-2c(eh+fg))}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((g + h\*x)\*(d + e\*x + f\*x^2))/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(c\*(2\*a\*e - b\*(d + (a\*f)/c)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x)\*(g + h\*x))/(c\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + ((4\*c^2\*d - 2\*b\*c\*e + 3\*b^2\*f - 8\*a\*c\*f)\*h\*Sqrt[a + b\*x + c\*x^2])/(c^2\*(b^2 - 4\*a\*c)) - ((3\*b\*f\*h - 2\*c\*(f\*g + e\*h))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*c^(5/2))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1644

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(
p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

Rubi steps

$$\begin{aligned}
\int \frac{(g + hx)(d + ex + fx^2)}{(a + bx + cx^2)^{3/2}} dx &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right) (g + hx)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} - 2 \int \frac{b^2fg + 2b(cd + a)}{\sqrt{a + bx + cx^2}} dx \\
&= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right) (g + hx)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{(4c^2d + 3b^2f - 2c^2g)x + (2cd + a)g}{\sqrt{a + bx + cx^2}} \\
&= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right) (g + hx)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{(4c^2d + 3b^2f - 2c^2g)x + (2cd + a)g}{\sqrt{a + bx + cx^2}} \\
&= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf)x \right) (g + hx)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{(4c^2d + 3b^2f - 2c^2g)x + (2cd + a)g}{\sqrt{a + bx + cx^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 205, normalized size = 1.10

$$\frac{2\sqrt{c} \left( 4c^2fh - ac(dh + e(g + hx) + fx(g - hx)) + c^2dgx \right) + b^2(cx(2eh + 2fg - fhx) - 3afh) + 2bc(aeh + af(g + 5hx) + cd(g - hx) - cegx) - 3b^2fhx}{\sqrt{a + x(b + cx)}} + \frac{(b^2 - 4ac) \log(2\sqrt{c} \sqrt{a + x(b + cx)} + b + 2cx) (3bfh - 2c(eh + fg))}{2c^{5/2} (4ac - b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x]
```

```
[Out] ((2*sqrt[c]*(-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g
+ 5*h*x)) + b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h
+ c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x))))/sqrt[a + x*(b + c*
x)] + (b^2 - 4*a*c)*(3*b*f*h - 2*c*(f*g + e*h))*Log[b + 2*c*x + 2*sqrt[c]*S
qrt[a + x*(b + c*x)])/(2*c^(5/2)*(-b^2 + 4*a*c))
```

**IntegrateAlgebraic [A]** time = 1.10, size = 245, normalized size = 1.32

$$\frac{\log\left(\frac{-2c^2\sqrt{a+bx+cx^2}+bc^2+2c^2x}{2c^2}\right)(3bfh-2ceh-2cfg) - \frac{-8a^2c^2fh+3ab^2fh-2abceh-2abcfh-10abcfx+4a^2dh+4a^2cg+4a^2eh+4a^2fgx-4a^2flx^2+3b^3flx-2b^2celx-2b^2cfxg+b^2cflx^2-2bc^2dg+2bc^2dlx+2bc^2egx-4c^3dgx}{c^2(4ac-b^2)\sqrt{a+bx+cx^2}}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x]
```

```
[Out] -((-2*b*c^2*d*g + 4*a*c^2*e*g - 2*a*b*c*f*g + 4*a*c^2*d*h - 2*a*b*c*e*h + 3
*a*b^2*f*h - 8*a^2*c*f*h - 4*c^3*d*g*x + 2*b*c^2*e*g*x - 2*b^2*c*f*g*x + 4*
a*c^2*f*g*x + 2*b*c^2*d*h*x - 2*b^2*c*e*h*x + 4*a*c^2*e*h*x + 3*b^3*f*h*x -
10*a*b*c*f*h*x + b^2*c*f*h*x^2 - 4*a*c^2*f*h*x^2)/(c^2*(-b^2 + 4*a*c)*sqrt
[a + b*x + c*x^2])) + ((-2*c*f*g - 2*c*e*h + 3*b*f*h)*Log[b*c^2 + 2*c^3*x -
2*c^(5/2)*sqrt[a + b*x + c*x^2]])/(2*c^(5/2))
```

**fricas [B]** time = 19.70, size = 905, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2), x, algorithm="fricas")
```

```
[Out] [-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*
c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^
2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c
- 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((b^2*
c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d
- 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^
2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 1
0*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^
4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c - 4*a^2*c^2)
*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4
*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)
*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b
^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-
c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d -
2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c
^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d -
```

$$2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)*\sqrt{c*x^2 + b*x + a})/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x)]$$

**giac [A]** time = 0.28, size = 271, normalized size = 1.46

$$\left(\frac{(b^2cfh-4a^2fh)x}{b^2c^2-4ac^3} - \frac{4c^3dg+2b^2cfh-4a^2fg-2bc^2dh-3b^3fh+10abcfh-2bc^2ge+2b^2che-4a^2he}{b^2c^2-4ac^3}\right)x - \frac{2bc^2dg+2abcfg-4a^2dh-3ab^2fh+8a^2cfh-4a^2ge+2abche}{b^2c^2-4ac^3} - \frac{(2c^2fg-3bfh+2che)\log\left(-2\left(\sqrt{cx^2+bx+a}\right)\sqrt{c-b}\right)}{2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] (((b^2\*c\*f\*h - 4\*a\*c^2\*f\*h)\*x/(b^2\*c^2 - 4\*a\*c^3) - (4\*c^3\*d\*g + 2\*b^2\*c\*f\*g - 4\*a\*c^2\*f\*g - 2\*b\*c^2\*d\*h - 3\*b^3\*f\*h + 10\*a\*b\*c\*f\*h - 2\*b\*c^2\*g\*e + 2\*b^2\*c\*h\*e - 4\*a\*c^2\*h\*e)/(b^2\*c^2 - 4\*a\*c^3))\*x - (2\*b\*c^2\*d\*g + 2\*a\*b\*c\*f\*g - 4\*a\*c^2\*d\*h - 3\*a\*b^2\*f\*h + 8\*a^2\*c\*f\*h - 4\*a\*c^2\*g\*e + 2\*a\*b\*c\*h\*e)/(b^2\*c^2 - 4\*a\*c^3))/sqrt(c\*x^2 + b\*x + a) - 1/2\*(2\*c\*f\*g - 3\*b\*f\*h + 2\*c\*h\*e)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(5/2)

**maple [B]** time = 0.01, size = 735, normalized size = 3.95

---

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x)

[Out] 1/c^(3/2)\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*f\*g-1/c/(c\*x^2+b\*x+a)^(1/2)\*d\*h-1/c/(c\*x^2+b\*x+a)^(1/2)\*e\*g+1/c^(3/2)\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))\*e\*h-3/2\*f\*h/c^2\*b^3/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x+2\*f\*h\*a/c^2\*b^2/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)+4\*f\*h\*a/c\*b/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x+1/c\*b^2/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x\*e\*h+1/c\*b^2/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x\*f\*g-x/c/(c\*x^2+b\*x+a)^(1/2)\*e\*h+2\*f\*h\*a/c^2/(c\*x^2+b\*x+a)^(1/2)-3/2\*f\*h/c^(5/2)\*b\*ln((c\*x+1/2\*b)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))+f\*h\*x^2/c/(c\*x^2+b\*x+a)^(1/2)-3/4\*f\*h/c^3\*b^2/(c\*x^2+b\*x+a)^(1/2)+2\*d\*g\*(2\*c\*x+b)/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)+1/2/c^2\*b/(c\*x^2+b\*x+a)^(1/2)\*f\*g-2\*b/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x\*e\*g-2\*b/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x\*d\*h+1/2/c^2\*b^3/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*f\*g+1/2/c^2\*b^3/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*e\*h+3/2\*f\*h/c^2\*b\*x/(c\*x^2+b\*x+a)^(1/2)-3/4\*f\*h/c^3\*b^4/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)-b^2/c/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*d\*h-b^2/c/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*e\*g-x/c/(c\*x^2+b\*x+a)^(1/2)\*f\*g+1/2/c^2\*b/(c\*x^2+b\*x+a)^(1/2)\*e\*h

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g + hx)(fx^2 + ex + d)}{(cx^2 + bx + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)`

[Out] `int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)(d + ex + fx^2)}{(a + bx + cx^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)`

[Out] `Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)`

$$3.232 \quad \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

**Rubi [A]** time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae - b\left(\frac{af}{c} + d\right)\right) - x(-2acf + b^2f - bce + 2c^2d)\right)}{c(b^2 - 4ac)\sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(c\*(2\*a\*e - b\*(d + (a\*f)/c)) - (2\*c^2\*d - b\*c\*e + b^2\*f - 2\*a\*c\*f)\*x))/(c\*(b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) + (f\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/c^(3/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}} dx &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} - \frac{2 \int -\frac{(b^2 - 4ac)f}{2c\sqrt{a + bx + cx^2}} dx}{b^2 - 4ac} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{f \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{c} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{(2f) \text{Subst} \left( \int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}} \right)}{c} \\ &= \frac{2 \left( c \left( 2ae - b \left( d + \frac{af}{c} \right) \right) - (2c^2d - bce + b^2f - 2acf) x \right)}{c(b^2 - 4ac) \sqrt{a + bx + cx^2}} + \frac{f \tanh^{-1} \left( \frac{b + 2cx}{2\sqrt{c} \sqrt{a + bx + cx^2}} \right)}{c^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.30, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}(abf - 2ac(e + fx) + b^2fx + bc(d - ex) + 2c^2dx)}{\sqrt{a + x(b + cx)}} - f(b^2 - 4ac) \log(2\sqrt{c} \sqrt{a + x(b + cx)} + b + 2cx)}{c^{3/2}(4ac - b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] ((2\*sqrt[c]\*(a\*b\*f + 2\*c^2\*d\*x + b^2\*f\*x + b\*c\*(d - e\*x) - 2\*a\*c\*(e + f\*x)))/sqrt[a + x\*(b + c\*x)] - (b^2 - 4\*a\*c)\*f\*Log[b + 2\*c\*x + 2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)]])/(c^(3/2)\*(-b^2 + 4\*a\*c))

**IntegrateAlgebraic [A]** time = 0.00, size = 111, normalized size = 1.00

$$\frac{2(abf - 2ace - 2acfx + b^2fx + bcd - bcex + 2c^2dx)}{c(4ac - b^2)\sqrt{a + bx + cx^2}} - \frac{f \log\left(-2c^{3/2}\sqrt{a + bx + cx^2} + bc + 2c^2x\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2), x]

[Out] (2\*(b\*c\*d - 2\*a\*c\*e + a\*b\*f + 2\*c^2\*d\*x - b\*c\*e\*x + b^2\*f\*x - 2\*a\*c\*f\*x))/(c\*(-b^2 + 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]) - (f\*Log[b\*c + 2\*c^2\*x - 2\*c^(3/2)\*Sqrt[a + b\*x + c\*x^2]])/c^(3/2)

**fricas [B]** time = 2.45, size = 429, normalized size = 3.86

$$\frac{\left(\frac{(b^2c - 4ac^2)f^2 + (b^2 - 4abc)fx + (ab^2 - 4a^2c^2)}{2(ab^2c - 4a^2c^2 + (b^2c - 4abc)^2)}\right)\sqrt{c} \log\left(\frac{-8c^2d - 8bcx - b^2 - 4\sqrt{a+bx+cx^2}(2cx + b)\sqrt{c} - 4a}{(b^2d - 2a^2c + abcf + (2c^2d - bc^2e + (b^2c - 2a^2c^2))\sqrt{a+bx+cx^2})}\right) - \frac{((b^2c - 4ac^2)f^2 + (b^2 - 4abc)fx + (ab^2 - 4a^2c^2))\sqrt{c} \arctan\left(\frac{\sqrt{a+bx+cx^2}}{2(ab^2c - 4a^2c^2 + (b^2c - 4abc)^2)}\right) + 2(b^2d - 2a^2c + abcf + (2c^2d - bc^2e + (b^2c - 2a^2c^2))\sqrt{a+bx+cx^2})}{ab^2c - 4a^2c^2 + (b^2c - 4ac^2)^2 + (b^2 - 4abc)^2}}{2(ab^2c - 4a^2c^2 + (b^2c - 4abc)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="fricas")

[Out] [1/2\*(((b^2\*c - 4\*a\*c^2)\*f\*x^2 + (b^3 - 4\*a\*b\*c)\*f\*x + (a\*b^2 - 4\*a^2\*c)\*f)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(b\*c^2\*d - 2\*a\*c^2\*e + a\*b\*c\*f + (2\*c^3\*d - b\*c^2\*e + (b^2\*c - 2\*a\*c^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x), -(((b^2\*c - 4\*a\*c^2)\*f\*x^2 + (b^3 - 4\*a\*b\*c)\*f\*x + (a\*b^2 - 4\*a^2\*c)\*f)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + 2\*(b\*c^2\*d - 2\*a\*c^2\*e + a\*b\*c\*f + (2\*c^3\*d - b\*c^2\*e + (b^2\*c - 2\*a\*c^2)\*f)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a\*b^2\*c^2 - 4\*a^2\*c^3 + (b^2\*c^3 - 4\*a\*c^4)\*x^2 + (b^3\*c^2 - 4\*a\*b\*c^3)\*x)]

**giac [A]** time = 0.27, size = 122, normalized size = 1.10

$$\frac{2\left(\frac{(2c^2d + b^2f - 2acf - bce)x}{b^2c - 4ac^2} + \frac{bcd + abf - 2ace}{b^2c - 4ac^2}\right)}{\sqrt{cx^2 + bx + a}} - \frac{f \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="giac")

[Out] -2\*((2\*c^2\*d + b^2\*f - 2\*a\*c\*f - b\*c\*e)\*x/(b^2\*c - 4\*a\*c^2) + (b\*c\*d + a\*b\*f - 2\*a\*c\*e)/(b^2\*c - 4\*a\*c^2))/sqrt(c\*x^2 + b\*x + a) - f\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(3/2)

**maple [B]** time = 0.01, size = 249, normalized size = 2.24

$$\frac{b^2 f x}{(4ac - b^2) \sqrt{cx^2 + bx + a} c} - \frac{2bex}{(4ac - b^2) \sqrt{cx^2 + bx + a}} + \frac{b^3 f}{2(4ac - b^2) \sqrt{cx^2 + bx + a} c^2} - \frac{b^2 e}{(4ac - b^2) \sqrt{cx^2 + bx + a} c} - \frac{fx}{\sqrt{cx^2 + bx + a} c} + \frac{2(2cx + b)d}{(4ac - b^2) \sqrt{cx^2 + bx + a}} + \frac{f \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{c^{\frac{3}{2}}} + \frac{bf}{2\sqrt{cx^2 + bx + a} c^2} - \frac{e}{\sqrt{cx^2 + bx + a} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x)

[Out]  $-f*x/c/(c*x^2+b*x+a)^{(1/2)} + 1/2*f/c^2*b/(c*x^2+b*x+a)^{(1/2)} + f/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x + 1/2*f/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} + f/c^{3/2} * \ln((c*x+1/2*b)/c^{1/2} + (c*x^2+b*x+a)^{(1/2)}) - e/c/(c*x^2+b*x+a)^{(1/2)} - 2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} * x - e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} + 2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 zero or nonzero?

**mupad [B]** time = 4.54, size = 143, normalized size = 1.29

$$\frac{f \ln\left(\frac{b}{2} + cx + \sqrt{cx^2 + bx + a}\right)}{c^{3/2}} - \frac{e(4a + 2bx)}{(4ac - b^2) \sqrt{cx^2 + bx + a}} + \frac{d\left(\frac{b}{2} + cx\right)}{\left(ac - \frac{b^2}{4}\right) \sqrt{cx^2 + bx + a}} + \frac{f\left(\frac{ab}{2} - x\left(ac - \frac{b^2}{2}\right)\right)}{c\left(ac - \frac{b^2}{4}\right) \sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/(a + b\*x + c\*x^2)^(3/2),x)

[Out]  $(f*\log((b/2 + c*x)/c^{1/2} + (a + b*x + c*x^2)^{(1/2)}))/c^{3/2} - (e*(4*a + 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^{(1/2)}) + (d*(b/2 + c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^{(1/2)}) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^{(1/2)})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

$$3.233 \quad \int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=225

$$\frac{2(-x(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ah^2-bgh+cg^2)}$$

**Rubi [A]** time = 0.27, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 12, 724, 206}

$$\frac{2(-x(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2dh)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ah^2-bgh+cg^2)} + \frac{(fg^2-h(eg-dh))\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{(ah^2-bgh+cg^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out] (2\*(b^2\*d\*h - b\*(c\*d\*g + a\*f\*g + a\*e\*h) + 2\*a\*(c\*e\*g - c\*d\*h + a\*f\*h) - (2\*c^2\*d\*g + b\*f\*(b\*g - a\*h) - c\*(b\*e\*g + 2\*a\*f\*g + b\*d\*h - 2\*a\*e\*h)\*x))/((b^2 - 4\*a\*c)\*(c\*g^2 - b\*g\*h + a\*h^2)\*Sqrt[a + b\*x + c\*x^2]) + ((f\*g^2 - h\*(e\*g - d\*h))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*Sqrt[c\*g^2 - b\*g\*h + a\*h^2]\*Sqrt[a + b\*x + c\*x^2])])/(c\*g^2 - b\*g\*h + a\*h^2)^(3/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d + ex + fx^2}{(g + hx)(a + bx + cx^2)^{3/2}} dx = \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - (b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}$$

$$= \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - (b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}$$

$$= \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - (b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}$$

$$= \frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - (b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}{(b^2 - 4ac)(cg^2 - bgh + ah^2)\sqrt{a + bx + cx^2}}$$

**Mathematica [A]** time = 0.49, size = 271, normalized size = 1.20

$$\frac{-b^2(afh^2 + 2cdh^2 + cfg(g - 2hx)) - 2bch(-aeh + af(g + hx) + c(-dg + dlx + egx)) + 4c^2(ah(dlh - eg + elx) + afg(g - lx) + cdghx) + b^3fgh}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(h(bg - ah) - cg^2)} - \frac{ch(h(dlh - eg) + fg^2) \tanh^{-1}\left(\frac{2ah - bg + bhx - 2cgx}{2\sqrt{a + x(b + cx)}\sqrt{h(ah - bg) + cg^2}}\right)}{(h(ah - bg) + cg^2)^{3/2}} - \frac{f}{\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out] (-f/Sqrt[a + x\*(b + c\*x)]) + (b^3\*f\*g\*h - b^2\*(2\*c\*d\*h^2 + a\*f\*h^2 + c\*f\*g\*(g - 2\*h\*x)) - 2\*b\*c\*h\*(-(a\*e\*h) + a\*f\*(g + h\*x) + c\*(-(d\*g) + e\*g\*x + d\*h



$x)) + 4*c^2*(c*d*g*h*x + a*f*g*(g - h*x) + a*h*(-(e*g) + d*h + e*h*x))/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)] - (c*h*(f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2))/(c*h)$

**IntegrateAlgebraic [A]** time = 1.22, size = 293, normalized size = 1.30

$$2 \tan^{-1} \left( \frac{-h\sqrt{b+bx+cx^2} + \sqrt{c}g + \sqrt{c}hx}{\sqrt{-a^2+bgh-cg^2}} \right) \frac{(dl^2\sqrt{-al^2+bgh-cg^2} - egh\sqrt{-al^2+bgh-cg^2} + fg^2\sqrt{-al^2+bgh-cg^2})}{(al^2-bgh+cg^2)^2} - \frac{2(2a^2fh - abeh - abfg + abfhx - 2acd - 2aceg - 2acehx + 2acfgx + b^2dh - b^2fgx - bcdg + bcdhx + bcegx - 2c^2dgx)}{(b^2-4ac)\sqrt{a+bx+cx^2}(-al^2+bgh-cg^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(-2*(-(b*c*d*g) + 2*a*c*e*g - a*b*f*g + b^2*d*h - 2*a*c*d*h - a*b*e*h + 2*a^2*f*h - 2*c^2*d*g*x + b*c*e*g*x - b^2*f*g*x + 2*a*c*f*g*x + b*c*d*h*x - 2*a*c*e*h*x + a*b*f*h*x))/((b^2 - 4*a*c)*(-(c*g^2) + b*g*h - a*h^2)*Sqrt[a + b*x + c*x^2]) + (2*(f*g^2*Sqrt[-(c*g^2) + b*g*h - a*h^2] - e*g*h*Sqrt[-(c*g^2) + b*g*h - a*h^2] + d*h^2*Sqrt[-(c*g^2) + b*g*h - a*h^2])*ArcTan[(Sqrt[c]*g + Sqrt[c]*h*x - h*Sqrt[a + b*x + c*x^2])/Sqrt[-(c*g^2) + b*g*h - a*h^2]])/(c*g^2 - b*g*h + a*h^2)^2$

**fricas [B]** time = 53.93, size = 1905, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+b\*x+a)^(3/2), x, algorithm="fricas")

[Out]  $[1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 - 4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c - a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 + ((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 + b*x + a)]/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*$

$$\begin{aligned}
& b^2c^2)g^2h^3 + (a^2b^2c - 4a^3c^2)h^4)x^2 + ((b^3c^2 - 4a^2b^2c^2)g^2h^3 + (b^5 - 2a^2b^3c - 8a^2b^2c^2)g^2h^2 \\
& - 2(a^2b^4 - 4a^2b^2c^2)g^2h^3 + (a^2b^3 - 4a^3b^2c)h^4)x, ((a^2b^2 - 4a^2c^2)fg^2 - (a^2b^2 - 4a^2c^2)eg^2h + (a^2b^2 - 4a^2c^2)d^2h^2 + ((b^2 \\
& 2c - 4a^2c^2)fg^2 - (b^2c - 4a^2c^2)eg^2h + (b^2c - 4a^2c^2)d^2h^2)x^2 + ((b^3 - 4a^2b^2c)fg^2 - (b^3 - 4a^2b^2c)eg^2h + (b^3 - 4a^2b^2c)d^2h^2 \\
& )x) \sqrt{-c^2g^2 + b^2g^2h - a^2h^2} \arctan(-1/2 \sqrt{-c^2g^2 + b^2g^2h - a^2h^2} \sqrt{cx^2 + bx + a} (b^2g^2 - 2a^2h + (2c^2g^2 - b^2h)x) / (a^2c^2g^2 - a^2b^2g^2h + \\
& a^2h^2 + (c^2g^2 - b^2c^2g^2h + a^2c^2h^2)x^2 + (b^2c^2g^2 - b^2c^2g^2h + a^2b^2h^2) \\
& x)) - 2((b^2c^2d - 2a^2c^2e + a^2b^2c^2f)g^3 + (3a^2b^2c^2e - 2(b^2c^2 - a^2c^2) \\
& ^2)d - (a^2b^2 + 2a^2c^2)fg^2h + (3a^2b^2c^2f + (b^3 - a^2b^2c)d - (a^2b^2 \\
& + 2a^2c^2)e)g^2h^2 + (a^2b^2e - 2a^2c^2f - (a^2b^2 - 2a^2c^2)d)h^3 + ((2c^2 \\
& ^3d - b^2c^2e + (b^2c^2 - 2a^2c^2)f)g^3 - (3b^2c^2d - (b^2c^2 + 2a^2c^2) \\
& e + (b^3 - a^2b^2c)f)g^2h - (3a^2b^2c^2e - (b^2c^2 + 2a^2c^2)d - 2(a^2b^2 - \\
& a^2c^2)f)g^2h^2 - (a^2b^2c^2d - 2a^2c^2e + a^2b^2f)h^3)x) \sqrt{cx^2 + bx + a} / ((a^2b^2c^2 - 4a^2c^2)g^4 - 2(a^2b^3c - 4a^2b^2c^2)g^3h + (a^2 \\
& ^4 - 2a^2b^2c - 8a^3c^2)g^2h^2 - 2(a^2b^3 - 4a^3b^2c)g^2h^3 + (a^2 \\
& 3b^2 - 4a^4c)h^4 + ((b^2c^3 - 4a^2c^4)g^4 - 2(b^3c^2 - 4a^2b^2c^2)g^3h + (b^4c - 2a^2b^2c^2 - 8a^2c^3)g^2h^2 - 2(a^2b^3c - 4a^2b^2c^2) \\
& )g^2h^3 + (a^2b^2c - 4a^3c^2)h^4)x^2 + ((b^3c^2 - 4a^2b^2c^2)g^4 - 2 \\
& (b^4c - 4a^2b^2c^2)g^3h + (b^5 - 2a^2b^3c - 8a^2b^2c^2)g^2h^2 - 2 \\
& (a^2b^4 - 4a^2b^2c)g^2h^3 + (a^2b^3 - 4a^3b^2c)h^4)x]
\end{aligned}$$

**giac [B]** time = 0.29, size = 719, normalized size = 3.20

$$\frac{2 \left( \frac{(2c^2g^2 - b^2c^2g^2h + a^2c^2h^2) \sqrt{cx^2 + bx + a} \arctan\left(\frac{\sqrt{cx^2 + bx + a} (b^2g^2 - 2a^2h + (2c^2g^2 - b^2h)x)}{\sqrt{-c^2g^2 + b^2g^2h - a^2h^2}}\right) + 2(fg^2 + d^2h^2 - g^2h^2) \sqrt{-c^2g^2 + b^2g^2h - a^2h^2}}{\sqrt{cx^2 + bx + a}} \right)}{\sqrt{cx^2 + bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out]  $-2((2c^3d^2g^3 + b^2c^2f^2g^3 - 2a^2c^2f^2g^3 - 3b^2c^2d^2g^2h - b^3f^2g^2h^2 + a^2b^2c^2f^2g^2h + b^2c^2d^2g^2h^2 + 2a^2c^2d^2g^2h^2 + 2a^2b^2f^2g^2h^2 - 2a^2c^2f^2g^2h^2 - a^2b^2c^2d^2h^3 - a^2b^2f^2h^3 - b^2c^2g^3e + b^2c^2g^2h^2e + 2a^2c^2g^2h^2e - 3a^2b^2c^2g^2h^2e + 2a^2c^2h^3e)x / (b^2c^2g^4 - 4a^2c^3g^4 - 2b^3c^2g^3h + 8a^2b^2c^2g^3h + b^4g^2h^2 - 2a^2b^2c^2g^2h^2 - 8a^2c^2g^2h^2 - 2a^2b^3g^2h^3 + 8a^2b^2c^2g^2h^3 + a^2b^2h^4 - 4a^3c^2h^4) + (b^2c^2d^2g^3 + a^2b^2c^2f^2g^3 - 2b^2c^2d^2g^2h + 2a^2c^2d^2g^2h - a^2b^2f^2g^2h - 2a^2c^2f^2g^2h + b^3d^2g^2h^2 - a^2b^2c^2d^2g^2h + 3a^2b^2f^2g^2h^2 - a^2b^2d^2h^3 + 2a^2c^2d^2h^3 - 2a^2c^2f^2h^3 - 2a^2c^2g^3e + 3a^2b^2c^2g^2h^2e - a^2b^2g^2h^2e - 2a^2c^2g^2h^2e + a^2b^2h^3e) / (b^2c^2g^4 - 4a^2c^3g^4 - 2b^3c^2g^3h + 8a^2b^2c^2g^3h + b^4g^2h^2 - 2a^2b^2c^2g^2h^2 - 8a^2c^2g^2h^2 - 2a^2b^3g^2h^3 + 8a^2b^2c^2g^2h^3 + a^2b^2h^4 - 4a^3c^2h^4)) / \sqrt{cx^2 + bx + a} + 2(fg^2 + d^2h^2 - g^2h^2) \arctan(-(\sqrt{cx^2 + bx + a})h + \sqrt{c}g) / \sqrt{-c^2g^2 + b^2g^2h - a^2h^2} / ((c^2g^2 - b^2g^2h + a^2h^2) \sqrt{-c^2g^2 + b^2g^2h - a^2h^2})$

**maple [B]** time = 0.01, size = 2079, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out] 
$$\begin{aligned} & -4/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a* \\ & h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*x*c^2*g^2*e+4/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/ \\ & ((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*x*c^2* \\ & g^3*f-2*h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/ \\ & h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*x*b*c*d+2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/ \\ & ((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*x*b*c*e*g \\ & -2/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a* \\ & h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*b*c*g^2*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2) \\ & /((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*b*c*g^3*f \\ & -1/h*f/c/(c*x^2+b*x+a)^{(1/2)}+h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g) \\ & )*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*d-1/(a*h^2-b*g*h+c*g^2)/((x+g/h) \\ & ^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*e*g-h/(a*h^2-b*g* \\ & h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2) \\ & /h^2)^{(1/2)}*b^2*d+1/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g) \\ & )*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*b^2*e*g-1/h/(a*h^2-b*g*h+c*g^2)/ \\ & ((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c* \\ & g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h) \\ & /h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+g/h))*f*g^2-2/h*f*b/(4*a*c-b^2)/(c*x^ \\ & 2+b*x+a)^{(1/2)}*x-1/h*f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}+4/h*e/(4*a*c-b \\ & ^2)/(c*x^2+b*x+a)^{(1/2)}*x*c-2/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*b-2/h \\ & /((a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2- \\ & b*g*h+c*g^2)/h^2)^{(1/2)}*x*b*c*f*g^2+2/h*e/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*b \\ & +1/h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c* \\ & g^2)/h^2)^{(1/2)}*f*g^2-h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)} \\ & *\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2) \\ & /h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}) \\ & /((x+g/h))*d+1/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b \\ & *h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)} \\ & *((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+ \\ & g/h))*e*g+2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h) \\ & )/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*b*c*g*d-4/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x \\ & +a)^{(1/2)}*x*c+4/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x \\ & +g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*x*c^2*g*d-1/h/(a*h^2-b*g*h+c*g^2)/(4 \\ & *a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)} \\ & *b^2*f*g^2 \end{aligned}$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume((b/h-(2\*c\*g)/h^2)^2>0)', see `assume?` for more details)Is (b/h-(2\*c\*g)/h^2)^2 - (4\*c \*((-b\*g)/h + (c\*g^2)/h^2+a)) /h^2 zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x) (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)),x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)\*(a + b\*x + c\*x^2)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2}{(g + h x) (a + b x + c x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/((g + h\*x)\*(a + b\*x + c\*x\*\*2)\*\*(3/2)), x)

$$3.234 \quad \int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}} dx$$

Optimal. Leaf size=421

$$\frac{2(c x(2 a^2 f h^2-c(2 a(d h^2-2 e g h+f g^2)+b g(2 d h+e g))-a b h(e h+2 f g)+b^2(d h^2+f g^2)+2 c^2 d g^2)+b(a^2-(b^2-4 a c) \sqrt{a+b x+c x^2})}{(b^2-4 a c) \sqrt{a+b x+c x^2}}$$

**Rubi [A]** time = 0.80, antiderivative size = 418, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 806, 724, 206}

$$\frac{2(c x(2 a^2 f h^2-c(2 a(d h^2-2 e g h+f g^2)+b g(2 d h+e g))-a b h(e h+2 f g)+b^2(d h^2+f g^2)+2 c^2 d g^2)+b(a^2-(b^2-4 a c) \sqrt{a+b x+c x^2})}{(b^2-4 a c) \sqrt{a+b x+c x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2)) + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2) - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x)/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^2*sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d*h))*sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*(f*g^3 - g*h*(2*e*g - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*g - e*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*sqrt[c*g^2 - b*g*h + a*h^2]*sqrt[a + b*x + c*x^2]])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))$

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

### Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\int \frac{d + ex + fx^2}{(g + hx)^2 (a + bx + cx^2)^{3/2}} dx = -\frac{2(b^3 dh^2 - b^2 h(2cdg + aeh) - 2ac(cg(eg - 2dh) + ah(2fg - eh)) + b(c^2 dg^2 + \dots)}{(g + hx)^2 (a + bx + cx^2)^{3/2}}$$

$$= -\frac{2(b^3 dh^2 - b^2 h(2cdg + aeh) - 2ac(cg(eg - 2dh) + ah(2fg - eh)) + b(c^2 dg^2 + \dots)}{(g + hx)^2 (a + bx + cx^2)^{3/2}}$$

$$= -\frac{2(b^3 dh^2 - b^2 h(2cdg + aeh) - 2ac(cg(eg - 2dh) + ah(2fg - eh)) + b(c^2 dg^2 + \dots)}{(g + hx)^2 (a + bx + cx^2)^{3/2}}$$

$$= -\frac{2(b^3 dh^2 - b^2 h(2cdg + aeh) - 2ac(cg(eg - 2dh) + ah(2fg - eh)) + b(c^2 dg^2 + \dots)}{(g + hx)^2 (a + bx + cx^2)^{3/2}}$$



$$\begin{aligned}
& f*g^3*x - 18*b*d*g^2*h*x + 20*a*e*g^2*h*x - 20*a*d*g*h^2*x - 20*b*f*g^3*x^2 \\
& + 18*b*e*g^2*h*x^2 + 16*a*f*g^2*h*x^2 - 24*b*d*g*h^2*x^2 - 4*a*e*g*h^2*x^2 \\
& - 4*b*f*g^2*h*x^3 - 4*b*e*g*h^2*x^3 + 16*a*f*g*h^2*x^3 + 12*b*d*h^3*x^3 - \\
& 8*a*e*h^3*x^3) + c^{(3/2)}*(-2*a*b*e*g^3 + 8*a^2*f*g^3 + 12*a*b*d*g^2*h - 12* \\
& a^2*e*g^2*h + 8*a^2*d*g*h^2 - 2*b^2*e*g^3*x + 24*a*b*f*g^3*x + 12*b^2*d*g^2 \\
& *h*x - 22*a*b*e*g^2*h*x - 16*a^2*f*g^2*h*x + 20*a*b*d*g*h^2*x + 4*a^2*e*g*h \\
& ^2*x + 19*b^2*f*g^3*x^2 - 11*b^2*e*g^2*h*x^2 - 20*a*b*f*g^2*h*x^2 + 7*b^2*d \\
& *g*h^2*x^2 + 18*a*b*e*g*h^2*x^2 - 20*a^2*f*g*h^2*x^2 - 16*a*b*d*h^3*x^2 + 8 \\
& *a^2*e*h^3*x^2 + 3*b^2*f*g^2*h*x^3 + 7*b^2*e*g*h^2*x^3 - 20*a*b*f*g*h^2*x^3 \\
& - 21*b^2*d*h^3*x^3 + 14*a*b*e*h^3*x^3 - 4*a^2*f*h^3*x^3) + c^{(7/2)}*(-4*d*g \\
& ^3*x^2 - 8*e*g^3*x^3 + 12*d*g^2*h*x^3 + 8*f*g^3*x^4 - 16*e*g^2*h*x^4 + 24*d \\
& *g*h^2*x^4) + c^{(5/2)}*(-4*a*d*g^3 - 4*b*d*g^3*x - 8*a*e*g^3*x + 12*a*d*g^2* \\
& h*x - 10*b*e*g^3*x^2 + 16*a*f*g^3*x^2 + 24*b*d*g^2*h*x^2 - 28*a*e*g^2*h*x^2 \\
& + 32*a*d*g*h^2*x^2 + 24*b*f*g^3*x^3 - 26*b*e*g^2*h*x^3 - 16*a*f*g^2*h*x^3 \\
& + 36*b*d*g*h^2*x^3 + 4*a*e*g*h^2*x^3 + 4*b*f*g^2*h*x^4 + 4*b*e*g*h^2*x^4 - \\
& 16*a*f*g*h^2*x^4 - 12*b*d*h^3*x^4 + 8*a*e*h^3*x^4)) / (-8*c^5*g^4*x^4*(g + h \\
& x) + (g + h*x)*(-(a*b^4*g^2*h^2) + 2*a^2*b^3*g*h^3 - a^3*b^2*h^4 - b^5*g^2* \\
& h^2*x + 2*a*b^4*g*h^3*x - a^2*b^3*h^4*x) + 8*c^{(9/2)}*g^4*x^3*(g + h*x)*\text{Sqrt} \\
& [a + b*x + c*x^2] + \text{Sqrt}[c]*(g + h*x)*(4*a*b^3*g^2*h^2 - 8*a^2*b^2*g*h^3 + \\
& 4*a^3*b*h^4 + 4*b^4*g^2*h^2*x - 8*a*b^3*g*h^3*x + 4*a^2*b^2*h^4*x)*\text{Sqrt}[a + \\
& b*x + c*x^2] + c^{(3/2)}*(g + h*x)*\text{Sqrt}[a + b*x + c*x^2]*(-8*a*b^2*g^3*h + 8 \\
& *a^2*b*g^2*h^2 - 8*b^3*g^3*h*x + 16*a*b^2*g^2*h^2*x - 16*a^2*b*g*h^3*x + 8* \\
& a^3*h^4*x + 12*b^3*g^2*h^2*x^2 - 24*a*b^2*g*h^3*x^2 + 12*a^2*b*h^4*x^2) + c \\
& *(g + h*x)*(2*a*b^3*g^3*h - 6*a^2*b^2*g^2*h^2 + 8*a^3*b*g*h^3 - 4*a^4*h^4 + \\
& 2*b^4*g^3*h*x - 14*a*b^3*g^2*h^2*x + 24*a^2*b^2*g*h^3*x - 12*a^3*b*h^4*x - \\
& 9*b^4*g^2*h^2*x^2 + 18*a*b^3*g*h^3*x^2 - 9*a^2*b^2*h^4*x^2) + c^{(7/2)}*(g + \\
& h*x)*\text{Sqrt}[a + b*x + c*x^2]*(8*a*g^4*x + 12*b*g^4*x^2 - 16*b*g^3*h*x^3 + 16 \\
& *a*g^2*h^2*x^3) + c^{(5/2)}*(g + h*x)*\text{Sqrt}[a + b*x + c*x^2]*(4*a*b*g^4 + 4*b^ \\
& 2*g^4*x - 16*a*b*g^3*h*x + 16*a^2*g^2*h^2*x - 24*b^2*g^3*h*x^2 + 24*a*b*g^2 \\
& *h^2*x^2 + 8*b^2*g^2*h^2*x^3 - 16*a*b*g*h^3*x^3 + 8*a^2*h^4*x^3) + c^2*(g + \\
& h*x)*(-(a*b^2*g^4) + 8*a^2*b*g^3*h - 8*a^3*g^2*h^2 - b^3*g^4*x + 24*a*b^2* \\
& g^3*h*x - 24*a^2*b*g^2*h^2*x + 18*b^3*g^3*h*x^2 - 30*a*b^2*g^2*h^2*x^2 + 24 \\
& *a^2*b*g*h^3*x^2 - 12*a^3*h^4*x^2 - 16*b^3*g^2*h^2*x^3 + 32*a*b^2*g*h^3*x^3 \\
& - 16*a^2*b*h^4*x^3) + c^4*(g + h*x)*(-12*a*g^4*x^2 - 16*b*g^4*x^3 + 16*b*g \\
& ^3*h*x^4 - 16*a*g^2*h^2*x^4) + c^3*(g + h*x)*(-4*a^2*g^4 - 12*a*b*g^4*x - 9 \\
& *b^2*g^4*x^2 + 24*a*b*g^3*h*x^2 - 24*a^2*g^2*h^2*x^2 + 32*b^2*g^3*h*x^3 - 3 \\
& 2*a*b*g^2*h^2*x^3 - 8*b^2*g^2*h^2*x^4 + 16*a*b*g*h^3*x^4 - 8*a^2*h^4*x^4)) \\
& + (3*b*f*g^2*h*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2] \\
& )/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]) / (\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - \\
& b*g*h + a*h^2)^2) - (3*b*e*g*h^2*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqr} \\
& \text{rt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]) / (\text{Sqrt}[-(c*g^2) + b*g* \\
& h - a*h^2]*(c*g^2 - b*g*h + a*h^2)^2) - (6*a*f*g*h^2*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \\
& \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]) / (S \\
& \text{qrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2)^2) + (3*b*d*h^3*\text{ArcTa} \\
& \text{n}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b
\end{aligned}$$



$$\frac{g*h - a*h^2]}{(\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2)^2) + (6*a*e*h^3*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2])]/(\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2)^2) - (6*a*d*h^4*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2])]/(g*\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2)^2) + (2*f*g*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2])]/(\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2)) - (4*e*h*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2])]/(\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2)) + (6*d*h^2*\text{ArcTan}[(-\text{Sqrt}[c]*g) - \text{Sqrt}[c]*h*x + h*\text{Sqrt}[a + b*x + c*x^2])/\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2])]/(g*\text{Sqrt}[-(c*g^2) + b*g*h - a*h^2]*(c*g^2 - b*g*h + a*h^2))$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] Timed out

**maple** [B] time = 0.02, size = 4930, normalized size = 11.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+b\*x+a)^(3/2),x)

[Out] 
$$-1/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*d-1/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*\ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)})/(x+g/h)*e+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^{(1/2)}*$$



$$\frac{a^2h^2 - b^2gh + c^2g^2}{(4ac - b^2)} \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 f g^2 + 6/h^2 c / (a^2h^2 - b^2gh + c^2g^2) / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot b^2 e^2 g - 8/h^2 c / (a^2h^2 - b^2gh + c^2g^2) / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot b^2 f g^2 + 12/h^2 c^2 / (a^2h^2 - b^2gh + c^2g^2) / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 e^2 g - 6/h / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 b^2 c^2 d + 12 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 b^2 c^2 g^2 e + 3/h^2 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 b^2 c^2 d + 12 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 b^2 c^2 g^2 e + 3 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 b^2 c^2 f g^2 - 3/2 h / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot b^3 e^2 g - 3/2 h / (a^2h^2 - b^2gh + c^2g^2)^2 / (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot \ln \left( \left( \frac{b^2h - 2c^2g}{h} \right) \frac{x+g}{h} + 2 \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} + 2 \left( \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} \right)^{1/2} \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \right) / (x+g/h) \cdot b^2 e^2 g + 12 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 c^3 g^2 d + 6 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot b^2 c^2 g^2 d + 6 / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot b^2 c^2 g^2 e - 3/h / (a^2h^2 - b^2gh + c^2g^2)^2 / \left( \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} \right)^{1/2} \cdot \ln \left( \left( \frac{b^2h - 2c^2g}{h} \right) \frac{x+g}{h} + 2 \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} + 2 \left( \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} \right)^{1/2} \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \right) / (x+g/h) \cdot c^2 g^2 d - 3/h / (a^2h^2 - b^2gh + c^2g^2)^2 / \left( \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} \right)^{1/2} \cdot \ln \left( \left( \frac{b^2h - 2c^2g}{h} \right) \frac{x+g}{h} + 2 \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} + 2 \left( \frac{a^2h^2 - b^2gh + c^2g^2}{h^2} \right)^{1/2} \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \right) / (x+g/h) \cdot c^2 g^3 f - 2 / (a^2h^2 - b^2gh + c^2g^2) / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 b^2 c^2 e + 2/h / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot b^2 f g - 12/h / (a^2h^2 - b^2gh + c^2g^2)^2 / (4ac - b^2) \left/ \left( \frac{x+g}{h} \right)^2 c + (b^2h - 2c^2g) \frac{x+g}{h} + (a^2h^2 - b^2gh + c^2g^2) / h^2 \right)^{1/2} \cdot x^2 c^3 g^3 e$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^2/(c\*x^2+b\*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume((b/h-(2\*c\*g)/h^2)^2>0)', see `assume`)

e?` for more details) Is  $(b/h - (2*c*g)/h^2)^2 - (4*c^2*(g^2/h^2 + a))$  zero or nonzero?  $*((-b*g)/h)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^2 (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2)), x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)^2\*(a + b\*x + c\*x^2)^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(3/2), x)

[Out] Timed out

$$3.235 \quad \int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=713

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2(8a^2fh^2+4abh(2fg-3eh)-(b^2(3h(eg-5dh)+fg^2)))\right)-4ch\left(ah(3dh^2-8(a^2-bgh+cg^2)^{7/2}\right)}{8(a^2-bgh+cg^2)^{7/2}}$$

**Rubi [A]** time = 2.67, antiderivative size = 707, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1646, 1650, 806, 724, 206}

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out] (2\*(b^4\*d\*h^3 - b^3\*h^2\*(3\*c\*d\*g + a\*e\*h) + b^2\*h\*(3\*c^2\*d\*g^2 + a^2\*f\*h^2 + a\*c\*h\*(3\*e\*g - 4\*d\*h)) - b\*c\*(c^2\*d\*g^3 + 3\*a^2\*h^2\*(f\*g - e\*h) + a\*c\*g\*(f\*g^2 + 3\*e\*g\*h - 9\*d\*h^2)) - 2\*a\*c\*(a^2\*f\*h^3 - c^2\*g^2\*(e\*g - 3\*d\*h) - a\*c\*h\*(3\*f\*g^2 - 3\*e\*g\*h + d\*h^2)) - c\*(2\*c^3\*d\*g^3 - b\*(b^2\*d - a\*b\*e + a^2\*f)\*h^3 - c^2\*g\*(2\*a\*f\*g^2 - 6\*a\*h\*(e\*g - d\*h) + b\*g\*(e\*g + 3\*d\*h)) + c\*(2\*a^2\*h^2\*(3\*f\*g - e\*h) + b^2\*(f\*g^3 + 3\*d\*g\*h^2) - 3\*a\*b\*h\*(f\*g^2 + h\*(e\*g - d\*h))))\*x)/((b^2 - 4\*a\*c)\*(c\*g^2 - b\*g\*h + a\*h^2)^3\*sqrt[a + b\*x + c\*x^2]) - (h\*(f\*g^2 - h\*(e\*g - d\*h))\*sqrt[a + b\*x + c\*x^2])/(2\*(c\*g^2 - b\*g\*h + a\*h^2)^2\*(g + h\*x)^2) - (h\*(6\*c\*f\*g^3 - 2\*c\*g\*h\*(5\*e\*g - 7\*d\*h) - 4\*a\*h^2\*(2\*f\*g - e\*h) + b\*h\*(f\*g^2 + h\*(3\*e\*g - 7\*d\*h)))\*sqrt[a + b\*x + c\*x^2])/(4\*(c\*g^2 - b\*g\*h + a\*h^2)^3\*(g + h\*x)) + ((8\*c^2\*g^2\*(f\*g^2 - 3\*e\*g\*h + 6\*d\*h^2) + 4\*c\*h\*(2\*b\*f\*g^3 + 3\*b\*g\*h\*(e\*g - 4\*d\*h) - a\*h\*(11\*f\*g^2 - 9\*e\*g\*h + 3\*d\*h^2)) + h^2\*(8\*a^2\*f\*h^2 + 4\*a\*b\*h\*(2\*f\*g - 3\*e\*h) - b^2\*(f\*g^2 + 3\*h\*(e\*g - 5\*d\*h))))\*ArcTanh[(b\*g - 2\*a\*h + (2\*c\*g - b\*h)\*x)/(2\*sqrt[c\*g^2 - b\*g\*h + a\*h^2])\*sqrt[a + b\*x + c\*x^2]]/(8\*(c\*g^2 - b\*g\*h + a\*h^2)^(7/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 724**

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$$\frac{a^2e - b^2d - (2cd - b^2e)x}{\sqrt{ax^2 + bx + c}}$$
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && NeQ[2cd - b^2e, 0]

### Rule 806

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4ac, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4ac, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2}{(g + hx)^3 (a + bx + cx^2)^{3/2}} dx &= \frac{2(b^4 dh^3 - b^3 h^2(3cdg + aeh) + b^2 h(3c^2 dg^2 + a^2 fh^2 + ach(3eg - 4dh)) - bc}{(g + hx)^3 (a + bx + cx^2)^{3/2}} \\
&= \frac{2(b^4 dh^3 - b^3 h^2(3cdg + aeh) + b^2 h(3c^2 dg^2 + a^2 fh^2 + ach(3eg - 4dh)) - bc}{(g + hx)^3 (a + bx + cx^2)^{3/2}} \\
&= \frac{2(b^4 dh^3 - b^3 h^2(3cdg + aeh) + b^2 h(3c^2 dg^2 + a^2 fh^2 + ach(3eg - 4dh)) - bc}{(g + hx)^3 (a + bx + cx^2)^{3/2}} \\
&= \frac{2(b^4 dh^3 - b^3 h^2(3cdg + aeh) + b^2 h(3c^2 dg^2 + a^2 fh^2 + ach(3eg - 4dh)) - bc}{(g + hx)^3 (a + bx + cx^2)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 5.29, size = 762, normalized size = 1.07



Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)),x]

[Out] 
$$\begin{aligned}
& \left( -\frac{f}{(g + hx)^2 \sqrt{a + x(b + cx)}} \right) + \frac{b^3 f g h - b^2 (6 c d h^2 + a f h^2 + c f g (g - 6 h x)) + 2 b^2 c h (3 a e h + a f (g - 3 h x) - 3 c (-d g + e g x + d h x)) + 4 c (-2 a^2 f h^2 + 3 c^2 d g h x + a c (f g (g - 3 h x) + 3 h (-e g + d h + e h x)))}{(b^2 - 4 a c) (-c g^2 + h (b g - a h)) (g + h x)^2 \sqrt{a + x(b + c x)}} \\
& + \frac{3 c h (-4 h (8 c^2 d g^2 + 8 a^2 f h^2 - 4 a b h (2 f g + e h) - 4 c (3 a f g^2 + b g (e g + 2 d h) + a h (-5 e g + 3 d h)) + b^2 (5 f g^2 + h (-e g + 5 d h))) \sqrt{a + x(b + c x)}}{(g + h x)^2 - (2 h (16 c^3 d g^3 - 8 c^2 g (5 a f g^2 + b g (e g + 3 d h) + a h (-11 e g + 13 d h)) + b h (-8 a^2 f h^2 + 4 a b h (-2 f g + 3 e h) + b^2 (f g^2 + 3 h (e g - 5 d h))) + 2 c (8 a^2 h^2 (5 f g - 2 e h) + 2 a b h (-7 f g^2 + h (-9 e g + 13 d h)) + b^2 (7 f g^3 + g h (-5 e g + 19 d h))) \sqrt{a + x(b + c x)}}{(c g^2 + h (-b g + a h)) (g + h x) + ((b^2 -
\end{aligned}$$

$$4*a*c)*(-8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2) - 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) + a*h*(-11*f*g^2 + 9*e*g*h - 3*d*h^2)) + h^2*(-8*a^2*f*h^2 + 4*a*b*h*(-2*f*g + 3*e*h) + b^2*(f*g^2 + 3*h*(e*g - 5*d*h))))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*sqrt[c*g^2 + h*(-(b*g) + a*h)]*sqrt[a + x*(b + c*x)])]/(c*g^2 + h*(-(b*g) + a*h))^(3/2))/(8*(b^2 - 4*a*c)*(c*g^2 + h*(-(b*g) + a*h))^2)/(3*c*h)$$

**IntegrateAlgebraic [F]** time = 180.04, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out] \$Aborted

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+b\*x+a)^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac [B]** time = 0.88, size = 5637, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)^3/(c\*x^2+b\*x+a)^(3/2),x, algorithm="giac")

[Out] 
$$-2*((2*c^7*d*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d*g^8*h - 3*b^3*c^4*f*g^8*h + 3*a*b*c^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 + 3*b^4*c^3*f*g^7*h^2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3 - b^5*c^2*f*g^6*h^3 - 13*a*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c^3*d*g^5*h^4 + 6*a*b^2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2*c^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*d*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*g^3*h^6 - 16*a^3*c^4*d*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7 + 24*a^3*b*c^3*d*g^2*h^7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 +$$



$$\begin{aligned}
& 3a^2b^4c^d * g^h^8 - 6a^3b^2c^2 * d * g^h^8 - 6a^4c^3 * d * g^h^8 + 3a^4b^2 \\
& 2c^f * g^h^8 + 6a^5c^2 * f * g^h^8 - a^3b^3c^d * h^9 + 3a^4b^2c^d * h^9 - a^5 \\
& * b^c * f * h^9 - b^c^6 * g^9 * e + 3b^2c^5 * g^8 * h * e + 6a * c^6 * g^8 * h * e - 3b^3c^4 * \\
& g^7 * h^2 * e - 24a * b * c^5 * g^7 * h^2 * e + b^4c^3 * g^6 * h^3 * e + 34a * b^2c^4 * g^6 * h^3 \\
& * e + 16a^2c^5 * g^6 * h^3 * e - 21a * b^3c^3 * g^5 * h^4 * e - 42a^2b^2c^4 * g^5 * h^4 * e \\
& + 6a * b^4c^2 * g^4 * h^5 * e + 36a^2b^2c^3 * g^4 * h^5 * e + 12a^3c^4 * g^4 * h^5 * e \\
& - a * b^5c * g^3 * h^6 * e - 13a^2b^3c^2 * g^3 * h^6 * e - 16a^3b^2c^3 * g^3 * h^6 * e + 3 \\
& * a^2b^4c * g^2 * h^7 * e + 6a^3b^2c^2 * g^2 * h^7 * e - 3a^3b^3c * g^h^8 * e + 3a^4 \\
& * b^2c^2 * g^h^8 * e + a^4b^2c^h^9 * e - 2a^5c^2 * h^9 * e) * x / (b^2c^6 * g^12 - 4a * a^ \\
& c^7 * g^12 - 6b^3c^5 * g^11 * h + 24a * a * b * c^6 * g^11 * h + 15b^4c^4 * g^10 * h^2 - 54a \\
& * a * b^2c^5 * g^10 * h^2 - 24a^2c^6 * g^10 * h^2 - 20b^5c^3 * g^9 * h^3 + 50a * a * b^3c^ \\
& 4 * g^9 * h^3 + 120a^2b^2c^5 * g^9 * h^3 + 15b^6c^2 * g^8 * h^4 - 225a^2b^2c^4 * g^ \\
& 8 * h^4 - 60a^3c^5 * g^8 * h^4 - 6b^7c * g^7 * h^5 - 36a * b^5c^2 * g^7 * h^5 + 180a^ \\
& ^2b^3c^3 * g^7 * h^5 + 240a^3b^2c^4 * g^7 * h^5 + b^8 * g^6 * h^6 + 26a * a * b^6c * g^6 * h \\
& ^6 - 30a^2b^4c^2 * g^6 * h^6 - 340a^3b^2c^3 * g^6 * h^6 - 80a^4c^4 * g^6 * h^6 \\
& - 6a * b^7 * g^5 * h^7 - 36a^2b^5c * g^5 * h^7 + 180a^3b^3c^2 * g^5 * h^7 + 240a^ \\
& 4 * b^2c^3 * g^5 * h^7 + 15a^2b^6 * g^4 * h^8 - 225a^4b^2c^2 * g^4 * h^8 - 60a^5c^3 \\
& * g^4 * h^8 - 20a^3b^5 * g^3 * h^9 + 50a^4b^3c * g^3 * h^9 + 120a^5b^2c^2 * g^3 * h^ \\
& 9 + 15a^4b^4 * g^2 * h^10 - 54a^5b^2c * g^2 * h^10 - 24a^6c^2 * g^2 * h^10 - 6a^ \\
& ^5b^3 * g^h^11 + 24a^6b^2c * g^h^11 + a^6b^2 * h^12 - 4a^7 * c * h^12) + (b^c^6 * d \\
& * g^9 + a * b * c^5 * f * g^9 - 6b^2c^5 * d * g^8 * h + 6a * c^6 * d * g^8 * h - 3a * b^2c^4 * f * \\
& g^8 * h - 6a^2c^5 * f * g^8 * h + 15b^3c^4 * d * g^7 * h^2 - 24a * a * b * c^5 * d * g^7 * h^2 + 3 \\
& * a * b^3c^3 * f * g^7 * h^2 + 24a^2b^2c^4 * f * g^7 * h^2 - 20b^4c^3 * d * g^6 * h^3 + 34a \\
& * b^2c^4 * d * g^6 * h^3 + 16a^2c^5 * d * g^6 * h^3 - a * b^4c^2 * f * g^6 * h^3 - 34a^2b^ \\
& 2c^3 * f * g^6 * h^3 - 16a^3c^4 * f * g^6 * h^3 + 15b^5c^2 * d * g^5 * h^4 - 15a * b^3c^ \\
& 3 * d * g^5 * h^4 - 54a^2b^2c^4 * d * g^5 * h^4 + 21a^2b^3c^2 * f * g^5 * h^4 + 42a^3b^ \\
& c^3 * f * g^5 * h^4 - 6b^6c * d * g^4 * h^5 - 9a * b^4c^2 * d * g^4 * h^5 + 66a^2b^2c^3 * \\
& d * g^4 * h^5 + 12a^3c^4 * d * g^4 * h^5 - 6a^2b^4c * f * g^4 * h^5 - 36a^3b^2c^2 * f \\
& * g^4 * h^5 - 12a^4c^3 * f * g^4 * h^5 + b^7 * d * g^3 * h^6 + 11a * a * b^5c * d * g^3 * h^6 - 31 \\
& * a^2b^3c^2 * d * g^3 * h^6 - 32a^3b^2c^3 * d * g^3 * h^6 + a^2b^5 * f * g^3 * h^6 + 13a^ \\
& 3 * b^3c * f * g^3 * h^6 + 16a^4b^2c^2 * f * g^3 * h^6 - 3a * a * b^6 * d * g^2 * h^7 + 30a^3b^2 \\
& * c^2 * d * g^2 * h^7 - 3a^3b^4 * f * g^2 * h^7 - 6a^4b^2c * f * g^2 * h^7 + 3a^2b^5 * d * \\
& g^h^8 - 9a^3b^3c * d * g^h^8 - 3a^4b^2c^2 * d * g^h^8 + 3a^4b^3 * f * g^h^8 - 3a^ \\
& ^5b^2c * f * g^h^8 - a^3b^4 * d * h^9 + 4a^4b^2c * d * h^9 - 2a^5c^2 * d * h^9 - a^5 * \\
& b^2 * f * h^9 + 2a^6c * f * h^9 - 2a * c^6 * g^9 * e + 9a * a * b * c^5 * g^8 * h * e - 18a * b^2c^ \\
& 4 * g^7 * h^2 * e + 21a * a * b^3c^3 * g^6 * h^3 * e - 15a * a * b^4c^2 * g^5 * h^4 * e - 6a^2b^2c^ \\
& ^3 * g^5 * h^4 * e + 12a^3c^4 * g^5 * h^4 * e + 6a * a * b^5c * g^4 * h^5 * e + 15a^2b^3c^2 * \\
& g^4 * h^5 * e - 30a^3b^2c^3 * g^4 * h^5 * e - a * b^6 * g^3 * h^6 * e - 12a^2b^4c * g^3 * h^6 \\
& * e + 18a^3b^2c^2 * g^3 * h^6 * e + 16a^4c^3 * g^3 * h^6 * e + 3a^2b^5 * g^2 * h^7 * e \\
& + 3a^3b^3c * g^2 * h^7 * e - 24a^4b^2c^2 * g^2 * h^7 * e - 3a^3b^4 * g^h^8 * e + 6a^ \\
& 4 * b^2c * g^h^8 * e + 6a^5c^2 * g^h^8 * e + a^4b^3 * h^9 * e - 3a^5b^2c * h^9 * e) / (b^2 \\
& * c^6 * g^12 - 4a * a * c^7 * g^12 - 6b^3c^5 * g^11 * h + 24a * a * b * c^6 * g^11 * h + 15b^4c^ \\
& 4 * g^10 * h^2 - 54a * a * b^2c^5 * g^10 * h^2 - 24a^2c^6 * g^10 * h^2 - 20b^5c^3 * g^9 * h^ \\
& ^3 + 50a * a * b^3c^4 * g^9 * h^3 + 120a^2b^2c^5 * g^9 * h^3 + 15b^6c^2 * g^8 * h^4 - 22 \\
& 5a^2b^2c^4 * g^8 * h^4 - 60a^3c^5 * g^8 * h^4 - 6b^7c * g^7 * h^5 - 36a * a * b^5c^2
\end{aligned}$$

$$\begin{aligned}
& *g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + \\
& 26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80* \\
& a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2* \\
& g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4* \\
& h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120* \\
& a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^ \\
& 2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c* \\
& h^12))/\sqrt{c*x^2 + b*x + a} + 1/4*(8*c^2*f*g^4 + 8*b*c*f*g^3*h + 48*c^2*d* \\
& g^2*h^2 - b^2*f*g^2*h^2 - 44*a*c*f*g^2*h^2 - 48*b*c*d*g*h^3 + 8*a*b*f*g*h^3 \\
& + 15*b^2*d*h^4 - 12*a*c*d*h^4 + 8*a^2*f*h^4 - 24*c^2*g^3*h*e + 12*b*c*g^2* \\
& h^2*e - 3*b^2*g*h^3*e + 36*a*c*g*h^3*e - 12*a*b*h^4*e)*\arctan(-(\sqrt{c)*x \\
& - \sqrt{c*x^2 + b*x + a})*h + \sqrt{c}*g)/\sqrt{-c*g^2 + b*g*h - a*h^2))/((c^3 \\
& *g^6 - 3*b*c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6* \\
& a*b*c*g^3*h^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6 \\
& )*\sqrt{-c*g^2 + b*g*h - a*h^2}) - 1/4*(8*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a} \\
& )^3*c^2*f*g^4*h + 24*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^3*c^2*d*g^2*h^3 - \\
& (\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^3*b^2*f*g^2*h^3 - 20*(\sqrt{c)*x - \sqrt{c \\
& *x^2 + b*x + a})^3*a*c*f*g^2*h^3 - 24*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^ \\
& 3*b*c*d*g*h^4 + 8*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^3*a*b*f*g*h^4 + 7*(\sqrt{ \\
& c)*x - \sqrt{c*x^2 + b*x + a})^3*b^2*d*h^5 - 4*(\sqrt{c)*x - \sqrt{c*x^2 + \\
& b*x + a})^3*a*c*d*h^5 - 16*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^3*c^2*g^3*h^ \\
& 2*e + 12*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^3*b*c*g^2*h^3*e - 3*(\sqrt{c)*x \\
& - \sqrt{c*x^2 + b*x + a})^3*b^2*g*h^4*e + 12*(\sqrt{c)*x - \sqrt{c*x^2 + b*x \\
& + a})^3*a*c*g*h^4*e - 4*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^3*a*b*h^5*e + 2 \\
& 4*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*c^(5/2)*f*g^5 - 8*(\sqrt{c)*x - \sqrt{ \\
& c*x^2 + b*x + a})^2*b*c^(3/2)*f*g^4*h + 56*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + \\
& a})^2*c^(5/2)*d*g^3*h^2 + 5*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*b^2*\sqrt{ \\
& c}*f*g^3*h^2 - 44*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*a*c^(3/2)*f*g^3*h^ \\
& 2 - 48*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*b*c^(3/2)*d*g^2*h^3 + 13*(\sqrt{ \\
& c)*x - \sqrt{c*x^2 + b*x + a})^2*b^2*\sqrt{c}*d*g*h^4 - 28*(\sqrt{c)*x - \sqrt{ \\
& c*x^2 + b*x + a})^2*a*c^(3/2)*d*g*h^4 + 16*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + \\
& a})^2*a^2*\sqrt{c}*f*g*h^4 + 8*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*a*b*\sqrt{ \\
& c}*d*h^5 - 40*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*c^(5/2)*g^4*h*e + 28 \\
& *(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*b*c^(3/2)*g^3*h^2*e - 9*(\sqrt{c)*x - \\
& \sqrt{c*x^2 + b*x + a})^2*b^2*\sqrt{c}*g^2*h^3*e + 36*(\sqrt{c)*x - \sqrt{c*x^ \\
& 2 + b*x + a})^2*a*c^(3/2)*g^2*h^3*e - 4*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a}) \\
& ^2*a*b*\sqrt{c}*g*h^4*e - 8*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})^2*a^2*\sqrt{c} \\
& )*h^5*e + 24*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})*b*c^2*f*g^5 - 4*(\sqrt{c)*x \\
& - \sqrt{c*x^2 + b*x + a})*b^2*c*f*g^4*h - 40*(\sqrt{c)*x - \sqrt{c*x^2 + b*x \\
& + a})*a*c^2*f*g^4*h + 56*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})*b*c^2*d*g^3*h^ \\
& 2 + (\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})*b^3*f*g^3*h^2 - 28*(\sqrt{c)*x - \sqrt{ \\
& c*x^2 + b*x + a})*a*b*c*f*g^3*h^2 - 44*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a} \\
& )*b^2*c*d*g^2*h^3 - 88*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})*a*c^2*d*g^2*h^3 \\
& + 7*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a})*a*b^2*f*g^2*h^3 + 44*(\sqrt{c)*x - \sqrt{ \\
& c*x^2 + b*x + a})*a^2*c*f*g^2*h^3 + 9*(\sqrt{c)*x - \sqrt{c*x^2 + b*x + a}
\end{aligned}$$

```

)) * b^3 * d * g * h^4 + 60 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * a * b * c * d * g * h^4 - 8 * (
sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * a^2 * b * f * g * h^4 - 9 * (sqrt(c) * x - sqrt(c * x^
2 + b * x + a)) * a * b^2 * d * h^5 - 4 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * a^2 * c * d * h
^5 - 40 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * b * c^2 * g^4 * h * e + 24 * (sqrt(c) * x -
sqrt(c * x^2 + b * x + a)) * b^2 * c * g^3 * h^2 * e + 64 * (sqrt(c) * x - sqrt(c * x^2 + b * x
+ a)) * a * c^2 * g^3 * h^2 * e - 5 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * b^3 * g^2 * h^3 * e
- 16 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * a * b * c * g^2 * h^3 * e + (sqrt(c) * x - sq
rt(c * x^2 + b * x + a)) * a * b^2 * g * h^4 * e - 20 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a))
* a^2 * c * g * h^4 * e + 4 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * a^2 * b * h^5 * e + 6 * b^2 *
c^(3/2) * f * g^5 + b^3 * sqrt(c) * f * g^4 * h - 20 * a * b * c^(3/2) * f * g^4 * h + 14 * b^2 * c^(3/
2) * d * g^3 * h^2 - 9 * a * b^2 * sqrt(c) * f * g^3 * h^2 + 12 * a^2 * c^(3/2) * f * g^3 * h^2 - 7 * b^3
* sqrt(c) * d * g^2 * h^3 - 44 * a * b * c^(3/2) * d * g^2 * h^3 + 24 * a^2 * b * sqrt(c) * f * g^2 * h^3
+ 23 * a * b^2 * sqrt(c) * d * g * h^4 + 28 * a^2 * c^(3/2) * d * g * h^4 - 16 * a^3 * sqrt(c) * f * g * h^
4 - 16 * a^2 * b * sqrt(c) * d * h^5 - 10 * b^2 * c^(3/2) * g^4 * h * e + 3 * b^3 * sqrt(c) * g^3 * h^2
* e + 32 * a * b * c^(3/2) * g^3 * h^2 * e - 7 * a * b^2 * sqrt(c) * g^2 * h^3 * e - 20 * a^2 * c^(3/2) *
g^2 * h^3 * e - 4 * a^2 * b * sqrt(c) * g * h^4 * e + 8 * a^3 * sqrt(c) * h^5 * e) / ((c^3 * g^6 - 3 * b *
c^2 * g^5 * h + 3 * b^2 * c * g^4 * h^2 + 3 * a * c^2 * g^4 * h^2 - b^3 * g^3 * h^3 - 6 * a * b * c * g^3 * h
^3 + 3 * a * b^2 * g^2 * h^4 + 3 * a^2 * c * g^2 * h^4 - 3 * a^2 * b * g * h^5 + a^3 * h^6) * ((sqrt(c)
* x - sqrt(c * x^2 + b * x + a))^2 * h + 2 * (sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * sqrt
(c) * g + b * g - a * h)^2)

```

**maple [B]** time = 0.02, size = 9126, normalized size = 12.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x)`

[Out] result too large to display

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a\*h^2-b\*g\*h>0)', see `assume?` for more details)Is a\*h^2-b\*g\*h +c\*g^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f x^2 + e x + d}{(g + h x)^3 (c x^2 + b x + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)),x)

[Out] int((d + e\*x + f\*x^2)/((g + h\*x)^3\*(a + b\*x + c\*x^2)^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Timed out

$$3.236 \quad \int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$$

**Optimal.** Leaf size=120

$$\frac{2}{15} \sqrt{3x^2 - x + 2} (2x+1)^4 + \frac{19}{60} \sqrt{3x^2 - x + 2} (2x+1)^3 + \frac{44}{135} \sqrt{3x^2 - x + 2} (2x+1)^2 - \frac{(6298x + 24897) \sqrt{3x^2 - x + 2}}{3240}$$

**Rubi [A]** time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1653, 832, 779, 619, 215}

$$\frac{2}{15} \sqrt{3x^2 - x + 2} (2x+1)^4 + \frac{19}{60} \sqrt{3x^2 - x + 2} (2x+1)^3 + \frac{44}{135} \sqrt{3x^2 - x + 2} (2x+1)^2 - \frac{(6298x + 24897) \sqrt{3x^2 - x + 2}}{3240} + \frac{9211 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2], x]

[Out] (44\*(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2])/135 + (19\*(1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2])/60 + (2\*(1 + 2\*x)^4\*Sqrt[2 - x + 3\*x^2])/15 - ((24897 + 6298\*x)\*Sqrt[2 - x + 3\*x^2])/3240 + (9211\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(1296\*Sqrt[3])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx &= \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{60} \int \frac{(1+2x)^3(-64+228x)}{\sqrt{2-x+3x^2}} dx \\
&= \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{720} \int \frac{(1+2x)^2(-339)}{\sqrt{2-x+3x^2}} dx \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} \\
&= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.50

$$\frac{6\sqrt{3x^2 - x + 2} \left(6912x^4 + 22032x^3 + 26904x^2 + 7538x - 22383\right) - 46055\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{19440}$$

Antiderivative was successfully verified.

[In] Integrate[(((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2]), x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(-22383 + 7538\*x + 26904\*x^2 + 22032\*x^3 + 6912\*x^4) - 46055\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/19440

**IntegrateAlgebraic [A]** time = 0.59, size = 75, normalized size = 0.62

$$\frac{9211 \log\left(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1\right)}{1296\sqrt{3}} + \frac{\sqrt{3x^2 - x + 2} \left(6912x^4 + 22032x^3 + 26904x^2 + 7538x - 22383\right)}{3240}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2]), x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(-22383 + 7538\*x + 26904\*x^2 + 22032\*x^3 + 6912\*x^4))/3240 + (9211\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(1296\*Sqrt[3])

**fricas [A]** time = 0.68, size = 73, normalized size = 0.61

$$\frac{1}{3240} \left(6912x^4 + 22032x^3 + 26904x^2 + 7538x - 22383\right) \sqrt{3x^2 - x + 2} + \frac{9211}{7776} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2), x, algorithm="fricas")

[Out] 1/3240\*(6912\*x^4 + 22032\*x^3 + 26904\*x^2 + 7538\*x - 22383)\*sqrt(3\*x^2 - x + 2) + 9211/7776\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac [A]** time = 0.24, size = 68, normalized size = 0.57

$$\frac{1}{3240} \left(2(12(18(16x + 51)x + 1121)x + 3769)x - 22383\right) \sqrt{3x^2 - x + 2} + \frac{9211}{3888} \sqrt{3} \log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2), x, algorithm="giac")

[Out] 1/3240\*(2\*(12\*(18\*(16\*x + 51)\*x + 1121)\*x + 3769)\*x - 22383)\*sqrt(3\*x^2 - x + 2) + 9211/3888\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple [A]** time = 0.02, size = 96, normalized size = 0.80

$$\frac{32\sqrt{3x^2-x+2}x^4}{15} + \frac{34\sqrt{3x^2-x+2}x^3}{5} + \frac{1121\sqrt{3x^2-x+2}x^2}{135} + \frac{3769\sqrt{3x^2-x+2}x}{1620} - \frac{9211\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888} - \frac{829\sqrt{3x^2-x+2}}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2), x)

[Out] 32/15\*x^4\*(3\*x^2-x+2)^(1/2)+34/5\*x^3\*(3\*x^2-x+2)^(1/2)+1121/135\*x^2\*(3\*x^2-x+2)^(1/2)+3769/1620\*x\*(3\*x^2-x+2)^(1/2)-829/120\*(3\*x^2-x+2)^(1/2)-9211/3888\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))

**maxima [A]** time = 0.95, size = 97, normalized size = 0.81

$$\frac{32}{15}\sqrt{3x^2-x+2}x^4 + \frac{34}{5}\sqrt{3x^2-x+2}x^3 + \frac{1121}{135}\sqrt{3x^2-x+2}x^2 + \frac{3769}{1620}\sqrt{3x^2-x+2}x - \frac{9211}{3888}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{829}{120}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2), x, algorithm="maxima")

[Out] 32/15\*sqrt(3\*x^2 - x + 2)\*x^4 + 34/5\*sqrt(3\*x^2 - x + 2)\*x^3 + 1121/135\*sqrt(3\*x^2 - x + 2)\*x^2 + 3769/1620\*sqrt(3\*x^2 - x + 2)\*x - 9211/3888\*sqrt(3)\*arcsinh(1/23\*sqrt(23)\*(6\*x - 1)) - 829/120\*sqrt(3\*x^2 - x + 2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3(4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)^3\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(1/2), x)

[Out] int(((2\*x + 1)^3\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(1/2), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3(4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*3\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2-x+2)\*\*(1/2), x)

[Out] Integral((2\*x + 1)\*\*3\*(4\*x\*\*2 + 3\*x + 1)/sqrt(3\*x\*\*2 - x + 2), x)



$$3.237 \quad \int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$$

**Optimal.** Leaf size=95

$$\frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3 + \frac{11}{27}\sqrt{3x^2-x+2}(2x+1)^2 - \frac{143}{324}(3-2x)\sqrt{3x^2-x+2} + \frac{4147 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1653, 832, 779, 619, 215}

$$\frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3 + \frac{11}{27}\sqrt{3x^2-x+2}(2x+1)^2 - \frac{143}{324}(3-2x)\sqrt{3x^2-x+2} + \frac{4147 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2], x]

[Out] (-143\*(3 - 2\*x)\*Sqrt[2 - x + 3\*x^2])/324 + (11\*(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2])/27 + ((1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2])/6 + (4147\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(648\*Sqrt[3])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g)\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx &= \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{48} \int \frac{(1+2x)^2(-44+176x)}{\sqrt{2-x+3x^2}} dx \\
&= \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{432} \int \frac{(1+2x)(-1716)}{\sqrt{2-x+3x^2}} dx \\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} \\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} \\
&= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 55, normalized size = 0.58

$$\frac{6\sqrt{3x^2 - x + 2} (432x^3 + 1176x^2 + 1138x - 243) - 4147\sqrt{3} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1944}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2], x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(-243 + 1138\*x + 1176\*x^2 + 432\*x^3) - 4147\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/1944

**IntegrateAlgebraic [A]** time = 0.51, size = 70, normalized size = 0.74

$$\frac{4147 \log\left(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1\right)}{648\sqrt{3}} + \frac{1}{324}\sqrt{3x^2 - x + 2} (432x^3 + 1176x^2 + 1138x - 243)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2], x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(-243 + 1138\*x + 1176\*x^2 + 432\*x^3))/324 + (4147\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(648\*Sqrt[3])

**fricas [A]** time = 1.49, size = 68, normalized size = 0.72

$$\frac{1}{324} (432x^3 + 1176x^2 + 1138x - 243)\sqrt{3x^2 - x + 2} + \frac{4147}{3888}\sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2), x, algorithm="fricas")

[Out] 1/324\*(432\*x^3 + 1176\*x^2 + 1138\*x - 243)\*sqrt(3\*x^2 - x + 2) + 4147/3888\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac [A]** time = 0.22, size = 63, normalized size = 0.66

$$\frac{1}{324} (2(12(18x + 49)x + 569)x - 243)\sqrt{3x^2 - x + 2} + \frac{4147}{1944}\sqrt{3} \log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2), x, algorithm="giac")

[Out] 1/324\*(2\*(12\*(18\*x + 49)\*x + 569)\*x - 243)\*sqrt(3\*x^2 - x + 2) + 4147/1944\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple [A]** time = 0.01, size = 79, normalized size = 0.83

$$\frac{4\sqrt{3x^2-x+2}x^3}{3} + \frac{98\sqrt{3x^2-x+2}x^2}{27} + \frac{569\sqrt{3x^2-x+2}x}{162} - \frac{4147\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944} - \frac{3\sqrt{3x^2-x+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)`

[Out] `4/3*(3*x^2-x+2)^(1/2)*x^3+98/27*(3*x^2-x+2)^(1/2)*x^2+569/162*(3*x^2-x+2)^(1/2)*x-3/4*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))`

**maxima [A]** time = 0.97, size = 80, normalized size = 0.84

$$\frac{4}{3}\sqrt{3x^2-x+2}x^3 + \frac{98}{27}\sqrt{3x^2-x+2}x^2 + \frac{569}{162}\sqrt{3x^2-x+2}x - \frac{4147}{1944}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{3}{4}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")`

[Out] `4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt(3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3/4*sqrt(3*x^2 - x + 2)`

**mpad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^2(4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)`

[Out] `int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2(4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)`

[Out] `Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)`

$$3.238 \quad \int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$$

**Optimal.** Leaf size=70

$$\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2 + \frac{1}{54}(62x+69)\sqrt{3x^2-x+2} + \frac{251 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {1653, 779, 619, 215}

$$\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2 + \frac{1}{54}(62x+69)\sqrt{3x^2-x+2} + \frac{251 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2], x]

[Out] (2\*(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2])/9 + ((69 + 62\*x)\*Sqrt[2 - x + 3\*x^2])/54 + (251\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(108\*Sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

#### Rule 1653

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx &= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{36} \int \frac{(1+2x)(-24+124x)}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} - \frac{251}{108} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} - \frac{251 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx \right)}{108\sqrt{69}} \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} + \frac{251 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{108\sqrt{3}}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 50, normalized size = 0.71

$$\frac{1}{324} \left( 6\sqrt{3x^2 - x + 2} (48x^2 + 110x + 81) - 251\sqrt{3} \sinh^{-1} \left( \frac{6x-1}{\sqrt{23}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2], x]

[Out] (6\*Sqrt[2 - x + 3\*x^2]\*(81 + 110\*x + 48\*x^2) - 251\*Sqrt[3]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/324

**IntegrateAlgebraic** [A] time = 0.36, size = 65, normalized size = 0.93

$$\frac{1}{54} \sqrt{3x^2 - x + 2} (48x^2 + 110x + 81) + \frac{251 \log \left( 2\sqrt{3} \sqrt{3x^2 - x + 2} - 6x + 1 \right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/Sqrt[2 - x + 3\*x^2],x]

[Out] (Sqrt[2 - x + 3\*x^2]\*(81 + 110\*x + 48\*x^2))/54 + (251\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(108\*Sqrt[3])

**fricas** [A] time = 0.69, size = 63, normalized size = 0.90

$$\frac{1}{54} (48x^2 + 110x + 81)\sqrt{3x^2 - x + 2} + \frac{251}{648} \sqrt{3} \log\left(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2),x, algorithm="fricas")

[Out] 1/54\*(48\*x^2 + 110\*x + 81)\*sqrt(3\*x^2 - x + 2) + 251/648\*sqrt(3)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25)

**giac** [A] time = 0.26, size = 58, normalized size = 0.83

$$\frac{1}{54} (2(24x + 55)x + 81)\sqrt{3x^2 - x + 2} + \frac{251}{324} \sqrt{3} \log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2),x, algorithm="giac")

[Out] 1/54\*(2\*(24\*x + 55)\*x + 81)\*sqrt(3\*x^2 - x + 2) + 251/324\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1)

**maple** [A] time = 0.01, size = 62, normalized size = 0.89

$$\frac{8\sqrt{3x^2 - x + 2}x^2}{9} + \frac{55\sqrt{3x^2 - x + 2}x}{27} - \frac{251\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324} + \frac{3\sqrt{3x^2 - x + 2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2),x)

[Out] 8/9\*(3\*x^2-x+2)^(1/2)\*x^2+55/27\*(3\*x^2-x+2)^(1/2)\*x+3/2\*(3\*x^2-x+2)^(1/2)-251/324\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))

**maxima** [A] time = 0.96, size = 63, normalized size = 0.90

$$\frac{8}{9} \sqrt{3x^2 - x + 2}x^2 + \frac{55}{27} \sqrt{3x^2 - x + 2}x - \frac{251}{324} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(6x - 1)\right) + \frac{3}{2} \sqrt{3x^2 - x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(1/2),x, algorithm="maxima")

[Out] 8/9\*sqrt(3\*x^2 - x + 2)\*x^2 + 55/27\*sqrt(3\*x^2 - x + 2)\*x - 251/324\*sqrt(3)\*arcsinh(1/23\*sqrt(23)\*(6\*x - 1)) + 3/2\*sqrt(3\*x^2 - x + 2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)(4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(1/2),x)

[Out] int(((2\*x + 1)\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)(4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2-x+2)\*\*(1/2),x)

[Out] Integral((2\*x + 1)\*(4\*x\*\*2 + 3\*x + 1)/sqrt(3\*x\*\*2 - x + 2), x)



$$3.239 \quad \int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}} dx$$

Optimal. Leaf size=78

$$\frac{2}{3}\sqrt{3x^2-x+2} - \frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}} - \frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{2}{3}\sqrt{3x^2-x+2} - \frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}} - \frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]), x]

[Out] (2\*Sqrt[2 - x + 3\*x^2])/3 - (5\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(6\*Sqrt[3]) - ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])]/(2\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
 \int \frac{1 + 3x + 4x^2}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx &= \frac{2}{3}\sqrt{2 - x + 3x^2} + \frac{1}{12} \int \frac{16 + 20x}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx \\
 &= \frac{2}{3}\sqrt{2 - x + 3x^2} + \frac{1}{2} \int \frac{1}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx + \frac{5}{6} \int \frac{1}{\sqrt{2 - x + 3x^2}} dx \\
 &= \frac{2}{3}\sqrt{2 - x + 3x^2} + \frac{5 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{23}} dx, x, -1 + 6x}\right)}{6\sqrt{69}} - \operatorname{Subst}\left(\int \frac{1}{52 - x^2} dx, x, \frac{1}{\sqrt{2 - x + 3x^2}}\right) \\
 &= \frac{2}{3}\sqrt{2 - x + 3x^2} - \frac{5 \sinh^{-1}\left(\frac{1 - 6x}{\sqrt{23}}\right)}{6\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{2 - x + 3x^2}}\right)}{2\sqrt{13}}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 1.00

$$\frac{2}{3}\sqrt{3x^2 - x + 2} - \frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}} + \frac{5\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]),x]

[Out] (2\*Sqrt[2 - x + 3\*x^2])/3 + (5\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/(6\*Sqrt[3]) - ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])]/(2\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.40, size = 101, normalized size = 1.29

$$\frac{2}{3}\sqrt{3x^2 - x + 2} - \frac{5\log\left(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1\right)}{6\sqrt{3}} + \frac{\tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]),x]

[Out] (2\*Sqrt[2 - x + 3\*x^2])/3 + ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]]/Sqrt[13] - (5\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(6\*Sqrt[3])

**fricas [A]** time = 1.53, size = 105, normalized size = 1.35

$$\frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+\frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(1/2),x, algorithm="fricas")

[Out] 5/36\*sqrt(3)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 1/52\*sqrt(13)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 2/3\*sqrt(3\*x^2 - x + 2)

**giac [A]** time = 0.57, size = 116, normalized size = 1.49

$$-\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{1}{26}\sqrt{13}\log\left(-\frac{|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}|}{2(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2})}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(1/2),x, algorithm="giac")

[Out]  $-5/18*\sqrt{3}*\log(-6*\sqrt{3}*x + \sqrt{3} + 6*\sqrt{3*x^2 - x + 2}) + 1/26*\sqrt{13}*\log(-1/2*\text{abs}(-4*\sqrt{3}*x - 2*\sqrt{13} - 2*\sqrt{3} + 4*\sqrt{3*x^2 - x + 2}))/ (2*\sqrt{3}*x - \sqrt{13} + \sqrt{3} - 2*\sqrt{3*x^2 - x + 2})) + 2/3*\sqrt{3*x^2 - x + 2}$

**maple** [A] time = 0.01, size = 60, normalized size = 0.77

$$\frac{5\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18} - \frac{\sqrt{13} \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{26} + \frac{2\sqrt{3x^2-x+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(1/2),x)`

[Out]  $2/3*(3*x^2-x+2)^(1/2)+5/18*3^(1/2)*\operatorname{arcsinh}(6/23*23^(1/2)*(x-1/6))-1/26*13^(1/2)*\operatorname{arctanh}(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))$

**maxima** [A] time = 0.97, size = 67, normalized size = 0.86

$$\frac{5}{18}\sqrt{3} \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{1}{26}\sqrt{13} \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{2}{3}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")`

[Out]  $5/18*\sqrt{3}*\operatorname{arcsinh}(6/23*\sqrt{23}*x - 1/23*\sqrt{23}) + 1/26*\sqrt{13}*\operatorname{arcsinh}(8/23*\sqrt{23}*x/\text{abs}(2*x + 1) - 9/23*\sqrt{23}/\text{abs}(2*x + 1)) + 2/3*\sqrt{3*x^2 - x + 2}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\sqrt{3x^2 - x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)`

[Out] `int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\sqrt{3x^2 - x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

$$3.240 \quad \int \frac{1+3x+4x^2}{(1+2x)^2 \sqrt{2-x+3x^2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)} + \frac{9 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}} - \frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)} + \frac{9 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}} - \frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]),x]

[Out] -Sqrt[2 - x + 3\*x^2]/(13\*(1 + 2\*x)) - ArcSinh[(1 - 6\*x)/Sqrt[23]]/Sqrt[3] + (9\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(26\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c,

d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 \sqrt{2 - x + 3x^2}} dx &= -\frac{\sqrt{2 - x + 3x^2}}{13(1 + 2x)} - \frac{1}{13} \int \frac{-\frac{17}{2} - 26x}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx \\ &= -\frac{\sqrt{2 - x + 3x^2}}{13(1 + 2x)} - \frac{9}{26} \int \frac{1}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx + \int \frac{1}{\sqrt{2 - x + 3x^2}} dx \\ &= -\frac{\sqrt{2 - x + 3x^2}}{13(1 + 2x)} + \frac{9}{13} \text{Subst} \left( \int \frac{1}{52 - x^2} dx, x, \frac{9 - 8x}{\sqrt{2 - x + 3x^2}} \right) + \text{Subst} \left( \int \frac{1}{\sqrt{1 + 2x}} dx, x, \frac{9 - 8x}{\sqrt{2 - x + 3x^2}} \right) \\ &= -\frac{\sqrt{2 - x + 3x^2}}{13(1 + 2x)} - \frac{\sinh^{-1} \left( \frac{1 - 6x}{\sqrt{23}} \right)}{\sqrt{3}} + \frac{9 \tanh^{-1} \left( \frac{9 - 8x}{2\sqrt{13}\sqrt{2 - x + 3x^2}} \right)}{26\sqrt{13}} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 82, normalized size = 0.99

$$-\frac{\sqrt{3x^2 - x + 2}}{13(2x + 1)} + \frac{9 \tanh^{-1} \left( \frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}} \right)}{26\sqrt{13}} + \frac{\sinh^{-1} \left( \frac{6x - 1}{\sqrt{23}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]),x]

[Out] -1/13\*Sqrt[2 - x + 3\*x^2]/(1 + 2\*x) + ArcSinh[(-1 + 6\*x)/Sqrt[23]]/Sqrt[3] + (9\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(26\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.42, size = 109, normalized size = 1.31

$$\frac{\sqrt{3x^2 - x + 2}}{13(2x + 1)} - \frac{\log\left(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1\right)}{\sqrt{3}} - \frac{9 \tanh^{-1}\left(-\frac{2\sqrt{3x^2 - x + 2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]),x]

[Out] -1/13\*Sqrt[2 - x + 3\*x^2]/(1 + 2\*x) - (9\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(13\*Sqrt[13]) - Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]]/Sqrt[3]

**fricas [A]** time = 0.99, size = 123, normalized size = 1.48

$$\frac{338\sqrt{3}(2x+1)\log(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25)+27\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)-156\sqrt{3x^2-x+2}}{2028(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2-x+2)^(1/2),x, algorithm="fricas")

[Out] 1/2028\*(338\*sqrt(3)\*(2\*x + 1)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 27\*sqrt(13)\*(2\*x + 1)\*log((4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) - 220\*x^2 + 196\*x - 185)/(4\*x^2 + 4\*x + 1)) - 156\*sqrt(3\*x^2 - x + 2))/(2\*x + 1)

**giac [A]** time = 0.28, size = 48, normalized size = 0.58

$$\frac{1}{26}\sqrt{3}\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{\sqrt{-\frac{8}{2x+1} + \frac{13}{(2x+1)^2} + 3}}{26\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2-x+2)^(1/2),x, algorithm="giac")

[Out] 1/26\*sqrt(3)\*sgn(1/(2\*x + 1)) - 1/26\*sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3)/sgn(1/(2\*x + 1))



**maple** [A] time = 0.01, size = 67, normalized size = 0.81

$$\frac{\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3} + \frac{9\sqrt{13} \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{338} - \frac{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{26\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(1/2),x)`

[Out]  $\frac{1}{3}3^{(1/2)}*\operatorname{arcsinh}(6/23*23^{(1/2)}*(x-1/6))+9/338*13^{(1/2)}*\operatorname{arctanh}(2/13*(-4*x+9/2)*13^{(1/2)/(-16*x+12*(x+1/2)^2+5)^{(1/2)})-1/26/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^{(1/2)}$

**maxima** [A] time = 0.98, size = 74, normalized size = 0.89

$$\frac{1}{3}\sqrt{3} \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{9}{338}\sqrt{13} \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) - \frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{1}{3}*\operatorname{sqrt}(3)*\operatorname{arcsinh}(6/23*\operatorname{sqrt}(23)*x - 1/23*\operatorname{sqrt}(23)) - 9/338*\operatorname{sqrt}(13)*\operatorname{arcsinh}(8/23*\operatorname{sqrt}(23)*x/\operatorname{abs}(2*x + 1) - 9/23*\operatorname{sqrt}(23)/\operatorname{abs}(2*x + 1)) - 1/13*\operatorname{sqrt}(3*x^2 - x + 2)/(2*x + 1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 \sqrt{3x^2 - x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)`

[Out] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 \sqrt{3x^2 - x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)`

[Out] `Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)`

$$3.241 \quad \int \frac{1+3x+4x^2}{(1+2x)^3 \sqrt{2-x+3x^2}} dx$$

**Optimal.** Leaf size=89

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)} - \frac{\sqrt{3x^2-x+2}}{26(2x+1)^2} - \frac{581 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

**Rubi [A]** time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1650, 806, 724, 206}

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)} - \frac{\sqrt{3x^2-x+2}}{26(2x+1)^2} - \frac{581 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2]),x]

[Out] -Sqrt[2 - x + 3\*x^2]/(26\*(1 + 2\*x)^2) + (7\*Sqrt[2 - x + 3\*x^2])/(169\*(1 + 2\*x)) - (581\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(676\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 806

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m +

2\*p + 3], 0]

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{1 + 3x + 4x^2}{(1 + 2x)^3 \sqrt{2 - x + 3x^2}} dx &= -\frac{\sqrt{2 - x + 3x^2}}{26(1 + 2x)^2} - \frac{1}{26} \int \frac{-\frac{35}{2} - 49x}{(1 + 2x)^2 \sqrt{2 - x + 3x^2}} dx \\ &= -\frac{\sqrt{2 - x + 3x^2}}{26(1 + 2x)^2} + \frac{7\sqrt{2 - x + 3x^2}}{169(1 + 2x)} + \frac{581}{676} \int \frac{1}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx \\ &= -\frac{\sqrt{2 - x + 3x^2}}{26(1 + 2x)^2} + \frac{7\sqrt{2 - x + 3x^2}}{169(1 + 2x)} - \frac{581}{338} \operatorname{Subst}\left(\int \frac{1}{52 - x^2} dx, x, \frac{9 - 8x}{\sqrt{2 - x + 3x^2}}\right) \\ &= -\frac{\sqrt{2 - x + 3x^2}}{26(1 + 2x)^2} + \frac{7\sqrt{2 - x + 3x^2}}{169(1 + 2x)} - \frac{581 \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{2 - x + 3x^2}}\right)}{676\sqrt{13}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 69, normalized size = 0.78

$$\frac{\frac{26(28x+1)\sqrt{3x^2-x+2}}{(2x+1)^2} - 581\sqrt{13} \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8788}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*Sqrt[2 - x + 3\*x^2]),x]

[Out] ((26\*(1 + 28\*x)\*Sqrt[2 - x + 3\*x^2])/((1 + 2\*x)^2 - 581\*Sqrt[13]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/8788

**IntegrateAlgebraic [A]** time = 0.47, size = 80, normalized size = 0.90

$$\frac{\sqrt{3x^2 - x + 2}(28x + 1)}{338(2x + 1)^2} + \frac{581 \tanh^{-1}\left(-\frac{2\sqrt{3x^2 - x + 2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{338\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*sqrt[2 - x + 3\*x^2]),x]

[Out] ((1 + 28\*x)\*sqrt[2 - x + 3\*x^2])/(338\*(1 + 2\*x)^2) + (581\*ArcTanh[sqrt[3/13] + 2\*sqrt[3/13]\*x - (2\*sqrt[2 - x + 3\*x^2])/sqrt[13]])/(338\*sqrt[13])

**fricas [A]** time = 1.50, size = 96, normalized size = 1.08

$$\frac{581\sqrt{13}(4x^2 + 4x + 1)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right) + 52\sqrt{3x^2-x+2}(28x+1)}{17576(4x^2+4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2-x+2)^(1/2),x, algorithm="fricas")

[Out] 1/17576\*(581\*sqrt(13)\*(4\*x^2 + 4\*x + 1)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2))\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 52\*sqrt(3\*x^2 - x + 2)\*(28\*x + 1))/(4\*x^2 + 4\*x + 1)

**giac [B]** time = 0.33, size = 204, normalized size = 2.29

$$\frac{581}{8788}\sqrt{13}\log\left(\frac{-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}}{2(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2})}\right) + \frac{190(\sqrt{3}x-\sqrt{3x^2-x+2})^3-53\sqrt{3}(\sqrt{3}x-\sqrt{3x^2-x+2})^2-489\sqrt{3}x+289\sqrt{3}+489\sqrt{3x^2-x+2}}{338(2(\sqrt{3}x-\sqrt{3x^2-x+2})^2+2\sqrt{3}(\sqrt{3}x-\sqrt{3x^2-x+2})-5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2-x+2)^(1/2),x, algorithm="giac")

[Out] 581/8788\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2))) + 1/338\*(190\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^3 - 53\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^2 - 489\*sqrt(3)\*x + 289\*sqrt(3) + 489\*sqrt(3\*x^2 - x + 2))/(2\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^2 + 2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) - 5)^2

**maple** [A] time = 0.04, size = 74, normalized size = 0.83

$$\frac{581\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{8788} + \frac{7\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}{338(x+\frac{1}{2})} - \frac{\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}{104(x+\frac{1}{2})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(1/2),x)`

[Out] `-581/8788*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))+7/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+5/4)^(1/2)`

**maxima** [A] time = 0.98, size = 82, normalized size = 0.92

$$\frac{581}{8788} \sqrt{13} \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) - \frac{\sqrt{3x^2-x+2}}{26(4x^2+4x+1)} + \frac{7\sqrt{3x^2-x+2}}{169(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")`

[Out] `581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x+1) - 9/23*sqrt(23)/abs(2*x+1)) - 1/26*sqrt(3*x^2-x+2)/(4*x^2+4*x+1) + 7/169*sqrt(3*x^2-x+2)/(2*x+1)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 \sqrt{3x^2 - x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x+4*x^2+1)/((2*x+1)^3*(3*x^2-x+2)^(1/2)),x)`

[Out] `int((3*x+4*x^2+1)/((2*x+1)^3*(3*x^2-x+2)^(1/2)),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 \sqrt{3x^2 - x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

$$3.242 \quad \int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx$$

Optimal. Leaf size=103

$$\frac{32}{27} \sqrt{3x^2 - x + 2} x^2 + \frac{412}{81} \sqrt{3x^2 - x + 2} x + \frac{746}{81} \sqrt{3x^2 - x + 2} + \frac{2(12839 - 3871x)}{1863 \sqrt{3x^2 - x + 2}} + \frac{353 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{81 \sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{32}{27} \sqrt{3x^2 - x + 2} x^2 + \frac{412}{81} \sqrt{3x^2 - x + 2} x + \frac{746}{81} \sqrt{3x^2 - x + 2} + \frac{2(12839 - 3871x)}{1863 \sqrt{3x^2 - x + 2}} + \frac{353 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{81 \sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (2\*(12839 - 3871\*x))/(1863\*Sqrt[2 - x + 3\*x^2]) + (746\*Sqrt[2 - x + 3\*x^2])/81 + (412\*x\*Sqrt[2 - x + 3\*x^2])/81 + (32\*x^2\*Sqrt[2 - x + 3\*x^2])/27 + (353\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(81\*Sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx &= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{2}{23} \int \frac{\frac{1127}{81} + \frac{7682x}{27} + \frac{2852x^2}{9} + \frac{368x^3}{3}}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{2}{207} \int \frac{\frac{1127}{9} + 2070x + \frac{9476x^2}{3}}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{1}{621} \int \frac{-5566}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} \\
&= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} \\
&= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2}
\end{aligned}$$



**Mathematica [A]** time = 0.04, size = 69, normalized size = 0.67

$$\frac{6(3312x^4 + 13110x^3 + 23207x^2 - 2974x + 29997) - 8119\sqrt{9x^2 - 3x + 6} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{5589\sqrt{3x^2 - x + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (6\*(29997 - 2974\*x + 23207\*x^2 + 13110\*x^3 + 3312\*x^4) - 8119\*Sqrt[6 - 3\*x + 9\*x^2]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]])/(5589\*Sqrt[2 - x + 3\*x^2])

**IntegrateAlgebraic [A]** time = 0.64, size = 75, normalized size = 0.73

$$\frac{353 \log\left(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1\right)}{81\sqrt{3}} + \frac{2(3312x^4 + 13110x^3 + 23207x^2 - 2974x + 29997)}{1863\sqrt{3x^2 - x + 2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (2\*(29997 - 2974\*x + 23207\*x^2 + 13110\*x^3 + 3312\*x^4))/(1863\*Sqrt[2 - x + 3\*x^2]) + (353\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(81\*Sqrt[3])

**fricas [A]** time = 0.87, size = 97, normalized size = 0.94

$$\frac{8119\sqrt{3}(3x^2 - x + 2)\log(4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25) + 12(3312x^4 + 13110x^3 + 23207x^2 - 2974x + 29997)\sqrt{3x^2 - x + 2}}{11178(3x^2 - x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2), x, algorithm="fricas")

[Out] 1/11178\*(8119\*sqrt(3)\*(3\*x^2 - x + 2)\*log(4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 12\*(3312\*x^4 + 13110\*x^3 + 23207\*x^2 - 2974\*x + 29997)\*sqrt(3\*x^2 - x + 2))/(3\*x^2 - x + 2)

**giac [A]** time = 0.21, size = 67, normalized size = 0.65

$$\frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right) + \frac{2((23(6(24x + 95)x + 1009)x - 2974)x + 29997)}{1863\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2), x, algorithm="giac")

[Out]  $353/243\sqrt{3}\log(-2\sqrt{3})(\sqrt{3}x - \sqrt{3x^2 - x + 2}) + 1) + 2/1863((23(6(24x + 95)x + 1009)x - 2974)x + 29997)/\sqrt{3x^2 - x + 2}$

**maple** [A] time = 0.01, size = 115, normalized size = 1.12

$$\frac{32x^4}{9\sqrt{3x^2-x+2}} + \frac{380x^3}{27\sqrt{3x^2-x+2}} + \frac{2018x^2}{81\sqrt{3x^2-x+2}} + \frac{353x}{81\sqrt{3x^2-x+2}} - \frac{353\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243} - \frac{521(6x-1)}{414\sqrt{3x^2-x+2}} + \frac{557}{18\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((2x+1)^3(4x^2+3x+1)/(3x^2-x+2)^{(3/2)}, x)$

[Out]  $32/9x^4/(3x^2-x+2)^{(1/2)}+380/27x^3/(3x^2-x+2)^{(1/2)}+2018/81x^2/(3x^2-x+2)^{(1/2)}+353/81x/(3x^2-x+2)^{(1/2)}-521/414(6x-1)/(3x^2-x+2)^{(1/2)}-353/2433^{(1/2)}\operatorname{arcsinh}(6/23\sqrt{23}^{(1/2)}(x-1/6))+557/18/(3x^2-x+2)^{(1/2)}$

**maxima** [A] time = 0.96, size = 97, normalized size = 0.94

$$\frac{32x^4}{9\sqrt{3x^2-x+2}} + \frac{380x^3}{27\sqrt{3x^2-x+2}} + \frac{2018x^2}{81\sqrt{3x^2-x+2}} - \frac{353}{243}\sqrt{3} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{5948x}{1863\sqrt{3x^2-x+2}} + \frac{2222}{69\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((1+2x)^3(4x^2+3x+1)/(3x^2-x+2)^{(3/2)}, x, \operatorname{algorithm}="maxima")$

[Out]  $32/9x^4/\sqrt{3x^2-x+2} + 380/27x^3/\sqrt{3x^2-x+2} + 2018/81x^2/\sqrt{3x^2-x+2} - 353/243\sqrt{3}\operatorname{arcsinh}(1/23\sqrt{23}(6x-1)) - 5948/1863x/\sqrt{3x^2-x+2} + 2222/69/\sqrt{3x^2-x+2}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3(4x^2+3x+1)}{(3x^2-x+2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}(((2x+1)^3(3x+4x^2+1))/(3x^2-x+2)^{(3/2)}, x)$

[Out]  $\operatorname{int}(((2x+1)^3(3x+4x^2+1))/(3x^2-x+2)^{(3/2)}, x)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3(4x^2+3x+1)}{(3x^2-x+2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*\*3\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2-x+2)\*\*(3/2),x)

[Out] Integral((2\*x + 1)\*\*3\*(4\*x\*\*2 + 3\*x + 1)/(3\*x\*\*2 - x + 2)\*\*(3/2), x)

$$3.243 \quad \int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx$$

**Optimal.** Leaf size=82

$$\frac{2(1249 - 2273x)}{621\sqrt{3x^2 - x + 2}} + \frac{8}{9}x\sqrt{3x^2 - x + 2} + \frac{112}{27}\sqrt{3x^2 - x + 2} - \frac{64 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{2(1249 - 2273x)}{621\sqrt{3x^2 - x + 2}} + \frac{8}{9}x\sqrt{3x^2 - x + 2} + \frac{112}{27}\sqrt{3x^2 - x + 2} - \frac{64 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (2\*(1249 - 2273\*x))/(621\*sqrt[2 - x + 3\*x^2]) + (112\*sqrt[2 - x + 3\*x^2])/27 + (8\*x\*sqrt[2 - x + 3\*x^2])/9 - (64\*ArcSinh[(1 - 6\*x)/sqrt[23]])/(9\*sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx &= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{2}{23} \int \frac{\frac{2116}{27} + \frac{1150x}{9} + \frac{184x^2}{3}}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{8}{9}x\sqrt{2-x+3x^2} + \frac{1}{69} \int \frac{\frac{3128}{9} + \frac{2576x}{3}}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} + \frac{64}{9} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} + \frac{64 \operatorname{Subst} \left[ \int \frac{1}{\sqrt{1+3x}} dx \right]}{9\sqrt{3}} \\
&= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} - \frac{64 \sinh^{-1} \left( \frac{1-6x}{\sqrt{23}} \right)}{9\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 61, normalized size = 0.74

$$\frac{2 \left( 828x^3 + 3588x^2 + 736\sqrt{9x^2 - 3x + 6} \sinh^{-1} \left( \frac{6x-1}{\sqrt{23}} \right) - 3009x + 3825 \right)}{621\sqrt{3x^2 - x + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2),x]

[Out] (2\*(3825 - 3009\*x + 3588\*x^2 + 828\*x^3 + 736\*Sqrt[6 - 3\*x + 9\*x^2]\*ArcSinh[(-1 + 6\*x)/Sqrt[23]]))/(621\*Sqrt[2 - x + 3\*x^2])

**IntegrateAlgebraic [A]** time = 0.64, size = 70, normalized size = 0.85

$$\frac{2(276x^3 + 1196x^2 - 1003x + 1275)}{207\sqrt{3x^2 - x + 2}} - \frac{64 \log(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2),x]

[Out] (2\*(1275 - 1003\*x + 1196\*x^2 + 276\*x^3))/(207\*Sqrt[2 - x + 3\*x^2]) - (64\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(9\*Sqrt[3])

**fricas [A]** time = 1.43, size = 92, normalized size = 1.12

$$\frac{2(368\sqrt{3}(3x^2 - x + 2)\log(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25) + 3(276x^3 + 1196x^2 - 1003x + 1275)\sqrt{3x^2 - x + 2})}{621(3x^2 - x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2),x, algorithm="fricas")

[Out] 2/621\*(368\*sqrt(3)\*(3\*x^2 - x + 2)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 3\*(276\*x^3 + 1196\*x^2 - 1003\*x + 1275)\*sqrt(3\*x^2 - x + 2))/(3\*x^2 - x + 2)

**giac [A]** time = 0.27, size = 62, normalized size = 0.76

$$-\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right) + \frac{2((92(3x + 13)x - 1003)x + 1275)}{207\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2),x, algorithm="giac")

[Out] -64/27\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1) + 2/207\*((92\*(3\*x + 13)\*x - 1003)\*x + 1275)/sqrt(3\*x^2 - x + 2)

**maple [A]** time = 0.01, size = 98, normalized size = 1.20

$$\frac{8x^3}{3\sqrt{3x^2 - x + 2}} + \frac{104x^2}{9\sqrt{3x^2 - x + 2}} - \frac{64x}{9\sqrt{3x^2 - x + 2}} + \frac{64\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x - \frac{1}{6}\right)}{23}\right)}{27} + \frac{107}{9\sqrt{3x^2 - x + 2}} - \frac{89(6x - 1)}{207\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)`

[Out]  $8/3/(3*x^2-x+2)^{(1/2)}*x^3+104/9/(3*x^2-x+2)^{(1/2)}*x^2-64/9/(3*x^2-x+2)^{(1/2)}*x+107/9/(3*x^2-x+2)^{(1/2)}-89/207*(6*x-1)/(3*x^2-x+2)^{(1/2)}+64/27*3^{(1/2)}*\operatorname{arcsinh}(6/23*23^{(1/2)}*(x-1/6))$

**maxima** [A] time = 0.97, size = 80, normalized size = 0.98

$$\frac{8x^3}{3\sqrt{3x^2-x+2}} + \frac{104x^2}{9\sqrt{3x^2-x+2}} + \frac{64}{27}\sqrt{3}\operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{2006x}{207\sqrt{3x^2-x+2}} + \frac{850}{69\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")`

[Out]  $8/3*x^3/\operatorname{sqrt}(3*x^2-x+2)+104/9*x^2/\operatorname{sqrt}(3*x^2-x+2)+64/27*\operatorname{sqrt}(3)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(6*x-1))-2006/207*x/\operatorname{sqrt}(3*x^2-x+2)+850/69/\operatorname{sqrt}(3*x^2-x+2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^2(4x^2+3x+1)}{(3x^2-x+2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x+1)^2*(3*x+4*x^2+1))/(3*x^2-x+2)^(3/2),x)`

[Out] `int(((2*x+1)^2*(3*x+4*x^2+1))/(3*x^2-x+2)^(3/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)`

[Out] `Integral((2*x+1)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)`

$$3.244 \quad \int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx$$

Optimal. Leaf size=63

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}} + \frac{8}{9}\sqrt{3x^2-x+2} - \frac{14 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {1660, 640, 619, 215}

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}} + \frac{8}{9}\sqrt{3x^2-x+2} - \frac{14 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (-2\*(73 + 367\*x))/(207\*sqrt[2 - x + 3\*x^2]) + (8\*sqrt[2 - x + 3\*x^2])/9 - (14\*ArcSinh[(1 - 6\*x)/sqrt[23]])/(3\*sqrt[3])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P



q, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]], Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}} dx &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}} + \frac{2}{23} \int \frac{\frac{437}{9} + \frac{92x}{3}}{\sqrt{2-x+3x^2}} dx \\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}} + \frac{8}{9}\sqrt{2-x+3x^2} + \frac{14}{3} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}} + \frac{8}{9}\sqrt{2-x+3x^2} + \frac{14 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+6x\right)}{3\sqrt{69}} \\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}} + \frac{8}{9}\sqrt{2-x+3x^2} - \frac{14 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 50, normalized size = 0.79

$$\frac{2(92x^2 - 153x + 37)}{69\sqrt{3x^2 - x + 2}} + \frac{14 \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (2\*(37 - 153\*x + 92\*x^2))/(69\*sqrt[2 - x + 3\*x^2]) + (14\*ArcSinh[(-1 + 6\*x)/sqrt[23]])/(3\*sqrt[3])

**IntegrateAlgebraic [A]** time = 0.56, size = 65, normalized size = 1.03

$$\frac{2(92x^2 - 153x + 37)}{69\sqrt{3x^2 - x + 2}} - \frac{14 \log(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(3/2), x]

[Out] (2\*(37 - 153\*x + 92\*x^2))/(69\*sqrt[2 - x + 3\*x^2]) - (14\*Log[1 - 6\*x + 2\*sqrt[3]\*sqrt[2 - x + 3\*x^2]])/(3\*sqrt[3])

**fricas** [A] time = 0.99, size = 87, normalized size = 1.38

$$\frac{161\sqrt{3}(3x^2 - x + 2)\log(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25) + 6(92x^2 - 153x + 37)\sqrt{3x^2 - x + 2}}{207(3x^2 - x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2), x, algorithm="fricas")

[Out] 1/207\*(161\*sqrt(3)\*(3\*x^2 - x + 2)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 6\*(92\*x^2 - 153\*x + 37)\*sqrt(3\*x^2 - x + 2))/(3\*x^2 - x + 2)

**giac** [A] time = 0.27, size = 57, normalized size = 0.90

$$-\frac{14}{9}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right) + \frac{2((92x - 153)x + 37)}{69\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2), x, algorithm="giac")

[Out] -14/9\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1) + 2/69\*((92\*x - 153)\*x + 37)/sqrt(3\*x^2 - x + 2)

**maple** [A] time = 0.01, size = 81, normalized size = 1.29

$$\frac{8x^2}{3\sqrt{3x^2 - x + 2}} - \frac{14x}{3\sqrt{3x^2 - x + 2}} + \frac{14\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x - \frac{1}{6}\right)}{23}\right)}{9} + \frac{10}{9\sqrt{3x^2 - x + 2}} + \frac{\frac{16x}{69} - \frac{8}{207}}{\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2), x)

[Out] 8/3/(3\*x^2-x+2)^(1/2)\*x^2-14/3/(3\*x^2-x+2)^(1/2)\*x+10/9/(3\*x^2-x+2)^(1/2)+8/207\*(6\*x-1)/(3\*x^2-x+2)^(1/2)+14/9\*3^(1/2)\*arcsinh(6/23\*23^(1/2)\*(x-1/6))

**maxima** [A] time = 0.94, size = 63, normalized size = 1.00

$$\frac{8x^2}{3\sqrt{3x^2 - x + 2}} + \frac{14}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x - 1)\right) - \frac{102x}{23\sqrt{3x^2 - x + 2}} + \frac{74}{69\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(3/2),x, algorithm="maxima")

[Out]  $8/3*x^2/\sqrt{3*x^2 - x + 2} + 14/9*\sqrt{3}*\operatorname{arcsinh}(1/23*\sqrt{23}*(6*x - 1)) - 102/23*x/\sqrt{3*x^2 - x + 2} + 74/69/\sqrt{3*x^2 - x + 2}$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(3/2), x)

[Out] int(((2\*x + 1)\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x\*\*2+3\*x+1)/(3\*x\*\*2-x+2)\*\*(3/2), x)

[Out] Integral((2\*x + 1)\*(4\*x\*\*2 + 3\*x + 1)/(3\*x\*\*2 - x + 2)\*\*(3/2), x)

$$3.245 \quad \int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{3/2}} dx$$

Optimal. Leaf size=62

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}} - \frac{2 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

**Rubi [A]** time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 12, 724, 206}

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}} - \frac{2 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)), x]

[Out] (-2\*(101 - 77\*x))/(299\*Sqrt[2 - x + 3\*x^2]) - (2\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(13\*Sqrt[13])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x

```

^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)(2 - x + 3x^2)^{3/2}} dx &= -\frac{2(101 - 77x)}{299\sqrt{2 - x + 3x^2}} + \frac{2}{23} \int \frac{23}{13(1 + 2x)\sqrt{2 - x + 3x^2}} dx \\
&= -\frac{2(101 - 77x)}{299\sqrt{2 - x + 3x^2}} + \frac{2}{13} \int \frac{1}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx \\
&= -\frac{2(101 - 77x)}{299\sqrt{2 - x + 3x^2}} - \frac{4}{13} \operatorname{Subst}\left(\int \frac{1}{52 - x^2} dx, x, \frac{9 - 8x}{\sqrt{2 - x + 3x^2}}\right) \\
&= -\frac{2(101 - 77x)}{299\sqrt{2 - x + 3x^2}} - \frac{2 \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{2 - x + 3x^2}}\right)}{13\sqrt{13}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 73, normalized size = 1.18

$$\frac{2\left(23\sqrt{13}\sqrt{3x^2 - x + 2} \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}}\right) - 1001x + 1313\right)}{3887\sqrt{3x^2 - x + 2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)), x]

[Out] (-2\*(1313 - 1001\*x + 23\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2]\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])]))/(3887\*Sqrt[2 - x + 3\*x^2])

**IntegrateAlgebraic [A]** time = 0.49, size = 73, normalized size = 1.18

$$\frac{2(77x - 101)}{299\sqrt{3x^2 - x + 2}} + \frac{4 \tanh^{-1}\left(-\frac{2\sqrt{3x^2 - x + 2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)),x]

[Out] (2\*(-101 + 77\*x))/(299\*sqrt[2 - x + 3\*x^2]) + (4\*ArcTanh[Sqrt[3/13] + 2\*sqrt[3/13]\*x - (2\*sqrt[2 - x + 3\*x^2])/sqrt[13]])/(13\*sqrt[13])

**fricas** [A] time = 1.42, size = 96, normalized size = 1.55

$$\frac{23\sqrt{13}(3x^2 - x + 2)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right) + 26\sqrt{3x^2-x+2}(77x-101)}{3887(3x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(3/2),x, algorithm="fricas")

[Out] 1/3887\*(23\*sqrt(13)\*(3\*x^2 - x + 2)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 26\*sqrt(3\*x^2 - x + 2)\*(77\*x - 101))/(3\*x^2 - x + 2)

**giac** [A] time = 0.58, size = 91, normalized size = 1.47

$$\frac{2}{169}\sqrt{13}\log\left(-\frac{|-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2-x+2}|}{2(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2-x+2})}\right) + \frac{2(77x-101)}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(3/2),x, algorithm="giac")

[Out] 2/169\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2))) + 2/299\*(77\*x - 101)/sqrt(3\*x^2 - x + 2)

**maple** [B] time = 0.01, size = 102, normalized size = 1.65

$$-\frac{2\sqrt{13}\operatorname{arctanh}\left(\frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{169} - \frac{2}{3\sqrt{3x^2-x+2}} + \frac{\frac{10x}{23} - \frac{5}{69}}{\sqrt{3x^2-x+2}} + \frac{1}{13\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} + \frac{\frac{24x}{299} - \frac{4}{299}}{\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(2\*x+1)/(3\*x^2-x+2)^(3/2),x)

[Out] -2/3/(3\*x^2-x+2)^(1/2)+5/69\*(6\*x-1)/(3\*x^2-x+2)^(1/2)+1/13/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)+4/299\*(6\*x-1)/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)-2/169\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))

**maxima** [A] time = 0.97, size = 64, normalized size = 1.03

$$\frac{2}{169} \sqrt{13} \operatorname{arsinh} \left( \frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|} \right) + \frac{154x}{299\sqrt{3x^2-x+2}} - \frac{202}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(3/2),x, algorithm="maxima")

[Out] 2/169\*sqrt(13)\*arcsinh(8/23\*sqrt(23)\*x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1)) + 154/299\*x/sqrt(3\*x^2 - x + 2) - 202/299/sqrt(3\*x^2 - x + 2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)(3x^2 - x + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 4\*x^2 + 1)/((2\*x + 1)\*(3\*x^2 - x + 2)^(3/2)),x)

[Out] int((3\*x + 4\*x^2 + 1)/((2\*x + 1)\*(3\*x^2 - x + 2)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)(3x^2 - x + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)/(1+2\*x)/(3\*x\*\*2-x+2)\*\*(3/2),x)

[Out] Integral((4\*x\*\*2 + 3\*x + 1)/((2\*x + 1)\*(3\*x\*\*2 - x + 2)\*\*(3/2)), x)

$$3.246 \quad \int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}} dx$$

Optimal. Leaf size=87

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}} - \frac{4\sqrt{3x^2-x+2}}{169(2x+1)} + \frac{2 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

**Rubi [A]** time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 806, 724, 206}

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}} - \frac{4\sqrt{3x^2-x+2}}{169(2x+1)} + \frac{2 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)),x]

[Out] (-2\*(197 - 837\*x))/(3887\*sqrt[2 - x + 3\*x^2]) - (4\*sqrt[2 - x + 3\*x^2])/(169\*(1 + 2\*x)) + (2\*ArcTanh[(9 - 8\*x)/(2\*sqrt[13]\*sqrt[2 - x + 3\*x^2])])/(169\*sqrt[13])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 806

Int[((d\_) + (e\_)\*(x\_)^m)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^p, x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m+1)\*(a + b\*x + c\*x^2)^(p+1))/(2\*(p+1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m+1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &



& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 (2 - x + 3x^2)^{3/2}} dx &= -\frac{2(197 - 837x)}{3887\sqrt{2 - x + 3x^2}} + \frac{2}{23} \int \frac{\frac{184}{169} - \frac{230x}{169}}{(1 + 2x)^2 \sqrt{2 - x + 3x^2}} dx \\ &= -\frac{2(197 - 837x)}{3887\sqrt{2 - x + 3x^2}} - \frac{4\sqrt{2 - x + 3x^2}}{169(1 + 2x)} - \frac{2}{169} \int \frac{1}{(1 + 2x)\sqrt{2 - x + 3x^2}} dx \\ &= -\frac{2(197 - 837x)}{3887\sqrt{2 - x + 3x^2}} - \frac{4\sqrt{2 - x + 3x^2}}{169(1 + 2x)} + \frac{4}{169} \text{Subst}\left(\int \frac{1}{52 - x^2} dx, x, \frac{9}{\sqrt{2 - x + 3x^2}}\right) \\ &= -\frac{2(197 - 837x)}{3887\sqrt{2 - x + 3x^2}} - \frac{4\sqrt{2 - x + 3x^2}}{169(1 + 2x)} + \frac{2 \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{2 - x + 3x^2}}\right)}{169\sqrt{13}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 74, normalized size = 0.85

$$\frac{2(1536x^2 + 489x - 289)}{3887(2x + 1)\sqrt{3x^2 - x + 2}} + \frac{2 \tanh^{-1}\left(\frac{9 - 8x}{2\sqrt{13}\sqrt{3x^2 - x + 2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)),x]

[Out] (2\*(-289 + 489\*x + 1536\*x^2))/(3887\*(1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]) + (2\*ArcTanH[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(169\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.51, size = 85, normalized size = 0.98

$$\frac{2(1536x^2 + 489x - 289)}{3887(2x + 1)\sqrt{3x^2 - x + 2}} - \frac{4 \tanh^{-1}\left(-\frac{2\sqrt{3x^2 - x + 2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)),x]

[Out] (2\*(-289 + 489\*x + 1536\*x^2))/(3887\*(1 + 2\*x)\*Sqrt[2 - x + 3\*x^2]) - (4\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(169\*Sqrt[13])

**fricas [A]** time = 1.05, size = 106, normalized size = 1.22

$$\frac{23\sqrt{13}(6x^3 + x^2 + 3x + 2) \log\left(\frac{4\sqrt{13}\sqrt{3x^2 - x + 2}(8x - 9) - 220x^2 + 196x - 185}{4x^2 + 4x + 1}\right) + 26(1536x^2 + 489x - 289)\sqrt{3x^2 - x + 2}}{50531(6x^3 + x^2 + 3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2-x+2)^(3/2),x, algorithm="fricas")

[Out] 1/50531\*(23\*sqrt(13)\*(6\*x^3 + x^2 + 3\*x + 2)\*log((4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) - 220\*x^2 + 196\*x - 185)/(4\*x^2 + 4\*x + 1)) + 26\*(1536\*x^2 + 489\*x - 289)\*sqrt(3\*x^2 - x + 2))/(6\*x^3 + x^2 + 3\*x + 2)

**giac [B]** time = 0.30, size = 168, normalized size = 1.93

$$-\frac{2}{50531}\sqrt{13}(256\sqrt{13}\sqrt{3} + 23\log(\sqrt{13}\sqrt{3} - 4))\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{2\left(\frac{\frac{1047}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)} + \frac{299}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1} - \frac{768}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1} + \frac{13}{(2x+1)^2} + 3}} + \frac{2\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1} + \frac{13}{(2x+1)^2} + 3} + \frac{\sqrt{13}}{2x+1}\right) - 4\right)}{2197\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2-x+2)^(3/2),x, algorithm="giac")

[Out] -2/50531\*sqrt(13)\*(256\*sqrt(13)\*sqrt(3) + 23\*log(sqrt(13)\*sqrt(3) - 4))\*sgn(1/(2\*x + 1)) - 2/3887\*((1047/sgn(1/(2\*x + 1)) + 299/((2\*x + 1)\*sgn(1/(2\*x + 1))))/(2\*x + 1) - 768/sgn(1/(2\*x + 1)))/sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + 2/2197\*sqrt(13)\*log(sqrt(13)\*(sqrt(-8/(2\*x + 1) + 13/(2\*x + 1)^2 + 3) + sqrt(13)/(2\*x + 1)) - 4)/sgn(1/(2\*x + 1))

**maple [A]** time = 0.01, size = 109, normalized size = 1.25

$$\frac{2\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{2197} + \frac{\frac{12x}{23} - \frac{2}{23}}{\sqrt{3x^2-x+2}} - \frac{1}{169\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{82(6x-1)}{3887\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{1}{26(x+\frac{1}{2})\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(3/2), x)`

[Out]  $2/23*(6*x-1)/(3*x^2-x+2)^{(1/2)} - 1/169/(-4*x+3*(x+1/2)^2+5/4)^{(1/2)} - 82/3887*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^{(1/2)} + 2/2197*13^{(1/2)}*\operatorname{arctanh}(2/13*(-4*x+9/2)*13^{(1/2)/(-16*x+12*(x+1/2)^2+5)^{(1/2)}) - 1/26/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^{(1/2)}$

**maxima [A]** time = 0.96, size = 96, normalized size = 1.10

$$-\frac{2}{2197}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{1536x}{3887\sqrt{3x^2-x+2}} - \frac{279}{3887\sqrt{3x^2-x+2}} - \frac{1}{13(2\sqrt{3x^2-x+2}x + \sqrt{3x^2-x+2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2), x, algorithm="maxima")`

[Out]  $-2/2197*\operatorname{sqrt}(13)*\operatorname{arcsinh}(8/23*\operatorname{sqrt}(23)*x/\operatorname{abs}(2*x+1) - 9/23*\operatorname{sqrt}(23)/\operatorname{abs}(2*x+1)) + 1536/3887*x/\operatorname{sqrt}(3*x^2-x+2) - 279/3887/\operatorname{sqrt}(3*x^2-x+2) - 1/13/(2*\operatorname{sqrt}(3*x^2-x+2)*x + \operatorname{sqrt}(3*x^2-x+2))$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 (3x^2 - x + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)`

[Out] `int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 (3x^2 - x + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2), x)`

[Out] `Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)`

$$3.247 \quad \int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}} dx$$

Optimal. Leaf size=112

$$\frac{2(3693x + 2363)}{50531\sqrt{3x^2 - x + 2}} - \frac{4\sqrt{3x^2 - x + 2}}{2197(2x + 1)} - \frac{2\sqrt{3x^2 - x + 2}}{169(2x + 1)^2} - \frac{487 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

**Rubi [A]** time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x + 2363)}{50531\sqrt{3x^2 - x + 2}} - \frac{4\sqrt{3x^2 - x + 2}}{2197(2x + 1)} - \frac{2\sqrt{3x^2 - x + 2}}{169(2x + 1)^2} - \frac{487 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2)),x]

[Out] (2\*(2363 + 3693\*x))/(50531\*sqrt[2 - x + 3\*x^2]) - (2\*sqrt[2 - x + 3\*x^2])/(169\*(1 + 2\*x)^2) - (4\*sqrt[2 - x + 3\*x^2])/(2197\*(1 + 2\*x)) - (487\*ArcTanh[(9 - 8\*x)/(2\*sqrt[13]\*sqrt[2 - x + 3\*x^2])])/(2197\*sqrt[13])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &

& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)^3 (2 - x + 3x^2)^{3/2}} dx &= \frac{2(2363 + 3693x)}{50531\sqrt{2 - x + 3x^2}} + \frac{2}{23} \int \frac{\frac{8349}{2197} + \frac{20838x}{2197} + \frac{23828x^2}{2197}}{(1 + 2x)^3 \sqrt{2 - x + 3x^2}} dx \\
&= \frac{2(2363 + 3693x)}{50531\sqrt{2 - x + 3x^2}} - \frac{2\sqrt{2 - x + 3x^2}}{169(1 + 2x)^2} - \frac{1}{299} \int \frac{-\frac{11615}{169} - \frac{22034x}{169}}{(1 + 2x)^2 \sqrt{2 - x + 3x^2}} dx \\
&= \frac{2(2363 + 3693x)}{50531\sqrt{2 - x + 3x^2}} - \frac{2\sqrt{2 - x + 3x^2}}{169(1 + 2x)^2} - \frac{4\sqrt{2 - x + 3x^2}}{2197(1 + 2x)} + \frac{487 \int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}}{2197} \\
&= \frac{2(2363 + 3693x)}{50531\sqrt{2 - x + 3x^2}} - \frac{2\sqrt{2 - x + 3x^2}}{169(1 + 2x)^2} - \frac{4\sqrt{2 - x + 3x^2}}{2197(1 + 2x)} - \frac{974 \operatorname{Subst}\left(\int \frac{1}{52-x^2}\right)}{2197} \\
&= \frac{2(2363 + 3693x)}{50531\sqrt{2 - x + 3x^2}} - \frac{2\sqrt{2 - x + 3x^2}}{169(1 + 2x)^2} - \frac{4\sqrt{2 - x + 3x^2}}{2197(1 + 2x)} - \frac{487 \tanh^{-1}\left(\frac{9}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 79, normalized size = 0.71

$$\frac{2(14496x^3 + 23281x^2 + 13306x + 1673)}{50531(2x + 1)^2\sqrt{3x^2 - x + 2}} - \frac{487 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2)), x]

[Out] (2\*(1673 + 13306\*x + 23281\*x^2 + 14496\*x^3))/(50531\*(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]) - (487\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(2197\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.59, size = 90, normalized size = 0.80

$$\frac{974 \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{2197\sqrt{13}} + \frac{2(14496x^3 + 23281x^2 + 13306x + 1673)}{50531(2x + 1)^2\sqrt{3x^2 - x + 2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(3/2)), x]

[Out] (2\*(1673 + 13306\*x + 23281\*x^2 + 14496\*x^3))/(50531\*(1 + 2\*x)^2\*Sqrt[2 - x + 3\*x^2]) + (974\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(2197\*Sqrt[13])

**fricas [A]** time = 1.03, size = 126, normalized size = 1.12

$$\frac{11201\sqrt{13}(12x^4 + 8x^3 + 7x^2 + 7x + 2) \log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right) + 52(14496x^3 + 23281x^2 + 13306x + 1673)\sqrt{3x^2-x+2}}{1313806(12x^4 + 8x^3 + 7x^2 + 7x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2-x+2)^(3/2),x, algorithm="fricas")

[Out] 1/1313806\*(11201\*sqrt(13)\*(12\*x^4 + 8\*x^3 + 7\*x^2 + 7\*x + 2)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 52\*(14496\*x^3 + 23281\*x^2 + 13306\*x + 1673)\*sqrt(3\*x^2 - x + 2))/(12\*x^4 + 8\*x^3 + 7\*x^2 + 7\*x + 2)

**giac [B]** time = 0.31, size = 223, normalized size = 1.99

$$\frac{487}{28561}\sqrt{13} \log\left(\frac{-4\sqrt{3}x-2\sqrt{3}-2\sqrt{3}+4\sqrt{3x^2-x+2}}{2(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2})}\right) + \frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}} + \frac{2(62(\sqrt{3}x-\sqrt{3x^2-x+2})^3-37\sqrt{3}(\sqrt{3}x-\sqrt{3x^2-x+2})^2+263\sqrt{3}x-71\sqrt{3}-263\sqrt{3x^2-x+2})}{2197(2(\sqrt{3}x-\sqrt{3x^2-x+2})^2+2\sqrt{3}(\sqrt{3}x-\sqrt{3x^2-x+2})-5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2-x+2)^(3/2),x, algorithm="giac")

[Out] 487/28561\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2))) + 2/50531\*(3693\*x + 2363)/sqrt(3\*x^2 - x + 2) + 2/2197\*(62\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^3 - 37\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^2 + 263\*sqrt(3)\*x - 71\*sqrt(3) - 263\*sqrt(3\*x^2 - x + 2))/(2\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^2 + 2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) - 5)^2

**maple [A]** time = 0.01, size = 111, normalized size = 0.99

$$\frac{487\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{28561} + \frac{487}{4394\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} + \frac{\frac{7248x}{50531} - \frac{1208}{50531}}{\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} + \frac{3}{338(x+\frac{1}{2})\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{1}{104(x+\frac{1}{2})^2\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(2\*x+1)^3/(3\*x^2-x+2)^(3/2),x)

[Out] 487/4394/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)+1208/50531\*(6\*x-1)/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)-487/28561\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))+3/338/(x+1/2)/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)

**maxima [A]** time = 0.97, size = 145, normalized size = 1.29

$$\frac{487}{28561}\sqrt{13} \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{7248x}{50531\sqrt{3x^2-x+2}} + \frac{8785}{101062\sqrt{3x^2-x+2}} - \frac{1}{26(4\sqrt{3x^2-x+2x^2+4\sqrt{3x^2-x+2x+3\sqrt{3x^2-x+2}})} + \frac{3}{169(2\sqrt{3x^2-x+2x+3\sqrt{3x^2-x+2}})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
[Out] 487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 - x
+ 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(3*x
^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
mupad [F] time = 0.00, size = -1, normalized size = -0.01
```

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 (3x^2 - x + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 (3x^2 - x + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```



$$3.248 \quad \int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=86

$$\frac{2(12839 - 3871x)}{5589(3x^2 - x + 2)^{3/2}} + \frac{32}{27}\sqrt{3x^2 - x + 2} - \frac{28(42240x + 35809)}{128547\sqrt{3x^2 - x + 2}} - \frac{296 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Rubi [A] time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1660, 640, 619, 215}

$$\frac{2(12839 - 3871x)}{5589(3x^2 - x + 2)^{3/2}} + \frac{32}{27}\sqrt{3x^2 - x + 2} - \frac{28(42240x + 35809)}{128547\sqrt{3x^2 - x + 2}} - \frac{296 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (2\*(12839 - 3871\*x))/(5589\*(2 - x + 3\*x^2)^(3/2)) - (28\*(35809 + 42240\*x))/(128547\*Sqrt[2 - x + 3\*x^2]) + (32\*Sqrt[2 - x + 3\*x^2])/27 - (296\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(27\*Sqrt[3])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx &= \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{4361}{81} + \frac{7682x}{9} + \frac{2852x^2}{3} + 368x^3}{(2-x+3x^2)^{3/2}} dx \\
&= \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{4 \int \frac{\frac{37030}{9} + \frac{4232x}{3}}{\sqrt{2-x+3x^2}} dx}{1587} \\
&= \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} + \frac{296}{27} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} + \frac{296 \operatorname{Subst}\left(\int \frac{1}{\sqrt{2-x+3x^2}} dx\right)}{27\sqrt{3}} \\
&= \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} - \frac{296 \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{27\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 71, normalized size = 0.83

$$\frac{2 \left( 228528x^4 - 743712x^3 + 25890x^2 + 78292\sqrt{3} (3x^2 - x + 2)^{3/2} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right) - 358377x - 134217 \right)}{42849(3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out]  $(2*(-134217 - 358377*x + 25890*x^2 - 743712*x^3 + 228528*x^4 + 78292*\sqrt{3}*(2 - x + 3*x^2)^{3/2})*\text{ArcSinh}[(-1 + 6*x)/\sqrt{23}])/(42849*(2 - x + 3*x^2)^{3/2})$

**IntegrateAlgebraic [A]** time = 0.86, size = 75, normalized size = 0.87

$$\frac{2(76176x^4 - 247904x^3 + 8630x^2 - 119459x - 44739)}{14283(3x^2 - x + 2)^{3/2}} - \frac{296 \log(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^3\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out]  $(2*(-44739 - 119459*x + 8630*x^2 - 247904*x^3 + 76176*x^4))/(14283*(2 - x + 3*x^2)^{3/2}) - (296*\text{Log}[1 - 6*x + 2*\sqrt{3}*\sqrt{2 - x + 3*x^2}])/(27*\text{Sqrt}[3])$

**fricas [A]** time = 1.01, size = 117, normalized size = 1.36

$$\frac{2(39146\sqrt{3}(9x^4 - 6x^3 + 13x^2 - 4x + 4)\log(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25) + 3(76176x^4 - 247904x^3 + 8630x^2 - 119459x - 44739)\sqrt{3x^2 - x + 2})}{42849(9x^4 - 6x^3 + 13x^2 - 4x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2), x, algorithm="fricas")

[Out]  $2/42849*(39146*\sqrt{3}*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*\log(-4*\sqrt{3}*\sqrt{3*x^2 - x + 2}*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904*x^3 + 8630*x^2 - 119459*x - 44739)*\sqrt{3*x^2 - x + 2})/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)$

**giac [A]** time = 0.21, size = 67, normalized size = 0.78

$$-\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right) + \frac{2((2(8(4761x - 15494)x + 4315)x - 119459)x - 44739)}{14283(3x^2 - x + 2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^3\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2), x, algorithm="giac")

[Out]  $-296/81*\sqrt{3}*\log(-2*\sqrt{3}*(\sqrt{3}*x - \sqrt{3*x^2 - x + 2})) + 1) + 2/14283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x + 2)^{3/2}$

**maple [B]** time = 0.01, size = 163, normalized size = 1.90

$$\frac{32x^4}{3(3x^2 - x + 2)^{3/2}} - \frac{296x^3}{27(3x^2 - x + 2)^{3/2}} + \frac{8x^2}{27(3x^2 - x + 2)^{3/2}} - \frac{296x}{27\sqrt{3x^2 - x + 2}} - \frac{461x}{81(3x^2 - x + 2)^{3/2}} + \frac{296\sqrt{3}\operatorname{arcsinh}\left(\frac{6\sqrt{23}(x-2)}{23}\right)}{81} + \frac{13763x}{5589} - \frac{13763}{33534} + \frac{130528x}{42849} - \frac{65264}{128547} - \frac{148}{81\sqrt{3x^2 - x + 2}} - \frac{1727}{1458(3x^2 - x + 2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2), x)`

[Out]  $13763/33534*(6*x-1)/(3*x^2-x+2)^{(3/2)} - 296/27/(3*x^2-x+2)^{(1/2)}*x + 65264/1285$   
 $47*(6*x-1)/(3*x^2-x+2)^{(1/2)} + 296/81*3^{(1/2)}*\operatorname{arcsinh}(6/23*23^{(1/2)}*(x-1/6)) -$   
 $148/81/(3*x^2-x+2)^{(1/2)} - 1727/1458/(3*x^2-x+2)^{(3/2)} + 32/3*x^4/(3*x^2-x+2)^{(3/2)}$   
 $- 296/27*x^3/(3*x^2-x+2)^{(3/2)} + 8/27*x^2/(3*x^2-x+2)^{(3/2)} - 461/81*x/(3*x^2-x+2)^{(3/2)}$

**maxima** [B] time = 0.96, size = 202, normalized size = 2.35

$$\frac{32x^4}{3(3x^2-x+2)^{5/2}} + \frac{296}{42849} \left( \frac{426x}{\sqrt{3x^2-x+2}} - \frac{4761x^2}{(3x^2-x+2)^{3/2}} - \frac{71}{\sqrt{3x^2-x+2}} + \frac{805x}{(3x^2-x+2)^{3/2}} - \frac{2162}{(3x^2-x+2)^{3/2}} \right) + \frac{296}{81} \sqrt{3} \operatorname{arcsinh}\left(\frac{1}{23} \sqrt{23}(6x-1)\right) - \frac{42032}{42849} \sqrt{3x^2-x+2} - \frac{47072x}{42849 \sqrt{3x^2-x+2}} + \frac{52x^2}{9(3x^2-x+2)^{3/2}} - \frac{23104}{14283 \sqrt{3x^2-x+2}} - \frac{7742x}{1863(3x^2-x+2)^{3/2}} + \frac{1666}{1863(3x^2-x+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2), x, algorithm="maxima")`

[Out]  $32/3*x^4/(3*x^2 - x + 2)^{(3/2)} + 296/42849*x*(426*x/\operatorname{sqrt}(3*x^2 - x + 2) - 4$   
 $761*x^2/(3*x^2 - x + 2)^{(3/2)} - 71/\operatorname{sqrt}(3*x^2 - x + 2) + 805*x/(3*x^2 - x +$   
 $2)^{(3/2)} - 2162/(3*x^2 - x + 2)^{(3/2)}) + 296/81*\operatorname{sqrt}(3)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}($   
 $23)*(6*x - 1)) - 42032/42849*\operatorname{sqrt}(3*x^2 - x + 2) - 47072/42849*x/\operatorname{sqrt}(3*x^2$   
 $- x + 2) + 52/9*x^2/(3*x^2 - x + 2)^{(3/2)} - 23104/14283/\operatorname{sqrt}(3*x^2 - x + 2$   
 $) - 7742/1863*x/(3*x^2 - x + 2)^{(3/2)} + 1666/1863/(3*x^2 - x + 2)^{(3/2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2-x+2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)`

[Out] `int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2-x+2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2), x)`

[Out] `Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)`

$$3.249 \quad \int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=68

$$\frac{2(1249 - 2273x)}{1863(3x^2 - x + 2)^{3/2}} - \frac{8(23257 - 1473x)}{42849\sqrt{3x^2 - x + 2}} - \frac{16 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1660, 12, 619, 215}

$$\frac{2(1249 - 2273x)}{1863(3x^2 - x + 2)^{3/2}} - \frac{8(23257 - 1473x)}{42849\sqrt{3x^2 - x + 2}} - \frac{16 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (2\*(1249 - 2273\*x))/(1863\*(2 - x + 3\*x^2)^(3/2)) - (8\*(23257 - 1473\*x))/(42849\*Sqrt[2 - x + 3\*x^2]) - (16\*ArcSinh[(1 - 6\*x)/Sqrt[23]])/(9\*Sqrt[3])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx &= \frac{2(1249-2273x)}{1863(2-x+3x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{1802}{27} + \frac{1150x}{3} + 184x^2}{(2-x+3x^2)^{3/2}} dx \\
&= \frac{2(1249-2273x)}{1863(2-x+3x^2)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} + \frac{4 \int \frac{2116}{3\sqrt{2-x+3x^2}} dx}{1587} \\
&= \frac{2(1249-2273x)}{1863(2-x+3x^2)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} + \frac{16}{9} \int \frac{1}{\sqrt{2-x+3x^2}} dx \\
&= \frac{2(1249-2273x)}{1863(2-x+3x^2)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} + \frac{16 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}} dx, x, -1+6\right)}{9\sqrt{69}} \\
&= \frac{2(1249-2273x)}{1863(2-x+3x^2)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} - \frac{16 \sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 66, normalized size = 0.97

$$\frac{2\left(5892x^3 - 94992x^2 + 4232\sqrt{3}(3x^2 - x + 2)^{3/2} \sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right) + 17511x - 52443\right)}{14283(3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (2\*(-52443 + 17511\*x - 94992\*x^2 + 5892\*x^3 + 4232\*sqrt(3)\*(2 - x + 3\*x^2)^(3/2)\*ArcSinh[(-1 + 6\*x)/sqrt(23)])/(14283\*(2 - x + 3\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.72, size = 70, normalized size = 1.03

$$\frac{2(1964x^3 - 31664x^2 + 5837x - 17481)}{4761(3x^2 - x + 2)^{3/2}} - \frac{16 \log(2\sqrt{3}\sqrt{3x^2 - x + 2} - 6x + 1)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)^2\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (2\*(-17481 + 5837\*x - 31664\*x^2 + 1964\*x^3))/(4761\*(2 - x + 3\*x^2)^(3/2)) - (16\*Log[1 - 6\*x + 2\*Sqrt[3]\*Sqrt[2 - x + 3\*x^2]])/(9\*Sqrt[3])

**fricas [B]** time = 0.94, size = 112, normalized size = 1.65

$$\frac{2(2116\sqrt{3}(9x^4 - 6x^3 + 13x^2 - 4x + 4)\log(-4\sqrt{3}\sqrt{3x^2 - x + 2}(6x - 1) - 72x^2 + 24x - 25) + 3(1964x^3 - 31664x^2 + 5837x - 17481)\sqrt{3x^2 - x + 2})}{14283(9x^4 - 6x^3 + 13x^2 - 4x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2), x, algorithm="fricas")

[Out] 2/14283\*(2116\*sqrt(3)\*(9\*x^4 - 6\*x^3 + 13\*x^2 - 4\*x + 4)\*log(-4\*sqrt(3)\*sqrt(3\*x^2 - x + 2)\*(6\*x - 1) - 72\*x^2 + 24\*x - 25) + 3\*(1964\*x^3 - 31664\*x^2 + 5837\*x - 17481)\*sqrt(3\*x^2 - x + 2))/(9\*x^4 - 6\*x^3 + 13\*x^2 - 4\*x + 4)

**giac [A]** time = 0.24, size = 62, normalized size = 0.91

$$-\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 - x + 2}\right) + 1\right) + \frac{2((4(491x - 7916)x + 5837)x - 17481)}{4761(3x^2 - x + 2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2), x, algorithm="giac")

[Out] -16/27\*sqrt(3)\*log(-2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) + 1) + 2/4761\*((4\*(491\*x - 7916)\*x + 5837)\*x - 17481)/(3\*x^2 - x + 2)^(3/2)

**maple [B]** time = 0.01, size = 146, normalized size = 2.15

$$-\frac{16x^3}{9(3x^2 - x + 2)^{3/2}} - \frac{92x^2}{9(3x^2 - x + 2)^{3/2}} - \frac{67x}{27(3x^2 - x + 2)^{3/2}} - \frac{16x}{9\sqrt{3x^2 - x + 2}} + \frac{16\sqrt{3} \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x - \frac{1}{6}\right)}{23}\right)}{27} - \frac{2653}{486(3x^2 - x + 2)^{3/2}} + \frac{4585x - 4585}{1863(3x^2 - x + 2)^{3/2}} + \frac{37784x - 18892}{14283\sqrt{3x^2 - x + 2}} - \frac{8}{27\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)^2\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2), x)

[Out]  $-16/9/(3*x^2-x+2)^{(3/2)}*x^3-92/9/(3*x^2-x+2)^{(3/2)}*x^2-67/27/(3*x^2-x+2)^{(3/2)}*x-2653/486/(3*x^2-x+2)^{(3/2)}+4585/11178*(6*x-1)/(3*x^2-x+2)^{(3/2)}+18892/42849*(6*x-1)/(3*x^2-x+2)^{(1/2)}-16/9/(3*x^2-x+2)^{(1/2)}*x-8/27/(3*x^2-x+2)^{(1/2)}+16/27*3^{(1/2)}*\operatorname{arcsinh}(6/23*23^{(1/2)}*(x-1/6))$

**maxima** [B] time = 0.97, size = 185, normalized size = 2.72

$$\frac{16}{14283} \left( \frac{426x}{\sqrt{3x^2-x+2}} - \frac{4761x^2}{(3x^2-x+2)^{3/2}} - \frac{71}{\sqrt{3x^2-x+2}} + \frac{805x}{(3x^2-x+2)^{3/2}} - \frac{2162}{(3x^2-x+2)^{3/2}} \right) + \frac{16}{27} \sqrt{3} \operatorname{arcsinh} \left( \frac{1}{23} \sqrt{23} (6x-1) \right) - \frac{2272}{14283} \sqrt{3x^2-x+2} + \frac{28184x}{14283 \sqrt{3x^2-x+2}} - \frac{28x^2}{3(3x^2-x+2)^{3/2}} - \frac{2956}{4761 \sqrt{3x^2-x+2}} - \frac{106x}{621(3x^2-x+2)^{3/2}} - \frac{3394}{621(3x^2-x+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")`

[Out]  $16/14283*x*(426*x/\sqrt{3*x^2-x+2}-4761*x^2/(3*x^2-x+2)^{(3/2)}-71/\sqrt{3*x^2-x+2}+805*x/(3*x^2-x+2)^{(3/2)}-2162/(3*x^2-x+2)^{(3/2)})+16/27*\sqrt{3}*\operatorname{arcsinh}(1/23*\sqrt{23}*(6*x-1))-2272/14283*\sqrt{3*x^2-x+2}+28184/14283*x/\sqrt{3*x^2-x+2}-28/3*x^2/(3*x^2-x+2)^{(3/2)}-2956/4761/\sqrt{3*x^2-x+2}-106/621*x/(3*x^2-x+2)^{(3/2)}-3394/621/(3*x^2-x+2)^{(3/2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{(3x^2-x+2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x+1)^2*(3*x+4*x^2+1))/(3*x^2-x+2)^(5/2),x)`

[Out] `int(((2*x+1)^2*(3*x+4*x^2+1))/(3*x^2-x+2)^(5/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{(3x^2-x+2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)`

[Out] `Integral((2*x+1)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)`



$$3.250 \quad \int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=47

$$-\frac{4(3889 - 4290x)}{14283\sqrt{3x^2 - x + 2}} - \frac{2(367x + 73)}{621(3x^2 - x + 2)^{3/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {1660, 636}

$$-\frac{4(3889 - 4290x)}{14283\sqrt{3x^2 - x + 2}} - \frac{2(367x + 73)}{621(3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (-2\*(73 + 367\*x))/(621\*(2 - x + 3\*x^2)^(3/2)) - (4\*(3889 - 4290\*x))/(14283\*  
Sqrt[2 - x + 3\*x^2])

Rule 636

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(3/2), x\_Symbol] :> Simp[(-2\*(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

Rubi steps

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}} dx = -\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{577}{9} + 92x}{(2-x+3x^2)^{3/2}} dx$$

$$= -\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}} - \frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

**Mathematica [A]** time = 0.06, size = 33, normalized size = 0.70

$$\frac{2(2860x^3 - 3546x^2 + 1833x - 1915)}{1587(3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (2\*(-1915 + 1833\*x - 3546\*x^2 + 2860\*x^3))/(1587\*(2 - x + 3\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.55, size = 33, normalized size = 0.70

$$\frac{2(2860x^3 - 3546x^2 + 1833x - 1915)}{1587(3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 2\*x)\*(1 + 3\*x + 4\*x^2))/(2 - x + 3\*x^2)^(5/2), x]

[Out] (2\*(-1915 + 1833\*x - 3546\*x^2 + 2860\*x^3))/(1587\*(2 - x + 3\*x^2)^(3/2))

**fricas [A]** time = 1.20, size = 51, normalized size = 1.09

$$\frac{2(2860x^3 - 3546x^2 + 1833x - 1915)\sqrt{3x^2 - x + 2}}{1587(9x^4 - 6x^3 + 13x^2 - 4x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2), x, algorithm="fricas")

[Out] 2/1587\*(2860\*x^3 - 3546\*x^2 + 1833\*x - 1915)\*sqrt(3\*x^2 - x + 2)/(9\*x^4 - 6\*x^3 + 13\*x^2 - 4\*x + 4)

**giac** [A] time = 0.20, size = 28, normalized size = 0.60

$$\frac{2((2(1430x - 1773)x + 1833)x - 1915)}{1587(3x^2 - x + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2),x, algorithm="giac")

[Out] 2/1587\*((2\*(1430\*x - 1773)\*x + 1833)\*x - 1915)/(3\*x^2 - x + 2)^(3/2)

**maple** [A] time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{5720}{1587}x^3 - \frac{2364}{529}x^2 + \frac{1222}{529}x - \frac{3830}{1587}}{(3x^2 - x + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x+1)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2),x)

[Out] 2/1587/(3\*x^2-x+2)^(3/2)\*(2860\*x^3-3546\*x^2+1833\*x-1915)

**maxima** [A] time = 0.44, size = 76, normalized size = 1.62

$$\frac{5720x}{4761\sqrt{3x^2-x+2}} - \frac{8x^2}{3(3x^2-x+2)^{\frac{3}{2}}} - \frac{2860}{14283\sqrt{3x^2-x+2}} - \frac{182x}{621(3x^2-x+2)^{\frac{3}{2}}} - \frac{1250}{621(3x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2\*x)\*(4\*x^2+3\*x+1)/(3\*x^2-x+2)^(5/2),x, algorithm="maxima")

[Out] 5720/4761\*x/sqrt(3\*x^2 - x + 2) - 8/3\*x^2/(3\*x^2 - x + 2)^(3/2) - 2860/14283/sqrt(3\*x^2 - x + 2) - 182/621\*x/(3\*x^2 - x + 2)^(3/2) - 1250/621/(3\*x^2 - x + 2)^(3/2)

**mupad** [B] time = 4.20, size = 49, normalized size = 1.04

$$\frac{442x - 5720x(3x^2 - x + 2) + 15556x^2 + 11490}{\sqrt{3x^2 - x + 2}(14283x^2 - 4761x + 9522)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 1)\*(3\*x + 4\*x^2 + 1))/(3\*x^2 - x + 2)^(5/2),x)

[Out]  $-(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^{(1/2)}*(14283*x^2 - 4761*x + 9522))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)`

[Out] `Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)`

$$3.251 \quad \int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=85

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}} - \frac{2(101-77x)}{897(3x^2-x+2)^{3/2}} - \frac{8 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

**Rubi** [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1646, 822, 12, 724, 206}

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}} - \frac{2(101-77x)}{897(3x^2-x+2)^{3/2}} - \frac{8 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 - x + 3\*x^2)^(5/2)), x]

[Out] (-2\*(101 - 77\*x))/(897\*(2 - x + 3\*x^2)^(3/2)) - (4\*(691 - 13668\*x))/(268203\*sqrt[2 - x + 3\*x^2]) - (8\*ArcTanh[(9 - 8\*x)/(2\*sqrt[13]\*sqrt[2 - x + 3\*x^2])])/(169\*sqrt[13])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 822

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

### Rule 1646

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}} dx &= -\frac{2(101-77x)}{897(2-x+3x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{223}{13} + \frac{308x}{13}}{(1+2x)(2-x+3x^2)^{3/2}} dx \\
&= -\frac{2(101-77x)}{897(2-x+3x^2)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{4 \int \frac{3174}{13(1+2x)\sqrt{2-x+3x^2}} dx}{20631} \\
&= -\frac{2(101-77x)}{897(2-x+3x^2)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{8}{169} \int \frac{1}{(1+2x)\sqrt{2-x+3x^2}} dx \\
&= -\frac{2(101-77x)}{897(2-x+3x^2)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{16}{169} \text{Subst} \left( \int \frac{1}{52-x^2} dx, x, \right. \\
&= -\frac{2(101-77x)}{897(2-x+3x^2)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{8 \tanh^{-1} \left( \frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}} \right)}{169\sqrt{13}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 72, normalized size = 0.85

$$\frac{2(82008x^3 - 31482x^2 + 79077x - 32963)}{268203(3x^2 - x + 2)^{3/2}} - \frac{8 \tanh^{-1} \left( \frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}} \right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 - x + 3\*x^2)^(5/2)),x]

[Out] (2\*(-32963 + 79077\*x - 31482\*x^2 + 82008\*x^3))/(268203\*(2 - x + 3\*x^2)^(3/2)) - (8\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(169\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.62, size = 83, normalized size = 0.98

$$\frac{16 \tanh^{-1} \left( -\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}} \right)}{169\sqrt{13}} + \frac{2(82008x^3 - 31482x^2 + 79077x - 32963)}{268203(3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)\*(2 - x + 3\*x^2)^(5/2)),x]

[Out] (2\*(-32963 + 79077\*x - 31482\*x^2 + 82008\*x^3))/(268203\*(2 - x + 3\*x^2)^(3/2)) + (16\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(169\*Sqrt[13])

**fricas** [A] time = 0.95, size = 126, normalized size = 1.48

$$\frac{2 \left( 3174 \sqrt{13} (9x^4 - 6x^3 + 13x^2 - 4x + 4) \log \left( -\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1} \right) + 13(82008x^3 - 31482x^2 + 79077x - 32963)\sqrt{3x^2-x+2} \right)}{3486639(9x^4 - 6x^3 + 13x^2 - 4x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(5/2),x, algorithm="fricas")

[Out] 2/3486639\*(3174\*sqrt(13)\*(9\*x^4 - 6\*x^3 + 13\*x^2 - 4\*x + 4)\*log(-4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1) + 13\*(82008\*x^3 - 31482\*x^2 + 79077\*x - 32963)\*sqrt(3\*x^2 - x + 2))/(9\*x^4 - 6\*x^3 + 13\*x^2 - 4\*x + 4)

**giac** [A] time = 0.43, size = 101, normalized size = 1.19

$$\frac{8}{2197} \sqrt{13} \log \left( -\frac{|-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2-x+2}|}{2(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2-x+2})} \right) + \frac{2(3(6(4556x - 1749)x + 26359)x - 32963)}{268203(3x^2 - x + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(5/2),x, algorithm="giac")

[Out] 8/2197\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2)) + 2/268203\*(3\*(6\*(4556\*x - 1749)\*x + 26359)\*x - 32963)/(3\*x^2 - x + 2)^(3/2)

**maple** [B] time = 0.01, size = 158, normalized size = 1.86

$$\frac{8\sqrt{13} \operatorname{arctanh} \left( \frac{2(-4x+\frac{9}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}} \right)}{2197} - \frac{2}{9(3x^2-x+2)^{\frac{3}{2}}} + \frac{10x}{69} - \frac{5}{207} + \frac{80x}{529} - \frac{40}{1587} + \frac{1}{39(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} + \frac{8x}{299} - \frac{4}{897} + \frac{4704x}{89401} - \frac{784}{89401} + \frac{4}{169\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+3\*x+1)/(2\*x+1)/(3\*x^2-x+2)^(5/2),x)

[Out] -2/9/(3\*x^2-x+2)^(3/2)+5/207\*(6\*x-1)/(3\*x^2-x+2)^(3/2)+40/1587\*(6\*x-1)/(3\*x^2-x+2)^(1/2)+1/39/(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)+4/897\*(6\*x-1)/(-4\*x+3\*(x+1/2)^2+5/4)^(3/2)+784/89401\*(6\*x-1)/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)+4/169/(-4\*x+3\*(x+1/2)^2+5/4)^(1/2)-8/2197\*13^(1/2)\*arctanh(2/13\*(-4\*x+9/2)\*13^(1/2)/(-16\*x+12\*(x+1/2)^2+5)^(1/2))

**maxima** [A] time = 0.96, size = 93, normalized size = 1.09

$$\frac{8}{2197} \sqrt{13} \operatorname{arsinh} \left( \frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|} \right) + \frac{18224x}{89401\sqrt{3x^2-x+2}} - \frac{2764}{268203\sqrt{3x^2-x+2}} + \frac{154x}{897(3x^2-x+2)^{\frac{3}{2}}} - \frac{202}{897(3x^2-x+2)^{\frac{3}{2}}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)/(3\*x^2-x+2)^(5/2),x, algorithm="maxima")

[Out] 8/2197\*sqrt(13)\*arcsinh(8/23\*sqrt(23)\*x/abs(2\*x + 1) - 9/23\*sqrt(23)/abs(2\*x + 1)) + 18224/89401\*x/sqrt(3\*x^2 - x + 2) - 2764/268203/sqrt(3\*x^2 - x + 2) + 154/897\*x/(3\*x^2 - x + 2)^(3/2) - 202/897/(3\*x^2 - x + 2)^(3/2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)(3x^2 - x + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x + 4\*x^2 + 1)/((2\*x + 1)\*(3\*x^2 - x + 2)^(5/2)),x)

[Out] int((3\*x + 4\*x^2 + 1)/((2\*x + 1)\*(3\*x^2 - x + 2)^(5/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)(3x^2 - x + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)/(1+2\*x)/(3\*x\*\*2-x+2)\*\*(5/2),x)

[Out] Integral((4\*x\*\*2 + 3\*x + 1)/((2\*x + 1)\*(3\*x\*\*2 - x + 2)\*\*(5/2)), x)

$$3.252 \quad \int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=110

$$\frac{24(841 - 6633x)}{1162213\sqrt{3x^2 - x + 2}} - \frac{16\sqrt{3x^2 - x + 2}}{2197(2x + 1)} - \frac{2(197 - 837x)}{11661(3x^2 - x + 2)^{3/2}} - \frac{56 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

**Rubi [A]** time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 806, 724, 206}

$$\frac{24(841 - 6633x)}{1162213\sqrt{3x^2 - x + 2}} - \frac{16\sqrt{3x^2 - x + 2}}{2197(2x + 1)} - \frac{2(197 - 837x)}{11661(3x^2 - x + 2)^{3/2}} - \frac{56 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2)), x]

[Out] (-2\*(197 - 837\*x))/(11661\*(2 - x + 3\*x^2)^(3/2)) - (24\*(841 - 6633\*x))/(1162213\*Sqrt[2 - x + 3\*x^2]) - (16\*Sqrt[2 - x + 3\*x^2])/(2197\*(1 + 2\*x)) - (56 \*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(2197\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 806

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &

& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1 + 3x + 4x^2}{(1 + 2x)^2 (2 - x + 3x^2)^{5/2}} dx &= -\frac{2(197 - 837x)}{11661 (2 - x + 3x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{2226}{169} + \frac{462x}{13} + \frac{6696x^2}{169}}{(1 + 2x)^2 (2 - x + 3x^2)^{3/2}} dx \\ &= -\frac{2(197 - 837x)}{11661 (2 - x + 3x^2)^{3/2}} - \frac{24(841 - 6633x)}{1162213\sqrt{2 - x + 3x^2}} + \frac{4 \int \frac{\frac{50784}{2197} + \frac{19044x}{2197}}{(1+2x)^2 \sqrt{2-x+3x^2}} dx}{1587} \\ &= -\frac{2(197 - 837x)}{11661 (2 - x + 3x^2)^{3/2}} - \frac{24(841 - 6633x)}{1162213\sqrt{2 - x + 3x^2}} - \frac{16\sqrt{2 - x + 3x^2}}{2197(1 + 2x)} + \frac{56 \int}{2197} \\ &= -\frac{2(197 - 837x)}{11661 (2 - x + 3x^2)^{3/2}} - \frac{24(841 - 6633x)}{1162213\sqrt{2 - x + 3x^2}} - \frac{16\sqrt{2 - x + 3x^2}}{2197(1 + 2x)} - \frac{112 \int}{2197} \\ &= -\frac{2(197 - 837x)}{11661 (2 - x + 3x^2)^{3/2}} - \frac{24(841 - 6633x)}{1162213\sqrt{2 - x + 3x^2}} - \frac{16\sqrt{2 - x + 3x^2}}{2197(1 + 2x)} - \frac{56 \tan^{-1}}{2197} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 111, normalized size = 1.01

$$\frac{26(1318464x^4 + 133308x^3 + 1021566x^2 + 569989x - 170239) - 88872\sqrt{13}\sqrt{3x^2 - x + 2}(6x^3 + x^2 + 3x + 2)\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{45326307(2x+1)(3x^2-x+2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2)),x]

[Out] (26\*(-170239 + 569989\*x + 1021566\*x^2 + 133308\*x^3 + 1318464\*x^4) - 88872\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2]\*(2 + 3\*x + x^2 + 6\*x^3)\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(45326307\*(1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2))

**IntegrateAlgebraic [A]** time = 0.72, size = 95, normalized size = 0.86

$$\frac{112 \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{2197\sqrt{13}} + \frac{2(1318464x^4 + 133308x^3 + 1021566x^2 + 569989x - 170239)}{3486639(2x+1)(3x^2-x+2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^2\*(2 - x + 3\*x^2)^(5/2)),x]

[Out] (2\*(-170239 + 569989\*x + 1021566\*x^2 + 133308\*x^3 + 1318464\*x^4))/(3486639\*(1 + 2\*x)\*(2 - x + 3\*x^2)^(3/2)) + (112\*ArcTanh[Sqrt[3/13] + 2\*Sqrt[3/13]\*x - (2\*Sqrt[2 - x + 3\*x^2])/Sqrt[13]])/(2197\*Sqrt[13])

**fricas [A]** time = 0.74, size = 141, normalized size = 1.28

$$\frac{2\left(22218\sqrt{13}(18x^5 - 3x^4 + 20x^3 + 5x^2 + 4x + 4)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right) + 13(1318464x^4 + 133308x^3 + 1021566x^2 + 569989x - 170239)\sqrt{3x^2-x+2}\right)}{45326307(18x^5 - 3x^4 + 20x^3 + 5x^2 + 4x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2-x+2)^(5/2),x, algorithm="fricas")

[Out] 2/45326307\*(22218\*sqrt(13)\*(18\*x^5 - 3\*x^4 + 20\*x^3 + 5\*x^2 + 4\*x + 4)\*log(-4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 13\*(1318464\*x^4 + 133308\*x^3 + 1021566\*x^2 + 569989\*x - 170239)\*sqrt(3\*x^2 - x + 2)/(18\*x^5 - 3\*x^4 + 20\*x^3 + 5\*x^2 + 4\*x + 4)

**giac [B]** time = 0.36, size = 233, normalized size = 2.12

$$\frac{\frac{56}{15108769}\sqrt{13}(872\sqrt{13}\sqrt{3}-529\log(\sqrt{13}\sqrt{3}-4))\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{56\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{28561\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{3486639\left(\frac{8}{2x+1}-\frac{13}{(2x+1)^2}-3\right)\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}} + \frac{8\left(\frac{13\left(\frac{77756}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}-\frac{20631}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{1399650}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{625905}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}-\frac{164808}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3486639\left(\frac{8}{2x+1}-\frac{13}{(2x+1)^2}-3\right)\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^2/(3\*x^2-x+2)^(5/2),x, algorithm="giac")

[Out]  $-56/15108769*\sqrt{13}*(872*\sqrt{13}*\sqrt{3} - 529*\log(\sqrt{13}*\sqrt{3}) - 4)$   
 $)*\operatorname{sgn}(1/(2*x + 1)) - 56/28561*\sqrt{13}*\log(\sqrt{13}*(\sqrt{-8/(2*x + 1)} + 13$   
 $/(2*x + 1)^2 + 3) + \sqrt{13}/(2*x + 1)) - 4)/\operatorname{sgn}(1/(2*x + 1)) + 8/3486639*($   
 $((13*(77756/\operatorname{sgn}(1/(2*x + 1))) + 20631/((2*x + 1)*\operatorname{sgn}(1/(2*x + 1))))/(2*x + 1$   
 $) - 1399650/\operatorname{sgn}(1/(2*x + 1)))/(2*x + 1) + 625905/\operatorname{sgn}(1/(2*x + 1)))/(2*x + 1$   
 $) - 164808/\operatorname{sgn}(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*\sqrt{-8/(2$   
 $*x + 1) + 13/(2*x + 1)^2 + 3))$

**maple [A]** time = 0.01, size = 165, normalized size = 1.50

$$-\frac{56\sqrt{13} \operatorname{arctanh}\left(\frac{2(-4x+\frac{5}{2})\sqrt{13}}{13\sqrt{-16x+12(x+\frac{1}{2})^2+5}}\right)}{28561} + \frac{4x-\frac{2}{69}}{(3x^2-x+2)^{\frac{3}{2}}} + \frac{\frac{96x-16}{387}-\frac{16}{387}}{\sqrt{3x^2-x+2}} + \frac{7}{507(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} - \frac{128(6x-1)}{11661(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} - \frac{10736(6x-1)}{1162213\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} + \frac{28}{2197\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{1}{26(x+\frac{1}{2})(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^{(5/2)}, x)$

[Out]  $2/69*(6*x-1)/(3*x^2-x+2)^{(3/2)}+16/529*(6*x-1)/(3*x^2-x+2)^{(1/2)}+7/507/(-4*x$   
 $+3*(x+1/2)^2+5/4)^{(3/2)}-128/11661*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^{(3/2)}-1073$   
 $6/1162213*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^{(1/2)}+28/2197/(-4*x+3*(x+1/2)^2+5/$   
 $4)^{(1/2)}-56/28561*13^{(1/2)}*\operatorname{arctanh}(2/13*(-4*x+9/2)*13^{(1/2)})/(-16*x+12*(x+1/$   
 $2)^2+5)^{(1/2)}-1/26/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^{(3/2)}$

**maxima [A]** time = 0.97, size = 125, normalized size = 1.14

$$\frac{56}{28561} \sqrt{13} \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{146496x}{1162213\sqrt{3x^2-x+2}} - \frac{9604}{1162213\sqrt{3x^2-x+2}} + \frac{420x}{3887(3x^2-x+2)^{\frac{3}{2}}} - \frac{1}{13\left(2(3x^2-x+2)^{\frac{3}{2}}x + (3x^2-x+2)^{\frac{3}{2}}\right)} - \frac{49}{11661(3x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^{(5/2)}, x, \operatorname{algorithm}="maxima")$

[Out]  $56/28561*\sqrt{13}*\operatorname{arcsinh}(8/23*\sqrt{23}*x/\operatorname{abs}(2*x + 1) - 9/23*\sqrt{23}/\operatorname{abs}($   
 $2*x + 1)) + 146496/1162213*x/\sqrt{3*x^2 - x + 2} - 9604/1162213/\sqrt{3*x^2$   
 $- x + 2} + 420/3887*x/(3*x^2 - x + 2)^{(3/2)} - 1/13/(2*(3*x^2 - x + 2)^{(3/2)}$   
 $*x + (3*x^2 - x + 2)^{(3/2)}) - 49/11661/(3*x^2 - x + 2)^{(3/2)}$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 (3x^2 - x + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^{(5/2)}), x)$

[Out]  $\operatorname{int}((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^{(5/2)}), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 (3x^2 - x + 2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+3\*x+1)/(1+2\*x)\*\*2/(3\*x\*\*2-x+2)\*\*(5/2), x)

[Out] Integral((4\*x\*\*2 + 3\*x + 1)/((2\*x + 1)\*\*2\*(3\*x\*\*2 - x + 2)\*\*(5/2)), x)

$$3.253 \quad \int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}} dx$$

Optimal. Leaf size=135

$$\frac{2(3693x + 2363)}{151593(3x^2 - x + 2)^{3/2}} - \frac{144\sqrt{3x^2 - x + 2}}{28561(2x + 1)} - \frac{8\sqrt{3x^2 - x + 2}}{2197(2x + 1)^2} + \frac{12(103526x + 25771)}{15108769\sqrt{3x^2 - x + 2}} - \frac{2084 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

**Rubi** [A] time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$ , Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x + 2363)}{151593(3x^2 - x + 2)^{3/2}} - \frac{144\sqrt{3x^2 - x + 2}}{28561(2x + 1)} - \frac{8\sqrt{3x^2 - x + 2}}{2197(2x + 1)^2} + \frac{12(103526x + 25771)}{15108769\sqrt{3x^2 - x + 2}} - \frac{2084 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2)), x]

[Out] (2\*(2363 + 3693\*x))/(151593\*(2 - x + 3\*x^2)^(3/2)) + (12\*(25771 + 103526\*x))/(15108769\*Sqrt[2 - x + 3\*x^2]) - (8\*Sqrt[2 - x + 3\*x^2])/(2197\*(1 + 2\*x)^2) - (144\*Sqrt[2 - x + 3\*x^2])/(28561\*(1 + 2\*x)) - (2084\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(28561\*Sqrt[13])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

### Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{1 + 3x + 4x^2}{(1 + 2x)^3 (2 - x + 3x^2)^{5/2}} dx &= \frac{2(2363 + 3693x)}{151593 (2 - x + 3x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{32433}{2197} + \frac{106830x}{2197} + \frac{160116x^2}{2197} + \frac{59088x^3}{2197}}{(1 + 2x)^3 (2 - x + 3x^2)^{3/2}} dx \\
&= \frac{2(2363 + 3693x)}{151593 (2 - x + 3x^2)^{3/2}} + \frac{12(25771 + 103526x)}{15108769\sqrt{2 - x + 3x^2}} + \frac{4 \int \frac{\frac{1434648}{28561} + \frac{3345396x}{28561} + \frac{30978x^2}{28561}}{(1+2x)^3 \sqrt{2-x+3x^2}}}{1587} \\
&= \frac{2(2363 + 3693x)}{151593 (2 - x + 3x^2)^{3/2}} + \frac{12(25771 + 103526x)}{15108769\sqrt{2 - x + 3x^2}} - \frac{8\sqrt{2 - x + 3x^2}}{2197(1 + 2x)^2} - \frac{2 \int \frac{1}{(1+2x)^3}}{(1+2x)^3} \\
&= \frac{2(2363 + 3693x)}{151593 (2 - x + 3x^2)^{3/2}} + \frac{12(25771 + 103526x)}{15108769\sqrt{2 - x + 3x^2}} - \frac{8\sqrt{2 - x + 3x^2}}{2197(1 + 2x)^2} - \frac{144\sqrt{2 - x + 3x^2}}{28561(1 + 2x)^2} \\
&= \frac{2(2363 + 3693x)}{151593 (2 - x + 3x^2)^{3/2}} + \frac{12(25771 + 103526x)}{15108769\sqrt{2 - x + 3x^2}} - \frac{8\sqrt{2 - x + 3x^2}}{2197(1 + 2x)^2} - \frac{144\sqrt{2 - x + 3x^2}}{28561(1 + 2x)^2} \\
&= \frac{2(2363 + 3693x)}{151593 (2 - x + 3x^2)^{3/2}} + \frac{12(25771 + 103526x)}{15108769\sqrt{2 - x + 3x^2}} - \frac{8\sqrt{2 - x + 3x^2}}{2197(1 + 2x)^2} - \frac{144\sqrt{2 - x + 3x^2}}{28561(1 + 2x)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 89, normalized size = 0.66

$$\frac{2(20304864x^5 + 20074356x^4 + 19381992x^3 + 21890266x^2 + 10777477x + 847141)}{45326307(2x + 1)^2 (3x^2 - x + 2)^{3/2}} - \frac{2084 \tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2)), x]

[Out] (2\*(847141 + 10777477\*x + 21890266\*x^2 + 19381992\*x^3 + 20074356\*x^4 + 20304864\*x^5))/(45326307\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)) - (2084\*ArcTanh[(9 - 8\*x)/(2\*Sqrt[13]\*Sqrt[2 - x + 3\*x^2])])/(28561\*Sqrt[13])

**IntegrateAlgebraic [A]** time = 0.83, size = 100, normalized size = 0.74

$$\frac{4168 \tanh^{-1}\left(-\frac{2\sqrt{3x^2-x+2}}{\sqrt{13}} + 2\sqrt{\frac{3}{13}}x + \sqrt{\frac{3}{13}}\right)}{28561\sqrt{13}} + \frac{2(20304864x^5 + 20074356x^4 + 19381992x^3 + 21890266x^2 + 10777477x + 847141)}{45326307(2x + 1)^2 (3x^2 - x + 2)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 3\*x + 4\*x^2)/((1 + 2\*x)^3\*(2 - x + 3\*x^2)^(5/2)),x]

[Out] (2\*(847141 + 10777477\*x + 21890266\*x^2 + 19381992\*x^3 + 20074356\*x^4 + 20304864\*x^5))/(45326307\*(1 + 2\*x)^2\*(2 - x + 3\*x^2)^(3/2)) + (4168\*ArcTanh[Sqrt[3/13] + 2\*sqrt[3/13]\*x - (2\*sqrt[2 - x + 3\*x^2])/sqrt[13]])/(28561\*sqrt[13])

**fricas** [A] time = 1.73, size = 156, normalized size = 1.16

$$\frac{2 \left( 826827 \sqrt{13} (36x^6 + 12x^5 + 37x^4 + 30x^3 + 13x^2 + 12x + 4) \log \left( -\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1} \right) + 13(20304864x^5 + 20074356x^4 + 19381992x^3 + 21890266x^2 + 10777477x + 847141)\sqrt{3x^2-x+2} \right)}{589241991(36x^6 + 12x^5 + 37x^4 + 30x^3 + 13x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2-x+2)^(5/2),x, algorithm="fricas")

[Out] 2/589241991\*(826827\*sqrt(13)\*(36\*x^6 + 12\*x^5 + 37\*x^4 + 30\*x^3 + 13\*x^2 + 12\*x + 4)\*log(-(4\*sqrt(13)\*sqrt(3\*x^2 - x + 2)\*(8\*x - 9) + 220\*x^2 - 196\*x + 185)/(4\*x^2 + 4\*x + 1)) + 13\*(20304864\*x^5 + 20074356\*x^4 + 19381992\*x^3 + 21890266\*x^2 + 10777477\*x + 847141)\*sqrt(3\*x^2 - x + 2))/(36\*x^6 + 12\*x^5 + 37\*x^4 + 30\*x^3 + 13\*x^2 + 12\*x + 4)

**giac** [B] time = 0.35, size = 233, normalized size = 1.73

$$\frac{2084}{371293} \sqrt{13} \log \left( \frac{-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2-x+2}}{2(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2-x+2})} \right) + \frac{2(3(6(310578x - 26213)x + 1455755)x + 1634293)}{45326307(3x^2-x+2)^{\frac{3}{2}}} - \frac{8(66(\sqrt{3}x - \sqrt{3x^2-x+2})^3 + 21\sqrt{3}(\sqrt{3}x - \sqrt{3x^2-x+2})^2 - 1015\sqrt{3}x + 431\sqrt{3} + 1015\sqrt{3x^2-x+2})}{28561(2(\sqrt{3}x - \sqrt{3x^2-x+2})^2 + 2\sqrt{3}(\sqrt{3}x - \sqrt{3x^2-x+2}) - 5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^2+3\*x+1)/(1+2\*x)^3/(3\*x^2-x+2)^(5/2),x, algorithm="giac")

[Out] 2084/371293\*sqrt(13)\*log(-1/2\*abs(-4\*sqrt(3)\*x - 2\*sqrt(13) - 2\*sqrt(3) + 4\*sqrt(3\*x^2 - x + 2))/(2\*sqrt(3)\*x - sqrt(13) + sqrt(3) - 2\*sqrt(3\*x^2 - x + 2))) + 2/45326307\*(3\*(6\*(310578\*x - 26213)\*x + 1455755)\*x + 1634293)/(3\*x^2 - x + 2)^(3/2) - 8/28561\*(66\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^3 + 21\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^2 - 1015\*sqrt(3)\*x + 431\*sqrt(3) + 1015\*sqrt(3\*x^2 - x + 2))/(2\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2))^2 + 2\*sqrt(3)\*(sqrt(3)\*x - sqrt(3\*x^2 - x + 2)) - 5)^2

**maple** [A] time = 0.01, size = 148, normalized size = 1.10

$$\frac{2084\sqrt{13} \operatorname{arctanh} \left( \frac{2(-4x+\frac{1}{2})\sqrt{13}}{13\sqrt{-16x+12(\frac{x+1}{2})^2+5}} \right)}{371293} + \frac{521}{13182(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} + \frac{17721-\frac{886}{151969}}{51531} + \frac{1128048x-\frac{188008}{15108769}}{15108769} + \frac{1042}{28561\sqrt{-4x+3(x+\frac{1}{2})^2+\frac{5}{4}}} - \frac{1}{338(x+\frac{1}{2})(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}} - \frac{1}{104(x+\frac{1}{2})^2(-4x+3(x+\frac{1}{2})^2+\frac{5}{4})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(5/2),x)`

[Out]  $521/13182/(-4*x+3*(x+1/2)^2+5/4)^{(3/2)}+886/151593*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^{(3/2)}+188008/15108769*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^{(1/2)}+1042/28561/(-4*x+3*(x+1/2)^2+5/4)^{(1/2)}-2084/371293*13^{(1/2)}*\operatorname{arctanh}(2/13*(-4*x+9/2)*13^{(1/2)}/(-16*x+12*(x+1/2)^2+5)^{(1/2)})-1/338/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^{(3/2)}-1/104/(x+1/2)^2/(-4*x+3*(x+1/2)^2+5/4)^{(3/2)}$

**maxima** [A] time = 0.99, size = 174, normalized size = 1.29

$$\frac{2084}{371293} \sqrt{13} \operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23(2x+1)} + \frac{9\sqrt{23}}{23(2x+1)}\right) + \frac{1128048x}{15108769\sqrt{3x^2-x+2}} + \frac{363210}{15108769\sqrt{3x^2-x+2}} + \frac{1772x}{50531(3x^2-x+2)^{3/2}} - \frac{1}{26(4(3x^2-x+2)^{3/2}x^2+4(3x^2-x+2)^{3/2}x+(3x^2-x+2)^{3/2})} - \frac{1}{169(2(3x^2-x+2)^{3/2}x+(3x^2-x+2)^{3/2})} + \frac{10211}{303186(3x^2-x+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")`

[Out]  $2084/371293*\operatorname{sqrt}(13)*\operatorname{arcsinh}(8/23*\operatorname{sqrt}(23)*x/\operatorname{abs}(2*x+1)-9/23*\operatorname{sqrt}(23)/\operatorname{abs}(2*x+1))+1128048/15108769*x/\operatorname{sqrt}(3*x^2-x+2)+363210/15108769/\operatorname{sqrt}(3*x^2-x+2)+1772/50531*x/(3*x^2-x+2)^{(3/2)}-1/26/(4*(3*x^2-x+2)^{(3/2)}*x^2+4*(3*x^2-x+2)^{(3/2)}*x+(3*x^2-x+2)^{(3/2)})-1/169/(2*(3*x^2-x+2)^{(3/2)}*x+(3*x^2-x+2)^{(3/2)})+10211/303186/(3*x^2-x+2)^{(3/2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 (3x^2 - x + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)`

[Out] `int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 (3x^2 - x + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)`

[Out] `Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)`

$$3.254 \quad \int \frac{d+ex+fx^2}{(g+hx)(-cg^2+bgh+bh^2x+ch^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=208

$$\frac{(b+2cx)(-3b^2fh^2+6bceh^2+4c^2(fg^2-h(2dh+eg)))}{3ch^2(2cg-bh)^3\sqrt{-g(CG-bh)+bh^2x+ch^2x^2}} + \frac{2(dh^2-egh+fg^2)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(CG-bh)+bh^2x+ch^2x^2}}$$

**Rubi [A]** time = 0.42, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 47,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.064$ , Rules used = {1638, 792, 613}

$$\frac{(b+2cx)(-3b^2fh^2+6bceh^2+4c^2(fg^2-h(2dh+eg)))}{3ch^2(2cg-bh)^3\sqrt{-g(CG-bh)+bh^2x+ch^2x^2}} + \frac{2(dh^2-egh+fg^2)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(CG-bh)+bh^2x+ch^2x^2}} - \frac{f}{ch^3\sqrt{-g(CG-bh)+bh^2x+ch^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)/((g + h\*x)\*(-(c\*g^2) + b\*g\*h + b\*h^2\*x + c\*h^2\*x^2)^(3/2)), x]

[Out] -(f/(c\*h^3\*Sqrt[-(g\*(c\*g - b\*h)) + b\*h^2\*x + c\*h^2\*x^2])) + ((6\*b\*c\*e\*h^2 - 3\*b^2\*f\*h^2 + 4\*c^2\*(f\*g^2 - h\*(e\*g + 2\*d\*h)))\*(b + 2\*c\*x))/(3\*c\*h^2\*(2\*c\*g - b\*h)^3\*Sqrt[-(g\*(c\*g - b\*h)) + b\*h^2\*x + c\*h^2\*x^2]) + (2\*(f\*g^2 - e\*g\*h + d\*h^2))/(3\*h^3\*(2\*c\*g - b\*h)\*(g + h\*x)\*Sqrt[-(g\*(c\*g - b\*h)) + b\*h^2\*x + c\*h^2\*x^2])

### Rule 613

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-3/2), x\_Symbol] := Simp[(-2\*(b + 2\*c\*x))/((b^2 - 4\*a\*c)\*Sqrt[a + b\*x + c\*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 792

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d\*g - e\*f)\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/((2\*c\*d - b\*e)\*(m + p + 1)), x] + Dist[(m\*(g\*(c\*d - b\*e) + c\*e\*f) + e\*(p + 1)\*(2\*c\*f - b\*g))/(e\*(2\*c\*d - b\*e)\*(m + p + 1)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && EqQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2\*p + 2, 0]) && NeQ[m + p + 1, 0]

### Rule 1638

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b*e)*x), x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

### Rubi steps

$$\int \frac{d + ex + fx^2}{(g + hx)(-cg^2 + bgh + bh^2x + ch^2x^2)^{3/2}} dx = -\frac{f}{ch^3\sqrt{-g(CG - bh) + bh^2x + ch^2x^2}} - \frac{\int \frac{\frac{1}{2}h^3(bfg - 2cdh) + \frac{1}{2}h^3(2cfg - 2)}{(g + hx)(-cg^2 + bgh + bh^2x + ch^2x^2)} dx}{ch^4}$$

$$= -\frac{f}{ch^3\sqrt{-g(CG - bh) + bh^2x + ch^2x^2}} + \frac{2(fg^2 - 2dgh + dh^2)}{3h^3(2cg - bh)(g + hx)\sqrt{-g(CG - bh) + bh^2x + ch^2x^2}}$$

$$= -\frac{f}{ch^3\sqrt{-g(CG - bh) + bh^2x + ch^2x^2}} + \frac{(6bceh^2 - 3b^2fh^2 + 4c^2d)}{3ch^2(2cg - bh)^3\sqrt{-g(CG - bh) + bh^2x + ch^2x^2}}$$

**Mathematica [A]** time = 0.51, size = 219, normalized size = 1.05

$$\frac{2h^2h^2(f(8g^2 + 12ghx + 3h^2x^2) - h(dh + 2eg + 3ehx)) - 4bch(h(e(g^2 + 2ghx + 3h^2x^2) - 2dh(2g + hx)) + 2fg^2(4g + 5hx)) + 8c^2(h(dh(-g^2 + 2ghx + 2h^2x^2) + eg(g^2 + ghx + h^2x^2)) + fg^2(2g^2 + 2ghx - h^2x^2))}{3h^3(g + hx)(bh - 2cg)^3\sqrt{(g + hx)(bh - cg + chx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)/((g + h\*x)\*(-(c\*g^2) + b\*g\*h + b\*h^2\*x + c\*h^2\*x^2)^(3/2)), x]

[Out] (2\*b^2\*h^2\*(-(h\*(2\*e\*g + d\*h + 3\*e\*h\*x)) + f\*(8\*g^2 + 12\*g\*h\*x + 3\*h^2\*x^2)) + 8\*c^2\*(f\*g^2\*(2\*g^2 + 2\*g\*h\*x - h^2\*x^2) + h\*(e\*g\*(g^2 + g\*h\*x + h^2\*x^2) + d\*h\*(-g^2 + 2\*g\*h\*x + 2\*h^2\*x^2))) - 4\*b\*c\*h\*(2\*f\*g^2\*(4\*g + 5\*h\*x) + h\*(-2\*d\*h\*(2\*g + h\*x) + e\*(g^2 + 2\*g\*h\*x + 3\*h^2\*x^2))))/(3\*h^3\*(-2\*c\*g + b\*h)^3\*(g + h\*x)\*Sqrt[(g + h\*x)\*(-(c\*g) + b\*h + c\*h\*x)])

**IntegrateAlgebraic [F]** time = 180.18, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)/((g + h\*x)\*(-c\*g^2) + b\*g\*h + b\*h^2\*x + c\*h^2\*x^2)^(3/2), x]

[Out] \$Aborted

**fricas** [B] time = 60.07, size = 465, normalized size = 2.24

$$\frac{2(8c^2fg^4 - b^2dh^4 + 4(c^2e - 4bcf)g^3h - 2(2c^2d + bce - 4b^2f)g^2h^2 + 2(4bcd - b^2e)gh^3 - (4c^2fg^2h^2 - 4c^2egh^3 - (8c^2d - 6bce + 3b^2f)h^4)x^2 + (8c^2fg^3h + 4(c^2e - 5bcf)g^2h^2 + 4(2c^2d - bce + 3b^2f)gh^3 + (4bcd - 3b^2e)h^4)x}{3(8c^4g^3h^3 - 20bc^2g^3h^4 + 18b^2c^2g^4h^5 - 7b^3cg^2h^6 + b^4g^2h^7 - (8c^4g^3h^6 - 12bc^2g^2h^7 + 6b^2c^2g^3h^8 - b^3ch^9)x^3 - (8c^4g^4h^5 - 4bc^2g^3h^6 - 6b^2c^2g^2h^7 + 5b^3cg^2h^8 - b^4h^9)x^2 + (8c^4g^5h^4 - 28bc^2g^4h^5 + 30b^2c^2g^3h^6 - 13b^3cg^2h^7 + 2b^4g^2h^8)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*h^2\*x^2+b\*h^2\*x+b\*g\*h-c\*g^2)^(3/2), x, algorithm="fricas")

[Out]  $\frac{2}{3}(8c^2f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*c*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*c^2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*(c^2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d - 3*b^2*e)*h^4)*x)*\sqrt{(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h^3 - 20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7 - (8*c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 - (8*c^4*g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4*h^9)*x^2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^3*c*g^2*h^7 + 2*b^4*g*h^8)*x}$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx^2 + ex + d}{(ch^2x^2 + bh^2x - cg^2 + bgh)^{\frac{3}{2}}(hx + g)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)/(h\*x+g)/(c\*h^2\*x^2+b\*h^2\*x+b\*g\*h-c\*g^2)^(3/2), x, algorithm="giac")

[Out] integrate((f\*x^2 + e\*x + d)/((c\*h^2\*x^2 + b\*h^2\*x - c\*g^2 + b\*g\*h)^(3/2)\*(h\*x + g)), x)

**maple** [A] time = 0.01, size = 324, normalized size = 1.56

$$\frac{2(dx + hb - cg)(-3b^2f h^4x^2 + 6bce h^4x^2 - 8c^2d h^4x^2 - 4c^2eg h^4x^2 + 4c^2fg^2h^2x^2 + 3b^2e h^4x - 12b^2fg h^3x - 4bcd h^4x + 4bcg h^3x + 20bcf g^2h^2x - 8c^2dg h^3x - 4c^2e g^2h^2x - 8c^2fg^3h^2x + b^2d h^4 + 2b^2eg h^3 - 8b^2fg^2h^2 - 8bcdg h^3 + 2bce g^2h^2 + 16bcf g^3h + 4c^2d g^2h^2 - 4c^2e g^2h - 8c^2fg^4)}{3(b^3h^3 - 6b^2cg h^2 + 12bc^2g^2h - 8c^3g^3)(c h^2x^2 + b h^2x + bgh - cg^2)^{\frac{3}{2}}h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^{(3/2)}, x)$

[Out] 
$$-2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c^2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*h^4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x-8*c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b*c*e*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^3*h^3-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^{(3/2)}$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(b\*h-2\*c\*g>0)', see `assume?` for more details)Is b\*h-2\*c\*g zero or nonzero?

**mupad** [B] time = 5.75, size = 1089, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(3/2)}), x)$

[Out] 
$$(16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 2*b^2*d*h^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 4*b^2*e*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 8*c^2*e*g^3*h*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 4*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 12*b*c*e*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 16*c^2*d*g*h^3*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 24*b^2*f*g*h^3*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)}$$

$$\begin{aligned}
& + 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} + 16*b*c*d* \\
& g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 32*b*c*f*g^3*h*(b*h^2*x \\
& - c*g^2 + c*h^2*x^2 + b*g*h)^{(1/2)} - 8*b*c*e*g*h^3*x*(b*h^2*x - c*g^2 + c* \\
& h^2*x^2 + b*g*h)^{(1/2)} - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + \\
& b*g*h)^{(1/2)})/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^9*x^2 - 60*b*c^3*g^ \\
& 5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*g^5*h^4*x + 54*b^2*c^2* \\
& g^4*h^5 - 24*c^4*g^4*h^5*x^2 - 24*c^4*g^3*h^6*x^3 + 6*b^4*g*h^8*x + 18*b^2* \\
& c^2*g^2*h^7*x^2 - 84*b*c^3*g^4*h^5*x - 39*b^3*c*g^2*h^7*x - 15*b^3*c*g*h^8* \\
& x^2 + 90*b^2*c^2*g^3*h^6*x + 12*b*c^3*g^3*h^6*x^2 + 36*b*c^3*g^2*h^7*x^3 - \\
& 18*b^2*c^2*g*h^8*x^3)
\end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{((g + hx)(bh - cg + chx))^{\frac{3}{2}}(g + hx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x\*\*2+e\*x+d)/(h\*x+g)/(c\*h\*\*2\*x\*\*2+b\*h\*\*2\*x+b\*g\*h-c\*g\*\*2)\*\*(3/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2)/(((g + h\*x)\*(b\*h - c\*g + c\*h\*x))\*\*(3/2)\*(g + h\*x)), x)



## 3.255

$$\int (d + fx^2)^p (2cdf + 2bf^2(3 + 2p)x + 2cf^2(3 + 2p)x^2) dx$$

Optimal. Leaf size=41

$$\frac{bf(2p + 3)(d + fx^2)^{p+1}}{p + 1} + 2cfx(d + fx^2)^{p+1}$$

**Rubi** [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 42,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {1815, 12, 261}

$$\frac{bf(2p + 3)(d + fx^2)^{p+1}}{p + 1} + 2cfx(d + fx^2)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[(d + f\*x^2)^p\*(2\*c\*d\*f + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] (b\*f\*(3 + 2\*p)\*(d + f\*x^2)^(1 + p))/(1 + p) + 2\*c\*f\*x\*(d + f\*x^2)^(1 + p)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 261

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1815

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x^2)^(p + 1))/(b\*(q + 2\*p + 1)), x] + Dist[1/(b\*(q + 2\*p + 1)), Int[(a + b\*x^2)^p\*ExpandToSum[b\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + 2\*p + 1)\*x^q, x], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int (d + fx^2)^p (2cdf + 2bf^2(3 + 2p)x + 2cf^2(3 + 2p)x^2) dx &= 2cfx(d + fx^2)^{1+p} + \frac{\int 2bf^3(3 + 2p)^2x(d + fx^2)^p}{f(3 + 2p)} \\
&= 2cfx(d + fx^2)^{1+p} + (2bf^2(3 + 2p)) \int x(d + fx^2)^p \\
&= \frac{bf(3 + 2p)(d + fx^2)^{1+p}}{1 + p} + 2cfx(d + fx^2)^{1+p}
\end{aligned}$$

**Mathematica [C]** time = 0.11, size = 119, normalized size = 2.90

$$\frac{f(d + fx^2)^p \left(\frac{fx^2}{d} + 1\right)^{-p} \left( (2p + 3) \left( 3b(d + fx^2) \left(\frac{fx^2}{d} + 1\right)^p + 2cf(p + 1)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{fx^2}{d}\right) \right) + 6cd(p + 1)x {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{fx^2}{d}\right) \right)}{3(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + f\*x^2)^p\*(2\*c\*d\*f + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] (f\*(d + f\*x^2)^p\*(6\*c\*d\*(1 + p)\*x\*Hypergeometric2F1[1/2, -p, 3/2, -((f\*x^2)/d)] + (3 + 2\*p)\*(3\*b\*(d + f\*x^2)\*(1 + (f\*x^2)/d)^p + 2\*c\*f\*(1 + p)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -((f\*x^2)/d)]))/((3\*(1 + p)\*(1 + (f\*x^2)/d)^p)

**IntegrateAlgebraic [F]** time = 0.29, size = 0, normalized size = 0.00

$$\int (d + fx^2)^p (2cdf + 2bf^2(3 + 2p)x + 2cf^2(3 + 2p)x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + f\*x^2)^p\*(2\*c\*d\*f + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] Defer[IntegrateAlgebraic][(d + f\*x^2)^p\*(2\*c\*d\*f + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

**fricas [A]** time = 0.89, size = 75, normalized size = 1.83

$$\frac{(2bdfp + 2(cf^2p + cf^2)x^3 + 3bdf + (2bf^2p + 3bf^2)x^2 + 2(cdfp + cdf)x)(fx^2 + d)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+d)^p\*(2\*c\*d\*f+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x, algorithm="fricas")

[Out] (2\*b\*d\*f\*p + 2\*(c\*f^2\*p + c\*f^2)\*x^3 + 3\*b\*d\*f + (2\*b\*f^2\*p + 3\*b\*f^2)\*x^2 + 2\*(c\*d\*f\*p + c\*d\*f)\*x)\*(f\*x^2 + d)^p/(p + 1)

**giac** [B] time = 0.18, size = 141, normalized size = 3.44

$$\frac{2(fx^2+d)^p c f^2 p x^3 + 2(fx^2+d)^p b f^2 p x^2 + 2(fx^2+d)^p c f^2 x^3 + 2(fx^2+d)^p c d f p x + 3(fx^2+d)^p b f^2 x^2 + 2(fx^2+d)^p b d f p + 2(fx^2+d)^p c d f x + 3(fx^2+d)^p b d f}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+d)^p\*(2\*c\*d\*f+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x, algorithm="giac")

[Out] (2\*(f\*x^2 + d)^p\*c\*f^2\*p\*x^3 + 2\*(f\*x^2 + d)^p\*b\*f^2\*p\*x^2 + 2\*(f\*x^2 + d)^p\*c\*f^2\*x^3 + 2\*(f\*x^2 + d)^p\*c\*d\*f\*p\*x + 3\*(f\*x^2 + d)^p\*b\*f^2\*x^2 + 2\*(f\*x^2 + d)^p\*b\*d\*f\*p + 2\*(f\*x^2 + d)^p\*c\*d\*f\*x + 3\*(f\*x^2 + d)^p\*b\*d\*f)/(p + 1)

**maple** [A] time = 0.00, size = 36, normalized size = 0.88

$$\frac{(2pcx + 2pb + 2cx + 3b) f (f x^2 + d)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+d)^p\*(2\*c\*d\*f+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x)

[Out] f\*(f\*x^2+d)^(1+p)\*(2\*c\*p\*x+2\*b\*p+2\*c\*x+3\*b)/(1+p)

**maxima** [A] time = 0.58, size = 59, normalized size = 1.44

$$\frac{(2cf^2(p+1)x^3 + bf^2(2p+3)x^2 + 2cdf(p+1)x + bdf(2p+3))(fx^2+d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+d)^p\*(2\*c\*d\*f+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x, algorithm="maxima")

[Out] (2\*c\*f^2\*(p + 1)\*x^3 + b\*f^2\*(2\*p + 3)\*x^2 + 2\*c\*d\*f\*(p + 1)\*x + b\*d\*f\*(2\*p + 3))\*(f\*x^2 + d)^p/(p + 1)

**mupad** [B] time = 4.25, size = 58, normalized size = 1.41

$$(f x^2 + d)^p \left( 2 c f^2 x^3 + 2 c d f x + \frac{b f^2 x^2 (2 p + 3)}{p + 1} + \frac{b d f (2 p + 3)}{p + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + f*x^2)^p*(2*c*d*f + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3)),x)`

[Out]  $(d + f*x^2)^p*(2*c*f^2*x^3 + 2*c*d*f*x + (b*f^2*x^2*(2*p + 3))/(p + 1) + (b*d*f*(2*p + 3))/(p + 1))$

**sympy [B]** time = 13.25, size = 221, normalized size = 5.39

$$\left\{ \begin{array}{ll} \frac{2bdfp(d+fx^2)^p}{p+1} + \frac{3bdf(d+fx^2)^p}{p+1} + \frac{2bf^2px^2(d+fx^2)^p}{p+1} + \frac{3bf^2x^2(d+fx^2)^p}{p+1} + \frac{2cdfpx(d+fx^2)^p}{p+1} + \frac{2cdfx(d+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+fx^2)^p}{p+1} + \frac{2cf^2x^3(d+fx^2)^p}{p+1} & \text{for } p \neq -1 \\ bf \log\left(-i\sqrt{d}\sqrt{\frac{1}{f}} + x\right) + bf \log\left(i\sqrt{d}\sqrt{\frac{1}{f}} + x\right) + 2cfx & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)`

[Out] `Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + f*x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(-I*sqrt(d)*sqrt(1/f) + x) + b*f*log(I*sqrt(d)*sqrt(1/f) + x) + 2*c*f*x, True))`

## 3.256

$$\int (d + ex + fx^2)^p (-2ce^2 + 2cdf - ce^2p + 2cf^2(3 + 2p)x^2) dx$$

Optimal. Leaf size=46

$$2cfx(d + ex + fx^2)^{p+1} - \frac{ce(p+2)(d + ex + fx^2)^{p+1}}{p+1}$$

**Rubi** [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {1661, 629}

$$2cfx(d + ex + fx^2)^{p+1} - \frac{ce(p+2)(d + ex + fx^2)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f - c\*e^2\*p + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] -((c\*e\*(2 + p)\*(d + e\*x + f\*x^2)^(1 + p))/(1 + p)) + 2\*c\*f\*x\*(d + e\*x + f\*x^2)^(1 + p)

Rule 629

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(d\*(a + b\*x + c\*x^2)^(p + 1))/(b\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\int (d + ex + fx^2)^p (-2ce^2 + 2cdf - ce^2p + 2cf^2(3 + 2p)x^2) dx = 2cfx (d + ex + fx^2)^{1+p} + \frac{\int (-ce^2f(2 + p)(3 + 2p)x^2) dx}{1 + p}$$

$$= -\frac{ce(2 + p)(d + ex + fx^2)^{1+p}}{1 + p} + 2cfx (d + ex + fx^2)^{1+p}$$

**Mathematica** [A] time = 0.13, size = 34, normalized size = 0.74

$$\frac{c(2f(p + 1)x - e(p + 2))(d + x(e + fx))^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f - c\*e^2\*p + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] (c\*(-(e\*(2 + p)) + 2\*f\*(1 + p)\*x)\*(d + x\*(e + f\*x))^(1 + p))/(1 + p)

**IntegrateAlgebraic** [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (d + ex + fx^2)^p (-2ce^2 + 2cdf - ce^2p + 2cf^2(3 + 2p)x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f - c\*e^2\*p + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] Defer[IntegrateAlgebraic] [(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f - c\*e^2\*p + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

**fricas** [A] time = 1.11, size = 83, normalized size = 1.80

$$\frac{(cefpx^2 - cdep + 2(cf^2p + cf^2)x^3 - 2cde - (2ce^2 - 2cdf + (ce^2 - 2cdf)p)x)(fx^2 + ex + d)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f-c\*e^2\*p+2\*c\*f^2\*(3+2\*p)\*x^2), x, algorithm="fricas")

[Out] (c\*e\*f\*p\*x^2 - c\*d\*e\*p + 2\*(c\*f^2\*p + c\*f^2)\*x^3 - 2\*c\*d\*e - (2\*c\*e^2 - 2\*c\*d\*f + (c\*e^2 - 2\*c\*d\*f)\*p)\*x)\*(f\*x^2 + e\*x + d)^p/(p + 1)

**giac [B]** time = 0.24, size = 191, normalized size = 4.15

$$\frac{2(fx^2 + xe + d)^p c f^2 p x^3 + 2(fx^2 + xe + d)^p c f^2 x^3 + (fx^2 + xe + d)^p c f p x^2 e + 2(fx^2 + xe + d)^p c d f p x + 2(fx^2 + xe + d)^p c d f x - (fx^2 + xe + d)^p c p x e^2 - (fx^2 + xe + d)^p c d p e - 2(fx^2 + xe + d)^p c x e^2 - 2(fx^2 + xe + d)^p c d e}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f-c\*e^2\*p+2\*c\*f^2\*(3+2\*p)\*x^2),x,  
algorithm="giac")

[Out] (2\*(f\*x^2 + x\*e + d)^p\*c\*f^2\*p\*x^3 + 2\*(f\*x^2 + x\*e + d)^p\*c\*f^2\*x^3 + (f\*x^2 + x\*e + d)^p\*c\*f\*p\*x^2\*e + 2\*(f\*x^2 + x\*e + d)^p\*c\*d\*f\*p\*x + 2\*(f\*x^2 + x\*e + d)^p\*c\*d\*f\*x - (f\*x^2 + x\*e + d)^p\*c\*p\*x\*e^2 - (f\*x^2 + x\*e + d)^p\*c\*d\*p\*e - 2\*(f\*x^2 + x\*e + d)^p\*c\*x\*e^2 - 2\*(f\*x^2 + x\*e + d)^p\*c\*d\*e)/(p + 1)

**maple [A]** time = 0.00, size = 39, normalized size = 0.85

$$\frac{(-2fpx + ep - 2fx + 2e)c(fx^2 + ex + d)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f-c\*e^2\*p+2\*c\*f^2\*(3+2\*p)\*x^2),x)

[Out] -c\*(f\*x^2+e\*x+d)^(p+1)\*(-2\*f\*p\*x+e\*p-2\*f\*x+2\*e)/(p+1)

**maxima [A]** time = 0.58, size = 66, normalized size = 1.43

$$\frac{(2cf^2(p+1)x^3 + cefpx^2 - cde(p+2) - (e^2(p+2) - 2df(p+1))cx)(fx^2 + ex + d)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f-c\*e^2\*p+2\*c\*f^2\*(3+2\*p)\*x^2),x,  
algorithm="maxima")

[Out] (2\*c\*f^2\*(p + 1)\*x^3 + c\*e\*f\*p\*x^2 - c\*d\*e\*(p + 2) - (e^2\*(p + 2) - 2\*d\*f\*(p + 1))\*c\*x)\*(f\*x^2 + e\*x + d)^p/(p + 1)

**mupad [B]** time = 4.39, size = 78, normalized size = 1.70

$$(fx^2 + ex + d)^p \left( 2cf^2x^3 + \frac{cx(2df - e^2p - 2e^2 + 2dfp)}{p + 1} - \frac{cde(p + 2)}{p + 1} + \frac{cefp x^2}{p + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(d + e*x + f*x^2)^p*(2*c*e^2 - 2*c*d*f + c*e^2*p - 2*c*f^2*x^2*(2*p + 3)), x)`

[Out]  $(d + e*x + f*x^2)^p*(2*c*f^2*x^3 + (c*x*(2*d*f - e^2*p - 2*e^2 + 2*d*f*p)) / (p + 1) - (c*d*e*(p + 2)) / (p + 1) + (c*e*f*p*x^2) / (p + 1))$

**sympy [A]** time = 173.95, size = 280, normalized size = 6.09

$$\begin{cases} -\frac{cdp(d+ex+fx^2)^p}{p+1} - \frac{2cde(d+ex+fx^2)^p}{p+1} + \frac{2cdfpx(d+ex+fx^2)^p}{p+1} + \frac{2cdfx(d+ex+fx^2)^p}{p+1} - \frac{ce^2px(d+ex+fx^2)^p}{p+1} - \frac{2ce^2x(d+ex+fx^2)^p}{p+1} + \frac{cefpx^2(d+ex+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+ex+fx^2)^p}{p+1} + \frac{2cf^2x^3(d+ex+fx^2)^p}{p+1} & \text{for } p \neq -1 \\ -ce \log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce \log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) + 2cfx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x**2), x)`

[Out] `Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))`



## 3.257

$$\int (d + ex + fx^2)^p (-2ce^2 + 2cdf + 3bef - ce^2p + 2befp + 2bf^2(3 + 2p)) dx$$

Optimal. Leaf size=57

$$2cfx(d + ex + fx^2)^{p+1} - \frac{(ce(p+2) - bf(2p+3))(d + ex + fx^2)^{p+1}}{p+1}$$

**Rubi** [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 69,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {1661, 629}

$$2cfx(d + ex + fx^2)^{p+1} - \frac{(ce(p+2) - bf(2p+3))(d + ex + fx^2)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f + 3\*b\*e\*f - c\*e^2\*p + 2\*b\*e\*f\*p + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] -(((c\*e\*(2 + p) - b\*f\*(3 + 2\*p))\*(d + e\*x + f\*x^2)^(1 + p))/(1 + p)) + 2\*c\*f\*x\*(d + e\*x + f\*x^2)^(1 + p)

Rule 629

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(d\*(a + b\*x + c\*x^2)^(p + 1))/(b\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

Int[(Pq\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

Rubi steps

$$\int (d + ex + fx^2)^p (-2ce^2 + 2cdf + 3bef - ce^2p + 2befp + 2bf^2(3 + 2p)x + 2cf^2(3 + 2p)x^2) dx = 2cfx(d + ex + fx^2)^p - \frac{(ce(2 + p) - 2cf^2(3 + 2p)x^2)(d + ex + fx^2)^{p+1}}{p+1}$$

**Mathematica [A]** time = 0.31, size = 43, normalized size = 0.75

$$\frac{(d + x(e + fx))^{p+1}(bf(2p + 3) - ce(p + 2) + 2cf(p + 1)x)}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f + 3\*b\*e\*f - c\*e^2\*p + 2\*b\*e\*f\*p + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] ((-(c\*e\*(2 + p)) + b\*f\*(3 + 2\*p) + 2\*c\*f\*(1 + p)\*x)\*(d + x\*(e + f\*x))^(1 + p))/(1 + p)

**IntegrateAlgebraic [F]** time = 0.31, size = 0, normalized size = 0.00

$$\int (d + ex + fx^2)^p (-2ce^2 + 2cdf + 3bef - ce^2p + 2befp + 2bf^2(3 + 2p)x + 2cf^2(3 + 2p)x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f + 3\*b\*e\*f - c\*e^2\*p + 2\*b\*e\*f\*p + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

[Out] Defer[IntegrateAlgebraic] [(d + e\*x + f\*x^2)^p\*(-2\*c\*e^2 + 2\*c\*d\*f + 3\*b\*e\*f - c\*e^2\*p + 2\*b\*e\*f\*p + 2\*b\*f^2\*(3 + 2\*p)\*x + 2\*c\*f^2\*(3 + 2\*p)\*x^2), x]

**fricas [B]** time = 0.81, size = 123, normalized size = 2.16

$$\frac{(2(cf^2p + cf^2)x^3 - 2cde + 3bdf + (3bf^2 + (cef + 2bf^2)p)x^2 - (cde - 2bdf)p - (2ce^2 - (2cd + 3be)f + (ce^2 - 2(cd + be)f)p)x)(fx^2 + ex + d)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f+3\*b\*e\*f-c\*e^2\*p+2\*b\*e\*f\*p+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2), x, algorithm="fricas")

[Out] (2\*(c\*f^2\*p + c\*f^2)\*x^3 - 2\*c\*d\*e + 3\*b\*d\*f + (3\*b\*f^2 + (c\*e\*f + 2\*b\*f^2)\*p)\*x^2 - (c\*d\*e - 2\*b\*d\*f)\*p - (2\*c\*e^2 - (2\*c\*d + 3\*b\*e)\*f + (c\*e^2 - 2\*(c\*d + b\*e)\*f)\*p)\*x\*(f\*x^2 + e\*x + d)^p/(p + 1)

**giac [B]** time = 0.31, size = 314, normalized size = 5.51

$$\frac{2(f^2 + xe + df)fp^3 + 2(f^2 + xe + df)bfp^2 + 2(f^2 + xe + df)cfp + (f^2 + xe + df)cp^2 + 2(f^2 + xe + df)afp + 3(f^2 + xe + df)bf + 2(f^2 + xe + df)af - (f^2 + xe + df)cp^2 - (f^2 + xe + df)afp + 3(f^2 + xe + df)bfp + 3(f^2 + xe + df)af - 2(f^2 + xe + df)cfp - 2(f^2 + xe + df)af}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f+3\*b\*e\*f-c\*e^2\*p+2\*b\*e\*f\*p+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x, algorithm="giac")

[Out]  $(2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2 + x*e + d)^p*c*d*f*p*x + 3*(f*x^2 + x*e + d)^p*b*f^2*x^2 + 2*(f*x^2 + x*e + d)^p*b*f*p*x*e + 2*(f*x^2 + x*e + d)^p*b*d*f*p + 2*(f*x^2 + x*e + d)^p*c*d*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*d*p*e + 3*(f*x^2 + x*e + d)^p*b*f*x*e + 3*(f*x^2 + x*e + d)^p*b*d*f - 2*(f*x^2 + x*e + d)^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1)$

**maple [A]** time = 0.01, size = 51, normalized size = 0.89

$$\frac{(2cfxp + 2bfp - cep + 2cfx + 3bf - 2ce)(fx^2 + ex + d)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f+3\*b\*e\*f-c\*e^2\*p+2\*b\*e\*f\*p+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x)

[Out]  $(f*x^2+e*x+d)^{(p+1)}*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)/(p+1)$

**maxima [A]** time = 0.60, size = 98, normalized size = 1.72

$$\frac{(2cf^2(p+1)x^3 + bdf(2p+3) - cde(p+2) + (bf^2(2p+3) + cefp)x^2 + (bef(2p+3) - (e^2(p+2) - 2df(p+1))c)x)(fx^2 + ex + d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x^2+e\*x+d)^p\*(-2\*c\*e^2+2\*c\*d\*f+3\*b\*e\*f-c\*e^2\*p+2\*b\*e\*f\*p+2\*b\*f^2\*(3+2\*p)\*x+2\*c\*f^2\*(3+2\*p)\*x^2),x, algorithm="maxima")

[Out]  $(2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*e*(p + 2) + (b*f^2*(2*p + 3) + c*e*f*p)*x^2 + (b*e*f*(2*p + 3) - (e^2*(p + 2) - 2*d*f*(p + 1))*c)*x)*(f*x^2 + e*x + d)^p/(p + 1)$

**mupad [B]** time = 4.46, size = 120, normalized size = 2.11

$$(fx^2 + ex + d)^p \left( \frac{x^2(3bf^2 + 2bf^2p + cefp)}{p+1} + 2cf^2x^3 + \frac{d(3bf - 2ce + 2bfp - cep)}{p+1} + \frac{x(3bef - 2ce^2 + 2cdf - ce^2p + 2befp + 2cdfp)}{p+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^p*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3) + 2*b*e*f*p),x)
```

```
[Out] (d + e*x + f*x^2)^p*((x^2*(3*b*f^2 + 2*b*f^2*p + c*e*f*p))/(p + 1) + 2*c*f^2*x^3 + (d*(3*b*f - 2*c*e + 2*b*f*p - c*e*p))/(p + 1) + (x*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*e*f*p + 2*c*d*f*p))/(p + 1))
```

**sympy [B]** time = 171.17, size = 483, normalized size = 8.47

$$\left\{ \begin{array}{l} \frac{2df(d+ex+f^2x^2)^p}{p+1} + \frac{3df(d+ex+f^2x^2)^p}{p+1} + \frac{2df(d+ex+f^2x^2)^p}{p+1} + \frac{3df(d+ex+f^2x^2)^p}{p+1} + \frac{3df^2(d+ex+f^2x^2)^p}{p+1} + \frac{3df^2(d+ex+f^2x^2)^p}{p+1} - \frac{cdp(d+ex+f^2x^2)^p}{p+1} - \frac{2bd(d+ex+f^2x^2)^p}{p+1} + \frac{2dfp(d+ex+f^2x^2)^p}{p+1} + \frac{2df(d+ex+f^2x^2)^p}{p+1} - \frac{c^2p(d+ex+f^2x^2)^p}{p+1} - \frac{2c^2(d+ex+f^2x^2)^p}{p+1} + \frac{c^2p^2(d+ex+f^2x^2)^p}{p+1} + \frac{2c^2p^2(d+ex+f^2x^2)^p}{p+1} + \frac{2c^2p^2(d+ex+f^2x^2)^p}{p+1} \\ b f \log\left(\frac{d}{2f} + x - \frac{\sqrt{-4df + e^2}}{2f}\right) + b f \log\left(\frac{d}{2f} + x + \frac{\sqrt{-4df + e^2}}{2f}\right) - c e \log\left(\frac{d}{2f} + x - \frac{\sqrt{-4df + e^2}}{2f}\right) - c e \log\left(\frac{d}{2f} + x + \frac{\sqrt{-4df + e^2}}{2f}\right) + 2c f x \end{array} \right. \begin{array}{l} \text{for } p \neq -1 \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

```
[Out] Piecewise(((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*x**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(d + e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))
```

$$3.258 \quad \int (d + ex)^3 (a + bx + cx^2)^5 (d(6bd + 5ae) + (12cd^2 + 17bde + 5ae^2)x + e(29cd + 11be)x^2 + 17ce^2x^3) dx$$

Optimal. Leaf size=20

$$(d + ex)^5 (a + bx + cx^2)^6$$

**Rubi [A]** time = 0.42, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 75,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$ , Rules used = {1624, 1590}

$$(d + ex)^5 (a + bx + cx^2)^6$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(a + b\*x + c\*x^2)^5\*(d\*(6\*b\*d + 5\*a\*e) + (12\*c\*d^2 + 17\*b\*d\*e + 5\*a\*e^2)\*x + e\*(29\*c\*d + 11\*b\*e)\*x^2 + 17\*c\*e^2\*x^3), x]

[Out] (d + e\*x)^5\*(a + b\*x + c\*x^2)^6

Rule 1590

Int[(Pp\_)\*(Qq\_)^(m\_.)\*(Rr\_)^(n\_.), x\_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]\*x^(p - q - r + 1)\*Qq^(m + 1)\*Rr^(n + 1)]/((p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]), x] /; NeQ[p + m\*q + n\*r + 1, 0] && EqQ[(p + m\*q + n\*r + 1)\*Coeff[Qq, x, q]\*Coeff[Rr, x, r]\*Pp, Coeff[Pp, x, p]\*x^(p - q - r)\*((p - q - r + 1)\*Qq\*Rr + (m + 1)\*x\*Rr\*D[Qq, x] + (n + 1)\*x\*Qq\*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rule 1624

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[(d + e\*x)^(m + 1)\*PolynomialQuotient[Pq, d + e\*x, x]\*(a + b\*x + c\*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemainder[Pq, d + e\*x, x], 0]

Rubi steps

$$\int (d + ex)^3 (a + bx + cx^2)^5 (d(6bd + 5ae) + (12cd^2 + 17bde + 5ae^2)x + e(29cd + 11be)x^2 + 17ce^2x^3) dx = \int (d + ex)^5 (a + bx + cx^2)^6 dx$$

**Mathematica [B]** time = 0.45, size = 167, normalized size = 8.35

$$x (a^6 e (5d^4 + 10d^3 e x + 10d^2 e^2 x^2 + 5d e^3 x^3 + e^4 x^4) + 6a^5 (b + c x)(d + e x)^5 + 15a^4 x (b + c x)^2 (d + e x)^5 + 20a^3 x^2 (b + c x)^3 (d + e x)^5 + 15a^2 x^3 (b + c x)^4 (d + e x)^5 + 6a x^4 (b + c x)^5 (d + e x)^5 + x^5 (b + c x)^6 (d + e x)^5)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + b\*x + c\*x^2)^5\*(d\*(6\*b\*d + 5\*a\*e) + (12\*c\*d^2 + 17\*b\*d\*e + 5\*a\*e^2)\*x + e\*(29\*c\*d + 11\*b\*e)\*x^2 + 17\*c\*e^2\*x^3), x]

[Out] x\*(6\*a^5\*(b + c\*x)\*(d + e\*x)^5 + 15\*a^4\*x\*(b + c\*x)^2\*(d + e\*x)^5 + 20\*a^3\*x^2\*(b + c\*x)^3\*(d + e\*x)^5 + 15\*a^2\*x^3\*(b + c\*x)^4\*(d + e\*x)^5 + 6\*a\*x^4\*(b + c\*x)^5\*(d + e\*x)^5 + x^5\*(b + c\*x)^6\*(d + e\*x)^5 + a^6\*e\*(5\*d^4 + 10\*d^3\*e\*x + 10\*d^2\*e^2\*x^2 + 5\*d\*e^3\*x^3 + e^4\*x^4))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + e x)^3 (a + b x + c x^2)^5 (d(6bd + 5ae) + (12cd^2 + 17bde + 5ae^2)x + e(29cd + 11be)x^2 + 17ce^2x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^3\*(a + b\*x + c\*x^2)^5\*(d\*(6\*b\*d + 5\*a\*e) + (12\*c\*d^2 + 17\*b\*d\*e + 5\*a\*e^2)\*x + e\*(29\*c\*d + 11\*b\*e)\*x^2 + 17\*c\*e^2\*x^3), x]

[Out] IntegrateAlgebraic[(d + e\*x)^3\*(a + b\*x + c\*x^2)^5\*(d\*(6\*b\*d + 5\*a\*e) + (12\*c\*d^2 + 17\*b\*d\*e + 5\*a\*e^2)\*x + e\*(29\*c\*d + 11\*b\*e)\*x^2 + 17\*c\*e^2\*x^3), x  
]

**fricas [B]** time = 0.78, size = 2467, normalized size = 123.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+b\*x+a)^5\*(d\*(5\*a\*e+6\*b\*d)+(5\*a\*e^2+17\*b\*d\*e+12\*c\*d^2)\*x+e\*(11\*b\*e+29\*c\*d)\*x^2+17\*c\*e^2\*x^3), x, algorithm="fricas")

[Out] x^17\*e^5\*c^6 + 5\*x^16\*e^4\*d\*c^6 + 6\*x^16\*e^5\*c^5\*b + 10\*x^15\*e^3\*d^2\*c^6 + 30\*x^15\*e^4\*d\*c^5\*b + 15\*x^15\*e^5\*c^4\*b^2 + 6\*x^15\*e^5\*c^5\*a + 10\*x^14\*e^2\*d^3\*c^6 + 60\*x^14\*e^3\*d^2\*c^5\*b + 75\*x^14\*e^4\*d\*c^4\*b^2 + 20\*x^14\*e^5\*c^3\*b^3 + 30\*x^14\*e^4\*d\*c^5\*a + 30\*x^14\*e^5\*c^4\*b\*a + 5\*x^13\*e\*d^4\*c^6 + 60\*x^13\*e^2\*d^3\*c^5\*b + 150\*x^13\*e^3\*d^2\*c^4\*b^2 + 100\*x^13\*e^4\*d\*c^3\*b^3 + 15\*x^13\*e^5\*c^2\*b^4 + 60\*x^13\*e^3\*d^2\*c^5\*a + 150\*x^13\*e^4\*d\*c^4\*b\*a + 60\*x^13\*e^5\*c^3\*b^2\*a + 15\*x^13\*e^5\*c^4\*a^2 + x^12\*d^5\*c^6 + 30\*x^12\*e\*d^4\*c^5\*b + 150\*x^12\*e^2\*d^3\*c^4\*b^2 + 200\*x^12\*e^3\*d^2\*c^3\*b^3 + 75\*x^12\*e^4\*d\*c^2\*b^4 + 6\*x^12\*e^5\*c\*b^5 + 60\*x^12\*e^2\*d^3\*c^5\*a + 300\*x^12\*e^3\*d^2\*c^4\*b\*a + 300\*x^12\*e^4\*d\*c^3\*b^2\*a + 60\*x^12\*e^5\*c^2\*b^3\*a + 75\*x^12\*e^4\*d\*c^4\*a^2 + 60\*x^11\*d^5\*c^5\*b + 75\*x^11\*e\*d^4\*c^4\*b^2 + 200\*x^11\*e^2

$$\begin{aligned}
& d^3c^3b^3 + 150x^{11}e^3d^2c^2b^4 + 30x^{11}e^4d^2c^2b^5 + x^{11}e^5b^6 \\
& + 30x^{11}e^6d^4c^5a + 300x^{11}e^2d^3c^4b^2a + 600x^{11}e^3d^2c^3b^2a \\
& + 300x^{11}e^4d^2c^2b^3a + 30x^{11}e^5c^2b^4a + 150x^{11}e^3d^2c^4a^2 \\
& + 300x^{11}e^4d^2c^3b^2a^2 + 90x^{11}e^5c^2b^2a^2 + 20x^{11}e^5c^3a^3 \\
& + 15x^{10}d^5c^4b^2 + 100x^{10}e^4d^4c^3b^3 + 150x^{10}e^2d^3c^2b^4 \\
& + 60x^{10}e^3d^2c^2b^5 + 5x^{10}e^4d^2b^6 + 6x^{10}d^5c^5a + 150x^{10} \\
& e^4d^4c^4b^2a + 600x^{10}e^2d^3c^3b^2a + 600x^{10}e^3d^2c^2b^3a \\
& + 150x^{10}e^4d^2c^2b^4a + 6x^{10}e^5b^5a + 150x^{10}e^2d^3c^4a^2 + 60 \\
& 0x^{10}e^3d^2c^3b^2a^2 + 450x^{10}e^4d^2c^2b^2a^2 + 60x^{10}e^5c^2b^3a^2 \\
& + 100x^{10}e^4d^2c^3a^3 + 60x^{10}e^5c^2b^2a^3 + 20x^9d^5c^3b^3 + \\
& 75x^9e^4d^4c^2b^4 + 60x^9e^2d^3c^2b^5 + 10x^9e^3d^2b^6 + 30x^9d^5 \\
& c^4b^2a + 300x^9e^4d^4c^3b^2a + 600x^9e^2d^3c^2b^3a + 300x^9e^3 \\
& d^2c^2b^4a + 30x^9e^4d^2b^5a + 75x^9e^4d^4c^4a^2 + 600x^9e^2d^3 \\
& c^3b^2a^2 + 900x^9e^3d^2c^2b^2a^2 + 300x^9e^4d^2c^2b^3a^2 + 15x^9 \\
& e^5b^4a^2 + 200x^9e^3d^2c^3a^3 + 300x^9e^4d^2c^2b^2a^3 + 60x^9 \\
& e^5c^2b^2a^3 + 15x^9e^5c^2a^4 + 15x^8d^5c^2b^4 + 30x^8e^4d^4c^2b^5 \\
& + 10x^8e^2d^3b^6 + 60x^8d^5c^3b^2a + 300x^8e^4d^4c^2b^3a + \\
& 300x^8e^2d^3c^2b^4a + 60x^8e^3d^2b^5a + 15x^8d^5c^4a^2 + 300x^8 \\
& e^4d^4c^3b^2a^2 + 900x^8e^2d^3c^2b^2a^2 + 600x^8e^3d^2c^2b^3a^2 \\
& + 75x^8e^4d^2b^4a^2 + 200x^8e^2d^3c^3a^3 + 600x^8e^3d^2c^2b^2a^3 \\
& + 300x^8e^4d^2c^2b^2a^3 + 20x^8e^5b^3a^3 + 75x^8e^4d^2c^2a^4 + \\
& 30x^8e^5c^2b^2a^4 + 6x^7d^5c^2b^5 + 5x^7e^4d^4b^6 + 60x^7d^5c^2b^3 \\
& a + 150x^7e^4d^4c^2b^4a + 60x^7e^2d^3b^5a + 60x^7d^5c^3b^2a^2 + \\
& 450x^7e^4d^4c^2b^2a^2 + 600x^7e^2d^3c^2b^3a^2 + 150x^7e^3d^2b^4 \\
& a^2 + 100x^7e^4d^4c^3a^3 + 600x^7e^2d^3c^2b^2a^3 + 600x^7e^3d^2 \\
& c^2b^2a^3 + 100x^7e^4d^2b^3a^3 + 150x^7e^3d^2c^2a^4 + 150x^7e^4d^4 \\
& c^2b^2a^4 + 15x^7e^5b^2a^4 + 6x^7e^5c^2a^5 + x^6d^5b^6 + 30x^6d^5 \\
& c^2b^4a + 30x^6e^4d^4b^5a + 90x^6d^5c^2b^2a^2 + 300x^6e^4d^4c^2b^3 \\
& a^2 + 150x^6e^2d^3b^4a^2 + 20x^6d^5c^3a^3 + 300x^6e^4d^4c^2b^2a^3 \\
& + 600x^6e^2d^3c^2b^2a^3 + 200x^6e^3d^2b^3a^3 + 150x^6e^2d^3 \\
& c^2a^4 + 300x^6e^3d^2c^2b^2a^4 + 75x^6e^4d^2b^2a^4 + 30x^6e^4d^4c^2 \\
& a^5 + 6x^6e^5b^2a^5 + 6x^5d^5b^5a + 60x^5d^5c^2b^3a^2 + 75x^5e^4d^4 \\
& b^4a^2 + 60x^5d^5c^2b^2a^3 + 300x^5e^4d^4c^2b^2a^3 + 200x^5e^2d^3 \\
& b^3a^3 + 75x^5e^4d^4c^2a^4 + 300x^5e^2d^3c^2b^2a^4 + 150x^5e^3d^2 \\
& b^2a^4 + 60x^5e^3d^2c^2a^5 + 30x^5e^4d^2b^2a^5 + x^5e^5a^6 + 15x^4 \\
& d^5b^4a^2 + 60x^4d^5c^2b^2a^3 + 100x^4e^4d^4b^3a^3 + 15x^4d^5c^2 \\
& a^4 + 150x^4e^4d^4c^2b^2a^4 + 150x^4e^2d^3b^2a^4 + 60x^4e^2d^3c^2 \\
& a^5 + 60x^4e^3d^2b^2a^5 + 5x^4e^4d^2a^6 + 20x^3d^5b^3a^3 + 30x^3 \\
& d^5c^2b^2a^4 + 75x^3e^4d^4b^2a^4 + 30x^3e^4d^4c^2a^5 + 60x^3e^2d^3 \\
& b^2a^5 + 10x^3e^3d^2a^6 + 15x^2d^5b^2a^4 + 6x^2d^5c^2a^5 + 30x^2 \\
& e^4d^4b^2a^5 + 10x^2e^2d^3a^6 + 6x^2d^5b^2a^5 + 5x^2e^4d^4a^6
\end{aligned}$$

**giac [B]** time = 0.23, size = 2383, normalized size = 119.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(c\*x^2+b\*x+a)^5\*(d\*(5\*a\*e+6\*b\*d)+(5\*a\*e^2+17\*b\*d\*e+12\*c\*d^2)\*x+e\*(11\*b\*e+29\*c\*d)\*x^2+17\*c\*e^2\*x^3),x, algorithm="giac")

[Out]  $c^6 x^{17} e^5 + 5 c^6 d x^{16} e^4 + 10 c^6 d^2 x^{15} e^3 + 10 c^6 d^3 x^{14} e^2 + 5 c^6 d^4 x^{13} e + c^6 d^5 x^{12} + 6 b^5 c^5 x^{16} e^5 + 30 b^5 c^5 d x^{15} e^4 + 60 b^5 c^5 d^2 x^{14} e^3 + 60 b^5 c^5 d^3 x^{13} e^2 + 30 b^5 c^5 d^4 x^{12} e + 6 b^5 c^5 d^5 x^{11} + 15 b^5 c^4 d^2 x^{15} e^5 + 6 a^5 c^5 x^{15} e^5 + 75 b^5 c^4 d x^{14} e^4 + 30 a^5 c^5 d x^{14} e^4 + 150 b^5 c^4 d^2 x^{13} e^3 + 60 a^5 c^5 d^2 x^{13} e^3 + 150 b^5 c^4 d^3 x^{12} e^2 + 60 a^5 c^5 d^3 x^{12} e^2 + 75 b^5 c^4 d^4 x^{11} e + 30 a^5 c^5 d^4 x^{11} e + 15 b^5 c^4 d^5 x^{10} + 6 a^5 c^5 d^5 x^{10} + 20 b^5 c^3 x^{14} e^5 + 30 a^5 b^3 c^4 x^{14} e^5 + 100 b^5 c^3 d x^{13} e^4 + 150 a^5 b^3 c^4 d x^{13} e^4 + 200 b^5 c^3 d^2 x^{12} e^3 + 300 a^5 b^3 c^4 d^2 x^{12} e^3 + 200 b^5 c^3 d^3 x^{11} e^2 + 300 a^5 b^3 c^4 d^3 x^{11} e^2 + 100 b^5 c^3 d^4 x^{10} e + 150 a^5 b^3 c^4 d^4 x^{10} e + 20 b^5 c^3 d^5 x^9 + 30 a^5 b^3 c^4 d^5 x^9 + 15 b^5 c^2 x^{13} e^5 + 60 a^5 b^2 c^3 x^{13} e^5 + 15 a^5 c^2 d^4 x^{13} e^5 + 75 b^5 c^2 d x^{12} e^4 + 300 a^5 b^2 c^3 d x^{12} e^4 + 75 a^5 c^2 d^2 x^{12} e^4 + 150 b^5 c^2 d^2 x^{11} e^3 + 600 a^5 b^2 c^3 d^2 x^{11} e^3 + 150 a^5 c^2 d^3 x^{11} e^3 + 150 b^5 c^2 d^3 x^{10} e^2 + 600 a^5 b^2 c^3 d^3 x^{10} e^2 + 150 a^5 c^2 d^4 x^{10} e^2 + 75 b^5 c^2 d^4 x^9 e + 300 a^5 b^2 c^3 d^4 x^9 e + 75 a^5 c^2 d^4 x^9 e + 15 b^5 c^2 d^5 x^8 + 60 a^5 b^2 c^3 d^5 x^8 + 15 a^5 c^2 d^5 x^8 + 6 b^5 c^5 x^{12} e^5 + 60 a^5 b^3 c^2 x^{12} e^5 + 60 a^5 c^2 b^3 c^3 x^{12} e^5 + 30 b^5 c^5 d x^{11} e^4 + 300 a^5 b^3 c^2 d x^{11} e^4 + 300 a^5 c^2 b^3 c^3 d x^{11} e^4 + 60 b^5 c^5 d^2 x^{10} e^3 + 600 a^5 b^3 c^2 d^2 x^{10} e^3 + 600 a^5 c^2 b^3 c^3 d^2 x^{10} e^3 + 60 b^5 c^5 d^3 x^9 e^2 + 600 a^5 b^3 c^2 d^3 x^9 e^2 + 600 a^5 c^2 b^3 c^3 d^3 x^9 e^2 + 30 b^5 c^5 d^4 x^8 e + 300 a^5 b^3 c^2 d^4 x^8 e + 300 a^5 c^2 b^3 c^3 d^4 x^8 e + 6 b^5 c^5 d^5 x^7 + 60 a^5 b^3 c^2 d^5 x^7 + 60 a^5 c^2 b^3 c^3 d^5 x^7 + b^6 x^{11} e^5 + 30 a^5 b^4 c^5 x^{11} e^5 + 90 a^5 c^2 b^2 c^2 x^{11} e^5 + 20 a^5 c^3 c^3 x^{11} e^5 + 5 b^6 d x^{10} e^4 + 150 a^5 b^4 c^5 d x^{10} e^4 + 450 a^5 c^2 b^2 c^2 d x^{10} e^4 + 100 a^5 c^3 c^3 d x^{10} e^4 + 10 b^6 d^2 x^9 e^3 + 300 a^5 b^4 c^5 d^2 x^9 e^3 + 900 a^5 c^2 b^2 c^2 d^2 x^9 e^3 + 200 a^5 c^3 c^3 d^2 x^9 e^3 + 10 b^6 d^3 x^8 e^2 + 300 a^5 b^4 c^5 d^3 x^8 e^2 + 900 a^5 c^2 b^2 c^2 d^3 x^8 e^2 + 200 a^5 c^3 c^3 d^3 x^8 e^2 + 5 b^6 d^4 x^7 e + 150 a^5 b^4 c^5 d^4 x^7 e + 450 a^5 c^2 b^2 c^2 d^4 x^7 e + 100 a^5 c^3 c^3 d^4 x^7 e + b^6 d^5 x^6 + 30 a^5 b^4 c^5 d^5 x^6 + 90 a^5 c^2 b^2 c^2 d^5 x^6 + 20 a^5 c^3 c^3 d^5 x^6 + 6 a^5 b^5 x^{10} e^5 + 60 a^5 c^2 b^3 c^3 x^{10} e^5 + 60 a^5 c^3 b^3 c^2 x^{10} e^5 + 30 a^5 b^5 d x^9 e^4 + 300 a^5 c^2 b^3 c^3 d x^9 e^4 + 300 a^5 c^3 b^3 c^2 d x^9 e^4 + 60 a^5 b^5 d^2 x^8 e^3 + 600 a^5 c^2 b^3 c^3 d^2 x^8 e^3 + 600 a^5 c^3 b^3 c^2 d^2 x^8 e^3 + 60 a^5 b^5 d^3 x^7 e^2 + 600 a^5 c^2 b^3 c^3 d^3 x^7 e^2 + 600 a^5 c^3 b^3 c^2 d^3 x^7 e^2 + 30 a^5 b^5 d^4 x^6 e + 300 a^5 c^2 b^3 c^3 d^4 x^6 e + 300 a^5 c^3 b^3 c^2 d^4 x^6 e + 6 a^5 b^5 d^5 x^5 + 60 a^5 c^2 b^3 c^3 d^5 x^5 + 60 a^5 c^3 b^3 c^2 d^5 x^5 + 15 a^5 c^2 b^4 x^9 e^5 + 60 a^5 c^3 b^2 c^2 x^9 e^5 + 15 a^5 c^4 c^2 x^9 e^5 + 75 a^5 c^2 b^4 d x^8 e^4 + 300 a^5 c^3 b^2 c^2 d x^8 e^4 + 75 a^5 c^4 c^2 d x^8 e^4 + 150 a^5 c^2 b^4 d^2 x^7 e^3 + 600 a^5 c^3 b^2 c^2 d^2 x^7 e^3 + 150 a^5 c^4 c^2 d^2 x^7 e^3 + 150 a^5 c^2 b^4 d^3 x^6 e^2 + 600 a^5 c^3 b^2 c^2 d^3 x^6 e^2 + 150 a^5 c^4 c^2 d^3 x^6 e^2 + 75 a^5$



$$2*b^4*d^4*x^5*e + 300*a^3*b^2*c*d^4*x^5*e + 75*a^4*c^2*d^4*x^5*e + 15*a^2*b^4*d^5*x^4 + 60*a^3*b^2*c*d^5*x^4 + 15*a^4*c^2*d^5*x^4 + 20*a^3*b^3*x^8*e^5 + 30*a^4*b*c*x^8*e^5 + 100*a^3*b^3*d*x^7*e^4 + 150*a^4*b*c*d*x^7*e^4 + 200*a^3*b^3*d^2*x^6*e^3 + 300*a^4*b*c*d^2*x^6*e^3 + 200*a^3*b^3*d^3*x^5*e^2 + 300*a^4*b*c*d^3*x^5*e^2 + 100*a^3*b^3*d^4*x^4*e + 150*a^4*b*c*d^4*x^4*e + 200*a^3*b^3*d^5*x^3 + 30*a^4*b*c*d^5*x^3 + 15*a^4*b^2*x^7*e^5 + 6*a^5*c*x^7*e^5 + 75*a^4*b^2*d*x^6*e^4 + 30*a^5*c*d*x^6*e^4 + 150*a^4*b^2*d^2*x^5*e^3 + 60*a^5*c*d^2*x^5*e^3 + 150*a^4*b^2*d^3*x^4*e^2 + 60*a^5*c*d^3*x^4*e^2 + 75*a^4*b^2*d^4*x^3*e + 30*a^5*c*d^4*x^3*e + 15*a^4*b^2*d^5*x^2 + 6*a^5*c*d^5*x^2 + 6*a^5*b*x^6*e^5 + 30*a^5*b*d*x^5*e^4 + 60*a^5*b*d^2*x^4*e^3 + 60*a^5*b*d^3*x^3*e^2 + 30*a^5*b*d^4*x^2*e + 6*a^5*b*d^5*x + a^6*x^5*e^5 + 5*a^6*d*x^4*e^4 + 10*a^6*d^2*x^3*e^3 + 10*a^6*d^3*x^2*e^2 + 5*a^6*d^4*x*e$$

**maple [B]** time = 0.00, size = 8419, normalized size = 420.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3), x)$

[Out] result too large to display

**maxima [B]** time = 0.50, size = 1779, normalized size = 88.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3), x, \text{algorithm}="maxima")$

[Out]  $c^6*e^5*x^{17} + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^{16} + (10*c^6*d^2*e^3 + 30*b*c^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^{15} + 5*(2*c^6*d^3*e^2 + 12*b*c^5*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*x^{14} + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 + 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^5)*x^{13} + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^{12} + (6*b*c^5*d^5 + 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)*x^{11} + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e + 150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^{10} + 5*(2*(2*b^3*c^3 + 3*a*b*c^4$

$$\begin{aligned}
& ) * d^5 + 15 * (b^4 * c^2 + 4 * a * b^2 * c^3 + a^2 * c^4) * d^4 * e + 12 * (b^5 * c + 10 * a * b^3 * c^2 + 10 * a^2 * b * c^3) * d^3 * e^2 + 2 * (b^6 + 30 * a * b^4 * c + 90 * a^2 * b^2 * c^2 + 20 * a^3 * c^3) * d^2 * e^3 + 6 * (a * b^5 + 10 * a^2 * b^3 * c + 10 * a^3 * b * c^2) * d * e^4 + 3 * (a^2 * b^4 + 4 * a^3 * b^2 * c + a^4 * c^2) * e^5 * x^9 + 5 * (3 * (b^4 * c^2 + 4 * a * b^2 * c^3 + a^2 * c^4) * d^5 + 6 * (b^5 * c + 10 * a * b^3 * c^2 + 10 * a^2 * b * c^3) * d^4 * e + 2 * (b^6 + 30 * a * b^4 * c + 90 * a^2 * b^2 * c^2 + 20 * a^3 * c^3) * d^3 * e^2 + 12 * (a * b^5 + 10 * a^2 * b^3 * c + 10 * a^3 * b * c^2) * d^2 * e^3 + 15 * (a^2 * b^4 + 4 * a^3 * b^2 * c + a^4 * c^2) * d * e^4 + 2 * (2 * a^3 * b^3 + 3 * a^4 * b * c) * e^5) * x^8 + (6 * (b^5 * c + 10 * a * b^3 * c^2 + 10 * a^2 * b * c^3) * d^5 + 5 * (b^6 + 30 * a * b^4 * c + 90 * a^2 * b^2 * c^2 + 20 * a^3 * c^3) * d^4 * e + 60 * (a * b^5 + 10 * a^2 * b^3 * c + 10 * a^3 * b * c^2) * d^3 * e^2 + 150 * (a^2 * b^4 + 4 * a^3 * b^2 * c + a^4 * c^2) * d^2 * e^3 + 50 * (2 * a^3 * b^3 + 3 * a^4 * b * c) * d * e^4 + 3 * (5 * a^4 * b^2 + 2 * a^5 * c) * e^5) * x^7 + (6 * a^5 * b * e^5 + (b^6 + 30 * a * b^4 * c + 90 * a^2 * b^2 * c^2 + 20 * a^3 * c^3) * d^5 + 30 * (a * b^5 + 10 * a^2 * b^3 * c + 10 * a^3 * b * c^2) * d^4 * e + 150 * (a^2 * b^4 + 4 * a^3 * b^2 * c + a^4 * c^2) * d^3 * e^2 + 100 * (2 * a^3 * b^3 + 3 * a^4 * b * c) * d^2 * e^3 + 15 * (5 * a^4 * b^2 + 2 * a^5 * c) * d * e^4) * x^6 + (30 * a^5 * b * d * e^4 + a^6 * e^5 + 6 * (a * b^5 + 10 * a^2 * b^3 * c + 10 * a^3 * b * c^2) * d^5 + 75 * (a^2 * b^4 + 4 * a^3 * b^2 * c + a^4 * c^2) * d^4 * e + 100 * (2 * a^3 * b^3 + 3 * a^4 * b * c) * d^3 * e^2 + 30 * (5 * a^4 * b^2 + 2 * a^5 * c) * d^2 * e^3) * x^5 + 5 * (12 * a^5 * b * d^2 * e^3 + a^6 * d * e^4 + 3 * (a^2 * b^4 + 4 * a^3 * b^2 * c + a^4 * c^2) * d^5 + 10 * (2 * a^3 * b^3 + 3 * a^4 * b * c) * d^4 * e + 6 * (5 * a^4 * b^2 + 2 * a^5 * c) * d^3 * e^2) * x^4 + 5 * (12 * a^5 * b * d^3 * e^2 + 2 * a^6 * d^2 * e^3 + 2 * (2 * a^3 * b^3 + 3 * a^4 * b * c) * d^5 + 3 * (5 * a^4 * b^2 + 2 * a^5 * c) * d^4 * e) * x^3 + (30 * a^5 * b * d^4 * e + 10 * a^6 * d^3 * e^2 + 3 * (5 * a^4 * b^2 + 2 * a^5 * c) * d^5) * x^2 + (6 * a^5 * b * d^5 + 5 * a^6 * d^4 * e) * x
\end{aligned}$$

**mupad [B]** time = 4.87, size = 2026, normalized size = 101.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x)^3 * (a + b*x + c*x^2)^5 * (d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c*d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3), x)$

[Out]  $x^6 * (b^6 * d^5 + 6 * a^5 * b * e^5 + 20 * a^3 * c^3 * d^5 + 75 * a^4 * b^2 * d * e^4 + 90 * a^2 * b^2 * c^2 * d^5 + 150 * a^2 * b^4 * d^3 * e^2 + 200 * a^3 * b^3 * d^2 * e^3 + 150 * a^4 * c^2 * d^3 * e^2 + 30 * a * b^4 * c * d^5 + 30 * a * b^5 * d^4 * e + 30 * a^5 * c * d * e^4 + 300 * a^2 * b^3 * c * d^4 * e + 300 * a^3 * b * c^2 * d^4 * e + 300 * a^4 * b * c * d^2 * e^3 + 600 * a^3 * b^2 * c * d^3 * e^2) + x^{11} * (b^6 * e^5 + 6 * b * c^5 * d^5 + 20 * a^3 * c^3 * e^5 + 75 * b^2 * c^4 * d^4 * e + 90 * a^2 * b^2 * c^2 * e^5 + 150 * a^2 * c^4 * d^2 * e^3 + 200 * b^3 * c^3 * d^3 * e^2 + 150 * b^4 * c^2 * d^2 * e^3 + 30 * a * b^4 * c * e^5 + 30 * a * c^5 * d^4 * e + 30 * b^5 * c * d * e^4 + 300 * a * b * c^4 * d^3 * e^2 + 300 * a * b^3 * c^2 * d * e^4 + 300 * a^2 * b * c^3 * d * e^4 + 600 * a * b^2 * c^3 * d^2 * e^3) + x^5 * (a^6 * e^5 + 6 * a * b^5 * d^5 + 60 * a^2 * b^3 * c * d^5 + 60 * a^3 * b * c^2 * d^5 + 75 * a^2 * b^4 * d^4 * e + 75 * a^4 * c^2 * d^4 * e + 60 * a^5 * c * d^2 * e^3 + 200 * a^3 * b^3 * d^3 * e^2 + 150 * a^4 * b^2 * d^2 * e^3 + 30 * a^5 * b * d * e^4 + 300 * a^3 * b^2 * c * d^4 * e + 300 * a^4 * b * c * d^3 * e^2) + x^3 * (20 * a^3 * b^3 * d^5 + 10 * a^6 * d^2 * e^3 + 75 * a^4 * b^2 * d^4 * e + 60 * a^5 * b * d^3 * e^2 + 30 * a^4 * b * c * d^5 + 30 * a^5 * c * d^4 * e) + x^{12} * (c^6 * d^5 + 6 * b^5 * c * e^5 + 60 * a * b^3 * c^2 * e^5 + 60 * a^2 * b * c^3 * e^5 + 60 * a * c^5 * d^3 * e^2 + 75 * a^2 * c^4 * d * e^4 + 75 * b^4 * c^2 * d * e^4)$

$$\begin{aligned}
& e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d^4*e + 300*a*b*c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b^5*c*d^5 + 5*b^6*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*d^5 + 60*a*b^5*d^3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b^4*d^2*e^3 + 150*a^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 + 450*a^2*b^2*c^2*d^4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 + 600*a^3*b^2*c*d^2*e^3) + x^{10}*(6*a*b^5*e^5 + 6*a*c^5*d^5 + 5*b^6*d*e^4 + 15*b^2*c^4*d^5 + 60*a^2*b^3*c*e^5 + 60*a^3*b*c^2*e^5 + 100*a^3*c^3*d*e^4 + 100*b^3*c^3*d^4*e + 60*b^5*c*d^2*e^3 + 150*a^2*c^4*d^3*e^2 + 150*b^4*c^2*d^3*e^2 + 150*a*b*c^4*d^4*e + 150*a*b^4*c*d*e^4 + 600*a*b^2*c^3*d^3*e^2 + 600*a*b^3*c^2*d^2*e^3 + 600*a^2*b*c^3*d^2*e^3 + 450*a^2*b^2*c^2*d*e^4) + x^8*(15*a^2*c^4*d^5 + 20*a^3*b^3*e^5 + 15*b^4*c^2*d^5 + 10*b^6*d^3*e^2 + 60*a*b^2*c^3*d^5 + 60*a*b^5*d^2*e^3 + 75*a^2*b^4*d*e^4 + 75*a^4*c^2*d*e^4 + 200*a^3*c^3*d^3*e^2 + 30*a^4*b*c*e^5 + 30*b^5*c*d^4*e + 900*a^2*b^2*c^2*d^3*e^2 + 300*a*b^3*c^2*d^4*e + 300*a*b^4*c*d^3*e^2 + 300*a^2*b*c^3*d^4*e + 300*a^3*b^2*c*d*e^4 + 600*a^2*b^3*c*d^2*e^3 + 600*a^3*b*c^2*d^2*e^3) + x^9*(15*a^2*b^4*e^5 + 15*a^4*c^2*e^5 + 20*b^3*c^3*d^5 + 10*b^6*d^2*e^3 + 60*a^3*b^2*c*e^5 + 75*a^2*c^4*d^4*e + 75*b^4*c^2*d^4*e + 60*b^5*c*d^3*e^2 + 200*a^3*c^3*d^2*e^3 + 30*a*b*c^4*d^5 + 30*a*b^5*d*e^4 + 900*a^2*b^2*c^2*d^2*e^3 + 300*a*b^2*c^3*d^4*e + 300*a*b^4*c*d^2*e^3 + 300*a^2*b^3*c*d*e^4 + 300*a^3*b*c^2*d*e^4 + 600*a*b^3*c^2*d^3*e^2 + 600*a^2*b*c^3*d^3*e^2) + x^4*(5*a^6*d*e^4 + 15*a^2*b^4*d^5 + 15*a^4*c^2*d^5 + 60*a^3*b^2*c*d^5 + 100*a^3*b^3*d^4*e + 60*a^5*b*d^2*e^3 + 60*a^5*c*d^3*e^2 + 150*a^4*b^2*d^3*e^2 + 150*a^4*b*c*d^4*e) + x^{13}*(5*c^6*d^4*e + 15*a^2*c^4*e^5 + 15*b^4*c^2*e^5 + 60*a*b^2*c^3*e^5 + 60*a*c^5*d^2*e^3 + 60*b*c^5*d^3*e^2 + 100*b^3*c^3*d*e^4 + 150*b^2*c^4*d^2*e^3 + 150*a*b*c^4*d*e^4) + c^6*e^5*x^{17} + a^5*d^4*x*(5*a*e + 6*b*d) + 5*c^3*e^2*x^{14}*(4*b^3*e^3 + 2*c^3*d^3 + 6*a*b*c*e^3 + 6*a*c^2*d*e^2 + 12*b*c^2*d^2*e + 15*b^2*c*d*e^2) + c^5*e^4*x^{16}*(6*b*e + 5*c*d) + a^4*d^3*x^2*(10*a^2*e^2 + 15*b^2*d^2 + 6*a*c*d^2 + 30*a*b*d*e) + c^4*e^3*x^{15}*(15*b^2*e^2 + 10*c^2*d^2 + 6*a*c*e^2 + 30*b*c*d*e)
\end{aligned}$$

**sympy [B]** time = 1.53, size = 2281, normalized size = 114.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(c\*x\*\*2+b\*x+a)\*\*5\*(d\*(5\*a\*e+6\*b\*d)+(5\*a\*e\*\*2+17\*b\*d\*e+12\*c\*d\*\*2)\*x+e\*(11\*b\*e+29\*c\*d)\*x\*\*2+17\*c\*e\*\*2\*x\*\*3),x)

[Out] c\*\*6\*e\*\*5\*x\*\*17 + x\*\*16\*(6\*b\*c\*\*5\*e\*\*5 + 5\*c\*\*6\*d\*e\*\*4) + x\*\*15\*(6\*a\*c\*\*5\*e\*\*5 + 15\*b\*\*2\*c\*\*4\*e\*\*5 + 30\*b\*c\*\*5\*d\*e\*\*4 + 10\*c\*\*6\*d\*\*2\*e\*\*3) + x\*\*14\*(30\*a\*b\*c\*\*4\*e\*\*5 + 30\*a\*c\*\*5\*d\*e\*\*4 + 20\*b\*\*3\*c\*\*3\*e\*\*5 + 75\*b\*\*2\*c\*\*4\*d\*e\*\*4 + 60\*b\*c\*\*5\*d\*\*2\*e\*\*3 + 10\*c\*\*6\*d\*\*3\*e\*\*2) + x\*\*13\*(15\*a\*\*2\*c\*\*4\*e\*\*5 + 60\*a\*b\*\*2\*c\*\*3\*e\*\*5 + 150\*a\*b\*c\*\*4\*d\*e\*\*4 + 60\*a\*c\*\*5\*d\*\*2\*e\*\*3 + 15\*b\*\*4\*c\*\*2\*e\*\*5 + 100\*b\*\*3\*c\*\*3\*d\*e\*\*4 + 150\*b\*\*2\*c\*\*4\*d\*\*2\*e\*\*3 + 60\*b\*c\*\*5\*d\*\*3\*e\*\*2)

$$\begin{aligned}
& *2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d**e**4 + 60 \\
& *a*b**3*c**2*e**5 + 300*a*b**2*c**3*d**e**4 + 300*a*b*c**4*d**2*e**3 + 60*a* \\
& c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d**e**4 + 200*b**3*c**3*d**2*e \\
& **3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x**11*(20*a \\
& **3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d**e**4 + 150*a**2* \\
& c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d**e**4 + 600*a*b**2*c** \\
& 3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6*e**5 + 30*b* \\
& *5*c*d**e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e**2 + 75*b**2*c \\
& **4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 100*a**3*c**3*d* \\
& e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d**e**4 + 600*a**2*b*c**3*d* \\
& *2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*b**4*c*d**e**4 + 6 \\
& 00*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 150*a*b*c**4*d**4*e \\
& + 6*a*c**5*d**5 + 5*b**6*d**e**4 + 60*b**5*c*d**2*e**3 + 150*b**4*c**2*d**3* \\
& e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x**9*(15*a**4*c**2*e**5 \\
& + 60*a**3*b**2*c*e**5 + 300*a**3*b*c**2*d**e**4 + 200*a**3*c**3*d**2*e**3 + \\
& 15*a**2*b**4*e**5 + 300*a**2*b**3*c*d**e**4 + 900*a**2*b**2*c**2*d**2*e**3 + \\
& 600*a**2*b*c**3*d**3*e**2 + 75*a**2*c**4*d**4*e + 30*a*b**5*d**e**4 + 300*a \\
& *b**4*c*d**2*e**3 + 600*a*b**3*c**2*d**3*e**2 + 300*a*b**2*c**3*d**4*e + 30 \\
& *a*b*c**4*d**5 + 10*b**6*d**2*e**3 + 60*b**5*c*d**3*e**2 + 75*b**4*c**2*d** \\
& 4*e + 20*b**3*c**3*d**5) + x**8*(30*a**4*b*c**e**5 + 75*a**4*c**2*d**e**4 + 2 \\
& 0*a**3*b**3*e**5 + 300*a**3*b**2*c*d**e**4 + 600*a**3*b*c**2*d**2*e**3 + 200 \\
& *a**3*c**3*d**3*e**2 + 75*a**2*b**4*d**e**4 + 600*a**2*b**3*c*d**2*e**3 + 90 \\
& 0*a**2*b**2*c**2*d**3*e**2 + 300*a**2*b*c**3*d**4*e + 15*a**2*c**4*d**5 + 6 \\
& 0*a*b**5*d**2*e**3 + 300*a*b**4*c*d**3*e**2 + 300*a*b**3*c**2*d**4*e + 60*a \\
& *b**2*c**3*d**5 + 10*b**6*d**3*e**2 + 30*b**5*c*d**4*e + 15*b**4*c**2*d**5) \\
& + x**7*(6*a**5*c*e**5 + 15*a**4*b**2*e**5 + 150*a**4*b*c*d**e**4 + 150*a**4 \\
& *c**2*d**2*e**3 + 100*a**3*b**3*d**e**4 + 600*a**3*b**2*c*d**2*e**3 + 600*a* \\
& **3*b*c**2*d**3*e**2 + 100*a**3*c**3*d**4*e + 150*a**2*b**4*d**2*e**3 + 600* \\
& a**2*b**3*c*d**3*e**2 + 450*a**2*b**2*c**2*d**4*e + 60*a**2*b*c**3*d**5 + 6 \\
& 0*a*b**5*d**3*e**2 + 150*a*b**4*c*d**4*e + 60*a*b**3*c**2*d**5 + 5*b**6*d** \\
& 4*e + 6*b**5*c*d**5) + x**6*(6*a**5*b*e**5 + 30*a**5*c*d**e**4 + 75*a**4*b** \\
& 2*d**e**4 + 300*a**4*b*c*d**2*e**3 + 150*a**4*c**2*d**3*e**2 + 200*a**3*b**3 \\
& *d**2*e**3 + 600*a**3*b**2*c*d**3*e**2 + 300*a**3*b*c**2*d**4*e + 20*a**3*c \\
& **3*d**5 + 150*a**2*b**4*d**3*e**2 + 300*a**2*b**3*c*d**4*e + 90*a**2*b**2* \\
& c**2*d**5 + 30*a*b**5*d**4*e + 30*a*b**4*c*d**5 + b**6*d**5) + x**5*(a**6*e \\
& **5 + 30*a**5*b*d**e**4 + 60*a**5*c*d**2*e**3 + 150*a**4*b**2*d**2*e**3 + 30 \\
& 0*a**4*b*c*d**3*e**2 + 75*a**4*c**2*d**4*e + 200*a**3*b**3*d**3*e**2 + 300* \\
& a**3*b**2*c*d**4*e + 60*a**3*b*c**2*d**5 + 75*a**2*b**4*d**4*e + 60*a**2*b* \\
& **3*c*d**5 + 6*a*b**5*d**5) + x**4*(5*a**6*d**e**4 + 60*a**5*b*d**2*e**3 + 60 \\
& *a**5*c*d**3*e**2 + 150*a**4*b**2*d**3*e**2 + 150*a**4*b*c*d**4*e + 15*a**4 \\
& *c**2*d**5 + 100*a**3*b**3*d**4*e + 60*a**3*b**2*c*d**5 + 15*a**2*b**4*d**5 \\
& ) + x**3*(10*a**6*d**2*e**3 + 60*a**5*b*d**3*e**2 + 30*a**5*c*d**4*e + 75*a \\
& **4*b**2*d**4*e + 30*a**4*b*c*d**5 + 20*a**3*b**3*d**5) + x**2*(10*a**6*d** \\
& 3*e**2 + 30*a**5*b*d**4*e + 6*a**5*c*d**5 + 15*a**4*b**2*d**5) + x*(5*a**6* \\
& d**4*e + 6*a**5*b*d**5)
\end{aligned}$$

$$3.259 \quad \int \frac{x^2+x^3}{-2+x+x^2} dx$$

Optimal. Leaf size=26

$$\frac{x^2}{2} + \frac{2}{3} \log(1-x) + \frac{4}{3} \log(x+2)$$

**Rubi** [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {1593, 800, 632, 31}

$$\frac{x^2}{2} + \frac{2}{3} \log(1-x) + \frac{4}{3} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(x^2 + x^3)/(-2 + x + x^2), x]

[Out] x^2/2 + (2\*Log[1 - x])/3 + (4\*Log[2 + x])/3

Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(c\*d - e\*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c\*x), x], x] - Dist[(c\*d - e\*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && NiceSqrtQ[b^2 - 4\*a\*c]

Rule 800

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

Rule 1593

Int[(u\_)\*((a\_)\*(x\_)^(p\_) + (b\_)\*(x\_)^(q\_))^(n\_), x\_Symbol] := Int[u\*x^(n\*p)\*(a + b\*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^2 + x^3}{-2 + x + x^2} dx &= \int \frac{x^2(1 + x)}{-2 + x + x^2} dx \\
&= \int \left( x + \frac{2x}{-2 + x + x^2} \right) dx \\
&= \frac{x^2}{2} + 2 \int \frac{x}{-2 + x + x^2} dx \\
&= \frac{x^2}{2} + \frac{2}{3} \int \frac{1}{-1 + x} dx + \frac{4}{3} \int \frac{1}{2 + x} dx \\
&= \frac{x^2}{2} + \frac{2}{3} \log(1 - x) + \frac{4}{3} \log(2 + x)
\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^2}{2} + \frac{2}{3} \log(1 - x) + \frac{4}{3} \log(x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2 + x^3)/(-2 + x + x^2), x]

[Out] x^2/2 + (2\*Log[1 - x])/3 + (4\*Log[2 + x])/3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x^3}{-2 + x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2 + x^3)/(-2 + x + x^2), x]

[Out] IntegrateAlgebraic[(x^2 + x^3)/(-2 + x + x^2), x]

**fricas [A]** time = 0.60, size = 18, normalized size = 0.69

$$\frac{1}{2} x^2 + \frac{4}{3} \log(x + 2) + \frac{2}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2)/(x^2+x-2), x, algorithm="fricas")

[Out]  $\frac{1}{2}x^2 + \frac{4}{3}\log(x + 2) + \frac{2}{3}\log(x - 1)$

**giac** [A] time = 0.16, size = 20, normalized size = 0.77

$$\frac{1}{2}x^2 + \frac{4}{3}\log(|x + 2|) + \frac{2}{3}\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")`

[Out]  $\frac{1}{2}x^2 + \frac{4}{3}\log(\text{abs}(x + 2)) + \frac{2}{3}\log(\text{abs}(x - 1))$

**maple** [A] time = 0.00, size = 19, normalized size = 0.73

$$\frac{x^2}{2} + \frac{4 \ln(x + 2)}{3} + \frac{2 \ln(x - 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+x^2)/(x^2+x-2),x)`

[Out]  $\frac{1}{2}x^2 + \frac{4}{3}\ln(x+2) + \frac{2}{3}\ln(x-1)$

**maxima** [A] time = 0.43, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2 + \frac{4}{3}\log(x + 2) + \frac{2}{3}\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")`

[Out]  $\frac{1}{2}x^2 + \frac{4}{3}\log(x + 2) + \frac{2}{3}\log(x - 1)$

**mupad** [B] time = 0.05, size = 18, normalized size = 0.69

$$\frac{2 \ln(x - 1)}{3} + \frac{4 \ln(x + 2)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^3)/(x + x^2 - 2),x)`

[Out]  $\frac{(2*\log(x - 1))}{3} + \frac{(4*\log(x + 2))}{3} + x^2/2$

**sympy** [A] time = 0.24, size = 20, normalized size = 0.77

$$\frac{x^2}{2} + \frac{2 \log(x - 1)}{3} + \frac{4 \log(x + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)/(x**2+x-2),x)
```

```
[Out] x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```



$$3.260 \quad \int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=346

$$\frac{x^2\sqrt{a+bx+cx^2}(-64acg+63b^2g-70bcf+80c^2e)}{240c^3} - \frac{\sqrt{a+bx+cx^2}(-2cx(-40c^2(9af+10be)+14bc(46ag+10c^2d-5ef)-40b^2(2d-5f)-40b^2(2d-5f)-40b^2(2d-5f)+48bc^2(4c-5g)-32ac^2(4d-3f)+70b^2f-63b^2g))}{240c^3}$$

**Rubi [A]** time = 0.81, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1653, 832, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}(-2cx(-40c^2(9af+10be)+14bc(46ag+10c^2d-5ef)-40b^2(2d-5f)-40b^2(2d-5f)+48bc^2(4c-5g)-32ac^2(4d-3f)+70b^2f-63b^2g))}{240c^3} + \frac{\sqrt{a+bx+cx^2}(-64acg+63b^2g-70bcf+80c^2e)}{240c^3} + \frac{\sqrt{a+bx+cx^2}(10c^2f-9b^2g)}{5c} - \frac{(1050b^3cf+40b^2c(36cd-55af)-945b^4g-60b^2c(20ce-49ag)+256a^2c^2(5ce-4ag)-2c(480c^3d-40c^2(10be+9af)-315b^3g+14bc(25bf+46ag)))*\sqrt{a+bx+cx^2}}{1920c^5} + \frac{(70b^4cf+48b^2c^2(2cd-5af)-32a^2c^3(4cd-3af)-63b^5g-40b^3c(2ce-7ag)+48a^2b^2c^2(4ce-5ag))*\text{ArcTanh}[(b+2cx)/(2\sqrt{c}\sqrt{a+bx+cx^2})]}{256c^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x + f\*x^2 + g\*x^3))/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((80\*c^2\*e - 70\*b\*c\*f + 63\*b^2\*g - 64\*a\*c\*g)\*x^2\*Sqrt[a + b\*x + c\*x^2])/(240\*c^3) + ((10\*c\*f - 9\*b\*g)\*x^3\*Sqrt[a + b\*x + c\*x^2])/(40\*c^2) + (g\*x^4\*Sqrt[a + b\*x + c\*x^2])/(5\*c) - ((1050\*b^3\*c\*f + 40\*b\*c^2\*(36\*c\*d - 55\*a\*f) - 945\*b^4\*g - 60\*b^2\*c\*(20\*c\*e - 49\*a\*g) + 256\*a^2\*c^2\*(5\*c\*e - 4\*a\*g) - 2\*c\*(480\*c^3\*d - 40\*c^2\*(10\*b\*e + 9\*a\*f) - 315\*b^3\*g + 14\*b\*c\*(25\*b\*f + 46\*a\*g)))\*Sqrt[a + b\*x + c\*x^2])/(1920\*c^5) + ((70\*b^4\*c\*f + 48\*b^2\*c^2\*(2\*c\*d - 5\*a\*f) - 32\*a^2\*c^3\*(4\*c\*d - 3\*a\*f) - 63\*b^5\*g - 40\*b^3\*c\*(2\*c\*e - 7\*a\*g) + 48\*a^2\*b^2\*c^2\*(4\*c\*e - 5\*a\*g))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(256\*c^(11/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 779**

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x

] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 832

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[(g\*(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + b\*x + c\*x^2)^p\*Simp[m\*(c\*d\*f - a\*e\*g) + d\*(2\*c\*f - b\*g)\*(p + 1) + (m\*(c\*e\*f + c\*d\*g - b\*e\*g) + e\*(p + 1)\*(2\*c\*f - b\*g))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{x^2(d + ex + fx^2 + gx^3)}{\sqrt{a + bx + cx^2}} dx &= \frac{gx^4\sqrt{a + bx + cx^2}}{5c} + \frac{\int \frac{x^2(5cd + (5ce - 4ag)x + \frac{1}{2}(10cf - 9bg)x^2)}{\sqrt{a + bx + cx^2}} dx}{5c} \\
&= \frac{(10cf - 9bg)x^3\sqrt{a + bx + cx^2}}{40c^2} + \frac{gx^4\sqrt{a + bx + cx^2}}{5c} + \frac{\int \frac{x^2(\frac{1}{2}(40c^2d - 30acf + 27abg) - \dots)}{\sqrt{a + bx + cx^2}} dx}{5c} \\
&= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)x^2\sqrt{a + bx + cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a + bx + cx^2}}{40c^2} \\
&= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)x^2\sqrt{a + bx + cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a + bx + cx^2}}{40c^2} \\
&= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)x^2\sqrt{a + bx + cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a + bx + cx^2}}{40c^2} \\
&= \frac{(80c^2e - 70bcf + 63b^2g - 64acg)x^2\sqrt{a + bx + cx^2}}{240c^3} + \frac{(10cf - 9bg)x^3\sqrt{a + bx + cx^2}}{40c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.74, size = 282, normalized size = 0.82

$$\frac{\sqrt{a + x(b + cx)} (16c^2(64a^2g - ac(80e + x(45f + 32gx)) + 2a^2x(90d + x(20e + 3(5f + 4gx))) + 4b^2c(-735ag + 300cx + 7cx(25f + 18gx)) - 8a^2(2x(90d + x(50e + 35fx + 27gx^2)) - a(275f + 161gx)) + 945b^2g - 210b^2c(5f + 3gx))}{1920c^5} - \frac{\operatorname{tanh}^{-1}\left(\frac{bx + 2cx}{2\sqrt{a + bx + cx^2}}\right) (40b^3(2bx - 7ag) - 48b^2(2ad - 5af) + 48ab^2(5bg - 4cx) + 32a^2(4ad - 3af) + 63b^2g - 70b^2f)}{256c^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x + f\*x^2 + g\*x^3))/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(945\*b^4\*g - 210\*b^3\*c\*(5\*f + 3\*g\*x) + 4\*b^2\*c\*(300\*c\*e - 735\*a\*g + 7\*c\*x\*(25\*f + 18\*g\*x)) - 8\*b\*c^2\*(-(a\*(275\*f + 161\*g\*x)) + 2\*c\*(90\*d + x\*(50\*e + 35\*f\*x + 27\*g\*x^2))) + 16\*c^2\*(64\*a^2\*g - a\*c\*(80\*e + x\*(45\*f + 32\*g\*x)) + 2\*c^2\*x\*(30\*d + x\*(20\*e + 3\*x\*(5\*f + 4\*g\*x)))))/(1920\*c^5) - (((-70\*b^4\*c\*f - 48\*b^2\*c^2\*(2\*c\*d - 5\*a\*f) + 32\*a\*c^3\*(4\*c\*d - 3\*a\*f) + 63\*b^5\*g + 40\*b^3\*c\*(2\*c\*e - 7\*a\*g) + 48\*a\*b\*c^2\*(-4\*c\*e + 5\*a\*g))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(256\*c^(11/2))

**IntegrateAlgebraic [A]** time = 1.33, size = 324, normalized size = 0.94

$$\frac{\sqrt{a + bx + c^2} (1024a^2c^2g - 2940a^2cg + 2520a^2cf + 1280a^2c^2g - 1280a^2c^2g - 720a^2c^2g + 512a^2c^2g + 945b^2g - 1050b^2c^2g - 630b^2c^2g + 1200b^2c^2g + 700b^2c^2g + 504b^2c^2g - 1440b^2c^2g - 800b^2c^2g - 560b^2c^2g - 432b^2c^2g + 960b^2c^2g + 640b^2c^2g + 880b^2c^2g + 384b^2c^2g)}{1920c^5} - \frac{\log\left(2\sqrt{a + bx + c^2} + b + 2cx\right) (240b^3c^2g - 96b^2c^2f - 280b^2c^2g + 240a^2c^2f - 192a^2c^2g + 128a^2c^2g + 63b^2g - 70b^2f + 80b^2c^2g - 96b^2c^2g)}{256c^{11/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^2\*(d + e\*x + f\*x^2 + g\*x^3))/Sqrt[a + b\*x + c\*x^2], x]

```
[Out] (Sqrt[a + b*x + c*x^2]*(-1440*b*c^3*d + 1200*b^2*c^2*e - 1280*a*c^3*e - 105
0*b^3*c*f + 2200*a*b*c^2*f + 945*b^4*g - 2940*a*b^2*c*g + 1024*a^2*c^2*g +
960*c^4*d*x - 800*b*c^3*e*x + 700*b^2*c^2*f*x - 720*a*c^3*f*x - 630*b^3*c*g
*x + 1288*a*b*c^2*g*x + 640*c^4*e*x^2 - 560*b*c^3*f*x^2 + 504*b^2*c^2*g*x^2
- 512*a*c^3*g*x^2 + 480*c^4*f*x^3 - 432*b*c^3*g*x^3 + 384*c^4*g*x^4))/(192
0*c^5) + ((-96*b^2*c^3*d + 128*a*c^4*d + 80*b^3*c^2*e - 192*a*b*c^3*e - 70*
b^4*c*f + 240*a*b^2*c^2*f - 96*a^2*c^3*f + 63*b^5*g - 280*a*b^3*c*g + 240*a
^2*b*c^2*g)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(256*c^(11/2)
)
```

**fricas** [A] time = 0.95, size = 701, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a
^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*(10*
c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a*c^4)
*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)*f + (
945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*c^4*e +
10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*sqrt(c*x^
2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2
- 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 2
80*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4 - 1440*b*c^4
*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3
- 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*
b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 4
00*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*
x)*sqrt(c*x^2 + b*x + a))/c^6]
```

**giac** [A] time = 0.31, size = 330, normalized size = 0.95

$$\frac{1}{1920} \sqrt{c^2 + b^2} \left( \frac{8d}{c} \left( \frac{8d}{c} - \frac{10cf - 9b^2}{c^2} \right) - \frac{70bf - 63f^2 + 64ac^2 - 80a^2}{c^2} \right) + \frac{480cf + 300f^2 - 360ac^2 - 315f^2 + 644ac^2 - 400b^2}{c^2} - \frac{1440b^2d + 1050f^2 - 2200ab^2f - 945f^2 + 2940ab^2g - 1024a^2c^2g - 1200b^2c^2g + 1280a^2c^2g}{256c^7} - \frac{(96b^2d - 128a^2f + 70f^2 - 240ab^2f + 96a^2f - 63f^2 + 280a^2g - 240ab^2g - 800b^2c + 192ab^2c) \log \left| \frac{2(\sqrt{c^2 + b^2} + c)}{c} \right|}{256c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^5
)*x - (70*b*c^3*f - 63*b^2*c^2*g + 64*a*c^3*g - 80*c^4*e)/c^5)*x + (480*c^4
```

```
*d + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c^2*g - 400*b*c^3*
e)/c^5)*x - (1440*b*c^3*d + 1050*b^3*c*f - 2200*a*b*c^2*f - 945*b^4*g + 294
0*a*b^2*c*g - 1024*a^2*c^2*g - 1200*b^2*c^2*e + 1280*a*c^3*e)/c^5) - 1/256*
(96*b^2*c^3*d - 128*a*c^4*d + 70*b^4*c*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f -
63*b^5*g + 280*a*b^3*c*g - 240*a^2*b*c^2*g - 80*b^3*c^2*e + 192*a*b*c^3*e)
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

**maple [B]** time = 0.01, size = 783, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 161/240*g/c^3*b*a*x*(c*x^2+b*x+a)^(1/2)+35/128*f/c^(9/2)*b^4*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*f*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+63/128*g/c^5*b^4*(c*x^2+b*x+a)^(1/2)-63/256*g/c^(11/2)*b^5
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/15*g*a^2/c^3*(c*x^2+b*x+a)^(
1/2)+1/2*d*x/c*(c*x^2+b*x+a)^(1/2)-3/4*d/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*d/c^
(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*d*a/c^(3/2)*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*e*x^2/c*(c*x^2+b*x+a)^(1/2)+5/8*
e/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*e/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))-2/3*e*a/c^2*(c*x^2+b*x+a)^(1/2)+1/4*f*x^3/c*(c*x^2+b*x+a)
^(1/2)-35/64*f/c^4*b^3*(c*x^2+b*x+a)^(1/2)+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-
3/8*f*a/c^2*x*(c*x^2+b*x+a)^(1/2)-9/40*g/c^2*b*x^3*(c*x^2+b*x+a)^(1/2)+21/8
0*g/c^3*b^2*x^2*(c*x^2+b*x+a)^(1/2)-21/64*g/c^4*b^3*x*(c*x^2+b*x+a)^(1/2)+3
5/32*g/c^(9/2)*b^3*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-49/32*g/c^
4*b^2*a*(c*x^2+b*x+a)^(1/2)-7/24*f/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)+35/96*f/c^
3*b^2*x*(c*x^2+b*x+a)^(1/2)-15/16*f/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))-4/15*g*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)+55/48*f/c^3*b*a*(c*
x^2+b*x+a)^(1/2)-15/16*g/c^(7/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+3/4*e/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/12*
e/c^2*b*x*(c*x^2+b*x+a)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is  $4ac - b^2$  zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (gx^3 + fx^2 + ex + d)}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(d + e\*x + f\*x^2 + g\*x^3))/(a + b\*x + c\*x^2)^(1/2), x)

[Out] int((x^2\*(d + e\*x + f\*x^2 + g\*x^3))/(a + b\*x + c\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + ex + fx^2 + gx^3)}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(g\*x\*\*3+f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral(x\*\*2\*(d + e\*x + f\*x\*\*2 + g\*x\*\*3)/sqrt(a + b\*x + c\*x\*\*2), x)

$$3.261 \quad \int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=245

$$\frac{\sqrt{a+bx+cx^2} \left( 2cx(-36acg + 35b^2g - 40bcf + 48c^2e) - 16c^2(8af + 9be) + 20bc(11ag + 6bf) - 105b^3g + 192cd \right)}{192c^4}$$

**Rubi [A]** time = 0.44, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2} \left( 2cx(-36acg + 35b^2g - 40bcf + 48c^2e) - 16c^2(8af + 9be) + 20bc(11ag + 6bf) - 105b^3g + 192cd \right)}{192c^4} - \frac{\tanh^{-1}\left(\frac{b+2x}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left( -24b^2c(2c-5ag) + 32b^2c(2cd-3af) + 16ac^2(4c-3ag) + 40b^3cf - 35b^4g \right)}{128c^{9/2}} + \frac{x^2\sqrt{a+bx+cx^2}(8cf-7bg)}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d + e\*x + f\*x^2 + g\*x^3))/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((8\*c\*f - 7\*b\*g)\*x^2\*Sqrt[a + b\*x + c\*x^2])/(24\*c^2) + (g\*x^3\*Sqrt[a + b\*x + c\*x^2])/(4\*c) + ((192\*c^3\*d - 16\*c^2\*(9\*b\*e + 8\*a\*f) - 105\*b^3\*g + 20\*b\*c\*(6\*b\*f + 11\*a\*g) + 2\*c\*(48\*c^2\*e - 40\*b\*c\*f + 35\*b^2\*g - 36\*a\*c\*g)\*x)\*Sqrt[a + b\*x + c\*x^2]/(192\*c^4) - ((40\*b^3\*c\*f + 32\*b\*c^2\*(2\*c\*d - 3\*a\*f) - 35\*b^4\*g - 24\*b^2\*c\*(2\*c\*e - 5\*a\*g) + 16\*a\*c^2\*(4\*c\*e - 3\*a\*g))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(9/2))

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 621**

Int[1/Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 779**

Int[((d\_) + (e\_)\*(x\_))\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^p, x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d}





```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7*b*g*x) - 8
*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^3*(12*d + x*
(6*e + 4*f*x + 3*g*x^2))))/(192*c^4) + ((-40*b^3*c*f + 32*b*c^2*(-2*c*d + 3
*a*f) + 35*b^4*g + 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(-4*c*e + 3*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

**IntegrateAlgebraic [A]** time = 0.86, size = 220, normalized size = 0.90

$$\frac{\log\left(-2\sqrt{c}\sqrt{bx+cx^2}+b+2c\right)\left(-48b^2c^2g+120ab^2cg-96abc^2f+64ac^3e-35b^4g+40b^3cf-48b^2c^2e+64bc^3d\right)}{128c^{9/2}}+\frac{\sqrt{bx+cx^2}\left(220abcg-128a^2f-72a^2gx+120b^2cf+70b^2c^2g-144bc^2e-80bc^2fx-56bc^2gx^2+192c^3d+96c^3ex+64c^3fx^2+48c^3gx^3\right)}{192c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(192*c^3*d - 144*b*c^2*e + 120*b^2*c*f - 128*a*c^2*f
- 105*b^3*g + 220*a*b*c*g + 96*c^3*e*x - 80*b*c^2*f*x + 70*b^2*c*g*x - 72*
a*c^2*g*x + 64*c^3*f*x^2 - 56*b*c^2*g*x^2 + 48*c^3*g*x^3))/(192*c^4) + ((64
*b*c^3*d - 48*b^2*c^2*e + 64*a*c^3*e + 40*b^3*c*f - 96*a*b*c^2*f - 35*b^4*g
+ 120*a*b^2*c*g - 48*a^2*c^2*g)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c
*x^2]])/(128*c^(9/2))
```

**fricas [A]** time = 0.76, size = 499, normalized size = 2.04

$$\frac{\log\left(\frac{b+2cx-2\sqrt{c}\sqrt{bx+cx^2}}{b+2cx+2\sqrt{c}\sqrt{bx+cx^2}}\right)\left(64bc^3d-48b^2c^2e+64ac^3e+40b^3cf-96abc^2f-35b^4g+120ab^2cg-48a^2c^2g\right)}{128c^{9/2}}+\frac{\sqrt{bx+cx^2}\left(192c^3d-144b^2c^2e+120b^2cf-128ac^2f-72a^2gx+120b^2cf+70b^2c^2g-144bc^2e-80bc^2fx-56bc^2gx^2+192c^3d+96c^3ex+64c^3fx^2+48c^3gx^3\right)}{192c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c
^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c
^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b
^2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*
f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(3*(64
*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4
- 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 192*c^4*d
- 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^3)*f -
5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^2*c^2 - 36*
a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

**giac [A]** time = 0.39, size = 228, normalized size = 0.93

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6gx}{c}+\frac{8cf-7bc^2}{c^2}\right)x-\frac{40bc^2f-35b^2cg+36ac^2g-48c^3e}{c^4}\right)x+\frac{192c^3d+120b^2cf-128ac^2f-105b^3g+220abcg-144bc^2e}{c^4}\right)+\frac{(64bc^3d+40b^3cf-35b^4g+120ab^2cg-48a^2c^2g-48b^2c^2e+64ac^3e)\log\left(\left|-2\left(\sqrt{cx-\sqrt{cx^2+bx+a}}\right)\sqrt{c-b}\right|\right)}{128c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{192}\sqrt{c x^2 + b x + a} \left( 2 \left( 4 \left( 6 g x / c + (8 c^3 f - 7 b c^2 g) / c^4 \right) x - (40 b c^2 f - 35 b^2 c g + 36 a c^2 g - 48 c^3 e) / c^4 \right) x + (192 c^3 d + 120 b^2 c f - 128 a c^2 f - 105 b^3 g + 220 a b c g - 144 b c^2 e) / c^4 \right) + \frac{1}{128} \left( 64 b c^3 d + 40 b^3 c f - 96 a b c^2 f - 35 b^4 g + 120 a b^2 c g - 48 a^2 c^2 g - 48 b^2 c^2 e + 64 a c^3 e \right) \log(\text{abs}(-2(\sqrt{c} x - \sqrt{c x^2 + b x + a})) \sqrt{c} - b) / c^{9/2}$

**maple [B]** time = 0.01, size = 532, normalized size = 2.17

-----

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $\frac{1}{4} g x^3 (c x^2 + b x + a)^{1/2} / c - \frac{7}{24} g / c^2 b x^2 (c x^2 + b x + a)^{1/2} + \frac{35}{96} g / c^3 b^2 x (c x^2 + b x + a)^{1/2} - \frac{35}{64} g / c^4 b^3 (c x^2 + b x + a)^{1/2} + \frac{35}{128} g / c^{9/2} b^4 \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - \frac{15}{16} g / c^{7/2} b^2 a \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + \frac{55}{48} g / c^3 b a (c x^2 + b x + a)^{1/2} - \frac{3}{8} g a / c^2 x (c x^2 + b x + a)^{1/2} + \frac{3}{8} g a^2 / c^{5/2} \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + \frac{1}{3} f x^2 / c (c x^2 + b x + a)^{1/2} - \frac{5}{12} f / c^2 b x (c x^2 + b x + a)^{1/2} + \frac{5}{8} f / c^3 b^2 (c x^2 + b x + a)^{1/2} - \frac{5}{16} f / c^{7/2} b^3 \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + \frac{3}{4} f / c^{5/2} b a \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - \frac{2}{3} f a / c^2 (c x^2 + b x + a)^{1/2} + \frac{1}{2} e x / c (c x^2 + b x + a)^{1/2} - \frac{3}{4} e / c^2 b (c x^2 + b x + a)^{1/2} + \frac{3}{8} e / c^{5/2} b^2 \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) - \frac{1}{2} e a / c^{3/2} \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2}) + d / c (c x^2 + b x + a)^{1/2} - \frac{1}{2} d b / c^{3/2} \ln((c x + 1/2 b) / c^{1/2} + (c x^2 + b x + a)^{1/2})$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 zero or nonzero?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x (g x^3 + f x^2 + e x + d)}{\sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)`

[Out] `int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d + ex + fx^2 + gx^3)}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral(x*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)`

$$3.262 \quad \int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=177

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}} + \frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{24c^3} + \frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

**Rubi [A]** time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}} + \frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{24c^3} + \frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((24\*c^2\*e - 18\*b\*c\*f + 15\*b^2\*g - 16\*a\*c\*g)\*Sqrt[a + b\*x + c\*x^2])/(24\*c^3) + ((6\*c\*f - 5\*b\*g)\*x\*Sqrt[a + b\*x + c\*x^2])/(12\*c^2) + (g\*x^2\*Sqrt[a + b\*x + c\*x^2])/(3\*c) + (((16\*c^3\*d - 8\*c^2\*(b\*e + a\*f) - 5\*b^3\*g + 6\*b\*c\*(b\*f + 2\*a\*g))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(16\*c^(7/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2 + gx^3}{\sqrt{a + bx + cx^2}} dx &= \frac{gx^2\sqrt{a + bx + cx^2}}{3c} + \frac{\int \frac{3cd + (3ce - 2ag)x + \frac{1}{2}(6cf - 5bg)x^2}{\sqrt{a + bx + cx^2}} dx}{3c} \\ &= \frac{(6cf - 5bg)x\sqrt{a + bx + cx^2}}{12c^2} + \frac{gx^2\sqrt{a + bx + cx^2}}{3c} + \frac{\int \frac{\frac{1}{2}(12c^2d - 6acf + 5abg) + \frac{1}{4}(24c^2e - 18bcf + 15b^2g - 16acg)}{\sqrt{a + bx + cx^2}} dx}{6c^2} \\ &= \frac{(24c^2e - 18bcf + 15b^2g - 16acg)\sqrt{a + bx + cx^2}}{24c^3} + \frac{(6cf - 5bg)x\sqrt{a + bx + cx^2}}{12c^2} + \frac{\int \frac{\frac{1}{4}(24c^2e - 18bcf + 15b^2g - 16acg)}{\sqrt{a + bx + cx^2}} dx}{6c^2} \\ &= \frac{(24c^2e - 18bcf + 15b^2g - 16acg)\sqrt{a + bx + cx^2}}{24c^3} + \frac{(6cf - 5bg)x\sqrt{a + bx + cx^2}}{12c^2} + \frac{\int \frac{\frac{1}{4}(24c^2e - 18bcf + 15b^2g - 16acg)}{\sqrt{a + bx + cx^2}} dx}{6c^2} \\ &= \frac{(24c^2e - 18bcf + 15b^2g - 16acg)\sqrt{a + bx + cx^2}}{24c^3} + \frac{(6cf - 5bg)x\sqrt{a + bx + cx^2}}{12c^2} + \frac{\int \frac{\frac{1}{4}(24c^2e - 18bcf + 15b^2g - 16acg)}{\sqrt{a + bx + cx^2}} dx}{6c^2} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 141, normalized size = 0.80

$$\frac{3 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) (-8c^2(af+be) + 6bc(2ag+bf) - 5b^3g + 16c^3d) + 2\sqrt{c}\sqrt{a+x(b+cx)} (-2c(8ag+9bf+5bgx) + 15b^2g + 4c^2(6e+x(3f+2gx)))}{48c^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3)/Sqrt[a + b\*x + c\*x^2], x]

[Out] (2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]\*(15\*b^2\*g - 2\*c\*(9\*b\*f + 8\*a\*g + 5\*b\*g\*x) + 4\*c^2\*(6\*e + x\*(3\*f + 2\*g\*x))) + 3\*(16\*c^3\*d - 8\*c^2\*(b\*e + a\*f) - 5\*b^3\*g + 6\*b\*c\*(b\*f + 2\*a\*g))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/(48\*c^(7/2))

**IntegrateAlgebraic [A]** time = 0.59, size = 144, normalized size = 0.81

$$\frac{\sqrt{a + bx + cx^2} (-16acg + 15b^2g - 18bcf - 10bcgx + 24c^2e + 12c^2fx + 8c^2gx^2)}{24c^3} + \frac{\log\left(-2\sqrt{c}\sqrt{a + bx + cx^2} + b + 2cx\right) (-12abcg + 8ac^2f + 5b^3g - 6b^2cf + 8bc^2e - 16c^3d)}{16c^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3)/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[a + b\*x + c\*x^2]\*(24\*c^2\*e - 18\*b\*c\*f + 15\*b^2\*g - 16\*a\*c\*g + 12\*c^2\*f\*x - 10\*b\*c\*g\*x + 8\*c^2\*g\*x^2))/(24\*c^3) + ((-16\*c^3\*d + 8\*b\*c^2\*e - 6\*b^2\*c\*f + 8\*a\*c^2\*f + 5\*b^3\*g - 12\*a\*b\*c\*g)\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/(16\*c^(7/2))

**fricas** [A] time = 1.04, size = 341, normalized size = 1.93

$$\frac{3(8c^2d - 8b^2e + 2(3e^2 - 4af) - (5b^3 - 12abg))\sqrt{c} \log\left(\frac{-8c^2d - 8bcx - f^2 - 4\sqrt{c^2 + bx + a}(2cx + b)\sqrt{c} - 4ae}{9c^4}\right) + 4(8c^2g^2 + 24c^2e - 18bc^2f + (5b^3c - 16ac^2)g + 2(b^2f - 5b^2g))\sqrt{c^2 + bx + a} - 3(8c^2d - 8b^2e + 2(3e^2 - 4af) - (5b^3 - 12abg))\sqrt{c} \operatorname{arctan}\left(\frac{\sqrt{c^2 + bx + a}}{2(b^2 + acx)}\right) - 2(8c^2g^2 + 24c^2e - 18bc^2f + (5b^3c - 16ac^2)g + 2(b^2f - 5b^2g))\sqrt{c^2 + bx + a}}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="fricas")

[Out] [1/96\*(3\*(16\*c^3\*d - 8\*b\*c^2\*e + 2\*(3\*b^2\*c - 4\*a\*c^2)\*f - (5\*b^3 - 12\*a\*b\*c)\*g)\*sqrt(c)\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + 4\*(8\*c^3\*g\*x^2 + 24\*c^3\*e - 18\*b\*c^2\*f + (15\*b^2\*c - 16\*a\*c^2)\*g + 2\*(6\*c^3\*f - 5\*b\*c^2\*g)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^4, - 1/48\*(3\*(16\*c^3\*d - 8\*b\*c^2\*e + 2\*(3\*b^2\*c - 4\*a\*c^2)\*f - (5\*b^3 - 12\*a\*b\*c)\*g)\*sqrt(-c)\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(8\*c^3\*g\*x^2 + 24\*c^3\*e - 18\*b\*c^2\*f + (15\*b^2\*c - 16\*a\*c^2)\*g + 2\*(6\*c^3\*f - 5\*b\*c^2\*g)\*x)\*sqrt(c\*x^2 + b\*x + a))/c^4]

**giac** [A] time = 0.27, size = 149, normalized size = 0.84

$$\frac{1}{24} \sqrt{cx^2 + bx + a} \left( 2 \left( \frac{4gx}{c} + \frac{6c^2f - 5bcg}{c^3} \right) x - \frac{18bcf - 15b^2g + 16acg - 24c^2e}{c^3} \right) - \frac{(16c^3d + 6b^2cf - 8ac^2f - 5b^3g + 12abcg - 8bc^2e) \log\left(\left| -2 \left( \sqrt{cx^2 + bx + a} \right) \sqrt{c} - b \right| \right)}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x, algorithm="giac")

[Out] 1/24\*sqrt(c\*x^2 + b\*x + a)\*(2\*(4\*g\*x/c + (6\*c^2\*f - 5\*b\*c\*g)/c^3)\*x - (18\*b\*c\*f - 15\*b^2\*g + 16\*a\*c\*g - 24\*c^2\*e)/c^3) - 1/16\*(16\*c^3\*d + 6\*b^2\*c\*f - 8\*a\*c^2\*f - 5\*b^3\*g + 12\*a\*b\*c\*g - 8\*b\*c^2\*e)\*log(abs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(7/2)

**maple** [B] time = 0.01, size = 333, normalized size = 1.88

$$\frac{\sqrt{cx^2 + bx + a} g x^2}{3c} + \frac{3b g \ln\left(\frac{\sqrt{c^2 + bx + a}}{\sqrt{c}}\right)}{4c^2} + \frac{e f \ln\left(\frac{\sqrt{c^2 + bx + a}}{\sqrt{c}}\right)}{2c^2} - \frac{5b^2 g \ln\left(\frac{\sqrt{c^2 + bx + a}}{\sqrt{c}}\right)}{16c^2} + \frac{3b^2 f \ln\left(\frac{\sqrt{c^2 + bx + a}}{\sqrt{c}}\right)}{8c^2} - \frac{b \ln\left(\frac{\sqrt{c^2 + bx + a}}{\sqrt{c}}\right)}{2c^2} + \frac{d \ln\left(\frac{\sqrt{c^2 + bx + a}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{5\sqrt{cx^2 + bx + a} b g x}{12c^2} + \frac{\sqrt{cx^2 + bx + a} f x}{2c} - \frac{2\sqrt{cx^2 + bx + a} e g}{3c^2} - \frac{5\sqrt{cx^2 + bx + a} b^2 g}{8c^2} - \frac{3\sqrt{cx^2 + bx + a} b f}{4c^2} + \frac{\sqrt{cx^2 + bx + a} e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x)

```
[Out] 1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c-5/12*g/c^2*b*x*(c*x^2+b*x+a)^(1/2)+5/8*g/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*g/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/4*g/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*g*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*(c*x^2+b*x+a)^(1/2)/c*f*x-3/4*(c*x^2+b*x+a)^(1/2)*b/c^2*f+3/8*b^2/c^(5/2)*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*a/c^(3/2)*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+(c*x^2+b*x+a)^(1/2)/c*e-1/2*b/c^(3/2)*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/c^(1/2)*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2), x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

$$3.263 \quad \int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=155

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(ag+bf)+3b^2g+8c^2e)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{4c^2} - \frac{d \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + gx$$

**Rubi [A]** time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1653, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(ag+bf)+3b^2g+8c^2e)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{4c^2} - \frac{d \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{gx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/(x\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] ((4\*c\*f - 3\*b\*g)\*Sqrt[a + b\*x + c\*x^2])/(4\*c^2) + (g\*x\*Sqrt[a + b\*x + c\*x^2])/(2\*c) - (d\*ArcTanh[(2\*a + b\*x)/(2\*Sqrt[a]\*Sqrt[a + b\*x + c\*x^2])])/Sqrt[a] + ((8\*c^2\*e + 3\*b^2\*g - 4\*c\*(b\*f + a\*g))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(8\*c^(5/2))

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 843



```
Int[((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_))*((a._) + (b._)*(x_) + (c
_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1653

```
Int[(Pq_)*((d._) + (e._)*(x_))^(m_)*((a._) + (b._)*(x_) + (c._)*(x_)^2)^(p
_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2 + gx^3}{x\sqrt{a + bx + cx^2}} dx &= \frac{gx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2cd + (2ce - ag)x + \frac{1}{2}(4cf - 3bg)x^2}{x\sqrt{a + bx + cx^2}} dx}{2c} \\ &= \frac{(4cf - 3bg)\sqrt{a + bx + cx^2}}{4c^2} + \frac{gx\sqrt{a + bx + cx^2}}{2c} + \frac{\int \frac{2c^2d + \frac{1}{4}(8c^2e + 3b^2g - 4c(bf + ag))x}{x\sqrt{a + bx + cx^2}} dx}{2c^2} \\ &= \frac{(4cf - 3bg)\sqrt{a + bx + cx^2}}{4c^2} + \frac{gx\sqrt{a + bx + cx^2}}{2c} + d \int \frac{1}{x\sqrt{a + bx + cx^2}} dx + \frac{(8c^2e}{2c^2} \\ &= \frac{(4cf - 3bg)\sqrt{a + bx + cx^2}}{4c^2} + \frac{gx\sqrt{a + bx + cx^2}}{2c} - (2d) \text{Subst} \left( \int \frac{1}{4a - x^2} dx, x, \frac{x}{\sqrt{a + bx + cx^2}} \right) \\ &= \frac{(4cf - 3bg)\sqrt{a + bx + cx^2}}{4c^2} + \frac{gx\sqrt{a + bx + cx^2}}{2c} - \frac{d \tanh^{-1} \left( \frac{2a + bx}{2\sqrt{a}\sqrt{a + bx + cx^2}} \right)}{\sqrt{a}} + \frac{(8c^2e}{2c^2} \end{aligned}$$

**Mathematica [A]** time = 0.37, size = 134, normalized size = 0.86

$$\frac{\tanh^{-1} \left( \frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}} \right) (-4c(ag + bf) + 3b^2g + 8c^2e)}{8c^{5/2}} + \frac{\sqrt{a + x(b + cx)} (-3bg + 4cf + 2cgx)}{4c^2} - \frac{d \tanh^{-1} \left( \frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)]/(4*c^2) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.64, size = 145, normalized size = 0.94

$$\frac{\log\left(-2c^{5/2}\sqrt{a+bx+cx^2}+bc^2+2c^3x\right)(4acg-3b^2g+4bcf-8c^2e)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}(-3bg+4cf+2cgx)}{4c^2} + \frac{2d \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}} - \frac{\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + b*x + c*x^2]/(4*c^2) + (2*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/Sqrt[a] + ((-8*c^2*e + 4*b*c*f - 3*b^2*g + 4*a*c*g)*Log[b*c^2 + 2*c^3*x - 2*c^(5/2)*Sqrt[a + b*x + c*x^2])/(8*c^(5/2))
```

**fricas [A]** time = 6.92, size = 733, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a))*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a)/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a))*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a)/(a*c^3), 1/16*(16*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a)/(a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c))
```

$2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*\sqrt{c*x^2 + b*x + a})/(a*c^3]$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x)::OUTPUT:Warning, replacing 0 by `u`, a substitution  
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu  
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs  
titution variable should perhaps be purged.index.cc index\_m operator + Erro  
r: Bad Argument Value

**maple** [A] time = 0.01, size = 220, normalized size = 1.42

$$-\frac{ag \ln\left(\frac{cx+\frac{b}{2}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}}-\frac{d \ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}}{x}\sqrt{a}\right)}{\sqrt{a}}+\frac{3b^2g \ln\left(\frac{cx+\frac{b}{2}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}-\frac{bf \ln\left(\frac{cx+\frac{b}{2}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}}+\frac{e \ln\left(\frac{cx+\frac{b}{2}+\sqrt{cx^2+bx+a}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+bx+a}gx}{2c}-\frac{3\sqrt{cx^2+bx+a}bg}{4c^2}+\frac{\sqrt{cx^2+bx+a}f}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x^3+f\*x^2+e\*x+d)/x/(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $\frac{1}{2}g*x*(c*x^2+b*x+a)^{(1/2)}/c-3/4*g/c^2*b*(c*x^2+b*x+a)^{(1/2)}+3/8*g/c^{(5/2)}$   
 $*b^2*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-1/2*g*a/c^{(3/2)}*\ln((c*x+1/$   
 $2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+f/c*(c*x^2+b*x+a)^{(1/2)}-1/2*f*b/c^{(3/2)}*l$   
 $n((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+e*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+$   
 $b*x+a)^{(1/2)})/c^{(1/2)}-d/a^{(1/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/$   
 $x)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a  
dditional constraints; using the 'assume' command before evaluation \*may\* h  
elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for mo  
re details)Is 4\*a\*c-b^2 zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g x^3 + f x^2 + e x + d}{x \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)`

[Out] `int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2 + g x^3}{x \sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)`

$$3.264 \quad \int \frac{d+ex+fx^2+gx^3}{x^2 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=139

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{g\sqrt{a+bx+cx^2}}{c}$$

**Rubi [A]** time = 0.24, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {1650, 1653, 843, 621, 206, 724}

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{g\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/(x^2\*sqrt[a + b\*x + c\*x^2]),x]

[Out] (g\*sqrt[a + b\*x + c\*x^2])/c - (d\*sqrt[a + b\*x + c\*x^2])/(a\*x) + ((b\*d - 2\*a\*e)\*ArcTanh[(2\*a + b\*x)/(2\*sqrt[a]\*sqrt[a + b\*x + c\*x^2])])/(2\*a^(3/2)) + ((2\*c\*f - b\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2])])/(2\*c^(3/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2 + gx^3}{x^2 \sqrt{a + bx + cx^2}} dx &= -\frac{d\sqrt{a + bx + cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}(bd-2ae) - afx - agx^2}{x\sqrt{a+bx+cx^2}} dx}{a} \\
&= \frac{g\sqrt{a + bx + cx^2}}{c} - \frac{d\sqrt{a + bx + cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}c(bd-2ae) - \frac{1}{2}a(2cf-bg)x}{x\sqrt{a+bx+cx^2}} dx}{ac} \\
&= \frac{g\sqrt{a + bx + cx^2}}{c} - \frac{d\sqrt{a + bx + cx^2}}{ax} - \frac{(bd - 2ae) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2a} + \frac{(2cf - bg) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2c} \\
&= \frac{g\sqrt{a + bx + cx^2}}{c} - \frac{d\sqrt{a + bx + cx^2}}{ax} + \frac{(bd - 2ae) \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{a} + \frac{(2cf - bg) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{2c} \\
&= \frac{g\sqrt{a + bx + cx^2}}{c} - \frac{d\sqrt{a + bx + cx^2}}{ax} + \frac{(bd - 2ae) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf - bg) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{2c}
\end{aligned}$$

**Mathematica [A]** time = 0.40, size = 127, normalized size = 0.91

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{3/2}} + \frac{(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2c^{3/2}} + \frac{\sqrt{a + x(b + cx)}(agx - cd)}{acx}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3)/(x^2\*sqrt[a + b\*x + c\*x^2]),x]

[Out] ((-(c\*d) + a\*g\*x)\*sqrt[a + x\*(b + c\*x)]/(a\*c\*x) + ((b\*d - 2\*a\*e)\*ArcTanh[(2\*a + b\*x)/(2\*sqrt[a]\*sqrt[a + x\*(b + c\*x)])])/(2\*a^(3/2)) + ((2\*c\*f - b\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + x\*(b + c\*x)])])/(2\*c^(3/2)))

**IntegrateAlgebraic [A]** time = 0.66, size = 127, normalized size = 0.91

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2} - \sqrt{cx}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(bg - 2cf) \log\left(-2c^{3/2}\sqrt{a + bx + cx^2} + bc + 2c^2x\right)}{2c^{3/2}} + \frac{\sqrt{a + bx + cx^2}(agx - cd)}{acx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3)/(x^2\*sqrt[a + b\*x + c\*x^2]),x]

[Out] ((-(c\*d) + a\*g\*x)\*sqrt[a + b\*x + c\*x^2]/(a\*c\*x) + ((b\*d - 2\*a\*e)\*ArcTanh[(-sqrt[c]\*x + sqrt[a + b\*x + c\*x^2])/sqrt[a]])/a^(3/2) + ((-2\*c\*f + b\*g)\*Log[b\*c + 2\*c^2\*x - 2\*c^(3/2)\*sqrt[a + b\*x + c\*x^2]])/(2\*c^(3/2)))

**fricas** [A] time = 6.16, size = 703, normalized size = 5.06

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/4\*((2\*a^2\*c\*f - a^2\*b\*g)\*sqrt(c)\*x\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) + (b\*c^2\*d - 2\*a\*c^2\*e)\*sqrt(a)\*x\*log(-(8\*a\*b\*x + (b^2 + 4\*a\*c)\*x^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a))\*sqrt(a) + 8\*a^2)/x^2) - 4\*(a^2\*c\*g\*x - a\*c^2\*d)\*sqrt(c\*x^2 + b\*x + a)/(a^2\*c^2\*x), -1/4\*(2\*(2\*a^2\*c\*f - a^2\*b\*g)\*sqrt(-c)\*x\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + (b\*c^2\*d - 2\*a\*c^2\*e)\*sqrt(a)\*x\*log(-(8\*a\*b\*x + (b^2 + 4\*a\*c)\*x^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a))\*sqrt(a) + 8\*a^2)/x^2) - 4\*(a^2\*c\*g\*x - a\*c^2\*d)\*sqrt(c\*x^2 + b\*x + a)/(a^2\*c^2\*x), -1/4\*(2\*(b\*c^2\*d - 2\*a\*c^2\*e)\*sqrt(-a)\*x\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(-a)/(a\*c\*x^2 + a\*b\*x + a^2)) + (2\*a^2\*c\*f - a^2\*b\*g)\*sqrt(c)\*x\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 + 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - 4\*(a^2\*c\*g\*x - a\*c^2\*d)\*sqrt(c\*x^2 + b\*x + a)/(a^2\*c^2\*x), -1/2\*((b\*c^2\*d - 2\*a\*c^2\*e)\*sqrt(-a)\*x\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(-a)/(a\*c\*x^2 + a\*b\*x + a^2)) + (2\*a^2\*c\*f - a^2\*b\*g)\*sqrt(-c)\*x\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) - 2\*(a^2\*c\*g\*x - a\*c^2\*d)\*sqrt(c\*x^2 + b\*x + a)/(a^2\*c^2\*x)]

**giac** [A] time = 0.37, size = 171, normalized size = 1.23

$$\frac{\sqrt{cx^2 + bx + a} g}{c} - \frac{(bd - 2ae) \arctan\left(\frac{-\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{(2cf - bg) \log\left(2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right)}{2c^{\frac{3}{2}}} + \frac{\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)bd + 2a\sqrt{c}d}{\left(\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^2/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] sqrt(c\*x^2 + b\*x + a)\*g/c - (b\*d - 2\*a\*e)\*arctan(-(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))/sqrt(-a))/sqrt(-a)\*a - 1/2\*(2\*c\*f - b\*g)\*log(abs(2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) + b))/c^(3/2) + ((sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*b\*d + 2\*a\*sqrt(c)\*d)/(((sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))^2 - a)\*a)

**maple** [A] time = 0.01, size = 173, normalized size = 1.24

$$-\frac{e \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a} \sqrt{a}}{x}\right)}{\sqrt{a}} + \frac{bd \ln\left(\frac{bx + 2a + 2\sqrt{cx^2 + bx + a} \sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}} - \frac{bg \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{2c^{\frac{3}{2}}} + \frac{f \ln\left(\frac{cx + \frac{b}{2}}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2 + bx + a} g}{c} - \frac{\sqrt{cx^2 + bx + a} d}{ax}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out]  $g*(c*x^2+b*x+a)^{(1/2)}/c-1/2*g*b/c^{(3/2)}*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})+f*\ln((c*x+1/2*b)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}-d*(c*x^2+b*x+a)^{(1/2)}/a/x+1/2*d*b/a^{(3/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)-e/a^{(1/2)}*\ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)}*a^{(1/2)})/x)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details)Is 4\*a\*c-b^2 zero or nonzero?

**mupad** [B] time = 4.46, size = 166, normalized size = 1.19

$$\frac{g\sqrt{cx^2+bx+a}}{c} - \frac{e\ln\left(\frac{b}{2} + \frac{a}{x} + \frac{\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)}{\sqrt{a}} + \frac{f\ln\left(\frac{b}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)}{\sqrt{c}} - \frac{bg\ln\left(\frac{b}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)}{2c^{3/2}} - \frac{d\sqrt{cx^2+bx+a}}{ax} + \frac{bd\operatorname{atanh}\left(\frac{a+\frac{bx}{2}}{\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{2a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^{(1/2)}), x)$

[Out]  $(g*(a + b*x + c*x^2)^{(1/2)})/c - (e*\log(b/2 + a/x + (a^{(1/2)}*(a + b*x + c*x^2)^{(1/2)})/x))/a^{(1/2)} + (f*\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))/c^{(1/2)} - (b*g*\log((b/2 + c*x)/c^{(1/2)} + (a + b*x + c*x^2)^{(1/2)}))/(2*c^{(3/2)}) - (d*(a + b*x + c*x^2)^{(1/2)})/(a*x) + (b*d*\operatorname{atanh}((a + (b*x)/2)/(a^{(1/2)}*(a + b*x + c*x^2)^{(1/2)})))/(2*a^{(3/2)})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^2\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2), x)$

[Out]  $\text{Integral}((d + e*x + f*x**2 + g*x**3)/(x**2*\text{sqrt}(a + b*x + c*x**2)), x)$

$$3.265 \quad \int \frac{d+ex+fx^2+gx^3}{x^3 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=159

$$\frac{\sqrt{a+bx+cx^2} (3bd-4ae)}{4a^2x} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) (8a^2f-4abe-4acd+3b^2d)}{8a^{5/2}} - \frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

**Rubi [A]** time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) (8a^2f-4abe-4acd+3b^2d)}{8a^{5/2}} + \frac{\sqrt{a+bx+cx^2} (3bd-4ae)}{4a^2x} - \frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/(x^3\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] -(d\*Sqrt[a + b\*x + c\*x^2])/(2\*a\*x^2) + ((3\*b\*d - 4\*a\*e)\*Sqrt[a + b\*x + c\*x^2])/(4\*a^2\*x) - ((3\*b^2\*d - 4\*a\*c\*d - 4\*a\*b\*e + 8\*a^2\*f)\*ArcTanh[(2\*a + b\*x)/(2\*Sqrt[a]\*Sqrt[a + b\*x + c\*x^2])])/(8\*a^(5/2)) + (g\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/Sqrt[c]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2 + gx^3}{x^3 \sqrt{a + bx + cx^2}} dx &= -\frac{d\sqrt{a + bx + cx^2}}{2ax^2} - \frac{\int \frac{\frac{1}{2}(3bd - 4ae) + (cd - 2af)x - 2agx^2}{x^2 \sqrt{a + bx + cx^2}} dx}{2a} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{2ax^2} + \frac{(3bd - 4ae)\sqrt{a + bx + cx^2}}{4a^2x} + \frac{\int \frac{\frac{1}{4}(3b^2d - 4abe - 4a(cd - 2af)) + 2a^2gx}{x \sqrt{a + bx + cx^2}} dx}{2a^2} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{2ax^2} + \frac{(3bd - 4ae)\sqrt{a + bx + cx^2}}{4a^2x} + \frac{(3b^2d - 4acd - 4abe + 8a^2f) \int \frac{dx}{x}}{8a^2} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{2ax^2} + \frac{(3bd - 4ae)\sqrt{a + bx + cx^2}}{4a^2x} - \frac{(3b^2d - 4acd - 4abe + 8a^2f) \operatorname{Subst}\left(\int \frac{dx}{x}\right)}{4a^2} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{2ax^2} + \frac{(3bd - 4ae)\sqrt{a + bx + cx^2}}{4a^2x} - \frac{(3b^2d - 4acd - 4abe + 8a^2f) \tan^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)}{8a^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.36, size = 137, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right)(4abe + 4a(cd - 2af) - 3b^2d)}{8a^{5/2}} + \frac{\sqrt{a + x(b + cx)}(3bdx - 2a(d + 2ex))}{4a^2x^2} + \frac{g \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3)/(x^3\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(3\*b\*d\*x - 2\*a\*(d + 2\*e\*x)))/(4\*a^2\*x^2) + ((-3\*b^2\*d + 4\*a\*b\*e + 4\*a\*(c\*d - 2\*a\*f))\*ArcTanh[(2\*a + b\*x)/(2\*Sqrt[a]\*Sqrt[a + x\*(b + c\*x)])])/(8\*a^(5/2)) + (g\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])])/Sqrt[c]

**IntegrateAlgebraic [A]** time = 0.73, size = 172, normalized size = 1.08

$$\frac{(be + cd) \tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a+bx+cx^2}(-2ad - 4aex + 3bdx)}{4a^2x^2} + \frac{(8a^2f + 3b^2d) \tanh^{-1}\left(\frac{\sqrt{c}x - \sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{4a^{5/2}} - \frac{g \log\left(-2\sqrt{c}\sqrt{a+bx+cx^2} + b + 2cx\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3)/(x^3\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] ((-2\*a\*d + 3\*b\*d\*x - 4\*a\*e\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*a^2\*x^2) + (((3\*b^2\*d + 8\*a^2\*f)\*ArcTanh[(Sqrt[c]\*x - Sqrt[a + b\*x + c\*x^2])/Sqrt[a]])/(4\*a^(5/2)) + ((c\*d + b\*e)\*ArcTanh[(-(Sqrt[c]\*x) + Sqrt[a + b\*x + c\*x^2])/Sqrt[a]])/a^(3/2) - (g\*Log[b + 2\*c\*x - 2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]])/Sqrt[c]

**fricas [A]** time = 8.04, size = 783, normalized size = 4.92

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/16\*(8\*a^3\*sqrt(c)\*g\*x^2\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - (4\*a\*b\*c\*e - 8\*a^2\*c\*f - (3\*b^2\*c - 4\*a\*c^2)\*d)\*sqrt(a)\*x^2\*log(-(8\*a\*b\*x + (b^2 + 4\*a\*c)\*x^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(a) + 8\*a^2)/x^2) - 4\*(2\*a^2\*c\*d - (3\*a\*b\*c\*d - 4\*a^2\*c\*e)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^3\*c\*x^2), -1/16\*(16\*a^3\*sqrt(-c)\*g\*x^2\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + (4\*a\*b\*c\*e - 8\*a^2\*c\*f - (3\*b^2\*c - 4\*a\*c^2)\*d)\*sqrt(a)\*x^2\*log(-(8\*a\*b\*x + (b^2 + 4\*a\*c)\*x^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(a) + 8\*a^2)/x^2) + 4\*(2\*a^2\*c\*d - (3\*a\*b\*c\*d - 4\*a^2\*c\*e)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^3\*c\*x^2), 1/8\*(4\*a^3\*sqrt(c)\*g\*x^2\*log(-8\*c^2\*x^2 - 8\*b\*c\*x - b^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(c) - 4\*a\*c) - (4\*a\*b\*c\*e - 8\*a^2\*c\*f - (3\*b^2\*c - 4\*a\*c^2)\*d)\*sqrt(-a)\*x^2\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(-a)/(a\*c\*x^2 + a\*b\*x + a^2)) - 2\*(2\*a^2\*c\*d - (3\*a\*b\*c\*d - 4\*a^2\*c\*e)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^3\*c\*x^2), -1/8\*(8\*a^3\*sqrt(-c)\*g\*x^2\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(2\*c\*x + b)\*sqrt(-c)/(c^2\*x^2 + b\*c\*x + a\*c)) + (4\*a\*b\*c\*e - 8\*a^2\*c\*f - (3\*b^2\*c - 4\*a\*c^2)\*d)\*sqrt(-a)\*x^2\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(-a)/(a\*c\*x^2 + a\*b\*x + a^2)) +

$2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*\sqrt{c*x^2 + b*x + a})/(a^3*c*x^2)]$

**giac [B]** time = 0.36, size = 352, normalized size = 2.21

$$\frac{g \log\left(\frac{-2(\sqrt{c x^2 + b x + a}) - b \sqrt{c}}{\sqrt{c}}\right) + \frac{(3 b^2 d - 4 a c d + 8 a^2 f - 4 a b e) \arctan\left(\frac{\sqrt{c x^2 + b x + a}}{\sqrt{c}}\right)}{4 \sqrt{c} a^2} + \frac{3(\sqrt{c x^2 + b x + a})^3 b^2 d - 4(\sqrt{c x^2 + b x + a})^3 a c d - 4(\sqrt{c x^2 + b x + a})^3 a b e - 8(\sqrt{c x^2 + b x + a})^2 a^2 \sqrt{c} e - 5(\sqrt{c x^2 + b x + a})^2 b^2 d - 4(\sqrt{c x^2 + b x + a})^2 c d + 4(\sqrt{c x^2 + b x + a})^2 b e - 8 a^2 b \sqrt{c} e + 8 a^2 \sqrt{c} e}{4(\sqrt{c x^2 + b x + a})^3 a^2}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out]  $-g*\log(\text{abs}(-2*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*c - b*\sqrt{c}))/\sqrt{c} + 1/4*(3*b^2*d - 4*a*c*d + 8*a^2*f - 4*a*b*e)*\arctan(-(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})/\sqrt{-a})/(\sqrt{-a}*a^2) - 1/4*(3*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*b^2*d - 4*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*c*d - 4*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*b*e - 8*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^2*\sqrt{c}*e - 5*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a*b^2*d - 4*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*c*d + 4*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*b*e - 8*a^2*b*\sqrt{c}*d + 8*a^3*\sqrt{c}*e)/(((\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2 - a)^2*a^2)$

**maple [A]** time = 0.01, size = 241, normalized size = 1.52

$$-\frac{f \ln\left(\frac{bx+2a+2\sqrt{c^2+bx+a}\sqrt{a}}{x}\right)}{\sqrt{a}} + \frac{be \ln\left(\frac{bx+2a+2\sqrt{c^2+bx+a}\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}} + \frac{cd \ln\left(\frac{bx+2a+2\sqrt{c^2+bx+a}\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2d \ln\left(\frac{bx+2a+2\sqrt{c^2+bx+a}\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}} + \frac{g \ln\left(\frac{cx+\frac{1}{2}}{\sqrt{c}} + \sqrt{cx^2+bx+a}\right)}{\sqrt{c}} - \frac{\sqrt{cx^2+bx+a}e}{ax} + \frac{3\sqrt{cx^2+bx+a}bd}{4a^2x} - \frac{\sqrt{cx^2+bx+a}d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x^3+f\*x^2+e\*x+d)/x^3/(c\*x^2+b\*x+a)^(1/2),x)

[Out]  $g*\ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+3/4*d*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*d*b^2/a^(5/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*d*c/a^(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-e/a/x*(c*x^2+b*x+a)^(1/2)+1/2*e*b/a^(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/a^(1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^3/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* h

elp (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is  $4ac - b^2$  zero or nonzero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{gx^3 + fx^2 + ex + d}{x^3 \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2 + g\*x^3)/(x^3\*(a + b\*x + c\*x^2)^(1/2)),x)

[Out] int((d + e\*x + f\*x^2 + g\*x^3)/(x^3\*(a + b\*x + c\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^3 \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x\*\*3+f\*x\*\*2+e\*x+d)/x\*\*3/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2 + g\*x\*\*3)/(x\*\*3\*sqrt(a + b\*x + c\*x\*\*2)), x)

$$3.266 \quad \int \frac{d+ex+fx^2+gx^3}{x^4 \sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=186

$$\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{12a^2x^2} + \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}} - \frac{\sqrt{a+bx}}{3ax^3}$$

**Rubi** [A] time = 0.32, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}} - \frac{\sqrt{a+bx+cx^2}\left(24a^2f-18abe-16acd+15b^2d\right)}{24a^3x} + \frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{12a^2x^2} - \frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/(x^4\*sqrt[a + b\*x + c\*x^2]), x]

[Out]  $-\frac{d\sqrt{a+bx+cx^2}}{(3ax^3)} + \frac{((5bd-6ae)\sqrt{a+bx+cx^2})}{(12a^2x^2)} - \frac{((15b^2d-16abc-18ab^2e+24a^2f)\sqrt{a+bx+cx^2})}{(24a^3x)} + \frac{((5b^3d-6ab^2e-4ab(3cd-2af))+8a^2(ce-2ag))\text{ArcTanh}\left[\frac{(2a+bx)}{(2\sqrt{a}\sqrt{a+bx+cx^2})}\right]}{(16a^{7/2})}$

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m+1)\*(a + b\*x + c\*x^2)^(p+1))/(2\*(p+1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m+1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &

& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{d + ex + fx^2 + gx^3}{x^4 \sqrt{a + bx + cx^2}} dx &= -\frac{d\sqrt{a + bx + cx^2}}{3ax^3} - \frac{\int \frac{\frac{1}{2}(5bd - 6ae) + (2cd - 3af)x - 3agx^2}{x^3 \sqrt{a + bx + cx^2}} dx}{3a} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{3ax^3} + \frac{(5bd - 6ae)\sqrt{a + bx + cx^2}}{12a^2x^2} + \frac{\int \frac{\frac{1}{4}(15b^2d - 16acd - 18abe + 24a^2f) + \frac{1}{2}(5bcd - 6ae^2)}{x^2 \sqrt{a + bx + cx^2}} dx}{6a^2} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{3ax^3} + \frac{(5bd - 6ae)\sqrt{a + bx + cx^2}}{12a^2x^2} - \frac{(15b^2d - 16acd - 18abe + 24a^2f)}{24a^3x} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{3ax^3} + \frac{(5bd - 6ae)\sqrt{a + bx + cx^2}}{12a^2x^2} - \frac{(15b^2d - 16acd - 18abe + 24a^2f)}{24a^3x} \\ &= -\frac{d\sqrt{a + bx + cx^2}}{3ax^3} + \frac{(5bd - 6ae)\sqrt{a + bx + cx^2}}{12a^2x^2} - \frac{(15b^2d - 16acd - 18abe + 24a^2f)}{24a^3x} \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 150, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)(8a^2(ce-2ag)-6ab^2e+4ab(2af-3cd)+5b^3d)}{16a^7/2} - \frac{\sqrt{a+x(b+cx)}(4a^2(2d+3x(e+2fx))-2ax(5bd+9bex+8cdx)+15b^2dx^2)}{24a^3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3)/(x^4\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] -1/24\*(Sqrt[a + x\*(b + c\*x)]\*(15\*b^2\*d\*x^2 - 2\*a\*x\*(5\*b\*d + 8\*c\*d\*x + 9\*b\*e\*x) + 4\*a^2\*(2\*d + 3\*x\*(e + 2\*f\*x)))/(a^3\*x^3) + ((5\*b^3\*d - 6\*a\*b^2\*e + 4



$*a*b*(-3*c*d + 2*a*f) + 8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])]/(16*a^(7/2))$

**IntegrateAlgebraic [A]** time = 1.17, size = 193, normalized size = 1.04

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{cx}}{\sqrt{a}}\right)(4abf+4ace-3b^2e-6bcd)}{4a^{5/2}} + \frac{(16a^3g-5b^3d)\tanh^{-1}\left(\frac{\sqrt{cx}-\sqrt{a+bx+cx^2}}{\sqrt{a}}\right)}{8a^{7/2}} + \frac{\sqrt{a+bx+cx^2}(-8a^2d-12a^2cx-24a^2fx^2+10abdx+18abx^2+16acdx^2-15b^2dx^2)}{24a^3x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3)/(x^4\*sqrt[a + b\*x + c\*x^2]),x]

[Out] (sqrt[a + b\*x + c\*x^2]\*(-8\*a^2\*d + 10\*a\*b\*d\*x - 12\*a^2\*e\*x - 15\*b^2\*d\*x^2 + 16\*a\*c\*d\*x^2 + 18\*a\*b\*e\*x^2 - 24\*a^2\*f\*x^2))/(24\*a^3\*x^3) + ((-5\*b^3\*d + 16\*a^3\*g)\*ArcTanh[(sqrt[c]\*x - sqrt[a + b\*x + c\*x^2])/sqrt[a]])/(8\*a^(7/2)) + ((-6\*b\*c\*d - 3\*b^2\*e + 4\*a\*c\*e + 4\*a\*b\*f)\*ArcTanh[(-sqrt[c]\*x) + sqrt[a + b\*x + c\*x^2])/sqrt[a]])/(4\*a^(5/2))

**fricas [A]** time = 7.65, size = 365, normalized size = 1.96

$$\frac{3(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} \log\left(\frac{8a^2f + 4a^2g + \sqrt{a+bx+cx^2}}{2(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} + bx + a}\right) + 4(8a^2d - (18a^2b - 24a^2f - (15ab^2 - 16a^2c)d)^2 - 2(8a^2b - 6a^2c))\sqrt{a+bx+cx^2}}{96a^{5/2}} + \frac{3(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} \arctan\left(\frac{\sqrt{a+bx+cx^2}}{2(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} + bx + a}\right) + 2(8a^2d - (18a^2b - 24a^2f - (15ab^2 - 16a^2c)d)^2 - 2(8a^2b - 6a^2c))\sqrt{a+bx+cx^2}}{48a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/96\*(3\*(8\*a^2\*b\*f - 16\*a^3\*g + (5\*b^3 - 12\*a\*b\*c)\*d - 2\*(3\*a\*b^2 - 4\*a^2\*c)\*e)\*sqrt(a)\*x^3\*log(-(8\*a\*b\*x + (b^2 + 4\*a\*c))\*x^2 - 4\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(a) + 8\*a^2)/x^2) + 4\*(8\*a^3\*d - (18\*a^2\*b\*e - 24\*a^3\*f - (15\*a\*b^2 - 16\*a^2\*c)\*d)\*x^2 - 2\*(5\*a^2\*b\*d - 6\*a^3\*e)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^4\*x^3), -1/48\*(3\*(8\*a^2\*b\*f - 16\*a^3\*g + (5\*b^3 - 12\*a\*b\*c)\*d - 2\*(3\*a\*b^2 - 4\*a^2\*c)\*e)\*sqrt(-a)\*x^3\*arctan(1/2\*sqrt(c\*x^2 + b\*x + a)\*(b\*x + 2\*a)\*sqrt(-a)/(a\*c\*x^2 + a\*b\*x + a^2)) + 2\*(8\*a^3\*d - (18\*a^2\*b\*e - 24\*a^3\*f - (15\*a\*b^2 - 16\*a^2\*c)\*d)\*x^2 - 2\*(5\*a^2\*b\*d - 6\*a^3\*e)\*x)\*sqrt(c\*x^2 + b\*x + a))/(a^4\*x^3)]

**giac [B]** time = 0.29, size = 689, normalized size = 3.70

$$\frac{3(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} \log\left(\frac{8a^2f + 4a^2g + \sqrt{a+bx+cx^2}}{2(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} + bx + a}\right) + 4(8a^2d - (18a^2b - 24a^2f - (15ab^2 - 16a^2c)d)^2 - 2(8a^2b - 6a^2c))\sqrt{a+bx+cx^2}}{96a^{5/2}} + \frac{3(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} \arctan\left(\frac{\sqrt{a+bx+cx^2}}{2(8a^2f - 16a^2g + (5b^3 - 12abc)d - 2(3ab^2 - 4a^2c))\sqrt{a} + bx + a}\right) + 2(8a^2d - (18a^2b - 24a^2f - (15ab^2 - 16a^2c)d)^2 - 2(8a^2b - 6a^2c))\sqrt{a+bx+cx^2}}{48a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^4/(c\*x^2+b\*x+a)^(1/2),x, algorithm="giac")

[Out] -1/8\*(5\*b^3\*d - 12\*a\*b\*c\*d + 8\*a^2\*b\*f - 16\*a^3\*g - 6\*a\*b^2\*e + 8\*a^2\*c\*e)\*arctan(-(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))/sqrt(-a))/(sqrt(-a)\*a^3) + 1/24

$$\begin{aligned} &*(15*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^5*b^3*d - 36*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^5*a*b*c*d + 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^5*a^2*b*f \\ &- 18*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^5*a*b^2*e + 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^5*a^2*c*e + 48*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^4*a^3* \\ &\sqrt{c}*f - 40*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a*b^3*d + 96*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a^2*b*c*d - 48*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a^3*b*f \\ &+ 48*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^3*a^2*b^2*e + 96*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*c^(3/2)*d - 96*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^4*\sqrt{c}*f \\ &+ 48*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2*a^3*b*\sqrt{c}*e + 33*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*a^2*b^3*d + 36*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*a^3*b*c*d \\ &+ 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*a^4*b*f - 30*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*a^3*b^2*e - 24*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*a^4*c*e \\ &+ 48*a^3*b^2*\sqrt{c}*d - 32*a^4*c^(3/2)*d + 48*a^5*\sqrt{c}*f - 48*a^4*b*\sqrt{c}*e)/(((\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})^2 - a)^3*a^3) \end{aligned}$$

**maple [B]** time = 0.01, size = 375, normalized size = 2.02

$$\frac{g \ln\left(\frac{bx+2a+\sqrt{b^2-4ac}}{a}\right)}{\sqrt{a}} + \frac{bf \ln\left(\frac{bx+2a+\sqrt{b^2-4ac}}{a}\right)}{2a^2} + \frac{ce \ln\left(\frac{bx+2a+\sqrt{b^2-4ac}}{a}\right)}{2a^2} - \frac{3d^2 \ln\left(\frac{bx+2a+\sqrt{b^2-4ac}}{a}\right)}{8a^2} - \frac{3bcd \ln\left(\frac{bx+2a+\sqrt{b^2-4ac}}{a}\right)}{4a^2} + \frac{5b^3d \ln\left(\frac{bx+2a+\sqrt{b^2-4ac}}{a}\right)}{16a^2} + \frac{\sqrt{cx^2+bx+a}f}{ax} + \frac{3\sqrt{cx^2+bx+a}bc}{4a^2x} + \frac{2\sqrt{cx^2+bx+a}cd}{3a^2x} - \frac{5\sqrt{cx^2+bx+a}d^2}{8a^2x} - \frac{\sqrt{cx^2+bx+a}e}{2a^2x^2} + \frac{5\sqrt{cx^2+bx+a}bd}{12a^2x^2} - \frac{\sqrt{cx^2+bx+a}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x^3+f\*x^2+e\*x+d)/x^4/(c\*x^2+b\*x+a)^(1/2), x)

[Out]  $-1/2*e/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*e*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*e*b^2/a^(5/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*e*c/a^(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/a/x*(c*x^2+b*x+a)^(1/2)+1/2*f*b/a^(3/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-g/a^(1/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/3*d*(c*x^2+b*x+a)^(1/2)/a/x^3+5/12*d*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*d*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/16*d*b^3/a^(7/2)*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-3/4*d*b/a^(5/2)*c*\ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*d*c/a^2/x*(c*x^2+b*x+a)^(1/2)$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^4/(c\*x^2+b\*x+a)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g x^3 + f x^2 + e x + d}{x^4 \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)`

[Out] `int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2 + g x^3}{x^4 \sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)`

$$3.267 \quad \int \frac{d+ex+fx^2+gx^3}{x^5 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=270

$$\frac{\sqrt{a+bx+cx^2} (7bd-8ae)}{24a^2x^3} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) \frac{(32a^2b(3ce-2ag) + 16a^2c(3cd-4af) - 40ab^3e - 24ab^2(5cd))}{128a^{9/2}}$$

**Rubi [A]** time = 0.49, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1650, 834, 806, 724, 206}

$$\frac{\sqrt{a+bx+cx^2} (64a^2(2ce-3ag) - 120ab^2e - 4ab(55cd-36af) + 105b^3d)}{192a^5x} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right) \frac{(32a^2b(3ce-2ag) + 16a^2c(3cd-4af) - 24ab^2(5cd-2af) - 40ab^3e + 35b^4d)}{128a^{9/2}} - \frac{\sqrt{a+bx+cx^2} (48a^2f - 40abc - 36acd + 35b^2d)}{96a^3x^2} + \frac{\sqrt{a+bx+cx^2} (7bd-8ae)}{24a^2x^3} - \frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/(x^5\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] -(d\*Sqrt[a + b\*x + c\*x^2])/(4\*a\*x^4) + ((7\*b\*d - 8\*a\*e)\*Sqrt[a + b\*x + c\*x^2])/(24\*a^2\*x^3) - ((35\*b^2\*d - 36\*a\*c\*d - 40\*a\*b\*e + 48\*a^2\*f)\*Sqrt[a + b\*x + c\*x^2])/(96\*a^3\*x^2) + ((105\*b^3\*d - 120\*a\*b^2\*e - 4\*a\*b\*(55\*c\*d - 36\*a\*f) + 64\*a^2\*(2\*c\*e - 3\*a\*g))\*Sqrt[a + b\*x + c\*x^2])/(192\*a^4\*x) - ((35\*b^4\*d - 40\*a\*b^3\*e + 16\*a^2\*c\*(3\*c\*d - 4\*a\*f) - 24\*a\*b^2\*(5\*c\*d - 2\*a\*f) + 32\*a^2\*b\*(3\*c\*e - 2\*a\*g))\*ArcTanh[(2\*a + b\*x)/(2\*Sqrt[a]\*Sqrt[a + b\*x + c\*x^2])])/(128\*a^(9/2))

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 724**

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

**Rule 806**

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m+1)\*(a + b\*x + c\*x^2)^(p+1))/(2\*(p+1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

### Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2 + gx^3}{x^5 \sqrt{a + bx + cx^2}} dx &= -\frac{d\sqrt{a + bx + cx^2}}{4ax^4} - \frac{\int \frac{\frac{1}{2}(7bd-8ae)+(3cd-4af)x-4agx^2}{x^4 \sqrt{a+bx+cx^2}} dx}{4a} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{4ax^4} + \frac{(7bd - 8ae)\sqrt{a + bx + cx^2}}{24a^2x^3} + \frac{\int \frac{\frac{1}{4}(35b^2d-40abe-12a(3cd-4af))+(7bcd-8a^2g)}{x^3 \sqrt{a+bx+cx^2}}}{12a^2} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{4ax^4} + \frac{(7bd - 8ae)\sqrt{a + bx + cx^2}}{24a^2x^3} - \frac{(35b^2d - 36acd - 40abe + 48a^2f)}{96a^3x^2} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{4ax^4} + \frac{(7bd - 8ae)\sqrt{a + bx + cx^2}}{24a^2x^3} - \frac{(35b^2d - 36acd - 40abe + 48a^2f)}{96a^3x^2} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{4ax^4} + \frac{(7bd - 8ae)\sqrt{a + bx + cx^2}}{24a^2x^3} - \frac{(35b^2d - 36acd - 40abe + 48a^2f)}{96a^3x^2} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{4ax^4} + \frac{(7bd - 8ae)\sqrt{a + bx + cx^2}}{24a^2x^3} - \frac{(35b^2d - 36acd - 40abe + 48a^2f)}{96a^3x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.52, size = 212, normalized size = 0.79

$$\frac{\sqrt{a+x(b+cx)}(-16a^3(3d+4ex+6x^2(f+2gx))+8a^2x(7bd+2bx(5e+9fx)+cx(9d+16cx))-10abx^2(7bd+12hex+22cdx)+105b^3dx^3)}{192a^4x^4} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx}}\right)(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e+24ab^2(2af-5cd)+35b^4d)}{128a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3)/(x^5\*sqrt[a + b\*x + c\*x^2]),x]

[Out] (sqrt[a + x\*(b + c\*x)]\*(105\*b^3\*d\*x^3 - 10\*a\*b\*x^2\*(7\*b\*d + 22\*c\*d\*x + 12\*b\*e\*x) + 8\*a^2\*x\*(7\*b\*d + c\*x\*(9\*d + 16\*e\*x)) + 2\*b\*x\*(5\*e + 9\*f\*x)) - 16\*a^3\*(3\*d + 4\*e\*x + 6\*x^2\*(f + 2\*g\*x)))/(192\*a^4\*x^4) - ((35\*b^4\*d - 40\*a\*b^3\*e + 16\*a^2\*c\*(3\*c\*d - 4\*a\*f) + 24\*a\*b^2\*(-5\*c\*d + 2\*a\*f) + 32\*a^2\*b\*(3\*c\*e - 2\*a\*g))\*ArcTanh[(2\*a + b\*x)/(2\*sqrt[a]\*sqrt[a + x\*(b + c\*x)])])/(128\*a^3)

**IntegrateAlgebraic [A]** time = 2.18, size = 278, normalized size = 1.03

$$\frac{35b^4d \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+bx+cx^2}}\right) + \frac{\sqrt{a+bx+cx^2}}{\sqrt{a}} \tanh^{-1}\left(\frac{\sqrt{a+bx+cx^2}-\sqrt{a}}{\sqrt{a}}\right)(8a^2bg + 8a^2cf - 6ab^2f - 12abce - 6a^2d + 5b^3e + 15b^2cd)}{64a^3} + \frac{\sqrt{a+bx+cx^2}(-48a^3d - 64a^2ex - 96a^2fx^2 - 192a^3gx^3 + 56a^2bds + 80a^2hex^2 + 144a^2bfx^3 + 72a^2cdx^2 + 128a^2cex^3 - 70ab^2dx^2 - 120ab^2ex^3 - 220abcdx^3 + 105b^3dx^4)}{192a^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3)/(x^5\*sqrt[a + b\*x + c\*x^2]),x]

```
[Out] (Sqrt[a + b*x + c*x^2]*(-48*a^3*d + 56*a^2*b*d*x - 64*a^3*e*x - 70*a*b^2*d*x^2 + 72*a^2*c*d*x^2 + 80*a^2*b*e*x^2 - 96*a^3*f*x^2 + 105*b^3*d*x^3 - 220*a*b*c*d*x^3 - 120*a*b^2*e*x^3 + 128*a^2*c*e*x^3 + 144*a^2*b*f*x^3 - 192*a^3*g*x^3))/(192*a^4*x^4) + ((15*b^2*c*d - 6*a*c^2*d + 5*b^3*e - 12*a*b*c*e - 6*a*b^2*f + 8*a^2*c*f + 8*a^2*b*g)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2])/Sqrt[a]])/(8*a^(7/2)) + (35*b^4*d*ArcTanh[(Sqrt[c]*x)/Sqrt[a] - Sqrt[a + b*x + c*x^2]/Sqrt[a]])/(64*a^(9/2))
```

**fricas [A]** time = 15.35, size = 525, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d - 8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 - 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d - 8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 - 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4)]
```

**giac [B]** time = 0.30, size = 1448, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d + 48*a^2*b^2*f - 64*a^3*c*f - 64*a^3*b*g - 40*a*b^3*e + 96*a^2*b*c*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*g - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt(c)*g - 385*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*x - sqrt(c
```





$\frac{2}{a^{5/2}} \ln\left(\frac{b^2x^2 + 2a^2(c^2x^2 + bx + a)^{1/2} a^{1/2}}{x}\right) - \frac{g}{a} \frac{1}{x} (c^2x^2 + bx + a)^{1/2} + \frac{1}{2} \frac{g^2b}{a^{3/2}} \ln\left(\frac{b^2x^2 + 2a^2(c^2x^2 + bx + a)^{1/2} a^{1/2}}{x}\right)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x^3+f\*x^2+e\*x+d)/x^5/(c\*x^2+b\*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive, negative or zero?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{gx^3 + fx^2 + ex + d}{x^5 \sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2 + g\*x^3)/(x^5\*(a + b\*x + c\*x^2)^(1/2)),x)

[Out] int((d + e\*x + f\*x^2 + g\*x^3)/(x^5\*(a + b\*x + c\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^5 \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x\*\*3+f\*x\*\*2+e\*x+d)/x\*\*5/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x + f\*x\*\*2 + g\*x\*\*3)/(x\*\*5\*sqrt(a + b\*x + c\*x\*\*2)), x)

$$3.268 \quad \int \frac{d+ex+fx^2+gx^3}{x^6 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=371

$$\frac{\sqrt{a+bx+cx^2} (9bd-10ae)}{40a^2x^4} - \frac{\sqrt{a+bx+cx^2} (40a^2b(55ce-36ag) + 256a^2c(4cd-5af) - 1050ab^3e - 60ab^2(49cd - 1920a^5x))}{1920a^5x}$$

**Rubi [A]** time = 0.82, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1650, 834, 806, 724, 206}

$$\frac{\sqrt{a+bx+cx^2} (40a^2b(55ce-36ag) + 256a^2c(4cd-5af) - 1050ab^3e - 60ab^2(49cd - 1920a^5x))}{1920a^5x} - \frac{\sqrt{a+bx+cx^2} (120a^2b(3ce-4g) - 350a^2c^2 - 4ab(4cd-10af) + 315a^2d)}{960a^4x^3} - \frac{\operatorname{tanh}^{-1}\left(\frac{2ax}{\sqrt{a+bx+cx^2}}\right) (48a^2b^2(3ce-2ag) + 48a^2b^2c(4cd-4af) - 32a^2b^2(3ce-4ag) - 40ab^3(7cd-2af) - 70ab^3e + 63a^2d)}{256a^5x} - \frac{\sqrt{a+bx+cx^2} (8a^2f - 70ab^2e - 64a^2cd + 63a^2d)}{240a^4x^3} - \frac{\sqrt{a+bx+cx^2} (8a^2f - 70ab^2e - 64a^2cd + 63a^2d)}{40a^4x^3} - \frac{d\sqrt{a+bx+cx^2}}{5a^2x}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3)/(x^6\*sqrt[a + b\*x + c\*x^2]),x]

[Out] -(d\*sqrt[a + b\*x + c\*x^2])/(5\*a\*x^5) + ((9\*b\*d - 10\*a\*e)\*sqrt[a + b\*x + c\*x^2])/(40\*a^2\*x^4) - ((63\*b^2\*d - 64\*a\*c\*d - 70\*a\*b\*e + 80\*a^2\*f)\*sqrt[a + b\*x + c\*x^2])/(240\*a^3\*x^3) + ((315\*b^3\*d - 350\*a\*b^2\*e - 4\*a\*b\*(161\*c\*d - 100\*a\*f) + 120\*a^2\*(3\*c\*e - 4\*a\*g))\*sqrt[a + b\*x + c\*x^2])/(960\*a^4\*x^2) - ((945\*b^4\*d - 1050\*a\*b^3\*e - 60\*a\*b^2\*(49\*c\*d - 20\*a\*f) + 256\*a^2\*c\*(4\*c\*d - 5\*a\*f) + 40\*a^2\*b\*(55\*c\*e - 36\*a\*g))\*sqrt[a + b\*x + c\*x^2])/(1920\*a^5\*x) + ((63\*b^5\*d - 70\*a\*b^4\*e + 48\*a^2\*b\*c\*(5\*c\*d - 4\*a\*f) - 40\*a\*b^3\*(7\*c\*d - 2\*a\*f) - 32\*a^3\*c\*(3\*c\*e - 4\*a\*g) + 48\*a^2\*b^2\*(5\*c\*e - 2\*a\*g))\*ArcTanh[(2\*a + b\*x)/(2\*sqrt[a]\*sqrt[a + b\*x + c\*x^2])])/(256\*a^(11/2))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\_)

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
+ d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

### Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{d + ex + fx^2 + gx^3}{x^6 \sqrt{a + bx + cx^2}} dx &= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} - \frac{\int \frac{\frac{1}{2}(9bd - 10ae) + (4cd - 5af)x - 5agx^2}{x^5 \sqrt{a + bx + cx^2}} dx}{5a} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} + \frac{(9bd - 10ae)\sqrt{a + bx + cx^2}}{40a^2x^4} + \frac{\int \frac{\frac{1}{4}(63b^2d - 64acd - 70abe + 80a^2f) + \frac{1}{2}(27bcd - 28ac^2)}{x^4 \sqrt{a + bx + cx^2}} dx}{20a^2} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} + \frac{(9bd - 10ae)\sqrt{a + bx + cx^2}}{40a^2x^4} - \frac{(63b^2d - 64acd - 70abe + 80a^2f)}{240a^3x^3} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} + \frac{(9bd - 10ae)\sqrt{a + bx + cx^2}}{40a^2x^4} - \frac{(63b^2d - 64acd - 70abe + 80a^2f)}{240a^3x^3} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} + \frac{(9bd - 10ae)\sqrt{a + bx + cx^2}}{40a^2x^4} - \frac{(63b^2d - 64acd - 70abe + 80a^2f)}{240a^3x^3} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} + \frac{(9bd - 10ae)\sqrt{a + bx + cx^2}}{40a^2x^4} - \frac{(63b^2d - 64acd - 70abe + 80a^2f)}{240a^3x^3} \\
&= -\frac{d\sqrt{a + bx + cx^2}}{5ax^5} + \frac{(9bd - 10ae)\sqrt{a + bx + cx^2}}{40a^2x^4} - \frac{(63b^2d - 64acd - 70abe + 80a^2f)}{240a^3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 299, normalized size = 0.81

$$\frac{\operatorname{tanh}^{-1}\left(\frac{2ax}{x^2 + \sqrt{a + bx + cx^2}}\right) \left( (32a^3(4ag - 3c) - 48a^2b(2ag - 5c) - 48a^2bc(4af - 5cd) - 70ab^2e + 40ab^2(2af - 7cd) + 63b^2d) \sqrt{a + bx + cx^2} + (32a^4(12d + 5c(3e + 4f + 6g^2)) - 16a^2b(9(27d + 5c(7e + 21(5f + 9g^2)) + c)(32d + 5c(9e + 16f))) + 4a^2c^2(p^2(126d + 25c(7e + 12f)) + 28c^2(16d + 275c) + 256c^2d^2) - 210ab^2c(3bd + 5ec + 14cd) + 945b^2e^2 \right)}{256a^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3)/(x^6\*sqrt[a + b\*x + c\*x^2]), x]

[Out]  $-\frac{1}{1920} \cdot (\sqrt{a + x(b + cx)}) \cdot (945b^4d x^4 - 210ab^2x^3(3bd + 14cd + 5be + 5bex) + 32a^4(12d + 5x(3e + 4fx + 6gx^2)) + 4a^2x^2(256c^2dx^2 + 2b^2cx(161d + 275ex) + b^2(126d + 25x(7e + 12fx))) - 16a^3x(c^2x(32d + 5x(9e + 16fx)) + b(27d + 5x(7e + 2x(5f + 9gx)))))/(a^5x^5) + ((63b^5d - 70ab^4e + 40ab^3(-7cd + 2af) - 48a^2b^2c(-5cd + 4af) - 48a^2b^2(-5ce + 2ag) + 32a^3c(-3ce + 4ag)) \cdot \operatorname{ArcTanh}[(2a + bx)/(2\sqrt{a}\sqrt{a + x(b + cx)})])/(256a^{11/2})$

**IntegrateAlgebraic [A]** time = 2.97, size = 391, normalized size = 1.05

$$\frac{(-126a^3g - 63b^2d) \operatorname{tanh}^{-1}\left(\frac{2ax}{x^2 + \sqrt{a + bx + cx^2}}\right) + \operatorname{tanh}^{-1}\left(\frac{2ax}{x^2 + \sqrt{a + bx + cx^2}}\right) \left( (-48a^2b^2e - 96a^2bcf - 48a^2c^2 + 40ab^2d + 120ab^2e + 120ab^2f - 35b^2 - 140b^2d) \sqrt{a + bx + cx^2} + (-384a^4d - 880a^3e - 640a^2f - 960a^2g + 432a^2bd + 560a^2ce + 800a^2fd + 1440a^2ge + 912a^2cd^2 + 720a^2ce^2 + 120a^2fd^2 - 504a^2fd^2 - 700a^2bd^2 - 120a^2bd^2 - 128a^2bd^2 - 220a^2cd^2 - 102a^2ce^2 + 630ab^2e^2 + 1050ab^2e^2 + 240ab^2e^2 - 945b^2e^2) \right)}{1920a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2 + g*x^3)/(x^6*sqrt[a + b*x + c*x^2]),x]
[Out] (sqrt[a + b*x + c*x^2]*(-384*a^4*d + 432*a^3*b*d*x - 480*a^4*e*x - 504*a^2*
b^2*d*x^2 + 512*a^3*c*d*x^2 + 560*a^3*b*e*x^2 - 640*a^4*f*x^2 + 630*a*b^3*d
*x^3 - 1288*a^2*b*c*d*x^3 - 700*a^2*b^2*e*x^3 + 720*a^3*c*e*x^3 + 800*a^3*b
*f*x^3 - 960*a^4*g*x^3 - 945*b^4*d*x^4 + 2940*a*b^2*c*d*x^4 - 1024*a^2*c^2*
d*x^4 + 1050*a*b^3*e*x^4 - 2200*a^2*b*c*e*x^4 - 1200*a^2*b^2*f*x^4 + 1280*a
^3*c*f*x^4 + 1440*a^3*b*g*x^4))/(1920*a^5*x^5) + ((-63*b^5*d - 128*a^4*c*g)
*ArcTanh[(sqrt[c]*x - sqrt[a + b*x + c*x^2])/sqrt[a]])/(128*a^(11/2)) + ((-
140*b^3*c*d + 120*a*b*c^2*d - 35*b^4*e + 120*a*b^2*c*e - 48*a^2*c^2*e + 40*
a*b^3*f - 96*a^2*b*c*f - 48*a^2*b^2*g)*ArcTanh[(-sqrt[c]*x) + sqrt[a + b*x
+ c*x^2])/sqrt[a]])/(64*a^(9/2))
```

**fricas [A]** time = 40.33, size = 727, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a
^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 -
4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 +
b*x + a))*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b*g -
(945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*
c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(
45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*
b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)
*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a*b^3*c +
240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*
b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(384
*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50
*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a
^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^
4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 4
8*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5)]
```

**giac [B]** time = 0.51, size = 2177, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] -1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d + 80*a^2*b^3*f - 192*a^3
*b*c*f - 96*a^3*b^2*g + 128*a^4*c*g - 70*a*b^4*e + 240*a^2*b^2*c*e - 96*a^3
*c^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5
) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^2*d + 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c*f - 1440*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^9*a^4*c*g - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e + 3600*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*a^3*c^2*e - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d - 16800*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d - 5600*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*g - 3840*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c*g + 4900*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^
2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e + 7680*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^2*b^5*d - 35840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*d + 3072
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*d + 10240*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^4*b^3*f - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*a^5*b*c*f - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*g - 896
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*e + 30720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^4*b^2*c*e + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^5*c^(5/2)*d + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt
(c)*f - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f - 11520*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*sqrt(c)*g + 20480*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/2)*e - 7110*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^3*b^5*d + 31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c
*d + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*d - 8480*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f + 1920*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^6*b*c*f + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*g
+ 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c*g + 7900*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^4*b^4*e - 13920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^3*a^5*b^2*c*e - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*e + 3
8400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d - 10240*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*d - 7680*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^6*b^2*sqrt(c)*f + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^7*c^(3/2)*f + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*sqrt
(c)*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*sqrt(c)*e - 2560
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(3/2)*e + 2895*(sqrt(c)*x -
```

$$\begin{aligned} & \sqrt{c*x^2 + b*x + a}) * a^4 * b^5 * d + 4200 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^5 * b^3 * c * d - 3600 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^6 * b * c^2 * d + 2640 \\ & * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^6 * b^3 * f + 2880 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^7 * b * c * f - 2400 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^7 * b^2 \\ & * g - 1920 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^8 * c * g - 2790 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^5 * b^4 * e - 3600 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * \\ & a^6 * b^2 * c * e + 1440 * (\sqrt{c} * x - \sqrt{c*x^2 + b*x + a}) * a^7 * c^2 * e + 3840 * a^5 \\ & * b^4 * \sqrt{c} * d - 7680 * a^6 * b^2 * c^{(3/2)} * d + 2048 * a^7 * c^{(5/2)} * d + 3840 * a^7 * b^2 \\ & * \sqrt{c} * f - 2560 * a^8 * c^{(3/2)} * f - 3840 * a^8 * b * \sqrt{c} * g - 3840 * a^6 * b^3 * \sqrt{c} \\ & * e + 5120 * a^7 * b * c^{(3/2)} * e) / (((\sqrt{c} * x - \sqrt{c*x^2 + b*x + a})^2 - a)^5 \\ & * a^5) \end{aligned}$$

**maple [B]** time = 0.02, size = 859, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out] 
$$\begin{aligned} & -1/4 * e/a/x^4 * (c*x^2+b*x+a)^{(1/2)} - 35/128 * e * b^4/a^{(9/2)} * \ln((b*x+2*a+2*(c*x^2+ \\ & b*x+a)^{(1/2)} * a^{(1/2)})/x) - 3/8 * e * c^2/a^{(5/2)} * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)} \\ & ) * a^{(1/2)})/x - 1/3 * f/a/x^3 * (c*x^2+b*x+a)^{(1/2)} + 5/16 * f * b^3/a^{(7/2)} * \ln((b*x+2* \\ & a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) + 63/256 * d * b^5/a^{(11/2)} * \ln((b*x+2*a+2*(c* \\ & x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) - 1/2 * g/a/x^2 * (c*x^2+b*x+a)^{(1/2)} - 3/8 * g * b^2/a^{(5 \\ & /2)} * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) + 1/2 * g * c/a^{(3/2)} * \ln((b*x+2 \\ & *a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) - 1/5 * d * (c*x^2+b*x+a)^{(1/2)}/a/x^5 - 161/24 \\ & 0 * d * b/a^3 * c/x^2 * (c*x^2+b*x+a)^{(1/2)} - 55/48 * e * b/a^3 * c/x * (c*x^2+b*x+a)^{(1/2)} + 4 \\ & 9/32 * d * b^2/a^4 * c/x * (c*x^2+b*x+a)^{(1/2)} + 3/8 * e * c/a^2/x^2 * (c*x^2+b*x+a)^{(1/2)} + \\ & 9/40 * d * b/a^2/x^4 * (c*x^2+b*x+a)^{(1/2)} - 21/80 * d * b^2/a^3/x^3 * (c*x^2+b*x+a)^{(1/2)} \\ & ) + 21/64 * d * b^3/a^4/x^2 * (c*x^2+b*x+a)^{(1/2)} - 63/128 * d * b^4/a^5/x * (c*x^2+b*x+a)^{(1/2)} \\ & - 35/32 * d * b^3/a^{(9/2)} * c * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) + 1 \\ & 5/16 * d * b/a^{(7/2)} * c^2 * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) + 4/15 * d * c \\ & /a^2/x^3 * (c*x^2+b*x+a)^{(1/2)} - 8/15 * d * c^2/a^3/x * (c*x^2+b*x+a)^{(1/2)} + 5/12 * f * b/ \\ & a^2/x^2 * (c*x^2+b*x+a)^{(1/2)} - 5/8 * f * b^2/a^3/x * (c*x^2+b*x+a)^{(1/2)} - 3/4 * f * b/a^{( \\ & 5/2)} * c * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)})/x) + 2/3 * f * c/a^2/x * (c*x^2+b \\ & *x+a)^{(1/2)} + 7/24 * e * b/a^2/x^3 * (c*x^2+b*x+a)^{(1/2)} - 35/96 * e * b^2/a^3/x^2 * (c*x^2 \\ & +b*x+a)^{(1/2)} + 3/4 * g * b/a^2/x * (c*x^2+b*x+a)^{(1/2)} + 35/64 * e * b^3/a^4/x * (c*x^2+b \\ & x+a)^{(1/2)} + 15/16 * e * b^2/a^{(7/2)} * c * \ln((b*x+2*a+2*(c*x^2+b*x+a)^{(1/2)} * a^{(1/2)}) \\ & /x) \end{aligned}$$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g x^3 + f x^2 + e x + d}{x^6 \sqrt{c x^2 + b x + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + e x + f x^2 + g x^3}{x^6 \sqrt{a + b x + c x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```



$$3.269 \quad \int (d + ex)^3 (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=258

$$\frac{(300d^2 + 85de + 17e^2)(d + ex)^8}{8e^7} - \frac{2(200d^3 + 85d^2e + 34de^2 + 2e^3)(d + ex)^7}{7e^7} + \frac{(300d^4 + 170d^3e + 102d^2e^2 + 12de^3 + 2e^4)(d + ex)^6}{6e^7} - \frac{(120d^5 + 85d^4e + 68d^3e^2 + 12d^2e^3 + 42de^4 - 7e^5)(d + ex)^5}{5e^7} + \frac{(5d^2 - 2de + 3e^2)(3d^2e + 5d^3e + 4d^4 - de^3 + 2e^4)(d + ex)^4}{4e^7} + \frac{2(d + ex)^{10}}{e^7} - \frac{(120d + 17e)(d + ex)^9}{9e^7} + \frac{2(d + ex)^{10}}{e^7}$$

**Rubi [A]** time = 0.26, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{(300d^2 + 85de + 17e^2)(d + ex)^8}{8e^7} - \frac{2(200d^3 + 85d^2e + 34de^2 + 2e^3)(d + ex)^7}{7e^7} + \frac{(300d^4 + 170d^3e + 102d^2e^2 + 12de^3 + 2e^4)(d + ex)^6}{6e^7} - \frac{(120d^5 + 85d^4e + 68d^3e^2 + 12d^2e^3 + 42de^4 - 7e^5)(d + ex)^5}{5e^7} + \frac{(5d^2 - 2de + 3e^2)(3d^2e + 5d^3e + 4d^4 - de^3 + 2e^4)(d + ex)^4}{4e^7} + \frac{2(d + ex)^{10}}{e^7} - \frac{(120d + 17e)(d + ex)^9}{9e^7} + \frac{2(d + ex)^{10}}{e^7}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*(d + e\*x)^4)/(4\*e^7) - ((120\*d^5 + 85\*d^4\*e + 68\*d^3\*e^2 + 12\*d^2\*e^3 + 42\*d\*e^4 - 7\*e^5)\*(d + e\*x)^5)/(5\*e^7) + ((300\*d^4 + 170\*d^3\*e + 102\*d^2\*e^2 + 12\*d\*e^3 + 21\*e^4)\*(d + e\*x)^6)/(6\*e^7) - (2\*(200\*d^3 + 85\*d^2\*e + 34\*d\*e^2 + 2\*e^3)\*(d + e\*x)^7)/(7\*e^7) + ((300\*d^2 + 85\*d\*e + 17\*e^2)\*(d + e\*x)^8)/(8\*e^7) - ((120\*d + 17\*e)\*(d + e\*x)^9)/(9\*e^7) + (2\*(d + e\*x)^10)/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_)\*(x\_))^(m\_)\*((a\_.) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int (d + ex)^3 (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int \left( \frac{(20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7de^5)}{e^6} \right) dx = \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{4e^7}$$

**Mathematica [A]** time = 0.04, size = 212, normalized size = 0.82

$$6d^3x + \frac{1}{8}ex^8(60d^2 - 51de + 17e^2) + dx^3(7d^2 + 7de + 6e^2) + \frac{1}{2}d^2x^2(7d + 18e) + \frac{1}{7}x^7(20d^3 - 51d^2e + 51de^2 - 4e^3) + \frac{1}{6}x^6(-17d^3 + 51d^2e - 12de^2 + 21e^3) + \frac{1}{5}x^5(17d^3 - 12d^2e + 63de^2 + 7e^3) + \frac{1}{4}x^4(-4d^3 + 63d^2e + 21de^2 + 6e^3) + \frac{1}{9}e^2x^9(60d - 17e) + 2e^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
[Out] 6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*d^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)*x^8)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3 (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
[Out] IntegrateAlgebraic[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
```

**fricas [A]** time = 0.74, size = 237, normalized size = 0.92

$$2x^{10}e^3 - \frac{17}{9}x^9e^3 + \frac{20}{3}x^9e^2d + \frac{17}{8}x^8e^3 - \frac{51}{8}x^8e^2d + \frac{15}{2}x^8e^2d^2 - \frac{4}{7}x^7e^3 + \frac{51}{7}x^7e^2d - \frac{51}{7}x^7e^2d^2 + \frac{20}{7}x^7e^2d^3 + \frac{7}{2}x^6e^3 - 2x^6e^2d + \frac{17}{2}x^6e^2d^2 - \frac{17}{6}x^6e^2d^3 + \frac{7}{5}x^5e^3 + \frac{63}{5}x^5e^2d - \frac{12}{5}x^5e^2d^2 + \frac{17}{5}x^5e^2d^3 + \frac{3}{2}x^4e^3 + \frac{21}{4}x^4e^2d + \frac{63}{4}x^4e^2d^2 - x^4e^2d^3 + 6x^3e^2d + 7x^3e^2d^2 + 7x^3e^2d^3 + 9x^2e^2d^2 + \frac{7}{2}x^2e^2d^3 + 6x^2e^2d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2), x, algorithm="fricas")
[Out] 2*x^10*e^3 - 17/9*x^9*e^3 + 20/3*x^9*e^2*d + 17/8*x^8*e^3 - 51/8*x^8*e^2*d + 15/2*x^8*e^2*d^2 - 4/7*x^7*e^3 + 51/7*x^7*e^2*d - 51/7*x^7*e^2*d^2 + 20/7*x^7*d^3 + 7/2*x^6*e^3 - 2*x^6*e^2*d + 17/2*x^6*e^2*d^2 - 17/6*x^6*d^3 + 7/5*x^5*e^3 + 63/5*x^5*e^2*d - 12/5*x^5*e^2*d^2 + 17/5*x^5*d^3 + 3/2*x^4*e^3 + 21/4*x^4*e^2*d + 63/4*x^4*e^2*d^2 - x^4*d^3 + 6*x^3*e^2*d + 7*x^3*e^2*d^2 + 7*x^3*d^3 + 9*x^2*e^2*d^2 + 7/2*x^2*d^3 + 6*x*d^3
```

**giac [A]** time = 0.17, size = 230, normalized size = 0.89

$$2x^{10}e^3 + \frac{20}{3}dx^9e^2 + \frac{15}{2}d^2x^8e^2 - \frac{17}{9}x^9e^3 - \frac{51}{8}dx^8e^2 - \frac{51}{7}d^2x^7e^2 - \frac{17}{6}d^3x^6e^2 + \frac{17}{8}x^8e^3 + \frac{51}{7}dx^7e^2 + \frac{17}{2}d^2x^6e^2 + \frac{17}{5}d^3x^5e^2 - \frac{4}{7}x^7e^3 - 2dx^6e^2 - \frac{12}{5}d^2x^5e^2 - d^3x^4e^2 + \frac{7}{2}x^6e^3 + \frac{63}{5}dx^5e^2 + \frac{63}{4}d^2x^4e^2 + 7d^3x^3e^2 + \frac{7}{2}d^3x^2e^2 + \frac{3}{2}x^4e^3 + 6dx^3e^2 + 9d^2x^2e^2 + 6d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2), x, algorithm="giac")
[Out] 2*x^10*e^3 + 20/3*d*x^9*e^2 + 15/2*d^2*x^8*e^2 + 20/7*d^3*x^7 - 17/9*x^9*e^3 - 51/8*d*x^8*e^2 - 51/7*d^2*x^7*e^2 - 17/6*d^3*x^6 + 17/8*x^8*e^3 + 51/7*d*x^7
```

$$7e^2 + 17/2*d^2*x^6*e + 17/5*d^3*x^5 - 4/7*x^7*e^3 - 2*d*x^6*e^2 - 12/5*d^2*x^5*e - d^3*x^4 + 7/2*x^6*e^3 + 63/5*d*x^5*e^2 + 63/4*d^2*x^4*e + 7*d^3*x^3 + 7/5*x^5*e^3 + 21/4*d*x^4*e^2 + 7*d^2*x^3*e + 7/2*d^3*x^2 + 3/2*x^4*e^3 + 6*d*x^3*e^2 + 9*d^2*x^2*e + 6*d^3*x$$

**maple [A]** time = 0.00, size = 208, normalized size = 0.81

$$2e^3x^{10} + \frac{(60d^2e - 17e^3)x^9}{9} + \frac{(60d^2e - 51d^2e + 17e^3)x^8}{8} + \frac{(20d^3 - 51d^2e + 51d^2e - 4e^3)x^7}{7} + \frac{(-17d^3 + 51d^2e - 12d^2e + 21e^3)x^6}{6} + \frac{(17d^3 - 12d^2e + 63d^2e + 7e^3)x^5}{5} + 6d^3x + \frac{(-4d^3 + 63d^2e + 21d^2e + 6e^3)x^4}{4} + \frac{(21d^3 + 21d^2e + 18d^2e)x^3}{3} + \frac{(7d^3 + 18d^2e)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x)

$$[Out] 2e^3x^{10} + 1/9*(60*d^2*e - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d^2*e + 17*e^3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d^2*e - 4*e^3)*x^7 + 1/6*(17*d^3 - 51*d^2*e + 12*d^2*e - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d^2*e + 7*e^3)*x^5 + 1/4*(4*d^3 - 63*d^2*e + 21*d^2*e + 6*e^3)*x^4 + 1/3*(21*d^3 + 21*d^2*e + 18*d^2*e)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2 + 6*d^3*x$$

**maxima [A]** time = 0.43, size = 206, normalized size = 0.80

$$2e^3x^{10} + \frac{1}{9}(60d^2e - 17e^3)x^9 + \frac{1}{8}(60d^2e - 51d^2e + 17e^3)x^8 + \frac{1}{7}(20d^3 - 51d^2e + 51d^2e - 4e^3)x^7 - \frac{1}{6}(17d^3 - 51d^2e + 12d^2e - 21e^3)x^6 + \frac{1}{5}(17d^3 - 12d^2e + 63d^2e + 7e^3)x^5 - \frac{1}{4}(4d^3 - 63d^2e - 21d^2e - 6e^3)x^4 + 6d^3x + (7d^3 + 7d^2e + 6d^2e)x^3 + \frac{1}{2}(7d^3 + 18d^2e)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="maxima")

$$[Out] 2e^3x^{10} + 1/9*(60*d^2*e - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d^2*e + 17*e^3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d^2*e - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*d^2*e + 12*d^2*e - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d^2*e + 7*e^3)*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d^2*e - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7*d^2*e + 6*d^2*e)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2$$

**mupad [B]** time = 4.20, size = 196, normalized size = 0.76

$$6d^3x + x^3 \left( \frac{15d^2e}{2} - \frac{51d^2e}{8} + \frac{17e^3}{8} \right) - x^6 \left( \frac{17d^3}{6} - \frac{17d^2e}{2} + 2d^2e - \frac{7e^3}{2} \right) + x^4 \left( -d^3 + \frac{63d^2e}{4} + \frac{21d^2e}{4} + \frac{3e^3}{2} \right) + x^5 \left( \frac{17d^3}{5} - \frac{12d^2e}{5} + \frac{63d^2e}{5} + \frac{7e^3}{5} \right) + x^2 \left( \frac{20d^3}{7} - \frac{51d^2e}{7} + \frac{51d^2e}{7} - \frac{4e^3}{7} \right) + 2e^3x^{10} + dx^3(7d^2 + 7de + 6e^2) + \frac{d^2x^2(7d + 18e)}{2} + \frac{e^2x^9(60d - 17e)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x)^3\*(2\*x + 5\*x^2 + 3)\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2), x)

$$[Out] 6*d^3*x + x^8*((15*d^2*e)/2 - (51*d^2*e)/8 + (17*e^3)/8) - x^6*(2*d^2*e - (17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d^2*e)/4 + (63*d^2*e)/4 - d^3 + (3*e^3)/2) + x^5*((63*d^2*e)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^3)/5) + x^7*((51*d^2*e)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3*x^10 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^9*(60*d - 17*e))/9$$

sympy [A] time = 0.52, size = 230, normalized size = 0.89

$$6d^3x + 2e^3x^{10} + x^9 \left( \frac{20de^2}{3} - \frac{17e^3}{9} \right) + x^8 \left( \frac{15d^2e}{2} - \frac{51de^2}{8} + \frac{17e^3}{8} \right) + x^7 \left( \frac{20d^3}{7} - \frac{51d^2e}{7} + \frac{51de^2}{7} - \frac{4e^3}{7} \right) + x^6 \left( -\frac{17d^3}{6} + \frac{17d^2e}{2} - 2de^2 + \frac{7e^3}{2} \right) + x^5 \left( \frac{17d^3}{5} - \frac{12d^2e}{5} + \frac{63de^2}{5} + \frac{7e^3}{5} \right) + x^4 \left( -d^3 + \frac{63d^2e}{4} + \frac{21de^2}{4} + \frac{3e^3}{2} \right) + x^3 (7d^3 + 7d^2e + 6de^2) + x^2 \left( \frac{7d^3}{2} + 9d^2e \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(5\*x\*\*2+2\*x+3)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2), x)

[Out] 6\*d\*\*3\*x + 2\*e\*\*3\*x\*\*10 + x\*\*9\*(20\*d\*e\*\*2/3 - 17\*e\*\*3/9) + x\*\*8\*(15\*d\*\*2\*e/2 - 51\*d\*e\*\*2/8 + 17\*e\*\*3/8) + x\*\*7\*(20\*d\*\*3/7 - 51\*d\*\*2\*e/7 + 51\*d\*e\*\*2/7 - 4\*e\*\*3/7) + x\*\*6\*(-17\*d\*\*3/6 + 17\*d\*\*2\*e/2 - 2\*d\*e\*\*2 + 7\*e\*\*3/2) + x\*\*5\*(17\*d\*\*3/5 - 12\*d\*\*2\*e/5 + 63\*d\*e\*\*2/5 + 7\*e\*\*3/5) + x\*\*4\*(-d\*\*3 + 63\*d\*\*2\*e/4 + 21\*d\*e\*\*2/4 + 3\*e\*\*3/2) + x\*\*3\*(7\*d\*\*3 + 7\*d\*\*2\*e + 6\*d\*e\*\*2) + x\*\*2\*(7\*d\*\*3/2 + 9\*d\*\*2\*e)

$$3.270 \quad \int (d + ex)^2 (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=157

$$\frac{1}{7}x^7(20d^2 - 34de + 17e^2) - \frac{1}{6}x^6(17d^2 - 34de + 4e^2) + \frac{1}{5}x^5(17d^2 - 8de + 21e^2) - \frac{1}{4}x^4(4d^2 - 42de - 7e^2) + \frac{1}{3}x^3(21d^2 - 14de + 6e^2) + 6d^2x + \frac{1}{8}ex^8(40d - 17e) + \frac{1}{2}dx^2(7d + 12e) + \frac{20e^2x^9}{9}$$

**Rubi [A]** time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{1}{7}x^7(20d^2 - 34de + 17e^2) - \frac{1}{6}x^6(17d^2 - 34de + 4e^2) + \frac{1}{5}x^5(17d^2 - 8de + 21e^2) - \frac{1}{4}x^4(4d^2 - 42de - 7e^2) + \frac{1}{3}x^3(21d^2 + 14de + 6e^2) + 6d^2x + \frac{1}{8}ex^8(40d - 17e) + \frac{1}{2}dx^2(7d + 12e) + \frac{20e^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 6\*d^2\*x + (d\*(7\*d + 12\*e)\*x^2)/2 + ((21\*d^2 + 14\*d\*e + 6\*e^2)\*x^3)/3 - ((4\*d^2 - 42\*d\*e - 7\*e^2)\*x^4)/4 + ((17\*d^2 - 8\*d\*e + 21\*e^2)\*x^5)/5 - ((17\*d^2 - 34\*d\*e + 4\*e^2)\*x^6)/6 + ((20\*d^2 - 34\*d\*e + 17\*e^2)\*x^7)/7 + ((40\*d - 17\*e)\*e\*x^8)/8 + (20\*e^2\*x^9)/9

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int (d + ex)^2 (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int (6d^2 + d(7d + 12e)x + (21d^2 + 14de + 6e^2)x^2 - (4d^2 - 42de - 7e^2)x^3 + (17d^2 - 8de + 21e^2)x^4 - (17d^2 - 34de + 4e^2)x^5 + (20d^2 - 34de + 17e^2)x^6 - (40d - 17e)ex^7 + 20e^2x^8) dx$$

$$= 6d^2x + \frac{1}{2}d(7d + 12e)x^2 + \frac{1}{3}(21d^2 + 14de + 6e^2)x^3 - \frac{1}{4}(4d^2 - 42de - 7e^2)x^4 + \frac{1}{5}(17d^2 - 8de + 21e^2)x^5 - \frac{1}{6}(17d^2 - 34de + 4e^2)x^6 + \frac{1}{7}(20d^2 - 34de + 17e^2)x^7 - \frac{1}{8}(40d - 17e)ex^8 + \frac{20e^2x^9}{9}$$

**Mathematica [A]** time = 0.03, size = 136, normalized size = 0.87

$$d^2 \left( \frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x \right) + de \left( 5x^8 - \frac{34x^7}{7} + \frac{17x^6}{3} - \frac{8x^5}{5} + \frac{21x^4}{2} + \frac{14x^3}{3} + 6x^2 \right) + \frac{e^2(5600x^6 - 5355x^5 + 6120x^4 - 1680x^3 + 10584x^2 + 4410x + 5040)x^3}{2520}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out]  $(e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 5600*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x^6)/3 - (34*x^7)/7 + 5*x^8)$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^2 (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas [A]** time = 0.88, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2 - \frac{17}{8}x^8e^2 + 5x^8ed + \frac{17}{7}x^7e^2 - \frac{34}{7}x^7ed + \frac{20}{7}x^7d^2 - \frac{2}{3}x^6e^2 + \frac{17}{3}x^6ed - \frac{17}{6}x^6d^2 + \frac{21}{5}x^5e^2 - \frac{8}{5}x^5ed + \frac{17}{5}x^5d^2 + \frac{7}{4}x^4e^2 + \frac{21}{2}x^4ed - x^4d^2 + 2x^3e^2 + \frac{14}{3}x^3ed + 7x^3d^2 + 6x^2ed + \frac{7}{2}x^2d^2 + 6xd^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out]  $20/9*x^9*e^2 - 17/8*x^8*e^2 + 5*x^8*e*d + 17/7*x^7*e^2 - 34/7*x^7*e*d + 20/7*x^7*d^2 - 2/3*x^6*e^2 + 17/3*x^6*e*d - 17/6*x^6*d^2 + 21/5*x^5*e^2 - 8/5*x^5*e*d + 17/5*x^5*d^2 + 7/4*x^4*e^2 + 21/2*x^4*e*d - x^4*d^2 + 2*x^3*e^2 + 14/3*x^3*e*d + 7*x^3*d^2 + 6*x^2*e*d + 7/2*x^2*d^2 + 6*x*d^2$

**giac [A]** time = 0.16, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2 + 5dx^8e + \frac{20}{7}d^2x^7 - \frac{17}{8}x^8e^2 - \frac{34}{7}dx^7e - \frac{17}{6}d^2x^6 + \frac{17}{7}x^7e^2 + \frac{17}{3}dx^6e + \frac{17}{5}d^2x^5 - \frac{2}{3}x^6e^2 - \frac{8}{5}dx^5e - d^2x^4 + \frac{21}{5}x^5e^2 + \frac{21}{2}dx^4e + 7d^2x^3 + \frac{7}{4}x^4e^2 + \frac{14}{3}dx^3e + \frac{7}{2}d^2x^2 + 2x^3e^2 + 6dx^2e + 6d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="giac")

[Out]  $20/9*x^9*e^2 + 5*d*x^8*e + 20/7*d^2*x^7 - 17/8*x^8*e^2 - 34/7*d*x^7*e - 17/6*d^2*x^6 + 17/7*x^7*e^2 + 17/3*d*x^6*e + 17/5*d^2*x^5 - 2/3*x^6*e^2 - 8/5*d*x^5*e - d^2*x^4 + 21/5*x^5*e^2 + 21/2*d*x^4*e + 7*d^2*x^3 + 7/4*x^4*e^2 + 14/3*d*x^3*e + 7/2*d^2*x^2 + 2*x^3*e^2 + 6*d*x^2*e + 6*d^2*x$

**maple [A]** time = 0.00, size = 146, normalized size = 0.93

$$\frac{20e^2x^9}{9} + \frac{(40de - 17e^2)x^8}{8} + \frac{(20d^2 - 34de + 17e^2)x^7}{7} + \frac{(-17d^2 + 34de - 4e^2)x^6}{6} + \frac{(17d^2 - 8de + 21e^2)x^5}{5} + \frac{(-4d^2 + 42de + 7e^2)x^4}{4} + 6d^2x + \frac{(21d^2 + 14de + 6e^2)x^3}{3} + \frac{(7d^2 + 12de)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2), x)$

[Out]  $20/9*e^2*x^9+1/8*(40*d*e-17*e^2)*x^8+1/7*(20*d^2-34*d*e+17*e^2)*x^7+1/6*(-17*d^2+34*d*e-4*e^2)*x^6+1/5*(17*d^2-8*d*e+21*e^2)*x^5+1/4*(-4*d^2+42*d*e+7*e^2)*x^4+1/3*(21*d^2+14*d*e+6*e^2)*x^3+1/2*(7*d^2+12*d*e)*x^2+6*d^2*x$

**maxima** [A] time = 0.43, size = 145, normalized size = 0.92

$$\frac{20}{9}e^2x^9 + \frac{1}{8}(40de - 17e^2)x^8 + \frac{1}{7}(20d^2 - 34de + 17e^2)x^7 - \frac{1}{6}(17d^2 - 34de + 4e^2)x^6 + \frac{1}{5}(17d^2 - 8de + 21e^2)x^5 - \frac{1}{4}(4d^2 - 42de - 7e^2)x^4 + \frac{1}{3}(21d^2 + 14de + 6e^2)x^3 + \frac{1}{2}(7d^2 + 12de)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2), x, \text{algorithm}="maxima")$

[Out]  $20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5 - 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2$

**mupad** [B] time = 4.11, size = 137, normalized size = 0.87

$$x^3\left(7d^2 + \frac{14de}{3} + 2e^2\right) + x^4\left(-d^2 + \frac{21de}{2} + \frac{7e^2}{4}\right) - x^6\left(\frac{17d^2}{6} - \frac{17de}{3} + \frac{2e^2}{3}\right) + x^5\left(\frac{17d^2}{5} - \frac{8de}{5} + \frac{21e^2}{5}\right) + x^7\left(\frac{20d^2}{7} - \frac{34de}{7} + \frac{17e^2}{7}\right) + 6d^2x + \frac{20e^2x^9}{9} + \frac{dx^2(7d+12e)}{2} + \frac{ex^8(40d-17e)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2), x)$

[Out]  $x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^6*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (21*e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2*x^9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8$

**sympy** [A] time = 0.15, size = 158, normalized size = 1.01

$$6d^2x + \frac{20e^2x^9}{9} + x^8\left(5de - \frac{17e^2}{8}\right) + x^7\left(\frac{20d^2}{7} - \frac{34de}{7} + \frac{17e^2}{7}\right) + x^6\left(-\frac{17d^2}{6} + \frac{17de}{3} - \frac{2e^2}{3}\right) + x^5\left(\frac{17d^2}{5} - \frac{8de}{5} + \frac{21e^2}{5}\right) + x^4\left(-d^2 + \frac{21de}{2} + \frac{7e^2}{4}\right) + x^3\left(7d^2 + \frac{14de}{3} + 2e^2\right) + x^2\left(\frac{7d^2}{2} + 6de\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2), x)$

[Out]  $6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 34*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*d**2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*(7*d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)$

$$3.271 \quad \int (d + ex) (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=93

$$\frac{1}{7}x^7(20d-17e) - \frac{17}{6}x^6(d-e) + \frac{1}{5}x^5(17d-4e) - \frac{1}{4}x^4(4d-21e) + \frac{7}{3}x^3(3d+e) + \frac{1}{2}x^2(7d+6e) + 6dx + \frac{5ex^8}{2}$$

**Rubi [A]** time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {1628}

$$\frac{1}{7}x^7(20d-17e) - \frac{17}{6}x^6(d-e) + \frac{1}{5}x^5(17d-4e) - \frac{1}{4}x^4(4d-21e) + \frac{7}{3}x^3(3d+e) + \frac{1}{2}x^2(7d+6e) + 6dx + \frac{5ex^8}{2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 6\*d\*x + ((7\*d + 6\*e)\*x^2)/2 + (7\*(3\*d + e)\*x^3)/3 - ((4\*d - 21\*e)\*x^4)/4 + ((17\*d - 4\*e)\*x^5)/5 - (17\*(d - e)\*x^6)/6 + ((20\*d - 17\*e)\*x^7)/7 + (5\*e\*x^8)/2

Rule 1628

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\int (d + ex) (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int (6d + (7d + 6e)x + 7(3d + e)x^2 - (4d - 21e)x^3 + (17d - 4e)x^4 - (17d - 4e)x^5 + (20d - 17e)x^6 + 5ex^7) dx$$

$$= 6dx + \frac{1}{2}(7d + 6e)x^2 + \frac{7}{3}(3d + e)x^3 - \frac{1}{4}(4d - 21e)x^4 + \frac{1}{5}(17d - 4e)x^5 - \frac{1}{6}(17d - 4e)x^6 + \frac{1}{7}(20d - 17e)x^7 + \frac{5ex^8}{2}$$

**Mathematica [A]** time = 0.01, size = 93, normalized size = 1.00

$$\frac{1}{7}x^7(20d-17e) - \frac{17}{6}x^6(d-e) + \frac{1}{5}x^5(17d-4e) + \frac{1}{4}x^4(21e-4d) + \frac{7}{3}x^3(3d+e) + \frac{1}{2}x^2(7d+6e) + 6dx + \frac{5ex^8}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]



[Out]  $6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4 + ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^8)/2$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)(3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x)\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas** [A] time = 0.76, size = 83, normalized size = 0.89

$$\frac{5}{2}x^8e - \frac{17}{7}x^7e + \frac{20}{7}x^7d + \frac{17}{6}x^6e - \frac{17}{6}x^6d - \frac{4}{5}x^5e + \frac{17}{5}x^5d + \frac{21}{4}x^4e - x^4d + \frac{7}{3}x^3e + 7x^3d + 3x^2e + \frac{7}{2}x^2d + 6xd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out]  $5/2*x^8*e - 17/7*x^7*e + 20/7*x^7*d + 17/6*x^6*e - 17/6*x^6*d - 4/5*x^5*e + 17/5*x^5*d + 21/4*x^4*e - x^4*d + 7/3*x^3*e + 7*x^3*d + 3*x^2*e + 7/2*x^2*d + 6*x*d$

**giac** [A] time = 0.15, size = 90, normalized size = 0.97

$$\frac{5}{2}x^8e + \frac{20}{7}dx^7 - \frac{17}{7}x^7e - \frac{17}{6}dx^6 + \frac{17}{6}x^6e + \frac{17}{5}dx^5 - \frac{4}{5}x^5e - dx^4 + \frac{21}{4}x^4e + 7dx^3 + \frac{7}{3}x^3e + \frac{7}{2}dx^2 + 3x^2e + 6dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="giac")

[Out]  $5/2*x^8*e + 20/7*d*x^7 - 17/7*x^7*e - 17/6*d*x^6 + 17/6*x^6*e + 17/5*d*x^5 - 4/5*x^5*e - d*x^4 + 21/4*x^4*e + 7*d*x^3 + 7/3*x^3*e + 7/2*d*x^2 + 3*x^2*e + 6*d*x$

**maple** [A] time = 0.00, size = 84, normalized size = 0.90

$$\frac{5ex^8}{2} + \frac{(20d - 17e)x^7}{7} + \frac{(-17d + 17e)x^6}{6} + \frac{(17d - 4e)x^5}{5} + \frac{(-4d + 21e)x^4}{4} + \frac{(21d + 7e)x^3}{3} + 6dx + \frac{(7d + 6e)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)`

[Out]  $5/2*e*x^8+1/7*(20*d-17*e)*x^7+1/6*(-17*d+17*e)*x^6+1/5*(17*d-4*e)*x^5+1/4*(-4*d+21*e)*x^4+1/3*(21*d+7*e)*x^3+1/2*(7*d+6*e)*x^2+6*d*x$

**maxima** [A] time = 0.43, size = 79, normalized size = 0.85

$$\frac{5}{2}ex^8 + \frac{1}{7}(20d - 17e)x^7 - \frac{17}{6}(d - e)x^6 + \frac{1}{5}(17d - 4e)x^5 - \frac{1}{4}(4d - 21e)x^4 + \frac{7}{3}(3d + e)x^3 + \frac{1}{2}(7d + 6e)x^2 + 6dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")`

[Out]  $5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x$

**mupad** [B] time = 0.05, size = 77, normalized size = 0.83

$$\frac{5ex^8}{2} + \left(\frac{20d}{7} - \frac{17e}{7}\right)x^7 + \left(\frac{17e}{6} - \frac{17d}{6}\right)x^6 + \left(\frac{17d}{5} - \frac{4e}{5}\right)x^5 + \left(\frac{21e}{4} - d\right)x^4 + \left(7d + \frac{7e}{3}\right)x^3 + \left(\frac{7d}{2} + 3e\right)x^2 + 6dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)`

[Out]  $x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6*((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 - x^4*(d - (21*e)/4)$

**sympy** [A] time = 1.91, size = 87, normalized size = 0.94

$$6dx + \frac{5ex^8}{2} + x^7\left(\frac{20d}{7} - \frac{17e}{7}\right) + x^6\left(-\frac{17d}{6} + \frac{17e}{6}\right) + x^5\left(\frac{17d}{5} - \frac{4e}{5}\right) + x^4\left(-d + \frac{21e}{4}\right) + x^3\left(7d + \frac{7e}{3}\right) + x^2\left(\frac{7d}{2} + 3e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)`

[Out]  $6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x**5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/2 + 3*e)$

$$3.272 \quad \int (3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Optimal. Leaf size=42

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

**Rubi** [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$ , Rules used = {1657}

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Int[(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 6\*x + (7\*x^2)/2 + 7\*x^3 - x^4 + (17\*x^5)/5 - (17\*x^6)/6 + (20\*x^7)/7

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4) dx &= \int (6 + 7x + 21x^2 - 4x^3 + 17x^4 - 17x^5 + 20x^6) dx \\ &= 6x + \frac{7x^2}{2} + 7x^3 - x^4 + \frac{17x^5}{5} - \frac{17x^6}{6} + \frac{20x^7}{7} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 6\*x + (7\*x^2)/2 + 7\*x^3 - x^4 + (17\*x^5)/5 - (17\*x^6)/6 + (20\*x^7)/7

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas** [A] time = 0.63, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out] 20/7\*x^7 - 17/6\*x^6 + 17/5\*x^5 - x^4 + 7\*x^3 + 7/2\*x^2 + 6\*x

**giac** [A] time = 0.15, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="giac")

[Out] 20/7\*x^7 - 17/6\*x^6 + 17/5\*x^5 - x^4 + 7\*x^3 + 7/2\*x^2 + 6\*x

**maple** [A] time = 0.00, size = 35, normalized size = 0.83

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2), x)

[Out] 6\*x+7/2\*x^2+7\*x^3-x^4+17/5\*x^5-17/6\*x^6+20/7\*x^7

**maxima** [A] time = 0.42, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="maxima")

[Out] 20/7\*x^7 - 17/6\*x^6 + 17/5\*x^5 - x^4 + 7\*x^3 + 7/2\*x^2 + 6\*x

**mupad** [B] time = 0.03, size = 34, normalized size = 0.81

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 5\*x^2 + 3)\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2),x)

[Out] 6\*x + (7\*x^2)/2 + 7\*x^3 - x^4 + (17\*x^5)/5 - (17\*x^6)/6 + (20\*x^7)/7

**sympy** [A] time = 0.29, size = 37, normalized size = 0.88

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2),x)

[Out] 20\*x\*\*7/7 - 17\*x\*\*6/6 + 17\*x\*\*5/5 - x\*\*4 + 7\*x\*\*3 + 7\*x\*\*2/2 + 6\*x

$$3.273 \quad \int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex} dx$$

**Optimal.** Leaf size=228

$$\frac{x^4(20d^2 + 17de + 17e^2)}{4e^3} - \frac{x^3(20d^3 + 17d^2e + 17de^2 + 4e^3)}{3e^4} + \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^7}$$

**Rubi [A]** time = 0.19, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{x^4(20d^2 + 17de + 17e^2)}{4e^3} - \frac{x^3(17d^2e + 20d^3 + 17de^2 + 4e^3)}{3e^4} + \frac{x^2(17d^2e^2 + 17d^3e + 20d^4 + 4d^2e + 21e^4)}{2e^5} - \frac{x(17d^3e^2 + 4d^2e^3 + 17d^4e + 20d^5 + 21de^4 - 7e^5)}{e^6} + \frac{(5d^2 - 2de + 3e^2)(3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4)\log(d+ex)}{e^7} - \frac{x^5(20d + 17e)}{5e^2} + \frac{10x^6}{3e}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x), x]

[Out] -(((20\*d^5 + 17\*d^4\*e + 17\*d^3\*e^2 + 4\*d^2\*e^3 + 21\*d\*e^4 - 7\*e^5)\*x)/e^6) + (((20\*d^4 + 17\*d^3\*e + 17\*d^2\*e^2 + 4\*d\*e^3 + 21\*e^4)\*x^2)/(2\*e^5) - ((20\*d^3 + 17\*d^2\*e + 17\*d\*e^2 + 4\*e^3)\*x^3)/(3\*e^4) + ((20\*d^2 + 17\*d\*e + 17\*e^2)\*x^4)/(4\*e^3) - ((20\*d + 17\*e)\*x^5)/(5\*e^2) + (10\*x^6)/(3\*e) + ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x])/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4)}{d + ex} dx = \int \left( \frac{-20d^5 - 17d^4e - 17d^3e^2 - 4d^2e^3 - 21de^4 + 7e^5}{e^6} + \frac{(20d^4 + 17d^3e + 17d^2e^2 + 4d^2e^3 + 21de^4 - 7e^5)x}{e^6} + \frac{(20d^4 + 17d^3e + 17d^2e^2 + 4d^2e^3 + 21de^4 - 7e^5)}{e^6} \right) dx$$

**Mathematica [A]** time = 0.06, size = 179, normalized size = 0.79

$$\frac{ex(-1200d^5 + 60d^4e(10x - 17) - 10d^3e^2(40x^2 - 51x + 102) + 10d^2e^3(30x^3 - 34x^2 + 51x - 24) - 5de^4(48x^4 - 51x^3 + 68x^2 - 24x + 252) + e^5(200x^5 - 204x^4 + 255x^3 - 80x^2 + 630x + 420)) + 60(20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7d^5 + 6e^6)\log(d+ex)}{60e^7}$$

Antiderivative was successfully verified.

[In] Integrate[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x), x]

[Out] (e\*x\*(-1200\*d^5 + 60\*d^4\*e\*(-17 + 10\*x) - 10\*d^3\*e^2\*(102 - 51\*x + 40\*x^2) + 10\*d^2\*e^3\*(-24 + 51\*x - 34\*x^2 + 30\*x^3) - 5\*d\*e^4\*(252 - 24\*x + 68\*x^2 - 51\*x^3 + 48\*x^4) + e^5\*(420 + 630\*x - 80\*x^2 + 255\*x^3 - 204\*x^4 + 200\*x^5)) + 60\*(20\*d^6 + 17\*d^5\*e + 17\*d^4\*e^2 + 4\*d^3\*e^3 + 21\*d^2\*e^4 - 7\*d\*e^5 + 6\*e^6)\*Log[d + e\*x])/(60\*e^7)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4)}{d + ex} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x), x]

[Out] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x), x]

**fricas** [A] time = 0.73, size = 230, normalized size = 1.01

$$\frac{200e^5x^6 - 12(20d^6 + 17e^6)x^5 + 15(20d^5e + 17d^6 + 17e^6)x^4 - 20(20d^4e^2 + 17d^5e + 17d^6 + 4e^6)x^3 + 30(20d^4e^2 + 17d^5e + 17d^6 + 4d^6 + 21e^6)x^2 - 60(20d^5e + 17d^4e^2 + 17d^5e + 4d^6 + 21d^6 - 7e^6)x + 60(20d^6 + 17d^5e + 17d^4e^2 + 4d^5e^3 + 21d^6e^4 - 7d^6 + 6e^6)\log(ex + d)}{60e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d), x, algorithm="fricas")

[Out] 1/60\*(200\*e^6\*x^6 - 12\*(20\*d\*e^5 + 17\*e^6)\*x^5 + 15\*(20\*d^2\*e^4 + 17\*d\*e^5 + 17\*e^6)\*x^4 - 20\*(20\*d^3\*e^3 + 17\*d^2\*e^4 + 17\*d\*e^5 + 4\*e^6)\*x^3 + 30\*(20\*d^4\*e^2 + 17\*d^3\*e^3 + 17\*d^2\*e^4 + 4\*d\*e^5 + 21\*e^6)\*x^2 - 60\*(20\*d^5\*e + 17\*d^4\*e^2 + 17\*d^3\*e^3 + 4\*d^2\*e^4 + 21\*d\*e^5 - 7\*e^6)\*x + 60\*(20\*d^6 + 17\*d^5\*e + 17\*d^4\*e^2 + 4\*d^3\*e^3 + 21\*d^2\*e^4 - 7\*d\*e^5 + 6\*e^6)\*log(e\*x + d))/e^7

**giac** [A] time = 0.16, size = 228, normalized size = 1.00

$$(20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7d^6 + 6e^6)\log(ex + d) + \frac{1}{60}(200e^5x^6 - 240d^6x^5 + 300d^5e^2x^4 - 400d^4e^3x^3 + 600d^3e^4x^2 - 1200d^2e^5x - 204e^6x^2 + 255d^6e^4x - 340d^5e^3x^2 + 510d^4e^4x^3 - 1020d^3e^5x^4 + 255e^6x^5 - 340d^6e^4x^6 + 510d^5e^3x^7 - 1020d^4e^4x^8 - 80e^6x^9 + 120d^6e^4x^{10} - 240d^5e^3x^{11} + 630d^4e^4x^{12} - 1260d^3e^5x^{13} + 420e^6x^{14})e^{-6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d), x, algorithm="giac")

[Out]  $(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6) * e^{-7} * \log(\text{abs}(x*e + d)) + 1/60*(200*x^6*e^5 - 240*d*x^5*e^4 + 300*d^2*x^4*e^3 - 400*d^3*x^3*e^2 + 600*d^4*x^2*e - 1200*d^5*x - 204*x^5*e^5 + 255*d*x^4*e^4 - 340*d^2*x^3*e^3 + 510*d^3*x^2*e^2 - 1020*d^4*x*e + 255*x^4*e^5 - 340*d*x^3*e^4 + 510*d^2*x^2*e^3 - 1020*d^3*x*e^2 - 80*x^3*e^5 + 120*d*x^2*e^4 - 240*d^2*x*e^3 + 630*x^2*e^5 - 1260*d*x*e^4 + 420*x*e^5) * e^{-6}$

**maple [A]** time = 0.01, size = 286, normalized size = 1.25

$\frac{10x^6}{3e} - \frac{4dx^5}{e^2} - \frac{17x^5}{5e} + \frac{5d^2x^4}{e^2} + \frac{17d^2x^4}{4e^2} + \frac{17x^4}{4e} - \frac{20d^2x^3}{3e} - \frac{17d^2x^3}{3e} - \frac{17dx^3}{3e} - \frac{4x^3}{3e} + \frac{10d^2x^2}{e^2} + \frac{17d^2x^2}{2d^2} + \frac{17d^2x^2}{2e^2} + \frac{2dx^2}{e^2} + \frac{21x^2}{2e} + \frac{20d^2 \ln(ex+d)}{e^2} - \frac{20dx}{e} + \frac{17d^2 \ln(ex+d)}{e^2} - \frac{17d^2x}{e^2} + \frac{17d^2 \ln(ex+d)}{e^2} - \frac{17d^2x}{e^2} + \frac{4d^2 \ln(ex+d)}{e^2} - \frac{4d^2x}{e^2} + \frac{21d^2 \ln(ex+d)}{e^2} - \frac{21dx}{e^2} - \frac{7d \ln(ex+d)}{e^2} + \frac{7x}{e} + \frac{6 \ln(ex+d)}{e}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d), x)$

[Out]  $21/2*x^2/e+10/3*x^6/e+6/e*\ln(e*x+d)+17/4/e*x^4-4/3/e*x^3+7/e*x-17/5/e*x^5+17/4/e^2*x^4*d-20/3/e^4*x^3*d^3-17/3/e^3*x^3*d^2-17/3/e^2*x^3*d+10/e^5*x^2*d^4+17/2/e^4*x^2*d^3+17/2/e^3*x^2*d^2-4/e^2*x^5*d+5/e^3*x^4*d^2+4/e^4*\ln(e*x+d)*d^3-17/e^4*x*d^3-4/e^3*x*d^2-21/e^2*x*d+2/e^2*x^2*d-20/e^6*x*d^5-17/e^5*x*d^4+21/e^3*\ln(e*x+d)*d^2-7/e^2*\ln(e*x+d)*d+20/e^7*\ln(e*x+d)*d^6+17/e^6*\ln(e*x+d)*d^5+17/e^5*\ln(e*x+d)*d^4$

**maxima [A]** time = 0.43, size = 228, normalized size = 1.00

$\frac{200e^5x^6 - 12(20d^6 + 17d^5)e^5 + 15(20d^6e^3 + 17d^5e^3)x^4 - 20(20d^6e^2 + 17d^5e^2 + 17d^4e^2 + 4d^3e^2 + 30(20d^6e + 17d^5e + 17d^4e + 4d^3e) + 60(20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^2 + 21d^2e - 7d^2)x + (20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^2 + 21d^2e - 7d^2 + 6e^6)\log(ex+d)}{60e^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d), x, \text{algorithm}="maxima")$

[Out]  $1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4 + 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*(20*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*\log(e*x + d)/e^7$

**mapad [B]** time = 4.14, size = 260, normalized size = 1.14

$\left( \frac{7}{e} - \frac{d \left( \frac{21}{e} + \frac{d \left( \frac{4}{e} + \frac{d \left( \frac{17}{e} + \frac{d \left( \frac{20d}{e^2} + \frac{17}{e} \right)}{e} \right)}{e} \right)}{e} \right)}{e} \right) - x^5 \left( \frac{4d}{e^2} + \frac{17}{5e} \right) + x^4 \left( \frac{17}{4e} + \frac{d \left( \frac{20d}{e^2} + \frac{17}{e} \right)}{4e} \right) - x^3 \left( \frac{4}{3e} + \frac{d \left( \frac{17}{e} + \frac{d \left( \frac{20d}{e^2} + \frac{17}{e} \right)}{e} \right)}{3e} \right) + x^2 \left( \frac{21}{2e} + \frac{d \left( \frac{4}{e} + \frac{d \left( \frac{17}{e} + \frac{d \left( \frac{20d}{e^2} + \frac{17}{e} \right)}{e} \right)}{e} \right)}{2e} \right) + \frac{10x^6}{3e} + \frac{\ln(dx+e) (20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7d^2e^5 + 6e^6)}{e^7}$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x), x)$

[Out]  $x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e)/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e))/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) + x^2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e)) + (10*x^6)/(3*e) + (\log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7$

sympy [A] time = 1.17, size = 235, normalized size = 1.03

$$x^5 \left( -\frac{4d}{e^2} - \frac{17}{5e} \right) + x^4 \left( \frac{5d^2}{e^3} + \frac{17d}{4e^2} + \frac{17}{4e} \right) + x^3 \left( -\frac{20d^3}{3e^4} - \frac{17d^2}{3e^3} - \frac{17d}{3e^2} - \frac{4}{3e} \right) + x^2 \left( \frac{10d^4}{e^5} + \frac{17d^3}{2e^4} + \frac{17d^2}{2e^3} + \frac{2d}{e^2} + \frac{21}{2e} \right) + x \left( -\frac{20d^5}{e^6} - \frac{17d^4}{e^5} - \frac{17d^3}{e^4} - \frac{4d^2}{e^3} - \frac{21d}{e^2} + \frac{7}{e} \right) + \frac{10x^6}{3e} + \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d), x)$

[Out]  $x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e)) + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e)) + x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21/(2*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 21*d/e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7$

$$3.274 \quad \int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$$

**Optimal.** Leaf size=228

$$\frac{x^3(60d^2 + 34de + 17e^2)}{3e^4} - \frac{x^2(80d^3 + 51d^2e + 34de^2 + 4e^3)}{2e^5} - \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^7(d+ex)} + \dots$$

**Rubi [A]** time = 0.19, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{x^3(60d^2 + 34de + 17e^2)}{3e^4} - \frac{x^2(51d^2e + 80d^3 + 34de^2 + 4e^3)}{2e^5} + \frac{x(51d^2e^2 + 68d^3e + 100d^4 + 8de^3 + 21e^4)}{e^6} - \frac{(5d^2 - 2de + 3e^2)(3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4)}{e^7(d+ex)} - \frac{(68d^3e^2 + 12d^2e^3 + 85d^4e + 120d^5 + 42de^4 - 7e^5)\log(d+ex)}{e^7} - \frac{x^4(40d + 17e)}{4e^5} + \frac{4x^5}{e^2}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^2,x]

[Out] ((100\*d^4 + 68\*d^3\*e + 51\*d^2\*e^2 + 8\*d\*e^3 + 21\*e^4)\*x)/e^6 - ((80\*d^3 + 51\*d^2\*e + 34\*d\*e^2 + 4\*e^3)\*x^2)/(2\*e^5) + ((60\*d^2 + 34\*d\*e + 17\*e^2)\*x^3)/(3\*e^4) - ((40\*d + 17\*e)\*x^4)/(4\*e^3) + (4\*x^5)/e^2 - ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(e^7\*(d + e\*x)) - ((120\*d^5 + 85\*d^4\*e + 68\*d^3\*e^2 + 12\*d^2\*e^3 + 42\*d\*e^4 - 7\*e^5)\*Log[d + e\*x])/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx = \int \left( \frac{100d^4 + 68d^3e + 51d^2e^2 + 8de^3 + 21e^4}{e^6} - \frac{(80d^3 + 51d^2e + 34de^2 + 4e^3)x^2}{e^5} + \frac{(60d^2 + 34de + 17e^2)x^3}{3e^4} - \frac{(40d + 17e)x^4}{4e^3} + \frac{4x^5}{e^2} - \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^7(d+ex)} - \frac{(120d^5 + 85d^4e + 68d^3e^2 + 12d^2e^3 + 42de^4 - 7e^5)\log(d+ex)}{e^7} \right) dx$$

**Mathematica [A]** time = 0.09, size = 223, normalized size = 0.98

$$\frac{4e^3x^3(60d^2 + 34de + 17e^2) - 6e^2x^2(80d^3 + 51d^2e + 34de^2 + 4e^3) + 12ex(100d^4 + 68d^3e + 51d^2e^2 + 8de^3 + 21e^4) - 12(120d^5 + 85d^4e + 68d^3e^2 + 12d^2e^3 + 42de^4 - 7e^5)\log(d+ex) - \frac{12(20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7d^5e^5)}{d+ex} - 3e^4x^4(40d + 17e) + 48e^3x^5}{12e^7}$$

Antiderivative was successfully verified.

[In] Integrate[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^2,x]

[Out] (12\*e\*(100\*d^4 + 68\*d^3\*e + 51\*d^2\*e^2 + 8\*d\*e^3 + 21\*e^4)\*x - 6\*e^2\*(80\*d^3 + 51\*d^2\*e + 34\*d\*e^2 + 4\*e^3)\*x^2 + 4\*e^3\*(60\*d^2 + 34\*d\*e + 17\*e^2)\*x^3 - 3\*e^4\*(40\*d + 17\*e)\*x^4 + 48\*e^5\*x^5 - (12\*(20\*d^6 + 17\*d^5\*e + 17\*d^4\*e^2 + 4\*d^3\*e^3 + 21\*d^2\*e^4 - 7\*d\*e^5 + 6\*e^6))/(d + e\*x) - 12\*(120\*d^5 + 85\*d^4\*e + 68\*d^3\*e^2 + 12\*d^2\*e^3 + 42\*d\*e^4 - 7\*e^5)\*Log[d + e\*x])/(12\*e^7)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^2,x]

[Out] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^2, x]

**fricas** [A] time = 0.47, size = 319, normalized size = 1.40

$\frac{48d^6e - 240d^5e^2 - 204d^4e^3 - 48d^3e^4 - 252d^2e^5 + 84d^2e^6 - 72d^2e^7 - 3(24d^2e^5 + 17e^6)^2 + (120d^2e^3 + 85d^2e^4 + 68d^2e^5 - 2(120d^2e^3 + 85d^2e^4 + 12e^6)^2 + 6(120d^2e^3 + 85d^2e^4 + 12e^6)^2 + 12(100d^2e^3 + 68d^2e^4 + 51d^2e^5 + 8d^2e^6 + 21d^2e^7) - 12(120d^2e^3 + 85d^2e^4 + 12e^6)^2 + 42d^2e^7 - 7d^2e^8 + (120d^2e^3 + 85d^2e^4 + 12e^6)^2 + 42d^2e^7 - 7d^2e^8)}{12(d^2e + d^2e^2)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2,x, algorithm="fricas")

[Out] 1/12\*(48\*e^6\*x^6 - 240\*d^6 - 204\*d^5\*e - 204\*d^4\*e^2 - 48\*d^3\*e^3 - 252\*d^2\*e^4 + 84\*d\*e^5 - 72\*e^6 - 3\*(24\*d\*e^5 + 17\*e^6)\*x^5 + (120\*d^2\*e^4 + 85\*d^2\*e^5 + 68\*e^6)\*x^4 - 2\*(120\*d^3\*e^3 + 85\*d^2\*e^4 + 68\*d\*e^5 + 12\*e^6)\*x^3 + 6\*(120\*d^4\*e^2 + 85\*d^3\*e^3 + 68\*d^2\*e^4 + 12\*d\*e^5 + 42\*e^6)\*x^2 + 12\*(100\*d^5\*e + 68\*d^4\*e^2 + 51\*d^3\*e^3 + 8\*d^2\*e^4 + 21\*d\*e^5)\*x - 12\*(120\*d^6 + 85\*d^5\*e + 68\*d^4\*e^2 + 12\*d^3\*e^3 + 42\*d^2\*e^4 - 7\*d\*e^5 + (120\*d^5\*e + 85\*d^4\*e^2 + 68\*d^3\*e^3 + 12\*d^2\*e^4 + 42\*d\*e^5 - 7\*e^6)\*x)\*log(e\*x + d))/(e^8\*x + d\*e^7)

**giac** [A] time = 0.17, size = 308, normalized size = 1.35

$\frac{1}{12} \frac{(ex+d)^3 \left( \frac{3(120de+17e^2)d^{(1)}}{ex+d} - \frac{4(300d^2e^2+85de^3+17e^4)d^{(2)}}{(ex+d)^2} + \frac{12(200d^3e^3+85d^2e^4+34de^5+2e^6)d^{(3)}}{(ex+d)^3} - \frac{12(300d^4e^4+170d^3e^5+102d^2e^6+12de^7+21e^8)d^{(4)}}{(ex+d)^4} - 48 \right) d^{(7)} + (120d^6+85d^5e+68d^4e^2+12d^3e^3+42d^2e^4-7e^5)d^{(7)}}{\log\left(\frac{ex+d}{ex+d}\right)} \left( \frac{20d^6e^6}{ex+d} + \frac{17d^5e^6}{ex+d} + \frac{17d^4e^6}{ex+d} + \frac{4d^3e^6}{ex+d} + \frac{21d^2e^6}{ex+d} + \frac{7de^{10}}{ex+d} + \frac{6e^{11}}{ex+d} \right) d^{(12)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2,x, algorithm="giac")

[Out]  $-1/12*(x*e + d)^5*(3*(120*d*e + 17*e^2)*e^{-1}/(x*e + d) - 4*(300*d^2*e^2 + 85*d*e^3 + 17*e^4)*e^{-2}/(x*e + d)^2 + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*d*e^5 + 2*e^6)*e^{-3}/(x*e + d)^3 - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2*e^6 + 12*d*e^7 + 21*e^8)*e^{-4}/(x*e + d)^4 - 48)*e^{-7} + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*e^{-7}*\log(\text{abs}(x*e + d)*e^{-1}/(x*e + d)^2) - (20*d^6*e^5/(x*e + d) + 17*d^5*e^6/(x*e + d) + 17*d^4*e^7/(x*e + d) + 4*d^3*e^8/(x*e + d) + 21*d^2*e^9/(x*e + d) - 7*d*e^{10}/(x*e + d) + 6*e^{11}/(x*e + d))*e^{-12}$

**maple [A]** time = 0.01, size = 313, normalized size = 1.37

$$\frac{4x^5}{e^2} - \frac{10dx^4}{e^3} - \frac{17d^2}{4e^2} + \frac{20d^2x^3}{e^4} + \frac{34dx^3}{3e^3} + \frac{17e^3}{3e^2} - \frac{40d^3x^2}{e^5} + \frac{51d^2x^2}{2e^4} - \frac{17dx^2}{e^3} - \frac{2e^2}{e^2} - \frac{20d^6}{(ex+d)e^2} - \frac{17d^5}{(ex+d)e^2} - \frac{120d^5 \ln(ex+d)}{e^2} - \frac{17d^4}{(ex+d)e^2} + \frac{100d^4x}{e^2} - \frac{85d^4 \ln(ex+d)}{e^2} - \frac{4d^3}{(ex+d)e^4} - \frac{68d^3x}{e^5} - \frac{68d^3 \ln(ex+d)}{e^5} - \frac{21d^2}{(ex+d)e^3} + \frac{51d^2x}{e^4} - \frac{12d^2 \ln(ex+d)}{e^4} + \frac{7d}{(ex+d)e^2} + \frac{8dx}{e^3} - \frac{42d \ln(ex+d)}{e^3} - \frac{6}{(ex+d)e} + \frac{21x}{e^2} + \frac{7 \ln(ex+d)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2,x)

[Out]  $21*x/e^2+4*x^5/e^2-2/e^2*x^2-6/e/(e*x+d)+7/e^2*\ln(e*x+d)+17/3/e^2*x^3-17/4/e^2*x^4-42/e^3*\ln(e*x+d)*d-120/e^7*\ln(e*x+d)*d^5-85/e^6*\ln(e*x+d)*d^4-68/e^5*\ln(e*x+d)*d^3-12/e^4*\ln(e*x+d)*d^2-21/e^3/(e*x+d)*d^2+7/e^2/(e*x+d)*d-20/e^7/(e*x+d)*d^6-17/e^6/(e*x+d)*d^5-17/e^5/(e*x+d)*d^4-4/e^4/(e*x+d)*d^3+8/e^3*x*d+34/3/e^3*x^3*d-40/e^5*x^2*d^3-51/2/e^4*x^2*d^2-17/e^3*x^2*d+100/e^6*d^4*x+68/e^5*x*d^3+51/e^4*x*d^2-10/e^3*x^4*d+20/e^4*x^3*d^2$

**maxima [A]** time = 0.43, size = 234, normalized size = 1.03

$$\frac{20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7de^5 + 6e^6}{e^8x + de^7} + \frac{48e^4x^5 - 3(40d^2e^3 + 17e^4)x^4 + 4(60d^2e^2 + 34de^3 + 17e^3)x^3 - 6(80d^2e + 51d^2e^2 + 34de^3 + 4e^4)x^2 + 12(100d^4 + 68d^3e + 51d^2e^2 + 8de^3 + 21e^4)x - (120d^5 + 85d^4e + 68d^3e^2 + 12d^2e^3 + 42de^4 - 7e^5)\log(ex+d)}{12e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*d^2*e^2 + 34*d*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*d*e^3 + 4*e^4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*d*e^3 + 21*e^4)*x)/e^6 - (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*d*e^4 - 7*e^5)*\log(e*x + d)/e^7$

**mupad [B]** time = 4.18, size = 363, normalized size = 1.59

$$x^3 \left( \frac{17}{3e^3} - \frac{20d^2}{3e^4} + \frac{2d \left( \frac{17d}{3e} + \frac{17}{3e} \right)}{3e} \right) - x^2 \left( \frac{2}{e} + \frac{d \left( \frac{17}{e} - \frac{20d^2}{e^4} + \frac{2d \left( \frac{17d}{3e} + \frac{17}{3e} \right)}{e} \right)}{e} - \frac{d^2 \left( \frac{17d}{3e} + \frac{17}{3e} \right)}{2e^2} \right) - x \left( \frac{10d}{e^3} + \frac{17}{4e} \right) + \frac{21}{e^2} + \frac{2d \left( \frac{2}{e} + \frac{d \left( \frac{17}{e} - \frac{20d^2}{e^4} + \frac{2d \left( \frac{17d}{3e} + \frac{17}{3e} \right)}{e} \right)}{e} - \frac{d^2 \left( \frac{17d}{3e} + \frac{17}{3e} \right)}{2e^2} \right)}{e} + \frac{d^2 \left( \frac{17}{e} - \frac{20d^2}{e^4} + \frac{2d \left( \frac{17d}{3e} + \frac{17}{3e} \right)}{e} \right)}{e^2} + \frac{4x^5}{e^2} - \frac{\ln(d+ex) (120d^6 + 85d^5e + 68d^4e^2 + 12d^3e^3 + 42d^2e^4 - 7e^5) - 20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7d^2e^5 + 6e^6}{e(xe^2 + d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 5\*x^2 + 3)\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(d + e\*x)^2,x)

[Out]  $x^3 \cdot \left( \frac{17}{3e^2} - \frac{(20d^2)}{(3e^4)} + \frac{(2d \cdot ((40d)/e^3 + 17/e^2))}{(3e)} \right) - x^2 \cdot \left( \frac{2}{e^2} + \frac{d \cdot \left( \frac{17}{e^2} - \frac{(20d^2)}{e^4} + \frac{(2d \cdot ((40d)/e^3 + 17/e^2))}{e} \right)}{e} - \frac{d^2 \cdot ((40d)/e^3 + 17/e^2)}{(2e^2)} - x^4 \cdot \left( \frac{(10d)}{e^3} + \frac{17}{(4e^2)} \right) + x \cdot \left( \frac{21}{e^2} + \frac{(2d \cdot (4/e^2 + (2d \cdot (17/e^2 - (20d^2)/e^4 + (2d \cdot ((40d)/e^3 + 17/e^2))/e))}{e} - \frac{d^2 \cdot ((40d)/e^3 + 17/e^2)}{e^2} \right)}{e} - \frac{d^2 \cdot (17/e^2 - (20d^2)/e^4 + (2d \cdot ((40d)/e^3 + 17/e^2))/e)}{e^2} + \frac{(4x^5)/e^2 - (\log(d + ex) \cdot (42d^6e^4 + 85d^5e + 120d^5 - 7e^5 + 12d^2e^3 + 68d^3e^2))}{e^7} - \frac{(17d^5e - 7d^4e^5 + 20d^6 + 6e^6 + 21d^2e^4 + 4d^3e^3 + 17d^4e^2)}{(e \cdot (d^2e^6 + e^7x))} \right)$

**sympy [A]** time = 1.13, size = 238, normalized size = 1.04

$$x^4 \left( -\frac{10d}{e^3} - \frac{17}{4e^2} \right) + x^3 \left( \frac{20d^2}{e^4} + \frac{34d}{3e^3} + \frac{17}{3e^2} \right) + x^2 \left( -\frac{40d^3}{e^5} - \frac{51d^2}{2e^4} - \frac{17d}{e^3} - \frac{2}{e^2} \right) + x \left( \frac{100d^4}{e^6} + \frac{68d^3}{e^5} + \frac{51d^2}{e^4} + \frac{8d}{e^3} + \frac{21}{e^2} \right) + \frac{-20d^6 - 17d^5e - 17d^4e^2 - 4d^3e^3 - 21d^2e^4 + 7d^5e - 6e^6}{de^7 + e^8x} + \frac{4x^5}{e^2} - \frac{(120d^6 + 85d^5e + 68d^4e^2 + 12d^3e^3 + 42d^2e^4 - 7e^5) \log(d + ex)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*2,x)

[Out]  $x^4 \cdot (-10d/e^3 - 17/(4e^2)) + x^3 \cdot (20d^2/e^4 + 34d/(3e^3) + 17/(3e^2)) + x^2 \cdot (-40d^3/e^5 - 51d^2/(2e^4) - 17d/e^3 - 2/e^2) + x \cdot (100d^4/e^6 + 68d^3/e^5 + 51d^2/e^4 + 8d/e^3 + 21/e^2) + (-20d^6 - 17d^5e - 17d^4e^2 - 4d^3e^3 - 21d^2e^4 + 7d^5e - 6e^6)/(d^2e^7 + e^8x) + 4x^5/e^2 - (120d^6 + 85d^5e + 68d^4e^2 + 12d^3e^3 + 42d^2e^4 - 7e^5) \cdot \log(d + ex)/e^7$

$$3.275 \quad \int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$$

**Optimal.** Leaf size=231

$$\frac{x^2(120d^2 + 51de + 17e^2)}{2e^5} - \frac{x(200d^3 + 102d^2e + 51de^2 + 4e^3)}{e^6} - \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{2e^7(d+ex)^2}$$

**Rubi [A]** time = 0.20, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{x^2(120d^2 + 51de + 17e^2)}{2e^5} - \frac{x(102d^2e + 200d^3 + 51de^2 + 4e^3)}{e^6} + \frac{68d^3e^2 + 12d^2e^3 + 85d^4e + 120d^5 + 42de^4 - 7e^5}{e^7(d+ex)} - \frac{(5d^2 - 2de + 3e^2)(3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4)}{2e^7(d+ex)^2} + \frac{(102d^2e^2 + 170d^3e + 300d^4 + 12de^3 + 21e^4) \log(d+ex)}{e^7} - \frac{x^2(60d + 17e)}{3e^4} + \frac{5x^4}{e^5}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3,x]

[Out] -(((200\*d^3 + 102\*d^2\*e + 51\*d\*e^2 + 4\*e^3)\*x)/e^6) + ((120\*d^2 + 51\*d\*e + 17\*e^2)\*x^2)/(2\*e^5) - ((60\*d + 17\*e)\*x^3)/(3\*e^4) + (5\*x^4)/e^3 - ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(2\*e^7\*(d + e\*x)^2) + (120\*d^5 + 85\*d^4\*e + 68\*d^3\*e^2 + 12\*d^2\*e^3 + 42\*d\*e^4 - 7\*e^5)/(e^7\*(d + e\*x)) + ((300\*d^4 + 170\*d^3\*e + 102\*d^2\*e^2 + 12\*d\*e^3 + 21\*e^4)\*Log[d + e\*x])/e^7

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^3} dx = \int \left( \frac{-200d^3 - 102d^2e - 51de^2 - 4e^3}{e^6} + \frac{(120d^2 + 51de + 17e^2)x}{e^5} \right) dx = -\frac{(200d^3 + 102d^2e + 51de^2 + 4e^3)x}{e^6} + \frac{(120d^2 + 51de + 17e^2)x}{2e^5}$$

**Mathematica [A]** time = 0.07, size = 204, normalized size = 0.88

$$\frac{660d^6 + d^4e(459 - 480x) - 51d^4e^2(40x^2 + 2x - 7) - 3d^3e^3(200x^3 + 357x^2 - 34x - 20) + d^2e^4(150x^4 - 340x^3 - 561x^2 + 48x + 189) + 6(300d^4 + 170d^3e + 102d^2e^2 + 12de^3 + 21e^4)(d + ex)^2 \log(d + ex) - de^5(60x^5 - 85x^4 + 204x^3 + 48x^2 - 252x + 21) + e^6(30x^6 - 34x^5 + 51x^4 - 24x^3 - 42x - 18)}{6e^7(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3,x]

[Out] (660\*d^6 + d^5\*e\*(459 - 480\*x) - 51\*d^4\*e^2\*(-7 + 2\*x + 40\*x^2) - 3\*d^3\*e^3\*(-20 - 34\*x + 357\*x^2 + 200\*x^3) + d^2\*e^4\*(189 + 48\*x - 561\*x^2 - 340\*x^3 + 150\*x^4) - d\*e^5\*(21 - 252\*x + 48\*x^2 + 204\*x^3 - 85\*x^4 + 60\*x^5) + e^6\*(-18 - 42\*x - 24\*x^3 + 51\*x^4 - 34\*x^5 + 30\*x^6) + 6\*(300\*d^4 + 170\*d^3\*e + 102\*d^2\*e^2 + 12\*d\*e^3 + 21\*e^4)\*(d + e\*x)^2\*Log[d + e\*x])/(6\*e^7\*(d + e\*x)^2)

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 + 2x + 5x^2)(2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3,x]

[Out] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3, x]

**fricas** [A] time = 0.77, size = 360, normalized size = 1.56

$\frac{30d^6e + 660d^6 + 459d^5e + 357d^4e^2 + 60d^3e^3 + 189d^2e^4 - 21d^2e^5 - 18e^6 - 2(30d^6 + 17d^5e + (150d^6 + 85d^5e + 51d^4e^2 - 4(150d^6 + 85d^5e + 51d^4e^2) - 3(680d^4e^2 + 357d^3e^3 + 187d^2e^4 + 16d^2e^5) - 6(80d^5e + 17d^4e^2 - 17d^3e^3 - 8d^2e^4 - 42d^2e^5 + 7e^6) * x + 6(300d^6 + 170d^5e + 102d^4e^2 + 12d^3e^3 + 21d^2e^4) * x^2 + 2(300d^5e + 170d^4e^2 + 102d^3e^3 + 12d^2e^4 + 21d^2e^5) * x) * \log(ex + d)}{6d^7e^7 + 2d^6e^6 + d^5e^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/6\*(30\*e^6\*x^6 + 660\*d^6 + 459\*d^5\*e + 357\*d^4\*e^2 + 60\*d^3\*e^3 + 189\*d^2\*e^4 - 21\*d\*e^5 - 18\*e^6 - 2\*(30\*d\*e^5 + 17\*e^6)\*x^5 + (150\*d^2\*e^4 + 85\*d\*e^5 + 51\*e^6)\*x^4 - 4\*(150\*d^3\*e^3 + 85\*d^2\*e^4 + 51\*d\*e^5 + 6\*e^6)\*x^3 - 3\*(680\*d^4\*e^2 + 357\*d^3\*e^3 + 187\*d^2\*e^4 + 16\*d^2\*e^5)\*x^2 - 6\*(80\*d^5\*e + 17\*d^4\*e^2 - 17\*d^3\*e^3 - 8\*d^2\*e^4 - 42\*d^2\*e^5 + 7\*e^6)\*x + 6\*(300\*d^6 + 170\*d^5\*e + 102\*d^4\*e^2 + 12\*d^3\*e^3 + 21\*d^2\*e^4 + (300\*d^4\*e^2 + 170\*d^3\*e^3 + 102\*d^2\*e^4 + 12\*d^2\*e^5 + 21\*e^6)\*x^2 + 2\*(300\*d^5\*e + 170\*d^4\*e^2 + 102\*d^3\*e^3 + 12\*d^2\*e^4 + 21\*d^2\*e^5)\*x)\*log(e\*x + d))/(e^9\*x^2 + 2\*d\*e^8\*x + d^2\*e^7)

**giac** [A] time = 0.18, size = 216, normalized size = 0.94

$\frac{(300d^6 + 170d^5e + 102d^4e^2 + 12d^3e^3 + 21d^2e^4) \log(ex + d) + \frac{1}{6}(30d^6e - 120d^5e^2 + 360d^4e^3e^2 - 1200d^3e^4e^2 - 34d^2e^5e^2 + 153d^2e^6 - 612d^2e^7 + 51d^2e^8 - 306d^2e^9 - 24d^2e^{10})}{2(dx + d)^7} + \frac{(220d^6 + 153d^5e + 119d^4e^2 + 20d^3e^3 + 63d^2e^4 + 2(120d^5e + 85d^4e^2 + 68d^3e^3 + 12d^2e^4 + 42d^2e^5 - 7d^6 - 7d^7 - 6d^8) * x - 7d^6 - 6d^7) * x^2}{2(dx + d)^7}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x, algorithm="giac")

[Out] (300\*d^4 + 170\*d^3\*e + 102\*d^2\*e^2 + 12\*d\*e^3 + 21\*e^4)\*e^(-7)\*log(abs(x\*e + d)) + 1/6\*(30\*x^4\*e^9 - 120\*d\*x^3\*e^8 + 360\*d^2\*x^2\*e^7 - 1200\*d^3\*x\*e^6 - 34\*x^3\*e^9 + 153\*d\*x^2\*e^8 - 612\*d^2\*x\*e^7 + 51\*x^2\*e^9 - 306\*d\*x\*e^8 - 24\*x\*e^9)\*e^(-12) + 1/2\*(220\*d^6 + 153\*d^5\*e + 119\*d^4\*e^2 + 20\*d^3\*e^3 + 63\*d^2\*e^4 + 2\*(120\*d^5\*e + 85\*d^4\*e^2 + 68\*d^3\*e^3 + 12\*d^2\*e^4 + 42\*d\*e^5 - 7\*e^6)\*x - 7\*d\*e^5 - 6\*e^6)\*e^(-7)/(x\*e + d)^2

**maple [A]** time = 0.01, size = 336, normalized size = 1.45

$$\frac{5x^4}{e^3} - \frac{20d^2x^3}{e^4} - \frac{17x^3}{3e^5} - \frac{10d^6}{(ex+d)^2e^2} - \frac{17d^6}{2(ex+d)^2e^3} - \frac{17d^6}{2(ex+d)^2e^4} - \frac{2d^6}{(ex+d)^2e^5} - \frac{21d^6}{2(ex+d)^2e^6} + \frac{60d^6x^2}{e^6} + \frac{7d}{2(ex+d)^2e^2} + \frac{51d^2}{2e^4} + \frac{3}{(ex+d)^2e^5} - \frac{17x^2}{2e^5} - \frac{120d^6}{(ex+d)^2e^6} + \frac{85d^6}{(ex+d)^2e^7} + \frac{300d^4 \ln(ex+d)}{e^2} + \frac{68d^6}{(ex+d)^2e^2} - \frac{200d^2x}{e^6} + \frac{170d^2 \ln(ex+d)}{(ex+d)^2e^6} + \frac{12d^6}{(ex+d)^2e^6} - \frac{102d^2x}{e^6} + \frac{102d^2 \ln(ex+d)}{e^6} + \frac{42d}{(ex+d)^2e^5} - \frac{51d^2}{e^4} + \frac{12d \ln(ex+d)}{e^4} - \frac{7}{(ex+d)^2e^5} - \frac{4x}{e^5} + \frac{21 \ln(ex+d)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x)

[Out] -200/e^6\*d^3\*x-102/e^5\*x\*d^2-51/e^4\*x\*d+5\*x^4/e^3-3/e/(e\*x+d)^2+21/e^3\*ln(e\*x+d)+17/2/e^3\*x^2-4/e^3\*x-7/e^2/(e\*x+d)-17/3/e^3\*x^3+120/e^7/(e\*x+d)\*d^5+85/e^6/(e\*x+d)\*d^4+68/e^5/(e\*x+d)\*d^3+12/e^4/(e\*x+d)\*d^2+42/e^3/(e\*x+d)\*d-20/e^4\*x^3\*d+60/e^5\*x^2\*d^2+51/2/e^4\*x^2\*d-10/e^7/(e\*x+d)^2\*d^6-17/2/e^6/(e\*x+d)^2\*d^5-17/2/e^5/(e\*x+d)^2\*d^4-2/e^4/(e\*x+d)^2\*d^3-21/2/e^3/(e\*x+d)^2\*d^2+7/2/e^2/(e\*x+d)^2\*d+300/e^7\*ln(e\*x+d)\*d^4+170/e^6\*ln(e\*x+d)\*d^3+102/e^5\*ln(e\*x+d)\*d^2+12/e^4\*ln(e\*x+d)\*d

**maxima [A]** time = 0.44, size = 240, normalized size = 1.04

$$\frac{220d^6 + 153d^5e + 119d^4e^2 + 20d^3e^3 + 63d^2e^4 - 7d^5 - 6e^6 + 2(120d^5e + 85d^4e^2 + 68d^3e^3 + 12d^2e^4 + 42de^5 - 7e^6)x}{2(e^2x^2 + 2de^2x + d^2e^2)} + \frac{30e^2x^4 - 2(60d^2e + 17e^2)x^3 + 3(120d^2e + 51d^2e + 17e^2)x^2 - 6(200d^3 + 102d^2e + 51d^2e + 4e^2)x}{6e^6} + \frac{(300d^4 + 170d^3e + 102d^2e^2 + 12de^3 + 21e^4) \log(ex+d)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x, algorithm="maxima")

[Out] 1/2\*(220\*d^6 + 153\*d^5\*e + 119\*d^4\*e^2 + 20\*d^3\*e^3 + 63\*d^2\*e^4 - 7\*d\*e^5 - 6\*e^6 + 2\*(120\*d^5\*e + 85\*d^4\*e^2 + 68\*d^3\*e^3 + 12\*d^2\*e^4 + 42\*d\*e^5 - 7\*e^6)\*x)/(e^9\*x^2 + 2\*d\*e^8\*x + d^2\*e^7) + 1/6\*(30\*e^3\*x^4 - 2\*(60\*d\*e^2 + 17\*e^3)\*x^3 + 3\*(120\*d^2\*e + 51\*d\*e^2 + 17\*e^3)\*x^2 - 6\*(200\*d^3 + 102\*d^2\*e + 51\*d\*e^2 + 4\*e^3)\*x)/e^6 + (300\*d^4 + 170\*d^3\*e + 102\*d^2\*e^2 + 12\*d\*e^3 + 21\*e^4)\*log(e\*x + d)/e^7

**mupad [B]** time = 0.09, size = 297, normalized size = 1.29

$$x^2 \left( \frac{17}{2e^3} - \frac{30d^2}{e^4} + \frac{3d \left( \frac{60d}{e} + \frac{17}{2} \right)}{2e} \right) - x^3 \left( \frac{20d}{e^4} + \frac{17}{3e^5} \right) + \frac{x \left( 120d^6 + 85d^5e + 68d^4e^2 + 12d^3e^3 + 42d^2e^4 - 7e^5 \right) + \frac{220d^6 + 153d^5e + 119d^4e^2 + 20d^3e^3 + 63d^2e^4 - 7d^5 - 6e^6}{2e^6}}{d^2e^6 + 2d^2e^2x + d^2e^2} - x \left( \frac{4}{e^3} + \frac{20d^3}{e^6} + \frac{3d \left( \frac{17}{e} - \frac{60d^2}{e^3} + \frac{3d \left( \frac{60d}{e} + \frac{17}{2} \right)}{e} \right)}{e} - \frac{3d^2 \left( \frac{60d}{e} + \frac{17}{2} \right)}{e^2} \right) + \frac{5x^4}{e^3} + \frac{\ln(d+ex) \left( 300d^4 + 170d^3e + 102d^2e^2 + 12de^3 + 21e^4 \right)}{e^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)`

[Out]  $x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*((20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e^4 + 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*(4/e^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3)))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2 + (5*x^4)/e^3 + (\log(d + e*x)*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7$

**sympy** [A] time = 2.63, size = 248, normalized size = 1.07

$$x^3 \left( -\frac{20d}{e^4} - \frac{17}{3e^3} \right) + x^2 \left( \frac{60d^2}{e^5} + \frac{51d}{2e^4} + \frac{17}{2e^3} \right) + x \left( -\frac{200d^3}{e^6} - \frac{102d^2}{e^5} - \frac{51d}{e^4} - \frac{4}{e^3} \right) + \frac{220d^6 + 153d^5e + 119d^4e^2 + 20d^3e^3 + 63d^2e^4 - 7de^5 - 6e^6 + x(240d^6e + 170d^5e^2 + 136d^4e^3 + 24d^3e^4 + 84d^2e^5 - 14e^6)}{2d^2e^7 + 4de^8x + 2e^9x^2} + \frac{5x^4}{e^3} + \frac{(300d^4 + 170d^3e + 102d^2e^2 + 12de^3 + 21e^4) \log(d + ex)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)`

[Out]  $x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/(2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220*d**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**5 - 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4 + 84*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/e**3 + (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*\log(d + e*x)/e**7$

$$3.276 \quad \int (d + ex)^3 (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=391

$$\frac{(2800d^2 + 315de + 111e^2)(d + ex)^{10}}{10e^9} - \frac{(5600d^3 + 945d^2e + 666de^2 + 37e^3)(d + ex)^9}{9e^9} + \frac{(7000d^4 + 1575d^3e + 1665d^2e^2 + 592d^2e^3 - 65e^5)(d + ex)^7}{7e^9} + \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)(d + ex)^4}{4e^9} - \frac{(5d^2 - 2de + 3e^2)(160d^5 + 127d^4e + 88d^3e^2 - 4d^2e^3 + 64d^2e^4 - 11e^5)(d + ex)^5}{5e^9} + \frac{(2800d^6 + 945d^5e + 1665d^4e^2 + 370d^3e^3 + 888d^2e^4 - 195d^2e^5 + 107e^6)(d + ex)^6}{6e^9} - \frac{(5600d^5 + 1575d^4e + 2220d^3e^2 + 370d^2e^3 + 592d^2e^4 - 65e^5)(d + ex)^7}{7e^9} + \frac{(7000d^4 + 1575d^3e + 1665d^2e^2 + 185d^2e^3 + 148e^4)(d + ex)^8}{8e^9} - \frac{(5600d^3 + 945d^2e + 666de^2 + 37e^3)(d + ex)^9}{9e^9} + \frac{(2800d^2 + 315de + 111e^2)(d + ex)^{10}}{10e^9} - \frac{5(160d + 9e)(d + ex)^{11}}{11e^9} + \frac{5(d + ex)^{12}}{3e^9}$$

**Rubi [A]** time = 0.39, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*(d + e\*x)^4)/(4\*e^9) - ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(160\*d^5 + 127\*d^4\*e + 88\*d^3\*e^2 - 4\*d^2\*e^3 + 64\*d^2\*e^4 - 11\*e^5)\*(d + e\*x)^5)/(5\*e^9) + ((2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d^2\*e^5 + 107\*e^6)\*(d + e\*x)^6)/(6\*e^9) - ((5600\*d^5 + 1575\*d^4\*e + 2220\*d^3\*e^2 + 370\*d^2\*e^3 + 592\*d^2\*e^4 - 65\*e^5)\*(d + e\*x)^7)/(7\*e^9) + ((7000\*d^4 + 1575\*d^3\*e + 1665\*d^2\*e^2 + 185\*d^2\*e^3 + 148\*e^4)\*(d + e\*x)^8)/(8\*e^9) - ((5600\*d^3 + 945\*d^2\*e + 666\*d^2\*e^2 + 37\*e^3)\*(d + e\*x)^9)/(9\*e^9) + ((2800\*d^2 + 315\*d\*e + 111\*e^2)\*(d + e\*x)^10)/(10\*e^9) - (5\*(160\*d + 9\*e)\*(d + e\*x)^11)/(11\*e^9) + (25\*(d + e\*x)^12)/(3\*e^9)

**Rule 1628**

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int (d + ex)^3 (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int \left( \frac{(5d^2 - 2de + 3e^2)^2 (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^8} \right) dx = \frac{(5d^2 - 2de + 3e^2)^2 (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{4e^9}$$

**Mathematica [A]** time = 0.04, size = 277, normalized size = 0.71

$$18d^3x + \frac{3}{10}d^{12}(100d^2 - 45d + 37e^3) + \frac{1}{2}d^2(107d^2 + 99de + 54e^2) + \frac{3}{2}d^2(11d + 18e) + \frac{1}{5}d^2(100d^2 - 135d^2e + 333d^2e^2 - 37e^3) + \frac{1}{5}d^2(-45d^3 + 333d^2e - 111d^2e^2 + 148e^3) + \frac{1}{2}d^2(111d^3 - 111d^2e + 444d^2e^2 + 65e^3) + \frac{1}{5}d^2(-37d^3 + 444d^2e + 195d^2e^2 + 107e^3) + \frac{1}{5}d^2(148d^3 + 195d^2e + 321d^2e^2 + 33e^3) + \frac{1}{4}d^2(65d^3 + 321d^2e + 99d^2e^2 + 18e^3) + \frac{15}{11}d^{11}(20d - 3e) + \frac{25d^{12}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 18\*d^3\*x + (3\*d^2\*(11\*d + 18\*e)\*x^2)/2 + (d\*(107\*d^2 + 99\*d\*e + 54\*e^2)\*x^3)/3 + ((65\*d^3 + 321\*d^2\*e + 99\*d\*e^2 + 18\*e^3)\*x^4)/4 + ((148\*d^3 + 195\*d^2\*e + 321\*d\*e^2 + 33\*e^3)\*x^5)/5 + ((-37\*d^3 + 444\*d^2\*e + 195\*d\*e^2 + 107\*e^3)\*x^6)/6 + ((111\*d^3 - 111\*d^2\*e + 444\*d\*e^2 + 65\*e^3)\*x^7)/7 + ((-45\*d^3 + 333\*d^2\*e - 111\*d\*e^2 + 148\*e^3)\*x^8)/8 + ((100\*d^3 - 135\*d^2\*e + 333\*d\*e^2 - 37\*e^3)\*x^9)/9 + (3\*e\*(100\*d^2 - 45\*d\*e + 37\*e^2)\*x^10)/10 + (15\*(20\*d - 3\*e)\*e^2\*x^11)/11 + (25\*e^3\*x^12)/3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^3 (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas [A]** time = 0.76, size = 305, normalized size = 0.78

$$\frac{25}{3}x^{12}e^3 - \frac{45}{11}x^{11}e^3 + \frac{300}{11}x^{11}e^2d + \frac{111}{10}x^{10}e^3 - \frac{27}{2}x^{10}e^2d + \frac{30}{11}x^{10}e^2d^2 - \frac{37}{9}x^9e^3 + \frac{37}{11}x^9e^2d - \frac{15}{8}x^9e^2d^2 + \frac{100}{9}x^9e^2d^3 + \frac{37}{2}x^8e^3 - \frac{111}{8}x^8e^2d + \frac{333}{8}x^8e^2d^2 - \frac{45}{8}x^8e^2d^3 + \frac{65}{7}x^7e^3 + \frac{444}{7}x^7e^2d - \frac{111}{7}x^7e^2d^2 + \frac{111}{7}x^7e^2d^3 + \frac{107}{6}x^6e^3 + \frac{65}{2}x^6e^2d + \frac{74}{6}x^6e^2d^2 - \frac{37}{6}x^6e^2d^3 + \frac{33}{5}x^5e^3 + \frac{321}{5}x^5e^2d + \frac{39}{4}x^5e^2d^2 + \frac{148}{5}x^5e^2d^3 + \frac{9}{2}x^4e^3 + \frac{99}{4}x^4e^2d + \frac{321}{4}x^4e^2d^2 + \frac{65}{4}x^4e^2d^3 + \frac{18}{3}x^3e^2d + \frac{33}{3}x^3e^2d^2 + \frac{107}{3}x^3e^2d^3 + \frac{27}{2}x^2e^2d^2 + \frac{33}{2}x^2e^2d^3 + 18x^2d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out] 25/3\*x^12\*e^3 - 45/11\*x^11\*e^3 + 300/11\*x^11\*e^2\*d + 111/10\*x^10\*e^3 - 27/2\*x^10\*e^2\*d + 30\*x^10\*e^2\*d^2 - 37/9\*x^9\*e^3 + 37\*x^9\*e^2\*d - 15\*x^9\*e^2\*d^2 + 100/9\*x^9\*e^2\*d^3 + 37/2\*x^8\*e^3 - 111/8\*x^8\*e^2\*d + 333/8\*x^8\*e^2\*d^2 - 45/8\*x^8\*e^2\*d^3 + 65/7\*x^7\*e^3 + 444/7\*x^7\*e^2\*d - 111/7\*x^7\*e^2\*d^2 + 111/7\*x^7\*e^2\*d^3 + 107/6\*x^6\*e^3 + 65/2\*x^6\*e^2\*d + 74\*x^6\*e^2\*d^2 - 37/6\*x^6\*e^2\*d^3 + 33/5\*x^5\*e^3 + 321/5\*x^5\*e^2\*d + 39\*x^5\*e^2\*d^2 + 148/5\*x^5\*e^2\*d^3 + 9/2\*x^4\*e^3 + 99/4\*x^4\*e^2\*d + 321/4\*x^4\*e^2\*d^2 + 65/4\*x^4\*e^2\*d^3 + 18\*x^3\*e^2\*d + 33\*x^3\*e^2\*d^2 + 107/3\*x^3\*e^2\*d^3 + 27\*x^2\*e^2\*d^2 + 33/2\*x^2\*e^2\*d^3 + 18\*x^2\*d^3

**giac** [A] time = 0.16, size = 296, normalized size = 0.76

$$\frac{25}{3}x^{12} + \frac{300}{11}d^3x^{11} + 30d^2x^{10}e + \frac{100}{9}d^3x^9 - \frac{45}{11}d^3x^8 + \frac{111}{10}d^3x^7 + \frac{37}{9}d^3x^6 - \frac{111}{8}d^3x^5 + \frac{37}{6}d^3x^4 + \frac{107}{6}d^3x^3 + \frac{321}{5}d^3x^2 + \frac{321}{4}d^3x + \frac{107}{3}d^3 + \frac{33}{5}d^2x^5e + \frac{99}{4}d^2x^4e^2 + \frac{33}{2}d^2x^3e + \frac{33}{2}d^2x^2 + \frac{9}{2}d^2x + 18d^2 + 27d^2 + 18d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="giac")

[Out]  $\frac{25}{3}x^{12}e^3 + \frac{300}{11}d^3x^{11}e^2 + 30d^2x^{10}e + \frac{100}{9}d^3x^9 - \frac{45}{11}d^3x^8 + \frac{111}{10}d^3x^7 + \frac{37}{9}d^3x^6 - \frac{111}{8}d^3x^5 + \frac{37}{6}d^3x^4 + \frac{107}{6}d^3x^3 + \frac{321}{5}d^3x^2 + \frac{321}{4}d^3x + \frac{107}{3}d^3 + \frac{33}{5}d^2x^5e + \frac{99}{4}d^2x^4e^2 + \frac{33}{2}d^2x^3e + \frac{33}{2}d^2x^2 + \frac{9}{2}d^2x + 18d^2 + 27d^2 + 18d^2$

**maple** [A] time = 0.00, size = 264, normalized size = 0.68

$$\frac{25e^3x^{12}}{3} + \frac{(300d^3-45e^3)x^{11}}{11} + \frac{(300d^2e-135d^2+111e^3)x^{10}}{10} + \frac{(100d^3-135d^2e+333d^2-37e^3)x^9}{9} + \frac{(-45d^3+333d^2e-111d^2+148e^3)x^8}{8} + \frac{(111d^3-111d^2e+444d^2+65e^3)x^7}{7} + \frac{(-37d^3+444d^2e+195d^2+107e^3)x^6}{6} + \frac{(148d^3+195d^2e+321d^2+33e^3)x^5}{5} + \frac{(65d^3+321d^2e+99d^2+18e^3)x^4}{4} + \frac{(107d^3+99d^2e+54d^2)x^3}{3} + \frac{(33d^3+54d^2e)x^2}{2} + 18d^2 + 27d^2 + 18d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2),x)

[Out]  $\frac{25}{3}e^3x^{12} + \frac{15}{11}(20d^3e^2 - 3e^3)x^{11} + \frac{3}{10}(100d^2e - 45d^3e^2 + 37e^3)x^{10} + \frac{1}{9}(100d^3 - 135d^2e + 333d^2e^2 - 37e^3)x^9 - \frac{1}{8}(45d^3 - 333d^2e + 111d^2e^2 - 148e^3)x^8 + \frac{1}{7}(111d^3 - 111d^2e + 444d^2e^2 + 65e^3)x^7 + \frac{1}{6}(-37d^3 + 444d^2e + 195d^2e^2 + 107e^3)x^6 + \frac{1}{5}(148d^3 + 195d^2e + 321d^2e^2 + 33e^3)x^5 + \frac{1}{4}(65d^3 + 321d^2e + 99d^2e^2 + 18e^3)x^4 + \frac{1}{3}(107d^3 + 99d^2e + 54d^2e^2)x^3 + \frac{1}{2}(33d^3 + 54d^2e)x^2 + 18d^3 + 27d^3 + 18d^3$

**maxima** [A] time = 0.43, size = 263, normalized size = 0.67

$$\frac{25}{3}e^3x^{12} + \frac{15}{11}(20d^3e^2 - 3e^3)x^{11} + \frac{3}{10}(100d^2e - 45d^3e^2 + 37e^3)x^{10} + \frac{1}{9}(100d^3 - 135d^2e + 333d^2e^2 - 37e^3)x^9 - \frac{1}{8}(45d^3 - 333d^2e + 111d^2e^2 - 148e^3)x^8 + \frac{1}{7}(111d^3 - 111d^2e + 444d^2e^2 + 65e^3)x^7 - \frac{1}{6}(37d^3 - 444d^2e - 195d^2e^2 - 107e^3)x^6 + \frac{1}{5}(148d^3 + 195d^2e + 321d^2e^2 + 33e^3)x^5 + \frac{1}{4}(65d^3 + 321d^2e + 99d^2e^2 + 18e^3)x^4 + \frac{1}{3}(107d^3 + 99d^2e + 54d^2e^2)x^3 + \frac{1}{2}(33d^3 + 54d^2e)x^2 + 18d^3 + 27d^3 + 18d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="maxima")

[Out]  $\frac{25}{3}e^3x^{12} + \frac{15}{11}(20d^3e^2 - 3e^3)x^{11} + \frac{3}{10}(100d^2e - 45d^3e^2 + 37e^3)x^{10} + \frac{1}{9}(100d^3 - 135d^2e + 333d^2e^2 - 37e^3)x^9 - \frac{1}{8}(45d^3 - 333d^2e + 111d^2e^2 - 148e^3)x^8 + \frac{1}{7}(111d^3 - 111d^2e + 444d^2e^2 + 65e^3)x^7 - \frac{1}{6}(37d^3 - 444d^2e - 195d^2e^2 - 107e^3)x^6 + \frac{1}{5}(148d^3 + 195d^2e + 321d^2e^2 + 33e^3)x^5 + \frac{1}{4}(65d^3 + 321d^2e + 99d^2e^2 + 18e^3)x^4 + \frac{1}{3}(107d^3 + 99d^2e + 54d^2e^2)x^3 + \frac{1}{2}(33d^3 + 54d^2e)x^2 + 18d^3 + 27d^3 + 18d^3$

$$d^2e + 99d^2e^2 + 18e^3)x^4 + 18d^3x + 1/3(107d^3 + 99d^2e + 54d^2e^2)x^3 + 3/2(11d^3 + 18d^2e)x^2$$

**mupad [B]** time = 4.25, size = 251, normalized size = 0.64

$$18d^2x + x^2\left(\frac{107d^3}{3} + 33d^2e + 18d^2e^2\right) + x^3\left(\frac{100d^3}{9} - 15d^2e + 37d^2e^2 - \frac{37d^3}{9}\right) + x^4\left(\frac{37d^3}{6} + 74d^2e + \frac{65d^2e^2}{2} + \frac{107d^3}{6}\right) + x^5\left(\frac{65d^3}{4} + \frac{321d^2e}{4} + \frac{99d^2e^2}{4} + \frac{9d^3}{2}\right) - x^6\left(\frac{45d^3}{8} - \frac{333d^2e}{8} + \frac{111d^2e^2}{8} - \frac{37d^3}{2}\right) + x^7\left(\frac{148d^3}{5} + 39d^2e + \frac{321d^2e^2}{5} + \frac{33d^3}{5}\right) + x^8\left(\frac{111d^3}{7} - \frac{111d^2e}{7} + \frac{444d^2e^2}{7} + \frac{65d^3}{7}\right) + \frac{25d^3x^{12}}{3} + \frac{3ex^{10}(100d^3 - 45d^2e + 37d^2e^2)}{10} + \frac{3d^2x^{11}(11d + 18e)}{2} + \frac{15d^2x^{11}(20d - 3e)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x)^3\*(2\*x + 5\*x^2 + 3)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2), x)

[Out]  $18d^3x + x^3(18d^2e^2 + 33d^2e + (107d^3)/3) + x^9(37d^2e^2 - 15d^2e + (100d^3)/9 - (37e^3)/9) + x^6((65d^2e^2)/2 + 74d^2e - (37d^3)/6 + (107e^3)/6) + x^4((99d^2e^2)/4 + (321d^2e)/4 + (65d^3)/4 + (9e^3)/2) - x^8((111d^2e^2)/8 - (333d^2e)/8 + (45d^3)/8 - (37e^3)/2) + x^5((321d^2e^2)/5 + 39d^2e + (148d^3)/5 + (33e^3)/5) + x^7((444d^2e^2)/7 - (111d^2e)/7 + (111d^3)/7 + (65e^3)/7) + (25e^3x^{12})/3 + (3ex^{10}(100d^2 - 45d^2e + 37e^2))/10 + (3d^2x^2(11d + 18e))/2 + (15e^2x^{11}(20d - 3e))/11$

**sympy [A]** time = 0.20, size = 298, normalized size = 0.76

$$18d^2x + \frac{25d^3x^{12}}{3} + x^{11}\left(\frac{300d^3}{11} - \frac{45e^3}{11}\right) + x^{10}\left(30d^2e - \frac{27d^2e^2}{2} + \frac{111d^3}{10}\right) + x^9\left(\frac{100d^3}{9} - 15d^2e + 37d^2e^2 - \frac{37d^3}{9}\right) + x^8\left(-\frac{45d^3}{8} + \frac{333d^2e}{8} - \frac{111d^2e^2}{8} + \frac{37d^3}{2}\right) + x^7\left(\frac{111d^3}{7} - \frac{111d^2e}{7} + \frac{444d^2e^2}{7} + \frac{65d^3}{7}\right) + x^6\left(\frac{37d^3}{6} + 74d^2e + \frac{65d^2e^2}{2} + \frac{107d^3}{6}\right) + x^5\left(\frac{148d^3}{5} + 39d^2e + \frac{321d^2e^2}{5} + \frac{33d^3}{5}\right) + x^4\left(\frac{65d^3}{4} + \frac{321d^2e}{4} + \frac{99d^2e^2}{4} + \frac{9d^3}{2}\right) + x^3\left(\frac{107d^3}{3} + 33d^2e + 18d^2e^2\right) + x^2\left(\frac{33d^3}{2} + 27d^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2), x)

[Out]  $18d^3x + 25e^3x^{12}/3 + x^{11}(300d^2e/11 - 45e^3/11) + x^{10}(30d^2e^2 - 27d^2e^2/2 + 111e^3/10) + x^9(100d^3/9 - 15d^2e + 37d^2e^2 - 37e^3/9) + x^8(-45d^3/8 + 333d^2e/8 - 111d^2e^2/8 + 37e^3/2) + x^7(111d^3/7 - 111d^2e/7 + 444d^2e^2/7 + 65e^3/7) + x^6(-37d^3/6 + 74d^2e + 65d^2e^2/2 + 107e^3/6) + x^5(148d^3/5 + 39d^2e + 321d^2e^2/5 + 33e^3/5) + x^4(65d^3/4 + 321d^2e/4 + 99d^2e^2/4 + 9e^3/2) + x^3(107d^3/3 + 33d^2e + 18d^2e^2) + x^2(33d^3/2 + 27d^2e)$

$$3.277 \quad \int (d + ex)^2 (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=201

$$\frac{1}{9}x^9(100d^2 - 90de + 111e^2) - \frac{1}{8}x^8(45d^2 - 222de + 37e^2) + \frac{37}{7}x^7(3d^2 - 2de + 4e^2) - \frac{1}{6}x^6(37d^2 - 296de - 65e^2) + \frac{1}{5}$$

**Rubi [A]** time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

$$\frac{1}{9}x^9(100d^2 - 90de + 111e^2) - \frac{1}{8}x^8(45d^2 - 222de + 37e^2) + \frac{37}{7}x^7(3d^2 - 2de + 4e^2) - \frac{1}{6}x^6(37d^2 - 296de - 65e^2) + \frac{1}{5}x^5(148d^2 + 130de + 107e^2) + \frac{1}{4}x^4(65d^2 + 214de + 33e^2) + \frac{1}{3}x^3(107d^2 + 66de + 18e^2) + 18d^2x + \frac{1}{2}ex^{10}(40d - 9e) + \frac{3}{2}dx^2(11d + 12e) + \frac{100e^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 18\*d^2\*x + (3\*d\*(11\*d + 12\*e)\*x^2)/2 + ((107\*d^2 + 66\*d\*e + 18\*e^2)\*x^3)/3 + ((65\*d^2 + 214\*d\*e + 33\*e^2)\*x^4)/4 + ((148\*d^2 + 130\*d\*e + 107\*e^2)\*x^5)/5 - ((37\*d^2 - 296\*d\*e - 65\*e^2)\*x^6)/6 + (37\*(3\*d^2 - 2\*d\*e + 4\*e^2)\*x^7)/7 - ((45\*d^2 - 222\*d\*e + 37\*e^2)\*x^8)/8 + ((100\*d^2 - 90\*d\*e + 111\*e^2)\*x^9)/9 + ((40\*d - 9\*e)\*e\*x^10)/2 + (100\*e^2\*x^11)/11

Rule 1628

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (d + ex)^2 (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx &= \int (18d^2 + 3d(11d + 12e)x + (107d^2 + 66de + 18e^2)) \\ &= 18d^2x + \frac{3}{2}d(11d + 12e)x^2 + \frac{1}{3}(107d^2 + 66de + 18e^2) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 201, normalized size = 1.00

$$\frac{1}{9}x^9(100d^2 - 90de + 111e^2) + \frac{1}{8}x^8(-45d^2 + 222de - 37e^2) + \frac{37}{7}x^7(3d^2 - 2de + 4e^2) + \frac{1}{6}x^6(-37d^2 + 296de + 65e^2) + \frac{1}{5}x^5(148d^2 + 130de + 107e^2) + \frac{1}{4}x^4(65d^2 + 214de + 33e^2) + \frac{1}{3}x^3(107d^2 + 66de + 18e^2) + 18d^2x + \frac{1}{2}ex^{10}(40d - 9e) + \frac{3}{2}dx^2(11d + 12e) + \frac{100e^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out]  $18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3 + ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)^2 (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas** [A] time = 0.46, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}d^2 - \frac{9}{2}x^{10}e^2 + 20x^{10}ed + \frac{37}{3}x^9e^2 - 10x^9ed + \frac{100}{9}x^9d^2 - \frac{37}{8}x^8e^2 + \frac{111}{4}x^8ed - \frac{45}{8}x^8d^2 + \frac{148}{7}x^7e^2 - \frac{74}{7}x^7ed + \frac{111}{7}x^7d^2 + \frac{65}{6}x^6e^2 + \frac{148}{3}x^6ed - \frac{37}{6}x^6d^2 + \frac{107}{5}x^5e^2 + 26x^5ed + \frac{148}{5}x^5d^2 + \frac{33}{4}x^4e^2 + \frac{107}{2}x^4ed + \frac{65}{4}x^4d^2 + 6x^3e^2 + 22x^3ed + \frac{107}{3}x^3d^2 + 18x^2e^2 + \frac{33}{2}x^2ed + 18xd^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out]  $100/11*x^{11}*e^2 - 9/2*x^{10}*e^2 + 20*x^{10}*e*d + 37/3*x^9*e^2 - 10*x^9*e*d + 100/9*x^9*d^2 - 37/8*x^8*e^2 + 111/4*x^8*e*d - 45/8*x^8*d^2 + 148/7*x^7*e^2 - 74/7*x^7*e*d + 111/7*x^7*d^2 + 65/6*x^6*e^2 + 148/3*x^6*e*d - 37/6*x^6*d^2 + 107/5*x^5*e^2 + 26*x^5*e*d + 148/5*x^5*d^2 + 33/4*x^4*e^2 + 107/2*x^4*e*d + 65/4*x^4*d^2 + 6*x^3*e^2 + 22*x^3*e*d + 107/3*x^3*d^2 + 18*x^2*e^2 + 33/2*x^2*d^2 + 18*x*d^2$

**giac** [A] time = 0.16, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}d^2 + 20dx^{10}e + \frac{100}{9}d^2x^9 - \frac{9}{2}x^{10}e^2 - 10dx^9e - \frac{45}{8}d^2x^8 + \frac{37}{3}x^9e^2 + \frac{111}{4}dx^8e + \frac{111}{7}d^2x^7 - \frac{37}{8}x^8e^2 - \frac{74}{7}dx^7e - \frac{37}{6}d^2x^6 + \frac{148}{3}x^6e^2 + \frac{148}{5}dx^6e + \frac{65}{6}x^6e^2 + 26dx^5e + \frac{65}{4}d^2x^5 + \frac{107}{5}x^5e^2 + \frac{107}{2}dx^5e + \frac{107}{3}d^2x^4 + \frac{33}{4}x^4e^2 + 22dx^3e + \frac{33}{2}d^2x^3 + 6x^3e^2 + 18dx^2e + 18d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="giac")

[Out]  $100/11*x^{11}*e^2 + 20*d*x^{10}*e + 100/9*d^2*x^9 - 9/2*x^{10}*e^2 - 10*d*x^9*e - 45/8*d^2*x^8 + 37/3*x^9*e^2 + 111/4*d*x^8*e + 111/7*d^2*x^7 - 37/8*x^8*e^2$

$$- 74/7*d*x^7*e - 37/6*d^2*x^6 + 148/7*x^7*e^2 + 148/3*d*x^6*e + 148/5*d^2*x^5 + 65/6*x^6*e^2 + 26*d*x^5*e + 65/4*d^2*x^4 + 107/5*x^5*e^2 + 107/2*d*x^4*e + 107/3*d^2*x^3 + 33/4*x^4*e^2 + 22*d*x^3*e + 33/2*d^2*x^2 + 6*x^3*e^2 + 18*d*x^2*e + 18*d^2*x$$

**maple [A]** time = 0.00, size = 186, normalized size = 0.93

$$\frac{100e^{2x^{11}}}{11} + \frac{(200de - 45e^2)x^{10}}{10} + \frac{(100d^2 - 90de + 111e^2)x^9}{9} + \frac{(-45d^2 + 222de - 37e^2)x^8}{8} + \frac{(111d^2 - 74de + 148e^2)x^7}{7} + \frac{(-37d^2 + 296de + 65e^2)x^6}{6} + \frac{(148d^2 + 130de + 107e^2)x^5}{5} + \frac{(65d^2 + 214de + 33e^2)x^4}{4} + 18d^2x + \frac{(107d^2 + 66de + 18e^2)x^3}{3} + \frac{(33d^2 + 36de)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x)

[Out] 100/11\*e^2\*x^11+1/10\*(200\*d\*e-45\*e^2)\*x^10+1/9\*(100\*d^2-90\*d\*e+111\*e^2)\*x^9+1/8\*(-45\*d^2+222\*d\*e-37\*e^2)\*x^8+1/7\*(111\*d^2-74\*d\*e+148\*e^2)\*x^7+1/6\*(-37\*d^2+296\*d\*e+65\*e^2)\*x^6+1/5\*(148\*d^2+130\*d\*e+107\*e^2)\*x^5+1/4\*(65\*d^2+214\*d\*e+33\*e^2)\*x^4+1/3\*(107\*d^2+66\*d\*e+18\*e^2)\*x^3+1/2\*(33\*d^2+36\*d\*e)\*x^2+18\*d^2\*x

**maxima [A]** time = 0.43, size = 185, normalized size = 0.92

$$\frac{100e^{2x^{11}}}{11} + \frac{1}{2}(40de - 9e^2)x^{10} + \frac{1}{9}(100d^2 - 90de + 111e^2)x^9 - \frac{1}{8}(45d^2 - 222de + 37e^2)x^8 + \frac{37}{7}(3d^2 - 2de + 4e^2)x^7 - \frac{1}{6}(37d^2 - 296de - 65e^2)x^6 + \frac{1}{5}(148d^2 + 130de + 107e^2)x^5 + \frac{1}{4}(65d^2 + 214de + 33e^2)x^4 + \frac{1}{3}(107d^2 + 66de + 18e^2)x^3 + 18d^2x + \frac{3}{2}(11d^2 + 12de)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="maxima")

[Out] 100/11\*e^2\*x^11 + 1/2\*(40\*d\*e - 9\*e^2)\*x^10 + 1/9\*(100\*d^2 - 90\*d\*e + 111\*e^2)\*x^9 - 1/8\*(45\*d^2 - 222\*d\*e + 37\*e^2)\*x^8 + 37/7\*(3\*d^2 - 2\*d\*e + 4\*e^2)\*x^7 - 1/6\*(37\*d^2 - 296\*d\*e - 65\*e^2)\*x^6 + 1/5\*(148\*d^2 + 130\*d\*e + 107\*e^2)\*x^5 + 1/4\*(65\*d^2 + 214\*d\*e + 33\*e^2)\*x^4 + 1/3\*(107\*d^2 + 66\*d\*e + 18\*e^2)\*x^3 + 18\*d^2\*x + 3/2\*(11\*d^2 + 12\*d\*e)\*x^2

**mupad [B]** time = 0.11, size = 175, normalized size = 0.87

$$x^3 \left( \frac{107d^2}{3} + 22de + 6e^2 \right) + x^9 \left( \frac{100d^2}{9} - 10de + \frac{37e^2}{3} \right) + x^8 \left( \frac{65d^2}{4} + \frac{107de}{2} + \frac{33e^2}{4} \right) - x^8 \left( \frac{45d^2}{8} - \frac{111de}{4} + \frac{37e^2}{8} \right) + x^6 \left( -\frac{37d^2}{6} + \frac{148de}{3} + \frac{65e^2}{6} \right) + x^5 \left( \frac{148d^2}{5} + 26de + \frac{107e^2}{5} \right) + x^7 \left( \frac{111d^2}{7} - \frac{74de}{7} + \frac{148e^2}{7} \right) + 18d^2x + \frac{100e^{2x^{11}}}{11} + \frac{3dx^2(11d+12e)}{2} + \frac{ex^{10}(40d-9e)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x)^2\*(2\*x + 5\*x^2 + 3)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2), x)

[Out] x^3\*(22\*d\*e + (107\*d^2)/3 + 6\*e^2) + x^9\*((100\*d^2)/9 - 10\*d\*e + (37\*e^2)/3) + x^4\*((107\*d\*e)/2 + (65\*d^2)/4 + (33\*e^2)/4) - x^8\*((45\*d^2)/8 - (111\*d\*e)/4 + (37\*e^2)/8) + x^6\*((148\*d\*e)/3 - (37\*d^2)/6 + (65\*e^2)/6) + x^5\*(26\*d\*e + (148\*d^2)/5 + (107\*e^2)/5) + x^7\*((111\*d^2)/7 - (74\*d\*e)/7 + (148\*e^2)/7) + 18\*d^2\*x + (100\*e^2\*x^11)/11 + (3\*d\*x^2\*(11\*d + 12\*e))/2 + (e\*x^10\*(40\*d - 9\*e))/2



sympy [A] time = 0.15, size = 206, normalized size = 1.02

$$18d^2x + \frac{100e^2x^{11}}{11} + x^{10}\left(20de - \frac{9e^2}{2}\right) + x^9\left(\frac{100d^2}{9} - 10de + \frac{37e^2}{3}\right) + x^8\left(-\frac{45d^2}{8} + \frac{111de}{4} - \frac{37e^2}{8}\right) + x^7\left(\frac{111d^2}{7} - \frac{74de}{7} + \frac{148e^2}{7}\right) + x^6\left(-\frac{37d^2}{6} + \frac{148de}{3} + \frac{65e^2}{6}\right) + x^5\left(\frac{148d^2}{5} + 26de + \frac{107e^2}{5}\right) + x^4\left(\frac{65d^2}{4} + \frac{107de}{2} + \frac{33e^2}{4}\right) + x^3\left(\frac{107d^2}{3} + 22de + 6e^2\right) + x^2\left(\frac{33d^2}{2} + 18de\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2), x)

[Out] 18\*d\*\*2\*x + 100\*e\*\*2\*x\*\*11/11 + x\*\*10\*(20\*d\*e - 9\*e\*\*2/2) + x\*\*9\*(100\*d\*\*2/9 - 10\*d\*e + 37\*e\*\*2/3) + x\*\*8\*(-45\*d\*\*2/8 + 111\*d\*e/4 - 37\*e\*\*2/8) + x\*\*7\*(111\*d\*\*2/7 - 74\*d\*e/7 + 148\*e\*\*2/7) + x\*\*6\*(-37\*d\*\*2/6 + 148\*d\*e/3 + 65\*e\*\*2/6) + x\*\*5\*(148\*d\*\*2/5 + 26\*d\*e + 107\*e\*\*2/5) + x\*\*4\*(65\*d\*\*2/4 + 107\*d\*e/2 + 33\*e\*\*2/4) + x\*\*3\*(107\*d\*\*2/3 + 22\*d\*e + 6\*e\*\*2) + x\*\*2\*(33\*d\*\*2/2 + 18\*d\*e)

$$3.278 \quad \int (d + ex) (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=121

$$\frac{5}{9}x^9(20d-9e) - \frac{3}{8}x^8(15d-37e) + \frac{37}{7}x^7(3d-e) - \frac{37}{6}x^6(d-4e) + \frac{1}{5}x^5(148d+65e) + \frac{1}{4}x^4(65d+107e) + \frac{1}{3}x^3(107d+33e) + \frac{3}{2}x^2(11d+6e) + 18dx + 10ex^{10}$$

**Rubi [A]** time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{5}{9}x^9(20d-9e) - \frac{3}{8}x^8(15d-37e) + \frac{37}{7}x^7(3d-e) - \frac{37}{6}x^6(d-4e) + \frac{1}{5}x^5(148d+65e) + \frac{1}{4}x^4(65d+107e) + \frac{1}{3}x^3(107d+33e) + \frac{3}{2}x^2(11d+6e) + 18dx + 10ex^{10}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 18\*d\*x + (3\*(11\*d + 6\*e)\*x^2)/2 + ((107\*d + 33\*e)\*x^3)/3 + ((65\*d + 107\*e)\*x^4)/4 + ((148\*d + 65\*e)\*x^5)/5 - (37\*(d - 4\*e)\*x^6)/6 + (37\*(3\*d - e)\*x^7)/7 - (3\*(15\*d - 37\*e)\*x^8)/8 + (5\*(20\*d - 9\*e)\*x^9)/9 + 10\*e\*x^10

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int (d + ex) (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int (18d + 3(11d + 6e)x + (107d + 33e)x^2 + (65d + 107e)x^3 + (148d + 65e)x^4 - 37(d - 4e)x^5 + 37(3d - e)x^6 - 3(15d - 37e)x^7 + 5(20d - 9e)x^8 + 10ex^9) dx$$

$$= 18dx + \frac{3}{2}(11d + 6e)x^2 + \frac{1}{3}(107d + 33e)x^3 + \frac{1}{4}(65d + 107e)x^4 - \frac{37}{5}(d - 4e)x^5 + \frac{37}{6}(3d - e)x^6 - \frac{3}{8}(15d - 37e)x^7 + \frac{5}{9}(20d - 9e)x^8 + 10ex^9$$

**Mathematica [A]** time = 0.02, size = 121, normalized size = 1.00

$$\frac{5}{9}x^9(20d-9e) - \frac{3}{8}x^8(15d-37e) + \frac{37}{7}x^7(3d-e) - \frac{37}{6}x^6(d-4e) + \frac{1}{5}x^5(148d+65e) + \frac{1}{4}x^4(65d+107e) + \frac{1}{3}x^3(107d+33e) + \frac{3}{2}x^2(11d+6e) + 18dx + 10ex^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out]  $18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^{10}$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d + ex)(3 + 2x + 5x^2)^2(2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(d + e\*x)\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas** [A] time = 0.81, size = 107, normalized size = 0.88

$$10x^{10}e - 5x^9e + \frac{100}{9}x^9d + \frac{111}{8}x^8e - \frac{45}{8}x^8d - \frac{37}{7}x^7e + \frac{111}{7}x^7d + \frac{74}{3}x^6e - \frac{37}{6}x^6d + 13x^5e + \frac{148}{5}x^5d + \frac{107}{4}x^4e + \frac{65}{4}x^4d + 11x^3e + \frac{107}{3}x^3d + 9x^2e + \frac{33}{2}x^2d + 18xd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out]  $10*x^{10}*e - 5*x^9*e + 100/9*x^9*d + 111/8*x^8*e - 45/8*x^8*d - 37/7*x^7*e + 111/7*x^7*d + 74/3*x^6*e - 37/6*x^6*d + 13*x^5*e + 148/5*x^5*d + 107/4*x^4*e + 65/4*x^4*d + 11*x^3*e + 107/3*x^3*d + 9*x^2*e + 33/2*x^2*d + 18*x*d$

**giac** [A] time = 0.16, size = 116, normalized size = 0.96

$$10x^{10}e + \frac{100}{9}dx^9 - 5x^9e - \frac{45}{8}dx^8 + \frac{111}{8}x^8e + \frac{111}{7}dx^7 - \frac{37}{7}x^7e - \frac{37}{6}dx^6 + \frac{74}{3}x^6e + \frac{148}{5}dx^5 + 13x^5e + \frac{65}{4}dx^4 + \frac{107}{4}x^4e + \frac{107}{3}dx^3 + 11x^3e + \frac{33}{2}dx^2 + 9x^2e + 18dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="giac")

[Out]  $10*x^{10}*e + 100/9*d*x^9 - 5*x^9*e - 45/8*d*x^8 + 111/8*x^8*e + 111/7*d*x^7 - 37/7*x^7*e - 37/6*d*x^6 + 74/3*x^6*e + 148/5*d*x^5 + 13*x^5*e + 65/4*d*x^4 + 107/4*x^4*e + 107/3*d*x^3 + 11*x^3*e + 33/2*d*x^2 + 9*x^2*e + 18*d*x$

**maple** [A] time = 0.00, size = 108, normalized size = 0.89

$$10ex^{10} + \frac{(100d - 45e)x^9}{9} + \frac{(-45d + 111e)x^8}{8} + \frac{(111d - 37e)x^7}{7} + \frac{(-37d + 148e)x^6}{6} + \frac{(148d + 65e)x^5}{5} + \frac{(65d + 107e)x^4}{4} + \frac{(107d + 33e)x^3}{3} + 18dx + \frac{(33d + 18e)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)`

[Out]  $10*e*x^{10} + 1/9*(100*d - 45*e)*x^9 + 1/8*(-45*d + 111*e)*x^8 + 1/7*(111*d - 37*e)*x^7 + 1/6*(-37*d + 148*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 + 1/3*(107*d + 33*e)*x^3 + 1/2*(33*d + 18*e)*x^2 + 18*d*x$

**maxima** [A] time = 0.43, size = 105, normalized size = 0.87

$$10ex^{10} + \frac{5}{9}(20d - 9e)x^9 - \frac{3}{8}(15d - 37e)x^8 + \frac{37}{7}(3d - e)x^7 - \frac{37}{6}(d - 4e)x^6 + \frac{1}{5}(148d + 65e)x^5 + \frac{1}{4}(65d + 107e)x^4 + \frac{1}{3}(107d + 33e)x^3 + \frac{3}{2}(11d + 6e)x^2 + 18dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")`

[Out]  $10*e*x^{10} + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 + 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x$

**mupad** [B] time = 4.17, size = 101, normalized size = 0.83

$$10ex^{10} + \left(\frac{100d}{9} - 5e\right)x^9 + \left(\frac{111e}{8} - \frac{45d}{8}\right)x^8 + \left(\frac{111d}{7} - \frac{37e}{7}\right)x^7 + \left(\frac{74e}{3} - \frac{37d}{6}\right)x^6 + \left(\frac{148d}{5} + 13e\right)x^5 + \left(\frac{65d}{4} + \frac{107e}{4}\right)x^4 + \left(\frac{107d}{3} + 11e\right)x^3 + \left(\frac{33d}{2} + 9e\right)x^2 + 18dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)`

[Out]  $x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^6*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13*e) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 10*e*x^{10}$

**sympy** [A] time = 0.14, size = 112, normalized size = 0.93

$$18dx + 10ex^{10} + x^9\left(\frac{100d}{9} - 5e\right) + x^8\left(-\frac{45d}{8} + \frac{111e}{8}\right) + x^7\left(\frac{111d}{7} - \frac{37e}{7}\right) + x^6\left(-\frac{37d}{6} + \frac{74e}{3}\right) + x^5\left(\frac{148d}{5} + 13e\right) + x^4\left(\frac{65d}{4} + \frac{107e}{4}\right) + x^3\left(\frac{107d}{3} + 11e\right) + x^2\left(\frac{33d}{2} + 9e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)`

[Out]  $18*d*x + 10*e*x^{10} + x^{9}*(100*d/9 - 5*e) + x^{8}*(-45*d/8 + 111*e/8) + x^{7}*(111*d/7 - 37*e/7) + x^{6}*(-37*d/6 + 74*e/3) + x^{5}*(148*d/5 + 13*e) + x^{4}*(65*d/4 + 107*e/4) + x^{3}*(107*d/3 + 11*e) + x^{2}*(33*d/2 + 9*e)$

$$3.279 \quad \int (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Optimal. Leaf size=60

$$\frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$ , Rules used = {1657}

$$\frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Int[(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] 18\*x + (33\*x^2)/2 + (107\*x^3)/3 + (65\*x^4)/4 + (148\*x^5)/5 - (37\*x^6)/6 + (111\*x^7)/7 - (45\*x^8)/8 + (100\*x^9)/9

Rule 1657

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[Expand Integrand[Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx &= \int (18 + 33x + 107x^2 + 65x^3 + 148x^4 - 37x^5 + 111x^6 - 45x^7 + 100x^8) dx \\ &= 18x + \frac{33x^2}{2} + \frac{107x^3}{3} + \frac{65x^4}{4} + \frac{148x^5}{5} - \frac{37x^6}{6} + \frac{111x^7}{7} - \frac{45x^8}{8} + \frac{100x^9}{9} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 60, normalized size = 1.00

$$\frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out]  $18x + (33x^2)/2 + (107x^3)/3 + (65x^4)/4 + (148x^5)/5 - (37x^6)/6 + (111x^7)/7 - (45x^8)/8 + (100x^9)/9$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] IntegrateAlgebraic[(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas** [A] time = 0.64, size = 44, normalized size = 0.73

$$\frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out]  $100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*x^3 + 33/2*x^2 + 18*x$

**giac** [A] time = 0.15, size = 44, normalized size = 0.73

$$\frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x, algorithm="giac")

[Out]  $100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*x^3 + 33/2*x^2 + 18*x$

**maple** [A] time = 0.00, size = 45, normalized size = 0.75

$$\frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2), x)

[Out]  $18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/9*x^9$

**maxima [A]** time = 0.42, size = 44, normalized size = 0.73

$$\frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="maxima")

[Out] 100/9\*x^9 - 45/8\*x^8 + 111/7\*x^7 - 37/6\*x^6 + 148/5\*x^5 + 65/4\*x^4 + 107/3\*x^3 + 33/2\*x^2 + 18\*x

**mupad [B]** time = 0.03, size = 44, normalized size = 0.73

$$\frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 5\*x^2 + 3)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2),x)

[Out] 18\*x + (33\*x^2)/2 + (107\*x^3)/3 + (65\*x^4)/4 + (148\*x^5)/5 - (37\*x^6)/6 + (111\*x^7)/7 - (45\*x^8)/8 + (100\*x^9)/9

**sympy [A]** time = 0.15, size = 56, normalized size = 0.93

$$\frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2),x)

[Out] 100\*x\*\*9/9 - 45\*x\*\*8/8 + 111\*x\*\*7/7 - 37\*x\*\*6/6 + 148\*x\*\*5/5 + 65\*x\*\*4/4 + 107\*x\*\*3/3 + 33\*x\*\*2/2 + 18\*x

$$3.280 \quad \int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex} dx$$

**Optimal.** Leaf size=352

$$\frac{x^6(100d^2 + 45de + 111e^2)}{6e^3} - \frac{x^5(100d^3 + 45d^2e + 111de^2 + 37e^3)}{5e^4} + \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 10e^4)}{e^9}$$

**Rubi [A]** time = 0.32, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

$$\frac{x^6(100d^2 + 45de + 111e^2)}{6e^3} - \frac{x^5(100d^3 + 45d^2e + 111de^2 + 37e^3)}{5e^4} + \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 10e^4)}{e^9}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x), x]

[Out] -((((100\*d^7 + 45\*d^6\*e + 111\*d^5\*e^2 + 37\*d^4\*e^3 + 148\*d^3\*e^4 - 65\*d^2\*e^5 + 107\*d\*e^6 - 33\*e^7)\*x)/e^8) + (((100\*d^6 + 45\*d^5\*e + 111\*d^4\*e^2 + 37\*d^3\*e^3 + 148\*d^2\*e^4 - 65\*d\*e^5 + 107\*e^6)\*x^2)/(2\*e^7) - ((100\*d^5 + 45\*d^4\*e + 111\*d^3\*e^2 + 37\*d^2\*e^3 + 148\*d\*e^4 - 65\*e^5)\*x^3)/(3\*e^6) + ((100\*d^4 + 45\*d^3\*e + 111\*d^2\*e^2 + 37\*d\*e^3 + 148\*e^4)\*x^4)/(4\*e^5) - ((100\*d^3 + 45\*d^2\*e + 111\*d\*e^2 + 37\*e^3)\*x^5)/(5\*e^4) + ((100\*d^2 + 45\*d\*e + 111\*e^2)\*x^6)/(6\*e^3) - (5\*(20\*d + 9\*e)\*x^7)/(7\*e^2) + (25\*x^8)/(2\*e) + ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x])/e^9

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3 + 2x + 5x^2)^2(2 + x + 3x^2 - 5x^3 + 4x^4)}{d + ex} dx = \int \left( \frac{-100d^7 - 45d^6e - 111d^5e^2 - 37d^4e^3 - 148d^3e^4 + 65d^2e^5 - 107de^6 + 33e^7}{e^8} \right. \\ \left. - \frac{(100d^7 + 45d^6e + 111d^5e^2 + 37d^4e^3 + 148d^3e^4 - 65d^2e^5 + 107de^6 - 33e^7)x}{2e^7} + \frac{(100d^5 + 45d^4e + 111d^3e^2 + 37d^2e^3 + 148de^4 - 65e^5)x^2}{3e^6} - \frac{(100d^4 + 45d^3e + 111d^2e^2 + 37de^3 + 148e^4)x^3}{4e^5} + \frac{(100d^3 + 45d^2e + 111de^2 + 37e^3)x^4}{5e^4} - \frac{(100d^2 + 45de + 111e^2)x^5}{6e^3} + \frac{5(20d + 9e)x^6}{7e^2} - \frac{25x^7}{2e} + (5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex) \right) dx$$





$$d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8) * \log(e*x + d) / e^9$$

**giac** [A] time = 0.17, size = 378, normalized size = 1.07

100\*d^8 + 45\*d^7\*e + 111\*d^6\*e^2 + 37\*d^5\*e^3 + 148\*d^4\*e^4 - 65\*d^3\*e^5 + 107\*d^2\*e^6 - 33\*d\*e^7 + 18\*e^8) \* log(e\*x + d) / e^9 + 1/420\*(5250\*x^8\*e^7 - 6000\*d\*x^7\*e^6 + 7000\*d^2\*x^6\*e^5 - 8400\*d^3\*x^5\*e^4 + 10500\*d^4\*x^4\*e^3 - 14000\*d^5\*x^3\*e^2 + 21000\*d^6\*x^2\*e - 42000\*d^7\*x - 2700\*x^7\*e^7 + 3150\*d\*x^6\*e^6 - 3780\*d^2\*x^5\*e^5 + 4725\*d^3\*x^4\*e^4 - 6300\*d^4\*x^3\*e^3 + 9450\*d^5\*x^2\*e^2 - 18900\*d^6\*x\*e + 7770\*x^6\*e^7 - 9324\*d\*x^5\*e^6 + 11655\*d^2\*x^4\*e^5 - 15540\*d^3\*x^3\*e^4 + 23310\*d^4\*x^2\*e^3 - 46620\*d^5\*x\*e^2 - 3108\*x^5\*e^7 + 3885\*d\*x^4\*e^6 - 5180\*d^2\*x^3\*e^5 + 7770\*d^3\*x^2\*e^4 - 15540\*d^4\*x\*e^3 + 15540\*x^4\*e^7 - 20720\*d\*x^3\*e^6 + 31080\*d^2\*x^2\*e^5 - 62160\*d^3\*x\*e^4 + 9100\*x^3\*e^7 - 13650\*d\*x^2\*e^6 + 27300\*d^2\*x\*e^5 + 22470\*x^2\*e^7 - 44940\*d\*x\*e^6 + 13860\*x\*e^7) \* e^(-8)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d),x, algorithm="giac")

[Out] (100\*d^8 + 45\*d^7\*e + 111\*d^6\*e^2 + 37\*d^5\*e^3 + 148\*d^4\*e^4 - 65\*d^3\*e^5 + 107\*d^2\*e^6 - 33\*d\*e^7 + 18\*e^8)\*e^(-9)\*log(abs(x\*e + d)) + 1/420\*(5250\*x^8\*e^7 - 6000\*d\*x^7\*e^6 + 7000\*d^2\*x^6\*e^5 - 8400\*d^3\*x^5\*e^4 + 10500\*d^4\*x^4\*e^3 - 14000\*d^5\*x^3\*e^2 + 21000\*d^6\*x^2\*e - 42000\*d^7\*x - 2700\*x^7\*e^7 + 3150\*d\*x^6\*e^6 - 3780\*d^2\*x^5\*e^5 + 4725\*d^3\*x^4\*e^4 - 6300\*d^4\*x^3\*e^3 + 9450\*d^5\*x^2\*e^2 - 18900\*d^6\*x\*e + 7770\*x^6\*e^7 - 9324\*d\*x^5\*e^6 + 11655\*d^2\*x^4\*e^5 - 15540\*d^3\*x^3\*e^4 + 23310\*d^4\*x^2\*e^3 - 46620\*d^5\*x\*e^2 - 3108\*x^5\*e^7 + 3885\*d\*x^4\*e^6 - 5180\*d^2\*x^3\*e^5 + 7770\*d^3\*x^2\*e^4 - 15540\*d^4\*x\*e^3 + 15540\*x^4\*e^7 - 20720\*d\*x^3\*e^6 + 31080\*d^2\*x^2\*e^5 - 62160\*d^3\*x\*e^4 + 9100\*x^3\*e^7 - 13650\*d\*x^2\*e^6 + 27300\*d^2\*x\*e^5 + 22470\*x^2\*e^7 - 44940\*d\*x\*e^6 + 13860\*x\*e^7)\*e^(-8)

**maple** [A] time = 0.01, size = 465, normalized size = 1.32

100\*d^8 + 45\*d^7\*e + 111\*d^6\*e^2 + 37\*d^5\*e^3 + 148\*d^4\*e^4 - 65\*d^3\*e^5 + 107\*d^2\*e^6 - 33\*d\*e^7 + 18\*e^8) \* log(e\*x + d) / e^9 + 1/420\*(5250\*x^8\*e^7 - 6000\*d\*x^7\*e^6 + 7000\*d^2\*x^6\*e^5 - 8400\*d^3\*x^5\*e^4 + 10500\*d^4\*x^4\*e^3 - 14000\*d^5\*x^3\*e^2 + 21000\*d^6\*x^2\*e - 42000\*d^7\*x - 2700\*x^7\*e^7 + 3150\*d\*x^6\*e^6 - 3780\*d^2\*x^5\*e^5 + 4725\*d^3\*x^4\*e^4 - 6300\*d^4\*x^3\*e^3 + 9450\*d^5\*x^2\*e^2 - 18900\*d^6\*x\*e + 7770\*x^6\*e^7 - 9324\*d\*x^5\*e^6 + 11655\*d^2\*x^4\*e^5 - 15540\*d^3\*x^3\*e^4 + 23310\*d^4\*x^2\*e^3 - 46620\*d^5\*x\*e^2 - 3108\*x^5\*e^7 + 3885\*d\*x^4\*e^6 - 5180\*d^2\*x^3\*e^5 + 7770\*d^3\*x^2\*e^4 - 15540\*d^4\*x\*e^3 + 15540\*x^4\*e^7 - 20720\*d\*x^3\*e^6 + 31080\*d^2\*x^2\*e^5 - 62160\*d^3\*x\*e^4 + 9100\*x^3\*e^7 - 13650\*d\*x^2\*e^6 + 27300\*d^2\*x\*e^5 + 22470\*x^2\*e^7 - 44940\*d\*x\*e^6 + 13860\*x\*e^7) \* e^(-8)

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d),x)

[Out] 107/2/e\*x^2+25/2\*x^8/e+37/2/e\*x^6-45/7/e\*x^7+18/e\*ln(e\*x+d)+37/e\*x^4+65/3/e\*x^3+33/e\*x-37/5/e\*x^5-15/e^5\*x^3\*d^4+25/e^5\*x^4\*d^4-20/e^4\*x^5\*d^3+45/4/e^4\*x^4\*d^3+50/3/e^3\*x^6\*d^2-9/e^3\*x^5\*d^2-100/7/e^2\*x^7\*d+15/2/e^2\*x^6\*d-100/e^8\*x\*d^7-45/e^7\*x\*d^6+45/e^8\*ln(e\*x+d)\*d^7-100/3/e^6\*x^3\*d^5+45/2/e^6\*x^2\*d^5+50/e^7\*x^2\*d^6+100/e^9\*ln(e\*x+d)\*d^8+37/4\*d/e^2\*x^4-37\*d^3/e^4\*x^3-37/3\*d^2/e^3\*x^3-148/3\*d/e^2\*x^3+111/2\*d^4/e^5\*x^2+37/2\*d^3/e^4\*x^2+74\*d^2/e^3\*x^2-111/5\*d/e^2\*x^5+111/4\*d^2/e^3\*x^4-65\*d^3/e^4\*ln(e\*x+d)-148\*d^3/e^4\*x+65\*d^2/e^3\*x-107\*d/e^2\*x-65/2\*d/e^2\*x^2-111\*d^5/e^6\*x-37\*d^4/e^5\*x+107\*d^2/e^3\*ln(e\*x+d)-33\*d/e^2\*ln(e\*x+d)+111\*d^6/e^7\*ln(e\*x+d)+37\*d^5/e^6\*ln(e\*x+d)+148\*d^4/e^5\*ln(e\*x+d)

**maxima** [A] time = 0.44, size = 366, normalized size = 1.04

100\*d^8 + 45\*d^7\*e + 111\*d^6\*e^2 + 37\*d^5\*e^3 + 148\*d^4\*e^4 - 65\*d^3\*e^5 + 107\*d^2\*e^6 - 33\*d\*e^7 + 18\*e^8) \* log(e\*x + d) / e^9 + 1/420\*(5250\*x^8\*e^7 - 6000\*d\*x^7\*e^6 + 7000\*d^2\*x^6\*e^5 - 8400\*d^3\*x^5\*e^4 + 10500\*d^4\*x^4\*e^3 - 14000\*d^5\*x^3\*e^2 + 21000\*d^6\*x^2\*e - 42000\*d^7\*x - 2700\*x^7\*e^7 + 3150\*d\*x^6\*e^6 - 3780\*d^2\*x^5\*e^5 + 4725\*d^3\*x^4\*e^4 - 6300\*d^4\*x^3\*e^3 + 9450\*d^5\*x^2\*e^2 - 18900\*d^6\*x\*e + 7770\*x^6\*e^7 - 9324\*d\*x^5\*e^6 + 11655\*d^2\*x^4\*e^5 - 15540\*d^3\*x^3\*e^4 + 23310\*d^4\*x^2\*e^3 - 46620\*d^5\*x\*e^2 - 3108\*x^5\*e^7 + 3885\*d\*x^4\*e^6 - 5180\*d^2\*x^3\*e^5 + 7770\*d^3\*x^2\*e^4 - 15540\*d^4\*x\*e^3 + 15540\*x^4\*e^7 - 20720\*d\*x^3\*e^6 + 31080\*d^2\*x^2\*e^5 - 62160\*d^3\*x\*e^4 + 9100\*x^3\*e^7 - 13650\*d\*x^2\*e^6 + 27300\*d^2\*x\*e^5 + 22470\*x^2\*e^7 - 44940\*d\*x\*e^6 + 13860\*x\*e^7) \* e^(-8)

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxima")`

[Out]  $\frac{1}{420} \cdot (5250 \cdot e^7 \cdot x^8 - 300 \cdot (20 \cdot d \cdot e^6 + 9 \cdot e^7) \cdot x^7 + 70 \cdot (100 \cdot d^2 \cdot e^5 + 45 \cdot d \cdot e^6 + 111 \cdot e^7) \cdot x^6 - 84 \cdot (100 \cdot d^3 \cdot e^4 + 45 \cdot d^2 \cdot e^5 + 111 \cdot d \cdot e^6 + 37 \cdot e^7) \cdot x^5 + 105 \cdot (100 \cdot d^4 \cdot e^3 + 45 \cdot d^3 \cdot e^4 + 111 \cdot d^2 \cdot e^5 + 37 \cdot d \cdot e^6 + 148 \cdot e^7) \cdot x^4 - 140 \cdot (100 \cdot d^5 \cdot e^2 + 45 \cdot d^4 \cdot e^3 + 111 \cdot d^3 \cdot e^4 + 37 \cdot d^2 \cdot e^5 + 148 \cdot d \cdot e^6 - 65 \cdot e^7) \cdot x^3 + 210 \cdot (100 \cdot d^6 \cdot e + 45 \cdot d^5 \cdot e^2 + 111 \cdot d^4 \cdot e^3 + 37 \cdot d^3 \cdot e^4 + 148 \cdot d^2 \cdot e^5 - 65 \cdot d \cdot e^6 + 107 \cdot e^7) \cdot x^2 - 420 \cdot (100 \cdot d^7 + 45 \cdot d^6 \cdot e + 111 \cdot d^5 \cdot e^2 + 37 \cdot d^4 \cdot e^3 + 148 \cdot d^3 \cdot e^4 - 65 \cdot d^2 \cdot e^5 + 107 \cdot d \cdot e^6 - 33 \cdot e^7) \cdot x) / e^8 + (100 \cdot d^8 + 45 \cdot d^7 \cdot e + 111 \cdot d^6 \cdot e^2 + 37 \cdot d^5 \cdot e^3 + 148 \cdot d^4 \cdot e^4 - 65 \cdot d^3 \cdot e^5 + 107 \cdot d^2 \cdot e^6 - 33 \cdot d \cdot e^7 + 18 \cdot e^8) \cdot \log(e \cdot x + d) / e^9$

**mupad [B]** time = 0.08, size = 434, normalized size = 1.23

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)`

[Out]  $x \cdot (33/e - (d \cdot (107/e - (d \cdot (65/e - (d \cdot (148/e + (d \cdot (37/e + (d \cdot (111/e + (d \cdot ((100 \cdot d)/e^2 + 45/e))/e))/e))/e))/e))/e) - x^7 \cdot ((100 \cdot d)/(7 \cdot e^2) + 45/(7 \cdot e)) + x^6 \cdot (37/(2 \cdot e) + (d \cdot ((100 \cdot d)/e^2 + 45/e))/(6 \cdot e)) - x^5 \cdot (37/(5 \cdot e) + (d \cdot (111/e + (d \cdot ((100 \cdot d)/e^2 + 45/e))/e))/(5 \cdot e)) + x^4 \cdot (37/e + (d \cdot (37/e + (d \cdot (111/$

$$e + (d*((100*d)/e^2 + 45/e))/e)/e)/(4*e)) + x^3*(65/(3*e) - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/e)/(3*e)) + x^2*(107/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/e)/(2*e)) + (25*x^8)/(2*e) + (\log(d + e*x)*(45*d^7*e - 33*d^6*e^2 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^4 + 37*d^5*e^3 + 111*d^6*e^2))/e^9$$

**sympy [A]** time = 1.00, size = 372, normalized size = 1.06

$$x^3 \left( \frac{107}{2e} - \frac{d(65/e - (d(148/e + (d(37/e + (d(111/e + (d((100d)/e^2 + 45/e))/e))/e))/e))/e}{2e} \right) + \frac{25x^8}{2e} + \frac{\log(d + ex)(45d^7e - 33d^6e^2 + 100d^8 + 18e^8 + 107d^2e^6 - 65d^3e^5 + 148d^4e^4 + 37d^5e^3 + 111d^6e^2)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d), x)

[Out] x\*\*7\*(-100\*d/(7\*e\*\*2) - 45/(7\*e)) + x\*\*6\*(50\*d\*\*2/(3\*e\*\*3) + 15\*d/(2\*e\*\*2) + 37/(2\*e)) + x\*\*5\*(-20\*d\*\*3/e\*\*4 - 9\*d\*\*2/e\*\*3 - 111\*d/(5\*e\*\*2) - 37/(5\*e)) + x\*\*4\*(25\*d\*\*4/e\*\*5 + 45\*d\*\*3/(4\*e\*\*4) + 111\*d\*\*2/(4\*e\*\*3) + 37\*d/(4\*e\*\*2) + 37/e) + x\*\*3\*(-100\*d\*\*5/(3\*e\*\*6) - 15\*d\*\*4/e\*\*5 - 37\*d\*\*3/e\*\*4 - 37\*d\*\*2/(3\*e\*\*3) - 148\*d/(3\*e\*\*2) + 65/(3\*e)) + x\*\*2\*(50\*d\*\*6/e\*\*7 + 45\*d\*\*5/(2\*e\*\*6) + 111\*d\*\*4/(2\*e\*\*5) + 37\*d\*\*3/(2\*e\*\*4) + 74\*d\*\*2/e\*\*3 - 65\*d/(2\*e\*\*2) + 107/(2\*e)) + x\*(-100\*d\*\*7/e\*\*8 - 45\*d\*\*6/e\*\*7 - 111\*d\*\*5/e\*\*6 - 37\*d\*\*4/e\*\*5 - 148\*d\*\*3/e\*\*4 + 65\*d\*\*2/e\*\*3 - 107\*d/e\*\*2 + 33/e) + 25\*x\*\*8/(2\*e) + (5\*d\*\*2 - 2\*d\*e + 3\*e\*\*2)\*\*2\*(4\*d\*\*4 + 5\*d\*\*3\*e + 3\*d\*\*2\*e\*\*2 - d\*e\*\*3 + 2\*e\*\*4)\*log(d + e\*x)/e\*\*9

$$3.281 \quad \int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$$

**Optimal.** Leaf size=353

$$\frac{3x^5(100d^2 + 30de + 37e^2)}{5e^4} - \frac{x^4(400d^3 + 135d^2e + 222de^2 + 37e^3)}{4e^5} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - d^2e^3)}{e^9(d+ex)}$$

**Rubi [A]** time = 0.33, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

$$\frac{3x^5(100d^2 + 30de + 37e^2)}{5e^4} - \frac{x^4(400d^3 + 135d^2e + 222de^2 + 37e^3)}{4e^5} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - d^2e^3)}{e^9(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^2,x]

[Out] ((700\*d^6 + 270\*d^5\*e + 555\*d^4\*e^2 + 148\*d^3\*e^3 + 444\*d^2\*e^4 - 130\*d\*e^5 + 107\*e^6)\*x)/e^8 - ((600\*d^5 + 225\*d^4\*e + 444\*d^3\*e^2 + 111\*d^2\*e^3 + 296\*d\*e^4 - 65\*e^5)\*x^2)/(2\*e^7) + ((500\*d^4 + 180\*d^3\*e + 333\*d^2\*e^2 + 74\*d\*e^3 + 148\*e^4)\*x^3)/(3\*e^6) - ((400\*d^3 + 135\*d^2\*e + 222\*d\*e^2 + 37\*e^3)\*x^4)/(4\*e^5) + (3\*(100\*d^2 + 30\*d\*e + 37\*e^2)\*x^5)/(5\*e^4) - (5\*(40\*d + 9\*e)\*x^6)/(6\*e^3) + (100\*x^7)/(7\*e^2) - ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(e^9\*(d + e\*x)) - ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(160\*d^5 + 127\*d^4\*e + 88\*d^3\*e^2 - 4\*d^2\*e^3 + 64\*d\*e^4 - 11\*e^5)\*Log[d + e\*x])/e^9

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3 + 2x + 5x^2)^2(2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^2} dx = \int \left( \frac{700d^6 + 270d^5e + 555d^4e^2 + 148d^3e^3 + 444d^2e^4 - 130de^5}{e^8} \right. \\ \left. - \frac{(700d^6 + 270d^5e + 555d^4e^2 + 148d^3e^3 + 444d^2e^4 - 130de^5 - \dots)}{e^8} \right) dx$$









$$\begin{aligned}
& (2*d*((200*d)/e^3 + 45/e^2))/e)/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2)/e \\
& - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2)/e \\
& + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e))/e \\
& - (d^2*((200*d)/e^3 + 45/e^2))/e^2)/e - 107/e^2 + (d^2*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e))/e \\
& - (d^2*((200*d)/e^3 + 45/e^2))/e^2)/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2) + (100*x^7)/(7*e^2) \\
& - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^4 + 37*d^5*e^3 + 111*d^6*e^2)/(e*(d*e^8 + e^9*x)) - (\log(d + e*x))*(2 \\
& 14*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d^4*e^3 + 666*d^5*e^2))/e^9
\end{aligned}$$

**sympy [A]** time = 2.31, size = 393, normalized size = 1.11

$$\frac{d^2 \left( \frac{100d^2}{3e^2} + \frac{15}{e^2} \right) + d \left( \frac{200d}{e^3} + \frac{15d}{e^2} + \frac{111}{e^2} \right) + d^2 \left( \frac{100d^2}{3e^2} + \frac{135d^2}{4e^2} + \frac{111d}{e^2} + \frac{37}{e^2} \right) + d^3 \left( \frac{500d^2}{3e^2} + \frac{60d^2}{e^2} + \frac{111d^2}{e^2} + \frac{74d}{e^2} + \frac{148}{e^2} \right) + d^4 \left( \frac{300d^2}{3e^2} + \frac{225d^2}{2e^2} + \frac{222d^2}{2e^2} + \frac{111d^2}{e^2} + \frac{148d}{e^2} + \frac{65}{e^2} \right) + d^5 \left( \frac{700d^2}{e^2} + \frac{270d^2}{e^2} + \frac{555d^2}{e^2} + \frac{148d^2}{e^2} + \frac{444d^2}{e^2} + \frac{130d}{e^2} + \frac{107}{e^2} \right) + \frac{200d^2 - 45d^2 - 111d^2 - 37d^2 - 148d^2 + 65d^2 - 107d^2 + 33d^2 - 18d^2}{e^2} - \frac{(d^2 - 2d + 2)(100d^2 + 122d^2 + 38d^2 - 4d^2 + 64d^2 - 11d^2) \log(d + ex)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*2,x)

[Out] x\*\*6\*(-100\*d/(3\*e\*\*3) - 15/(2\*e\*\*2)) + x\*\*5\*(60\*d\*\*2/e\*\*4 + 18\*d/e\*\*3 + 111/(5\*e\*\*2)) + x\*\*4\*(-100\*d\*\*3/e\*\*5 - 135\*d\*\*2/(4\*e\*\*4) - 111\*d/(2\*e\*\*3) - 37/(4\*e\*\*2)) + x\*\*3\*(500\*d\*\*4/(3\*e\*\*6) + 60\*d\*\*3/e\*\*5 + 111\*d\*\*2/e\*\*4 + 74\*d/(3\*e\*\*3) + 148/(3\*e\*\*2)) + x\*\*2\*(-300\*d\*\*5/e\*\*7 - 225\*d\*\*4/(2\*e\*\*6) - 222\*d\*\*3/e\*\*5 - 111\*d\*\*2/(2\*e\*\*4) - 148\*d/e\*\*3 + 65/(2\*e\*\*2)) + x\*(700\*d\*\*6/e\*\*8 + 270\*d\*\*5/e\*\*7 + 555\*d\*\*4/e\*\*6 + 148\*d\*\*3/e\*\*5 + 444\*d\*\*2/e\*\*4 - 130\*d/e\*\*3 + 107/e\*\*2) + (-100\*d\*\*8 - 45\*d\*\*7\*e - 111\*d\*\*6\*e\*\*2 - 37\*d\*\*5\*e\*\*3 - 148\*d\*\*4\*e\*\*4 + 65\*d\*\*3\*e\*\*5 - 107\*d\*\*2\*e\*\*6 + 33\*d\*e\*\*7 - 18\*e\*\*8)/(d\*e\*\*9 + e\*\*10\*x) + 100\*x\*\*7/(7\*e\*\*2) - (5\*d\*\*2 - 2\*d\*e + 3\*e\*\*2)\*(160\*d\*\*5 + 127\*d\*\*4\*e + 88\*d\*\*3\*e\*\*2 - 4\*d\*\*2\*e\*\*3 + 64\*d\*e\*\*4 - 11\*e\*\*5)\*log(d + e\*x)/e\*\*9

$$3.282 \quad \int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$$

**Optimal.** Leaf size=354

$$\frac{3x^4(200d^2 + 45de + 37e^2)}{4e^5} - \frac{x^3(1000d^3 + 270d^2e + 333de^2 + 37e^3)}{3e^6} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3)}{2e^9(d+ex)^2}$$

**Rubi [A]** time = 0.34, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

$$\frac{3x^4(200d^2 + 45de + 37e^2)}{4e^5} - \frac{x^3(1000d^3 + 270d^2e + 333de^2 + 37e^3)}{3e^6} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3)}{2e^9(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3,x]

[Out] -(((2100\*d^5 + 675\*d^4\*e + 1110\*d^3\*e^2 + 222\*d^2\*e^3 + 444\*d\*e^4 - 65\*e^5)\*x)/e^8) + ((1500\*d^4 + 450\*d^3\*e + 666\*d^2\*e^2 + 111\*d\*e^3 + 148\*e^4)\*x^2)/(2\*e^7) - ((1000\*d^3 + 270\*d^2\*e + 333\*d\*e^2 + 37\*e^3)\*x^3)/(3\*e^6) + (3\*(200\*d^2 + 45\*d\*e + 37\*e^2)\*x^4)/(4\*e^5) - (3\*(20\*d + 3\*e)\*x^5)/e^4 + (50\*x^6)/(3\*e^3) - ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(2\*e^9\*(d + e\*x)^2) + ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(160\*d^5 + 127\*d^4\*e + 88\*d^3\*e^2 - 4\*d^2\*e^3 + 64\*d\*e^4 - 11\*e^5))/(e^9\*(d + e\*x)) + ((2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*Log[d + e\*x])/e^9

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3 + 2x + 5x^2)^2(2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^3} dx = \int \left( \frac{-2100d^5 - 675d^4e - 1110d^3e^2 - 222d^2e^3 - 444de^4 + 65e^5}{e^8} \right. \\ \left. - \frac{(2100d^5 + 675d^4e + 1110d^3e^2 + 222d^2e^3 + 444de^4 - 65e^5)x}{e^8} \right) dx$$

**Mathematica [A]** time = 0.10, size = 311, normalized size = 0.88

Antiderivative was successfully verified.

[In] Integrate[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3, x]

[Out] (9000\*d^8 - 390\*d^7\*e\*(-9 + 40\*x) - 18\*d^6\*e^2\*(-407 + 240\*x + 2300\*x^2) - 2\*d^5\*e^3\*(-999 + 2664\*x + 6750\*x^2 + 5600\*x^3) + 4\*d^4\*e^4\*(1554 - 111\*x - 5661\*x^2 - 945\*x^3 + 700\*x^4) - d^3\*e^5\*(1950 - 1776\*x + 4662\*x^2 + 6660\*x^3 - 945\*x^4 + 1120\*x^5) + d^2\*e^6\*(1926 - 1560\*x - 9768\*x^2 - 1480\*x^3 + 1665\*x^4 - 378\*x^5 + 560\*x^6) + d\*e^7\*(-198 + 2568\*x + 1560\*x^2 - 3552\*x^3 + 370\*x^4 - 666\*x^5 + 189\*x^6 - 320\*x^7) + e^8\*(-108 - 396\*x + 780\*x^3 + 888\*x^4 - 148\*x^5 + 333\*x^6 - 108\*x^7 + 200\*x^8) + 12\*(2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*(d + e\*x)^2 \*Log[d + e\*x])/(12\*e^9\*(d + e\*x)^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3, x]

[Out] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^3, x]

**fricas [A]** time = 1.56, size = 545, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x, algorithm="fricas")

[Out] 1/12\*(200\*e^8\*x^8 + 9000\*d^8 + 3510\*d^7\*e + 7326\*d^6\*e^2 + 1998\*d^5\*e^3 + 6216\*d^4\*e^4 - 1950\*d^3\*e^5 + 1926\*d^2\*e^6 - 198\*d\*e^7 - 108\*e^8 - 4\*(80\*d\*e^7 + 27\*e^8)\*x^7 + (560\*d^2\*e^6 + 189\*d\*e^7 + 333\*e^8)\*x^6 - 2\*(560\*d^3\*e^5 + 189\*d^2\*e^6 + 333\*d\*e^7 + 74\*e^8)\*x^5 + (2800\*d^4\*e^4 + 945\*d^3\*e^5 + 1665\*d^2\*e^6 + 370\*d\*e^7 + 888\*e^8)\*x^4 - 4\*(2800\*d^5\*e^3 + 945\*d^4\*e^4 + 166

$$5*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2 + 2250*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^2 - 12*(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5 + 130*d^2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^6*e^2 + 370*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 + 888*d^3*e^5 - 195*d^2*e^6 + 107*d*e^7)*x)*\log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e^9)$$

**giac** [A] time = 0.16, size = 354, normalized size = 1.00

2800\*d^8 + 945\*d^7\*e + 1665\*d^6\*e^2 + 370\*d^5\*e^3 + 888\*d^4\*e^4 - 195\*d^3\*e^5 + 107\*d^2\*e^6 + (2800\*d^6\*e^2 + 945\*d^5\*e^3 + 1665\*d^4\*e^4 + 370\*d^3\*e^5 + 888\*d^2\*e^6 - 195\*d\*e^7 + 107\*e^8)\*x^2 + 2\*(2800\*d^7\*e + 945\*d^6\*e^2 + 1665\*d^5\*e^3 + 370\*d^4\*e^4 + 888\*d^3\*e^5 - 195\*d^2\*e^6 + 107\*d\*e^7)\*x)\*log(e\*x + d))/(e^11\*x^2 + 2\*d\*e^10\*x + d^2\*e^9)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x, algorithm="giac")

[Out] (2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*e^(-9)\*log(abs(x\*e + d)) + 1/12\*(200\*x^6\*e^15 - 720\*d\*x^5\*e^14 + 1800\*d^2\*x^4\*e^13 - 4000\*d^3\*x^3\*e^12 + 9000\*d^4\*x^2\*e^11 - 25200\*d^5\*x\*e^10 - 108\*x^5\*e^15 + 405\*d\*x^4\*e^14 - 1080\*d^2\*x^3\*e^13 + 2700\*d^3\*x^2\*e^12 - 8100\*d^4\*x\*e^11 + 333\*x^4\*e^15 - 1332\*d\*x^3\*e^14 + 3996\*d^2\*x^2\*e^13 - 13320\*d^3\*x\*e^12 - 148\*x^3\*e^15 + 666\*d\*x^2\*e^14 - 2664\*d^2\*x\*e^13 + 888\*x^2\*e^15 - 5328\*d\*x\*e^14 + 780\*x\*e^15)\*e^(-18) + 1/2\*(1500\*d^8 + 585\*d^7\*e + 1221\*d^6\*e^2 + 333\*d^5\*e^3 + 1036\*d^4\*e^4 - 325\*d^3\*e^5 + 321\*d^2\*e^6 + 2\*(800\*d^7\*e + 315\*d^6\*e^2 + 666\*d^5\*e^3 + 185\*d^4\*e^4 + 592\*d^3\*e^5 - 195\*d^2\*e^6 + 214\*d\*e^7 - 33\*e^8)\*x - 33\*d\*e^7 - 18\*e^8)\*e^(-9)/(x\*e + d)^2

**maple** [A] time = 0.01, size = 531, normalized size = 1.50

2800\*d^8 + 945\*d^7\*e + 1665\*d^6\*e^2 + 370\*d^5\*e^3 + 888\*d^4\*e^4 - 195\*d^3\*e^5 + 107\*d^2\*e^6 + (2800\*d^6\*e^2 + 945\*d^5\*e^3 + 1665\*d^4\*e^4 + 370\*d^3\*e^5 + 888\*d^2\*e^6 - 195\*d\*e^7 + 107\*e^8)\*x^2 + 2\*(2800\*d^7\*e + 945\*d^6\*e^2 + 1665\*d^5\*e^3 + 370\*d^4\*e^4 + 888\*d^3\*e^5 - 195\*d^2\*e^6 + 214\*d\*e^7 - 33\*e^8)\*x - 33\*d\*e^7 - 18\*e^8)\*e^(-9)/(x\*e + d)^2

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x)

[Out] -1110\*d^3/e^6\*x-222\*d^2/e^5\*x-444\*d/e^4\*x+50/3\*x^6/e^3+111/4/e^3\*x^4-9/e^3\*x^5-9/(e\*x+d)^2/e+107/e^3\*ln(e\*x+d)+74/e^3\*x^2+65/e^3\*x-33/(e\*x+d)/e^2-37/3/e^3\*x^3-50/e^9/(e\*x+d)^2\*d^8-45/2/e^8/(e\*x+d)^2\*d^7+2800/e^9\*ln(e\*x+d)\*d^6+945/e^8\*ln(e\*x+d)\*d^5-60/e^4\*x^5\*d+150/e^5\*x^4\*d^2+135/4/e^4\*x^4\*d-1000/3/e^6\*x^3\*d^3-90/e^5\*x^3\*d^2+750/e^7\*x^2\*d^4+225/e^6\*x^2\*d^3-2100/e^8\*x\*d^5-675/e^7\*x\*d^4+800/e^9/(e\*x+d)\*d^7+315/e^8/(e\*x+d)\*d^6+666/(e\*x+d)\*d^5/e^7+185/(e\*x+d)\*d^4/e^6+592/(e\*x+d)\*d^3/e^5-195/(e\*x+d)\*d^2/e^4+214/(e\*x+d)\*d/e^3-111\*d/e^4\*x^3+333\*d^2/e^5\*x^2+111/2\*d/e^4\*x^2-111/2/(e\*x+d)^2\*d^6/e^7-37/2

$$\frac{1}{(e*x+d)^2*d^5/e^6-74/(e*x+d)^2*d^4/e^5+65/2/(e*x+d)^2*d^3/e^4-107/2/(e*x+d)^2*d^2/e^3+33/2/(e*x+d)^2*d/e^2+1665*d^4/e^7*\ln(e*x+d)+370*d^3/e^6*\ln(e*x+d)+888*d^2/e^5*\ln(e*x+d)-195*d/e^4*\ln(e*x+d)}$$

**maxima [A]** time = 0.45, size = 378, normalized size = 1.07

1500\*d^8 + 585\*d^7\*e + 1221\*d^6\*e^2 + 333\*d^5\*e^3 + 1036\*d^4\*e^4 - 325\*d^3\*e^5 + 321\*d^2\*e^6 - 33\*d\*e^7 - 18\*e^8 + 2\*(800\*d^7\*e + 315\*d^6\*e^2 + 666\*d^5\*e^3 + 185\*d^4\*e^4 + 592\*d^3\*e^5 - 195\*d^2\*e^6 + 214\*d\*e^7 - 33\*e^8)\*x)/(e^11\*x^2 + 2\*d\*e^10\*x + d^2\*e^9) + 1/12\*(200\*e^5\*x^6 - 36\*(20\*d\*e^4 + 3\*e^5)\*x^5 + 9\*(200\*d^2\*e^3 + 45\*d\*e^4 + 37\*e^5)\*x^4 - 4\*(1000\*d^3\*e^2 + 270\*d^2\*e^3 + 333\*d\*e^4 + 37\*e^5)\*x^3 + 6\*(1500\*d^4\*e + 450\*d^3\*e^2 + 666\*d^2\*e^3 + 111\*d\*e^4 + 148\*e^5)\*x^2 - 12\*(2100\*d^5 + 675\*d^4\*e + 1110\*d^3\*e^2 + 222\*d^2\*e^3 + 444\*d\*e^4 - 65\*e^5)\*x)/e^8 + (2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*log(e\*x + d)/e^9

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3,x, algorithm="maxima")

[Out] 1/2\*(1500\*d^8 + 585\*d^7\*e + 1221\*d^6\*e^2 + 333\*d^5\*e^3 + 1036\*d^4\*e^4 - 325\*d^3\*e^5 + 321\*d^2\*e^6 - 33\*d\*e^7 - 18\*e^8 + 2\*(800\*d^7\*e + 315\*d^6\*e^2 + 666\*d^5\*e^3 + 185\*d^4\*e^4 + 592\*d^3\*e^5 - 195\*d^2\*e^6 + 214\*d\*e^7 - 33\*e^8)\*x)/(e^11\*x^2 + 2\*d\*e^10\*x + d^2\*e^9) + 1/12\*(200\*e^5\*x^6 - 36\*(20\*d\*e^4 + 3\*e^5)\*x^5 + 9\*(200\*d^2\*e^3 + 45\*d\*e^4 + 37\*e^5)\*x^4 - 4\*(1000\*d^3\*e^2 + 270\*d^2\*e^3 + 333\*d\*e^4 + 37\*e^5)\*x^3 + 6\*(1500\*d^4\*e + 450\*d^3\*e^2 + 666\*d^2\*e^3 + 111\*d\*e^4 + 148\*e^5)\*x^2 - 12\*(2100\*d^5 + 675\*d^4\*e + 1110\*d^3\*e^2 + 222\*d^2\*e^3 + 444\*d\*e^4 - 65\*e^5)\*x)/e^8 + (2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*log(e\*x + d)/e^9

**mupad [B]** time = 0.13, size = 771, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 5\*x^2 + 3)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(d + e\*x)^3,x)

[Out] x^4\*(111/(4\*e^3) - (75\*d^2)/e^5 + (3\*d\*((300\*d)/e^4 + 45/e^3))/(4\*e)) - x^3\*(37/(3\*e^3) + (100\*d^3)/(3\*e^6) + (d\*(111/e^3 - (300\*d^2)/e^5 + (3\*d\*((300\*d)/e^4 + 45/e^3))/e))/e - (d^2\*((300\*d)/e^4 + 45/e^3))/e^2 - x^5\*((60\*d)/e^4 + 9/e^3) + x\*(65/e^3 - (3\*d\*(148/e^3 + (3\*d\*(37/e^3 + (100\*d^3)/e^6 + (3\*d\*(111/e^3 - (300\*d^2)/e^5 + (3\*d\*((300\*d)/e^4 + 45/e^3))/e))/e) - (3\*d^2\*((300\*d)/e^4 + 45/e^3))/e^2))/e - (3\*d^2\*((300\*d)/e^4 + 45/e^3))/e^2) + (d^3\*((300\*d)/e^4 + 45/e^3))/e^3) + (3\*d^2\*(37/e^3 + (100\*d^3)/e^6 + (3\*d\*(111/e^3 - (300\*d^2)/e^5 + (3\*d\*((300\*d)/e^4 + 45/e^3))/e))/e - (3\*d^2\*((300\*d)/e^4 + 45/e^3))/e^2))/e^2 - (d^3\*(111/e^3 - (300\*d^2)/e^5 + (3\*d\*((300\*d)/e^4 + 45/e^3))/e))/e^3) + (x\*(214\*d\*e^6 + 315\*d^6\*e + 800\*d^7 - 33\*e^7 - 195\*d^2\*e^5 + 592\*d^3\*e^4 + 185\*d^4\*e^3 + 666\*d^5\*e^2) + (585\*d^7\*e - 33\*d\*e^7 + 1500\*d^8 - 18\*e^8 + 321\*d^2\*e^6 - 325\*d^3\*e^5 + 1036\*d^4\*e^4 + 333\*d^5\*e^3 + 1221\*d^6\*e^2)/(2\*e))/(d^2\*e^8 + e^10\*x^2 + 2\*d\*e^9\*x) + (50\*x^6)/(3\*e^3) + x^2\*(74/e^3 + (3\*d\*(37/e^3 + (10

$$0*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2)/(2*e) - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3*((300*d)/e^4 + 45/e^3))/(2*e^3)) + (\log(d + e*x)*(945*d^5*e - 195*d*e^5 + 2800*d^6 + 107*e^6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9$$

**sympy [A]** time = 4.87, size = 394, normalized size = 1.11

$$-\left(\frac{9d}{e^6} + \frac{3}{e^6}\right) + \left(\frac{150d^2}{e^5} + \frac{135d}{e^5} + \frac{111}{e^5}\right) + \left(\frac{1000d^3}{e^4} + \frac{90d^2}{e^4} + \frac{111d}{e^4} + \frac{37}{e^4}\right) + \left(\frac{750d^4}{e^3} + \frac{225d^3}{e^3} + \frac{333d^2}{e^3} + \frac{111d}{e^3} + \frac{71}{e^3}\right) + \left(\frac{2100d^5}{e^2} + \frac{675d^4}{e^2} + \frac{1110d^3}{e^2} + \frac{222d^2}{e^2} + \frac{444d}{e^2} + \frac{65}{e^2}\right) + \frac{1500d^6 + 585d^5 + 1221d^4 + 333d^3 + 1036d^2 - 325d + 321d^2e - 33d^3e + 1600d^4 + 630d^3e + 1332d^2e^2 + 370d^4 + 1184d^3e - 390d^2e^2 - 66d^3e^2}{2d^2 + 4d^3 + 2d^4} + \frac{50d^6}{e^9} + \frac{(2800d^6 + 945d^5 + 1665d^4 + 370d^3 + 888d^2 - 195d + 107)\log(d + ex)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*3,x)

[Out] x\*\*5\*(-60\*d/e\*\*4 - 9/e\*\*3) + x\*\*4\*(150\*d\*\*2/e\*\*5 + 135\*d/(4\*e\*\*4) + 111/(4\*e\*\*3)) + x\*\*3\*(-1000\*d\*\*3/(3\*e\*\*6) - 90\*d\*\*2/e\*\*5 - 111\*d/e\*\*4 - 37/(3\*e\*\*3)) + x\*\*2\*(750\*d\*\*4/e\*\*7 + 225\*d\*\*3/e\*\*6 + 333\*d\*\*2/e\*\*5 + 111\*d/(2\*e\*\*4) + 74/e\*\*3) + x\*(-2100\*d\*\*5/e\*\*8 - 675\*d\*\*4/e\*\*7 - 1110\*d\*\*3/e\*\*6 - 222\*d\*\*2/e\*\*5 - 444\*d/e\*\*4 + 65/e\*\*3) + (1500\*d\*\*8 + 585\*d\*\*7\*e + 1221\*d\*\*6\*e\*\*2 + 333\*d\*\*5\*e\*\*3 + 1036\*d\*\*4\*e\*\*4 - 325\*d\*\*3\*e\*\*5 + 321\*d\*\*2\*e\*\*6 - 33\*d\*e\*\*7 - 18\*e\*\*8 + x\*(1600\*d\*\*7\*e + 630\*d\*\*6\*e\*\*2 + 1332\*d\*\*5\*e\*\*3 + 370\*d\*\*4\*e\*\*4 + 1184\*d\*\*3\*e\*\*5 - 390\*d\*\*2\*e\*\*6 + 428\*d\*e\*\*7 - 66\*e\*\*8))/(2\*d\*\*2\*e\*\*9 + 4\*d\*e\*\*10\*x + 2\*e\*\*11\*x\*\*2) + 50\*x\*\*6/(3\*e\*\*3) + (2800\*d\*\*6 + 945\*d\*\*5\*e + 1665\*d\*\*4\*e\*\*2 + 370\*d\*\*3\*e\*\*3 + 888\*d\*\*2\*e\*\*4 - 195\*d\*e\*\*5 + 107\*e\*\*6)\*log(d + e\*x)/e\*\*9

$$3.283 \quad \int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} dx$$

**Optimal.** Leaf size=360

$$\frac{x^3(1000d^2 + 180de + 111e^2)}{3e^6} - \frac{x^2(2000d^3 + 450d^2e + 444de^2 + 37e^3)}{2e^7} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2)}{3e^9(d+ex)^3}$$

**Rubi [A]** time = 0.36, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

$$\frac{x^3(1000d^2 + 180de + 111e^2)}{3e^6} - \frac{x^2(2000d^3 + 450d^2e + 444de^2 + 37e^3)}{2e^7} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2)}{3e^9(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^4,x]

[Out] (2\*(1750\*d^4 + 450\*d^3\*e + 555\*d^2\*e^2 + 74\*d\*e^3 + 74\*e^4)\*x)/e^8 - ((2000\*d^3 + 450\*d^2\*e + 444\*d\*e^2 + 37\*e^3)\*x^2)/(2\*e^7) + ((1000\*d^2 + 180\*d\*e + 111\*e^2)\*x^3)/(3\*e^6) - (5\*(80\*d + 9\*e)\*x^4)/(4\*e^5) + (20\*x^5)/e^4 - ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(3\*e^9\*(d + e\*x)^3) + ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(160\*d^5 + 127\*d^4\*e + 88\*d^3\*e^2 - 4\*d^2\*e^3 + 64\*d\*e^4 - 11\*e^5))/(2\*e^9\*(d + e\*x)^2) - (2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)/(e^9\*(d + e\*x)) - ((5600\*d^5 + 1575\*d^4\*e + 2220\*d^3\*e^2 + 370\*d^2\*e^3 + 592\*d\*e^4 - 65\*e^5)\*Log[d + e\*x])/e^9

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int \frac{(3 + 2x + 5x^2)^2(2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^4} dx = \int \left( \frac{2(1750d^4 + 450d^3e + 555d^2e^2 + 74de^3 + 74e^4)}{e^8} - \frac{(2000d^3 + 450d^2e + 444de^2 + 37e^3)x^2}{2e^7} + \frac{(1000d^2 + 180de + 111e^2)x^3}{3e^6} - \frac{5(80d + 9e)x^4}{4e^5} + \frac{20x^5}{e^4} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{3e^9(d+ex)^3} + \frac{(5d^2 - 2de + 3e^2)(160d^5 + 127d^4e + 88d^3e^2 - 4d^2e^3 + 64de^4 - 11e^5)}{2e^9(d+ex)^2} - \frac{2800d^6 + 945d^5e + 1665d^4e^2 + 370d^3e^3 + 888d^2e^4 - 195de^5 + 107e^6}{e^9(d+ex)} - \frac{(5600d^5 + 1575d^4e + 2220d^3e^2 + 370d^2e^3 + 592de^4 - 65e^5)\text{Log}[d+ex]}{e^9} \right) dx$$

$$= \frac{2(1750d^4 + 450d^3e + 555d^2e^2 + 74de^3 + 74e^4)x}{e^8} - \frac{(2000d^3 + 450d^2e + 444de^2 + 37e^3)x^2}{2e^7} + \frac{(1000d^2 + 180de + 111e^2)x^3}{3e^6} - \frac{5(80d + 9e)x^4}{4e^5} + \frac{20x^5}{e^4} - \frac{(5d^2 - 2de + 3e^2)^2(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{3e^9(d+ex)^3} + \frac{(5d^2 - 2de + 3e^2)(160d^5 + 127d^4e + 88d^3e^2 - 4d^2e^3 + 64de^4 - 11e^5)}{2e^9(d+ex)^2} - \frac{2800d^6 + 945d^5e + 1665d^4e^2 + 370d^3e^3 + 888d^2e^4 - 195de^5 + 107e^6}{e^9(d+ex)} - \frac{(5600d^5 + 1575d^4e + 2220d^3e^2 + 370d^2e^3 + 592de^4 - 65e^5)\text{Log}[d+ex]}{e^9}$$

**Mathematica [A]** time = 0.12, size = 344, normalized size = 0.96

$$\frac{4^5 x^3 (1000 d^3 + 180 d e + 111 e^2) - 6 e^2 d^2 (2000 d^3 + 450 d^2 e + 444 d e^2 + 37 e^3) + 24 c x (1750 d^4 + 450 d^3 e + 555 d^2 e^2 + 74 d e^3 + 74 e^4)}{12 (5600 d^5 + 1575 d^4 e + 2220 d^3 e^2 + 370 d^2 e^3 + 592 d e^4 - 65 e^5) \log(d + e x)} - \frac{11 (2800 d^6 + 945 d^5 e + 1665 d^4 e^2 + 370 d^3 e^3 + 888 d^2 e^4 - 195 d e^5 + 107 e^6)}{(d + e x)^2} - \frac{6 (800 d^7 + 315 d^6 e + 666 d^5 e^2 + 185 d^4 e^3 + 592 d^3 e^4 - 195 d^2 e^5 + 214 d e^6 - 33 e^7)}{(d + e x)^4} - 12 (5600 d^5 + 1575 d^4 e + 2220 d^3 e^2 + 370 d^2 e^3 + 592 d e^4 - 65 e^5) \operatorname{Log}[d + e x] / (12 e^9)$$

Antiderivative was successfully verified.

[In] Integrate[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^4, x]

[Out] (24\*e\*(1750\*d^4 + 450\*d^3\*e + 555\*d^2\*e^2 + 74\*d\*e^3 + 74\*e^4)\*x - 6\*e^2\*(2000\*d^3 + 450\*d^2\*e + 444\*d\*e^2 + 37\*e^3)\*x^2 + 4\*e^3\*(1000\*d^2 + 180\*d\*e + 111\*e^2)\*x^3 - 15\*e^4\*(80\*d + 9\*e)\*x^4 + 240\*e^5\*x^5 - (4\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(d + e\*x)^3 + (6\*(800\*d^7 + 315\*d^6\*e + 666\*d^5\*e^2 + 185\*d^4\*e^3 + 592\*d^3\*e^4 - 195\*d^2\*e^5 + 214\*d\*e^6 - 33\*e^7))/(d + e\*x)^2 - (12\*(2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6))/(d + e\*x) - 12\*(5600\*d^5 + 1575\*d^4\*e + 2220\*d^3\*e^2 + 370\*d^2\*e^3 + 592\*d\*e^4 - 65\*e^5)\*Log[d + e\*x])/(12\*e^9)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4)}{(d + ex)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^4, x]

[Out] IntegrateAlgebraic[((3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(d + e\*x)^4, x]

**fricas [A]** time = 0.93, size = 587, normalized size = 1.63

$$\frac{1}{12} (240 e^8 x^8 - 29200 d^8 - 9630 d^7 e - 16428 d^6 e^2 - 3478 d^5 e^3 - 7696 d^4 e^4 + 1430 d^3 e^5 - 428 d^2 e^6 - 66 d e^7 - 72 e^8 - 15 (32 d e^7 + 9 e^8) x^7 + (1120 d^2 e^6 + 315 d e^7 + 444 e^8) x^6 - 3 (1120 d^3 e^5 + 315 d^2 e^6 + 444 d e^7 + 74 e^8) x^5 + 3 (5600 d^4 e^4 + 1575 d^3 e^5 + 2220 d^2 e^6 + 370 d e^7 + 592 e^8) x^4 + 2 (47000 d^5 e^3 + 12510 d^4 e^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^4,x, algorithm="fricas")

[Out] 1/12\*(240\*e^8\*x^8 - 29200\*d^8 - 9630\*d^7\*e - 16428\*d^6\*e^2 - 3478\*d^5\*e^3 - 7696\*d^4\*e^4 + 1430\*d^3\*e^5 - 428\*d^2\*e^6 - 66\*d\*e^7 - 72\*e^8 - 15\*(32\*d\*e^7 + 9\*e^8)\*x^7 + (1120\*d^2\*e^6 + 315\*d\*e^7 + 444\*e^8)\*x^6 - 3\*(1120\*d^3\*e^5 + 315\*d^2\*e^6 + 444\*d\*e^7 + 74\*e^8)\*x^5 + 3\*(5600\*d^4\*e^4 + 1575\*d^3\*e^5 + 2220\*d^2\*e^6 + 370\*d\*e^7 + 592\*e^8)\*x^4 + 2\*(47000\*d^5\*e^3 + 12510\*d^4\*e^7





$$\frac{3}{(e*x+d)^2*d+60/e^5*x^3*d-1000/e^7*x^2*d^3-225/e^6*x^2*d^2-222/e^5*x^2*d+3500/e^8*d^4*x+900/e^7*x*d^3+1110/e^6*x*d^2+148/e^5*x*d-2800/e^9/(e*x+d)*d^6-945/e^8/(e*x+d)*d^5-1665/e^7/(e*x+d)*d^4-370/e^6/(e*x+d)*d^3-888/e^5/(e*x+d)*d^2+195/e^4/(e*x+d)*d-100/3/e^9/(e*x+d)^3*d^8-15/e^8/(e*x+d)^3*d^7-37/e^7/(e*x+d)^3*d^6}$$

**maxima** [A] time = 0.46, size = 390, normalized size = 1.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^4,x, algorithm="maxima")

[Out] 
$$-1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4 - 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^{12}*x^3 + 3*d*e^{11}*x^2 + 3*d^2*e^{10}*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(1000*d^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 444*d*e^3 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*log(e*x + d)/e^9$$

**mupad** [B] time = 4.28, size = 560, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 5\*x^2 + 3)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(d + e\*x)^4,x)

[Out] 
$$x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(37/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2 - (x*(107*d*e^6 + (3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4 + (1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 + 36*e^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d^6*e^2))/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*d^3*e^4 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^{11}*x^3 + 3*d^2*e^9*x + 3*d*e^{10}*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e^8 + (4*d*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6*d^2*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*d^3*((400*d)$$

$$/e^5 + 45/e^4)/e^3) + (20*x^5)/e^4 - (\log(d + e*x)*(592*d*e^4 + 1575*d^4*e + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9$$

sympy [A] time = 8.09, size = 401, normalized size = 1.11

$$\frac{x^4 \left( \frac{100d}{e^5} - \frac{45}{4e^4} \right) + x^3 \left( \frac{1000d^2}{3e^6} + \frac{60d}{e^5} + \frac{37}{2e^4} \right) + x^2 \left( -\frac{1000d^3}{e^7} - \frac{225d^2}{e^6} - \frac{222d}{e^5} - \frac{37}{2e^4} \right) + x \left( \frac{3500d^4}{e^8} + \frac{900d^3}{e^7} + \frac{1110d^2}{e^6} + \frac{148d}{e^5} + \frac{148}{e^4} \right) + \left( -\frac{14600d^8}{e^9} - \frac{4815d^7}{e^8} - \frac{8214d^6}{e^7} - \frac{1739d^5}{e^6} - \frac{3848d^4}{e^5} - \frac{715d^3}{e^4} - \frac{214d^2}{e^3} - \frac{33d}{e^2} - \frac{36}{e} + x^2 \left( -\frac{16800d^6}{e^7} - \frac{5670d^5}{e^6} - \frac{9990d^4}{e^5} - \frac{2220d^3}{e^4} - \frac{5328d^2}{e^3} + \frac{1170d}{e^2} - \frac{642}{e} \right) + x \left( -\frac{31200d^7}{e^8} - \frac{10395d^6}{e^7} - \frac{17982d^5}{e^6} - \frac{3885d^4}{e^5} - \frac{8880d^3}{e^4} + \frac{1755d^2}{e^3} - \frac{642d}{e^2} - \frac{99}{e} \right) \right) / (6d^3e^9 + 18d^2e^{10}x + 18de^{11}x^2 + 6e^{12}x^3) + \frac{20x^5}{e^4} - \frac{(5600d^5 + 1575d^4e + 2220d^3e^2 + 370d^2e^3 + 592de^4 - 65e^5) \log(d + ex)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*4,x)

[Out] x\*\*4\*(-100\*d/e\*\*5 - 45/(4\*e\*\*4)) + x\*\*3\*(1000\*d\*\*2/(3\*e\*\*6) + 60\*d/e\*\*5 + 37/e\*\*4) + x\*\*2\*(-1000\*d\*\*3/e\*\*7 - 225\*d\*\*2/e\*\*6 - 222\*d/e\*\*5 - 37/(2\*e\*\*4)) + x\*(3500\*d\*\*4/e\*\*8 + 900\*d\*\*3/e\*\*7 + 1110\*d\*\*2/e\*\*6 + 148\*d/e\*\*5 + 148/e\*\*4) + (-14600\*d\*\*8 - 4815\*d\*\*7\*e - 8214\*d\*\*6\*e\*\*2 - 1739\*d\*\*5\*e\*\*3 - 3848\*d\*\*4\*e\*\*4 + 715\*d\*\*3\*e\*\*5 - 214\*d\*\*2\*e\*\*6 - 33\*d\*e\*\*7 - 36\*e\*\*8 + x\*\*2\*(-16800\*d\*\*6\*e\*\*2 - 5670\*d\*\*5\*e\*\*3 - 9990\*d\*\*4\*e\*\*4 - 2220\*d\*\*3\*e\*\*5 - 5328\*d\*\*2\*e\*\*6 + 1170\*d\*e\*\*7 - 642\*e\*\*8) + x\*(-31200\*d\*\*7\*e - 10395\*d\*\*6\*e\*\*2 - 17982\*d\*\*5\*e\*\*3 - 3885\*d\*\*4\*e\*\*4 - 8880\*d\*\*3\*e\*\*5 + 1755\*d\*\*2\*e\*\*6 - 642\*d\*e\*\*7 - 99\*e\*\*8))/(6\*d\*\*3\*e\*\*9 + 18\*d\*\*2\*e\*\*10\*x + 18\*d\*e\*\*11\*x\*\*2 + 6\*e\*\*12\*x\*\*3) + 20\*x\*\*5/e\*\*4 - (5600\*d\*\*5 + 1575\*d\*\*4\*e + 2220\*d\*\*3\*e\*\*2 + 370\*d\*\*2\*e\*\*3 + 592\*d\*e\*\*4 - 65\*e\*\*5)\*log(d + e\*x)/e\*\*9

$$3.284 \quad \int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$$

**Optimal.** Leaf size=221

$$\frac{3}{500}ex^4(100d^2 - 165de + 27e^2) + \frac{x^3(500d^3 - 2475d^2e + 1215de^2 + 458e^3)}{1875} - \frac{x^2(4125d^3 - 6075d^2e - 6870de^2 + 881e^3)}{6250}$$

**Rubi [A]** time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1628, 634, 618, 204, 628}

$$\frac{3}{500}e^{x^4}(100d^2 - 165de + 27e^2) + \frac{x^3(-2475d^2e + 500d^3 + 1215de^2 + 458e^3)}{1875} - \frac{x^2(-6075d^2e + 4125d^3 - 6870d^2e + 881e^3)}{6250} + \frac{(-66075d^2e + 57250d^3 - 76620d^2e + 23431e^3)\log(5x^2 + 2x + 3)}{156250} + \frac{x(34350d^2e + 10125d^3 - 13215d^2e - 5108e^3)}{15625} - \frac{(449175d^2e + 52875d^3 - 274845d^2e - 53189e^3)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{78125\sqrt{14}} + \frac{3}{125}e^{-2x^5}(20d - 11e) + \frac{2d^3e}{15}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

[Out] ((10125\*d^3 + 34350\*d^2\*e - 13215\*d\*e^2 - 5108\*e^3)\*x)/15625 - ((4125\*d^3 - 6075\*d^2\*e - 6870\*d\*e^2 + 881\*e^3)\*x^2)/6250 + ((500\*d^3 - 2475\*d^2\*e + 1215\*d\*e^2 + 458\*e^3)\*x^3)/1875 + (3\*e\*(100\*d^2 - 165\*d\*e + 27\*e^2)\*x^4)/500 + (3\*(20\*d - 11\*e)\*e^2\*x^5)/125 + (2\*e^3\*x^6)/15 - ((52875\*d^3 + 449175\*d^2\*e - 274845\*d\*e^2 - 53189\*e^3)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(78125\*Sqrt[14]) + ((57250\*d^3 - 66075\*d^2\*e - 76620\*d\*e^2 + 23431\*e^3)\*Log[3 + 2\*x + 5\*x^2])/156250

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\int \frac{(d + ex)^3 (2 + x + 3x^2 - 5x^3 + 4x^4)}{3 + 2x + 5x^2} dx = \int \left( \frac{10125d^3 + 34350d^2e - 13215de^2 - 5108e^3}{15625} - \frac{(4125d^3 - 6075d^2}{15625} \right. \\ = \frac{(10125d^3 + 34350d^2e - 13215de^2 - 5108e^3)x}{15625} - \frac{(4125d^3 - 6075d^2}{15625} \\ = \frac{(10125d^3 + 34350d^2e - 13215de^2 - 5108e^3)x}{15625} - \frac{(4125d^3 - 6075d^2}{15625} \\ = \frac{(10125d^3 + 34350d^2e - 13215de^2 - 5108e^3)x}{15625} - \frac{(4125d^3 - 6075d^2}{15625} \\ = \frac{(10125d^3 + 34350d^2e - 13215de^2 - 5108e^3)x}{15625} - \frac{(4125d^3 - 6075d^2}{15625}$$

**Mathematica [A]** time = 0.12, size = 178, normalized size = 0.81

$42(57250d^3 - 66075d^2e - 76620d^2 + 23431e^2) \log(5x^2 + 2x + 3) + 35x(250d^3(200x^2 - 495x + 486) + 450d^2e(250x^3 - 550x^2 + 405x + 916) + 45d^2(2000x^4 - 4125x^3 + 2700x^2 + 4580x - 3524) + e^2(25000x^5 - 49500x^4 + 30375x^3 + 45800x^2 - 26430x - 61296)) - 6\sqrt{14}(52875d^3 + 449175d^2e - 274845d^2 - 53189e^3) \tan^{-1}\left(\frac{5x}{\sqrt{14}}\right)$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2), x]
```

```
[Out] (35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2 + 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) + e^3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) - 6*Sqrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d^2 - 53189*e^3)*ArcTan[(1 + 5
```

\*x)/Sqrt[14]] + 42\*(57250\*d^3 - 66075\*d^2\*e - 76620\*d\*e^2 + 23431\*e^3)\*Log[3 + 2\*x + 5\*x^2])/6562500

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3 (2 + x + 3x^2 - 5x^3 + 4x^4)}{3 + 2x + 5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

**fricas** [A] time = 0.77, size = 206, normalized size = 0.93

$\frac{2}{15}e^{3x^6} + \frac{3}{125}(20d^2e^2 - 11e^3)x^5 + \frac{3}{500}(100d^2e - 165d^2e^2 + 27e^3)x^4 + \frac{1}{1875}(500d^3 - 2475d^2e + 1215d^2e^2 + 458e^3)x^3 - \frac{1}{6250}(4125d^3 - 6075d^2e - 6870d^2e^2 + 881e^3)x^2 - \frac{1}{1093750}\sqrt{14}(52875d^3 + 449175d^2e - 274845d^2e^2 - 53189e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{15625}(10125d^3 + 34350d^2e - 13215d^2e^2 - 5108e^3)x + \frac{1}{156250}(57250d^3 - 66075d^2e - 76620d^2e^2 + 23431e^3)\log(5x^2 + 2x + 3)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3),x, algorithm="fricas")

[Out] 2/15\*e^3\*x^6 + 3/125\*(20\*d\*e^2 - 11\*e^3)\*x^5 + 3/500\*(100\*d^2\*e - 165\*d^2\*e^2 + 27\*e^3)\*x^4 + 1/1875\*(500\*d^3 - 2475\*d^2\*e + 1215\*d^2\*e^2 + 458\*e^3)\*x^3 - 1/6250\*(4125\*d^3 - 6075\*d^2\*e - 6870\*d^2\*e^2 + 881\*e^3)\*x^2 - 1/1093750\*sqrt(14)\*(52875\*d^3 + 449175\*d^2\*e - 274845\*d^2\*e^2 - 53189\*e^3)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 1/15625\*(10125\*d^3 + 34350\*d^2\*e - 13215\*d^2\*e^2 - 5108\*e^3)\*x + 1/156250\*(57250\*d^3 - 66075\*d^2\*e - 76620\*d^2\*e^2 + 23431\*e^3)\*log(5\*x^2 + 2\*x + 3)

**giac** [A] time = 0.17, size = 212, normalized size = 0.96

$\frac{2}{15}e^{3x^6} + \frac{12}{25}d^2e^2 + \frac{3}{5}d^2e^2 + \frac{4}{15}d^2e^2 - \frac{33}{125}e^3 - \frac{99}{100}d^2e^2 - \frac{33}{25}d^2e^2 - \frac{33}{50}d^2e^2 + \frac{81}{500}d^2e^2 - \frac{81}{125}d^2e^2 + \frac{243}{250}d^2e^2 + \frac{81}{125}d^2e^2 + \frac{458}{1875}d^2e^2 + \frac{687}{625}d^2e^2 + \frac{1374}{625}d^2e^2 - \frac{881}{6250}d^2e^2 - \frac{2643}{3125}d^2e^2 - \frac{1}{1093750}\sqrt{14}(52875d^3 + 449175d^2e - 274845d^2e^2 - 53189e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{5108}{15625}e^3 + \frac{1}{156250}(57250d^3 - 66075d^2e - 76620d^2e^2 + 23431e^3)\log(5x^2 + 2x + 3)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3),x, algorithm="giac")

[Out] 2/15\*x^6\*e^3 + 12/25\*d\*x^5\*e^2 + 3/5\*d^2\*x^4\*e + 4/15\*d^3\*x^3 - 33/125\*x^5\*e^3 - 99/100\*d\*x^4\*e^2 - 33/25\*d^2\*x^3\*e - 33/50\*d^3\*x^2 + 81/500\*x^4\*e^3 + 81/125\*d\*x^3\*e^2 + 243/250\*d^2\*x^2\*e + 81/125\*d^3\*x + 458/1875\*x^3\*e^3 + 687/625\*d\*x^2\*e^2 + 1374/625\*d^2\*x\*e - 881/6250\*x^2\*e^3 - 2643/3125\*d\*x\*e^2 - 1/1093750\*sqrt(14)\*(52875\*d^3 + 449175\*d^2\*e - 274845\*d^2\*e^2 - 53189\*e^3)\*

$$\arctan(1/14*\sqrt{14}*(5*x + 1)) - 5108/15625*x*e^3 + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*\log(5*x^2 + 2*x + 3)$$

**maple [A]** time = 0.01, size = 291, normalized size = 1.32

$$\frac{2}{15}e^3 + \frac{3}{125}(20d^2 - 11e^3)e^2 + \frac{3}{500}(100d^2e - 165d^2 + 27e^3)e + \frac{1}{1875}(800d^3 - 2475d^2e + 1215d^2 + 458e^3)e^2 - \frac{1}{6250}(4125d^3 - 6075d^2e - 6870d^2 + 881e^3)e^2 - \frac{1}{1093750}\sqrt{14}(52875d^3 + 449175d^2e - 274845d^2 - 53189e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{156250}(10125d^3 + 34350d^2e - 13215d^2 - 5108e^3)x + \frac{1}{156250}(57250d^3 - 66075d^2e - 76620d^2 + 23431e^3)\log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x)

$$-881/6250*e^3*x^2+2/15*e^3*x^6+81/500*e^3*x^4-17967/43750*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*d^2*e+54969/218750*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*d*e^2+4/15*x^3*d^3-5108/15625*x*e^3-33/125*x^5*e^3-33/50*x^2*d^3+458/1875*x^3*e^3+229/625*\ln(5*x^2+2*x+3)*d^3+23431/156250*\ln(5*x^2+2*x+3)*e^3+81/125*d^3*x-2643/3125*x*d*e^2-423/8750*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*d^3+53189/1093750*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*e^3+3/5*x^4*d^2*e-99/100*x^4*d*e^2-33/25*x^3*d^2*e+687/625*x^2*d*e^2+1374/625*x*d^2*e+81/125*x^3*d*e^2+243/250*x^2*d^2*e-7662/15625*\ln(5*x^2+2*x+3)*d*e^2+12/25*x^5*d*e^2-2643/6250*\ln(5*x^2+2*x+3)*d^2*e$$

**maxima [A]** time = 0.96, size = 206, normalized size = 0.93

$$\frac{2}{15}e^3 + \frac{3}{125}(20d^2 - 11e^3)e^2 + \frac{3}{500}(100d^2e - 165d^2 + 27e^3)e + \frac{1}{1875}(800d^3 - 2475d^2e + 1215d^2 + 458e^3)e^2 - \frac{1}{6250}(4125d^3 - 6075d^2e - 6870d^2 + 881e^3)e^2 - \frac{1}{1093750}\sqrt{14}(52875d^3 + 449175d^2e - 274845d^2 - 53189e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{156250}(10125d^3 + 34350d^2e - 13215d^2 - 5108e^3)x + \frac{1}{156250}(57250d^3 - 66075d^2e - 76620d^2 + 23431e^3)\log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="maxima")

$$2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 - 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*\sqrt{14}*(52875*d^3 + 449175*d^2*e - 274845*d^2 - 53189*e^3)*\arctan(1/14*\sqrt{14}*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d^2 - 5108*e^3)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d^2 + 23431*e^3)*\log(5*x^2 + 2*x + 3)$$

**mupad [B]** time = 4.18, size = 397, normalized size = 1.80

$$\frac{2}{15}e^3 + \frac{3}{125}(20d^2 - 11e^3)e^2 + \frac{3}{500}(100d^2e - 165d^2 + 27e^3)e + \frac{1}{1875}(800d^3 - 2475d^2e + 1215d^2 + 458e^3)e^2 - \frac{1}{6250}(4125d^3 - 6075d^2e - 6870d^2 + 881e^3)e^2 - \frac{1}{1093750}\sqrt{14}(52875d^3 + 449175d^2e - 274845d^2 - 53189e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{156250}(10125d^3 + 34350d^2e - 13215d^2 - 5108e^3)x + \frac{1}{156250}(57250d^3 - 66075d^2e - 76620d^2 + 23431e^3)\log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^3\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(2\*x + 5\*x^2 + 3), x)

$$x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e^2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d -$$

$$\begin{aligned}
& 5e)) / 375 + (2e(4d^2 - 5de + e^2)) / 25 - (3de^2) / 5 + d^2e - (4d^3) / \\
& 15 - (111e^3) / 625 + x^5((e^2(12d - 5e)) / 25 - (8e^3) / 125) - \log(2x + \\
& 5x^2 + 3)((7662de^2) / 15625 + (2643d^2e) / 6250 - (229d^3) / 625 - (2343 \\
& 1e^3) / 156250) - x^4((e^2(12d - 5e)) / 50 - (3e(4d^2 - 5de + e^2)) / 2 \\
& 0 + (11e^3) / 125) + (2e^3x^6) / 15 + x((61e^2(12d - 5e)) / 3125 + (3d( \\
& de + d^2 + 2e^2)) / 5 + (156e(4d^2 - 5de + e^2)) / 625 - (129de^2) / 125 \\
& + (3d^2e) / 5 + (6d^3) / 125 - (7483e^3) / 15625) + (14^{(1/2)} \operatorname{atan}(((14^{(1/2)} \\
& )(274845de^2 - 449175d^2e - 52875d^3 + 53189e^3)) / 1093750 + (14^{(1/2)} \\
& ) * x(274845de^2 - 449175d^2e - 52875d^3 + 53189e^3)) / 218750) / ((54969 * \\
& de^2) / 15625 - (17967d^2e) / 3125 - (423d^3) / 625 + (53189e^3) / 78125)) * (27 \\
& 4845de^2 - 449175d^2e - 52875d^3 + 53189e^3) / 1093750
\end{aligned}$$

**sympy [C]** time = 2.58, size = 450, normalized size = 2.04

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3), x)

[Out]  $2e^{**3}x^{**6}/15 + x^{**5}(12de^{**2}/25 - 33e^{**3}/125) + x^{**4}(3d^{**2}e/5 - 99d^{**2}e^{**2}/100 + 81e^{**3}/500) + x^{**3}(4d^{**3}/15 - 33d^{**2}e/25 + 81d^{**2}e^{**2}/125 + 458e^{**3}/1875) + x^{**2}(-33d^{**3}/50 + 243d^{**2}e/250 + 687d^{**2}e^{**2}/625 - 881e^{**3}/6250) + x(81d^{**3}/125 + 1374d^{**2}e/625 - 2643de^{**2}/3125 - 5108e^{**3}/15625) + (229d^{**3}/625 - 2643d^{**2}e/6250 - 7662de^{**2}/15625 + 23431e^{**3}/156250 - \sqrt{14}I(52875d^{**3} + 449175d^{**2}e - 274845d^{**2}e^{**2} - 53189e^{**3})/2187500) * \log(x + (10575d^{**3} + 89835d^{**2}e - 54969d^{**2}e^{**2} - 53189e^{**3})/5 + \sqrt{14}I(52875d^{**3} + 449175d^{**2}e - 274845d^{**2}e^{**2} - 53189e^{**3})/5) / (52875d^{**3} + 449175d^{**2}e - 274845d^{**2}e^{**2} - 53189e^{**3}) + (229d^{**3}/625 - 2643d^{**2}e/6250 - 7662d^{**2}e^{**2}/15625 + 23431e^{**3}/156250 + \sqrt{14}I(52875d^{**3} + 449175d^{**2}e - 274845d^{**2}e^{**2} - 53189e^{**3})/2187500) * \log(x + (10575d^{**3} + 89835d^{**2}e - 54969d^{**2}e^{**2} - 53189e^{**3})/5 - \sqrt{14}I(52875d^{**3} + 449175d^{**2}e - 274845d^{**2}e^{**2} - 53189e^{**3})/5) / (52875d^{**3} + 449175d^{**2}e - 274845d^{**2}e^{**2} - 53189e^{**3}))$



$$3.285 \quad \int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$$

Optimal. Leaf size=156

$$\frac{1}{375}x^3(100d^2 - 330de + 81e^2) - \frac{x^2(825d^2 - 810de - 458e^2)}{1250} + \frac{(5725d^2 - 4405de - 2554e^2)\log(5x^2 + 2x + 3)}{15625} + \dots$$

**Rubi** [A] time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{375}x^3(100d^2 - 330de + 81e^2) - \frac{x^2(825d^2 - 810de - 458e^2)}{1250} + \frac{(5725d^2 - 4405de - 2554e^2)\log(5x^2 + 2x + 3)}{15625} + \frac{x(2025d^2 + 4580de - 881e^2)}{3125} - \frac{(10575d^2 + 59890de - 18323e^2)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{15625\sqrt{14}} + \frac{1}{100}ex^4(40d - 33e) + \frac{4e^2x^5}{25}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2),x]

[Out] ((2025\*d^2 + 4580\*d\*e - 881\*e^2)\*x)/3125 - ((825\*d^2 - 810\*d\*e - 458\*e^2)\*x^2)/1250 + ((100\*d^2 - 330\*d\*e + 81\*e^2)\*x^3)/375 + ((40\*d - 33\*e)\*e\*x^4)/100 + (4\*e^2\*x^5)/25 - ((10575\*d^2 + 59890\*d\*e - 18323\*e^2)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(15625\*Sqrt[14]) + ((5725\*d^2 - 4405\*d\*e - 2554\*e^2)\*Log[3 + 2\*x + 5\*x^2])/15625

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex)^2 (2 + x + 3x^2 - 5x^3 + 4x^4)}{3 + 2x + 5x^2} dx &= \int \left( \frac{2025d^2 + 4580de - 881e^2}{3125} - \frac{1}{625} (825d^2 - 810de - 458e^2)x + \frac{1}{1250} (825d^2 - 810de - 458e^2)x^2 \right) dx \\ &= \frac{(2025d^2 + 4580de - 881e^2)x}{3125} - \frac{(825d^2 - 810de - 458e^2)x^2}{1250} + \frac{1}{375} (825d^2 - 810de - 458e^2)x^3 \\ &= \frac{(2025d^2 + 4580de - 881e^2)x}{3125} - \frac{(825d^2 - 810de - 458e^2)x^2}{1250} + \frac{1}{375} (825d^2 - 810de - 458e^2)x^3 \\ &= \frac{(2025d^2 + 4580de - 881e^2)x}{3125} - \frac{(825d^2 - 810de - 458e^2)x^2}{1250} + \frac{1}{375} (825d^2 - 810de - 458e^2)x^3 \\ &= \frac{(2025d^2 + 4580de - 881e^2)x}{3125} - \frac{(825d^2 - 810de - 458e^2)x^2}{1250} + \frac{1}{375} (825d^2 - 810de - 458e^2)x^3 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 130, normalized size = 0.83

$$\frac{84(5725d^2 - 4405de - 2554e^2) \log(5x^2 + 2x + 3) + 35x(50d^2(200x^2 - 495x + 486) + 60de(250x^3 - 550x^2 + 405x + 916) + 3e^2(2000x^4 - 4125x^3 + 2700x^2 + 4580x - 3524)) - 6\sqrt{14}(10575d^2 + 59890de - 18323e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1312500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2), x]
```

```
[Out] (35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 250*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt[14]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2 (2 + x + 3x^2 - 5x^3 + 4x^4)}{3 + 2x + 5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

**fricas** [A] time = 0.78, size = 141, normalized size = 0.90

$$\frac{4}{25} e^2 x^5 + \frac{1}{100} (40de - 33e^2) x^4 + \frac{1}{375} (100d^2 - 330de + 81e^2) x^3 - \frac{1}{1250} (825d^2 - 810de - 458e^2) x^2 - \frac{1}{218750} \sqrt{14} (10575d^2 + 59890de - 18323e^2) \arctan\left(\frac{1}{14} \sqrt{14}(5x+1)\right) + \frac{1}{3125} (2025d^2 + 4580de - 881e^2) x + \frac{1}{15625} (5725d^2 - 4405de - 2554e^2) \log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="fricas")

[Out] 4/25\*e^2\*x^5 + 1/100\*(40\*d\*e - 33\*e^2)\*x^4 + 1/375\*(100\*d^2 - 330\*d\*e + 81\*e^2)\*x^3 - 1/1250\*(825\*d^2 - 810\*d\*e - 458\*e^2)\*x^2 - 1/218750\*sqrt(14)\*(10575\*d^2 + 59890\*d\*e - 18323\*e^2)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 1/3125\*(2025\*d^2 + 4580\*d\*e - 881\*e^2)\*x + 1/15625\*(5725\*d^2 - 4405\*d\*e - 2554\*e^2)\*log(5\*x^2 + 2\*x + 3)

**giac** [A] time = 0.16, size = 145, normalized size = 0.93

$$\frac{4}{25} x^5 e^2 + \frac{2}{5} d x^4 e + \frac{4}{15} d^2 x^3 - \frac{33}{100} x^4 e^2 - \frac{22}{25} d x^3 e - \frac{33}{50} d^2 x^2 + \frac{27}{125} x^3 e^2 + \frac{81}{125} d x^2 e + \frac{81}{125} d^2 x + \frac{229}{625} x^2 e^2 + \frac{916}{625} d x e - \frac{1}{218750} \sqrt{14} (10575d^2 + 59890de - 18323e^2) \arctan\left(\frac{1}{14} \sqrt{14}(5x+1)\right) - \frac{881}{3125} x e^2 + \frac{1}{15625} (5725d^2 - 4405de - 2554e^2) \log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="giac")

[Out] 4/25\*x^5\*e^2 + 2/5\*d\*x^4\*e + 4/15\*d^2\*x^3 - 33/100\*x^4\*e^2 - 22/25\*d\*x^3\*e - 33/50\*d^2\*x^2 + 27/125\*x^3\*e^2 + 81/125\*d\*x^2\*e + 81/125\*d^2\*x + 229/625\*x^2\*e^2 + 916/625\*d\*x\*e - 1/218750\*sqrt(14)\*(10575\*d^2 + 59890\*d\*e - 18323\*e^2)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) - 881/3125\*x\*e^2 + 1/15625\*(5725\*d^2 - 4405\*d\*e - 2554\*e^2)\*log(5\*x^2 + 2\*x + 3)

**maple** [A] time = 0.01, size = 191, normalized size = 1.22

$$\frac{4e^2x^5}{25} + \frac{2dex^4}{5} + \frac{33d^2x^3}{100} + \frac{4d^2x^3}{15} - \frac{22de^2x^3}{25} + \frac{27d^2x^3}{125} - \frac{33d^2x^2}{50} + \frac{81dex^2}{125} + \frac{229e^2x^2}{625} + \frac{81d^2x}{125} + \frac{423\sqrt{14}d^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750} + \frac{229d^2\ln(5x^2+2x+3)}{625} + \frac{916dex}{625} - \frac{5989\sqrt{14}de\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21875} - \frac{881de\ln(5x^2+2x+3)}{3125} - \frac{881e^2x}{3125} + \frac{18323\sqrt{14}e^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{218750} - \frac{2554e^2\ln(5x^2+2x+3)}{15625}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)`

[Out]  $4e^{2x}x^5/25 + x^4(2de/5 - 33e^2/100) + x^3(4d^2/15 - 22de/25 + 27e^2/125) + x^2(-33d^2/50 + 81de/125 + 229e^2/625) + x(81d^2/125 + 916de/625 - 881e^2/3125) + (229d^2/625 - 881de/3125 - 2554e^2/15625 - \sqrt{14}I(10575d^2 + 59890de - 18323e^2)/437500)\log(x + (2115d^2 + 11978de - 18323e^2/5 + \sqrt{14}I(10575d^2 + 59890de - 18323e^2)/5)/(10575d^2 + 59890de - 18323e^2)) + (229d^2/625 - 881de/3125 - 2554e^2/15625 + \sqrt{14}I(10575d^2 + 59890de - 18323e^2)/437500)\log(x + (2115d^2 + 11978de - 18323e^2/5 - \sqrt{14}I(10575d^2 + 59890de - 18323e^2)/5)/(10575d^2 + 59890de - 18323e^2))$

$$3.286 \quad \int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$$

**Optimal.** Leaf size=99

$$\frac{1}{75}x^3(20d-33e) - \frac{3}{250}x^2(55d-27e) + \frac{(2290d-881e)\log(5x^2+2x+3)}{6250} + \frac{1}{625}x(405d+458e) - \frac{(2115d+5989e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3125\sqrt{14}}$$

**Rubi [A]** time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$ , Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{75}x^3(20d-33e) - \frac{3}{250}x^2(55d-27e) + \frac{(2290d-881e)\log(5x^2+2x+3)}{6250} + \frac{1}{625}x(405d+458e) - \frac{(2115d+5989e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3125\sqrt{14}} + \frac{ex^4}{5}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

[Out] ((405\*d + 458\*e)\*x)/625 - (3\*(55\*d - 27\*e)\*x^2)/250 + ((20\*d - 33\*e)\*x^3)/75 + (e\*x^4)/5 - ((2115\*d + 5989\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(3125\*Sqrt[14]) + ((2290\*d - 881\*e)\*Log[3 + 2\*x + 5\*x^2])/6250

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

### Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_)*(x_))^{(m_)}*((a_.) + (b_)*(x_.) + (c_)*(x_.)^2)^{(p_)}], x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + ex)^m * Pq * (a + bx + cx^2)^p], x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex)(2 + x + 3x^2 - 5x^3 + 4x^4)}{3 + 2x + 5x^2} dx &= \int \left( \frac{1}{625}(405d + 458e) - \frac{3}{125}(55d - 27e)x + \frac{1}{25}(20d - 33e)x^2 + \frac{4ex}{5} \right. \\ &= \frac{1}{625}(405d + 458e)x - \frac{3}{250}(55d - 27e)x^2 + \frac{1}{75}(20d - 33e)x^3 + \frac{ex^4}{5} + \\ &= \frac{1}{625}(405d + 458e)x - \frac{3}{250}(55d - 27e)x^2 + \frac{1}{75}(20d - 33e)x^3 + \frac{ex^4}{5} + \\ &= \frac{1}{625}(405d + 458e)x - \frac{3}{250}(55d - 27e)x^2 + \frac{1}{75}(20d - 33e)x^3 + \frac{ex^4}{5} + \\ &= \frac{1}{625}(405d + 458e)x - \frac{3}{250}(55d - 27e)x^2 + \frac{1}{75}(20d - 33e)x^3 + \frac{ex^4}{5} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 0.87

$$\frac{21(2290d - 881e) \log(5x^2 + 2x + 3) + 35x(5d(200x^2 - 495x + 486) + 3e(250x^3 - 550x^2 + 405x + 916)) - 3\sqrt{14}(2115d + 5989e) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{131250}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

[Out] (35\*x\*(5\*d\*(486 - 495\*x + 200\*x^2) + 3\*e\*(916 + 405\*x - 550\*x^2 + 250\*x^3)) - 3\*Sqrt[14]\*(2115\*d + 5989\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]] + 21\*(2290\*d - 881\*e)\*Log[3 + 2\*x + 5\*x^2])/131250

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)(2 + x + 3x^2 - 5x^3 + 4x^4)}{3 + 2x + 5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2), x]

**fricas** [A] time = 0.80, size = 84, normalized size = 0.85

$$\frac{1}{5}ex^4 + \frac{1}{75}(20d - 33e)x^3 - \frac{3}{250}(55d - 27e)x^2 - \frac{1}{43750}\sqrt{14}(2115d + 5989e)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{1}{625}(405d + 458e)x + \frac{1}{6250}(2290d - 881e)\log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="fricas")

[Out] 1/5\*e\*x^4 + 1/75\*(20\*d - 33\*e)\*x^3 - 3/250\*(55\*d - 27\*e)\*x^2 - 1/43750\*sqrt(14)\*(2115\*d + 5989\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 1/625\*(405\*d + 458\*e)\*x + 1/6250\*(2290\*d - 881\*e)\*log(5\*x^2 + 2\*x + 3)

**giac** [A] time = 0.16, size = 88, normalized size = 0.89

$$\frac{1}{5}x^4e + \frac{4}{15}dx^3 - \frac{11}{25}x^3e - \frac{33}{50}dx^2 + \frac{81}{250}x^2e - \frac{1}{43750}\sqrt{14}(2115d + 5989e)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{81}{125}dx + \frac{458}{625}xe + \frac{1}{6250}(2290d - 881e)\log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="giac")

[Out] 1/5\*x^4\*e + 4/15\*d\*x^3 - 11/25\*x^3\*e - 33/50\*d\*x^2 + 81/250\*x^2\*e - 1/43750\*sqrt(14)\*(2115\*d + 5989\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 81/125\*d\*x + 458/625\*x\*e + 1/6250\*(2290\*d - 881\*e)\*log(5\*x^2 + 2\*x + 3)

**maple** [A] time = 0.00, size = 102, normalized size = 1.03

$$\frac{ex^4}{5} + \frac{4dx^3}{15} - \frac{11ex^3}{25} - \frac{33dx^2}{50} + \frac{81ex^2}{250} + \frac{81dx}{125} - \frac{423\sqrt{14}d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750} + \frac{229d\ln(5x^2 + 2x + 3)}{625} + \frac{458ex}{625} - \frac{5989\sqrt{14}e\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{43750} - \frac{881e\ln(5x^2 + 2x + 3)}{6250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x)

[Out] 1/5\*e\*x^4+4/15\*x^3\*d-11/25\*x^3\*e-33/50\*x^2\*d+81/250\*e\*x^2+81/125\*d\*x+458/625\*e\*x+229/625\*ln(5\*x^2+2\*x+3)\*d-881/6250\*e\*ln(5\*x^2+2\*x+3)-423/8750\*14^(1/2)\*arctan(1/28\*(10\*x+2)\*14^(1/2))\*d-5989/43750\*14^(1/2)\*arctan(1/28\*(10\*x+2)\*14^(1/2))\*e



**maxima [A]** time = 0.96, size = 84, normalized size = 0.85

$$\frac{1}{5}ex^4 + \frac{1}{75}(20d - 33e)x^3 - \frac{3}{250}(55d - 27e)x^2 - \frac{1}{43750}\sqrt{14}(2115d + 5989e)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{1}{625}(405d + 458e)x + \frac{1}{6250}(2290d - 881e)\log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3),x, algorithm="maxima")

[Out] 1/5\*e\*x^4 + 1/75\*(20\*d - 33\*e)\*x^3 - 3/250\*(55\*d - 27\*e)\*x^2 - 1/43750\*sqrt(14)\*(2115\*d + 5989\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 1/625\*(405\*d + 458\*e)\*x + 1/6250\*(2290\*d - 881\*e)\*log(5\*x^2 + 2\*x + 3)

**mupad [B]** time = 0.07, size = 107, normalized size = 1.08

$$x^3\left(\frac{4d}{15} - \frac{11e}{25}\right) - x^2\left(\frac{33d}{50} - \frac{81e}{250}\right) + \ln(5x^2 + 2x + 3)\left(\frac{229d}{625} - \frac{881e}{6250}\right) + \frac{ex^4}{5} + x\left(\frac{81d}{125} + \frac{458e}{625}\right) - \frac{\sqrt{14}\operatorname{atan}\left(\frac{\frac{\sqrt{14}(2115d+5989e)}{43750} + \frac{\sqrt{14}x(2115d+5989e)}{8750}}{\frac{423d}{625} + \frac{5989e}{3125}}\right)}{43750}(2115d + 5989e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(2\*x + 5\*x^2 + 3),x)

[Out] x^3\*((4\*d)/15 - (11\*e)/25) - x^2\*((33\*d)/50 - (81\*e)/250) + log(2\*x + 5\*x^2 + 3)\*((229\*d)/625 - (881\*e)/6250) + (e\*x^4)/5 + x\*((81\*d)/125 + (458\*e)/625) - (14^(1/2)\*atan(((14^(1/2)\*(2115\*d + 5989\*e))/43750 + (14^(1/2)\*x\*(2115\*d + 5989\*e))/8750)/((423\*d)/625 + (5989\*e)/3125))\*(2115\*d + 5989\*e)/43750

**sympy [C]** time = 0.85, size = 163, normalized size = 1.65

$$\frac{ex^4}{5} + x^3\left(\frac{4d}{15} - \frac{11e}{25}\right) + x^2\left(-\frac{33d}{50} + \frac{81e}{250}\right) + x\left(\frac{81d}{125} + \frac{458e}{625}\right) + \left(\frac{229d}{625} - \frac{881e}{6250} - \frac{\sqrt{14}i(2115d + 5989e)}{87500}\right)\log\left(x + \frac{423d + \frac{5989e}{5} + \frac{\sqrt{14}i(2115d + 5989e)}{5}}{2115d + 5989e}\right) + \left(\frac{229d}{625} - \frac{881e}{6250} + \frac{\sqrt{14}i(2115d + 5989e)}{87500}\right)\log\left(x + \frac{423d + \frac{5989e}{5} - \frac{\sqrt{14}i(2115d + 5989e)}{5}}{2115d + 5989e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3),x)

[Out] e\*x\*\*4/5 + x\*\*3\*(4\*d/15 - 11\*e/25) + x\*\*2\*(-33\*d/50 + 81\*e/250) + x\*(81\*d/125 + 458\*e/625) + (229\*d/625 - 881\*e/6250 - sqrt(14)\*I\*(2115\*d + 5989\*e)/87500)\*log(x + (423\*d + 5989\*e/5 + sqrt(14)\*I\*(2115\*d + 5989\*e)/5)/(2115\*d + 5989\*e)) + (229\*d/625 - 881\*e/6250 + sqrt(14)\*I\*(2115\*d + 5989\*e)/87500)\*log(x + (423\*d + 5989\*e/5 - sqrt(14)\*I\*(2115\*d + 5989\*e)/5)/(2115\*d + 5989\*e))

$$3.287 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} dx$$

Optimal. Leaf size=56

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{229}{625} \log(5x^2 + 2x + 3) + \frac{81x}{125} - \frac{423 \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

**Rubi [A]** time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{229}{625} \log(5x^2 + 2x + 3) + \frac{81x}{125} - \frac{423 \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2), x]

[Out] (81\*x)/125 - (33\*x^2)/50 + (4\*x^3)/15 - (423\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(625\*Sqrt[14]) + (229\*Log[3 + 2\*x + 5\*x^2])/625

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$\text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 1657

$\text{Int}[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow \text{Int}[\text{Expand}[\text{Integrand}[Pq*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

### Rubi steps

$$\begin{aligned} \int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{3 + 2x + 5x^2} dx &= \int \left( \frac{81}{125} - \frac{33x}{25} + \frac{4x^2}{5} + \frac{7 + 458x}{125(3 + 2x + 5x^2)} \right) dx \\ &= \frac{81x}{125} - \frac{33x^2}{50} + \frac{4x^3}{15} + \frac{1}{125} \int \frac{7 + 458x}{3 + 2x + 5x^2} dx \\ &= \frac{81x}{125} - \frac{33x^2}{50} + \frac{4x^3}{15} + \frac{229}{625} \int \frac{2 + 10x}{3 + 2x + 5x^2} dx - \frac{423}{625} \int \frac{1}{3 + 2x + 5x^2} dx \\ &= \frac{81x}{125} - \frac{33x^2}{50} + \frac{4x^3}{15} + \frac{229}{625} \log(3 + 2x + 5x^2) + \frac{846}{625} \text{Subst} \left( \int \frac{1}{-56 - x^2} dx, x, \right. \\ &= \frac{81x}{125} - \frac{33x^2}{50} + \frac{4x^3}{15} - \frac{423 \tan^{-1} \left( \frac{1+5x}{\sqrt{14}} \right)}{625\sqrt{14}} + \frac{229}{625} \log(3 + 2x + 5x^2) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 0.89

$$\frac{35x(200x^2 - 495x + 486) + 9618 \log(5x^2 + 2x + 3) - 1269\sqrt{14} \tan^{-1} \left( \frac{5x+1}{\sqrt{14}} \right)}{26250}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2), x]

[Out] (35\*x\*(486 - 495\*x + 200\*x^2) - 1269\*sqrt[14]\*ArcTan[(1 + 5\*x)/sqrt[14]] + 9618\*Log[3 + 2\*x + 5\*x^2])/26250

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{3 + 2x + 5x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2), x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2), x]

**fricas** [A] time = 0.75, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{33}{50}x^2 - \frac{423}{8750}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{81}{125}x + \frac{229}{625}\log(5x^2+2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="fricas")

[Out] 4/15\*x^3 - 33/50\*x^2 - 423/8750\*sqrt(14)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 81/125\*x + 229/625\*log(5\*x^2 + 2\*x + 3)

**giac** [A] time = 0.16, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{33}{50}x^2 - \frac{423}{8750}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{81}{125}x + \frac{229}{625}\log(5x^2+2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x, algorithm="giac")

[Out] 4/15\*x^3 - 33/50\*x^2 - 423/8750\*sqrt(14)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 81/125\*x + 229/625\*log(5\*x^2 + 2\*x + 3)

**maple** [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125} - \frac{423\sqrt{14}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750} + \frac{229\ln(5x^2+2x+3)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3), x)

[Out] 4/15\*x^3-33/50\*x^2+81/125\*x+229/625\*ln(5\*x^2+2\*x+3)-423/8750\*14^(1/2)\*arctan(1/28\*(10\*x+2)\*14^(1/2))

**maxima** [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}x^3 - \frac{33}{50}x^2 - \frac{423}{8750}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{81}{125}x + \frac{229}{625}\log(5x^2+2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3),x, algorithm="maxima")

[Out]  $\frac{4}{15}x^3 - \frac{33}{50}x^2 - \frac{423}{8750}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{81}{125}x + \frac{229}{625}\log(5x^2+2x+3)$

**mupad [B]** time = 0.04, size = 45, normalized size = 0.80

$$\frac{81x}{125} + \frac{229 \ln(5x^2 + 2x + 3)}{625} - \frac{423\sqrt{14} \operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{8750} - \frac{33x^2}{50} + \frac{4x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2)/(2\*x + 5\*x^2 + 3),x)

[Out]  $\frac{(81*x)}{125} + \frac{(229*\log(2*x + 5*x^2 + 3))}{625} - \frac{(423*14^{(1/2)}*\operatorname{atan}((5*14^{(1/2)})*x)/14 + 14^{(1/2)}/14))}{8750} - \frac{(33*x^2)}{50} + \frac{(4*x^3)}{15}$

**sympy [A]** time = 0.23, size = 61, normalized size = 1.09

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125} + \frac{229 \log\left(x^2 + \frac{2x}{5} + \frac{3}{5}\right)}{625} - \frac{423\sqrt{14} \operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3),x)

[Out]  $4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*\log(x**2 + 2*x/5 + 3/5)/625 - 423*\operatorname{sqrt}(14)*\operatorname{atan}(5*\operatorname{sqrt}(14)*x/14 + \operatorname{sqrt}(14)/14)/8750$

$$3.288 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)} dx$$

**Optimal.** Leaf size=168

$$\frac{(458d - 7e) \log(5x^2 + 2x + 3)}{250(5d^2 - 2de + 3e^2)} - \frac{(423d - 1367e) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}(5d^2 - 2de + 3e^2)} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)} - \frac{x(20d + 33e)}{25e^2} + \frac{2x^2}{5e}$$

**Rubi [A]** time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1628, 634, 618, 204, 628}

$$\frac{(458d - 7e) \log(5x^2 + 2x + 3)}{250(5d^2 - 2de + 3e^2)} + \frac{(3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)} - \frac{(423d - 1367e) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}(5d^2 - 2de + 3e^2)} - \frac{x(20d + 33e)}{25e^2} + \frac{2x^2}{5e}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)), x]

[Out] -((20\*d + 33\*e)\*x)/(25\*e^2) + (2\*x^2)/(5\*e) - ((423\*d - 1367\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(125\*Sqrt[14]\*(5\*d^2 - 2\*d\*e + 3\*e^2)) + ((4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x])/(e^3\*(5\*d^2 - 2\*d\*e + 3\*e^2)) + ((458\*d - 7\*e)\*Log[3 + 2\*x + 5\*x^2])/(250\*(5\*d^2 - 2\*d\*e + 3\*e^2))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned} \int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)(3 + 2x + 5x^2)} dx &= \int \left( \frac{-20d - 33e}{25e^2} + \frac{4x}{5e} + \frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{e^2(5d^2 - 2de + 3e^2)(d + ex)} + \frac{7d + 272e + (423d - 1367e)}{25(5d^2 - 2de + 3e^2)} \right) dx \\ &= -\frac{(20d + 33e)x}{25e^2} + \frac{2x^2}{5e} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)} + \frac{\int \frac{7d + 272e + (423d - 1367e)}{25(5d^2 - 2de + 3e^2)} dx}{25(5d^2 - 2de + 3e^2)} \\ &= -\frac{(20d + 33e)x}{25e^2} + \frac{2x^2}{5e} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)} - \frac{(423d - 1367e) \tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}(5d^2 - 2de + 3e^2)} \\ &= -\frac{(20d + 33e)x}{25e^2} + \frac{2x^2}{5e} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)} + \frac{(458d - 7e) \log(5x^2 + 2x + 3)}{25e^3(5d^2 - 2de + 3e^2)} \\ &= -\frac{(20d + 33e)x}{25e^2} + \frac{2x^2}{5e} - \frac{(423d - 1367e) \tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}(5d^2 - 2de + 3e^2)} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 146, normalized size = 0.87

$$\frac{70ex(5d^2 - 2de + 3e^2)(e(10x - 33) - 20d) + 1750(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(d + ex) + 7e^3(458d - 7e) \log(5x^2 + 2x + 3) - \sqrt{14}e^3(423d - 1367e) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1750e^3(5d^2 - 2de + 3e^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)), x]
[Out] (70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
- 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^2])
/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)(3 + 2x + 5x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)), x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)), x]

**fricas [A]** time = 0.80, size = 171, normalized size = 1.02

$$\frac{700(5d^2e^2 - 2de^3 + 3e^4)x^2 - \sqrt{14}(423de^3 - 1367e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - 70(100d^3e + 125d^2e^2 - 6de^3 + 99e^4)x + 1750(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log(ex+d) + 7(458de^3 - 7e^4)\log(5x^2 + 2x + 3)}{1750(5d^2e^3 - 2de^4 + 3e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)/(5\*x^2+2\*x+3), x, algorithm="fricas")

[Out] 1/1750\*(700\*(5\*d^2\*e^2 - 2\*d\*e^3 + 3\*e^4)\*x^2 - sqrt(14)\*(423\*d\*e^3 - 1367\*e^4)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) - 70\*(100\*d^3\*e + 125\*d^2\*e^2 - 6\*d\*e^3 + 99\*e^4)\*x + 1750\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*log(e\*x + d) + 7\*(458\*d\*e^3 - 7\*e^4)\*log(5\*x^2 + 2\*x + 3))/(5\*d^2\*e^3 - 2\*d\*e^4 + 3\*e^5)

**giac [A]** time = 0.22, size = 158, normalized size = 0.94

$$\frac{1}{25}(10x^2e - 20dx - 33xe)e^{(-2)} - \frac{\sqrt{14}(423d - 1367e)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{1750(5d^2 - 2de + 3e^2)} + \frac{(458d - 7e)\log(5x^2 + 2x + 3)}{250(5d^2 - 2de + 3e^2)} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)\log((xe + d))}{5d^2e^3 - 2de^4 + 3e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)/(5\*x^2+2\*x+3), x, algorithm="giac")

[Out] 1/25\*(10\*x^2\*e - 20\*d\*x - 33\*x\*e)\*e^(-2) - 1/1750\*sqrt(14)\*(423\*d - 1367\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1))/(5\*d^2 - 2\*d\*e + 3\*e^2) + 1/250\*(458\*d - 7\*e)\*log(5\*x^2 + 2\*x + 3)/(5\*d^2 - 2\*d\*e + 3\*e^2) + (4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*log(abs(x\*e + d))/(5\*d^2\*e^3 - 2\*d\*e^4 + 3\*e^5)

**maple [A]** time = 0.01, size = 298, normalized size = 1.77

$$\frac{4d^4 \ln(ex+d)}{(5d^2 - 2de + 3e^2)e^3} + \frac{5d^3 \ln(ex+d)}{(5d^2 - 2de + 3e^2)e^2} + \frac{3d^2 \ln(ex+d)}{(5d^2 - 2de + 3e^2)e} - \frac{423\sqrt{14}d \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{70(125d^2 - 50de + 75e^2)} - \frac{d \ln(ex+d)}{5d^2 - 2de + 3e^2} + \frac{229d \ln(5x^2 + 2x + 3)}{5(125d^2 - 50de + 75e^2)} + \frac{1367\sqrt{14}e \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{70(125d^2 - 50de + 75e^2)} + \frac{2e \ln(ex+d)}{5d^2 - 2de + 3e^2} - \frac{7e \ln(5x^2 + 2x + 3)}{10(125d^2 - 50de + 75e^2)} + \frac{2x^2}{5e} - \frac{4dx}{5e^2} - \frac{33x}{25e}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3), x)$

[Out]  $\frac{2}{5}e^{-x}d^{-4} - \frac{4}{5}d^{-5}e^{-2x} - \frac{33}{25}e^{-x} + \frac{229}{5} \ln(125d^2 - 50de + 75e^2) \ln(5x^2 + 2x + 3) - \frac{7}{10} \ln(125d^2 - 50de + 75e^2) \ln(5x^2 + 2x + 3) - \frac{423}{70} \ln(125d^2 - 50de + 75e^2) \ln(14^{1/2}) \arctan(1/28(10x+2)14^{1/2})d + \frac{1367}{70} \ln(125d^2 - 50de + 75e^2) \ln(14^{1/2}) \arctan(1/28(10x+2)14^{1/2})e + \frac{4}{e^3} (5d^2 - 2de + 3e^2) \ln(e*x+d) d^4 + \frac{5}{e^2} (5d^2 - 2de + 3e^2) \ln(e*x+d) d^3 + \frac{3}{e} (5d^2 - 2de + 3e^2) \ln(e*x+d) d^2 - \frac{1}{(5d^2 - 2de + 3e^2) \ln(e*x+d)} d + \frac{2e}{(5d^2 - 2de + 3e^2) \ln(e*x+d)}$

**maxima [A]** time = 0.96, size = 160, normalized size = 0.95

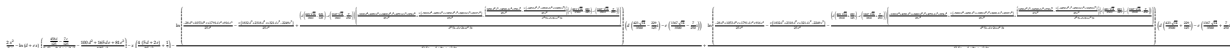
$$-\frac{\sqrt{14}(423d - 1367e) \arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{1750(5d^2 - 2de + 3e^2)} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(ex + d)}{5d^2e^3 - 2de^4 + 3e^5} + \frac{(458d - 7e) \log(5x^2 + 2x + 3)}{250(5d^2 - 2de + 3e^2)} + \frac{10ex^2 - (20d + 33e)x}{25e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3), x, \text{algorithm}="maxima")$

[Out]  $-\frac{1}{1750} \sqrt{14} (423d - 1367e) \arctan(1/14 \sqrt{14} (5x + 1)) / (5d^2 - 2de + 3e^2) + (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(ex + d) / (5d^2e^3 - 2de^4 + 3e^5) + 1/250 (458d - 7e) \log(5x^2 + 2x + 3) / (5d^2 - 2de + 3e^2) + 1/25 (10ex^2 - (20d + 33e)x) / e^2$

**mupad [B]** time = 6.39, size = 713, normalized size = 4.24



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)), x)$

[Out]  $\frac{2x^2}{5e} - \log(d + ex) \left( \frac{(458d)/125 - (7e)/125}{(5d^2 - 2de + 3e^2)} - \frac{(165de + 100d^2 + 81e^2)/(125e^3)}{(5d^2 - 2de + 3e^2)} - \frac{x((4(5d + 2e))/(25e^2) + 1/e)}{(5d^2 - 2de + 3e^2)} - \frac{\log((1791de^2 + 1053d^2e - 28d^3 + 916e^3)/(25e^2))}{(5d^2 - 2de + 3e^2)} - \frac{(x(321de^2 + 2318d^2e + 1832d^3 - 2249e^3))/(25e^2) + (d((423 \cdot 14^{1/2}))/3500 - 229i/125) - e((1367 \cdot 14^{1/2}))/3500 - 7i/250}{(5d^2 - 2de + 3e^2)} \right) + \frac{(4751de^3 + 4350d^3e - 1000d^4 + 874e^4 + 8490d^2e^2)/(25e^2) + (x(8200de^3 - 6250d^3e - 5000d^4 + 2917e^4 + 1850d^2e^2))/(25e^2) - ((750e^5 - 14500de^4 + 1250d^2e^3)/(25e^2) - (x(2500de^4 + 10250e^5 - 6250d^2e^3))/(25e^2)) \cdot (d((423 \cdot 14^{1/2}))/3500 - 229i/125) - e((1367 \cdot 14^{1/2}))/3500 - 7i/250}{(d^2 \cdot 5i - d \cdot e \cdot 2i + e^2 \cdot 3i)} + \frac{\log((1791de^2 + 1053d^2e - 28d^3 + 916e^3)/$

$$(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) - ((d*((423*14^{(1/2)})/3500 + 229i/125) - e*((1367*14^{(1/2)})/3500 + 7i/250))*((4751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) + ((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^{(1/2)})/3500 + 229i/125) - e*((1367*14^{(1/2)})/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^2*3i))*((423*14^{(1/2)})/3500 + 229i/125) - e*((1367*14^{(1/2)})/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)/(5\*x\*\*2+2\*x+3),x)

[Out] Timed out

$$3.289 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)} dx$$

**Optimal.** Leaf size=233

$$\frac{(229d^2 - 7de - 136e^2) \log(5x^2 + 2x + 3)}{25(5d^2 - 2de + 3e^2)^2} - \frac{(423d^2 - 2734de + 293e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2 - 2de + 3e^2)^2} - \frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + e^4}{e^3(5d^2 - 2de + 3e^2)(d + ex)}$$

**Rubi [A]** time = 0.25, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1628, 634, 618, 204, 628}

$$\frac{(229d^2 - 7de - 136e^2) \log(5x^2 + 2x + 3)}{25(5d^2 - 2de + 3e^2)^2} - \frac{3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4}{e^3(5d^2 - 2de + 3e^2)(d + ex)} - \frac{(28d^3e^2 + 44d^2e^3 + d^4e + 40d^5 - 2de^4 + e^5) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)^2} - \frac{(423d^2 - 2734de + 293e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2 - 2de + 3e^2)^2} + \frac{4x}{5e^2}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)),x]

[Out] (4\*x)/(5\*e^2) - (4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)/(e^3\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(d + e\*x)) - ((423\*d^2 - 2734\*d\*e + 293\*e^2)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(25\*Sqrt[14]\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2) - ((40\*d^5 + d^4\*e + 28\*d^3\*e^2 + 44\*d^2\*e^3 - 2\*d\*e^4 + e^5)\*Log[d + e\*x])/(e^3\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2) + ((229\*d^2 - 7\*d\*e - 136\*e^2)\*Log[3 + 2\*x + 5\*x^2])/(25\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2)

#### Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)} dx &= \int \left( \frac{4}{5e^2} + \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^2} + \frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4}{e^2(5d^2-2de+3e^2)^2(d+ex)} \right) dx \\ &= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4)}{e^3(5d^2-2de+3e^2)^2} \\ &= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4)}{e^3(5d^2-2de+3e^2)^2} \\ &= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4)}{e^3(5d^2-2de+3e^2)^2} \\ &= \frac{4x}{5e^2} - \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(423d^2-2734de+293e^2) \tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 233, normalized size = 1.00

$$\frac{(229d^2-7de-136e^2) \log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2} + \frac{(-423d^2+2734de-293e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2} + \frac{-4d^4-5d^3e-3d^2e^2-de^3-2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} + \frac{(-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5) \log(d+ex)}{e^3(5d^2-2de+3e^2)^2} + \frac{4x}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)), x]
```

[Out]  $(4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2 - 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^4*e - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2 - 2*d*e + 3*e^2)^2)$

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^2 (3 + 2x + 5x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)), x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)), x]

**fricas** [A] time = 0.98, size = 416, normalized size = 1.79

$\frac{7000e^6 + 5950e^5 + 5950e^4 + 1400e^3 + 7350e^2 - 2450e + 2100 - 280(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)\sqrt{14} + \sqrt{14}(423d^3e^3 - 2734d^2e^4 + 293de^5 + (423d^2e^4 - 2734de^5 + 293e^6)x)\arctan\left(\frac{1}{14}\sqrt{14}(5d + e)\right) - 280(25d^5e - 20d^4e^2 + 34d^3e^3 - 12d^2e^4 + 9de^5)x + 350(40d^6 + d^5e + 28d^4e^2 + 44d^3e^3 - 2d^2e^4 + de^5 + (40d^5e + d^4e^2 + 28d^3e^3 + 44d^2e^4 - 2de^5 + e^6)x)\log(ex + d) - 14(229d^3e^3 - 7d^2e^4 - 136de^5 + (229d^2e^4 - 7de^5 - 136e^6)x)\log(5x^2 + 2x + 3)}{25(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3), x, algorithm="fricas")

[Out]  $-1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4 - 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*e^5 + 9*e^6)*x^2 + \text{sqrt}(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (423*d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*\arctan(1/14*\text{sqrt}(14)*(5*x + 1)) - 280*(25*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*d^6 + d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*\log(e*x + d) - 14*(229*d^3*e^3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*\log(5*x^2 + 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9*d*e^7 + (25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)$

**giac** [A] time = 0.18, size = 355, normalized size = 1.52

$\frac{1}{25}(40d + 33e)^{e^{-3}} \log\left(\frac{(xe + d)e^{d-1}}{(xe + d)^2}\right) - \frac{\sqrt{14}(423d^3e^3 - 2734d^2e^4 + 293de^5)\arctan\left(\frac{1}{14}\sqrt{14}\left(5d + \frac{5d^2}{xe+d} + \frac{2d^2}{xe+d} - \frac{3d^2}{xe+d} - e\right)e^{d-1}\right)}{350(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)} + \frac{4}{5}(xe + d)e^{e^{-3}} + \frac{(229d^2 - 7de - 136e^2)\log\left(\frac{10d}{xe+d} + \frac{5d^2}{(xe+d)^2} + \frac{2e}{xe+d} - \frac{2de}{(xe+d)^2} + \frac{3e^2}{(xe+d)^2} + 5\right)}{25(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)} - \frac{4d^2e^2 + \frac{3d^3e^3}{xe+d} + \frac{3d^2e^2}{xe+d} - \frac{de^3}{xe+d} + \frac{2e^2}{xe+d}}{5d^2e^2 - 2de^3 + 3e^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3),x, algorithm="giac")

[Out]  $\frac{1}{25}(40d + 33e)e^{-3} \log(\text{abs}(xe + d))e^{-1} / (xe + d)^2 - \frac{1}{350} \sqrt{14} (423d^2e^2 - 2734d^3e + 293e^4) \arctan\left(\frac{1}{14} \sqrt{14} (5d - 5d^2 / (xe + d) + 2de / (xe + d) - 3e^2 / (xe + d) - e) e^{-1}\right) e^{-2} / (25d^4 - 20d^3e + 34d^2e^2 - 12d^3e + 9e^4) + \frac{4}{5} (xe + d) e^{-3} + \frac{1}{25} (229d^2 - 7de - 136e^2) \log(-10d / (xe + d) + 5d^2 / (xe + d)^2 + 2e / (xe + d) - 2de / (xe + d)^2 + 3e^2 / (xe + d)^2 + 5) / (25d^4 - 20d^3e + 34d^2e^2 - 12d^3e + 9e^4) - (4d^4e^3 / (xe + d) + 5d^3e^4 / (xe + d) + 3d^2e^5 / (xe + d) - de^6 / (xe + d) + 2e^7 / (xe + d)) / (5d^2e^6 - 2de^7 + 3e^8)$

**maple [B]** time = 0.02, size = 538, normalized size = 2.31

$$\frac{4d^2 \ln(x+d)}{(d^2-2d+e)^2} - \frac{d \ln(x+d)}{(d^2-2d+e)^2} - \frac{2d^2 \ln(x+d)}{(d^2-2d+e)^2} - \frac{423\sqrt{14} \arctan\left(\frac{14d-2d^2}{14d-2d^2+2d+1}\right)}{350(d^2-2d+e)^2} - \frac{44d^2 \ln(x+d)}{(d^2-2d+e)^2} - \frac{229d^2 \ln(5d^2+2x+3)}{25(d^2-2d+e)^2} - \frac{120\sqrt{14} \arctan\left(\frac{14d-2d^2}{14d-2d^2+2d+1}\right)}{175(d^2-2d+e)^2} - \frac{3d \ln(x+d)}{(d^2-2d+e)^2} - \frac{3d \ln(5d^2+2x+3)}{25(d^2-2d+e)^2} - \frac{20\sqrt{14} \arctan\left(\frac{14d-2d^2}{14d-2d^2+2d+1}\right)}{350(d^2-2d+e)^2} - \frac{d^2 \ln(x+d)}{(d^2-2d+e)^2} - \frac{136d \ln(5d^2+2x+3)}{25(d^2-2d+e)^2} - \frac{4e^4}{(d^2-2d+e)(d+e)^2} - \frac{e^4}{(d^2-2d+e)(d+e)^2} - \frac{e^4}{(d^2-2d+e)(d+e)^2} - \frac{e}{(d^2-2d+e)(d+e)} - \frac{4}{(d^2-2d+e)(d+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3),x)

[Out]  $\frac{4}{5} e^{-2} x + \frac{229}{25} (5d^2 - 2d^2e + 3e^2)^{-2} \ln(5x^2 + 2x + 3) d^2 - \frac{7}{25} (5d^2 - 2d^2e + 3e^2)^{-2} \ln(5x^2 + 2x + 3) d^2 e - \frac{136}{25} (5d^2 - 2d^2e + 3e^2)^{-2} \ln(5x^2 + 2x + 3) e^2 - \frac{423}{350} (5d^2 - 2d^2e + 3e^2)^{-2} 14^{1/2} \arctan\left(\frac{1}{28} (10x + 2) 14^{1/2}\right) d^2 + \frac{1367}{175} (5d^2 - 2d^2e + 3e^2)^{-2} 14^{1/2} \arctan\left(\frac{1}{28} (10x + 2) 14^{1/2}\right) d^2 e - \frac{293}{350} (5d^2 - 2d^2e + 3e^2)^{-2} 14^{1/2} \arctan\left(\frac{1}{28} (10x + 2) 14^{1/2}\right) e^2 - \frac{4}{e^3} (5d^2 - 2d^2e + 3e^2) / (ex + d) d^4 - \frac{5}{e^2} (5d^2 - 2d^2e + 3e^2) / (ex + d) d^3 - \frac{3}{e} (5d^2 - 2d^2e + 3e^2) / (ex + d) d^2 + \frac{1}{(5d^2 - 2d^2e + 3e^2)} / (ex + d) d - \frac{2}{(5d^2 - 2d^2e + 3e^2)} / (ex + d) - \frac{40}{e^3} (5d^2 - 2d^2e + 3e^2)^{-2} \ln(ex + d) d^5 - \frac{1}{e^2} (5d^2 - 2d^2e + 3e^2)^{-2} \ln(ex + d) d^4 - \frac{28}{e} (5d^2 - 2d^2e + 3e^2)^{-2} \ln(ex + d) d^3 - \frac{44}{(5d^2 - 2d^2e + 3e^2)^{-2} \ln(ex + d) d^2 + \frac{2}{(5d^2 - 2d^2e + 3e^2)^{-2} \ln(ex + d) d - \frac{e^2}{(5d^2 - 2d^2e + 3e^2)^{-2} \ln(ex + d)}$

**maxima [A]** time = 0.98, size = 294, normalized size = 1.26

$$\frac{\sqrt{14} (423d^2 - 2734de + 293e^2) \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right)}{350 (25d^4 - 20d^3e + 34d^2e^2 - 12d^3e + 9e^4)} - \frac{(40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5) \log(ex + d)}{25d^4e^3 - 20d^3e^4 + 34d^2e^5 - 12de^6 + 9e^7} + \frac{(229d^2 - 7de - 136e^2) \log(5x^2 + 2x + 3)}{25 (25d^4 - 20d^3e + 34d^2e^2 - 12d^3e + 9e^4)} - \frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{5d^3e^3 - 2d^2e^4 + 3de^5 + (5d^2e^4 - 2de^5 + 3e^6)x} + \frac{4x}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3),x, algorithm="maxima")

[Out]  $-\frac{1}{350} \sqrt{14} (423d^2 - 2734d^3e + 293e^4) \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) / (25d^4 - 20d^3e + 34d^2e^2 - 12d^3e + 9e^4) - (40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2d^4e + e^5) \log(ex + d) / (25d^4e^3 - 20d^3e^4 + 34d^2e^5 - 12d^3e^6 + 9e^7) + \frac{1}{25} (229d^2 - 7de - 136e^2) \log(5x^2 + 2x + 3) / (25d^4 - 20d^3e + 34d^2e^2 - 12d^3e + 9e^4) - (4$

$$*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 + 3*d*e^5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2$$

**mupad [B]** time = 4.67, size = 312, normalized size = 1.34

$$\frac{4x}{5e^2} \frac{\ln\left(x + \frac{1}{5} - \frac{\sqrt{11}}{5}\right) \left(\frac{423\sqrt{11}}{700} - \frac{229}{25}\right) d^2 + \left(\frac{1367\sqrt{11}}{350} + \frac{7}{25}\right) de + \left(\frac{293\sqrt{11}}{700} + \frac{136}{25}\right) e^2}{d^4 25i - d^3 e 20i + d^2 e^2 34i - d e^3 12i + e^4 9i} + \frac{\ln\left(x + \frac{1}{5} + \frac{\sqrt{11}}{5}\right) \left(\frac{423\sqrt{11}}{700} + \frac{229}{25}\right) d^2 + \left(\frac{1367\sqrt{11}}{350} - \frac{7}{25}\right) de + \left(\frac{293\sqrt{11}}{700} - \frac{136}{25}\right) e^2}{d^4 25i - d^3 e 20i + d^2 e^2 34i - d e^3 12i + e^4 9i} - \frac{5(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e(5xe^3 + 5de^2)} \frac{\ln(dx)}{e^3(5d^2 - 2de + 3e^2)} \frac{(40d^4 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5)}{e^3(5d^2 - 2de + 3e^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2)/((d + e\*x)^2\*(2\*x + 5\*x^2 + 3)),x)

[Out] (4\*x)/(5\*e^2) - (log(x - (14^(1/2)\*1i)/5 + 1/5)\*(d^2\*((423\*14^(1/2))/700 - 229i/25) + e^2\*((293\*14^(1/2))/700 + 136i/25) - d\*e\*((1367\*14^(1/2))/350 - 7i/25)))/(d^4\*25i - d^3\*e\*20i - d\*e^3\*12i + e^4\*9i + d^2\*e^2\*34i) + (log(x + (14^(1/2)\*1i)/5 + 1/5)\*(d^2\*((423\*14^(1/2))/700 + 229i/25) + e^2\*((293\*14^(1/2))/700 - 136i/25) - d\*e\*((1367\*14^(1/2))/350 + 7i/25)))/(d^4\*25i - d^3\*e\*20i - d\*e^3\*12i + e^4\*9i + d^2\*e^2\*34i) - (5\*(5\*d^3\*e - d\*e^3 + 4\*d^4 + 2\*e^4 + 3\*d^2\*e^2))/(e\*(5\*d\*e^2 + 5\*e^3\*x)\*(5\*d^2 - 2\*d\*e + 3\*e^2)) - (log(d + e\*x)\*(d^4\*e - 2\*d\*e^4 + 40\*d^5 + e^5 + 44\*d^2\*e^3 + 28\*d^3\*e^2))/(e^3\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*2/(5\*x\*\*2+2\*x+3),x)

[Out] Timed out

$$3.290 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)} dx$$

**Optimal.** Leaf size=317

$$\frac{(458d^3 - 21d^2e - 816de^2 + 113e^3) \log(5x^2 + 2x + 3)}{10(5d^2 - 2de + 3e^2)^3} - \frac{(423d^3 - 4101d^2e + 879de^2 + 703e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{5\sqrt{14}(5d^2 - 2de + 3e^2)^3} - \frac{4d^4 + 2e^5}{2e^3(5d^2 - 2de + 3e^2)^3}$$

**Rubi [A]** time = 0.29, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1628, 634, 618, 204, 628}

$$\frac{(-21d^2e + 458d^3 - 816d^2e + 113e^3) \log(5x^2 + 2x + 3)}{10(5d^2 - 2de + 3e^2)^3} - \frac{3d^2e^2 + 5d^3e + 4d^4 - d^2e^3 + 2e^4}{2e^3(5d^2 - 2de + 3e^2)(d + ex)^2} + \frac{28d^2e^2 + 44d^2e^3 + d^4e + 40d^5 - 2de^4 + e^5}{e^3(5d^2 - 2de + 3e^2)(d + ex)} + \frac{(228d^4e^2 - 242d^3e^3 + 141d^2e^4 - 120d^2e + 100d^6 + 120de^5 - e^6) \log(d + ex)}{e^3(5d^2 - 2de + 3e^2)^3} - \frac{(-4101d^2e + 423d^3 + 879de^2 + 703e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{5\sqrt{14}(5d^2 - 2de + 3e^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^3\*(3 + 2\*x + 5\*x^2)), x]

[Out]  $-(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e^3*(5*d^2 - 2*d*e + 3*e^2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*d*e + 3*e^2)^3)$

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]



Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rule 1628

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^3(3 + 2x + 5x^2)} dx &= \int \left( \frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{e^2(5d^2 - 2de + 3e^2)(d + ex)^3} + \frac{-40d^5 - d^4e - 28d^3e^2 - 44d^2e^3 + 2de^4 - e^5}{e^2(5d^2 - 2de + 3e^2)^2(d + ex)^2} \right. \\ &= -\frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{2e^3(5d^2 - 2de + 3e^2)(d + ex)^2} + \frac{40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5}{e^3(5d^2 - 2de + 3e^2)^2(d + ex)} \\ &= -\frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{2e^3(5d^2 - 2de + 3e^2)(d + ex)^2} + \frac{40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5}{e^3(5d^2 - 2de + 3e^2)^2(d + ex)} \\ &= -\frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{2e^3(5d^2 - 2de + 3e^2)(d + ex)^2} + \frac{40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5}{e^3(5d^2 - 2de + 3e^2)^2(d + ex)} \\ &= -\frac{4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4}{2e^3(5d^2 - 2de + 3e^2)(d + ex)^2} + \frac{40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5}{e^3(5d^2 - 2de + 3e^2)^2(d + ex)} \end{aligned}$$

**Mathematica [A]** time = 0.43, size = 278, normalized size = 0.88

$$\frac{-7(458d^3 - 21d^2e - 816de^2 + 113e^3) \log(5x^2 + 2x + 3) + \sqrt{14} (423d^3 - 4101d^2e + 879de^2 + 703e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) + \frac{35(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)(5d^2 - 2de + 3e^2)^2}{e^3(d+ex)^2} - \frac{70(40d^5 + d^4e + 28d^3e^2 + 44d^2e^3 - 2de^4 + e^5)(5d^2 - 2de + 3e^2)}{e^3(d+ex)} + \frac{70(-100d^6 + 120d^5e - 228d^4e^2 + 242d^3e^3 - 141d^2e^4 - 120de^5 + e^6) \log(d+ex)}{e^6}}{70(5d^2 - 2de + 3e^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^3\*(3 + 2\*x + 5\*x^2)), x]

```
[Out] -1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(423*d
^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70*(-1
00*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e^5 +
e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3
+ 2*x + 5*x^2)]/(5*d^2 - 2*d*e + 3*e^2)^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^3 (3 + 2x + 5x^2)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x +
5*x^2)), x]
```

```
[Out] IntegrateAlgebraic[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x +
5*x^2)), x]
```

**fricas [B]** time = 1.27, size = 698, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3), x, algorithm="fri
cas")
```

```
[Out] 1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^4
+ 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^5*
e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2*e^
6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*e^6
+ 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d^6*e^
2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 + 3*e^8)
*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e^4 + 12
0*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 242*d^3*e^
5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^2 + 228*d
^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(e*x + d) +
7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*d^3*e^5 - 2
1*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*e^5 - 816*d^
2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150*d^7*e^4 + 28
5*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e^9 + (125*d^6*
e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9 - 54*d*e^10 + 2
7*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 - 188*d^4*e^7 + 17
1*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

**giac [A]** time = 0.18, size = 406, normalized size = 1.28

$$\frac{\sqrt{14}(423d^3 - 4101d^2e + 879d^2e^2 + 703e^3) \arctan\left(\frac{1}{14}\sqrt{14}(x+1)\right)}{70(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54d^2e^5 + 27e^6)} \frac{(408d^3 - 21d^2e - 816d^2 + 113e^3) \log(5x^2 + 2x + 3)}{10(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54d^2e^5 + 27e^6)} \frac{(100d^6 - 120d^5e + 228d^4e^2 - 242d^3e^3 + 141d^2e^4 + 120d^2e^5 - e^6) \log(|xe + d|)}{125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54d^2e^5 + 27e^6} \frac{(5(200d^7 - 75d^6e + 258d^5e^2 + 167d^4e^3 - 14d^3e^4 + 141d^2e^5 - 8d^2e^6 + 3e^7) + (300d^8 - 195d^7e + 405d^6e^2 + 297d^5e^3 - 176d^4e^4 + 347d^3e^5 - 123d^2e^6 + 39d^2e^7 - 18d^2e^8) e^{-1}) e^{-2}}{2(5d^2 - 2de + 3e^2)^3 (xe + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3/(5\*x^2+2\*x+3),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/70*\sqrt{14}*(423*d^3 - 4101*d^2*e + 879*d^2*e^2 + 703*e^3)*\arctan(1/14*\sqrt{14}* \\ & \text{t}(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2 \\ & *e^4 - 54*d^2*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d^2*e^2 + 113*e^3) \\ & * \log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17 \\ & 1*d^2*e^4 - 54*d^2*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d \\ & ^3*e^3 + 141*d^2*e^4 + 120*d^2*e^5 - e^6)*\log(\text{abs}(x*e + d))/(125*d^6*e^3 - 15 \\ & 0*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d^2*e^8 + 27*e^9) + \\ & 1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4 + 141*d \\ & ^2*e^5 - 8*d^2*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 + 297*d^5* \\ & e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d^2*e^7 - 18*e^8)*e^{-1})* \\ & e^{-2}/((5*d^2 - 2*d*e + 3*e^2)^3*(x*e + d)^2) \end{aligned}$$

**maple [B]** time = 0.02, size = 819, normalized size = 2.58

$$\frac{-423}{70} \frac{(5d^2 - 2de + 3e^2)^3 14^{1/2} \arctan(1/28(10x+2)14^{1/2}) d^{3-7}}{(5d^2 - 2de + 3e^2)^3 14^{1/2} \arctan(1/28(10x+2)14^{1/2}) e^{3-2}} \frac{e^{-3-2}}{(5d^2 - 2de + 3e^2) (ex+d)^2 d^4 - 5/2 e^2 / (5d^2 - 2de + 3e^2) / (ex+d)^2} \frac{d^3 - 3/2 e}{(5d^2 - 2de + 3e^2) / (ex+d)^2 d^2 + 40/e^3 / (5d^2 - 2de + 3e^2)^2 / (ex+d) d^5 + 1/e^2 / (5d^2 - 2de + 3e^2)^2 / (ex+d) d^4 - 2e / (5d^2 - 2de + 3e^2)^2} \frac{d^2 - 21/10}{(5d^2 - 2de + 3e^2)^3 \ln(5x^2 + 2x + 3) d^2 e - 408/5 / (5d^2 - 2de + 3e^2)^3 \ln(5x^2 + 2x + 3) d^2 e + 120 / (5d^2 - 2de + 3e^2)^3 e^2 \ln(ex+d) d + 100 / (5d^2 - 2de + 3e^2)^3 e^3 \ln(ex+d) d^6 - 120 / (5d^2 - 2de + 3e^2)^3 e^2 \ln(ex+d) d^5 + 228 / (5d^2 - 2de + 3e^2)^3 e \ln(ex+d) d^4 + 141 / (5d^2 - 2de + 3e^2)^3 e \ln(ex+d) d^2 + 28/e / (5d^2 - 2de + 3e^2)^2 / (ex+d) d^3 + 44 / (5d^2 - 2de + 3e^2)^2 / (ex+d) d^2 + 1/2 / (5d^2 - 2de + 3e^2) / (ex+d)^2 d - e / (5d^2 - 2de + 3e^2) / (ex+d)^2 + e^2 / (5d^2 - 2de + 3e^2)^2 / (ex+d) - 242 / (5d^2 - 2de + 3e^2)^3 \ln(ex+d) d^3 - 1 / (5d^2 - 2de + 3e^2)^3 e^3 \ln(ex+d) + 229/5 / (5d^2 - 2de + 3e^2)^3 \ln(5x^2 + 2x + 3) d^3 + 113/10 / (5d^2 - 2de + 3e^2)^3 \ln(5x^2 + 2x + 3) e^3 + 4101/70 / (5d^2 - 2de + 3e^2)^3 14^{1/2} \arctan(1/28(10x+2)14^{1/2}) d^2 e - 879/70 / (5d^2 - 2de + 3e^2)^3 14^{1/2} \arctan(1/28(10x+2)14^{1/2}) d^2 e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3/(5\*x^2+2\*x+3),x)

[Out] 
$$\begin{aligned} & -423/70/(5*d^2-2*d*e+3*e^2)^3*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*d^{3-7} \\ & 03/70/(5*d^2-2*d*e+3*e^2)^3*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*e^{3-2}/e \\ & ^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^4-5/2/e^2/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2* \\ & d^3-3/2/e/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^2+40/e^3/(5*d^2-2*d*e+3*e^2)^2/(e \\ & *x+d)*d^5+1/e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^4-2*e/(5*d^2-2*d*e+3*e^2)^2 \\ & /(e*x+d)*d-21/10/(5*d^2-2*d*e+3*e^2)^3*\ln(5*x^2+2*x+3)*d^2*e-408/5/(5*d^2-2 \\ & *d*e+3*e^2)^3*\ln(5*x^2+2*x+3)*d^2*e+120/(5*d^2-2*d*e+3*e^2)^3*e^2*\ln(e*x+d) \\ & *d+100/(5*d^2-2*d*e+3*e^2)^3/e^3*\ln(e*x+d)*d^6-120/(5*d^2-2*d*e+3*e^2)^3/e^ \\ & 2*\ln(e*x+d)*d^5+228/(5*d^2-2*d*e+3*e^2)^3/e*\ln(e*x+d)*d^4+141/(5*d^2-2*d*e+ \\ & 3*e^2)^3*e*\ln(e*x+d)*d^2+28/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^3+44/(5*d^2-2 \\ & *d*e+3*e^2)^2/(e*x+d)*d^2+1/2/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d-e/(5*d^2-2*d* \\ & e+3*e^2)/(e*x+d)^2+e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)-242/(5*d^2-2*d*e+3*e^2 \\ & )^3*\ln(e*x+d)*d^3-1/(5*d^2-2*d*e+3*e^2)^3*e^3*\ln(e*x+d)+229/5/(5*d^2-2*d*e+ \\ & 3*e^2)^3*\ln(5*x^2+2*x+3)*d^3+113/10/(5*d^2-2*d*e+3*e^2)^3*\ln(5*x^2+2*x+3)*e \\ & ^3+4101/70/(5*d^2-2*d*e+3*e^2)^3*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*d^ \\ & 2*e-879/70/(5*d^2-2*d*e+3*e^2)^3*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})*d^ \\ & e^2 \end{aligned}$$

**maxima [A]** time = 1.00, size = 498, normalized size = 1.57

$$\frac{\sqrt{14}(423d^3 - 410d^2e + 879de^2 + 703e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + (100d^6 - 120d^5e + 228d^4e^2 - 242d^3e^3 + 141d^2e^4 + 120de^5 - e^6)\log(ex+d) + \frac{(458d^3 - 21d^2e - 816de^2 + 113e^3)\log(5x^2+2x+3)}{10(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6)} + \frac{60d^6 - 15d^5e + 39d^4e^2 + 84d^3e^3 - 25d^2e^4 + 9de^5 - 6e^6 + 2(40d^5e + d^4e^2 + 28d^3e^3 + 44d^2e^4 - 2de^5 + e^6)x}{2(25d^6 - 20d^5e + 34d^4e^2 - 12d^3e^3 + 9d^2e^4 + 25d^5e^2 - 20d^4e^3 + 34d^3e^4 - 12d^2e^5 + 9d^2e^6)} + 2(25d^6 - 20d^5e + 34d^4e^2 - 12d^3e^3 + 9d^2e^4 + 25d^5e^2 - 20d^4e^3 + 34d^3e^4 - 12d^2e^5 + 9d^2e^6)x}{70(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3/(5\*x^2+2\*x+3),x, algorithm="maxima")

[Out] -1/70\*sqrt(14)\*(423\*d^3 - 410\*d^2\*e + 879\*d\*e^2 + 703\*e^3)\*arctan(1/14\*sqrt(14)\*(5\*x + 1))/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d\*e^5 + 27\*e^6) + (100\*d^6 - 120\*d^5\*e + 228\*d^4\*e^2 - 242\*d^3\*e^3 + 141\*d^2\*e^4 + 120\*d\*e^5 - e^6)\*log(e\*x + d)/(125\*d^6\*e^3 - 150\*d^5\*e^4 + 285\*d^4\*e^5 - 188\*d^3\*e^6 + 171\*d^2\*e^7 - 54\*d\*e^8 + 27\*e^9) + 1/10\*(458\*d^3 - 21\*d^2\*e - 816\*d\*e^2 + 113\*e^3)\*log(5\*x^2 + 2\*x + 3)/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d\*e^5 + 27\*e^6) + 1/2\*(60\*d^6 - 15\*d^5\*e + 39\*d^4\*e^2 + 84\*d^3\*e^3 - 25\*d^2\*e^4 + 9\*d\*e^5 - 6\*e^6 + 2\*(40\*d^5\*e + d^4\*e^2 + 28\*d^3\*e^3 + 44\*d^2\*e^4 - 2\*d\*e^5 + e^6)\*x)/(25\*d^6\*e^3 - 20\*d^5\*e^4 + 34\*d^4\*e^5 - 12\*d^3\*e^6 + 9\*d^2\*e^7 + (25\*d^4\*e^5 - 20\*d^3\*e^6 + 34\*d^2\*e^7 - 12\*d\*e^8 + 9\*e^9)\*x^2 + 2\*(25\*d^5\*e^4 - 20\*d^4\*e^5 + 34\*d^3\*e^6 - 12\*d^2\*e^7 + 9\*d\*e^8)\*x)

**mupad [B]** time = 4.76, size = 493, normalized size = 1.56

$$\frac{60d^6 - 15d^5e + 39d^4e^2 + 84d^3e^3 - 25d^2e^4 + 9de^5 - 6e^6 + 2(40d^5e + d^4e^2 + 28d^3e^3 + 44d^2e^4 - 2de^5 + e^6)x}{2(25d^6 - 20d^5e + 34d^4e^2 - 12d^3e^3 + 9d^2e^4 + 25d^5e^2 - 20d^4e^3 + 34d^3e^4 - 12d^2e^5 + 9d^2e^6)} \cdot \ln\left(x + \frac{1}{2} + \frac{\sqrt{14}}{2}\right) \left(\frac{458\sqrt{14} - 21d}{140}d^3 + \left(\frac{458\sqrt{14} - 21d}{140}d^2 + \frac{113}{140}\right)de + \left(\frac{458\sqrt{14} - 21d}{140}d + \frac{113}{140}\right)e\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{14}}{2}\right) \left(\frac{458\sqrt{14} - 21d}{140}d^3 + \left(\frac{458\sqrt{14} - 21d}{140}d^2 + \frac{113}{140}\right)de + \left(\frac{458\sqrt{14} - 21d}{140}d + \frac{113}{140}\right)e\right) + \ln(dx) \frac{(100d^6 - 120d^5e + 228d^4e^2 - 242d^3e^3 + 141d^2e^4 - 54de^5 + 27e^6)}{e^3(5d^6 - 2d^5e + 3e^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2)/((d + e\*x)^3\*(2\*x + 5\*x^2 + 3)),x)

[Out] ((9\*d\*e^5 - 15\*d^5\*e + 60\*d^6 - 6\*e^6 - 25\*d^2\*e^4 + 84\*d^3\*e^3 + 39\*d^4\*e^2)/(2\*e^3\*(25\*d^4 - 20\*d^3\*e - 12\*d\*e^3 + 9\*e^4 + 34\*d^2\*e^2)) + (x\*(d^4\*e - 2\*d\*e^4 + 40\*d^5 + e^5 + 44\*d^2\*e^3 + 28\*d^3\*e^2))/(e^2\*(25\*d^4 - 20\*d^3\*e - 12\*d\*e^3 + 9\*e^4 + 34\*d^2\*e^2)))/(d^2 + e^2\*x^2 + 2\*d\*e\*x) - (log(x - (14^(1/2)\*1i)/5 + 1/5)\*(d^3\*((423\*14^(1/2))/140 - 229i/5) + e^3\*((703\*14^(1/2))/140 - 113i/10) + d\*e^2\*((879\*14^(1/2))/140 + 408i/5) - d^2\*e\*((4101\*14^(1/2))/140 - 21i/10)))/(d^6\*125i - d^5\*e\*150i - d\*e^5\*54i + e^6\*27i + d^2\*e^4\*171i - d^3\*e^3\*188i + d^4\*e^2\*285i) + (log(x + (14^(1/2)\*1i)/5 + 1/5)\*(d^3\*((423\*14^(1/2))/140 + 229i/5) + e^3\*((703\*14^(1/2))/140 + 113i/10) + d\*e^2\*((879\*14^(1/2))/140 - 408i/5) - d^2\*e\*((4101\*14^(1/2))/140 + 21i/10)))/(d^6\*125i - d^5\*e\*150i - d\*e^5\*54i + e^6\*27i + d^2\*e^4\*171i - d^3\*e^3\*188i + d^4\*e^2\*285i) + (log(d + e\*x)\*(120\*d\*e^5 - 120\*d^5\*e + 100\*d^6 - e^6 + 141\*d^2\*e^4 - 242\*d^3\*e^3 + 228\*d^4\*e^2))/(e^3\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

$$3.291 \quad \int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$

**Optimal.** Leaf size=189

$$\frac{ex^2(840d^2 - 1722de + 373e^2)}{3500} - \frac{(1025d^3 - 1545d^2e - 2601de^2 + 832e^3) \log(5x^2 + 2x + 3)}{6250} + \frac{x(2800d^3 - 17220d^2e + 9921d^2e^2 + 6053e^3)}{17500} + \frac{(317565d^2e + 32825d^2 - 221643d^2 - 67499e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) + \frac{1}{375}e^2x^3(60d - 41e) - \frac{(423x + 1367)(d + ex)^3}{3500(5x^2 + 2x + 3)} + \frac{e^3x^4}{25}}{87500\sqrt{14}}$$

**Rubi [A]** time = 0.26, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{ex^2(840d^2 - 1722de + 373e^2)}{3500} - \frac{(-1545d^2e + 1025d^3 - 2601d^2e + 832e^3) \log(5x^2 + 2x + 3)}{6250} + \frac{x(-17220d^2e + 2800d^3 + 9921d^2e + 6053e^3)}{17500} + \frac{(317565d^2e + 32825d^2 - 221643d^2 - 67499e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) + \frac{1}{375}e^2x^3(60d - 41e) - \frac{(423x + 1367)(d + ex)^3}{3500(5x^2 + 2x + 3)} + \frac{e^3x^4}{25}}{87500\sqrt{14}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2,x]

[Out] ((2800\*d^3 - 17220\*d^2\*e + 9921\*d\*e^2 + 6053\*e^3)\*x)/17500 + (e\*(840\*d^2 - 1722\*d\*e + 373\*e^2)\*x^2)/3500 + ((60\*d - 41\*e)\*e^2\*x^3)/375 + (e^3\*x^4)/25 - ((1367 + 423\*x)\*(d + e\*x)^3)/(3500\*(3 + 2\*x + 5\*x^2)) + ((32825\*d^3 + 317565\*d^2\*e - 221643\*d\*e^2 - 67499\*e^3)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(87500\*Sqrt[14]) - ((1025\*d^3 - 1545\*d^2\*e - 2601\*d\*e^2 + 832\*e^3)\*Log[3 + 2\*x + 5\*x^2])/6250

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx &= -\frac{(1367+423x)(d+ex)^3}{3500(3+2x+5x^2)} + \frac{1}{56} \int \frac{(d+ex)^2 \left( \frac{6}{125}(615d+1367e) - \frac{12}{125} \right)}{3} \\
&= -\frac{(1367+423x)(d+ex)^3}{3500(3+2x+5x^2)} + \frac{1}{56} \int \left( \frac{2}{625} (2800d^3 - 17220d^2e + 9921de^2 + 6053e^3) \right. \\
&= \frac{(2800d^3 - 17220d^2e + 9921de^2 + 6053e^3)x}{17500} + \frac{e(840d^2 - 1722de + 3)}{3500} \\
&= \frac{(2800d^3 - 17220d^2e + 9921de^2 + 6053e^3)x}{17500} + \frac{e(840d^2 - 1722de + 3)}{3500} \\
&= \frac{(2800d^3 - 17220d^2e + 9921de^2 + 6053e^3)x}{17500} + \frac{e(840d^2 - 1722de + 3)}{3500} \\
&= \frac{(2800d^3 - 17220d^2e + 9921de^2 + 6053e^3)x}{17500} + \frac{e(840d^2 - 1722de + 3)}{3500}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 209, normalized size = 1.11

$$\frac{14700ex^2(300d^2 - 615de + 103e^2) - \frac{e(125d^2(423+1367e+5989e-1269-15e^{10323+17967e-54989-53189}))}{56^2 \cdot 2e^{13}} + 2940(-1025d^3 + 1545d^2e + 2601de^2 - 832e^3) \log(5x^2 + 2x + 3) + 5880x(500d^3 - 3075d^2e + 1545d^2e + 867e^3) + 15\sqrt{14}(32825d^3 + 317565d^2e - 221643d^2e - 67499e^3) \tan^{-1}\left(\frac{5x}{\sqrt{14}}\right) + 49000e^2(60d - 41e) + 735000e^3x^4}{18375000}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] (5880\*(500\*d^3 - 3075\*d^2\*e + 1545\*d\*e^2 + 867\*e^3)\*x + 14700\*e\*(300\*d^2 - 615\*d\*e + 103\*e^2)\*x^2 + 49000\*(60\*d - 41\*e)\*e^2\*x^3 + 735000\*e^3\*x^4 - (42\*(e^3\*(54969 - 53189\*x) + 125\*d^3\*(1367 + 423\*x) + 75\*d^2\*e\*(-1269 + 5989\*x) - 15\*d\*e^2\*(17967 + 18323\*x)))/(3 + 2\*x + 5\*x^2) + 15\*sqrt[14]\*(32825\*d^3 + 317565\*d^2\*e - 221643\*d\*e^2 - 67499\*e^3)\*ArcTan[(1 + 5\*x)/sqrt[14]] + 2940\*(-1025\*d^3 + 1545\*d^2\*e + 2601\*d\*e^2 - 832\*e^3)\*Log[3 + 2\*x + 5\*x^2])/18375000

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$



Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

**fricas** [B] time = 0.59, size = 350, normalized size = 1.85

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="fricas")

[Out]  $\frac{1}{18375000} \cdot (3675000 \cdot e^3 \cdot x^6 + 1225000 \cdot (12 \cdot d \cdot e^2 - 7 \cdot e^3) \cdot x^5 + 122500 \cdot (180 \cdot d^2 \cdot e - 321 \cdot d \cdot e^2 + 47 \cdot e^3) \cdot x^4 + 147000 \cdot (100 \cdot d^3 - 555 \cdot d^2 \cdot e + 246 \cdot d \cdot e^2 + 153 \cdot e^3) \cdot x^3 - 7176750 \cdot d^3 + 3997350 \cdot d^2 \cdot e + 11319210 \cdot d \cdot e^2 - 2308698 \cdot e^3 + 2940 \cdot (2000 \cdot d^3 - 7800 \cdot d^2 \cdot e - 3045 \cdot d \cdot e^2 + 5013 \cdot e^3) \cdot x^2 + 15 \cdot \sqrt{14} \cdot (9 \cdot 8475 \cdot d^3 + 952695 \cdot d^2 \cdot e - 664929 \cdot d \cdot e^2 - 202497 \cdot e^3 + 5 \cdot (32825 \cdot d^3 + 317565 \cdot d^2 \cdot e - 221643 \cdot d \cdot e^2 - 67499 \cdot e^3) \cdot x^2 + 2 \cdot (32825 \cdot d^3 + 317565 \cdot d^2 \cdot e - 221643 \cdot d \cdot e^2 - 67499 \cdot e^3) \cdot x) \cdot \arctan\left(\frac{1}{14} \cdot \sqrt{14} \cdot (5 \cdot x + 1)\right) + 42 \cdot (157125 \cdot d^3 - 1740675 \cdot d^2 \cdot e + 923745 \cdot d \cdot e^2 + 417329 \cdot e^3) \cdot x - 2940 \cdot (3075 \cdot d^3 - 4635 \cdot d^2 \cdot e - 7803 \cdot d \cdot e^2 + 2496 \cdot e^3 + 5 \cdot (1025 \cdot d^3 - 1545 \cdot d^2 \cdot e - 2601 \cdot d \cdot e^2 + 832 \cdot e^3) \cdot x^2 + 2 \cdot (1025 \cdot d^3 - 1545 \cdot d^2 \cdot e - 2601 \cdot d \cdot e^2 + 832 \cdot e^3) \cdot x) \cdot \log(5 \cdot x^2 + 2 \cdot x + 3)) / (5 \cdot x^2 + 2 \cdot x + 3)$

**giac** [A] time = 0.16, size = 206, normalized size = 1.09

$\frac{1}{25} d^3 x^4 + \frac{4}{25} d^2 x^3 + \frac{6}{25} d^2 x^2 + \frac{4}{25} d^2 x + \frac{41}{375} d^2 - \frac{123}{250} d^2 x^2 - \frac{123}{125} d^2 x + \frac{103}{1250} d^2 x^2 + \frac{309}{625} d^2 x + \frac{1}{122500} \sqrt{14} (32825 d^3 + 317565 d^2 e - 221643 d^2 e - 67499 e^3) \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) + \frac{867}{3125} d^3 x^3 - \frac{1}{6250} (1025 d^3 - 1545 d^2 e - 2601 d \cdot e^2 + 832 e^3) \log(5x^2 + 2x + 3) - \frac{170875 d^3 - 95175 d^2 e + (52875 d^3 + 449175 d^2 e - 274845 d^2 e - 53189 e^3) x - 269505 d^2 e + 54969 e^3}{437500 (5x^2 + 2x + 3)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="giac")

[Out]  $\frac{1}{25} x^4 e^3 + \frac{4}{25} d x^3 e^2 + \frac{6}{25} d^2 x^2 e + \frac{4}{25} d^2 x^3 e - \frac{41}{375} x^3 e^3 - \frac{123}{250} d x^2 e^2 - \frac{123}{125} d^2 x^2 e + \frac{103}{1250} x^2 e^3 + \frac{309}{625} d x e^2 + \frac{1}{122500} \sqrt{14} \cdot (32825 \cdot d^3 + 317565 \cdot d^2 \cdot e - 221643 \cdot d \cdot e^2 - 67499 \cdot e^3) \cdot \arctan\left(\frac{1}{14} \cdot \sqrt{14} \cdot (5 \cdot x + 1)\right) + \frac{867}{3125} x^3 e^3 - \frac{1}{6250} \cdot (1025 \cdot d^3 - 1545 \cdot d^2 \cdot e - 2601 \cdot d \cdot e^2 + 832 \cdot e^3) \cdot \log(5 \cdot x^2 + 2 \cdot x + 3) - \frac{1}{437500} \cdot (170875 \cdot d^3 - 95175 \cdot d^2 \cdot e + (52875 \cdot d^3 + 449175 \cdot d^2 \cdot e - 274845 \cdot d \cdot e^2 - 53189 \cdot e^3) \cdot x - 269505 \cdot d \cdot e^2 + 54969 \cdot e^3) / (5 \cdot x^2 + 2 \cdot x + 3)$

**maple** [A] time = 0.02, size = 283, normalized size = 1.50

$\frac{d^3 x^4}{25} + \frac{4 d^2 x^3}{25} + \frac{6 d^2 x^2}{25} + \frac{4 d^2 x}{25} + \frac{41 d^2}{375} - \frac{123 d^2 x^2}{250} - \frac{123 d^2 x}{125} + \frac{103 d^2 x^2}{1250} + \frac{309 d^2 x}{625} + \frac{1}{122500} \sqrt{14} d^3 \arctan\left(\frac{\sqrt{14} (5x + 1)}{14}\right) + \frac{867 d^3 x^3}{3125} - \frac{1}{6250} (1025 d^3 - 1545 d^2 e - 2601 d \cdot e^2 + 832 e^3) \log(5x^2 + 2x + 3) - \frac{170875 d^3 - 95175 d^2 e + (52875 d^3 + 449175 d^2 e - 274845 d \cdot e^2 - 53189 e^3) x - 269505 d \cdot e^2 + 54969 e^3}{437500 (5x^2 + 2x + 3)}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2, x)$

[Out]  $\frac{1}{25}e^3x^4 + \frac{4}{25}d^2e^2x^3 - \frac{41}{375}e^3x^3 + \frac{6}{25}d^2e^2x^2 - \frac{123}{250}d^2e^2x + \frac{103}{1250}e^3x^2 + \frac{4}{25}d^3x - \frac{123}{125}d^2e^2x + \frac{309}{625}d^2e^2x + \frac{867}{3125}e^3x - \frac{1}{3125}((\frac{2115}{28}d^3 + \frac{17967}{28}d^2e - \frac{54969}{140}d^2e^2 - \frac{53189}{700}e^3)x + \frac{6835}{28}d^3 - \frac{3807}{28}d^2e - \frac{53901}{140}d^2e^2 + \frac{54969}{700}e^3) / (x^2 + \frac{2}{5}x + \frac{3}{5}) - \frac{41}{250}d^3 \ln(5x^2 + 2x + 3) + \frac{309}{1250}d^2e \ln(5x^2 + 2x + 3) + \frac{2601}{6250}d^2e^2 \ln(5x^2 + 2x + 3) - \frac{416}{3125}e^3 \ln(5x^2 + 2x + 3) + \frac{1313}{49000}14^{(1/2)}d^3 \arctan(\frac{1}{28}(10x+2)*14^{(1/2)}) + \frac{63513}{245000}14^{(1/2)}d^2e \arctan(\frac{1}{28}(10x+2)*14^{(1/2)}) - \frac{221643}{1225000}14^{(1/2)}d^2e^2 \arctan(\frac{1}{28}(10x+2)*14^{(1/2)}) - \frac{67499}{1225000}14^{(1/2)}e^3 \arctan(\frac{1}{28}(10x+2)*14^{(1/2)})$

**maxima** [A] time = 0.96, size = 212, normalized size = 1.12

$$\frac{1}{25}e^3x^4 + \frac{4}{25}d^2e^2x^3 + \frac{1}{1250}(300d^2e - 615d^2e^2 + 103e^3)x^2 + \frac{1}{1225000}\sqrt{14}(32825d^3 + 317565d^2e - 221643d^2e^2 - 67499e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{3125}(500d^3 - 3075d^2e + 1545d^2e^2 + 867e^3)x - \frac{1}{6250}(1025d^3 - 1545d^2e - 2601d^2e^2 + 832e^3)\log(5x^2 + 2x + 3) - \frac{170875d^3 - 95175d^2e - 269505d^2e^2 + 54969e^3 + (32075d^3 + 449175d^2e - 274845d^2e^2 - 53189e^3)}{437500(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2, x, \text{algorithm}="m\text{axima}"))$

[Out]  $\frac{1}{25}e^3x^4 + \frac{1}{375}(60d^2e^2 - 41e^3)x^3 + \frac{1}{1250}(300d^2e - 615d^2e^2 + 103e^3)x^2 + \frac{1}{1225000}\sqrt{14}(32825d^3 + 317565d^2e - 221643d^2e^2 - 67499e^3)\arctan(\frac{1}{14}\sqrt{14}(5x+1)) + \frac{1}{3125}(500d^3 - 3075d^2e + 1545d^2e^2 + 867e^3)x - \frac{1}{6250}(1025d^3 - 1545d^2e - 2601d^2e^2 + 832e^3)\log(5x^2 + 2x + 3) - \frac{1}{437500}(170875d^3 - 95175d^2e - 269505d^2e^2 + 54969e^3 + (52875d^3 + 449175d^2e - 274845d^2e^2 - 53189e^3)x) / (5x^2 + 2x + 3)$

**mupad** [B] time = 0.15, size = 333, normalized size = 1.76

$$\frac{1025d^3 + 3075d^2e + 1545d^2e^2 + 867e^3}{15625x^2 + 4250x + 9375} + \frac{1}{1225000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{3125}(500d^3 - 3075d^2e + 1545d^2e^2 + 867e^3)x - \frac{1}{6250}(1025d^3 - 1545d^2e - 2601d^2e^2 + 832e^3)\log(5x^2 + 2x + 3) - \frac{1}{437500}(170875d^3 - 95175d^2e - 269505d^2e^2 + 54969e^3 + (52875d^3 + 449175d^2e - 274845d^2e^2 - 53189e^3)x) / (5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)$

[Out]  $((\frac{53901d^2e^2}{28} + \frac{19035d^2e^2}{28} + x((\frac{54969d^2e^2}{28} - \frac{89835d^2e^2}{28} - \frac{10575d^3}{28} + \frac{53189e^3}{140}) - \frac{34175d^3}{28} - \frac{54969e^3}{140}) / (6250x + 15625x^2 + 9375) + x^3((\frac{e^2(12d - 5e)}{75} - \frac{16e^3}{375}) - x((\frac{18e^2(12d - 5e)}{625} + \frac{12e(4d^2 - 5d^2e + e^2)}{125} - \frac{9d^2e^2}{25} + \frac{3d^2e^2}{5} - \frac{4d^3}{25} - \frac{717e^3}{3125}) + \log(2x + 5x^2 + 3)((\frac{601d^2e^2}{6250} + \frac{309d^2e^2}{1250} - \frac{41d^3}{250} - \frac{416e^3}{3125}) - x^2((\frac{2e^2(12d - 5e)}{125} - \frac{3e(4d^2 - 5d^2e + e^2)}{50} + \frac{36e^3}{625}) + (e^3x^4)/25 - 14^{(1/2)}\text{atan}((14^{(1/2)}(221643d^2e^2 - 317565d^2e - 32$

$$\frac{825d^3 + 67499e^3}{1225000} + (14^{1/2}xx(221643de^2 - 317565d^2e - 32825d^3 + 67499e^3))/245000 / ((221643de^2)/87500 - (63513d^2e)/17500 - (1313d^3)/3500 + (67499e^3)/87500) * (221643de^2 - 317565d^2e - 32825d^3 + 67499e^3) / 1225000$$

**sympy [C]** time = 2.77, size = 444, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3)\*\*2,x)

[Out]  $e^{3x}x^4/25 + x^3(4de^{2x}/25 - 41e^{3x}/375) + x^2(6d^2e/25 - 123de^{2x}/250 + 103e^{3x}/1250) + x(4d^3/25 - 123d^2e/125 + 309de^{2x}/625 + 867e^{3x}/3125) + (-41d^3/250 + 309d^2e/1250 + 2601de^{2x}/6250 - 416e^{3x}/3125 - \sqrt{14}I(32825d^3 + 317565d^2e - 221643de^{2x} - 67499e^{3x})/2450000) \log(x + (6565d^3 + 63513d^2e - 221643de^{2x}/5 - 67499e^{3x}/5 - \sqrt{14}I(32825d^3 + 317565d^2e - 221643de^{2x} - 67499e^{3x})/5) / (32825d^3 + 317565d^2e - 221643de^{2x} - 67499e^{3x})) + (-41d^3/250 + 309d^2e/1250 + 2601de^{2x}/6250 - 416e^{3x}/3125 + \sqrt{14}I(32825d^3 + 317565d^2e - 221643de^{2x} - 67499e^{3x})/2450000) \log(x + (6565d^3 + 63513d^2e - 221643de^{2x}/5 - 67499e^{3x}/5 + \sqrt{14}I(32825d^3 + 317565d^2e - 221643de^{2x} - 67499e^{3x})/5) / (32825d^3 + 317565d^2e - 221643de^{2x} - 67499e^{3x})) + (-170875d^3 + 95175d^2e + 269505de^{2x} - 54969e^{3x} + x(-52875d^3 - 449175d^2e + 274845de^{2x} + 53189e^{3x})) / (2187500x^2 + 875000x + 1312500)$

$$3.292 \quad \int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$

**Optimal.** Leaf size=140

$$-\frac{(1025d^2 - 1030de - 867e^2) \log(5x^2 + 2x + 3)}{6250} + \frac{x(2800d^2 - 11480de + 3307e^2)}{17500} + \frac{(32825d^2 + 211710de - 73881e^2) \operatorname{ArcTan}\left[\frac{5x+1}{\sqrt{14}}\right]}{87500\sqrt{14}}$$

**Rubi [A]** time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1644, 1628, 634, 618, 204, 628}

$$-\frac{(1025d^2 - 1030de - 867e^2) \log(5x^2 + 2x + 3)}{6250} + \frac{x(2800d^2 - 11480de + 3307e^2)}{17500} + \frac{(32825d^2 + 211710de - 73881e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{87500\sqrt{14}} + \frac{1}{250}ex^2(40d - 41e) - \frac{(423x + 1367)(d + ex)^2}{3500(5x^2 + 2x + 3)} + \frac{4e^2x^3}{75}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] ((2800\*d^2 - 11480\*d\*e + 3307\*e^2)\*x)/17500 + ((40\*d - 41\*e)\*e\*x^2)/250 + (4\*e^2\*x^3)/75 - ((1367 + 423\*x)\*(d + e\*x)^2)/(3500\*(3 + 2\*x + 5\*x^2)) + ((32825\*d^2 + 211710\*d\*e - 73881\*e^2)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(87500\*Sqrt[14]) - ((1025\*d^2 - 1030\*d\*e - 867\*e^2)\*Log[3 + 2\*x + 5\*x^2])/6250

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx &= -\frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)} + \frac{1}{56} \int \frac{(d+ex) \left( \frac{2}{125}(1845d+2734e) - \frac{6}{125} \right)}{(3+2x+5x^2)^2} dx \\
&= -\frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)} + \frac{1}{56} \int \left( \frac{2}{625}(2800d^2-11480de+3307e^2) \right. \\
&\quad \left. + \frac{(2800d^2-11480de+3307e^2)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)} \right) dx \\
&= \frac{(2800d^2-11480de+3307e^2)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)} \\
&= \frac{(2800d^2-11480de+3307e^2)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)} \\
&= \frac{(2800d^2-11480de+3307e^2)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)} \\
&= \frac{(2800d^2-11480de+3307e^2)x}{17500} + \frac{1}{250}(40d-41e)ex^2 + \frac{4e^2x^3}{75} - \frac{(1367+423x)(d+ex)^2}{3500(3+2x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 150, normalized size = 1.07

$$\frac{-\frac{42(25d^2(423x+1367)+10de(5989x-1269)-e^2(18323x+17967))}{5x^2+2x+3} + 588(-1025d^2+1030de+867e^2)\log(5x^2+2x+3) + 5880x(100d^2-410de+103e^2) + 3\sqrt{14}(32825d^2+211710de-73881e^2)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) + 14700ex^2(40d-41e) + 196000e^2x^3}{3675000}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] (5880\*(100\*d^2 - 410\*d\*e + 103\*e^2)\*x + 14700\*(40\*d - 41\*e)\*e\*x^2 + 196000\*e^2\*x^3 - (42\*(25\*d^2\*(1367 + 423\*x) + 10\*d\*e\*(-1269 + 5989\*x) - e^2\*(17967 + 18323\*x)))/(3 + 2\*x + 5\*x^2) + 3\*sqrt[14]\*(32825\*d^2 + 211710\*d\*e - 73881\*e^2)\*ArcTan[(1 + 5\*x)/sqrt[14]] + 588\*(-1025\*d^2 + 1030\*d\*e + 867\*e^2)\*Log[3 + 2\*x + 5\*x^2])/3675000

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

**fricas** [A] time = 0.82, size = 245, normalized size = 1.75

$\frac{98000e^2d^2 + 24500(120de - 107e^2) + 58800(50d^2 - 185de + 41e^2) + 2940(400d^2 - 1040de - 203e^2) + 3\sqrt{14}(32825d^2 + 211710de - 73881e^2) + 98475d^2 + 635130de - 21643e^2 + 2(32825d^2 + 211710de - 73881e^2)x}{367500(5x^2 + 2x + 3)}$   $\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) - \frac{1435350d^2 + 532980de + 754614e^2 + 42(31425d^2 - 232090de + 61583e^2)x - 588(1025d^2 - 1030de - 867e^2)x^2 + 3075d^2 - 3090de - 2601e^2 + 2(1025d^2 - 1030de - 867e^2)x}{87500(5x^2 + 2x + 3)} \log(5x^2 + 2x + 3)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="fricas")

[Out]  $\frac{1}{3675000} \cdot (980000e^2x^5 + 24500(120de - 107e^2)x^4 + 58800(50d^2 - 185de + 41e^2)x^3 + 2940(400d^2 - 1040de - 203e^2)x^2 + 3\sqrt{14}(32825d^2 + 211710de - 73881e^2)x^2 + 98475d^2 + 635130de - 21643e^2 + 2(32825d^2 + 211710de - 73881e^2)x) \cdot \arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) - \frac{1435350d^2 + 532980de + 754614e^2 + 42(31425d^2 - 232090de + 61583e^2)x - 588(1025d^2 - 1030de - 867e^2)x^2 + 3075d^2 - 3090de - 2601e^2 + 2(1025d^2 - 1030de - 867e^2)x}{87500(5x^2 + 2x + 3)} \log(5x^2 + 2x + 3)$

**giac** [A] time = 0.16, size = 145, normalized size = 1.04

$\frac{4}{75}x^3e^2 + \frac{4}{25}dx^2e + \frac{4}{25}d^2x - \frac{41}{250}x^2e^2 - \frac{82}{125}dex + \frac{1}{1225000}\sqrt{14}(32825d^2 + 211710de - 73881e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{103}{625}xe^2 - \frac{1}{6250}(1025d^2 - 1030de - 867e^2)\log(5x^2 + 2x + 3) - \frac{34175d^2 + (10575d^2 + 59890de - 18323e^2)x - 12690de - 17967e^2}{87500(5x^2 + 2x + 3)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="giac")

[Out]  $\frac{4}{75}x^3e^2 + \frac{4}{25}dx^2e + \frac{4}{25}d^2x - \frac{41}{250}x^2e^2 - \frac{82}{125}dex + \frac{1}{1225000}\sqrt{14}(32825d^2 + 211710de - 73881e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{103}{625}xe^2 - \frac{1}{6250}(1025d^2 - 1030de - 867e^2)\log(5x^2 + 2x + 3) - \frac{1}{87500}(34175d^2 + (10575d^2 + 59890de - 18323e^2)x - 12690de - 17967e^2)/(5x^2 + 2x + 3)$

**maple** [A] time = 0.01, size = 189, normalized size = 1.35

$\frac{4e^2x^3}{75} + \frac{4dex^2}{25} - \frac{41e^2x^2}{250} + \frac{4d^2x}{25} + \frac{1313\sqrt{14}d^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) - 41d^2\ln(5x^2+2x+3) - 82dex}{49000} + \frac{21171\sqrt{14}d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) + 103d\ln(5x^2+2x+3)}{122500} + \frac{103e^2x}{625} - \frac{73881\sqrt{14}e^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) + 867e^2\ln(5x^2+2x+3)}{122500} - \frac{1367e^2 - 12690de - 17967e^2}{700} + \frac{(423d^2 + 9890de - 18323e^2)x}{625\left(x^2 + \frac{2}{5}x + \frac{3}{5}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x)

[Out]  $4/75*e^2*x^3+4/25*d*e*x^2-41/250*e^2*x^2+4/25*d^2*x-82/125*d*e*x+103/625*e^2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d*e-17967/700*e^2)/(x^2+2/5*x+3/5)-41/250*d^2*\ln(5*x^2+2*x+3)+103/625*d*e*\ln(5*x^2+2*x+3)+867/6250*e^2*\ln(5*x^2+2*x+3)+1313/49000*14^{(1/2)}*d^2*\arctan(1/28*(10*x+2)*14^{(1/2)})+21171/122500*14^{(1/2)}*d*e*\arctan(1/28*(10*x+2)*14^{(1/2)})-73881/1225000*14^{(1/2)}*e^2*\arctan(1/28*(10*x+2)*14^{(1/2)})$

**maxima [A]** time = 0.96, size = 147, normalized size = 1.05

$$\frac{4}{75}e^{2x^3} + \frac{1}{250}(40de - 41e^2)x^2 + \frac{1}{122500}\sqrt{14}(32825d^2 + 211710de - 73881e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{625}(100d^2 - 410de + 103e^2)x - \frac{1}{6250}(1025d^2 - 1030de - 867e^2)\log(5x^2 + 2x + 3) - \frac{34175d^2 - 12690de - 17967e^2 + (10575d^2 + 59890de - 18323e^2)x}{87500(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="maxima")

[Out]  $4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*\text{sqrt}(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*\arctan(1/14*\text{sqrt}(14)*(5*x + 1)) + 1/625*(100*d^2 - 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*\log(5*x^2 + 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 59890*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)$

**mupad [B]** time = 0.11, size = 211, normalized size = 1.51

$$\ln(5x^2 + 2x + 3) \left( -\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250} \right) - x \left( \frac{2de}{5} + \frac{4e(8d-5e)}{125} - \frac{4d^2}{25} - \frac{3e^2}{625} \right) + x^2 \left( \frac{e(8d-5e)}{50} - \frac{8e^2}{125} \right) + \frac{1269de}{14} - x \left( \frac{2115d^2}{28} + \frac{5989de}{14} - \frac{18323e^2}{140} \right) - \frac{6835d^2}{28} + \frac{17967e^2}{140} + \frac{4e^2x^3}{75} + \frac{\sqrt{14} \operatorname{atan}\left(\frac{\sqrt{14}(32825d^2 + 211710de - 73881e^2)}{1225000}\right)}{1225000} \left( \frac{113d^2 + 211710de - 73881e^2}{87500} \right) (32825d^2 + 211710de - 73881e^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(2\*x + 5\*x^2 + 3)^2,x)

[Out]  $\log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*((2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8*d - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d^2)/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 3125*x^2 + 1875) + (4*e^2*x^3)/75 + (14^{(1/2)}*\operatorname{atan}(((14^{(1/2)}*(211710*d*e + 32825*d^2 - 73881*e^2))/1225000 + (14^{(1/2)}*x*(211710*d*e + 32825*d^2 - 73881*e^2))/245000))/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))*(211710*d*e + 32825*d^2 - 73881*e^2)/1225000$

**sympy [C]** time = 1.96, size = 298, normalized size = 2.13

$$\frac{4e^{2x^3} + x^2\left(\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250}\right) + \left(\frac{4d^2}{25} + \frac{4e(8d-5e)}{125} - \frac{4d^2}{25} - \frac{3e^2}{625}\right)x^2 + \left(\frac{e(8d-5e)}{50} - \frac{8e^2}{125}\right)x^2 + \frac{1269de}{14} - x\left(\frac{2115d^2}{28} + \frac{5989de}{14} - \frac{18323e^2}{140}\right) - \frac{6835d^2}{28} + \frac{17967e^2}{140} + \frac{4e^2x^3}{75} + \frac{\sqrt{14}\operatorname{atan}\left(\frac{\sqrt{14}(32825d^2 + 211710de - 73881e^2)}{1225000}\right)}{1225000} \left(\frac{113d^2 + 211710de - 73881e^2}{87500}\right) (32825d^2 + 211710de - 73881e^2)}{437500x^2 + 175000x + 262500}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3)\*\*2,x)



```
[Out] 4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
+ 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*I*
(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 42342*
d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/5)/(
32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 + 867*
e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(
x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 + 211710
*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-34175*d**
2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e**2))/(437
500*x**2 + 175000*x + 262500)
```

$$3.293 \quad \int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$

**Optimal.** Leaf size=97

$$-\frac{(423x+1367)(d+ex)}{3500(5x^2+2x+3)} - \frac{(205d-103e)\log(5x^2+2x+3)}{1250} + \frac{1}{125}x(20d-41e) + \frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}} + \frac{2ex^2}{25}$$

**Rubi [A]** time = 0.19, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1644, 1657, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{3500(5x^2+2x+3)} - \frac{(205d-103e)\log(5x^2+2x+3)}{1250} + \frac{1}{125}x(20d-41e) + \frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}} + \frac{2ex^2}{25}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] ((20\*d - 41\*e)\*x)/125 + (2\*e\*x^2)/25 - ((1367 + 423\*x)\*(d + e\*x))/(3500\*(3 + 2\*x + 5\*x^2)) + ((6565\*d + 21171\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(17500\*Sqrt[14]) - ((205\*d - 103\*e)\*Log[3 + 2\*x + 5\*x^2])/1250

#### Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx &= -\frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} + \frac{1}{56} \int \frac{\frac{2}{125}(1845d+1367e) - \frac{168}{125}(55d-27e)}{3+2x+5x^2} dx \\
&= -\frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} + \frac{1}{56} \int \left( \frac{56}{125}(20d-41e) + \frac{224ex}{25} + \frac{2(165d-27e)}{3+2x+5x^2} \right) dx \\
&= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} + \frac{\int \frac{165d+4811e-28(20d-41e)x}{3+2x+5x^2}}{3500} dx \\
&= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} + \frac{(-205d+103e) \log(3+2x+5x^2)}{12500} \\
&= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} - \frac{(205d-103e) \log(3+2x+5x^2)}{12500} \\
&= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} + \frac{(6565d+21171e) \operatorname{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 96, normalized size = 0.99

$$\frac{-\frac{14(5d(423x+1367)+e(5989x-1269))}{5x^2+2x+3} + 196(103e-205d) \log(5x^2+2x+3) + 1960x(20d-41e) + \sqrt{14}(6565d+21171e) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) + 19600ex^2}{245000}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] (1960\*(20\*d - 41\*e)\*x + 19600\*e\*x^2 - (14\*(5\*d\*(1367 + 423\*x) + e\*(-1269 + 5989\*x)))/(3 + 2\*x + 5\*x^2) + Sqrt[14]\*(6565\*d + 21171\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]] + 196\*(-205\*d + 103\*e)\*Log[3 + 2\*x + 5\*x^2])/245000

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^2, x]

**fricas** [A] time = 0.63, size = 147, normalized size = 1.52

$$\frac{98000ex^4 + 9800(20d - 37e)x^3 + 7840(10d - 13e)x^2 + \sqrt{14}(5(6565d + 21171e)x^2 + 2(6565d + 21171e)x + 19695d + 63513e)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + 14(6285d - 23209e)x - 196(5(205d - 103e)x^2 + 2(205d - 103e)x + 615d - 309e)\log(5x^2 + 2x + 3) - 95690d + 17766e}{245000(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="fricas")

[Out] 1/245000\*(98000\*e\*x^4 + 9800\*(20\*d - 37\*e)\*x^3 + 7840\*(10\*d - 13\*e)\*x^2 + sqrt(14)\*(5\*(6565\*d + 21171\*e)\*x^2 + 2\*(6565\*d + 21171\*e)\*x + 19695\*d + 63513\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 14\*(6285\*d - 23209\*e)\*x - 196\*(5\*(205\*d - 103\*e)\*x^2 + 2\*(205\*d - 103\*e)\*x + 615\*d - 309\*e)\*log(5\*x^2 + 2\*x + 3) - 95690\*d + 17766\*e)/(5\*x^2 + 2\*x + 3)

**giac** [A] time = 0.16, size = 94, normalized size = 0.97

$$\frac{2}{25}x^2e + \frac{1}{245000}\sqrt{14}(6565d + 21171e)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{4}{25}dx - \frac{41}{125}xe - \frac{1}{1250}(205d - 103e)\log(5x^2 + 2x + 3) - \frac{(2115d + 5989e)x + 6835d - 1269e}{17500(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="giac")

[Out] 2/25\*x^2\*e + 1/245000\*sqrt(14)\*(6565\*d + 21171\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 4/25\*d\*x - 41/125\*x\*e - 1/1250\*(205\*d - 103\*e)\*log(5\*x^2 + 2\*x + 3) - 1/17500\*((2115\*d + 5989\*e)\*x + 6835\*d - 1269\*e)/(5\*x^2 + 2\*x + 3)

**maple** [A] time = 0.01, size = 106, normalized size = 1.09

$$\frac{2ex^2}{25} + \frac{4dx}{25} + \frac{1313\sqrt{14}d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000} - \frac{41d\ln(5x^2+2x+3)}{250} - \frac{41ex}{125} + \frac{21171\sqrt{14}e\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{245000} + \frac{103e\ln(5x^2+2x+3)}{1250} - \frac{\frac{1367d}{140} - \frac{1269e}{700} + \left(\frac{423d}{140} + \frac{5989e}{700}\right)x}{125\left(x^2 + \frac{2}{5}x + \frac{3}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2, x)

[Out] 2/25\*e\*x^2+4/25\*d\*x-41/125\*e\*x-1/125\*((423/140\*d+5989/700\*e)\*x+1367/140\*d-1269/700\*e)/(x^2+2/5\*x+3/5)-41/250\*d\*ln(5\*x^2+2\*x+3)+103/1250\*e\*ln(5\*x^2+2\*x+3)+1313/49000\*14^(1/2)\*d\*arctan(1/28\*(10\*x+2)\*14^(1/2))+21171/245000\*14^(1/2)\*e\*arctan(1/28\*(10\*x+2)\*14^(1/2))

**maxima** [A] time = 0.96, size = 90, normalized size = 0.93

$$\frac{2}{25}ex^2 + \frac{1}{245000}\sqrt{14}(6565d + 21171e)\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{1}{125}(20d - 41e)x - \frac{1}{1250}(205d - 103e)\log(5x^2 + 2x + 3) - \frac{(2115d + 5989e)x + 6835d - 1269e}{17500(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="maxima")

[Out]  $\frac{2}{25}e*x^2 + \frac{1}{245000}\sqrt{14}*(6565*d + 21171*e)*\arctan(1/14*\sqrt{14}*(5*x + 1)) + \frac{1}{125}*(20*d - 41*e)*x - \frac{1}{1250}*(205*d - 103*e)*\log(5*x^2 + 2*x + 3) - \frac{1}{17500}*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)$

**mupad [B]** time = 4.15, size = 115, normalized size = 1.19

$$\frac{2ex^2}{25} - \ln(5x^2 + 2x + 3) \left( \frac{41d}{250} - \frac{103e}{1250} \right) + x \left( \frac{4d}{25} - \frac{41e}{125} \right) - \frac{\frac{1367d}{28} - \frac{1269e}{140} + x \left( \frac{423d}{28} + \frac{5989e}{140} \right)}{625x^2 + 250x + 375} + \frac{\sqrt{14} \operatorname{atan} \left( \frac{\frac{\sqrt{14}(6565d+21171e)}{245000} + \frac{\sqrt{14}x(6565d+21171e)}{49000}}{\frac{1313d}{3500} + \frac{21171e}{17500}} \right)}{245000} (6565d + 21171e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(2\*x + 5\*x^2 + 3)^2,x)

[Out]  $\frac{2e*x^2}{25} - \log(2*x + 5*x^2 + 3)*\left(\frac{41*d}{250} - \frac{103*e}{1250}\right) + x*\left(\frac{4*d}{25} - \frac{41*e}{125}\right) - \left(\frac{1367*d}{28} - \frac{1269*e}{140} + x*\left(\frac{423*d}{28} + \frac{5989*e}{140}\right)\right)/(250*x + 625*x^2 + 375) + \left(14^{1/2}*\operatorname{atan}\left(\frac{14^{1/2}*(6565*d + 21171*e)}{245000} + \frac{14^{1/2}*x*(6565*d + 21171*e)}{49000}\right)\right)/\left(\frac{1313*d}{3500} + \frac{21171*e}{17500}\right)*(6565*d + 21171*e)/245000$

**sympy [C]** time = 1.02, size = 165, normalized size = 1.70

$$\frac{2x^2}{25} + x \left( \frac{4d}{25} - \frac{41e}{125} \right) + \frac{-6835d + 1269e + x(-2115d - 5989e)}{87500x^2 + 35000x + 52500} + \left( \frac{41d}{250} + \frac{103e}{1250} - \frac{\sqrt{14}i(6565d + 21171e)}{490000} \right) \log \left( x + \frac{1313d + \frac{21171e}{5} - \frac{\sqrt{14}i(6565d + 21171e)}{5}}{6565d + 21171e} \right) + \left( -\frac{41d}{250} + \frac{103e}{1250} + \frac{\sqrt{14}i(6565d + 21171e)}{490000} \right) \log \left( x + \frac{1313d + \frac{21171e}{5} + \frac{\sqrt{14}i(6565d + 21171e)}{5}}{6565d + 21171e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3)\*\*2,x)

[Out]  $2e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 5989*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - \sqrt{14}*I*(6565*d + 21171*e)/490000)*\log(x + (1313*d + 21171*e/5 - \sqrt{14}*I*(6565*d + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + \sqrt{14}*I*(6565*d + 21171*e)/490000)*\log(x + (1313*d + 21171*e/5 + \sqrt{14}*I*(6565*d + 21171*e)/5)/(6565*d + 21171*e))$

$$3.294 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx$$

Optimal. Leaf size=63

$$-\frac{423x+1367}{3500(5x^2+2x+3)} - \frac{41}{250} \log(5x^2+2x+3) + \frac{4x}{25} + \frac{1313 \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

**Rubi [A]** time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {1660, 1657, 634, 618, 204, 628}

$$-\frac{423x+1367}{3500(5x^2+2x+3)} - \frac{41}{250} \log(5x^2+2x+3) + \frac{4x}{25} + \frac{1313 \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^2,x]

[Out] (4\*x)/25 - (1367 + 423\*x)/(3500\*(3 + 2\*x + 5\*x^2)) + (1313\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(3500\*Sqrt[14]) - (41\*Log[3 + 2\*x + 5\*x^2])/250

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ  
 $[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 1657

$\text{Int}[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x\_Symbol] := \text{Int}[\text{Expand}$   
 $\text{Integrand}[Pq*(a + b*x + c*x^2)^p, x], x] /;$  FreeQ[{a, b, c}, x] && PolyQ[Pq,  
 x] && IGtQ[p, -2]

### Rule 1660

$\text{Int}[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x\_Symbol] := \text{With}[\{Q =$   
 $\text{PolynomialQuotient}[Pq, a + b*x + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[Pq,$   
 $a + b*x + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x +$   
 $c*x^2, x], x, 1]\}, \text{Simp}[\frac{(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^($   
 $p + 1)}{(p + 1)*(b^2 - 4*a*c)}, x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[\frac{1}{(p + 1)}$   
 $(a + b*x + c*x^2)^(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*($   
 $2*c*f - b*g), x], x]] /;$  FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 -  
 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(3 + 2x + 5x^2)^2} dx &= -\frac{1367 + 423x}{3500(3 + 2x + 5x^2)} + \frac{1}{56} \int \frac{\frac{738}{25} - \frac{1848x}{25} + \frac{224x^2}{5}}{3 + 2x + 5x^2} dx \\ &= -\frac{1367 + 423x}{3500(3 + 2x + 5x^2)} + \frac{1}{56} \int \left( \frac{224}{25} + \frac{2(33 - 1148x)}{25(3 + 2x + 5x^2)} \right) dx \\ &= \frac{4x}{25} - \frac{1367 + 423x}{3500(3 + 2x + 5x^2)} + \frac{1}{700} \int \frac{33 - 1148x}{3 + 2x + 5x^2} dx \\ &= \frac{4x}{25} - \frac{1367 + 423x}{3500(3 + 2x + 5x^2)} - \frac{41}{250} \int \frac{2 + 10x}{3 + 2x + 5x^2} dx + \frac{1313 \int \frac{1}{3 + 2x + 5x^2} dx}{3500} \\ &= \frac{4x}{25} - \frac{1367 + 423x}{3500(3 + 2x + 5x^2)} - \frac{41}{250} \log(3 + 2x + 5x^2) - \frac{1313 \text{Subst}\left(\int \frac{1}{-56-x^2} dx\right)}{1750} \\ &= \frac{4x}{25} - \frac{1367 + 423x}{3500(3 + 2x + 5x^2)} + \frac{1313 \tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}} - \frac{41}{250} \log(3 + 2x + 5x^2) \end{aligned}$$



**Mathematica [A]** time = 0.04, size = 59, normalized size = 0.94

$$\frac{-\frac{14(423x+1367)}{5x^2+2x+3} - 8036 \log(5x^2 + 2x + 3) + 7840x + 1313\sqrt{14} \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{49000}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^2,x]

[Out] (7840\*x - (14\*(1367 + 423\*x))/(3 + 2\*x + 5\*x^2) + 1313\*sqrt[14]\*ArcTan[(1 + 5\*x)/sqrt[14]] - 8036\*Log[3 + 2\*x + 5\*x^2])/49000

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(3 + 2x + 5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^2,x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^2, x]

**fricas [A]** time = 0.39, size = 78, normalized size = 1.24

$$\frac{39200x^3 + 1313\sqrt{14}(5x^2 + 2x + 3) \arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + 15680x^2 - 8036(5x^2 + 2x + 3) \log(5x^2 + 2x + 3) + 17598x - 19138}{49000(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="fricas")

[Out] 1/49000\*(39200\*x^3 + 1313\*sqrt(14)\*(5\*x^2 + 2\*x + 3)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 15680\*x^2 - 8036\*(5\*x^2 + 2\*x + 3)\*log(5\*x^2 + 2\*x + 3) + 17598\*x - 19138)/(5\*x^2 + 2\*x + 3)

**giac [A]** time = 0.15, size = 52, normalized size = 0.83

$$\frac{1313}{49000} \sqrt{14} \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) + \frac{4}{25} x - \frac{423x + 1367}{3500(5x^2 + 2x + 3)} - \frac{41}{250} \log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^2,x, algorithm="giac")

[Out]  $1313/49000*\sqrt{14}*\arctan(1/14*\sqrt{14}*(5*x + 1)) + 4/25*x - 1/3500*(423*x + 1367)/(5*x^2 + 2*x + 3) - 41/250*\log(5*x^2 + 2*x + 3)$

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25} + \frac{1313\sqrt{14} \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000} - \frac{41 \ln(5x^2 + 2x + 3)}{250} - \frac{\frac{423x}{700} + \frac{1367}{700}}{25\left(x^2 + \frac{2}{5}x + \frac{3}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)`

[Out]  $4/25*x - 1/25*(423/700*x + 1367/700)/(x^2 + 2/5*x + 3/5) - 41/250*\ln(5*x^2 + 2*x + 3) + 1313/49000*14^{(1/2)}*\arctan(1/28*(10*x+2)*14^{(1/2)})$

maxima [A] time = 0.95, size = 52, normalized size = 0.83

$$\frac{1313}{49000} \sqrt{14} \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) + \frac{4}{25} x - \frac{423x + 1367}{3500(5x^2 + 2x + 3)} - \frac{41}{250} \log(5x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")`

[Out]  $1313/49000*\sqrt{14}*\arctan(1/14*\sqrt{14}*(5*x + 1)) + 4/25*x - 1/3500*(423*x + 1367)/(5*x^2 + 2*x + 3) - 41/250*\log(5*x^2 + 2*x + 3)$

mupad [B] time = 4.15, size = 52, normalized size = 0.83

$$\frac{4x}{25} - \frac{41 \ln(5x^2 + 2x + 3)}{250} - \frac{\frac{423x}{17500} + \frac{1367}{17500}}{x^2 + \frac{2x}{5} + \frac{3}{5}} + \frac{1313 \sqrt{14} \operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)`

[Out]  $(4*x)/25 - (41*\log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((2*x)/5 + x^2 + 3/5) + (1313*14^{(1/2)}*\operatorname{atan}((5*14^{(1/2)}*x)/14 + 14^{(1/2)}/14))/49000$

sympy [A] time = 0.19, size = 65, normalized size = 1.03

$$\frac{4x}{25} + \frac{-423x - 1367}{17500x^2 + 7000x + 10500} - \frac{41 \log\left(x^2 + \frac{2x}{5} + \frac{3}{5}\right)}{250} + \frac{1313\sqrt{14} \operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x/5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

$$3.295 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2} dx$$

Optimal. Leaf size=224

$$\frac{x(423d - 1367e) + 1367d - 293e}{700(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)} - \frac{(205d^3 - 61d^2e + 23de^2 + 14e^3) \log(5x^2 + 2x + 3)}{50(5d^2 - 2de + 3e^2)^2} + \frac{(6565d^3 - 26423d^2e + 11089de^2 - 6623e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{700\sqrt{14}(5d^2 - 2de + 3e^2)^2}$$

**Rubi [A]** time = 0.34, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{x(423d - 1367e) + 1367d - 293e}{700(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)} - \frac{(-61d^2e + 205d^3 + 23de^2 + 14e^3) \log(5x^2 + 2x + 3)}{50(5d^2 - 2de + 3e^2)^2} + \frac{(3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4) \log(d + ex)}{e(5d^2 - 2de + 3e^2)^2} + \frac{(-26423d^2e + 6565d^3 + 11089de^2 - 6623e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{700\sqrt{14}(5d^2 - 2de + 3e^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)^2), x]

[Out] -(1367\*d - 293\*e + (423\*d - 1367\*e)\*x)/(700\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(3 + 2\*x + 5\*x^2)) + ((6565\*d^3 - 26423\*d^2\*e + 11089\*d\*e^2 - 6623\*e^3)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(700\*Sqrt[14]\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2) + ((4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x])/(e\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2) - ((205\*d^3 - 61\*d^2\*e + 23\*d\*e^2 + 14\*e^3)\*Log[3 + 2\*x + 5\*x^2])/(50\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2)

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2} dx &= -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{1}{56} \int \frac{\frac{2(369d^2-421de+280e^2)}{5(5d^2-2de+3e^2)} - \frac{2(924d^2-285e^2)}{5(5d^2-2de+3e^2)}}{(d+ex)(3+2x+5x^2)} dx \\
&= -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{1}{56} \int \left( \frac{56(4d^4+5d^3e+3d^2e^2-de^3-de^4)}{(5d^2-2de+3e^2)^2(d+ex)} \right) dx \\
&= -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4) \log(d+ex)}{e(5d^2-2de+3e^2)^2} \\
&= -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4) \log(d+ex)}{e(5d^2-2de+3e^2)^2} \\
&= -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4) \log(d+ex)}{e(5d^2-2de+3e^2)^2} \\
&= -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{(6565d^3-26423d^2e+11089de^2-6623e^3)}{700\sqrt{14}(5d^2-2de+3e^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 186, normalized size = 0.83

$$\frac{\frac{14(5d^2-2de+3e^2)e(1367x+293)-d(423x+1367)}{5x^2+2x+3} - 196(205d^3-61d^2e+23de^2+14e^3) \log(5x^2+2x+3) + \sqrt{14}(6565d^3-26423d^2e+11089de^2-6623e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) + \frac{9800(4d^4+5d^3e+3d^2e^2-d^3+2e^4) \log(d+ex)}{e}}{9800(5d^2-2de+3e^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)^2), x]


[Out] ((14\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(-(d\*(1367 + 423\*x)) + e\*(293 + 1367\*x)))/(3 + 2\*x + 5\*x^2) + Sqrt[14]\*(6565\*d^3 - 26423\*d^2\*e + 11089\*d\*e^2 - 6623\*e^3)\*ArcTan[(1 + 5\*x)/Sqrt[14]] + (9800\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x])/e - 196\*(205\*d^3 - 61\*d^2\*e + 23\*d\*e^2 + 14\*e^3)\*Log[3 + 2\*x + 5\*x^2])/(9800\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2} dx$$



**maple [B]** time = 0.02, size = 691, normalized size = 3.08



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x)`

[Out] 
$$\begin{aligned} & -423/700/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3*x+7681/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*x*d^2*e-4003/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*x*d*e^2+4101/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*x*e^3-1367/700/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3+4199/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^2*e-4687/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d*e^2+879/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*e^3-41/10/(5*d^2-2*d*e+3*e^2)^2*\ln(5*x^2+2*x+3)*d^3+61/50/(5*d^2-2*d*e+3*e^2)^2*\ln(5*x^2+2*x+3)*d^2*e-23/50/(5*d^2-2*d*e+3*e^2)^2*\ln(5*x^2+2*x+3)*d*e^2-7/25/(5*d^2-2*d*e+3*e^2)^2*\ln(5*x^2+2*x+3)*e^3+1313/1960/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*\arctan(1/28*(10*x+2)*14^(1/2))*d^3-26423/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*\arctan(1/28*(10*x+2)*14^(1/2))*d^2*e+11089/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*\arctan(1/28*(10*x+2)*14^(1/2))*d*e^2-6623/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*\arctan(1/28*(10*x+2)*14^(1/2))*e^3+4/(5*d^2-2*d*e+3*e^2)^2/e*\ln(e*x+d)*d^4+5/(5*d^2-2*d*e+3*e^2)^2*\ln(e*x+d)*d^3+3/(5*d^2-2*d*e+3*e^2)^2*e*\ln(e*x+d)*d^2-1/(5*d^2-2*d*e+3*e^2)^2*e^2*\ln(e*x+d)*d+2/(5*d^2-2*d*e+3*e^2)^2*e^3*\ln(e*x+d) \end{aligned}$$

**maxima [A]** time = 0.98, size = 289, normalized size = 1.29

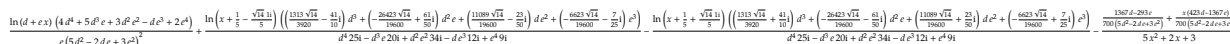
$$\frac{\sqrt{14}(6565d^3 - 26423d^2e + 11089de^2 - 6623e^3) \arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{9800(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)} + \frac{(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4) \log(ex+d)}{25d^4e - 20d^3e^2 + 34d^2e^3 - 12de^4 + 9e^5} - \frac{(205d^3 - 61d^2e + 23de^2 + 14e^3) \log(5x^2 + 2x + 3)}{50(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)} - \frac{(423d - 1367e)x + 1367d - 293e}{700(5(5d^2 - 2de + 3e^2)x^2 + 15d^2 - 6de + 9e^2 + 2(5d^2 - 2de + 3e^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 1/9800*\sqrt{14}*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*\arctan(1/14*\sqrt{14}*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) \\ & + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*\log(e*x + d)/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*\log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) \\ & - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2 - 2*d*e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*x) \end{aligned}$$

**mupad [B]** time = 4.61, size = 330, normalized size = 1.47





Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3*
((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50) -
d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
+ e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^2))
+ (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3) - (
log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) - e^3*(
(6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i/50) -
d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
+ e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 + 2*e^4 +
3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

$$3.296 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2} dx$$

**Optimal.** Leaf size=313

$$\frac{x(423d^2 - 2734de + 293e^2) + 1367d^2 - 586de - 703e^2}{140(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)^2} \frac{(41d^4 - 8d^3e - 60d^2e^2 + 24de^3 - 5e^4) \log(5x^2 + 2x + 3)}{2(5d^2 - 2de + 3e^2)^3}$$

**Rubi [A]** time = 0.50, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{x(423d^2 - 2734de + 293e^2) + 1367d^2 - 586de - 703e^2}{140(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)^2} - \frac{(-60d^2e^2 - 8d^3e + 41d^4 + 24de^3 - 5e^4) \log(5x^2 + 2x + 3)}{2(5d^2 - 2de + 3e^2)^3} - \frac{3d^2e^2 + 5d^3e + 4d^4 - de^3 + 2e^4}{e(5d^2 - 2de + 3e^2)^2(d + ex)} + \frac{(-60d^2e^2 - 8d^3e + 41d^4 + 24de^3 - 5e^4) \log(d + ex)}{(5d^2 - 2de + 3e^2)^3} + \frac{(4290d^2e^2 - 10044d^3e + 1313d^4 + 156de^3 - 271e^4) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{28\sqrt{14}(5d^2 - 2de + 3e^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^2), x]

[Out] -((4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)/(e\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(d + e\*x))) - (1367\*d^2 - 586\*d\*e - 703\*e^2 + (423\*d^2 - 2734\*d\*e + 293\*e^2)\*x)/(140\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(3 + 2\*x + 5\*x^2)) + ((1313\*d^4 - 10044\*d^3\*e + 4290\*d^2\*e^2 + 156\*d\*e^3 - 271\*e^4)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(28\*Sqrt[14]\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3) + ((41\*d^4 - 8\*d^3\*e - 60\*d^2\*e^2 + 24\*d\*e^3 - 5\*e^4)\*Log[d + e\*x])/(5\*d^2 - 2\*d\*e + 3\*e^2)^3 - ((41\*d^4 - 8\*d^3\*e - 60\*d^2\*e^2 + 24\*d\*e^3 - 5\*e^4)\*Log[3 + 2\*x + 5\*x^2])/(2\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3)

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 1628

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2} dx &= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} + \frac{1}{56} \int \frac{2(369d^4-842d^3e+567d^2e^2-196de^3+27e^4)}{(5d^2-2de+3e^2)^3} dx \\
&= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} + \frac{1}{56} \int \left( \frac{56(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} \right) dx \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e(5d^2-2de+3e^2)^2(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e(5d^2-2de+3e^2)^2(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e(5d^2-2de+3e^2)^2(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e(5d^2-2de+3e^2)^2(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 270, normalized size = 0.86

$$\frac{14(5d^2-2de+3e^2)(d^4(423+1367)-2d(1367+293)+2(293-703))}{5e^4(2e+3)} + 980(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)\log(5x^2+2x+3) - \frac{1960(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{40(5d^2-2de+3e^2)^3} + 1960(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(d+ex) + 5\sqrt{14}(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^2), x]

[Out] ((-1960\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(e\*(d + e\*x)) - (14\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(e^2\*(-703 + 293\*x) + d^2\*(1367 + 423\*x) - 2\*d\*e\*(293 + 1367\*x)))/(3 + 2\*x + 5\*x^2) + 5\*sqrt[14]\*(1313\*d^4 - 10044\*d^3\*e + 4290\*d^2\*e^2 + 156\*d\*e^3 - 271\*e^4)\*ArcTan[(1 + 5\*x)/sqrt[14]] + 1960\*(41\*d^4 - 8\*d^3\*e - 60\*d^2\*e^2 + 24\*d\*e^3 - 5\*e^4)\*Log[d + e\*x] + 980\*(-41\*d^4 + 8\*d^3\*e + 60\*d^2\*e^2 - 24\*d\*e^3 + 5\*e^4)\*Log[3 + 2\*x + 5\*x^2])/(1960\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2), x]
```

```
[Out] IntegrateAlgebraic[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2), x]
```

**fricas** [B] time = 1.10, size = 910, normalized size = 2.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] -1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 118552*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*d^4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*sqrt(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d*e^5 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*x^3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*d*e^5 - 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182*d^2*e^4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(5600*d^6 + 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*e^5 - 429*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(e*x + d) + 980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2 + 2*x + 3))/(375*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^3*e^5 - 162*d^2*e^6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^4 - 188*d^3*e^5 + 171*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 500*d^6*e^2 + 1125*d^5*e^3 - 370*d^4*e^4 + 479*d^3*e^5 + 72*d^2*e^6 + 27*d*e^7 + 54*e^8)*x^2 + (250*d^7*e + 75*d^6*e^2 + 120*d^5*e^3 + 479*d^4*e^4 - 222*d^3*e^5 + 405*d^2*e^6 - 108*d*e^7 + 81*e^8)*x)
```

**giac** [A] time = 0.20, size = 571, normalized size = 1.82

$$\frac{\sqrt{14}(1313d^4e^2 - 10044d^3e^3 + 4290d^2e^4 - 271e^6) \arctan\left(\frac{1}{14}\sqrt{14}\left(5d - \frac{5d^2}{e+d} + \frac{2d}{e+d} - \frac{3d^2}{e+d} - e\right)e^{d-2}\right) - (41d^4 - 8d^3e - 60d^2e^2 + 24de^3 - 5e^4) \log\left(\frac{10d}{e+d} + \frac{5d^2}{(e+d)^2} + \frac{2d}{e+d} - \frac{2d}{(e+d)^2} + \frac{3d^2}{(e+d)^2} + 5\right) - \frac{42d^4e^2 + 5d^3e^2 + 3d^2e^2 - d^2e^2 + 2d^2}{25d^4e^2 - 20d^3e^2 + 34d^2e^2 - 12de^2 + 9e^2}}{392(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6)} + \frac{423d^4e^2 - 410d^3e^2 + 379d^2e^2 - 701e^4}{5d^6 - 2de^3e^2} - \frac{(423d^4e^2 - 5468d^3e^2 + 1758d^2e^4 + 28124d^2e^5 - 457d^4e^{d-1})}{(5d^6 - 2de^3e^2)(e+d)}}{28(5d^6 - 2de + 3e^2) \left(\frac{10d}{e+d} - \frac{5d}{(e+d)^2} + \frac{2d}{e+d} + \frac{2d}{(e+d)^2} - \frac{3d^2}{(e+d)^2} - 5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3)^2,x, algorithm="giac")

[Out]  $\frac{1}{392}\sqrt{14}\left(1313d^4e^2 - 10044d^3e^3 + 4290d^2e^4 + 156d^5e^5 - 271e^6\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5d - \frac{5d^2}{xe+d} + \frac{2de}{xe+d} - \frac{3e^2}{xe+d} - e\right)e^{-1}\right)e^{-2} / \left(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6\right) - \frac{1}{2}\left(41d^4 - 8d^3e - 60d^2e^2 + 24de^3 - 5e^4\right)\log\left(-\frac{10d}{xe+d} + \frac{5d^2}{xe+d} + \frac{2e}{xe+d} - \frac{2de}{xe+d} + \frac{3e^2}{xe+d} + 5\right) / \left(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6\right) - \left(\frac{4d^4e^3}{xe+d} + \frac{5d^3e^4}{xe+d} + \frac{3d^2e^5}{xe+d} - \frac{de^6}{xe+d} + \frac{2e^7}{xe+d}\right) / \left(25d^4e^4 - 20d^3e^5 + 34d^2e^6 - 12de^7 + 9e^8\right) + \frac{1}{28}\left(\frac{423d^3e - 4101d^2e^2 + 879de^3 + 703e^4}{5d^2 - 2de + 3e^2} - \frac{423d^4e^2 - 5468d^3e^3 + 1758d^2e^4 + 2812de^5 - 457e^6}{(5d^2 - 2de + 3e^2)(xe+d)}\right) / \left((5d^2 - 2de + 3e^2)^2\left(\frac{10d}{xe+d} - \frac{5d^2}{xe+d} - \frac{2e}{xe+d} + \frac{2de}{xe+d} - \frac{3e^2}{xe+d} - 5\right)\right)$

**maple [B]** time = 0.02, size = 986, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3)^2,x)

[Out]  $\frac{1}{(5d^2-2de+3e^2)^2e^2(e*x+d)d-8/(5d^2-2de+3e^2)^3\ln(e*x+d)d^3e-60/(5d^2-2de+3e^2)^3\ln(e*x+d)d^2e^2+24/(5d^2-2de+3e^2)^3\ln(e*x+d)de^3-423/140/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)d^4x-879/700/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)xe^4+1416/175/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)d^3e-879/350/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)d^2e^2+88/175/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)de^3+4/(5d^2-2de+3e^2)^3\ln(5x^2+2x+3)d^3e+30/(5d^2-2de+3e^2)^3\ln(5x^2+2x+3)d^2e^2-12/(5d^2-2de+3e^2)^3\ln(5x^2+2x+3)de^3+1313/392/(5d^2-2de+3e^2)^3\frac{1}{2}\arctan\left(\frac{1}{28}(10x+2)\sqrt{14}\right)d^4-271/392/(5d^2-2de+3e^2)^3\frac{1}{2}\arctan\left(\frac{1}{28}(10x+2)\sqrt{14}\right)e^4-4/(5d^2-2de+3e^2)^2/e/(e*x+d)d^4-3/(5d^2-2de+3e^2)^2e/(e*x+d)d^2-5/(5d^2-2de+3e^2)^2/(e*x+d)d^3-1367/140/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)d^4+2109/700/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)e^4-41/2/(5d^2-2de+3e^2)^3\ln(5x^2+2x+3)d^4+5/2/(5d^2-2de+3e^2)^3\ln(5x^2+2x+3)e^4-2/(5d^2-2de+3e^2)^2e^3/(e*x+d)+41/(5d^2-2de+3e^2)^3\ln(e*x+d)d^4-5/(5d^2-2de+3e^2)^3\ln(e*x+d)e^4+3629/175/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)xd^3e-4101/350/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)xd^2e^2+2197/175/(5d^2-2de+3e^2)^3/(x^2+2/5x+3/5)xd^3e-2511/98/(5d^2-2de+3e^2)^3\frac{1}{2}\arctan\left(\frac{1}{28}(10x+2)\sqrt{14}\right)d^3e+2145/196/(5d^2-2de+3e^2)^3\frac{1}{2}\arctan\left(\frac{1}{28}(10x+2)\sqrt{14}\right)d^2e^2+39/98/(5d^2-2de+3e^2)^3\frac{1}{2}\arctan\left(\frac{1}{28}(10x+2)\sqrt{14}\right)de^3}$

**maxima [A]** time = 1.02, size = 548, normalized size = 1.75

$$\frac{\sqrt{14} \sqrt{133d^2 - 10044de + 4290d^2 + 156d^2 - 271e^2} \arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{(41d^4 - 8d^3e - 60d^2e^2 + 24d^2 - 5e^4) \log(ex+d)}{125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54d^2e^5 + 27e^6} + \frac{(41d^4 - 8d^3e - 60d^2e^2 + 24d^2 - 5e^4) \log(5x^2+2x+3)}{2(125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54d^2e^5 + 27e^6)} + \frac{1680d^4 + 3467d^3e + 674d^2e^2 - 1123d^2e^3 + 840e^4 + (2800d^4 + 3500d^3e + 2523d^2e^2 - 3434d^2e^3 + 1693e^4)x^2 + (1120d^4 + 1823d^3e - 527d^2e^2 - 573d^2e^3 - 143e^4)x}{140(75d^5e - 60d^4e^2 + 102d^3e^3 - 36d^2e^4 + 27d^2e^5 + 5(25d^4e^2 - 20d^3e^3 + 34d^2e^4 - 12d^2e^5 + 9e^6)x^3 + (125d^5e - 50d^4e^2 + 130d^3e^3 + 8d^2e^4 + 21d^2e^5 + 18e^6)x^2 + (50d^5e + 35d^4e^2 + 8d^3e^3 + 78d^2e^4 - 18d^2e^5 + 27e^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3)^2,x, algorithm="maxima")

[Out] 1/392\*sqrt(14)\*(1313\*d^4 - 10044\*d^3\*e + 4290\*d^2\*e^2 + 156\*d\*e^3 - 271\*e^4)\*arctan(1/14\*sqrt(14)\*(5\*x + 1))/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d^2\*e^5 + 27\*e^6) + (41\*d^4 - 8\*d^3\*e - 60\*d^2\*e^2 + 24\*d^2\*e^3 - 5\*e^4)\*log(e\*x + d)/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d^2\*e^5 + 27\*e^6) - 1/2\*(41\*d^4 - 8\*d^3\*e - 60\*d^2\*e^2 + 24\*d^2\*e^3 - 5\*e^4)\*log(5\*x^2 + 2\*x + 3)/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d^2\*e^5 + 27\*e^6) - 1/140\*(1680\*d^4 + 3467\*d^3\*e + 674\*d^2\*e^2 - 1123\*d^2\*e^3 + 840\*e^4 + (2800\*d^4 + 3500\*d^3\*e + 2523\*d^2\*e^2 - 3434\*d^2\*e^3 + 1693\*e^4)\*x^2 + (1120\*d^4 + 1823\*d^3\*e - 527\*d^2\*e^2 - 573\*d^2\*e^3 - 143\*e^4)\*x)/(75\*d^5\*e - 60\*d^4\*e^2 + 102\*d^3\*e^3 - 36\*d^2\*e^4 + 27\*d^2\*e^5 + 5\*(25\*d^4\*e^2 - 20\*d^3\*e^3 + 34\*d^2\*e^4 - 12\*d^2\*e^5 + 9\*e^6)\*x^3 + (125\*d^5\*e - 50\*d^4\*e^2 + 130\*d^3\*e^3 + 8\*d^2\*e^4 + 21\*d^2\*e^5 + 18\*e^6)\*x^2 + (50\*d^5\*e + 35\*d^4\*e^2 + 8\*d^3\*e^3 + 78\*d^2\*e^4 - 18\*d^2\*e^5 + 27\*e^6)\*x)

**mupad [B]** time = 4.84, size = 601, normalized size = 1.92

$$\frac{\log(d+ex) \left( \frac{41}{25(5d^2-2de+3e^2)} - \frac{4e^3(423d-1367e)}{125(5d^2-2de+3e^2)^3} + \frac{2e(310d-1323e)}{125(5d^2-2de+3e^2)^2} \right) - \frac{(3467d^3e-1123d^2e^3+1680d^4+840e^4+674d^2e^2)}{140e(25d^4-20d^3e-12d^2e^3+9e^4+34d^2e^2)} - \frac{(x(573d^3e-1823d^3e-1120d^4+143e^4+527d^2e^2))}{140e(25d^4-20d^3e-12d^2e^3+9e^4+34d^2e^2)} + \frac{(x^2(3500d^3e-3434d^2e^3+2800d^4+1693e^4+2523d^2e^2))}{140e(25d^4-20d^3e-12d^2e^3+9e^4+34d^2e^2)}}{(3d+x^2(5d+2e)+5ex^3+x(2d+3e)) + \left( \log\left(x - \frac{14^{1/2}i}{5} + \frac{1}{5}\right) \left( \frac{d^4((1313 \cdot 14^{1/2})/784 - 41i/2) - e^4((271 \cdot 14^{1/2})/784 - 5i/2)}{d^6 \cdot 125i - d^5e \cdot 150i - d^4e^2 \cdot 271 + d^3e^3 \cdot 188i + d^2e^4 \cdot 171i - d^3e^3 \cdot 188i + d^4e^2 \cdot 285i} \right) - \log\left(x + \frac{14^{1/2}i}{5} + \frac{1}{5}\right) \left( \frac{d^4((1313 \cdot 14^{1/2})/784 + 41i/2) - e^4((271 \cdot 14^{1/2})/784 + 5i/2)}{d^6 \cdot 125i - d^5e \cdot 150i - d^4e^2 \cdot 271 + d^3e^3 \cdot 188i + d^2e^4 \cdot 171i - d^3e^3 \cdot 188i + d^4e^2 \cdot 285i} \right) + \frac{d^2e^2((2145 \cdot 14^{1/2})/392 + 30i)}{d^6 \cdot 125i - d^5e \cdot 150i - d^4e^2 \cdot 271 + d^3e^3 \cdot 188i + d^2e^4 \cdot 171i - d^3e^3 \cdot 188i + d^4e^2 \cdot 285i} + \frac{de^3((39 \cdot 14^{1/2})/196 - 12i)}{d^6 \cdot 125i - d^5e \cdot 150i - d^4e^2 \cdot 271 + d^3e^3 \cdot 188i + d^2e^4 \cdot 171i - d^3e^3 \cdot 188i + d^4e^2 \cdot 285i} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2)/((d + e\*x)^2\*(2\*x + 5\*x^2 + 3)^2),x)

[Out] log(d + e\*x)\*(41/(25\*(5\*d^2 - 2\*d\*e + 3\*e^2)) - (4\*e^3\*(423\*d - 1367\*e))/(125\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3) + (2\*e\*(310\*d - 1323\*e))/(125\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2)) - ((3467\*d^3\*e - 1123\*d^2\*e^3 + 1680\*d^4 + 840\*e^4 + 674\*d^2\*e^2)/(140\*e\*(25\*d^4 - 20\*d^3\*e - 12\*d^2\*e^3 + 9\*e^4 + 34\*d^2\*e^2)) - (x\*(573\*d^3e - 1823\*d^3e - 1120\*d^4 + 143\*e^4 + 527\*d^2\*e^2))/(140\*e\*(25\*d^4 - 20\*d^3e - 12\*d^2e^3 + 9\*e^4 + 34\*d^2e^2))) / (3\*d + x^2\*(5\*d + 2\*e) + 5\*e\*x^3 + x\*(2\*d + 3\*e)) + (log(x - (14^(1/2)\*i)/5 + 1/5)\*(d^4\*((1313\*14^(1/2))/784 - 41i/2) - e^4\*((271\*14^(1/2))/784 - 5i/2)) / (d^6\*125i - d^5\*e\*150i - d^4\*e^2\*271 + d^3\*e^3\*188i + d^2\*e^4\*171i - d^3\*e^3\*188i + d^4\*e^2\*285i) - (log(x + (14^(1/2)\*i)/5 + 1/5)\*(d^4\*((1313\*14^(1/2))/784 + 41i/2) - e^4\*((271\*14^(1/2))/784 + 5i/2)) + d^2\*e^2\*((2145\*14^(1/2))/392 - 30i) + d\*e^3\*((39\*14^(1/2))/196 - 12i) - d^3\*e^3\*((2511\*14^(1/2))/196 - 4i)) / (d^6\*125i - d^5\*e\*150i - d^4\*e^2\*271 + d^3\*e^3\*188i + d^2\*e^4\*171i - d^3\*e^3\*188i + d^4\*e^2\*285i))

$9 \cdot 14^{(1/2)}/196 + 12i) - d^3 \cdot e^{((2511 \cdot 14^{(1/2)})/196 + 4i)}) / (d^6 \cdot 125i - d^5 \cdot e \cdot 150i - d \cdot e^5 \cdot 54i + e^6 \cdot 27i + d^2 \cdot e^4 \cdot 171i - d^3 \cdot e^3 \cdot 188i + d^4 \cdot e^2 \cdot 285i)$   
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*2/(5\*x\*\*2+2\*x+3)\*\*2,x)

[Out] Timed out



$$3.297 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2} dx$$

Optimal. Leaf size=412

$$\frac{1367d^3 - 879d^2e + x(423d^3 - 4101d^2e + 879de^2 + 703e^3) - 2109de^2 + 457e^3}{28(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)^3} - \frac{41d^4 - 8d^3e - 60d^2e^2 + 24de^3}{(5d^2 - 2de + 3e^2)^3 (d + ex)}$$

**Rubi [A]** time = 0.71, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{x(-401d^2e + 423d^3 + 879d^2e + 703e^3) - 879d^2e + 457e^3}{28(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)} - \frac{(-846d^2e + 396d^2e^2 - 19d^4e + 205d^4 - 21e^2) \log(5x^2 + 2x + 3)}{2(5d^2 - 2de + 3e^2)} - \frac{-60d^2e - 8d^3e + 41d^4 + 24d^4 - 5e^4}{(5d^2 - 2de + 3e^2)(d + ex)} - \frac{3d^2e + 5d^2e + 4d^4 - d^2e + 2e^4}{2(5d^2 - 2de + 3e^2)(d + ex)^2} + \frac{(-846d^2e + 396d^2e^2 - 19d^4e + 205d^4 - 21e^2) \log(d + ex)}{(5d^2 - 2de + 3e^2)} + \frac{(38022d^2e^2 + 42888d^2e^3 - 74017d^4e + 6565d^4 - 17247d^4e + 579e^2)}{28\sqrt{14}(5d^2 - 2de + 3e^2)^4} \arctan\left(\frac{1+5x}{\sqrt{14}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2), x]

[Out]  $-(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*\text{ArcTan}[(1 + 5*x)/\text{Sqrt}[14]])/(28*\text{Sqrt}[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*\text{Log}[d + e*x])/((5*d^2 - 2*d*e + 3*e^2)^4) - ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*\text{Log}[3 + 2*x + 5*x^2])/((2*(5*d^2 - 2*d*e + 3*e^2)^4)$

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2} dx &= -\frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+703e^3)}{28(5d^2-2de+3e^2)^3(3+2x+5x^2)} \\
&= -\frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+703e^3)}{28(5d^2-2de+3e^2)^3(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e(5d^2-2de+3e^2)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{13}{3(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e(5d^2-2de+3e^2)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{13}{3(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e(5d^2-2de+3e^2)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{13}{3(3+2x+5x^2)} \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e(5d^2-2de+3e^2)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{13}{3(3+2x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.40, size = 363, normalized size = 0.88

$$\frac{145d^4e^4(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4) + 196(5d^2-2de+3e^2)^3(41d^4-8d^3e-60d^2e^2+24de^3-5e^4) + 392(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4) + 196(-205d^5+19d^4e+846d^3e^2-396d^2e^3+57d^4+21e^5)\log(5d^2+2x+3) + 392(205d^5-19d^4e-846d^3e^2+396d^2e^3+57d^4+21e^5)\log(d+ex) + \sqrt{14}(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247d^4+579e^5)\operatorname{ArcTan}\left[\frac{d+ex}{\sqrt{14}}\right] + 392(205d^5-19d^4e-846d^3e^2+396d^2e^3+57d^4+21e^5)\operatorname{Log}[3+2x+5x^2]}{392(5d^2-2de+3e^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2), x]

[Out] ((-196\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(e\*(d + e\*x)^2) + (392\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(-41\*d^4 + 8\*d^3\*e + 60\*d^2\*e^2 - 24\*d\*e^3 + 5\*e^4))/(d + e\*x) - (14\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(3\*d^3\*e^2\*(-703 + 293\*x) + d^3\*(1367 + 423\*x) + e^3\*(457 + 703\*x) - 3\*d^2\*e\*(293 + 1367\*x)))/(3 + 2\*x + 5\*x^2) + Sqrt[14]\*(6565\*d^5 - 74017\*d^4\*e + 35022\*d^3\*e^2 + 42858\*d^2\*e^3 - 17247\*d^4 + 579\*e^5)\*ArcTan[(1 + 5\*x)/Sqrt[14]] + 392\*(205\*d^5 - 19\*d^4\*e - 846\*d^3\*e^2 + 396\*d^2\*e^3 + 57\*d^4 + 21\*e^5)\*Log[d + e\*x] + 196\*(-205\*d^5 + 19\*d^4\*e + 846\*d^3\*e^2 - 396\*d^2\*e^3 - 57\*d^4 + 21\*e^5)\*Log[3 + 2\*x + 5\*x^2)]/(392\*(5\*d^2 - 2\*d\*e + 3\*e^2)^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^3 (3 + 2x + 5x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2), x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^3\*(3 + 2\*x + 5\*x^2)^2), x]

fricas [B] time = 1.44, size = 1499, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^3/(5\*x^2+2\*x+3)^2,x, algorithm="fricas")

[Out] -1/392\*(58800\*d^8 + 363230\*d^7\*e - 178010\*d^6\*e^2 - 233184\*d^5\*e^3 + 395164\*d^4\*e^4 - 437122\*d^3\*e^5 + 178542\*d^2\*e^6 - 37044\*d\*e^7 + 10584\*e^8 + 14\*(28700\*d^6\*e^2 - 14965\*d^5\*e^3 - 43891\*d^4\*e^4 + 44106\*d^3\*e^5 - 45966\*d^2\*e^6 + 12711\*d\*e^7 + 9\*e^8)\*x^3 + 14\*(7000\*d^8 + 31850\*d^7\*e + 6400\*d^6\*e^2 - 62649\*d^5\*e^3 + 52187\*d^4\*e^4 - 53652\*d^3\*e^5 + 11130\*d^2\*e^6 - 2841\*d\*e^7 + 1791\*e^8)\*x^2 - sqrt(14)\*(19695\*d^7\*e - 222051\*d^6\*e^2 + 105066\*d^5\*e^3 + 128574\*d^4\*e^4 - 51741\*d^3\*e^5 + 1737\*d^2\*e^6 + 5\*(6565\*d^5\*e^3 - 74017\*d^4\*e^4 + 35022\*d^3\*e^5 + 42858\*d^2\*e^6 - 17247\*d\*e^7 + 579\*e^8)\*x^4 + 2\*(32825\*d^6\*e^2 - 363520\*d^5\*e^3 + 101093\*d^4\*e^4 + 249312\*d^3\*e^5 - 43377\*d^2\*e^6 - 14352\*d\*e^7 + 579\*e^8)\*x^3 + (32825\*d^7\*e - 343825\*d^6\*e^2 - 101263\*d^5\*e^3 + 132327\*d^4\*e^4 + 190263\*d^3\*e^5 + 62481\*d^2\*e^6 - 49425\*d\*e^7 + 1737\*e^8)\*x^2 + 2\*(6565\*d^7\*e - 54322\*d^6\*e^2 - 187029\*d^5\*e^3 + 147924\*d^4\*e^4 + 111327\*d^3\*e^5 - 51162\*d^2\*e^6 + 1737\*d\*e^7)\*x)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 14\*(2800\*d^8 + 14855\*d^7\*e + 5815\*d^6\*e^2 - 18620\*d^5\*e^3 - 17202\*d^4\*e^4 + 11119\*d^3\*e^5 - 26037\*d^2\*e^6 + 7866\*d\*e^7 - 756\*e^8)\*x - 392\*(615\*d^7\*e - 57\*d^6\*e^2 - 2538\*d^5\*e^3 + 1188\*d^4\*e^4 + 171\*d^3\*e^5 - 63\*d^2\*e^6 + 5\*(205\*d^5\*e^3 - 19\*d^4\*e^4 - 846\*d^3\*e^5 + 396\*d^2\*e^6 + 57\*d\*e^7 - 21\*e^8)\*x^4 + 2\*(1025\*d^6\*e^2 + 110\*d^5\*e^3 - 4249\*d^4\*e^4 + 1134\*d^3\*e^5 + 681\*d^2\*e^6 - 48\*d\*e^7 - 21\*e^8)\*x^3 + (1025\*d^7\*e + 725\*d^6\*e^2 - 3691\*d^5\*e^3 - 1461\*d^4\*e^4 - 669\*d^3\*e^5 + 1311\*d^2\*e^6 + 87\*d\*e^7 - 63\*e^8)\*x^2 + 2\*(205\*d^7\*e + 596\*d^6\*e^2 - 903\*d^5\*e^3 - 2142\*d^4\*e^4 + 1245\*d^3\*e^5 + 150\*d^2\*e^6 - 63\*d\*e^7)\*x)\*log(e\*x + d) + 196\*(615\*d^7\*e - 57\*d^6\*e^2 - 2538\*d^5\*e^3 + 1188\*d^4\*e^4 + 171\*d^3\*e^5 - 63\*d^2\*e^6 + 5\*(205\*d^5\*e^3 - 19\*d^4\*e^4 - 846\*d^3\*e^5 + 396\*d^2\*e^6 + 57\*d\*e^7 - 21\*e^8)\*x^4 + 2\*(1025\*d^6

```
*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 -
  21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 66
  9*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^2 + 2*(205*d^7*e + 596*d^6*
  e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5 + 150*d^2*e^6 - 63*d*e^7)*x
  )*log(5*x^2 + 2*x + 3))/(1875*d^10*e - 3000*d^9*e^2 + 6300*d^8*e^3 - 5880*d
  ^7*e^4 + 6258*d^6*e^5 - 3528*d^5*e^6 + 2268*d^4*e^7 - 648*d^3*e^8 + 243*d^2
  *e^9 + 5*(625*d^8*e^3 - 1000*d^7*e^4 + 2100*d^6*e^5 - 1960*d^5*e^6 + 2086*d
  ^4*e^7 - 1176*d^3*e^8 + 756*d^2*e^9 - 216*d*e^10 + 81*e^11)*x^4 + 2*(3125*d
  ^9*e^2 - 4375*d^8*e^3 + 9500*d^7*e^4 - 7700*d^6*e^5 + 8470*d^5*e^6 - 3794*d
  ^4*e^7 + 2604*d^3*e^8 - 324*d^2*e^9 + 189*d*e^10 + 81*e^11)*x^3 + (3125*d^1
  0*e - 2500*d^9*e^2 + 8375*d^8*e^3 - 4400*d^7*e^4 + 8890*d^6*e^5 - 3416*d^5*
  e^6 + 5334*d^4*e^7 - 1584*d^3*e^8 + 1809*d^2*e^9 - 324*d*e^10 + 243*e^11)*x
  ^2 + 2*(625*d^10*e + 875*d^9*e^2 - 900*d^8*e^3 + 4340*d^7*e^4 - 3794*d^6*e^
  5 + 5082*d^5*e^6 - 2772*d^4*e^7 + 2052*d^3*e^8 - 567*d^2*e^9 + 243*d*e^10)*
  x)
```

**giac [A]** time = 0.20, size = 595, normalized size = 1.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
  247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
  + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
  216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
  + 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
  e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
  + 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57*d*e^5
  - 21*e^6)*log(abs(x*e + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960
  *d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9)
  - 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 + 28226*d^4*
  e^4 + (28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 4596
  6*d^2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 - 31223*d^3*e^5 + (7000*d^8 + 31850*d^
  7*e + 6400*d^6*e^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*
  d^2*e^6 - 2841*d*e^7 + 1791*e^8)*x^2 + 12753*d^2*e^6 + (2800*d^8 + 14855*d^
  7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*
  d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 2646*d*e^7 + 756*e^8)*e^(-1)/((5*d^2 -
  2*d*e + 3*e^2)^4*(5*x^2 + 2*x + 3)*(x*e + d)^2)
```

**maple [B]** time = 0.03, size = 1314, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x)$

[Out] 
$$-3/2/(5*d^2-2*d*e+3*e^2)^2*e/(e*x+d)^2*d^2+1/2/(5*d^2-2*d*e+3*e^2)^2*e^2/(e*x+d)^2*d+8/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^3*e+60/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^2*e^2-24/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d*e^3-19/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)*d^4*e-846/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)*d^3*e^2+396/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)*d^2*e^3+57/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)*d*e^4-423/28/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^5-2109/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*e^5+7129/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^4*e+2343/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^3*e^2-1933/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^2*e^3+7241/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d*e^4+19/2/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+2*x+3)*d^4*e+423/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+2*x+3)*d^3*e^2-198/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+2*x+3)*d^2*e^3-57/2/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+2*x+3)*d*e^4+6565/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^5+579/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^5-2/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)^2*d^4-5/2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^2*d^3+205/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)*d^5-21/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)*e^5-1367/28/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^5-1371/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*e^5-205/2/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+2*x+3)*d^5+21/2/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+2*x+3)*e^5-1/(5*d^2-2*d*e+3*e^2)^2*e^3/(e*x+d)^2-41/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^4+5/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*e^4+21429/196/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^3-17247/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^4+21351/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^4*e-6933/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^3*e^2+5273/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^2*e^3-1231/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d*e^4-74017/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4*e+17511/196/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^2$$

**maxima** [B] time = 1.09, size = 851, normalized size = 2.07

---

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, \text{algorithm}=\text{"maxima"})$

[Out] 
$$1/392*\text{sqrt}(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*\text{arctan}(1/14*\text{sqrt}(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57$$

```

*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d
^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) -
1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*lo
g(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 20
86*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/28*(840*d
^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714*d*e^5 + 2
52*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^5 + 3*e^6)*
x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2349*d^2*e^4
- 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*e^2 - 4799*d
^3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e - 450*d^7*e^2 +
855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d^2*e^7 + 5*(125
*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8
+ 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e^4 - 655*d^4*e^5
+ 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (625*d^8*e - 250*d^7*
e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3*e^6 + 432*d^2*e^7
- 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^2 - 165*d^6*e^3 + 667*d
^5*e^4 - 393*d^4*e^5 + 459*d^3*e^6 - 135*d^2*e^7 + 81*d*e^8)*x)

```

**mupad [B]** time = 4.94, size = 887, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x + 3x^2 - 5x^3 + 4x^4 + 2)/((d + ex)^3(2x + 5x^2 + 3)^2), x)$

[Out]  $\log(d + ex) * (((41d)/5 + (29e)/5)/(5d^2 - 2de + 3e^2)^2 + (168e^4(458d - 7e))/(125(5d^2 - 2de + 3e^2)^4) - (2e^2(12610d + 1329e))/(125(5d^2 - 2de + 3e^2)^3)) - ((5525d^5e - 714de^5 + 840d^6 + 252e^6 + 3355d^2e^4 - 6981d^3e^3 - 837d^4e^2)/(28e(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) + (x^3(4239de^4 + 5740d^4e + 3e^5 - 12501d^2e^3 - 697d^3e^2))/(28(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) + (x^2(6930d^5e - 549de^5 + 1400d^6 + 597e^6 + 2349d^2e^4 - 15403d^3e^3 + 3212d^4e^2))/(28e(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) + (x(2454de^5 + 3195d^5e + 560d^6 - 252e^6 - 6623d^2e^4 - 4799d^3e^3 + 2105d^4e^2))/(28e(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2))))/(x^2(4de + 5d^2 + 3e^2) + x(6de + 2d^2) + 3d^2 + x^3(10de + 2e^2) + 5e^2x^4) + (\log(x - (14^{(1/2)}*1i)/5 + 1/5)*(d^5*((6565*14^{(1/2)})/784 - 205i/2) + e^5*((579*14^{(1/2)})/784 + 21i/2) + d^3e^2*((17511*14^{(1/2)})/392 + 423i) + d^2e^3*((21429*14^{(1/2)})/392 - 198i) - de^4*((17247*14^{(1/2)})/784 + 57i/2) - d^4e*((74017*14^{(1/2)})/784 - 19i/2)))/(d^8*625i - d^7e*1000i - de^7*216i + e^8*81i + d^2e^6*756i - d^3e^5*1176i + d^4e^4*2086i - d^5e^3*1960i + d^6e^2*2100i) - (\log(x + (14^{(1/2)}*1i)/5 + 1/5)*(d^5*((6565*14^{(1/2)})/784 + 205i/2) + e^5*((579*14^{(1/2)})/784 - 21i/2) + d^3e^2*((17511*14^{(1/2)})/392 + 423i) + d^2e^3*((21429*14^{(1/2)})/392 - 198i) - de^4*((17247*14^{(1/2)})/784 + 57i/2) - d^4e*((74017*14^{(1/2)})/784 - 19i/2)))/(d^8*625i - d^7e*1000i - de^7*216i + e^8*81i + d^2e^6*756i - d^3e^5*1176i + d^4e^4*2086i - d^5e^3*1960i + d^6e^2*2100i)$

$$\frac{3e^{2\sqrt{14}}(17511\sqrt{14} - 423i) + d^2e^{3\sqrt{14}}(21429\sqrt{14} + 198i) - d^4e^{4\sqrt{14}}(17247\sqrt{14} - 57i/2) - d^4e^{4\sqrt{14}}(74017\sqrt{14} + 19i/2)}{(d^8e^{625i} - d^7e^{1000i} - d^7e^{216i} + e^{8*81i} + d^2e^{6*756i} - d^3e^{5*1176i} + d^4e^{4*2086i} - d^5e^{3*1960i} + d^6e^{2*2100i})}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(e\*x+d)\*\*3/(5\*x\*\*2+2\*x+3)\*\*2,x)

[Out] Timed out



$$3.298 \quad \int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$$

**Optimal.** Leaf size=171

$$\frac{3e(100d^2 - 245de + 47e^2) \log(5x^2 + 2x + 3)}{6250} + \frac{3(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}$$

**Rubi [A]** time = 0.34, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{3e(100d^2 - 245de + 47e^2) \log(5x^2 + 2x + 3)}{6250} + \frac{3(-855175d^2e + 353125d^3 + 74085d^2e + 556349e^3) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}} + \frac{e^2x(83065d - 126009e)}{980000} + \frac{(d+ex)^2(x(11015d + 49177e) + 3(11449d - 2105e))}{196000(5x^2 + 2x + 3)} - \frac{(423x + 1367)(d+ex)^3}{7000(5x^2 + 2x + 3)^2} + \frac{2e^3x^2}{125}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3,x]

[Out] ((83065\*d - 126009\*e)\*e^2\*x)/980000 + (2\*e^3\*x^2)/125 - ((1367 + 423\*x)\*(d + e\*x)^3)/(7000\*(3 + 2\*x + 5\*x^2)^2) + ((d + e\*x)^2\*(3\*(11449\*d - 2105\*e) + (11015\*d + 49177\*e)\*x))/(196000\*(3 + 2\*x + 5\*x^2)) + (3\*(353125\*d^3 - 855175\*d^2\*e + 74085\*d\*e^2 + 556349\*e^3)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(4900000\*Sqrt[14]) + (3\*e\*(100\*d^2 - 245\*d\*e + 47\*e^2)\*Log[3 + 2\*x + 5\*x^2])/6250

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 618**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

**Rule 628**

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

**Rule 634**

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx &= -\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{1}{112} \int \frac{(d+ex)^2 \left( \frac{6}{125}(1089d+1367e) - \right.}{(3+2x+5x^2)^2} \\
&= -\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{(d+ex)^2(3(11449d-2105e) + (11015a}{196000(3+2x+5x^2)} \\
&= -\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{(d+ex)^2(3(11449d-2105e) + (11015a}{196000(3+2x+5x^2)} \\
&= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{(d+}{(3+2x+5x^2)} \\
&= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{(d+}{(3+2x+5x^2)} \\
&= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{(d+}{(3+2x+5x^2)} \\
&= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2} + \frac{(d+}{(3+2x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 209, normalized size = 1.22

$$\frac{164640e(100d^2 - 245de + 47e^2) \log(5x^2 + 2x + 3) - \frac{392(125d^2(423+1367)+75d^2(9899-1269-15d^2(8323+17967)+2(5969-53189)x) + 14(125d^2(1015+34347)+75d^2(181765-44399-15d^2(647195+809167)+2(257969-3109005)x) + 15\sqrt{14}(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3) \tan^{-1}\left(\frac{5x}{\sqrt{14}}\right) + 548800e^2x(60d - 49e) + 5488000e^2x^2}{5^7 2^{21} 3^7}}{343000000}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3 ,x]

[Out] (548800\*(60\*d - 49\*e)\*e^2\*x + 5488000\*e^3\*x^2 - (392\*(e^3\*(54969 - 53189\*x) + 125\*d^3\*(1367 + 423\*x) + 75\*d^2\*e\*(-1269 + 5989\*x) - 15\*d\*e^2\*(17967 + 18323\*x)))/(3 + 2\*x + 5\*x^2)^2 + (14\*(e^3\*(2639639 - 3109005\*x) + 125\*d^3\*(34347 + 11015\*x) + 75\*d^2\*e\*(-44399 + 181765\*x) - 15\*d\*e^2\*(809167 + 647195\*x)))/(3 + 2\*x + 5\*x^2) + 15\*sqrt[14]\*(353125\*d^3 - 855175\*d^2\*e + 74085\*d\*e^2 + 556349\*e^3)\*ArcTan[(1 + 5\*x)/sqrt[14]] + 164640\*e\*(100\*d^2 - 245\*d\*e + 47\*e^2)\*Log[3 + 2\*x + 5\*x^2])/343000000

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3 (2 + x + 3x^2 - 5x^3 + 4x^4)}{(3 + 2x + 5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[((d + e\*x)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

**fricas** [B] time = 0.77, size = 441, normalized size = 2.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="fricas")

[Out] 1/68600000\*(27440000\*e^3\*x^6 + 2744000\*(60\*d\*e^2 - 41\*e^3)\*x^5 + 8780800\*(15\*d\*e^2 - 8\*e^3)\*x^4 + 70\*(275375\*d^3 + 2726475\*d^2\*e + 1257135\*d\*e^2 - 3045929\*e^3)\*x^3 + 22667750\*d^3 - 20509650\*d^2\*e - 80825850\*d\*e^2 + 17863398\*e^3 + 14\*(4844125\*d^3 + 2123025\*d^2\*e - 10375875\*d\*e^2 - 2508283\*e^3)\*x^2 + 3\*sqrt(14)\*(25\*(353125\*d^3 - 855175\*d^2\*e + 74085\*d\*e^2 + 556349\*e^3)\*x^4 + 20\*(353125\*d^3 - 855175\*d^2\*e + 74085\*d\*e^2 + 556349\*e^3)\*x^3 + 3178125\*d^3 - 7696575\*d^2\*e + 666765\*d\*e^2 + 5007141\*e^3 + 34\*(353125\*d^3 - 855175\*d^2\*e + 74085\*d\*e^2 + 556349\*e^3)\*x^2 + 12\*(353125\*d^3 - 855175\*d^2\*e + 74085\*d\*e^2 + 556349\*e^3)\*x)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 42\*(749125\*d^3 + 1444025\*d^2\*e - 1635675\*d\*e^2 - 1323043\*e^3)\*x + 32928\*(25\*(100\*d^2\*e - 245\*d\*e^2 + 47\*e^3)\*x^4 + 20\*(100\*d^2\*e - 245\*d\*e^2 + 47\*e^3)\*x^3 + 900\*d^2\*e - 2205\*d\*e^2 + 423\*e^3 + 34\*(100\*d^2\*e - 245\*d\*e^2 + 47\*e^3)\*x^2 + 12\*(100\*d^2\*e - 245\*d\*e^2 + 47\*e^3)\*x)\*log(5\*x^2 + 2\*x + 3))/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

**giac** [A] time = 0.21, size = 201, normalized size = 1.18

$$\frac{1}{68600000} \left( \frac{27440000}{e^3} x^6 + \frac{2744000}{(60 d e^2 - 41 e^3)} x^5 + \frac{8780800}{(15 d e^2 - 8 e^3)} x^4 + 70 (275375 d^3 + 2726475 d^2 e + 1257135 d e^2 - 3045929 e^3) x^3 + 22667750 d^3 - 20509650 d^2 e - 80825850 d e^2 + 17863398 e^3 + 14 (4844125 d^3 + 2123025 d^2 e - 10375875 d e^2 - 2508283 e^3) x^2 + 3 \sqrt{14} (25 (353125 d^3 - 855175 d^2 e + 74085 d e^2 + 556349 e^3) x^4 + 20 (353125 d^3 - 855175 d^2 e + 74085 d e^2 + 556349 e^3) x^3 + 3178125 d^3 - 7696575 d^2 e + 666765 d e^2 + 5007141 e^3 + 34 (353125 d^3 - 855175 d^2 e + 74085 d e^2 + 556349 e^3) x^2 + 12 (353125 d^3 - 855175 d^2 e + 74085 d e^2 + 556349 e^3) x) \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) + 42 (749125 d^3 + 1444025 d^2 e - 1635675 d e^2 - 1323043 e^3) x + 32928 (25 (100 d^2 e - 245 d e^2 + 47 e^3) x^4 + 20 (100 d^2 e - 245 d e^2 + 47 e^3) x^3 + 900 d^2 e - 2205 d e^2 + 423 e^3 + 34 (100 d^2 e - 245 d e^2 + 47 e^3) x^2 + 12 (100 d^2 e - 245 d e^2 + 47 e^3) x) \log(5x^2 + 2x + 3) \right) / (25x^4 + 20x^3 + 34x^2 + 12x + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="giac")

[Out]  $2/125*x^2*e^3 + 12/125*d*x*e^2 + 3/68600000*\sqrt{14}*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*\arctan(1/14*\sqrt{14}*(5*x + 1)) - 49/625*x*e^3 + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*\log(5*x^2 + 2*x + 3) + 1/4900000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 1619125*d^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 - 1464975*d^2*e + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e^3)*x - 5773275*d*e^2 + 1275957*e^3)/(5*x^2 + 2*x + 3)^2$

**maple [A]** time = 0.02, size = 267, normalized size = 1.56

$$\frac{2}{125}x^2e^3 + \frac{12}{125}dxe^2 + \frac{3}{68600000}\sqrt{14}(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{49}{625}xe^3 + \frac{3}{6250}(100d^2e - 245de^2 + 47e^3)\log(5x^2 + 2x + 3) + \frac{1}{4900000}(5(275375d^3 + 2726475d^2e - 1941585de^2 - 621801e^3)x^3 + 1619125d^3 + (4844125d^3 + 2123025d^2e - 16020675de^2 + 1396037e^3)x^2 - 1464975d^2e + 3(749125d^3 + 1444025d^2e - 3046875de^2 - 170563e^3)x - 5773275de^2 + 1275957e^3)/(5x^2 + 2x + 3)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3, x)$

[Out]  $2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/1568*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840*d^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/7840*d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d^2*e-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+6/125*d^2*e*\ln(5*x^2+2*x+3)-147/1250*d*e^2*\ln(5*x^2+2*x+3)+141/6250*e^3*\ln(5*x^2+2*x+3)+339/21952*14^(1/2)*d^3*\arctan(1/28*(10*x+2)*14^(1/2))-102621/2744000*14^(1/2)*d^2*e*\arctan(1/28*(10*x+2)*14^(1/2))+44451/13720000*14^(1/2)*d*e^2*\arctan(1/28*(10*x+2)*14^(1/2))+1669047/68600000*14^(1/2)*e^3*\arctan(1/28*(10*x+2)*14^(1/2))$

**maxima [A]** time = 0.97, size = 222, normalized size = 1.30

$$\frac{2}{125}x^2e^3 + \frac{12}{125}dxe^2 + \frac{3}{68600000}\sqrt{14}(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{49}{625}xe^3 + \frac{3}{6250}(100d^2e - 245de^2 + 47e^3)\log(5x^2 + 2x + 3) + \frac{1}{4900000}(5(275375d^3 + 2726475d^2e - 1941585de^2 - 621801e^3)x^3 + 1619125d^3 + (4844125d^3 + 2123025d^2e - 16020675de^2 + 1396037e^3)x^2 + 3(749125d^3 + 1444025d^2e - 3046875de^2 - 170563e^3)x - 5773275de^2 + 1275957e^3)/(5x^2 + 2x + 3)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3, x, \text{algorithm}="maxima")$

[Out]  $2/125*e^3*x^2 + 3/68600000*\sqrt{14}*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*\arctan(1/14*\sqrt{14}*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^3)*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*\log(5*x^2 + 2*x + 3) + 1/4900000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 1619125*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)$

**mupad [B]** time = 0.15, size = 299, normalized size = 1.75

$$\frac{2}{125}x^2e^3 + \frac{3}{68600000}\sqrt{14}(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{625}(60d^2e - 49e^3)x + \frac{3}{6250}(100d^2e - 245de^2 + 47e^3)\log(5x^2 + 2x + 3) + \frac{1}{4900000}(5(275375d^3 + 2726475d^2e - 1941585de^2 - 621801e^3)x^3 + 1619125d^3 - 1464975d^2e - 5773275de^2 + 1275957e^3 + (4844125d^3 + 2123025d^2e - 16020675de^2 + 1396037e^3)x^2 + 3(749125d^3 + 1444025d^2e - 3046875de^2 - 170563e^3)x)/(25x^4 + 20x^3 + 34x^2 + 12x + 9)$$



$$3.299 \quad \int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$$

Optimal. Leaf size=134

$$\frac{(211875d^2 - 342070de + 14817e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}} + \frac{(d+ex)(5x(2203d + 8553e) + 34347d - 6413e)}{196000(5x^2 + 2x + 3)} - \frac{(423x + 1367)}{7000(5x^2 + 2x + 3)}$$

Rubi [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1644, 1657, 634, 618, 204, 628}

$$\frac{(211875d^2 - 342070de + 14817e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}} + \frac{(d+ex)(5x(2203d + 8553e) + 34347d - 6413e)}{196000(5x^2 + 2x + 3)} - \frac{(423x + 1367)(d+ex)^2}{7000(5x^2 + 2x + 3)^2} + \frac{e(40d - 49e) \log(5x^2 + 2x + 3)}{1250} + \frac{4e^2x}{125}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3,x]

[Out] (4\*e^2\*x)/125 - ((1367 + 423\*x)\*(d + e\*x)^2)/(7000\*(3 + 2\*x + 5\*x^2)^2) + ((d + e\*x)\*(34347\*d - 6413\*e + 5\*(2203\*d + 8553\*e)\*x))/(196000\*(3 + 2\*x + 5\*x^2)) + ((211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(980000\*Sqrt[14]) + ((40\*d - 49\*e)\*e\*Log[3 + 2\*x + 5\*x^2])/1250

Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m+1)*(a + b*x + c*x^2)^(p+1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p+1)*(b^2 - 4*a*c)), x] + Dist[1/((p+1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m-1)*(a + b*x + c*x^2)^(p+1)*ExpandToSum[(p+1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p+3)) - f*(b*e*m + 2*c*d*(2*p+3)) - e*(2*c*f - b*g)*(m+2*p+3)*x, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rule 1657

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx &= -\frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{1}{112} \int \frac{(d+ex)\left(\frac{2}{125}(3267d+2734e) - \right.}{(3+2x+5x^2)^2} \\
&= -\frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+2734e))}{196000(3+2x+5x^2)} \\
&= -\frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+2734e))}{196000(3+2x+5x^2)} \\
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+2734e))}{196000(3+2x+5x^2)} \\
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+2734e))}{196000(3+2x+5x^2)} \\
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+2734e))}{196000(3+2x+5x^2)} \\
&= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+2734e))}{196000(3+2x+5x^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 146, normalized size = 1.09

$$\frac{70 \left( \frac{5(5d^2(11015x^3+38753x^2+17979x+12953)+2d(181765x^3+28307x^2+57761x-19533))+e^2(156800x^5+125440x^4+83809x^3-138345x^2-65427x-76977)}{(5x^2+2x+3)^2} + 784e(40d-49e) \log(5x^2+2x+3) \right) + 5\sqrt{14} (211875d^2-342070de+14817e^2) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{68600000}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

[Out] (5\*sqrt(14)\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*ArcTan[(1 + 5\*x)/sqrt(14)] + 70\*((5\*(5\*d^2\*(12953 + 17979\*x + 38753\*x^2 + 11015\*x^3) + 2\*d\*e\*(-19533 + 57761\*x + 28307\*x^2 + 181765\*x^3) + e^2\*(-76977 - 65427\*x - 138345\*x^2 + 83809\*x^3 + 125440\*x^4 + 156800\*x^5)))/(3 + 2\*x + 5\*x^2)^2 + 784\*(40\*d - 49\*e)\*e\*Log[3 + 2\*x + 5\*x^2]))/68600000

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2 (2 + x + 3x^2 - 5x^3 + 4x^4)}{(3 + 2x + 5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3,x]

[Out] IntegrateAlgebraic[((d + e\*x)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

**fricas [B]** time = 0.81, size = 302, normalized size = 2.25

10976000\*d^2\*e^2\*x^5 + 8780800\*d^2\*e\*x^4 + 70\*(55075\*d^2 + 363530\*d\*e + 83809\*e^2)\*x^3 + 70\*(193765\*d^2 + 56614\*d\*e - 138345\*e^2)\*x^2 + sqrt(14)\*(25\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x^4 + 20\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x^3 + 34\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x^2 + 1906875\*d^2 - 3078630\*d\*e + 133353\*e^2 + 12\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 4533550\*d^2 - 2734620\*d\*e - 5388390\*e^2 + 70\*(89895\*d^2 + 115522\*d\*e - 65427\*e^2)\*x + 10976\*(25\*(40\*d\*e - 49\*e^2)\*x^4 + 20\*(40\*d\*e - 49\*e^2)\*x^3 + 34\*(40\*d\*e - 49\*e^2)\*x^2 + 360\*d\*e - 441\*e^2 + 12\*(40\*d\*e - 49\*e^2)\*x)\*log(5\*x^2 + 2\*x + 3))/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="fricas")

[Out] 1/13720000\*(10976000\*e^2\*x^5 + 8780800\*e^2\*x^4 + 70\*(55075\*d^2 + 363530\*d\*e + 83809\*e^2)\*x^3 + 70\*(193765\*d^2 + 56614\*d\*e - 138345\*e^2)\*x^2 + sqrt(14)\*(25\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x^4 + 20\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x^3 + 34\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x^2 + 1906875\*d^2 - 3078630\*d\*e + 133353\*e^2 + 12\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*x)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 4533550\*d^2 - 2734620\*d\*e - 5388390\*e^2 + 70\*(89895\*d^2 + 115522\*d\*e - 65427\*e^2)\*x + 10976\*(25\*(40\*d\*e - 49\*e^2)\*x^4 + 20\*(40\*d\*e - 49\*e^2)\*x^3 + 34\*(40\*d\*e - 49\*e^2)\*x^2 + 360\*d\*e - 441\*e^2 + 12\*(40\*d\*e - 49\*e^2)\*x)\*log(5\*x^2 + 2\*x + 3))/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

**giac [A]** time = 0.16, size = 144, normalized size = 1.07

1/13720000\*sqrt(14)\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 4/125\*x^2 + 1/1250\*(40\*d\*e - 49\*e^2)\*log(5\*x^2 + 2\*x + 3) + (55075\*d^2 + 363530\*d\*e - 129439\*e^2)\*x^3 + (193765\*d^2 + 56614\*d\*e - 213609\*e^2)\*x^2 + 64765\*d^2 + (89895\*d^2 + 115522\*d\*e - 121875\*e^2)\*x - 39066\*d\*e - 76977\*e^2/196000\*(5\*x^2 + 2\*x + 3)^2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="giac")

[Out] 1/13720000\*sqrt(14)\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 4/125\*x^2 + 1/1250\*(40\*d\*e - 49\*e^2)\*log(5\*x^2 + 2\*x + 3) + 1/196000\*((55075\*d^2 + 363530\*d\*e - 129439\*e^2)\*x^3 + (193765\*d^2 + 566

$$14*d*e - 213609*e^2)*x^2 + 64765*d^2 + (89895*d^2 + 115522*d*e - 121875*e^2) * x - 39066*d*e - 76977*e^2)/(5*x^2 + 2*x + 3)^2$$

**maple [A]** time = 0.01, size = 179, normalized size = 1.34

$$\frac{339\sqrt{14}d^2 \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) - 34207\sqrt{14}de \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) + 4de \ln(5x^2+2x+3) + \frac{4e^2x}{125} + \frac{14817\sqrt{14}e^2 \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) - 49e^2 \ln(5x^2+2x+3)}{13720000} + \frac{\left(\frac{2203}{1568}d^2 + \frac{36353}{3920}de - \frac{129439}{39200}e^2\right)x^3 + \frac{12953d^2}{7840} - \frac{19533de}{19600} - \frac{76977e^2}{39200} + \frac{38753}{7840}d^2 + \frac{28307}{19600}de - \frac{213609}{39200}e^2}{5(5x^2+2x+3)^2} + \frac{\left(\frac{17979}{7840}d^2 + \frac{57761}{19600}de - \frac{4875}{1568}e^2\right)x}{13720000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x)

$$[Out] \frac{4}{125}e^2x + \frac{1}{5} \left( \frac{2203}{1568}d^2 + \frac{36353}{3920}de - \frac{129439}{39200}e^2 \right) x^3 + \frac{38753}{7840}d^2 + \frac{28307}{19600}de - \frac{213609}{39200}e^2 x^2 + \frac{17979}{7840}d^2 + \frac{57761}{19600}de - \frac{4875}{1568}e^2 x + \frac{12953}{7840}d^2 - \frac{19533}{19600}de - \frac{76977}{39200}e^2 / (5x^2+2x+3)^2 + \frac{4}{125}d^2 \ln(5x^2+2x+3) - \frac{49}{1250}e^2 \ln(5x^2+2x+3) + \frac{339}{21952}14^{1/2}d^2 \arctan(1/28*(10*x+2)*14^{1/2}) - \frac{34207}{13720000}14^{1/2}d^2 e \arctan(1/28*(10*x+2)*14^{1/2}) + \frac{14817}{13720000}14^{1/2}e^2 \arctan(1/28*(10*x+2)*14^{1/2})$$

**maxima [A]** time = 0.96, size = 155, normalized size = 1.16

$$\frac{4}{125}e^2x + \frac{1}{13720000}\sqrt{14}(211875d^2 - 342070de + 14817e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{1250}(40de - 49e^2)\log(5x^2+2x+3) + \frac{(55075d^2 + 363530de - 129439e^2)x^3 + (193765d^2 + 56614de - 213609e^2)x^2 + 64765d^2 - 39066de - 76977e^2 + (89895d^2 + 115522de - 121875e^2)x}{196000(25x^4 + 20x^3 + 34x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="maxima")

$$[Out] \frac{4}{125}e^2x + \frac{1}{13720000}\sqrt{14}(211875d^2 - 342070de + 14817e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{1250}(40de - 49e^2)\log(5x^2+2x+3) + \frac{1}{196000} \left( (55075d^2 + 363530de - 129439e^2)x^3 + (193765d^2 + 56614de - 213609e^2)x^2 + 64765d^2 - 39066de - 76977e^2 + (89895d^2 + 115522de - 121875e^2)x \right) / (25x^4 + 20x^3 + 34x^2 + 12x + 9)$$

**mupad [B]** time = 4.21, size = 203, normalized size = 1.51

$$\frac{x^3 \left( \frac{55075d^2 + 363530de - 129439e^2}{1568} \right) + x^2 \left( \frac{193765d^2 + 56614de - 213609e^2}{784} - \frac{19533de}{1568} + x \left( \frac{89895d^2 + 57761de - 121875e^2}{784} + \frac{64765d^2 - 76977e^2}{1568} \right) \right) + \frac{4e^2x}{125} + \ln(5x^2+2x+3) \left( \frac{4de}{125} - \frac{49e^2}{1250} \right) + \frac{\sqrt{14} \operatorname{atan}\left(\frac{\sqrt{14}(211875d^2 - 342070de + 14817e^2)}{13720000}, \frac{\sqrt{14}(211875d^2 - 342070de + 14817e^2)}{13720000}\right)}{13720000} (211875d^2 - 342070de + 14817e^2)}{13720000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)^2\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(2\*x + 5\*x^2 + 3)^3,x)

$$[Out] \frac{x^3 \left( (181765de)/784 + (55075d^2)/1568 - (129439e^2)/1568 \right) + x^2 \left( (28307de)/784 + (193765d^2)/1568 - (213609e^2)/1568 - (19533de)/784 + x \left( (57761de)/784 + (89895d^2)/1568 - (121875e^2)/1568 + (64765d^2)/1568 - (76977e^2)/1568 \right) / (1500x + 4250x^2 + 2500x^3 + 3125x^4 + 1125) + \frac{4e^2x}{125} + \log(2x + 5x^2 + 3) \left( \frac{4de}{125} - \frac{49e^2}{1250} \right) + \frac{14^{1/2}d^2 \operatorname{atan}\left(\frac{14^{1/2}(211875d^2 - 342070de + 14817e^2)}{13720000}\right)}{13720000}$$

atan(((14^(1/2)\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2))/13720000 + (14^(1/2)\*x\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2))/2744000)/((339\*d^2)/1568 - (34207\*d\*e)/98000 + (14817\*e^2)/980000))\*(211875\*d^2 - 342070\*d\*e + 14817\*e^2))/13720000

**sympy** [C] time = 3.96, size = 304, normalized size = 2.27

$$\frac{e^{\frac{1}{2}} \left( \frac{(40d - 49e) \sqrt{14(211875d^2 - 342070de + 14817e^2)}}{27440000} \right) \log \left( \frac{42375d^2 - 244030de + 218093e^2 + \sqrt{14(211875d^2 - 342070de + 14817e^2)}}{211875d^2 - 342070de + 14817e^2} \right) + \frac{(40d - 49e) \sqrt{14(211875d^2 - 342070de + 14817e^2)}}{27440000} \log \left( \frac{42375d^2 - 244030de + 218093e^2 + \sqrt{14(211875d^2 - 342070de + 14817e^2)}}{211875d^2 - 342070de + 14817e^2} \right) + \frac{64765d^2 - 39066de - 76977e^2 + x^3(55075d^2 + 363530de - 129439e^2) + x^2(193765d^2 + 56614de - 213609e^2) + x(89895d^2 + 115522de - 121875e^2)}{(4900000x^4 + 3920000x^3 + 6664000x^2 + 2352000x + 1764000)}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3)\*\*3,x)

[Out] 4\*e\*\*2\*x/125 + (e\*(40\*d - 49\*e)/1250 - sqrt(14)\*I\*(211875\*d\*\*2 - 342070\*d\*e + 14817\*e\*\*2)/27440000)\*log(x + (42375\*d\*\*2 - 244030\*d\*e + 218093\*e\*\*2 + 21952\*e\*(40\*d - 49\*e)/5 - sqrt(14)\*I\*(211875\*d\*\*2 - 342070\*d\*e + 14817\*e\*\*2)/5)/(211875\*d\*\*2 - 342070\*d\*e + 14817\*e\*\*2)) + (e\*(40\*d - 49\*e)/1250 + sqrt(14)\*I\*(211875\*d\*\*2 - 342070\*d\*e + 14817\*e\*\*2)/27440000)\*log(x + (42375\*d\*\*2 - 244030\*d\*e + 218093\*e\*\*2 + 21952\*e\*(40\*d - 49\*e)/5 + sqrt(14)\*I\*(211875\*d\*\*2 - 342070\*d\*e + 14817\*e\*\*2)/5)/(211875\*d\*\*2 - 342070\*d\*e + 14817\*e\*\*2)) + (64765\*d\*\*2 - 39066\*d\*e - 76977\*e\*\*2 + x\*\*3\*(55075\*d\*\*2 + 363530\*d\*e - 129439\*e\*\*2) + x\*\*2\*(193765\*d\*\*2 + 56614\*d\*e - 213609\*e\*\*2) + x\*(89895\*d\*\*2 + 115522\*d\*e - 121875\*e\*\*2))/(4900000\*x\*\*4 + 3920000\*x\*\*3 + 6664000\*x\*\*2 + 2352000\*x + 1764000)

$$3.300 \quad \int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$$

Optimal. Leaf size=103

$$-\frac{(423x+1367)(d+ex)}{7000(5x^2+2x+3)^2} + \frac{x(11015d+36353e)+34347d-6511e}{196000(5x^2+2x+3)} + \frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}} + \frac{2}{125}e \log\left(\frac{5x+1}{\sqrt{14}}\right)$$

**Rubi [A]** time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {1644, 1660, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{7000(5x^2+2x+3)^2} + \frac{x(11015d+36353e)+34347d-6511e}{196000(5x^2+2x+3)} + \frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}} + \frac{2}{125}e \log(5x^2+2x+3)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3,x]

[Out] -((1367 + 423\*x)\*(d + e\*x))/(7000\*(3 + 2\*x + 5\*x^2)^2) + (34347\*d - 6511\*e + (11015\*d + 36353\*e)\*x)/(196000\*(3 + 2\*x + 5\*x^2)) + ((42375\*d - 34207\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(196000\*Sqrt[14]) + (2\*e\*Log[3 + 2\*x + 5\*x^2])/125

#### Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 628

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[p] || !IntegerQ[m] || !RationalQ[a, b, c, d, e]) && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx &= -\frac{(1367+423x)(d+ex)}{7000(3+2x+5x^2)^2} + \frac{1}{112} \int \frac{\frac{2}{125}(3267d+1367e) - \frac{12}{25}(308d-)}{(3+2x+5x^2)^3} dx \\
&= -\frac{(1367+423x)(d+ex)}{7000(3+2x+5x^2)^2} + \frac{34347d-6511e+(11015d+36353e)x}{196000(3+2x+5x^2)} + \dots \\
&= -\frac{(1367+423x)(d+ex)}{7000(3+2x+5x^2)^2} + \frac{34347d-6511e+(11015d+36353e)x}{196000(3+2x+5x^2)} + \dots \\
&= -\frac{(1367+423x)(d+ex)}{7000(3+2x+5x^2)^2} + \frac{34347d-6511e+(11015d+36353e)x}{196000(3+2x+5x^2)} + \dots \\
&= -\frac{(1367+423x)(d+ex)}{7000(3+2x+5x^2)^2} + \frac{34347d-6511e+(11015d+36353e)x}{196000(3+2x+5x^2)} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 107, normalized size = 1.04

$$\frac{-2115dx - 6835d - 5989ex + 1269e}{35000(5x^2 + 2x + 3)^2} + \frac{55075dx + 171735d + 181765ex - 44399e}{980000(5x^2 + 2x + 3)} + \frac{(42375d - 34207e) \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}} + \frac{2}{125}e \log(5x^2 + 2x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

[Out] (-6835\*d + 1269\*e - 2115\*d\*x - 5989\*e\*x)/(35000\*(3 + 2\*x + 5\*x^2)^2) + (171735\*d - 44399\*e + 55075\*d\*x + 181765\*e\*x)/(980000\*(3 + 2\*x + 5\*x^2)) + ((42375\*d - 34207\*e)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(196000\*Sqrt[14]) + (2\*e\*Log[3 + 2\*x + 5\*x^2])/125

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[((d + e\*x)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4))/(3 + 2\*x + 5\*x^2)^3, x]

**fricas** [A] time = 0.59, size = 172, normalized size = 1.67

$$\frac{70(11015d + 36353e)x^3 + 14(193765d + 28307e)x^2 + \sqrt{14}(25(42375d - 34207e)x^4 + 20(42375d - 34207e)x^3 + 34(42375d - 34207e)x^2 + 12(42375d - 34207e)x + 381375d - 307863e) \arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + 14(89895d + 57761e)x + 43904(25ex^4 + 20ex^3 + 34ex^2 + 12ex + 9e) \log(5x^2 + 2x + 3) + 906710d - 273462e}{2744000(25x^4 + 20x^3 + 34x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="fricas")

[Out] 1/2744000\*(70\*(11015\*d + 36353\*e)\*x^3 + 14\*(193765\*d + 28307\*e)\*x^2 + sqrt(14)\*(25\*(42375\*d - 34207\*e)\*x^4 + 20\*(42375\*d - 34207\*e)\*x^3 + 34\*(42375\*d - 34207\*e)\*x^2 + 12\*(42375\*d - 34207\*e)\*x + 381375\*d - 307863\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 14\*(89895\*d + 57761\*e)\*x + 43904\*(25\*e\*x^4 + 20\*e\*x^3 + 34\*e\*x^2 + 12\*e\*x + 9\*e)\*log(5\*x^2 + 2\*x + 3) + 906710\*d - 273462\*e)/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

**giac** [A] time = 0.19, size = 97, normalized size = 0.94

$$\frac{1}{2744000} \sqrt{14} (42375d - 34207e) \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) + \frac{2}{125} e \log(5x^2 + 2x + 3) + \frac{5(11015d + 36353e)x^3 + (193765d + 28307e)x^2 + (89895d + 57761e)x + 64765d - 19533e}{196000(5x^2 + 2x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="giac")

[Out] 1/2744000\*sqrt(14)\*(42375\*d - 34207\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 2/125\*e\*log(5\*x^2 + 2\*x + 3) + 1/196000\*(5\*(11015\*d + 36353\*e)\*x^3 + (193765\*d + 28307\*e)\*x^2 + (89895\*d + 57761\*e)\*x + 64765\*d - 19533\*e)/(5\*x^2 + 2\*x + 3)^2

**maple** [A] time = 0.01, size = 102, normalized size = 0.99

$$\frac{339\sqrt{14}d \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right) - 34207\sqrt{14}e \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952} + \frac{2e \ln(5x^2 + 2x + 3)}{125} + \frac{25\left(\frac{2203d}{196000} + \frac{36353e}{980000}\right)x^3 + 25\left(\frac{38753d}{980000} + \frac{28307e}{4900000}\right)x^2 + \frac{12953d}{39200} - \frac{19533e}{196000} + 25\left(\frac{17979d}{980000} + \frac{57761e}{4900000}\right)x}{(5x^2 + 2x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x)

[Out] 25\*((36353/980000\*e+2203/196000\*d)\*x^3+(28307/4900000\*e+38753/980000\*d)\*x^2+(57761/4900000\*e+17979/980000\*d)\*x+12953/980000\*d-19533/4900000\*e)/(5\*x^2+2\*x+3)^2+2/125\*e\*ln(5\*x^2+2\*x+3)+339/21952\*14^(1/2)\*d\*arctan(1/28\*(10\*x+2)\*14^(1/2))-34207/2744000\*14^(1/2)\*e\*arctan(1/28\*(10\*x+2)\*14^(1/2))



**maxima [A]** time = 0.96, size = 101, normalized size = 0.98

$$\frac{1}{2744000} \sqrt{14} (42375d - 34207e) \arctan\left(\frac{1}{14} \sqrt{14} (5x + 1)\right) + \frac{2}{125} e \log(5x^2 + 2x + 3) + \frac{5(11015d + 36353e)x^3 + (193765d + 28307e)x^2 + (89895d + 57761e)x + 64765d - 19533e}{196000(25x^4 + 20x^3 + 34x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="maxima")

[Out] 1/2744000\*sqrt(14)\*(42375\*d - 34207\*e)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 2/125\*e\*log(5\*x^2 + 2\*x + 3) + 1/196000\*(5\*(11015\*d + 36353\*e)\*x^3 + (193765\*d + 28307\*e)\*x^2 + (89895\*d + 57761\*e)\*x + 64765\*d - 19533\*e)/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

**mupad [B]** time = 0.12, size = 125, normalized size = 1.21

$$\left(\frac{2203d + 36353e}{7840 + 39200}x^3 + \left(\frac{38753d + 28307e}{39200} + \frac{17979d + 57761e}{196000}\right)x^2 + \left(\frac{12953d}{39200} - \frac{19533e}{196000}\right)x + \frac{2e \ln(5x^2 + 2x + 3)}{125} + \frac{\sqrt{14} \operatorname{atan}\left(\frac{\sqrt{14}(42375d - 34207e)}{2744000} + \frac{\sqrt{14}(42375d - 34207e)}{548800}\right)}{2744000}\right)(42375d - 34207e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x)\*(x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2))/(2\*x + 5\*x^2 + 3)^3,x)

[Out] ((12953\*d)/39200 - (19533\*e)/196000 + x^3\*((2203\*d)/7840 + (36353\*e)/39200) + x^2\*((38753\*d)/39200 + (28307\*e)/196000) + x\*((17979\*d)/39200 + (57761\*e)/196000))/(12\*x + 34\*x^2 + 20\*x^3 + 25\*x^4 + 9) + (2\*e\*log(2\*x + 5\*x^2 + 3))/125 + (14^(1/2)\*atan((14^(1/2)\*(42375\*d - 34207\*e))/2744000 + (14^(1/2)\*x\*(42375\*d - 34207\*e))/548800)/((339\*d)/1568 - (34207\*e)/196000))\*(42375\*d - 34207\*e)/2744000

**sympy [C]** time = 2.31, size = 163, normalized size = 1.58

$$\left(\frac{2e}{125} - \frac{\sqrt{14}(42375d - 34207e)}{5488000}\right) \log\left(x + \frac{8475d - 34207e - \sqrt{14}(42375d - 34207e)}{42375d - 34207e}\right) + \left(\frac{2e}{125} + \frac{\sqrt{14}(42375d - 34207e)}{5488000}\right) \log\left(x + \frac{8475d - 34207e + \sqrt{14}(42375d - 34207e)}{42375d - 34207e}\right) + \frac{64765d - 19533e + x^3(55075d + 181765e) + x^2(193765d + 28307e) + x(89895d + 57761e)}{4900000x^4 + 3920000x^3 + 6664000x^2 + 2352000x + 1764000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3)\*\*3,x)

[Out] (2\*e/125 - sqrt(14)\*I\*(42375\*d - 34207\*e)/5488000)\*log(x + (8475\*d - 34207\*e)/5 - sqrt(14)\*I\*(42375\*d - 34207\*e)/5)/(42375\*d - 34207\*e) + (2\*e/125 + sqrt(14)\*I\*(42375\*d - 34207\*e)/5488000)\*log(x + (8475\*d - 34207\*e)/5 + sqrt(14)\*I\*(42375\*d - 34207\*e)/5)/(42375\*d - 34207\*e) + (64765\*d - 19533\*e + x\*\*3\*(55075\*d + 181765\*e) + x\*\*2\*(193765\*d + 28307\*e) + x\*(89895\*d + 57761\*e))/(4900000\*x\*\*4 + 3920000\*x\*\*3 + 6664000\*x\*\*2 + 2352000\*x + 1764000)

$$3.301 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3} dx$$

Optimal. Leaf size=64

$$-\frac{423x + 1367}{7000(5x^2 + 2x + 3)^2} + \frac{11015x + 34347}{196000(5x^2 + 2x + 3)} + \frac{339 \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {1660, 12, 618, 204}

$$-\frac{423x + 1367}{7000(5x^2 + 2x + 3)^2} + \frac{11015x + 34347}{196000(5x^2 + 2x + 3)} + \frac{339 \tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^3,x]

[Out] -(1367 + 423\*x)/(7000\*(3 + 2\*x + 5\*x^2)^2) + (34347 + 11015\*x)/(196000\*(3 + 2\*x + 5\*x^2)) + (339\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(1568\*Sqrt[14])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

q, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 1]], Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[(p + 1)\*(b^2 - 4\*a\*c)\*Q - (2\*p + 3)\*(2\*c\*f - b\*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(3 + 2x + 5x^2)^3} dx &= -\frac{1367 + 423x}{7000(3 + 2x + 5x^2)^2} + \frac{1}{112} \int \frac{\frac{6534}{125} - \frac{3696x}{25} + \frac{448x^2}{5}}{(3 + 2x + 5x^2)^2} dx \\
 &= -\frac{1367 + 423x}{7000(3 + 2x + 5x^2)^2} + \frac{34347 + 11015x}{196000(3 + 2x + 5x^2)} + \frac{\int \frac{1356}{3+2x+5x^2} dx}{6272} \\
 &= -\frac{1367 + 423x}{7000(3 + 2x + 5x^2)^2} + \frac{34347 + 11015x}{196000(3 + 2x + 5x^2)} + \frac{339 \int \frac{1}{3+2x+5x^2} dx}{1568} \\
 &= -\frac{1367 + 423x}{7000(3 + 2x + 5x^2)^2} + \frac{34347 + 11015x}{196000(3 + 2x + 5x^2)} - \frac{339}{784} \text{Subst} \left( \int \frac{1}{-56 - x^2} dx \right) \\
 &= -\frac{1367 + 423x}{7000(3 + 2x + 5x^2)^2} + \frac{34347 + 11015x}{196000(3 + 2x + 5x^2)} + \frac{339 \tan^{-1} \left( \frac{1+5x}{\sqrt{14}} \right)}{1568\sqrt{14}}
 \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 53, normalized size = 0.83

$$\frac{14(11015x^3 + 38753x^2 + 17979x + 12953)}{(5x^2 + 2x + 3)^2} + 8475\sqrt{14} \tan^{-1} \left( \frac{5x+1}{\sqrt{14}} \right)$$


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548800

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^3, x]

[Out] ((14\*(12953 + 17979\*x + 38753\*x^2 + 11015\*x^3))/(3 + 2\*x + 5\*x^2)^2 + 8475\*  
Sqrt[14]\*ArcTan[(1 + 5\*x)/Sqrt[14]])/548800

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(3 + 2x + 5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^3, x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/(3 + 2\*x + 5\*x^2)^3, x]

**fricas** [A] time = 0.81, size = 75, normalized size = 1.17

$$\frac{154210x^3 + 8475\sqrt{14}(25x^4 + 20x^3 + 34x^2 + 12x + 9) \arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + 542542x^2 + 251706x + 181342}{548800(25x^4 + 20x^3 + 34x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="fricas")

[Out] 1/548800\*(154210\*x^3 + 8475\*sqrt(14)\*(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 542542\*x^2 + 251706\*x + 181342)/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

**giac** [A] time = 0.16, size = 46, normalized size = 0.72

$$\frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{11015x^3 + 38753x^2 + 17979x + 12953}{39200(5x^2 + 2x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="giac")

[Out] 339/21952\*sqrt(14)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 1/39200\*(11015\*x^3 + 38753\*x^2 + 17979\*x + 12953)/(5\*x^2 + 2\*x + 3)^2

**maple** [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{339\sqrt{14}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952} + \frac{\frac{2203}{7840}x^3 + \frac{38753}{39200}x^2 + \frac{17979}{39200}x + \frac{12953}{39200}}{(5x^2 + 2x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x)

[Out] 25\*(2203/196000\*x^3+38753/980000\*x^2+17979/980000\*x+12953/980000)/(5\*x^2+2\*x+3)^2+339/21952\*14^(1/2)\*arctan(1/28\*(10\*x+2)\*14^(1/2))

**maxima** [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x + 1)\right) + \frac{11015x^3 + 38753x^2 + 17979x + 12953}{39200(25x^4 + 20x^3 + 34x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(5\*x^2+2\*x+3)^3,x, algorithm="maxima")

[Out] 339/21952\*sqrt(14)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 1/39200\*(11015\*x^3 + 38753\*x^2 + 17979\*x + 12953)/(25\*x^4 + 20\*x^3 + 34\*x^2 + 12\*x + 9)

**mupad [B]** time = 0.05, size = 55, normalized size = 0.86

$$\frac{339\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{21952} + \frac{\frac{2203x^3}{196000} + \frac{38753x^2}{980000} + \frac{17979x}{980000} + \frac{12953}{980000}}{x^4 + \frac{4x^3}{5} + \frac{34x^2}{25} + \frac{12x}{25} + \frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - 5\*x^3 + 4\*x^4 + 2)/(2\*x + 5\*x^2 + 3)^3,x)

[Out] (339\*14^(1/2)\*atan((5\*14^(1/2)\*x)/14 + 14^(1/2)/14))/21952 + ((17979\*x)/980000 + (38753\*x^2)/980000 + (2203\*x^3)/196000 + 12953/980000)/((12\*x)/25 + (34\*x^2)/25 + (4\*x^3)/5 + x^4 + 9/25)

**sympy [A]** time = 0.20, size = 61, normalized size = 0.95

$$\frac{11015x^3 + 38753x^2 + 17979x + 12953}{980000x^4 + 784000x^3 + 1332800x^2 + 470400x + 352800} + \frac{339\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{21952}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2)/(5\*x\*\*2+2\*x+3)\*\*3,x)

[Out] (11015\*x\*\*3 + 38753\*x\*\*2 + 17979\*x + 12953)/(980000\*x\*\*4 + 784000\*x\*\*3 + 1332800\*x\*\*2 + 470400\*x + 352800) + 339\*sqrt(14)\*atan(5\*sqrt(14)\*x/14 + sqrt(14)/14)/21952

$$3.302 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3} dx$$

Optimal. Leaf size=329

$$\frac{x(423d - 1367e) + 1367d - 293e}{1400(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)} + \frac{171735d^3 - 92989d^2e + 25x(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)}{39200(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)^2}$$

**Rubi [A]** time = 0.50, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1646, 800, 634, 618, 204, 628}

$$\frac{-x(423d - 1367e) + 1367d - 293e}{1400(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)} + \frac{25x(-9033d^2e + 2203d^3 + 3635d^2e^2 - 1829e^3) - 92989d^2e + 171735d^3 + 36207d^2e + 1831e^3}{39200(5x^2 + 2x + 3)(5d^2 - 2de + 3e^2)^2} + \frac{e(3d^2 + 5d^2e + 4d^3 - d^3 + 2d^3) \log(5x^2 + 2x + 3)}{2(5d^2 - 2de + 3e^2)} + \frac{e(3d^2 + 5d^2e + 4d^3 - d^3 + 2d^3) \log(d + ex)}{(5d^2 - 2de + 3e^2)^2} + \frac{(58530d^3e^2 - 56058d^2e^2 - 16643d^2e + 42375d^3 + 31811de^4 - 8623e^5) \tan^{-1}\left(\frac{5x}{\sqrt{14}}\right)}{1568\sqrt{14}(5d^2 - 2de + 3e^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)^3), x]

[Out] -(1367\*d - 293\*e + (423\*d - 1367\*e)\*x)/(1400\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(3 + 2\*x + 5\*x^2)^2) + (171735\*d^3 - 92989\*d^2\*e + 36207\*d\*e^2 + 1831\*e^3 + 25\*(2203\*d^3 - 9033\*d^2\*e + 3635\*d\*e^2 - 1829\*e^3)\*x)/(39200\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(3 + 2\*x + 5\*x^2)) + ((42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(1568\*Sqrt[14]\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3) + (e\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x])/(5\*d^2 - 2\*d\*e + 3\*e^2)^3 - (e\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[3 + 2\*x + 5\*x^2])/(2\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3)

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 800

Int[(((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_)^m)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)(3 + 2x + 5x^2)^3} dx &= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{1}{112} \int \frac{2(3267d^2 - 2843de + 2800e^2)}{25(5d^2 - 2de + 3e^2)} - \frac{6(308)}{2} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)} \\
&= -\frac{1367d - 293e + (423d - 1367e)x}{1400(5d^2 - 2de + 3e^2)(3 + 2x + 5x^2)^2} + \frac{171735d^3 - 92989d^2e + 36207de^2 + \dots}{39200(5d^2 - 2de + 3e^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 282, normalized size = 0.86

$$\frac{392(5d^2 - 2de + 3e^2)^2(1367d - 293e + (423d - 1367e)x)}{(5d^2 - 2de + 3e^2)^3} + \frac{14(5d^2 - 2de + 3e^2)(1367d - 293e + (423d - 1367e)x)}{5d^2 - 2de + 3e^2} - \frac{274400e(4d^3 + 5d^2e + 3d^2e^2 - d^2 + 2d^4) \log(5d^2 + 2x + 3) + 548800e(4d^3 + 5d^2e + 3d^2e^2 - d^2 + 2d^4) \log(d + ex) + 25\sqrt{14}(42375d^5 - 16643d^4e + 58530d^3e^2 - 56058d^2e^3 + 31811de^4 - 8623e^5) \arctan\left(\frac{5d + x}{\sqrt{14}}\right)}{548800(5d^2 - 2de + 3e^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)^3), x]

[Out] ((392\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(-(d\*(1367 + 423\*x)) + e\*(293 + 1367\*x)))/(3 + 2\*x + 5\*x^2)^2 + (14\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(e^3\*(1831 - 45725\*x) + 5\*d^3\*(34347 + 11015\*x) + d\*e^2\*(36207 + 90875\*x) - d^2\*e\*(92989 + 225825\*x)))/(3 + 2\*x + 5\*x^2) + 25\*sqrt[14]\*(42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*ArcTan[(1 + 5\*x)/sqrt[14]] + 5488



00\*e\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[d + e\*x] - 274400\*e\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*Log[3 + 2\*x + 5\*x^2])/(548800\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)(3 + 2x + 5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)^3), x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)\*(3 + 2\*x + 5\*x^2)^3), x]

**fricas [B]** time = 1.54, size = 1052, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)/(5\*x^2+2\*x+3)^3,x, algorithm="fricas")

[Out] 1/109760\*(4533550\*d^5 - 4072950\*d^4\*e + 3307332\*d^3\*e^2 - 807604\*d^2\*e^3 - 358554\*d\*e^4 + 252882\*e^5 + 350\*(11015\*d^5 - 49571\*d^4\*e + 42850\*d^3\*e^2 - 43514\*d^2\*e^3 + 14563\*d\*e^4 - 5487\*e^5)\*x^3 + 14\*(968825\*d^5 - 1304125\*d^4\*e + 1310718\*d^3\*e^2 - 777366\*d^2\*e^3 + 250589\*d\*e^4 - 49377\*e^5)\*x^2 + 5\*sqrt(14)\*(381375\*d^5 - 149787\*d^4\*e + 526770\*d^3\*e^2 - 504522\*d^2\*e^3 + 286299\*d\*e^4 - 77607\*e^5 + 25\*(42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*x^4 + 20\*(42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*x^3 + 34\*(42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*x^2 + 12\*(42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*x)\*arctan(1/14\*sqrt(14)\*(5\*x + 1)) + 14\*(449475\*d^5 - 828175\*d^4\*e + 761994\*d^3\*e^2 - 500898\*d^2\*e^3 + 147247\*d\*e^4 - 11211\*e^5)\*x + 109760\*(36\*d^4\*e + 45\*d^3\*e^2 + 27\*d^2\*e^3 - 9\*d\*e^4 + 18\*e^5 + 25\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x^4 + 20\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x^3 + 34\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x^2 + 12\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x)\*log(e\*x + d) - 54880\*(36\*d^4\*e + 45\*d^3\*e^2 + 27\*d^2\*e^3 - 9\*d\*e^4 + 18\*e^5 + 25\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x^4 + 20\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x^3 + 34\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x^2 + 12\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*x)\*log(5\*x^2 + 2\*x + 3))/(1125\*d^6 - 1350\*d^5\*e + 2565\*d^4\*e^2 - 1692\*d^3\*e^3



$$\begin{aligned}
& e^2)^3 \cdot 14^{1/2} \cdot \arctan(1/28 \cdot (10x+2) \cdot 14^{1/2}) \cdot d^{5-5/2} / (5d^2-2de+3e^2) \\
& ^3 \ln(5x^2+2x+3) \cdot d^3 e^2 - 3/2 / (5d^2-2de+3e^2)^3 \ln(5x^2+2x+3) \cdot d^2 e^ \\
& 3+1/2 / (5d^2-2de+3e^2)^3 \ln(5x^2+2x+3) \cdot d e^4 - 2 / (5d^2-2de+3e^2)^3 \ln \\
& (5x^2+2x+3) \cdot d^4 e + 55075/1568 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x^3 \cdot d \\
& ^5 - 27435/1568 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x^3 e^5 + 4e / (5d^2-2de \\
& +3e^2)^3 \ln(ex+d) \cdot d^4 + 64765/1568 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 \cdot d \\
& ^5 + 18063/7840 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 e^5 - 1 / (5d^2-2de+3e^ \\
& 2)^3 \ln(5x^2+2x+3) \cdot e^5 + 2e^5 / (5d^2-2de+3e^2)^3 \ln(ex+d) - 260825/1568 / \\
& (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x^2 \cdot d^4 e + 655359/3920 / (5d^2-2de+3e \\
& ^2)^3 / (5x^2+2x+3)^2 x^2 \cdot d^3 e^2 - 388683/3920 / (5d^2-2de+3e^2)^3 / (5x^2 \\
& +2x+3)^2 x^2 \cdot d^2 e^3 + 250589/7840 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x^2 \\
& \cdot d e^4 - 165635/1568 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x \cdot d^4 e + 380997/392 \\
& 0 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x \cdot d^3 e^2 - 250449/3920 / (5d^2-2de+ \\
& 3e^2)^3 / (5x^2+2x+3)^2 x \cdot d^2 e^3 + 147247/7840 / (5d^2-2de+3e^2)^3 / (5x^2 \\
& +2x+3)^2 x \cdot d e^4 - 16643/21952 / (5d^2-2de+3e^2)^3 \cdot 14^{1/2} \cdot \arctan(1/28 \cdot (1 \\
& 0x+2) \cdot 14^{1/2}) \cdot d^4 e + 29265/10976 / (5d^2-2de+3e^2)^3 \cdot 14^{1/2} \cdot \arctan(1/ \\
& 28 \cdot (10x+2) \cdot 14^{1/2}) \cdot d^3 e^2 - 28029/10976 / (5d^2-2de+3e^2)^3 \cdot 14^{1/2} \cdot \ar \\
& ctan(1/28 \cdot (10x+2) \cdot 14^{1/2}) \cdot d^2 e^3 + 31811/21952 / (5d^2-2de+3e^2)^3 \cdot 14^{( \\
& 1/2) \cdot \arctan(1/28 \cdot (10x+2) \cdot 14^{1/2}) \cdot d e^4 - 247855/1568 / (5d^2-2de+3e^2)^3 \\
& / (5x^2+2x+3)^2 x^3 \cdot d^4 e - 108785/784 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 \\
& x^3 \cdot d^2 e^3 + 107125/784 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x^3 \cdot d^3 e^2 + 7 \\
& 2815/1568 / (5d^2-2de+3e^2)^3 / (5x^2+2x+3)^2 x^3 \cdot d e^4
\end{aligned}$$

**maxima [A]** time = 1.03, size = 571, normalized size = 1.74

$\sqrt{14}(42375d^5 - 16643d^4e + 58530d^3e^2 - 56058d^2e^3 + 31811de^4 - 8623e^5) \arctan(1/14\sqrt{14}(5x+1)) / (125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6) + (4d^4e + 5d^3e^2 + 3d^2e^3 - de^4 + 2e^5) \log(ex+d) / (125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6) - 1/2(4d^4e + 5d^3e^2 + 3d^2e^3 - de^4 + 2e^5) \log(5x^2+2x+3) / (125d^6 - 150d^5e + 285d^4e^2 - 188d^3e^3 + 171d^2e^4 - 54de^5 + 27e^6) + 1/7840(25(2203d^3 - 9033d^2e + 3635de^2 - 1829e^3)x^3 + 64765d^3 - 32279d^2e - 4523de^2 + 6021e^3 + (193765d^3 - 183319d^2e + 72557de^2 - 16459e^3)x^2 + (89895d^3 - 129677d^2e + 46591de^2 - 3737e^3)x) / (25(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)x^4 + 225d^4 - 180d^3e + 306d^2e^2 - 108de^3 + 81e^4 + 20(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)x^3 + 34(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)x^2 + 34(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4)x + 34(25d^4 - 20d^3e + 34d^2e^2 - 12de^3 + 9e^4))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)/(5\*x^2+2\*x+3)^3,x, algorithm="maxima")

[Out] 1/21952\*sqrt(14)\*(42375\*d^5 - 16643\*d^4\*e + 58530\*d^3\*e^2 - 56058\*d^2\*e^3 + 31811\*d\*e^4 - 8623\*e^5)\*arctan(1/14\*sqrt(14)\*(5\*x + 1))/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d\*e^5 + 27\*e^6) + (4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*log(e\*x + d)/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d\*e^5 + 27\*e^6) - 1/2\*(4\*d^4\*e + 5\*d^3\*e^2 + 3\*d^2\*e^3 - d\*e^4 + 2\*e^5)\*log(5\*x^2 + 2\*x + 3)/(125\*d^6 - 150\*d^5\*e + 285\*d^4\*e^2 - 188\*d^3\*e^3 + 171\*d^2\*e^4 - 54\*d\*e^5 + 27\*e^6) + 1/7840\*(25\*(2203\*d^3 - 9033\*d^2\*e + 3635\*d\*e^2 - 1829\*e^3)\*x^3 + 64765\*d^3 - 32279\*d^2\*e - 4523\*d\*e^2 + 6021\*e^3 + (193765\*d^3 - 183319\*d^2\*e + 72557\*d\*e^2 - 16459\*e^3)\*x^2 + (89895\*d^3 - 129677\*d^2\*e + 46591\*d\*e^2 - 3737\*e^3)\*x)/(25\*(25\*d^4 - 20\*d^3\*e + 34\*d^2\*e^2 - 12\*d\*e^3 + 9\*e^4)\*x^4 + 225\*d^4 - 180\*d^3\*e + 306\*d^2\*e^2 - 108\*d\*e^3 + 81\*e^4 + 20\*(25\*d^4 - 20\*d^3\*e + 34\*d^2\*e^2 - 12\*d\*e^3 + 9\*e^4)\*x^3 + 34\*(25\*d^4 - 20\*d^3\*e + 34\*d^2\*e^2 - 12\*d\*e^3 + 9\*e^4)\*x^2 + 34\*(25\*d^4 - 20\*d^3\*e + 34\*d^2\*e^2 - 12\*d\*e^3 + 9\*e^4)\*x + 34\*(25\*d^4 - 20\*d^3\*e + 34\*d^2\*e^2 - 12\*d\*e^3 + 9\*e^4))



$$3.303 \quad \int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3} dx$$

Optimal. Leaf size=443

$$\frac{x(423d^2 - 2734de + 293e^2) + 1367d^2 - 586de - 703e^2}{280(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)^2} + \frac{171735d^4 - 117284d^3e - 200502d^2e^2 + 5x(11015d^4 - 85924d^3e + 34698d^2e^2 + 10348d^2e^3 - 3589e^4) + 5x(11015d^4 - 85924d^3e + 34698d^2e^2 + 10348d^2e^3 - 3589e^4)}{7840(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)^2}$$

Rubi [A] time = 0.89, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {1646, 1628, 634, 618, 204, 628}

$\frac{5x(11015d^4 - 85924d^3e + 34698d^2e^2 + 10348d^2e^3 - 3589e^4) + 5x(11015d^4 - 85924d^3e + 34698d^2e^2 + 10348d^2e^3 - 3589e^4)}{7840(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)^2} + \frac{x(423d^2 - 2734de + 293e^2) + 1367d^2 - 586de - 703e^2}{280(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)^2} + \frac{171735d^4 - 117284d^3e - 200502d^2e^2 + 5x(11015d^4 - 85924d^3e + 34698d^2e^2 + 10348d^2e^3 - 3589e^4)}{7840(5x^2 + 2x + 3)^2(5d^2 - 2de + 3e^2)^2}$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^3), x]

[Out] -((e\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/((5\*d^2 - 2\*d\*e + 3\*e^2)^3\*(d + e\*x)) - (1367\*d^2 - 586\*d\*e - 703\*e^2 + (423\*d^2 - 2734\*d\*e + 293\*e^2)\*x)/(280\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(3 + 2\*x + 5\*x^2)^2) + (171735\*d^4 - 117284\*d^3\*e - 200502\*d^2\*e^2 + 104428\*d\*e^3 - 23189\*e^4 + 5\*(11015\*d^4 - 85924\*d^3\*e + 34698\*d^2\*e^2 + 10348\*d^2e^3 - 3589\*e^4)\*x)/(7840\*(5\*d^2 - 2\*d\*e + 3\*e^2)^3\*(3 + 2\*x + 5\*x^2)) + ((211875\*d^6 + 3070\*d^5\*e + 209039\*d^4\*e^2 - 921444\*d^3\*e^3 + 380621\*d^2\*e^4 - 49586\*d\*e^5 - 43695\*e^6)\*ArcTan[(1 + 5\*x)/Sqrt[14]])/(1568\*Sqrt[14]\*(5\*d^2 - 2\*d\*e + 3\*e^2)^4) + (e\*(40\*d^5 + 83\*d^4\*e + 12\*d^3\*e^2 - 76\*d^2\*e^3 + 46\*d\*e^4 - 9\*e^5)\*Log[d + e\*x])/(5\*d^2 - 2\*d\*e + 3\*e^2)^4 - (e\*(40\*d^5 + 83\*d^4\*e + 12\*d^3\*e^2 - 76\*d^2\*e^3 + 46\*d\*e^4 - 9\*e^5)\*Log[3 + 2\*x + 5\*x^2])/(2\*(5\*d^2 - 2\*d\*e + 3\*e^2)^4)

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1628

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 1646

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3} dx &= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} + \frac{1}{112} \int \frac{2(3267d^4-5e^4)}{(d+ex)^2(3+2x+5x^2)^3} dx \\
&= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} + \frac{171735d^4-1172e^4}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&= -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} + \frac{171735d^4-1172e^4}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\
&= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.53, size = 389, normalized size = 0.88

$$\frac{109760 e^4 (5 d^2 - 2 d e + 3 e^2)^3 (d + e x) - (392 (5 d^2 - 2 d e + 3 e^2)^2 (e^2 (-703 + 293 x) + d^2 (1367 + 423 x) - 2 d e (293 + 1367 x))) (3 + 2 x + 5 x^2)^2 + (14 (5 d^2 - 2 d e + 3 e^2) (5 d^4 (34347 + 11015 x) + 4 d^3 e (26107 + 12935 x) - e^4 (23189 + 17945 x) + 6 d^2 e^2 (-33417 + 28915 x) - 4 d^3 e (29321 + 107405$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^3), x]

[Out] ((-109760\*e\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(d + e\*x) - (392\*(5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(e^2\*(-703 + 293\*x) + d^2\*(1367 + 423\*x) - 2\*d\*e\*(293 + 1367\*x)))/(3 + 2\*x + 5\*x^2)^2 + (14\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(5\*d^4\*(34347 + 11015\*x) + 4\*d^3\*e\*(26107 + 12935\*x) - e^4\*(23189 + 17945\*x) + 6\*d^2\*e^2\*(-33417 + 28915\*x) - 4\*d^3\*e\*(29321 + 107405

$$\frac{\text{*(x))}}{(3 + 2*x + 5*x^2) + 5*\text{Sqrt}[14]*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*\text{ArcTan}[(1 + 5*x)/\text{Sqrt}[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*\text{Log}[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*\text{Log}[3 + 2*x + 5*x^2]})/(109760*(5*d^2 - 2*d*e + 3*e^2)^4)$$

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^2 (3 + 2x + 5x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^3), x]

[Out] IntegrateAlgebraic[(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4)/((d + e\*x)^2\*(3 + 2\*x + 5\*x^2)^3), x]

**fricas [B]** time = 1.59, size = 1734, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3)^3,x, algorithm="fricas")

[Out] 
$$\frac{1}{21952}*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 - 8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(10172*5*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e^5 - 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 474395*d^5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*e^6 - 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 - 279581*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 394767*e^7)*x^2 + \text{sqrt}(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 8292996*d^4*e^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*d^6*e + 3070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 - 49586*d*e^6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5*e^2 - 3771064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6 - 174780*e^7)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 5660777*d^4*e^3 - 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815*e^7)*x^3 + 2*(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^4*e^3 + 941893*d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2 + 3*(847500*d^7 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 1241848*d^3*e^4 + 943519*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*\text{arctan}(1/14*\text{sqrt}(14)*(5*x + 1))$$



$$\begin{aligned}
& + 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325*d^4*e^3 - 14261*d^3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x + 21952*(360*d^6*e + 747*d^5*e^2 + 108*d^4*e^3 - 684*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5*e^2 + 83*d^4*e^3 + 12*d^3*e^4 - 76*d^2*e^5 + 46*d*e^6 - 9*e^7))*x^5 + 5*(200*d^6*e + 575*d^5*e^2 + 392*d^4*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6 - 36*e^7)*x^4 + 2*(400*d^6*e + 1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 - 832*d^2*e^5 + 692*d*e^6 - 153*e^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*d^4*e^3 - 1220*d^3*e^4 + 326*d^2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6*e + 452*d^5*e^2 + 297*d^4*e^3 - 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*e^7)*x*log(e*x + d) - 10976*(360*d^6*e + 747*d^5*e^2 + 108*d^4*e^3 - 684*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5*e^2 + 83*d^4*e^3 + 12*d^3*e^4 - 76*d^2*e^5 + 46*d*e^6 - 9*e^7))*x^5 + 5*(200*d^6*e + 575*d^5*e^2 + 392*d^4*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6 - 36*e^7)*x^4 + 2*(400*d^6*e + 1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 - 832*d^2*e^5 + 692*d*e^6 - 153*e^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*d^4*e^3 - 1220*d^3*e^4 + 326*d^2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6*e + 452*d^5*e^2 + 297*d^4*e^3 - 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*e^7)*x*log(5*x^2 + 2*x + 3))/(5625*d^9 - 9000*d^8*e + 18900*d^7*e^2 - 17640*d^6*e^3 + 18774*d^5*e^4 - 10584*d^4*e^5 + 6804*d^3*e^6 - 1944*d^2*e^7 + 729*d*e^8 + 25*(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9))*x^5 + 5*(3125*d^9 - 2500*d^8*e + 6500*d^7*e^2 - 1400*d^6*e^3 + 2590*d^5*e^4 + 2464*d^4*e^5 - 924*d^3*e^6 + 1944*d^2*e^7 - 459*d*e^8 + 324*e^9)*x^4 + 2*(6250*d^9 + 625*d^8*e + 4000*d^7*e^2 + 16100*d^6*e^3 - 12460*d^5*e^4 + 23702*d^4*e^5 - 12432*d^3*e^6 + 10692*d^2*e^7 - 2862*d*e^8 + 1377*e^9)*x^3 + 2*(10625*d^9 - 13250*d^8*e + 29700*d^7*e^2 - 20720*d^6*e^3 + 23702*d^5*e^4 - 7476*d^4*e^5 + 5796*d^3*e^6 + 864*d^2*e^7 + 81*d*e^8 + 486*e^9)*x^2 + 3*(2500*d^9 - 2125*d^8*e + 5400*d^7*e^2 - 1540*d^6*e^3 + 2464*d^5*e^4 + 1554*d^4*e^5 - 504*d^3*e^6 + 1404*d^2*e^7 - 324*d*e^8 + 243*e^9)*x)
\end{aligned}$$

**giac** [A] time = 0.26, size = 762, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3)^3,x, algorithm="giac")

[Out] 1/21952\*sqrt(14)\*(211875\*d^6\*e^2 + 3070\*d^5\*e^3 + 209039\*d^4\*e^4 - 921444\*d^3\*e^5 + 380621\*d^2\*e^6 - 49586\*d\*e^7 - 43695\*e^8)\*arctan(1/14\*sqrt(14)\*(5\*d - 5\*d^2/(x\*e + d) + 2\*d\*e/(x\*e + d) - 3\*e^2/(x\*e + d) - e)\*e^(-1))\*e^(-2)/(625\*d^8 - 1000\*d^7\*e + 2100\*d^6\*e^2 - 1960\*d^5\*e^3 + 2086\*d^4\*e^4 - 1176\*d^3\*e^5 + 756\*d^2\*e^6 - 216\*d\*e^7 + 81\*e^8) - 1/2\*(40\*d^5\*e + 83\*d^4\*e^2 + 12\*d^3\*e^3 - 76\*d^2\*e^4 + 46\*d\*e^5 - 9\*e^6)\*log(-10\*d/(x\*e + d) + 5\*d^2/(x\*e + d)^2 + 2\*e/(x\*e + d) - 2\*d\*e/(x\*e + d)^2 + 3\*e^2/(x\*e + d)^2 + 5)/(625\*



$$\begin{aligned} & x+3)^2 x^2 d^4 e^2 + 5109/392 / (5d^2 - 2de + 3e^2)^4 / (5x^2 + 2x + 3)^2 x^2 d^3 e \\ & ^3 + 1891915/1568 / (5d^2 - 2de + 3e^2)^4 / (5x^2 + 2x + 3)^2 x^3 d^4 e^2 - 344285/39 \\ & 2 / (5d^2 - 2de + 3e^2)^4 / (5x^2 + 2x + 3)^2 x^3 d^3 e^3 + 327265/1568 / (5d^2 - 2de \\ & e + 3e^2)^4 / (5x^2 + 2x + 3)^2 x^3 d^2 e^4 - 1129125/784 / (5d^2 - 2de + 3e^2)^4 / (5 \\ & *x^2 + 2*x+3)^2 *x^3*d^5*e-434995/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x \\ & *d^2*e^4+323825/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6-6309/1568/(5 \\ & *d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*e^6+9/2/(5*d^2-2*d*e+3*e^2)^4*\ln(5*x^2+ \\ & 2*x+3)*e^6-9*e^6/(5*d^2-2*d*e+3*e^2)^4*\ln(e*x+d)-2*e^5/(5*d^2-2*d*e+3*e^2)^ \\ & 3/(e*x+d)+208007/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d*e^5+606287/1 \\ & 568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^4*e^2-3993/392/(5*d^2-2*d*e+3 \\ & *e^2)^4/(5*x^2+2*x+3)^2*x*d^3*e^3+1535/10976/(5*d^2-2*d*e+3*e^2)^4*14^(1/2) \\ & *arctan(1/28*(10*x+2)*14^(1/2))*d^5*e+209039/21952/(5*d^2-2*d*e+3*e^2)^4*14 \\ & ^{(1/2)}*arctan(1/28*(10*x+2)*14^(1/2))*d^4*e^2-230361/5488/(5*d^2-2*d*e+3*e^ \\ & 2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^3+380621/21952/(5*d^2-2* \\ & d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^4-24793/10976/(5 \\ & *d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^5 \end{aligned}$$

**maxima [B]** time = 1.13, size = 916, normalized size = 2.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x^4-5\*x^3+3\*x^2+x+2)/(e\*x+d)^2/(5\*x^2+2\*x+3)^3,x, algorithm="maxima")

[Out] 1/21952\*sqrt(14)\*(211875\*d^6 + 3070\*d^5\*e + 209039\*d^4\*e^2 - 921444\*d^3\*e^3 + 380621\*d^2\*e^4 - 49586\*d\*e^5 - 43695\*e^6)\*arctan(1/14\*sqrt(14)\*(5\*x + 1))/(625\*d^8 - 1000\*d^7\*e + 2100\*d^6\*e^2 - 1960\*d^5\*e^3 + 2086\*d^4\*e^4 - 1176\*d^3\*e^5 + 756\*d^2\*e^6 - 216\*d\*e^7 + 81\*e^8) + (40\*d^5\*e + 83\*d^4\*e^2 + 12\*d^3\*e^3 - 76\*d^2\*e^4 + 46\*d\*e^5 - 9\*e^6)\*log(e\*x + d)/(625\*d^8 - 1000\*d^7\*e + 2100\*d^6\*e^2 - 1960\*d^5\*e^3 + 2086\*d^4\*e^4 - 1176\*d^3\*e^5 + 756\*d^2\*e^6 - 216\*d\*e^7 + 81\*e^8) - 1/2\*(40\*d^5\*e + 83\*d^4\*e^2 + 12\*d^3\*e^3 - 76\*d^2\*e^4 + 46\*d\*e^5 - 9\*e^6)\*log(5\*x^2 + 2\*x + 3)/(625\*d^8 - 1000\*d^7\*e + 2100\*d^6\*e^2 - 1960\*d^5\*e^3 + 2086\*d^4\*e^4 - 1176\*d^3\*e^5 + 756\*d^2\*e^6 - 216\*d\*e^7 + 81\*e^8) + 1/1568\*(64765\*d^5 - 95100\*d^4\*e - 200706\*d^3\*e^2 + 22292\*d^2\*e^3 + 12009\*d\*e^4 - 28224\*e^5 - 5\*(20345\*d^4\*e + 125124\*d^3\*e^2 - 11178\*d^2\*e^3 - 18188\*d\*e^4 + 19269\*e^5)\*x^4 + (55075\*d^5 - 361295\*d^4\*e - 272442\*d^3\*e^2 - 173446\*d^2\*e^3 + 138539\*d\*e^4 - 93087\*e^5)\*x^3 + (193765\*d^5 - 412485\*d^4\*e - 621062\*d^3\*e^2 - 56850\*d^2\*e^3 + 144973\*d\*e^4 - 131589\*e^5)\*x^2 + 3\*(29965\*d^5 - 77965\*d^4\*e - 51590\*d^3\*e^2 - 21522\*d^2\*e^3 + 19493\*d\*e^4 - 13245\*e^5)\*x)/(1125\*d^7 - 1350\*d^6\*e + 2565\*d^5\*e^2 - 1692\*d^4\*e^3 + 1539\*d^3\*e^4 - 486\*d^2\*e^5 + 243\*d\*e^6 + 25\*(125\*d^6\*e - 150\*d^5\*e^2 + 285\*d^4\*e^3 - 188\*d^3\*e^4 + 171\*d^2\*e^5 - 54\*d\*e^6 + 27\*e^7)\*x^5 + 5\*(625\*d^7 - 250\*d^6\*e + 825\*d^5\*e^2 + 200\*d^4\*e^3 + 103\*d^3\*e^4 + 414\*d^2\*e^5 - 81\*d\*e^6 + 108\*e^7)\*x^4 + 2\*(1250\*d^7 + 625\*d^6\*e + 300\*d^5\*e^2 + 2965\*d^4\*e^3 - 1486\*

$$d^3e^4 + 2367d^2e^5 - 648de^6 + 459e^7)x^3 + 2(2125d^7 - 1800d^6e + 3945d^5e^2 - 1486d^4e^3 + 1779d^3e^4 + 108d^2e^5 + 135de^6 + 162e^7)x^2 + 3(500d^7 - 225d^6e + 690d^5e^2 + 103d^4e^3 + 120d^3e^4 + 297d^2e^5 - 54de^6 + 81e^7)x$$

**mupad [B]** time = 4.99, size = 965, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x + 3x^2 - 5x^3 + 4x^4 + 2)/((d + ex)^2(2x + 5x^2 + 3)^3), x)$

[Out]  $\log(d + ex) \cdot \frac{(2e^3(620d - 2417e))}{(125(5d^2 - 2de + 3e^2)^3)} - (6e^5(423d - 1367e)) / (125(5d^2 - 2de + 3e^2)^4) + (e(8d + 23e)) / (5(5d^2 - 2de + 3e^2)^2) - ((3x(77965d^4e - 19493de^4 - 29965d^5 + 13245e^5 + 21522d^2e^3 + 51590d^3e^2)) / (1568(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) - (12009de^4 - 95100d^4e + 64765d^5 - 28224e^5 + 22292d^2e^3 - 200706d^3e^2) / (1568(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) + (5x^4(20345d^4e - 18188de^4 + 19269e^5 - 11178d^2e^3 + 125124d^3e^2)) / (1568(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) + (x^3(361295d^4e - 138539de^4 - 55075d^5 + 93087e^5 + 173446d^2e^3 + 272442d^3e^2)) / (1568(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) + (x^2(412485d^4e - 144973de^4 - 193765d^5 + 131589e^5 + 56850d^2e^3 + 621062d^3e^2)) / (1568(125d^6 - 150d^5e - 54de^5 + 27e^6 + 171d^2e^4 - 188d^3e^3 + 285d^4e^2)) / (9d + x^2(34d + 12e) + x^4(25d + 20e) + x^3(20d + 34e) + 25ex^5 + x(12d + 9e)) + (\log(x - (14^{1/2}i)/5 + 1/5) * ((211875 * 14^{1/2} * d^6) / 43904 - e^6 * ((43695 * 14^{1/2}) / 43904 - 9i/2) - d^3e^3 * ((230361 * 14^{1/2}) / 10976 + 6i) + d^4e^2 * ((209039 * 14^{1/2}) / 43904 - 83i/2) + d^2e^4 * ((380621 * 14^{1/2}) / 43904 + 38i) + d^5e * ((1535 * 14^{1/2}) / 21952 - 20i) - d * e^5 * ((24793 * 14^{1/2}) / 21952 + 23i))) / (d^8 * 625i - d^7 * e * 1000i - d * e^7 * 216i + e^8 * 81i + d^2 * e^6 * 756i - d^3 * e^5 * 1176i + d^4 * e^4 * 2086i - d^5 * e^3 * 1960i + d^6 * e^2 * 2100i) - (\log(x + (14^{1/2}i)/5 + 1/5) * ((211875 * 14^{1/2} * d^6) / 43904 - e^6 * ((43695 * 14^{1/2}) / 43904 + 9i/2) - d^3e^3 * ((230361 * 14^{1/2}) / 10976 - 6i) + d^4e^2 * ((209039 * 14^{1/2}) / 43904 + 83i/2) + d^2e^4 * ((380621 * 14^{1/2}) / 43904 - 38i) + d^5e * ((1535 * 14^{1/2}) / 21952 + 20i) - d * e^5 * ((24793 * 14^{1/2}) / 21952 - 23i))) / (d^8 * 625i - d^7 * e * 1000i - d * e^7 * 216i + e^8 * 81i + d^2 * e^6 * 756i - d^3 * e^5 * 1176i + d^4 * e^4 * 2086i - d^5 * e^3 * 1960i + d^6 * e^2 * 2100i)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

$$3.304 \quad \int (5 + 2x) \sqrt{3 - x + 2x^2} (2 + x + 3x^2 - x^3 + 5x^4) dx$$

Optimal. Leaf size=143

$$\frac{5}{112} (2x^2 - x + 3)^{3/2} (2x+5)^4 - \frac{823 (2x^2 - x + 3)^{3/2} (2x+5)^3}{1344} + \frac{11433 (2x^2 - x + 3)^{3/2} (2x+5)^2}{4480} - \frac{(295276x + 1005757) (2x^2 - x + 3)^{3/2}}{71680}$$

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{112} (2x^2 - x + 3)^{3/2} (2x+5)^4 - \frac{823 (2x^2 - x + 3)^{3/2} (2x+5)^3}{1344} + \frac{11433 (2x^2 - x + 3)^{3/2} (2x+5)^2}{4480} - \frac{(295276x + 1005757) (2x^2 - x + 3)^{3/2}}{71680} - \frac{51435(1-4x)\sqrt{2x^2-x+3}}{32768} - \frac{1183005 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 2\*x)\*Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (-51435\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/32768 + (11433\*(5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2))/4480 - (823\*(5 + 2\*x)^3\*(3 - x + 2\*x^2)^(3/2))/1344 + (5\*(5 + 2\*x)^4\*(3 - x + 2\*x^2)^(3/2))/112 - ((1005757 + 295276\*x)\*(3 - x + 2\*x^2)^(3/2))/71680 - (1183005\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(65536\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

$2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x$   
 $] + \text{Dist}[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), \text{Int}[(a + b*x + c*x^2)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 1653

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x\_Symbol] :=$  With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned} \int (5 + 2x)\sqrt{3 - x + 2x^2} (2 + x + 3x^2 - x^3 + 5x^4) dx &= \frac{5}{112}(5 + 2x)^4 (3 - x + 2x^2)^{3/2} + \frac{1}{224} \int (5 + 2x)\sqrt{3 - x + 2x^2} dx \\ &= -\frac{823(5 + 2x)^3 (3 - x + 2x^2)^{3/2}}{1344} + \frac{5}{112}(5 + 2x)^4 (3 - x + 2x^2)^{3/2} \\ &= \frac{11433(5 + 2x)^2 (3 - x + 2x^2)^{3/2}}{4480} - \frac{823(5 + 2x)^3 (3 - x + 2x^2)^{3/2}}{1344} \\ &= \frac{11433(5 + 2x)^2 (3 - x + 2x^2)^{3/2}}{4480} - \frac{823(5 + 2x)^3 (3 - x + 2x^2)^{3/2}}{1344} \\ &= -\frac{51435(1 - 4x)\sqrt{3 - x + 2x^2}}{32768} + \frac{11433(5 + 2x)^2 (3 - x + 2x^2)^{3/2}}{4480} \\ &= -\frac{51435(1 - 4x)\sqrt{3 - x + 2x^2}}{32768} + \frac{11433(5 + 2x)^2 (3 - x + 2x^2)^{3/2}}{4480} \\ &= -\frac{51435(1 - 4x)\sqrt{3 - x + 2x^2}}{32768} + \frac{11433(5 + 2x)^2 (3 - x + 2x^2)^{3/2}}{4480} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 70, normalized size = 0.49

$$\frac{4\sqrt{2x^2 - x + 3} (4915200x^6 + 12984320x^5 + 1390592x^4 + 20304768x^3 + 11357024x^2 + 14742332x + 6231117) - 124215525\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{13762560}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 2\*x)\*Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(6231117 + 14742332\*x + 11357024\*x^2 + 20304768\*x^3 + 1390592\*x^4 + 12984320\*x^5 + 4915200\*x^6) - 124215525\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/13762560

**IntegrateAlgebraic [A]** time = 1.29, size = 85, normalized size = 0.59

$$\frac{\sqrt{2x^2 - x + 3} (4915200x^6 + 12984320x^5 + 1390592x^4 + 20304768x^3 + 11357024x^2 + 14742332x + 6231117)}{3440640} - \frac{1183005 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(5 + 2\*x)\*Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(6231117 + 14742332\*x + 11357024\*x^2 + 20304768\*x^3 + 1390592\*x^4 + 12984320\*x^5 + 4915200\*x^6))/3440640 - (1183005\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(65536\*Sqrt[2])

**fricas [A]** time = 0.86, size = 83, normalized size = 0.58

$$\frac{1}{3440640} (4915200x^6 + 12984320x^5 + 1390592x^4 + 20304768x^3 + 11357024x^2 + 14742332x + 6231117)\sqrt{2x^2 - x + 3} + \frac{1183005}{262144} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/3440640\*(4915200\*x^6 + 12984320\*x^5 + 1390592\*x^4 + 20304768\*x^3 + 11357024\*x^2 + 14742332\*x + 6231117)\*sqrt(2\*x^2 - x + 3) + 1183005/262144\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.20, size = 78, normalized size = 0.55

$$\frac{1}{3440640} (4(8(4(16(20(120x + 317)x + 679)x + 158631)x + 354907)x + 3685583)x + 6231117)\sqrt{2x^2 - x + 3} - \frac{1183005}{131072} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2), x, algorithm="giac")



[Out]  $1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*x + 3685583)*x + 6231117)*\sqrt{2*x^2 - x + 3} - 1183005/131072*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})) + 1)$

**maple [A]** time = 0.02, size = 115, normalized size = 0.80

$$\frac{5(2x^2-x+3)^{\frac{3}{2}}x^4}{7} + \frac{377(2x^2-x+3)^{\frac{3}{2}}x^3}{168} + \frac{283(2x^2-x+3)^{\frac{3}{2}}x^2}{1120} - \frac{5179(2x^2-x+3)^{\frac{3}{2}}x}{17920} + \frac{1183005\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-\frac{1}{4})}{23}\right)}{131072} + \frac{51435(4x-1)\sqrt{2x^2-x+3}}{32768} + \frac{242329(2x^2-x+3)^{\frac{3}{2}}}{215040}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^{(1/2)}, x)$

[Out]  $5/7*x^4*(2*x^2-x+3)^{(3/2)}+377/168*x^3*(2*x^2-x+3)^{(3/2)}+283/1120*x^2*(2*x^2-x+3)^{(3/2)}-5179/17920*x*(2*x^2-x+3)^{(3/2)}+1183005/131072*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+51435/32768*(4*x-1)*(2*x^2-x+3)^{(1/2)}+242329/215040*(2*x^2-x+3)^{(3/2)}$

**maxima [A]** time = 0.97, size = 126, normalized size = 0.88

$$\frac{5}{7}(2x^2-x+3)^{\frac{3}{2}}x^4 + \frac{377}{168}(2x^2-x+3)^{\frac{3}{2}}x^3 + \frac{283}{1120}(2x^2-x+3)^{\frac{3}{2}}x^2 - \frac{5179}{17920}(2x^2-x+3)^{\frac{3}{2}}x + \frac{242329}{215040}(2x^2-x+3)^{\frac{3}{2}} + \frac{51435}{8192}\sqrt{2x^2-x+3}x + \frac{1183005}{131072}\sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{51435}{32768}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^{(1/2)}, x, \operatorname{algorithm}="maxima")$

[Out]  $5/7*(2*x^2 - x + 3)^{(3/2)}*x^4 + 377/168*(2*x^2 - x + 3)^{(3/2)}*x^3 + 283/1120*(2*x^2 - x + 3)^{(3/2)}*x^2 - 5179/17920*(2*x^2 - x + 3)^{(3/2)}*x + 242329/215040*(2*x^2 - x + 3)^{(3/2)} + 51435/8192*\sqrt{2*x^2 - x + 3}*x + 1183005/131072*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) - 51435/32768*\sqrt{2*x^2 - x + 3}$

**mupad [B]** time = 1.72, size = 170, normalized size = 1.19

$$\frac{283x^2(2x^2-x+3)^{3/2}}{1120} + \frac{377x^3(2x^2-x+3)^{3/2}}{168} + \frac{5x^4(2x^2-x+3)^{3/2}}{7} + \frac{4478951\sqrt{2} \ln\left(\frac{\sqrt{2x^2-x+3} + \sqrt{2x^2-x+3}}{2}\right)}{573440} + \frac{194737\left(\frac{5}{2} - \frac{1}{8}\right)\sqrt{2x^2-x+3}}{17920} + \frac{242329\sqrt{2x^2-x+3}(32x^2-4x+45)}{3440640} - \frac{5179x(2x^2-x+3)^{3/2}}{17920} + \frac{5573567\sqrt{2} \ln\left(2\sqrt{2x^2-x+3} + \frac{\sqrt{2(4x-1)}}{2}\right)}{4587520}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((2*x + 5)*(2*x^2 - x + 3)^{(1/2)}*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)$

[Out]  $(283*x^2*(2*x^2 - x + 3)^{(3/2)})/1120 + (377*x^3*(2*x^2 - x + 3)^{(3/2)})/168 + (5*x^4*(2*x^2 - x + 3)^{(3/2)})/7 + (4478951*2^{(1/2)}*\log((2*x^2 - x + 3)^{(1/2)} + (2^{(1/2)}*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x + 3)^{(1/2)})/17920 + (242329*(2*x^2 - x + 3)^{(1/2)}*(32*x^2 - 4*x + 45))/3440640 - (5179*x*(2*x^2 - x + 3)^{(3/2)})/17920 + (5573567*2^{(1/2)}*\log(2*(2*x^2 - x + 3)^{(1/2)} + (2^{(1/2)}*(4*x - 1))/2))/4587520$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 5) \sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2), x)
```

```
[Out] Integral((2*x + 5)*sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

$$3.305 \quad \int \sqrt{3 - x + 2x^2} (2 + x + 3x^2 - x^3 + 5x^4) dx$$

Optimal. Leaf size=124

$$\frac{7}{80} (2x^2 - x + 3)^{3/2} x^2 - \frac{71 (2x^2 - x + 3)^{3/2} x}{1280} + \frac{287 (2x^2 - x + 3)^{3/2}}{5120} - \frac{4609(1 - 4x)\sqrt{2x^2 - x + 3}}{16384} + \frac{5}{12} (2x^2 - x + 3)$$

**Rubi [A]** time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12} (2x^2 - x + 3)^{3/2} x^3 + \frac{7}{80} (2x^2 - x + 3)^{3/2} x^2 - \frac{71 (2x^2 - x + 3)^{3/2} x}{1280} + \frac{287 (2x^2 - x + 3)^{3/2}}{5120} - \frac{4609(1 - 4x)\sqrt{2x^2 - x + 3}}{16384} - \frac{106007 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (-4609\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/16384 + (287\*(3 - x + 2\*x^2)^(3/2))/5120 - (71\*x\*(3 - x + 2\*x^2)^(3/2))/1280 + (7\*x^2\*(3 - x + 2\*x^2)^(3/2))/80 + (5\*x^3\*(3 - x + 2\*x^2)^(3/2))/12 - (106007\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32768\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$*e)/(2*c)$ , Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]  
&& NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q =  
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x +  
c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a +  
b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*  
e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c,  
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4) dx &= \frac{5}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{12} \int \sqrt{3-x+2x^2} (24+12x-9x^2+2x^3) dx \\
 &= \frac{7}{80}x^2(3-x+2x^2)^{3/2} + \frac{5}{12}x^3(3-x+2x^2)^{3/2} + \frac{1}{120} \int (240+5x^3) dx \\
 &= -\frac{71x(3-x+2x^2)^{3/2}}{1280} + \frac{7}{80}x^2(3-x+2x^2)^{3/2} + \frac{5}{12}x^3(3-x+2x^2)^{3/2} \\
 &= \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280} + \frac{7}{80}x^2(3-x+2x^2)^{3/2} \\
 &= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384} + \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280} \\
 &= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384} + \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280} \\
 &= -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384} + \frac{287(3-x+2x^2)^{3/2}}{5120} - \frac{71x(3-x+2x^2)^{3/2}}{1280}
 \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3} (204800x^5 - 59392x^4 + 258432x^3 + 105696x^2 + 221868x - 27807) - 1590105\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{983040}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-27807 + 221868\*x + 105696\*x^2 + 258432\*x^3 - 59392\*x^4 + 204800\*x^5) - 1590105\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/983040

**IntegrateAlgebraic [A]** time = 0.60, size = 80, normalized size = 0.65

$$\frac{\sqrt{2x^2 - x + 3} (204800x^5 - 59392x^4 + 258432x^3 + 105696x^2 + 221868x - 27807)}{245760} - \frac{106007 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{32768\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-27807 + 221868\*x + 105696\*x^2 + 258432\*x^3 - 59392\*x^4 + 204800\*x^5))/245760 - (106007\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(32768\*Sqrt[2])

**fricas [A]** time = 0.82, size = 78, normalized size = 0.63

$$\frac{1}{245760} (204800x^5 - 59392x^4 + 258432x^3 + 105696x^2 + 221868x - 27807)\sqrt{2x^2 - x + 3} + \frac{106007}{131072} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/245760\*(204800\*x^5 - 59392\*x^4 + 258432\*x^3 + 105696\*x^2 + 221868\*x - 27807)\*sqrt(2\*x^2 - x + 3) + 106007/131072\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.20, size = 73, normalized size = 0.59

$$\frac{1}{245760} (4(8(4(16(100x - 29)x + 2019)x + 3303)x + 55467)x - 27807)\sqrt{2x^2 - x + 3} - \frac{106007}{65536} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] 1/245760\*(4\*(8\*(4\*(16\*(100\*x - 29)\*x + 2019)\*x + 3303)\*x + 55467)\*x - 27807)\*sqrt(2\*x^2 - x + 3) - 106007/65536\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple [A]** time = 0.01, size = 98, normalized size = 0.79

$$\frac{5(2x^2 - x + 3)^{\frac{3}{2}} x^3}{12} + \frac{7(2x^2 - x + 3)^{\frac{3}{2}} x^2}{80} - \frac{71(2x^2 - x + 3)^{\frac{3}{2}} x}{1280} + \frac{106007\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{65536} + \frac{287(2x^2 - x + 3)^{\frac{3}{2}}}{5120} + \frac{4609(4x - 1)\sqrt{2x^2 - x + 3}}{16384}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)`

[Out]  $5/12*(2*x^2-x+3)^{(3/2)}*x^3+7/80*(2*x^2-x+3)^{(3/2)}*x^2-71/1280*(2*x^2-x+3)^{(3/2)}*x+287/5120*(2*x^2-x+3)^{(3/2)}+4609/16384*(4*x-1)*(2*x^2-x+3)^{(1/2)}+106007/65536*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima** [A] time = 0.95, size = 109, normalized size = 0.88

$$\frac{5}{12}(2x^2-x+3)^{\frac{3}{2}}x^3 + \frac{7}{80}(2x^2-x+3)^{\frac{3}{2}}x^2 - \frac{71}{1280}(2x^2-x+3)^{\frac{3}{2}}x + \frac{287}{5120}(2x^2-x+3)^{\frac{3}{2}} + \frac{4609}{4096}\sqrt{2x^2-x+3} + \frac{106007}{65536}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{4609}{16384}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $5/12*(2*x^2-x+3)^{(3/2)}*x^3 + 7/80*(2*x^2-x+3)^{(3/2)}*x^2 - 71/1280*(2*x^2-x+3)^{(3/2)}*x + 287/5120*(2*x^2-x+3)^{(3/2)} + 4609/4096*\operatorname{sqrt}(2*x^2-x+3)*x + 106007/65536*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x-1)) - 4609/16384*\operatorname{sqrt}(2*x^2-x+3)$

**mupad** [B] time = 0.77, size = 153, normalized size = 1.23

$$\frac{7x^2(2x^2-x+3)^{3/2}}{80} + \frac{5x^3(2x^2-x+3)^{3/2}}{12} + \frac{63779\sqrt{2}\ln\left(\sqrt{2x^2-x+3} + \frac{\sqrt{2}(2x-1)}{2}\right)}{40960} + \frac{2773\left(\frac{x}{2} - \frac{1}{8}\right)\sqrt{2x^2-x+3}}{1280} + \frac{287\sqrt{2x^2-x+3}(32x^2-4x+45)}{81920} - \frac{71x(2x^2-x+3)^{3/2}}{1280} + \frac{19803\sqrt{2}\ln\left(2\sqrt{2x^2-x+3} + \frac{\sqrt{2}(4x-1)}{2}\right)}{327680}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(1/2)*(x+3*x^2-x^3+5*x^4+2),x)`

[Out]  $(7*x^2*(2*x^2-x+3)^{(3/2)})/80 + (5*x^3*(2*x^2-x+3)^{(3/2)})/12 + (63779*2^{(1/2)}*\log((2*x^2-x+3)^{(1/2)} + (2^{(1/2)}*(2*x-1/2))/2))/40960 + (2773*(x/2-1/8)*(2*x^2-x+3)^{(1/2)})/1280 + (287*(2*x^2-x+3)^{(1/2)}*(32*x^2-4*x+45))/81920 - (71*x*(2*x^2-x+3)^{(3/2)})/1280 + (19803*2^{(1/2)}*\log(2*(2*x^2-x+3)^{(1/2)} + (2^{(1/2)}*(4*x-1))/2))/327680$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2-x+3} (5x^4-x^3+3x^2+x+2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)`

[Out] `Integral(sqrt(2*x**2-x+3)*(5*x**4-x**3+3*x**2+x+2),x)`

$$3.306 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{5+2x} dx$$

**Optimal.** Leaf size=149

$$\frac{1}{16} (2x^2 - x + 3)^{3/2} (2x+5)^2 - \frac{127}{128} (2x^2 - x + 3)^{3/2} (2x+5) + \frac{4535}{768} (2x^2 - x + 3)^{3/2} + \frac{(489587 - 80844x)\sqrt{2x^2 - x}}{4096}$$

**Rubi [A]** time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{1}{16} (2x^2 - x + 3)^{3/2} (2x+5)^2 - \frac{127}{128} (2x^2 - x + 3)^{3/2} (2x+5) + \frac{4535}{768} (2x^2 - x + 3)^{3/2} + \frac{(489587 - 80844x)\sqrt{2x^2 - x}}{4096} - \frac{11001 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{16\sqrt{2}} + \frac{5627989 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x), x]

[Out] ((489587 - 80844\*x)\*Sqrt[3 - x + 2\*x^2])/4096 + (4535\*(3 - x + 2\*x^2)^(3/2))/768 - (127\*(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2))/128 + ((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2))/16 + (5627989\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(8192\*Sqrt[2]) - (11001\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(16\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{5+2x} dx &= \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= -\frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= \frac{4535}{768} (3-x+2x^2)^{3/2} - \frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768} (3-x+2x^2)^{3/2} - \frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768} (3-x+2x^2)^{3/2} - \frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768} (3-x+2x^2)^{3/2} - \frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768} (3-x+2x^2)^{3/2} - \frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx \\
&= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768} (3-x+2x^2)^{3/2} - \frac{127}{128}(5+2x)(3-x+2x^2)^{3/2} + \frac{1}{16}(5+2x)^2 (3-x+2x^2)^{3/2} + \frac{1}{160} \int \frac{\sqrt{3-x+2x^2} (-805-6x)}{5+2x} dx
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 91, normalized size = 0.61

$$\frac{-16897536\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 4\sqrt{2x^2-x+3} (6144x^4 - 21120x^3 + 79840x^2 - 300404x + 1561161) + 16883967\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{49152}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(1561161 - 300404\*x + 79840\*x^2 - 21120\*x^3 + 6144\*x^4) + 16883967\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] - 16897536\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/49152

**IntegrateAlgebraic [A]** time = 0.61, size = 117, normalized size = 0.79

$$\frac{5627989 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{8192\sqrt{2}} + \frac{11001 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{8\sqrt{2}} + \frac{\sqrt{2x^2-x+3} (6144x^4 - 21120x^3 + 79840x^2 - 300404x + 1561161)}{12288}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(1561161 - 300404\*x + 79840\*x^2 - 21120\*x^3 + 6144\*x^4))/12288 + (11001\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(8\*Sqrt[2]) + (5627989\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(8192\*Sqrt[2])

**fricas** [A] time = 0.78, size = 125, normalized size = 0.84

$$\frac{1}{12288} (6144x^4 - 21120x^3 + 79840x^2 - 300404x + 1561161)\sqrt{2x^2 - x + 3} + \frac{5627989}{32768} \sqrt{2} \log\left(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right) + \frac{11001}{64} \sqrt{2} \log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) + 1060x^2 - 1036x + 1153}{4x^2 + 20x + 25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x), x, algorithm="fricas")

[Out] 1/12288\*(6144\*x^4 - 21120\*x^3 + 79840\*x^2 - 300404\*x + 1561161)\*sqrt(2\*x^2 - x + 3) + 5627989/32768\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 11001/64\*sqrt(2)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25))

**giac** [A] time = 0.22, size = 129, normalized size = 0.87

$$\frac{1}{12288} (4(8(12(16x - 55)x + 2495)x - 75101)x + 1561161)\sqrt{2x^2 - x + 3} + \frac{5627989}{16384} \sqrt{2} \log(-4\sqrt{2}x + \sqrt{2} + 4\sqrt{2x^2 - x + 3}) - \frac{11001}{32} \sqrt{2} \log\left(\frac{-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}}{-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}}\right) + \frac{11001}{32} \sqrt{2} \log\left(\frac{-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}}{-2\sqrt{2}x + \sqrt{2} + 4\sqrt{2x^2 - x + 3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x), x, algorithm="giac")

[Out] 1/12288\*(4\*(8\*(12\*(16\*x - 55)\*x + 2495)\*x - 75101)\*x + 1561161)\*sqrt(2\*x^2 - x + 3) + 5627989/16384\*sqrt(2)\*log(-4\*sqrt(2)\*x + sqrt(2) + 4\*sqrt(2\*x^2 - x + 3)) - 11001/32\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 11001/32\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3)))

**maple** [A] time = 0.01, size = 127, normalized size = 0.85

$$\frac{(2x^2 - x + 3)^{\frac{3}{2}} x^2}{4} - \frac{47(2x^2 - x + 3)^{\frac{3}{2}} x}{64} - \frac{5627989\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{2}\right)}{23}\right)}{16384} - \frac{11001\sqrt{2} \operatorname{arctanh}\left(\frac{\left(-11x + \frac{7}{2}\right)\sqrt{2}}{12\sqrt{-11x + 2}\left(x + \frac{5}{2}\right) - \frac{19}{2}}\right)}{32} + \frac{1925(2x^2 - x + 3)^{\frac{3}{2}}}{768} - \frac{20211(4x - 1)\sqrt{2x^2 - x + 3}}{4096} + \frac{3667\sqrt{-11x + 2}\left(x + \frac{5}{2}\right) - \frac{19}{2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x), x)

[Out] 1/4\*(2\*x^2-x+3)^(3/2)\*x^2-47/64\*(2\*x^2-x+3)^(3/2)\*x+1925/768\*(2\*x^2-x+3)^(3/2)-20211/4096\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)-5627989/16384\*2^(1/2)\*arcsinh(4/23)

$*23^{(1/2)}*(x-1/4))+3667/32*(2*(x+5/2)^2-11*x-19/2)^{(1/2)}-11001/32*2^{(1/2)}*arctanh(1/12*(17/2-11*x)*2^{(1/2)}/(2*(x+5/2)^2-11*x-19/2)^{(1/2)})$

**maxima** [A] time = 1.01, size = 128, normalized size = 0.86

$$\frac{1}{4}(2x^2-x+3)^{\frac{3}{2}}x^2 - \frac{47}{64}(2x^2-x+3)^{\frac{3}{2}}x + \frac{1925}{768}(2x^2-x+3)^{\frac{3}{2}} - \frac{20211}{1024}\sqrt{2x^2-x+3}x - \frac{5627989}{16384}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{11001}{32}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{489587}{4096}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x),x, algorithm="maxima")

[Out]  $1/4*(2*x^2 - x + 3)^{(3/2)}*x^2 - 47/64*(2*x^2 - x + 3)^{(3/2)}*x + 1925/768*(2*x^2 - x + 3)^{(3/2)} - 20211/1024*\sqrt{2*x^2 - x + 3}*x - 5627989/16384*\sqrt{2}*\operatorname{arcsinh}(4/23*\sqrt{23}*x - 1/23*\sqrt{23}) + 11001/32*\sqrt{2}*\operatorname{arcsinh}(22/23*\sqrt{23}*x/abs(2*x + 5) - 17/23*\sqrt{23}/abs(2*x + 5)) + 489587/4096*\sqrt{2*x^2 - x + 3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2-x+3} (5x^4-x^3+3x^2+x+2)}{2x+5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5),x)

[Out] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3} (5x^4-x^3+3x^2+x+2)}{2x+5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)\*(2\*x\*\*2-x+3)\*\*(1/2)/(5+2\*x),x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5), x)

$$3.307 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx$$

**Optimal.** Leaf size=149

$$\frac{5}{64}(2x+5)(2x^2-x+3)^{3/2} - \frac{3667(2x^2-x+3)^{3/2}}{576(2x+5)} - \frac{541}{384}(2x^2-x+3)^{3/2} - \frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432} + \dots$$

**Rubi [A]** time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{64}(2x+5)(2x^2-x+3)^{3/2} - \frac{3667(2x^2-x+3)^{3/2}}{576(2x+5)} - \frac{541}{384}(2x^2-x+3)^{3/2} - \frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432} + \frac{239201 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{384\sqrt{2}} - \frac{2551847 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^2,x]

[Out] -((1996953 - 333380\*x)\*Sqrt[3 - x + 2\*x^2])/18432 - (541\*(3 - x + 2\*x^2)^(3/2))/384 - (3667\*(3 - x + 2\*x^2)^(3/2))/(576\*(5 + 2\*x)) + (5\*(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2))/64 - (2551847\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4096\*Sqrt[2]) + (239201\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(384\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$\frac{a*e - b*d - (2*c*d - b*e)*x}{\sqrt{a + b*x + c*x^2}}$ , x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} - \frac{1}{72} \int \frac{\sqrt{3-x+2x^2} \left(\frac{19341}{16} - \frac{6313x}{2} + 4x^2\right)}{5+2x} dx \\
 &= -\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)(3-x+2x^2)^{3/2} - \int \frac{\sqrt{3-x+2x^2}}{5+2x} dx \\
 &= -\frac{541}{384}(3-x+2x^2)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)(3-x+2x^2)^{3/2} - \int \frac{\sqrt{3-x+2x^2}}{5+2x} dx \\
 &= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}(3-x+2x^2)^{3/2} - \int \frac{\sqrt{3-x+2x^2}}{5+2x} dx \\
 &= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}(3-x+2x^2)^{3/2} - \int \frac{\sqrt{3-x+2x^2}}{5+2x} dx \\
 &= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}(3-x+2x^2)^{3/2} - \int \frac{\sqrt{3-x+2x^2}}{5+2x} dx \\
 &= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}(3-x+2x^2)^{3/2} - \int \frac{\sqrt{3-x+2x^2}}{5+2x} dx
 \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 98, normalized size = 0.66

$$\frac{7654432\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{4\sqrt{2x^2-x+3}(3840x^4-17344x^3+94936x^2-728410x-3539439)}{2x+5} - 7655541\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2, x
]
```

[Out]  $((4*\sqrt{3-x+2*x^2})*(-3539439-728410*x+94936*x^2-17344*x^3+3840*x^4))/(5+2*x)-765541*\sqrt{2}*\text{ArcSinh}[(1-4*x)/\sqrt{23}]+7654432*\sqrt{2}*\text{ArcTanh}[(17-22*x)/(12*\sqrt{6-2*x+4*x^2})])/24576$

**IntegrateAlgebraic [A]** time = 0.67, size = 124, normalized size = 0.83

$$\frac{2551847 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{4096\sqrt{2}} - \frac{239201 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{192\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(3840x^4-17344x^3+94936x^2-728410x-3539439)}{6144(2x+5)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[3-x+2\*x^2]\*(2+x+3\*x^2-x^3+5\*x^4))/(5+2\*x)^2,x]

[Out]  $(\sqrt{3-x+2*x^2}*(-3539439-728410*x+94936*x^2-17344*x^3+3840*x^4))/(6144*(5+2*x))-(239201*\text{ArcTanh}[5/6+x/3-\sqrt{3-x+2*x^2}]/(3*\sqrt{2}))/((192*\sqrt{2})-(2551847*\log[1-4*x+2*\sqrt{2}*\sqrt{3-x+2*x^2}]))/(4096*\sqrt{2})$

**fricas [A]** time = 0.87, size = 143, normalized size = 0.96

$$\frac{765541\sqrt{2}(2x+5)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+7654432\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+8(3840x^4-17344x^3+94936x^2-728410x-3539439)\sqrt{2x^2-x+3}}{49152(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^2,x, algorithm="fricas")

[Out]  $1/49152*(765541*\sqrt{2}*(2*x+5)*\log(-4*\sqrt{2}*\sqrt{2*x^2-x+3}*(4*x-1)-32*x^2+16*x-25)+7654432*\sqrt{2}*(2*x+5)*\log((24*\sqrt{2}*\sqrt{2*x^2-x+3}*(22*x-17)-1060*x^2+1036*x-1153)/(4*x^2+20*x+25))+8*(3840*x^4-17344*x^3+94936*x^2-728410*x-3539439)*\sqrt{2*x^2-x+3}))/((2*x+5)$

**giac [B]** time = 0.53, size = 531, normalized size = 3.56

$$\frac{1}{24576}\sqrt{2}\left(7654432\log\left(\frac{12\sqrt{-11/(2x+5)+36/(2x+5)^2+1}+72/(2x+5)-11}{\sqrt{-11/(2x+5)+36/(2x+5)^2+1}}\right)+7655541\log\left(\frac{\sqrt{-11/(2x+5)+36/(2x+5)^2+1}+6/(2x+5)+1}{\sqrt{-11/(2x+5)+36/(2x+5)^2+1}}\right)-7655541\log\left(\frac{\sqrt{-11/(2x+5)+36/(2x+5)^2+1}+6/(2x+5)-1}{\sqrt{-11/(2x+5)+36/(2x+5)^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^2,x, algorithm="giac")

[Out]  $1/24576*\sqrt{2}*(7654432*\log(12*\sqrt{-11/(2*x+5)+36/(2*x+5)^2+1}+72/(2*x+5)-11)*\text{sgn}(1/(2*x+5))+7655541*\log(\text{abs}(\sqrt{-11/(2*x+5)+36/(2*x+5)^2+1}+6/(2*x+5)+1))*\text{sgn}(1/(2*x+5))-7655541*\log(\text{abs}(\sqrt{-11/(2*x+5)+36/(2*x+5)^2+1}+6/(2*x+5)-1))*\text{sgn}(1/(2*x+5)$

)) - 1408128\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1)\*sgn(1/(2\*x + 5)) + 2\*(16367883\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^7\*sgn(1/(2\*x + 5)) - 34896384\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^6\*sgn(1/(2\*x + 5)) - 93395\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^5\*sgn(1/(2\*x + 5)) + 25574400\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^4\*sgn(1/(2\*x + 5)) + 19752365\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^3\*sgn(1/(2\*x + 5)) - 31921920\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^2\*sgn(1/(2\*x + 5)) - 2445813\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))\*sgn(1/(2\*x + 5)) + 7663104\*sgn(1/(2\*x + 5)))/((sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^2 - 1)^4

**maple [A]** time = 0.01, size = 152, normalized size = 1.02

$$\frac{5(2x^2-x+3)^{\frac{3}{2}}x}{32} + \frac{2551847\sqrt{2}\arcsinh\left(\frac{4\sqrt{23}(x-1)}{23}\right)}{8192} + \frac{239201\sqrt{2}\operatorname{arctanh}\left(\frac{(-11x+\frac{19}{2})\sqrt{2}}{12\sqrt{-11x+2(x+\frac{5}{2})^2-\frac{19}{2}}}\right)}{768} - \frac{391(2x^2-x+3)^{\frac{3}{2}}}{384} + \frac{6001(4x-1)\sqrt{2x^2-x+3}}{2048} - \frac{3667(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{1152(x+\frac{5}{2})} - \frac{239201\sqrt{-11x+2(x+\frac{5}{2})^2-\frac{19}{2}}}{2304} + \frac{3667(4x-1)\sqrt{-11x+2(x+\frac{5}{2})^2-\frac{19}{2}}}{2304}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^2,x)

[Out] 5/32\*(2\*x^2-x+3)^(3/2)\*x-391/384\*(2\*x^2-x+3)^(3/2)+6001/2048\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+2551847/8192\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))-3667/1152/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-239201/2304\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+239201/768\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))+3667/2304\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)

**maxima [A]** time = 1.00, size = 132, normalized size = 0.89

$$\frac{5}{32}(2x^2-x+3)^{\frac{3}{2}}x - \frac{391}{384}(2x^2-x+3)^{\frac{3}{2}} + \frac{6001}{512}\sqrt{2x^2-x+3} + \frac{2551847}{8192}\sqrt{2}\arcsinh\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{239201}{768}\sqrt{2}\arcsinh\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{182769}{2048}\sqrt{2x^2-x+3} - \frac{3667\sqrt{2x^2-x+3}}{32(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^2,x, algorithm="maxima")

[Out] 5/32\*(2\*x^2 - x + 3)^(3/2)\*x - 391/384\*(2\*x^2 - x + 3)^(3/2) + 6001/512\*sqrt(2\*x^2 - x + 3)\*x + 2551847/8192\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) - 239201/768\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) - 182769/2048\*sqrt(2\*x^2 - x + 3) - 3667/32\*sqrt(2\*x^2 - x + 3)/(2\*x + 5)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**2, x)
```

$$3.308 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx$$

**Optimal.** Leaf size=151

$$\frac{357391 (2x^2 - x + 3)^{3/2}}{82944(2x + 5)} - \frac{3667 (2x^2 - x + 3)^{3/2}}{1152(2x + 5)^2} + \frac{5}{48} (2x^2 - x + 3)^{3/2} + \frac{5(661065 - 110099x)\sqrt{2x^2 - x + 3}}{82944} - \frac{12670805}{82944}$$

**Rubi [A]** time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{357391 (2x^2 - x + 3)^{3/2}}{82944(2x + 5)} - \frac{3667 (2x^2 - x + 3)^{3/2}}{1152(2x + 5)^2} + \frac{5}{48} (2x^2 - x + 3)^{3/2} + \frac{5(661065 - 110099x)\sqrt{2x^2 - x + 3}}{82944} - \frac{12670805 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{55296\sqrt{2}} + \frac{117315 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^3,x]

[Out] (5\*(661065 - 110099\*x)\*Sqrt[3 - x + 2\*x^2])/82944 + (5\*(3 - x + 2\*x^2)^(3/2))/48 - (3667\*(3 - x + 2\*x^2)^(3/2))/(1152\*(5 + 2\*x)^2) + (357391\*(3 - x + 2\*x^2)^(3/2))/(82944\*(5 + 2\*x)) + (117315\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(512\*Sqrt[2]) - (12670805\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(55296\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*

$d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; \text{GtQ}[q, 1] \&\& \text{NeQ}[m + q + 2*p + 1, 0]] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !( \text{IGtQ}[m, 0] \&\& \text{RationalQ}[a, b, c, d, e] \&\& (\text{IntegerQ}[p] || \text{ILtQ}[p + 1/2, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} - \frac{1}{144} \int \frac{\sqrt{3-x+2x^2} \left( \frac{27681}{16} - \frac{14251x}{4} + \dots \right)}{(5+2x)^2} dx \\ &= -\frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \int \frac{\sqrt{3-x+2x^2}}{\dots} dx \\ &= \frac{5}{48} (3-x+2x^2)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} \\ &= \frac{5(661065 - 110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48} (3-x+2x^2)^{3/2} - \frac{3667}{1152} \frac{(3-x+2x^2)^{3/2}}{(5+2x)^2} \\ &= \frac{5(661065 - 110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48} (3-x+2x^2)^{3/2} - \frac{3667}{1152} \frac{(3-x+2x^2)^{3/2}}{(5+2x)^2} \\ &= \frac{5(661065 - 110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48} (3-x+2x^2)^{3/2} - \frac{3667}{1152} \frac{(3-x+2x^2)^{3/2}}{(5+2x)^2} \\ &= \frac{5(661065 - 110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48} (3-x+2x^2)^{3/2} - \frac{3667}{1152} \frac{(3-x+2x^2)^{3/2}}{(5+2x)^2} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 98, normalized size = 0.65

$$\frac{-12670805\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}(3840x^4-25632x^3+272520x^2+2959330x+4880551)}{(2x+5)^2} + 12670020\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{110592}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^3, x]

[Out]  $((24*\sqrt{3-x+2*x^2})*(4880551+2959330*x+272520*x^2-25632*x^3+3840*x^4))/(5+2*x)^2+12670020*\sqrt{2}*\text{ArcSinh}[(1-4*x)/\sqrt{23}]-12670805*\sqrt{2}*\text{ArcTanh}[(17-22*x)/(12*\sqrt{6-2*x+4*x^2})])/110592$

**IntegrateAlgebraic [A]** time = 0.78, size = 124, normalized size = 0.82

$$\frac{117315 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{512\sqrt{2}} + \frac{12670805 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{27648\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(3840x^4-25632x^3+272520x^2+2959330x+4880551)}{4608(2x+5)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[3-x+2\*x^2]\*(2+x+3\*x^2-x^3+5\*x^4))/(5+2\*x)^3,x]

[Out]  $(\sqrt{3-x+2*x^2}*(4880551+2959330*x+272520*x^2-25632*x^3+3840*x^4))/(4608*(5+2*x)^2)+(12670805*\text{ArcTanh}[5/6+x/3-\sqrt{3-x+2*x^2}]/(3*\sqrt{2}))/27648*\sqrt{2}+(117315*\text{Log}[1-4*x+2*\sqrt{2}*\sqrt{3-x+2*x^2}])/512*\sqrt{2}$

**fricas [A]** time = 0.81, size = 159, normalized size = 1.05

$$\frac{12670020\sqrt{2}(4x^2+20x+25)\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+12670805\sqrt{2}(4x^2+20x+25)\log\left(\frac{-24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+48(3840x^4-25632x^3+272520x^2+2959330x+4880551)\sqrt{2x^2-x+3}}{221184(4x^2+20x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^3,x, algorithm="fricas")

[Out]  $1/221184*(12670020*\sqrt{2}*(4*x^2+20*x+25)*\log(4*\sqrt{2}*\sqrt{2*x^2-x+3}*(4*x-1)-32*x^2+16*x-25)+12670805*\sqrt{2}*(4*x^2+20*x+25)*\log(-24*\sqrt{2}*\sqrt{2*x^2-x+3}*(22*x-17)+1060*x^2-1036*x+1153)/(4*x^2+20*x+25))+48*(3840*x^4-25632*x^3+272520*x^2+2959330*x+4880551)*\sqrt{2*x^2-x+3})/(4*x^2+20*x+25)$

**giac [B]** time = 0.24, size = 258, normalized size = 1.71

$$\frac{1}{768}(4(40x-467)x+19695)\sqrt{2x^2-x+3}+\frac{117315}{1024}\sqrt{2}\log(-2\sqrt{2}\sqrt{2x^2-x+3}+1)+\frac{12670805}{110592}\sqrt{2}\log(-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3})+\frac{12670805}{110592}\sqrt{2}\log(-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3})+\frac{\sqrt{2}(10693526\sqrt{2}(\sqrt{2x^2-x+3})^3+79895946(\sqrt{2x^2-x+3})^2-124044603\sqrt{2}(\sqrt{2x^2-x+3})+80334011)}{9216(2(\sqrt{2x^2-x+3})^2+10\sqrt{2}(\sqrt{2x^2-x+3})-11)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^3,x, algorithm="giac")

[Out]  $1/768*(4*(40*x-467)*x+19695)*\sqrt{2*x^2-x+3}+117315/1024*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x-\sqrt{2*x^2-x+3})+1)-12670805/110592*\sqrt{2}*\log(2*\log(\text{abs}(-2*\sqrt{2}*x+\sqrt{2}+2*\sqrt{2*x^2-x+3})))$

0592\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 1/9216\*sqrt(2)\*(10693526\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 + 79895946\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 - 124044603\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 80334011)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^2

**maple [A]** time = 0.02, size = 158, normalized size = 1.05

$$\frac{117315\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\sqrt{x+1}}{23}\right) - 12670805\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+\frac{19}{2})\sqrt{2}}{12\sqrt{-11x+2}\sqrt{x+\frac{5}{2}}-\frac{19}{2}}\right) + 5(2x^2-x+3)^{\frac{3}{2}} - 149(4x-1)\sqrt{2x^2-x+3} - 357391\left(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}} + 12670805\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}} - 357391(4x-1)\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}} - 3667\left(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}}}{1024 - 110592 - 48 - 256 - 165888\left(x+\frac{5}{2}\right) + 331776 - 331776 - 4608\left(x+\frac{5}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^3,x)

[Out] 5/48\*(2\*x^2-x+3)^(3/2)-149/256\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)-117315/1024\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+357391/165888/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+12670805/331776\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-12670805/110592\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))-357391/331776\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/4608/(x+5/2)^2\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)

**maxima [A]** time = 1.00, size = 143, normalized size = 0.95

$$\frac{5}{48}(2x^2-x+3)^{\frac{3}{2}} - \frac{149}{64}\sqrt{2x^2-x+3} - \frac{117315}{1024}\sqrt{2} \operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{12670805}{110592}\sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{3877}{144}\sqrt{2x^2-x+3} - \frac{3667(2x^2-x+3)^{\frac{3}{2}}}{1152(4x^2+20x+25)} + \frac{357391\sqrt{2x^2-x+3}}{4608(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^3,x, algorithm="maxima")

[Out] 5/48\*(2\*x^2 - x + 3)^(3/2) - 149/64\*sqrt(2\*x^2 - x + 3)\*x - 117315/1024\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 12670805/110592\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 3877/144\*sqrt(2\*x^2 - x + 3) - 3667/1152\*(2\*x^2 - x + 3)^(3/2)/(4\*x^2 + 20\*x + 25) + 357391/4608\*sqrt(2\*x^2 - x + 3)/(2\*x + 5)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2-x+3} (5x^4-x^3+3x^2+x+2)}{(2x+5)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^3,x)

[Out] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)\*(2\*x\*\*2-x+3)\*\*(1/2)/(5+2\*x)\*\*3,x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*3, x)

$$3.309 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx$$

**Optimal.** Leaf size=158

$$-\frac{6467659 (2x^2 - x + 3)^{3/2}}{5971968(2x + 5)} + \frac{158527 (2x^2 - x + 3)^{3/2}}{82944(2x + 5)^2} - \frac{3667 (2x^2 - x + 3)^{3/2}}{1728(2x + 5)^3} - \frac{(44378877 - 7400779x)\sqrt{2x^2 - x + 3}}{5971968}$$

**Rubi [A]** time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{6467659 (2x^2 - x + 3)^{3/2}}{5971968(2x + 5)} + \frac{158527 (2x^2 - x + 3)^{3/2}}{82944(2x + 5)^2} - \frac{3667 (2x^2 - x + 3)^{3/2}}{1728(2x + 5)^3} - \frac{(44378877 - 7400779x)\sqrt{2x^2 - x + 3}}{5971968} + \frac{170114729 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3981312\sqrt{2}} - \frac{10939 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^4,x]

[Out] -((44378877 - 7400779\*x)\*Sqrt[3 - x + 2\*x^2])/5971968 - (3667\*(3 - x + 2\*x^2)^(3/2))/(1728\*(5 + 2\*x)^3) + (158527\*(3 - x + 2\*x^2)^(3/2))/(82944\*(5 + 2\*x)^2) - (6467659\*(3 - x + 2\*x^2)^(3/2))/(5971968\*(5 + 2\*x)) - (10939\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(256\*Sqrt[2]) + (170114729\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(3981312\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2



$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} - \frac{1}{216} \int \frac{\sqrt{3-x+2x^2} \left(\frac{36021}{16} - 3969x + \dots\right)}{(5+2x)^3} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} + \frac{\int \frac{\sqrt{3-x+2x^2}}{\dots}}{\dots} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} - \frac{6467659(3-x+2x^2)^{3/2}}{5971968} \\
&= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} \\
&= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} \\
&= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} \\
&= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 98, normalized size = 0.62

$$\frac{170114729\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}(414720x^4-5453568x^3-97682900x^2-329667508x-327735797)}{(2x+5)^3} - 170123328\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{3}}\right)}{7962624}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^4, x]

[Out] ((24\*Sqrt[3 - x + 2\*x^2]\*(-327735797 - 329667508\*x - 97682900\*x^2 - 5453568\*x^3 + 414720\*x^4))/(5 + 2\*x)^3 - 170123328\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[3]] + 170114729\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/7962624

**IntegrateAlgebraic [A]** time = 0.88, size = 124, normalized size = 0.78

$$-\frac{10939 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{256\sqrt{2}} - \frac{170114729 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{1990656\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(414720x^4-5453568x^3-97682900x^2-329667508x-327735797)}{331776(2x+5)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^4,x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-327735797 - 329667508\*x - 97682900\*x^2 - 5453568\*x^3 + 414720\*x^4))/(331776\*(5 + 2\*x)^3) - (170114729\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(1990656\*Sqrt[2]) - (10939\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(256\*Sqrt[2])

**fricas** [A] time = 0.83, size = 173, normalized size = 1.09

$$\frac{170123328 \sqrt{2} (8x^3 + 60x^2 + 150x + 125) \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 170114729 \sqrt{2} (8x^3 + 60x^2 + 150x + 125) \log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) - 1060x^2 + 1036x - 1153}{4x^2 + 20x + 25}\right) + 48(414720x^4 - 5453568x^3 - 97682900x^2 - 329667508x - 327735797)\sqrt{2x^2 - x + 3}}{15925248(8x^3 + 60x^2 + 150x + 125)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^4,x, algorithm="fricas")

[Out] 1/15925248\*(170123328\*sqrt(2)\*(8\*x^3 + 60\*x^2 + 150\*x + 125)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 170114729\*sqrt(2)\*(8\*x^3 + 60\*x^2 + 150\*x + 125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(414720\*x^4 - 5453568\*x^3 - 97682900\*x^2 - 329667508\*x - 327735797)\*sqrt(2\*x^2 - x + 3))/(8\*x^3 + 60\*x^2 + 150\*x + 125)

**giac** [B] time = 0.26, size = 304, normalized size = 1.92

$$\frac{\frac{1}{128}\sqrt{2x^2-x+3}(20x-413) - 10939/512\sqrt{2}\log(-2\sqrt{2}\sqrt{2x^2-x+3}) + 170114729/7962624\sqrt{2}\log(\text{abs}(-2\sqrt{2}\sqrt{2x^2-x+3})) - 170114729/7962624\sqrt{2}\log(\text{abs}(-2\sqrt{2}\sqrt{2x^2-x+3})) - 1/663552\sqrt{2}\log(\text{abs}(-2\sqrt{2}\sqrt{2x^2-x+3})) + 9206213116(\sqrt{2x^2-x+3})^4 + 9688786604\sqrt{2}(\sqrt{2x^2-x+3})^3 - 73157325092(\sqrt{2x^2-x+3})^2 + 49481952947\sqrt{2}(\sqrt{2x^2-x+3}) - 20269228621}{2*(\sqrt{2x^2-x+3})^2 + 10\sqrt{2}(\sqrt{2x^2-x+3}) - 11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^4,x, algorithm="giac")

[Out] 1/128\*sqrt(2\*x^2 - x + 3)\*(20\*x - 413) - 10939/512\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 170114729/7962624\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 170114729/7962624\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 1/663552\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - sqrt(2\*x^2 - x + 3)))^5 + 9206213116\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^4 + 9688786604\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 - 73157325092\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 49481952947\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 20269228621/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^3

**maple [A]** time = 0.01, size = 165, normalized size = 1.04

$$\frac{10939\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x+\frac{1}{2}\right)}{23}\right) + \frac{170114729\sqrt{2} \operatorname{arctanh}\left(\frac{\left(-11x+\frac{19}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{7962624} + \frac{5(4x-1)\sqrt{2x^2-x+3}}{128} - \frac{6467659\left(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}}}{11943936\left(x+\frac{5}{2}\right)} - \frac{170114729\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{23887872} + \frac{6467659(4x-1)\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{23887872} + \frac{158527\left(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}}}{331776\left(x+\frac{5}{2}\right)^2} - \frac{3667\left(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}}}{13824\left(x+\frac{5}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^4, x)

[Out] 5/128\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)+10939/512\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))-6467659/11943936/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-170114729/2387872\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+170114729/7962624\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))+6467659/23887872\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+158527/331776/(x+5/2)^2\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-3667/13824/(x+5/2)^3\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)

**maxima [A]** time = 0.98, size = 160, normalized size = 1.01

$$\frac{5}{32}\sqrt{2x^2-x+3} + \frac{10939}{512}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{170114729}{7962624}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{693775}{165888}\sqrt{2x^2-x+3} - \frac{3667(2x^2-x+3)^{\frac{3}{2}}}{1728(8x^3+60x^2+150x+125)} + \frac{158527(2x^2-x+3)^{\frac{3}{2}}}{82944(4x^2+20x+25)} - \frac{6467659\sqrt{2x^2-x+3}}{331776(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^4, x, algorithm="maxima")

[Out] 5/32\*sqrt(2\*x^2 - x + 3)\*x + 10939/512\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) - 170114729/7962624\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) - 693775/165888\*sqrt(2\*x^2 - x + 3) - 3667/1728\*(2\*x^2 - x + 3)^(3/2)/(8\*x^3 + 60\*x^2 + 150\*x + 125) + 158527/82944\*(2\*x^2 - x + 3)^(3/2)/(4\*x^2 + 20\*x + 25) - 6467659/331776\*sqrt(2\*x^2 - x + 3)/(2\*x + 5)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^4, x)

[Out] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^4, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4, x)
```

$$3.310 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx$$

**Optimal.** Leaf size=165

$$-\frac{9363383 (2x^2 - x + 3)^{3/2}}{23887872(2x + 5)^2} + \frac{593771 (2x^2 - x + 3)^{3/2}}{497664(2x + 5)^3} - \frac{3667 (2x^2 - x + 3)^{3/2}}{2304(2x + 5)^4} + \frac{7(9616196x + 52836655)\sqrt{2x^2 - x}}{95551488(2x + 5)}$$

**Rubi [A]** time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{9363383 (2x^2 - x + 3)^{3/2}}{23887872(2x + 5)^2} + \frac{593771 (2x^2 - x + 3)^{3/2}}{497664(2x + 5)^3} - \frac{3667 (2x^2 - x + 3)^{3/2}}{2304(2x + 5)^4} + \frac{7(9616196x + 52836655)\sqrt{2x^2 - x + 3}}{95551488(2x + 5)} - \frac{4640586097 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1146617856\sqrt{2}} + \frac{259 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^5,x]

[Out] (7\*(52836655 + 9616196\*x)\*Sqrt[3 - x + 2\*x^2])/(95551488\*(5 + 2\*x)) - (3667\*(3 - x + 2\*x^2)^(3/2))/(2304\*(5 + 2\*x)^4) + (593771\*(3 - x + 2\*x^2)^(3/2))/(497664\*(5 + 2\*x)^3) - (9363383\*(3 - x + 2\*x^2)^(3/2))/(23887872\*(5 + 2\*x)^2) + (259\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(64\*Sqrt[2]) - (4640586097\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(1146617856\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 812

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(e\*f\*(m + 2\*p + 2) - d\*g\*(2\*p + 1) + e\*g\*(m + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(e^2\*(m + 1)\*(m + 2\*p + 2)), x] + Dist[p/(e^2\*(m + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[g\*(b\*d + 2\*a\*e + 2\*a\*e\*m + 2\*b\*d\*p) - f\*b\*e\*(m + 2\*p + 2) + (g\*(2\*c\*d + b\*e + b\*e\*m + 4\*c\*d\*p) - 2\*c\*e\*f\*(m + 2\*p + 2))\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2\*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} - \frac{1}{288} \int \frac{\sqrt{3-x+2x^2} \left(\frac{44361}{16} - \frac{17501x}{4} + \dots\right)}{(5+2x)^4} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} + \frac{\int \frac{\sqrt{3-x+2x^2}}{(5+2x)^4}}{497664} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} \\
&= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} \\
&= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} \\
&= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} \\
&= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 98, normalized size = 0.59

$$\frac{-4640586097\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}(238878720x^4+6105343976x^3+31323229164x^2+62847867486x+44676885233)}{(2x+5)^4} + 4640219136\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2293235712}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^5, x]

[Out] ((24\*Sqrt[3 - x + 2\*x^2]\*(44676885233 + 62847867486\*x + 31323229164\*x^2 + 6105343976\*x^3 + 238878720\*x^4))/(5 + 2\*x)^4 + 4640219136\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] - 4640586097\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/2293235712

**IntegrateAlgebraic [A]** time = 0.83, size = 124, normalized size = 0.75

$$\frac{259 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{64\sqrt{2}} + \frac{4640586097 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{573308928\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(238878720x^4+6105343976x^3+31323229164x^2+62847867486x+44676885233)}{95551488(2x+5)^4}$$



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^5,x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(44676885233 + 62847867486\*x + 31323229164\*x^2 + 6105343976\*x^3 + 238878720\*x^4))/(95551488\*(5 + 2\*x)^4) + (4640586097\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(573308928\*Sqrt[2]) + (259\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(64\*Sqrt[2])

**fricas** [A] time = 1.21, size = 189, normalized size = 1.15

$$\frac{4640219136\sqrt{2}\sqrt{16x^4+160x^3+600x^2+1000x+625}\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+4640586097\sqrt{2}\sqrt{16x^4+160x^3+600x^2+1000x+625}\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+48(238878720x^4+6105343976x^3+31323229164x^2+62847867486x+44676885233)\sqrt{2x^2-x+3}}{4586471424(16x^4+160x^3+600x^2+1000x+625)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^5,x, algorithm="fricas")

[Out] 1/4586471424\*(4640219136\*sqrt(2)\*(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 4640586097\*sqrt(2)\*(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(238878720\*x^4 + 6105343976\*x^3 + 31323229164\*x^2 + 62847867486\*x + 44676885233)\*sqrt(2\*x^2 - x + 3))/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)

**giac** [B] time = 0.37, size = 327, normalized size = 1.98

$$\frac{1}{25023572}\sqrt{2}\left(4640586097\log\left(2\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}+\frac{72}{2x+5}-11}\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+4640219136\log\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}+\frac{6}{2x+5}}\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+4640219136\log\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}+\frac{6}{2x+5}}\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+12\left(\frac{24\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}+\frac{6}{2x+5}}\operatorname{sgn}\left(\frac{1}{2x+5}\right)+57384361\operatorname{sgn}\left(\frac{1}{2x+5}\right)}{2x+5}\right)+4640586097\operatorname{sgn}\left(\frac{1}{2x+5}\right)\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}}+\frac{179159040\left(11\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}+\frac{6}{2x+5}}\operatorname{sgn}\left(\frac{1}{2x+5}\right)-12\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)}{\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2+1}+\frac{6}{2x+5}}\right)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^5,x, algorithm="giac")

[Out] -1/2293235712\*sqrt(2)\*(4640586097\*log(12\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 72/(2\*x + 5) - 11)\*sgn(1/(2\*x + 5)) + 4640219136\*log(abs(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5) - 1))\*sgn(1/(2\*x + 5)) - 4640219136\*log(abs(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5) - 1))\*sgn(1/(2\*x + 5)) + 12\*(24\*(144\*(792072\*sgn(1/(2\*x + 5)))/(2\*x + 5) - 835793\*sgn(1/(2\*x + 5)))/(2\*x + 5) + 57384361\*sgn(1/(2\*x + 5)))/(2\*x + 5) - 464569597\*sgn(1/(2\*x + 5)))\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 179159040\*(11\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))\*sgn(1/(2\*x + 5)) - 12\*sgn(1/(2\*x + 5)))/((sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^2 - 1))

**maple [A]** time = 0.01, size = 167, normalized size = 1.01

$$\frac{259\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}(x+\frac{1}{2})}{23}\right)}{128} - \frac{4640586097\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+\frac{17}{2})\sqrt{2}}{23\sqrt{-11x+2(x+\frac{5}{2})-\frac{19}{2}}}\right)}{2293235712} + \frac{4640586097\sqrt{-11x+2(x+\frac{5}{2})-\frac{19}{2}}}{6879707136} + \frac{593771(-11x+2(x+\frac{5}{2})-\frac{19}{2})^{\frac{3}{2}}}{3981312(x+\frac{5}{2})} - \frac{3667(-11x+2(x+\frac{5}{2})-\frac{19}{2})^{\frac{3}{2}}}{36864(x+\frac{5}{2})^4} - \frac{9363383(-11x+2(x+\frac{5}{2})-\frac{19}{2})^{\frac{3}{2}}}{9551488(x+\frac{5}{2})^2} - \frac{201573155(4x-1)\sqrt{-11x+2(x+\frac{5}{2})-\frac{19}{2}}}{6879707136} + \frac{201573155(-11x+2(x+\frac{5}{2})-\frac{19}{2})^{\frac{3}{2}}}{3439853568(x+\frac{5}{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)`

[Out]  $4640586097/6879707136*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+593771/3981312/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-9363383/9551488/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-201573155/6879707136*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+201573155/3439853568/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-4640586097/2293235712*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)})-259/128*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima [A]** time = 1.02, size = 181, normalized size = 1.10

$$\frac{259}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{4640586097}{2293235712}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x-17\sqrt{23}}{23|2x+5|}\right)+\frac{16828343}{47775744}\sqrt{2x^2-x+3}-\frac{3667(2x^2-x+3)^{\frac{3}{2}}}{2304(16x^4+160x^3+600x^2+1000x+625)}+\frac{593771(2x^2-x+3)^{\frac{3}{2}}}{497664(8x^3+60x^2+150x+125)}-\frac{9363383(2x^2-x+3)^{\frac{3}{2}}}{23887872(4x^2+20x+25)}+\frac{201573155\sqrt{2x^2-x+3}}{95551488(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="maxima")`

[Out]  $-259/128*\sqrt{2}*\operatorname{arcsinh}(4/23*\sqrt{23}*x-1/23*\sqrt{23})+4640586097/2293235712*\sqrt{2}*\operatorname{arcsinh}(22/23*\sqrt{23}*x/\operatorname{abs}(2*x+5)-17/23*\sqrt{23}/\operatorname{abs}(2*x+5))+16828343/47775744*\sqrt{2*x^2-x+3}-3667/2304*(2*x^2-x+3)^{(3/2)}/(16*x^4+160*x^3+600*x^2+1000*x+625)+593771/497664*(2*x^2-x+3)^{(3/2)}/(8*x^3+60*x^2+150*x+125)-9363383/23887872*(2*x^2-x+3)^{(3/2)}/(4*x^2+20*x+25)+201573155/95551488*\sqrt{2*x^2-x+3}/(2*x+5)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2-x+3} (5x^4-x^3+3x^2+x+2)}{(2x+5)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x^2-x+3)^(1/2)*(x+3*x^2-x^3+5*x^4+2))/(2*x+5)^5,x)`

[Out] `int(((2*x^2-x+3)^(1/2)*(x+3*x^2-x^3+5*x^4+2))/(2*x+5)^5,x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)\*(2\*x\*\*2-x+3)\*\*(1/2)/(5+2\*x)\*\*5,x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*5, x)

$$3.311 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx$$

**Optimal.** Leaf size=165

$$-\frac{38732321 (2x^2 - x + 3)^{3/2}}{179159040(2x + 5)^3} + \frac{711961 (2x^2 - x + 3)^{3/2}}{829440(2x + 5)^4} - \frac{3667 (2x^2 - x + 3)^{3/2}}{2880(2x + 5)^5} - \frac{(3174439702x + 4583087983)\sqrt{2x^2 - x + 3}}{6879707136(2x + 5)^2}$$

**Rubi [A]** time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{38732321 (2x^2 - x + 3)^{3/2}}{179159040(2x + 5)^3} + \frac{711961 (2x^2 - x + 3)^{3/2}}{829440(2x + 5)^4} - \frac{3667 (2x^2 - x + 3)^{3/2}}{2880(2x + 5)^5} - \frac{(3174439702x + 4583087983)\sqrt{2x^2 - x + 3}}{6879707136(2x + 5)^2} + \frac{12895597463 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{82556485632\sqrt{2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^6,x]

[Out] -((4583087983 + 3174439702\*x)\*Sqrt[3 - x + 2\*x^2])/(6879707136\*(5 + 2\*x)^2) - (3667\*(3 - x + 2\*x^2)^(3/2))/(2880\*(5 + 2\*x)^5) + (711961\*(3 - x + 2\*x^2)^(3/2))/(829440\*(5 + 2\*x)^4) - (38732321\*(3 - x + 2\*x^2)^(3/2))/(179159040\*(5 + 2\*x)^3) - (5\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32\*Sqrt[2]) + (12895597463\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(82556485632\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/ (e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} - \frac{1}{360} \int \frac{\sqrt{3-x+2x^2} \left(\frac{52701}{16} - \frac{9563x}{2} + \dots\right)}{(5+2x)^5} dx \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} + \frac{\int \frac{\sqrt{3-x+2x^2}}{(5+2x)^5} dx}{829440} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} - \frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3} \\
&= -\frac{(4583087983 + 3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} \\
&= -\frac{(4583087983 + 3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} \\
&= -\frac{(4583087983 + 3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} \\
&= -\frac{(4583087983 + 3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 98, normalized size = 0.59

$$\frac{64477987315\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - \frac{24\sqrt{2x^2-x+3}(186470433136x^4+1285267446304x^3+3919478861832x^2+5608297138216x+3110673952831)}{(2x+5)^5} - 64497254400\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{825564856320}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^6, x]

[Out] ((-24\*Sqrt[3 - x + 2\*x^2]\*(3110673952831 + 5608297138216\*x + 3919478861832\*x^2 + 1285267446304\*x^3 + 186470433136\*x^4))/(5 + 2\*x)^5 - 64497254400\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] + 64477987315\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/825564856320

**IntegrateAlgebraic [A]** time = 0.95, size = 124, normalized size = 0.75

$$\frac{5 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{32\sqrt{2}} - \frac{12895597463 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{41278242816\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(-186470433136x^4-1285267446304x^3-3919478861832x^2-5608297138216x-3110673952831)}{34398535680(2x+5)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-3110673952831 - 5608297138216*x - 3919478861832*x^2 - 1285267446304*x^3 - 186470433136*x^4))/(34398535680*(5 + 2*x)^5) - (12895 597463*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2*x^2]/(3*Sqrt[2])])/(41278242816*Sqrt[2]) - (5*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(32*Sqrt[2])
```

**fricas [A]** time = 1.07, size = 203, normalized size = 1.23

64497254400\*sqrt(2)\*sqrt(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 64477987315\*sqrt(2)\*sqrt(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) - 48\*(186470433136\*x^4 + 1285267446304\*x^3 + 3919478861832\*x^2 + 5608297138216\*x + 3110673952831)\*sqrt(2\*x^2 - x + 3))/(32\*sqrt(2)\*sqrt(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="fricas")
```

```
[Out] 1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 1285267446304*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^2 - x + 3))/(32*sqrt(2)*sqrt(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125))
```

**giac [B]** time = 0.28, size = 387, normalized size = 2.35

1/1651129712640\*(64497254400\*sqrt(2)\*sqrt(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 64477987315\*sqrt(2)\*sqrt(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) - 48\*(186470433136\*x^4 + 1285267446304\*x^3 + 3919478861832\*x^2 + 5608297138216\*x + 3110673952831)\*sqrt(2\*x^2 - x + 3))/(32\*sqrt(2)\*sqrt(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="giac")
```

```
[Out] -5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 12895 597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 37631893001870 16*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 2507056315485
```

767)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^5

**maple [A]** time = 0.01, size = 188, normalized size = 1.14

$$\frac{5\sqrt{2} \operatorname{arcsinh}\left(\frac{x+\sqrt{2}}{2}\right)}{64} - \frac{1289597463\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+2)\sqrt{2}}{2\sqrt{-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2}}}\right)}{165112971264} - \frac{1289597463\sqrt{-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2}}}{495338913792} - \frac{3667(-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2})^{\frac{3}{2}}}{92160\left(x+\frac{1}{2}\right)^2} - \frac{38732321(-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2})^{\frac{3}{2}}}{1433272320\left(x+\frac{1}{2}\right)^3} + \frac{71961(-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2})^{\frac{3}{2}}}{13271040\left(x+\frac{1}{2}\right)^4} - \frac{46569601(-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2})^{\frac{3}{2}}}{6879707136\left(x+\frac{1}{2}\right)^5} + \frac{562688629(4x-1)\sqrt{-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2}}}{495338913792} - \frac{562688629\left(-11x+2\left(x+\frac{1}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}}}{247669456896\left(x+\frac{1}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^6,x)

[Out] -1289597463/495338913792\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/92160/(x+5/2)^5\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-38732321/1433272320/(x+5/2)^3\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+711961/13271040/(x+5/2)^4\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+46569601/6879707136/(x+5/2)^2\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+562688629/495338913792\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-562688629/247669456896/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+1289597463/165112971264\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))+5/64\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima [A]** time = 1.04, size = 222, normalized size = 1.35

$$\frac{5}{64}\sqrt{2} \operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{1289597463}{165112971264}\sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23(2x+5)} - \frac{17\sqrt{23}}{23(2x+5)}\right) - \frac{46569601}{3439853568}\sqrt{2x^2-x+3} - \frac{3667(2x^2-x+3)^{\frac{3}{2}}}{2880(32x^2+400x^4+2000x^3+5000x^2+6250x+3125)} - \frac{71961(2x^2-x+3)^{\frac{3}{2}}}{829440(16x^4+160x^3+600x^2+1000x+625)} - \frac{38732321(2x^2-x+3)^{\frac{3}{2}}}{179159040(8x^3+60x^2+150x+125)} - \frac{46569601(2x^2-x+3)^{\frac{3}{2}}}{1719926784(4x^2+20x+25)} - \frac{562688629\sqrt{2x^2-x+3}}{6879707136(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^6,x, algorithm="maxima")

[Out] 5/64\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) - 1289597463/165112971264\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) - 46569601/3439853568\*sqrt(2\*x^2 - x + 3) - 3667/2880\*(2\*x^2 - x + 3)^(3/2)/(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125) + 711961/829440\*(2\*x^2 - x + 3)^(3/2)/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625) - 38732321/179159040\*(2\*x^2 - x + 3)^(3/2)/(8\*x^3 + 60\*x^2 + 150\*x + 125) + 46569601/1719926784\*(2\*x^2 - x + 3)^(3/2)/(4\*x^2 + 20\*x + 25) - 562688629/6879707136\*sqrt(2\*x^2 - x + 3)/(2\*x + 5)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^6,x)



[Out] `int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)`

[Out] `Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**6, x)`

$$3.312 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx$$

**Optimal.** Leaf size=169

$$\frac{87677717 (2x^2 - x + 3)^{3/2}}{8599633920(2x + 5)^3} - \frac{5703277 (2x^2 - x + 3)^{3/2}}{39813120(2x + 5)^4} + \frac{92239 (2x^2 - x + 3)^{3/2}}{138240(2x + 5)^5} - \frac{3667 (2x^2 - x + 3)^{3/2}}{3456(2x + 5)^6} - \frac{1172725(3 - x + 2x^2)^{3/2}}{330225942528(2x + 5)^2}$$

**Rubi [A]** time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1650, 806, 720, 724, 206}

$$\frac{87677717 (2x^2 - x + 3)^{3/2}}{8599633920(2x + 5)^3} - \frac{5703277 (2x^2 - x + 3)^{3/2}}{39813120(2x + 5)^4} + \frac{92239 (2x^2 - x + 3)^{3/2}}{138240(2x + 5)^5} - \frac{3667 (2x^2 - x + 3)^{3/2}}{3456(2x + 5)^6} - \frac{1172725(17 - 22x)\sqrt{2x^2 - x + 3}}{330225942528(2x + 5)^2} - \frac{26972675 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{3962711310336\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^7,x]

[Out] (-1172725\*(17 - 22\*x)\*Sqrt[3 - x + 2\*x^2])/((330225942528\*(5 + 2\*x)^2) - (367\*(3 - x + 2\*x^2)^(3/2))/(3456\*(5 + 2\*x)^6) + (92239\*(3 - x + 2\*x^2)^(3/2))/(138240\*(5 + 2\*x)^5) - (5703277\*(3 - x + 2\*x^2)^(3/2))/(39813120\*(5 + 2\*x)^4) + (87677717\*(3 - x + 2\*x^2)^(3/2))/(8599633920\*(5 + 2\*x)^3) - (26972675\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(3962711310336\*Sqrt[2]))

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 720

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(d\*b - 2\*a\*e + (2\*c\*d - b\*e)\*x)\*(a + b\*x + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[(p\*(b^2 - 4\*a\*c))/(2\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && EqQ[m + 2\*p + 2, 0] && GtQ[p, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$\frac{a e - b d - (2 c d - b e) x}{\sqrt{a + b x + c x^2}}$ , x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4 a c, 0] && NeQ[2 c d - b e, 0]

### Rule 806

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} - \frac{1}{432} \int \frac{\sqrt{3-x+2x^2} \left( \frac{61041}{16} - \frac{20751x}{4} + \dots \right)}{(5+2x)^6} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} + \frac{\int \frac{\sqrt{3-x+2x^2} \left( \frac{8}{12\sqrt{4x^2-2x+6}} \right)}{(5+2x)^6}}{138240(5+2x)^5} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4} \\
&= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} \\
&= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} \\
&= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 91, normalized size = 0.54

$$\frac{24\sqrt{2x^2-x+3} (271409942624x^5 + 12256250416x^4 + 397498825328x^3 + 158340720344x^2 + 27245373694x - 219337079305) - 134863375\sqrt{2} (2x+5)^6 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)}{39627113103360(2x+5)^6}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^7, x]

[Out] (24\*Sqrt[3 - x + 2\*x^2]\*(-219337079305 + 27245373694\*x + 158340720344\*x^2 + 397498825328\*x^3 + 12256250416\*x^4 + 271409942624\*x^5) - 134863375\*Sqrt[2]\*(5 + 2\*x)^6\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2]]))/(39627113103360\*(5 + 2\*x)^6)

**IntegrateAlgebraic [A]** time = 1.00, size = 93, normalized size = 0.55

$$\frac{26972675 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{198135565168\sqrt{2}} + \frac{\sqrt{2x^2-x+3} (271409942624x^5 + 12256250416x^4 + 397498825328x^3 + 158340720344x^2 + 27245373694x - 219337079305)}{1651129712640(2x+5)^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^7, x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-219337079305 + 27245373694\*x + 158340720344\*x^2 + 397498825328\*x^3 + 12256250416\*x^4 + 271409942624\*x^5))/(1651129712640\*(5 + 2\*x)^6) + (26972675\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(1981355655168\*Sqrt[2])

**fricas** [A] time = 0.53, size = 156, normalized size = 0.92

$$\frac{134863375\sqrt{2}(64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625)\log\left(\frac{-24\sqrt{2}\sqrt{x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right) + 48(271409942624x^5 + 12256250416x^4 + 397498825328x^3 + 158340720344x^2 + 27245373694x - 219337079305)\sqrt{2x^2-x+3}}{79254226206720(64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^7,x, algorithm="fricas")

[Out] 1/79254226206720\*(134863375\*sqrt(2)\*(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(271409942624\*x^5 + 12256250416\*x^4 + 397498825328\*x^3 + 158340720344\*x^2 + 27245373694\*x - 219337079305)\*sqrt(2\*x^2 - x + 3))/(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625)

**giac** [B] time = 0.26, size = 405, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^7,x, algorithm="giac")

[Out] -26972675/7925422620672\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 26972675/7925422620672\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 1/3302259425280\*sqrt(2)\*(16506981498400\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^11 + 389429252643040\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^10 + 2263923918689840\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^9 + 11663651054548560\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^8 + 902212326134736\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^7 - 84192729519861840\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^6 - 4317200555009448\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^5 + 351543414066518760\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^4 - 376787166452923830\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 + 356306707647610982\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 - 82348353128195465\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 15499394004553969)/(2\*(sqrt(2)\*x

- sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^6

**maple [A]** time = 0.02, size = 195, normalized size = 1.15

$$\frac{26972675\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+\frac{17}{2})x}{(x+\frac{5}{2})\sqrt{-11x+2(x+\frac{5}{2})^2-\frac{19}{2}}}\right)}{7925422620672} + \frac{26972675\sqrt{-11x+2(x+\frac{5}{2})^2-\frac{19}{2}}}{23776267862016} - \frac{3667(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{221184(x+\frac{5}{2})^2} + \frac{92239(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{4423680(x+\frac{5}{2})^2} + \frac{87677717(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{68797071360(x+\frac{5}{2})^2} - \frac{5703277(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{637009920(x+\frac{5}{2})^2} - \frac{1172725(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{330225942528(x+\frac{5}{2})^2} - \frac{12899975(4x-1)\sqrt{-11x+2(x+\frac{5}{2})^2-\frac{19}{2}}}{23776267862016} - \frac{12899975(-11x+2(x+\frac{5}{2})^2-\frac{19}{2})^{\frac{3}{2}}}{11888133931008(x+\frac{5}{2})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^7, x)

[Out] 26972675/23776267862016\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/221184/(x+5/2)^6\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+92239/4423680/(x+5/2)^5\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+87677717/68797071360/(x+5/2)^3\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-5703277/637009920/(x+5/2)^4\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-1172725/330225942528/(x+5/2)^2\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+12899975/23776267862016\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-12899975/11888133931008/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-26972675/7925422620672\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))

**maxima [A]** time = 1.05, size = 250, normalized size = 1.48

$$\frac{26972675}{7925422620672} \sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23(2x+5)} - \frac{37\sqrt{23}}{23(2x+5)}\right) + \frac{1172725}{165112971264} \sqrt{2x^2-x+3} - \frac{3667(2x^2-x+3)^{\frac{3}{2}}}{3456(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625)} + \frac{92239(2x^2-x+3)^{\frac{3}{2}}}{138240(32x^5+400x^4+2000x^3+5000x^2+6250x+3125)} - \frac{5703277(2x^2-x+3)^{\frac{3}{2}}}{39813120(16x^4+160x^3+600x^2+1000x+625)} + \frac{87677717(2x^2-x+3)^{\frac{3}{2}}}{8599633920(8x^3+60x^2+150x+125)} - \frac{1172725(2x^2-x+3)^{\frac{3}{2}}}{82556485632(4x^2+20x+25)} - \frac{12899975\sqrt{2x^2-x+3}}{330225942528(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^7, x, algorithm="maxima")

[Out] 26972675/7925422620672\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 1172725/165112971264\*sqrt(2\*x^2 - x + 3) - 3667/3456\*(2\*x^2 - x + 3)^(3/2)/(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625) + 92239/138240\*(2\*x^2 - x + 3)^(3/2)/(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125) - 5703277/39813120\*(2\*x^2 - x + 3)^(3/2)/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625) + 87677717/8599633920\*(2\*x^2 - x + 3)^(3/2)/(8\*x^3 + 60\*x^2 + 150\*x + 125) - 1172725/82556485632\*(2\*x^2 - x + 3)^(3/2)/(4\*x^2 + 20\*x + 25) - 12899975/330225942528\*sqrt(2\*x^2 - x + 3)/(2\*x + 5)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)`

[Out] `int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)`

[Out] `Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**7, x)`

$$3.313 \quad \int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx$$

**Optimal.** Leaf size=194

$$\frac{246159769 (2x^2 - x + 3)^{3/2}}{866843099136(2x + 5)^3} + \frac{19414831 (2x^2 - x + 3)^{3/2}}{4013162496(2x + 5)^4} - \frac{1464037 (2x^2 - x + 3)^{3/2}}{13934592(2x + 5)^5} + \frac{948341 (2x^2 - x + 3)^{3/2}}{1741824(2x + 5)^6} - \frac{3667 (2x^2 - x + 3)^{3/2}}{4032(2x + 5)^7} - \frac{12568315(17 - 22x)\sqrt{2x^2 - x + 3}}{23776267862016(2x + 5)^2} - \frac{289071245 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{285315214344192\sqrt{2}}$$

**Rubi [A]** time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{246159769 (2x^2 - x + 3)^{3/2}}{866843099136(2x + 5)^3} + \frac{19414831 (2x^2 - x + 3)^{3/2}}{4013162496(2x + 5)^4} - \frac{1464037 (2x^2 - x + 3)^{3/2}}{13934592(2x + 5)^5} + \frac{948341 (2x^2 - x + 3)^{3/2}}{1741824(2x + 5)^6} - \frac{3667 (2x^2 - x + 3)^{3/2}}{4032(2x + 5)^7} - \frac{12568315(17 - 22x)\sqrt{2x^2 - x + 3}}{23776267862016(2x + 5)^2} - \frac{289071245 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{285315214344192\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^8,x]

[Out] (-12568315\*(17 - 22\*x)\*Sqrt[3 - x + 2\*x^2])/(23776267862016\*(5 + 2\*x)^2) - (3667\*(3 - x + 2\*x^2)^(3/2))/(4032\*(5 + 2\*x)^7) + (948341\*(3 - x + 2\*x^2)^(3/2))/(1741824\*(5 + 2\*x)^6) - (1464037\*(3 - x + 2\*x^2)^(3/2))/(13934592\*(5 + 2\*x)^5) + (19414831\*(3 - x + 2\*x^2)^(3/2))/(4013162496\*(5 + 2\*x)^4) + (246159769\*(3 - x + 2\*x^2)^(3/2))/(866843099136\*(5 + 2\*x)^3) - (289071245\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(285315214344192\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 720

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((d + e\*x)^(m + 1)\*(d\*b - 2\*a\*e + (2\*c\*d - b\*e)\*x)\*(a + b\*x + c\*x^2)^p)/(2\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[(p\*(b^2 - 4\*a\*c))/(2\*(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 2)\*(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && EqQ[m + 2\*p + 2, 0] && GtQ[p, 0]

### Rule 724



```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

### Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(m + 1)*(c*d^2 - b*d*e + a*e^2), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(m + 1)*(c*d^2 - b*d*e + a*e^2), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x+2x^2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx &= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} - \frac{1}{504} \int \frac{\sqrt{3-x+2x^2} \left(\frac{69381}{16} - 5594x + \dots\right)}{(5+2x)^7} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} + \frac{\int \frac{\sqrt{3-x+2x^2}}{\dots}}{\dots} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{1393459(5+2x)^5} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{1393459(5+2x)^5} \\
&= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464037(3-x+2x^2)^{3/2}}{1393459(5+2x)^5} \\
&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} \\
&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} \\
&= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 96, normalized size = 0.49

$$\frac{24\sqrt{2x^2-x+3} (1574342277056x^6 + 27976951397184x^5 + 4982916071952x^4 + 41058010262368x^3 + 14716683780036x^2 + 590492177460x - 20465234808721) - 2023498715\sqrt{2}(2x+5)^7 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)}{3994413000818688(2x+5)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[3 - x + 2\*x^2]\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^8, x]

[Out] (24\*Sqrt[3 - x + 2\*x^2]\*(-20465234808721 + 590492177460\*x + 14716683780036\*x^2 + 41058010262368\*x^3 + 4982916071952\*x^4 + 27976951397184\*x^5 + 1574342277056\*x^6) - 2023498715\*Sqrt[2]\*(5 + 2\*x)^7\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2]])]/(3994413000818688\*(5 + 2\*x)^7)

**IntegrateAlgebraic [A]** time = 1.30, size = 98, normalized size = 0.51

$$\frac{289071245 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right) + \sqrt{2x^2-x+3} (1574342277056x^6 + 27976951397184x^5 + 4982916071952x^4 + 41058010262368x^3 + 14716683780036x^2 + 590492177460x - 20465234808721)}{142657607172096\sqrt{2}} + \frac{166433875034112(2x+5)^7}{166433875034112(2x+5)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 14716683780036*x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 1574342277056*x^6))/(166433875034112*(5 + 2*x)^7) + (289071245*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2*x^2]/(3*Sqrt[2])])/(142657607172096*Sqrt[2])
```

**fricas [A]** time = 0.90, size = 171, normalized size = 0.88

$$\frac{2023498715 \sqrt{2} (128x^7 + 2240x^6 + 16800x^5 + 70000x^4 + 175000x^3 + 262500x^2 + 218750x + 78125) \log\left(\frac{24\sqrt{2}\sqrt{x^2-x+3}-17x+1060x^2-1036x+1153}{4x^2+20x+25}\right) + 48(1574342277056x^6 + 27976951397184x^5 + 4982916071952x^4 + 41058010262368x^3 + 14716683780036x^2 + 590492177460x - 20465234808721)\sqrt{2x^2-x+3}}{7988826001637376(128x^7 + 2240x^6 + 16800x^5 + 70000x^4 + 175000x^3 + 262500x^2 + 218750x + 78125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="fricas")
```

```
[Out] 1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 41058010262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*sqrt(2*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)
```

**giac [B]** time = 0.29, size = 456, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="giac")
```

```
[Out] -289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(129503917760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 796135370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 2531523139171005408*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
```

- 21873571601855032556\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 + 16204706960604668100\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 - 3196254593191113265\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 536799032216117911/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^7

**maple [A]** time = 0.02, size = 216, normalized size = 1.11

$$\frac{289071245 \sqrt{2} \operatorname{arctanh}\left(\frac{(x+5)\sqrt{2}}{2\sqrt{-11x+2(x+5/2)^2}}\right)}{5763042668384} + \frac{289071245 \sqrt{-11x+2(x+5/2)^2}}{1711891286065152} - \frac{3667(-11x+2(x+5/2)^2)^{3/2}}{516096(x+5/2)} + \frac{948341(-11x+2(x+5/2)^2)^{3/2}}{111476736(x+5/2)^2} - \frac{1464037(-11x+2(x+5/2)^2)^{3/2}}{44900844(x+5/2)^3} + \frac{246159769(-11x+2(x+5/2)^2)^{3/2}}{6934744793088(x+5/2)^4} - \frac{19414831(-11x+2(x+5/2)^2)^{3/2}}{64210599936(x+5/2)^5} + \frac{12568315(-11x+2(x+5/2)^2)^{3/2}}{23776267862016(x+5/2)^6} + \frac{138251465(4x-1)\sqrt{-11x+2(x+5/2)^2}}{1711891286065152} - \frac{138251465(-11x+2(x+5/2)^2)^{3/2}}{855945643032576(x+5/2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^8,x)

[Out] 289071245/1711891286065152\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/516096/(x+5/2)^7\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+948341/111476736/(x+5/2)^6\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-1464037/445906944/(x+5/2)^5\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+246159769/6934744793088/(x+5/2)^4\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+19414831/64210599936/(x+5/2)^3\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-12568315/23776267862016/(x+5/2)^2\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+138251465/1711891286065152\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-138251465/855945643032576/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-289071245/570630428688384\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))

**maxima [A]** time = 1.04, size = 301, normalized size = 1.55

$$\frac{289071245 \sqrt{2} \operatorname{arcsinh}\left(\frac{22\sqrt{23}}{23\sqrt{-11x+2(x+5/2)^2}}\right)}{5763042668384} + \frac{12568315 \sqrt{-11x+2(x+5/2)^2}}{11888133931008} - \frac{3667(2x^2-x+3)^{3/2}}{4032(128x^7+2240x^6+16800x^5+70000x^4+175000x^3+262500x^2+218750x+78125)} + \frac{948341(2x^2-x+3)^{3/2}}{1741824(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625)} - \frac{1464037(2x^2-x+3)^{3/2}}{13934592(32x^5+400x^4+2000x^3+5000x^2+6250x+3125)} + \frac{19414831(2x^2-x+3)^{3/2}}{4013162496(16x^4+160x^3+600x^2+1000x+625)} + \frac{246159769(2x^2-x+3)^{3/2}}{866843099136(8x^3+60x^2+150x+125)} - \frac{12568315(2x^2-x+3)^{3/2}}{5944066965504(4x^2+20x+25)} - \frac{138251465 \sqrt{2} \sqrt{-11x+2(x+5/2)^2}}{855945643032576}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)\*(2\*x^2-x+3)^(1/2)/(5+2\*x)^8,x, algorithm="maxima")

[Out] 289071245/570630428688384\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 12568315/11888133931008\*sqrt(2\*x^2 - x + 3) - 3667/4032\*(2\*x^2 - x + 3)^(3/2)/(128\*x^7 + 2240\*x^6 + 16800\*x^5 + 70000\*x^4 + 175000\*x^3 + 262500\*x^2 + 218750\*x + 78125) + 948341/1741824\*(2\*x^2 - x + 3)^(3/2)/(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625) - 1464037/13934592\*(2\*x^2 - x + 3)^(3/2)/(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125) + 19414831/4013162496\*(2\*x^2 - x + 3)^(3/2)/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625) + 246159769/866843099136\*(2\*x^2 - x + 3)^(3/2)/(8\*x^3 + 60\*x^2 + 150\*x + 125) - 12568315/5944066965504\*(2\*x^2 - x + 3)^(3/2)/(4\*x^2 + 20\*x + 25) - 138251465/23776267862016\*sqrt(2\*x^2 - x + 3)/(2\*x + 5)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^8,x)

[Out] int(((2\*x^2 - x + 3)^(1/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^8, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)\*(2\*x\*\*2-x+3)\*\*(1/2)/(5+2\*x)\*\*8,x)

[Out] Integral(sqrt(2\*x\*\*2 - x + 3)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*8, x)

$$3.314 \quad \int (5 + 2x) (3 - x + 2x^2)^{3/2} (2 + x + 3x^2 - x^3 + 5x^4) dx$$

Optimal. Leaf size=166

$$\frac{5}{144} (2x^2 - x + 3)^{5/2} (2x+5)^4 - \frac{1121 (2x^2 - x + 3)^{5/2} (2x+5)^3}{2304} + \frac{69415 (2x^2 - x + 3)^{5/2} (2x+5)^2}{32256} - \frac{3(215900x + 661397) (2x^2 - x + 3)^{5/2}}{143360} - \frac{92727(1-4x)(2x^2-x+3)^{3/2}}{131072} - \frac{6398163(1-4x)\sqrt{2x^2-x+3}}{2097152} - \frac{147157749 \operatorname{arcsinh}^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

**Rubi [A]** time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{144} (2x^2 - x + 3)^{5/2} (2x+5)^4 - \frac{1121 (2x^2 - x + 3)^{5/2} (2x+5)^3}{2304} + \frac{69415 (2x^2 - x + 3)^{5/2} (2x+5)^2}{32256} - \frac{3(215900x + 661397) (2x^2 - x + 3)^{5/2}}{143360} - \frac{92727(1-4x)(2x^2-x+3)^{3/2}}{131072} - \frac{6398163(1-4x)\sqrt{2x^2-x+3}}{2097152} - \frac{147157749 \operatorname{arcsinh}^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (-6398163\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/2097152 - (92727\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/131072 + (69415\*(5 + 2\*x)^2\*(3 - x + 2\*x^2)^(5/2))/32256 - (1121\*(5 + 2\*x)^3\*(3 - x + 2\*x^2)^(5/2))/2304 + (5\*(5 + 2\*x)^4\*(3 - x + 2\*x^2)^(5/2))/144 - (3\*(661397 + 215900\*x)\*(3 - x + 2\*x^2)^(5/2))/143360 - (147157749\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4194304\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) -

```

2*c*e*g*(p + 1)*x*(a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rule 1653

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :=> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int (5+2x)(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4) dx &= \frac{5}{144}(5+2x)^4(3-x+2x^2)^{5/2} + \frac{1}{288} \int (5+2x)(3-x+2x^2)^{5/2} dx \\
&= -\frac{1121(5+2x)^3(3-x+2x^2)^{5/2}}{2304} + \frac{5}{144}(5+2x)^4(3-x+2x^2)^{5/2} \\
&= \frac{69415(5+2x)^2(3-x+2x^2)^{5/2}}{32256} - \frac{1121(5+2x)^3(3-x+2x^2)^{5/2}}{2304} \\
&= \frac{69415(5+2x)^2(3-x+2x^2)^{5/2}}{32256} - \frac{1121(5+2x)^3(3-x+2x^2)^{5/2}}{2304} \\
&= -\frac{92727(1-4x)(3-x+2x^2)^{3/2}}{131072} + \frac{69415(5+2x)^2(3-x+2x^2)^{3/2}}{32256} \\
&= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)(3-x+2x^2)^{3/2}}{131072} \\
&= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)(3-x+2x^2)^{3/2}}{131072} \\
&= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)(3-x+2x^2)^{3/2}}{131072}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 80, normalized size = 0.48

$$\frac{4\sqrt{2x^2-x+3}(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x+1592737263)-46354690935\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2642411520}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(1592737263 + 12357760788\*x + 4870637856\*x^2 + 12669290112\*x^3 + 379086848\*x^4 + 12117893120\*x^5 + 1033175040\*x^6 + 2926837760\*x^7 + 1468006400\*x^8) - 46354690935\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/2642411520

**IntegrateAlgebraic [A]** time = 0.96, size = 95, normalized size = 0.57

$$\frac{\sqrt{2x^2-x+3}(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x+1592737263)-147157749\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{660602880\sqrt{2}}$$

Antiderivative was successfully verified.



[In] IntegrateAlgebraic[(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(1592737263 + 12357760788\*x + 4870637856\*x^2 + 12669290112\*x^3 + 379086848\*x^4 + 12117893120\*x^5 + 1033175040\*x^6 + 2926837760\*x^7 + 1468006400\*x^8))/660602880 - (147157749\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(4194304\*Sqrt[2])

**fricas** [A] time = 0.86, size = 93, normalized size = 0.56

$$\frac{1}{660602880} (1468006400x^8 + 2926837760x^7 + 1033175040x^6 + 12117893120x^5 + 379086848x^4 + 12669290112x^3 + 4870637856x^2 + 12357760788x + 1592737263) \sqrt{2x^2 - x + 3} + \frac{147157749}{16777216} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2), x, algorithm="fricas")

[Out] 1/660602880\*(1468006400\*x^8 + 2926837760\*x^7 + 1033175040\*x^6 + 12117893120\*x^5 + 379086848\*x^4 + 12669290112\*x^3 + 4870637856\*x^2 + 12357760788\*x + 1592737263)\*sqrt(2\*x^2 - x + 3) + 147157749/16777216\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.19, size = 88, normalized size = 0.53

$$\frac{1}{660602880} (4(8(4(16(20(8(28(160x + 319)x + 3153)x + 295847)x + 185101)x + 98978829)x + 152207433)x + 3089440197)x + 1592737263) \sqrt{2x^2 - x + 3} - \frac{147157749}{8388608} \sqrt{2} \log\left(-2\sqrt{2}(\sqrt{2x^2 - x + 3}) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2), x, algorithm="giac")

[Out] 1/660602880\*(4\*(8\*(4\*(16\*(20\*(8\*(28\*(160\*x + 319)\*x + 3153)\*x + 295847)\*x + 185101)\*x + 98978829)\*x + 152207433)\*x + 3089440197)\*x + 1592737263)\*sqrt(2\*x^2 - x + 3) - 147157749/8388608\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.02, size = 134, normalized size = 0.81

$$\frac{5(2x^2 - x + 3)^{\frac{5}{2}}x^4}{9} + \frac{479(2x^2 - x + 3)^{\frac{5}{2}}x^3}{288} + \frac{2005(2x^2 - x + 3)^{\frac{5}{2}}x^2}{8064} + \frac{5645(2x^2 - x + 3)^{\frac{5}{2}}x}{21504} + \frac{147157749\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{8388608} + \frac{120809(2x^2 - x + 3)^{\frac{5}{2}}}{143360} + \frac{92727(4x - 1)(2x^2 - x + 3)^{\frac{5}{2}}}{131072} + \frac{6398163(4x - 1)\sqrt{2x^2 - x + 3}}{2097152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+2\*x)\*(2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2), x)

[Out] 120809/143360\*(2\*x^2-x+3)^(5/2)+5/9\*x^4\*(2\*x^2-x+3)^(5/2)+479/288\*x^3\*(2\*x^2-x+3)^(5/2)+2005/8064\*x^2\*(2\*x^2-x+3)^(5/2)+5645/21504\*x\*(2\*x^2-x+3)^(5/2)+92727/131072\*(4\*x-1)\*(2\*x^2-x+3)^(3/2)+147157749/8388608\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+6398163/2097152\*(4\*x-1)\*(2\*x^2-x+3)^(1/2)

**maxima** [A] time = 0.98, size = 155, normalized size = 0.93

$$\frac{5}{9}(2x^2-x+3)^{\frac{5}{2}}x^4 + \frac{479}{288}(2x^2-x+3)^{\frac{5}{2}}x^3 + \frac{2005}{8064}(2x^2-x+3)^{\frac{5}{2}}x^2 + \frac{5645}{21504}(2x^2-x+3)^{\frac{5}{2}}x + \frac{120809}{143360}(2x^2-x+3)^{\frac{5}{2}} + \frac{92727}{32768}(2x^2-x+3)^{\frac{3}{2}}x - \frac{92727}{131072}(2x^2-x+3)^{\frac{3}{2}} + \frac{6398163}{524288}\sqrt{2x^2-x+3}x + \frac{147157749}{8388608}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6398163}{2097152}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2),x, algorithm="maxima")

[Out] 5/9\*(2\*x^2 - x + 3)^(5/2)\*x^4 + 479/288\*(2\*x^2 - x + 3)^(5/2)\*x^3 + 2005/8064\*(2\*x^2 - x + 3)^(5/2)\*x^2 + 5645/21504\*(2\*x^2 - x + 3)^(5/2)\*x + 120809/143360\*(2\*x^2 - x + 3)^(5/2) + 92727/32768\*(2\*x^2 - x + 3)^(3/2)\*x - 92727/131072\*(2\*x^2 - x + 3)^(3/2) + 6398163/524288\*sqrt(2\*x^2 - x + 3)\*x + 147157749/8388608\*sqrt(2)\*arsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 6398163/2097152\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x + 5) (2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x + 5)\*(2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2),x)

[Out] int((2\*x + 5)\*(2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 5) (2x^2 - x + 3)^{\frac{3}{2}} (5x^4 - x^3 + 3x^2 + x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2),x)

[Out] Integral((2\*x + 5)\*(2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2), x)

$$3.315 \quad \int (3 - x + 2x^2)^{3/2} (2 + x + 3x^2 - x^3 + 5x^4) dx$$

Optimal. Leaf size=147

$$\frac{23}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{125(2x^2 - x + 3)^{5/2} x}{3584} + \frac{1167(2x^2 - x + 3)^{5/2}}{14336} - \frac{8597(1 - 4x)(2x^2 - x + 3)^{3/2}}{65536} - \frac{593193(1 - 4x)\sqrt{2x^2 - x + 3}}{1048576} - \frac{13643439 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16} (2x^2 - x + 3)^{5/2} x^3 + \frac{23}{448} (2x^2 - x + 3)^{5/2} x^2 + \frac{125(2x^2 - x + 3)^{5/2} x}{3584} + \frac{1167(2x^2 - x + 3)^{5/2}}{14336} - \frac{8597(1 - 4x)(2x^2 - x + 3)^{3/2}}{65536} - \frac{593193(1 - 4x)\sqrt{2x^2 - x + 3}}{1048576} - \frac{13643439 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4), x]

[Out] (-593193\*(1 - 4\*x)\*Sqrt[3 - x + 2\*x^2])/1048576 - (8597\*(1 - 4\*x)\*(3 - x + 2\*x^2)^(3/2))/65536 + (1167\*(3 - x + 2\*x^2)^(5/2))/14336 + (125\*x\*(3 - x + 2\*x^2)^(5/2))/3584 + (23\*x^2\*(3 - x + 2\*x^2)^(5/2))/448 + (5\*x^3\*(3 - x + 2\*x^2)^(5/2))/16 - (13643439\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2097152\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$\ast e)/(2\ast c)$ , Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]  
&& NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q =  
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x +  
c\*x^2)^(p + 1))/(c\*(q + 2\*p + 1)), x] + Dist[1/(c\*(q + 2\*p + 1)), Int[(a +  
b\*x + c\*x^2)^p\*ExpandToSum[c\*(q + 2\*p + 1)\*Pq - a\*e\*(q - 1)\*x^(q - 2) - b\*  
e\*(q + p)\*x^(q - 1) - c\*e\*(q + 2\*p + 1)\*x^q, x], x]] /; FreeQ[{a, b, c,  
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int (3 - x + 2x^2)^{3/2} (2 + x + 3x^2 - x^3 + 5x^4) dx &= \frac{5}{16}x^3 (3 - x + 2x^2)^{5/2} + \frac{1}{16} \int (3 - x + 2x^2)^{3/2} (32 + 16x + 3x^2) dx \\
 &= \frac{23}{448}x^2 (3 - x + 2x^2)^{5/2} + \frac{5}{16}x^3 (3 - x + 2x^2)^{5/2} + \frac{1}{224} \int (3 - x + 2x^2)^{3/2} (32 + 16x + 3x^2) dx \\
 &= \frac{125x (3 - x + 2x^2)^{5/2}}{3584} + \frac{23}{448}x^2 (3 - x + 2x^2)^{5/2} + \frac{5}{16}x^3 (3 - x + 2x^2)^{5/2} \\
 &= \frac{1167 (3 - x + 2x^2)^{5/2}}{14336} + \frac{125x (3 - x + 2x^2)^{5/2}}{3584} + \frac{23}{448}x^2 (3 - x + 2x^2)^{5/2} \\
 &= -\frac{8597(1 - 4x) (3 - x + 2x^2)^{3/2}}{65536} + \frac{1167 (3 - x + 2x^2)^{5/2}}{14336} + \frac{125x^3 (3 - x + 2x^2)^{5/2}}{1048576} \\
 &= -\frac{593193(1 - 4x)\sqrt{3 - x + 2x^2}}{1048576} - \frac{8597(1 - 4x) (3 - x + 2x^2)^{3/2}}{65536} \\
 &= -\frac{593193(1 - 4x)\sqrt{3 - x + 2x^2}}{1048576} - \frac{8597(1 - 4x) (3 - x + 2x^2)^{3/2}}{65536} \\
 &= -\frac{593193(1 - 4x)\sqrt{3 - x + 2x^2}}{1048576} - \frac{8597(1 - 4x) (3 - x + 2x^2)^{3/2}}{65536}
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2 - x + 3} (9175040x^7 - 7667712x^6 + 29335552x^5 - 7497728x^4 + 27023744x^3 + 3845856x^2 + 27845612x - 1663407) - 95504073\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{29360128}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4),x]

[Out] (4\*sqrt(3 - x + 2\*x^2)\*(-1663407 + 27845612\*x + 3845856\*x^2 + 27023744\*x^3 - 7497728\*x^4 + 29335552\*x^5 - 7667712\*x^6 + 9175040\*x^7) - 95504073\*sqrt(2)\*ArcSinh[(1 - 4\*x)/sqrt(23)]) / 29360128

**IntegrateAlgebraic [A]** time = 0.87, size = 90, normalized size = 0.61

$$\frac{\sqrt{2x^2 - x + 3} (9175040x^7 - 7667712x^6 + 29335552x^5 - 7497728x^4 + 27023744x^3 + 3845856x^2 + 27845612x - 1663407)}{7340032} - \frac{13643439 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{2097152\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4),x]

[Out] (sqrt(3 - x + 2\*x^2)\*(-1663407 + 27845612\*x + 3845856\*x^2 + 27023744\*x^3 - 7497728\*x^4 + 29335552\*x^5 - 7667712\*x^6 + 9175040\*x^7)) / 7340032 - (13643439 \* Log[1 - 4\*x + 2\*sqrt(2)\*sqrt(3 - x + 2\*x^2)]) / (2097152\*sqrt(2))

**fricas [A]** time = 0.88, size = 88, normalized size = 0.60

$$\frac{1}{7340032} (9175040x^7 - 7667712x^6 + 29335552x^5 - 7497728x^4 + 27023744x^3 + 3845856x^2 + 27845612x - 1663407)\sqrt{2x^2 - x + 3} + \frac{13643439}{8388608} \sqrt{2} \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2),x, algorithm="fricas")

[Out] 1/7340032\*(9175040\*x^7 - 7667712\*x^6 + 29335552\*x^5 - 7497728\*x^4 + 27023744\*x^3 + 3845856\*x^2 + 27845612\*x - 1663407)\*sqrt(2\*x^2 - x + 3) + 13643439/8388608\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac [A]** time = 0.33, size = 83, normalized size = 0.56

$$\frac{1}{7340032} (4(8(4(16(4(140x - 117)x + 3581)x - 3661)x + 211123)x + 120183)x + 6961403)x - 1663407)\sqrt{2x^2 - x + 3} - \frac{13643439}{4194304} \sqrt{2} \log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2),x, algorithm="giac")

[Out] 1/7340032\*(4\*(8\*(4\*(16\*(4\*(8\*(140\*x - 117)\*x + 3581)\*x - 3661)\*x + 211123)\*x + 120183)\*x + 6961403)\*x - 1663407)\*sqrt(2\*x^2 - x + 3) - 13643439/4194304\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple [A]** time = 0.00, size = 117, normalized size = 0.80

$$\frac{5(2x^2 - x + 3)^{\frac{5}{2}} x^3}{16} + \frac{23(2x^2 - x + 3)^{\frac{5}{2}} x^2}{448} + \frac{125(2x^2 - x + 3)^{\frac{5}{2}} x}{3584} + \frac{13643439\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304} + \frac{1167(2x^2 - x + 3)^{\frac{5}{2}}}{14336} + \frac{8597(4x - 1)(2x^2 - x + 3)^{\frac{3}{2}}}{65536} + \frac{593193(4x - 1)\sqrt{2x^2 - x + 3}}{1048576}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2), x)`

[Out]  $1167/14336*(2*x^2-x+3)^{(5/2)}+5/16*(2*x^2-x+3)^{(5/2)}*x^3+23/448*(2*x^2-x+3)^{(5/2)}*x^2+125/3584*(2*x^2-x+3)^{(5/2)}*x+8597/65536*(4*x-1)*(2*x^2-x+3)^{(3/2)}+13643439/4194304*2^{(1/2)}*\operatorname{arsinh}(4/23*23^{(1/2)}*(x-1/4))+593193/1048576*(4*x-1)*(2*x^2-x+3)^{(1/2)}$

**maxima** [A] time = 1.00, size = 138, normalized size = 0.94

$\frac{5}{16}(2x^2-x+3)^{\frac{5}{2}}x^3 + \frac{23}{448}(2x^2-x+3)^{\frac{5}{2}}x^2 + \frac{125}{3584}(2x^2-x+3)^{\frac{5}{2}}x + \frac{1167}{14336}(2x^2-x+3)^{\frac{5}{2}} + \frac{8597}{16384}(2x^2-x+3)^{\frac{3}{2}}x - \frac{8597}{65536}(2x^2-x+3)^{\frac{3}{2}} + \frac{593193}{262144}\sqrt{2x^2-x+3}x + \frac{13643439}{4194304}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{593193}{1048576}\sqrt{2x^2-x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2), x, algorithm="maxima")`

[Out]  $5/16*(2*x^2 - x + 3)^{(5/2)}*x^3 + 23/448*(2*x^2 - x + 3)^{(5/2)}*x^2 + 125/3584*(2*x^2 - x + 3)^{(5/2)}*x + 1167/14336*(2*x^2 - x + 3)^{(5/2)} + 8597/16384*(2*x^2 - x + 3)^{(3/2)}*x - 8597/65536*(2*x^2 - x + 3)^{(3/2)} + 593193/262144*\operatorname{sqrt}(2*x^2 - x + 3)*x + 13643439/4194304*\operatorname{sqrt}(2)*\operatorname{arsinh}(1/23*\operatorname{sqrt}(23)*(4*x - 1)) - 593193/1048576*\operatorname{sqrt}(2*x^2 - x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)`

[Out] `int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 - x + 3)^{\frac{3}{2}} (5x^4 - x^3 + 3x^2 + x + 2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2), x)`

[Out] `Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)`

$$3.316 \quad \int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x} dx$$

Optimal. Leaf size=172

$$\frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2} - \frac{311}{448}(2x+5)(2x^2-x+3)^{5/2} + \frac{3505}{896}(2x^2-x+3)^{5/2} + \frac{(500141-123060x)(2x^2-x+3)^{5/2}}{12288}$$

**Rubi [A]** time = 0.27, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2} - \frac{311}{448}(2x+5)(2x^2-x+3)^{5/2} + \frac{3505}{896}(2x^2-x+3)^{5/2} + \frac{(500141-123060x)(2x^2-x+3)^{3/2}}{12288} + \frac{(141051019-23482924x)\sqrt{2x^2-x+3}}{65536} - \frac{99009 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2x^2-x+3}}\right)}{8\sqrt{2}} + \frac{1622009981 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x), x]

[Out] ((141051019 - 23482924\*x)\*Sqrt[3 - x + 2\*x^2])/65536 + ((500141 - 123060\*x)\*(3 - x + 2\*x^2)^(3/2))/12288 + (3505\*(3 - x + 2\*x^2)^(5/2))/896 - (311\*(5 + 2\*x)\*(3 - x + 2\*x^2)^(5/2))/448 + (5\*(5 + 2\*x)^2\*(3 - x + 2\*x^2)^(5/2))/112 + (1622009981\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(131072\*Sqrt[2]) - (99009\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(8\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps



$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{5+2x} dx &= \frac{5}{112} (5+2x)^2 (3-x+2x^2)^{5/2} + \frac{1}{224} \int \frac{(3-x+2x^2)^{3/2} (573-10x+2x^2)}{5+2x} dx \\
&= -\frac{311}{448} (5+2x) (3-x+2x^2)^{5/2} + \frac{5}{112} (5+2x)^2 (3-x+2x^2)^{5/2} \\
&= \frac{3505}{896} (3-x+2x^2)^{5/2} - \frac{311}{448} (5+2x) (3-x+2x^2)^{5/2} + \frac{5}{112} (5+2x)^2 (3-x+2x^2)^{5/2} \\
&= \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} + \frac{3505}{896} (3-x+2x^2)^{5/2} \\
&= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} \\
&= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} \\
&= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} \\
&= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} \\
&= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 101, normalized size = 0.59

$$\frac{-34065432576\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 4\sqrt{2x^2-x+3} (983040x^6 - 3710976x^5 + 14493696x^4 - 46476672x^3 + 159973408x^2 - 609499532x + 3149403255) + 34062209601\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5505024}$$

Antiderivative was successfully verified.

[In] Integrate[(((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x), x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(3149403255 - 609499532\*x + 159973408\*x^2 - 46476672\*x^3 + 14493696\*x^4 - 3710976\*x^5 + 983040\*x^6) + 34062209601\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] - 34065432576\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/5505024

**IntegrateAlgebraic [A]** time = 0.89, size = 127, normalized size = 0.74

$$\frac{1622009981 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{131072\sqrt{2}} + \frac{99009 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{4\sqrt{2}} + \frac{\sqrt{2x^2-x+3} (983040x^6 - 3710976x^5 + 14493696x^4 - 46476672x^3 + 159973408x^2 - 609499532x + 3149403255)}{1376256}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(3149403255 - 609499532\*x + 159973408\*x^2 - 46476672\*x^3 + 14493696\*x^4 - 3710976\*x^5 + 983040\*x^6))/1376256 + (99009\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(4\*Sqrt[2]) + (1622009981\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(131072\*Sqrt[2])

**fricas** [A] time = 0.83, size = 135, normalized size = 0.78

$$\frac{1}{1376256} (983040x^6 - 3710976x^5 + 14493696x^4 - 46476672x^3 + 159973408x^2 - 609499532x + 3149403255)\sqrt{2x^2 - x + 3} + \frac{1622009981}{524288} \sqrt{2} \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + \frac{99009}{32} \sqrt{2} \log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) + 1060x^2 - 1036x + 1153}{4x^2 + 20x + 25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x), x, algorithm="fricas")

[Out] 1/1376256\*(983040\*x^6 - 3710976\*x^5 + 14493696\*x^4 - 46476672\*x^3 + 159973408\*x^2 - 609499532\*x + 3149403255)\*sqrt(2\*x^2 - x + 3) + 1622009981/524288\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 99009/32\*sqrt(2)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25))

**giac** [A] time = 0.23, size = 139, normalized size = 0.81

$$\frac{1}{1376256} (4(8(12(16(4(40x - 151)x + 2359)x - 121033)x + 4999169)x - 152374883)x + 3149403255)\sqrt{2x^2 - x + 3} + \frac{1622009981}{262144} \sqrt{2} \log(-4\sqrt{2}x + \sqrt{2} + 4\sqrt{2x^2 - x + 3}) - \frac{99009}{16} \sqrt{2} \log(-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3})) + \frac{99009}{16} \sqrt{2} \log\left(\frac{-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}}{-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x), x, algorithm="giac")

[Out] 1/1376256\*(4\*(8\*(12\*(16\*(4\*(40\*x - 151)\*x + 2359)\*x - 121033)\*x + 4999169)\*x - 152374883)\*x + 3149403255)\*sqrt(2\*x^2 - x + 3) + 1622009981/262144\*sqrt(2)\*log(-4\*sqrt(2)\*x + sqrt(2) + 4\*sqrt(2\*x^2 - x + 3)) - 99009/16\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 99009/16\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3)))

**maple** [A] time = 0.01, size = 183, normalized size = 1.06

$$\frac{5(2x^2 - x + 3)^{\frac{3}{2}} x^2}{28} - \frac{111(2x^2 - x + 3)^{\frac{3}{2}} x}{224} - \frac{1622009981\sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{2x^2 - x + 3}}{2}\right)}{262144} - \frac{99009\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x + 2)\sqrt{2}}{2\sqrt{2x^2 - x + 3}}\right)}{16} + \frac{1395(2x^2 - x + 3)^{\frac{3}{2}}}{896} - \frac{10255(4x - 1)(2x^2 - x + 3)^{\frac{3}{2}}}{4096} - \frac{702595(4x - 1)\sqrt{2x^2 - x + 3}}{65536} + \frac{3667\left(-11x + 2\left(x + \frac{3}{2}\right)\right)^{\frac{3}{2}}}{96} - \frac{40337(4x - 1)\sqrt{-11x + 2\left(x + \frac{3}{2}\right)} - \frac{11}{2}}{512} + \frac{33003\sqrt{-11x + 2\left(x + \frac{3}{2}\right)} - \frac{11}{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x), x)

```
[Out] 5/28*(2*x^2-x+3)^(5/2)*x^2-111/224*(2*x^2-x+3)^(5/2)*x+1395/896*(2*x^2-x+3)^(5/2)-10255/4096*(4*x-1)*(2*x^2-x+3)^(3/2)-707595/65536*(4*x-1)*(2*x^2-x+3)^(1/2)-1622009981/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+3667/96*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-40337/512*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+33003/16*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-99009/16*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima** [A] time = 1.00, size = 157, normalized size = 0.91

$$\frac{5}{28}(2x^2-x+3)^{5/2} - \frac{111}{224}(2x^2-x+3)^{5/2}x + \frac{1395}{896}(2x^2-x+3)^{5/2} - \frac{10255}{1024}(2x^2-x+3)^{3/2}x + \frac{500141}{12288}(2x^2-x+3)^{3/2} - \frac{5870731}{16384}\sqrt{2x^2-x+3} - \frac{1622009981}{262144}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{99009}{16}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{141051019}{65536}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="maxima")
```

```
[Out] 5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/896*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/12288*(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981/262144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 141051019/65536*sqrt(2*x^2 - x + 3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{2x + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{2x + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5), x)
```

$$3.317 \quad \int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx$$

Optimal. Leaf size=172

$$\frac{5}{96}(2x+5)(2x^2-x+3)^{5/2} - \frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)} - \frac{839}{960}(2x^2-x+3)^{5/2} - \frac{(909513-226052x)(2x^2-x+3)^{3/2}}{18432} - \dots$$

**Rubi [A]** time = 0.28, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{96}(2x+5)(2x^2-x+3)^{5/2} - \frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)} - \frac{839}{960}(2x^2-x+3)^{5/2} - \frac{(909513-226052x)(2x^2-x+3)^{3/2}}{18432} - \frac{(85448933-14243732x)\sqrt{2x^2-x+3}}{32768} + \frac{959625 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{64\sqrt{2}} - \frac{982669459 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^2,x]

[Out] -((85448933 - 14243732\*x)\*Sqrt[3 - x + 2\*x^2])/32768 - ((909513 - 226052\*x)\*(3 - x + 2\*x^2)^(3/2))/18432 - (839\*(3 - x + 2\*x^2)^(5/2))/960 - (3667\*(3 - x + 2\*x^2)^(5/2))/(576\*(5 + 2\*x)) + (5\*(5 + 2\*x)\*(3 - x + 2\*x^2)^(5/2))/96 - (982669459\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(65536\*Sqrt[2]) + (959625\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(64\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*

$d^2(m + q + 2p + 1) - e(2cd - b^2)(m + q + p)x$ ,  $x$ ,  $x$  /;  $GtQ[q, 1] \&\& NeQ[m + q + 2p + 1, 0]$  /;  $FreeQ[\{a, b, c, d, e, m, p\}, x]$  &&  $PolyQ[Pq, x]$  &&  $NeQ[b^2 - 4ac, 0]$  &&  $NeQ[c^2d^2 - b^2de + ae^2, 0]$  &&  $!(IGtQ[m, 0] \&\& RationalQ[a, b, c, d, e] \&\& (IntegerQ[p] || ILtQ[p + 1/2, 0]))$

### Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} - \frac{1}{72} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{26675}{16} - 4990\right)}{5+2x} \\
 &= -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)(3-x+2x^2)^{5/2} - \int \frac{(3-x+2x^2)^{3/2}}{5+2x} \\
 &= -\frac{839}{960}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)(3-x+2x^2)^{5/2} \\
 &= -\frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} - \frac{839}{960}(3-x+2x^2)^{5/2} \\
 &= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} \\
 &= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} \\
 &= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} \\
 &= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} \\
 &= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} \\
 &= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432}
 \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 108, normalized size = 0.63

$$\frac{14739840000\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{4\sqrt{2x^2-x+3}(409600x^6-1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6814208295)}{2x+5} - 14740041885\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1966080}$$

Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^2, x]

[Out]  $((4*\sqrt{3-x+2*x^2})*(-6814208295-1404323114*x+182033816*x^2-35369408*x^3+8283904*x^4-1798144*x^5+409600*x^6))/(5+2*x)-14740041885*\sqrt{2}*\text{ArcSinh}[(1-4*x)/\sqrt{23}]+14739840000*\sqrt{2}*\text{ArcTanh}[(17-22*x)/(12*\sqrt{6-2*x+4*x^2})])/1966080$

**IntegrateAlgebraic [A]** time = 1.06, size = 134, normalized size = 0.78

$$\frac{982669459 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{65536\sqrt{2}} - \frac{959625 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{32\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(409600x^6-1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6814208295)}{491520(2x+5)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3-x+2\*x^2)^(3/2)\*(2+x+3\*x^2-x^3+5\*x^4))/(5+2\*x)^2,x]

[Out]  $(\sqrt{3-x+2*x^2})*(-6814208295-1404323114*x+182033816*x^2-35369408*x^3+8283904*x^4-1798144*x^5+409600*x^6)/(491520*(5+2*x))-(959625*\text{ArcTanh}[5/6+x/3-\sqrt{3-x+2*x^2}/(3*\sqrt{2})])/(32*\sqrt{2})-(982669459*\text{Log}[1-4*x+2*\sqrt{2}*\sqrt{3-x+2*x^2}])/(65536*\sqrt{2})$

**fricas [A]** time = 1.00, size = 153, normalized size = 0.89

$$\frac{14740041885\sqrt{2}(2x+5)\log(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+14739840000\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+8(409600x^6-1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6814208295)\sqrt{2x^2-x+3}}{3932160(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2,x, algorithm="fricas")

[Out]  $1/3932160*(14740041885*\sqrt{2}*(2*x+5)*\log(-4*\sqrt{2}*\sqrt{2*x^2-x+3}*(4*x-1)-32*x^2+16*x-25)+14739840000*\sqrt{2}*(2*x+5)*\log((24*\sqrt{2}*\sqrt{2*x^2-x+3}*(22*x-17)-1060*x^2+1036*x-1153)/(4*x^2+20*x+25))+8*(409600*x^6-1798144*x^5+8283904*x^4-35369408*x^3+182033816*x^2-1404323114*x-6814208295)*\sqrt{2*x^2-x+3})/(2*x+5)$

**giac [B]** time = 0.45, size = 707, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2,x, algorithm="giac")

[Out]  $1/1966080*\sqrt{2}*(14739840000*\log(12*\sqrt{-11/(2*x+5)}+36/(2*x+5)^2+1)+72/(2*x+5)-11)*\text{sgn}(1/(2*x+5))+14740041885*\log(\text{abs}(\sqrt{-11/(2*x+5)}+36/(2*x+5)^2+1)+6/(2*x+5)+1))*\text{sgn}(1/(2*x+5))-14740041885*\log(\text{abs}(\sqrt{-11/(2*x+5)}+36/(2*x+5)^2+1)+6/(2*x+5)-1))*\text{sgn}(1/(2*x+5))-2027704320*\sqrt{-11/(2*x+5)}+36/(2*x+5)^2+1)*\text{sgn}(\sqrt{-11/(2*x+5)}+36/(2*x+5)^2+1)$

$$\begin{aligned} & 1/(2*x + 5)) + 2*(45496763235*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6 \\ & / (2*x + 5))^11*\text{sgn}(1/(2*x + 5)) - 126553743360*(\text{sqrt}(-11/(2*x + 5) + 36/(2* \\ & x + 5)^2 + 1) + 6/(2*x + 5))^10*\text{sgn}(1/(2*x + 5)) + 44062768335*(\text{sqrt}(-11/(2 \\ & *x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*\text{sgn}(1/(2*x + 5)) + 331789824 \\ & 00*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*\text{sgn}(1/(2*x + \\ & 5)) + 294206421582*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5)) \\ & ^7*\text{sgn}(1/(2*x + 5)) - 463672074240*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1 \\ & ) + 6/(2*x + 5))^6*\text{sgn}(1/(2*x + 5)) + 35099942478*(\text{sqrt}(-11/(2*x + 5) + 36/ \\ & (2*x + 5)^2 + 1) + 6/(2*x + 5))^5*\text{sgn}(1/(2*x + 5)) + 171324610560*(\text{sqrt}(-11 \\ & / (2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*\text{sgn}(1/(2*x + 5)) + 600592 \\ & 81615*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*\text{sgn}(1/(2*x \\ & + 5)) - 105051009024*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + \\ & 5))^2*\text{sgn}(1/(2*x + 5)) - 5210329245*(\text{sqrt}(-11/(2*x + 5) + 36/(2*x + 5)^2 + \\ & 1) + 6/(2*x + 5))*\text{sgn}(1/(2*x + 5)) + 17058392064*\text{sgn}(1/(2*x + 5)))/((\text{sqrt}(- \\ & 11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^6 \end{aligned}$$

**maple [A]** time = 0.01, size = 208, normalized size = 1.21

$$\frac{5(2x^2-x+3)^{\frac{5}{2}}x}{48} - \frac{982669459\sqrt{2}}{131072} \operatorname{arcsinh}\left(\frac{4\sqrt{23}}{23}\right) - \frac{959625\sqrt{2}}{128} \operatorname{arctanh}\left(\frac{(-11x+2)\sqrt{2}}{12\sqrt{-11x+2}\sqrt{x+\frac{5}{2}}}\right) - \frac{589(2x^2-x+3)^{\frac{5}{2}}}{960} - \frac{9059(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{6144} - \frac{208357(4x-1)\sqrt{2x^2-x+3}}{32768} - \frac{3667(-11x+2)\sqrt{x+\frac{5}{2}}}{1152\sqrt{x+\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2,x)

[Out]  $\frac{5}{48}(2x^2-x+3)^{5/2}x - 589/960(2x^2-x+3)^{5/2} + 9059/6144(4x-1)(2x^2-x+3)^{3/2} + 208357/32768(4x-1)(2x^2-x+3)^{1/2} + 982669459/131072\sqrt{2}\operatorname{arcsinh}(4/23\sqrt{23}) - 3667/1152(x+5/2)^{-1/2}(-11x+2)(x+5/2)^{-2-19/2} - 106625/2304(-11x+2)(x+5/2)^{-2-19/2} - 1637/16(4x-1)(-11x+2)(x+5/2)^{-2-19/2} - 319875/128(-11x+2)(x+5/2)^{-2-19/2} + 959625/128\sqrt{2}\operatorname{arctanh}(1/12(-11x+17/2)\sqrt{2}) - 67/2304(4x-1)(-11x+2)(x+5/2)^{-2-19/2}$

**maxima [A]** time = 1.01, size = 161, normalized size = 0.94

$$\frac{5}{48}(2x^2-x+3)^{\frac{5}{2}}x - \frac{589}{960}(2x^2-x+3)^{\frac{5}{2}} + \frac{9059}{1536}(2x^2-x+3)^{\frac{3}{2}}x - \frac{185827}{6144}(2x^2-x+3)^{\frac{3}{2}} + \frac{3560933}{8192}\sqrt{2x^2-x+3} + \frac{982669459}{131072}\sqrt{2}\operatorname{arcsinh}\left(\frac{4}{23}\sqrt{23}\right) - \frac{959625}{128}\sqrt{2}\operatorname{arcsinh}\left(\frac{22\sqrt{23}x}{23(2x+5)} - \frac{17\sqrt{23}}{23(2x+5)}\right) - \frac{85448933}{32768}\sqrt{2x^2-x+3} - \frac{3667(2x^2-x+3)^{\frac{3}{2}}}{32(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2,x, algorithm="maxima")

[Out]  $\frac{5}{48}(2x^2-x+3)^{5/2}x - 589/960(2x^2-x+3)^{5/2} + 9059/1536(2x^2-x+3)^{3/2}x - 185827/6144(2x^2-x+3)^{3/2} + 3560933/8192\sqrt{2x^2-x+3} + 982669459/131072\sqrt{2}\operatorname{arcsinh}(4/23\sqrt{23}) - 1/23\sqrt{23} - 959625/128\sqrt{2}\operatorname{arcsinh}(22/23\sqrt{23})x/\operatorname{abs}(2x+5) - 17/23\sqrt{23}/\operatorname{abs}(2x+5) - 85448933/32768\sqrt{2x^2-x+3} - 3667/32(2x^2-x+3)^{3/2}/(2x+5)$



mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^2,x)

[Out] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*2,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*2, x)

$$3.318 \quad \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx$$

Optimal. Leaf size=174

$$\frac{438065(2x^2-x+3)^{5/2}}{82944(2x+5)} - \frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2} + \frac{1}{16}(2x^2-x+3)^{5/2} + \frac{(2154633-534617x)(2x^2-x+3)^{3/2}}{82944} + \frac{(33741483-5623292x)\sqrt{2x^2-x+3}}{24576} - \frac{8083915 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2x^2-x+3}}\right)}{1024\sqrt{2}} + \frac{129342063 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

**Rubi [A]** time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{438065(2x^2-x+3)^{5/2}}{82944(2x+5)} - \frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2} + \frac{1}{16}(2x^2-x+3)^{5/2} + \frac{(2154633-534617x)(2x^2-x+3)^{3/2}}{82944} + \frac{(33741483-5623292x)\sqrt{2x^2-x+3}}{24576} - \frac{8083915 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2x^2-x+3}}\right)}{1024\sqrt{2}} + \frac{129342063 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^3,x]

[Out] ((33741483 - 5623292\*x)\*Sqrt[3 - x + 2\*x^2])/24576 + ((2154633 - 534617\*x)\*(3 - x + 2\*x^2)^(3/2))/82944 + (3 - x + 2\*x^2)^(5/2)/16 - (3667\*(3 - x + 2\*x^2)^(5/2))/(1152\*(5 + 2\*x)^2) + (438065\*(3 - x + 2\*x^2)^(5/2))/(82944\*(5 + 2\*x)) + (129342063\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(16384\*Sqrt[2]) - (8083915\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(1024\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 814

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(c\*e\*f\*(m + 2\*p + 2) - g\*(c\*d + 2\*c\*d\*p - b\*e\*p) + g\*c\*e\*(m + 2\*p + 1)\*x)\*(a + b\*x + c\*x^2)^p)/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), x] - Dist[p/(c\*e^2\*(m + 2\*p + 1)\*(m + 2\*p + 2)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p - 1)\*Simp[c\*e\*f\*(b\*d - 2\*a\*e)\*(m + 2\*p + 2) + g\*(a\*e\*(b\*e - 2\*c\*d\*m + b\*e\*m) + b\*d\*(b\*e\*p - c\*d - 2\*c\*d\*p)) + (c\*e\*f\*(2\*c\*d - b\*e)\*(m + 2\*p + 2) + g\*(b^2\*e^2\*(p + m + 1) - 2\*c^2\*d^2\*(1 + 2\*p) - c\*e\*(b\*d\*(m - 2\*p) + 2\*a\*e\*(m + 2\*p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2\*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p])

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*

```
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} - \frac{1}{144} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{35015}{16} - \frac{2158}{4}\right)}{(5+2x)^2} dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} + \int \frac{(3-x+2x^2)^{3/2}}{(5+2x)} dx \\
&= \frac{1}{16} (3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} \\
&= \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944} + \frac{1}{16} (3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} \\
&= \frac{(33741483 - 5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944} \\
&= \frac{(33741483 - 5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944} \\
&= \frac{(33741483 - 5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944} \\
&= \frac{(33741483 - 5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944} \\
&= \frac{(33741483 - 5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944} \\
&= \frac{(33741483 - 5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633 - 534617x)(3-x+2x^2)^{3/2}}{82944}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 108, normalized size = 0.62

$$\frac{-129342640\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{4\sqrt{2x^2-x+3}(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+298966737)}{(2x+5)^2} + 129342063\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768}$$

Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^3, x]

[Out] ((4\*sqrt[3 - x + 2\*x^2]\*(298966737 + 181223072\*x + 16667188\*x^2 - 1620944\*x^3 + 253312\*x^4 - 43520\*x^5 + 8192\*x^6))/(5 + 2\*x)^2 + 129342063\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]] - 129342640\*sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*sqrt[6 - 2\*x + 4\*x^2])])/32768

**IntegrateAlgebraic [A]** time = 1.00, size = 134, normalized size = 0.77

$$\frac{129342063 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{16384\sqrt{2}} + \frac{8083915 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{512\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+298966737)}{8192(2x+5)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^3, x]

[Out] (sqrt[3 - x + 2\*x^2]\*(298966737 + 181223072\*x + 16667188\*x^2 - 1620944\*x^3 + 253312\*x^4 - 43520\*x^5 + 8192\*x^6))/(8192\*(5 + 2\*x)^2) + (8083915\*ArcTanh[5/6 + x/3 - sqrt[3 - x + 2\*x^2]/(3\*sqrt[2])])/(512\*sqrt[2]) + (129342063\*log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(16384\*sqrt[2])

**fricas [A]** time = 0.88, size = 169, normalized size = 0.97

$$\frac{129342063\sqrt{2}(4x^2+20x+25)\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+129342640\sqrt{2}(4x^2+20x+25)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+8(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+298966737)\sqrt{2x^2-x+3}}{65536(4x^2+20x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3,x, algorithm="fricas")

[Out] 1/65536\*(129342063\*sqrt(2)\*(4\*x^2 + 20\*x + 25)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 129342640\*sqrt(2)\*(4\*x^2 + 20\*x + 25)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 8\*(8192\*x^6 - 43520\*x^5 + 253312\*x^4 - 1620944\*x^3 + 16667188\*x^2 + 181223072\*x + 298966737)\*sqrt(2\*x^2 - x + 3))/(4\*x^2 + 20\*x + 25)

**giac [A]** time = 0.30, size = 268, normalized size = 1.54

$$\frac{1}{8192}(4(8(4(16x-165)+4879)+263469)+8460377)\sqrt{2x^2-x+3} + \frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2x^2-x+3}\right)+1\right) - \frac{8083915}{2048}\sqrt{2}\log\left(-2\sqrt{2x^2-x+3}\right) + \frac{8083915}{2048}\sqrt{2}\log\left(-2\sqrt{2x^2-x+3}\right) + \frac{\sqrt{2}\left(14243182\sqrt{2}\left(\sqrt{2x^2-x+3}\right)^2+10996674\left(\sqrt{2x^2-x+3}\right)-17998671\sqrt{2}\left(\sqrt{2x^2-x+3}\right)+11096087\right)}{512\left(2\left(\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2x^2-x+3}\right)-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3,x, algorithm="giac")

[Out]  $1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*\sqrt{2*x^2 - x + 3} + 129342063/32768*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3}) + 1) - 8083915/2048*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2}*x + \sqrt{2} + 2*\sqrt{2*x^2 - x + 3})) + 8083915/2048*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2}*x - 11*\sqrt{2} + 2*\sqrt{2*x^2 - x + 3})) + 1/512*\sqrt{2}*(14243182*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})^3 + 109906674*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})^2 - 170996871*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3}) + 110506087)/(2*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})^2 + 10*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3}) - 11)^2$

**maple** [A] time = 0.02, size = 214, normalized size = 1.23

$$\frac{129342063\sqrt{2}\operatorname{arcsinh}\left(\frac{\sqrt{2}(x-1)}{2}\right)}{32768} - \frac{8083915\sqrt{2}\operatorname{arcsinh}\left(\frac{(x+1)\sqrt{2}}{2}\right)}{2048} + \frac{46691(2x^2-x+3)^{\frac{3}{2}}}{4608} - \frac{3667(2x^2-x+3)^{\frac{5}{2}}}{1152(4x^2+20x+25)} - \frac{1405823}{6144}\sqrt{2x^2-x+3} - \frac{129342063}{32768}\sqrt{2}\operatorname{arcsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{8083915}{2048}\sqrt{2}\operatorname{arcsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{11247161}{8192}\sqrt{2x^2-x+3} + \frac{438065(2x^2-x+3)^{\frac{3}{2}}}{4608(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((2*x^2-x+3)^{(3/2)}*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3, x)$

[Out]  $1/16*(2*x^2-x+3)^{(5/2)}+8083915/6144*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+8083915/331776*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-3667/4608/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}-438065/331776*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}+438065/165888/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}-343745/6144*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-8083915/2048*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)})/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-149/512*(4*x-1)*(2*x^2-x+3)^{(3/2)}-129342063/32768*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))-10281/8192*(4*x-1)*(2*x^2-x+3)^{(1/2)}$

**maxima** [A] time = 1.00, size = 172, normalized size = 0.99

$$\frac{1}{16}(2x^2-x+3)^{\frac{5}{2}} - \frac{149}{128}(2x^2-x+3)^{\frac{3}{2}} + \frac{46691}{4608}(2x^2-x+3)^{\frac{3}{2}} - \frac{3667(2x^2-x+3)^{\frac{5}{2}}}{1152(4x^2+20x+25)} - \frac{1405823}{6144}\sqrt{2x^2-x+3} - \frac{129342063}{32768}\sqrt{2}\operatorname{arcsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{8083915}{2048}\sqrt{2}\operatorname{arcsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{11247161}{8192}\sqrt{2x^2-x+3} + \frac{438065(2x^2-x+3)^{\frac{3}{2}}}{4608(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((2*x^2-x+3)^{(3/2)}*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3, x, \text{algorithm}="m\text{axima}"))$

[Out]  $1/16*(2*x^2 - x + 3)^{(5/2)} - 149/128*(2*x^2 - x + 3)^{(3/2)}*x + 46691/4608*(2*x^2 - x + 3)^{(3/2)} - 3667/1152*(2*x^2 - x + 3)^{(5/2)}/(4*x^2 + 20*x + 25) - 1405823/6144*\sqrt{2*x^2 - x + 3}*x - 129342063/32768*\sqrt{2}*\operatorname{arcsinh}(4/23*\sqrt{23}*x - 1/23*\sqrt{23}) + 8083915/2048*\sqrt{2}*\operatorname{arcsinh}(22/23*\sqrt{23}*x/\text{abs}(2*x + 5) - 17/23*\sqrt{23}/\text{abs}(2*x + 5)) + 11247161/8192*\sqrt{2*x^2 - x + 3} + 438065/4608*(2*x^2 - x + 3)^{(3/2)}/(2*x + 5)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)`

[Out] `int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)`

[Out] `Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**3, x)`

$$3.319 \quad \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx$$

Optimal. Leaf size=181

$$-\frac{32865365 (2x^2 - x + 3)^{5/2}}{17915904(2x + 5)} + \frac{556255 (2x^2 - x + 3)^{5/2}}{248832(2x + 5)^2} - \frac{3667 (2x^2 - x + 3)^{5/2}}{1728(2x + 5)^3} - \frac{(138006843 - 34265045x) (2x^2 - x + 3)^{3/2}}{17915904}$$

**Rubi [A]** time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{32865365 (2x^2 - x + 3)^{5/2}}{17915904(2x + 5)} + \frac{556255 (2x^2 - x + 3)^{5/2}}{248832(2x + 5)^2} - \frac{3667 (2x^2 - x + 3)^{5/2}}{1728(2x + 5)^3} - \frac{(138006843 - 34265045x) (2x^2 - x + 3)^{3/2}}{17915904} - \frac{(135068604 - 22512089x)\sqrt{2x^2 - x + 3}}{331776} + \frac{517762327 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{221184\sqrt{2}} - \frac{19176431 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^4,x]

[Out] -((135068604 - 22512089\*x)\*Sqrt[3 - x + 2\*x^2])/331776 - ((138006843 - 34265045\*x)\*(3 - x + 2\*x^2)^(3/2))/17915904 - (3667\*(3 - x + 2\*x^2)^(5/2))/(1728\*(5 + 2\*x)^3) + (556255\*(3 - x + 2\*x^2)^(5/2))/(248832\*(5 + 2\*x)^2) - (32865365\*(3 - x + 2\*x^2)^(5/2))/(17915904\*(5 + 2\*x)) - (19176431\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(8192\*Sqrt[2]) + (517762327\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(221184\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724



```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} - \frac{1}{216} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{43355}{16} - \frac{1160}{2}\right)}{(5+2x)} \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} + \frac{556255(3-x+2x^2)^{5/2}}{248832(5+2x)^2} + \frac{\int \frac{(3-x+2x^2)^{3/2}}{(5+2x)}}{216} \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} + \frac{556255(3-x+2x^2)^{5/2}}{248832(5+2x)^2} - \frac{32865365}{1791504} \\
&= -\frac{(138006843 - 34265045x)(3-x+2x^2)^{3/2}}{1791504} - \frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} \\
&= -\frac{(135068604 - 22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843 - 34265045x)(3-x+2x^2)^{3/2}}{1791504} \\
&= -\frac{(135068604 - 22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843 - 34265045x)(3-x+2x^2)^{3/2}}{1791504} \\
&= -\frac{(135068604 - 22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843 - 34265045x)(3-x+2x^2)^{3/2}}{1791504} \\
&= -\frac{(135068604 - 22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843 - 34265045x)(3-x+2x^2)^{3/2}}{1791504}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 108, normalized size = 0.60

$$\frac{517762327\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{12\sqrt{2x^2-x+3}(46080x^6-315648x^5+2626848x^4-33595416x^3-594798908x^2-2006873194x-1994650739)}{(2x+5)^3} - 517763637\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{442368}$$

Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^4, x]

[Out] ((12\*Sqrt[3 - x + 2\*x^2]\*(-1994650739 - 2006873194\*x - 594798908\*x^2 - 33595416\*x^3 + 2626848\*x^4 - 315648\*x^5 + 46080\*x^6))/(5 + 2\*x)^3 - 517763637\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] + 517762327\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/442368





[Out] `int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)`

[Out] `Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4, x)`

$$3.320 \quad \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx$$

Optimal. Leaf size=188

$$-\frac{14477995(2x^2-x+3)^{5/2}}{23887872(2x+5)^2} + \frac{224815(2x^2-x+3)^{5/2}}{165888(2x+5)^3} - \frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4} + \frac{(67865260x+762984903)(2x^2-x+3)^{3/2}}{95551488(2x+5)}$$

**Rubi [A]** time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{14477995(2x^2-x+3)^{5/2}}{23887872(2x+5)^2} + \frac{224815(2x^2-x+3)^{5/2}}{165888(2x+5)^3} - \frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4} + \frac{(67865260x+762984903)(2x^2-x+3)^{3/2}}{95551488(2x+5)} + \frac{(2339916063-389975609x)\sqrt{2x^2-x+3}}{31850496} - \frac{8969688643 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{21233664\sqrt{2}} + \frac{432565 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^5, x]

[Out] ((2339916063 - 389975609\*x)\*Sqrt[3 - x + 2\*x^2])/31850496 + ((762984903 + 67865260\*x)\*(3 - x + 2\*x^2)^(3/2))/(95551488\*(5 + 2\*x)) - (3667\*(3 - x + 2\*x^2)^(5/2))/(2304\*(5 + 2\*x)^4) + (224815\*(3 - x + 2\*x^2)^(5/2))/(165888\*(5 + 2\*x)^3) - (14477995\*(3 - x + 2\*x^2)^(5/2))/(23887872\*(5 + 2\*x)^2) + (432565\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1024\*Sqrt[2]) - (8969688643\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(21233664\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x]
+ Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x]
- Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

lRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} - \frac{1}{288} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{51695}{16} - \frac{2483}{4}\right)}{(5+2x)} \\
 &= -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} + \frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} + \int \frac{(3-x+2x^2)^{3/2}}{(5+2x)} \\
 &= -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} + \frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} - \frac{14477995}{23887} \\
 &= \frac{(762984903 + 67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)} \\
 &= \frac{(2339916063 - 389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903 + 6}{955} \\
 &= \frac{(2339916063 - 389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903 + 6}{955} \\
 &= \frac{(2339916063 - 389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903 + 6}{955} \\
 &= \frac{(2339916063 - 389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903 + 6}{955}
 \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 108, normalized size = 0.57

$$\frac{-8969688643\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}(2949120x^6-29270016x^5+468043776x^4+11761910072x^3+60528581892x^2+121473790266x+86386856771)}{(2x+5)^4} + 8969667840\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{42467328}$$



Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^5, x]

[Out] ((24\*sqrt(3 - x + 2\*x^2)\*(86386856771 + 121473790266\*x + 60528581892\*x^2 + 11761910072\*x^3 + 468043776\*x^4 - 29270016\*x^5 + 2949120\*x^6))/(5 + 2\*x)^4 + 8969667840\*sqrt(2)\*ArcSinh[(1 - 4\*x)/sqrt(23)] - 8969688643\*sqrt(2)\*ArcTanh[(17 - 22\*x)/(12\*sqrt(6 - 2\*x + 4\*x^2))])/42467328

**IntegrateAlgebraic [A]** time = 0.97, size = 134, normalized size = 0.71

$$\frac{432565 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{1024\sqrt{2}} + \frac{8969688643 \tanh^{-1}\left(-\frac{\sqrt{2x^2 - x + 3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{10616832\sqrt{2}} + \frac{\sqrt{2x^2 - x + 3} (2949120x^6 - 29270016x^5 + 468043776x^4 + 11761910072x^3 + 60528581892x^2 + 121473790266x + 86386856771)}{1769472(2x + 5)^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^5, x]

[Out] (sqrt(3 - x + 2\*x^2)\*(86386856771 + 121473790266\*x + 60528581892\*x^2 + 11761910072\*x^3 + 468043776\*x^4 - 29270016\*x^5 + 2949120\*x^6))/(1769472\*(5 + 2\*x)^4) + (8969688643\*ArcTanh[5/6 + x/3 - sqrt(3 - x + 2\*x^2)/(3\*sqrt(2))])/ (10616832\*sqrt(2)) + (432565\*Log[1 - 4\*x + 2\*sqrt(2)\*sqrt(3 - x + 2\*x^2)])/(1024\*sqrt(2))

**fricas [A]** time = 1.30, size = 199, normalized size = 1.06

$$\frac{896967840\sqrt{2}(16x^4 + 160x^3 + 600x^2 + 1000x + 625)\log(\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8969688643\sqrt{2}(16x^4 + 160x^3 + 600x^2 + 1000x + 625)\log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3} - 175 - 1000x^2 - 3396x + 1153}{49 - 231x + 25x^2}\right) + 48(2949120x^6 - 29270016x^5 + 468043776x^4 + 11761910072x^3 + 60528581892x^2 + 121473790266x + 86386856771)\sqrt{2x^2 - x + 3}}{84934656(16x^4 + 160x^3 + 600x^2 + 1000x + 625)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^5,x, algorithm="fricas")

[Out] 1/84934656\*(8969667840\*sqrt(2)\*(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8969688643\*sqrt(2)\*(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(2949120\*x^6 - 29270016\*x^5 + 468043776\*x^4 + 11761910072\*x^3 + 60528581892\*x^2 + 121473790266\*x + 86386856771)\*sqrt(2\*x^2 - x + 3))/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)

**giac [B]** time = 0.42, size = 503, normalized size = 2.68

$$\frac{1}{84934656} \left( 8969667840 \sqrt{2} (16x^4 + 160x^3 + 600x^2 + 1000x + 625) \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8969688643 \sqrt{2} (16x^4 + 160x^3 + 600x^2 + 1000x + 625) \log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3} - 175 - 1000x^2 - 3396x + 1153}{49 - 231x + 25x^2}\right) + 48(2949120x^6 - 29270016x^5 + 468043776x^4 + 11761910072x^3 + 60528581892x^2 + 121473790266x + 86386856771)\sqrt{2x^2 - x + 3} \right) / (16x^4 + 160x^3 + 600x^2 + 1000x + 625)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^5,x, algorithm="g iac")

[Out]  $-1/42467328*\sqrt{2}*(8969688643*\log(12*\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)*\operatorname{sgn}(1/(2*x + 5)) + 8969667840*\log(\operatorname{abs}(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*\operatorname{sgn}(1/(2*x + 5)) - 8969667840*\log(\operatorname{abs}(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*\operatorname{sgn}(1/(2*x + 5)) + 12*(24*(1296*(29336*\operatorname{sgn}(1/(2*x + 5)))/(2*x + 5) - 42907*\operatorname{sgn}(1/(2*x + 5)))/(2*x + 5) + 39923563*\operatorname{sgn}(1/(2*x + 5)))/(2*x + 5) - 541312039*\operatorname{sgn}(1/(2*x + 5)))*\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 13824*(806241*(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*\operatorname{sgn}(1/(2*x + 5)) - 1152288*(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*\operatorname{sgn}(1/(2*x + 5)) - 957352*(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*\operatorname{sgn}(1/(2*x + 5)) + 1529280*(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*\operatorname{sgn}(1/(2*x + 5)) + 394431*(\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*\operatorname{sgn}(1/(2*x + 5)) - 620352*\operatorname{sgn}(1/(2*x + 5)))/((\sqrt{-11/(2*x + 5)} + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3$

**maple [A]** time = 0.02, size = 204, normalized size = 1.09

$$\frac{43256\sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{-11/(2x+5)}}{2}\right)}{2048} - \frac{8969688643\sqrt{2} \operatorname{arcsinh}\left(\frac{-11/(2x+5)}{\sqrt{-11/(2x+5)} + 36/(2x+5)^2 + 1}\right)}{42467328} + \frac{8969688643\sqrt{-11/(2x+5)} + 36/(2x+5)^2 + 1}{127401984} - \frac{8969688643(-11/(2x+5) + 36/(2x+5)^2 + 1)}{6879707136} - \frac{3667(-11/(2x+5) + 36/(2x+5)^2 + 1)}{30864(x+1/2)} - \frac{224815(-11/(2x+5) + 36/(2x+5)^2 + 1)}{1327104(x+1/2)} - \frac{14477995(-11/(2x+5) + 36/(2x+5)^2 + 1)}{9551488(x+1/2)} - \frac{593321753(4x-1)(-11/(2x+5) + 36/(2x+5)^2 + 1)}{6879707136} - \frac{593321753(-11/(2x+5) + 36/(2x+5)^2 + 1)}{3439853568(x+1/2)} - \frac{389975609(4x-1)\sqrt{-11/(2x+5)} + 36/(2x+5)^2 + 1}{127401984}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^5,x)

[Out]  $8969688643/127401984*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+8969688643/6879707136*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}+224815/1327104/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}-14477995/9551488/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}-593321753/6879707136*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}+593321753/3439853568/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}-389975609/127401984*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-8969688643/42467328*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)})/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-432565/2048*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima [A]** time = 1.04, size = 210, normalized size = 1.12

$$\frac{16966315}{47775744}(2x^2-x+3)^{3/2} - \frac{3667(2x^2-x+3)^{5/2}}{2304(16x^4+160x^3+600x^2+1000x+625)} + \frac{224815(2x^2-x+3)^{5/2}}{165888(8x^2+60x^2+150x+125)} - \frac{14477995(2x^2-x+3)^{5/2}}{23887872(4x^2+20x+25)} - \frac{389975609}{31850496}\sqrt{2x^2-x+3} - \frac{432565}{2048}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{8969688643}{42467328}\sqrt{2}\operatorname{arcsinh}\left(\frac{22\sqrt{23}x - 17\sqrt{23}}{23(2x+5)}\right) + \frac{779972021}{10616832}\sqrt{2x^2-x+3} + \frac{593321753(2x^2-x+3)^{3/2}}{9551488(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^5,x, algorithm="maxima")

[Out]  $16966315/47775744*(2*x^2 - x + 3)^{(3/2)} - 3667/2304*(2*x^2 - x + 3)^{(5/2)}/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3)^{(5/2)}/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)^{(5/2)}/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*\sqrt{2x^2-x+3} - 432565/2048*\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + 8969688643/42467328*\sqrt{2}\operatorname{arcsinh}\left(\frac{22\sqrt{23}x - 17\sqrt{23}}{23(2x+5)}\right) + 779972021/10616832*\sqrt{2x^2-x+3} + 593321753(2x^2-x+3)^{3/2}/9551488(2x+5)$

$$\frac{1}{2} / (4x^2 + 20x + 25) - \frac{389975609}{31850496} \sqrt{2x^2 - x + 3} x - \frac{432565}{2048 \sqrt{2}} \operatorname{arcsinh}\left(\frac{4}{23} \sqrt{23} x - \frac{1}{23} \sqrt{23}\right) + \frac{8969688643}{4246732} \sqrt{2} \operatorname{arcsinh}\left(\frac{22}{23} \sqrt{23} x / \operatorname{abs}(2x + 5) - \frac{17}{23} \sqrt{23} / \operatorname{abs}(2x + 5)\right) + \frac{779972021}{10616832} \sqrt{2x^2 - x + 3} + \frac{593321753}{95551488} (2x^2 - x + 3)^{3/2} / (2x + 5)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^5, x)

[Out] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*5, x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*5, x)

$$3.321 \quad \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx$$

Optimal. Leaf size=195

$$-\frac{3730507(2x^2-x+3)^{5/2}}{11943936(2x+5)^3} + \frac{158527(2x^2-x+3)^{5/2}}{165888(2x+5)^4} - \frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5} + \frac{(44773976x+246012435)(2x^2-x+3)^{3/2}}{95551488(2x+5)^2}$$

**Rubi [A]** time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{3730507(2x^2-x+3)^{5/2}}{11943936(2x+5)^3} + \frac{158527(2x^2-x+3)^{5/2}}{165888(2x+5)^4} - \frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5} + \frac{(44773976x+246012435)(2x^2-x+3)^{3/2}}{95551488(2x+5)^2} - \frac{(1028823716x+5658774871)\sqrt{2x^2-x+3}}{127401984(2x+5)} + \frac{70991525167 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2x^2-x+3}}\right)}{1528823808\sqrt{2}} - \frac{23775 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^6, x]

[Out] -((5658774871 + 1028823716\*x)\*Sqrt[3 - x + 2\*x^2])/(127401984\*(5 + 2\*x)) + ((246012435 + 44773976\*x)\*(3 - x + 2\*x^2)^(3/2))/(95551488\*(5 + 2\*x)^2) - (3667\*(3 - x + 2\*x^2)^(5/2))/(2880\*(5 + 2\*x)^5) + (158527\*(3 - x + 2\*x^2)^(5/2))/(165888\*(5 + 2\*x)^4) - (3730507\*(3 - x + 2\*x^2)^(5/2))/(11943936\*(5 + 2\*x)^3) - (23775\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(512\*Sqrt[2]) + (70991525167\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(1528823808\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x]
+ Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} - \frac{1}{360} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{60035}{16} - 661\right)}{(5+2x)} \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} + \frac{\int \frac{(3-x+2x^2)^{3/2}}{(5+2x)}}{165888} \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507}{11943} \\
&= \frac{(246012435 + 44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2} - \frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} \\
&= -\frac{(5658774871 + 1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435 + 44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2} \\
&= -\frac{(5658774871 + 1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435 + 44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2} \\
&= -\frac{(5658774871 + 1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435 + 44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2} \\
&= -\frac{(5658774871 + 1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435 + 44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2} \\
&= -\frac{(5658774871 + 1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435 + 44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 108, normalized size = 0.55

$$\frac{354957625835\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}(1592524800x^6-30496849920x^5-1023534029552x^4-7117092892448x^3-21590439797064x^2-30872393829992x-17093312738327)}{(2x+5)^5} - 354958848000\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{15288238080}$$

Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^6, x]

[Out] ((24\*sqrt[3 - x + 2\*x^2]\*(-17093312738327 - 30872393829992\*x - 21590439797064\*x^2 - 7117092892448\*x^3 - 1023534029552\*x^4 - 30496849920\*x^5 + 1592524800\*x^6))/(5 + 2\*x)^5 - 354958848000\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]] + 3

54957625835\* $\sqrt{2}$ \* $\text{ArcTanh}[(17 - 22*x)/(12*\sqrt{6 - 2*x + 4*x^2})]/152882$   
38080

**IntegrateAlgebraic [A]** time = 1.20, size = 134, normalized size = 0.69

$$\frac{23775 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{512\sqrt{2}} - \frac{70991525167 \tanh^{-1}\left(\frac{\sqrt{2x^2 - x + 3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{764411904\sqrt{2}} + \frac{\sqrt{2x^2 - x + 3} (1592524800x^6 - 30496849920x^5 - 1023534029552x^4 - 7117092892448x^3 - 21590439797064x^2 - 30872393829992x - 17093312738327)}{637009920(2x + 5)^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^6, x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-17093312738327 - 30872393829992\*x - 21590439797064\*x^2 - 7117092892448\*x^3 - 1023534029552\*x^4 - 30496849920\*x^5 + 1592524800\*x^6))/(637009920\*(5 + 2\*x)^5) - (70991525167\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(764411904\*Sqrt[2]) - (23775\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(512\*Sqrt[2])

**fricas [A]** time = 0.81, size = 213, normalized size = 1.09

$$\frac{35495884800\sqrt{2}(32x^5 + 400x^4 + 2000x^3 + 5000x^2 + 6250x + 3125)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) - 354957625835\sqrt{2}(32x^5 + 400x^4 + 2000x^3 + 5000x^2 + 6250x + 3125)\log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) - 1060x^2 + 1036x - 1153}{(4x^2 + 20x + 25)}\right) + 48(1592524800x^6 - 30496849920x^5 - 1023534029552x^4 - 7117092892448x^3 - 21590439797064x^2 - 30872393829992x - 17093312738327)\sqrt{2x^2 - x + 3}}{30576476160(32x^5 + 400x^4 + 2000x^3 + 5000x^2 + 6250x + 3125)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^6,x, algorithm="fricas")

[Out] 1/30576476160\*(354958848000\*sqrt(2)\*(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 354957625835\*sqrt(2)\*(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(1592524800\*x^6 - 30496849920\*x^5 - 1023534029552\*x^4 - 7117092892448\*x^3 - 21590439797064\*x^2 - 30872393829992\*x - 17093312738327)\*sqrt(2\*x^2 - x + 3))/(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125)

**giac [B]** time = 0.32, size = 406, normalized size = 2.08

$$\frac{1}{256}\sqrt{2x^2 - x + 3}(20x - 633) - \frac{23775}{1024}\sqrt{2}\log(-2\sqrt{2}(\sqrt{2}x - \sqrt{2x^2 - x + 3}) + 1) + \frac{70991525167}{3057647616}\sqrt{2}\log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) - 1060x^2 + 1036x - 1153}{(4x^2 + 20x + 25)}\right) + \frac{48(1592524800x^6 - 30496849920x^5 - 1023534029552x^4 - 7117092892448x^3 - 21590439797064x^2 - 30872393829992x - 17093312738327)\sqrt{2x^2 - x + 3}}{30576476160(32x^5 + 400x^4 + 2000x^3 + 5000x^2 + 6250x + 3125)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^6,x, algorithm="giac")

[Out] 1/256\*sqrt(2\*x^2 - x + 3)\*(20\*x - 633) - 23775/1024\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 70991525167/3057647616\*sqrt(2)\*log

$$\begin{aligned} & (\text{abs}(-2*\text{sqrt}(2)*x + \text{sqrt}(2) + 2*\text{sqrt}(2*x^2 - x + 3))) - 70991525167/3057647 \\ & 616*\text{sqrt}(2)*\log(\text{abs}(-2*\text{sqrt}(2)*x - 11*\text{sqrt}(2) + 2*\text{sqrt}(2*x^2 - x + 3))) - 1 \\ & /1274019840*\text{sqrt}(2)*(8281387393360*\text{sqrt}(2)*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3)) \\ & )^9 + 275661428628240*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3))^8 + 156038270334576 \\ & 0*\text{sqrt}(2)*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3))^7 + 4938646760855520*(\text{sqrt}(2)*x \\ & - \text{sqrt}(2*x^2 - x + 3))^6 - 9673562837036232*\text{sqrt}(2)*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 \\ & - x + 3))^5 - 30647310393849000*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3))^4 + 700 \\ & 60241036847960*\text{sqrt}(2)*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3))^3 - 97730658088823 \\ & 880*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3))^2 + 30180638363071845*\text{sqrt}(2)*(\text{sqrt}(2) \\ & )*x - \text{sqrt}(2*x^2 - x + 3)) - 7096913381268319)/(2*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - \\ & x + 3))^2 + 10*\text{sqrt}(2)*(\text{sqrt}(2)*x - \text{sqrt}(2*x^2 - x + 3)) - 11)^5 \end{aligned}$$

**maple [A]** time = 0.02, size = 225, normalized size = 1.15

$$\frac{23775 \sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{2}(x+1)}{2}\right)}{1024} + \frac{20995525167 \sqrt{2} \operatorname{arcsinh}\left(\frac{-(11+2)x+17}{2\sqrt{2}(x+1)}\right)}{3057647616} + \frac{70991525167 \sqrt{2} \operatorname{arcsinh}\left(\frac{-(11+2)\sqrt{x^2-x+3}}{2}\right)}{9172942848} + \frac{20995525167 \sqrt{2} \operatorname{arcsinh}\left(\frac{-(11+2)\sqrt{x^2-x+3}}{2}\right)}{495338913792} + \frac{3667 \sqrt{2} (11+2)(x+1)^2 - 19}{92160(x+1)} + \frac{158527 \sqrt{2} (11+2)(x+1)^2 - 19}{2654208(x+1)} + \frac{3730507 \sqrt{2} (11+2)(x+1)^2 - 19}{95551488(x+1)} + \frac{134077495 \sqrt{2} (11+2)(x+1)^2 - 19}{6879707136(x+1)} + \frac{4995707(44-1)\sqrt{2}(11+2)(x+1)^2 - 19}{495338913792} + \frac{4995707(44-1)\sqrt{2}(11+2)(x+1)^2 - 19}{247669456896(x+1)} + \frac{3086715581(44-1)\sqrt{2}(11+2)(x+1)^2 - 19}{9172942848}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)`

[Out]  $-70991525167/9172942848*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-70991525167/495338913792*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-3667/92160/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}+158527/2654208/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}-3730507/95551488/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}+134077495/6879707136/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}+4698578717/495338913792*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(3/2)}-4698578717/247669456896/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(5/2)}+3086715581/9172942848*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+70991525167/3057647616*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)})/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+23775/1024*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima [A]** time = 1.06, size = 251, normalized size = 1.29

$$\frac{134077495}{349853568} (2x^2-x+3)^{5/2} - \frac{3667(2x^2-x+3)^{3/2}}{2880(32x^5+400x^4+2000x^3+5000x^2+6250x+3125)} + \frac{158527(2x^2-x+3)^{5/2}}{165888(16x^4+160x^3+600x^2+1000x+625)} + \frac{3730507(2x^2-x+3)^{5/2}}{11943936(8x^3+60x^2+150x+125)} + \frac{134077495(2x^2-x+3)^{5/2}}{1719926784(4x^2+20x+25)} + \frac{308671581}{229323572} \sqrt{2x^2-x+3} + \frac{23775}{1024} \sqrt{2} \operatorname{arcsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{20995525167}{3057647616} \sqrt{2} \operatorname{arcsinh}\left(\frac{22\sqrt{23}x - 17\sqrt{23}}{23(2x+5)}\right) + \frac{4995707(44-1)\sqrt{2}(2x^2-x+3)^{1/2}}{76411904} - \frac{4995707(44-1)\sqrt{2}(2x^2-x+3)^{1/2}}{6879707136(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="maxima")`

[Out]  $-134077495/349853568*(2*x^2 - x + 3)^{(3/2)} - 3667/2880*(2*x^2 - x + 3)^{(5/2)}/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/165888*(2*x^2 - x + 3)^{(5/2)}/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 3730507/11943936*(2*x^2 - x + 3)^{(5/2)}/(8*x^3 + 60*x^2 + 150*x + 125) + 134077495/1719926784*(2*x^2 - x + 3)^{(5/2)}/(4*x^2 + 20*x + 25) + 3086715581/229323572*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*\operatorname{arcsinh}(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 70991525167/3057647616*sqrt(2)*\operatorname{arcsinh}(22/23*sqrt(23)*x/abs(2$



$*x + 5) - 17/23*\text{sqrt}(23)/\text{abs}(2*x + 5)) - 6173186729/764411904*\text{sqrt}(2*x^2 - x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^{(3/2)}/(2*x + 5)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^6,x)

[Out] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*6,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*6, x)

$$3.322 \quad \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx$$

Optimal. Leaf size=195

$$-\frac{14087245(2x^2-x+3)^{5/2}}{71663616(2x+5)^4} + \frac{182165(2x^2-x+3)^{5/2}}{248832(2x+5)^5} - \frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6} - \frac{(6793718806x+9802984711)(2x+5)^{3/2}}{13759414272(2x+5)^3}$$

**Rubi [A]** time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1650, 810, 812, 843, 619, 215, 724, 206}

$$-\frac{14087245(2x^2-x+3)^{5/2}}{71663616(2x+5)^4} + \frac{182165(2x^2-x+3)^{5/2}}{248832(2x+5)^5} - \frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6} - \frac{(6793718806x+9802984711)(2x^2-x+3)^{3/2}}{13759414272(2x+5)^3} + \frac{(27596573612x+151764102421)\sqrt{2x^2-x+3}}{55037657088(2x+5)} - \frac{1903976002333 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{660451885056\sqrt{2}} + \frac{369 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^7,x]

[Out] ((151764102421 + 27596573612\*x)\*Sqrt[3 - x + 2\*x^2])/(55037657088\*(5 + 2\*x)) - ((9802984711 + 6793718806\*x)\*(3 - x + 2\*x^2)^(3/2))/(13759414272\*(5 + 2\*x)^3) - (3667\*(3 - x + 2\*x^2)^(5/2))/(3456\*(5 + 2\*x)^6) + (182165\*(3 - x + 2\*x^2)^(5/2))/(248832\*(5 + 2\*x)^5) - (14087245\*(3 - x + 2\*x^2)^(5/2))/(71663616\*(5 + 2\*x)^4) + (369\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(128\*Sqrt[2]) - (1903976002333\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(660451885056\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[Rt[b, 2]\*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x)/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

### Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```

lRemainder[Pq, d + e*x, x], Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} - \frac{1}{432} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{68375}{16} - \frac{2808}{4}\right)}{(5+2x)^5} dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} + \int \frac{(3-x+2x^2)^{3/2}}{(5+2x)^4} dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} - \frac{14087245}{71663} \int \frac{(3-x+2x^2)^{3/2}}{(5+2x)^3} dx \\
&= -\frac{(9802984711 + 6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} - \frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^2} \\
&= \frac{(151764102421 + 27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711 + 6793718806x)\sqrt{3-x+2x^2}}{13759414272(5+2x)^3} \\
&= \frac{(151764102421 + 27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711 + 6793718806x)\sqrt{3-x+2x^2}}{13759414272(5+2x)^3} \\
&= \frac{(151764102421 + 27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711 + 6793718806x)\sqrt{3-x+2x^2}}{13759414272(5+2x)^3} \\
&= \frac{(151764102421 + 27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711 + 6793718806x)\sqrt{3-x+2x^2}}{13759414272(5+2x)^3}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 108, normalized size = 0.55

$$\frac{-1903976002333\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}(275188285440x^6+11854023276320x^5+103803827945872x^4+422554114856528x^3+910256842473992x^2+1011372787716826x+458411625354581)}{(2x+5)^6} + 1903958949888\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1320903770112}$$

Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^7, x]

[Out] ((24\*sqrt[3 - x + 2\*x^2]\*(458411625354581 + 1011372787716826\*x + 910256842473992\*x^2 + 422554114856528\*x^3 + 103803827945872\*x^4 + 11854023276320\*x^5 + 275188285440\*x^6))/(5 + 2\*x)^6 + 1903958949888\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]] - 1903976002333\*sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*sqrt[6 - 2\*x + 4\*x^2])])/1320903770112

**IntegrateAlgebraic [A]** time = 1.36, size = 134, normalized size = 0.69

$$\frac{369 \log(\sqrt{2} \sqrt{2x^2 - x + 3} - 4x + 1)}{128\sqrt{2}} + \frac{1903976002333 \tanh^{-1}\left(\frac{\sqrt{2x^2 - x + 3}}{3\sqrt{2}} + \frac{x}{3} + \frac{2}{3}\right)}{330225942528\sqrt{2}} + \frac{\sqrt{2x^2 - x + 3} (275188285440x^6 + 11854023276320x^5 + 103803827945872x^4 + 422554114856528x^3 + 910256842473992x^2 + 1011372787716826x + 458411625354581)}{55037657088(2x + 5)^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^7, x]

[Out] (sqrt[3 - x + 2\*x^2]\*(458411625354581 + 1011372787716826\*x + 910256842473992\*x^2 + 422554114856528\*x^3 + 103803827945872\*x^4 + 11854023276320\*x^5 + 275188285440\*x^6))/(55037657088\*(5 + 2\*x)^6) + (1903976002333\*ArcTanh[5/6 + x/3 - sqrt[3 - x + 2\*x^2]/(3\*sqrt[2])])/(330225942528\*sqrt[2]) + (369\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(128\*sqrt[2])

**fricas [A]** time = 0.79, size = 229, normalized size = 1.17

$$\frac{19039589888 \sqrt{2} (64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625) \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 1903976002333 \sqrt{2} (64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625) \log\left(\frac{2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1}{\sqrt{2}\sqrt{2x^2 - x + 3}}\right) + 48 (275188285440x^6 + 11854023276320x^5 + 103803827945872x^4 + 422554114856528x^3 + 910256842473992x^2 + 1011372787716826x + 458411625354581) \sqrt{2x^2 - x + 3}}{244807540224 (64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^7,x, algorithm="fricas")

[Out] 1/2641807540224\*(1903958949888\*sqrt(2)\*(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 1903976002333\*sqrt(2)\*(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(275188285440\*x^6 + 11854023276320\*x^5 + 103803827945872\*x^4 + 422554114856528\*x^3 + 910256842473992\*x^2 + 1011372787716826\*x + 458411625354581)\*sqrt(2\*x^2 - x + 3))/(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625)

**giac [B]** time = 0.34, size = 452, normalized size = 2.32

$$\frac{19039589888 \sqrt{2} (64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625) \log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 1903976002333 \sqrt{2} (64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625) \log\left(\frac{2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1}{\sqrt{2}\sqrt{2x^2 - x + 3}}\right) + 48 (275188285440x^6 + 11854023276320x^5 + 103803827945872x^4 + 422554114856528x^3 + 910256842473992x^2 + 1011372787716826x + 458411625354581) \sqrt{2x^2 - x + 3}}{244807540224 (64x^6 + 960x^5 + 6000x^4 + 20000x^3 + 37500x^2 + 37500x + 15625)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^7,x, algorithm="giac")

[Out] 369/256\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) - 1903976002333/1320903770112\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 1903976002333/1320903770112\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 5/64\*sqrt(2\*x^2 - x + 3) + 1/110075314176\*sqrt(2)\*(159278433934432\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^11 + 6347903280912544\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^10 + 48544526840833424\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^9 + 305716670132783088\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^8 + 88313821135911024\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^7 - 2423668581998843376\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^6 - 397211131697032056\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^5 + 11708897232532299576\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^4 - 12803484860728491138\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 + 12593033197867577234\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 - 3042533760672408875\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 589526263249780195)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^6

**maple [A]** time = 0.02, size = 246, normalized size = 1.26

$$\frac{369\sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{2}x-1}{\sqrt{2x^2-x+3}}\right) - 1903976002333\sqrt{2} \log\left(\frac{-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}}{-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}}\right) + \frac{5}{64}\sqrt{2x^2-x+3} + \frac{1}{110075314176}\sqrt{2}\left(159278433934432\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11} + 6347903280912544\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10} + 48544526840833424\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^9 + 305716670132783088\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^8 + 88313821135911024\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^7 - 2423668581998843376\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^6 - 397211131697032056\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5 + 11708897232532299576\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4 - 12803484860728491138\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3 + 12593033197867577234\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2 - 3042533760672408875\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right) + 589526263249780195\right)}{2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2 + 10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right) - 11}^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^7,x)

[Out] 1903976002333/3962711310336\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+1903976002333/213986410758144\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-3667/221184/(x+5/2)^6\*(-11\*x+2\*(x+5/2)^2-19/2)^(5/2)+182165/7962624/(x+5/2)^5\*(-11\*x+2\*(x+5/2)^2-19/2)^(5/2)-14087245/1146617856/(x+5/2)^4\*(-11\*x+2\*(x+5/2)^2-19/2)^(5/2)+149610673/41278242816/(x+5/2)^3\*(-11\*x+2\*(x+5/2)^2-19/2)^(5/2)-3607708597/2972033482752/(x+5/2)^2\*(-11\*x+2\*(x+5/2)^2-19/2)^(5/2)-125860542215/213986410758144\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+125860542215/106993205379072/(x+5/2)\*(-11\*x+2\*(x+5/2)^2-19/2)^(5/2)-82772668391/3962711310336\*(4\*x-1)\*(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-1903976002333/1320903770112\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))-369/256\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima [A]** time = 1.07, size = 297, normalized size = 1.52

$$\frac{1903976002333\sqrt{2} \operatorname{arcsinh}\left(\frac{\sqrt{2}x-1}{\sqrt{2x^2-x+3}}\right) + 1903976002333\sqrt{2} \log\left(\frac{-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}}{-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}}\right) - \frac{3667}{221184}\sqrt{2x^2-x+3}^{-6}\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{5/2} + \frac{182165}{7962624}\sqrt{2x^2-x+3}^{-5}\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{5/2} - \frac{14087245}{1146617856}\sqrt{2x^2-x+3}^{-4}\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{5/2} + \frac{149610673}{41278242816}\sqrt{2x^2-x+3}^{-3}\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{5/2} - \frac{3607708597}{2972033482752}\sqrt{2x^2-x+3}^{-2}\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{5/2} - \frac{125860542215}{213986410758144}\left(4x-1\right)\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{3/2} + \frac{125860542215}{106993205379072}\sqrt{2x^2-x+3}\left(-11x+2\sqrt{2x^2-x+3}-\frac{19}{2}\right)^{5/2} - \frac{82772668391}{3962711310336}\left(4x-1\right)\sqrt{2x^2-x+3}-\frac{1903976002333}{1320903770112}\sqrt{2} \operatorname{arctanh}\left(\frac{1}{12}\frac{-11x+17/2}{\sqrt{2x^2-x+3}}\right) - \frac{369}{256}\sqrt{2} \operatorname{arcsinh}\left(\frac{4}{23}\sqrt{2}x-\frac{1}{4}\right)}{2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2 + 10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right) - 11}^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^7,x, algorithm="maxima")

[Out] 3607708597/1486016741376\*(2\*x^2 - x + 3)^(3/2) - 3667/3456\*(2\*x^2 - x + 3)^(5/2)/(64\*x^6 + 960\*x^5 + 6000\*x^4 + 20000\*x^3 + 37500\*x^2 + 37500\*x + 15625) + 182165/248832\*(2\*x^2 - x + 3)^(5/2)/(32\*x^5 + 400\*x^4 + 2000\*x^3 + 5000\*x^2 + 6250\*x + 3125) - 14087245/71663616\*(2\*x^2 - x + 3)^(5/2)/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625) + 149610673/5159780352\*(2\*x^2 - x + 3)^(5/2)/(8\*x^3 + 60\*x^2 + 150\*x + 125) - 3607708597/743008370688\*(2\*x^2 - x + 3)^(5/2)/(4\*x^2 + 20\*x + 25) - 82772668391/990677827584\*sqrt(2\*x^2 - x + 3)\*x - 369/256\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 190397600233/1320903770112\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 165562389227/330225942528\*sqrt(2\*x^2 - x + 3) + 125860542215/2972033482752\*(2\*x^2 - x + 3)^(3/2)/(2\*x + 5)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^7,x)

[Out] int(((2\*x^2 - x + 3)^(3/2)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x + 5)^7, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-x+3)\*\*(3/2)\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*7,x)

[Out] Integral((2\*x\*\*2 - x + 3)\*\*(3/2)\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x + 5)\*\*7, x)

$$3.323 \quad \int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx$$

Optimal. Leaf size=195

$$\frac{1930441 (2x^2 - x + 3)^{5/2}}{13934592(2x + 5)^5} + \frac{114335 (2x^2 - x + 3)^{5/2}}{193536(2x + 5)^6} - \frac{3667 (2x^2 - x + 3)^{5/2}}{4032(2x + 5)^7} - \frac{(411822458x + 463558457) (2x^2 - x + 3)^{3/2}}{2293235712(2x + 5)^4}$$

**Rubi [A]** time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$\frac{1930441 (2x^2 - x + 3)^{5/2}}{13934592(2x + 5)^5} + \frac{114335 (2x^2 - x + 3)^{5/2}}{193536(2x + 5)^6} - \frac{3667 (2x^2 - x + 3)^{5/2}}{4032(2x + 5)^7} - \frac{(411822458x + 463558457) (2x^2 - x + 3)^{3/2}}{2293235712(2x + 5)^4} - \frac{(101679102454x + 146583836191)\sqrt{2x^2 - x + 3}}{440301256704(2x + 5)^2} + \frac{412760561351 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}}\right)}{5283615080448\sqrt{2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^8,x]

[Out] -((146583836191 + 101679102454\*x)\*Sqrt[3 - x + 2\*x^2])/(440301256704\*(5 + 2\*x)^2) - ((463558457 + 411822458\*x)\*(3 - x + 2\*x^2)^(3/2))/(2293235712\*(5 + 2\*x)^4) - (3667\*(3 - x + 2\*x^2)^(5/2))/(4032\*(5 + 2\*x)^7) + (114335\*(3 - x + 2\*x^2)^(5/2))/(193536\*(5 + 2\*x)^6) - (1930441\*(3 - x + 2\*x^2)^(5/2))/(13934592\*(5 + 2\*x)^5) - (5\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(64\*Sqrt[2]) + (412760561351\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(5283615080448\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724



```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x)/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m - 2*p) + e*f*(m + 2*p + 2)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] && !ILtQ[m + 2*p + 3, 0]
```

### Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^8} dx &= -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} - \frac{1}{504} \int \frac{(3-x+2x^2)^{3/2} \left(\frac{76715}{16} - \frac{1485}{2}\right)}{(5+2x)^7} dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} + \frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6} + \int \frac{(3-x+2x^2)^{3/2}}{(5+2x)^6} dx \\
&= -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} + \frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6} - \frac{1930441(3-x+2x^2)^{3/2}}{139344(5+2x)^5} \\
&= -\frac{(463558457 + 411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4} - \frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} \\
&= -\frac{(146583836191 + 101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457 + 411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4} \\
&= -\frac{(146583836191 + 101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457 + 411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4} \\
&= -\frac{(146583836191 + 101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457 + 411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4} \\
&= -\frac{(146583836191 + 101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457 + 411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4} \\
&= -\frac{(146583836191 + 101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558457 + 411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 108, normalized size = 0.55

$$\frac{2889323929457\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - \frac{24\sqrt{2x^2-x+3}(38463671680832x^6+402255822731712x^5+2069947287085104x^4+5966329646300704x^3+9976065367498188x^2+9065154700300572x+3479517268702637)}{(2x+5)^7} - 2889476997120\sqrt{2} \sinh^{-1}\left(\frac{1-x}{\sqrt{23}}\right)}{73970611126272}$$

Antiderivative was successfully verified.

[In] Integrate[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^8, x]

[Out] ((-24\*Sqrt[3 - x + 2\*x^2]\*(3479517268702637 + 9065154700300572\*x + 9976065367498188\*x^2 + 5966329646300704\*x^3 + 2069947287085104\*x^4 + 402255822731712\*x^5 + 38463671680832\*x^6))/(5 + 2\*x)^7 - 2889476997120\*Sqrt[2]\*ArcSinh[(1

- 4\*x)/Sqrt[23]] + 2889323929457\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])]/73970611126272

**IntegrateAlgebraic [A]** time = 1.89, size = 134, normalized size = 0.69

$$\frac{5 \log(2\sqrt{2}\sqrt{x^2-x+3}-4x+1)}{64\sqrt{2}} - \frac{412760561351 \tanh^{-1}\left(\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{2641807540224\sqrt{2}} + \frac{\sqrt{2x^2-x+3}(-38463671680832x^6 - 402255822731712x^5 - 2069947287085104x^4 - 5966329646300704x^3 - 9976065367498188x^2 - 9065154700300572x - 3479517268702637)}{3082108796928(2x+5)^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((3 - x + 2\*x^2)^(3/2)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(5 + 2\*x)^8,x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-3479517268702637 - 9065154700300572\*x - 9976065367498188\*x^2 - 5966329646300704\*x^3 - 2069947287085104\*x^4 - 402255822731712\*x^5 - 38463671680832\*x^6))/(3082108796928\*(5 + 2\*x)^7) - (412760561351\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(2641807540224\*Sqrt[2]) - (5\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(64\*Sqrt[2])

**fricas [A]** time = 2.06, size = 243, normalized size = 1.25

$$\frac{2889476997120\sqrt{2}\sqrt{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)/(5+2x)^8} + 2240x^6 + 16800x^5 + 70000x^4 + 175000x^3 + 262500x^2 + 218750x + 78125}{147941222252544}\log\left(\frac{\sqrt{2x^2-x+3}(22x-17) - 1060x^2 + 1036x - 1153}{(4x^2+20x+25)}\right) - \frac{48(38463671680832x^6 + 402255822731712x^5 + 2069947287085104x^4 + 5966329646300704x^3 + 9976065367498188x^2 + 9065154700300572x + 3479517268702637)\sqrt{2x^2-x+3}}{147941222252544(2x+5)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^8,x, algorithm="fricas")

[Out] 1/147941222252544\*(2889476997120\*sqrt(2)\*(128\*x^7 + 2240\*x^6 + 16800\*x^5 + 70000\*x^4 + 175000\*x^3 + 262500\*x^2 + 218750\*x + 78125)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 2889323929457\*sqrt(2)\*(128\*x^7 + 2240\*x^6 + 16800\*x^5 + 70000\*x^4 + 175000\*x^3 + 262500\*x^2 + 218750\*x + 78125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) - 48\*(38463671680832\*x^6 + 402255822731712\*x^5 + 2069947287085104\*x^4 + 5966329646300704\*x^3 + 9976065367498188\*x^2 + 9065154700300572\*x + 3479517268702637)\*sqrt(2\*x^2 - x + 3))/(128\*x^7 + 2240\*x^6 + 16800\*x^5 + 70000\*x^4 + 175000\*x^3 + 262500\*x^2 + 218750\*x + 78125)

**giac [B]** time = 0.33, size = 489, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2-x+3)^(3/2)\*(5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^8,x, algorithm="giac")

```
[Out] -5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 4127
60561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x -
11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(11218973984
12224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 -
17132661028483948080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130120947372
46112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 106515880136064432096*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x - sqrt(2*x^2
- x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 9229208
0735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 15161716093827501
349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^7
```

**maple [A]** time = 0.02, size = 267, normalized size = 1.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

```
[Out] -412760561351/31701690482688*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-412760561351/17
11891286065152*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/516096/(x+5/2)^7*(-11*x+
2*(x+5/2)^2-19/2)^(5/2)+114335/12386304/(x+5/2)^6*(-11*x+2*(x+5/2)^2-19/2)^(
5/2)-1930441/445906944/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+7861079/91
72942848/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-32967491/330225942528/(x+
5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+769352975/23776267862016/(x+5/2)^2*(-
11*x+2*(x+5/2)^2-19/2)^(5/2)+27452157541/1711891286065152*(4*x-1)*(-11*x+2*
(x+5/2)^2-19/2)^(3/2)-27452157541/855945643032576/(x+5/2)*(-11*x+2*(x+5/2)^
2-19/2)^(5/2)+17957520133/31701690482688*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(
1/2)+412760561351/10567230160896*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/
(-11*x+2*(x+5/2)^2-19/2)^(1/2))+5/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)
```

**maxima [B]** time = 1.07, size = 348, normalized size = 1.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] -769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3)^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/13934592*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230160896*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23776267862016*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{3/2} (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**8, x)
```

$$3.324 \quad \int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=120

$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4 - \frac{105}{128}\sqrt{2x^2-x+3}(2x+5)^3 + \frac{761}{256}\sqrt{2x^2-x+3}(2x+5)^2 - \frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}$$

**Rubi [A]** time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1653, 779, 619, 215}

$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4 - \frac{105}{128}\sqrt{2x^2-x+3}(2x+5)^3 + \frac{761}{256}\sqrt{2x^2-x+3}(2x+5)^2 - \frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048} - \frac{85429 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/Sqrt[3 - x + 2\*x^2], x]

[Out] (761\*(5 + 2\*x)^2\*Sqrt[3 - x + 2\*x^2])/256 - (105\*(5 + 2\*x)^3\*Sqrt[3 - x + 2\*x^2])/128 + ((5 + 2\*x)^4\*Sqrt[3 - x + 2\*x^2])/16 - ((19227 + 4676\*x)\*Sqrt[3 - x + 2\*x^2])/2048 - (85429\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4096\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 779

Int[((d\_.) + (e\_.)\*(x\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((b\*e\*g\*(p + 2) - c\*(e\*f + d\*g))\*(2\*p + 3) - 2\*c\*e\*g\*(p + 1)\*x\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c^2\*(p + 1)\*(2\*p + 3)), x] + Dist[(b^2\*e\*g\*(p + 2) - 2\*a\*c\*e\*g + c\*(2\*c\*d\*f - b\*(e\*f + d\*g))\*(2\*p + 3))/(2\*c^2\*(2\*p + 3)), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rule 1653

```

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
+ 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}} dx &= \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{1}{160} \int \frac{(5+2x)(-5055-4390x-5580x^2)}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{1}{160} \int \frac{(5+2x)(-5055-4390x-5580x^2)}{\sqrt{3-x+2x^2}} dx \\
&= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} \\
&= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} \\
&= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} \\
&= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 60, normalized size = 0.50

$$\frac{4\sqrt{2x^2-x+3}(2048x^4+7040x^3+352x^2-6916x+2973)-85429\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192}$$

Antiderivative was successfully verified.

[In] Integrate[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/Sqrt[3 - x + 2\*x^2], x]

[Out]  $(4\sqrt{3-x+2x^2})(2973-6916x+352x^2+7040x^3+2048x^4) - 85429\sqrt{2}\operatorname{ArcSinh}\left(\frac{1-4x}{\sqrt{23}}\right)/8192$

**IntegrateAlgebraic [A]** time = 0.60, size = 75, normalized size = 0.62

$$\frac{\sqrt{2x^2-x+3}(2048x^4+7040x^3+352x^2-6916x+2973)}{2048} - \frac{85429\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((5+2\*x)\*(2+x+3\*x^2-x^3+5\*x^4))/sqrt(3-x+2\*x^2),x)

[Out]  $(\sqrt{3-x+2x^2})(2973-6916x+352x^2+7040x^3+2048x^4)/2048 - (85429\log[1-4x+2\sqrt{2}\sqrt{3-x+2x^2}])/(4096\sqrt{2})$

**fricas [A]** time = 0.71, size = 73, normalized size = 0.61

$$\frac{1}{2048}(2048x^4+7040x^3+352x^2-6916x+2973)\sqrt{2x^2-x+3} + \frac{85429}{16384}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out]  $1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*\operatorname{sqrt}(2*x^2 - x + 3) + 85429/16384*\operatorname{sqrt}(2)*\log(-4*\operatorname{sqrt}(2)*\operatorname{sqrt}(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)$

**giac [A]** time = 0.19, size = 68, normalized size = 0.57

$$\frac{1}{2048}(4(8(4(16x+55)x+11)x-1729)x+2973)\sqrt{2x^2-x+3} - \frac{85429}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out]  $1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*\operatorname{sqrt}(2*x^2 - x + 3) - 85429/8192*\operatorname{sqrt}(2)*\log(-2*\operatorname{sqrt}(2)*(\operatorname{sqrt}(2)*x - \operatorname{sqrt}(2*x^2 - x + 3)) + 1)$

**maple [A]** time = 0.01, size = 95, normalized size = 0.79

$$\sqrt{2x^2-x+3}x^4 + \frac{55\sqrt{2x^2-x+3}x^3}{16} + \frac{11\sqrt{2x^2-x+3}x^2}{64} - \frac{1729\sqrt{2x^2-x+3}x}{512} + \frac{85429\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192} + \frac{2973\sqrt{2x^2-x+3}}{2048}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)`

[Out]  $x^4(2x^2-x+3)^{1/2} + 55/16x^3(2x^2-x+3)^{1/2} + 11/64x^2(2x^2-x+3)^{1/2} - 1729/512x(2x^2-x+3)^{1/2} + 2973/2048(2x^2-x+3)^{1/2} + 85429/8192 \cdot 2^{1/2} \operatorname{arcsinh}(4/23 \cdot 23^{1/2} \cdot (x-1/4))$

**maxima** [A] time = 0.97, size = 96, normalized size = 0.80

$$\sqrt{2x^2-x+3}x^4 + \frac{55}{16}\sqrt{2x^2-x+3}x^3 + \frac{11}{64}\sqrt{2x^2-x+3}x^2 - \frac{1729}{512}\sqrt{2x^2-x+3}x + \frac{85429}{8192}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2973}{2048}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $\sqrt{2x^2-x+3}x^4 + 55/16\sqrt{2x^2-x+3}x^3 + 11/64\sqrt{2x^2-x+3}x^2 - 1729/512\sqrt{2x^2-x+3}x + 85429/8192\sqrt{2}\operatorname{arcsinh}(1/23\sqrt{23}(4x-1)) + 2973/2048\sqrt{2x^2-x+3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{\sqrt{2x^2-x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x+5)*(x+3*x^2-x^3+5*x^4+2))/(2*x^2-x+3)^(1/2),x)`

[Out] `int(((2*x+5)*(x+3*x^2-x^3+5*x^4+2))/(2*x^2-x+3)^(1/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{\sqrt{2x^2-x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)`

[Out] `Integral((2*x+5)*(5*x**4-x**3+3*x**2+x+2)/sqrt(2*x**2-x+3),x)`

$$3.325 \quad \int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=101

$$\frac{19}{96}\sqrt{2x^2-x+3}x^2 - \frac{409}{768}\sqrt{2x^2-x+3}x - \frac{505\sqrt{2x^2-x+3}}{1024} + \frac{5}{8}\sqrt{2x^2-x+3}x^3 - \frac{6863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

**Rubi [A]** time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}x^3 + \frac{19}{96}\sqrt{2x^2-x+3}x^2 - \frac{409}{768}\sqrt{2x^2-x+3}x - \frac{505\sqrt{2x^2-x+3}}{1024} - \frac{6863 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/Sqrt[3 - x + 2\*x^2], x]

[Out] (-505\*Sqrt[3 - x + 2\*x^2])/1024 - (409\*x\*Sqrt[3 - x + 2\*x^2])/768 + (19\*x^2\*Sqrt[3 - x + 2\*x^2])/96 + (5\*x^3\*Sqrt[3 - x + 2\*x^2])/8 - (6863\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(2048\*Sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x +

$c*x^2)^{(p+1)}/(c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a+b*x+c*x^2)^p \text{ExpandToSum}[c*(q+2*p+1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /;$  FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && !LeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}} dx &= \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8} \int \frac{16+8x-21x^2+\frac{19x^3}{2}}{\sqrt{3-x+2x^2}} dx \\ &= \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48} \int \frac{96-9x-\frac{409x^2}{4}}{\sqrt{3-x+2x^2}} dx \\ &= -\frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192} \int \frac{\frac{2763}{4}}{\sqrt{3-x+2x^2}} dx \\ &= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} \\ &= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} \\ &= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}(1920x^3+608x^2-1636x-1515)-20589\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/Sqrt[3 - x + 2\*x^2], x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(-1515 - 1636\*x + 608\*x^2 + 1920\*x^3) - 20589\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/12288

**IntegrateAlgebraic [A]** time = 0.54, size = 70, normalized size = 0.69

$$\frac{\sqrt{2x^2-x+3}(1920x^3+608x^2-1636x-1515)}{3072} - \frac{6863\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/Sqrt[3 - x + 2\*x^2],x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-1515 - 1636\*x + 608\*x^2 + 1920\*x^3))/3072 - (6863\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(2048\*Sqrt[2])

**fricas** [A] time = 0.68, size = 68, normalized size = 0.67

$$\frac{1}{3072} (1920x^3 + 608x^2 - 1636x - 1515)\sqrt{2x^2 - x + 3} + \frac{6863}{8192} \sqrt{2} \log\left(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/3072\*(1920\*x^3 + 608\*x^2 - 1636\*x - 1515)\*sqrt(2\*x^2 - x + 3) + 6863/8192\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25)

**giac** [A] time = 0.20, size = 63, normalized size = 0.62

$$\frac{1}{3072} (4(8(60x + 19)x - 409)x - 1515)\sqrt{2x^2 - x + 3} - \frac{6863}{4096} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out] 1/3072\*(4\*(8\*(60\*x + 19)\*x - 409)\*x - 1515)\*sqrt(2\*x^2 - x + 3) - 6863/4096\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1)

**maple** [A] time = 0.01, size = 79, normalized size = 0.78

$$\frac{5\sqrt{2x^2 - x + 3}x^3}{8} + \frac{19\sqrt{2x^2 - x + 3}x^2}{96} - \frac{409\sqrt{2x^2 - x + 3}x}{768} + \frac{6863\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096} - \frac{505\sqrt{2x^2 - x + 3}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(1/2),x)

[Out] 5/8\*(2\*x^2-x+3)^(1/2)\*x^3+19/96\*(2\*x^2-x+3)^(1/2)\*x^2-409/768\*(2\*x^2-x+3)^(1/2)\*x-505/1024\*(2\*x^2-x+3)^(1/2)+6863/4096\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))

**maxima** [A] time = 0.96, size = 80, normalized size = 0.79

$$\frac{5}{8} \sqrt{2x^2 - x + 3}x^3 + \frac{19}{96} \sqrt{2x^2 - x + 3}x^2 - \frac{409}{768} \sqrt{2x^2 - x + 3}x + \frac{6863}{4096} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23}(4x - 1)\right) - \frac{505}{1024} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 5/8\*sqrt(2\*x^2 - x + 3)\*x^3 + 19/96\*sqrt(2\*x^2 - x + 3)\*x^2 - 409/768\*sqrt(2\*x^2 - x + 3)\*x + 6863/4096\*sqrt(2)\*arcsinh(1/23\*sqrt(23)\*(4\*x - 1)) - 505/1024\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/(2\*x^2 - x + 3)^(1/2),x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/(2\*x^2 - x + 3)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{\sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/sqrt(2\*x\*\*2 - x + 3), x)

$$3.326 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=126

$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2 - \frac{337}{192}\sqrt{2x^2-x+3}(2x+5) + \frac{1669}{128}\sqrt{2x^2-x+3} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}} + \frac{9657 \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}$$

**Rubi [A]** time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2 - \frac{337}{192}\sqrt{2x^2-x+3}(2x+5) + \frac{1669}{128}\sqrt{2x^2-x+3} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}} + \frac{9657 \operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*Sqrt[3 - x + 2\*x^2]), x]

[Out] (1669\*Sqrt[3 - x + 2\*x^2])/128 - (337\*(5 + 2\*x)\*Sqrt[3 - x + 2\*x^2])/192 + (5\*(5 + 2\*x)^2\*Sqrt[3 - x + 2\*x^2])/48 + (9657\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(256\*Sqrt[2]) - (3667\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(96\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}} dx &= \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{1}{96} \int \frac{-2183-3054x-4092x^2-2696x^3}{(5+2x)\sqrt{3-x+2x^2}} dx \\
&= -\frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{\int \frac{24504+128736x+160224x^2}{(5+2x)\sqrt{3-x+2x^2}}}{3072} \\
&= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \dots \\
&= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} - \frac{9}{2} \\
&= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 81, normalized size = 0.64

$$\frac{4\sqrt{2x^2-x+3}(160x^2-548x+2637) - 29336\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 28971\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1536}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (4\*Sqrt[3 - x + 2\*x^2]\*(2637 - 548\*x + 160\*x^2) + 28971\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] - 29336\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/1536

**IntegrateAlgebraic [A]** time = 0.52, size = 107, normalized size = 0.85

$$\frac{1}{384}\sqrt{2x^2-x+3}(160x^2-548x+2637) + \frac{9657 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{256\sqrt{2}} + \frac{3667 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{48\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*Sqrt[3 - x + 2\*x^2]),x]



[Out]  $(\sqrt{3-x+2x^2}*(2637-548x+160x^2))/384 + (3667*\text{ArcTanh}[5/6 + x/3 - \sqrt{3-x+2x^2}/(3*\sqrt{2})])/(48*\sqrt{2}) + (9657*\text{Log}[1-4x+2*\sqrt{2}*\sqrt{3-x+2x^2}])/(256*\sqrt{2})$

**fricas** [A] time = 0.84, size = 115, normalized size = 0.91

$$\frac{1}{384}(160x^2 - 548x + 2637)\sqrt{2x^2 - x + 3} + \frac{9657}{1024}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right) + \frac{3667}{384}\sqrt{2}\log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) + 1060x^2 - 1036x + 1153}{4x^2 + 20x + 25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")`

[Out]  $1/384*(160*x^2 - 548*x + 2637)*\text{sqrt}(2*x^2 - x + 3) + 9657/1024*\text{sqrt}(2)*\log(4*\text{sqrt}(2)*\text{sqrt}(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*\text{sqrt}(2)*\log(-(24*\text{sqrt}(2)*\text{sqrt}(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))$

**giac** [A] time = 0.23, size = 119, normalized size = 0.94

$$\frac{1}{384}(4(40x - 137)x + 2637)\sqrt{2x^2 - x + 3} + \frac{9657}{512}\sqrt{2}\log(-4\sqrt{2}x + \sqrt{2} + 4\sqrt{2x^2 - x + 3}) - \frac{3667}{192}\sqrt{2}\log(|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}|) + \frac{3667}{192}\sqrt{2}\log(|-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="giac")`

[Out]  $1/384*(4*(40*x - 137)*x + 2637)*\text{sqrt}(2*x^2 - x + 3) + 9657/512*\text{sqrt}(2)*\log(-4*\text{sqrt}(2)*x + \text{sqrt}(2) + 4*\text{sqrt}(2*x^2 - x + 3)) - 3667/192*\text{sqrt}(2)*\log(\text{abs}(-2*\text{sqrt}(2)*x + \text{sqrt}(2) + 2*\text{sqrt}(2*x^2 - x + 3))) + 3667/192*\text{sqrt}(2)*\log(\text{abs}(-2*\text{sqrt}(2)*x - 11*\text{sqrt}(2) + 2*\text{sqrt}(2*x^2 - x + 3)))$

**maple** [A] time = 0.01, size = 92, normalized size = 0.73

$$\frac{5\sqrt{2x^2 - x + 3}x^2}{12} - \frac{137\sqrt{2x^2 - x + 3}x}{96} - \frac{9657\sqrt{2}\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{512} - \frac{3667\sqrt{2}\text{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{192} + \frac{879\sqrt{2x^2 - x + 3}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)`

[Out]  $5/12*(2*x^2-x+3)^(1/2)*x^2-137/96*(2*x^2-x+3)^(1/2)*x+879/128*(2*x^2-x+3)^(1/2)-9657/512*2^(1/2)*\text{arcsinh}(4/23*23^(1/2)*(x-1/4))-3667/192*2^(1/2)*\text{arctanh}(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))$

**maxima** [A] time = 0.98, size = 99, normalized size = 0.79

$$\frac{5}{12} \sqrt{2x^2 - x + 3} x^2 - \frac{137}{96} \sqrt{2x^2 - x + 3} x - \frac{9657}{512} \sqrt{2} \operatorname{arsinh}\left(\frac{4}{23} \sqrt{23} x - \frac{1}{23} \sqrt{23}\right) + \frac{3667}{192} \sqrt{2} \operatorname{arsinh}\left(\frac{22 \sqrt{23} x}{23|2x+5|} - \frac{17 \sqrt{23}}{23|2x+5|}\right) + \frac{879}{128} \sqrt{2x^2 - x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)/(2\*x^2-x+3)^(1/2),x, algorithm="maxima")

[Out] 5/12\*sqrt(2\*x^2 - x + 3)\*x^2 - 137/96\*sqrt(2\*x^2 - x + 3)\*x - 9657/512\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 3667/192\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 879/128\*sqrt(2\*x^2 - x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5) \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)\*(2\*x^2 - x + 3)^(1/2)),x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)\*(2\*x^2 - x + 3)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5) \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)/(2\*x\*\*2-x+3)\*\*(1/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*sqrt(2\*x\*\*2 - x + 3)), x)

$$3.327 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2 \sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=126

$$\frac{5}{32} \sqrt{2x^2 - x + 3} (2x+5) - \frac{243}{64} \sqrt{2x^2 - x + 3} - \frac{3667 \sqrt{2x^2 - x + 3}}{576(2x+5)} + \frac{158527 \tanh^{-1} \left( \frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}} \right)}{6912\sqrt{2}} - \frac{2943 \sinh^{-1}}{128\sqrt{2}}$$

**Rubi [A]** time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{5}{32} \sqrt{2x^2 - x + 3} (2x+5) - \frac{243}{64} \sqrt{2x^2 - x + 3} - \frac{3667 \sqrt{2x^2 - x + 3}}{576(2x+5)} + \frac{158527 \tanh^{-1} \left( \frac{17-22x}{12\sqrt{2} \sqrt{2x^2-x+3}} \right)}{6912\sqrt{2}} - \frac{2943 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (-243\*Sqrt[3 - x + 2\*x^2])/64 - (3667\*Sqrt[3 - x + 2\*x^2])/(576\*(5 + 2\*x)) + (5\*(5 + 2\*x)\*Sqrt[3 - x + 2\*x^2])/32 - (2943\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(128\*Sqrt[2]) + (158527\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(6912\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}} dx &= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{1}{72} \int \frac{\frac{12007}{16} - 1323x + 486x^2 - 180x^3}{(5+2x)\sqrt{3-x+2x^2}} dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{30314-27216x+34992x^2}{(5+2x)\sqrt{3-x+2x^2}} dx}{2304} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{41747}{(5+2x)}}{1} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{2943}{128} \int \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{158527}{576} \\
&= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{2943}{128} \int
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 88, normalized size = 0.70

$$\frac{48\sqrt{2x^2-x+3}(180x^2-1287x-6176)}{2x+5} + 158527\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - 158922\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

13824

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*Sqrt[3 - x + 2\*x^2]), x]

[Out] ((48\*Sqrt[3 - x + 2\*x^2]\*(-6176 - 1287\*x + 180\*x^2))/(5 + 2\*x) - 158922\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] + 158527\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/13824

**IntegrateAlgebraic [A]** time = 0.77, size = 114, normalized size = 0.90

$$\frac{\sqrt{2x^2-x+3}(180x^2-1287x-6176)}{288(2x+5)} - \frac{2943 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{128\sqrt{2}} - \frac{158527 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{3456\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*Sqrt[3 - x + 2\*x^2]), x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-6176 - 1287\*x + 180\*x^2))/((288\*(5 + 2\*x)) - (158527\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(3456\*Sqrt[2]) - (294\*3\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(128\*Sqrt[2])

**fricas** [A] time = 1.32, size = 133, normalized size = 1.06

$$\frac{158922\sqrt{2}(2x+5)\log(-4\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+158527\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+96(180x^2-1287x-6176)\sqrt{2x^2-x+3}}{27648(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/27648\*(158922\*sqrt(2)\*(2\*x + 5)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 158527\*sqrt(2)\*(2\*x + 5)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) + 96\*(180\*x^2 - 1287\*x - 6176)\*sqrt(2\*x^2 - x + 3))/(2\*x + 5)

**giac** [B] time = 0.40, size = 339, normalized size = 2.69

$$\frac{1}{13824}\sqrt{2}\left(\frac{158527\log\left(12\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{158922\log\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}+1\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{158922\log\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}-1\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{44004\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{108\left(3393\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^3-4896\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^2-743\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}-\frac{4458}{2x+5}+2256\right)}{\left(\left(\sqrt{\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^3-1\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] 1/13824\*sqrt(2)\*(158527\*log(12\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 72/(2\*x + 5) - 11)/sgn(1/(2\*x + 5)) + 158922\*log(abs(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5) + 1))/sgn(1/(2\*x + 5)) - 158922\*log(abs(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5) - 1))/sgn(1/(2\*x + 5)) - 44004\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1)/sgn(1/(2\*x + 5)) + 108\*(3393\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^3 - 4896\*(sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^2 - 743\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) - 4458/(2\*x + 5) + 2256)/(((sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 6/(2\*x + 5))^3 - 1)^2\*sgn(1/(2\*x + 5))))

**maple** [A] time = 0.01, size = 96, normalized size = 0.76

$$\frac{5\sqrt{2x^2-x+3}}{16} + \frac{2943\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256} + \frac{158527\sqrt{2}\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{13824} - \frac{193\sqrt{2x^2-x+3}}{64} - \frac{3667\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{1152\left(x+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)`

[Out]  $5/16*(2*x^2-x+3)^(1/2)*x-193/64*(2*x^2-x+3)^(1/2)+2943/256*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+158527/13824*2^(1/2)*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))$

**maxima** [A] time = 0.98, size = 103, normalized size = 0.82

$$\frac{5}{16}\sqrt{2x^2-x+3} + \frac{2943}{256}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{158527}{13824}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{193}{64}\sqrt{2x^2-x+3} - \frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $5/16*\operatorname{sqrt}(2*x^2 - x + 3)*x + 2943/256*\operatorname{sqrt}(2)*\operatorname{arcsinh}(4/23*\operatorname{sqrt}(23)*x - 1/23*\operatorname{sqrt}(23)) - 158527/13824*\operatorname{sqrt}(2)*\operatorname{arcsinh}(22/23*\operatorname{sqrt}(23)*x/\operatorname{abs}(2*x + 5) - 17/23*\operatorname{sqrt}(23)/\operatorname{abs}(2*x + 5)) - 193/64*\operatorname{sqrt}(2*x^2 - x + 3) - 3667/576*\operatorname{sqrt}(2*x^2 - x + 3)/(2*x + 5)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)`

[Out] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)`

[Out] `Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x + 3)), x)`

$$3.328 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3 \sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=128

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)} - \frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2} + \frac{5}{16}\sqrt{2x^2-x+3} - \frac{1546507 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}} + \frac{149 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

**Rubi [A]** time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)} - \frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2} + \frac{5}{16}\sqrt{2x^2-x+3} - \frac{1546507 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}} + \frac{149 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (5\*Sqrt[3 - x + 2\*x^2])/16 - (3667\*Sqrt[3 - x + 2\*x^2])/((1152\*(5 + 2\*x)^2) + (92239\*Sqrt[3 - x + 2\*x^2])/(27648\*(5 + 2\*x)) + (149\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32\*Sqrt[2]) - (1546507\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(331776\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2



$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}} dx &= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} - \frac{1}{144} \int \frac{\frac{20347}{16} - \frac{6917x}{4} + 972x^2 - 360x^3}{(5+2x)^2\sqrt{3-x+2x^2}} dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{647841}{16} - 67392x + 12960x^2}{(5+2x)\sqrt{3-x+2x^2}} dx}{10368} \\
&= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{777441}{2} - 772416x}{(5+2x)\sqrt{3-x+2x^2}}}{82944} \\
&= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{149}{32} \int \frac{1}{\sqrt{3-x+2x^2}} \\
&= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{1546507 \text{Subst}}{663552} \\
&= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{149 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 88, normalized size = 0.69

$$\frac{24\sqrt{2x^2-x+3}(34560x^2+357278x+589187)}{(2x+5)^2} - 1546507\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 1544832\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

663552

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*Sqrt[3 - x + 2\*x^2]), x]

[Out] ((24\*Sqrt[3 - x + 2\*x^2]\*(589187 + 357278\*x + 34560\*x^2))/(5 + 2\*x)^2 + 1544832\*Sqrt[2]\*ArcSinh[(1 - 4\*x)/Sqrt[23]] - 1546507\*Sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/663552

**IntegrateAlgebraic [A]** time = 0.70, size = 114, normalized size = 0.89

$$\frac{\sqrt{2x^2-x+3}(34560x^2+357278x+589187)}{27648(2x+5)^2} + \frac{149 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{32\sqrt{2}} + \frac{1546507 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{165888\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*sqrt[3 - x + 2\*x^2]), x]

[Out] (sqrt[3 - x + 2\*x^2]\*(589187 + 357278\*x + 34560\*x^2))/(27648\*(5 + 2\*x)^2) + (1546507\*ArcTanh[5/6 + x/3 - sqrt[3 - x + 2\*x^2]/(3\*sqrt[2])])/(165888\*sqrt[2]) + (149\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(32\*sqrt[2])

**fricas** [A] time = 1.51, size = 149, normalized size = 1.16

$$\frac{1544832\sqrt{2}(4x^2+20x+25)\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+1546507\sqrt{2}(4x^2+20x+25)\log\left(\frac{-24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+48(34560x^2+357278x+589187)\sqrt{2x^2-x+3}}{1327104(4x^2+20x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/1327104\*(1544832\*sqrt(2)\*(4\*x^2 + 20\*x + 25)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 1546507\*sqrt(2)\*(4\*x^2 + 20\*x + 25)\*log(-24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(34560\*x^2 + 357278\*x + 589187)\*sqrt(2\*x^2 - x + 3))/(4\*x^2 + 20\*x + 25)

**giac** [B] time = 0.26, size = 248, normalized size = 1.94

$$\frac{149}{64}\sqrt{2}\log(-2\sqrt{2}\sqrt{2x^2-x+3})+1-\frac{1546507}{663552}\sqrt{2}\log(-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3})+\frac{1546507}{663552}\sqrt{2}\log(-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3})+\frac{5}{16}\sqrt{2x^2-x+3}+\frac{\sqrt{2}(2381290\sqrt{2}\sqrt{2x^2-x+3}+16628406(\sqrt{2x^2-x+3})^2-25697445\sqrt{2}\sqrt{2x^2-x+3}+16720645)}{55296(2(\sqrt{2x^2-x+3})^2+10\sqrt{2}\sqrt{2x^2-x+3}-11)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3/(2\*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] 149/64\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) - 1546507/663552\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 1546507/663552\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 5/16\*sqrt(2\*x^2 - x + 3) + 1/55296\*sqrt(2)\*(2381290\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 + 16628406\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 - 25697445\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 16720645)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^2

**maple** [A] time = 0.01, size = 102, normalized size = 0.80

$$-\frac{149\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}-\frac{1546507\sqrt{2}\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{663552}+\frac{5\sqrt{2x^2-x+3}}{16}+\frac{92239\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{55296\left(x+\frac{5}{2}\right)}-\frac{3667\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{4608\left(x+\frac{5}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2), x)`

[Out]  $5/16*(2*x^2-x+3)^{(1/2)}-149/64*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))+92239/55296/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-1546507/663552*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)}/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)})-3667/4608/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}$

**maxima** [A] time = 0.98, size = 114, normalized size = 0.89

$$-\frac{149}{64}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{1546507}{663552}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{1152(4x^2+20x+25)}+\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2), x, algorithm="maxima")`

[Out]  $-149/64*\sqrt{2}*\operatorname{arcsinh}(4/23*\sqrt{23}*x - 1/23*\sqrt{23}) + 1546507/663552*\sqrt{2}*\operatorname{arcsinh}(22/23*\sqrt{23}*x/\operatorname{abs}(2*x + 5) - 17/23*\sqrt{23}/\operatorname{abs}(2*x + 5)) + 5/16*\sqrt{2*x^2 - x + 3} - 3667/1152*\sqrt{2*x^2 - x + 3}/(4*x^2 + 20*x + 25) + 92239/27648*\sqrt{2*x^2 - x + 3}/(2*x + 5)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^3 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)`

[Out] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^3 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2), x)`

[Out] `Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x + 3)), x)`

$$3.329 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4 \sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=135

$$\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)} + \frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2} - \frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3} + \frac{22389491 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

**Rubi [A]** time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {1650, 843, 619, 215, 724, 206}

$$\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)} + \frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2} - \frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3} + \frac{22389491 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (-3667\*Sqrt[3 - x + 2\*x^2])/(1728\*(5 + 2\*x)^3) + (394907\*Sqrt[3 - x + 2\*x^2])/(248832\*(5 + 2\*x)^2) - (3163415\*Sqrt[3 - x + 2\*x^2])/(5971968\*(5 + 2\*x)) - (5\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(16\*Sqrt[2]) + (22389491\*ArcTanh[(17 - 2\*2\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(71663616\*Sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}} dx &= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} - \frac{1}{216} \int \frac{\frac{28687}{16} - \frac{4271x}{2} + 1458x^2 - 540x^3}{(5+2x)^3\sqrt{3-x+2x^2}} dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} + \frac{\int \frac{\frac{1464275}{16} - \frac{413797x}{4} + 38880x^2}{(5+2x)^2\sqrt{3-x+2x^2}} dx}{31104} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{\int \frac{\frac{1118}{5}}{(5+2x)} dx}{16} \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} + \frac{5}{16} \int \frac{1}{5+2x} dx \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} + \frac{22389491}{143327232} \operatorname{ArcTan}\left[\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right] \\
&= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} - \frac{5 \operatorname{ArcSinh}\left[\frac{1-4x}{\sqrt{23}}\right]}{143327232}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 88, normalized size = 0.65

$$\frac{-\frac{24\sqrt{2x^2-x+3}(12653660x^2+44312764x+44369687)}{(2x+5)^3} + 22389491\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - 22394880\sqrt{2} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{143327232}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*sqrt[3 - x + 2\*x^2]), x]

[Out] ((-24\*sqrt[3 - x + 2\*x^2]\*(44369687 + 44312764\*x + 12653660\*x^2))/(5 + 2\*x)^3 - 22394880\*sqrt[2]\*ArcSinh[(1 - 4\*x)/sqrt[23]] + 22389491\*sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*sqrt[6 - 2\*x + 4\*x^2])])/143327232

**IntegrateAlgebraic [A]** time = 0.76, size = 114, normalized size = 0.84

$$\frac{\sqrt{2x^2-x+3}(-12653660x^2-44312764x-44369687)}{5971968(2x+5)^3} - \frac{5 \log(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1)}{16\sqrt{2}} - \frac{22389491 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{35831808\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*Sqrt[3 - x + 2\*x^2]), x]

[Out] ((-44369687 - 44312764\*x - 12653660\*x^2)\*Sqrt[3 - x + 2\*x^2])/((5971968\*(5 + 2\*x)^3 - (22389491\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])]))/(35831808\*Sqrt[2]) - (5\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(16\*Sqrt[2]))

**fricas** [A] time = 1.70, size = 163, normalized size = 1.21

$$\frac{22394880\sqrt{2}(8x^3 + 60x^2 + 150x + 125)\log(-4\sqrt{2}x^2 - x + 3(4x - 1) - 32x^2 + 16x - 25) + 22389491\sqrt{2}(8x^3 + 60x^2 + 150x + 125)\log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) - 1060x^2 + 1036x - 1153}{4x^2 + 20x + 25}\right) - 48(12653660x^2 + 44312764x + 44369687)\sqrt{2x^2 - x + 3}}{286654464(8x^3 + 60x^2 + 150x + 125)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(1/2), x, algorithm="fricas")

[Out] 1/286654464\*(22394880\*sqrt(2)\*(8\*x^3 + 60\*x^2 + 150\*x + 125)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 22389491\*sqrt(2)\*(8\*x^3 + 60\*x^2 + 150\*x + 125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) - 48\*(12653660\*x^2 + 44312764\*x + 44369687)\*sqrt(2\*x^2 - x + 3))/(8\*x^3 + 60\*x^2 + 150\*x + 125)

**giac** [B] time = 0.26, size = 285, normalized size = 2.11

$$\frac{\frac{5}{32}\sqrt{2}\log(-2\sqrt{2}(\sqrt{2x^2-x+3}+1)) + \frac{22389491}{143327232}\sqrt{2}\log(-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2-x+3}) - \frac{22389491}{143327232}\sqrt{2}\log(-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2-x+3}) - \frac{\sqrt{2}(215012404\sqrt{2}(\sqrt{2x^2-x+3}) + 3010410772(\sqrt{2x^2-x+3}) + 2740802468\sqrt{2}(\sqrt{2x^2-x+3}) - 21459328844(\sqrt{2x^2-x+3}) + 14434519361\sqrt{2}(\sqrt{2x^2-x+3}) - 5957650879)}{11943936(2(\sqrt{2x^2-x+3})^2 + 10\sqrt{2}(\sqrt{2x^2-x+3}) - 11)}}{11943936(2(\sqrt{2x^2-x+3})^2 + 10\sqrt{2}(\sqrt{2x^2-x+3}) - 11)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(1/2), x, algorithm="giac")

[Out] -5/32\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 22389491/143327232\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 22389491/143327232\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 1/11943936\*sqrt(2)\*(215012404\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^5 + 3010410772\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^4 + 2740802468\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 - 21459328844\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 14434519361\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 5957650879)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^3

**maple** [A] time = 0.01, size = 109, normalized size = 0.81

$$\frac{5\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32} + \frac{22389491\sqrt{2} \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{143327232} - \frac{3163415\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{11943936\left(x+\frac{5}{2}\right)} + \frac{394907\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{995328\left(x+\frac{5}{2}\right)^2} - \frac{3667\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{13824\left(x+\frac{5}{2}\right)^3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)`

[Out]  $5/32*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))-3163415/11943936/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+22389491/143327232*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)}/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)})+394907/995328/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}$

**maxima** [A] time = 1.00, size = 131, normalized size = 0.97

$$\frac{5}{32} \sqrt{2} \operatorname{arsinh}\left(\frac{4}{23} \sqrt{23}x - \frac{1}{23} \sqrt{23}\right) - \frac{22389491}{143327232} \sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{3667\sqrt{2x^2-x+3}}{1728(8x^3+60x^2+150x+125)} + \frac{394907\sqrt{2x^2-x+3}}{248832(4x^2+20x+25)} - \frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")`

[Out]  $5/32*\sqrt{2}*\operatorname{arcsinh}(4/23*\sqrt{23}*x - 1/23*\sqrt{23}) - 22389491/143327232*\sqrt{2}*\operatorname{arcsinh}(22/23*\sqrt{23}*x/\operatorname{abs}(2*x + 5) - 17/23*\sqrt{23}/\operatorname{abs}(2*x + 5)) - 3667/1728*\sqrt{2*x^2 - x + 3}/(8*x^3 + 60*x^2 + 150*x + 125) + 394907/248832*\sqrt{2*x^2 - x + 3}/(4*x^2 + 20*x + 25) - 3163415/5971968*\sqrt{2*x^2 - x + 3}/(2*x + 5)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^4 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)`

[Out] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^4 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)`

[Out] `Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x + 3)), x)`

$$3.330 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5 \sqrt{3-x+2x^2}} dx$$

**Optimal.** Leaf size=139

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)} - \frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2} + \frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3} - \frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4} + \frac{2053207 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{20639121408\sqrt{2}}$$

**Rubi [A]** time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1650, 806, 724, 206}

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)} - \frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2} + \frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3} - \frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4} + \frac{2053207 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{20639121408\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^5\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (-3667\*Sqrt[3 - x + 2\*x^2])/(2304\*(5 + 2\*x)^4) + (513097\*Sqrt[3 - x + 2\*x^2])/(497664\*(5 + 2\*x)^3) - (16295969\*Sqrt[3 - x + 2\*x^2])/(71663616\*(5 + 2\*x)^2) + (26800085\*Sqrt[3 - x + 2\*x^2])/(1719926784\*(5 + 2\*x)) + (2053207\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(20639121408\*Sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m +

$2*p + 3], 0]$

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}} dx &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} - \frac{1}{288} \int \frac{\frac{37027}{16} - \frac{10167x}{4} + 1944x^2 - 720x^3}{(5+2x)^4\sqrt{3-x+2x^2}} dx \\ &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} + \frac{\int \frac{\frac{2607829}{16} - \frac{295607x}{2} + 77760x^2}{(5+2x)^3\sqrt{3-x+2x^2}} dx}{62208} \\ &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} - \frac{\int \frac{19}{(5+2x)^2} dx}{17} \\ &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{2680}{17} \\ &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{2680}{17} \\ &= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{2680}{17} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 81, normalized size = 0.58

$$\frac{2053207\sqrt{2}(2x+5)^4 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 24\sqrt{2x^2-x+3}(214400680x^3 + 43592076x^2 - 255525906x - 298655447)}{41278242816(2x+5)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^5\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (24\*Sqrt[3 - x + 2\*x^2]\*(-298655447 - 255525906\*x + 43592076\*x^2 + 214400680\*x^3) + 2053207\*Sqrt[2]\*(5 + 2\*x)^4\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/(41278242816\*(5 + 2\*x)^4)

**IntegrateAlgebraic [A]** time = 0.78, size = 83, normalized size = 0.60

$$\frac{\sqrt{2x^2 - x + 3} (214400680x^3 + 43592076x^2 - 255525906x - 298655447)}{1719926784(2x + 5)^4} - \frac{2053207 \tanh^{-1}\left(-\frac{\sqrt{2x^2 - x + 3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{10319560704\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^5\*Sqrt[3 - x + 2\*x^2]),x]

[Out] (Sqrt[3 - x + 2\*x^2]\*(-298655447 - 255525906\*x + 43592076\*x^2 + 214400680\*x^3))/(1719926784\*(5 + 2\*x)^4) - (2053207\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(10319560704\*Sqrt[2])

**fricas [A]** time = 0.86, size = 125, normalized size = 0.90

$$\frac{2053207 \sqrt{2} (16x^4 + 160x^3 + 600x^2 + 1000x + 625) \log\left(\frac{24 \sqrt{2} \sqrt{2x^2 - x + 3} (22x - 17) - 1060x^2 + 1036x - 1153}{4x^2 + 20x + 25}\right) + 48 (214400680x^3 + 43592076x^2 - 255525906x - 298655447) \sqrt{2x^2 - x + 3}}{82556485632 (16x^4 + 160x^3 + 600x^2 + 1000x + 625)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^5/(2\*x^2-x+3)^(1/2),x, algorithm="fricas")

[Out] 1/82556485632\*(2053207\*sqrt(2)\*(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(214400680\*x^3 + 43592076\*x^2 - 255525906\*x - 298655447)\*sqrt(2\*x^2 - x + 3))/(16\*x^4 + 160\*x^3 + 600\*x^2 + 1000\*x + 625)

**giac [A]** time = 0.28, size = 164, normalized size = 1.18

$$\frac{1}{41278242816} \sqrt{2} \left( 12 \left( \frac{24 \left( \frac{144 \left( \frac{513097}{2x+5} - \frac{792072}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)} \right)}{2x+5} - \frac{16295969}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)} \right)}{2x+5} + \frac{26800085}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)} \right) \sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{2053207 \log\left(12 \sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{72}{2x+5} - 11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)} - 321601020 \operatorname{sgn}\left(\frac{1}{2x+5}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^5/(2\*x^2-x+3)^(1/2),x, algorithm="giac")

[Out]  $1/41278242816*\sqrt{2}*(12*(24*(144*(513097/\text{sgn}(1/(2*x + 5))) - 792072/((2*x + 5)*\text{sgn}(1/(2*x + 5))))/(2*x + 5) - 16295969/\text{sgn}(1/(2*x + 5)))/(2*x + 5) + 26800085/\text{sgn}(1/(2*x + 5))*\sqrt{-11/(2*x + 5) + 36/(2*x + 5)^2 + 1} + 2053207*\log(12*\sqrt{-11/(2*x + 5) + 36/(2*x + 5)^2 + 1} + 72/(2*x + 5) - 11)/\text{sgn}(1/(2*x + 5)) - 321601020*\text{sgn}(1/(2*x + 5)))$

**maple [A]** time = 0.01, size = 116, normalized size = 0.83

$$\frac{2053207\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+\frac{17}{2})\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{41278242816} + \frac{26800085\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{3439853568\left(x+\frac{5}{2}\right)} - \frac{16295969\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{286654464\left(x+\frac{5}{2}\right)} - \frac{3667\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{36864\left(x+\frac{5}{2}\right)^4} + \frac{513097\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{3981312\left(x+\frac{5}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^{(1/2)}, x)$

[Out]  $26800085/3439853568/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+2053207/41278242816*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)}/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)})-16295969/286654464/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}+513097/3981312/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}$

**maxima [A]** time = 1.02, size = 149, normalized size = 1.07

$$-\frac{2053207}{41278242816}\sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{3667\sqrt{2x^2-x+3}}{2304(16x^4+160x^3+600x^2+1000x+625)} + \frac{513097\sqrt{2x^2-x+3}}{497664(8x^3+60x^2+150x+125)} - \frac{16295969\sqrt{2x^2-x+3}}{71663616(4x^2+20x+25)} + \frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $-2053207/41278242816*\sqrt{2}*\operatorname{arcsinh}(22/23*\sqrt{23}*x/\text{abs}(2*x + 5) - 17/23*\sqrt{23}/\text{abs}(2*x + 5)) - 3667/2304*\sqrt{2*x^2 - x + 3}/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 513097/497664*\sqrt{2*x^2 - x + 3}/(8*x^3 + 60*x^2 + 150*x + 125) - 16295969/71663616*\sqrt{2*x^2 - x + 3}/(4*x^2 + 20*x + 25) + 26800085/1719926784*\sqrt{2*x^2 - x + 3}/(2*x + 5)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^5 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^{(1/2)}), x)$

[Out]  $\text{int}((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^{(1/2)}), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^5 \sqrt{2x^2 - x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x + 3)), x)
```

$$3.331 \quad \int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=124

$$\frac{153}{16} \sqrt{2x^2 - x + 3} x^2 + \frac{2645}{128} \sqrt{2x^2 - x + 3} x - \frac{13153}{512} \sqrt{2x^2 - x + 3} - \frac{4(346 - 533x)}{23\sqrt{2x^2 - x + 3}} + \frac{5}{4} \sqrt{2x^2 - x + 3} x^3 + \frac{144217}{1024\sqrt{2}}$$

Rubi [A] time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{4} \sqrt{2x^2 - x + 3} x^3 + \frac{153}{16} \sqrt{2x^2 - x + 3} x^2 + \frac{2645}{128} \sqrt{2x^2 - x + 3} x - \frac{13153}{512} \sqrt{2x^2 - x + 3} - \frac{4(346 - 533x)}{23\sqrt{2x^2 - x + 3}} + \frac{144217 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((5 + 2\*x)^2\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-4\*(346 - 533\*x))/(23\*Sqrt[3 - x + 2\*x^2]) - (13153\*Sqrt[3 - x + 2\*x^2])/512 + (2645\*x\*Sqrt[3 - x + 2\*x^2])/128 + (153\*x^2\*Sqrt[3 - x + 2\*x^2])/16 + (5\*x^3\*Sqrt[3 - x + 2\*x^2])/4 + (144217\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(1024\*Sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx &= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{-759 - \frac{575x}{2} + 805x^2 + \frac{1219x^3}{2} + 115x^4}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{1}{92} \int \frac{-6072 - 2300x + 5}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{1}{\sqrt{3-x+2x^2}} dx \\
&= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{1}{552} \int \frac{1}{\sqrt{3-x+2x^2}} dx
\end{aligned}$$

**Mathematica [A]** time = 0.47, size = 74, normalized size = 0.60

$$\frac{3316991\sqrt{4x^2-2x+6} \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + 4(29440x^5 + 210496x^4 + 418232x^3 - 510554x^2 + 2124123x - 1616165)}{47104\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] Integrate[((5 + 2\*x)^2\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(3/2), x]

[Out] (4\*(-1616165 + 2124123\*x - 510554\*x^2 + 418232\*x^3 + 210496\*x^4 + 29440\*x^5) + 3316991\*sqrt[6 - 2\*x + 4\*x^2]\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(47104\*sqrt[3 - x + 2\*x^2])

**IntegrateAlgebraic [A]** time = 0.65, size = 80, normalized size = 0.65

$$\frac{144217 \log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{1024\sqrt{2}} + \frac{29440x^5 + 210496x^4 + 418232x^3 - 510554x^2 + 2124123x - 1616165}{11776\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((5 + 2\*x)^2\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-1616165 + 2124123\*x - 510554\*x^2 + 418232\*x^3 + 210496\*x^4 + 29440\*x^5)/(11776\*sqrt[3 - x + 2\*x^2]) + (144217\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(1024\*sqrt[2])

**fricas** [A] time = 0.94, size = 102, normalized size = 0.82

$$\frac{3316991\sqrt{2}(2x^2-x+3)\log(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25)+8(29440x^5+210496x^4+418232x^3-510554x^2+2124123x-1616165)\sqrt{2x^2-x+3}}{94208(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2), x, algorithm="fricas")

[Out] 1/94208\*(3316991\*sqrt(2)\*(2\*x^2 - x + 3)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(29440\*x^5 + 210496\*x^4 + 418232\*x^3 - 510554\*x^2 + 2124123\*x - 1616165)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac** [A] time = 0.22, size = 72, normalized size = 0.58

$$\frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(8(20x+143)x+2273)x-11099)x+2124123)x-1616165)}{11776\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2), x, algorithm="giac")

[Out] 144217/2048\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/11776\*((46\*(4\*(8\*(20\*x + 143)\*x + 2273)\*x - 11099)\*x + 2124123)\*x - 1616165)/sqrt(2\*x^2 - x + 3)

**maple** [A] time = 0.02, size = 132, normalized size = 1.06

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}+\frac{144217x}{1024\sqrt{2x^2-x+3}}-\frac{144217\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}+\frac{\frac{931255x}{2352}-\frac{931255}{94208}}{\sqrt{2x^2-x+3}}-\frac{521655}{4096\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2), x)

[Out] 931255/94208\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-144217/2048\*2^(1/2)\*arcsinh(4/23\*23^(1/2)\*(x-1/4))+5/2\*x^5/(2\*x^2-x+3)^(1/2)+143/8\*x^4/(2\*x^2-x+3)^(1/2)+2273/6

$4x^3/(2x^2-x+3)^{(1/2)}-11099/256x^2/(2x^2-x+3)^{(1/2)}+144217/1024x/(2x^2-x+3)^{(1/2)}-521655/4096/(2x^2-x+3)^{(1/2)}$

**maxima** [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{5x^5}{2\sqrt{2x^2-x+3}} + \frac{143x^4}{8\sqrt{2x^2-x+3}} + \frac{2273x^3}{64\sqrt{2x^2-x+3}} - \frac{11099x^2}{256\sqrt{2x^2-x+3}} - \frac{144217}{2048}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2124123x}{11776\sqrt{2x^2-x+3}} - \frac{1616165}{11776\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out]  $5/2x^5/\sqrt{2x^2-x+3} + 143/8x^4/\sqrt{2x^2-x+3} + 2273/64x^3/\sqrt{2x^2-x+3} - 11099/256x^2/\sqrt{2x^2-x+3} - 144217/2048\sqrt{2}\operatorname{arsinh}(1/23\sqrt{23}(4x-1)) + 2124123/11776x/\sqrt{2x^2-x+3} - 1616165/11776/\sqrt{2x^2-x+3}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)^2(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 5)^2\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x^2 - x + 3)^(3/2),x)

[Out] int(((2\*x + 5)^2\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x^2 - x + 3)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*\*2\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((2\*x + 5)\*\*2\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x\*\*2 - x + 3)\*\*(3/2), x)

$$3.332 \quad \int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=103

$$\frac{5}{6}\sqrt{2x^2-x+3}x^2 + \frac{193}{48}\sqrt{2x^2-x+3}x + \frac{33}{64}\sqrt{2x^2-x+3} + \frac{373x-53}{23\sqrt{2x^2-x+3}} + \frac{3111 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{6}\sqrt{2x^2-x+3}x^2 + \frac{193}{48}\sqrt{2x^2-x+3}x + \frac{33}{64}\sqrt{2x^2-x+3} - \frac{53-373x}{23\sqrt{2x^2-x+3}} + \frac{3111 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(3/2), x]

[Out] -(53 - 373\*x)/(23\*sqrt[3 - x + 2\*x^2]) + (33\*sqrt[3 - x + 2\*x^2])/64 + (193\*x\*sqrt[3 - x + 2\*x^2])/48 + (5\*x^2\*sqrt[3 - x + 2\*x^2])/6 + (3111\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(128\*sqrt[2])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(5 + 2x)(2 + x + 3x^2 - x^3 + 5x^4)}{(3 - x + 2x^2)^{3/2}} dx &= -\frac{53 - 373x}{23\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{-\frac{575}{4} + 161x^2 + \frac{115x^3}{2}}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{53 - 373x}{23\sqrt{3 - x + 2x^2}} + \frac{5}{6}x^2\sqrt{3 - x + 2x^2} + \frac{1}{69} \int \frac{-\frac{1725}{2} - 345x + \frac{4439x^2}{4}}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{53 - 373x}{23\sqrt{3 - x + 2x^2}} + \frac{193}{48}x\sqrt{3 - x + 2x^2} + \frac{5}{6}x^2\sqrt{3 - x + 2x^2} + \frac{1}{276} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
&= -\frac{53 - 373x}{23\sqrt{3 - x + 2x^2}} + \frac{33}{64}\sqrt{3 - x + 2x^2} + \frac{193}{48}x\sqrt{3 - x + 2x^2} + \frac{5}{6}x^2\sqrt{3 - x + 2x^2} \\
&= -\frac{53 - 373x}{23\sqrt{3 - x + 2x^2}} + \frac{33}{64}\sqrt{3 - x + 2x^2} + \frac{193}{48}x\sqrt{3 - x + 2x^2} + \frac{5}{6}x^2\sqrt{3 - x + 2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 60, normalized size = 0.58

$$\frac{7360x^4 + 31832x^3 - 2162x^2 + 122607x - 3345}{4416\sqrt{2x^2 - x + 3}} - \frac{3111 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-3345 + 122607\*x - 2162\*x^2 + 31832\*x^3 + 7360\*x^4)/(4416\*Sqrt[3 - x + 2\*x^2]) - (3111\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(128\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.60, size = 75, normalized size = 0.73

$$\frac{3111 \log\left(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1\right)}{128\sqrt{2}} + \frac{7360x^4 + 31832x^3 - 2162x^2 + 122607x - 3345}{4416\sqrt{2x^2 - x + 3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(3/2), x]

[Out] (-3345 + 122607\*x - 2162\*x^2 + 31832\*x^3 + 7360\*x^4)/(4416\*Sqrt[3 - x + 2\*x^2]) + (3111\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(128\*Sqrt[2])

**fricas [A]** time = 1.44, size = 97, normalized size = 0.94

$$\frac{214659\sqrt{2}(2x^2 - x + 3)\log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(7360x^4 + 31832x^3 - 2162x^2 + 122607x - 3345)\sqrt{2x^2 - x + 3}}{35328(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2), x, algorithm="fricas")

[Out] 1/35328\*(214659\*sqrt(2)\*(2\*x^2 - x + 3)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(7360\*x^4 + 31832\*x^3 - 2162\*x^2 + 122607\*x - 3345)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac [A]** time = 0.22, size = 67, normalized size = 0.65

$$\frac{3111}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(4(40x + 173)x - 47)x + 122607)x - 3345}{4416\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out]  $3111/256*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3}) + 1) + 1/4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/\sqrt{2*x^2 - x + 3}$

**maple [A]** time = 0.01, size = 115, normalized size = 1.12

$$\frac{5x^4}{3\sqrt{2x^2-x+3}} + \frac{173x^3}{24\sqrt{2x^2-x+3}} - \frac{47x^2}{96\sqrt{2x^2-x+3}} + \frac{3111x}{128\sqrt{2x^2-x+3}} - \frac{3111\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256} + \frac{\frac{10185x}{2944} - \frac{10185}{11776}}{\sqrt{2x^2-x+3}} + \frac{55}{512\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2),x)

[Out]  $10185/11776*(4*x-1)/(2*x^2-x+3)^{(1/2)} - 3111/256*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4)) + 5/3/(2*x^2-x+3)^{(1/2)}*x^4 + 173/24/(2*x^2-x+3)^{(1/2)}*x^3 - 47/96/(2*x^2-x+3)^{(1/2)}*x^2 + 3111/128/(2*x^2-x+3)^{(1/2)}*x + 55/512/(2*x^2-x+3)^{(1/2)}$

**maxima [A]** time = 0.97, size = 97, normalized size = 0.94

$$\frac{5x^4}{3\sqrt{2x^2-x+3}} + \frac{173x^3}{24\sqrt{2x^2-x+3}} - \frac{47x^2}{96\sqrt{2x^2-x+3}} - \frac{3111}{256}\sqrt{2} \operatorname{arcsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{40869x}{1472\sqrt{2x^2-x+3}} - \frac{1115}{1472\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out]  $5/3*x^4/\sqrt{2*x^2 - x + 3} + 173/24*x^3/\sqrt{2*x^2 - x + 3} - 47/96*x^2/\sqrt{2*x^2 - x + 3} - 3111/256*\sqrt{2}*\operatorname{arcsinh}(1/23*\sqrt{23}*(4*x - 1)) + 40869/1472*x/\sqrt{2*x^2 - x + 3} - 1115/1472/\sqrt{2*x^2 - x + 3}$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 5)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x^2 - x + 3)^(3/2),x)

[Out] int(((2\*x + 5)\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x^2 - x + 3)^(3/2),x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```



$$3.333 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=82

$$\frac{5}{8}\sqrt{2x^2-x+3}x + \frac{27}{32}\sqrt{2x^2-x+3} + \frac{219x+89}{92\sqrt{2x^2-x+3}} + \frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}x + \frac{27}{32}\sqrt{2x^2-x+3} + \frac{219x+89}{92\sqrt{2x^2-x+3}} + \frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/(3 - x + 2\*x^2)^(3/2), x]

[Out] (89 + 219\*x)/(92\*Sqrt[3 - x + 2\*x^2]) + (27\*Sqrt[3 - x + 2\*x^2])/32 + (5\*x\*Sqrt[3 - x + 2\*x^2])/8 + (213\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(64\*Sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[P

```

q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]], Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(3 - x + 2x^2)^{3/2}} dx &= \frac{89 + 219x}{92\sqrt{3 - x + 2x^2}} + \frac{2}{23} \int \frac{-\frac{345}{16} + \frac{69x}{8} + \frac{115x^2}{4}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{89 + 219x}{92\sqrt{3 - x + 2x^2}} + \frac{5}{8}x\sqrt{3 - x + 2x^2} + \frac{1}{46} \int \frac{-\frac{345}{2} + \frac{621x}{8}}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{89 + 219x}{92\sqrt{3 - x + 2x^2}} + \frac{27}{32}\sqrt{3 - x + 2x^2} + \frac{5}{8}x\sqrt{3 - x + 2x^2} - \frac{213}{64} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
&= \frac{89 + 219x}{92\sqrt{3 - x + 2x^2}} + \frac{27}{32}\sqrt{3 - x + 2x^2} + \frac{5}{8}x\sqrt{3 - x + 2x^2} - \frac{213 \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{23}}} dx \right)}{64\sqrt{46}} \\
&= \frac{89 + 219x}{92\sqrt{3 - x + 2x^2}} + \frac{27}{32}\sqrt{3 - x + 2x^2} + \frac{5}{8}x\sqrt{3 - x + 2x^2} + \frac{213 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{64\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 55, normalized size = 0.67

$$\frac{920x^3 + 782x^2 + 2511x + 2575}{736\sqrt{2x^2 - x + 3}} - \frac{213 \sinh^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/(3 - x + 2\*x^2)^(3/2),x]

[Out] (2575 + 2511\*x + 782\*x^2 + 920\*x^3)/(736\*sqrt[3 - x + 2\*x^2]) - (213\*ArcSinh[(-1 + 4\*x)/sqrt[23]])/(64\*sqrt[2])

**IntegrateAlgebraic [A]** time = 0.60, size = 70, normalized size = 0.85

$$\frac{213 \log\left(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1\right)}{64\sqrt{2}} + \frac{920x^3 + 782x^2 + 2511x + 2575}{736\sqrt{2x^2 - x + 3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/(3 - x + 2\*x^2)^(3/2),x]

[Out] (2575 + 2511\*x + 782\*x^2 + 920\*x^3)/(736\*sqrt[3 - x + 2\*x^2]) + (213\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(64\*sqrt[2])

**fricas [A]** time = 1.65, size = 92, normalized size = 1.12

$$\frac{4899\sqrt{2}(2x^2 - x + 3)\log\left(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25\right) + 8(920x^3 + 782x^2 + 2511x + 2575)\sqrt{2x^2 - x + 3}}{5888(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="fricas")

[Out] 1/5888\*(4899\*sqrt(2)\*(2\*x^2 - x + 3)\*log(4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(920\*x^3 + 782\*x^2 + 2511\*x + 2575)\*sqrt(2\*x^2 - x + 3))/(2\*x^2 - x + 3)

**giac [A]** time = 0.22, size = 62, normalized size = 0.76

$$\frac{213}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(20x + 17)x + 2511)x + 2575}{736\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(3/2),x, algorithm="giac")

[Out] 213/128\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/736\*((46\*(20\*x + 17)\*x + 2511)\*x + 2575)/sqrt(2\*x^2 - x + 3)

**maple [A]** time = 0.01, size = 98, normalized size = 1.20

$$\frac{5x^3}{4\sqrt{2x^2 - x + 3}} + \frac{17x^2}{16\sqrt{2x^2 - x + 3}} + \frac{213x}{64\sqrt{2x^2 - x + 3}} - \frac{213\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128} + \frac{901}{256\sqrt{2x^2 - x + 3}} + \frac{\frac{123x}{1472} - \frac{123}{5888}}{\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)`

[Out]  $5/4/(2*x^2-x+3)^{(1/2)}*x^3+17/16/(2*x^2-x+3)^{(1/2)}*x^2+213/64/(2*x^2-x+3)^{(1/2)}*x+901/256/(2*x^2-x+3)^{(1/2)}+123/5888*(4*x-1)/(2*x^2-x+3)^{(1/2)}-213/128*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima** [A] time = 0.95, size = 80, normalized size = 0.98

$$\frac{5x^3}{4\sqrt{2x^2-x+3}} + \frac{17x^2}{16\sqrt{2x^2-x+3}} - \frac{213}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2511x}{736\sqrt{2x^2-x+3}} + \frac{2575}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")`

[Out]  $5/4*x^3/\operatorname{sqrt}(2*x^2-x+3)+17/16*x^2/\operatorname{sqrt}(2*x^2-x+3)-213/128*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x-1))+2511/736*x/\operatorname{sqrt}(2*x^2-x+3)+2575/736/\operatorname{sqrt}(2*x^2-x+3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+3*x^2-x^3+5*x^4+2)/(2*x^2-x+3)^(3/2),x)`

[Out] `int((x+3*x^2-x^3+5*x^4+2)/(2*x^2-x+3)^(3/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)`

[Out] `Integral((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)`

$$3.334 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=101

$$\frac{917x + 1191}{3312\sqrt{2x^2 - x + 3}} + \frac{5}{8}\sqrt{2x^2 - x + 3} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}} + \frac{39 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

**Rubi [A]** time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1646, 1653, 843, 619, 215, 724, 206}

$$\frac{917x + 1191}{3312\sqrt{2x^2 - x + 3}} + \frac{5}{8}\sqrt{2x^2 - x + 3} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}} + \frac{39 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)),x]

[Out] (1191 + 917\*x)/(3312\*sqrt[3 - x + 2\*x^2]) + (5\*sqrt[3 - x + 2\*x^2])/8 + (39\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(16\*sqrt[2]) - (3667\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(1728\*sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rule 1653

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + b\*x + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(b\*d\*e\*(p + 1) + a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - e\*(2\*c\*d - b\*e)\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}} dx &= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{-\frac{6739}{576} + \frac{69x}{8} + \frac{115x^2}{4}}{(5+2x)\sqrt{3-x+2x^2}} dx \\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{1}{92} \int \frac{\frac{3611}{72} - \frac{897x}{2}}{(5+2x)\sqrt{3-x+2x^2}} dx \\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{39}{16} \int \frac{1}{\sqrt{3-x+2x^2}} dx + \frac{3667}{288} \int \frac{1}{(5+2x)\sqrt{3-x+2x^2}} dx \\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{3667}{144} \text{Subst} \left( \int \frac{1}{288-x^2} dx, x, \frac{17-x}{\sqrt{3-x+2x^2}} \right) \\
&= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{39 \sinh^{-1} \left( \frac{1-4x}{\sqrt{23}} \right)}{16\sqrt{2}} - \frac{3667 \tanh^{-1} \left( \frac{17-x}{12\sqrt{2}} \right)}{1728\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 86, normalized size = 0.85

$$\frac{12(4140x^2-1153x+7401)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}} - 3667 \log(12\sqrt{4x^2-2x+6}-22x+17) + 3667 \log(2x+5) - 4212 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)$$


---


$$1728\sqrt{2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] ((12\*(7401 - 1153\*x + 4140\*x^2))/(23\*Sqrt[3/2 - x/2 + x^2]) - 4212\*ArcSinh[(-1 + 4\*x)/Sqrt[23]] + 3667\*Log[5 + 2\*x] - 3667\*Log[17 - 22\*x + 12\*Sqrt[6 - 2\*x + 4\*x^2]])/(1728\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.53, size = 107, normalized size = 1.06

$$\frac{4140x^2 - 1153x + 7401}{3312\sqrt{2x^2 - x + 3}} + \frac{39 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{16\sqrt{2}} + \frac{3667 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{864\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)), x]

[Out]  $(7401 - 1153x + 4140x^2)/(3312\sqrt{3 - x + 2x^2}) + (3667\text{ArcTanh}[5/6 + x/3 - \sqrt{3 - x + 2x^2}/(3\sqrt{2})])/(864\sqrt{2}) + (39\text{Log}[1 - 4x + 2\sqrt{2}\sqrt{3 - x + 2x^2}])/(16\sqrt{2})$

**fricas** [A] time = 1.17, size = 149, normalized size = 1.48

$$\frac{96876\sqrt{2}(2x^2 - x + 3)\log(4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 84341\sqrt{2}(2x^2 - x + 3)\log\left(\frac{-24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) + 1060x^2 - 1036x + 1153}{4x^2 + 20x + 25}\right) + 48(4140x^2 - 1153x + 7401)\sqrt{2x^2 - x + 3}}{158976(2x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")`

[Out]  $1/158976*(96876*\sqrt{2}*(2x^2 - x + 3)*\log(4*\sqrt{2}*\sqrt{2x^2 - x + 3}*(4x - 1) - 32*x^2 + 16*x - 25) + 84341*\sqrt{2}*(2x^2 - x + 3)*\log(-24*\sqrt{2}*\sqrt{2x^2 - x + 3}*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*\sqrt{2x^2 - x + 3})/(2x^2 - x + 3)$

**giac** [A] time = 0.40, size = 118, normalized size = 1.17

$$\frac{39}{32}\sqrt{2}\log(-4\sqrt{2}x + \sqrt{2} + 4\sqrt{2x^2 - x + 3}) - \frac{3667}{3456}\sqrt{2}\log(-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}) + \frac{3667}{3456}\sqrt{2}\log(-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}) + \frac{(4140x - 1153)x + 7401}{3312\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="giac")`

[Out]  $39/32*\sqrt{2}*\log(-4*\sqrt{2}*x + \sqrt{2} + 4*\sqrt{2x^2 - x + 3}) - 3667/3456*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2}*x + \sqrt{2} + 2*\sqrt{2x^2 - x + 3})) + 3667/3456*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2}*x - 11*\sqrt{2} + 2*\sqrt{2x^2 - x + 3})) + 1/3312*((4140*x - 1153)*x + 7401)/\sqrt{2x^2 - x + 3}$

**maple** [A] time = 0.01, size = 148, normalized size = 1.47

$$\frac{5x^2}{4\sqrt{2x^2 - x + 3}} + \frac{39x}{16\sqrt{2x^2 - x + 3}} - \frac{39\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{32} - \frac{3667\sqrt{2}\operatorname{arctanh}\left(\frac{\left(-11x + \frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x + 2\left(x + \frac{5}{2}\right) - \frac{19}{2}}}\right)}{3456} - \frac{309}{64\sqrt{2x^2 - x + 3}} - \frac{5507(4x - 1)}{1472\sqrt{2x^2 - x + 3}} + \frac{3667}{576\sqrt{-11x + 2\left(x + \frac{5}{2}\right) - \frac{19}{2}}} + \frac{\frac{40337x}{3312} - \frac{40337}{13248}}{\sqrt{-11x + 2\left(x + \frac{5}{2}\right) - \frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)`

[Out]  $5/4/(2x^2-x+3)^(1/2)*x^2+39/16/(2x^2-x+3)^(1/2)*x-309/64/(2x^2-x+3)^(1/2)-5507/1472*(4*x-1)/(2x^2-x+3)^(1/2)-39/32*2^(1/2)*\operatorname{arcsinh}(4/23*23^(1/2)*(x-1/4))+3667/576/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+40337/13248*(4*x-1)/(-11*x+$



$$2*(x+5/2)^2-19/2)^{(1/2)}-3667/3456*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)})/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)})$$

**maxima [A]** time = 0.98, size = 99, normalized size = 0.98

$$\frac{5x^2}{4\sqrt{2x^2-x+3}} - \frac{39}{32}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{3667}{3456}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{1153x}{3312\sqrt{2x^2-x+3}} + \frac{2467}{1104\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 5/4\*x^2/sqrt(2\*x^2 - x + 3) - 39/32\*sqrt(2)\*arcsinh(4/23\*sqrt(23)\*x - 1/23\*sqrt(23)) + 3667/3456\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) - 1153/3312\*x/sqrt(2\*x^2 - x + 3) + 2467/1104/sqrt(2\*x^2 - x + 3)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)(2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)\*(2\*x^2 - x + 3)^(3/2)),x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)\*(2\*x^2 - x + 3)^(3/2)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)(2x^2 - x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*(2\*x\*\*2 - x + 3)\*\*(3/2)), x)

$$3.335 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=108

$$\frac{2203x + 9897}{119232\sqrt{2x^2 - x + 3}} - \frac{3667\sqrt{2x^2 - x + 3}}{10368(2x + 5)} + \frac{25951 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

**Rubi [A]** time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {1646, 1650, 843, 619, 215, 724, 206}

$$\frac{2203x + 9897}{119232\sqrt{2x^2 - x + 3}} - \frac{3667\sqrt{2x^2 - x + 3}}{10368(2x + 5)} + \frac{25951 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] (9897 + 2203\*x)/(119232\*sqrt[3 - x + 2\*x^2]) - (3667\*sqrt[3 - x + 2\*x^2])/(10368\*(5 + 2\*x)) - (5\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(8\*sqrt[2]) + (25951\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(41472\*sqrt[2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2

$*a*e - b*d - (2*c*d - b*e)*x/\text{Sqrt}[a + b*x + c*x^2], x] /;$  FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 843

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x, 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rule 1650

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e\*x, x], R = PolynomialRemainder[Pq, d + e\*x, x]}, Simp[(e\*R\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p\*ExpandToSum[(m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Q + c\*d\*R\*(m + 1) - b\*e\*R\*(m + p + 2) - c\*e\*R\*(m + 2\*p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}} dx &= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{-\frac{33649}{20736} + \frac{131215x}{10368} + \frac{115x^2}{4}}{(5+2x)^2\sqrt{3-x+2x^2}} dx \\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} - \frac{1}{828} \int \frac{\frac{100073}{192} - 1035x}{(5+2x)\sqrt{3-x+2x^2}} dx \\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} + \frac{5}{8} \int \frac{1}{\sqrt{3-x+2x^2}} dx - \frac{25951}{41472\sqrt{2}} \\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} + \frac{25951 \operatorname{Subst}\left(\int \frac{1}{288-x^2} dx, x, \frac{17}{\sqrt{3-x+2x^2}}\right)}{3456} \\
&= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}} + \frac{25951 \tanh^{-1}\left(\frac{17}{\sqrt{3-x+2x^2}}\right)}{41472\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.32, size = 104, normalized size = 0.96

$$\frac{\frac{8(2203x+9897)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}} - \frac{14668\sqrt{4x^2-2x+6}}{2x+5} + 25951 \log\left(12\sqrt{4x^2-2x+6} - 22x + 17\right) - 25951 \log(2x+5) + 25920 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{41472\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] ((8\*(9897 + 2203\*x))/(23\*Sqrt[3/2 - x/2 + x^2]) - (14668\*Sqrt[6 - 2\*x + 4\*x^2])/(5 + 2\*x) + 25920\*ArcSinh[(-1 + 4\*x)/Sqrt[23]] - 25951\*Log[5 + 2\*x] + 25951\*Log[17 - 22\*x + 12\*Sqrt[6 - 2\*x + 4\*x^2]])/(41472\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.63, size = 114, normalized size = 1.06

$$\frac{-53290x^2 + 48653x - 51351}{79488(2x+5)\sqrt{2x^2-x+3}} - \frac{5 \log\left(2\sqrt{2}\sqrt{2x^2-x+3} - 4x + 1\right)}{8\sqrt{2}} - \frac{25951 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{20736\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2)), x]

[Out]  $(-51351 + 48653x - 53290x^2)/(79488(5 + 2x)\sqrt{3 - x + 2x^2}) - (25951 \operatorname{ArcTanh}[5/6 + x/3 - \sqrt{3 - x + 2x^2}/(3\sqrt{2})])/(20736\sqrt{2}) - (5 \operatorname{Log}[1 - 4x + 2\sqrt{2}\sqrt{3 - x + 2x^2}])/(8\sqrt{2})$

**fricas** [A] time = 1.15, size = 157, normalized size = 1.45

$$\frac{596160\sqrt{2}(4x^3 + 8x^2 + x + 15)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 596873\sqrt{2}(4x^3 + 8x^2 + x + 15)\log\left(\frac{24\sqrt{2}\sqrt{2x^2 - x + 3}(22x - 17) - 1060x^2 + 1036x - 1153}{4x^2 + 20x + 25}\right) - 48(53290x^2 - 48653x + 51351)\sqrt{2x^2 - x + 3}}{3815424(4x^3 + 8x^2 + x + 15)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")`

[Out]  $1/3815424*(596160*\sqrt{2}*(4*x^3 + 8*x^2 + x + 15)*\log(-4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*\sqrt{2}*(4*x^3 + 8*x^2 + x + 15)*\log((24*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*\sqrt{2*x^2 - x + 3})/(4*x^3 + 8*x^2 + x + 15)$

**giac** [B] time = 0.42, size = 225, normalized size = 2.08

$$\frac{1}{1907712}\sqrt{2}\left(\frac{12\left(\frac{315103}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\frac{1012092}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)} - \frac{26645}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{\sqrt{\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1}} + \frac{596873\log\left(12\sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{72}{2x+5} - 11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)} + \frac{596160\log\left(\sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{6}{2x+5} + 1\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)} - \frac{596160\log\left(\sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{6}{2x+5} - 1\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")`

[Out]  $1/1907712*\sqrt{2}*(12*((315103/\operatorname{sgn}(1/(2*x + 5))) - 1012092/((2*x + 5)*\operatorname{sgn}(1/(2*x + 5))))/(2*x + 5) - 26645/\operatorname{sgn}(1/(2*x + 5)))/\sqrt{-11/(2*x + 5) + 36/(2*x + 5)^2 + 1} + 596873*\log(12*\sqrt{-11/(2*x + 5) + 36/(2*x + 5)^2 + 1} + 72/(2*x + 5) - 11)/\operatorname{sgn}(1/(2*x + 5)) + 596160*\log(\operatorname{abs}(\sqrt{-11/(2*x + 5) + 36/(2*x + 5)^2 + 1} + 6/(2*x + 5) + 1))/\operatorname{sgn}(1/(2*x + 5)) - 596160*\log(\operatorname{abs}(\sqrt{-11/(2*x + 5) + 36/(2*x + 5)^2 + 1} + 6/(2*x + 5) - 1))/\operatorname{sgn}(1/(2*x + 5)))$

**maple** [A] time = 0.01, size = 152, normalized size = 1.41

$$-\frac{5x}{8\sqrt{2x^2-x+3}} + \frac{5\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}(x-\frac{1}{2})}{23}\right)}{16} + \frac{25951\sqrt{2}\operatorname{arctanh}\left(\frac{(-11x+\frac{19}{2})\sqrt{2}}{12\sqrt{-11x+2}\left(x+\frac{5}{2}\right)-\frac{19}{2}}\right)}{82944} + \frac{99}{32\sqrt{2x^2-x+3}} + \frac{\frac{1529x}{184} - \frac{1529}{736}}{\sqrt{2x^2-x+3}} - \frac{3667}{1152\left(x+\frac{5}{2}\right)\sqrt{-11x+2}\left(x+\frac{5}{2}\right)-\frac{19}{2}} - \frac{25951}{13824\sqrt{-11x+2}\left(x+\frac{5}{2}\right)-\frac{19}{2}} - \frac{637493(4x-1)}{317952\sqrt{-11x+2}\left(x+\frac{5}{2}\right)-\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)`

[Out] 
$$-5/8/(2x^2-x+3)^{(1/2)} * x + 99/32/(2x^2-x+3)^{(1/2)} + 1529/736 * (4x-1)/(2x^2-x+3)^{(1/2)} + 5/16 * 2^{(1/2)} * \operatorname{arcsinh}(4/23 * 23^{(1/2)} * (x-1/4)) - 3667/1152/(x+5/2)/(-11 * x + 2 * (x+5/2)^2 - 19/2)^{(1/2)} - 25951/13824/(-11 * x + 2 * (x+5/2)^2 - 19/2)^{(1/2)} - 637493/317952 * (4x-1)/(-11 * x + 2 * (x+5/2)^2 - 19/2)^{(1/2)} + 25951/82944 * 2^{(1/2)} * \operatorname{arctanh}(1/12 * (-11 * x + 17/2) * 2^{(1/2)}/(-11 * x + 2 * (x+5/2)^2 - 19/2)^{(1/2)})$$

**maxima** [A] time = 1.01, size = 116, normalized size = 1.07

$$\frac{5}{16} \sqrt{2} \operatorname{arsinh}\left(\frac{4}{23} \sqrt{23} x - \frac{1}{23} \sqrt{23}\right) - \frac{25951}{82944} \sqrt{2} \operatorname{arsinh}\left(\frac{22 \sqrt{23} x}{23 |2x+5|} - \frac{17 \sqrt{23}}{23 |2x+5|}\right) - \frac{26645 x}{79488 \sqrt{2x^2-x+3}} + \frac{30313}{26496 \sqrt{2x^2-x+3}} - \frac{3667}{576 (2 \sqrt{2x^2-x+3} x + 5 \sqrt{2x^2-x+3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 
$$5/16 * \sqrt{2} * \operatorname{arcsinh}(4/23 * \sqrt{23} * x - 1/23 * \sqrt{23}) - 25951/82944 * \sqrt{2} * \operatorname{arcsinh}(22/23 * \sqrt{23} * x / \operatorname{abs}(2 * x + 5) - 17/23 * \sqrt{23} / \operatorname{abs}(2 * x + 5)) - 26645/79488 * x / \sqrt{2 * x^2 - x + 3} + 30313/26496 / \sqrt{2 * x^2 - x + 3} - 3667/576 / (2 * \sqrt{2 * x^2 - x + 3} * x + 5 * \sqrt{2 * x^2 - x + 3})$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 (2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^2\*(2\*x^2 - x + 3)^(3/2)),x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^2\*(2\*x^2 - x + 3)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 (2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*2/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*\*2\*(2\*x\*\*2 - x + 3)\*\*(3/2)), x)

$$3.336 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=112

$$\frac{65991 - 8779x}{4292352\sqrt{2x^2 - x + 3}} + \frac{115369\sqrt{2x^2 - x + 3}}{1492992(2x + 5)} - \frac{3667\sqrt{2x^2 - x + 3}}{20736(2x + 5)^2} - \frac{52631 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 1650, 806, 724, 206}

$$\frac{65991 - 8779x}{4292352\sqrt{2x^2 - x + 3}} + \frac{115369\sqrt{2x^2 - x + 3}}{1492992(2x + 5)} - \frac{3667\sqrt{2x^2 - x + 3}}{20736(2x + 5)^2} - \frac{52631 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(3/2)),x]

[Out] (65991 - 8779\*x)/(4292352\*sqrt[3 - x + 2\*x^2]) - (3667\*sqrt[3 - x + 2\*x^2])/((20736\*(5 + 2\*x)^2) + (115369\*sqrt[3 - x + 2\*x^2])/(1492992\*(5 + 2\*x)) - (52631\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(5971968\*sqrt[2]))

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &

& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}} dx &= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{\frac{5168261}{746496} + \frac{3637795x}{186624} + \frac{5620625x^2}{186624}}{(5+2x)^3\sqrt{3-x+2x^2}} dx \\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} - \frac{\int \frac{\frac{842237}{1296} - \frac{4102487x}{2592}}{(5+2x)^2\sqrt{3-x+2x^2}} dx}{1656} \\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} + \frac{52631}{52631} \\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631}{52631} \\
&= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631}{52631}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 84, normalized size = 0.75

$$\frac{-52631 \log\left(12\sqrt{4x^2-2x+6}-22x+17\right) + \frac{12(3444340x^3+3263288x^2+5842933x+11594283)}{23(2x+5)^2\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}} + 52631 \log(2x+5)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] ((12\*(11594283 + 5842933\*x + 3263288\*x^2 + 3444340\*x^3))/(23\*(5 + 2\*x)^2\*Sqrt[3/2 - x/2 + x^2]) + 52631\*Log[5 + 2\*x] - 52631\*Log[17 - 22\*x + 12\*Sqrt[6 - 2\*x + 4\*x^2]])/(5971968\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.59, size = 83, normalized size = 0.74

$$\frac{52631 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{2985984\sqrt{2}} + \frac{3444340x^3 + 3263288x^2 + 5842933x + 11594283}{11446272(2x+5)^2\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(3/2)), x]

[Out]  $(11594283 + 5842933x + 3263288x^2 + 3444340x^3)/(11446272(5 + 2x)^2 \sqrt{3 - x + 2x^2}) + (52631 \operatorname{ArcTanh}[5/6 + x/3 - \sqrt{3 - x + 2x^2}]/(3 \sqrt{2})))/(2985984 \sqrt{2})$

**fricas [A]** time = 1.20, size = 126, normalized size = 1.12

$$\frac{1210513 \sqrt{2} (8x^4 + 36x^3 + 42x^2 + 35x + 75) \log\left(-\frac{24 \sqrt{2} \sqrt{2x^2 - x + 3} (22x - 17) + 1060x^2 - 1036x + 1153}{4x^2 + 20x + 25}\right) + 48 (3444340x^3 + 3263288x^2 + 5842933x + 11594283) \sqrt{2x^2 - x + 3}}{549421056 (8x^4 + 36x^3 + 42x^2 + 35x + 75)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")`

[Out]  $1/549421056 * (1210513 * \sqrt{2}) * (8x^4 + 36x^3 + 42x^2 + 35x + 75) * \log(-24 * \sqrt{2} * \sqrt{2x^2 - x + 3} * (22x - 17) + 1060x^2 - 1036x + 1153) / (4x^2 + 20x + 25) + 48 * (3444340x^3 + 3263288x^2 + 5842933x + 11594283) * \sqrt{2x^2 - x + 3} / (8x^4 + 36x^3 + 42x^2 + 35x + 75)$

**giac [B]** time = 0.25, size = 220, normalized size = 1.96

$$\frac{-\frac{52631}{11943936} \sqrt{2} \log\left(-2\sqrt{2x} + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right) + \frac{52631}{11943936} \sqrt{2} \log\left(-2\sqrt{2x} - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}\right) - \frac{8779x - 65991}{4292352 \sqrt{2x^2 - x + 3}} + \frac{\sqrt{2} (3594214 \sqrt{2} (\sqrt{2x} - \sqrt{2x^2 - x + 3})^3 + 19874490 (\sqrt{2x} - \sqrt{2x^2 - x + 3})^2 - 30140067 \sqrt{2} (\sqrt{2x} - \sqrt{2x^2 - x + 3}) + 19989859)}{2985984 (2 (\sqrt{2x} - \sqrt{2x^2 - x + 3})^2 + 10 \sqrt{2} (\sqrt{2x} - \sqrt{2x^2 - x + 3}) - 11)}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")`

[Out]  $-52631/11943936 * \sqrt{2} * \log(\operatorname{abs}(-2 * \sqrt{2} * x + \sqrt{2} + 2 * \sqrt{2x^2 - x + 3})) + 52631/11943936 * \sqrt{2} * \log(\operatorname{abs}(-2 * \sqrt{2} * x - 11 * \sqrt{2} + 2 * \sqrt{2x^2 - x + 3})) - 1/4292352 * (8779x - 65991) / \sqrt{2x^2 - x + 3} + 1/2985984 * \sqrt{2} * (3594214 * \sqrt{2} * (\sqrt{2} * x - \sqrt{2x^2 - x + 3})^3 + 19874490 * (\sqrt{2} * x - \sqrt{2x^2 - x + 3})^2 - 30140067 * \sqrt{2} * (\sqrt{2} * x - \sqrt{2x^2 - x + 3}) + 19989859) / (2 * (\sqrt{2} * x - \sqrt{2x^2 - x + 3})^2 + 10 * \sqrt{2} * (\sqrt{2} * x - \sqrt{2x^2 - x + 3}) - 11)^2$

**maple [A]** time = 0.01, size = 144, normalized size = 1.29

$$\frac{52631 \sqrt{2} \operatorname{arctanh}\left(\frac{(-11x + \frac{17}{2}) \sqrt{2}}{12 \sqrt{-11x + 2(x + \frac{5}{2})^2 - \frac{19}{2}}}\right)}{11943936} - \frac{5}{16 \sqrt{2x^2 - x + 3}} - \frac{149(4x - 1)}{368 \sqrt{2x^2 - x + 3}} + \frac{196043}{165888 \left(x + \frac{5}{2}\right) \sqrt{-11x + 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}}} + \frac{52631}{1990656 \sqrt{-11x + 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}}} + \frac{19399069}{11446272} - \frac{19399069}{4578088} - \frac{3667}{4608 \left(x + \frac{5}{2}\right)^2 \sqrt{-11x + 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)`

[Out]  $-5/16 / (2x^2 - x + 3)^{1/2} - 149/368 * (4x - 1) / (2x^2 - x + 3)^{1/2} + 196043/165888 / (x + 5/2) / (-11x + 2(x + 5/2)^2 - 19/2)^{1/2} + 52631/1990656 / (-11x + 2(x + 5/2)^2 - 19/2)^{1/2}$

$(1/2)+19399069/45785088*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}-52631/11943936*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)}/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)})-3667/4608/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)}$

**maxima [A]** time = 0.98, size = 149, normalized size = 1.33

$$\frac{52631}{11943936} \sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{861085x}{11446272\sqrt{2x^2-x+3}} - \frac{1163201}{3815424\sqrt{2x^2-x+3}} - \frac{3667}{1152(4\sqrt{2x^2-x+3}x^2+20\sqrt{2x^2-x+3}x+25\sqrt{2x^2-x+3})} + \frac{196043}{82944(2\sqrt{2x^2-x+3}x+5\sqrt{2x^2-x+3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3/(2\*x^2-x+3)^(3/2),x, algorithm="maxima")

[Out] 52631/11943936\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 861085/11446272\*x/sqrt(2\*x^2 - x + 3) - 1163201/3815424/sqrt(2\*x^2 - x + 3) - 3667/1152/(4\*sqrt(2\*x^2 - x + 3)\*x^2 + 20\*sqrt(2\*x^2 - x + 3)\*x + 25\*sqrt(2\*x^2 - x + 3)) + 196043/82944/(2\*sqrt(2\*x^2 - x + 3)\*x + 5\*sqrt(2\*x^2 - x + 3))

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^3 (2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^3\*(2\*x^2 - x + 3)^(3/2)),x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^3\*(2\*x^2 - x + 3)^(3/2)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^3 (2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*3/(2\*x\*\*2-x+3)\*\*(3/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*\*3\*(2\*x\*\*2 - x + 3)\*\*(3/2)), x)

$$3.337 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}} dx$$

Optimal. Leaf size=137

$$\frac{369609 - 175877x}{154524672\sqrt{2x^2 - x + 3}} + \frac{430799\sqrt{2x^2 - x + 3}}{107495424(2x + 5)} + \frac{152885\sqrt{2x^2 - x + 3}}{4478976(2x + 5)^2} - \frac{3667\sqrt{2x^2 - x + 3}}{31104(2x + 5)^3} - \frac{3505819 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1289945088\sqrt{2}}$$

**Rubi [A]** time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 1650, 806, 724, 206}

$$\frac{369609 - 175877x}{154524672\sqrt{2x^2 - x + 3}} + \frac{430799\sqrt{2x^2 - x + 3}}{107495424(2x + 5)} + \frac{152885\sqrt{2x^2 - x + 3}}{4478976(2x + 5)^2} - \frac{3667\sqrt{2x^2 - x + 3}}{31104(2x + 5)^3} - \frac{3505819 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1289945088\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] (369609 - 175877\*x)/(154524672\*sqrt[3 - x + 2\*x^2]) - (3667\*sqrt[3 - x + 2\*x^2])/(31104\*(5 + 2\*x)^3) + (152885\*sqrt[3 - x + 2\*x^2])/(4478976\*(5 + 2\*x)^2) + (430799\*sqrt[3 - x + 2\*x^2])/(107495424\*(5 + 2\*x)) - (3505819\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(1289945088\*sqrt[2])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 806

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &

& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}} dx &= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} + \frac{2}{23} \int \frac{\frac{348877271}{26873856} + \frac{119871055x}{4478976} + \frac{73960295x^2}{2239488} + \frac{1302559x^3}{3359232}}{(5+2x)^4\sqrt{3-x+2x^2}} dx \\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} - \frac{\int \frac{\frac{79609325}{124416} - \frac{71248733x}{31104} - \frac{1302559x^2}{31104}}{(5+2x)^3\sqrt{3-x+2x^2}} dx}{2484} \\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{\int \frac{2}{(5+2x)^4} dx}{23} \\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{430}{10} \\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{430}{10} \\
&= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{430}{10}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 95, normalized size = 0.69

$$\frac{24(56754760x^4 + 572739684x^3 + 441046842x^2 + 1257975811x + 1873786587) - 80633837(2x+5)^3\sqrt{4x^2-2x+6} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)}{59337474048(2x+5)^3\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] (24\*(1873786587 + 1257975811\*x + 441046842\*x^2 + 572739684\*x^3 + 56754760\*x^4) - 80633837\*(5 + 2\*x)^3\*Sqrt[6 - 2\*x + 4\*x^2]\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[6 - 2\*x + 4\*x^2])])/(59337474048\*(5 + 2\*x)^3\*Sqrt[3 - x + 2\*x^2])

**IntegrateAlgebraic [A]** time = 0.68, size = 88, normalized size = 0.64

$$\frac{3505819 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{644972544\sqrt{2}} + \frac{56754760x^4 + 572739684x^3 + 441046842x^2 + 1257975811x + 1873786587}{2472394752(2x+5)^3\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*(3 - x + 2\*x^2)^(3/2)), x]

[Out] (1873786587 + 1257975811\*x + 441046842\*x^2 + 572739684\*x^3 + 56754760\*x^4)/(2472394752\*(5 + 2\*x)^3\*Sqrt[3 - x + 2\*x^2]) + (3505819\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(644972544\*Sqrt[2])

**fricas** [A] time = 1.07, size = 141, normalized size = 1.03

$$\frac{80633837\sqrt{2}(16x^5 + 112x^4 + 264x^3 + 280x^2 + 325x + 375)\log\left(\frac{-24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right) + 48(56754760x^4 + 572739684x^3 + 441046842x^2 + 1257975811x + 1873786587)\sqrt{2x^2-x+3}}{118674948096(16x^5 + 112x^4 + 264x^3 + 280x^2 + 325x + 375)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(3/2), x, algorithm="fricas")

[Out] 1/118674948096\*(80633837\*sqrt(2)\*(16\*x^5 + 112\*x^4 + 264\*x^3 + 280\*x^2 + 325\*x + 375)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(56754760\*x^4 + 572739684\*x^3 + 441046842\*x^2 + 1257975811\*x + 1873786587)\*sqrt(2\*x^2 - x + 3))/(16\*x^5 + 112\*x^4 + 264\*x^3 + 280\*x^2 + 325\*x + 375)

**giac** [B] time = 0.29, size = 271, normalized size = 1.98

$$\frac{\frac{3505819}{2579890176}\sqrt{2}\log\left(\frac{-2\sqrt{2x^2-x+3}}{-2\sqrt{2x^2-x+3}}\right) + \frac{3505819}{2579890176}\sqrt{2}\log\left(\frac{-2\sqrt{2x^2-x+3}}{-2\sqrt{2x^2-x+3}}\right) - \frac{175877x-369609}{154524672\sqrt{2x^2-x+3}} - \frac{\sqrt{2}\left(10398764\sqrt{2}\left(\sqrt{2x^2-x+3}\right)^3 - 303070900\left(\sqrt{2x^2-x+3}\right)^4 - 529738052\sqrt{2}\left(\sqrt{2x^2-x+3}\right)^5 + 3644644652\left(\sqrt{2x^2-x+3}\right)^6 - 2612608649\sqrt{2}\left(\sqrt{2x^2-x+3}\right)^7 + 1052284471\right)}{214990848\left(2\left(\sqrt{2x^2-x+3}\right)^2 + 10\sqrt{2}\left(\sqrt{2x^2-x+3}\right) - 11\right)}}{214990848\left(2\left(\sqrt{2x^2-x+3}\right)^2 + 10\sqrt{2}\left(\sqrt{2x^2-x+3}\right) - 11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(3/2), x, algorithm="giac")

[Out] -3505819/2579890176\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 3505819/2579890176\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 1/154524672\*(175877\*x - 369609)/sqrt(2\*x^2 - x + 3) - 1/214990848\*sqrt(2)\*(10398764\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^5 - 303070900\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^4 - 529738052\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 + 3644644652\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 - 2612608649\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1052284471)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^3

**maple** [A] time = 0.01, size = 151, normalized size = 1.10

$$\frac{3505819\sqrt{2}\operatorname{arctanh}\left(\frac{\left(-11+\frac{5}{2}\right)\sqrt{2}}{12\sqrt{-11+2\left(x+\frac{5}{2}\right)\frac{19}{2}}}\right)}{2579890176} + \frac{\frac{5x}{6} - \frac{5}{181}}{\sqrt{2x^2-x+3}} - \frac{3127169}{35831808\left(x+\frac{5}{2}\right)\sqrt{-11x+2\left(x+\frac{5}{2}\right)\frac{19}{2}}} + \frac{3505819}{429981696\sqrt{-11x+2\left(x+\frac{5}{2}\right)\frac{19}{2}}} - \frac{261644215(4x-1)}{9889579008\sqrt{-11x+2\left(x+\frac{5}{2}\right)\frac{19}{2}}} + \frac{314233}{995328\left(x+\frac{5}{2}\right)^2\sqrt{-11x+2\left(x+\frac{5}{2}\right)\frac{19}{2}}} - \frac{3667}{13824\left(x+\frac{5}{2}\right)^3\sqrt{-11x+2\left(x+\frac{5}{2}\right)\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(3/2), x)

[Out] 5/184\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-3127169/35831808/(x+5/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+3505819/429981696/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-261644215/9889579008\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3505819/2579890176\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))+314233/995328/(x+5/2)^2/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)

**maxima** [A] time = 1.01, size = 217, normalized size = 1.58

$$\frac{3505819}{2579890176} \sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23(2x+5)} - \frac{17\sqrt{23}}{23(2x+5)}\right) + \frac{7094345x}{2472394752\sqrt{2x^2-x+3}} + \frac{6128291}{824131584\sqrt{2x^2-x+3}} - \frac{3667}{1728(8\sqrt{2x^2-x+3}x^3+60\sqrt{2x^2-x+3}x^2+150\sqrt{2x^2-x+3}x+125\sqrt{2x^2-x+3})} + \frac{314233}{248832(4\sqrt{2x^2-x+3}x^2+20\sqrt{2x^2-x+3}x+25\sqrt{2x^2-x+3})} - \frac{3127169}{17915904(2\sqrt{2x^2-x+3}x+5\sqrt{2x^2-x+3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(3/2), x, algorithm="maxima")

[Out] 3505819/2579890176\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 7094345/2472394752\*x/sqrt(2\*x^2 - x + 3) + 6128291/824131584/sqrt(2\*x^2 - x + 3) - 3667/1728/(8\*sqrt(2\*x^2 - x + 3)\*x^3 + 60\*sqrt(2\*x^2 - x + 3)\*x^2 + 150\*sqrt(2\*x^2 - x + 3)\*x + 125\*sqrt(2\*x^2 - x + 3)) + 314233/248832/(4\*sqrt(2\*x^2 - x + 3)\*x^2 + 20\*sqrt(2\*x^2 - x + 3)\*x + 25\*sqrt(2\*x^2 - x + 3)) - 3127169/17915904/(2\*sqrt(2\*x^2 - x + 3)\*x + 5\*sqrt(2\*x^2 - x + 3))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^4 (2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^4\*(2\*x^2 - x + 3)^(3/2)), x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^4\*(2\*x^2 - x + 3)^(3/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^4 (2x^2 - x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*4/(2\*x\*\*2-x+3)\*\*(3/2), x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*\*4\*(2\*x\*\*2 - x + 3)\*\*(3/2)), x)



$$3.338 \quad \int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=105

$$\frac{4(18982 - 20383x)}{1587\sqrt{2x^2 - x + 3}} + \frac{5}{4}x\sqrt{2x^2 - x + 3} + \frac{247}{16}\sqrt{2x^2 - x + 3} - \frac{4(346 - 533x)}{69(2x^2 - x + 3)^{3/2}} - \frac{1471 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{4(18982 - 20383x)}{1587\sqrt{2x^2 - x + 3}} + \frac{5}{4}x\sqrt{2x^2 - x + 3} + \frac{247}{16}\sqrt{2x^2 - x + 3} - \frac{4(346 - 533x)}{69(2x^2 - x + 3)^{3/2}} - \frac{1471 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((5 + 2\*x)^2\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-4\*(346 - 533\*x))/(69\*(3 - x + 2\*x^2)^(3/2)) + (4\*(18982 - 20383\*x))/(1587\*sqrt[3 - x + 2\*x^2]) + (247\*sqrt[3 - x + 2\*x^2])/16 + (5\*x\*sqrt[3 - x + 2\*x^2])/4 - (1471\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(32\*sqrt[2])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} dx &= -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{-145 - \frac{1725x}{2} + 2415x^2 + \frac{3657x^3}{2} + 345}{(3-x+2x^2)^{3/2}} \\
&= -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{4 \int \frac{\frac{33327}{2} + \frac{46023x}{4} + \frac{7935x^2}{4}}{\sqrt{3-x+2x^2}}}{1587} \\
&= -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{5}{4} x \sqrt{3-x+2x^2} + \int \\
&= -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16} \sqrt{3-x+2x^2} + \frac{5}{4} \\
&= -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16} \sqrt{3-x+2x^2} + \frac{5}{4} \\
&= -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16} \sqrt{3-x+2x^2} + \frac{5}{4}
\end{aligned}$$

**Mathematica [A]** time = 0.69, size = 65, normalized size = 0.62

$$\frac{126960x^5 + 1440996x^4 - 3764360x^3 + 8639625x^2 - 6410082x + 6663133}{25392(2x^2 - x + 3)^{3/2}} - \frac{1471 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[((5 + 2\*x)^2\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(5/2), x]

[Out] (6663133 - 6410082\*x + 8639625\*x^2 - 3764360\*x^3 + 1440996\*x^4 + 126960\*x^5)/(25392\*(3 - x + 2\*x^2)^(3/2)) - (1471\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(32\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.68, size = 80, normalized size = 0.76

$$\frac{126960x^5 + 1440996x^4 - 3764360x^3 + 8639625x^2 - 6410082x + 6663133}{25392(2x^2 - x + 3)^{3/2}} - \frac{1471 \log\left(2\sqrt{2} \sqrt{2x^2 - x + 3} - 4x + 1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((5 + 2\*x)^2\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(5/2), x]

[Out] (6663133 - 6410082\*x + 8639625\*x^2 - 3764360\*x^3 + 1440996\*x^4 + 126960\*x^5)/(25392\*(3 - x + 2\*x^2)^(3/2)) - (1471\*Log[1 - 4\*x + 2\*sqrt[2]\*sqrt[3 - x + 2\*x^2]])/(32\*sqrt[2])

**fricas** [A] time = 1.08, size = 122, normalized size = 1.16

$$\frac{2334477\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(126960x^5 + 1440996x^4 - 3764360x^3 + 8639625x^2 - 6410082x + 6663133)\sqrt{2x^2 - x + 3}}{203136(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 1/203136\*(2334477\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) + 8\*(126960\*x^5 + 1440996\*x^4 - 3764360\*x^3 + 8639625\*x^2 - 6410082\*x + 6663133)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac** [A] time = 0.22, size = 71, normalized size = 0.68

$$-\frac{1471}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2x-x+3}\right)+1\right)+\frac{\left(\left(4(1587(20x+227)x-941090)x+8639625\right)x-6410082\right)x+6663133}{25392(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="giac")

[Out] -1471/64\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) + 1/25392\*((4\*(1587\*(20\*x + 227)\*x - 941090)\*x + 8639625)\*x - 6410082)\*x + 6663133)/(2\*x^2 - x + 3)^(3/2)

**maple** [B] time = 0.02, size = 180, normalized size = 1.71

$$\frac{5x^5}{(2x^2-x+3)^{\frac{3}{2}}} + \frac{227x^4}{4(2x^2-x+3)^{\frac{3}{2}}} - \frac{1471x^3}{48(2x^2-x+3)^{\frac{3}{2}}} + \frac{19073x^2}{64(2x^2-x+3)^{\frac{3}{2}}} - \frac{32257x}{512(2x^2-x+3)^{\frac{3}{2}}} - \frac{1471x}{32\sqrt{2x^2-x+3}} + \frac{1471\sqrt{2}\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(-\frac{1}{4}\right)}{23}\right)}{64} - \frac{162931(4x-1)}{50784\sqrt{2x^2-x+3}} - \frac{753223(4x-1)}{141312(2x^2-x+3)^{\frac{3}{2}}} + \frac{577397}{2048(2x^2-x+3)^{\frac{3}{2}}} - \frac{1471}{128\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x)

[Out] -162931/50784\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-753223/141312\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+5\*x^5/(2\*x^2-x+3)^(3/2)+227/4\*x^4/(2\*x^2-x+3)^(3/2)-1471/48\*x^3/(2\*x^2

$-x+3)^{3/2}+19073/64*x^2/(2*x^2-x+3)^{3/2}-32257/512*x/(2*x^2-x+3)^{3/2}+1471/64*2^{1/2}*arcsinh(4/23*23^{1/2}*(x-1/4))-1471/32/(2*x^2-x+3)^{1/2}*x+577397/2048/(2*x^2-x+3)^{3/2}-1471/128/(2*x^2-x+3)^{1/2}$

**maxima** [B] time = 0.98, size = 219, normalized size = 2.09

$$\frac{5x^5}{(2x^2-x+3)^{3/2}} + \frac{227x^4}{4(2x^2-x+3)^{3/2}} + \frac{1471}{50784} \left( \frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{3/2}} - \frac{71}{\sqrt{2x^2-x+3}} + \frac{805x}{(2x^2-x+3)^{3/2}} - \frac{3243}{(2x^2-x+3)^{3/2}} \right) + \frac{1471}{64} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{104441}{25392} \sqrt{2x^2-x+3} - \frac{383581x}{12696\sqrt{2x^2-x+3}} + \frac{321x^2}{(2x^2-x+3)^{3/2}} - \frac{15965}{4232\sqrt{2x^2-x+3}} - \frac{4147x}{46(2x^2-x+3)^{3/2}} + \frac{42883}{138(2x^2-x+3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)^2\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out]  $5*x^5/(2*x^2 - x + 3)^{3/2} + 227/4*x^4/(2*x^2 - x + 3)^{3/2} + 1471/50784*x*(284*x/\sqrt{2*x^2 - x + 3} - 3174*x^2/(2*x^2 - x + 3)^{3/2} - 71/\sqrt{2*x^2 - x + 3} + 805*x/(2*x^2 - x + 3)^{3/2} - 3243/(2*x^2 - x + 3)^{3/2}) + 1471/64*\sqrt{2}*arcsinh(1/23*\sqrt{23}*(4*x - 1)) - 104441/25392*\sqrt{2*x^2 - x + 3} - 383581/12696*x/\sqrt{2*x^2 - x + 3} + 321*x^2/(2*x^2 - x + 3)^{3/2} - 15965/4232/\sqrt{2*x^2 - x + 3} - 4147/46*x/(2*x^2 - x + 3)^{3/2} + 42883/138/(2*x^2 - x + 3)^{3/2}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)^2 (5x^4 - x^3 + 3x^2 + x + 2)}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2\*x + 5)^2\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x^2 - x + 3)^(5/2),x)

[Out] int(((2\*x + 5)^2\*(x + 3\*x^2 - x^3 + 5\*x^4 + 2))/(2\*x^2 - x + 3)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2 (5x^4 - x^3 + 3x^2 + x + 2)}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*\*2\*(5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((2\*x + 5)\*\*2\*(5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/(2\*x\*\*2 - x + 3)\*\*(5/2), x)

$$3.339 \quad \int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=86

$$\frac{6055 - 28981x}{3174\sqrt{2x^2 - x + 3}} + \frac{5}{4}\sqrt{2x^2 - x + 3} + \frac{373x - 53}{69(2x^2 - x + 3)^{3/2}} - \frac{71 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

**Rubi [A]** time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {1660, 640, 619, 215}

$$\frac{6055 - 28981x}{3174\sqrt{2x^2 - x + 3}} + \frac{5}{4}\sqrt{2x^2 - x + 3} - \frac{53 - 373x}{69(2x^2 - x + 3)^{3/2}} - \frac{71 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(5/2), x]

[Out] -(53 - 373\*x)/(69\*(3 - x + 2\*x^2)^(3/2)) + (6055 - 28981\*x)/(3174\*sqrt[3 - x + 2\*x^2]) + (5\*sqrt[3 - x + 2\*x^2])/4 - (71\*ArcSinh[(1 - 4\*x)/sqrt[23]])/(8\*sqrt[2])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} dx &= -\frac{53-373x}{69(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{233}{4} + 483x^2 + \frac{345x^3}{2}}{(3-x+2x^2)^{3/2}} dx \\
&= -\frac{53-373x}{69(3-x+2x^2)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{4 \int \frac{\frac{52371}{16} + \frac{7935x}{8}}{\sqrt{3-x+2x^2}} dx}{1587} \\
&= -\frac{53-373x}{69(3-x+2x^2)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} + \frac{71}{8} \int \dots \\
&= -\frac{53-373x}{69(3-x+2x^2)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} + \dots \\
&= -\frac{53-373x}{69(3-x+2x^2)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 60, normalized size = 0.70

$$\frac{31740x^4 - 147664x^3 + 185337x^2 - 199290x + 102869}{6348(2x^2 - x + 3)^{3/2}} + \frac{71 \sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```

[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2), x
]

```

[Out]  $(102869 - 199290x + 185337x^2 - 147664x^3 + 31740x^4)/(6348(3 - x + 2x^2)^{(3/2)}) + (71 \operatorname{ArcSinh}[(-1 + 4x)/\sqrt{23}])/(8\sqrt{2})$

**IntegrateAlgebraic [A]** time = 0.87, size = 75, normalized size = 0.87

$$\frac{31740x^4 - 147664x^3 + 185337x^2 - 199290x + 102869}{6348(2x^2 - x + 3)^{3/2}} - \frac{71 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((5 + 2\*x)\*(2 + x + 3\*x^2 - x^3 + 5\*x^4))/(3 - x + 2\*x^2)^(5/2), x]

[Out]  $(102869 - 199290x + 185337x^2 - 147664x^3 + 31740x^4)/(6348(3 - x + 2x^2)^{(3/2)}) - (71 \operatorname{Log}[1 - 4x + 2\sqrt{2}\sqrt{3 - x + 2x^2}])/(8\sqrt{2})$

**fricas [A]** time = 0.92, size = 117, normalized size = 1.36

$$\frac{112677\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) + 8(31740x^4 - 147664x^3 + 185337x^2 - 199290x + 102869)\sqrt{2x^2 - x + 3}}{50784(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out]  $1/50784*(112677*\sqrt{2}*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(-4*\sqrt{2}*\sqrt{2*x^2 - x + 3}*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 147664*x^3 + 185337*x^2 - 199290*x + 102869)*\sqrt{2*x^2 - x + 3})/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$

**giac [A]** time = 0.21, size = 66, normalized size = 0.77

$$-\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{((4(7935x - 36916)x + 185337)x - 199290)x + 102869}{6348(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2\*x)\*(5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="giac")

[Out]  $-71/16*\sqrt{2}*\log(-2*\sqrt{2}*(\sqrt{2}*x - \sqrt{2*x^2 - x + 3})) + 1) + 1/6348*((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x + 3)^{(3/2)}$



**maple [B]** time = 0.01, size = 163, normalized size = 1.90

$$\frac{5x^4}{(2x^2-x+3)^{\frac{5}{2}}} - \frac{71x^3}{12(2x^2-x+3)^{\frac{3}{2}}} + \frac{401x^2}{16(2x^2-x+3)^{\frac{3}{2}}} - \frac{945x}{128(2x^2-x+3)^{\frac{3}{2}}} - \frac{71x}{8\sqrt{2x^2-x+3}} + \frac{71\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16} + \frac{\frac{643x}{3174} - \frac{643}{12696}}{\sqrt{2x^2-x+3}} - \frac{2327(4x-1)}{35328(2x^2-x+3)^{\frac{3}{2}}} + \frac{11749}{512(2x^2-x+3)^{\frac{3}{2}}} - \frac{71}{32\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)`

[Out] `643/12696*(4*x-1)/(2*x^2-x+3)^(1/2)-2327/35328*(4*x-1)/(2*x^2-x+3)^(3/2)+5/(2*x^2-x+3)^(3/2)*x^4-71/12/(2*x^2-x+3)^(3/2)*x^3+401/16/(2*x^2-x+3)^(3/2)*x^2-945/128/(2*x^2-x+3)^(3/2)*x+71/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-71/8/(2*x^2-x+3)^(1/2)*x+11749/512/(2*x^2-x+3)^(3/2)-71/32/(2*x^2-x+3)^(1/2)`

**maxima [B]** time = 0.97, size = 202, normalized size = 2.35

$$\frac{5x^4}{(2x^2-x+3)^{\frac{5}{2}}} + \frac{71}{12696} \left( \frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2-x+3}} + \frac{805x}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2x^2-x+3)^{\frac{3}{2}}} \right) + \frac{71}{16} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{5041}{6348} \sqrt{2x^2-x+3} - \frac{10007x}{3174\sqrt{2x^2-x+3}} + \frac{59x^2}{2(2x^2-x+3)^{\frac{3}{2}}} - \frac{2959}{2116\sqrt{2x^2-x+3}} - \frac{807x}{92(2x^2-x+3)^{\frac{3}{2}}} + \frac{7603}{276(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")`

[Out] `5*x^4/(2*x^2-x+3)^(3/2)+71/12696*x*(284*x/sqrt(2*x^2-x+3)-3174*x^2/(2*x^2-x+3)^(3/2)-71/sqrt(2*x^2-x+3)+805*x/(2*x^2-x+3)^(3/2)-3243/(2*x^2-x+3)^(3/2))+71/16*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x-1))-5041/6348*sqrt(2*x^2-x+3)-10007/3174*x/sqrt(2*x^2-x+3)+59/2*x^2/(2*x^2-x+3)^(3/2)-2959/2116/sqrt(2*x^2-x+3)-807/92*x/(2*x^2-x+3)^(3/2)+7603/276/(2*x^2-x+3)^(3/2)`

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((2*x+5)*(x+3*x^2-x^3+5*x^4+2))/(2*x^2-x+3)^(5/2),x)`

[Out] `int(((2*x+5)*(x+3*x^2-x^3+5*x^4+2))/(2*x^2-x+3)^(5/2),x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

$$3.340 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=68

$$\frac{219x + 89}{276(2x^2 - x + 3)^{3/2}} - \frac{2604x + 1465}{2116\sqrt{2x^2 - x + 3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {1660, 12, 619, 215}

$$\frac{219x + 89}{276(2x^2 - x + 3)^{3/2}} - \frac{2604x + 1465}{2116\sqrt{2x^2 - x + 3}} - \frac{5 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/(3 - x + 2\*x^2)^(5/2), x]

[Out] (89 + 219\*x)/(276\*(3 - x + 2\*x^2)^(3/2)) - (1465 + 2604\*x)/(2116\*Sqrt[3 - x + 2\*x^2]) - (5\*ArcSinh[(1 - 4\*x)/Sqrt[23]])/(4\*Sqrt[2])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 1660

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b\*x + c\*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b\*x + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b\*x +

$c*x^2, x], x, 1] \}, \text{Simp}[\{(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^{(p + 1)} / ((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)} * \text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1]$

### Rubi steps

$$\begin{aligned}
 \int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(3 - x + 2x^2)^{5/2}} dx &= \frac{89 + 219x}{276(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{159}{16} + \frac{207x}{8} + \frac{345x^2}{4}}{(3 - x + 2x^2)^{3/2}} dx \\
 &= \frac{89 + 219x}{276(3 - x + 2x^2)^{3/2}} - \frac{1465 + 2604x}{2116\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{7935}{16\sqrt{3 - x + 2x^2}} dx}{1587} \\
 &= \frac{89 + 219x}{276(3 - x + 2x^2)^{3/2}} - \frac{1465 + 2604x}{2116\sqrt{3 - x + 2x^2}} + \frac{5}{4} \int \frac{1}{\sqrt{3 - x + 2x^2}} dx \\
 &= \frac{89 + 219x}{276(3 - x + 2x^2)^{3/2}} - \frac{1465 + 2604x}{2116\sqrt{3 - x + 2x^2}} + \frac{5 \text{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{23}}} dx, x, -1 + 4x \right)}{4\sqrt{46}} \\
 &= \frac{89 + 219x}{276(3 - x + 2x^2)^{3/2}} - \frac{1465 + 2604x}{2116\sqrt{3 - x + 2x^2}} - \frac{5 \sinh^{-1} \left( \frac{1 - 4x}{\sqrt{23}} \right)}{4\sqrt{2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 55, normalized size = 0.81

$$\frac{5 \sinh^{-1} \left( \frac{4x-1}{\sqrt{23}} \right)}{4\sqrt{2}} - \frac{7812x^3 + 489x^2 + 7002x + 5569}{3174(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/(3 - x + 2\*x^2)^(5/2), x]

[Out] -1/3174\*(5569 + 7002\*x + 489\*x^2 + 7812\*x^3)/(3 - x + 2\*x^2)^(3/2) + (5\*ArcSinh[(-1 + 4\*x)/Sqrt[23]])/(4\*Sqrt[2])

**IntegrateAlgebraic [A]** time = 0.61, size = 70, normalized size = 1.03

$$\frac{-7812x^3 - 489x^2 - 7002x - 5569}{3174(2x^2 - x + 3)^{3/2}} - \frac{5 \log(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/(3 - x + 2\*x^2)^(5/2), x]

[Out] (-5569 - 7002\*x - 489\*x^2 - 7812\*x^3)/(3174\*(3 - x + 2\*x^2)^(3/2)) - (5\*Log[1 - 4\*x + 2\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2]])/(4\*Sqrt[2])

**fricas [B]** time = 0.93, size = 112, normalized size = 1.65

$$\frac{7935\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9) \log(-4\sqrt{2}\sqrt{2x^2 - x + 3}(4x - 1) - 32x^2 + 16x - 25) - 8(7812x^3 + 489x^2 + 7002x + 5569)\sqrt{2x^2 - x + 3}}{25392(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 1/25392\*(7935\*sqrt(2)\*(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)\*log(-4\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(4\*x - 1) - 32\*x^2 + 16\*x - 25) - 8\*(7812\*x^3 + 489\*x^2 + 7002\*x + 5569)\*sqrt(2\*x^2 - x + 3))/(4\*x^4 - 4\*x^3 + 13\*x^2 - 6\*x + 9)

**giac [A]** time = 0.30, size = 62, normalized size = 0.91

$$-\frac{5}{8}\sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) - \frac{3((2604x + 163)x + 2334)x + 5569}{3174(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x, algorithm="giac")

[Out] -5/8\*sqrt(2)\*log(-2\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) + 1) - 1/3174\*(3\*((2604\*x + 163)\*x + 2334)\*x + 5569)/(2\*x^2 - x + 3)^(3/2)

**maple [B]** time = 0.01, size = 146, normalized size = 2.15

$$-\frac{5x^3}{6(2x^2 - x + 3)^{3/2}} - \frac{x^2}{8(2x^2 - x + 3)^{3/2}} - \frac{47x}{64(2x^2 - x + 3)^{3/2}} - \frac{5x}{4\sqrt{2x^2 - x + 3}} + \frac{5\sqrt{2} \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x - \frac{1}{4}\right)}{23}\right)}{8} - \frac{271}{768(2x^2 - x + 3)^{3/2}} + \frac{\frac{2423x}{4416} - \frac{2423}{17664}}{(2x^2 - x + 3)^{3/2}} + \frac{\frac{692x}{1587} - \frac{173}{1587}}{\sqrt{2x^2 - x + 3}} - \frac{5}{16\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)/(2\*x^2-x+3)^(5/2), x)

[Out]  $-5/6/(2*x^2-x+3)^{(3/2)}*x^3-1/8/(2*x^2-x+3)^{(3/2)}*x^2-47/64/(2*x^2-x+3)^{(3/2)}*x-271/768/(2*x^2-x+3)^{(3/2)}+2423/17664*(4*x-1)/(2*x^2-x+3)^{(3/2)}+173/1587*(4*x-1)/(2*x^2-x+3)^{(1/2)}-5/4/(2*x^2-x+3)^{(1/2)}*x-5/16/(2*x^2-x+3)^{(1/2)}+5/8*2^{(1/2)}*\operatorname{arcsinh}(4/23*23^{(1/2)}*(x-1/4))$

**maxima** [B] time = 0.98, size = 185, normalized size = 2.72

$$\frac{5}{6348} \left( \frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{3/2}} - \frac{71}{\sqrt{2x^2-x+3}} + \frac{805x}{(2x^2-x+3)^{3/2}} - \frac{3243}{(2x^2-x+3)^{3/2}} \right) + \frac{5}{8} \sqrt{2} \operatorname{arcsinh} \left( \frac{1}{23} \sqrt{23} (4x-1) \right) - \frac{355}{3174} \sqrt{2x^2-x+3} - \frac{58x}{1587 \sqrt{2x^2-x+3}} + \frac{x^2}{2(2x^2-x+3)^{3/2}} - \frac{1897}{6348 \sqrt{2x^2-x+3}} - \frac{95x}{276(2x^2-x+3)^{3/2}} + \frac{41}{276(2x^2-x+3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")`

[Out]  $5/6348*x*(284*x/\operatorname{sqrt}(2*x^2-x+3)-3174*x^2/(2*x^2-x+3)^{(3/2)}-71/\operatorname{sqrt}(2*x^2-x+3)+805*x/(2*x^2-x+3)^{(3/2)}-3243/(2*x^2-x+3)^{(3/2)})+5/8*\operatorname{sqrt}(2)*\operatorname{arcsinh}(1/23*\operatorname{sqrt}(23)*(4*x-1))-355/3174*\operatorname{sqrt}(2*x^2-x+3)-58/1587*x/\operatorname{sqrt}(2*x^2-x+3)+1/2*x^2/(2*x^2-x+3)^{(3/2)}-1897/6348/\operatorname{sqrt}(2*x^2-x+3)-95/276*x/(2*x^2-x+3)^{(3/2)}+41/276/(2*x^2-x+3)^{(3/2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+3*x^2-x^3+5*x^4+2)/(2*x^2-x+3)^(5/2),x)`

[Out] `int((x+3*x^2-x^3+5*x^4+2)/(2*x^2-x+3)^(5/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)`

[Out] `Integral((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)`

$$3.341 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=85

$$\frac{917x + 1191}{9936 (2x^2 - x + 3)^{3/2}} - \frac{146729x + 335337}{1371168\sqrt{2x^2 - x + 3}} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

**Rubi** [A] time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1646, 12, 724, 206}

$$\frac{917x + 1191}{9936 (2x^2 - x + 3)^{3/2}} - \frac{146729x + 335337}{1371168\sqrt{2x^2 - x + 3}} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*(3 - x + 2\*x^2)^(5/2)),x]

[Out] (1191 + 917\*x)/(9936\*(3 - x + 2\*x^2)^(3/2)) - (335337 + 146729\*x)/(1371168\*Sqrt[3 - x + 2\*x^2]) - (3667\*ArcTanh[(17 - 22\*x)/(12\*Sqrt[2]\*Sqrt[3 - x + 2\*x^2])])/(31104\*Sqrt[2])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 1646

```

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(5 + 2x)(3 - x + 2x^2)^{5/2}} dx &= \frac{1191 + 917x}{9936(3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{-\frac{1877}{576} + \frac{695x}{18} + \frac{345x^2}{4}}{(5 + 2x)(3 - x + 2x^2)^{3/2}} dx \\
&= \frac{1191 + 917x}{9936(3 - x + 2x^2)^{3/2}} - \frac{335337 + 146729x}{1371168\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{1939843}{6912(5+2x)\sqrt{3-x+2x^2}} dx}{1587} \\
&= \frac{1191 + 917x}{9936(3 - x + 2x^2)^{3/2}} - \frac{335337 + 146729x}{1371168\sqrt{3 - x + 2x^2}} + \frac{3667 \int \frac{1}{(5+2x)\sqrt{3-x+2x^2}} dx}{5184} \\
&= \frac{1191 + 917x}{9936(3 - x + 2x^2)^{3/2}} - \frac{335337 + 146729x}{1371168\sqrt{3 - x + 2x^2}} - \frac{3667 \operatorname{Subst}\left(\int \frac{1}{288-x^2} dx, x, \frac{1}{\sqrt{3-x+2x^2}}\right)}{2592} \\
&= \frac{1191 + 917x}{9936(3 - x + 2x^2)^{3/2}} - \frac{335337 + 146729x}{1371168\sqrt{3 - x + 2x^2}} - \frac{3667 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.47, size = 80, normalized size = 0.94

$$\frac{-3667 \log\left(12\sqrt{4x^2 - 2x + 6} - 22x + 17\right) - \frac{12\sqrt{2}(293458x^3 + 523945x^2 - 21696x + 841653)}{529(2x^2 - x + 3)^{3/2}} + 3667 \log(2x + 5)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```

[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)), x
]

```



[Out]  $((-12*\text{Sqrt}[2]*(841653 - 21696*x + 523945*x^2 + 293458*x^3))/(529*(3 - x + 2*x^2)^{(3/2)}) + 3667*\text{Log}[5 + 2*x] - 3667*\text{Log}[17 - 22*x + 12*\text{Sqrt}[6 - 2*x + 4*x^2]])/(31104*\text{Sqrt}[2])$

**IntegrateAlgebraic [A]** time = 0.66, size = 76, normalized size = 0.89

$$\frac{3667 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{15552\sqrt{2}} + \frac{-293458x^3 - 523945x^2 + 21696x - 841653}{1371168(2x^2 - x + 3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)\*(3 - x + 2\*x^2)^(5/2)),x]

[Out]  $(-841653 + 21696*x - 523945*x^2 - 293458*x^3)/(1371168*(3 - x + 2*x^2)^{(3/2)}) + (3667*\text{ArcTanh}[5/6 + x/3 - \text{Sqrt}[3 - x + 2*x^2]/(3*\text{Sqrt}[2])])/(15552*\text{Sqrt}[2])$

**fricas [A]** time = 0.84, size = 126, normalized size = 1.48

$$\frac{1939843\sqrt{2}(4x^4 - 4x^3 + 13x^2 - 6x + 9)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right) - 48(293458x^3 + 523945x^2 - 21696x + 841653)\sqrt{2x^2-x+3}}{65816064(4x^4 - 4x^3 + 13x^2 - 6x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)/(2\*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out]  $1/65816064*(1939843*\text{sqrt}(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*\log(-(24*\text{sqrt}(2)*\text{sqrt}(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*\text{sqrt}(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)$

**giac [A]** time = 0.23, size = 92, normalized size = 1.08

$$-\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2-x+3}\right|\right) + \frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2-x+3}\right|\right) - \frac{((293458x + 523945)x - 21696)x + 841653}{1371168(2x^2 - x + 3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out]  $-3667/62208*\text{sqrt}(2)*\log(\text{abs}(-2*\text{sqrt}(2)*x + \text{sqrt}(2) + 2*\text{sqrt}(2*x^2 - x + 3))) + 3667/62208*\text{sqrt}(2)*\log(\text{abs}(-2*\text{sqrt}(2)*x - 11*\text{sqrt}(2) + 2*\text{sqrt}(2*x^2 - x$

+ 3))) - 1/1371168\*((293458\*x + 523945)\*x - 21696)\*x + 841653)/(2\*x^2 - x + 3)^(3/2)

**maple [B]** time = 0.01, size = 190, normalized size = 2.24

$$\frac{5x^2}{4(2x^2-x+3)^{3/2}} + \frac{59x}{32(2x^2-x+3)^{3/2}} - \frac{3667\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+\frac{17}{2})\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{62208} - \frac{1597}{384(2x^2-x+3)^{3/2}} - \frac{3817(4x-1)}{2944(2x^2-x+3)^{3/2}} - \frac{3817(4x-1)}{4232\sqrt{2x^2-x+3}} + \frac{3667}{1728(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2})^{3/2}} + \frac{40337}{9936} - \frac{48137}{39744} + \frac{4800103}{1371168} - \frac{4800103}{5484672} + \frac{3667}{10368\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)/(2\*x^2-x+3)^(5/2), x)

[Out] -5/4/(2\*x^2-x+3)^(3/2)\*x^2+59/32/(2\*x^2-x+3)^(3/2)\*x-1597/384/(2\*x^2-x+3)^(3/2)-3817/2944\*(4\*x-1)/(2\*x^2-x+3)^(3/2)-3817/4232\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+3667/1728/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+40337/39744\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+4800103/5484672\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+3667/10368/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/62208\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))

**maxima [A]** time = 0.97, size = 110, normalized size = 1.29

$$\frac{3667}{62208} \sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{146729x}{1371168\sqrt{2x^2-x+3}} - \frac{5x^2}{4(2x^2-x+3)^{3/2}} + \frac{173881}{457056\sqrt{2x^2-x+3}} + \frac{7127x}{9936(2x^2-x+3)^{3/2}} - \frac{5813}{3312(2x^2-x+3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)/(2\*x^2-x+3)^(5/2), x, algorithm="maxima")

[Out] 3667/62208\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) - 146729/1371168\*x/sqrt(2\*x^2 - x + 3) - 5/4\*x^2/(2\*x^2 - x + 3)^(3/2) + 173881/457056/sqrt(2\*x^2 - x + 3) + 7127/9936\*x/(2\*x^2 - x + 3)^(3/2) - 5813/3312/(2\*x^2 - x + 3)^(3/2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)(2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)\*(2\*x^2 - x + 3)^(5/2)), x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)\*(2\*x^2 - x + 3)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)(2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*(2\*x\*\*2 - x + 3)\*\*(5/2)), x)

$$3.342 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=110

$$\frac{1255878 - 62021x}{24681024\sqrt{2x^2 - x + 3}} - \frac{3667\sqrt{2x^2 - x + 3}}{186624(2x + 5)} + \frac{2203x + 9897}{357696(2x^2 - x + 3)^{3/2}} - \frac{2821 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

**Rubi [A]** time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {1646, 806, 724, 206}

$$\frac{1255878 - 62021x}{24681024\sqrt{2x^2 - x + 3}} - \frac{3667\sqrt{2x^2 - x + 3}}{186624(2x + 5)} + \frac{2203x + 9897}{357696(2x^2 - x + 3)^{3/2}} - \frac{2821 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(5/2)), x]

[Out] (9897 + 2203\*x)/(357696\*(3 - x + 2\*x^2)^(3/2)) - (1255878 - 62021\*x)/(24681024\*sqrt[3 - x + 2\*x^2]) - (3667\*sqrt[3 - x + 2\*x^2])/(186624\*(5 + 2\*x)) - (2821\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(2239488\*sqrt[2])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 806

Int[((d\_.) + (e\_.)\*(x\_)^m)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^p, x\_Symbol] :> -Simp[((e\*f - d\*g)\*(d + e\*x)^(m+1)\*(a + b\*x + c\*x^2)^(p+1))/(2\*(p+1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^m

+ 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &  
 & NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m +  
 2\*p + 3], 0]

### Rule 1646

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p  
 \_), x\_Symbol] :> With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + b\*x + c\*x  
 ^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x],  
 x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + b\*x + c\*x^2, x], x  
 , 1]}, Simp[((b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*(a + b\*x + c\*x^2)^(p + 1))/((p  
 + 1)\*(b^2 - 4\*a\*c)), x] + Dist[1/((p + 1)\*(b^2 - 4\*a\*c)), Int[(d + e\*x)^m\*  
 (a + b\*x + c\*x^2)^(p + 1)\*ExpandToSum[((p + 1)\*(b^2 - 4\*a\*c)\*Q)/(d + e\*x)^m  
 - ((2\*p + 3)\*(2\*c\*f - b\*g))/(d + e\*x)^m, x], x]] /; FreeQ[{a, b, c, d,  
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2  
 , 0] && LtQ[p, -1] && ILtQ[m, 0]

### Rubi steps

$$\int \frac{2 + x + 3x^2 - x^3 + 5x^4}{(5 + 2x)^2 (3 - x + 2x^2)^{5/2}} dx = \frac{9897 + 2203x}{357696 (3 - x + 2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{119353}{20736} + \frac{481765x}{10368} + \frac{113983x^2}{1296}}{(5 + 2x)^2 (3 - x + 2x^2)^{3/2}} dx$$

$$= \frac{9897 + 2203x}{357696 (3 - x + 2x^2)^{3/2}} - \frac{1255878 - 62021x}{24681024\sqrt{3 - x + 2x^2}} + \frac{4 \int \frac{\frac{10109719}{124416} - \frac{4961491x}{62208}}{(5+2x)^2 \sqrt{3-x+2x^2}} dx}{1587}$$

$$= \frac{9897 + 2203x}{357696 (3 - x + 2x^2)^{3/2}} - \frac{1255878 - 62021x}{24681024\sqrt{3 - x + 2x^2}} - \frac{3667\sqrt{3 - x + 2x^2}}{186624(5 + 2x)} + \frac{2}{186624(5 + 2x)}$$

$$= \frac{9897 + 2203x}{357696 (3 - x + 2x^2)^{3/2}} - \frac{1255878 - 62021x}{24681024\sqrt{3 - x + 2x^2}} - \frac{3667\sqrt{3 - x + 2x^2}}{186624(5 + 2x)} - \frac{2}{186624(5 + 2x)}$$

$$= \frac{9897 + 2203x}{357696 (3 - x + 2x^2)^{3/2}} - \frac{1255878 - 62021x}{24681024\sqrt{3 - x + 2x^2}} - \frac{3667\sqrt{3 - x + 2x^2}}{186624(5 + 2x)} - \frac{2}{186624(5 + 2x)}$$

**Mathematica [A]** time = 0.41, size = 92, normalized size = 0.84

$$\frac{-2821 \log(12\sqrt{4x^2 - 2x + 6} - 22x + 17) - \frac{12\sqrt{2}(6767036x^4 + 10350004x^3 + 63941915x^2 - 18840090x + 79153407)}{529(2x+5)(2x^2-x+3)^{3/2}} + 2821 \log(2x + 5)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(5/2)), x]

[Out] ((-12\*sqrt[2]\*(79153407 - 18840090\*x + 63941915\*x^2 + 10350004\*x^3 + 6767036\*x^4))/(529\*(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)) + 2821\*Log[5 + 2\*x] - 2821\*Log[17 - 22\*x + 12\*sqrt[6 - 2\*x + 4\*x^2]])/(2239488\*sqrt[2])

**IntegrateAlgebraic [A]** time = 0.80, size = 88, normalized size = 0.80

$$\frac{2821 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{1119744\sqrt{2}} + \frac{-6767036x^4 - 10350004x^3 - 63941915x^2 + 18840090x - 79153407}{98724096(2x+5)(2x^2-x+3)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^2\*(3 - x + 2\*x^2)^(5/2)), x]

[Out] (-79153407 + 18840090\*x - 63941915\*x^2 - 10350004\*x^3 - 6767036\*x^4)/(98724096\*(5 + 2\*x)\*(3 - x + 2\*x^2)^(3/2)) + (2821\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*sqrt[2])])/(1119744\*sqrt[2])

**fricas [A]** time = 1.24, size = 141, normalized size = 1.28

$$\frac{1492309\sqrt{2}(8x^5 + 12x^4 + 6x^3 + 53x^2 - 12x + 45)\log\left(\frac{-24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right) - 48(6767036x^4 + 10350004x^3 + 63941915x^2 - 18840090x + 79153407)\sqrt{2x^2-x+3}}{4738756608(8x^5 + 12x^4 + 6x^3 + 53x^2 - 12x + 45)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 1/4738756608\*(1492309\*sqrt(2)\*(8\*x^5 + 12\*x^4 + 6\*x^3 + 53\*x^2 - 12\*x + 45)\*log(-(24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) + 1060\*x^2 - 1036\*x + 1153)/(4\*x^2 + 20\*x + 25)) - 48\*(6767036\*x^4 + 10350004\*x^3 + 63941915\*x^2 - 18840090\*x + 79153407)\*sqrt(2\*x^2 - x + 3))/(8\*x^5 + 12\*x^4 + 6\*x^3 + 53\*x^2 - 12\*x + 45)

**giac [B]** time = 0.40, size = 206, normalized size = 1.87

$$-\frac{1}{2369378304} \sqrt{2} \left( \frac{1492309 \log \left( 12 \sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{72}{2x+5} - 11 \right)}{\operatorname{sgn} \left( \frac{1}{2x+5} \right)} + \frac{12 \left( \frac{48 \left( \frac{23642785}{\operatorname{sgn} \left( \frac{1}{2x+5} \right)} - \frac{52375761}{(2x+5) \operatorname{sgn} \left( \frac{1}{2x+5} \right)} \right)}{2x+5} - \frac{240080735}{\operatorname{sgn} \left( \frac{1}{2x+5} \right)} + \frac{28660178}{\operatorname{sgn} \left( \frac{1}{2x+5} \right)} - \frac{1691759}{\operatorname{sgn} \left( \frac{1}{2x+5} \right)} \right)}{\left( \frac{11}{2x+5} - \frac{36}{(2x+5)^2} - 1 \right) \sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1}} - 20301108 \operatorname{sgn} \left( \frac{1}{2x+5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] -1/2369378304\*sqrt(2)\*(1492309\*log(12\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1) + 72/(2\*x + 5) - 11)/sgn(1/(2\*x + 5)) + 12\*(((48\*(23642785/sgn(1/(2\*x + 5))) - 52375761/((2\*x + 5)\*sgn(1/(2\*x + 5))))/(2\*x + 5) - 240080735/sgn(1/(2\*x + 5)))/(2\*x + 5) + 28660178/sgn(1/(2\*x + 5)))/(2\*x + 5) - 1691759/sgn(1/(2\*x + 5)))/((11/(2\*x + 5) - 36/(2\*x + 5)^2 - 1)\*sqrt(-11/(2\*x + 5) + 36/(2\*x + 5)^2 + 1)) - 20301108\*sgn(1/(2\*x + 5)))

**maple [B]** time = 0.01, size = 194, normalized size = 1.76

$$\frac{5x}{16(2x^2-x+3)^2} - \frac{2821\sqrt{2} \operatorname{arctanh} \left( \frac{(-11+\frac{x}{2})\sqrt{2}}{12\sqrt{-11+2(x+\frac{5}{2})^2-\frac{19}{2}}} \right)}{4478976} + \frac{203}{192(2x^2-x+3)^2} + \frac{3173}{1152} - \frac{3173}{2304} + \frac{3173}{1857} - \frac{3173}{2328} - \frac{3667}{1152(x+\frac{5}{2})\sqrt{-11+2(x+\frac{5}{2})^2-\frac{19}{2}}} + \frac{2821}{124416\sqrt{-11+2(x+\frac{5}{2})^2-\frac{19}{2}}} - \frac{2081161(4x-1)}{2861568\sqrt{-11+2(x+\frac{5}{2})^2-\frac{19}{2}}} - \frac{19907743(4x-1)}{394896384\sqrt{-11+2(x+\frac{5}{2})^2-\frac{19}{2}}} + \frac{2821}{746496\sqrt{-11+2(x+\frac{5}{2})^2-\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(5/2),x)

[Out] -5/16/(2\*x^2-x+3)^(3/2)\*x+203/192/(2\*x^2-x+3)^(3/2)+3173/4416\*(4\*x-1)/(2\*x^2-x+3)^(3/2)+3173/6348\*(4\*x-1)/(2\*x^2-x+3)^(1/2)-3667/1152/(x+5/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+2821/124416/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-2081161/2861568\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-19907743/394896384\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)+2821/746496/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-2821/4478976\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))

**maxima [A]** time = 1.01, size = 127, normalized size = 1.15

$$\frac{2821}{4478976} \sqrt{2} \operatorname{arsinh} \left( \frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|} \right) - \frac{1691759x}{98724096\sqrt{2x^2-x+3}} + \frac{265339}{32908032\sqrt{2x^2-x+3}} - \frac{248617x}{715392(2x^2-x+3)^2} - \frac{3667}{576(2(2x^2-x+3)^2x+5(2x^2-x+3)^2)} + \frac{259621}{238464(2x^2-x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^2/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] 2821/4478976\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) - 1691759/98724096\*x/sqrt(2\*x^2 - x + 3) + 265339/32908032/sqrt(2\*x^2 - x + 3) - 248617/715392\*x/(2\*x^2 - x + 3)^(3/2) - 3667/576/(2\*(2\*x^2 - x + 3)^(3/2)\*x + 5\*(2\*x^2 - x + 3)^(3/2)) + 259621/238464/(2\*x^2 - x + 3)^(3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 (2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^2\*(2\*x^2 - x + 3)^(5/2)),x)

[Out] int((x + 3\*x^2 - x^3 + 5\*x^4 + 2)/((2\*x + 5)^2\*(2\*x^2 - x + 3)^(5/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^2 (2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*2/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*\*2\*(2\*x\*\*2 - x + 3)\*\*(5/2)), x)



$$3.343 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=135

$$\frac{4679797 - 2148263x}{592344576\sqrt{2x^2 - x + 3}} - \frac{45979\sqrt{2x^2 - x + 3}}{26873856(2x + 5)} - \frac{3667\sqrt{2x^2 - x + 3}}{373248(2x + 5)^2} + \frac{65991 - 8779x}{12877056(2x^2 - x + 3)^{3/2}} + \frac{774079 \tan^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{322486272\sqrt{2}}$$

Rubi [A] time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 1650, 806, 724, 206}

$$\frac{4679797 - 2148263x}{592344576\sqrt{2x^2 - x + 3}} - \frac{45979\sqrt{2x^2 - x + 3}}{26873856(2x + 5)} - \frac{3667\sqrt{2x^2 - x + 3}}{373248(2x + 5)^2} + \frac{65991 - 8779x}{12877056(2x^2 - x + 3)^{3/2}} + \frac{774079 \tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{322486272\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(5/2)),x]

[Out] (65991 - 8779\*x)/(12877056\*(3 - x + 2\*x^2)^(3/2)) - (4679797 - 2148263\*x)/(592344576\*sqrt[3 - x + 2\*x^2]) - (3667\*sqrt[3 - x + 2\*x^2])/(373248\*(5 + 2\*x)^2) - (45979\*sqrt[3 - x + 2\*x^2])/(26873856\*(5 + 2\*x)) + (774079\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(322486272\*sqrt[2])

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &

& NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && EqQ[Simplify[m + 2\*p + 3], 0]

### Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}} dx &= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{11115283}{746496} + \frac{3198845x}{62208} + \frac{605005x^2}{6912} - \frac{8779x^3}{23328}}{(5+2x)^3(3-x+2x^2)^{3/2}} dx \\
&= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} + \frac{4}{1587} \int \frac{\frac{171639869}{2985984} - \frac{1423925}{74649}}{(5+2x)^3\sqrt{3-x+2x^2}} dx \\
&= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} \\
&= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} \\
&= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} \\
&= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} \\
&= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 97, normalized size = 0.72

$$\frac{774079 \log(12\sqrt{4x^2-2x+6}-22x+17) + \frac{12\sqrt{2}(217883368x^5+107028732x^4-1503926130x^3-5919924791x^2+2280511668x-8953831359)}{529(2x+5)^2(2x^2-x+3)^{3/2}} - 774079 \log(2x+5)}{322486272\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(5/2)), x]

[Out] ((12\*sqrt[2]\*(-8953831359 + 2280511668\*x - 5919924791\*x^2 - 1503926130\*x^3 + 107028732\*x^4 + 217883368\*x^5))/(529\*(5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2)) - 774079\*Log[5 + 2\*x] + 774079\*Log[17 - 22\*x + 12\*sqrt[6 - 2\*x + 4\*x^2]])/(322486272\*sqrt[2])

**IntegrateAlgebraic [A]** time = 0.76, size = 93, normalized size = 0.69

$$\frac{217883368x^5 + 107028732x^4 - 1503926130x^3 - 5919924791x^2 + 2280511668x - 8953831359}{14216269824(2x+5)^2(2x^2-x+3)^{3/2}} - \frac{774079 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{161243136\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(5/2)), x]

[Out] (-8953831359 + 2280511668\*x - 5919924791\*x^2 - 1503926130\*x^3 + 107028732\*x^4 + 217883368\*x^5)/(14216269824\*(5 + 2\*x)^2\*(3 - x + 2\*x^2)^(3/2)) - (774079\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(161243136\*Sqrt[2])

**fricas [A]** time = 1.10, size = 155, normalized size = 1.15

$$\frac{409487791 \sqrt{2} (16x^6 + 64x^5 + 72x^4 + 136x^3 + 241x^2 + 30x + 225) \log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right) + 48(217883368x^5 + 107028732x^4 - 1503926130x^3 - 5919924791x^2 + 2280511668x - 8953831359)\sqrt{2x^2-x+3}}{682380951552(16x^6 + 64x^5 + 72x^4 + 136x^3 + 241x^2 + 30x + 225)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3/(2\*x^2-x+3)^(5/2), x, algorithm="fricas")

[Out] 1/682380951552\*(409487791\*sqrt(2)\*(16\*x^6 + 64\*x^5 + 72\*x^4 + 136\*x^3 + 241\*x^2 + 30\*x + 225)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(217883368\*x^5 + 107028732\*x^4 - 1503926130\*x^3 - 5919924791\*x^2 + 2280511668\*x - 8953831359)\*sqrt(2\*x^2 - x + 3))/(16\*x^6 + 64\*x^5 + 72\*x^4 + 136\*x^3 + 241\*x^2 + 30\*x + 225)

**giac [B]** time = 0.27, size = 228, normalized size = 1.69

$$\frac{774079}{644972544} \sqrt{2} \log\left(-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2-x+3}\right) - \frac{774079}{644972544} \sqrt{2} \log\left(-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2-x+3}\right) + \frac{\sqrt{2}\left(44588\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right)^3 - 10136238\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right)^2 + 16812201\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right) - 10182217\right)}{53747712\left(2\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right)^2 + 10\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right) - 11\right)^2} + \frac{\left(4296526x - 11507857\right)x + 10720752x - 11003805}{592344576\left(2x^2 - x + 3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^3/(2\*x^2-x+3)^(5/2), x, algorithm="giac")

[Out] 774079/644972544\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 774079/644972544\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 1/53747712\*sqrt(2)\*(44558\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 - 10136238\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 16812201\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 10182217)/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^2 + 1/592344576\*((4296526\*x - 11507857)\*x + 10720752)\*x - 11003805)/(2\*x^2 - x + 3)^(3/2)

**maple [A]** time = 0.01, size = 200, normalized size = 1.48

$$\frac{774079\sqrt{2} \operatorname{arctanh}\left(\frac{-11x+\sqrt{2}}{12\sqrt{-11x+\frac{5}{2}}+\frac{11}{2}}\right)}{644972544} - \frac{5}{48(2x^2-x+3)^{\frac{3}{2}}} - \frac{149(4x-1)}{1104(2x^2-x+3)^{\frac{3}{2}}} - \frac{149(4x-1)}{1587\sqrt{2x^2-x+3}} + \frac{115369}{165888\left(x+\frac{5}{2}\right)\left(-11x+2\left(x+\frac{5}{2}\right)-\frac{11}{2}\right)^{\frac{3}{2}}} - \frac{774079}{17915904\left(-11x+2\left(x+\frac{5}{2}\right)-\frac{11}{2}\right)^{\frac{3}{2}}} + \frac{\frac{5297925}{1292644} - \frac{2702675}{4126762}}{\left(-11x+2\left(x+\frac{5}{2}\right)-\frac{11}{2}\right)^{\frac{3}{2}}} + \frac{\frac{536174813x}{141268952} - \frac{536174813}{566672626}}{\sqrt{-11x+2\left(x+\frac{5}{2}\right)-\frac{11}{2}}} + \frac{774079}{107495424\sqrt{-11x+2\left(x+\frac{5}{2}\right)-\frac{11}{2}}} - \frac{3667}{4608\left(x+\frac{5}{2}\right)\left(-11x+2\left(x+\frac{5}{2}\right)-\frac{11}{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)`

[Out] 
$$\begin{aligned} & -5/48/(2*x^2-x+3)^{(3/2)} - 149/1104*(4*x-1)/(2*x^2-x+3)^{(3/2)} - 149/1587*(4*x-1) \\ & / (2*x^2-x+3)^{(1/2)} + 115369/165888/(x+5/2)/(-11*x+2*(x+5/2)^2-19/2)^{(3/2)} - 774 \\ & 079/17915904/(-11*x+2*(x+5/2)^2-19/2)^{(3/2)} + 57937675/412065792*(4*x-1)/(-11 \\ & *x+2*(x+5/2)^2-19/2)^{(3/2)} + 5366174813/56865079296*(4*x-1)/(-11*x+2*(x+5/2)^ \\ & 2-19/2)^{(1/2)} - 774079/107495424/(-11*x+2*(x+5/2)^2-19/2)^{(1/2)} + 774079/644972 \\ & 544*2^{(1/2)}*\operatorname{arctanh}(1/12*(-11*x+17/2)*2^{(1/2)}/(-11*x+2*(x+5/2)^2-19/2)^{(1/2} \\ & )) - 3667/4608/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^{(3/2)} \end{aligned}$$

**maxima** [A] time = 1.00, size = 178, normalized size = 1.32

$$\frac{-774079}{644972544} \sqrt{2} \operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23(2x+5)} - \frac{17\sqrt{23}}{23(2x+5)}\right) + \frac{27235421x}{14216269824\sqrt{2x^2-x+3}} - \frac{36393601}{4738756608\sqrt{2x^2-x+3}} + \frac{2323723x}{103016448(2x^2-x+3)^{3/2}} - \frac{3667}{1152(4(2x^2-x+3)^{3/2}x^2 + 20(2x^2-x+3)^{3/2}x + 25(2x^2-x+3)^{3/2})} + \frac{115369}{82944(2(2x^2-x+3)^{3/2}x + 5(2x^2-x+3)^{3/2})} - \frac{5254255}{34338816(2x^2-x+3)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -774079/644972544*\operatorname{sqrt}(2)*\operatorname{arcsinh}(22/23*\operatorname{sqrt}(23)*x/\operatorname{abs}(2*x + 5) - 17/23*\operatorname{sqrt} \\ & t(23)/\operatorname{abs}(2*x + 5)) + 27235421/14216269824*x/\operatorname{sqrt}(2*x^2 - x + 3) - 36393601 \\ & /4738756608/\operatorname{sqrt}(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^{(3/2)} \\ & - 3667/1152/(4*(2*x^2 - x + 3)^{(3/2)}*x^2 + 20*(2*x^2 - x + 3)^{(3/2)}*x + 25 \\ & *(2*x^2 - x + 3)^{(3/2})) + 115369/82944/(2*(2*x^2 - x + 3)^{(3/2)}*x + 5*(2*x^ \\ & 2 - x + 3)^{(3/2})) - 5254255/34338816/(2*x^2 - x + 3)^{(3/2)} \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^3 (2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)`

[Out] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^3 (2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x\*\*4-x\*\*3+3\*x\*\*2+x+2)/(5+2\*x)\*\*3/(2\*x\*\*2-x+3)\*\*(5/2),x)

[Out] Integral((5\*x\*\*4 - x\*\*3 + 3\*x\*\*2 + x + 2)/((2\*x + 5)\*\*3\*(2\*x\*\*2 - x + 3)\*\*(5/2)), x)

$$3.344 \quad \int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}} dx$$

Optimal. Leaf size=160

$$\frac{27754539 - 31190998x}{31986607104\sqrt{2x^2 - x + 3}} + \frac{475357\sqrt{2x^2 - x + 3}}{1934917632(2x + 5)} - \frac{89137\sqrt{2x^2 - x + 3}}{80621568(2x + 5)^2} - \frac{3667\sqrt{2x^2 - x + 3}}{559872(2x + 5)^3} + \frac{369609 - 175877x}{463574016(2x^2 - x + 3)^{3/2}} + \frac{4778789 \operatorname{tanh}^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{7739670528\sqrt{2}}$$

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {1646, 1650, 806, 724, 206}

$$\frac{27754539 - 31190998x}{31986607104\sqrt{2x^2 - x + 3}} + \frac{475357\sqrt{2x^2 - x + 3}}{1934917632(2x + 5)} - \frac{89137\sqrt{2x^2 - x + 3}}{80621568(2x + 5)^2} - \frac{3667\sqrt{2x^2 - x + 3}}{559872(2x + 5)^3} + \frac{369609 - 175877x}{463574016(2x^2 - x + 3)^{3/2}} + \frac{4778789 \operatorname{tanh}^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{7739670528\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*(3 - x + 2\*x^2)^(5/2)),x]

[Out] (369609 - 175877\*x)/(463574016\*(3 - x + 2\*x^2)^(3/2)) - (27754539 - 31190998\*x)/(31986607104\*sqrt[3 - x + 2\*x^2]) - (3667\*sqrt[3 - x + 2\*x^2])/(559872\*(5 + 2\*x)^3) - (89137\*sqrt[3 - x + 2\*x^2])/(80621568\*(5 + 2\*x)^2) + (475357\*sqrt[3 - x + 2\*x^2])/(1934917632\*(5 + 2\*x)) + (4778789\*ArcTanh[(17 - 22\*x)/(12\*sqrt[2]\*sqrt[3 - x + 2\*x^2])])/(7739670528\*sqrt[2])

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 806

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Simp[((e\*f - d\*g)\*(d + e\*x)^(m + 1)\*(a + b\*x + c\*x^2)^(p + 1))/(2\*(p + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] - Dist[(b\*(e\*f + d\*g) - 2\*(c\*d\*f + a\*e\*g))/(2\*(c\*d^2 - b\*d\*e + a\*e^2)), Int[(d + e\*x)^(m

```
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

### Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
+ 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
- ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x]] /; FreeQ[{a, b, c, d,
e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(
p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
+ 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}} dx &= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} + \frac{2}{69} \int \frac{\frac{606939313}{26873856} + \frac{727085495x}{13436928} + \frac{186705485x^2}{2239488} - \frac{107}{3}}{(5+2x)^4(3-x+2x^2)^3} dx \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} + \frac{4 \int \frac{\frac{4811736919}{40310784}}{(5+2x)^4(3-x+2x^2)^3} dx}{(5+2x)^4(3-x+2x^2)^3} \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^4} \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^4} \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^4} \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^4} \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^4} \\
&= \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^4}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 89, normalized size = 0.56

$$\frac{2527979381\sqrt{2} \tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24(6664404208x^6+34872810880x^5+46210466520x^4+27484986184x^3-6702882569x^2+73621973154x-95241881529)}{(2x+5)^3(2x^2-x+3)^{3/2}}}{8188571418624}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*(3 - x + 2\*x^2)^(5/2)), x]

[Out] ((24\*(-95241881529 + 73621973154\*x - 6702882569\*x^2 + 27484986184\*x^3 + 46210466520\*x^4 + 34872810880\*x^5 + 6664404208\*x^6))/((5 + 2\*x)^3\*(3 - x + 2\*x^2)^(3/2)) + 2527979381\*sqrt[2]\*ArcTanh[(17 - 22\*x)/(12\*sqrt[6 - 2\*x + 4\*x^2]])]/8188571418624

**IntegrateAlgebraic [A]** time = 0.86, size = 98, normalized size = 0.61

$$\frac{6664404208x^6 + 34872810880x^5 + 46210466520x^4 + 27484986184x^3 - 6702882569x^2 + 73621973154x - 95241881529}{341190475776(2x+5)^3(2x^2-x+3)^{3/2}} - \frac{4778789 \tanh^{-1}\left(-\frac{\sqrt{2x^2-x+3}}{3\sqrt{2}} + \frac{x}{3} + \frac{5}{6}\right)}{3869835264\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x + 3\*x^2 - x^3 + 5\*x^4)/((5 + 2\*x)^4\*(3 - x + 2\*x^2)^(5/2)),x]

[Out] (-95241881529 + 73621973154\*x - 6702882569\*x^2 + 27484986184\*x^3 + 46210466520\*x^4 + 34872810880\*x^5 + 6664404208\*x^6)/(341190475776\*(5 + 2\*x)^3\*(3 - x + 2\*x^2)^(3/2)) - (4778789\*ArcTanh[5/6 + x/3 - Sqrt[3 - x + 2\*x^2]/(3\*Sqrt[2])])/(3869835264\*Sqrt[2])

**fricas [A]** time = 0.72, size = 170, normalized size = 1.06

$$\frac{2527979381 \sqrt{2} (32x^7 + 208x^6 + 464x^5 + 632x^4 + 1162x^3 + 1265x^2 + 600x + 1125) \log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right) + 48(6664404208x^6 + 34872810880x^5 + 46210466520x^4 + 27484986184x^3 - 6702882569x^2 + 73621973154x - 95241881529)\sqrt{2x^2-x+3}}{16377142837248(32x^7 + 208x^6 + 464x^5 + 632x^4 + 1162x^3 + 1265x^2 + 600x + 1125)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/16377142837248\*(2527979381\*sqrt(2)\*(32\*x^7 + 208\*x^6 + 464\*x^5 + 632\*x^4 + 1162\*x^3 + 1265\*x^2 + 600\*x + 1125)\*log((24\*sqrt(2)\*sqrt(2\*x^2 - x + 3)\*(22\*x - 17) - 1060\*x^2 + 1036\*x - 1153)/(4\*x^2 + 20\*x + 25)) + 48\*(6664404208\*x^6 + 34872810880\*x^5 + 46210466520\*x^4 + 27484986184\*x^3 - 6702882569\*x^2 + 73621973154\*x - 95241881529)\*sqrt(2\*x^2 - x + 3))/(32\*x^7 + 208\*x^6 + 464\*x^5 + 632\*x^4 + 1162\*x^3 + 1265\*x^2 + 600\*x + 1125)

**giac [B]** time = 0.28, size = 279, normalized size = 1.74

$$\frac{4778789}{15479341056} \sqrt{2} \log\left(\left| \frac{2\sqrt{2x^2-x+3}}{2\sqrt{2x^2-x+3}} \right| \right) - \frac{4778789}{15479341056} \sqrt{2} \log\left(\left| \frac{2\sqrt{2x^2-x+3}}{2\sqrt{2x^2-x+3}} \right| \right) + \frac{(11595499x - 21675019) * x + 27298005}{799651776(2x^2-x+3)^2} \sqrt{2} \left( \frac{38030012\sqrt{2}(\sqrt{2x^2-x+3})^5 + 734231900(\sqrt{2x^2-x+3})^4 + 122834956\sqrt{2}(\sqrt{2x^2-x+3})^3 - 215499536(\sqrt{2x^2-x+3})^2 + 1659431083\sqrt{2}(\sqrt{2x^2-x+3}) - 760577420}{3869835264(\sqrt{2x^2-x+3})^2 + 10\sqrt{2}(\sqrt{2x^2-x+3}) - 11 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] 4778789/15479341056\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x + sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) - 4778789/15479341056\*sqrt(2)\*log(abs(-2\*sqrt(2)\*x - 11\*sqrt(2) + 2\*sqrt(2\*x^2 - x + 3))) + 1/7996651776\*((11595499\*x - 21675019)\*x + 27298005)\*x - 14440149)/(2\*x^2 - x + 3)^(3/2) + 1/3869835264\*sqrt(2)\*(38030012\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^5 + 734231900\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^4 + 122834956\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^3 - 2

154595396\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3))^2 + 1659431083\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 760577429/(2\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)))^2 + 10\*sqrt(2)\*(sqrt(2)\*x - sqrt(2\*x^2 - x + 3)) - 11)^3

**maple [A]** time = 0.01, size = 207, normalized size = 1.29

$$\frac{4778789\sqrt{2} \operatorname{arctanh}\left(\frac{(-11x+2)\sqrt{x+5/2}}{12\sqrt{-11x+2}\sqrt{x+5/2}-\frac{19}{2}}\right)}{15479341056} - \frac{4778789}{429981696(-11x+2(x+5/2)^2-19/2)^{3/2}} - \frac{4778789}{2579890176\sqrt{-11x+2}\sqrt{x+5/2}-\frac{19}{2}} - \frac{3667}{13824(x+5/2)^3(-11x+2(x+5/2)^2-19/2)^{3/2}} - \frac{25951}{110592(x+5/2)^2(-11x+2(x+5/2)^2-19/2)^{3/2}} - \frac{34861}{3981312(x+5/2)(-11x+2(x+5/2)^2-19/2)^{3/2}} - \frac{72646615(4x-1)}{9889579008(-11x+2(x+5/2)^2-19/2)^{3/2}} + \frac{10}{\sqrt{2x^2-x+3}} + \frac{2x-5}{(2x^2-x+3)^2} - \frac{8183108657(4x-1)}{1364761903104\sqrt{-11x+2}\sqrt{x+5/2}-\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(5/2),x)

[Out] -4778789/429981696/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-4778789/2579890176/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+25951/110592/(x+5/2)^2/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-34861/3981312/(x+5/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)-72646615/9889579008\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(3/2)+4778789/15479341056\*2^(1/2)\*arctanh(1/12\*(-11\*x+17/2)\*2^(1/2)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2))+10/1587\*(4\*x-1)/(2\*x^2-x+3)^(1/2)+5/55\*2\*(4\*x-1)/(2\*x^2-x+3)^(3/2)-8183108657/1364761903104\*(4\*x-1)/(-11\*x+2\*(x+5/2)^2-19/2)^(1/2)

**maxima [A]** time = 1.01, size = 246, normalized size = 1.54

$$\frac{4778789}{15479341056} \sqrt{2} \operatorname{arcsinh}\left(\frac{22\sqrt{23}x-17\sqrt{23}}{23(2x+5)}\right) + \frac{416525263x}{341190475776\sqrt{2x^2-x+3}} - \frac{245375387}{113730158592\sqrt{2x^2-x+3}} - \frac{16932905x}{2472394752(2x^2-x+3)^{3/2}} - \frac{3667}{1728(8(2x^2-x+3)^2x^3+60(2x^2-x+3)^3/2x^2+150(2x^2-x+3)^3/2x+125(2x^2-x+3)^3/2)} + \frac{25951}{27648(4(2x^2-x+3)^3/2x^2+20(2x^2-x+3)^3/2x+25(2x^2-x+3)^3/2)} - \frac{34861}{1990656(2(2x^2-x+3)^3/2x+5(2x^2-x+3)^3/2)} - \frac{10570421}{824131584(2x^2-x+3)^3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5\*x^4-x^3+3\*x^2+x+2)/(5+2\*x)^4/(2\*x^2-x+3)^(5/2),x, algorithm="maxima")

[Out] -4778789/15479341056\*sqrt(2)\*arcsinh(22/23\*sqrt(23)\*x/abs(2\*x + 5) - 17/23\*sqrt(23)/abs(2\*x + 5)) + 416525263/341190475776\*x/sqrt(2\*x^2 - x + 3) - 245375387/113730158592/sqrt(2\*x^2 - x + 3) + 16932905/2472394752\*x/(2\*x^2 - x + 3)^(3/2) - 3667/1728/(8\*(2\*x^2 - x + 3)^(3/2)\*x^3 + 60\*(2\*x^2 - x + 3)^(3/2)\*x^2 + 150\*(2\*x^2 - x + 3)^(3/2)\*x + 125\*(2\*x^2 - x + 3)^(3/2)) + 25951/27648/(4\*(2\*x^2 - x + 3)^(3/2)\*x^2 + 20\*(2\*x^2 - x + 3)^(3/2)\*x + 25\*(2\*x^2 - x + 3)^(3/2)) - 34861/1990656/(2\*(2\*x^2 - x + 3)^(3/2)\*x + 5\*(2\*x^2 - x + 3)^(3/2)) - 10570421/824131584/(2\*x^2 - x + 3)^(3/2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^4 (2x^2 - x + 3)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)`

[Out] `int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)^4 (2x^2 - x + 3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2), x)`

[Out] `Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(5/2)), x)`

$$3.345 \quad \int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}} dx$$

Optimal. Leaf size=354

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab^2)}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

Rubi [A] time = 0.38, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1660, 12, 621, 206}

$$\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+b)-c^3(88c-8ab)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+2c^2f)+24a^2c^2i+2b^2c^2(2c^2g-3ai)-b^3c(8h-10aj)+b^4ci+b^5(-j))}{3c^3(b^2-4ac)^2\sqrt{a+bx+cx^2}} + \frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)+ab^2ci-ab^3j+2ac^2(cg-ai))}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}} + \frac{j \operatorname{tanh}^{-1}\left(\frac{bx+cx^2}{\sqrt{a+bx+cx^2}}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2 + i\*x^3 + j\*x^4)/(a + b\*x + c\*x^2)^(5/2), x]

[Out] (2\*(a\*b^2\*c\*i + 2\*a\*c^2\*(c\*g - a\*i) - a\*b^3\*j - b\*c\*(c^2\*f + a\*c\*h - 3\*a^2\*j) - (2\*c^4\*f - c^3\*(b\*g + 2\*a\*h) + b^4\*j - b^2\*c\*(b\*i + 4\*a\*j) + c^2\*(b^2\*h + 3\*a\*b\*i + 2\*a^2\*j))\*x)/(3\*c^3\*(b^2 - 4\*a\*c)\*(a + b\*x + c\*x^2)^(3/2)) - (2\*(b^4\*c\*i + 24\*a^2\*c^3\*i + 2\*b^2\*c^2\*(2\*c\*g - 3\*a\*i) - b^5\*j - b^3\*c\*(c\*h - 10\*a\*j) - 4\*b\*c^2\*(2\*c^2\*f + a\*c\*h + 8\*a^2\*j) - c\*(16\*c^4\*f - c^3\*(8\*b\*g - 8\*a\*h) - 4\*b^4\*j + b^2\*c\*(b\*i + 28\*a\*j) + 2\*c^2\*(b^2\*h - 6\*a\*b\*i - 16\*a^2\*j))\*x)/(3\*c^3\*(b^2 - 4\*a\*c)^2\*sqrt[a + b\*x + c\*x^2]) + (j\*ArcTanh[(b + 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x + c\*x^2])])/c^(5/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

## Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

## Rubi steps

$$\int \frac{f + gx + hx^2 + 345x^3 + jx^4}{(a + bx + cx^2)^{5/2}} dx = \frac{2 \left( c^3 \left( bf + \frac{3a^2(230c - bj)}{c^2} - \frac{a(345b^2c + 2c^3g - bc^2h - b^3j)}{c^3} \right) - (345b^3c - bc^2(1035a - cg)) \right)}{3c^3(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{2 \left( c^3 \left( bf + \frac{3a^2(230c - bj)}{c^2} - \frac{a(345b^2c + 2c^3g - bc^2h - b^3j)}{c^3} \right) - (345b^3c - bc^2(1035a - cg)) \right)}{3c^3(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{2 \left( c^3 \left( bf + \frac{3a^2(230c - bj)}{c^2} - \frac{a(345b^2c + 2c^3g - bc^2h - b^3j)}{c^3} \right) - (345b^3c - bc^2(1035a - cg)) \right)}{3c^3(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{2 \left( c^3 \left( bf + \frac{3a^2(230c - bj)}{c^2} - \frac{a(345b^2c + 2c^3g - bc^2h - b^3j)}{c^3} \right) - (345b^3c - bc^2(1035a - cg)) \right)}{3c^3(b^2 - 4ac)(a + bx + cx^2)}$$

$$= \frac{2 \left( c^3 \left( bf + \frac{3a^2(230c - bj)}{c^2} - \frac{a(345b^2c + 2c^3g - bc^2h - b^3j)}{c^3} \right) - (345b^3c - bc^2(1035a - cg)) \right)}{3c^3(b^2 - 4ac)(a + bx + cx^2)}$$

**Mathematica [A]** time = 1.14, size = 316, normalized size = 0.89

$$\frac{2(bc(-3a^2 + ac(3a + c) + c^2(f - gx)) + 2c^2(a^2(i + jx) - ac(g + hx) + c^2fx) + b^3(aj - ci) + b^2c(chx - a(i + 4jx)) + b^4ix) + 2(4bc^2(8a^2j + ac(3a + c)(-gx)) + 8c^3(-a^2(3i + 4jx) + achx + 2c^2fx) + b^3c(i(3a + c)(10aj) + 2b^2c^2(3ai + 14ajx - 2cg + chx) + b^5j - b^4c(i + 4jx)) + 3\sqrt{c} \log(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx))}{(b^2 - 4ac)(a + x(b + cx))^{3/2} \cdot 3c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2 + i\*x^3 + j\*x^4)/(a + b\*x + c\*x^2)^(5/2), x]

```
[Out] ((-2*(b^4*j*x + b^3*(a*j - c*i*x) + b*c*(-3*a^2*j + c^2*(f - g*x) + a*c*(h + 3*i*x)) + 2*c^2*(c^2*f*x - a*c*(g + h*x) + a^2*(i + j*x)) + b^2*c*(c*h*x - a*(i + 4*j*x))))/(b^2 - 4*a*c)*(a + x*(b + c*x))^(3/2)) + (2*(b^5*j - b^4*c*(i + 4*j*x) + 2*b^2*c^2*(-2*c*g + 3*a*i + c*h*x + 14*a*j*x) + 4*b*c^2*(8*a^2*j + 2*c^2*(f - g*x) + a*c*(h - 3*i*x)) + b^3*c*(-10*a*j + c*(h + i*x)) + 8*c^3*(2*c^2*f*x + a*c*h*x - a^2*(3*i + 4*j*x)))/(b^2 - 4*a*c)^2*Sqrt[a + x*(b + c*x)] + 3*Sqrt[c]*j*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]/(3*c^3)
```

**IntegrateAlgebraic [A]** time = 2.61, size = 425, normalized size = 1.20

$\frac{2(-20b^5j + 16b^4j + 24b^3j + 5b^2j) - 42b^2j + 8b^2i + 24b^2i + 8b^2i + 24b^2i + 32b^2i + 64b^2i - 18ab^2i + 24b^2i - 12ab^2i + 64b^2i - 24ab^2i - 12ab^2i + 12ab^2i - 12ab^2i - 24ab^2i - 24ab^2i - 8ab^2i + 3b^2i + 4b^2i + 3b^2i + 3b^2i - 3b^2i - 6b^2i + 12b^2i - 2b^2i - 24b^2i + 8b^2i - 16b^2i}{3^2(4c - b^2)(a + bx + cx)^2} \frac{3 \log\left(\frac{-20b^5j + 16b^4j + 24b^3j + 5b^2j}{20}\right)}{20}$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x]
```

```
[Out] (-2*(b^3*c^2*f - 12*a*b*c^3*f + 2*a*b^2*c^2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20*a^3*b*c*j - 6*b^2*c^3*f*x - 24*a*c^4*f*x + 3*b^3*c^2*g*x + 12*a*b*c^3*g*x - 12*a*b^2*c^2*h*x + 24*a^2*b*c^2*i*x + 6*a*b^4*j*x - 42*a^2*b^2*c*j*x + 24*a^3*c^2*j*x - 24*b*c^4*f*x^2 + 12*b^2*c^3*g*x^2 - 3*b^3*c^2*h*x^2 - 12*a*b*c^3*h*x^2 + 6*a*b^2*c^2*i*x^2 + 24*a^2*c^3*i*x^2 + 3*b^5*j*x^2 - 18*a*b^3*c*j*x^2 - 16*c^5*f*x^3 + 8*b*c^4*g*x^3 - 2*b^2*c^3*h*x^3 - 8*a*c^4*h*x^3 - b^3*c^2*i*x^3 + 12*a*b*c^3*i*x^3 + 4*b^4*c*j*x^3 - 28*a*b^2*c^2*j*x^3 + 32*a^2*c^3*j*x^3))/(3*c^2*(-b^2 + 4*a*c))^2*(a + b*x + c*x^2)^(3/2) - (j*Log[b*c^2 + 2*c^3*x - 2*c^(5/2)*Sqrt[a + b*x + c*x^2]])/c^(5/2)
```

**fricas [B]** time = 95.07, size = 1373, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2), x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 + 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c - 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5)*f - (b^3
```

```

*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt(c*x
x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 +
(b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j
*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f
- 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c
c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c
^3 + 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c
- 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^
5)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*
c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*
b^3*c^4 + 16*a^3*b*c^5)*x)]

```

**giac [A]** time = 0.31, size = 465, normalized size = 1.31

$$2 \left( \frac{\left( \frac{(16c^5f - 8bc^4g + 2b^2c^3h + 8a^2c^4h + b^3c^2i - 12a^2b^2c^3i - 4b^4c^2j + 28a^2b^2c^2j - 32a^2c^3j)x}{b^4c^2 - 8a^2b^2c^3 + 16a^2c^4} + 3(8bc^4f - 4b^2c^3g + b^3c^2h + 4a^2b^2c^3h - 2a^2b^2c^2i - 8a^2c^3i - b^5j + 6a^2b^3c^2j)}{b^4c^2 - 8a^2b^2c^3 + 16a^2c^4} \right) x + \frac{3(2b^2c^3f + 8a^2c^4f - b^3c^2g - 4a^2b^2c^3g + 4a^2b^2c^2h - 8a^2b^2c^2i - 2a^2b^4j + 14a^2b^2c^2j - 8a^3c^2j)}{b^4c^2 - 8a^2b^2c^3 + 16a^2c^4} \right) x - \frac{b^3c^2f - 12a^2b^2c^3f + 2a^2b^2c^2g + 8a^2c^3g - 8a^2b^2c^2h + 16a^3c^2i + 3a^2b^3j - 20a^3bc^2j}{b^4c^2 - 8a^2b^2c^3 + 16a^2c^4} \right) / (c^2x^2 + b^2cx + a^2) \sqrt{c} - j \log \left( \frac{-2(\sqrt{cx^2 + bx + a})\sqrt{c} - b}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j\*x^4+i\*x^3+h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] 2/3\*(((16\*c^5\*f - 8\*b\*c^4\*g + 2\*b^2\*c^3\*h + 8\*a^2\*c^4\*h + b^3\*c^2\*i - 12\*a^2\*b^2\*c^3\*i - 4\*b^4\*c^2\*j + 28\*a^2\*b^2\*c^2\*j - 32\*a^2\*c^3\*j)\*x/(b^4\*c^2 - 8\*a^2\*b^2\*c^3 + 16\*a^2\*c^4) + 3\*(8\*b\*c^4\*f - 4\*b^2\*c^3\*g + b^3\*c^2\*h + 4\*a^2\*b^2\*c^3\*h - 2\*a^2\*b^2\*c^2\*i - 8\*a^2\*c^3\*i - b^5\*j + 6\*a^2\*b^3\*c^2\*j)/(b^4\*c^2 - 8\*a^2\*b^2\*c^3 + 16\*a^2\*c^4))\*x + 3\*(2\*b^2\*c^3\*f + 8\*a^2\*c^4\*f - b^3\*c^2\*g - 4\*a^2\*b^2\*c^3\*g + 4\*a^2\*b^2\*c^2\*h - 8\*a^2\*b^2\*c^2\*i - 2\*a^2\*b^4\*j + 14\*a^2\*b^2\*c^2\*j - 8\*a^3\*c^2\*j)/(b^4\*c^2 - 8\*a^2\*b^2\*c^3 + 16\*a^2\*c^4))\*x - (b^3\*c^2\*f - 12\*a^2\*b^2\*c^3\*f + 2\*a^2\*b^2\*c^2\*g + 8\*a^2\*c^3\*g - 8\*a^2\*b^2\*c^2\*h + 16\*a^3\*c^2\*i + 3\*a^2\*b^3\*j - 20\*a^3\*b\*c^2\*j)/(b^4\*c^2 - 8\*a^2\*b^2\*c^3 + 16\*a^2\*c^4))/(c\*x^2 + b\*x + a)^(3/2) - j\*log(a\*bs(-2\*(sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*sqrt(c) - b))/c^(5/2)

**maple [B]** time = 0.02, size = 1406, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((j\*x^4+i\*x^3+h\*x^2+g\*x+f)/(c\*x^2+b\*x+a)^(5/2),x)



```
[Out] j/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/24*j/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*j/c^2*b^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/4*j/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*j/c^2*b^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+2/3*i/c*b^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/3*h*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b+16/3*h*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-1/2*i/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/12*i/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-16/3*g*c*b/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-8*i*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-4*i/c*b^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/6*h/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+2/3*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-i*x^2/c/(c*x^2+b*x+a)^(3/2)+1/24*i/c^3*b^2/(c*x^2+b*x+a)^(3/2)-2/3*i*a/c^2/(c*x^2+b*x+a)^(3/2)-1/2*h*x/c/(c*x^2+b*x+a)^(3/2)+1/12*h/c^2*b/(c*x^2+b*x+a)^(3/2)-8/3*g*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-1/3*j*x^3/c/(c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^3/(c*x^2+b*x+a)^(3/2)-j/c^2*x/(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b/(c*x^2+b*x+a)^(1/2)+4*j/c*b^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-i/c*b*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*j/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-2/3*g*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*g/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+4/3*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*c+32/3*f*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*f*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+1/12*h/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+4/3*h*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+2/3*h/c*b^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+2/3*h*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+8/3*h*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+1/2*j/c^2*b*x^2/(c*x^2+b*x+a)^(3/2)+1/8*j/c^3*b^2*x/(c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/6*j/c^3*b^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-1/4*i/c^2*b*x/(c*x^2+b*x+a)^(3/2)+1/24*i/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/3*i/c^2*b^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/3*g/c/(c*x^2+b*x+a)^(3/2)+j/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*j/c^3*b*a/(c*x^2+b*x+a)^(3/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{jx^4 + ix^3 + hx^2 + gx + f}{(cx^2 + bx + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)`

[Out] `int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2), x)`

[Out] Timed out

$$3.346 \quad \int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}} dx$$

Optimal. Leaf size=353

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c\left(4aj+bi\right)+c^3\left(2ah+bg\right)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c\right)}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {1660, 12, 621, 204}

$$\frac{2(-ca(2c^2(-16a^2j-6abi+b^2h)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f)+4b^2(8a^2j-ach+2c^2f)+24a^2c^3i+2b^2c^2(3ai+2cj)+b^2c(10aj+ci)+b^4ci+b^6j)}{3c^3(4ac+b^2)\sqrt{a+bx-cx^2}} + \frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c\left(4aj+bi\right)+c^3\left(2ah+bg\right)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c\right)}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}} - \frac{j \tan^{-1}\left(\frac{b-2cx}{2c\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x + h\*x^2 + i\*x^3 + j\*x^4)/(a + b\*x - c\*x^2)^(5/2), x]

[Out] (2\*(a\*b^2\*c\*i + 2\*a\*c^2\*(c\*g + a\*i) + a\*b^3\*j - b\*c\*(c^2\*f - a\*c\*h - 3\*a^2\*j) + (2\*c^4\*f + c^3\*(b\*g + 2\*a\*h) + b^4\*j + b^2\*c\*(b\*i + 4\*a\*j) + c^2\*(b^2\*h + 3\*a\*b\*i + 2\*a^2\*j))\*x)/(3\*c^3\*(b^2 + 4\*a\*c)\*(a + b\*x - c\*x^2)^(3/2)) - (2\*(b^4\*c\*i + 24\*a^2\*c^3\*i + 2\*b^2\*c^2\*(2\*c\*g + 3\*a\*i) + b^5\*j + b^3\*c\*(c\*h + 10\*a\*j) + 4\*b\*c^2\*(2\*c^2\*f - a\*c\*h + 8\*a^2\*j) - c\*(16\*c^4\*f + 8\*c^3\*(b\*g - a\*h) - 4\*b^4\*j - b^2\*c\*(b\*i + 28\*a\*j) + 2\*c^2\*(b^2\*h - 6\*a\*b\*i - 16\*a^2\*j))\*x)/(3\*c^3\*(b^2 + 4\*a\*c)^2\*sqrt[a + b\*x - c\*x^2]) - (j\*ArcTan[(b - 2\*c\*x)/(2\*sqrt[c]\*sqrt[a + b\*x - c\*x^2])])/c^(5/2)

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 621

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[2, Subst[Int[1/(4\*c - x^2), x], x, (b + 2\*c\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

## Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq,
a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

## Rubi steps

$$\int \frac{f + gx + hx^2 + 346x^3 + jx^4}{(a + bx - cx^2)^{5/2}} dx = -\frac{2\left(c^3\left(bf - \frac{a^2(692c+3bj)}{c^2} - \frac{a(346b^2c+2c^3g+bc^2h+b^3j)}{c^3}\right) - (346b^3c + bc^2(1038a + cg))\right)}{3c^3(b^2 + 4ac)(a + bx - cx^2)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf - \frac{a^2(692c+3bj)}{c^2} - \frac{a(346b^2c+2c^3g+bc^2h+b^3j)}{c^3}\right) - (346b^3c + bc^2(1038a + cg))\right)}{3c^3(b^2 + 4ac)(a + bx - cx^2)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf - \frac{a^2(692c+3bj)}{c^2} - \frac{a(346b^2c+2c^3g+bc^2h+b^3j)}{c^3}\right) - (346b^3c + bc^2(1038a + cg))\right)}{3c^3(b^2 + 4ac)(a + bx - cx^2)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf - \frac{a^2(692c+3bj)}{c^2} - \frac{a(346b^2c+2c^3g+bc^2h+b^3j)}{c^3}\right) - (346b^3c + bc^2(1038a + cg))\right)}{3c^3(b^2 + 4ac)(a + bx - cx^2)^{3/2}}$$

$$= -\frac{2\left(c^3\left(bf - \frac{a^2(692c+3bj)}{c^2} - \frac{a(346b^2c+2c^3g+bc^2h+b^3j)}{c^3}\right) - (346b^3c + bc^2(1038a + cg))\right)}{3c^3(b^2 + 4ac)(a + bx - cx^2)^{3/2}}$$

**Mathematica [C]** time = 1.11, size = 319, normalized size = 0.90

$$\frac{2\left(b^2(3a^2+18ac)x^2+c^2(f+3g-(x^2(3h+3j)))\right)+2b^2c(21f^2x+ac(g+x(-6h+3ix-14j^2))+c^2x(3f+x(hx-6g))) + 4bc(5a^2-2a^2c(3-3ix)+3ac^2(f-x(x-hx+ix^2))+2c^3a^2(gx-3f))+8c^2(a^2(2i+3ix)-a^2c(g+x^2(3h+4ix))-ac^2x(3f+hx^2)+2c^2fx^2)+b^4(692c-4jx^2)+3b^5x^2}{3c^2(4ac+b^2)(a+xb-cx)^{3/2}} + \frac{ij \log\left(\frac{2\sqrt{a+xb-cx} + \frac{6b-2cx}{c}}{c}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x + h\*x^2 + i\*x^3 + j\*x^4)/(a + b\*x - c\*x^2)^(5/2), x]

```
[Out] (-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
+ c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*j*
x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a^3*
j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g - h*x
+ i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*(g +
x*(-6*h + 3*i*x - 14*j*x^2))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*x))^(3
/2)) + (I*j*Log[(I*(b - 2*c*x))/Sqrt[c] + 2*Sqrt[a + x*(b - c*x)])]/c^(5/2)
```

**IntegrateAlgebraic [F]** time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2
),x]
```

```
[Out] $Aborted
```

**fricas [B]** time = 78.23, size = 1385, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5
+ 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)
*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c*x -
b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f + 8*b*c
^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^2 + 7*
a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^3 - 4*a
*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*x^2 - (b
^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*c + 20*a^
3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*c^5)*f - (
b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt
(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 +
8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*a^2*b^3*c^4
+ 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 -
2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^
3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^
2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x + a)*(2*c*x - b)*
sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^
6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b
```

$$\begin{aligned} & \left( 4c^2 + 7ab^2c^3 + 8a^2c^4 \right) j \cdot x^3 + 3 \left( 8b^5c^5f + 4b^2c^4g + (b^3c^3 - 4ab^2c^4)h - 2(ab^2c^3 - 4a^2c^4)i - (b^5c + 6ab^3c^2)j \right) \cdot x^2 \\ & - (b^3c^3 + 12ab^2c^4)f - 2(ab^2c^3 - 4a^2c^4)g - (3a^2b^3c + 20a^3b^2c^2)j + 3(4ab^2c^3h - 8a^2b^2c^3i - 2(b^2c^4 - 4a^3c^5)f \\ & - (b^3c^3 - 4ab^2c^4)g - 2(ab^4c + 7a^2b^2c^2 + 4a^3c^3)j) \cdot x \cdot \sqrt{-cx^2 + bx + a} \\ & / (a^2b^4c^3 + 8a^3b^2c^4 + 16a^4c^5 + (b^4c^5 + 8ab^2c^6 + 16a^2c^7) \cdot x^4 - 2(b^5c^4 + 8ab^3c^5 + 16a^2b^2c^6) \cdot x^3 \\ & + (b^6c^3 + 6ab^4c^4 - 32a^3c^6) \cdot x^2 + 2(ab^5c^3 + 8a^2b^3c^4 + 16a^3b^2c^5) \cdot x) \end{aligned}$$

**giac [A]** time = 0.45, size = 488, normalized size = 1.38

$$\frac{2\sqrt{-cx^2 + bx + a} \left( \frac{16b^2f + 8b^4g + 2b^2h - 8ab^2i - 12ab^2j - 4b^3k - 28a^2b^2l - 32a^2m}{b^2 + 8ab^2 + 16a^2} \right) + \frac{3(8b^4f + 4b^2g + 8b^2h - 4ab^2i - 2a^2b^2j + 8a^2k - 6a^2l)}{b^2 + 8ab^2 + 16a^2} \cdot x + \frac{3(2b^2f - 8ab^2g + 4ab^2h - 4a^2b^2i + 8a^2b^2j + 14a^2k - 14a^2l)}{b^2 + 8ab^2 + 16a^2} \cdot x^2 + \frac{b^2f + 12ab^2g + 2a^2b^2h - 8a^2b^2i - 8a^2b^2j + 16a^2k - 20a^2l}{b^2 + 8ab^2 + 16a^2} \cdot x^3}{3(cx^2 - bx - a)^2} \cdot \frac{\log\left(\frac{b(\sqrt{-cx^2 + bx + a} + \sqrt{-c})}{\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j\*x^4+i\*x^3+h\*x^2+g\*x+f)/(-c\*x^2+b\*x+a)^(5/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2/3 \cdot \sqrt{-cx^2 + bx + a} \cdot \left( \frac{(16c^5f + 8b^2c^4g + 2b^2c^3h - 8a^2c^4h - b^3c^2i - 12ab^2c^3i - 4b^4c^2j - 28a^2b^2c^2j - 32a^2c^3j) \cdot x}{(b^4c^2 + 8ab^2c^3 + 16a^2c^4)} - 3(8b^2c^4f + 4b^2c^3g + b^3c^2h - 4ab^2c^3h - 2ab^2c^2i + 8a^2c^3i - b^5j - 6ab^3c^2j) \right. \\ & \left. / (b^4c^2 + 8ab^2c^3 + 16a^2c^4) \cdot x + 3(2b^2c^3f - 8a^2c^4f + b^3c^2g - 4ab^2c^3g - 4ab^2c^2h + 8a^2b^2c^2i + 2ab^4j + 14a^2b^2c^2j + 8a^3c^2j) \right. \\ & \left. / (b^4c^2 + 8ab^2c^3 + 16a^2c^4) \cdot x + (b^3c^2f + 12a^2b^2c^3f + 2ab^2c^2g - 8a^2c^3g - 8a^2b^2c^2h + 16a^3c^2i + 3a^2b^3j + 20a^3b^2c^2j) \right. \\ & \left. / (b^4c^2 + 8ab^2c^3 + 16a^2c^4) \right) / (cx^2 - bx - a)^2 - j \cdot \log\left(\frac{2(\sqrt{-c} \cdot x - \sqrt{-cx^2 + bx + a}) \cdot \sqrt{-c} + b}{(\sqrt{-c}) \cdot c^2}\right) \end{aligned}$$

**maple [B]** time = 0.02, size = 1453, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((j\*x^4+i\*x^3+h\*x^2+g\*x+f)/(-c\*x^2+b\*x+a)^(5/2),x)

[Out] 
$$\begin{aligned} & 1/3 \cdot g/c / (-cx^2 + bx + a)^{(3/2)} + j/c^{(5/2)} \cdot \arctan\left(\frac{c^{(1/2)} \cdot (x - 1/2/c \cdot b)}{(-cx^2 + bx + a)^{(1/2)}}\right) + 16/3 \cdot g \cdot c \cdot b / (-4ac - b^2)^2 / (-cx^2 + bx + a)^{(1/2)} \cdot x + 1/12 \cdot i/c^2 \cdot b^3 / (-4ac - b^2) / (-cx^2 + bx + a)^{(3/2)} \cdot x - 2/3 \cdot i/c \cdot b^3 / (-4ac - b^2)^2 / (-cx^2 + bx + a)^{(1/2)} \cdot x - 1/2 \cdot i/c^2 \cdot b^2 \cdot a / (-4ac - b^2) / (-cx^2 + bx + a)^{(3/2)} - 8 \cdot i \cdot b \cdot a / (-4ac - b^2)^2 / (-cx^2 + bx + a)^{(1/2)} \cdot x + 4 \cdot i/c \cdot b^2 \cdot a / (-4ac - b^2)^2 / (-cx^2 + bx + a)^{(1/2)} - 1/6 \cdot h/c \cdot b^2 / (-4ac - b^2) / (-cx^2 + bx + a)^{(3/2)} \cdot x - 1/3 \cdot h \cdot a/c / (-4ac - b^2) / (-cx^2 + bx + a)^{(3/2)} \cdot b - 16/3 \cdot h \cdot a \cdot c / (-4ac - b^2)^2 / (-cx^2 + bx + a)^{(1/2)} \cdot x + 2 \cdot j/c^2 \cdot b^3 \cdot a / (-4ac - b^2)^2 / (-cx^2 + bx + a)^{(1/2)} + j/c^2 \cdot b^2 / (-4ac - b^2) / (- \end{aligned}$$

$$\begin{aligned}
& c*x^2+b*x+a)^{(1/2)}*x+1/24*j/c^3*b^4/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}*x-1/3 \\
& *j/c^2*b^4/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^{(1/2)}*x-1/4*j/c^3*b^3*a/(-4*a*c-b^ \\
& 2)/(-c*x^2+b*x+a)^{(3/2)}+1/2*h*x/c/(-c*x^2+b*x+a)^{(3/2)}+1/12*h/c^2*b/(-c*x^2 \\
& +b*x+a)^{(3/2)}-8/3*g*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^{(1/2)}+2/3*f/(-4*a*c-b \\
& ^2)/(-c*x^2+b*x+a)^{(3/2)}*b+i*x^2/c/(-c*x^2+b*x+a)^{(3/2)}-1/24*i/c^3*b^2/(-c* \\
& x^2+b*x+a)^{(3/2)}-2/3*i*a/c^2/(-c*x^2+b*x+a)^{(3/2)}+1/3*j*x^3/c/(-c*x^2+b*x+a \\
& )^{(3/2)}-1/48*j/c^4*b^3/(-c*x^2+b*x+a)^{(3/2)}-j/c^2*x/(-c*x^2+b*x+a)^{(1/2)}-1/ \\
& 2*j/c^3*b/(-c*x^2+b*x+a)^{(1/2)}+i/c*b*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}*x+ \\
& 1/2*j/c^2*b^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}*x-4*j/c*b^2*a/(-4*a*c-b^2 \\
& )^2/(-c*x^2+b*x+a)^{(1/2)}*x-1/3*j/c^3*b*a/(-c*x^2+b*x+a)^{(3/2)}-1/2*j/c^3*b^3 \\
& /(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(1/2)}-1/4*i/c^2*b*x/(-c*x^2+b*x+a)^{(3/2)}-1/24* \\
& i/c^3*b^4/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}+1/3*i/c^2*b^4/(-4*a*c-b^2)^2/(- \\
& c*x^2+b*x+a)^{(1/2)}-4/3*f/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}*x*c+32/3*f*c^2/( \\
& -4*a*c-b^2)^2/(-c*x^2+b*x+a)^{(1/2)}*x-16/3*f*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a) \\
& ^{(1/2)}*b-2/3*g*b/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}*x+1/3*g/c*b^2/(-4*a*c-b^ \\
& 2)/(-c*x^2+b*x+a)^{(3/2)}+1/12*h/c^2*b^3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}+4/ \\
& 3*h*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^{(1/2)}*x-2/3*h/c*b^3/(-4*a*c-b^2)^2/(- \\
& c*x^2+b*x+a)^{(1/2)}+2/3*h*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^{(3/2)}*x+8/3*h*a/(-4* \\
& a*c-b^2)^2/(-c*x^2+b*x+a)^{(1/2)}*b+1/2*j/c^2*b*x^2/(-c*x^2+b*x+a)^{(3/2)}-1/8* \\
& j/c^3*b^2*x/(-c*x^2+b*x+a)^{(3/2)}-1/48*j/c^4*b^5/(-4*a*c-b^2)/(-c*x^2+b*x+a) \\
& ^{(3/2)}+1/6*j/c^3*b^5/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^{(1/2)}
\end{aligned}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((j\*x^4+i\*x^3+h\*x^2+g\*x+f)/(-c\*x^2+b\*x+a)^(5/2),x, algorithm="maxi  
ma")

[Out] 
$$\begin{aligned}
& -1/3*i*(32*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 16*a*b^2/(sqrt( \\
& -c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + b^3*x/((-c*x^2 + b*x + a)^{(3/2)}*(b^2 \\
& + 4*a*c)*c^2) + 2*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^ \\
& 2*c) + 6*a*b*x/((-c*x^2 + b*x + a)^{(3/2)}*(b^2 + 4*a*c)*c) - 3*x^2/((-c*x^2 \\
& + b*x + a)^{(3/2)}*c) - (b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a* \\
& c)^2*c^2) - a*b^2/((-c*x^2 + b*x + a)^{(3/2)}*(b^2 + 4*a*c)*c^2) + 2*a/((-c*x \\
& ^2 + b*x + a)^{(3/2)}*c^2)) + 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + \\
& 4*a*c)^2) - 8*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/((-c*x^2 \\
& + b*x + a)^{(3/2)}*(b^2 + 4*a*c)) - b^2/((-c*x^2 + b*x + a)^{(3/2)}*(b^2 + 4*a \\
& *c)*c) + 1/((-c*x^2 + b*x + a)^{(3/2)}*c)) + 2/3*f*(16*c^2*x/(sqrt(-c*x^2 + b \\
& *x + a)*(b^2 + 4*a*c)^2) - 8*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + \\
& 2*c*x/((-c*x^2 + b*x + a)^{(3/2)}*(b^2 + 4*a*c)) - b/((-c*x^2 + b*x + a)^{(3/ \\
& 2)}*(b^2 + 4*a*c))) + 2/3*h*(2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b*x + a)*(b^2 \\
& + 4*a*c)^2) + 2*a*x/((-c*x^2 + b*x + a)^{(3/2)}*(b^2 + 4*a*c)) + b^2*x/((-c*x \\
& ^2 + b*x + a)^{(3/2)}*(b^2 + 4*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt(-c*x^2 + b*x +
\end{aligned}$$

```
a)*(b^2 + 4*a*c)^2*c) + a*b/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c)) +
j*integrate(x^4/((c^2*x^4 - 2*b*c*x^3 + 2*a*b*x + (b^2 - 2*a*c)*x^2 + a^2)*
sqrt(-c*x^2 + b*x + a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{jx^4 + ix^3 + hx^2 + gx + f}{(-cx^2 + bx + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

```
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2), x)
```

```
[Out] Timed out
```



## 3.347

$$\int (d + ex)^m (3 + 2x + 5x^2)^3 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=588

$$\frac{45(500d^2 + 5de + 17e^2)(d + ex)^{m+9}}{e^{11(m+9)}} - \frac{2(30000d^3 + 450d^2e + 3060de^2 + 49e^3)(d + ex)^{m+8}}{e^{11(m+8)}} + \frac{(5d^2 - 2de + 3e^2)^3}{e^{11(m+8)}}$$

**Rubi [A]** time = 0.36, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(3 + 2\*x + 5\*x^2)^3\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] ((5\*d^2 - 2\*d\*e + 3\*e^2)^3\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*(d + e\*x)^(1 + m))/(e^11\*(1 + m)) - ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(200\*d^5 + 169\*d^4\*e + 108\*d^3\*e^2 - 20\*d^2\*e^3 + 86\*d\*e^4 - 15\*e^5)\*(d + e\*x)^(2 + m))/(e^11\*(2 + m)) + (3\*(5\*d^2 - 2\*d\*e + 3\*e^2)\*(1500\*d^6 + 660\*d^5\*e + 792\*d^4\*e^2 + 58\*d^3\*e^3 + 547\*d^2\*e^4 - 156\*d\*e^5 + 53\*e^6)\*(d + e\*x)^(3 + m))/(e^11\*(3 + m)) - (2\*(30000\*d^7 + 1050\*d^6\*e + 21420\*d^5\*e^2 + 1715\*d^4\*e^3 + 9990\*d^3\*e^4 - 2550\*d^2\*e^5 + 2218\*d\*e^6 - 287\*e^7)\*(d + e\*x)^(4 + m))/(e^11\*(4 + m)) + ((105000\*d^6 + 3150\*d^5\*e + 53550\*d^4\*e^2 + 3430\*d^3\*e^3 + 14985\*d^2\*e^4 - 2550\*d\*e^5 + 1109\*e^6)\*(d + e\*x)^(5 + m))/(e^11\*(5 + m)) - (6\*(21000\*d^5 + 525\*d^4\*e + 7140\*d^3\*e^2 + 343\*d^2\*e^3 + 999\*d\*e^4 - 85\*e^5)\*(d + e\*x)^(6 + m))/(e^11\*(6 + m)) + ((105000\*d^4 + 2100\*d^3\*e + 21420\*d^2\*e^2 + 686\*d\*e^3 + 999\*e^4)\*(d + e\*x)^(7 + m))/(e^11\*(7 + m)) - (2\*(30000\*d^3 + 450\*d^2\*e + 3060\*d\*e^2 + 49\*e^3)\*(d + e\*x)^(8 + m))/(e^11\*(8 + m)) + (45\*(500\*d^2 + 5\*d\*e + 17\*e^2)\*(d + e\*x)^(9 + m))/(e^11\*(9 + m)) - (25\*(200\*d + e)\*(d + e\*x)^(10 + m))/(e^11\*(10 + m)) + (500\*(d + e\*x)^(11 + m))/(e^11\*(11 + m))

### Rule 1628

```
Int[(Pq_)*((d_.) + (e_)*(x_))^(m_)*((a_.) + (b_)*(x_) + (c_)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\int (d + ex)^m (3 + 2x + 5x^2)^3 (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int \left( \frac{(5d^2 - 2de + 3e^2)^3 (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^{10}} \right) dx$$

$$= \frac{(5d^2 - 2de + 3e^2)^3 (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^{11}(1 + m)}$$

**Mathematica [A]** time = 0.38, size = 537, normalized size = 0.91

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^4)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3 + 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m) + (500*(d + e*x)^10)/(11 + m))/e^11
```

**IntegrateAlgebraic [F]** time = 0.23, size = 0, normalized size = 0.00

$$\int (d + ex)^m (3 + 2x + 5x^2)^3 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
```

```
[Out] Defer[IntegrateAlgebraic] [(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
```

**fricas** [B] time = 0.94, size = 4795, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^m\*(5\*x^2+2\*x+3)^3\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="fricas")

[Out] (54\*d\*e^10\*m^10 + 500\*(e^11\*m^10 + 55\*e^11\*m^9 + 1320\*e^11\*m^8 + 18150\*e^11\*m^7 + 157773\*e^11\*m^6 + 902055\*e^11\*m^5 + 3416930\*e^11\*m^4 + 8409500\*e^11\*m^3 + 12753576\*e^11\*m^2 + 10628640\*e^11\*m + 3628800\*e^11)\*x^11 + 181440000\*d^11 + 99792000\*d^10\*e + 3392928000\*d^9\*e^2 + 488980800\*d^8\*e^3 + 5696697600\*d^7\*e^4 - 3392928000\*d^6\*e^5 + 8853546240\*d^5\*e^6 - 5728060800\*d^4\*e^7 + 6346771200\*d^3\*e^8 - 2694384000\*d^2\*e^9 + 2155507200\*d\*e^10 - 25\*(3991680\*e^11 - (20\*d\*e^10 - e^11)\*m^10 - 4\*(225\*d\*e^10 - 14\*e^11)\*m^9 - 15\*(1160\*d\*e^10 - 91\*e^11)\*m^8 - 60\*(3150\*d\*e^10 - 317\*e^11)\*m^7 - 21\*(60260\*d\*e^10 - 7963\*e^11)\*m^6 - 84\*(64125\*d\*e^10 - 11492\*e^11)\*m^5 - 5\*(2894720\*d\*e^10 - 737251\*e^11)\*m^4 - 20\*(1172700\*d\*e^10 - 456659\*e^11)\*m^3 - 36\*(570320\*d\*e^10 - 386841\*e^11)\*m^2 - 144\*(50400\*d\*e^10 - 80939\*e^11)\*m\*x^10 - 135\*(d^2\*e^9 - 26\*d\*e^10)\*m^9 + 5\*(678585600\*e^11 - (5\*d\*e^10 - 153\*e^11)\*m^10 - (1000\*d^2\*e^9 + 235\*d\*e^10 - 8721\*e^11)\*m^9 - 6\*(6000\*d^2\*e^9 + 785\*d\*e^10 - 36006\*e^11)\*m^8 - 6\*(91000\*d^2\*e^9 + 8785\*d\*e^10 - 509031\*e^11)\*m^7 - 105\*(43200\*d^2\*e^9 + 3445\*d\*e^10 - 259029\*e^11)\*m^6 - 21\*(1069000\*d^2\*e^9 + 74815\*d\*e^10 - 7560189\*e^11)\*m^5 - 2\*(33642000\*d^2\*e^9 + 2145620\*d\*e^10 - 306036567\*e^11)\*m^4 - 4\*(29531000\*d^2\*e^9 + 1761185\*d\*e^10 - 382172121\*e^11)\*m^3 - 72\*(1522000\*d^2\*e^9 + 86510\*d\*e^10 - 32587351\*e^11)\*m^2 - 1440\*(28000\*d^2\*e^9 + 1540\*d\*e^10 - 1370727\*e^11)\*m\*x^9 + 9\*(106\*d^3\*e^8 - 945\*d^2\*e^9 + 11160\*d\*e^10)\*m^8 - (488980800\*e^11 - (765\*d\*e^10 - 98\*e^11)\*m^10 - (225\*d^2\*e^9 + 37485\*d\*e^10 - 5684\*e^11)\*m^9 - 3\*(15000\*d^3\*e^8 + 2925\*d^2\*e^9 + 260100\*d\*e^10 - 47726\*e^11)\*m^8 - 42\*(30000\*d^3\*e^8 + 3375\*d^2\*e^9 + 214965\*d\*e^10 - 48958\*e^11)\*m^7 - 63\*(230000\*d^3\*e^8 + 19650\*d^2\*e^9 + 1012095\*d\*e^10 - 294882\*e^11)\*m^6 - 63\*(1400000\*d^3\*e^8 + 101175\*d^2\*e^9 + 4503555\*d\*e^10 - 1743812\*e^11)\*m^5 - (304605000\*d^3\*e^8 + 19707975\*d^2\*e^9 + 790573950\*d\*e^10 - 428393182\*e^11)\*m^4 - 4\*(147735000\*d^3\*e^8 + 8860500\*d^2\*e^9 + 329712705\*d\*e^10 - 270109021\*e^11)\*m^3 - 36\*(16335000\*d^3\*e^8 + 929925\*d^2\*e^9 + 32795550\*d\*e^10 - 46438966\*e^11)\*m^2 - 5040\*(45000\*d^3\*e^8 + 2475\*d^2\*e^9 + 84150\*d\*e^10 - 280861\*e^11)\*m\*x^8 - 6\*(574\*d^4\*e^7 - 9540\*d^3\*e^8 + 39015\*d^2\*e^9 - 277290\*d\*e^10)\*m^7 + (5696697600\*e^11 - (98\*d\*e^10 - 999\*e^11)\*m^10 - 3\*(2040\*d^2\*e^9 + 1666\*d\*e^10 - 19647\*e^11)\*m^9 - 24\*(75\*d^3\*e^8 + 10710\*d^2\*e^9 + 4508\*d\*e^10 - 62937\*e^11)\*m^8 - 6\*(60000\*d^4\*e^7 + 9600\*d^3\*e^8 + 740520\*d^2\*e^9 + 216482\*d\*e^10 - 3677319\*e^11)\*m^7 - 3\*(2520000\*d^4\*e^7 + 243600\*d^3\*e^8 + 13708800\*d^2\*e^9 + 3161774\*d\*e^10 - 67539393\*e^11)\*m^6 - 21\*(3000000\*d^4\*e^7 + 228000\*d^3\*e^8 + 10581480\*d^2\*e^9 + 2069662\*d\*e^10 - 57933009\*e^11)\*m^5 - 2\*(132300000\*d^4\*e^7 + 8738100\*d^3\*e^8 + 357157080\*d^2\*e^9 + 62076434\*d\*e^10 - 2405021571\*e^11)\*m^4 - 36\*(16240000\*d^4\*e^7 +

$$\begin{aligned}
& 981400*d^3*e^8 + 36788680*d^2*e^9 + 5871278*d*e^{10} - 341095341*e^{11})*m^3 - \\
& 72*(8820000*d^4*e^7 + 503100*d^3*e^8 + 17778600*d^2*e^9 + 2670010*d*e^{10} - \\
& 266622111*e^{11})*m^2 - 12960*(20000*d^4*e^7 + 1100*d^3*e^8 + 37400*d^2*e^9 \\
& + 5390*d*e^{10} - 1264623*e^{11})*m)*x^7 + 6*(4436*d^5*e^6 - 32144*d^4*e^7 + 24 \\
& 7086*d^3*e^8 - 615195*d^2*e^9 + 2939517*d*e^{10})*m^6 + (3392928000*e^{11} + 3* \\
& (333*d*e^{10} + 170*e^{11})*m^{10} + (686*d^2*e^9 + 52947*d*e^{10} + 30600*e^{11})*m^9 \\
& + 6*(7140*d^3*e^8 + 5145*d^2*e^9 + 198801*d*e^{10} + 133025*e^{11})*m^8 + 6*( \\
& 2100*d^4*e^7 + 257040*d^3*e^8 + 95354*d^2*e^9 + 2484513*d*e^{10} + 1978800*e^{11})*m^7 \\
& + 3*(840000*d^5*e^6 + 109200*d^4*e^7 + 7282800*d^3*e^8 + 1886500*d^2*e^9 \\
& + 37725237*d*e^{10} + 37016310*e^{11})*m^6 + 3*(12600000*d^5*e^6 + 105000 \\
& 0*d^4*e^7 + 52264800*d^3*e^8 + 10813418*d^2*e^9 + 179179641*d*e^{10} + 226287 \\
& 000*e^{11})*m^5 + 42*(5100000*d^5*e^6 + 348000*d^4*e^7 + 14635980*d^3*e^8 + 2 \\
& 609495*d^2*e^9 + 37733562*d*e^{10} + 64999925*e^{11})*m^4 + 4*(141750000*d^5*e^6 \\
& + 8659350*d^4*e^7 + 327983040*d^3*e^8 + 52869334*d^2*e^9 + 692643663*d*e^{10} \\
& + 1769460300*e^{11})*m^3 + 120*(5754000*d^5*e^6 + 329070*d^4*e^7 + 1165962 \\
& 0*d^3*e^8 + 1755817*d^2*e^9 + 21444534*d*e^{10} + 93454763*e^{11})*m^2 + 7200*( \\
& 42000*d^5*e^6 + 2310*d^4*e^7 + 78540*d^3*e^8 + 11319*d^2*e^9 + 131868*d*e^{10} \\
& + 1344547*e^{11})*m)*x^6 - 3*(20400*d^6*e^5 - 452472*d^5*e^6 + 1526840*d^4* \\
& e^7 - 7212240*d^3*e^8 + 12236805*d^2*e^9 - 41597010*d*e^{10})*m^5 + (88535462 \\
& 40*e^{11} + (510*d*e^{10} + 1109*e^{11})*m^{10} - (5994*d^2*e^9 - 28050*d*e^{10} - 67 \\
& 649*e^{11})*m^9 - 12*(343*d^3*e^8 + 23976*d^2*e^9 - 54825*d*e^{10} - 149715*e^{11})*m^8 \\
& - 6*(42840*d^4*e^7 + 27440*d^3*e^8 + 953046*d^2*e^9 - 1430550*d*e^{10} \\
& - 4541355*e^{11})*m^7 - 3*(25200*d^5*e^6 + 2656080*d^4*e^7 + 869848*d^3*e^8 \\
& + 20283696*d^2*e^9 - 22710810*d*e^{10} - 86713819*e^{11})*m^6 - 3*(5040000*d^6* \\
& e^5 + 529200*d^5*e^6 + 30416400*d^4*e^7 + 6969760*d^3*e^8 + 124932942*d^2*e^9 \\
& - 112732950*d*e^{10} - 541448179*e^{11})*m^5 - 2*(75600000*d^6*e^5 + 5481000 \\
& *d^5*e^6 + 242260200*d^4*e^7 + 45047562*d^3*e^8 + 675619704*d^2*e^9 - 51950 \\
& 1300*d*e^{10} - 3335910815*e^{11})*m^4 - 4*(132300000*d^6*e^5 + 8221500*d^5*e^6 \\
& + 316416240*d^4*e^7 + 51779280*d^3*e^8 + 688165146*d^2*e^9 - 470707050*d*e^{10} \\
& - 4412539105*e^{11})*m^3 - 72*(10500000*d^6*e^5 + 602700*d^5*e^6 + 214342 \\
& 80*d^4*e^7 + 3239978*d^3*e^8 + 39724236*d^2*e^9 - 25005980*d*e^{10} - 3955614 \\
& 47*e^{11})*m^2 - 288*(1260000*d^6*e^5 + 69300*d^5*e^6 + 2356200*d^4*e^7 + 339 \\
& 570*d^3*e^8 + 3956040*d^2*e^9 - 2356200*d*e^{10} - 86687203*e^{11})*m)*x^5 + 3* \\
& (239760*d^7*e^4 - 918000*d^6*e^5 + 9537400*d^5*e^6 - 19929280*d^4*e^7 + 648 \\
& 36702*d^3*e^8 - 79518915*d^2*e^9 + 198514620*d*e^{10})*m^4 + (5728060800*e^{11} \\
& + (1109*d*e^{10} + 574*e^{11})*m^{10} - (2550*d^2*e^9 - 63213*d*e^{10} - 35588*e^{11})*m^9 \\
& + 6*(4995*d^3*e^8 - 21675*d^2*e^9 + 257288*d*e^{10} + 160433*e^{11})*m^8 \\
& + 6*(3430*d^4*e^7 + 219780*d^3*e^8 - 461550*d^2*e^9 + 3512203*d*e^{10} + 248 \\
& 3698*e^{11})*m^7 + 15*(85680*d^5*e^6 + 49392*d^4*e^7 + 1554444*d^3*e^8 - 2122 \\
& 620*d^2*e^9 + 11723239*d*e^{10} + 9703470*e^{11})*m^6 + 3*(126000*d^6*e^5 + 115 \\
& 66800*d^5*e^6 + 3361400*d^4*e^7 + 70329600*d^3*e^8 - 71101650*d^2*e^9 + 306 \\
& 983399*d*e^{10} + 310583364*e^{11})*m^5 + 2*(37800000*d^7*e^4 + 3213000*d^6*e^5 \\
& + 158722200*d^5*e^6 + 32104800*d^4*e^7 + 515019465*d^3*e^8 - 418887225*d^2 \\
& *e^9 + 1494010421*d*e^{10} + 1964946361*e^{11})*m^4 + 4*(113400000*d^7*e^4 + 72 \\
& 76500*d^6*e^5 + 288206100*d^5*e^6 + 48409305*d^4*e^7 + 659010330*d^3*e^8 -
\end{aligned}$$

$$\begin{aligned}
& 460978800*d^2*e^9 + 1424518263*d*e^10 + 2670494533*e^11)*m^3 + 72*(1155000 \\
& *d^7*e^4 + 666750*d^6*e^5 + 23847600*d^5*e^6 + 3625510*d^4*e^7 + 44710245*d \\
& ^3*e^8 - 28312225*d^2*e^9 + 79001833*d*e^10 + 245697543*e^11)*m^2 + 720*(63 \\
& 0000*d^7*e^4 + 34650*d^6*e^5 + 1178100*d^5*e^6 + 169785*d^4*e^7 + 1978020*d \\
& ^3*e^8 - 1178100*d^2*e^9 + 3074148*d*e^10 + 22036147*e^11)*m)*x^4 + 12*(411 \\
& 60*d^8*e^3 + 2277720*d^7*e^4 - 4105500*d^6*e^5 + 26582730*d^5*e^6 - 3858686 \\
& 3*d^4*e^7 + 91855890*d^3*e^8 - 84312180*d^2*e^9 + 157352130*d*e^10)*m^3 + ( \\
& 6346771200*e^11 + (574*d*e^10 + 477*e^11)*m^10 - (4436*d^2*e^9 - 33866*d*e^ \\
& 10 - 30051*e^11)*m^9 + 24*(425*d^3*e^8 - 9981*d^2*e^9 + 35875*d*e^10 + 3450 \\
& 3*e^11)*m^8 - 6*(19980*d^4*e^7 - 81600*d^3*e^8 + 909380*d^2*e^9 - 2053198*d \\
& *e^10 - 2183229*e^11)*m^7 - 3*(27440*d^5*e^6 + 1638360*d^4*e^7 - 3202800*d^ \\
& 3*e^8 + 22641344*d^2*e^9 - 36198162*d*e^10 - 43730883*e^11)*m^6 - 3*(171360 \\
& 0*d^6*e^5 + 905520*d^5*e^6 + 26173800*d^4*e^7 - 32844000*d^3*e^8 + 16654074 \\
& 8*d^2*e^9 - 201988878*d*e^10 - 288179073*e^11)*m^5 - 2*(756000*d^7*e^4 + 61 \\
& 689600*d^6*e^5 + 16093560*d^5*e^6 + 304195500*d^4*e^7 - 278811900*d^3*e^8 + \\
& 1092467028*d^2*e^9 - 1055996410*d*e^10 - 1884673269*e^11)*m^4 - 4*(7560000 \\
& 0*d^8*e^3 + 5292000*d^7*e^4 + 224910000*d^6*e^5 + 40069260*d^5*e^6 + 573745 \\
& 680*d^4*e^7 - 419556600*d^3*e^8 + 1349320300*d^2*e^9 - 1086499918*d*e^10 - \\
& 2657980899*e^11)*m^3 - 24*(37800000*d^8*e^3 + 2205000*d^7*e^4 + 79682400*d^ \\
& 6*e^5 + 12238240*d^5*e^6 + 152467380*d^4*e^7 - 97540900*d^3*e^8 + 275018692 \\
& *d^2*e^9 - 193842670*d*e^10 - 763013811*e^11)*m^2 - 4320*(140000*d^8*e^3 + \\
& 7700*d^7*e^4 + 261800*d^6*e^5 + 37730*d^5*e^6 + 439560*d^4*e^7 - 261800*d^3 \\
& *e^8 + 683144*d^2*e^9 - 441980*d*e^10 - 3946963*e^11)*m)*x^3 + 12*(2570400* \\
& d^9*e^2 + 1234800*d^8*e^3 + 32307660*d^7*e^4 - 36490500*d^6*e^5 + 165294232 \\
& *d^5*e^6 - 177258088*d^4*e^7 + 320238402*d^3*e^8 - 224755965*d^2*e^9 + 3163 \\
& 09212*d*e^10)*m^2 + 3*(898128000*e^11 + 3*(53*d*e^10 + 15*e^11)*m^10 - (574 \\
& *d^2*e^9 - 9699*d*e^10 - 2880*e^11)*m^9 + (4436*d^3*e^8 - 32718*d^2*e^9 + 2 \\
& 56626*d*e^10 + 80865*e^11)*m^8 - 2*(5100*d^4*e^7 - 115336*d^3*e^8 + 397782* \\
& d^2*e^9 - 1926603*d*e^10 - 654210*e^11)*m^7 + (119880*d^5*e^6 - 469200*d^4* \\
& e^7 + 4994936*d^3*e^8 - 10728060*d^2*e^9 + 36024471*d*e^10 + 13467195*e^11) \\
& *m^6 + (82320*d^6*e^5 + 4675320*d^5*e^6 - 8670000*d^4*e^7 + 57934160*d^3*e^ \\
& 8 - 87138366*d^2*e^9 + 216130131*d*e^10 + 91755720*e^11)*m^5 + (5140800*d^7 \\
& *e^4 + 2551920*d^6*e^5 + 69170760*d^5*e^6 - 81192000*d^4*e^7 + 383753924*d^ \\
& 3*e^8 - 431689902*d^2*e^9 + 824188584*d*e^10 + 416767635*e^11)*m^4 + 4*(378 \\
& 000*d^8*e^3 + 28274400*d^7*e^4 + 6770820*d^6*e^5 + 117512370*d^5*e^6 - 9880 \\
& 9950*d^4*e^7 + 354356552*d^3*e^8 - 312153254*d^2*e^9 + 473899341*d*e^10 + 3 \\
& 09068145*e^11)*m^3 + 12*(25200000*d^9*e^2 + 1512000*d^8*e^3 + 56120400*d^7* \\
& e^4 + 8842540*d^6*e^5 + 112906980*d^5*e^6 - 73978900*d^4*e^7 + 213535732*d^ \\
& 3*e^8 - 154064470*d^2*e^9 + 192742980*d*e^10 + 188672355*e^11)*m^2 + 2160*( \\
& 140000*d^9*e^2 + 7700*d^8*e^3 + 261800*d^7*e^4 + 37730*d^6*e^5 + 439560*d^5 \\
& *e^6 - 261800*d^4*e^7 + 683144*d^3*e^8 - 441980*d^2*e^9 + 489720*d*e^10 + 1 \\
& 047765*e^11)*m)*x^2 + 144*(63000*d^10*e + 4498200*d^9*e^2 + 1025570*d^8*e^3 \\
& + 16893090*d^7*e^4 - 13427450*d^6*e^5 + 45284906*d^5*e^6 - 37254035*d^4*e^ \\
& 7 + 52296690*d^3*e^8 - 28438425*d^2*e^9 + 30235140*d*e^10)*m + 3*(718502400 \\
& *e^11 + 9*(5*d*e^10 + 2*e^11)*m^10 - 3*(106*d^2*e^9 - 945*d*e^10 - 390*e^11
\end{aligned}$$

```

)*m^9 + 2*(574*d^3*e^8 - 9540*d^2*e^9 + 39015*d*e^10 + 16740*e^11)*m^8 - 2*
(4436*d^4*e^7 - 32144*d^3*e^8 + 247086*d^2*e^9 - 615195*d*e^10 - 277290*e^1
1)*m^7 + (20400*d^5*e^6 - 452472*d^4*e^7 + 1526840*d^3*e^8 - 7212240*d^2*e^
9 + 12236805*d*e^10 + 5879034*e^11)*m^6 - (239760*d^6*e^5 - 918000*d^5*e^6
+ 9537400*d^4*e^7 - 19929280*d^3*e^8 + 64836702*d^2*e^9 - 79518915*d*e^10 -
41597010*e^11)*m^5 - 4*(41160*d^7*e^4 + 2277720*d^6*e^5 - 4105500*d^5*e^6
+ 26582730*d^4*e^7 - 38586863*d^3*e^8 + 91855890*d^2*e^9 - 84312180*d*e^10
- 49628655*e^11)*m^4 - 4*(2570400*d^8*e^3 + 1234800*d^7*e^4 + 32307660*d^6*
e^5 - 36490500*d^5*e^6 + 165294232*d^4*e^7 - 177258088*d^3*e^8 + 320238402*
d^2*e^9 - 224755965*d*e^10 - 157352130*e^11)*m^3 - 48*(63000*d^9*e^2 + 4498
200*d^8*e^3 + 1025570*d^7*e^4 + 16893090*d^6*e^5 - 13427450*d^5*e^6 + 45284
906*d^4*e^7 - 37254035*d^3*e^8 + 52296690*d^2*e^9 - 28438425*d*e^10 - 26359
101*e^11)*m^2 - 8640*(70000*d^10*e + 3850*d^9*e^2 + 130900*d^8*e^3 + 18865*
d^7*e^4 + 219780*d^6*e^5 - 130900*d^5*e^6 + 341572*d^4*e^7 - 220990*d^3*e^8
+ 244860*d^2*e^9 - 103950*d*e^10 - 167973*e^11)*m)*x*(e*x + d)^m/(e^11*m^
11 + 66*e^11*m^10 + 1925*e^11*m^9 + 32670*e^11*m^8 + 357423*e^11*m^7 + 2637
558*e^11*m^6 + 13339535*e^11*m^5 + 45995730*e^11*m^4 + 105258076*e^11*m^3 +
150917976*e^11*m^2 + 120543840*e^11*m + 39916800*e^11)

```

**giac [B]** time = 0.62, size = 10960, normalized size = 18.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (500*(x*e + d)^m*m^10*x^11*e^11 + 500*(x*e + d)^m*d*m^10*x^10*e^10 - 25*(x*
e + d)^m*m^10*x^10*e^11 + 27500*(x*e + d)^m*m^9*x^11*e^11 - 25*(x*e + d)^m*
d*m^10*x^9*e^10 + 22500*(x*e + d)^m*d*m^9*x^10*e^10 - 5000*(x*e + d)^m*d^2*
m^9*x^9*e^9 + 765*(x*e + d)^m*m^10*x^9*e^11 - 1400*(x*e + d)^m*m^9*x^10*e^1
1 + 66000*(x*e + d)^m*m^8*x^11*e^11 + 765*(x*e + d)^m*d*m^10*x^8*e^10 - 11
75*(x*e + d)^m*d*m^9*x^9*e^10 + 435000*(x*e + d)^m*d*m^8*x^10*e^10 + 225*(x
*e + d)^m*d^2*m^9*x^8*e^9 - 180000*(x*e + d)^m*d^2*m^8*x^9*e^9 + 45000*(x*e
+ d)^m*d^3*m^8*x^8*e^8 - 98*(x*e + d)^m*m^10*x^8*e^11 + 43605*(x*e + d)^m*
m^9*x^9*e^11 - 34125*(x*e + d)^m*m^8*x^10*e^11 + 9075000*(x*e + d)^m*m^7*x^
11*e^11 - 98*(x*e + d)^m*d*m^10*x^7*e^10 + 37485*(x*e + d)^m*d*m^9*x^8*e^10
- 23550*(x*e + d)^m*d*m^8*x^9*e^10 + 4725000*(x*e + d)^m*d*m^7*x^10*e^10 -
6120*(x*e + d)^m*d^2*m^9*x^7*e^9 + 8775*(x*e + d)^m*d^2*m^8*x^8*e^9 - 2730
000*(x*e + d)^m*d^2*m^7*x^9*e^9 - 1800*(x*e + d)^m*d^3*m^8*x^7*e^8 + 126000
0*(x*e + d)^m*d^3*m^7*x^8*e^8 - 360000*(x*e + d)^m*d^4*m^7*x^7*e^7 + 999*(x
*e + d)^m*m^10*x^7*e^11 - 5684*(x*e + d)^m*m^9*x^8*e^11 + 1080180*(x*e + d)
^m*m^8*x^9*e^11 - 475500*(x*e + d)^m*m^7*x^10*e^11 + 78886500*(x*e + d)^m*m
^6*x^11*e^11 + 999*(x*e + d)^m*d*m^10*x^6*e^10 - 4998*(x*e + d)^m*d*m^9*x^7
*e^10 + 780300*(x*e + d)^m*d*m^8*x^8*e^10 - 263550*(x*e + d)^m*d*m^7*x^9*e^

```

$$\begin{aligned}
& 10 + 31636500*(x*e + d)^m*d*m^6*x^10*e^10 + 686*(x*e + d)^m*d^2*m^9*x^6*e^9 \\
& - 257040*(x*e + d)^m*d^2*m^8*x^7*e^9 + 141750*(x*e + d)^m*d^2*m^7*x^8*e^9 \\
& - 22680000*(x*e + d)^m*d^2*m^6*x^9*e^9 + 42840*(x*e + d)^m*d^3*m^8*x^6*e^8 \\
& - 57600*(x*e + d)^m*d^3*m^7*x^7*e^8 + 14490000*(x*e + d)^m*d^3*m^6*x^8*e^8 \\
& + 12600*(x*e + d)^m*d^4*m^7*x^6*e^7 - 7560000*(x*e + d)^m*d^4*m^6*x^7*e^7 + \\
& 2520000*(x*e + d)^m*d^5*m^6*x^6*e^6 + 510*(x*e + d)^m*m^10*x^6*e^11 + 5894 \\
& 1*(x*e + d)^m*m^9*x^7*e^11 - 143178*(x*e + d)^m*m^8*x^8*e^11 + 15270930*(x* \\
& e + d)^m*m^7*x^9*e^11 - 4180575*(x*e + d)^m*m^6*x^10*e^11 + 451027500*(x*e \\
& + d)^m*m^5*x^11*e^11 + 510*(x*e + d)^m*d*m^10*x^5*e^10 + 52947*(x*e + d)^m* \\
& d*m^9*x^6*e^10 - 108192*(x*e + d)^m*d*m^8*x^7*e^10 + 9028530*(x*e + d)^m*d* \\
& m^7*x^8*e^10 - 1808625*(x*e + d)^m*d*m^6*x^9*e^10 + 134662500*(x*e + d)^m*d \\
& *m^5*x^10*e^10 - 5994*(x*e + d)^m*d^2*m^9*x^5*e^9 + 30870*(x*e + d)^m*d^2*m \\
& ^8*x^6*e^9 - 4443120*(x*e + d)^m*d^2*m^7*x^7*e^9 + 1237950*(x*e + d)^m*d^2* \\
& m^6*x^8*e^9 - 112245000*(x*e + d)^m*d^2*m^5*x^9*e^9 - 4116*(x*e + d)^m*d^3* \\
& m^8*x^5*e^8 + 1542240*(x*e + d)^m*d^3*m^7*x^6*e^8 - 730800*(x*e + d)^m*d^3* \\
& m^6*x^7*e^8 + 88200000*(x*e + d)^m*d^3*m^5*x^8*e^8 - 257040*(x*e + d)^m*d^4 \\
& *m^7*x^5*e^7 + 327600*(x*e + d)^m*d^4*m^6*x^6*e^7 - 63000000*(x*e + d)^m*d^ \\
& 4*m^5*x^7*e^7 - 75600*(x*e + d)^m*d^5*m^6*x^5*e^6 + 37800000*(x*e + d)^m*d^ \\
& 5*m^5*x^6*e^6 - 15120000*(x*e + d)^m*d^6*m^5*x^5*e^5 + 1109*(x*e + d)^m*m^1 \\
& 0*x^5*e^11 + 30600*(x*e + d)^m*m^9*x^6*e^11 + 1510488*(x*e + d)^m*m^8*x^7*e \\
& ^11 - 2056236*(x*e + d)^m*m^7*x^8*e^11 + 135990225*(x*e + d)^m*m^6*x^9*e^11 \\
& - 24133200*(x*e + d)^m*m^5*x^10*e^11 + 1708465000*(x*e + d)^m*m^4*x^11*e^1 \\
& 1 + 1109*(x*e + d)^m*d*m^10*x^4*e^10 + 28050*(x*e + d)^m*d*m^9*x^5*e^10 + 1 \\
& 192806*(x*e + d)^m*d*m^8*x^6*e^10 - 1298892*(x*e + d)^m*d*m^7*x^7*e^10 + 63 \\
& 761985*(x*e + d)^m*d*m^6*x^8*e^10 - 7855575*(x*e + d)^m*d*m^5*x^9*e^10 + 36 \\
& 1840000*(x*e + d)^m*d*m^4*x^10*e^10 - 2550*(x*e + d)^m*d^2*m^9*x^4*e^9 - 28 \\
& 7712*(x*e + d)^m*d^2*m^8*x^5*e^9 + 572124*(x*e + d)^m*d^2*m^7*x^6*e^9 - 411 \\
& 26400*(x*e + d)^m*d^2*m^6*x^7*e^9 + 6374025*(x*e + d)^m*d^2*m^5*x^8*e^9 - 3 \\
& 36420000*(x*e + d)^m*d^2*m^4*x^9*e^9 + 29970*(x*e + d)^m*d^3*m^8*x^4*e^8 - \\
& 164640*(x*e + d)^m*d^3*m^7*x^5*e^8 + 21848400*(x*e + d)^m*d^3*m^6*x^6*e^8 - \\
& 4788000*(x*e + d)^m*d^3*m^5*x^7*e^8 + 304605000*(x*e + d)^m*d^3*m^4*x^8*e^ \\
& 8 + 20580*(x*e + d)^m*d^4*m^7*x^4*e^7 - 7968240*(x*e + d)^m*d^4*m^6*x^5*e^7 \\
& + 3150000*(x*e + d)^m*d^4*m^5*x^6*e^7 - 264600000*(x*e + d)^m*d^4*m^4*x^7* \\
& e^7 + 1285200*(x*e + d)^m*d^5*m^6*x^4*e^6 - 1587600*(x*e + d)^m*d^5*m^5*x^5 \\
& *e^6 + 214200000*(x*e + d)^m*d^5*m^4*x^6*e^6 + 378000*(x*e + d)^m*d^6*m^5*x \\
& ^4*e^5 - 151200000*(x*e + d)^m*d^6*m^4*x^5*e^5 + 75600000*(x*e + d)^m*d^7*m \\
& ^4*x^4*e^4 + 574*(x*e + d)^m*m^10*x^4*e^11 + 67649*(x*e + d)^m*m^9*x^5*e^11 \\
& + 798150*(x*e + d)^m*m^8*x^6*e^11 + 22063914*(x*e + d)^m*m^7*x^7*e^11 - 18 \\
& 577566*(x*e + d)^m*m^6*x^8*e^11 + 793819845*(x*e + d)^m*m^5*x^9*e^11 - 9215 \\
& 6375*(x*e + d)^m*m^4*x^10*e^11 + 4204750000*(x*e + d)^m*m^3*x^11*e^11 + 574 \\
& *(x*e + d)^m*d*m^10*x^3*e^10 + 63213*(x*e + d)^m*d*m^9*x^4*e^10 + 657900*(x \\
& *e + d)^m*d*m^8*x^5*e^10 + 14907078*(x*e + d)^m*d*m^7*x^6*e^10 - 9485322*(x \\
& *e + d)^m*d*m^6*x^7*e^10 + 283723965*(x*e + d)^m*d*m^5*x^8*e^10 - 21456200* \\
& (x*e + d)^m*d*m^4*x^9*e^10 + 586350000*(x*e + d)^m*d*m^3*x^10*e^10 - 4436*( \\
& x*e + d)^m*d^2*m^9*x^3*e^9 - 130050*(x*e + d)^m*d^2*m^8*x^4*e^9 - 5718276*(
\end{aligned}$$

$$\begin{aligned}
& x^e + d)^{m*d^2*m^7*x^5*e^9} + 5659500*(x^e + d)^{m*d^2*m^6*x^6*e^9} - 22221108 \\
& 0*(x^e + d)^{m*d^2*m^5*x^7*e^9} + 19707975*(x^e + d)^{m*d^2*m^4*x^8*e^9} - 5906 \\
& 20000*(x^e + d)^{m*d^2*m^3*x^9*e^9} + 10200*(x^e + d)^{m*d^3*m^8*x^3*e^8} + 131 \\
& 8680*(x^e + d)^{m*d^3*m^7*x^4*e^8} - 2609544*(x^e + d)^{m*d^3*m^6*x^5*e^8} + 15 \\
& 6794400*(x^e + d)^{m*d^3*m^5*x^6*e^8} - 17476200*(x^e + d)^{m*d^3*m^4*x^7*e^8} \\
& + 590940000*(x^e + d)^{m*d^3*m^3*x^8*e^8} - 119880*(x^e + d)^{m*d^4*m^7*x^3*e^7} \\
& + 740880*(x^e + d)^{m*d^4*m^6*x^4*e^7} - 91249200*(x^e + d)^{m*d^4*m^5*x^5*e^7} \\
& + 14616000*(x^e + d)^{m*d^4*m^4*x^6*e^7} - 584640000*(x^e + d)^{m*d^4*m^3*x^7*e^7} \\
& - 82320*(x^e + d)^{m*d^5*m^6*x^3*e^6} + 34700400*(x^e + d)^{m*d^5*m^5*x^4*e^6} \\
& - 10962000*(x^e + d)^{m*d^5*m^4*x^5*e^6} + 567000000*(x^e + d)^{m*d^5*m^3*x^6*e^6} \\
& - 5140800*(x^e + d)^{m*d^6*m^5*x^3*e^5} + 6426000*(x^e + d)^{m*d^6*m^4*x^4*e^5} \\
& - 529200000*(x^e + d)^{m*d^6*m^3*x^5*e^5} - 1512000*(x^e + d)^{m*d^7*m^4*x^3*e^4} \\
& + 453600000*(x^e + d)^{m*d^7*m^3*x^4*e^4} - 302400000*(x^e + d)^{m*d^8*m^3*x^3*e^3} \\
& + 477*(x^e + d)^{m*m^10*x^3*e^11} + 35588*(x^e + d)^{m*m^9*x^4*e^11} \\
& + 1796580*(x^e + d)^{m*m^8*x^5*e^11} + 11872800*(x^e + d)^{m*m^7*x^6*e^11} \\
& + 202618179*(x^e + d)^{m*m^6*x^7*e^11} - 109860156*(x^e + d)^{m*m^5*x^8*e^11} \\
& + 3060365670*(x^e + d)^{m*m^4*x^9*e^11} - 228329500*(x^e + d)^{m*m^3*x^10*e^11} \\
& + 6376788000*(x^e + d)^{m*m^2*x^11*e^11} + 477*(x^e + d)^{m*d*m^10*x^2*e^10} \\
& + 33866*(x^e + d)^{m*d*m^9*x^3*e^10} + 1543728*(x^e + d)^{m*d*m^8*x^4*e^10} \\
& + 8583300*(x^e + d)^{m*d*m^7*x^5*e^10} + 113175711*(x^e + d)^{m*d*m^6*x^6*e^10} \\
& - 43462902*(x^e + d)^{m*d*m^5*x^7*e^10} + 790573950*(x^e + d)^{m*d*m^4*x^8*e^10} \\
& - 35223700*(x^e + d)^{m*d*m^3*x^9*e^10} + 513288000*(x^e + d)^{m*d*m^2*x^10*e^10} \\
& - 1722*(x^e + d)^{m*d^2*m^9*x^2*e^9} - 239544*(x^e + d)^{m*d^2*m^8*x^3*e^9} \\
& - 2769300*(x^e + d)^{m*d^2*m^7*x^4*e^9} - 60851088*(x^e + d)^{m*d^2*m^6*x^5*e^9} \\
& + 32440254*(x^e + d)^{m*d^2*m^5*x^6*e^9} - 714314160*(x^e + d)^{m*d^2*m^4*x^7*e^9} \\
& + 35442000*(x^e + d)^{m*d^2*m^3*x^8*e^9} - 547920000*(x^e + d)^{m*d^2*m^2*x^9*e^9} \\
& + 13308*(x^e + d)^{m*d^3*m^8*x^2*e^8} + 489600*(x^e + d)^{m*d^3*m^7*x^3*e^8} \\
& + 23316660*(x^e + d)^{m*d^3*m^6*x^4*e^8} - 20909280*(x^e + d)^{m*d^3*m^5*x^5*e^8} \\
& + 614711160*(x^e + d)^{m*d^3*m^4*x^6*e^8} - 35330400*(x^e + d)^{m*d^3*m^3*x^7*e^8} \\
& + 588060000*(x^e + d)^{m*d^3*m^2*x^8*e^8} - 30600*(x^e + d)^{m*d^4*m^7*x^2*e^7} \\
& - 4915080*(x^e + d)^{m*d^4*m^6*x^3*e^7} + 10084200*(x^e + d)^{m*d^4*m^5*x^4*e^7} \\
& - 484520400*(x^e + d)^{m*d^4*m^4*x^5*e^7} + 34637400*(x^e + d)^{m*d^4*m^3*x^6*e^7} \\
& - 635040000*(x^e + d)^{m*d^4*m^2*x^7*e^7} + 359640*(x^e + d)^{m*d^5*m^6*x^2*e^6} \\
& - 2716560*(x^e + d)^{m*d^5*m^5*x^3*e^6} + 317444400*(x^e + d)^{m*d^5*m^4*x^4*e^6} \\
& - 32886000*(x^e + d)^{m*d^5*m^3*x^5*e^6} + 690480000*(x^e + d)^{m*d^5*m^2*x^6*e^6} \\
& + 246960*(x^e + d)^{m*d^6*m^5*x^2*e^5} - 123379200*(x^e + d)^{m*d^6*m^4*x^3*e^5} \\
& + 29106000*(x^e + d)^{m*d^6*m^3*x^4*e^5} - 756000000*(x^e + d)^{m*d^6*m^2*x^5*e^5} \\
& + 15422400*(x^e + d)^{m*d^7*m^4*x^2*e^4} - 21168000*(x^e + d)^{m*d^7*m^3*x^3*e^4} \\
& + 831600000*(x^e + d)^{m*d^7*m^2*x^4*e^4} + 4536000*(x^e + d)^{m*d^8*m^3*x^2*e^3} \\
& - 907200000*(x^e + d)^{m*d^8*m^2*x^3*e^3} + 907200000*(x^e + d)^{m*d^9*m^2*x^2*e^2} \\
& + 135*(x^e + d)^{m*m^10*x^2*e^11} + 30051*(x^e + d)^{m*m^9*x^3*e^11} + 962598*(x^e + d)^{m*m^8*x^4*e^11} \\
& + 27248130*(x^e + d)^{m*m^7*x^5*e^11} + 111048930*(x^e + d)^{m*m^6*x^6*e^11} \\
& + 1216593189*(x^e + d)^{m*m^5*x^7*e^11} - 428393182*(x^e + d)^{m*m^4*x^8*e^11} \\
& + 7643442420*(x^e + d)^{m*m^3*x^9*e^11} - 348156900*(x^e + d)^{m*m^2*x^10*e^11}
\end{aligned}$$



$$\begin{aligned}
& + 5314320000*(x*e + d)^m*m*x^{11}*e^{11} + 135*(x*e + d)^m*d*m^{10}*x*e^{10} + 2909 \\
& 7*(x*e + d)^m*d*m^9*x^2*e^{10} + 861000*(x*e + d)^m*d*m^8*x^3*e^{10} + 21073218 \\
& *(x*e + d)^m*d*m^7*x^4*e^{10} + 68132430*(x*e + d)^m*d*m^6*x^5*e^{10} + 5375389 \\
& 23*(x*e + d)^m*d*m^5*x^6*e^{10} - 124152868*(x*e + d)^m*d*m^4*x^7*e^{10} + 1318 \\
& 850820*(x*e + d)^m*d*m^3*x^8*e^{10} - 31143600*(x*e + d)^m*d*m^2*x^9*e^{10} + 1 \\
& 81440000*(x*e + d)^m*d*m*x^{10}*e^{10} - 954*(x*e + d)^m*d^2*m^9*x*e^9 - 98154* \\
& (x*e + d)^m*d^2*m^8*x^2*e^9 - 5456280*(x*e + d)^m*d^2*m^7*x^3*e^9 - 3183930 \\
& 0*(x*e + d)^m*d^2*m^6*x^4*e^9 - 374798826*(x*e + d)^m*d^2*m^5*x^5*e^9 + 109 \\
& 598790*(x*e + d)^m*d^2*m^4*x^6*e^9 - 1324392480*(x*e + d)^m*d^2*m^3*x^7*e^9 \\
& + 33477300*(x*e + d)^m*d^2*m^2*x^8*e^9 - 201600000*(x*e + d)^m*d^2*m*x^9*e \\
& ^9 + 3444*(x*e + d)^m*d^3*m^8*x*e^8 + 692016*(x*e + d)^m*d^3*m^7*x^2*e^8 + \\
& 9608400*(x*e + d)^m*d^3*m^6*x^3*e^8 + 210988800*(x*e + d)^m*d^3*m^5*x^4*e^8 \\
& - 90095124*(x*e + d)^m*d^3*m^4*x^5*e^8 + 1311932160*(x*e + d)^m*d^3*m^3*x^ \\
& 6*e^8 - 36223200*(x*e + d)^m*d^3*m^2*x^7*e^8 + 226800000*(x*e + d)^m*d^3*m* \\
& x^8*e^8 - 26616*(x*e + d)^m*d^4*m^7*x*e^7 - 1407600*(x*e + d)^m*d^4*m^6*x^2 \\
& *e^7 - 78521400*(x*e + d)^m*d^4*m^5*x^3*e^7 + 64209600*(x*e + d)^m*d^4*m^4* \\
& x^4*e^7 - 1265664960*(x*e + d)^m*d^4*m^3*x^5*e^7 + 39488400*(x*e + d)^m*d^4 \\
& *m^2*x^6*e^7 - 259200000*(x*e + d)^m*d^4*m*x^7*e^7 + 61200*(x*e + d)^m*d^5* \\
& m^6*x*e^6 + 14025960*(x*e + d)^m*d^5*m^5*x^2*e^6 - 32187120*(x*e + d)^m*d^5 \\
& *m^4*x^3*e^6 + 1152824400*(x*e + d)^m*d^5*m^3*x^4*e^6 - 43394400*(x*e + d)^ \\
& m*d^5*m^2*x^5*e^6 + 302400000*(x*e + d)^m*d^5*m*x^6*e^6 - 719280*(x*e + d)^ \\
& m*d^6*m^5*x*e^5 + 7655760*(x*e + d)^m*d^6*m^4*x^2*e^5 - 899640000*(x*e + d) \\
& ^m*d^6*m^3*x^3*e^5 + 48006000*(x*e + d)^m*d^6*m^2*x^4*e^5 - 362880000*(x*e \\
& + d)^m*d^6*m*x^5*e^5 - 493920*(x*e + d)^m*d^7*m^4*x*e^4 + 339292800*(x*e + \\
& d)^m*d^7*m^3*x^2*e^4 - 52920000*(x*e + d)^m*d^7*m^2*x^3*e^4 + 453600000*(x* \\
& e + d)^m*d^7*m*x^4*e^4 - 30844800*(x*e + d)^m*d^8*m^3*x*e^3 + 54432000*(x*e \\
& + d)^m*d^8*m^2*x^2*e^3 - 604800000*(x*e + d)^m*d^8*m*x^3*e^3 - 9072000*(x* \\
& e + d)^m*d^9*m^2*x*e^2 + 907200000*(x*e + d)^m*d^9*m*x^2*e^2 - 1814400000*( \\
& x*e + d)^m*d^10*m*x*e + 54*(x*e + d)^m*m^{10}*x*e^{11} + 8640*(x*e + d)^m*m^9*x \\
& ^2*e^{11} + 828072*(x*e + d)^m*m^8*x^3*e^{11} + 14902188*(x*e + d)^m*m^7*x^4*e^{ \\
& 11} + 260141457*(x*e + d)^m*m^6*x^5*e^{11} + 678861000*(x*e + d)^m*m^5*x^6*e^{1 \\
& 1} + 4810043142*(x*e + d)^m*m^4*x^7*e^{11} - 1080436084*(x*e + d)^m*m^3*x^8*e^{ \\
& 11} + 11731446360*(x*e + d)^m*m^2*x^9*e^{11} - 291380400*(x*e + d)^m*m*x^{10}*e^{ \\
& 11} + 1814400000*(x*e + d)^m*x^{11}*e^{11} + 54*(x*e + d)^m*d*m^{10}*e^{10} + 8505*( \\
& x*e + d)^m*d*m^9*x*e^{10} + 769878*(x*e + d)^m*d*m^8*x^2*e^{10} + 12319188*(x*e \\
& + d)^m*d*m^7*x^3*e^{10} + 175848585*(x*e + d)^m*d*m^6*x^4*e^{10} + 338198850*( \\
& x*e + d)^m*d*m^5*x^5*e^{10} + 1584809604*(x*e + d)^m*d*m^4*x^6*e^{10} - 2113660 \\
& 08*(x*e + d)^m*d*m^3*x^7*e^{10} + 1180639800*(x*e + d)^m*d*m^2*x^8*e^{10} - 110 \\
& 88000*(x*e + d)^m*d*m*x^9*e^{10} - 135*(x*e + d)^m*d^2*m^9*e^9 - 57240*(x*e + \\
& d)^m*d^2*m^8*x*e^9 - 2386692*(x*e + d)^m*d^2*m^7*x^2*e^9 - 67924032*(x*e + \\
& d)^m*d^2*m^6*x^3*e^9 - 213304950*(x*e + d)^m*d^2*m^5*x^4*e^9 - 1351239408* \\
& (x*e + d)^m*d^2*m^4*x^5*e^9 + 211477336*(x*e + d)^m*d^2*m^3*x^6*e^9 - 12800 \\
& 59200*(x*e + d)^m*d^2*m^2*x^7*e^9 + 12474000*(x*e + d)^m*d^2*m*x^8*e^9 + 95 \\
& 4*(x*e + d)^m*d^3*m^8*e^8 + 192864*(x*e + d)^m*d^3*m^7*x*e^8 + 14984808*(x* \\
& e + d)^m*d^3*m^6*x^2*e^8 + 98532000*(x*e + d)^m*d^3*m^5*x^3*e^8 + 103003893
\end{aligned}$$

$0*(x*e + d)^m*d^3*m^4*x^4*e^8 - 207117120*(x*e + d)^m*d^3*m^3*x^5*e^8 + 139$   
 $9154400*(x*e + d)^m*d^3*m^2*x^6*e^8 - 14256000*(x*e + d)^m*d^3*m*x^7*e^8 -$   
 $3444*(x*e + d)^m*d^4*m^7*e^7 - 1357416*(x*e + d)^m*d^4*m^6*x*e^7 - 26010000$   
 $*(x*e + d)^m*d^4*m^5*x^2*e^7 - 608391000*(x*e + d)^m*d^4*m^4*x^3*e^7 + 1936$   
 $37220*(x*e + d)^m*d^4*m^3*x^4*e^7 - 1543268160*(x*e + d)^m*d^4*m^2*x^5*e^7$   
 $+ 16632000*(x*e + d)^m*d^4*m*x^6*e^7 + 26616*(x*e + d)^m*d^5*m^6*e^6 + 2754$   
 $000*(x*e + d)^m*d^5*m^5*x*e^6 + 207512280*(x*e + d)^m*d^5*m^4*x^2*e^6 - 160$   
 $277040*(x*e + d)^m*d^5*m^3*x^3*e^6 + 1717027200*(x*e + d)^m*d^5*m^2*x^4*e^6$   
 $- 19958400*(x*e + d)^m*d^5*m*x^5*e^6 - 61200*(x*e + d)^m*d^6*m^5*e^5 - 273$   
 $32640*(x*e + d)^m*d^6*m^4*x*e^5 + 81249840*(x*e + d)^m*d^6*m^3*x^2*e^5 - 19$   
 $12377600*(x*e + d)^m*d^6*m^2*x^3*e^5 + 24948000*(x*e + d)^m*d^6*m*x^4*e^5 +$   
 $719280*(x*e + d)^m*d^7*m^4*e^4 - 14817600*(x*e + d)^m*d^7*m^3*x*e^4 + 2020$   
 $334400*(x*e + d)^m*d^7*m^2*x^2*e^4 - 33264000*(x*e + d)^m*d^7*m*x^3*e^4 + 4$   
 $93920*(x*e + d)^m*d^8*m^3*e^3 - 647740800*(x*e + d)^m*d^8*m^2*x*e^3 + 49896$   
 $000*(x*e + d)^m*d^8*m*x^2*e^3 + 30844800*(x*e + d)^m*d^9*m^2*e^2 - 99792000$   
 $*(x*e + d)^m*d^9*m*x*e^2 + 9072000*(x*e + d)^m*d^10*m*e + 1814400000*(x*e +$   
 $d)^m*d^11 + 3510*(x*e + d)^m*m^9*x*e^11 + 242595*(x*e + d)^m*m^8*x^2*e^11$   
 $+ 13099374*(x*e + d)^m*m^7*x^3*e^11 + 145552050*(x*e + d)^m*m^6*x^4*e^11 +$   
 $1624344537*(x*e + d)^m*m^5*x^5*e^11 + 2729996850*(x*e + d)^m*m^4*x^6*e^11 +$   
 $12279432276*(x*e + d)^m*m^3*x^7*e^11 - 1671802776*(x*e + d)^m*m^2*x^8*e^11$   
 $+ 9869234400*(x*e + d)^m*m*x^9*e^11 - 99792000*(x*e + d)^m*x^10*e^11 + 351$   
 $0*(x*e + d)^m*d*m^9*e^10 + 234090*(x*e + d)^m*d*m^8*x*e^10 + 11559618*(x*e$   
 $+ d)^m*d*m^7*x^2*e^10 + 108594486*(x*e + d)^m*d*m^6*x^3*e^10 + 920950197*(x$   
 $e + d)^m*d*m^5*x^4*e^10 + 1039002600*(x*e + d)^m*d*m^4*x^5*e^10 + 27705746$   
 $52*(x*e + d)^m*d*m^3*x^6*e^10 - 192240720*(x*e + d)^m*d*m^2*x^7*e^10 + 4241$   
 $16000*(x*e + d)^m*d*m*x^8*e^10 - 8505*(x*e + d)^m*d^2*m^8*e^9 - 1482516*(x*$   
 $e + d)^m*d^2*m^7*x*e^9 - 32184180*(x*e + d)^m*d^2*m^6*x^2*e^9 - 499622244*($   
 $x*e + d)^m*d^2*m^5*x^3*e^9 - 837774450*(x*e + d)^m*d^2*m^4*x^4*e^9 - 275266$   
 $0584*(x*e + d)^m*d^2*m^3*x^5*e^9 + 210698040*(x*e + d)^m*d^2*m^2*x^6*e^9 -$   
 $484704000*(x*e + d)^m*d^2*m*x^7*e^9 + 57240*(x*e + d)^m*d^3*m^7*e^8 + 45805$   
 $20*(x*e + d)^m*d^3*m^6*x*e^8 + 173802480*(x*e + d)^m*d^3*m^5*x^2*e^8 + 5576$   
 $23800*(x*e + d)^m*d^3*m^4*x^3*e^8 + 2636041320*(x*e + d)^m*d^3*m^3*x^4*e^8$   
 $- 233278416*(x*e + d)^m*d^3*m^2*x^5*e^8 + 565488000*(x*e + d)^m*d^3*m*x^6*$   
 $e^8 - 192864*(x*e + d)^m*d^4*m^6*e^7 - 28612200*(x*e + d)^m*d^4*m^5*x*e^7 -$   
 $243576000*(x*e + d)^m*d^4*m^4*x^2*e^7 - 2294982720*(x*e + d)^m*d^4*m^3*x^3*$   
 $e^7 + 261036720*(x*e + d)^m*d^4*m^2*x^4*e^7 - 678585600*(x*e + d)^m*d^4*m*x$   
 $^5*e^7 + 1357416*(x*e + d)^m*d^5*m^5*e^6 + 49266000*(x*e + d)^m*d^5*m^4*x*e$   
 $^6 + 1410148440*(x*e + d)^m*d^5*m^3*x^2*e^6 - 293717760*(x*e + d)^m*d^5*m^2$   
 $*x^3*e^6 + 848232000*(x*e + d)^m*d^5*m*x^4*e^6 - 2754000*(x*e + d)^m*d^6*m^$   
 $4*e^5 - 387691920*(x*e + d)^m*d^6*m^3*x*e^5 + 318331440*(x*e + d)^m*d^6*m^2$   
 $*x^2*e^5 - 1130976000*(x*e + d)^m*d^6*m*x^3*e^5 + 27332640*(x*e + d)^m*d^7*$   
 $m^3*e^4 - 147682080*(x*e + d)^m*d^7*m^2*x*e^4 + 1696464000*(x*e + d)^m*d^7*$   
 $m*x^2*e^4 + 14817600*(x*e + d)^m*d^8*m^2*e^3 - 3392928000*(x*e + d)^m*d^8*m$   
 $*x*e^3 + 647740800*(x*e + d)^m*d^9*m*e^2 + 99792000*(x*e + d)^m*d^10*e + 10$   
 $0440*(x*e + d)^m*m^8*x*e^11 + 3925260*(x*e + d)^m*m^7*x^2*e^11 + 131192649*$

$$\begin{aligned}
& (x^e + d)^m m^6 x^3 e^{11} + 931750092 (x^e + d)^m m^5 x^4 e^{11} + 6671821630 (x^e + d)^m m^4 x^5 e^{11} + 7077841200 (x^e + d)^m m^3 x^6 e^{11} + 1919679199 \\
& 2 (x^e + d)^m m^2 x^7 e^{11} - 1415539440 (x^e + d)^m m x^8 e^{11} + 3392928000 (x^e + d)^m x^9 e^{11} + 100440 (x^e + d)^m d m^8 e^{10} + 3691170 (x^e + d)^m \\
& d m^7 x e^{10} + 108073413 (x^e + d)^m d m^6 x^2 e^{10} + 605966634 (x^e + d)^m d m^5 x^3 e^{10} + 2988020842 (x^e + d)^m d m^4 x^4 e^{10} + 1882828200 (x^e + d)^m d m^3 x^5 e^{10} + 2573344080 (x^e + d)^m d m^2 x^6 e^{10} - 69854400 (x^e + d)^m d m x^7 e^{10} - 234090 (x^e + d)^m d^2 m^7 e^9 - 21636720 (x^e + d)^m d^2 m^6 x e^9 - 261415098 (x^e + d)^m d^2 m^5 x^2 e^9 - 2184934056 (x^e + d)^m d^2 m^4 x^3 e^9 - 1843915200 (x^e + d)^m d^2 m^3 x^4 e^9 - 28601449 \\
& 92 (x^e + d)^m d^2 m^2 x^5 e^9 + 81496800 (x^e + d)^m d^2 m x^6 e^9 + 14825 \\
& 16 (x^e + d)^m d^3 m^6 e^8 + 59787840 (x^e + d)^m d^3 m^5 x e^8 + 115126177 \\
& 2 (x^e + d)^m d^3 m^4 x^2 e^8 + 1678226400 (x^e + d)^m d^3 m^3 x^3 e^8 + 32 \\
& 19137640 (x^e + d)^m d^3 m^2 x^4 e^8 - 97796160 (x^e + d)^m d^3 m x^5 e^8 - \\
& 4580520 (x^e + d)^m d^4 m^5 e^7 - 318992760 (x^e + d)^m d^4 m^4 x e^7 - 11 \\
& 85719400 (x^e + d)^m d^4 m^3 x^2 e^7 - 3659217120 (x^e + d)^m d^4 m^2 x^3 e^7 + 122245200 (x^e + d)^m d^4 m x^4 e^7 + 28612200 (x^e + d)^m d^5 m^4 e^6 + 437886000 (x^e + d)^m d^5 m^3 x e^6 + 4064651280 (x^e + d)^m d^5 m^2 x^2 e^6 - 162993600 (x^e + d)^m d^5 m x^3 e^6 - 49266000 (x^e + d)^m d^6 m^3 e^5 - 2432604960 (x^e + d)^m d^6 m^2 x e^5 + 244490400 (x^e + d)^m d^6 m x^2 e^5 + 387691920 (x^e + d)^m d^7 m^2 e^4 - 488980800 (x^e + d)^m d^7 m x e^4 + 147682080 (x^e + d)^m d^8 m e^3 + 3392928000 (x^e + d)^m d^9 e^2 + 1663 \\
& 740 (x^e + d)^m m^7 x e^{11} + 40401585 (x^e + d)^m m^6 x^2 e^{11} + 864537219 (x^e + d)^m m^5 x^3 e^{11} + 3929892722 (x^e + d)^m m^4 x^4 e^{11} + 1765015642 \\
& 0 (x^e + d)^m m^3 x^5 e^{11} + 11214571560 (x^e + d)^m m^2 x^6 e^{11} + 1638951 \\
& 4080 (x^e + d)^m m x^7 e^{11} - 488980800 (x^e + d)^m x^8 e^{11} + 1663740 (x^e + d)^m d m^7 e^{10} + 36710415 (x^e + d)^m d m^6 x e^{10} + 648390393 (x^e + d)^m d m^5 x^2 e^{10} + 2111992820 (x^e + d)^m d m^4 x^3 e^{10} + 5698073052 (x^e + d)^m d m^3 x^4 e^{10} + 1800430560 (x^e + d)^m d m^2 x^5 e^{10} + 949449600 (x^e + d)^m d m x^6 e^{10} - 3691170 (x^e + d)^m d^2 m^6 e^9 - 194510106 (x^e + d)^m d^2 m^5 x e^9 - 1295069706 (x^e + d)^m d^2 m^4 x^2 e^9 - 539728120 \\
& 0 (x^e + d)^m d^2 m^3 x^3 e^9 - 2038480200 (x^e + d)^m d^2 m^2 x^4 e^9 - 11 \\
& 39339520 (x^e + d)^m d^2 m x^5 e^9 + 21636720 (x^e + d)^m d^3 m^5 e^8 + 463 \\
& 042356 (x^e + d)^m d^3 m^4 x e^8 + 4252278624 (x^e + d)^m d^3 m^3 x^2 e^8 + \\
& 2340981600 (x^e + d)^m d^3 m^2 x^3 e^8 + 1424174400 (x^e + d)^m d^3 m x^4 e^8 - 59787840 (x^e + d)^m d^4 m^4 e^7 - 1983530784 (x^e + d)^m d^4 m^3 x e^7 - 2663240400 (x^e + d)^m d^4 m^2 x^2 e^7 - 1898899200 (x^e + d)^m d^4 m x^3 e^7 + 318992760 (x^e + d)^m d^5 m^3 e^6 + 1933552800 (x^e + d)^m d^5 m^2 x e^6 + 2848348800 (x^e + d)^m d^5 m x^2 e^6 - 437886000 (x^e + d)^m d^6 m^2 e^5 - 5696697600 (x^e + d)^m d^6 m x e^5 + 2432604960 (x^e + d)^m d^7 m e^4 + 488980800 (x^e + d)^m d^8 e^3 + 17637102 (x^e + d)^m m^6 x e^{11} + 27 \\
& 5267160 (x^e + d)^m m^5 x^2 e^{11} + 3769346538 (x^e + d)^m m^4 x^3 e^{11} + 10 \\
& 681978132 (x^e + d)^m m^3 x^4 e^{11} + 28480424184 (x^e + d)^m m^2 x^5 e^{11} + \\
& 9680738400 (x^e + d)^m m x^6 e^{11} + 5696697600 (x^e + d)^m x^7 e^{11} + 1763 \\
& 7102 (x^e + d)^m d m^6 e^{10} + 238556745 (x^e + d)^m d m^5 x e^{10} + 24725657
\end{aligned}$$

$$\begin{aligned}
& 52*(x*e + d)^m*d*m^4*x^2*e^{10} + 4345999672*(x*e + d)^m*d*m^3*x^3*e^{10} + 568 \\
& 8131976*(x*e + d)^m*d*m^2*x^4*e^{10} + 678585600*(x*e + d)^m*d*m*x^5*e^{10} - 3 \\
& 6710415*(x*e + d)^m*d^2*m^5*e^9 - 1102270680*(x*e + d)^m*d^2*m^4*x*e^9 - 37 \\
& 45839048*(x*e + d)^m*d^2*m^3*x^2*e^9 - 6600448608*(x*e + d)^m*d^2*m^2*x^3*e \\
& ^9 - 848232000*(x*e + d)^m*d^2*m*x^4*e^9 + 194510106*(x*e + d)^m*d^3*m^4*e^ \\
& 8 + 2127097056*(x*e + d)^m*d^3*m^3*x*e^8 + 7687286352*(x*e + d)^m*d^3*m^2*x \\
& ^2*e^8 + 1130976000*(x*e + d)^m*d^3*m*x^3*e^8 - 463042356*(x*e + d)^m*d^4*m \\
& ^3*e^7 - 6521026464*(x*e + d)^m*d^4*m^2*x*e^7 - 1696464000*(x*e + d)^m*d^4* \\
& m*x^2*e^7 + 1983530784*(x*e + d)^m*d^5*m^2*e^6 + 3392928000*(x*e + d)^m*d^5 \\
& *m*x*e^6 - 1933552800*(x*e + d)^m*d^6*m*e^5 + 5696697600*(x*e + d)^m*d^7*e^ \\
& 4 + 124791030*(x*e + d)^m*m^5*x*e^{11} + 1250302905*(x*e + d)^m*m^4*x^2*e^{11} \\
& + 10631923596*(x*e + d)^m*m^3*x^3*e^{11} + 17690223096*(x*e + d)^m*m^2*x^4*e^ \\
& 11 + 24965914464*(x*e + d)^m*m*x^5*e^{11} + 3392928000*(x*e + d)^m*x^6*e^{11} + \\
& 124791030*(x*e + d)^m*d*m^5*e^{10} + 1011746160*(x*e + d)^m*d*m^4*x*e^{10} + 5 \\
& 686792092*(x*e + d)^m*d*m^3*x^2*e^{10} + 4652224080*(x*e + d)^m*d*m^2*x^3*e^{1 \\
& 0} + 2213386560*(x*e + d)^m*d*m*x^4*e^{10} - 238556745*(x*e + d)^m*d^2*m^4*e^9 \\
& - 3842860824*(x*e + d)^m*d^2*m^3*x*e^9 - 5546320920*(x*e + d)^m*d^2*m^2*x^ \\
& 2*e^9 - 2951182080*(x*e + d)^m*d^2*m*x^3*e^9 + 1102270680*(x*e + d)^m*d^3*m \\
& ^3*e^8 + 5364581040*(x*e + d)^m*d^3*m^2*x*e^8 + 4426773120*(x*e + d)^m*d^3* \\
& m*x^2*e^8 - 2127097056*(x*e + d)^m*d^4*m^2*e^7 - 8853546240*(x*e + d)^m*d^4 \\
& *m*x*e^7 + 6521026464*(x*e + d)^m*d^5*m*e^6 - 3392928000*(x*e + d)^m*d^6*e^ \\
& 5 + 595543860*(x*e + d)^m*m^4*x*e^{11} + 3708817740*(x*e + d)^m*m^3*x^2*e^{11} \\
& + 18312331464*(x*e + d)^m*m^2*x^3*e^{11} + 15866025840*(x*e + d)^m*m*x^4*e^{11} \\
& + 8853546240*(x*e + d)^m*x^5*e^{11} + 595543860*(x*e + d)^m*d*m^4*e^{10} + 269 \\
& 7071580*(x*e + d)^m*d*m^3*x*e^{10} + 6938747280*(x*e + d)^m*d*m^2*x^2*e^{10} + \\
& 1909353600*(x*e + d)^m*d*m*x^3*e^{10} - 1011746160*(x*e + d)^m*d^2*m^3*e^9 - \\
& 7530723360*(x*e + d)^m*d^2*m^2*x*e^9 - 2864030400*(x*e + d)^m*d^2*m*x^2*e^9 \\
& + 3842860824*(x*e + d)^m*d^3*m^2*e^8 + 5728060800*(x*e + d)^m*d^3*m*x*e^8 \\
& - 5364581040*(x*e + d)^m*d^4*m*e^7 + 8853546240*(x*e + d)^m*d^5*e^6 + 18882 \\
& 25560*(x*e + d)^m*m^3*x*e^{11} + 6792204780*(x*e + d)^m*m^2*x^2*e^{11} + 170508 \\
& 80160*(x*e + d)^m*m*x^3*e^{11} + 5728060800*(x*e + d)^m*x^4*e^{11} + 1888225560 \\
& *(x*e + d)^m*d*m^3*e^{10} + 4095133200*(x*e + d)^m*d*m^2*x*e^{10} + 3173385600* \\
& (x*e + d)^m*d*m*x^2*e^{10} - 2697071580*(x*e + d)^m*d^2*m^2*e^9 - 6346771200* \\
& (x*e + d)^m*d^2*m*x*e^9 + 7530723360*(x*e + d)^m*d^3*m*e^8 - 5728060800*(x* \\
& e + d)^m*d^4*e^7 + 3795710544*(x*e + d)^m*m^2*x*e^{11} + 6789517200*(x*e + d) \\
& ^m*m*x^2*e^{11} + 6346771200*(x*e + d)^m*x^3*e^{11} + 3795710544*(x*e + d)^m*d* \\
& m^2*e^{10} + 2694384000*(x*e + d)^m*d*m*x*e^{10} - 4095133200*(x*e + d)^m*d^2*m \\
& *e^9 + 6346771200*(x*e + d)^m*d^3*e^8 + 4353860160*(x*e + d)^m*m*x*e^{11} + 2 \\
& 694384000*(x*e + d)^m*x^2*e^{11} + 4353860160*(x*e + d)^m*d*m*e^{10} - 26943840 \\
& 00*(x*e + d)^m*d^2*e^9 + 2155507200*(x*e + d)^m*x*e^{11} + 2155507200*(x*e + \\
& d)^m*d*e^{10})/(m^{11}*e^{11} + 66*m^{10}*e^{11} + 1925*m^9*e^{11} + 32670*m^8*e^{11} + 3 \\
& 57423*m^7*e^{11} + 2637558*m^6*e^{11} + 13339535*m^5*e^{11} + 45995730*m^4*e^{11} + \\
& 105258076*m^3*e^{11} + 150917976*m^2*e^{11} + 120543840*m*e^{11} + 39916800*e^{11} \\
& )
\end{aligned}$$

**maple [B]** time = 0.08, size = 5924, normalized size = 10.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2), x)$

[Out] result too large to display

**maxima [B]** time = 0.74, size = 2292, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2), x, \text{algorithm}="maxima")$

[Out]  $135*(e^{2*(m+1)}x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 54*(e*x + d)^{(m+1)}/(e*(m+1)) + 477*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 574*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 1109*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 510*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 98*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 - 7*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)*d^6*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8) + 765*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 67$

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69*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*
m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5
+ 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35
*m^3 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e
^4*x^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2
- 40320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450
*m^6 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 3628
80)*e^9) - 25*((m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4
+ 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*e^10*x^10 + (m^9 + 36*m^8
+ 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 118124*m^3 + 109584*m^2 + 40
320*m)*d*e^9*x^9 - 9*(m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*
m^3 + 13068*m^2 + 5040*m)*d^2*e^8*x^8 + 72*(m^7 + 21*m^6 + 175*m^5 + 735*m^
4 + 1624*m^3 + 1764*m^2 + 720*m)*d^3*e^7*x^7 - 504*(m^6 + 15*m^5 + 85*m^4 +
225*m^3 + 274*m^2 + 120*m)*d^4*e^6*x^6 + 3024*(m^5 + 10*m^4 + 35*m^3 + 50*
m^2 + 24*m)*d^5*e^5*x^5 - 15120*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^6*e^4*x^4 +
60480*(m^3 + 3*m^2 + 2*m)*d^7*e^3*x^3 - 181440*(m^2 + m)*d^8*e^2*x^2 + 3628
80*d^9*e*m*x - 362880*d^10)*(e*x + d)^m/((m^10 + 55*m^9 + 1320*m^8 + 18150*
m^7 + 157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 +
10628640*m + 3628800)*e^10) + 500*((m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 +
157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 + 106286
40*m + 3628800)*e^11*x^11 + (m^10 + 45*m^9 + 870*m^8 + 9450*m^7 + 63273*m^6
+ 269325*m^5 + 723680*m^4 + 1172700*m^3 + 1026576*m^2 + 362880*m)*d*e^10*x
^10 - 10*(m^9 + 36*m^8 + 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 11812
4*m^3 + 109584*m^2 + 40320*m)*d^2*e^9*x^9 + 90*(m^8 + 28*m^7 + 322*m^6 + 19
60*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d^3*e^8*x^8 - 720*(m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^4*e^7*x^7 + 5
040*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^5*e^6*x^6 - 30240
*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^6*e^5*x^5 + 151200*(m^4 + 6*m^3
+ 11*m^2 + 6*m)*d^7*e^4*x^4 - 604800*(m^3 + 3*m^2 + 2*m)*d^8*e^3*x^3 + 1814
400*(m^2 + m)*d^9*e^2*x^2 - 3628800*d^10*e*m*x + 3628800*d^11)*(e*x + d)^m/
((m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 133395
35*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916
800)*e^11)

```

**mupad [B]** time = 8.39, size = 4341, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2), x)$

[Out]  $(500*x^{11}*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^{10} + 3628800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^{10} + m^{11} + 39$

$$\begin{aligned}
& 916800) + ((d + e*x)^m*(2155507200*d*e^{10} + 99792000*d^{10}*e + 1814400000*d^{11} \\
& - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8853546240*d^5*e^6 \\
& - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^8*e^3 + 3392928000*d^9*e^2 \\
& - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m - 5364581040*d^4*e^7*m + 6521026464*d^5*e^6*m \\
& - 1933552800*d^6*e^5*m + 2432604960*d^7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*d*e^{10}*m^2 \\
& + 1888225560*d*e^{10}*m^3 + 595543860*d*e^{10}*m^4 + 124791030*d*e^{10}*m^5 + 17637102*d*e^{10}*m^6 \\
& + 1663740*d*e^{10}*m^7 + 100440*d*e^{10}*m^8 + 3510*d*e^{10}*m^9 + 54*d*e^{10}*m^{10} - 2697071580*d^2*e^9*m^2 \\
& + 3842860824*d^3*e^8*m^2 - 2127097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e^5*m^2 + 387691920*d^7*e^4*m^2 \\
& + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m^2 - 1011746160*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 \\
& - 463042356*d^4*e^7*m^3 + 318992760*d^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 + 493920*d^8*e^3*m^3 \\
& - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 59787840*d^4*e^7*m^4 + 28612200*d^5*e^6*m^4 \\
& - 2754000*d^6*e^5*m^4 + 719280*d^7*e^4*m^4 - 36710415*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 \\
& - 4580520*d^4*e^7*m^5 + 1357416*d^5*e^6*m^5 - 61200*d^6*e^5*m^5 - 3691170*d^2*e^9*m^6 + 1482516*d^3*e^8*m^6 \\
& - 192864*d^4*e^7*m^6 + 26616*d^5*e^6*m^6 - 234090*d^2*e^9*m^7 + 57240*d^3*e^8*m^7 - 3444*d^4*e^7*m^7 \\
& - 8505*d^2*e^9*m^8 + 954*d^3*e^8*m^8 - 135*d^2*e^9*m^9 + 4353860160*d*e^{10}*m + 9072000*d^{10}*e*m)) / (e^{11}*(120543840*m + 150917976*m^2 \\
& + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 \\
& + 66*m^{10} + m^{11} + 39916800)) + (x*(d + e*x)^m*(4353860160*e^{11}*m + 2155507200*e^{11} \\
& + 3795710544*e^{11}*m^2 + 1888225560*e^{11}*m^3 + 595543860*e^{11}*m^4 + 124791030*e^{11}*m^5 \\
& + 17637102*e^{11}*m^6 + 1663740*e^{11}*m^7 + 100440*e^{11}*m^8 + 3510*e^{11}*m^9 + 54*e^{11}*m^{10} - 6346771200*d^2*e^9*m \\
& + 5728060800*d^3*e^8*m - 8853546240*d^4*e^7*m + 3392928000*d^5*e^6*m - 5696697600*d^6*e^5*m \\
& - 488980800*d^7*e^4*m - 3392928000*d^8*e^3*m - 99792000*d^9*e^2*m + 4095133200*d*e^{10}*m^2 \\
& + 2697071580*d*e^{10}*m^3 + 1011746160*d*e^{10}*m^4 + 238556745*d*e^{10}*m^5 + 36710415*d*e^{10}*m^6 \\
& + 3691170*d*e^{10}*m^7 + 234090*d*e^{10}*m^8 + 8505*d*e^{10}*m^9 + 135*d*e^{10}*m^{10} - 7530723360*d^2*e^9*m^2 \\
& + 5364581040*d^3*e^8*m^2 - 6521026464*d^4*e^7*m^2 + 1933552800*d^5*e^6*m^2 - 2432604960*d^6*e^5*m^2 \\
& - 147682080*d^7*e^4*m^2 - 647740800*d^8*e^3*m^2 - 9072000*d^9*e^2*m^2 - 3842860824*d^2*e^9*m^3 \\
& + 2127097056*d^3*e^8*m^3 - 1983530784*d^4*e^7*m^3 + 437886000*d^5*e^6*m^3 - 387691920*d^6*e^5*m^3 \\
& - 14817600*d^7*e^4*m^3 - 30844800*d^8*e^3*m^3 - 1102270680*d^2*e^9*m^4 + 463042356*d^3*e^8*m^4 \\
& - 318992760*d^4*e^7*m^4 + 49266000*d^5*e^6*m^4 - 27332640*d^6*e^5*m^4 - 493920*d^7*e^4*m^4 \\
& - 194510106*d^2*e^9*m^5 + 59787840*d^3*e^8*m^5 - 28612200*d^4*e^7*m^5 + 2754000*d^5*e^6*m^5 \\
& - 719280*d^6*e^5*m^5 - 21636720*d^2*e^9*m^6 + 4580520*d^3*e^8*m^6 - 1357416*d^4*e^7*m^6 + 61200*d^5*e^6*m^6 \\
& - 1482516*d^2*e^9*m^7 + 192864*d^3*e^8*m^7 - 26616*d^4*e^7*m^7 - 57240*d^2*e^9*m^8 + 3444*d^3*e^8*m^8 \\
& - 954*d^2*e^9*m^9 + 2694384000*d*e^{10}*m - 1814400000*d^{10}*e*m)) / (e^{11}*(120543840*m + 150917976*m^2 \\
& + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 \\
& + 66*m^{10} + m^{11} + 39916800)) + (x^8*(d + e*x)^m*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 \\
& + 322*m^5 + 28*m^6 + m^7 + 5040)*(45000*d^3*m - 29
\end{aligned}$$

$$\begin{aligned}
& 302e^3m - 97020e^3 - 2940e^3m^2 - 98e^3m^3 + 16065d^2e^2m^2 + 225d^2e^2m^2 + 765d^2e^2m^3 + 84150d^2e^2m + 2475d^2e^2m) / (e^3(120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800)) + (3x^2(m + 1)(d + e^x)^m(302400000d^9m + 1365044400e^9m + 898128000e^9 + 899023860e^9m^2 + 337248720e^9m^3 + 79518915e^9m^4 + 12236805e^9m^5 + 1230390e^9m^6 + 78030e^9m^7 + 2835e^9m^8 + 45e^9m^9 - 954676800d^2e^7m + 1475591040d^3e^6m - 565488000d^4e^5m + 949449600d^5e^4m + 81496800d^6e^3m + 565488000d^7e^2m + 1255120560d^8e^1m + 1512000d^8e^1m^2 + 640476804d^8e^1m^3 + 183711780d^8e^1m^4 + 32418351d^8e^1m^5 + 3606120d^8e^1m^6 + 247086d^8e^1m^7 + 9540d^8e^1m^8 + 159d^8e^1m^9 - 894096840d^2e^7m^2 + 1086837744d^3e^6m^2 - 322258800d^4e^5m^2 + 405434160d^5e^4m^2 + 24613680d^6e^3m^2 + 107956800d^7e^2m^2 - 354516176d^2e^7m^3 + 330588464d^3e^6m^3 - 72981000d^4e^5m^3 + 64615320d^5e^4m^3 + 2469600d^6e^3m^3 + 5140800d^7e^2m^3 - 77173726d^2e^7m^4 + 53165460d^3e^6m^4 - 8211000d^4e^5m^4 + 4555440d^5e^4m^4 + 82320d^6e^3m^4 - 9964640d^2e^7m^5 + 4768700d^3e^6m^5 - 459000d^4e^5m^5 + 119880d^5e^4m^5 - 763420d^2e^7m^6 + 226236d^3e^6m^6 - 10200d^4e^5m^6 - 32144d^2e^7m^7 + 4436d^3e^6m^7 - 574d^2e^7m^8 + 1057795200d^8e^1m + 16632000d^8e^1m) / (e^9(120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800)) + (x^6(d + e^x)^m(274m + 225m^2 + 85m^3 + 15m^4 + m^5 + 120)(2520000d^5m + 16112940e^5m + 28274400e^5 + 3649050e^5m^2 + 410550e^5m^3 + 22950e^5m^4 + 510e^5m^5 + 679140d^2e^3m + 4712400d^3e^2m + 3378618d^4e^1m^2 + 12600d^4e^1m^2 + 538461d^4e^1m^3 + 37962d^4e^1m^4 + 999d^4e^1m^5 + 205114d^2e^3m^2 + 899640d^3e^2m^2 + 20580d^2e^3m^3 + 42840d^3e^2m^3 + 686d^2e^3m^4 + 7912080d^4e^1m + 138600d^4e^1m) / (e^5(120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800)) + (x^3(d + e^x)^m(3m + m^2 + 2)(3765361680e^8m - 302400000d^8m + 3173385600e^8 + 1921430412e^8m^2 + 551135340e^8m^3 + 97255053e^8m^4 + 10818360e^8m^5 + 741258e^8m^6 + 28620e^8m^7 + 477e^8m^8 - 1475591040d^2e^6m + 565488000d^3e^5m - 949449600d^4e^4m - 81496800d^5e^3m - 565488000d^6e^2m + 894096840d^7e^1m^2 - 1512000d^7e^1m^2 + 354516176d^7e^1m^3 + 77173726d^7e^1m^4 + 9964640d^7e^1m^5 + 763420d^7e^1m^6 + 32144d^7e^1m^7 + 574d^7e^1m^8 - 1086837744d^2e^6m^2 + 322258800d^3e^5m^2 - 405434160d^4e^4m^2 - 24613680d^5e^3m^2 - 107956800d^6e^2m^2 - 330588464d^2e^6m^3 + 72981000d^3e^5m^3 - 64615320d^4e^4m^3 - 2469600d^5e^3m^3 - 5140800d^6e^2m^3 - 53165460d^2e^6m^4 + 8211000d^3e^5m^4 - 4555440d^4e^4m^4 - 82320d^5e^3m^4 - 4768700d^2e^6m^5 + 459000d^3e^5m^5 - 119880d^4e^4m^5 - 226236d^2e^6m^6 + 10200d^3e^5m^6 - 4436d^2e^6m^7 + 954676800d^7e^1m - 16632000d^7e^1m) / (e^8(120543840m + 150917976m^2 + 105258076m^3 + 45995730m^4 + 13339535m^5 + 2637558m^6 + 357423m^7 + 32670m^8 + 1925m^9 + 66m^{10} + m^{11} + 39916800)) + (x^4(d + e^x)
\end{aligned}$$



$$\begin{aligned} & m*(11*m + 6*m^2 + m^3 + 6)*(75600000*d^7*m + 894096840*e^7*m + 954676800*e \\ & ^7 + 354516176*e^7*m^2 + 77173726*e^7*m^3 + 9964640*e^7*m^4 + 763420*e^7*m^ \\ & ^5 + 32144*e^7*m^6 + 574*e^7*m^7 - 141372000*d^2*e^5*m + 237362400*d^3*e^4*m \\ & + 20374200*d^4*e^3*m + 141372000*d^5*e^2*m + 271709436*d*e^6*m^2 + 378000* \\ & d^6*e*m^2 + 82647116*d*e^6*m^3 + 13291365*d*e^6*m^4 + 1192175*d*e^6*m^5 + 5 \\ & 6559*d*e^6*m^6 + 1109*d*e^6*m^7 - 80564700*d^2*e^5*m^2 + 101358540*d^3*e^4* \\ & m^2 + 6153420*d^4*e^3*m^2 + 26989200*d^5*e^2*m^2 - 18245250*d^2*e^5*m^3 + 1 \\ & 6153830*d^3*e^4*m^3 + 617400*d^4*e^3*m^3 + 1285200*d^5*e^2*m^3 - 2052750*d^ \\ & ^2*e^5*m^4 + 1138860*d^3*e^4*m^4 + 20580*d^4*e^3*m^4 - 114750*d^2*e^5*m^5 + \\ & 29970*d^3*e^4*m^5 - 2550*d^2*e^5*m^6 + 368897760*d*e^6*m + 4158000*d^6*e*m) \\ & )/(e^7*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133395 \\ & 35*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + \\ & 39916800)) - (25*x^10*(d + e*x)^m*(11*e - 20*d*m + e*m)*(1026576*m + 11727 \\ & 00*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 \\ & + m^9 + 362880))/(e*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730 \\ & *m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66* \\ & m^10 + m^11 + 39916800)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + \\ & 24)*(15120000*d^6*m - 271709436*e^6*m - 368897760*e^6 - 82647116*e^6*m^2 - \\ & 13291365*e^6*m^3 - 1192175*e^6*m^4 - 56559*e^6*m^5 - 1109*e^6*m^6 + 474724 \\ & 80*d^2*e^4*m + 4074840*d^3*e^3*m + 28274400*d^4*e^2*m - 16112940*d*e^5*m^2 \\ & + 75600*d^5*e*m^2 - 3649050*d*e^5*m^3 - 410550*d*e^5*m^4 - 22950*d*e^5*m^5 \\ & - 510*d*e^5*m^6 + 20271708*d^2*e^4*m^2 + 1230684*d^3*e^3*m^2 + 5397840*d^4* \\ & e^2*m^2 + 3230766*d^2*e^4*m^3 + 123480*d^3*e^3*m^3 + 257040*d^4*e^2*m^3 + 2 \\ & 27772*d^2*e^4*m^4 + 4116*d^3*e^3*m^4 + 5994*d^2*e^4*m^5 - 28274400*d*e^5*m \\ & + 831600*d^5*e*m))/(e^6*(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599 \\ & 5730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + \\ & 66*m^10 + m^11 + 39916800)) - (x^7*(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^ \\ & ^3 + 175*m^4 + 21*m^5 + m^6 + 720)*(360000*d^4*m - 3378618*e^4*m - 7912080*e \\ & ^4 - 538461*e^4*m^2 - 37962*e^4*m^3 - 999*e^4*m^4 + 673200*d^2*e^2*m + 2930 \\ & 2*d*e^3*m^2 + 1800*d^3*e*m^2 + 2940*d*e^3*m^3 + 98*d*e^3*m^4 + 128520*d^2*e \\ & ^2*m^2 + 6120*d^2*e^2*m^3 + 97020*d*e^3*m + 19800*d^3*e*m))/(e^4*(120543840 \\ & *m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558* \\ & m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) - (5* \\ & x^9*(d + e*x)^m*(1000*d^2*m - 3213*e^2*m - 16830*e^2 - 153*e^2*m^2 + 55*d*e \\ & *m + 5*d*e*m^2)*(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + \\ & 546*m^6 + 36*m^7 + m^8 + 40320))/(e^2*(120543840*m + 150917976*m^2 + 10525 \\ & 8076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m \\ & ^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(5\*x\*\*2+2\*x+3)\*\*3\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2),x)

[Out] Timed out

## 3.348

$$\int (d + ex)^m (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Optimal. Leaf size=432

$$\frac{(2800d^2 + 315de + 111e^2)(d + ex)^{m+7}}{e^9(m+7)} - \frac{(5600d^3 + 945d^2e + 666de^2 + 37e^3)(d + ex)^{m+6}}{e^9(m+6)} + \frac{(5d^2 - 2de + 3e^2)^2 (4d + 5e)}{e^9(m+5)}$$

**Rubi** [A] time = 0.24, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {1628}

$\frac{(d^2 - 2de + 3e^2)^2 (4d + 5e)}{e^9(m+5)}$   $\frac{(2800d^2 + 315de + 111e^2)(d + ex)^{m+7}}{e^9(m+7)}$   $\frac{(5600d^3 + 945d^2e + 666de^2 + 37e^3)(d + ex)^{m+6}}{e^9(m+6)}$   $\frac{(5d^2 - 2de + 3e^2)^2 (4d + 5e)}{e^9(m+5)}$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] ((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*(d + e\*x)^(1 + m))/(e^9\*(1 + m)) - ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(160\*d^5 + 127\*d^4\*e + 88\*d^3\*e^2 - 4\*d^2\*e^3 + 64\*d\*e^4 - 11\*e^5)\*(d + e\*x)^(2 + m))/(e^9\*(2 + m)) + ((2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*(d + e\*x)^(3 + m))/(e^9\*(3 + m)) - ((5600\*d^5 + 1575\*d^4\*e + 2220\*d^3\*e^2 + 370\*d^2\*e^3 + 592\*d\*e^4 - 65\*e^5)\*(d + e\*x)^(4 + m))/(e^9\*(4 + m)) + ((7000\*d^4 + 1575\*d^3\*e + 1665\*d^2\*e^2 + 185\*d\*e^3 + 148\*e^4)\*(d + e\*x)^(5 + m))/(e^9\*(5 + m)) - ((5600\*d^3 + 945\*d^2\*e + 666\*d\*e^2 + 37\*e^3)\*(d + e\*x)^(6 + m))/(e^9\*(6 + m)) + ((2800\*d^2 + 315\*d\*e + 111\*e^2)\*(d + e\*x)^(7 + m))/(e^9\*(7 + m)) - (5\*(160\*d + 9\*e)\*(d + e\*x)^(8 + m))/(e^9\*(8 + m)) + (100\*(d + e\*x)^(9 + m))/(e^9\*(9 + m))

### Rule 1628

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rubi steps

$$\int (d + ex)^m (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int \left( \frac{(5d^2 - 2de + 3e^2)^2 (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^8} \right) dx$$

$$= \frac{(5d^2 - 2de + 3e^2)^2 (4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)}{e^9(1 + m)}$$

**Mathematica [A]** time = 0.24, size = 391, normalized size = 0.91

$$(d + ex)^m \left( \frac{(2800d^6 + 945d^5e + 1665d^4e^2 + 370d^3e^3 + 888d^2e^4 - 195de^5 + 107e^6)(d + ex)^2}{(3 + m)} - \frac{(5600d^5 + 1575d^4e + 2220d^3e^2 + 370d^2e^3 + 592de^4 - 65e^5)(d + ex)^3}{(4 + m)} + \frac{(7000d^4 + 1575d^3e + 1665d^2e^2 + 185de^3 + 148e^4)(d + ex)^4}{(5 + m)} - \frac{(5600d^3 + 945d^2e + 666de^2 + 37e^3)(d + ex)^5}{(6 + m)} + \frac{(2800d^2 + 315de + 111e^2)(d + ex)^6}{(7 + m)} - \frac{5(160d + 9e)(d + ex)^7}{(8 + m)} + \frac{100(d + ex)^8}{(9 + m)} \right) / e^9$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^m\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] ((d + e\*x)^(1 + m)\*(((5\*d^2 - 2\*d\*e + 3\*e^2)^2\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4))/(1 + m) - ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(160\*d^5 + 127\*d^4\*e + 88\*d^3\*e^2 - 4\*d^2\*e^3 + 64\*d\*e^4 - 11\*e^5)\*(d + e\*x))/(2 + m) + ((2800\*d^6 + 945\*d^5\*e + 1665\*d^4\*e^2 + 370\*d^3\*e^3 + 888\*d^2\*e^4 - 195\*d\*e^5 + 107\*e^6)\*(d + e\*x)^2)/(3 + m) - ((5600\*d^5 + 1575\*d^4\*e + 2220\*d^3\*e^2 + 370\*d^2\*e^3 + 592\*d\*e^4 - 65\*e^5)\*(d + e\*x)^3)/(4 + m) + ((7000\*d^4 + 1575\*d^3\*e + 1665\*d^2\*e^2 + 185\*d\*e^3 + 148\*e^4)\*(d + e\*x)^4)/(5 + m) - ((5600\*d^3 + 945\*d^2\*e + 666\*d\*e^2 + 37\*e^3)\*(d + e\*x)^5)/(6 + m) + ((2800\*d^2 + 315\*d\*e + 111\*e^2)\*(d + e\*x)^6)/(7 + m) - (5\*(160\*d + 9\*e)\*(d + e\*x)^7)/(8 + m) + (100\*(d + e\*x)^8)/(9 + m))/e^9

**IntegrateAlgebraic [F]** time = 0.17, size = 0, normalized size = 0.00

$$\int (d + ex)^m (3 + 2x + 5x^2)^2 (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x)^m\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] Defer[IntegrateAlgebraic] [(d + e\*x)^m\*(3 + 2\*x + 5\*x^2)^2\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

**fricas [B]** time = 1.37, size = 2796, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^m\*(5\*x^2+2\*x+3)^2\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="ricas")

[Out]  $(18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 22449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*x^9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10741248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 6531840*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^9)*m^7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7*(19340*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(1210*d*e^8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^7 - 24*d*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*e^7 + 675*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9)*m^6 - 14*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^7 + 17655*d*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 326229*e^9)*m^3 - 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(4000*d^2*e^7 + 2025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7 + 7434*d*e^8)*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^7 + 1184*d*e^8 - 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e^8 - 5883*e^9)*m^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 34743*e^9)*m^5 - (476000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^9)*m^4 - 3*(420000*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*m^3 - 2*(767200*d^3*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)*m^2 - 24*(28000*d^3*e^6 + 14175*d^2*e^7 + 39960*d*e^8 - 248233*e^9)*m)*x^6 - 6*(65*d^4*e^5 - 1391*d^3*e^6 + 4081*d^2*e^7 - 25872*d*e^8)*m^5 + (10741248*e^9 - 37*(d*e^8 - 4*e^9)*m^8 - 74*(9*d^2*e^7 + 17*d*e^8 - 80*e^9)*m^7 - 2*(945*d^3*e^6 + 8991*d^2*e^7 + 8621*d*e^8 - 49580*e^9)*m^6 - 2*(16800*d^4*e^5 + 17955*d^3*e^6 + 92241*d^2*e^7 + 61124*d*e^8 - 451400*e^9)*m^5 - (336000*d^4*e^5 + 236250*d^3*e^6 + 909090*d^2*e^7 + 479113*d*e^8 - 4850404*e^9)*m^4 - 2*(588000*d^4*e^5 + 344925*d^3*e^6 + 1130202*d^2*e^7 + 513671*d*e^8 - 7804040*e^9)*m^3 - 12*(140000*d^4*e^5 + 74655*d^3*e^6 + 222444*d^2*e^7 + 91834*d*e^8 - 2422020*e^9)*m^2 - 144*(5600*d^4*e^5 + 2835*d^3*e^6 + 7992*d^2*e^7 + 3108*d*e^8 - 196100*e^9)*m)*x^5 + 2*(1776*d^5*e^4 - 6825*d^4*e^5 + 66875*d^3*e^6 - 117810*d^2*e^7 + 491841*d*e^8)*m^4 + (5896800*e^9 + (148*d*e^8 + 65*e^9)*m^8 + (185*d^2*e^7 + 5328*d*e^8 + 2665*e^9)*m^7 + 2*(1665*d^3*e^6 + 2775*d^2*e^7 + 38924*d*e^8 + 22945*e^9)*m^6 + 2*(4725*d^4*e^5 + 38295*d^3*e^6 + 32005*d^2*e^7 + 295704*d*e^8 + 215345*e^9)*m^5 + (168000*d^5*e^4 + 141750*d^4*e^5 + 616050*d^3*e^6 + 355200*d^2*e^7 + 2484772*d*e^8 + 2389985*e^9)*m^4 + (1008000*d^5*e^4 + 614250*d^4*e^5 + 2081250*d^3*e^6 + 974765*d^2*e^7 + 5668992*d*e^8 + 7946185*e^9)*m^3 + 6*(308000*d^5*e^4 + 165375*d^4*e^5 + 496170*d^3*e^6 + 206275*d^2*e^7 + 1064712*d*e^8 + 2542410*e^9)*m^2 + 36*(28000*d^5*e^4 + 14175*d^4*e^5 + 39960*d^3*e^6 + 15540*d^2*e^7 + 74592*d*e^8 + 422435*e^9)*m)*x^4 + 3*(1480*d^6*e^3 + 35520*d^5*e^4 - 63050*d^4*e^5 + 375570*d^3*e^6 - 444059*d^2*e^7 + 1288056*d*e^8)*m^3 + (12942720*e^9 + (65*d*e^8 + 107*e^9)*m^8 - 2*(296*d^2*e^7 - 1235*d*e^8 - 2247*e^9)*m^7 - 4*(185*d^3*e^6 + 4884*d^2$

```

*e^7 - 9620*d*e^8 - 19902*e^9)*m^6 - 2*(6660*d^4*e^5 + 9990*d^3*e^6 + 12639
2*d^2*e^7 - 157625*d*e^8 - 386163*e^9)*m^5 - (37800*d^5*e^4 + 266400*d^4*e^
5 + 196100*d^3*e^6 + 1607280*d^2*e^7 - 1444235*d*e^8 - 4453233*e^9)*m^4 - 4
*(168000*d^6*e^3 + 113400*d^5*e^4 + 416250*d^4*e^5 + 208125*d^3*e^6 + 12793
12*d^2*e^7 - 903370*d*e^8 - 3864519*e^9)*m^3 - 4*(504000*d^6*e^3 + 274050*d
^5*e^4 + 832500*d^4*e^5 + 350390*d^3*e^6 + 1831056*d^2*e^7 - 1103505*d*e^8
- 7764883*e^9)*m^2 - 48*(28000*d^6*e^3 + 14175*d^5*e^4 + 39960*d^4*e^5 + 15
540*d^3*e^6 + 74592*d^2*e^7 - 40950*d*e^8 - 672923*e^9)*m)*x^3 + 2*(39960*d
^7*e^2 + 53280*d^6*e^3 + 594960*d^5*e^4 - 648375*d^4*e^5 + 2629418*d^3*e^6
- 2209977*d^2*e^7 + 4581036*d*e^8)*m^2 + (5987520*e^9 + (107*d*e^8 + 33*e^9
)*m^8 - (195*d^2*e^7 - 4280*d*e^8 - 1419*e^9)*m^7 + 4*(444*d^3*e^6 - 1755*d
^2*e^7 + 17762*d*e^8 + 6468*e^9)*m^6 + 2*(1110*d^4*e^5 + 27528*d^3*e^6 - 50
700*d^2*e^7 + 315115*d*e^8 + 130053*e^9)*m^5 + (39960*d^5*e^4 + 55500*d^4*e
^5 + 648240*d^3*e^6 - 742950*d^2*e^7 + 3192773*d*e^8 + 1567797*e^9)*m^4 + (
113400*d^6*e^3 + 719280*d^5*e^4 + 477300*d^4*e^5 + 3525360*d^3*e^6 - 284680
5*d^2*e^7 + 9072530*d*e^8 + 5752131*e^9)*m^3 + 6*(336000*d^7*e^2 + 189000*d
^6*e^3 + 592740*d^5*e^4 + 257150*d^4*e^5 + 1383504*d^3*e^6 - 857805*d^2*e^7
+ 2152412*d*e^8 + 2062863*e^9)*m^2 + 72*(28000*d^7*e^2 + 14175*d^6*e^3 + 3
9960*d^5*e^4 + 15540*d^4*e^5 + 74592*d^3*e^6 - 40950*d^2*e^7 + 89880*d*e^8
+ 193677*e^9)*m)*x^2 + 12*(18900*d^8*e + 113220*d^7*e^2 + 70670*d^6*e^3 + 4
88400*d^5*e^4 - 366405*d^4*e^5 + 1073852*d^3*e^6 - 663102*d^2*e^7 + 995544*
d*e^8)*m + (6531840*e^9 + 3*(11*d*e^8 + 6*e^9)*m^8 - 2*(107*d^2*e^7 - 693*d
*e^8 - 396*e^9)*m^7 + 6*(65*d^3*e^6 - 1391*d^2*e^7 + 4081*d*e^8 + 2478*e^9)
)*m^6 - 2*(1776*d^4*e^5 - 6825*d^3*e^6 + 66875*d^2*e^7 - 117810*d*e^8 - 7761
6*e^9)*m^5 - 3*(1480*d^5*e^4 + 35520*d^4*e^5 - 63050*d^3*e^6 + 375570*d^2*e
^7 - 444059*d*e^8 - 327894*e^9)*m^4 - 2*(39960*d^6*e^3 + 53280*d^5*e^4 + 59
4960*d^4*e^5 - 648375*d^3*e^6 + 2629418*d^2*e^7 - 2209977*d*e^8 - 1932084*e
^9)*m^3 - 12*(18900*d^7*e^2 + 113220*d^6*e^3 + 70670*d^5*e^4 + 488400*d^4*e
^5 - 366405*d^3*e^6 + 1073852*d^2*e^7 - 663102*d*e^8 - 763506*e^9)*m^2 - 14
4*(28000*d^8*e + 14175*d^7*e^2 + 39960*d^6*e^3 + 15540*d^5*e^4 + 74592*d^4*
e^5 - 40950*d^3*e^6 + 89880*d^2*e^7 - 41580*d*e^8 - 82962*e^9)*m)*x*(e*x +
d)^m/(e^9*m^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 +
269325*e^9*m^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*
e^9)

```

**giac [B]** time = 0.39, size = 6223, normalized size = 14.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (100*(x*e + d)^m*m^8*x^9*e^9 + 100*(x*e + d)^m*d*m^8*x^8*e^8 - 45*(x*e + d)
^m*m^8*x^8*e^9 + 3600*(x*e + d)^m*m^7*x^9*e^9 - 45*(x*e + d)^m*d*m^8*x^7*e^
```

$$\begin{aligned}
& 8 + 2800*(x*e + d)^m*d*m^7*x^8*e^8 - 800*(x*e + d)^m*d^2*m^7*x^7*e^7 + 111* \\
& (x*e + d)^m*m^8*x^7*e^9 - 1665*(x*e + d)^m*m^7*x^8*e^9 + 54600*(x*e + d)^m* \\
& m^6*x^9*e^9 + 111*(x*e + d)^m*d*m^8*x^6*e^8 - 1350*(x*e + d)^m*d*m^7*x^7*e^ \\
& 8 + 32200*(x*e + d)^m*d*m^6*x^8*e^8 + 315*(x*e + d)^m*d^2*m^7*x^6*e^7 - 168 \\
& 00*(x*e + d)^m*d^2*m^6*x^7*e^7 + 5600*(x*e + d)^m*d^3*m^6*x^6*e^6 - 37*(x*e \\
& + d)^m*m^8*x^6*e^9 + 4218*(x*e + d)^m*m^7*x^7*e^9 - 25830*(x*e + d)^m*m^6* \\
& x^8*e^9 + 453600*(x*e + d)^m*m^5*x^9*e^9 - 37*(x*e + d)^m*d*m^8*x^5*e^8 + 3 \\
& 552*(x*e + d)^m*d*m^7*x^6*e^8 - 16380*(x*e + d)^m*d*m^6*x^7*e^8 + 196000*(x \\
& *e + d)^m*d*m^5*x^8*e^8 - 666*(x*e + d)^m*d^2*m^7*x^5*e^7 + 7560*(x*e + d)^ \\
& m*d^2*m^6*x^6*e^7 - 140000*(x*e + d)^m*d^2*m^5*x^7*e^7 - 1890*(x*e + d)^m*d \\
& ^3*m^6*x^5*e^6 + 84000*(x*e + d)^m*d^3*m^5*x^6*e^6 - 33600*(x*e + d)^m*d^4* \\
& m^5*x^5*e^5 + 148*(x*e + d)^m*m^8*x^5*e^9 - 1443*(x*e + d)^m*m^7*x^6*e^9 + \\
& 67044*(x*e + d)^m*m^6*x^7*e^9 - 218610*(x*e + d)^m*m^5*x^8*e^9 + 2244900*(x \\
& *e + d)^m*m^4*x^9*e^9 + 148*(x*e + d)^m*d*m^8*x^4*e^8 - 1258*(x*e + d)^m*d* \\
& m^7*x^5*e^8 + 45732*(x*e + d)^m*d*m^6*x^6*e^8 - 103950*(x*e + d)^m*d*m^5*x^ \\
& 7*e^8 + 676900*(x*e + d)^m*d*m^4*x^8*e^8 + 185*(x*e + d)^m*d^2*m^7*x^4*e^7 \\
& - 17982*(x*e + d)^m*d^2*m^6*x^5*e^7 + 69300*(x*e + d)^m*d^2*m^5*x^6*e^7 - 5 \\
& 88000*(x*e + d)^m*d^2*m^4*x^7*e^7 + 3330*(x*e + d)^m*d^3*m^6*x^4*e^6 - 3591 \\
& 0*(x*e + d)^m*d^3*m^5*x^5*e^6 + 476000*(x*e + d)^m*d^3*m^4*x^6*e^6 + 9450*( \\
& x*e + d)^m*d^4*m^5*x^4*e^5 - 336000*(x*e + d)^m*d^4*m^4*x^5*e^5 + 168000*(x \\
& *e + d)^m*d^5*m^4*x^4*e^4 + 65*(x*e + d)^m*m^8*x^4*e^9 + 5920*(x*e + d)^m*m \\
& ^7*x^5*e^9 - 23532*(x*e + d)^m*m^6*x^6*e^9 + 579642*(x*e + d)^m*m^5*x^7*e^9 \\
& - 1098405*(x*e + d)^m*m^4*x^8*e^9 + 6728400*(x*e + d)^m*m^3*x^9*e^9 + 65*( \\
& x*e + d)^m*d*m^8*x^3*e^8 + 5328*(x*e + d)^m*d*m^7*x^4*e^8 - 17242*(x*e + d) \\
& ^m*d*m^6*x^5*e^8 + 305250*(x*e + d)^m*d*m^5*x^6*e^8 - 370755*(x*e + d)^m*d* \\
& m^4*x^7*e^8 + 1313200*(x*e + d)^m*d*m^3*x^8*e^8 - 592*(x*e + d)^m*d^2*m^7*x \\
& ^3*e^7 + 5550*(x*e + d)^m*d^2*m^6*x^4*e^7 - 184482*(x*e + d)^m*d^2*m^5*x^5* \\
& e^7 + 311850*(x*e + d)^m*d^2*m^4*x^6*e^7 - 1299200*(x*e + d)^m*d^2*m^3*x^7* \\
& e^7 - 740*(x*e + d)^m*d^3*m^6*x^3*e^6 + 76590*(x*e + d)^m*d^3*m^5*x^4*e^6 - \\
& 236250*(x*e + d)^m*d^3*m^4*x^5*e^6 + 1260000*(x*e + d)^m*d^3*m^3*x^6*e^6 - \\
& 13320*(x*e + d)^m*d^4*m^5*x^3*e^5 + 141750*(x*e + d)^m*d^4*m^4*x^4*e^5 - 1 \\
& 176000*(x*e + d)^m*d^4*m^3*x^5*e^5 - 37800*(x*e + d)^m*d^5*m^4*x^3*e^4 + 10 \\
& 08000*(x*e + d)^m*d^5*m^3*x^4*e^4 - 672000*(x*e + d)^m*d^6*m^3*x^3*e^3 + 10 \\
& 7*(x*e + d)^m*m^8*x^3*e^9 + 2665*(x*e + d)^m*m^7*x^4*e^9 + 99160*(x*e + d)^ \\
& m*m^6*x^5*e^9 - 208458*(x*e + d)^m*m^5*x^6*e^9 + 2965809*(x*e + d)^m*m^4*x^ \\
& 7*e^9 - 3332385*(x*e + d)^m*m^3*x^8*e^9 + 11812400*(x*e + d)^m*m^2*x^9*e^9 \\
& + 107*(x*e + d)^m*d*m^8*x^2*e^8 + 2470*(x*e + d)^m*d*m^7*x^3*e^8 + 77848*(x \\
& *e + d)^m*d*m^6*x^4*e^8 - 122248*(x*e + d)^m*d*m^5*x^5*e^8 + 1134309*(x*e + \\
& d)^m*d*m^4*x^6*e^8 - 737100*(x*e + d)^m*d*m^3*x^7*e^8 + 1306800*(x*e + d)^ \\
& m*d*m^2*x^8*e^8 - 195*(x*e + d)^m*d^2*m^7*x^2*e^7 - 19536*(x*e + d)^m*d^2*m \\
& ^6*x^3*e^7 + 64010*(x*e + d)^m*d^2*m^5*x^4*e^7 - 909090*(x*e + d)^m*d^2*m^4 \\
& *x^5*e^7 + 724185*(x*e + d)^m*d^2*m^3*x^6*e^7 - 1411200*(x*e + d)^m*d^2*m^2 \\
& *x^7*e^7 + 1776*(x*e + d)^m*d^3*m^6*x^2*e^6 - 19980*(x*e + d)^m*d^3*m^5*x^3 \\
& *e^6 + 616050*(x*e + d)^m*d^3*m^4*x^4*e^6 - 689850*(x*e + d)^m*d^3*m^3*x^5* \\
& e^6 + 1534400*(x*e + d)^m*d^3*m^2*x^6*e^6 + 2220*(x*e + d)^m*d^4*m^5*x^2*e^
\end{aligned}$$

$$\begin{aligned}
& 5 - 266400*(x*e + d)^m*d^4*m^4*x^3*e^5 + 614250*(x*e + d)^m*d^4*m^3*x^4*e^5 \\
& - 1680000*(x*e + d)^m*d^4*m^2*x^5*e^5 + 39960*(x*e + d)^m*d^5*m^4*x^2*e^4 \\
& - 453600*(x*e + d)^m*d^5*m^3*x^3*e^4 + 1848000*(x*e + d)^m*d^5*m^2*x^4*e^4 \\
& + 113400*(x*e + d)^m*d^6*m^3*x^2*e^3 - 2016000*(x*e + d)^m*d^6*m^2*x^3*e^3 \\
& + 2016000*(x*e + d)^m*d^7*m^2*x^2*e^2 + 33*(x*e + d)^m*m^8*x^2*e^9 + 4494*(x*e + d)^m*m^7*x^3*e^9 \\
& + 45890*(x*e + d)^m*m^6*x^4*e^9 + 902800*(x*e + d)^m*m^5*x^5*e^9 - 1090353*(x*e + d)^m*m^4*x^6*e^9 \\
& + 9134412*(x*e + d)^m*m^3*x^7*e^9 - 5906520*(x*e + d)^m*m^2*x^8*e^9 + 10958400*(x*e + d)^m*m*x^9*e^9 + \\
& 33*(x*e + d)^m*d*m^8*x*e^8 + 4280*(x*e + d)^m*d*m^7*x^2*e^8 + 38480*(x*e + d)^m*d*m^6*x^3*e^8 \\
& + 591408*(x*e + d)^m*d*m^5*x^4*e^8 - 479113*(x*e + d)^m*d*m^4*x^5*e^8 + 2328558*(x*e + d)^m*d*m^3*x^6*e^8 \\
& - 746820*(x*e + d)^m*d*m^2*x^7*e^8 + 504000*(x*e + d)^m*d*m*x^8*e^8 - 214*(x*e + d)^m*d^2*m^7*x*e^7 \\
& - 7020*(x*e + d)^m*d^2*m^6*x^2*e^7 - 252784*(x*e + d)^m*d^2*m^5*x^3*e^7 + 355200*(x*e + d)^m*d^2*m^4*x^4*e^7 \\
& - 2260404*(x*e + d)^m*d^2*m^3*x^5*e^7 + 814590*(x*e + d)^m*d^2*m^2*x^6*e^7 - 576000*(x*e + d)^m*d^2*m*x^7*e^7 + 390*(x*e + d)^m*d^3*m^6*x*e^6 \\
& + 55056*(x*e + d)^m*d^3*m^5*x^2*e^6 - 196100*(x*e + d)^m*d^3*m^4*x^3*e^6 + 2081250*(x*e + d)^m*d^3*m^3*x^4*e^6 \\
& - 895860*(x*e + d)^m*d^3*m^2*x^5*e^6 + 672000*(x*e + d)^m*d^3*m*x^6*e^6 - 3552*(x*e + d)^m*d^4*m^5*x*e^5 \\
& + 55500*(x*e + d)^m*d^4*m^4*x^2*e^5 - 1665000*(x*e + d)^m*d^4*m^3*x^3*e^5 + 992250*(x*e + d)^m*d^4*m^2*x^4*e^5 \\
& - 806400*(x*e + d)^m*d^4*m*x^5*e^5 - 4440*(x*e + d)^m*d^5*m^4*x*e^4 + 719280*(x*e + d)^m*d^5*m^3*x^2*e^4 \\
& - 1096200*(x*e + d)^m*d^5*m^2*x^3*e^4 + 1008000*(x*e + d)^m*d^5*m*x^4*e^4 - 79920*(x*e + d)^m*d^6*m^3*x*e^3 \\
& + 1134000*(x*e + d)^m*d^6*m^2*x^2*e^3 - 1344000*(x*e + d)^m*d^6*m*x^3*e^3 - 226800*(x*e + d)^m*d^7*m^2*x*e^2 \\
& + 2016000*(x*e + d)^m*d^7*m*x^2*e^2 - 4032000*(x*e + d)^m*d^8*m*x*e + 18*(x*e + d)^m*m^8*x*e^9 \\
& + 1419*(x*e + d)^m*m^7*x^2*e^9 + 79608*(x*e + d)^m*m^6*x^3*e^9 + 430690*(x*e + d)^m*m^5*x^4*e^9 \\
& + 4850404*(x*e + d)^m*m^4*x^5*e^9 - 3422907*(x*e + d)^m*m^3*x^6*e^9 + 16387596*(x*e + d)^m*m^2*x^7*e^9 - 5519340 \\
& *(x*e + d)^m*m*x^8*e^9 + 4032000*(x*e + d)^m*x^9*e^9 + 18*(x*e + d)^m*d*m^8*e^8 + 1386*(x*e + d)^m*d*m^7*x*e^8 \\
& + 71048*(x*e + d)^m*d*m^6*x^2*e^8 + 315250*(x*e + d)^m*d*m^5*x^3*e^8 + 2484772*(x*e + d)^m*d*m^4*x^4*e^8 - 1027342 \\
& *(x*e + d)^m*d*m^3*x^5*e^8 + 2416248*(x*e + d)^m*d*m^2*x^6*e^8 - 291600*(x*e + d)^m*d*m*x^7*e^8 - 33*(x*e + d)^m*d^2*m^7*e^7 \\
& - 8346*(x*e + d)^m*d^2*m^6*x*e^7 - 101400*(x*e + d)^m*d^2*m^5*x^2*e^7 - 1607280*(x*e + d)^m*d^2*m^4*x^3*e^7 \\
& + 974765*(x*e + d)^m*d^2*m^3*x^4*e^7 - 2669328*(x*e + d)^m*d^2*m^2*x^5*e^7 + 340200*(x*e + d)^m*d^2*m*x^6*e^7 \\
& + 214*(x*e + d)^m*d^3*m^6*e^6 + 13650*(x*e + d)^m*d^3*m^5*x*e^6 + 648240*(x*e + d)^m*d^3*m^4*x^2*e^6 - 832500 \\
& *(x*e + d)^m*d^3*m^3*x^3*e^6 + 2977020*(x*e + d)^m*d^3*m^2*x^4*e^6 - 408240*(x*e + d)^m*d^3*m*x^5*e^6 - 390 \\
& *(x*e + d)^m*d^4*m^5*e^5 - 106560*(x*e + d)^m*d^4*m^4*x*e^5 + 477300*(x*e + d)^m*d^4*m^3*x^2*e^5 - 3330000*(x*e + d)^m \\
& *d^4*m^2*x^3*e^5 + 510300*(x*e + d)^m*d^4*m*x^4*e^5 + 3552*(x*e + d)^m*d^5*m^4*e^4 - 106560*(x*e + d)^m*d^5*m^3*x*e^4 \\
& + 3556440*(x*e + d)^m*d^5*m^2*x^2*e^4 - 680400*(x*e + d)^m*d^5*m*x^3*e^4 + 4440*(x*e + d)^m*d^6*m^3*e^3 - 1358640 \\
& *(x*e + d)^m*d^6*m^2*x*e^3 + 1020600*(x*e + d)^m*d^6*m*x^2*e^3 + 79920*(x*e + d)^m*d^7*m^2*e^2 - 2041200*(x*e + d)^m \\
& *d^7*m*x*e^2 + 226800*(
\end{aligned}$$



$$\begin{aligned}
& x^*e + d)^m*d^8*m^*e + 4032000*(x^*e + d)^m*d^9 + 792*(x^*e + d)^m*m^7*x^*e^9 + \\
& 25872*(x^*e + d)^m*m^6*x^2*e^9 + 772326*(x^*e + d)^m*m^5*x^3*e^9 + 2389985*(x \\
& *e + d)^m*m^4*x^4*e^9 + 15608080*(x^*e + d)^m*m^3*x^5*e^9 - 6238718*(x^*e + d \\
& )^m*m^2*x^6*e^9 + 15456528*(x^*e + d)^m*m*x^7*e^9 - 2041200*(x^*e + d)^m*x^8* \\
& e^9 + 792*(x^*e + d)^m*d*m^7*e^8 + 24486*(x^*e + d)^m*d*m^6*x^*e^8 + 630230*(x \\
& *e + d)^m*d*m^5*x^2*e^8 + 1444235*(x^*e + d)^m*d*m^4*x^3*e^8 + 5668992*(x^*e \\
& + d)^m*d*m^3*x^4*e^8 - 1102008*(x^*e + d)^m*d*m^2*x^5*e^8 + 959040*(x^*e + d) \\
& ^m*d*m*x^6*e^8 - 1386*(x^*e + d)^m*d^2*m^6*e^7 - 133750*(x^*e + d)^m*d^2*m^5* \\
& x^*e^7 - 742950*(x^*e + d)^m*d^2*m^4*x^2*e^7 - 5117248*(x^*e + d)^m*d^2*m^3*x^ \\
& 3*e^7 + 1237650*(x^*e + d)^m*d^2*m^2*x^4*e^7 - 1150848*(x^*e + d)^m*d^2*m*x^5 \\
& *e^7 + 8346*(x^*e + d)^m*d^3*m^5*e^6 + 189150*(x^*e + d)^m*d^3*m^4*x^*e^6 + 35 \\
& 25360*(x^*e + d)^m*d^3*m^3*x^2*e^6 - 1401560*(x^*e + d)^m*d^3*m^2*x^3*e^6 + 1 \\
& 438560*(x^*e + d)^m*d^3*m*x^4*e^6 - 13650*(x^*e + d)^m*d^4*m^4*e^5 - 1189920* \\
& (x^*e + d)^m*d^4*m^3*x^*e^5 + 1542900*(x^*e + d)^m*d^4*m^2*x^2*e^5 - 1918080*( \\
& x^*e + d)^m*d^4*m*x^3*e^5 + 106560*(x^*e + d)^m*d^5*m^3*e^4 - 848040*(x^*e + d) \\
& )^m*d^5*m^2*x^*e^4 + 2877120*(x^*e + d)^m*d^5*m*x^2*e^4 + 106560*(x^*e + d)^m* \\
& d^6*m^2*e^3 - 5754240*(x^*e + d)^m*d^6*m*x^*e^3 + 1358640*(x^*e + d)^m*d^7*m^*e \\
& ^2 + 2041200*(x^*e + d)^m*d^8*e + 14868*(x^*e + d)^m*m^6*x^*e^9 + 260106*(x^*e \\
& + d)^m*m^5*x^2*e^9 + 4453233*(x^*e + d)^m*m^4*x^3*e^9 + 7946185*(x^*e + d)^m* \\
& m^3*x^4*e^9 + 29064240*(x^*e + d)^m*m^2*x^5*e^9 - 5957592*(x^*e + d)^m*m*x^6* \\
& e^9 + 5754240*(x^*e + d)^m*x^7*e^9 + 14868*(x^*e + d)^m*d*m^6*e^8 + 235620*(x \\
& *e + d)^m*d*m^5*x^*e^8 + 3192773*(x^*e + d)^m*d*m^4*x^2*e^8 + 3613480*(x^*e + \\
& d)^m*d*m^3*x^3*e^8 + 6388272*(x^*e + d)^m*d*m^2*x^4*e^8 - 447552*(x^*e + d)^m \\
& *d*m*x^5*e^8 - 24486*(x^*e + d)^m*d^2*m^5*e^7 - 1126710*(x^*e + d)^m*d^2*m^4* \\
& x^*e^7 - 2846805*(x^*e + d)^m*d^2*m^3*x^2*e^7 - 7324224*(x^*e + d)^m*d^2*m^2*x \\
& ^3*e^7 + 559440*(x^*e + d)^m*d^2*m*x^4*e^7 + 133750*(x^*e + d)^m*d^3*m^4*e^6 \\
& + 1296750*(x^*e + d)^m*d^3*m^3*x^*e^6 + 8301024*(x^*e + d)^m*d^3*m^2*x^2*e^6 - \\
& 745920*(x^*e + d)^m*d^3*m*x^3*e^6 - 189150*(x^*e + d)^m*d^4*m^3*e^5 - 586080 \\
& 0*(x^*e + d)^m*d^4*m^2*x^*e^5 + 1118880*(x^*e + d)^m*d^4*m*x^2*e^5 + 1189920*( \\
& x^*e + d)^m*d^5*m^2*e^4 - 2237760*(x^*e + d)^m*d^5*m*x^*e^4 + 848040*(x^*e + d) \\
& ^m*d^6*m^*e^3 + 5754240*(x^*e + d)^m*d^7*e^2 + 155232*(x^*e + d)^m*m^5*x^*e^9 + \\
& 1567797*(x^*e + d)^m*m^4*x^2*e^9 + 15458076*(x^*e + d)^m*m^3*x^3*e^9 + 15254 \\
& 460*(x^*e + d)^m*m^2*x^4*e^9 + 28238400*(x^*e + d)^m*m*x^5*e^9 - 2237760*(x^*e \\
& + d)^m*x^6*e^9 + 155232*(x^*e + d)^m*d*m^5*e^8 + 1332177*(x^*e + d)^m*d*m^4* \\
& x^*e^8 + 9072530*(x^*e + d)^m*d*m^3*x^2*e^8 + 4414020*(x^*e + d)^m*d*m^2*x^3*e \\
& ^8 + 2685312*(x^*e + d)^m*d*m*x^4*e^8 - 235620*(x^*e + d)^m*d^2*m^4*e^7 - 525 \\
& 8836*(x^*e + d)^m*d^2*m^3*x^*e^7 - 5146830*(x^*e + d)^m*d^2*m^2*x^2*e^7 - 3580 \\
& 416*(x^*e + d)^m*d^2*m*x^3*e^7 + 1126710*(x^*e + d)^m*d^3*m^3*e^6 + 4396860*( \\
& x^*e + d)^m*d^3*m^2*x^*e^6 + 5370624*(x^*e + d)^m*d^3*m*x^2*e^6 - 1296750*(x^*e \\
& + d)^m*d^4*m^2*e^5 - 10741248*(x^*e + d)^m*d^4*m*x^*e^5 + 5860800*(x^*e + d)^ \\
& m*d^5*m^*e^4 + 2237760*(x^*e + d)^m*d^6*e^3 + 983682*(x^*e + d)^m*m^4*x^*e^9 + \\
& 5752131*(x^*e + d)^m*m^3*x^2*e^9 + 31059532*(x^*e + d)^m*m^2*x^3*e^9 + 152076 \\
& 60*(x^*e + d)^m*m*x^4*e^9 + 10741248*(x^*e + d)^m*x^5*e^9 + 983682*(x^*e + d)^ \\
& m*d*m^4*e^8 + 4419954*(x^*e + d)^m*d*m^3*x^*e^8 + 12914472*(x^*e + d)^m*d*m^2* \\
& x^2*e^8 + 1965600*(x^*e + d)^m*d*m*x^3*e^8 - 1332177*(x^*e + d)^m*d^2*m^3*e^7
\end{aligned}$$

$$\begin{aligned}
& - 12886224*(x*e + d)^m*d^2*m^2*x*e^7 - 2948400*(x*e + d)^m*d^2*m*x^2*e^7 + \\
& 5258836*(x*e + d)^m*d^3*m^2*e^6 + 5896800*(x*e + d)^m*d^3*m*x*e^6 - 439686 \\
& 0*(x*e + d)^m*d^4*m*e^5 + 10741248*(x*e + d)^m*d^5*e^4 + 3864168*(x*e + d)^ \\
& m*m^3*x*e^9 + 12377178*(x*e + d)^m*m^2*x^2*e^9 + 32300304*(x*e + d)^m*m*x^3 \\
& *e^9 + 5896800*(x*e + d)^m*x^4*e^9 + 3864168*(x*e + d)^m*d*m^3*e^8 + 795722 \\
& 4*(x*e + d)^m*d*m^2*x*e^8 + 6471360*(x*e + d)^m*d*m*x^2*e^8 - 4419954*(x*e \\
& + d)^m*d^2*m^2*e^7 - 12942720*(x*e + d)^m*d^2*m*x*e^7 + 12886224*(x*e + d)^ \\
& m*d^3*m*e^6 - 5896800*(x*e + d)^m*d^4*e^5 + 9162072*(x*e + d)^m*m^2*x*e^9 + \\
& 13944744*(x*e + d)^m*m*x^2*e^9 + 12942720*(x*e + d)^m*x^3*e^9 + 9162072*(x \\
& *e + d)^m*d*m^2*e^8 + 5987520*(x*e + d)^m*d*m*x*e^8 - 7957224*(x*e + d)^m*d \\
& ^2*m*e^7 + 12942720*(x*e + d)^m*d^3*e^6 + 11946528*(x*e + d)^m*m*x*e^9 + 59 \\
& 87520*(x*e + d)^m*x^2*e^9 + 11946528*(x*e + d)^m*d*m*e^8 - 5987520*(x*e + d \\
& )^m*d^2*e^7 + 6531840*(x*e + d)^m*x*e^9 + 6531840*(x*e + d)^m*d*e^8)/(m^9*e \\
& ^9 + 45*m^8*e^9 + 870*m^7*e^9 + 9450*m^6*e^9 + 63273*m^5*e^9 + 269325*m^4*e \\
& ^9 + 723680*m^3*e^9 + 1172700*m^2*e^9 + 1026576*m*e^9 + 362880*e^9)
\end{aligned}$$

**maple [B]** time = 0.04, size = 3222, normalized size = 7.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2), x)$

[Out]  $(e*x+d)^{(1+m)}*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*e^8*m^7*x^8-800*d*e^7*m^7*x^7+1111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m^6*x^8+315*d*e^7*m^7*x^6-22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x^6-25830*e^8*m^6*x^7+453600*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7*x^5+9450*d*e^7*m^6*x^6-257600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*x^5+67044*e^8*m^6*x^6-218610*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*m^6*x^5+117600*d^2*e^6*m^5*x^6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+114660*d*e^7*m^5*x^6-1568000*d*e^7*m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-23532*e^8*m^6*x^5+579642*e^8*m^5*x^6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-33600*d^3*e^5*m^5*x^5+3330*d^2*e^6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2*e^6*m^4*x^6-592*d*e^7*m^7*x^3+6290*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727650*d*e^7*m^4*x^6-5415200*d*e^7*m^3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+99160*e^8*m^6*x^4-208458*e^8*m^5*x^5+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^7+11812400*e^8*m^2*x^8+9450*d^3*e^5*m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-415800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*m^6*x^3+86210*d*e^7*m^5*x^4-1831500*d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-10505600*d*e^7*m^2*x^7+33*e^8*m^8*x+4494*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*e^8*m^5*x^4-1090353*e^8*m^4*x^5+9134412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+10958400*e^8*m*x^8+168000*d^4*e^4*m^4*x^4-13320*d^3*e^5*m^5*x^3+179550*d^3*e^5*m^4*x^4-2856000*d^3*e^5*m^3*x^5+1776*d^2*e^6*m^6*x^2-22200*d^2*e^6*m^5*x^3+922410*d^2*e^6*m^4*x^4-1871100*d^2*e^6*m^3*x^5+9094400*d^2*e^6*m^2*x^6-214*d*e^7*m^7*x-7410*d*e^7*m^6*x^2-311392*d*e^7*m^5*x^3+611240*d*e^7*m^4*x^4-6$

805854\*d\*e^7\*m^3\*x^5+5159700\*d\*e^7\*m^2\*x^6-10454400\*d\*e^7\*m\*x^7+18\*e^8\*m^8+1419\*e^8\*m^7\*x+79608\*e^8\*m^6\*x^2+430690\*e^8\*m^5\*x^3+4850404\*e^8\*m^4\*x^4-3422907\*e^8\*m^3\*x^5+16387596\*e^8\*m^2\*x^6-5519340\*e^8\*m\*x^7+4032000\*e^8\*x^8-37800\*d^4\*e^4\*m^4\*x^3+1680000\*d^4\*e^4\*m^3\*x^4+2220\*d^3\*e^5\*m^5\*x^2-306360\*d^3\*e^5\*m^4\*x^3+1181250\*d^3\*e^5\*m^3\*x^4-7560000\*d^3\*e^5\*m^2\*x^5+390\*d^2\*e^6\*m^6\*x+58608\*d^2\*e^6\*m^5\*x^2-256040\*d^2\*e^6\*m^4\*x^3+4545450\*d^2\*e^6\*m^3\*x^4-4345110\*d^2\*e^6\*m^2\*x^5+9878400\*d^2\*e^6\*m\*x^6-33\*d\*e^7\*m^7-8560\*d\*e^7\*m^6\*x-115440\*d\*e^7\*m^5\*x^2-2365632\*d\*e^7\*m^4\*x^3+2395565\*d\*e^7\*m^3\*x^4-13971348\*d\*e^7\*m^2\*x^5+5227740\*d\*e^7\*m\*x^6-4032000\*d\*e^7\*x^7+792\*e^8\*m^7+25872\*e^8\*m^6\*x+772326\*e^8\*m^5\*x^2+2389985\*e^8\*m^4\*x^3+15608080\*e^8\*m^3\*x^4-6238718\*e^8\*m^2\*x^5+15456528\*e^8\*m\*x^6-2041200\*e^8\*x^7-672000\*d^5\*e^3\*m^3\*x^3+39960\*d^4\*e^4\*m^4\*x^2-567000\*d^4\*e^4\*m^3\*x^3+5880000\*d^4\*e^4\*m^2\*x^4-3552\*d^3\*e^5\*m^5\*x+59940\*d^3\*e^5\*m^4\*x^2-2464200\*d^3\*e^5\*m^3\*x^3+3449250\*d^3\*e^5\*m^2\*x^4-9206400\*d^3\*e^5\*m\*x^5+214\*d^2\*e^6\*m^6+14040\*d^2\*e^6\*m^5\*x+758352\*d^2\*e^6\*m^4\*x^2-1420800\*d^2\*e^6\*m^3\*x^3+11302020\*d^2\*e^6\*m^2\*x^4-4887540\*d^2\*e^6\*m\*x^5+4032000\*d^2\*e^6\*x^6-1386\*d\*e^7\*m^6-142096\*d\*e^7\*m^5\*x-945750\*d\*e^7\*m^4\*x^2-9939088\*d\*e^7\*m^3\*x^3+5136710\*d\*e^7\*m^2\*x^4-14497488\*d\*e^7\*m\*x^5+2041200\*d\*e^7\*x^6+14868\*e^8\*m^6+260106\*e^8\*m^5\*x+4453233\*e^8\*m^4\*x^2+7946185\*e^8\*m^3\*x^3+29064240\*e^8\*m^2\*x^4-5957592\*e^8\*m\*x^5+5754240\*e^8\*x^6+113400\*d^5\*e^3\*m^3\*x^2-4032000\*d^5\*e^3\*m^2\*x^3-4440\*d^4\*e^4\*m^4\*x+799200\*d^4\*e^4\*m^3\*x^2-2457000\*d^4\*e^4\*m^2\*x^3+8400000\*d^4\*e^4\*m\*x^4-390\*d^3\*e^5\*m^5-110112\*d^3\*e^5\*m^4\*x+588300\*d^3\*e^5\*m^3\*x^2-8325000\*d^3\*e^5\*m^2\*x^3+4479300\*d^3\*e^5\*m\*x^4-4032000\*d^3\*e^5\*x^5+8346\*d^2\*e^6\*m^5+202800\*d^2\*e^6\*m^4\*x+4821840\*d^2\*e^6\*m^3\*x^2-3899060\*d^2\*e^6\*m^2\*x^3+13346640\*d^2\*e^6\*m\*x^4-2041200\*d^2\*e^6\*x^5-24486\*d\*e^7\*m^5-1260460\*d\*e^7\*m^4\*x-4332705\*d\*e^7\*m^3\*x^2-22675968\*d\*e^7\*m^2\*x^3+5510040\*d\*e^7\*m\*x^4-5754240\*d\*e^7\*x^5+155232\*e^8\*m^5+1567797\*e^8\*m^4\*x+15458076\*e^8\*m^3\*x^2+15254460\*e^8\*m^2\*x^3+28238400\*e^8\*m\*x^4-2237760\*e^8\*x^5+2016000\*d^6\*e^2\*m^2\*x^2-79920\*d^5\*e^3\*m^3\*x+1360800\*d^5\*e^3\*m^2\*x^2-7392000\*d^5\*e^3\*m\*x^3+3552\*d^4\*e^4\*m^4-111000\*d^4\*e^4\*m^3\*x+4995000\*d^4\*e^4\*m^2\*x^2-3969000\*d^4\*e^4\*m\*x^3+4032000\*d^4\*e^4\*x^4-13650\*d^3\*e^5\*m^4-1296480\*d^3\*e^5\*m^3\*x+2497500\*d^3\*e^5\*m^2\*x^2-11908080\*d^3\*e^5\*m\*x^3+2041200\*d^3\*e^5\*x^4+133750\*d^2\*e^6\*m^4+1485900\*d^2\*e^6\*m^3\*x+15351744\*d^2\*e^6\*m^2\*x^2-4950600\*d^2\*e^6\*m\*x^3+5754240\*d^2\*e^6\*x^4-235620\*d\*e^7\*m^4-6385546\*d\*e^7\*m^3\*x-10840440\*d\*e^7\*m^2\*x^2-25553088\*d\*e^7\*m\*x^3+2237760\*d\*e^7\*x^4+983682\*e^8\*m^4+5752131\*e^8\*m^3\*x+31059532\*e^8\*m^2\*x^2+15207660\*e^8\*m\*x^3+10741248\*e^8\*x^4-226800\*d^6\*e^2\*m^2\*x+6048000\*d^6\*e^2\*m\*x^2+4440\*d^5\*e^3\*m^3-1438560\*d^5\*e^3\*m^2\*x+3288600\*d^5\*e^3\*m\*x^2-4032000\*d^5\*e^3\*x^3+106560\*d^4\*e^4\*m^3-954600\*d^4\*e^4\*m^2\*x+9990000\*d^4\*e^4\*m\*x^2-2041200\*d^4\*e^4\*x^3-189150\*d^3\*e^5\*m^3-7050720\*d^3\*e^5\*m^2\*x+4204680\*d^3\*e^5\*m\*x^2-5754240\*d^3\*e^5\*x^3+1126710\*d^2\*e^6\*m^3+5693610\*d^2\*e^6\*m^2\*x+21972672\*d^2\*e^6\*m\*x^2-2237760\*d^2\*e^6\*x^3-1332177\*d\*e^7\*m^3-18145060\*d\*e^7\*m^2\*x-13242060\*d\*e^7\*m\*x^2-10741248\*d\*e^7\*x^3+3864168\*e^8\*m^3+12377178\*e^8\*m^2\*x+32300304\*e^8\*m\*x^2+5896800\*e^8\*x^3-4032000\*d^7\*e\*m\*x+79920\*d^6\*e^2\*m^2-2268000\*d^6\*e^2\*m\*x+4032000\*d^6\*e^2\*x^2+106560\*d^5\*e^3\*m^2-7112880\*d^5\*e^3\*m\*x+2041200\*d^5\*e^3\*x^2+1189920\*d^4\*e^4\*m^2-3085800\*d^4\*e^4\*m\*x+5754240\*d^4\*e^4\*x^2-1296750\*d^3\*e^5\*m^2-16602048\*d^

```

3*e^5*m*x+2237760*d^3*e^5*x^2+5258836*d^2*e^6*m^2+10293660*d^2*e^6*m*x+1074
1248*d^2*e^6*x^2-4419954*d*e^7*m^2-25828944*d*e^7*m*x-5896800*d*e^7*x^2+916
2072*e^8*m^2+13944744*e^8*m*x+12942720*e^8*x^2+226800*d^7*e*m-4032000*d^7*e
*x+1358640*d^6*e^2*m-2041200*d^6*e^2*x+848040*d^5*e^3*m-5754240*d^5*e^3*x+5
860800*d^4*e^4*m-2237760*d^4*e^4*x-4396860*d^3*e^5*m-10741248*d^3*e^5*x+128
86224*d^2*e^6*m+5896800*d^2*e^6*x-7957224*d*e^7*m-12942720*d*e^7*x+11946528
*e^8*m+5987520*e^8*x+4032000*d^8+2041200*d^7*e+5754240*d^6*e^2+2237760*d^5*
e^3+10741248*d^4*e^4-5896800*d^3*e^5+12942720*d^2*e^6-5987520*d*e^7+6531840
*e^8)/e^9/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+723680*m^3+1172
700*m^2+1026576*m+362880)

```

**maxima** [B] time = 0.64, size = 1414, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")

```

```

[Out] 33*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 18
*(e*x + d)^(m + 1)/(e*(m + 1)) + 107*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
+ 65*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*
(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24)*e^4) + 148*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
+ (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^
3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 37*((m^5 + 15*m^4 + 85*m^3 +
225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d
*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2
*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x
+ d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) +
111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7
+ (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6
*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*
e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 19
60*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 45*((m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 - 7*(
m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^2 +
6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)*d^6
*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*m^6
+ 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8) +
100*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^

```

$$\begin{aligned} & 2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m \\ & ^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*m^5 \\ & + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5 + 8 \\ & 5*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35*m^3 \\ & + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e^4*x \\ & ^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2 - 4 \\ & 0320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450*m^6 \\ & + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)* \\ & e^9) \end{aligned}$$

**mupad [B]** time = 6.05, size = 2625, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2), x)$

[Out] 
$$\begin{aligned} & ((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^7 \\ & + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^3 \\ & + 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4*e^ \\ & 5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 9162072*d* \\ & e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14868*d \\ & *e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 5258836*d^3 \\ & *e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^3*m^2 + \\ & 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 189150*d^4 \\ & *e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*m^4 + 133 \\ & 750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^2*e^7*m^5 \\ & + 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^3*e^6*m^6 - \\ & 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m)/(e^9*(1026576*m + 117 \\ & 2700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^ \\ & 8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*m^2 + 67284*m^ \\ & 3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/(1026576*m + 11 \\ & 72700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m \\ & ^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531840*e^9 + 9162072 \\ & *e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*m^5 + 14868*e^9*m^ \\ & 6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896800*d^3*e^6*m - 107 \\ & 41248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^3*m - 2041200*d^7*e^2*m \\ & + 7957224*d*e^8*m^2 + 4419954*d*e^8*m^3 + 1332177*d*e^8*m^4 + 235620*d*e^8 \\ & *m^5 + 24486*d*e^8*m^6 + 1386*d*e^8*m^7 + 33*d*e^8*m^8 - 12886224*d^2*e^7*m \\ & ^2 + 4396860*d^3*e^6*m^2 - 5860800*d^4*e^5*m^2 - 848040*d^5*e^4*m^2 - 13586 \\ & 40*d^6*e^3*m^2 - 226800*d^7*e^2*m^2 - 5258836*d^2*e^7*m^3 + 1296750*d^3*e^6 \\ & *m^3 - 1189920*d^4*e^5*m^3 - 106560*d^5*e^4*m^3 - 79920*d^6*e^3*m^3 - 11267 \\ & 10*d^2*e^7*m^4 + 189150*d^3*e^6*m^4 - 106560*d^4*e^5*m^4 - 4440*d^5*e^4*m^4 \\ & - 133750*d^2*e^7*m^5 + 13650*d^3*e^6*m^5 - 3552*d^4*e^5*m^5 - 8346*d^2*e^7 \\ & *m^6 + 390*d^3*e^6*m^6 - 214*d^2*e^7*m^7 + 5987520*d*e^8*m - 4032000*d^8*e* \end{aligned}$$

$$\begin{aligned}
& m)) / (e^9(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(33600*d^4*m - 244200*e^4*m - 447552*e^4 - 49580*e^4*m^2 - 4440*e^4*m^3 - 148*e^4*m^4 + 47952*d^2*e^2*m + 7067*d*e^3*m^2 + 1890*d^3*e*m^2 + 888*d*e^3*m^3 + 37*d*e^3*m^4 + 11322*d^2*e^2*m^2 + 666*d^2*e^2*m^3 + 18648*d*e^3*m + 17010*d^3*e*m)) / (e^4*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^7*(d + e*x)^m*(800*d^2*m - 1887*e^2*m - 7992*e^2 - 111*e^2*m^2 + 405*d*e*m + 45*d*e*m^2)*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) / (e^2*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^6*(d + e*x)^m*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(5600*d^3*m - 7067*e^3*m - 18648*e^3 - 888*e^3*m^2 - 37*e^3*m^3 + 1887*d*e^2*m^2 + 315*d^2*e*m^2 + 111*d*e^2*m^3 + 7992*d*e^2*m + 2835*d^2*e*m)) / (e^3*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(168000*d^5*m + 732810*e^5*m + 982800*e^5 + 216125*e^5*m^2 + 31525*e^5*m^3 + 2275*e^5*m^4 + 65*e^5*m^5 + 93240*d^2*e^3*m + 239760*d^3*e^2*m + 244200*d*e^4*m^2 + 9450*d^4*e*m^2 + 49580*d*e^4*m^3 + 4440*d*e^4*m^4 + 148*d*e^4*m^5 + 35335*d^2*e^3*m^2 + 56610*d^3*e^2*m^2 + 4440*d^2*e^3*m^3 + 3330*d^3*e^2*m^3 + 185*d^2*e^3*m^4 + 447552*d*e^4*m + 85050*d^4*e*m)) / (e^5*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (5*x^8*(d + e*x)^m*(81*e - 20*d*m + 9*e*m)*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) / (e*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^2*(m + 1)*(d + e*x)^m*(2016000*d^7*m + 7957224*e^7*m + 5987520*e^7 + 4419954*e^7*m^2 + 1332177*e^7*m^3 + 235620*e^7*m^4 + 24486*e^7*m^5 + 1386*e^7*m^6 + 33*e^7*m^7 - 2948400*d^2*e^5*m + 5370624*d^3*e^4*m + 1118880*d^4*e^3*m + 2877120*d^5*e^2*m + 6443112*d*e^6*m^2 + 113400*d^6*e*m^2 + 2629418*d*e^6*m^3 + 563355*d*e^6*m^4 + 66875*d*e^6*m^5 + 4173*d*e^6*m^6 + 107*d*e^6*m^7 - 2198430*d^2*e^5*m^2 + 2930400*d^3*e^4*m^2 + 424020*d^4*e^3*m^2 + 679320*d^5*e^2*m^2 - 648375*d^2*e^5*m^3 + 594960*d^3*e^4*m^3 + 53280*d^4*e^3*m^3 + 39960*d^5*e^2*m^3 - 94575*d^2*e^5*m^4 + 53280*d^3*e^4*m^4 + 2220*d^4*e^3*m^4 - 6825*d^2*e^5*m^5 + 1776*d^3*e^4*m^5 - 195*d^2*e^5*m^6 + 6471360*d*e^6*m + 1020600*d^6*e*m)) / (e^7*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(672000*d^6*m - 6443112*e^6*m - 6471360*e^6 - 2629418*e^6*m^2 - 563355*e^6*m^3 - 66875*e^6*m^4 - 4173*e^6*m^5 - 107*e^6*m^6 + 1790208*d^2*e^4*m + 372960*d^3*e^3*m + 959040*d^4*e^2*m - 732810*d*e^5*m^2 + 37800*d^5*e*m^2 - 216125*d*e^5*m^3 - 31525*d*e^5*m^4 - 2275*d*e^5*m^5 - 65*d*e^5*m^6 + 976800*d^2*e^4*m^2 + 141340*d^3*e^3*m^2 + 226440*d^4*e^2*m^2 + 198320*d^2*e^4*m^3 + 17760*d^3*e^3*m^3 + 13320*d^4*e^2*m^3 + 17760*d^2*e^4*m^4 + 740*d^3*e^3*m^4 + 592*d^2*e^4*m^5 - 982800*d*e^5*m + 340200*d^5*e*m)) / (e^6*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(5\*x\*\*2+2\*x+3)\*\*2\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2),x)

[Out] Timed out

$$3.349 \quad \int (d + ex)^m (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

**Optimal.** Leaf size=292

$$\frac{(300d^2 + 85de + 17e^2)(d + ex)^{m+5}}{e^7(m+5)} - \frac{2(200d^3 + 85d^2e + 34de^2 + 2e^3)(d + ex)^{m+4}}{e^7(m+4)} + \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)(d + ex)^{m+3}}{e^7(m+3)}$$

**Rubi [A]** time = 0.19, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {1628}

$$\frac{(5d^2 - 2de + 3e^2)(3d^2e + 5d^2e + 4d^2 - de^3 + 2e^4)(d + ex)^{m+1}}{e^7(m+1)} - \frac{(68d^2e^2 + 12d^2e^3 + 85d^4e + 120d^5 + 42de^4 - 7e^5)(d + ex)^{m+2}}{e^7(m+2)} + \frac{(102d^2e^2 + 170d^3e + 300d^4 + 12d^5 + 21e^6)(d + ex)^{m+3}}{e^7(m+3)} - \frac{2(85d^2e + 200d^3 + 34de^2 + 2e^3)(d + ex)^{m+4}}{e^7(m+4)} + \frac{(300d^2 + 85de + 17e^2)(d + ex)^{m+5}}{e^7(m+5)} - \frac{(120d + 17e)(d + ex)^{m+6}}{e^7(m+6)} + \frac{20(d + ex)^{m+7}}{e^7(m+7)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(3 + 2\*x + 5\*x^2)\*(2 + x + 3\*x^2 - 5\*x^3 + 4\*x^4), x]

[Out] ((5\*d^2 - 2\*d\*e + 3\*e^2)\*(4\*d^4 + 5\*d^3\*e + 3\*d^2\*e^2 - d\*e^3 + 2\*e^4)\*(d + e\*x)^(1 + m))/(e^7\*(1 + m)) - ((120\*d^5 + 85\*d^4\*e + 68\*d^3\*e^2 + 12\*d^2\*e^3 + 42\*d\*e^4 - 7\*e^5)\*(d + e\*x)^(2 + m))/(e^7\*(2 + m)) + ((300\*d^4 + 170\*d^3\*e + 102\*d^2\*e^2 + 12\*d\*e^3 + 21\*e^4)\*(d + e\*x)^(3 + m))/(e^7\*(3 + m)) - (2\*(200\*d^3 + 85\*d^2\*e + 34\*d\*e^2 + 2\*e^3)\*(d + e\*x)^(4 + m))/(e^7\*(4 + m)) + ((300\*d^2 + 85\*d\*e + 17\*e^2)\*(d + e\*x)^(5 + m))/(e^7\*(5 + m)) - ((120\*d + 17\*e)\*(d + e\*x)^(6 + m))/(e^7\*(6 + m)) + (20\*(d + e\*x)^(7 + m))/(e^7\*(7 + m))

**Rule 1628**

Int[(Pq\_)\*((d\_.) + (e\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*Pq\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

**Rubi steps**

$$\int (d + ex)^m (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx = \int \left( \frac{(20d^6 + 17d^5e + 17d^4e^2 + 4d^3e^3 + 21d^2e^4 - 7de^5)}{e^6} \right. \\ \left. = \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)(d + ex)^{m+3}}{e^7(1 + m)} \right)$$

**Mathematica [A]** time = 0.17, size = 261, normalized size = 0.89

$$\frac{(d + ex)^{m+1} \left( \frac{(300d^2 + 85de + 17e^2)(d + ex)^4}{m+5} - \frac{2(200d^3 + 85d^2e + 34de^2 + 2e^3)(d + ex)^3}{m+4} + \frac{(300d^4 + 170d^3e + 102d^2e^2 + 12d^5 + 21e^6)(d + ex)^2}{m+3} + \frac{(5d^2 - 2de + 3e^2)(4d^4 + 5d^3e + 3d^2e^2 - de^3 + 2e^4)(d + ex)}{m+1} - \frac{(120d^5 + 85d^4e + 68d^3e^2 + 12d^2e^3 + 42de^4 - 7e^5)(d + ex)}{m+2} + \frac{20(d + ex)^6}{m+7} - \frac{(120d + 17e)(d + ex)^5}{m+6} \right)}{e^7}$$



Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4), x]
[Out] ((d + e*x)^(1 + m)*((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 +
42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2
+ 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 34*d*
e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^
4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/(7 + m
)))/e^7
```

**IntegrateAlgebraic** [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (d + ex)^m (3 + 2x + 5x^2) (2 + x + 3x^2 - 5x^3 + 4x^4) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4
*x^4), x]
```

```
[Out] Defer[IntegrateAlgebraic] [(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*
x^3 + 4*x^4), x]
```

**fricas** [B] time = 0.78, size = 1448, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2), x, algorithm="fri
cas")
```

```
[Out] (6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624*
e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5*e
^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280*e^
7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e^6 -
19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)*m^2
- 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (17136*
e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5 - 3*(4
00*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^6 - 3145
*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(240*d^2*e^5
+ 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 1770*d*e^6)*m
^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^6 - 96*e^7)
*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4 - (3600*d^3*
e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*e^4 + 1411*d^
2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2*e^5 + 714*d*e
```

$$\begin{aligned} &^6 - 1968e^7)m)x^4 + (24d^4e^3 + 924d^3e^4 - 1715d^2e^5 + 9990d^1e^6) \\ &^6)m^3 + (35280e^7 - (4d^4e^3 - 21e^7)m^6 - (68d^2e^5 + 84d^1e^6 - 52 \\ &5e^7)m^5 - (340d^3e^4 + 1088d^2e^5 + 652d^1e^6 - 5187e^7)m^4 - (240 \\ &0d^4e^3 + 3400d^3e^4 + 5644d^2e^5 + 2268d^1e^6 - 25599e^7)m^3 - 4*( \\ &1800d^4e^3 + 1955d^3e^4 + 2584d^2e^5 + 844d^1e^6 - 16338e^7)m^2 - 4 \\ &*(1200d^4e^3 + 1190d^3e^4 + 1428d^2e^5 + 420d^1e^6 - 19929e^7)m)x^3 \\ &+ (408d^5e^2 + 432d^4e^3 + 7518d^3e^4 - 8225d^2e^5 + 30624d^1e^6) \\ &^6)m^2 + (17640e^7 + 7*(3d^3e^6 + e^7)m^6 + (12d^2e^5 + 483d^1e^6 + 182e^7) \\ &^6)m^5 + 3*(68d^3e^4 + 76d^2e^5 + 1407d^1e^6 + 630e^7)m^4 + (1020d^4 \\ &4e^3 + 2856d^3e^4 + 1500d^2e^5 + 17157d^1e^6 + 9940e^7)m^3 + (7200d^5 \\ &^5e^2 + 8160d^4e^3 + 11220d^3e^4 + 3804d^2e^5 + 31038d^1e^6 + 27503e^7) \\ &^6)m^2 + 6*(1200d^5e^2 + 1190d^4e^3 + 1428d^3e^4 + 420d^2e^5 + 29 \\ &40d^1e^6 + 6153e^7)m)x^2 + 6*(340d^6e + 884d^5e^2 + 428d^4e^3 + 44 \\ &66d^3e^4 - 3213d^2e^5 + 8028d^1e^6)m + (30240e^7 + (7d^6e + 6e^7)m^6 - \\ &(42d^2e^5 - 175d^1e^6 - 162e^7)m^5 - (24d^3e^4 + 924d^2e^5 - 1715d^1e^6 - \\ &1770e^7)m^4 - (408d^4e^3 + 432d^3e^4 + 7518d^2e^5 - 8225d^1e^6 - 9990e^7) \\ &^6)m^3 - 6*(340d^5e^2 + 884d^4e^3 + 428d^3e^4 + 4466d^2e^5 - 3213d^1e^6 - \\ &5104e^7)m^2 - 24*(600d^6e + 595d^5e^2 + 714d^4e^3 + 210d^3e^4 + 1470d^2e^5 - \\ &735d^1e^6 - 2007e^7)m)x*(e*x + d)^m/(e^7m^7 + 28e^7m^6 + 322e^7m^5 + \\ &1960e^7m^4 + 6769e^7m^3 + 13132e^7m^2 + 13068e^7m + 5040e^7) \end{aligned}$$

**giac [B]** time = 0.27, size = 3098, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^m\*(5\*x^2+2\*x+3)\*(4\*x^4-5\*x^3+3\*x^2+x+2),x, algorithm="giac")

[Out] (20\*(x\*e + d)^m\*m^6\*x^7\*e^7 + 20\*(x\*e + d)^m\*d\*m^6\*x^6\*e^6 - 17\*(x\*e + d)^m\*m^6\*x^6\*e^7 + 420\*(x\*e + d)^m\*m^5\*x^7\*e^7 - 17\*(x\*e + d)^m\*d\*m^6\*x^5\*e^6 + 300\*(x\*e + d)^m\*d\*m^5\*x^6\*e^6 - 120\*(x\*e + d)^m\*d^2\*m^5\*x^5\*e^5 + 17\*(x\*e + d)^m\*m^6\*x^5\*e^7 - 374\*(x\*e + d)^m\*m^5\*x^6\*e^7 + 3500\*(x\*e + d)^m\*m^4\*x^7\*e^7 + 17\*(x\*e + d)^m\*d\*m^6\*x^4\*e^6 - 289\*(x\*e + d)^m\*d\*m^5\*x^5\*e^6 + 1700\*(x\*e + d)^m\*d\*m^4\*x^6\*e^6 + 85\*(x\*e + d)^m\*d^2\*m^5\*x^4\*e^5 - 1200\*(x\*e + d)^m\*d^2\*m^4\*x^5\*e^5 + 600\*(x\*e + d)^m\*d^3\*m^4\*x^4\*e^4 - 4\*(x\*e + d)^m\*m^6\*x^4\*e^7 + 391\*(x\*e + d)^m\*m^5\*x^5\*e^7 - 3230\*(x\*e + d)^m\*m^4\*x^6\*e^7 + 14700\*(x\*e + d)^m\*m^3\*x^7\*e^7 - 4\*(x\*e + d)^m\*d\*m^6\*x^3\*e^6 + 323\*(x\*e + d)^m\*d\*m^5\*x^4\*e^6 - 1785\*(x\*e + d)^m\*d\*m^4\*x^5\*e^6 + 4500\*(x\*e + d)^m\*d\*m^3\*x^6\*e^6 - 68\*(x\*e + d)^m\*d^2\*m^5\*x^3\*e^5 + 1105\*(x\*e + d)^m\*d^2\*m^4\*x^4\*e^5 - 4200\*(x\*e + d)^m\*d^2\*m^3\*x^5\*e^5 - 340\*(x\*e + d)^m\*d^3\*m^4\*x^3\*e^4 + 3600\*(x\*e + d)^m\*d^3\*m^3\*x^4\*e^4 - 2400\*(x\*e + d)^m\*d^4\*m^3\*x^3\*e^3 + 21\*(x\*e + d)^m\*m^6\*x^3\*e^7 - 96\*(x\*e + d)^m\*m^5\*x^4\*e^7 + 3519\*(x\*e + d)^m\*m^4\*x^5\*e^7 - 13940\*(x\*e + d)^m\*m^3\*x^6\*e^7 + 32480\*(x\*e + d)^m\*m^2\*x^7\*e^7 + 21\*(x\*e + d)

$$\begin{aligned}
& )^m d^m x^2 e^6 - 84(xe + d)^m d^m x^3 e^6 + 2227(xe + d)^m d^m x^4 e^6 - 5015(xe + d)^m d^m x^5 e^6 + 5480(xe + d)^m d^m x^6 e^6 + \\
& 12(xe + d)^m d^2 m^5 x^2 e^5 - 1088(xe + d)^m d^2 m^4 x^3 e^5 + 4505(xe + d)^m d^2 m^3 x^4 e^5 - 6000(xe + d)^m d^2 m^2 x^5 e^5 + 204(xe + d)^m d^3 m^4 x^2 e^4 - \\
& 3400(xe + d)^m d^3 m^3 x^3 e^4 + 6600(xe + d)^m d^3 m^2 x^4 e^4 + 1020(xe + d)^m d^4 m^3 x^2 e^3 - 7200(xe + d)^m d^4 m^2 x^3 e^3 + 7200(xe + d)^m d^5 m^2 x^2 e^2 + \\
& 7(xe + d)^m m^6 x^2 e^7 + 525(xe + d)^m m^5 x^3 e^7 - 904(xe + d)^m m^4 x^4 e^7 + 15725(xe + d)^m m^3 x^5 e^7 - 31433(xe + d)^m m^2 x^6 e^7 + 35280(xe + d)^m m x^7 e^7 + \\
& 7(xe + d)^m d^m x^6 e^6 + 483(xe + d)^m d^m x^5 e^6 - 652(xe + d)^m d^m x^4 e^6 + 6817(xe + d)^m d^m x^3 e^6 - 6358(xe + d)^m d^m x^2 e^6 + 2400(xe + d)^m d^m x e^6 - \\
& 42(xe + d)^m d^2 m^5 x e^5 + 228(xe + d)^m d^2 m^4 x^2 e^5 - 5644(xe + d)^m d^2 m^3 x^3 e^5 + 7055(xe + d)^m d^2 m^2 x^4 e^5 - 2880(xe + d)^m d^2 m x^5 e^5 - \\
& 24(xe + d)^m d^3 m^4 x e^4 + 2856(xe + d)^m d^3 m^3 x^2 e^4 - 7820(xe + d)^m d^3 m^2 x^3 e^4 + 3600(xe + d)^m d^3 m x^4 e^4 - 408(xe + d)^m d^4 m^3 x e^3 + \\
& 8160(xe + d)^m d^4 m^2 x^2 e^3 - 4800(xe + d)^m d^4 m x^3 e^3 - 2040(xe + d)^m d^5 m^2 x e^2 + 7200(xe + d)^m d^5 m x^2 e^2 - 14400(xe + d)^m d^6 m x e + \\
& 6(xe + d)^m m^6 x e^7 + 182(xe + d)^m m^5 x^2 e^7 + 5187(xe + d)^m m^4 x^3 e^7 - 4224(xe + d)^m m^3 x^4 e^7 + 36448(xe + d)^m m^2 x^5 e^7 - 34646(xe + d)^m m x^6 e^7 + 14400(xe + d)^m x^7 e^7 + \\
& 6(xe + d)^m d^m x^6 e^6 + 175(xe + d)^m d^m x^5 e^6 + 4221(xe + d)^m d^m x^4 e^6 - 2268(xe + d)^m d^m x^3 e^6 + 9180(xe + d)^m d^m x^2 e^6 - 2856(xe + d)^m d^m x e^6 - \\
& 7(xe + d)^m d^2 m^5 x e^5 - 924(xe + d)^m d^2 m^4 x^2 e^5 + 1500(xe + d)^m d^2 m^3 x^3 e^5 - 10336(xe + d)^m d^2 m^2 x^4 e^5 + 3570(xe + d)^m d^2 m x^5 e^5 + \\
& 42(xe + d)^m d^3 m^4 x e^4 - 432(xe + d)^m d^3 m^3 x^2 e^4 + 11220(xe + d)^m d^3 m^2 x^3 e^4 - 4760(xe + d)^m d^3 m x^4 e^4 + 24(xe + d)^m d^4 m^3 x^3 e^3 - \\
& 5304(xe + d)^m d^4 m^2 x^4 e^3 + 7140(xe + d)^m d^4 m x^5 e^3 + 408(xe + d)^m d^5 m^2 x^4 e^2 - 14280(xe + d)^m d^5 m x^5 e^2 + 2040(xe + d)^m d^6 m^2 x^4 e^2 + \\
& 14400(xe + d)^m d^7 m x^5 e^2 + 162(xe + d)^m m^5 x^4 e^7 + 1890(xe + d)^m m^4 x^5 e^7 + 25599(xe + d)^m m^3 x^6 e^7 - 10180(xe + d)^m m^2 x^7 e^7 + \\
& 41004(xe + d)^m m x^8 e^7 - 14280(xe + d)^m x^9 e^7 + 162(xe + d)^m d^m x^8 e^6 + 1715(xe + d)^m d^m x^7 e^6 + 17157(xe + d)^m d^m x^6 e^6 - 3376(xe + d)^m d^m x^5 e^6 + \\
& 4284(xe + d)^m d^m x^4 e^6 - 175(xe + d)^m d^2 m^4 x^3 e^5 - 7518(xe + d)^m d^2 m^3 x^4 e^5 + 3804(xe + d)^m d^2 m^2 x^5 e^5 - 5712(xe + d)^m d^2 m x^6 e^5 + \\
& 924(xe + d)^m d^3 m^3 x^5 e^4 - 2568(xe + d)^m d^3 m^2 x^6 e^4 + 8568(xe + d)^m d^3 m x^7 e^4 + 432(xe + d)^m d^4 m^2 x^6 e^3 - 17136(xe + d)^m d^4 m x^7 e^3 + \\
& 5304(xe + d)^m d^5 m^2 x^6 e^2 + 14280(xe + d)^m d^6 m x^7 e^2 + 1770(xe + d)^m m^4 x^6 e^7 + 9940(xe + d)^m m^3 x^7 e^7 + 65352(xe + d)^m m^2 x^8 e^7 - \\
& 11808(xe + d)^m m x^9 e^7 + 17136(xe + d)^m x^{10} e^7 + 1770(xe + d)^m d^m x^9 e^6 + 8225(xe + d)^m d^m x^8 e^6 + 31038(xe + d)^m d^m x^7 e^6 - 1680(xe + d)^m d^m x^6 e^6 - \\
& 1715(xe + d)^m d^2 m^3 x^5 e^5 - 26796(xe + d)^m d^2 m^2 x^6 e^5 + 2520(xe + d)^m d^2 m x^7 e^5 + 7518(xe + d)^m d^3 m^2 x^6 e^4 - 5040(xe + d)^m d^3 m x^7 e^4
\end{aligned}$$

$$\begin{aligned} & 4 + 2568*(x*e + d)^m*d^4*m*e^3 + 17136*(x*e + d)^m*d^5*e^2 + 9990*(x*e + d) \\ & ^m*m^3*x*e^7 + 27503*(x*e + d)^m*m^2*x^2*e^7 + 79716*(x*e + d)^m*m*x^3*e^7 \\ & - 5040*(x*e + d)^m*x^4*e^7 + 9990*(x*e + d)^m*d*m^3*e^6 + 19278*(x*e + d)^m \\ & *d*m^2*x*e^6 + 17640*(x*e + d)^m*d*m*x^2*e^6 - 8225*(x*e + d)^m*d^2*m^2*e^5 \\ & - 35280*(x*e + d)^m*d^2*m*x*e^5 + 26796*(x*e + d)^m*d^3*m*e^4 + 5040*(x*e \\ & + d)^m*d^4*e^3 + 30624*(x*e + d)^m*m^2*x*e^7 + 36918*(x*e + d)^m*m*x^2*e^7 \\ & + 35280*(x*e + d)^m*x^3*e^7 + 30624*(x*e + d)^m*d*m^2*e^6 + 17640*(x*e + d) \\ & ^m*d*m*x*e^6 - 19278*(x*e + d)^m*d^2*m*e^5 + 35280*(x*e + d)^m*d^3*e^4 + 48 \\ & 168*(x*e + d)^m*m*x*e^7 + 17640*(x*e + d)^m*x^2*e^7 + 48168*(x*e + d)^m*d*m \\ & *e^6 - 17640*(x*e + d)^m*d^2*e^5 + 30240*(x*e + d)^m*x*e^7 + 30240*(x*e + d) \\ & )^m*d*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^ \\ & 7 + 13132*m^2*e^7 + 13068*m*e^7 + 5040*e^7) \end{aligned}$$

**maple [B]** time = 0.02, size = 1504, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2), x)$

[Out]  $(e*x+d)^{(1+m)}*(20*e^6*m^6*x^6-17*e^6*m^6*x^5+420*e^6*m^5*x^6-120*d*e^5*m^5*x^5+17*e^6*m^6*x^4-374*e^6*m^5*x^5+3500*e^6*m^4*x^6+85*d*e^5*m^5*x^4-1800*d*e^5*m^4*x^5-4*e^6*m^6*x^3+391*e^6*m^5*x^4-3230*e^6*m^4*x^5+14700*e^6*m^3*x^6+600*d^2*e^4*m^4*x^4-68*d*e^5*m^5*x^3+1445*d*e^5*m^4*x^4-10200*d*e^5*m^3*x^5+21*e^6*m^6*x^2-96*e^6*m^5*x^3+3519*e^6*m^4*x^4-13940*e^6*m^3*x^5+32480*e^6*m^2*x^6-340*d^2*e^4*m^4*x^3+6000*d^2*e^4*m^3*x^4+12*d*e^5*m^5*x^2-1292*d*e^5*m^4*x^3+8925*d*e^5*m^3*x^4-27000*d*e^5*m^2*x^5+7*e^6*m^6*x+525*e^6*m^5*x^2-904*e^6*m^4*x^3+15725*e^6*m^3*x^4-31433*e^6*m^2*x^5+35280*e^6*m*x^6-2400*d^3*e^3*m^3*x^3+204*d^2*e^4*m^4*x^2-4420*d^2*e^4*m^3*x^3+21000*d^2*e^4*m^2*x^4-42*d*e^5*m^5*x+252*d*e^5*m^4*x^2-8908*d*e^5*m^3*x^3+25075*d*e^5*m^2*x^4-32880*d*e^5*m*x^5+6*e^6*m^6+182*e^6*m^5*x+5187*e^6*m^4*x^2-4224*e^6*m^3*x^3+36448*e^6*m^2*x^4-34646*e^6*m*x^5+14400*e^6*x^6+1020*d^3*e^3*m^3*x^2-14400*d^3*e^3*m^2*x^3-24*d^2*e^4*m^4*x+3264*d^2*e^4*m^3*x^2-18020*d^2*e^4*m^2*x^3+30000*d^2*e^4*m*x^4-7*d*e^5*m^5-966*d*e^5*m^4*x+1956*d*e^5*m^3*x^2-27268*d*e^5*m^2*x^3+31790*d*e^5*m*x^4-14400*d*e^5*x^5+162*e^6*m^5+1890*e^6*m^4*x+25599*e^6*m^3*x^2-10180*e^6*m^2*x^3+41004*e^6*m*x^4-14280*e^6*x^5+7200*d^4*e^2*m^2*x^2-408*d^3*e^3*m^3*x+10200*d^3*e^3*m^2*x^2-26400*d^3*e^3*m*x^3+42*d^2*e^4*m^4-456*d^2*e^4*m^3*x+16932*d^2*e^4*m^2*x^2-28220*d^2*e^4*m*x^3+14400*d^2*e^4*x^4-175*d*e^5*m^4-8442*d*e^5*m^3*x+6804*d*e^5*m^2*x^2-36720*d*e^5*m*x^3+14280*d*e^5*x^4+1770*e^6*m^4+9940*e^6*m^3*x+65352*e^6*m^2*x^2-11808*e^6*m*x^3+17136*e^6*x^4-2040*d^4*e^2*m^2*x+21600*d^4*e^2*m*x^2+24*d^3*e^3*m^3-5712*d^3*e^3*m^2*x+23460*d^3*e^3*m*x^2-14400*d^3*e^3*x^3+924*d^2*e^4*m^3-3000*d^2*e^4*m^2*x+31008*d^2*e^4*m*x^2-14280*d^2*e^4*x^3-1715*d*e^5*m^3-34314*d*e^5*m^2*x+10128*d*e^5*m*x^2-17136*d*e^5*x^3+9990*e^6*m^3+27503*e^6*m^2*x+79716*e^6*m*x^2-5040*e^6*x^3-14400*d^5*e*m*x+408*d^4*e^2*m^2-1632$

```
0*d^4*e^2*m*x+14400*d^4*e^2*x^2+432*d^3*e^3*m^2-22440*d^3*e^3*m*x+14280*d^3
*e^3*x^2+7518*d^2*e^4*m^2-7608*d^2*e^4*m*x+17136*d^2*e^4*x^2-8225*d*e^5*m^2
-62076*d*e^5*m*x+5040*d*e^5*x^2+30624*e^6*m^2+36918*e^6*m*x+35280*e^6*x^2+2
040*d^5*e*m-14400*d^5*e*x+5304*d^4*e^2*m-14280*d^4*e^2*x+2568*d^3*e^3*m-171
36*d^3*e^3*x+26796*d^2*e^4*m-5040*d^2*e^4*x-19278*d*e^5*m-35280*d*e^5*x+481
68*e^6*m+17640*e^6*x+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280
*d^2*e^4-17640*d*e^5+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1
3132*m^2+13068*m+5040)
```

**maxima [B]** time = 0.55, size = 788, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 7*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 6*(
e*x + d)^(m + 1)/(e*(m + 1)) + 21*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3) - 4
*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
+ m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^
2 + 50*m + 24)*e^4) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^
4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 1
2*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 15*m^4
+ 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 17*((m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*
x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d
^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^
m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 20*(
(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6
+ 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4
+ 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^
3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2
- 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4
+ 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

**mupad [B]** time = 5.09, size = 1425, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 35280
*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4*m
```

$$\begin{aligned}
& + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 + 177 \\
& 0*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3*e^4 \\
& *m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*e^4*m \\
& ^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^5 + 48 \\
& 168*d*e^6*m + 2040*d^6*e*m))/ (e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^ \\
& 4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m + 1624*m^ \\
& 2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^ \\
& 3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(35280*d^2*e \\
& ^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 - 162*e^7*m^ \\
& 5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m + 14280*d^5* \\
& e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 175*d*e^6*m^5 - \\
& 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^4*e^3*m^2 + 20 \\
& 40*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^4*e^3*m^3 + 924 \\
& *d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^6*m + 14400*d^6* \\
& e*m))/ (e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + \\
& m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4*m - 13398*e^4*m - \\
& 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2856*d^2*e^2*m + 428 \\
& *d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*m^4 + 884*d^2*e^2*m^2 + \\
& 68*d^2*e^2*m^3 + 840*d*e^3*m + 2380*d^3*e*m))/ (e^4*(13068*m + 13132*m^2 + \\
& 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^5*(d + e*x)^m*(5 \\
& 0*m + 35*m^2 + 10*m^3 + m^4 + 24)*(120*d^2*m - 221*e^2*m - 714*e^2 - 17*e^2 \\
& *m^2 + 119*d*e*m + 17*d*e*m^2))/ (e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960 \\
& *m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m \\
& ^3 + 6)*(600*d^3*m - 428*e^3*m - 840*e^3 - 72*e^3*m^2 - 4*e^3*m^3 + 221*d*e \\
& ^2*m^2 + 85*d^2*e*m^2 + 17*d*e^2*m^3 + 714*d*e^2*m + 595*d^2*e*m))/ (e^3*(13 \\
& 068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + \\
& (x^2*(m + 1)*(d + e*x)^m*(7200*d^5*m + 19278*e^5*m + 17640*e^5 + 8225*e^5* \\
& m^2 + 1715*e^5*m^3 + 175*e^5*m^4 + 7*e^5*m^5 + 2520*d^2*e^3*m + 8568*d^3*e^ \\
& 2*m + 13398*d*e^4*m^2 + 1020*d^4*e*m^2 + 3759*d*e^4*m^3 + 462*d*e^4*m^4 + 2 \\
& 1*d*e^4*m^5 + 1284*d^2*e^3*m^2 + 2652*d^3*e^2*m^2 + 216*d^2*e^3*m^3 + 204*d \\
& ^3*e^2*m^3 + 12*d^2*e^3*m^4 + 17640*d*e^4*m + 7140*d^4*e*m))/ (e^5*(13068*m \\
& + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^6* \\
& (d + e*x)^m*(119*e - 20*d*m + 17*e*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + \\
& m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^ \\
& 6 + m^7 + 5040))
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(5\*x\*\*2+2\*x+3)\*(4\*x\*\*4-5\*x\*\*3+3\*x\*\*2+x+2), x)

[Out] Timed out

$$3.350 \quad \int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3} dx$$

**Optimal.** Leaf size=528

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) \left(2c^3(6a^2h-3abg+b^2f) - 30a^2bc^2i + 10ab^3ci - c^4(6be-4af) + b^5(-i) + 12c^5d\right) x \left(c^3(2a^2h - 3abg + b^2f) - 30a^2bc^2i + 10ab^3ci - c^4(6be - 4af) + b^5(-i) + 12c^5d\right)}{c^3(b^2 - 4ac)^{5/2}}$$

**Rubi [A]** time = 1.31, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$ , Rules used = {1660, 634, 618, 206, 628}

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + i\*x^5)/(a + b\*x + c\*x^2)^3,x]

[Out]  $-(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g - 4*a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) + c^3*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2*g + 4*a*b*h + 5*a^2*i))*x)/(2*c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*c*h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i - b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*c*g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g - 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x)/(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (i*Log[a + b*x + c*x^2])/(2*c^3)$

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1660

```
Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rubi steps





IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx + cx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + i\*x^5)/(a + b\*x + c\*x^2)^3,x]

[Out] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + i\*x^5)/(a + b\*x + c\*x^2)^3, x]

fricas [B] time = 0.87, size = 3480, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i\*x^5+h\*x^4+g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^3,x, algorithm="fricas")

[Out] [1/2\*(2\*(6\*(b^2\*c^6 - 4\*a\*c^7)\*d - 3\*(b^3\*c^5 - 4\*a\*b\*c^6)\*e + (b^4\*c^4 - 2\*a\*b^2\*c^5 - 8\*a^2\*c^6)\*f - 3\*(a\*b^3\*c^4 - 4\*a^2\*b\*c^5)\*g - (b^6\*c^2 - 12\*a\*b^4\*c^3 + 42\*a^2\*b^2\*c^4 - 40\*a^3\*c^5)\*h + (2\*b^7\*c - 23\*a\*b^5\*c^2 + 85\*a^2\*b^3\*c^3 - 100\*a^3\*b\*c^4)\*i)\*x^3 + (18\*(b^3\*c^5 - 4\*a\*b\*c^6)\*d - 9\*(b^4\*c^4 - 4\*a\*b^2\*c^5)\*e + 3\*(b^5\*c^3 - 2\*a\*b^3\*c^4 - 8\*a^2\*b\*c^5)\*f - (b^6\*c^2 - 3\*a\*b^4\*c^3 + 12\*a^2\*b^2\*c^4 - 64\*a^3\*c^5)\*g - (b^7\*c - 12\*a\*b^5\*c^2 + 30\*a^2\*b^3\*c^3 + 8\*a^3\*b\*c^4)\*h + (3\*b^8 - 31\*a\*b^6\*c + 87\*a^2\*b^4\*c^2 - 12\*a^3\*b^2\*c^3 - 128\*a^4\*c^4)\*i)\*x^2 - (12\*a^2\*c^5\*d - 6\*a^2\*b\*c^4\*e - 6\*a^3\*b\*c^3\*g + 12\*a^4\*c^3\*h + (12\*c^7\*d - 6\*b\*c^6\*e - 6\*a\*b\*c^5\*g + 12\*a^2\*c^5\*h + 2\*(b^2\*c^5 + 2\*a\*c^6)\*f - (b^5\*c^2 - 10\*a\*b^3\*c^3 + 30\*a^2\*b\*c^4)\*i)\*x^4 + 2\*(12\*b\*c^6\*d - 6\*b^2\*c^5\*e - 6\*a\*b^2\*c^4\*g + 12\*a^2\*b\*c^4\*h + 2\*(b^3\*c^4 + 2\*a\*b\*c^5)\*f - (b^6\*c - 10\*a\*b^4\*c^2 + 30\*a^2\*b^2\*c^3)\*i)\*x^3 + (12\*(b^2\*c^5 + 2\*a\*c^6)\*d - 6\*(b^3\*c^4 + 2\*a\*b\*c^5)\*e + 2\*(b^4\*c^3 + 4\*a\*b^2\*c^4 + 4\*a^2\*c^5)\*f - 6\*(a\*b^3\*c^3 + 2\*a^2\*b\*c^4)\*g + 12\*(a^2\*b^2\*c^3 + 2\*a^3\*c^4)\*h - (b^7 - 8\*a\*b^5\*c + 10\*a^2\*b^3\*c^2 + 60\*a^3\*b\*c^3)\*i)\*x^2 + 2\*(a^2\*b^2\*c^3 + 2\*a^3\*c^4)\*f - (a^2\*b^5 - 10\*a^3\*b^3\*c + 30\*a^4\*b\*c^2)\*i + 2\*(12\*a\*b\*c^5\*d - 6\*a\*b^2\*c^4\*e - 6\*a^2\*b^2\*c^3\*g + 12\*a^3\*b\*c^3\*h + 2\*(a\*b^3\*c^3 + 2\*a^2\*b\*c^4)\*f - (a\*b^6 - 10\*a^2\*b^4\*c + 30\*a^3\*b^2\*c^2)\*i)\*x)\*sqrt(b^2 - 4\*a\*c)\*log((2\*c^2\*x^2 + 2\*b\*c\*x + b^2 - 2\*a\*c + sqrt(b^2 - 4\*a\*c)\*(2\*c\*x + b))/(c\*x^2 + b\*x + a)) - (b^5\*c^3 - 14\*a\*b^3\*c^4 + 40\*a^2\*b\*c^5)\*d - (a\*b^4\*c^3 + 4\*a^2\*b^2\*c^4 - 32\*a^3\*c^5)\*e + 6\*(a^2\*b^3\*c^3 - 4\*a^3\*b\*c^4)\*f - (a^2\*b^4\*c^2 + 4\*a^3\*b^2\*c^3 - 32\*a^4\*c^4)\*g - (a^2\*b^5\*c - 14\*a^3\*b^3\*c^2 + 40\*a^4\*b\*c^3)\*h + 3\*(a^2\*b^6 - 11\*a^3\*b^4\*c + 36\*a^4\*b^2\*c^2 - 32\*a^5\*c^3)\*i + 2\*(2\*(b^4\*c^4 + a\*b^2\*c^5 - 20\*a^2\*c^6)\*d - (b^5\*c^3 + a\*b^3\*c^4 - 20\*a^2\*b

$$\begin{aligned}
& *c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)* \\
& x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*i)*\log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x), 1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^4*c^4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2 + 30*a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - 2*(12*a^2*c^5*d - 6*a^2*b*c^4*e - 6*a^3*b*c^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2*c^5*h + 2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*i)*x^4 + 2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2*(b^3*c^4 + 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 + (12*(b^2*c^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b^2*c^4 + 4*a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2*a^3*c^4)*h - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*(a^2*b^2*c^3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2*(12*a*b*c^5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^3*c^3 + 2*a^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*\sqrt{(-b^2 + 4*a*c)*\arctan(-\sqrt{-b^2 + 4*a*c})*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3 + 4*a^2*b^2*c^4 - 32*a^3*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a^4*b*c^3)*h + 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i + 2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b*c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*i)*\log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2
\end{aligned}$$

$$*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x]$$

**giac [A]** time = 0.22, size = 657, normalized size = 1.24

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i\*x^5+h\*x^4+g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^3,x, algorithm="giac")

[Out] (12\*c^5\*d\*i + 2\*b^2\*c^3\*f\*i + 4\*a\*c^4\*f\*i - 6\*a\*b\*c^3\*g\*i + 12\*a^2\*c^3\*h\*i - 6\*b\*c^4\*i\*e + b^5 - 10\*a\*b^3\*c + 30\*a^2\*b\*c^2)\*arctan((2\*c\*x + b)/sqrt(-b^2 + 4\*a\*c))/((b^4\*c^3\*i - 8\*a\*b^2\*c^4\*i + 16\*a^2\*c^5\*i)\*sqrt(-b^2 + 4\*a\*c)) + 1/2\*i\*log(c\*x^2 + b\*x + a)/c^3 - 1/2\*(b^3\*c^3\*d - 10\*a\*b\*c^4\*d - 6\*a^2\*b\*c^3\*f + a^2\*b^2\*c^2\*g + 8\*a^3\*c^3\*g + a^2\*b^3\*c\*h - 10\*a^3\*b\*c^2\*h - 3\*a^2\*b^4\*i + 21\*a^3\*b^2\*c\*i - 24\*a^4\*c^2\*i + a\*b^2\*c^3\*e + 8\*a^2\*c^4\*e - 2\*(6\*c^6\*d + b^2\*c^4\*f + 2\*a\*c^5\*f - 3\*a\*b\*c^4\*g - b^4\*c^2\*h + 8\*a\*b^2\*c^3\*h - 10\*a^2\*c^4\*h + 2\*b^5\*c\*i - 15\*a\*b^3\*c^2\*i + 25\*a^2\*b\*c^3\*i - 3\*b\*c^5\*e)\*x^3 - (18\*b\*c^5\*d + 3\*b^3\*c^3\*f + 6\*a\*b\*c^4\*f - b^4\*c^2\*g - a\*b^2\*c^3\*g - 16\*a^2\*c^4\*g - b^5\*c\*h + 8\*a\*b^3\*c^2\*h + 2\*a^2\*b\*c^3\*h + 3\*b^6\*i - 19\*a\*b^4\*c\*i + 11\*a^2\*b^2\*c^2\*i + 32\*a^3\*c^3\*i - 9\*b^2\*c^4\*e)\*x^2 - 2\*(2\*b^2\*c^4\*d + 10\*a\*c^5\*d + 5\*a\*b^2\*c^3\*f - 2\*a^2\*c^4\*f - a\*b^3\*c^2\*g - 5\*a^2\*b\*c^3\*g - a\*b^4\*c\*h + 10\*a^2\*b^2\*c^2\*h - 6\*a^3\*c^3\*h + 3\*a\*b^5\*i - 22\*a^2\*b^3\*c\*i + 31\*a^3\*b\*c^2\*i - b^3\*c^3\*e - 5\*a\*b\*c^4\*e)\*x)/((c\*x^2 + b\*x + a)^2\*(b^2 - 4\*a\*c)^2\*c^3)

**maple [B]** time = 0.02, size = 1244, normalized size = 2.36

---

Verification of antiderivative is not currently implemented for this CAS.

[In] int((i\*x^5+h\*x^4+g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a)^3,x)

[Out] ((25\*a^2\*b\*c^2\*i-10\*a^2\*c^3\*h-15\*a\*b^3\*c\*i+8\*a\*b^2\*c^2\*h-3\*a\*b\*c^3\*g+2\*a\*c^4\*f+2\*b^5\*i-b^4\*c\*h+b^2\*c^3\*f-3\*b\*c^4\*e+6\*c^5\*d)/c^2/(16\*a^2\*c^2-8\*a\*b^2\*c+b^4)\*x^3+1/2\*(32\*a^3\*c^3\*i+11\*a^2\*b^2\*c^2\*i+2\*a^2\*b\*c^3\*h-16\*a^2\*c^4\*g-19\*a\*b^4\*c\*i+8\*a\*b^3\*c^2\*h-a\*b^2\*c^3\*g+6\*a\*b\*c^4\*f+3\*b^6\*i-b^5\*c\*h-b^4\*c^2\*g+3\*b^3\*c^3\*f-9\*b^2\*c^4\*e+18\*b\*c^5\*d)/(16\*a^2\*c^2-8\*a\*b^2\*c+b^4)/c^3\*x^2+(31\*a^3\*b\*c^2\*i-6\*a^3\*c^3\*h-22\*a^2\*b^3\*c\*i+10\*a^2\*b^2\*c^2\*h-5\*a^2\*b\*c^3\*g-2\*a^2\*c^4\*f+3\*a\*b^5\*i-a\*b^4\*c\*h-a\*b^3\*c^2\*g+5\*a\*b^2\*c^3\*f-5\*a\*b\*c^4\*e+10\*a\*c^5\*d-b^3\*c^3\*e+2\*b^2\*c^4\*d)/(16\*a^2\*c^2-8\*a\*b^2\*c+b^4)/c^3\*x+1/2/c^3\*(24\*a^4\*c^2\*i-21\*a^3\*b^2\*c\*i+10\*a^3\*b\*c^2\*h-8\*a^3\*c^3\*g+3\*a^2\*b^4\*i-a^2\*b^3\*c\*h-a^2\*b^2

```
*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/(c*x^2+b*x+a)^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a^2*i-4/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2*i+1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4*i-30/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*i+1/2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*h+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*i-6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g+4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*f-6*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e+12*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d-1/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*i
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

**mupad** [B] time = 6.17, size = 1027, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x)
```

```
[Out] (atan((x*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(c^2*(4*a*c - b^2)^5) + ((32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^2)^(5/2))*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(12*c^5*d - b^5*i + 2*b^2*c^3*f + 12*a^2*c^3*h + 4*a*c^4*f - 6*b*c^4*e - 6*a*b*c^3*g + 10*a*b^3*c*i - 30*a^2*b*c^2*i))/(c^3*(4*a*c - b^2)^(5/2)) - (log(a + b*x + c*x^2)*(b^10*i - 1024*a^5*c^5*i + 160*a^2*b^6*c^2*i - 640*a^3*b^4*c^3*i + 1280*a^4*b^2*c^4*i - 20*a*b^8*c*i))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b
```

$$\begin{aligned}
& ^4c^6 - 1280a^4b^2c^7)) - ((8a^2c^4e + b^3c^3d + 8a^3c^3g - 3a^2b^4i - 24a^4c^2i + a^2b^2c^2g - 10ab^3c^4d + ab^2c^3e - 6a^2b^3c^3f + a^2b^3c^3h - 10a^3b^2c^2h + 21a^3b^2c^2i)/(2c^3(b^4 + 16a^2c^2 - 8ab^2c)) - (x^2(3b^6i - 9b^2c^4e - 16a^2c^4g + 3b^3c^3f - b^4c^2g + 32a^3c^3i + 18b^5c^5d - b^5c^5h + 11a^2b^2c^2i + 6a^2b^2c^4f - 19a^2b^4c^2i - ab^2c^3g + 8a^2b^3c^2h + 2a^2b^3c^3h)))/(2c^3(b^4 + 16a^2c^2 - 8ab^2c)) + (x(2a^2c^4f - 2b^2c^4d + b^3c^3e + 6a^3c^3h - 10a^2c^5d - 3ab^5i - 10a^2b^2c^2h + 5ab^3c^4e + ab^4c^3h - 5ab^2c^3f + ab^3c^2g + 5a^2b^2c^3g + 22a^2b^3c^2i - 31a^3b^2c^2i))/(c^3(b^4 + 16a^2c^2 - 8ab^2c)) - (x^3(6c^5d + 2b^5i + b^2c^3f - 10a^2c^3h + 2a^2c^4f - 3b^3c^4e - b^4c^3h - 3ab^3c^3g - 15ab^3c^3i + 8ab^2c^2h + 25a^2b^2c^2i))/(c^2(b^4 + 16a^2c^2 - 8ab^2c)))/(x^2(2ac + b^2) + a^2 + c^2x^4 + 2abx + 2b^2cx^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((i\*x\*\*5+h\*x\*\*4+g\*x\*\*3+f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*3,x)

[Out] Timed out

$$3.351 \quad \int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=765

$$\frac{x^3 \left( c^2 (a^2 m + 2abl + b^2 k) - b^2 c (3am + bl) - c^3 (ak + bj) + b^4 m + c^4 h \right)}{3c^5} + \frac{x^2 \left( c^3 (a^2 l + 2abk + b^2 j) - bc^2 (3a^2 m + \dots) \right)}{3c^5}$$

**Rubi [A]** time = 5.83, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 53,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {1657, 634, 618, 206, 628}

Antiderivative was successfully verified.

[In] Int[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + j\*x^5 + k\*x^6 + l\*x^7 + m\*x^8)/(a + b\*x + c\*x^2), x]

[Out] ((c^6\*f - c^5\*(b\*g + a\*h) + c^4\*(b^2\*h + 2\*a\*b\*j + a^2\*k) + b^6\*m - b^4\*c\*(b\*l + 5\*a\*m) + b^2\*c^2\*(b^2\*k + 4\*a\*b\*l + 6\*a^2\*m) - c^3\*(b^3\*j + 3\*a\*b^2\*k + 3\*a^2\*b\*l + a^3\*m))\*x)/c^7 + ((c^5\*g - c^4\*(b\*h + a\*j) + c^3\*(b^2\*j + 2\*a\*b\*k + a^2\*l) - b^5\*m + b^3\*c\*(b\*l + 4\*a\*m) - b\*c^2\*(b^2\*k + 3\*a\*b\*l + 3\*a^2\*m))\*x^2)/(2\*c^6) + ((c^4\*h - c^3\*(b\*j + a\*k) + b^4\*m - b^2\*c\*(b\*l + 3\*a\*m) + c^2\*(b^2\*k + 2\*a\*b\*l + a^2\*m))\*x^3)/(3\*c^5) + ((c^3\*j - c^2\*(b\*k + a\*l) - b^3\*m + b\*c\*(b\*l + 2\*a\*m))\*x^4)/(4\*c^4) + ((c^2\*k + b^2\*m - c\*(b\*l + a\*m))\*x^5)/(5\*c^3) + ((c\*l - b\*m)\*x^6)/(6\*c^2) + (m\*x^7)/(7\*c) - ((2\*c^8\*d - c^7\*(b\*e + 2\*a\*f) + c^6\*(b^2\*f + 3\*a\*b\*g + 2\*a^2\*h) - c^5\*(b^3\*g + 4\*a\*b^2\*h + 5\*a^2\*b\*j + 2\*a^3\*k) + b^8\*m - b^6\*c\*(b\*l + 8\*a\*m) + b^4\*c^2\*(b^2\*k + 7\*a\*b\*l + 20\*a^2\*m) - b^2\*c^3\*(b^3\*j + 6\*a\*b^2\*k + 14\*a^2\*b\*l + 16\*a^3\*m) + c^4\*(b^4\*h + 5\*a\*b^3\*j + 9\*a^2\*b^2\*k + 7\*a^3\*b\*l + 2\*a^4\*m))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/(c^8\*Sqrt[b^2 - 4\*a\*c]) + ((c^7\*e - c^6\*(b\*f + a\*g) + c^5\*(b^2\*g + 2\*a\*b\*h + a^2\*j) - c^4\*(b^3\*h + 3\*a\*b^2\*j + 3\*a^2\*b\*k + a^3\*l) - b^7\*m + b^5\*c\*(b\*l + 6\*a\*m) - b^3\*c^2\*(b^2\*k + 5\*a\*b\*l + 10\*a^2\*m) + b\*c^3\*(b^3\*j + 4\*a\*b^2\*k + 6\*a^2\*b\*l + 4\*a^3\*m))\*Log[a + b\*x + c\*x^2])/(2\*c^8)

**Rule 206**

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 618**

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rubi steps

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{a + bx + cx^2} dx = \int \left( \frac{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d} \right) dx$$

$$= \frac{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}$$

$$= \frac{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}$$

$$= \frac{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}{c^6 f - c^5(bg + ah) + c^4(b^2h + 2abj + a^2k) + b^6m - b^5d}$$



**Mathematica [A]** time = 0.73, size = 754, normalized size = 0.99

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + j\*x^5 + k\*x^6 + l\*x^7 + m\*x^8)/(a + b\*x + c\*x^2), x]

[Out] (420\*c\*(c^6\*f - c^5\*(b\*g + a\*h) + c^4\*(b^2\*h + 2\*a\*b\*j + a^2\*k) + b^6\*m - b^4\*c\*(b\*1 + 5\*a\*m) + b^2\*c^2\*(b^2\*k + 4\*a\*b\*1 + 6\*a^2\*m) - c^3\*(b^3\*j + 3\*a\*b^2\*k + 3\*a^2\*b\*1 + a^3\*m))\*x + 210\*c^2\*(c^5\*g - c^4\*(b\*h + a\*j) + c^3\*(b^2\*j + 2\*a\*b\*k + a^2\*1) - b^5\*m + b^3\*c\*(b\*1 + 4\*a\*m) - b\*c^2\*(b^2\*k + 3\*a\*b\*1 + 3\*a^2\*m))\*x^2 + 140\*c^3\*(c^4\*h - c^3\*(b\*j + a\*k) + b^4\*m - b^2\*c\*(b\*1 + 3\*a\*m) + c^2\*(b^2\*k + 2\*a\*b\*1 + a^2\*m))\*x^3 + 105\*c^4\*(c^3\*j - c^2\*(b\*k + a\*1) - b^3\*m + b\*c\*(b\*1 + 2\*a\*m))\*x^4 + 84\*c^5\*(c^2\*k + b^2\*m - c\*(b\*1 + a\*m))\*x^5 + 70\*c^6\*(c\*1 - b\*m)\*x^6 + 60\*c^7\*m\*x^7 + (420\*(2\*c^8\*d - c^7\*(b\*e + 2\*a\*f) + c^6\*(b^2\*f + 3\*a\*b\*g + 2\*a^2\*h) - c^5\*(b^3\*g + 4\*a\*b^2\*h + 5\*a^2\*b\*j + 2\*a^3\*k) + b^8\*m - b^6\*c\*(b\*1 + 8\*a\*m) + b^4\*c^2\*(b^2\*k + 7\*a\*b\*1 + 20\*a^2\*m) - b^2\*c^3\*(b^3\*j + 6\*a\*b^2\*k + 14\*a^2\*b\*1 + 16\*a^3\*m) + c^4\*(b^4\*h + 5\*a\*b^3\*j + 9\*a^2\*b^2\*k + 7\*a^3\*b\*1 + 2\*a^4\*m))\*ArcTan[(b + 2\*c\*x)/Sqrt[-b^2 + 4\*a\*c]]/Sqrt[-b^2 + 4\*a\*c] + 210\*(c^7\*e - c^6\*(b\*f + a\*g) + c^5\*(b^2\*g + 2\*a\*b\*h + a^2\*j) - c^4\*(b^3\*h + 3\*a\*b^2\*j + 3\*a^2\*b\*k + a^3\*1) - b^7\*m + b^5\*c\*(b\*1 + 6\*a\*m) - b^3\*c^2\*(b^2\*k + 5\*a\*b\*1 + 10\*a^2\*m) + b\*c^3\*(b^3\*j + 4\*a\*b^2\*k + 6\*a^2\*b\*1 + 4\*a^3\*m))\*Log[a + x\*(b + c\*x)]/(420\*c^8)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{a + bx + cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + j\*x^5 + k\*x^6 + l\*x^7 + m\*x^8)/(a + b\*x + c\*x^2), x]

[Out] IntegrateAlgebraic[(d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + j\*x^5 + k\*x^6 + l\*x^7 + m\*x^8)/(a + b\*x + c\*x^2), x]

**fricas [A]** time = 1.75, size = 2643, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((m\*x^8+l\*x^7+k\*x^6+j\*x^5+h\*x^4+g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a), x, algorithm="fricas")



$$\begin{aligned} &^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*l + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*m)*x + 210*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7)*f + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3*c^3 + 16*a^4*b*c^4)*m)*\log(c*x^2 + b*x + a))/(b^2*c^8 - 4*a*c^9)] \end{aligned}$$

**giac** [A] time = 0.18, size = 982, normalized size = 1.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((m\*x^8+l\*x^7+k\*x^6+j\*x^5+h\*x^4+g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a), x, algorithm="giac")

[Out]  $\frac{1}{420}*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*c^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*k*x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*c^4*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^5*k*x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*a*b^2*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 210*b^2*c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2 + 210*b^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*m*x^2 + 840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^5*g*x + 420*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*x + 420*b^4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x + 1680*a*b^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*x + 2520*a^2*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 - \frac{1}{2}*(b*c^6*f - b^2*c^5*g + a*c^6*g + b^3*c^4*h - 2*a*b*c^5*h - b^4*c^3*j + 3*a*b^2*c^4*j - a^2*c^5*j + b^5*c^2*k - 4*a*b^3*c^3*k + 3*a^2*b*c^4*k - b^6*c*l + 5*a*b^4*c^2*l - 6*a^2*b^2*c^3*l + a^3*c^4*l + b^7*m - 6*a*b^5*c*m + 10*a^2*b^3*c^2*m - 4*a^3*b*c^3*m - c^7*e)*\log(c*x^2 + b*x + a)/c^8 + (2*c^8*d + b^2*c^6*f - 2*a*c^7*f - b^3*c^5*g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2*a^2*c^6*h - b^5*c^3*j + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b^4*c^3*k + 9*a^2*b^2*c^4*k - 2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14*a^2*b^3*c^3*l + 7*a^3*b*c^4*l + b^8*m - 8*a*b^6*c*m + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 2*a^4*c^4*m - b*c^7*e)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c})*c^8)$

**maple** [B] time = 0.01, size = 1960, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a), x)$

[Out]  $\frac{1}{7}m*x^7/c - \frac{1}{2}/c^2*\ln(c*x^2+b*x+a)*b*f - \frac{1}{2}/c^4*\ln(c*x^2+b*x+a)*a^3*l + \frac{1}{2}/c^3*\ln(c*x^2+b*x+a)*a^2*j - \frac{1}{2}/c^2*\ln(c*x^2+b*x+a)*a*g - \frac{1}{2}/c^8*\ln(c*x^2+b*x+a)*b^7*m + \frac{1}{2}/c^7*\ln(c*x^2+b*x+a)*b^6*l - \frac{1}{6}/c^2*x^6*b*m + \frac{1}{c^7}*b^6*m*x - \frac{1}{4}/c^2*x^4*a*l - \frac{1}{4}/c^4*x^4*b^3*m + \frac{1}{5}/c^3*x^5*b^2*m - \frac{1}{5}/c^2*x^5*b*l - \frac{1}{5}/c^2*x^5*a*m + \frac{1}{c^5}*b^4*k*x - \frac{1}{c^4}*b^3*j*x + \frac{1}{c^3}*b^2*h*x - \frac{1}{c^2}*b*g*x + \frac{1}{2}/c^5*x^2*b^4*l - \frac{1}{2}/c^4*x^2*b^3*k - \frac{1}{2}/c^2*x^2*a*j - \frac{1}{2}/c^6*x^2*b^5*m - \frac{1}{3}/c^2*x^3*a*k + \frac{1}{3}/c^5*x^3*b^4*m - \frac{1}{3}/c^4*x^3*b^3*l + \frac{1}{3}/c^3*x^3*b^2*k - \frac{1}{3}/c^2*x^3*b*j + \frac{1}{2}/c^3*x^2*a^2*l - \frac{1}{4}/c^2*x^4*b*k + \frac{1}{3}/c^3*x^3*a^2*m + \frac{1}{4}/c^3*x^4*b^2*l + \frac{1}{2}/c^3*x^2*b^2*j - \frac{1}{2}/c^2*x^2*b*h - \frac{1}{c^4}*a^3*m*x + \frac{1}{c^3}*a^2*k*x - \frac{1}{c^2}*a*h*x - \frac{1}{c^6}*b^5*l*x - \frac{1}{2}/c^6*\ln(c*x^2+b*x+a)*b^5*k + \frac{1}{2}/c^5*\ln(c*x^2+b*x+a)*b^4*j - \frac{1}{2}/c^4*\ln(c*x^2+b*x+a)*b^3*h + \frac{1}{2}/c^3*\ln(c*x^2+b*x+a)*b^2*g + \frac{20}{c^6}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^4*m - \frac{14}{c^5}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3*l + \frac{9}{c^4}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2*k - \frac{16}{c^5}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b^2*m + \frac{7}{c^4}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*b*l - \frac{5}{c^3}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*j + \frac{7}{c^6}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^5*l - \frac{6}{c^5}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*k + \frac{2}{(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d + \frac{1}{2}/c*\ln(c*x^2+b*x+a)*e + \frac{1}{c}*f*x + \frac{1}{2}/c*x^2*g + \frac{1}{3}/c*x^3*h + \frac{1}{6}/c*x^6*l + \frac{1}{5}/c*x^5*k + \frac{1}{4}/c*x^4*j + \frac{3}{c^5}*l*\ln(c*x^2+b*x+a)*a^2*b^2*l - \frac{3}{2}/c^4*\ln(c*x^2+b*x+a)*a^2*b*k + \frac{3}{c^7}*l*\ln(c*x^2+b*x+a)*a*b^5*m - \frac{5}{2}/c^6*\ln(c*x^2+b*x+a)*a*b^4*l + \frac{2}{c^5}*l*\ln(c*x^2+b*x+a)*a*b^3*k - \frac{1}{c^7}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7*l + \frac{2}{c^5}*l*\ln(c*x^2+b*x+a)*a^3*b*m - \frac{5}{c^6}*l*\ln(c*x^2+b*x+a)*a^2*b^3*m - \frac{3}{2}/c^4*\ln(c*x^2+b*x+a)*a*b^2*j + \frac{1}{c^3}*l*\ln(c*x^2+b*x+a)*a*b*h + \frac{1}{c^2}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*f - \frac{1}{c}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e + \frac{1}{c^8}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^8*m - \frac{3}{c^4}*a*b^2*k*x + \frac{2}{c^3}*a*b*j*x + \frac{6}{c^5}*a^2*b^2*m*x - \frac{3}{c^4}*a^2*b*l*x + \frac{2}{c^4}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^4*m - \frac{2}{c}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f + \frac{1}{c^6}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6*k - \frac{1}{c^5}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*j + \frac{1}{c^4}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*h - \frac{1}{c^3}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*g + \frac{1}{c^3}*x^2*a*b*k - \frac{5}{c^6}*a*b^4*m*x + \frac{4}{c^5}*a*b^3*l*x - \frac{3}{2}/c^4*x^2*a^2*b*m + \frac{2}{c^5}*x^2*a*b^3*m - \frac{3}{2}/c^4*x^2*a*b^2*l - \frac{1}{c^4}*x^3*a*b^2*m + \frac{2}{3}/c^3*x^3*a*b*l + \frac{1}{2}/c^3*x^4*a*b*m + \frac{5}{c^4}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*j - \frac{4}{c^3}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*h + \frac{3}{c^2}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g - \frac{8}{c^7}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^6*m - \frac{2}{c^3}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*k + \frac{2}{c^2}/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*h$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((m\*x^8+l\*x^7+k\*x^6+j\*x^5+h\*x^4+g\*x^3+f\*x^2+e\*x+d)/(c\*x^2+b\*x+a), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see `assume?` for more details) Is 4\*a\*c-b^2 positive or negative?

**mupad [B]** time = 7.26, size = 2779, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x + f\*x^2 + g\*x^3 + h\*x^4 + j\*x^5 + k\*x^6 + l\*x^7 + m\*x^8)/(a + b\*x + c\*x^2), x)

[Out] 
$$\begin{aligned} & x^6 \left( \frac{1}{6c} - \frac{bm}{6c^2} \right) + x \left( \frac{f}{c} + \frac{b \left( \frac{a(j}{c} - \left( \frac{a}{c} - \frac{bm}{c^2} \right) \right)}{c} \right) / c + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} / c - \frac{g}{c} + \frac{b \left( \frac{h}{c} - \frac{b \left( \frac{j}{c} - \left( \frac{a}{c} - \frac{bm}{c^2} \right) \right)}{c} + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right)}{c} / c + \frac{a \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right) / c - \frac{a \left( \frac{h}{c} - \frac{b \left( \frac{j}{c} - \left( \frac{a}{c} - \frac{bm}{c^2} \right) \right)}{c} + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right)}{c} / c + \frac{a \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right) / c + x^4 \left( \frac{j}{4c} - \frac{a \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{4c} + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{4c} \right) - x^2 \left( \frac{a \left( \frac{j}{c} - \left( \frac{a}{c} - \frac{bm}{c^2} \right) \right)}{c} + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right) / (2c) - \frac{g}{2c} + \frac{b \left( \frac{h}{c} - \frac{b \left( \frac{j}{c} - \left( \frac{a}{c} - \frac{bm}{c^2} \right) \right)}{c} + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right)}{c} / (2c) + \frac{a \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right) / (2c) + x^3 \left( \frac{h}{3c} - \frac{b \left( \frac{j}{c} - \left( \frac{a}{c} - \frac{bm}{c^2} \right) \right)}{c} + \frac{b \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{c} \right) / (3c) + \frac{a \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{c} - \frac{k}{c} + \frac{am}{c^2} \right)}{3c} \right) - x^5 \left( \frac{b \left( \frac{1}{c} - \frac{bm}{c^2} \right)}{5c} - \frac{k}{5c} + \frac{am}{5c^2} \right) + \log \left( \frac{2c^9 x x \left( - \left( 2c^8 d + b^8 m + b^2 c^6 f + 2a^2 c^6 h - b^3 c^5 g + b^4 c^4 h - 2a^3 c^5 k - b^5 c^3 j + b^6 c^2 k + 2a^4 c^4 m - 2a^2 c^7 f - b^7 c^7 e - b^7 c^7 l + 9a^2 b^2 c^4 k - 14a^2 b^3 c^3 l + 20a^2 b^4 c^2 m - 16a^3 b^2 c^3 m + 3a^2 b^3 c^6 g - 8a^2 b^6 c^6 m - 4a^2 b^2 c^5 h + 5a^2 b^3 c^4 j - 5a^2 b^2 c^5 j - 6a^2 b^4 c^3 k + 7a^2 b^5 c^2 l + 7a^3 b^3 c^4 l \right)^2}{c^{16} (4ac - b^2)} \right)^{1/2} - b^8 m - 2c^8 d - b^2 c^6 f - 2a^2 c^6 h + b^3 c^5 g - b^4 c^4 h + 2a^3 c^5 k + b^5 c^3 j - b^6 c^2 k - 2a^4 c^4 m + 2a^2 c^7 f + b^7 c^7 e + b^7 c^7 l + b^7 c^8 \left( - \left( 2c^8 d + b^8 m + b^2 c^6 f + 2a^2 c^6 h - b^3 c^5 g + b^4 c^4 h - 2a^3 c^5 k - b^5 c^3 j + b^6 c^2 k + 2a^4 c^4 m - 2a^2 c^7 f - b^7 c^7 e - b^7 c^7 l + 9a^2 b^2 c^4 k - \right. \right. \end{aligned}$$

$$\begin{aligned}
& 14a^2b^3c^3 + 20a^2b^4c^2m - 16a^3b^2c^3m + 3ab^6c^6g - 8ab^6c^6m - 4ab^2c^5h + 5ab^3c^4j - 5a^2b^5c^5j - 6ab^4c^3k + 7 \\
& ab^5c^2 + 7a^3b^4c^4)^2 / (c^{16}(4ac - b^2))^{1/2} - 9a^2b^2c^4 \\
& k + 14a^2b^3c^3 + 20a^2b^4c^2m + 16a^3b^2c^3m - 3ab^6c^6g + \\
& 8ab^6c^6m + 4ab^2c^5h - 5ab^3c^4j + 5a^2b^5c^5j + 6ab^4c^3k - 7ab^5c^2 \\
& - 7a^3b^4c^4)(2c^8d + b^8m + 2c^9x * (- (2c^8d + \\
& b^8m + b^2c^6f + 2a^2c^6h - b^3c^5g + b^4c^4h - 2a^3c^5k - b^5 \\
& c^3j + b^6c^2k + 2a^4c^4m - 2ac^7f - bc^7e - b^7c^1 + 9a^2b^ \\
& 2c^4k - 14a^2b^3c^3 + 20a^2b^4c^2m - 16a^3b^2c^3m + 3ab^6c^6g - 8ab^6c^6m - 4ab^2c^5h + 5ab^3c^4j - 5a^2b^5c^5j - 6ab^4c^3k + 7ab^5c^2 \\
& + 7a^3b^4c^4)^2 / (c^{16}(4ac - b^2))^{1/2} + b^2 \\
& c^6f + 2a^2c^6h - b^3c^5g + b^4c^4h - 2a^3c^5k - b^5c^3j + b^ \\
& 6c^2k + 2a^4c^4m - 2ac^7f - bc^7e - b^7c^1 + bc^8 * (- (2c^8d + \\
& b^8m + b^2c^6f + 2a^2c^6h - b^3c^5g + b^4c^4h - 2a^3c^5k - b^5 \\
& c^3j + b^6c^2k + 2a^4c^4m - 2ac^7f - bc^7e - b^7c^1 + 9a^2b^ \\
& 2c^4k - 14a^2b^3c^3 + 20a^2b^4c^2m - 16a^3b^2c^3m + 3ab^6c^6g - 8ab^6c^6m - 4ab^2c^5h + 5ab^3c^4j - 5a^2b^5c^5j - 6ab^4c^3k + 7ab^5c^2 \\
& + 7a^3b^4c^4)^2 / (c^{16}(4ac - b^2))^{1/2} + 9a \\
& ^2b^2c^4k - 14a^2b^3c^3 + 20a^2b^4c^2m - 16a^3b^2c^3m + 3a \\
& ab^6c^6g - 8ab^6c^6m - 4ab^2c^5h + 5ab^3c^4j - 5a^2b^5c^5j - 6 \\
& ab^4c^3k + 7ab^5c^2 + 7a^3b^4c^4)(b^9m - b^2c^7e - 4a^2c^ \\
& 7g + b^3c^6f - b^4c^5g + b^5c^4h + 4a^3c^6j - b^6c^3j - 4a^4c \\
& ^5 + b^7c^2k + 4ac^8e - b^8c^1 - 13a^2b^2c^5j + 19a^2b^3c^4 \\
& k - 26a^2b^4c^3 + 25a^3b^2c^4 + 34a^2b^5c^2m - 44a^3b^3c^3 \\
& m - 4ab^6c^6g - 10ab^7c^6m + 5ab^2c^6g - 6ab^3c^5h + 8a^2b^6c^6h + 7ab^4c^4j - 8ab^5c^3k - 12a^3b^5c^5k + 9ab^6c^2 + 16 \\
& a^4b^4c^4m) / (2(4ac^9 - b^2c^8)) + (mx^7) / (7c) + (atan(b / (4ac - b^ \\
& 2))^{1/2} + (2cx) / (4ac - b^2)^{1/2}) * (2c^8d + b^8m + b^2c^6f + 2a^ \\
& 2c^6h - b^3c^5g + b^4c^4h - 2a^3c^5k - b^5c^3j + b^6c^2k + 2a \\
& ^4c^4m - 2ac^7f - bc^7e - b^7c^1 + 9a^2b^2c^4k - 14a^2b^3c^3 \\
& * 1 + 20a^2b^4c^2m - 16a^3b^2c^3m + 3ab^6c^6g - 8ab^6c^6m - 4ab^ \\
& 2c^5h + 5ab^3c^4j - 5a^2b^5c^5j - 6ab^4c^3k + 7ab^5c^2 + \\
& 7a^3b^4c^4) / (c^8(4ac - b^2)^{1/2})
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((m\*x\*\*8+1\*x\*\*7+k\*x\*\*6+j\*x\*\*5+h\*x\*\*4+g\*x\*\*3+f\*x\*\*2+e\*x+d)/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

$$3.352 \quad \int (1 + 4x - 7x^2)^3 (2 + 5x + x^2) \sqrt{3 + 2x + 5x^2} dx$$

Optimal. Leaf size=208

$$\frac{98060877(5x^2 + 2x + 3)^{3/2} x^2}{4375000} + \frac{1045360143(5x^2 + 2x + 3)^{3/2} x}{43750000} - \frac{1968340667(5x^2 + 2x + 3)^{3/2}}{131250000} - \frac{77159983(5x^2 + 2x + 3)^{3/2}}{31250000}$$

Rubi [A] time = 0.35, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{343}{50} (5x^2 + 2x + 3)^{3/2} x^7 - \frac{50519(5x^2 + 2x + 3)^{3/2} x^6}{2250} - \frac{190939(5x^2 + 2x + 3)^{3/2} x^5}{3000} - \frac{888751(5x^2 + 2x + 3)^{3/2} x^4}{105000} - \frac{90960857(5x^2 + 2x + 3)^{3/2} x^3}{1575000} - \frac{98060877(5x^2 + 2x + 3)^{3/2} x^2}{4375000} - \frac{1045360143(5x^2 + 2x + 3)^{3/2} x}{43750000} - \frac{1968340667(5x^2 + 2x + 3)^{3/2}}{131250000} - \frac{77159983(5x^2 + 2x + 3)^{3/2}}{31250000} - \frac{540119881 \operatorname{arcsinh}^{-1}\left(\frac{5x+1}{\sqrt{5x^2+2x+3}}\right)}{15625000\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-77159983\*(1 + 5\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/31250000 - (1968340667\*(3 + 2\*x + 5\*x^2)^(3/2))/131250000 + (1045360143\*x\*(3 + 2\*x + 5\*x^2)^(3/2))/43750000 + (98060877\*x^2\*(3 + 2\*x + 5\*x^2)^(3/2))/4375000 - (90960857\*x^3\*(3 + 2\*x + 5\*x^2)^(3/2))/1575000 - (888751\*x^4\*(3 + 2\*x + 5\*x^2)^(3/2))/105000 + (190939\*x^5\*(3 + 2\*x + 5\*x^2)^(3/2))/3000 - (50519\*x^6\*(3 + 2\*x + 5\*x^2)^(3/2))/2250 - (343\*x^7\*(3 + 2\*x + 5\*x^2)^(3/2))/50 - (540119881\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(15625000\*Sqrt[5])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps



$$\begin{aligned}
\int (1 + 4x - 7x^2)^3 (2 + 5x + x^2) \sqrt{3 + 2x + 5x^2} dx &= -\frac{343}{50} x^7 (3 + 2x + 5x^2)^{3/2} + \frac{1}{50} \int \sqrt{3 + 2x + 5x^2} (100 + \\
&= -\frac{50519x^6 (3 + 2x + 5x^2)^{3/2}}{2250} - \frac{343}{50} x^7 (3 + 2x + 5x^2)^{3/2} + \\
&= \frac{190939x^5 (3 + 2x + 5x^2)^{3/2}}{3000} - \frac{50519x^6 (3 + 2x + 5x^2)^{3/2}}{2250} \\
&= -\frac{888751x^4 (3 + 2x + 5x^2)^{3/2}}{105000} + \frac{190939x^5 (3 + 2x + 5x^2)^{3/2}}{3000} \\
&= -\frac{90960857x^3 (3 + 2x + 5x^2)^{3/2}}{1575000} - \frac{888751x^4 (3 + 2x + 5x^2)^{3/2}}{105000} \\
&= \frac{98060877x^2 (3 + 2x + 5x^2)^{3/2}}{4375000} - \frac{90960857x^3 (3 + 2x + 5x^2)^{3/2}}{1575000} \\
&= \frac{1045360143x (3 + 2x + 5x^2)^{3/2}}{43750000} + \frac{98060877x^2 (3 + 2x + 5x^2)^{3/2}}{4375000} \\
&= -\frac{1968340667 (3 + 2x + 5x^2)^{3/2}}{131250000} + \frac{1045360143x (3 + 2x + 5x^2)^{3/2}}{43750000} \\
&= -\frac{77159983(1 + 5x)\sqrt{3 + 2x + 5x^2}}{31250000} - \frac{1968340667 (3 + 2x + 5x^2)^{3/2}}{131250000} \\
&= -\frac{77159983(1 + 5x)\sqrt{3 + 2x + 5x^2}}{31250000} - \frac{1968340667 (3 + 2x + 5x^2)^{3/2}}{131250000} \\
&= -\frac{77159983(1 + 5x)\sqrt{3 + 2x + 5x^2}}{31250000} - \frac{1968340667 (3 + 2x + 5x^2)^{3/2}}{131250000}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 85, normalized size = 0.41

$$\frac{-5\sqrt{5x^2 + 2x + 3} (67528125000x^9 + 248031875000x^8 - 497593468750x^7 - 34674656250x^6 + 225922362500x^5 + 56757413000x^4 + 17642392275x^3 - 78839046795x^2 - 57768004650x + 93436408944) - 68055105006\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{5}}\right)}{9843750000}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-5\*Sqrt[3 + 2\*x + 5\*x^2]\*(93436408944 - 57768004650\*x - 78839046795\*x^2 + 17642392275\*x^3 + 56757413000\*x^4 + 225922362500\*x^5 - 34674656250\*x^6 - 49

$7593468750*x^7 + 248031875000*x^8 + 67528125000*x^9) - 68055105006*\text{Sqrt}[5]*\text{ArcSinh}[(1 + 5*x)/\text{Sqrt}[14]]/9843750000$

**IntegrateAlgebraic [A]** time = 1.34, size = 99, normalized size = 0.48

$$\frac{540119881 \log(\sqrt{5}\sqrt{5x^2+2x+3}-5x-1)}{15625000\sqrt{5}} + \frac{\sqrt{5x^2+2x+3}(-67528125000x^9 - 248031875000x^8 + 497593468750x^7 + 34674656250x^6 - 225922362500x^5 - 56757413000x^4 - 17642392275x^3 + 78839046795x^2 + 57768004650x - 93436408944)}{1968750000}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-93436408944 + 57768004650\*x + 78839046795\*x^2 - 17642392275\*x^3 - 56757413000\*x^4 - 225922362500\*x^5 + 34674656250\*x^6 + 497593468750\*x^7 - 248031875000\*x^8 - 67528125000\*x^9))/1968750000 + (540119881\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(15625000\*Sqrt[5])

**fricas [A]** time = 0.76, size = 97, normalized size = 0.47

$$\frac{1}{1968750000} (67528125000x^9 + 248031875000x^8 - 497593468750x^7 - 34674656250x^6 + 225922362500x^5 + 56757413000x^4 + 17642392275x^3 - 78839046795x^2 - 57768004650x + 93436408944)\sqrt{5x^2+2x+3} + \frac{540119881}{156250000} \sqrt{5} \log(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1) - 25x^2 - 10x - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2), x, algorithm="fricas")

[Out] -1/1968750000\*(67528125000\*x^9 + 248031875000\*x^8 - 497593468750\*x^7 - 34674656250\*x^6 + 225922362500\*x^5 + 56757413000\*x^4 + 17642392275\*x^3 - 78839046795\*x^2 - 57768004650\*x + 93436408944)\*sqrt(5\*x^2 + 2\*x + 3) + 540119881/156250000\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.21, size = 92, normalized size = 0.44

$$\frac{1}{1968750000} (5(10(25(5(49(140(315x+1157)x-324959)x-1109589)x+36147578)x+227029652)x+705695691)x-15767809359)x-11553600930)x+93436408944)\sqrt{5x^2+2x+3} + \frac{540119881}{78125000} \sqrt{5} \log(-\sqrt{5}(\sqrt{5x^2+2x+3})-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2), x, algorithm="giac")

[Out] -1/1968750000\*(5\*((5\*(10\*(25\*(5\*(49\*(140\*(315\*x + 1157)\*x - 324959)\*x - 1109589)\*x + 36147578)\*x + 227029652)\*x + 705695691)\*x - 15767809359)\*x - 11553600930)\*x + 93436408944)\*sqrt(5\*x^2 + 2\*x + 3) + 540119881/78125000\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)

**maple [A]** time = 0.03, size = 166, normalized size = 0.80

$$\frac{343(5x^2+2x+3)^{\frac{3}{2}}x^7}{50} - \frac{50519(5x^2+2x+3)^{\frac{3}{2}}x^6}{2250} + \frac{190939(5x^2+2x+3)^{\frac{3}{2}}x^5}{3000} - \frac{888751(5x^2+2x+3)^{\frac{3}{2}}x^4}{105000} - \frac{90960857(5x^2+2x+3)^{\frac{3}{2}}x^3}{1575000} + \frac{98060877(5x^2+2x+3)^{\frac{3}{2}}x^2}{4375000} + \frac{1045360143(5x^2+2x+3)^{\frac{3}{2}}x}{43750000} - \frac{540119881\sqrt{5}\operatorname{arcsinh}\left(\frac{\sqrt{5x^2+2x+3}}{14}\right)}{78125000} - \frac{1968340667(5x^2+2x+3)^{\frac{3}{2}}}{131250000} - \frac{77159983(10x+2)\sqrt{5x^2+2x+3}}{62500000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^{(1/2)}, x)$

[Out]  $-1968340667/131250000*(5*x^2+2*x+3)^{(3/2)}-343/50*x^7*(5*x^2+2*x+3)^{(3/2)}-50519/2250*x^6*(5*x^2+2*x+3)^{(3/2)}+190939/3000*x^5*(5*x^2+2*x+3)^{(3/2)}-888751/105000*x^4*(5*x^2+2*x+3)^{(3/2)}-90960857/1575000*x^3*(5*x^2+2*x+3)^{(3/2)}+98060877/4375000*x^2*(5*x^2+2*x+3)^{(3/2)}+1045360143/43750000*x*(5*x^2+2*x+3)^{(3/2)}-540119881/78125000*5^{(1/2)}*\text{arcsinh}(5/14*14^{(1/2)}*(x+1/5))-77159983/62500000*(10*x+2)*(5*x^2+2*x+3)^{(1/2)}$

**maxima [A]** time = 0.99, size = 177, normalized size = 0.85

$\frac{343}{50}(5x^2+2x+3)^{3/2} - \frac{50519}{2250}(5x^2+2x+3)^{3/2}x^6 + \frac{190939}{3000}(5x^2+2x+3)^{3/2}x^5 - \frac{888751}{105000}(5x^2+2x+3)^{3/2}x^4 - \frac{90960857}{1575000}(5x^2+2x+3)^{3/2}x^3 + \frac{98060877}{4375000}(5x^2+2x+3)^{3/2}x^2 + \frac{1045360143}{43750000}(5x^2+2x+3)^{3/2}x - \frac{1968340667}{131250000}(5x^2+2x+3)^{3/2} - \frac{77159983}{62500000}\sqrt{5x^2+2x+3} - \frac{540119881}{78125000}\sqrt{5}\text{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{77159983}{31250000}\sqrt{5x^2+2x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out]  $-343/50*(5*x^2 + 2*x + 3)^{(3/2)}*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^{(3/2)}*x^6 + 190939/3000*(5*x^2 + 2*x + 3)^{(3/2)}*x^5 - 888751/105000*(5*x^2 + 2*x + 3)^{(3/2)}*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^{(3/2)}*x^3 + 98060877/4375000*(5*x^2 + 2*x + 3)^{(3/2)}*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)^{(3/2)}*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^{(3/2)} - 77159983/6250000*\text{sqrt}(5*x^2 + 2*x + 3)*x - 540119881/78125000*\text{sqrt}(5)*\text{arcsinh}(1/14*\text{sqrt}(14)*(5*x + 1)) - 77159983/31250000*\text{sqrt}(5*x^2 + 2*x + 3)$

**mupad [B]** time = 6.31, size = 221, normalized size = 1.06

$\frac{98060877}{4375000}(5x^2+2x+3)^{3/2} - \frac{90960857}{1575000}(5x^2+2x+3)^{3/2}x^6 + \frac{888751}{105000}(5x^2+2x+3)^{3/2}x^5 - \frac{190939}{3000}(5x^2+2x+3)^{3/2}x^4 - \frac{50519}{2250}(5x^2+2x+3)^{3/2}x^3 + \frac{343}{50}(5x^2+2x+3)^{3/2}x^2 + \frac{3048580429}{156250000}\sqrt{5}\ln\left(\sqrt{5x^2+2x+3} + \frac{\sqrt{5x^2+2x+3}}{5}\right) - \frac{3048580429}{43750000}\sqrt{5}\ln\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{1968340667}{525000000}\sqrt{5x^2+2x+3} - \frac{1045360143}{43750000}(5x^2+2x+3)^{3/2}x - \frac{1968340667}{131250000}(5x^2+2x+3)^{3/2} - \frac{77159983}{31250000}\sqrt{5x^2+2x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^{(1/2)}*(4*x - 7*x^2 + 1)^3, x)$

[Out]  $(98060877*x^2*(2*x + 5*x^2 + 3)^{(3/2)})/4375000 - (90960857*x^3*(2*x + 5*x^2 + 3)^{(3/2)})/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^{(3/2)})/105000 + (190939*x^5*(2*x + 5*x^2 + 3)^{(3/2)})/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^{(3/2)})/2250 - (343*x^7*(2*x + 5*x^2 + 3)^{(3/2)})/50 - (3048580429*5^{(1/2)}*\log((2*x + 5*x^2 + 3)^{(1/2)} + (5^{(1/2)}*(5*x + 1))/5))/156250000 - (3048580429*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^{(1/2)})/43750000 - (1968340667*(2*x + 5*x^2 + 3)^{(1/2)}*(20*x + 200*x^2 + 108))/525000000 + (1045360143*x*(2*x + 5*x^2 + 3)^{(3/2)})/43750000 + (1968340667*5^{(1/2)}*\log(2*(2*x + 5*x^2 + 3)^{(1/2)} + (5^{(1/2)}*(10*x + 2))/5))/156250000$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$-\int(-29x\sqrt{5x^2+2x+3})dx - \int(-115x^2\sqrt{5x^2+2x+3})dx - \int 61x^3\sqrt{5x^2+2x+3}dx - \int 871x^4\sqrt{5x^2+2x+3}dx - \int(-127x^5\sqrt{5x^2+2x+3})dx - \int(-2065x^6\sqrt{5x^2+2x+3})dx - \int 1127x^7\sqrt{5x^2+2x+3}dx - \int 343x^8\sqrt{5x^2+2x+3}dx - \int(-2\sqrt{5x^2+2x+3})dx$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2*sqrt(5*x**2  
+ 2*x + 3), x) - Integral(61*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(87  
1*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5*sqrt(5*x**2 + 2*x +  
3), x) - Integral(-2065*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**  
7*sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8*sqrt(5*x**2 + 2*x + 3), x)  
- Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

$$3.353 \quad \int (1 + 4x - 7x^2)^2 (2 + 5x + x^2) \sqrt{3 + 2x + 5x^2} dx$$

Optimal. Leaf size=166

$$\frac{77509(5x^2 + 2x + 3)^{3/2} x^2}{25000} + \frac{1781669(5x^2 + 2x + 3)^{3/2} x}{250000} + \frac{198439(5x^2 + 2x + 3)^{3/2}}{750000} - \frac{2521723(5x + 1)\sqrt{5x^2 + 3}}{1250000}$$

**Rubi [A]** time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{40}(5x^2 + 2x + 3)^{3/2} x^5 + \frac{989}{200}(5x^2 + 2x + 3)^{3/2} x^4 - \frac{25277(5x^2 + 2x + 3)^{3/2} x^3}{3000} - \frac{77509(5x^2 + 2x + 3)^{3/2} x^2}{25000} + \frac{1781669(5x^2 + 2x + 3)^{3/2} x}{250000} + \frac{198439(5x^2 + 2x + 3)^{3/2}}{750000} - \frac{2521723(5x + 1)\sqrt{5x^2 + 3}}{1250000} - \frac{17652061 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-2521723\*(1 + 5\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/1250000 + (198439\*(3 + 2\*x + 5\*x^2)^(3/2))/750000 + (1781669\*x\*(3 + 2\*x + 5\*x^2)^(3/2))/250000 - (77509\*x^2\*(3 + 2\*x + 5\*x^2)^(3/2))/25000 - (25277\*x^3\*(3 + 2\*x + 5\*x^2)^(3/2))/3000 + (989\*x^4\*(3 + 2\*x + 5\*x^2)^(3/2))/200 + (49\*x^5\*(3 + 2\*x + 5\*x^2)^(3/2))/40 - (17652061\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(625000\*Sqrt[5])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x]$   
 $\&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[p, -1]$

### Rule 1661

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x\_Symbol] := \text{With}[\{q =$   
 $\text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e*x^(q - 1)*(a + b*x +$   
 $c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + \text{Dist}[1/(c*(q + 2*p + 1)), \text{Int}[(a +$   
 $b*x + c*x^2)^p*\text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*$   
 $e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; \text{FreeQ}[\{a, b, c,$   
 $p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int (1 + 4x - 7x^2)^2 (2 + 5x + x^2) \sqrt{3 + 2x + 5x^2} dx &= \frac{49}{40}x^5 (3 + 2x + 5x^2)^{3/2} + \frac{1}{40} \int \sqrt{3 + 2x + 5x^2} (80 + 840x \\ &= \frac{989}{200}x^4 (3 + 2x + 5x^2)^{3/2} + \frac{49}{40}x^5 (3 + 2x + 5x^2)^{3/2} + \frac{\int \sqrt{3}}{4} \\ &= -\frac{25277x^3 (3 + 2x + 5x^2)^{3/2}}{3000} + \frac{989}{200}x^4 (3 + 2x + 5x^2)^{3/2} + \frac{4}{4} \\ &= -\frac{77509x^2 (3 + 2x + 5x^2)^{3/2}}{25000} - \frac{25277x^3 (3 + 2x + 5x^2)^{3/2}}{3000} + \\ &= \frac{1781669x (3 + 2x + 5x^2)^{3/2}}{250000} - \frac{77509x^2 (3 + 2x + 5x^2)^{3/2}}{25000} + \\ &= \frac{198439 (3 + 2x + 5x^2)^{3/2}}{750000} + \frac{1781669x (3 + 2x + 5x^2)^{3/2}}{250000} - \\ &= -\frac{2521723(1 + 5x)\sqrt{3 + 2x + 5x^2}}{1250000} + \frac{198439 (3 + 2x + 5x^2)}{750000} \\ &= -\frac{2521723(1 + 5x)\sqrt{3 + 2x + 5x^2}}{1250000} + \frac{198439 (3 + 2x + 5x^2)}{750000} \\ &= -\frac{2521723(1 + 5x)\sqrt{3 + 2x + 5x^2}}{1250000} + \frac{198439 (3 + 2x + 5x^2)}{750000} \end{aligned}$$

**Mathematica [A]** time = 0.19, size = 75, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3} (22968750x^7 + 101906250x^6 - 107112500x^5 - 65693000x^4 + 15583725x^3 + 23531995x^2 + 44333650x - 4588584) - 105912366\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{18750000}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (5\*Sqrt[3 + 2\*x + 5\*x^2]\*(-4588584 + 44333650\*x + 23531995\*x^2 + 15583725\*x^3 - 65693000\*x^4 - 107112500\*x^5 + 101906250\*x^6 + 22968750\*x^7) - 105912366\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/18750000

**IntegrateAlgebraic [A]** time = 0.87, size = 89, normalized size = 0.54

$$\frac{17652061 \log(\sqrt{5}\sqrt{5x^2+2x+3} - 5x - 1)}{625000\sqrt{5}} + \frac{\sqrt{5x^2+2x+3} (22968750x^7 + 101906250x^6 - 107112500x^5 - 65693000x^4 + 15583725x^3 + 23531995x^2 + 44333650x - 4588584)}{3750000}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-4588584 + 44333650\*x + 23531995\*x^2 + 15583725\*x^3 - 65693000\*x^4 - 107112500\*x^5 + 101906250\*x^6 + 22968750\*x^7))/3750000 + (17652061\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(625000\*Sqrt[5])

**fricas [A]** time = 0.82, size = 87, normalized size = 0.52

$$\frac{1}{3750000} (22968750x^7 + 101906250x^6 - 107112500x^5 - 65693000x^4 + 15583725x^3 + 23531995x^2 + 44333650x - 4588584)\sqrt{5x^2+2x+3} + \frac{17652061}{6250000}\sqrt{5} \log(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1) - 25x^2 - 10x - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2), x, algorithm="fricas")

[Out] 1/3750000\*(22968750\*x^7 + 101906250\*x^6 - 107112500\*x^5 - 65693000\*x^4 + 15583725\*x^3 + 23531995\*x^2 + 44333650\*x - 4588584)\*sqrt(5\*x^2 + 2\*x + 3) + 17652061/6250000\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.20, size = 82, normalized size = 0.49

$$\frac{1}{3750000} (5((5(10(25(15(245x+1087)x-17138)x-262772)x+623349)x+4706399)x+8866730)x-4588584)\sqrt{5x^2+2x+3} + \frac{17652061}{3125000}\sqrt{5} \log(-\sqrt{5}(\sqrt{5x-\sqrt{5x^2+2x+3}}-1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2), x, algorithm="giac")

[Out]  $\frac{1}{31250000} * (5 * ((5 * (10 * (25 * (15 * (245 * x + 1087) * x - 17138) * x - 262772) * x + 623349) * x + 4706399) * x + 8866730) * x - 4588584) * \sqrt{5 * x^2 + 2 * x + 3} + 17652061 / 3125000 * \sqrt{5} * \log(-\sqrt{5} * (\sqrt{5} * x - \sqrt{5 * x^2 + 2 * x + 3})) - 1)$

**maple [A]** time = 0.01, size = 132, normalized size = 0.80

$$\frac{49(5x^2+2x+3)^{\frac{3}{2}}x^5}{40} + \frac{989(5x^2+2x+3)^{\frac{3}{2}}x^4}{200} - \frac{25277(5x^2+2x+3)^{\frac{3}{2}}x^3}{3000} - \frac{77509(5x^2+2x+3)^{\frac{3}{2}}x^2}{25000} + \frac{1781669(5x^2+2x+3)^{\frac{3}{2}}x}{250000} - \frac{17652061\sqrt{5}\operatorname{arcsinh}\left(\frac{5\sqrt{14}(x+1)}{14}\right)}{3125000} + \frac{198439(5x^2+2x+3)^{\frac{3}{2}}}{750000} - \frac{2521723(10x+2)\sqrt{5x^2+2x+3}}{2500000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((-7 * x^2 + 4 * x + 1)^2 * (x^2 + 5 * x + 2) * (5 * x^2 + 2 * x + 3)^{(1/2)}, x)$

[Out]  $\frac{198439}{750000} * (5 * x^2 + 2 * x + 3)^{(3/2)} + \frac{49}{40} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^5 + \frac{989}{200} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^4 - \frac{25277}{3000} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^3 - \frac{77509}{25000} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^2 + \frac{1781669}{250000} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x - \frac{17652061}{3125000} * 5^{(1/2)} * \operatorname{arcsinh}\left(\frac{5}{14} * \sqrt{14} * (x + 1/5)\right) - \frac{2521723}{2500000} * (10 * x + 2) * (5 * x^2 + 2 * x + 3)^{(1/2)}$

**maxima [A]** time = 0.97, size = 143, normalized size = 0.86

$$\frac{49}{40} (5x^2+2x+3)^{\frac{3}{2}}x^5 + \frac{989}{200} (5x^2+2x+3)^{\frac{3}{2}}x^4 - \frac{25277}{3000} (5x^2+2x+3)^{\frac{3}{2}}x^3 - \frac{77509}{25000} (5x^2+2x+3)^{\frac{3}{2}}x^2 + \frac{1781669}{250000} (5x^2+2x+3)^{\frac{3}{2}}x + \frac{198439}{750000} (5x^2+2x+3)^{\frac{3}{2}} - \frac{2521723}{250000} \sqrt{5x^2+2x+3} - \frac{17652061}{3125000} \sqrt{5} \operatorname{arsinh}\left(\frac{1}{14} \sqrt{14} (5x+1)\right) - \frac{2521723}{1250000} \sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((-7 * x^2 + 4 * x + 1)^2 * (x^2 + 5 * x + 2) * (5 * x^2 + 2 * x + 3)^{(1/2)}, x, \operatorname{algorithm}="maxima")$

[Out]  $\frac{49}{40} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^5 + \frac{989}{200} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^4 - \frac{25277}{3000} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^3 - \frac{77509}{25000} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x^2 + \frac{1781669}{250000} * (5 * x^2 + 2 * x + 3)^{(3/2)} * x + \frac{198439}{750000} * (5 * x^2 + 2 * x + 3)^{(3/2)} - \frac{2521723}{250000} * \sqrt{5 * x^2 + 2 * x + 3} * x - \frac{17652061}{3125000} * \sqrt{5} * \operatorname{arcsinh}\left(\frac{1}{14} * \sqrt{14} * (5 * x + 1)\right) - \frac{2521723}{1250000} * \sqrt{5 * x^2 + 2 * x + 3}$

**mupad [B]** time = 6.01, size = 187, normalized size = 1.13

$$\frac{989x^4(5x^2+2x+3)^{\frac{3}{2}}}{200} - \frac{25277x^3(5x^2+2x+3)^{\frac{3}{2}}}{3000} - \frac{77509x^2(5x^2+2x+3)^{\frac{3}{2}}}{25000} + \frac{49x^5(5x^2+2x+3)^{\frac{3}{2}}}{40} - \frac{33915049\sqrt{5}\ln\left(\frac{\sqrt{5x^2+2x+3} + \sqrt{5(5x+1)}}{9}\right)}{6250000} - \frac{4845007\left(\frac{x}{2} + \frac{1}{10}\right)\sqrt{5x^2+2x+3}}{250000} + \frac{198439\sqrt{5x^2+2x+3}(200x^2+20x+108)}{3000000} - \frac{1781669x(5x^2+2x+3)^{\frac{3}{2}}}{250000} - \frac{1389073\sqrt{5}\ln\left(2\sqrt{5x^2+2x+3} + \frac{\sqrt{5(10x+2)}}{9}\right)}{6250000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((5 * x + x^2 + 2) * (2 * x + 5 * x^2 + 3)^{(1/2)} * (4 * x - 7 * x^2 + 1)^2, x)$

[Out]  $\frac{989 * x^4 * (2 * x + 5 * x^2 + 3)^{(3/2)}}{200} - \frac{(25277 * x^3 * (2 * x + 5 * x^2 + 3)^{(3/2)})}{3000} - \frac{(77509 * x^2 * (2 * x + 5 * x^2 + 3)^{(3/2)})}{25000} + \frac{(49 * x^5 * (2 * x + 5 * x^2 + 3)^{(3/2)})}{40} - \frac{(33915049 * 5^{(1/2)} * \log((2 * x + 5 * x^2 + 3)^{(1/2)} + (5^{(1/2)} * (5 * x + 1)) / 5))}{6250000} - \frac{(4845007 * (x / 2 + 1 / 10) * (2 * x + 5 * x^2 + 3)^{(1/2)})}{250000} + \frac{(198439 * (2 * x + 5 * x^2 + 3)^{(1/2)} * (20 * x + 200 * x^2 + 108))}{30000000} + (1781$



669\*x\*(2\*x + 5\*x^2 + 3)^(3/2)/250000 - (1389073\*5^(1/2)\*log(2\*(2\*x + 5\*x^2 + 3)^(1/2) + (5^(1/2)\*(10\*x + 2))/5))/6250000

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + 5x + 2) \sqrt{5x^2 + 2x + 3} (7x^2 - 4x - 1)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x\*\*2+4\*x+1)\*\*2\*(x\*\*2+5\*x+2)\*(5\*x\*\*2+2\*x+3)\*\*(1/2),x)

[Out] Integral((x\*\*2 + 5\*x + 2)\*sqrt(5\*x\*\*2 + 2\*x + 3)\*(7\*x\*\*2 - 4\*x - 1)\*\*2, x)

$$3.354 \quad \int (1 + 4x - 7x^2) (2 + 5x + x^2) \sqrt{3 + 2x + 5x^2} dx$$

Optimal. Leaf size=124

$$-\frac{289}{250} (5x^2 + 2x + 3)^{3/2} x^2 + \frac{2149 (5x^2 + 2x + 3)^{3/2} x}{2500} + \frac{7819 (5x^2 + 2x + 3)^{3/2}}{7500} - \frac{4633(5x + 1)\sqrt{5x^2 + 2x + 3}}{12500} - \frac{7}{30}$$

**Rubi [A]** time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{30} (5x^2 + 2x + 3)^{3/2} x^3 - \frac{289}{250} (5x^2 + 2x + 3)^{3/2} x^2 + \frac{2149 (5x^2 + 2x + 3)^{3/2} x}{2500} + \frac{7819 (5x^2 + 2x + 3)^{3/2}}{7500} - \frac{4633(5x + 1)\sqrt{5x^2 + 2x + 3}}{12500} - \frac{32431 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{6250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-4633\*(1 + 5\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/12500 + (7819\*(3 + 2\*x + 5\*x^2)^(3/2))/7500 + (2149\*x\*(3 + 2\*x + 5\*x^2)^(3/2))/2500 - (289\*x^2\*(3 + 2\*x + 5\*x^2)^(3/2))/250 - (7\*x^3\*(3 + 2\*x + 5\*x^2)^(3/2))/30 - (32431\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(6250\*Sqrt[5])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$*e)/(2*c), \text{Int}[(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\}$   
 $\&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[p, -1]$

### Rule 1661

$\text{Int}[(Pq_)*(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_)}, x\_Symbol] :> \text{With}\{q = \text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(e*x^{(q-1)}*(a + b*x + c*x^2)^{(p+1)})/(c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a + b*x + c*x^2)^p * \text{ExpandToSum}[c*(q+2*p+1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int (1 + 4x - 7x^2)(2 + 5x + x^2) \sqrt{3 + 2x + 5x^2} dx &= -\frac{7}{30}x^3(3 + 2x + 5x^2)^{3/2} + \frac{1}{30} \int \sqrt{3 + 2x + 5x^2} (60 + 390x - 289x^2) dx \\ &= -\frac{289}{250}x^2(3 + 2x + 5x^2)^{3/2} - \frac{7}{30}x^3(3 + 2x + 5x^2)^{3/2} + \frac{1}{750} \int (2149x - 289x^2) \sqrt{3 + 2x + 5x^2} dx \\ &= \frac{2149x(3 + 2x + 5x^2)^{3/2}}{2500} - \frac{289}{250}x^2(3 + 2x + 5x^2)^{3/2} - \frac{7}{30}x^3(3 + 2x + 5x^2)^{3/2} \\ &= \frac{7819(3 + 2x + 5x^2)^{3/2}}{7500} + \frac{2149x(3 + 2x + 5x^2)^{3/2}}{2500} - \frac{289}{250}x^3(3 + 2x + 5x^2)^{3/2} \\ &= -\frac{4633(1 + 5x)\sqrt{3 + 2x + 5x^2}}{12500} + \frac{7819(3 + 2x + 5x^2)^{3/2}}{7500} \\ &= -\frac{4633(1 + 5x)\sqrt{3 + 2x + 5x^2}}{12500} + \frac{7819(3 + 2x + 5x^2)^{3/2}}{7500} \\ &= -\frac{4633(1 + 5x)\sqrt{3 + 2x + 5x^2}}{12500} + \frac{7819(3 + 2x + 5x^2)^{3/2}}{7500} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 65, normalized size = 0.52

$$\frac{5\sqrt{5x^2 + 2x + 3} \left(-43750x^5 - 234250x^4 + 48225x^3 + 129895x^2 + 105400x + 103386\right) - 194586\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{187500}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (5\*Sqrt[3 + 2\*x + 5\*x^2]\*(103386 + 105400\*x + 129895\*x^2 + 48225\*x^3 - 234250\*x^4 - 43750\*x^5) - 194586\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/187500

**IntegrateAlgebraic [A]** time = 0.67, size = 79, normalized size = 0.64

$$\frac{32431 \log\left(\sqrt{5} \sqrt{5x^2 + 2x + 3} - 5x - 1\right)}{6250\sqrt{5}} + \frac{\sqrt{5x^2 + 2x + 3}(-43750x^5 - 234250x^4 + 48225x^3 + 129895x^2 + 105400x + 103386)}{37500}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(103386 + 105400\*x + 129895\*x^2 + 48225\*x^3 - 234250\*x^4 - 43750\*x^5))/37500 + (32431\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(6250\*Sqrt[5])

**fricas [A]** time = 0.65, size = 77, normalized size = 0.62

$$-\frac{1}{37500}(43750x^5 + 234250x^4 - 48225x^3 - 129895x^2 - 105400x - 103386)\sqrt{5x^2 + 2x + 3} + \frac{32431}{62500}\sqrt{5} \log\left(\sqrt{5} \sqrt{5x^2 + 2x + 3}(5x + 1) - 25x^2 - 10x - 8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2), x, algorithm="fricas")

[Out] -1/37500\*(43750\*x^5 + 234250\*x^4 - 48225\*x^3 - 129895\*x^2 - 105400\*x - 103386)\*sqrt(5\*x^2 + 2\*x + 3) + 32431/62500\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.19, size = 72, normalized size = 0.58

$$-\frac{1}{37500}(5((5(10(175x + 937)x - 1929)x - 25979)x - 21080)x - 103386)\sqrt{5x^2 + 2x + 3} + \frac{32431}{31250}\sqrt{5} \log\left(-\sqrt{5}\left(\sqrt{5x} - \sqrt{5x^2 + 2x + 3}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2), x, algorithm="giac")

[Out] -1/37500\*(5\*((5\*(10\*(175\*x + 937)\*x - 1929)\*x - 25979)\*x - 21080)\*x - 103386)\*sqrt(5\*x^2 + 2\*x + 3) + 32431/31250\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)

**maple [A]** time = 0.01, size = 98, normalized size = 0.79

$$-\frac{7(5x^2 + 2x + 3)^{\frac{3}{2}}x^3}{30} - \frac{289(5x^2 + 2x + 3)^{\frac{3}{2}}x^2}{250} + \frac{2149(5x^2 + 2x + 3)^{\frac{3}{2}}x}{2500} - \frac{32431\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x + \frac{1}{5}\right)}{14}\right)}{31250} + \frac{7819(5x^2 + 2x + 3)^{\frac{3}{2}}}{7500} - \frac{4633(10x + 2)\sqrt{5x^2 + 2x + 3}}{25000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^{(1/2)}, x)$

[Out]  $-7/30*(5*x^2+2*x+3)^{(3/2)}*x^3-289/250*(5*x^2+2*x+3)^{(3/2)}*x^2+2149/2500*(5*x^2+2*x+3)^{(3/2)}*x+7819/7500*(5*x^2+2*x+3)^{(3/2)}-4633/25000*(10*x+2)*(5*x^2+2*x+3)^{(1/2)}-32431/31250*5^{(1/2)}*\text{arcsinh}(5/14*14^{(1/2)}*(x+1/5))$

**maxima** [A] time = 0.96, size = 109, normalized size = 0.88

$-\frac{7}{30}(5x^2+2x+3)^{\frac{3}{2}}x^3 - \frac{289}{250}(5x^2+2x+3)^{\frac{3}{2}}x^2 + \frac{2149}{2500}(5x^2+2x+3)^{\frac{3}{2}}x + \frac{7819}{7500}(5x^2+2x+3)^{\frac{3}{2}} - \frac{4633}{2500}\sqrt{5x^2+2x+3} - \frac{32431}{31250}\sqrt{5}\text{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{4633}{12500}\sqrt{5x^2+2x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $-7/30*(5*x^2 + 2*x + 3)^{(3/2)}*x^3 - 289/250*(5*x^2 + 2*x + 3)^{(3/2)}*x^2 + 2149/2500*(5*x^2 + 2*x + 3)^{(3/2)}*x + 7819/7500*(5*x^2 + 2*x + 3)^{(3/2)} - 4633/2500*\text{sqrt}(5*x^2 + 2*x + 3)*x - 32431/31250*\text{sqrt}(5)*\text{arcsinh}(1/14*\text{sqrt}(14)*(5*x + 1)) - 4633/12500*\text{sqrt}(5*x^2 + 2*x + 3)$

**mupad** [B] time = 5.38, size = 153, normalized size = 1.23

$\frac{7819\sqrt{5x^2+2x+3}(200x^2+20x+108)}{300000} - \frac{7x^3(5x^2+2x+3)^{3/2}}{30} - \frac{10129\sqrt{5}\ln(\sqrt{5x^2+2x+3} + \frac{\sqrt{5(5x+1)}}{5})}{62500} - \frac{1447(\frac{x}{2} + \frac{1}{10})\sqrt{5x^2+2x+3}}{2500} - \frac{289x^2(5x^2+2x+3)^{3/2}}{250} + \frac{2149x(5x^2+2x+3)^{3/2}}{2500} - \frac{54733\sqrt{5}\ln(2\sqrt{5x^2+2x+3} + \frac{\sqrt{5(10x+2)}}{5})}{62500}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^{(1/2)}*(4*x - 7*x^2 + 1), x)$

[Out]  $(7819*(2*x + 5*x^2 + 3)^{(1/2)}*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x + 5*x^2 + 3)^{(3/2)})/30 - (10129*5^{(1/2)}*\log((2*x + 5*x^2 + 3)^{(1/2)} + (5^{(1/2)}*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^{(1/2)})/2500 - (289*x^2*(2*x + 5*x^2 + 3)^{(3/2)})/250 + (2149*x*(2*x + 5*x^2 + 3)^{(3/2)})/2500 - (54733*5^{(1/2)}*\log(2*(2*x + 5*x^2 + 3)^{(1/2)} + (5^{(1/2)}*(10*x + 2))/5))/62500$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$-\int(-13x\sqrt{5x^2+2x+3})dx - \int(-7x^2\sqrt{5x^2+2x+3})dx - \int 31x^3\sqrt{5x^2+2x+3}dx - \int 7x^4\sqrt{5x^2+2x+3}dx - \int(-2\sqrt{5x^2+2x+3})dx$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2), x)$

[Out]  $-\text{Integral}(-13*x*\text{sqrt}(5*x**2 + 2*x + 3), x) - \text{Integral}(-7*x**2*\text{sqrt}(5*x**2 + 2*x + 3), x) - \text{Integral}(31*x**3*\text{sqrt}(5*x**2 + 2*x + 3), x) - \text{Integral}(7*x**4*\text{sqrt}(5*x**2 + 2*x + 3), x) - \text{Integral}(-2*\text{sqrt}(5*x**2 + 2*x + 3), x)$

$$3.355 \quad \int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2} dx$$

Optimal. Leaf size=187

$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397) - \frac{3}{343}\sqrt{\frac{1}{11}(497041-146555\sqrt{11})} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)$$

**Rubi [A]** time = 0.35, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397) - \frac{3}{343}\sqrt{\frac{1}{11}(497041-146555\sqrt{11})} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right) + \frac{3}{343}\sqrt{\frac{1}{11}(497041+146555\sqrt{11})} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right) - \frac{8233 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1715\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2), x]

[Out] -((397 + 35\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/490 - (8233\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(1715\*Sqrt[5]) - (3\*Sqrt[(497041 - 146555\*Sqrt[11])/11]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/343 + (3\*Sqrt[(497041 + 146555\*Sqrt[11])/11]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/343

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:= Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:= With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 1066

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:= Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

### Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:= Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2} dx &= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{1}{490} \int \frac{-3442-13408x-16466x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}} dx \\
&= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} + \frac{\int \frac{40560+159720x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}} dx}{3430} - \frac{8233 \int \frac{1}{\sqrt{3+2x+5x^2}} dx}{1715\sqrt{5}} \\
&= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}} dx}, x, 2+10x\right)}{3430\sqrt{70}} \\
&= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233 \sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{(24(14641-5024x+14x^2))\sqrt{5}}{1715\sqrt{5}} \\
&= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233 \sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{3\sqrt{5467451-16x^2}}{1715\sqrt{5}}
\end{aligned}$$

**Mathematica [A]** time = 0.89, size = 189, normalized size = 1.01

$$\frac{-385\sqrt{5x^2+2x+3}(35x+397) - 75\sqrt{250-34\sqrt{11}}(61\sqrt{11}-143)\operatorname{tanh}^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right) + 75\sqrt{250+34\sqrt{11}}(143+61\sqrt{11})\operatorname{tanh}^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{250+34\sqrt{11}}\sqrt{5x^2+2x+3}}\right) - 181126\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{188650}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2), x]

[Out] (-385\*(397 + 35\*x)\*Sqrt[3 + 2\*x + 5\*x^2] - 181126\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]] - 75\*Sqrt[250 - 34\*Sqrt[11]]\*(-143 + 61\*Sqrt[11])\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])] + 75\*Sqrt[250 + 34\*Sqrt[11]]\*(143 + 61\*Sqrt[11])\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[250 + 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])]) / 188650

**IntegrateAlgebraic [C]** time = 0.60, size = 234, normalized size = 1.25

$$\frac{6}{343}\operatorname{RootSum}\left[7\#1^4 + 8\sqrt{5}\#1^3 - 70\#1^2 - 16\sqrt{5}\#1 + 83&e, \frac{-1331\#1^2 \log(-\#1 + \sqrt{5x^2+2x+3} - \sqrt{5}x) + 676\sqrt{5}\#1 \log(-\#1 + \sqrt{5x^2+2x+3} - \sqrt{5}x) + 3317 \log(-\#1 + \sqrt{5x^2+2x+3} - \sqrt{5}x)}{7\#1^3 + 6\sqrt{5}\#1^2 - 35\#1 - 4\sqrt{5}}\right] + \frac{1}{490}\sqrt{5x^2+2x+3}(-35x-397) + \frac{8233 \log(\sqrt{5}\sqrt{5x^2+2x+3} - 5x - 1)}{1715\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2), x]



```
[Out] ((-397 - 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 + (8233*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]]/(1715*Sqrt[5])) + (6*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (3317*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 676*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 1331*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/343
```

**fricas** [B] time = 1.02, size = 304, normalized size = 1.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x + 3) + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 117306*x + 195510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log((sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) - 39102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

**giac** [A] time = 0.27, size = 144, normalized size = 0.77

```

$$\frac{1}{350} \sqrt{5x^2 + 2x + 3} (35x + 397) + \frac{8233}{3500} \sqrt{5} \log(-5\sqrt{5}x - \sqrt{5} + 5\sqrt{5x^2 + 2x + 3}) + 2.61475869687464 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} + 4.41924736459000) - 0.276245077121866 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} + 1.25295163054000) - 2.61475869687464 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} - 1.02258038113000) + 0.276245077121866 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} - 2.09411235400000)$$

```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] -1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.09, size = 403, normalized size = 2.16

$$\frac{\sqrt{5} \operatorname{arcsinh}\left(\frac{\sqrt{11}(x+1)}{11}\right)}{25} - \frac{3(-61+13\sqrt{11})\sqrt{11}}{140} - \frac{3\sqrt{11}(61+13\sqrt{11})}{154} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x)`

[Out]  $-1/140*(10*x+2)*(5*x^2+2*x+3)^{(1/2)}-1/25*5^{(1/2)}*\operatorname{arcsinh}(5/14*14^{(1/2)}*(x+1/5))-3/154*(-61+13*11^{(1/2)})*11^{(1/2)}*(1/49*(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}+1/70*(34/7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}-3/154*11^{(1/2)}*(61+13*11^{(1/2)})*(1/49*(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}+1/70*(34/7+10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))$

**maxima [B]** time = 1.17, size = 500, normalized size = 2.67

$$\frac{1}{188650}\sqrt{11}(975\sqrt{11}\sqrt{2}\sqrt{17\sqrt{11}+125}\operatorname{arcsinh}(5/7\sqrt{11}\sqrt{7}\sqrt{2})x/\operatorname{abs}(14x-2\sqrt{11}-4)+17/7\sqrt{7}\sqrt{2}\sqrt{2})x/\operatorname{abs}(14x-2\sqrt{11}-4)+1/7\sqrt{11}\sqrt{7}\sqrt{2}/\operatorname{abs}(14x-2\sqrt{11}-4)+23/7\sqrt{7}\sqrt{2}/\operatorname{abs}(14x-2\sqrt{11}-4))-1225\sqrt{11}\sqrt{2}\sqrt{5x^2+2x+3}x-16466\sqrt{11}\sqrt{5}\operatorname{arcsinh}(5/14\sqrt{11}\sqrt{7}\sqrt{2})x+1/14\sqrt{7}\sqrt{2})-6825\sqrt{11}\sqrt{-34/49\sqrt{11}+250/49}\operatorname{arcsinh}(5/7\sqrt{11}\sqrt{7}\sqrt{2})x/\operatorname{abs}(14x+2\sqrt{11}-4)-17/7\sqrt{7}\sqrt{2}\sqrt{2})x/\operatorname{abs}(14x+2\sqrt{11}-4)+1/7\sqrt{11}\sqrt{7}\sqrt{2}/\operatorname{abs}(14x+2\sqrt{11}-4)-23/7\sqrt{7}\sqrt{2}/\operatorname{abs}(14x+2\sqrt{11}-4)+4575\sqrt{2}\sqrt{17\sqrt{11}+125}\operatorname{arcsinh}(5/7\sqrt{11}\sqrt{7}\sqrt{2})x/\operatorname{abs}(14x-2\sqrt{11}-4)+17/7\sqrt{7}\sqrt{2}\sqrt{2})x/\operatorname{abs}(14x-2\sqrt{11}-4)+1/7\sqrt{11}\sqrt{7}\sqrt{2}/\operatorname{abs}(14x-2\sqrt{11}-4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="maxima")`

[Out]  $1/188650*\sqrt{11}(975*\sqrt{11}*\sqrt{2}*\sqrt{17*\sqrt{11}+125}*\operatorname{arcsinh}(5/7*\sqrt{11}*\sqrt{7}*\sqrt{2})*x/\operatorname{abs}(14*x-2*\sqrt{11}-4)+17/7*\sqrt{7}*\sqrt{2}*\sqrt{2})*x/\operatorname{abs}(14*x-2*\sqrt{11}-4)+1/7*\sqrt{11}*\sqrt{7}*\sqrt{2}/\operatorname{abs}(14*x-2*\sqrt{11}-4)+23/7*\sqrt{7}*\sqrt{2}/\operatorname{abs}(14*x-2*\sqrt{11}-4))-1225*\sqrt{11}*\sqrt{2}*\sqrt{5*x^2+2*x+3}*x-16466*\sqrt{11}*\sqrt{5}*\operatorname{arcsinh}(5/14*\sqrt{11}*\sqrt{7}*\sqrt{2})*x+1/14*\sqrt{7}*\sqrt{2})-6825*\sqrt{11}*\sqrt{-34/49*\sqrt{11}+250/49}*\operatorname{arcsinh}(5/7*\sqrt{11}*\sqrt{7}*\sqrt{2})*x/\operatorname{abs}(14*x+2*\sqrt{11}-4)-17/7*\sqrt{7}*\sqrt{2}*\sqrt{2})*x/\operatorname{abs}(14*x+2*\sqrt{11}-4)+1/7*\sqrt{11}*\sqrt{7}*\sqrt{2}/\operatorname{abs}(14*x+2*\sqrt{11}-4)-23/7*\sqrt{7}*\sqrt{2}/\operatorname{abs}(14*x+2*\sqrt{11}-4)+4575*\sqrt{2}*\sqrt{17*\sqrt{11}+125}*\operatorname{arcsinh}(5/7*\sqrt{11}*\sqrt{7}*\sqrt{2})*x/\operatorname{abs}(14*x-2*\sqrt{11}-4)+17/7*\sqrt{7}*\sqrt{2}*\sqrt{2})*x/\operatorname{abs}(14*x-2*\sqrt{11}-4)+1/7*\sqrt{11}*\sqrt{7}*\sqrt{2}/\operatorname{abs}(14*x-2*\sqrt{11}-4)$

+ 23/7\*sqrt(7)\*sqrt(2)/abs(14\*x - 2\*sqrt(11) - 4)) + 32025\*sqrt(-34/49\*sqrt(11) + 250/49)\*arcsinh(5/7\*sqrt(11)\*sqrt(7)\*sqrt(2)\*x/abs(14\*x + 2\*sqrt(11) - 4) - 17/7\*sqrt(7)\*sqrt(2)\*x/abs(14\*x + 2\*sqrt(11) - 4) + 1/7\*sqrt(11)\*sqrt(7)\*sqrt(2)/abs(14\*x + 2\*sqrt(11) - 4) - 23/7\*sqrt(7)\*sqrt(2)/abs(14\*x + 2\*sqrt(11) - 4)) - 13895\*sqrt(11)\*sqrt(5\*x^2 + 2\*x + 3))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2) \sqrt{5x^2 + 2x + 3}}{-7x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(1/2))/(4\*x - 7\*x^2 + 1), x)

[Out] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(1/2))/(4\*x - 7\*x^2 + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx - \int \frac{5x\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx - \int \frac{x^2\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+5\*x+2)\*(5\*x\*\*2+2\*x+3)\*\*(1/2)/(-7\*x\*\*2+4\*x+1), x)

[Out] -Integral(2\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x) - Integral(5\*x\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x) - Integral(x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x)

$$3.356 \quad \int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2} dx$$

**Optimal.** Leaf size=199

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)} - \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156} + \frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156} + \frac{1}{49}\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

**Rubi [A]** time = 0.25, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)} - \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156} + \frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156} + \frac{1}{49}\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2)^2, x]

[Out] (3\*(3 + 61\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(154\*(1 + 4\*x - 7\*x^2)) + (Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731\*Sqrt[11])/1397]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/2156 + (Sqrt[(325022311 - 39132731\*Sqrt[11])/1397]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/2156

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2)))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps



Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] (-3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(-1 - 4*x + 7*x^2)) - (Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/49 - RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-314239*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 28462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 11221*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/4802 - (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (599633*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 391895*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 21462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/(26411*Sqrt[5])
```

**fricas** [B] time = 0.96, size = 378, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425*x + 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) + 78359860425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sqrt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(11)*(x + 3) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 26119953475*sqrt(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2)/(-7\*x^2+4\*x+1)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{184473632, [8]%%}%+%%{%%{421654016, 0} : [1, 0, -5]%%}, [7]%%}%+%%{-2484746880, [6]%%}%+%%{%%{-5059848192, 0} : [1, 0, -5]%%}, [5]%%}%+%%{18003120576, [4]%%}%+%%{%%{13432692224, 0} : [1, 0, -5]%%}, [3]%%}%+%%{-38927701120, [2]%%}%+%%{%%{-9999223808, 0} : [1, 0, -5]%%}, [1]%%}%+%%{25935486752, [0]%%}% / %%{245, [8]%%}%+%%{%%{poly1[560, 0] : [1, 0, -5]%%}, [7]%%}%+%%{-3300, [6]%%}%+%%{%%{poly1[-6720, 0] : [1, 0, -5]%%}, [5]%%}%+%%{23910, [4]%%}%+%%{%%{poly1[17840, 0] : [1, 0, -5]%%}, [3]%%}%+%%{-51700, [2]%%}%+%%{%%{poly1[-13280, 0] : [1, 0, -5]%%}, [1]%%}%+%%{34445, [0]%%}% Error: Bad Argument Value

**maple [B]** time = 0.03, size = 1084, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2)/(-7\*x^2+4\*x+1)^2,x)

[Out] 
$$\begin{aligned} & (183/44+39/44*11^{(1/2)})*(-1/49/(250/49+34/49*11^{(1/2)})/(x-2/7-1/7*11^{(1/2)})) \\ & * (5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49 \\ & +34/49*11^{(1/2)})^{(3/2)}+1/98*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(1 \\ & /7*(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)}) \\ & +250+34*11^{(1/2)})^{(1/2)}+1/10*(34/7+10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)})/( \\ & 250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49 \\ & +34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)} \\ & )+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)}))/(250+34*11^{(1/2)})^{(1/2)})/(245*( \\ & x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*1 \\ & 1^{(1/2)})^{(1/2)}))+10/49/(250/49+34/49*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7-1/7 \\ & *11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)} \\ & ))^{(1/2)}+1/200*(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arc} \\ & \operatorname{sinh}(5^{(1/2)})/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1 \\ & /5)))+161/484*11^{(1/2)}*(1/49*(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)} \\ & (1/2))*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}+1/70*(34/7-10/7*11^{(1/2)} \\ & )*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)})/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2 \\ & )^{(1/2)}*(x+1/5))-(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(4 \\ & 9/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))/(250- \\ & 34*11^{(1/2)})^{(1/2)})/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2 \\ & /7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}))+183/44-39/44*11^{(1/2)})*(-1/49/(2 \\ & 50/49-34/49*11^{(1/2)})/(x-2/7+1/7*11^{(1/2)})*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7- \\ & 10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(3/2)}+1/98*(34/7 \\ & -10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)})^2+49 \\ & *(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}+1/10*(34/ \\ & 7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)})/(250/49-34/49*11^{(1/2)}-1/20*(34/7- \end{aligned}$$



$$\frac{10/7 \cdot 11^{(1/2)} \cdot (x+1/5) - 7 \cdot (250/49 - 34/49 \cdot 11^{(1/2)}) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(49/2 \cdot (500/49 - 68/49 \cdot 11^{(1/2)} + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250 - 34 \cdot 11^{(1/2)})^{(1/2)}) + 10/49 / (250/49 - 34/49 \cdot 11^{(1/2)}) \cdot (1/20 \cdot (10 \cdot x + 2) \cdot (5 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250/49 - 34/49 \cdot 11^{(1/2)})^{(1/2)} + 1/200 \cdot (5000/49 - 680/49 \cdot 11^{(1/2)} - (34/7 - 10/7 \cdot 11^{(1/2)})^2 \cdot 5^{(1/2)} \cdot \operatorname{arcsinh}(5^{(1/2)} / (250/49 - 34/49 \cdot 11^{(1/2)})) - 1/20 \cdot (34/7 - 10/7 \cdot 11^{(1/2)})^2 \cdot (x+1/5)) - 161/484 \cdot 11^{(1/2)} \cdot (1/49 \cdot (245 \cdot (x - 2/7 - 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7 + 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 - 1/7 \cdot 11^{(1/2)}) + 250 + 34 \cdot 11^{(1/2)})^{(1/2)} + 1/70 \cdot (34/7 + 10/7 \cdot 11^{(1/2)}) \cdot 5^{(1/2)} \cdot \operatorname{arcsinh}(5^{(1/2)} / (250/49 + 34/49 \cdot 11^{(1/2)} - 1/20 \cdot (34/7 + 10/7 \cdot 11^{(1/2)})^2 \cdot (x+1/5)) - (250/49 + 34/49 \cdot 11^{(1/2)}) / (250 + 34 \cdot 11^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(49/2 \cdot (500/49 + 68/49 \cdot 11^{(1/2)} + (34/7 + 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 - 1/7 \cdot 11^{(1/2)})) / (250 + 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x - 2/7 - 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7 + 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 - 1/7 \cdot 11^{(1/2)}) + 250 + 34 \cdot 11^{(1/2)})^{(1/2)})$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5x^2 + 2x + 3} (x^2 + 5x + 2)}{(7x^2 - 4x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2)/(-7\*x^2+4\*x+1)^2,x, algorithm="maxima")

[Out] integrate(sqrt(5\*x^2 + 2\*x + 3)\*(x^2 + 5\*x + 2)/(7\*x^2 - 4\*x - 1)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2) \sqrt{5x^2 + 2x + 3}}{(-7x^2 + 4x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(1/2))/(4\*x - 7\*x^2 + 1)^2,x)

[Out] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(1/2))/(4\*x - 7\*x^2 + 1)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 5x + 2) \sqrt{5x^2 + 2x + 3}}{(7x^2 - 4x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

$$3.357 \quad \int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3} dx$$

Optimal. Leaf size=213

$$\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{1721104(-7x^2+4x+1)} + \frac{3(61x+3)\sqrt{5x^2+2x+3}}{308(-7x^2+4x+1)^2} - \frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17-\sqrt{2(125-17\sqrt{11})})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{491744}$$

**Rubi [A]** time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1054, 1060, 1032, 724, 206}

$$-\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{1721104(-7x^2+4x+1)} + \frac{3(61x+3)\sqrt{5x^2+2x+3}}{308(-7x^2+4x+1)^2} - \frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{491744} + \frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{491744}$$

Antiderivative was successfully verified.

[In] Int[((2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2)^3, x]

[Out] (3\*(3 + 61\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(308\*(1 + 4\*x - 7\*x^2)^2) - ((272941 - 813113\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(1721104\*(1 + 4\*x - 7\*x^2)) - (Sqrt[(6492253020949 - 11879169071\*Sqrt[11])/1397]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/491744 + (Sqrt[(6492253020949 + 11879169071\*Sqrt[11])/1397]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/491744

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

Rule 1032

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

### Rule 1054

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2)))]*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))]*x^2, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

```

### Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f))]*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))]*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))]*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))]*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))]*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(2 + 5x + x^2) \sqrt{3 + 2x + 5x^2}}{(1 + 4x - 7x^2)^3} dx &= \frac{3(3 + 61x)\sqrt{3 + 2x + 5x^2}}{308(1 + 4x - 7x^2)^2} - \frac{1}{616} \int \frac{-3012 - 1564x - 3220x^2}{(1 + 4x - 7x^2)^2 \sqrt{3 + 2x + 5x^2}} dx \\
&= \frac{3(3 + 61x)\sqrt{3 + 2x + 5x^2}}{308(1 + 4x - 7x^2)^2} - \frac{(272941 - 813113x)\sqrt{3 + 2x + 5x^2}}{1721104(1 + 4x - 7x^2)} + \int \frac{47}{(1+} \\
&= \frac{3(3 + 61x)\sqrt{3 + 2x + 5x^2}}{308(1 + 4x - 7x^2)^2} - \frac{(272941 - 813113x)\sqrt{3 + 2x + 5x^2}}{1721104(1 + 4x - 7x^2)} + \frac{(1391} \\
&= \frac{3(3 + 61x)\sqrt{3 + 2x + 5x^2}}{308(1 + 4x - 7x^2)^2} - \frac{(272941 - 813113x)\sqrt{3 + 2x + 5x^2}}{1721104(1 + 4x - 7x^2)} + \frac{(-139} \\
&= \frac{3(3 + 61x)\sqrt{3 + 2x + 5x^2}}{308(1 + 4x - 7x^2)^2} - \frac{(272941 - 813113x)\sqrt{3 + 2x + 5x^2}}{1721104(1 + 4x - 7x^2)} - \frac{\sqrt{649}}{1721104}
\end{aligned}$$

**Mathematica [A]** time = 1.40, size = 334, normalized size = 1.57

$$\frac{-\sqrt{\frac{22}{125-17\sqrt{11}}}(126542\sqrt{11}-1740003)\log(49x^2+14(\sqrt{11}-2)x-4\sqrt{11}+15)+2\sqrt{\frac{22}{125+17\sqrt{11}}}(1740003+126542\sqrt{11})\log(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}+(55+17\sqrt{11})x+23\sqrt{11}+11)}{10818368}-\frac{\sqrt{\frac{22}{125-17\sqrt{11}}}(126542\sqrt{11}-1740003)\operatorname{arctanh}\left(\frac{\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}}{\sqrt{125-17\sqrt{11}}}\right)+\frac{\sqrt{22}\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}}{\sqrt{125-17\sqrt{11}}}}{10818368}-2\sqrt{\frac{22}{125+17\sqrt{11}}}(1740003+126542\sqrt{11})\log(-7x+\sqrt{11}+2)+\sqrt{\frac{22}{125+17\sqrt{11}}}(126542\sqrt{11}-1740003)\log(7x+\sqrt{11}-2)}}{10818368}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 5\*x + x^2)\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2)^3,x]

[Out] ((-44\*Sqrt[3 + 2\*x + 5\*x^2]\*(31807 - 106279\*x - 737577\*x^2 + 813113\*x^3))/(1 + 4\*x - 7\*x^2)^2 - 2\*Sqrt[22/(125 - 17\*Sqrt[11])]\*(-1740003 + 126542\*Sqrt[11])\*ArcTanh[(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])/(-23 + Sqrt[11] + (-17 + 5\*Sqrt[11])\*x)] - 2\*Sqrt[22/(125 + 17\*Sqrt[11])]\*(1740003 + 126542\*Sqrt[11])\*Log[2 + Sqrt[11] - 7\*x] + Sqrt[22/(125 - 17\*Sqrt[11])]\*(-1740003 + 126542\*Sqrt[11])\*Log[(-2 + Sqrt[11] + 7\*x)^2] - Sqrt[22/(125 - 17\*Sqrt[11])]\*(-1740003 + 126542\*Sqrt[11])\*Log[15 - 4\*Sqrt[11] + 14\*(-2 + Sqrt[11])\*x + 49\*x^2] + 2\*Sqrt[22/(125 + 17\*Sqrt[11])]\*(1740003 + 126542\*Sqrt[11])\*Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]])/10818368

**IntegrateAlgebraic [C]** time = 1.06, size = 608, normalized size = 2.85

$$\frac{-\sqrt{\frac{22}{125-17\sqrt{11}}}(126542\sqrt{11}-1740003)\log(49x^2+14(\sqrt{11}-2)x-4\sqrt{11}+15)+2\sqrt{\frac{22}{125+17\sqrt{11}}}(1740003+126542\sqrt{11})\log(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}+(55+17\sqrt{11})x+23\sqrt{11}+11)}{10818368}-\frac{\sqrt{\frac{22}{125-17\sqrt{11}}}(126542\sqrt{11}-1740003)\operatorname{arctanh}\left(\frac{\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}}{\sqrt{125-17\sqrt{11}}}\right)+\frac{\sqrt{22}\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}}{\sqrt{125-17\sqrt{11}}}}{10818368}-2\sqrt{\frac{22}{125+17\sqrt{11}}}(1740003+126542\sqrt{11})\log(-7x+\sqrt{11}+2)+\sqrt{\frac{22}{125+17\sqrt{11}}}(126542\sqrt{11}-1740003)\log(7x+\sqrt{11}-2)}}{10818368}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-31807 + 106279*x + 737577*x^2 - 813113*x^3))/(2458
72*(-1 - 4*x + 7*x^2)^2) - (2752069162747*RootSum[83 - 16*Sqrt[5]*#1 - 70*#
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2]
- #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/64427415976 + (5*
RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-465*Lo
g[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) +
Sqrt[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#
1^3) & ])/343 + RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#
1^4 & , (3751778663030*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #
1]*#1 + 2597308755559*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/
(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/64427415976 - (3*RootSum
[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-1164877805727
1*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 13372446682211*Sqrt[5]*L
og[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 9645047011740*Log[-(Sqrt
[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*
#1^2 + 7*#1^3) & ])/708701575736
```

**fricas** [B] time = 0.99, size = 390, normalized size = 1.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169
071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11
879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690716
98870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) - sqr
t(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11) + 649
2253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*sqrt(11
) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*sqrt(11)
*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(49*x^4 -
56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log(
-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-1187
9169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x + 3) - 1707
215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2
+ 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqr
t(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-11879169071*sqrt(11)
+ 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 1707215096611365*x -
```

2845358494352275)/x) + 5588\*(813113\*x^3 - 737577\*x^2 - 106279\*x + 31807)\*sqrt(5\*x^2 + 2\*x + 3))/(49\*x^4 - 56\*x^3 + 2\*x^2 + 8\*x + 1)

**giac [B]** time = 0.26, size = 378, normalized size = 1.77

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2)/(-7\*x^2+4\*x+1)^3,x, algorithm="giac")

[Out] 1/430276\*(6200558\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^7 - 835775\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^6 - 190947036\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^5 - 92732607\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^4 + 816321374\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^3 + 419437335\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^2 - 765111048\*sqrt(5)\*x - 376983161\*sqrt(5) + 765111048\*sqrt(5\*x^2 + 2\*x + 3))/(7\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^4 - 8\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^3 - 70\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^2 + 16\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) + 83)^2 + 0.139051039089329\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 4.41924736459000) - 0.138209741946100\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 1.25295163054000) - 0.139051039089329\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 1.02258038113000) + 0.138209741946100\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 2.09411235400000)

**maple [B]** time = 0.03, size = 2342, normalized size = 11.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(1/2)/(-7\*x^2+4\*x+1)^3,x)

[Out] -21/968\*(61+13\*11^(1/2))\*11^(1/2)\*(-1/686/(250/49+34/49\*11^(1/2)))/(x-2/7-1/7\*11^(1/2))^2\*(5\*(x-2/7-1/7\*11^(1/2))^2+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250/49+34/49\*11^(1/2))^3/2-1/1372\*(34/7+10/7\*11^(1/2))/(250/49+34/49\*11^(1/2))\*(-1/(250/49+34/49\*11^(1/2)))/(x-2/7-1/7\*11^(1/2))\*(5\*(x-2/7-1/7\*11^(1/2))^2+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250/49+34/49\*11^(1/2))^3/2+1/2\*(34/7+10/7\*11^(1/2))/(250/49+34/49\*11^(1/2))\*(1/7\*(245\*(x-2/7-1/7\*11^(1/2))^2+49\*(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250+34\*11^(1/2))^1/2+1/10\*(34/7+10/7\*11^(1/2))\*5^(1/2)\*arcsinh(5^(1/2)/(250/49+34/49\*11^(1/2)-1/20\*(34/7+10/7\*11^(1/2))^2)^(1/2)\*(x+1/5))-7\*(250/49+34/49\*11^(1/2))/(250+34\*11^(1/2))^1/2\*arctanh(49/2\*(500/49+68/49\*11^(1/2)+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2)))/(250+34\*11^(1/2))^1/2)/(245\*(x-2/7-1/7\*11^(1/2))^2+49\*(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250+34\*11^(1/2))^1/2)

$$\begin{aligned}
&)+10/(250/49+34/49*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7 \\
&+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}+1/200*(50 \\
&00/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/ \\
&49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))) +5/686/(250/ \\
&49+34/49*11^{(1/2)})*(1/7*(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)}) \\
&*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}+1/10*(34/7+10/7*11^{(1/2)})*5^{(1 \\
&/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)} \\
&*(x+1/5))-7*(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2* \\
&(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*1 \\
&1^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1 \\
&/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)})))+3535/21296*11^{(1/2)}*(1/49*(245*(x-2/ \\
&7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1 \\
&/2)})^{(1/2)}+1/70*(34/7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49* \\
&11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-(250/49-34/49*11^{(1/2)} \\
&)/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11 \\
&^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1 \\
&/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)} \\
&))-21/968*(-61+13*11^{(1/2)})*11^{(1/2)}*(-1/686/(250/49-34/49*11^{(1/2)}))/(x-2/7+ \\
&1/7*11^{(1/2)})^2*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*1 \\
&1^{(1/2)})+250/49-34/49*11^{(1/2)})^{(3/2)}-1/1372*(34/7-10/7*11^{(1/2)})/(250/49-3 \\
&4/49*11^{(1/2)})*(-1/(250/49-34/49*11^{(1/2)}))/(x-2/7+1/7*11^{(1/2)})*(5*(x-2/7+1 \\
&/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1 \\
&/2)})^{(3/2)}+1/2*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/ \\
&7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1 \\
&/2)})^{(1/2)}+1/10*(34/7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49* \\
&11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1/2)} \\
&)/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7* \\
&11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1 \\
&/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)} \\
&)))+10/(250/49-34/49*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34 \\
&/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}+1/200*( \\
&5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(25 \\
&0/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))) +5/686/(25 \\
&0/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)} \\
&))*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}+1/10*(34/7-10/7*11^{(1/2)})*5^{( \\
&1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{( \\
&1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/ \\
&2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34 \\
&*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7 \\
&+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)})))-(-3535/1936+273/1936*11^{(1/2)})*(-1 \\
&/49/(250/49-34/49*11^{(1/2)}))/(x-2/7+1/7*11^{(1/2)})*(5*(x-2/7+1/7*11^{(1/2)})^2+ \\
&(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(3/2)}+1/98 \\
&*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)} \\
&)^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}+1/1 \\
&0*(34/7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*
\end{aligned}$$





Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)`

[Out] `int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx - \int \frac{5x\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx - \int \frac{x^2\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3, x)`

[Out] `-Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x)`

$$3.358 \quad \int (1 + 4x - 7x^2)^3 (2 + 5x + x^2) (3 + 2x + 5x^2)^{3/2} dx$$

Optimal. Leaf size=231

$$\frac{2173004363 (5x^2 + 2x + 3)^{5/2} x^2}{173250000} + \frac{837379699 (5x^2 + 2x + 3)^{5/2} x}{72187500} - \frac{6133820867 (5x^2 + 2x + 3)^{5/2}}{1203125000} - \frac{22840599(5x^2 + 2x + 3)^{3/2}}{62500000}$$

**Rubi [A]** time = 0.36, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 35, number of rules / integrand size = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{343}{60} (5x^2 + 2x + 3)^{5/2} x^2 - \frac{61103 (5x^2 + 2x + 3)^{5/2} x^2}{3300} - \frac{1031177 (5x^2 + 2x + 3)^{5/2} x^2}{20625} - \frac{796559 (5x^2 + 2x + 3)^{5/2} x^2}{123750} - \frac{190236913 (5x^2 + 2x + 3)^{5/2} x^2}{4950000} - \frac{2173004363 (5x^2 + 2x + 3)^{5/2} x^2}{173250000} - \frac{837379699 (5x^2 + 2x + 3)^{5/2} x}{72187500} - \frac{6133820867 (5x^2 + 2x + 3)^{5/2}}{1203125000} - \frac{22840599(5x^2 + 2x + 3)^{3/2}}{62500000} - \frac{479652579(5x + 1)\sqrt{5x^2 + 2x + 3}}{312500000} - \frac{3357568053 \operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{5x^2+2x+3}}\right)}{156250000\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-479652579\*(1 + 5\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/312500000 - (22840599\*(1 + 5\*x)\*(3 + 2\*x + 5\*x^2)^(3/2))/62500000 - (6133820867\*(3 + 2\*x + 5\*x^2)^(5/2))/1203125000 + (837379699\*x\*(3 + 2\*x + 5\*x^2)^(5/2))/72187500 + (2173004363\*x^2\*(3 + 2\*x + 5\*x^2)^(5/2))/173250000 - (190236913\*x^3\*(3 + 2\*x + 5\*x^2)^(5/2))/4950000 - (796559\*x^4\*(3 + 2\*x + 5\*x^2)^(5/2))/123750 + (1031177\*x^5\*(3 + 2\*x + 5\*x^2)^(5/2))/20625 - (61103\*x^6\*(3 + 2\*x + 5\*x^2)^(5/2))/3300 - (343\*x^7\*(3 + 2\*x + 5\*x^2)^(5/2))/60 - (3357568053\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(156250000\*Sqrt[5])

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (1 + 4x - 7x^2)^3 (2 + 5x + x^2) (3 + 2x + 5x^2)^{3/2} dx &= -\frac{343}{60} x^7 (3 + 2x + 5x^2)^{5/2} + \frac{1}{60} \int (3 + 2x + 5x^2)^{3/2} (1 \\
&= -\frac{61103x^6 (3 + 2x + 5x^2)^{5/2}}{3300} - \frac{343}{60} x^7 (3 + 2x + 5x^2)^{5/2} \\
&= \frac{1031177x^5 (3 + 2x + 5x^2)^{5/2}}{20625} - \frac{61103x^6 (3 + 2x + 5x^2)^{5/2}}{3300} \\
&= -\frac{796559x^4 (3 + 2x + 5x^2)^{5/2}}{123750} + \frac{1031177x^5 (3 + 2x + 5x^2)^{5/2}}{20625} \\
&= -\frac{190236913x^3 (3 + 2x + 5x^2)^{5/2}}{4950000} - \frac{796559x^4 (3 + 2x + 5x^2)^{5/2}}{123750} \\
&= \frac{2173004363x^2 (3 + 2x + 5x^2)^{5/2}}{173250000} - \frac{190236913x^3 (3 + 2x + 5x^2)^{5/2}}{4950000} \\
&= \frac{837379699x (3 + 2x + 5x^2)^{5/2}}{72187500} + \frac{2173004363x^2 (3 + 2x + 5x^2)^{5/2}}{173250000} \\
&= -\frac{6133820867 (3 + 2x + 5x^2)^{5/2}}{1203125000} + \frac{837379699x (3 + 2x + 5x^2)^{5/2}}{72187500} \\
&= -\frac{22840599(1 + 5x) (3 + 2x + 5x^2)^{3/2}}{62500000} - \frac{6133820867 (3 + 2x + 5x^2)^{5/2}}{1203125000} \\
&= -\frac{479652579(1 + 5x)\sqrt{3 + 2x + 5x^2}}{312500000} - \frac{22840599(1 + 5x)\sqrt{3 + 2x + 5x^2}}{62500000} \\
&= -\frac{479652579(1 + 5x)\sqrt{3 + 2x + 5x^2}}{312500000} - \frac{22840599(1 + 5x)\sqrt{3 + 2x + 5x^2}}{62500000} \\
&= -\frac{479652579(1 + 5x)\sqrt{3 + 2x + 5x^2}}{312500000} - \frac{22840599(1 + 5x)\sqrt{3 + 2x + 5x^2}}{62500000}
\end{aligned}$$

**Mathematica [A]** time = 0.42, size = 95, normalized size = 0.41

$-\frac{4653589321458\sqrt{5} \operatorname{sinh}^{-1}\left(\frac{5x+1}{\sqrt{3x^2+2x+3}}\right) - 5\sqrt{3x^2+2x+3} (30950390625000x^{11} + 125007421875000x^{10} - 148393743750000x^9 - 30505457500000x^8 - 72918247281250x^7 + 52106830406250x^6 + 85130334087500x^5 - 2573089891000x^4 - 19041688239675x^3 - 15865844408685x^2 - 635277129950x + 10506617068392)}{108281250000}$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out]  $(-5*\text{Sqrt}[3 + 2*x + 5*x^2]*(10506617068392 - 6352777129950*x - 15865844408685*x^2 - 19041688239675*x^3 - 2573089891000*x^4 + 85130334087500*x^5 + 52106830406250*x^6 - 72918247281250*x^7 - 30505457500000*x^8 - 148393743750000*x^9 + 125007421875000*x^{10} + 30950390625000*x^{11}) - 4653589321458*\text{Sqrt}[5]*\text{ArcSinh}[(1 + 5*x)/\text{Sqrt}[14]])/1082812500000$

**IntegrateAlgebraic [A]** time = 1.29, size = 109, normalized size = 0.47

$\frac{3357568053 \log(\sqrt{5}\sqrt{x^2+2x+3}-5x-1)}{156250000\sqrt{5}} + \frac{\sqrt{5x^2+2x+3}(-30950390625000x^{11}-125007421875000x^{10}+148393743750000x^9+30505457500000x^8+72918247281250x^7-52106830406250x^6-85130334087500x^5+2573089891000x^4+19041688239675x^3+15865844408685x^2+6352777129950x-10506617068392)}{21656250000}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out]  $(\text{Sqrt}[3 + 2*x + 5*x^2]*(-10506617068392 + 6352777129950*x + 15865844408685*x^2 + 19041688239675*x^3 + 2573089891000*x^4 - 85130334087500*x^5 - 52106830406250*x^6 + 72918247281250*x^7 + 30505457500000*x^8 + 148393743750000*x^9 - 125007421875000*x^{10} - 30950390625000*x^{11}))/216562500000 + (3357568053*\text{Log}[-1 - 5*x + \text{Sqrt}[5]*\text{Sqrt}[3 + 2*x + 5*x^2]])/(156250000*\text{Sqrt}[5])$

**fricas [A]** time = 0.87, size = 107, normalized size = 0.46

$\frac{1}{21656250000}(30950390625000x^{11}+125007421875000x^{10}-148393743750000x^9-30505457500000x^8-72918247281250x^7+52106830406250x^6+85130334087500x^5-2573089891000x^4-19041688239675x^3-15865844408685x^2-6352777129950x+10506617068392)\sqrt{5x^2+2x+3} + \frac{3357568053}{156250000}\sqrt{5}\log(\sqrt{5}\sqrt{x^2+2x+3}(5x+1)-25x^2-10x-8)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="fricas")

[Out]  $-1/216562500000*(30950390625000*x^{11} + 125007421875000*x^{10} - 148393743750000*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 85130334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 15865844408685*x^2 - 6352777129950*x + 10506617068392)*\text{sqrt}(5*x^2 + 2*x + 3) + 3357568053/156250000*\text{sqrt}(5)*\log(\text{sqrt}(5)*\text{sqrt}(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)$

**giac [A]** time = 0.23, size = 102, normalized size = 0.44

$\frac{1}{216562500000}(5*(5*(10(25(5(7(20(105(875(77*x+311)*x-323034)*x-6972676)*x-333340559)*x+1667418573)*x+13620853454)*x-10292359564)*x-761667529587)*x-3173168881737)*x-1270555425990)*x+10506617068392)\sqrt{5x^2+2x+3} + \frac{3357568053}{781250000}\sqrt{5}\log(-\sqrt{5}(x-\sqrt{5x^2+2x+3})-1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="giac")

[Out]  $-1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x+311)*x-323034)*x-6972676)*x-333340559)*x+1667418573)*x+13620853454)*x-10292359564)*x-761667529587)*x-3173168881737)*x-1270555425990)*x+10506617068392)\sqrt{5x^2+2x+3} + \frac{3357568053}{781250000}\sqrt{5}\log(-\sqrt{5}(x-\sqrt{5x^2+2x+3})-1)$

64)\*x - 761667529587)\*x - 3173168881737)\*x - 1270555425990)\*x + 10506617068  
392)\*sqrt(5\*x^2 + 2\*x + 3) + 3357568053/781250000\*sqrt(5)\*log(-sqrt(5)\*(sqrt  
t(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)

**maple [A]** time = 0.04, size = 185, normalized size = 0.80

$\frac{343}{60} (5x^2 + 2x + 3)^{\frac{3}{2}} x^7 - \frac{61103}{3300} (5x^2 + 2x + 3)^{\frac{5}{2}} x^6 + \frac{1031177}{20625} (5x^2 + 2x + 3)^{\frac{5}{2}} x^5 - \frac{796559}{123750} (5x^2 + 2x + 3)^{\frac{5}{2}} x^4 - \frac{190236913}{4950000} (5x^2 + 2x + 3)^{\frac{5}{2}} x^3 + \frac{2173004363}{173250000} (5x^2 + 2x + 3)^{\frac{5}{2}} x^2 + \frac{837379699}{72187500} (5x^2 + 2x + 3)^{\frac{5}{2}} x - \frac{3357568053\sqrt{5} \operatorname{arcsinh}\left(\frac{\sqrt{14}(x+1)}{14}\right)}{781250000} - \frac{6133820867}{1203125000} (5x^2 + 2x + 3)^{\frac{3}{2}} x - \frac{479652579(10x+2)\sqrt{5x^2+2x+3}}{62500000} - \frac{22840599(10x+2)(5x^2+2x+3)^{\frac{3}{2}}}{12500000}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x)

[Out] -6133820867/1203125000\*(5\*x^2+2\*x+3)^(5/2)-343/60\*x^7\*(5\*x^2+2\*x+3)^(5/2)-6  
1103/3300\*x^6\*(5\*x^2+2\*x+3)^(5/2)+1031177/20625\*x^5\*(5\*x^2+2\*x+3)^(5/2)-796  
559/123750\*x^4\*(5\*x^2+2\*x+3)^(5/2)-190236913/4950000\*x^3\*(5\*x^2+2\*x+3)^(5/2  
) + 2173004363/173250000\*x^2\*(5\*x^2+2\*x+3)^(5/2) - 3357568053/781250000\*5^(1/2)  
\*arcsinh(5/14\*sqrt(14)\*(x+1/5)) - 479652579/625000000\*(10\*x+2)\*(5\*x^2+2\*x+3)^(  
1/2) + 837379699/72187500\*x\*(5\*x^2+2\*x+3)^(5/2) - 22840599/125000000\*(10\*x+2)\*  
(5\*x^2+2\*x+3)^(3/2)

**maxima [A]** time = 1.01, size = 206, normalized size = 0.89

$\frac{343}{60} (5x^2 + 2x + 3)^{\frac{3}{2}} x^7 - \frac{61103}{3300} (5x^2 + 2x + 3)^{\frac{5}{2}} x^6 + \frac{1031177}{20625} (5x^2 + 2x + 3)^{\frac{5}{2}} x^5 - \frac{796559}{123750} (5x^2 + 2x + 3)^{\frac{5}{2}} x^4 - \frac{190236913}{4950000} (5x^2 + 2x + 3)^{\frac{5}{2}} x^3 + \frac{2173004363}{173250000} (5x^2 + 2x + 3)^{\frac{5}{2}} x^2 + \frac{837379699}{72187500} (5x^2 + 2x + 3)^{\frac{5}{2}} x - \frac{3357568053\sqrt{5} \operatorname{arcsinh}\left(\frac{1}{14}\sqrt{14}(x+1)\right)}{781250000} - \frac{479652579\sqrt{5x^2+2x+3}}{62500000} - \frac{22840599(10x+2)(5x^2+2x+3)^{\frac{3}{2}}}{12500000}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="ma  
xima")

[Out] -343/60\*(5\*x^2 + 2\*x + 3)^(5/2)\*x^7 - 61103/3300\*(5\*x^2 + 2\*x + 3)^(5/2)\*x^  
6 + 1031177/20625\*(5\*x^2 + 2\*x + 3)^(5/2)\*x^5 - 796559/123750\*(5\*x^2 + 2\*x  
+ 3)^(5/2)\*x^4 - 190236913/4950000\*(5\*x^2 + 2\*x + 3)^(5/2)\*x^3 + 2173004363  
/173250000\*(5\*x^2 + 2\*x + 3)^(5/2)\*x^2 + 837379699/72187500\*(5\*x^2 + 2\*x +  
3)^(5/2)\*x - 6133820867/1203125000\*(5\*x^2 + 2\*x + 3)^(5/2) - 22840599/12500  
000\*(5\*x^2 + 2\*x + 3)^(3/2)\*x - 22840599/62500000\*(5\*x^2 + 2\*x + 3)^(3/2) -  
479652579/62500000\*sqrt(5\*x^2 + 2\*x + 3)\*x - 3357568053/781250000\*sqrt(5)\*  
arcsinh(1/14\*sqrt(14)\*(5\*x + 1)) - 479652579/312500000\*sqrt(5\*x^2 + 2\*x + 3  
)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (x^2 + 5x + 2) (5x^2 + 2x + 3)^{3/2} (-7x^2 + 4x + 1)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2)\*(4\*x - 7\*x^2 + 1)^3, x)

[Out] `int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$-\int (-91x\sqrt{5x^2+2x+3}) dx - \int (-413x^2\sqrt{5x^2+2x+3}) dx - \int (-192x^3\sqrt{5x^2+2x+3}) dx - \int (2160x^4\sqrt{5x^2+2x+3}) dx - \int (1666x^5\sqrt{5x^2+2x+3}) dx - \int (-2094x^6\sqrt{5x^2+2x+3}) dx - \int (-1384x^7\sqrt{5x^2+2x+3}) dx - \int (-7042x^8\sqrt{5x^2+2x+3}) dx - \int (6321x^9\sqrt{5x^2+2x+3}) dx - \int (-6\sqrt{5x^2+2x+3}) dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2), x)`

[Out] `-Integral(-91*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-413*x**2*sqrt(5*x**2 + 2*x + 3), x) - Integral(-192*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(2160*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(1666*x**5*sqrt(5*x**2 + 2*x + 3), x) - Integral(-2094*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-1384*x**7*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7042*x**8*sqrt(5*x**2 + 2*x + 3), x) - Integral(6321*x**9*sqrt(5*x**2 + 2*x + 3), x) - Integral(1715*x**10*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2 + 2*x + 3), x)`



$$3.359 \quad \int (1 + 4x - 7x^2)^2 (2 + 5x + x^2) (3 + 2x + 5x^2)^{3/2} dx$$

Optimal. Leaf size=189

$$\frac{219271 (5x^2 + 2x + 3)^{5/2} x^2}{105000} + \frac{86721 (5x^2 + 2x + 3)^{5/2} x}{21875} + \frac{505667 (5x^2 + 2x + 3)^{5/2}}{2187500} - \frac{690561(5x + 1) (5x^2 + 2x + 3)^{3/2}}{1250000}$$

**Rubi [A]** time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, number of rules / integrand size = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{50} (5x^2 + 2x + 3)^{5/2} x^2 + \frac{581}{150} (5x^2 + 2x + 3)^{5/2} x^4 - \frac{18379 (5x^2 + 2x + 3)^{5/2} x^3}{3000} - \frac{219271 (5x^2 + 2x + 3)^{5/2} x^2}{105000} + \frac{86721 (5x^2 + 2x + 3)^{5/2} x}{21875} + \frac{505667 (5x^2 + 2x + 3)^{5/2}}{2187500} - \frac{690561(5x + 1) (5x^2 + 2x + 3)^{3/2}}{1250000} - \frac{14501781(5x + 1)\sqrt{5x^2 + 2x + 3}}{6250000} - \frac{101512467 \operatorname{sinh}^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3125000\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-14501781\*(1 + 5\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/6250000 - (690561\*(1 + 5\*x)\*(3 + 2\*x + 5\*x^2)^(3/2))/1250000 + (505667\*(3 + 2\*x + 5\*x^2)^(5/2))/2187500 + (86721\*x\*(3 + 2\*x + 5\*x^2)^(5/2))/21875 - (219271\*x^2\*(3 + 2\*x + 5\*x^2)^(5/2))/105000 - (18379\*x^3\*(3 + 2\*x + 5\*x^2)^(5/2))/3000 + (581\*x^4\*(3 + 2\*x + 5\*x^2)^(5/2))/150 + (49\*x^5\*(3 + 2\*x + 5\*x^2)^(5/2))/50 - (101512467\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(3125000\*Sqrt[5])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int (1 + 4x - 7x^2)^2 (2 + 5x + x^2) (3 + 2x + 5x^2)^{3/2} dx &= \frac{49}{50}x^5 (3 + 2x + 5x^2)^{5/2} + \frac{1}{50} \int (3 + 2x + 5x^2)^{3/2} (100 \\
&= \frac{581}{150}x^4 (3 + 2x + 5x^2)^{5/2} + \frac{49}{50}x^5 (3 + 2x + 5x^2)^{5/2} + \int \\
&= -\frac{18379x^3 (3 + 2x + 5x^2)^{5/2}}{3000} + \frac{581}{150}x^4 (3 + 2x + 5x^2)^{5/2} \\
&= -\frac{219271x^2 (3 + 2x + 5x^2)^{5/2}}{105000} - \frac{18379x^3 (3 + 2x + 5x^2)^{5/2}}{3000} \\
&= \frac{86721x (3 + 2x + 5x^2)^{5/2}}{21875} - \frac{219271x^2 (3 + 2x + 5x^2)^{5/2}}{105000} \\
&= \frac{505667 (3 + 2x + 5x^2)^{5/2}}{2187500} + \frac{86721x (3 + 2x + 5x^2)^{5/2}}{21875} \\
&= -\frac{690561(1 + 5x) (3 + 2x + 5x^2)^{3/2}}{1250000} + \frac{505667 (3 + 2x + 5x^2)^{3/2}}{2187500} \\
&= -\frac{14501781(1 + 5x)\sqrt{3 + 2x + 5x^2}}{6250000} - \frac{690561(1 + 5x) (3 + 2x + 5x^2)^{3/2}}{1250000} \\
&= -\frac{14501781(1 + 5x)\sqrt{3 + 2x + 5x^2}}{6250000} - \frac{690561(1 + 5x) (3 + 2x + 5x^2)^{3/2}}{1250000} \\
&= -\frac{14501781(1 + 5x)\sqrt{3 + 2x + 5x^2}}{6250000} - \frac{690561(1 + 5x) (3 + 2x + 5x^2)^{3/2}}{1250000}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 85, normalized size = 0.45

$$\frac{5\sqrt{5x^2 + 2x + 3} (3215625000x^9 + 15281875000x^8 - 5561281250x^7 - 4105593750x^6 - 12554262500x^5 - 3227597000x^4 + 5959365525x^3 + 3721040355x^2 + 2291675850x - 249003936) - 4263523614\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{656250000}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (5\*sqrt[3 + 2\*x + 5\*x^2]\*(-249003936 + 2291675850\*x + 3721040355\*x^2 + 5959365525\*x^3 - 3227597000\*x^4 - 12554262500\*x^5 - 4105593750\*x^6 - 5561281250\*x^7 + 15281875000\*x^8 + 3215625000\*x^9) - 4263523614\*sqrt[5]\*ArcSinh[(1 + 5\*x)/sqrt[14]])/656250000

**IntegrateAlgebraic [A]** time = 1.05, size = 99, normalized size = 0.52

$$\frac{101512467 \log(\sqrt{5} \sqrt{5x^2 + 2x + 3} - 5x - 1)}{3125000\sqrt{5}} + \frac{\sqrt{5x^2 + 2x + 3} (3215625000x^9 + 15281875000x^8 - 5561281250x^7 - 4105593750x^6 - 12554262500x^5 - 3227597000x^4 + 5959365525x^3 + 3721040355x^2 + 2291675850x - 249003936)}{131250000}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-249003936 + 2291675850\*x + 3721040355\*x^2 + 5959365525\*x^3 - 3227597000\*x^4 - 12554262500\*x^5 - 4105593750\*x^6 - 5561281250\*x^7 + 15281875000\*x^8 + 3215625000\*x^9))/131250000 + (101512467\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(3125000\*Sqrt[5])

**fricas [A]** time = 0.82, size = 97, normalized size = 0.51

$$\frac{1}{131250000} (3215625000x^9 + 15281875000x^8 - 5561281250x^7 - 4105593750x^6 - 12554262500x^5 - 3227597000x^4 + 5959365525x^3 + 3721040355x^2 + 2291675850x - 249003936) \sqrt{5x^2 + 2x + 3} + \frac{101512467}{31250000} \sqrt{5} \log(\sqrt{5} \sqrt{5x^2 + 2x + 3} (5x + 1) - 25x^2 - 10x - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="fricas")

[Out] 1/131250000\*(3215625000\*x^9 + 15281875000\*x^8 - 5561281250\*x^7 - 4105593750\*x^6 - 12554262500\*x^5 - 3227597000\*x^4 + 5959365525\*x^3 + 3721040355\*x^2 + 2291675850\*x - 249003936)\*sqrt(5\*x^2 + 2\*x + 3) + 101512467/31250000\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.22, size = 92, normalized size = 0.49

$$\frac{1}{131250000} (5(5(10(25(5(7(140(105x + 499)x - 25423)x - 131379)x - 2008682)x - 12910388)x + 238374621)x + 744208071)x + 458335170)x - 249003936) \sqrt{5x^2 + 2x + 3} + \frac{101512467}{15625000} \sqrt{5} \log(-\sqrt{5}(\sqrt{5x - \sqrt{5x^2 + 2x + 3}}) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="giac")

[Out] 1/131250000\*(5\*((5\*(10\*(25\*(5\*(7\*(140\*(105\*x + 499)\*x - 25423)\*x - 131379)\*x - 2008682)\*x - 12910388)\*x + 238374621)\*x + 744208071)\*x + 458335170)\*x - 249003936)\*sqrt(5\*x^2 + 2\*x + 3) + 101512467/15625000\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)

**maple [A]** time = 0.01, size = 151, normalized size = 0.80

$$\frac{49(5x^2 + 2x + 3)^{\frac{5}{2}}x^5}{50} + \frac{581(5x^2 + 2x + 3)^{\frac{5}{2}}x^4}{150} - \frac{18379(5x^2 + 2x + 3)^{\frac{5}{2}}x^3}{3000} - \frac{219271(5x^2 + 2x + 3)^{\frac{5}{2}}x^2}{105000} + \frac{86721(5x^2 + 2x + 3)^{\frac{5}{2}}x}{21875} - \frac{101512467\sqrt{5} \operatorname{arcsinh}\left(\frac{\sqrt{5x^2 + 2x + 3}}{14}\right)}{15625000} + \frac{505667(5x^2 + 2x + 3)^{\frac{5}{2}}}{2187500} - \frac{14501781(10x + 2)\sqrt{5x^2 + 2x + 3}}{12500000} - \frac{690561(10x + 2)(5x^2 + 2x + 3)^{\frac{3}{2}}}{2500000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2), x)`

[Out]  $505667/2187500*(5*x^2+2*x+3)^{(5/2)}+49/50*(5*x^2+2*x+3)^{(5/2)}*x^5+581/150*(5*x^2+2*x+3)^{(5/2)}*x^4-18379/3000*(5*x^2+2*x+3)^{(5/2)}*x^3-219271/105000*(5*x^2+2*x+3)^{(5/2)}*x^2-101512467/15625000*5^{(1/2)}*\operatorname{arcsinh}(5/14*14^{(1/2)}*(x+1/5))-14501781/12500000*(10*x+2)*(5*x^2+2*x+3)^{(1/2)}+86721/21875*(5*x^2+2*x+3)^{(5/2)}*x-690561/2500000*(10*x+2)*(5*x^2+2*x+3)^{(3/2)}$

**maxima** [A] time = 0.98, size = 172, normalized size = 0.91

$\frac{49}{50}(5x^2+2x+3)^{\frac{5}{2}}x^5 + \frac{581}{150}(5x^2+2x+3)^{\frac{5}{2}}x^4 - \frac{18379}{3000}(5x^2+2x+3)^{\frac{5}{2}}x^3 - \frac{219271}{105000}(5x^2+2x+3)^{\frac{5}{2}}x^2 + \frac{86721}{21875}(5x^2+2x+3)^{\frac{5}{2}}x + \frac{505667}{2187500}(5x^2+2x+3)^{\frac{5}{2}} - \frac{690561}{250000}(5x^2+2x+3)^{\frac{3}{2}}x - \frac{690561}{1250000}(5x^2+2x+3)^{\frac{3}{2}} - \frac{14501781}{1250000}\sqrt{5x^2+2x+3}x - \frac{101512467}{15625000}\sqrt{5}\operatorname{arcsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{14501781}{6250000}\sqrt{5x^2+2x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2), x, algorithm="maxima")`

[Out]  $49/50*(5*x^2 + 2*x + 3)^{(5/2)}*x^5 + 581/150*(5*x^2 + 2*x + 3)^{(5/2)}*x^4 - 18379/3000*(5*x^2 + 2*x + 3)^{(5/2)}*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^{(5/2)}*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^{(5/2)}*x + 505667/2187500*(5*x^2 + 2*x + 3)^{(5/2)} - 690561/250000*(5*x^2 + 2*x + 3)^{(3/2)}*x - 690561/1250000*(5*x^2 + 2*x + 3)^{(3/2)} - 14501781/1250000*\operatorname{sqrt}(5*x^2 + 2*x + 3)*x - 101512467/15625000*\operatorname{sqrt}(5)*\operatorname{arcsinh}(1/14*\operatorname{sqrt}(14)*(5*x + 1)) - 14501781/6250000*\operatorname{sqrt}(5*x^2 + 2*x + 3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + 5x + 2) (5x^2 + 2x + 3)^{3/2} (-7x^2 + 4x + 1)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)`

[Out] `int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + 5x + 2) (5x^2 + 2x + 3)^{\frac{3}{2}} (7x^2 - 4x - 1)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2), x)`

[Out] `Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2, x)`

$$3.360 \quad \int (1 + 4x - 7x^2) (2 + 5x + x^2) (3 + 2x + 5x^2)^{3/2} dx$$

**Optimal.** Leaf size=147

$$-\frac{1163(5x^2 + 2x + 3)^{5/2} x^2}{1400} + \frac{2809(5x^2 + 2x + 3)^{5/2} x}{5250} + \frac{149509(5x^2 + 2x + 3)^{5/2}}{262500} - \frac{18397(5x + 1)(5x^2 + 2x + 3)^3}{150000}$$

**Rubi [A]** time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{40}(5x^2 + 2x + 3)^{5/2} x^3 - \frac{1163(5x^2 + 2x + 3)^{5/2} x^2}{1400} + \frac{2809(5x^2 + 2x + 3)^{5/2} x}{5250} + \frac{149509(5x^2 + 2x + 3)^{5/2}}{262500} - \frac{18397(5x + 1)(5x^2 + 2x + 3)^{3/2}}{150000} - \frac{128779(5x + 1)\sqrt{5x^2 + 2x + 3}}{250000} - \frac{901453 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125000\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-128779\*(1 + 5\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/250000 - (18397\*(1 + 5\*x)\*(3 + 2\*x + 5\*x^2)^(3/2))/150000 + (149509\*(3 + 2\*x + 5\*x^2)^(5/2))/262500 + (2809\*x\*(3 + 2\*x + 5\*x^2)^(5/2))/5250 - (1163\*x^2\*(3 + 2\*x + 5\*x^2)^(5/2))/1400 - (7\*x^3\*(3 + 2\*x + 5\*x^2)^(5/2))/40 - (901453\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(125000\*Sqrt[5])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 612

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((b + 2\*c\*x)\*(a + b\*x + c\*x^2)^p)/(2\*c\*(2\*p + 1)), x] - Dist[(p\*(b^2 - 4\*a\*c))/(2\*c\*(2\*p + 1)), Int[(a + b\*x + c\*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0] && GtQ[p, 0] && IntegerQ[4\*p]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b

$*e)/(2*c)$ ,  $\text{Int}[(a + b*x + c*x^2)^p, x]$ ,  $x]$  /;  $\text{FreeQ}\{a, b, c, d, e, p\}, x]$   
 $\&\& \text{NeQ}[2*c*d - b*e, 0]$   $\&\& \text{NeQ}[p, -1]$

### Rule 1661

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_)}], x\_Symbol]$   $\rightarrow$   $\text{With}\{q = \text{Expon}[Pq, x], e = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}$ ,  $\text{Simp}[(e*x^{(q - 1)}*(a + b*x + c*x^2)^{(p + 1)})/(c*(q + 2*p + 1)), x] + \text{Dist}[1/(c*(q + 2*p + 1)), \text{Int}[(a + b*x + c*x^2)^p * \text{ExpandToSum}[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^{(q - 2)} - b*e*(q + p)*x^{(q - 1)} - c*e*(q + 2*p + 1)*x^q, x], x]]$  /;  $\text{FreeQ}\{a, b, c, p\}, x]$   $\&\& \text{PolyQ}[Pq, x]$   $\&\& \text{NeQ}[b^2 - 4*a*c, 0]$   $\&\& !\text{LeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int (1 + 4x - 7x^2)(2 + 5x + x^2)(3 + 2x + 5x^2)^{3/2} dx &= -\frac{7}{40}x^3(3 + 2x + 5x^2)^{5/2} + \frac{1}{40} \int (3 + 2x + 5x^2)^{3/2} (80 - \\ &= -\frac{1163x^2(3 + 2x + 5x^2)^{5/2}}{1400} - \frac{7}{40}x^3(3 + 2x + 5x^2)^{5/2} + \dots \\ &= \frac{2809x(3 + 2x + 5x^2)^{5/2}}{5250} - \frac{1163x^2(3 + 2x + 5x^2)^{5/2}}{1400} - \dots \\ &= \frac{149509(3 + 2x + 5x^2)^{5/2}}{262500} + \frac{2809x(3 + 2x + 5x^2)^{5/2}}{5250} - \dots \\ &= -\frac{18397(1 + 5x)(3 + 2x + 5x^2)^{3/2}}{150000} + \frac{149509(3 + 2x + 5x^2)^{5/2}}{262500} \\ &= -\frac{128779(1 + 5x)\sqrt{3 + 2x + 5x^2}}{250000} - \frac{18397(1 + 5x)(3 + 2x + 5x^2)^{3/2}}{150000} \\ &= -\frac{128779(1 + 5x)\sqrt{3 + 2x + 5x^2}}{250000} - \frac{18397(1 + 5x)(3 + 2x + 5x^2)^{3/2}}{150000} \\ &= -\frac{128779(1 + 5x)\sqrt{3 + 2x + 5x^2}}{250000} - \frac{18397(1 + 5x)(3 + 2x + 5x^2)^{3/2}}{150000} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 75, normalized size = 0.51

$$\frac{-5\sqrt{5x^2 + 2x + 3} (22968750x^7 + 127406250x^6 + 48237500x^5 + 28373000x^4 - 78608475x^3 - 86464445x^2 - 36695150x - 22275576) - 37861026\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{26250000}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-5\*Sqrt[3 + 2\*x + 5\*x^2]\*(-22275576 - 36695150\*x - 86464445\*x^2 - 78608475\*x^3 + 28373000\*x^4 + 48237500\*x^5 + 127406250\*x^6 + 22968750\*x^7) - 37861026\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/26250000

**IntegrateAlgebraic [A]** time = 0.90, size = 89, normalized size = 0.61

$$\frac{901453 \log(\sqrt{5}\sqrt{5x^2+2x+3}-5x-1)}{125000\sqrt{5}} + \frac{\sqrt{5x^2+2x+3}(-22968750x^7-127406250x^6-48237500x^5-28373000x^4+78608475x^3+86464445x^2+36695150x+22275576)}{5250000}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(22275576 + 36695150\*x + 86464445\*x^2 + 78608475\*x^3 - 28373000\*x^4 - 48237500\*x^5 - 127406250\*x^6 - 22968750\*x^7))/5250000 + (901453\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(125000\*Sqrt[5])

**fricas [A]** time = 0.82, size = 87, normalized size = 0.59

$$-\frac{1}{5250000}(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-36695150x-22275576)\sqrt{5x^2+2x+3}+\frac{901453}{1250000}\sqrt{5}\log(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="fricas")

[Out] -1/5250000\*(22968750\*x^7 + 127406250\*x^6 + 48237500\*x^5 + 28373000\*x^4 - 78608475\*x^3 - 86464445\*x^2 - 36695150\*x - 22275576)\*sqrt(5\*x^2 + 2\*x + 3) + 901453/1250000\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.20, size = 82, normalized size = 0.56

$$-\frac{1}{5250000}(5((5(10(25(15(245x+1359)x+7718)x+113492)x-3144339)x-17292889)x-7339030)x-22275576)\sqrt{5x^2+2x+3}+\frac{901453}{625000}\sqrt{5}\log(-\sqrt{5}(\sqrt{5x-\sqrt{5x^2+2x+3}})-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2), x, algorithm="giac")

[Out] -1/5250000\*(5\*((5\*(10\*(25\*(15\*(245\*x + 1359)\*x + 7718)\*x + 113492)\*x - 3144339)\*x - 17292889)\*x - 7339030)\*x - 22275576)\*sqrt(5\*x^2 + 2\*x + 3) + 901453/625000\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)



**maple [A]** time = 0.01, size = 117, normalized size = 0.80

$$\frac{7(5x^2+2x+3)^{\frac{5}{2}}x^3}{40} - \frac{1163(5x^2+2x+3)^{\frac{5}{2}}x^2}{1400} + \frac{2809(5x^2+2x+3)^{\frac{5}{2}}x}{5250} - \frac{901453\sqrt{5}\operatorname{arcsinh}\left(\frac{5\sqrt{14}(x+\frac{1}{5})}{14}\right)}{625000} + \frac{149509(5x^2+2x+3)^{\frac{5}{2}}}{262500} - \frac{128779(10x+2)\sqrt{5x^2+2x+3}}{500000} - \frac{18397(10x+2)(5x^2+2x+3)^{\frac{3}{2}}}{300000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2), x)`

[Out]  $149509/262500*(5*x^2+2*x+3)^{(5/2)}-7/40*(5*x^2+2*x+3)^{(5/2)}*x^3-1163/1400*(5*x^2+2*x+3)^{(5/2)}*x^2-901453/625000*5^{(1/2)}*\operatorname{arcsinh}(5/14*14^{(1/2)}*(x+1/5))-128779/500000*(10*x+2)*(5*x^2+2*x+3)^{(1/2)}+2809/5250*(5*x^2+2*x+3)^{(5/2)}*x-18397/300000*(10*x+2)*(5*x^2+2*x+3)^{(3/2)}$

**maxima [A]** time = 0.96, size = 138, normalized size = 0.94

$$-\frac{7}{40}(5x^2+2x+3)^{\frac{5}{2}}x^3 - \frac{1163}{1400}(5x^2+2x+3)^{\frac{5}{2}}x^2 + \frac{2809}{5250}(5x^2+2x+3)^{\frac{5}{2}}x + \frac{149509}{262500}(5x^2+2x+3)^{\frac{5}{2}} - \frac{18397}{30000}(5x^2+2x+3)^{\frac{3}{2}}x - \frac{18397}{150000}(5x^2+2x+3)^{\frac{3}{2}} - \frac{128779}{50000}\sqrt{5x^2+2x+3}x - \frac{901453}{625000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{128779}{250000}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2), x, algorithm="maxima")`

[Out]  $-7/40*(5*x^2+2*x+3)^{(5/2)}*x^3 - 1163/1400*(5*x^2+2*x+3)^{(5/2)}*x^2 + 2809/5250*(5*x^2+2*x+3)^{(5/2)}*x + 149509/262500*(5*x^2+2*x+3)^{(5/2)} - 18397/30000*(5*x^2+2*x+3)^{(3/2)}*x - 18397/150000*(5*x^2+2*x+3)^{(3/2)} - 128779/50000*\operatorname{sqrt}(5*x^2+2*x+3)*x - 901453/625000*\operatorname{sqrt}(5)*\operatorname{arcsinh}(1/14*\operatorname{sqrt}(14)*(5*x+1)) - 128779/250000*\operatorname{sqrt}(5*x^2+2*x+3)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + 5x + 2) (5x^2 + 2x + 3)^{3/2} (-7x^2 + 4x + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)`

[Out] `int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int (-43x\sqrt{5x^2+2x+3}) dx - \int (-57x^2\sqrt{5x^2+2x+3}) dx - \int 14x^3\sqrt{5x^2+2x+3} dx - \int 48x^4\sqrt{5x^2+2x+3} dx - \int 169x^5\sqrt{5x^2+2x+3} dx - \int 35x^6\sqrt{5x^2+2x+3} dx - \int (-6\sqrt{5x^2+2x+3}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2), x)`

```
[Out] -Integral(-43*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-57*x**2*sqrt(5*x**2 + 2*x + 3), x) - Integral(14*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(48*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(169*x**5*sqrt(5*x**2 + 2*x + 3), x) - Integral(35*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2 + 2*x + 3), x)
```

$$3.361 \quad \int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2} dx$$

Optimal. Leaf size=210

$$-\frac{1}{980}(35x+267)(5x^2+2x+3)^{3/2} - \frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100} - \frac{6\sqrt{\frac{2}{11}}(8098902607-2434122235\sqrt{11})}{16807}$$

Rubi [A] time = 0.30, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{980}(35x+267)(5x^2+2x+3)^{3/2} - \frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100} - \frac{6\sqrt{\frac{2}{11}}(8098902607-2434122235\sqrt{11}) \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{16807} + \frac{6\sqrt{\frac{2}{11}}(8098902607+2434122235\sqrt{11}) \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{16807} - \frac{34425687 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{840350\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2), x]

[Out] (-3\*(571621 + 196105\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/240100 - ((267 + 35\*x)\*(3 + 2\*x + 5\*x^2)^(3/2))/980 - (34425687\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(840350\*Sqrt[5]) - (6\*Sqrt[(2\*(8098902607 - 2434122235\*Sqrt[11]))/11]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])]/16807 + (6\*Sqrt[(2\*(8098902607 + 2434122235\*Sqrt[11]))/11]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])]/16807

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^(p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2} dx &= -\frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2} - \int \frac{(-20358-79272x-100854x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2} \\
&= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x+5x^2)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.94, size = 202, normalized size = 0.96

$$\frac{-5\left(600\sqrt{1572625-425459\sqrt{11}}(61\sqrt{11}-143)\tanh^{-1}\left(\frac{-3\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)-600(143+61\sqrt{11})\sqrt{1572625+425459\sqrt{11}}\tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{250+34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)+77\sqrt{5x^2+2x+3}(42875x^3+344225x^2+744870x+1911108)\right)-757365114\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{92438500}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2), x]

[Out] (-757365114\*sqrt[5]\*ArcSinh[(1 + 5\*x)/sqrt[14]] - 5\*(77\*sqrt[3 + 2\*x + 5\*x^2]\*(1911108 + 744870\*x + 344225\*x^2 + 42875\*x^3) + 600\*sqrt[1572625 - 425459\*sqrt[11]]\*(-143 + 61\*sqrt[11])\*ArcTanh[(23 - sqrt[11] + 17\*x - 5\*sqrt[11]\*x)/(sqrt[250 - 34\*sqrt[11]]\*sqrt[3 + 2\*x + 5\*x^2])] - 600\*(143 + 61\*sqrt[11])\*sqrt[1572625 + 425459\*sqrt[11]]\*ArcTanh[(23 + sqrt[11] + (17 + 5\*sqrt[11])\*x)/(sqrt[250 + 34\*sqrt[11]]\*sqrt[3 + 2\*x + 5\*x^2])]))/92438500

**IntegrateAlgebraic [C]** time = 0.87, size = 254, normalized size = 1.21

$$\frac{12\text{RootSum}\left[7\theta^4 + 8\sqrt{5}\theta^3 - 70\theta^2 - 16\sqrt{5}\theta + 83\sqrt{5}, \frac{251851\sqrt{5}\theta^2 \log(-\theta + \sqrt{5x^2+2x+3} - \sqrt{5}) - 533850\theta \log(-\theta + \sqrt{5x^2+2x+3} - \sqrt{5}) - 648783\sqrt{5} \log(-\theta + \sqrt{5x^2+2x+3} - \sqrt{5})}{7\theta^3 + 6\sqrt{5}\theta^2 - 35\theta - 4\sqrt{5}}\right] + 34425687 \log(\sqrt{5}\sqrt{5x^2+2x+3} - 5x - 1) + \sqrt{5x^2+2x+3}(-42875x^3 - 344225x^2 - 744870x - 1911108)}{16807\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2), x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-1911108 - 744870\*x - 344225\*x^2 - 42875\*x^3))/240100 + (34425687\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(840350\*Sqrt[5]) - (12\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (-648783\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] - 533850\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 251851\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/(16807\*Sqrt[5])

**fricas** [B] time = 0.64, size = 326, normalized size = 1.55

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1), x, algorithm="fricas")

[Out] 3/184877\*sqrt(11)\*sqrt(2)\*sqrt(2434122235\*sqrt(11) + 8098902607)\*log(12\*(sqrt(2)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(2434122235\*sqrt(11) + 8098902607)\*(7690\*sqrt(11) - 24697) + 40555291\*sqrt(11)\*(x + 3) + 121665873\*x - 202776455)/x) - 3/184877\*sqrt(11)\*sqrt(2)\*sqrt(2434122235\*sqrt(11) + 8098902607)\*log(-12\*(sqrt(2)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(2434122235\*sqrt(11) + 8098902607)\*(7690\*sqrt(11) - 24697) - 40555291\*sqrt(11)\*(x + 3) - 121665873\*x + 202776455)/x) - 1/739508\*sqrt(11)\*sqrt(-701027203680\*sqrt(11) + 2332483950816)\*log(-(sqrt(5\*x^2 + 2\*x + 3)\*(7690\*sqrt(11) + 24697)\*sqrt(-701027203680\*sqrt(11) + 2332483950816) + 486663492\*sqrt(11)\*(x + 3) - 1459990476\*x + 2433317460)/x) + 1/739508\*sqrt(11)\*sqrt(-701027203680\*sqrt(11) + 2332483950816)\*log((sqrt(5\*x^2 + 2\*x + 3)\*(7690\*sqrt(11) + 24697)\*sqrt(-701027203680\*sqrt(11) + 2332483950816) - 486663492\*sqrt(11)\*(x + 3) + 1459990476\*x - 2433317460)/x) - 1/240100\*(42875\*x^3 + 344225\*x^2 + 744870\*x + 1911108)\*sqrt(5\*x^2 + 2\*x + 3) + 34425687/8403500\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac** [A] time = 0.28, size = 154, normalized size = 0.73

$\frac{1}{240100} (35 (35 (35 x + 281) x + 21282) x + 1911108) \sqrt{5 x^2 + 2 x + 3} + \frac{34425687}{4201750} \sqrt{5} \log(-5 \sqrt{5} x - \sqrt{5} + 5 \sqrt{5 x^2 + 2 x + 3}) + 19.386021168561 \log(\sqrt{5} x + \sqrt{5 x^2 + 2 x + 3} + 4.4012473649000) - 0.77368216442404 \log(\sqrt{5} x + \sqrt{5 x^2 + 2 x + 3} + 1.229516305400) - 19.386021168561 \log(\sqrt{5} x + \sqrt{5 x^2 + 2 x + 3} - 1.022980301300) + 0.77368216442454 \log(\sqrt{5} x + \sqrt{5 x^2 + 2 x + 3} - 2.09412544000)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1), x, algorithm="giac")

[Out] -1/240100\*(35\*(35\*(35\*x + 281)\*x + 21282)\*x + 1911108)\*sqrt(5\*x^2 + 2\*x + 3) + 34425687/4201750\*sqrt(5)\*log(-5\*sqrt(5)\*x - sqrt(5) + 5\*sqrt(5\*x^2 + 2\*x + 3))

$x + 3)) + 19.3580321168561 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) + 4.41924736459000) - 0.773682164624264 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) + 1.25295163054000) - 19.3580321168561 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) - 1.02258038113000) + 0.773682164625454 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) - 2.09411235400000)$

**maple [B]** time = 0.02, size = 730, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^2+5x+2) \cdot (5x^2+2x+3)^{(3/2)} / (-7x^2+4x+1), x)$

[Out] 
$$\begin{aligned} & -1/280 \cdot (10x+2) \cdot (5x^2+2x+3)^{(3/2)} - 3/200 \cdot (10x+2) \cdot (5x^2+2x+3)^{(1/2)} - 21/2 \\ & 50 \cdot 5^{(1/2)} \cdot \text{arcsinh}(5/14 \cdot 14^{(1/2)} \cdot (x+1/5)) - 3/154 \cdot (-61+13 \cdot 11^{(1/2)}) \cdot 11^{(1/2)} \cdot \\ & (1/21 \cdot (5 \cdot (x-2/7+1/7 \cdot 11^{(1/2)})^2 + (34/7-10/7 \cdot 11^{(1/2)}) \cdot (x-2/7+1/7 \cdot 11^{(1/2)})) + 2 \\ & 50/49 - 34/49 \cdot 11^{(1/2)})^{(3/2)} + 1/14 \cdot (34/7-10/7 \cdot 11^{(1/2)}) \cdot (1/20 \cdot (10x+2) \cdot (5 \cdot (x- \\ & 2/7+1/7 \cdot 11^{(1/2)})^2 + (34/7-10/7 \cdot 11^{(1/2)}) \cdot (x-2/7+1/7 \cdot 11^{(1/2)})) + 250/49 - 34/49 \cdot \\ & 11^{(1/2)})^{(1/2)} + 1/200 \cdot (5000/49 - 680/49 \cdot 11^{(1/2)} - (34/7-10/7 \cdot 11^{(1/2)})^2) \cdot 5^{(1/2)} \cdot \\ & \text{arcsinh}(5^{(1/2)} / (250/49 - 34/49 \cdot 11^{(1/2)} - 1/20 \cdot (34/7-10/7 \cdot 11^{(1/2)})^2)^{(1/2)} \cdot \\ & (x+1/5)) + 1/7 \cdot (250/49 - 34/49 \cdot 11^{(1/2)}) \cdot (1/7 \cdot (245 \cdot (x-2/7+1/7 \cdot 11^{(1/2)})^2 + 4 \\ & 9 \cdot (34/7-10/7 \cdot 11^{(1/2)}) \cdot (x-2/7+1/7 \cdot 11^{(1/2)})) + 250 - 34 \cdot 11^{(1/2)})^{(1/2)} + 1/10 \cdot (34 \\ & /7 - 10/7 \cdot 11^{(1/2)}) \cdot 5^{(1/2)} \cdot \text{arcsinh}(5^{(1/2)} / (250/49 - 34/49 \cdot 11^{(1/2)} - 1/20 \cdot (34/7 \\ & - 10/7 \cdot 11^{(1/2)})^2)^{(1/2)} \cdot (x+1/5)) - 7 \cdot (250/49 - 34/49 \cdot 11^{(1/2)}) / (250 - 34 \cdot 11^{(1/2)}) \\ & )^{(1/2)} \cdot \text{arctanh}(49/2 \cdot (500/49 - 68/49 \cdot 11^{(1/2)} + (34/7-10/7 \cdot 11^{(1/2)}) \cdot (x-2/7+1/ \\ & 7 \cdot 11^{(1/2)})) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x-2/7+1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7-1 \\ & 0/7 \cdot 11^{(1/2)}) \cdot (x-2/7+1/7 \cdot 11^{(1/2)})) + 250 - 34 \cdot 11^{(1/2)})^{(1/2)})) - 3/154 \cdot 11^{(1/2)} \\ & \cdot (61+13 \cdot 11^{(1/2)}) \cdot (1/21 \cdot (5 \cdot (x-2/7-1/7 \cdot 11^{(1/2)})^2 + (34/7+10/7 \cdot 11^{(1/2)}) \cdot (x-2 \\ & /7-1/7 \cdot 11^{(1/2)}) + 250/49 + 34/49 \cdot 11^{(1/2)})^{(3/2)} + 1/14 \cdot (34/7+10/7 \cdot 11^{(1/2)}) \cdot (1/ \\ & 20 \cdot (10x+2) \cdot (5 \cdot (x-2/7-1/7 \cdot 11^{(1/2)})^2 + (34/7+10/7 \cdot 11^{(1/2)}) \cdot (x-2/7-1/7 \cdot 11^{(1/2)})) \\ & + 250/49 + 34/49 \cdot 11^{(1/2)})^{(1/2)} + 1/200 \cdot (5000/49 + 680/49 \cdot 11^{(1/2)} - (34/7+10/7 \\ & \cdot 11^{(1/2)})^2) \cdot 5^{(1/2)} \cdot \text{arcsinh}(5^{(1/2)} / (250/49 + 34/49 \cdot 11^{(1/2)} - 1/20 \cdot (34/7+10/ \\ & 7 \cdot 11^{(1/2)})^2)^{(1/2)} \cdot (x+1/5)) + 1/7 \cdot (250/49 + 34/49 \cdot 11^{(1/2)}) \cdot (1/7 \cdot (245 \cdot (x-2/7 \\ & - 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7+10/7 \cdot 11^{(1/2)}) \cdot (x-2/7-1/7 \cdot 11^{(1/2)})) + 250 + 34 \cdot 11^{(1/2)}) \\ & )^{(1/2)} + 1/10 \cdot (34/7+10/7 \cdot 11^{(1/2)}) \cdot 5^{(1/2)} \cdot \text{arcsinh}(5^{(1/2)} / (250/49 + 34/49 \cdot 1 \\ & 1^{(1/2)} - 1/20 \cdot (34/7+10/7 \cdot 11^{(1/2)})^2)^{(1/2)} \cdot (x+1/5)) - 7 \cdot (250/49 + 34/49 \cdot 11^{(1/2)}) \\ & ) / (250 + 34 \cdot 11^{(1/2)})^{(1/2)} \cdot \text{arctanh}(49/2 \cdot (500/49 + 68/49 \cdot 11^{(1/2)} + (34/7+10/7 \cdot 1 \\ & 1^{(1/2)}) \cdot (x-2/7-1/7 \cdot 11^{(1/2)})) / (250 + 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x-2/7-1/7 \cdot 11^{(1/2)} \\ & )^2 + 49 \cdot (34/7+10/7 \cdot 11^{(1/2)}) \cdot (x-2/7-1/7 \cdot 11^{(1/2)})) + 250 + 34 \cdot 11^{(1/2)})^{(1/2)})) \end{aligned}$$

**maxima [B]** time = 1.31, size = 535, normalized size = 2.55

maxima [B] time = 1.31, size = 535, normalized size = 2.55

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1),x, algorithm="maxima")

[Out] 1/92438500\*sqrt(11)\*(19500\*sqrt(11)\*sqrt(2)\*(17\*sqrt(11) + 125)^(3/2)\*arcsinh(5/7\*sqrt(11)\*sqrt(7)\*sqrt(2)\*x/abs(14\*x - 2\*sqrt(11) - 4) + 17/7\*sqrt(7)\*sqrt(2)\*x/abs(14\*x - 2\*sqrt(11) - 4) + 1/7\*sqrt(11)\*sqrt(7)\*sqrt(2)/abs(14\*x - 2\*sqrt(11) - 4) + 23/7\*sqrt(7)\*sqrt(2)/abs(14\*x - 2\*sqrt(11) - 4)) - 300125\*sqrt(11)\*(5\*x^2 + 2\*x + 3)^(3/2)\*x - 3344250\*sqrt(11)\*(-34/49\*sqrt(11) + 250/49)^(3/2)\*arcsinh(5/7\*sqrt(11)\*sqrt(7)\*sqrt(2)\*x/abs(14\*x + 2\*sqrt(11) - 4) - 17/7\*sqrt(7)\*sqrt(2)\*x/abs(14\*x + 2\*sqrt(11) - 4) + 1/7\*sqrt(11)\*sqrt(7)\*sqrt(2)/abs(14\*x + 2\*sqrt(11) - 4) - 23/7\*sqrt(7)\*sqrt(2)/abs(14\*x + 2\*sqrt(11) - 4)) + 91500\*sqrt(2)\*(17\*sqrt(11) + 125)^(3/2)\*arcsinh(5/7\*sqrt(11)\*sqrt(7)\*sqrt(2)\*x/abs(14\*x - 2\*sqrt(11) - 4) + 17/7\*sqrt(7)\*sqrt(2)\*x/abs(14\*x - 2\*sqrt(11) - 4) + 1/7\*sqrt(11)\*sqrt(7)\*sqrt(2)/abs(14\*x - 2\*sqrt(11) - 4) + 23/7\*sqrt(7)\*sqrt(2)/abs(14\*x - 2\*sqrt(11) - 4)) + 15692250\*(-34/49\*sqrt(11) + 250/49)^(3/2)\*arcsinh(5/7\*sqrt(11)\*sqrt(7)\*sqrt(2)\*x/abs(14\*x + 2\*sqrt(11) - 4) - 17/7\*sqrt(7)\*sqrt(2)\*x/abs(14\*x + 2\*sqrt(11) - 4) + 1/7\*sqrt(11)\*sqrt(7)\*sqrt(2)/abs(14\*x + 2\*sqrt(11) - 4) - 23/7\*sqrt(7)\*sqrt(2)/abs(14\*x + 2\*sqrt(11) - 4)) - 2289525\*sqrt(11)\*(5\*x^2 + 2\*x + 3)^(3/2) - 20591025\*sqrt(11)\*sqrt(5\*x^2 + 2\*x + 3)\*x - 68851374\*sqrt(11)\*sqrt(5)\*arcsinh(5/14\*sqrt(7)\*sqrt(2)\*x + 1/14\*sqrt(7)\*sqrt(2)) - 60020205\*sqrt(11)\*sqrt(5\*x^2 + 2\*x + 3))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^2 + 5x + 2)(5x^2 + 2x + 3)^{3/2}}{-7x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2))/(4\*x - 7\*x^2 + 1),x)

[Out] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2))/(4\*x - 7\*x^2 + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx - \int \frac{19x\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx - \int \frac{23x^2\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx - \int \frac{27x^3\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx - \int \frac{5x^4\sqrt{5x^2 + 2x + 3}}{7x^2 - 4x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+5\*x+2)\*(5\*x\*\*2+2\*x+3)\*\*(3/2)/(-7\*x\*\*2+4\*x+1),x)

[Out] -Integral(6\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x) - Integral(19\*x\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x) - Integral(23\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x) - Integral(27\*x\*\*3\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x) - Integral(5\*x\*\*4\*sqrt(5\*x\*\*2 + 2\*x + 3)/(7\*x\*\*2 - 4\*x - 1), x)



$$3.362 \quad \int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} dx$$

**Optimal.** Leaf size=222

$$\frac{3(61x+3)(5x^2+2x+3)^{3/2}}{154(-7x^2+4x+1)} + \frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773} - \frac{\sqrt{\frac{1}{22}(52175400311-13155376531\sqrt{11})} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2}(125-17\sqrt{11})\sqrt{5x^2+2x+3}}\right)}{26411} - \frac{\sqrt{\frac{1}{22}(52175400311+13155376531\sqrt{11})} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2}(125+17\sqrt{11})\sqrt{5x^2+2x+3}}\right)}{26411} + \frac{16691 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{2401\sqrt{5}}$$

**Rubi [A]** time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, number of rules / integrand size = 0.229, Rules used = {1054, 1066, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)(5x^2+2x+3)^{3/2}}{154(-7x^2+4x+1)} + \frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773} - \frac{\sqrt{\frac{1}{22}(52175400311-13155376531\sqrt{11})} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2}(125-17\sqrt{11})\sqrt{5x^2+2x+3}}\right)}{26411} - \frac{\sqrt{\frac{1}{22}(52175400311+13155376531\sqrt{11})} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2}(125+17\sqrt{11})\sqrt{5x^2+2x+3}}\right)}{26411} + \frac{16691 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{2401\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2)^2,x]

[Out] ((5826 + 3395\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/3773 + (3\*(3 + 61\*x)\*(3 + 2\*x + 5\*x^2)^(3/2))/(154\*(1 + 4\*x - 7\*x^2)) + (16691\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(2401\*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531\*Sqrt[11])/22]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531\*Sqrt[11])/22]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/26411

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2)))]*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol]
:> Simp[((B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3)))] + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3)))]*x^2,
```

x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2\*p + 2\*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]

### Rule 1076

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e\*x + f\*x^2], x], x] + Dist[1/c, Int[(A\*c - a\*C + (B\*c - b\*C)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(2 + 5x + x^2)(3 + 2x + 5x^2)^{3/2}}{(1 + 4x - 7x^2)^2} dx &= \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{154(1 + 4x - 7x^2)} - \frac{1}{308} \int \frac{\sqrt{3 + 2x + 5x^2}(-912 + 724x + 1669x^2)}{1 + 4x - 7x^2} dx \\
 &= \frac{(5826 + 3395x)\sqrt{3 + 2x + 5x^2}}{3773} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{154(1 + 4x - 7x^2)} + \frac{\int \frac{700x^2}{1 + 4x - 7x^2} dx}{154} \\
 &= \frac{(5826 + 3395x)\sqrt{3 + 2x + 5x^2}}{3773} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{154(1 + 4x - 7x^2)} - \frac{\int \frac{2x^2}{1 + 4x - 7x^2} dx}{154} \\
 &= \frac{(5826 + 3395x)\sqrt{3 + 2x + 5x^2}}{3773} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{154(1 + 4x - 7x^2)} + \frac{1669x^2}{154} \\
 &= \frac{(5826 + 3395x)\sqrt{3 + 2x + 5x^2}}{3773} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{154(1 + 4x - 7x^2)} + \frac{1669x^2}{154} \\
 &= \frac{(5826 + 3395x)\sqrt{3 + 2x + 5x^2}}{3773} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{154(1 + 4x - 7x^2)} + \frac{1669x^2}{154}
 \end{aligned}$$

**Mathematica [A]** time = 1.83, size = 354, normalized size = 1.59

$\frac{1}{308} \sqrt{\frac{3}{5}} \frac{(74307\sqrt{11} - 1701489) \log(4x^2 + 14(\sqrt{11} - 2)x - 4\sqrt{11} + 15) - 10 \sqrt{\frac{3}{5}} (1701489 + 74307\sqrt{11}) \log(\sqrt{3 + 2x + 5x^2} \sqrt{5x^2 + 2x + 3} + (5 + 17\sqrt{11})x + 21\sqrt{11} + 15)}{154(1 + 4x - 7x^2)^2} - \frac{1}{308} \sqrt{\frac{3}{5}} \frac{(74307\sqrt{11} - 1701489) \log(\frac{2(2 + 5x + x^2)\sqrt{3 + 2x + 5x^2}}{1 + 4x - 7x^2})}{154(1 + 4x - 7x^2)^2} + \frac{1}{308} \sqrt{\frac{3}{5}} \frac{(74307\sqrt{11} - 1701489) \log(\frac{2(2 + 5x + x^2)\sqrt{3 + 2x + 5x^2}}{1 + 4x - 7x^2})}{154(1 + 4x - 7x^2)^2} + 10 \sqrt{\frac{3}{5}} \frac{(1701489 + 74307\sqrt{11}) \log(-7x + \sqrt{11} + 2) - 5 \sqrt{\frac{3}{5}} (1701489 + 74307\sqrt{11}) \log(7x + \sqrt{11} - 2)}{154(1 + 4x - 7x^2)^2} - \frac{1669x^2}{154}$

Antiderivative was successfully verified.

[In] Integrate[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2)^2,x]

[Out] ((770\*Sqrt[3 + 2\*x + 5\*x^2]\*(-12975 - 81181\*x + 34265\*x^2 + 2695\*x^3))/(-1 - 4\*x + 7\*x^2) + 8078444\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]] + 10\*Sqrt[22/(125 - 17\*Sqrt[11])]\*(-1701489 + 743879\*Sqrt[11])\*ArcTanh[(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])/(-23 + Sqrt[11] + (-17 + 5\*Sqrt[11])\*x)] + 10\*Sqrt[22/(125 + 17\*Sqrt[11])]\*(1701489 + 743879\*Sqrt[11])\*Log[2 + Sqrt[11] - 7\*x] - 5\*Sqrt[22/(125 - 17\*Sqrt[11])]\*(-1701489 + 743879\*Sqrt[11])\*Log[(-2 + Sqrt[11] + 7\*x)^2] + 5\*Sqrt[22/(125 - 17\*Sqrt[11])]\*(-1701489 + 743879\*Sqrt[11])\*Log[15 - 4\*Sqrt[11] + 14\*(-2 + Sqrt[11])\*x + 49\*x^2] - 10\*Sqrt[22/(125 + 17\*Sqrt[11])]\*(1701489 + 743879\*Sqrt[11])\*Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]])/5810420

**IntegrateAlgebraic [C]** time = 1.03, size = 451, normalized size = 2.03

$$\frac{28005 \sum \left[ \frac{781^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836}{117649\sqrt{5}} \frac{27709\sqrt{5}^2 \log(41 - \sqrt{5}227 - \sqrt{5}) - 100000 \log(41 - \sqrt{5}227 - \sqrt{5}) - 200000 \log(41 - \sqrt{5}227 - \sqrt{5})}{701^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836} \right]}{1294139\sqrt{5}} + \frac{38005 \sum \left[ \frac{781^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836}{1294139\sqrt{5}} \frac{27709\sqrt{5}^2 \log(41 - \sqrt{5}227 - \sqrt{5}) - 100000 \log(41 - \sqrt{5}227 - \sqrt{5}) - 200000 \log(41 - \sqrt{5}227 - \sqrt{5})}{701^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836} \right]}{1294139\sqrt{5}} + \frac{16691 \log(\sqrt{5}\sqrt{2^2 + 2^2 + 3 - 5x - 1})}{2401\sqrt{5}} + \frac{\sqrt{5}^2 + 2\sqrt{3} (2695x^3 + 34265x^2 - 81181x - 12975)}{7546(7x^2 - 4x - 1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2)^2,x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-12975 - 81181\*x + 34265\*x^2 + 2695\*x^3))/(7546\*(-1 - 4\*x + 7\*x^2)) - (16691\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(2401\*Sqrt[5]) + (2\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (25954129\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] - 19416530\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 2717099\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/(117649\*Sqrt[5]) - (3\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (225782939\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] - 137400830\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 7775369\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/(1294139\*Sqrt[5])

**fricas [B]** time = 0.70, size = 378, normalized size = 1.70

$$\frac{28005 \sum \left[ \frac{781^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836}{117649\sqrt{5}} \frac{27709\sqrt{5}^2 \log(41 - \sqrt{5}227 - \sqrt{5}) - 100000 \log(41 - \sqrt{5}227 - \sqrt{5}) - 200000 \log(41 - \sqrt{5}227 - \sqrt{5})}{701^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836} \right]}{1294139\sqrt{5}} + \frac{38005 \sum \left[ \frac{781^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836}{1294139\sqrt{5}} \frac{27709\sqrt{5}^2 \log(41 - \sqrt{5}227 - \sqrt{5}) - 100000 \log(41 - \sqrt{5}227 - \sqrt{5}) - 200000 \log(41 - \sqrt{5}227 - \sqrt{5})}{701^4 + 8\sqrt{5}81^3 - 7081^2 - 16\sqrt{5}81 + 836} \right]}{1294139\sqrt{5}} + \frac{16691 \log(\sqrt{5}\sqrt{2^2 + 2^2 + 3 - 5x - 1})}{2401\sqrt{5}} + \frac{\sqrt{5}^2 + 2\sqrt{3} (2695x^3 + 34265x^2 - 81181x - 12975)}{7546(7x^2 - 4x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1)^2,x, algorithm="fricas")

```
[Out] 1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 5385575055*x + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11) + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3) - 5385575055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 770*(2695*x^3 + 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4*x - 1)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error
%%{63274455776, [8]%%}+%%{%%{ [144627327488, 0] : [1, 0, -5]%%}, [7]%%}+%%{-852268179840, [6]%%}+%%{%%{-1735527929856, 0] : [1, 0, -5]%%}, [5]%%}+%%{6175070357568, [4]%%}+%%{%%{ [4607413432832, 0] : [1, 0, -5]%%}, [3]%%}+%%{-13352201484160, [2]%%}+%%{%%{-3429733766144, 0] : [1, 0, -5]%%}, [1]%%}+%%{8895871955936, [0]%%} / %%{245, [8]%%}+%%{%%{poly1[560, 0] : [1, 0, -5]%%}, [7]%%}+%%{-3300, [6]%%}+%%{%%{poly1[-6720, 0] : [1, 0, -5]%%}, [5]%%}+%%{23910, [4]%%}+%%{%%{poly1[17840, 0] : [1, 0, -5]%%}, [3]%%}+%%{-51700, [2]%%}+%%{%%{poly1[-13280, 0] : [1, 0, -5]%%}, [1]%%}+%%{34445, [0]%%} Error: Bad Argument Value
```

**maple** [B] time = 0.02, size = 1828, normalized size = 8.23

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] 161/484*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^3/2+1/14*(34/7-10/7*11^(1/2))*(1/2
```

$$\begin{aligned}
& 0*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})) \\
& +250/49-34/49*11^{(1/2)}\big)^{(1/2)}+1/200*(5000/49-680/49*11^{(1/2)}-(34/7-10/7* \\
& 11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7 \\
& *11^{(1/2)})^2)^{(1/2)}*(x+1/5)))+1/7*(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+ \\
& 1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250-34*11^{(1/2)} \\
& )\big)^{(1/2)}+1/10*(34/7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11 \\
& ^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1/2)} \\
& )/(250-34*11^{(1/2)})\big)^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11 \\
& ^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})\big)^{(1/2)}/(245*(x-2/7+1/7*11^{(1 \\
& /2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250-34*11^{(1/2)}) \\
& )+(183/44-39/44*11^{(1/2)})*(-1/49/(250/49-34/49*11^{(1/2)}))/(x-2/7+1/7*11^{(1/2)}) \\
& )*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250 \\
& /49-34/49*11^{(1/2)}\big)^{(5/2)}+3/98*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)}) \\
& *(1/3*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+2 \\
& 50/49-34/49*11^{(1/2)})\big)^{(3/2)}+1/2*(34/7-10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2 \\
& /7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250/49-34/49*1 \\
& 1^{(1/2)})\big)^{(1/2)}+1/200*(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)} \\
& *\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)} \\
& )*(x+1/5)))+(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34 \\
& /7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250-34*11^{(1/2)})\big)^{(1/2)}+1/10*(34/7-10 \\
& /7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7 \\
& *11^{(1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})\big)^{(1 \\
& /2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)} \\
& )))/(250-34*11^{(1/2)})\big)^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*1 \\
& 1^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250-34*11^{(1/2)})\big)^{(1/2)})))+20/49/(250/49-34/49 \\
& *11^{(1/2)})*(1/40*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x \\
& -2/7+1/7*11^{(1/2)}))+250/49-34/49*11^{(1/2)})\big)^{(3/2)}+3/80*(5000/49-680/49*11^{(1/2)} \\
& -(34/7-10/7*11^{(1/2)})^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-1 \\
& 0/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))+250/49-34/49*11^{(1/2)})\big)^{(1/2)}+1/200*(5000 \\
& /49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49 \\
& -34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))+183/44+39/4 \\
& 4*11^{(1/2)})*(-1/49/(250/49+34/49*11^{(1/2)}))/(x-2/7-1/7*11^{(1/2)})*(5*(x-2/7-1 \\
& /7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)}))+250/49+34/49*11^{(1 \\
& /2)})\big)^{(5/2)}+3/98*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(1/3*(5*(x-2/7 \\
& -1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)}))+250/49+34/49*11^{(1 \\
& /2)})\big)^{(3/2)}+1/2*(34/7+10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)} \\
& ))^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)}))+250/49+34/49*11^{(1/2)})\big)^{(1/2)}+ \\
& 1/200*(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1 \\
& /2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))+(25 \\
& 0/49+34/49*11^{(1/2)})*(1/7*(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)} \\
& ))*(x-2/7-1/7*11^{(1/2)}))+250+34*11^{(1/2)})\big)^{(1/2)}+1/10*(34/7+10/7*11^{(1/2)})*5^{( \\
& 1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{( \\
& 1/2)}*(x+1/5))-7*(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})\big)^{(1/2)}*\operatorname{arctanh}(49/ \\
& 2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34 \\
& *11^{(1/2)})\big)^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7
\end{aligned}$$

$$\begin{aligned}
& -1/7*11^{(1/2)}+250+34*11^{(1/2)})^{(1/2)})))+20/49/(250/49+34/49*11^{(1/2)})*(1/4 \\
& 0*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)}) \\
& )+250/49+34/49*11^{(1/2)})^{(3/2)}+3/80*(5000/49+680/49*11^{(1/2)}-(34/7+10/7*1 \\
& 1^{(1/2)})^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*( \\
& x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}+1/200*(5000/49+680/49*11^{( \\
& 1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)} \\
& -1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))))-161/484*11^{(1/2)}*(1/21*(5*( \\
& x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/4 \\
& 9*11^{(1/2)})^{(3/2)}+1/14*(34/7+10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7-1/7*11 \\
& ^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{( \\
& 1/2)}+1/200*(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsin} \\
& h(5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5) \\
& ))+1/7*(250/49+34/49*11^{(1/2)})*(1/7*(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10 \\
& /7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}+1/10*(34/7+10/7*11 \\
& ^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{( \\
& 1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*a \\
& rctanh(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)} \\
& ))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/ \\
& 2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 2x + 3)^{\frac{3}{2}}(x^2 + 5x + 2)}{(7x^2 - 4x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1)^2,x, algorithm="maxima")

[Out] integrate((5\*x^2 + 2\*x + 3)^(3/2)\*(x^2 + 5\*x + 2)/(7\*x^2 - 4\*x - 1)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^2 + 5x + 2)(5x^2 + 2x + 3)^{3/2}}{(-7x^2 + 4x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2))/(4\*x - 7\*x^2 + 1)^2,x)

[Out] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2))/(4\*x - 7\*x^2 + 1)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 5x + 2)(5x^2 + 2x + 3)^{\frac{3}{2}}}{(7x^2 - 4x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+5\*x+2)\*(5\*x\*\*2+2\*x+3)\*\*(3/2)/(-7\*x\*\*2+4\*x+1)\*\*2,x)

[Out] Integral((x\*\*2 + 5\*x + 2)\*(5\*x\*\*2 + 2\*x + 3)\*\*(3/2)/(7\*x\*\*2 - 4\*x - 1)\*\*2, x)



$$3.363 \quad \int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3} dx$$

**Optimal.** Leaf size=234

$$\frac{3(61x+3)(5x^2+2x+3)^{3/2}}{308(-7x^2+4x+1)^2} - \frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716(-7x^2+4x+1)} - \frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})-(-\sqrt{11}+23)}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{332024}$$

**Rubi [A]** time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)(5x^2+2x+3)^{3/2}}{308(-7x^2+4x+1)^2} - \frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716(-7x^2+4x+1)} - \frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})-(-\sqrt{11}+23)}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{332024} + \frac{\sqrt{\frac{62294197250171+2085440742055\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})+(-\sqrt{11}+23)}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{332024} - \frac{5}{343} \sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

Antiderivative was successfully verified.

[In] Int[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2)^3,x]

[Out] -((9495 - 37088\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(23716\*(1 + 4\*x - 7\*x^2)) + (3\*(3 + 61\*x)\*(3 + 2\*x + 5\*x^2)^(3/2))/(308\*(1 + 4\*x - 7\*x^2)^2) - (5\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055\*Sqrt[11])/2794]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])]/332024 + (Sqrt[(62294197250171 + 2085440742055\*Sqrt[11])/2794]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])]/332024

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

**Rule 215**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

**Rule 619**

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b

+ 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 1032

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x] - Dist[(2\*c\*g - h\*(b + q))/q, Int[1/((b + q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && PosQ[b^2 - 4\*a\*c]

### Rule 1054

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_)\*((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)\*((d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2)^(q\_), x\_Symbol] := Simp[((A\*b\*c - 2\*a\*B\*c + a\*b\*C - (c\*(b\*B - 2\*A\*c) - C\*(b^2 - 2\*a\*c))\*x)\*(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^q)/(c\*(b^2 - 4\*a\*c)\*(p + 1)), x] - Dist[1/(c\*(b^2 - 4\*a\*c)\*(p + 1)), Int[(a + b\*x + c\*x^2)^(p + 1)\*(d + e\*x + f\*x^2)^(q - 1)\*Simp[e\*q\*(A\*b\*c - 2\*a\*B\*c + a\*b\*C) - d\*(c\*(b\*B - 2\*A\*c)\*(2\*p + 3) + C\*(2\*a\*c - b^2\*(p + 2))) + (2\*f\*q\*(A\*b\*c - 2\*a\*B\*c + a\*b\*C) - e\*(c\*(b\*B - 2\*A\*c)\*(2\*p + q + 3) + C\*(2\*a\*c\*(q + 1) - b^2\*(p + q + 2)))]\*x - f\*(c\*(b\*B - 2\*A\*c)\*(2\*p + 2\*q + 3) + C\*(2\*a\*c\*(2\*q + 1) - b^2\*(p + 2\*q + 2)))\*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0] && LtQ[p, -1] && GtQ[q, 0] && !IGtQ[q, 0]

### Rule 1076

Int[((A\_.) + (B\_.)\*(x\_) + (C\_.)\*(x\_)^2)/(((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[C/c, Int[1/Sqrt[d + e\*x + f\*x^2], x], x] + Dist[1/c, Int[(A\*c - a\*C + (B\*c - b\*C)\*x)/((a + b\*x + c\*x^2)\*Sqrt[d + e\*x + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[e^2 - 4\*d\*f, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(2 + 5x + x^2)(3 + 2x + 5x^2)^{3/2}}{(1 + 4x - 7x^2)^3} dx &= \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{308(1 + 4x - 7x^2)^2} - \frac{1}{616} \int \frac{\sqrt{3 + 2x + 5x^2}(-2976 - 652x)}{(1 + 4x - 7x^2)^2} \\
&= -\frac{(9495 - 37088x)\sqrt{3 + 2x + 5x^2}}{23716(1 + 4x - 7x^2)} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{308(1 + 4x - 7x^2)^2} + \int \frac{1}{68} \\
&= -\frac{(9495 - 37088x)\sqrt{3 + 2x + 5x^2}}{23716(1 + 4x - 7x^2)} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{308(1 + 4x - 7x^2)^2} + \int \frac{1}{68} \\
&= -\frac{(9495 - 37088x)\sqrt{3 + 2x + 5x^2}}{23716(1 + 4x - 7x^2)} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{308(1 + 4x - 7x^2)^2} - \frac{1}{68} \\
&= -\frac{(9495 - 37088x)\sqrt{3 + 2x + 5x^2}}{23716(1 + 4x - 7x^2)} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{308(1 + 4x - 7x^2)^2} - \frac{5}{34} \\
&= -\frac{(9495 - 37088x)\sqrt{3 + 2x + 5x^2}}{23716(1 + 4x - 7x^2)} + \frac{3(3 + 61x)(3 + 2x + 5x^2)^{3/2}}{308(1 + 4x - 7x^2)^2} - \frac{5}{34}
\end{aligned}$$

**Mathematica [A]** time = 2.24, size = 376, normalized size = 1.61

$$\frac{\frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} - \frac{5}{34}}{1}$$

Antiderivative was successfully verified.

```

[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
[Out] ((11616*(655 + 5028*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (88*(13
8372 - 189161*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 212960*Sqrt[5]
*ArcSinh[(1 + 5*x)/Sqrt[14]] - 2*Sqrt[22/(125 - 17*Sqrt[11])] * (-7706073 + 6
74221*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-2
3 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])] * (770
6073 + 674221*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11
])] * (-7706073 + 674221*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(12
5 - 17*Sqrt[11])] * (-7706073 + 674221*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2
+ Sqrt[11])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])] * (7706073 + 674221

```

\*Sqrt[11])\*Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]]/14609056

**IntegrateAlgebraic [C]** time = 1.18, size = 636, normalized size = 2.72

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((2 + 5\*x + x^2)\*(3 + 2\*x + 5\*x^2)^(3/2))/(1 + 4\*x - 7\*x^2)^3,x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-7416 + 42767\*x + 246464\*x^2 - 189161\*x^3))/(23716\*(-1 - 4\*x + 7\*x^2)^2) + (5\*Sqrt[5]\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/343 - RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (4506829\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] - 1320270\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 64435\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ]/(33614\*Sqrt[5]) + RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (-16323208013227\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 151120773150070\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 21832390993791\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ]/(71748713246\*Sqrt[5]) - (3\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (-4192656948824863\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 24518831643829090\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 3523608887504055\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/(4726377211064\*Sqrt[5])

**fricas [B]** time = 0.74, size = 447, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1)^3,x, algorithm="fricas")

[Out] -1/1855350112\*(sqrt(2794)\*(49\*x^4 - 56\*x^3 + 2\*x^2 + 8\*x + 1)\*sqrt(2085440742055\*sqrt(11) + 62294197250171)\*log((sqrt(2794)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(2085440742055\*sqrt(11) + 62294197250171)\*(11840590\*sqrt(11) - 83479737) + 5426671202560069\*sqrt(11)\*(x + 3) + 16280013607680207\*x - 27133356012800345)/x) - sqrt(2794)\*(49\*x^4 - 56\*x^3 + 2\*x^2 + 8\*x + 1)\*sqrt(2085440742055\*sqrt(11) + 62294197250171)\*log(-(sqrt(2794)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(2085440742055\*sqrt(11) + 62294197250171)\*(11840590\*sqrt(11) - 83479737) - 5426671202560069\*sqrt(11)\*(x + 3) - 16280013607680207\*x + 27133356012800345)/x) +

```

sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11)
+ 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11)
) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 542667120256
0069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) - sqrt(
2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11) + 62
294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 5426671202560069*s
qrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/x) - 13522960*sq
rt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)
*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246464*x^2 - 42767*x
+ 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)

```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%{-1771684761728, [12]%%}+%%{-6074347754496, 0} : [1, 0, -5]%%, [11]%%
}+%%{-18439984254720, [10]%%}+%%{-120412580657152, 0} : [1, 0, -5]%%, [9]%%
}+%%{-108578966111616, [8]%%}+%%{-915119084156928, 0} : [1, 0, -5]%%, [7]
%%}+%%{-1093279290575360, [6]%%}+%%{-2784778529734656, 0} : [1, 0, -5]%%, [
5]%%}+%%{-4014694487954304, [4]%%}+%%{-3629195511796736, 0} : [1, 0, -5]%%
}, [3]%%}+%%{-5826260235237120, [2]%%}+%%{-1708007415539712, 0} : [1, 0, -5
]%%, [1]%%}+%%{-2953429489370752, [0]%%} / %%{-1715, 0} : [1, 0, -5]%%, [1
2]%%}+%%{29400, [11]%%}+%%{-17850, 0} : [1, 0, -5]%%, [10]%%}+%%{-58280
0, [9]%%}+%%{-105105, 0} : [1, 0, -5]%%, [8]%%}+%%{4429200, [7]%%}+%%{-1058300, 0} : [1, 0, -5]%%, [6]%%}+%%{-13478400, [5]%%}+%%{-3886245, 0} : [
1, 0, -5]%%, [4]%%}+%%{17565400, [3]%%}+%%{-5639850, 0} : [1, 0, -5]%%, [2]
%%}+%%{-8266800, [1]%%}+%%{-2858935, 0} : [1, 0, -5]%%, [0]%%} Error: Bad
Argument Value
```

**maple** [B] time = 0.02, size = 3828, normalized size = 16.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] 3535/21296*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(
```

$$\begin{aligned}
& 1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)} \\
& +250/49-34/49*11^{(1/2)})^{(1/2)}+1/200*(5000/49-680/49*11^{(1/2)}-(34/7-10 \\
& /7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10 \\
& 0/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))+1/7*(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2 \\
& /7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250-34*11^{(1/2)}) \\
& ^{(1/2)}+1/10*(34/7-10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49 \\
& *11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1 \\
& /2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7 \\
& *11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))/((250-34*11^{(1/2)})^{(1/2)})/(245*(x-2/7+1/7*11 \\
& ^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250-34*11^{(1/2)})^{(1/ \\
& 2)})))-21/968*(-61+13*11^{(1/2)})*11^{(1/2)}*(-1/686/(250/49-34/49*11^{(1/2)}))/(x- \\
& 2/7+1/7*11^{(1/2)})^2*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1 \\
& /7*11^{(1/2)}+250/49-34/49*11^{(1/2)})^{(5/2)}+1/1372*(34/7-10/7*11^{(1/2)})/(250/ \\
& 49-34/49*11^{(1/2)})*(-1/(250/49-34/49*11^{(1/2)}))/(x-2/7+1/7*11^{(1/2)})*(5*(x-2 \\
& /7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250/49-34/49*1 \\
& 1^{(1/2)})^{(5/2)}+3/2*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(1/3*(5*(x- \\
& 2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250/49-34/49* \\
& 11^{(1/2)})^{(3/2)}+1/2*(34/7-10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1 \\
& /2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250/49-34/49*11^{(1/2)})^{(1/ \\
& 2)}+1/200*(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5 \\
& ^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))+ \\
& (250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)} \\
& ^{(1/2)}*(x-2/7+1/7*11^{(1/2)}+250-34*11^{(1/2)})^{(1/2)}+1/10*(34/7-10/7*11^{(1/2)}) \\
& *5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2 \\
& )^{(1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}( \\
& 49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}))/((250 \\
& -34*11^{(1/2)})^{(1/2)})/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x- \\
& 2/7+1/7*11^{(1/2)}+250-34*11^{(1/2)})^{(1/2)})))+20/(250/49-34/49*11^{(1/2)})*(1/4 \\
& 0*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/ \\
& 2)}+250/49-34/49*11^{(1/2)})^{(3/2)}+3/80*(5000/49-680/49*11^{(1/2)}-(34/7-10/7*1 \\
& 1^{(1/2)})^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*( \\
& x-2/7+1/7*11^{(1/2)}+250/49-34/49*11^{(1/2)})^{(1/2)}+1/200*(5000/49-680/49*11^{( \\
& 1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)} \\
& -1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))))+15/686/(250/49-34/49*11^{(1/ \\
& 2)})*(1/3*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)} \\
& )+250/49-34/49*11^{(1/2)})^{(3/2)}+1/2*(34/7-10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*( \\
& x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250/49-34/4 \\
& 9*11^{(1/2)})^{(1/2)}+1/200*(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{( \\
& 1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{( \\
& 1/2)}*(x+1/5)))+(250/49-34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)})^2+49* \\
& (34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}+250-34*11^{(1/2)})^{(1/2)}+1/10*(34/7 \\
& -10/7*11^{(1/2)})*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10 \\
& 0/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))-7*(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)}) \\
& ^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7* \\
& 11^{(1/2)}))/((250-34*11^{(1/2)})^{(1/2)})/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/
\end{aligned}$$

$$\begin{aligned}
& 7*11^{(1/2)}*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)})))-(-3535/1936-273 \\
& /1936*11^{(1/2)}*(-1/49/(250/49+34/49*11^{(1/2)})/(x-2/7-1/7*11^{(1/2)})*(5*(x-2 \\
& /7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*1 \\
& 1^{(1/2)})^{(5/2)}+3/98*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(1/3*(5*(x \\
& -2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49 \\
& *11^{(1/2)})^{(3/2)}+1/2*(34/7+10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)} \\
& (1/2))^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1 \\
& /2)}+1/200*(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}( \\
& 5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5))) \\
& +(250/49+34/49*11^{(1/2)})*(1/7*(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)} \\
& (1/2))*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}+1/10*(34/7+10/7*11^{(1/2)} \\
& )*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)}-1/20*(34/7+10/7*11^{(1/2)})^2 \\
& )^{(1/2)}*(x+1/5))-7*(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh} \\
& (49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(25 \\
& 0+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x \\
& -2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)})))+20/49/(250/49+34/49*11^{(1/2)})* \\
& (1/40*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11 \\
& ^{(1/2)})+250/49+34/49*11^{(1/2)})^{(3/2)}+3/80*(5000/49+680/49*11^{(1/2)}-(34/7+10 \\
& /7*11^{(1/2)})^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)} \\
& ))*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}+1/200*(5000/49+680/49* \\
& 11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49+34/49*11^{(1/2)} \\
& (1/2)-1/20*(34/7+10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))))-(-3535/1936+273/1936*11 \\
& ^{(1/2)}*(-1/49/(250/49-34/49*11^{(1/2)})/(x-2/7+1/7*11^{(1/2)})*(5*(x-2/7+1/7*1 \\
& 1^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)}) \\
& ^{(5/2)}+3/98*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(1/3*(5*(x-2/7+1/7 \\
& *11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)} \\
& ))^{(3/2)}+1/2*(34/7-10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+ \\
& (34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}+1/20 \\
& 0*(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/ \\
& (250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))+(250/49 \\
& -34/49*11^{(1/2)})*(1/7*(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*( \\
& x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}+1/10*(34/7-10/7*11^{(1/2)})*5^{(1/2)} \\
& )*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/20*(34/7-10/7*11^{(1/2)})^2)^{(1/2)} \\
& *(x+1/5))-7*(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(5 \\
& 00/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)} \\
& ^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7 \\
& *11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)})))+20/49/(250/49-34/49*11^{(1/2)})*(1/40*(1 \\
& 0*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+ \\
& 250/49-34/49*11^{(1/2)})^{(3/2)}+3/80*(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)} \\
& (1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/ \\
& 7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}+1/200*(5000/49-680/49*11^{(1/2)} \\
& -(34/7-10/7*11^{(1/2)})^2)*5^{(1/2)}*\operatorname{arcsinh}(5^{(1/2)}/(250/49-34/49*11^{(1/2)}-1/2 \\
& 0*(34/7-10/7*11^{(1/2)})^2)^{(1/2)}*(x+1/5)))))-3535/21296*11^{(1/2)}*(1/21*(5*(x \\
& -2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49 \\
& *11^{(1/2)})^{(3/2)}+1/14*(34/7+10/7*11^{(1/2)})*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& (1/2))^2 + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(1/2)} \\
& + 1/200 * (5000/49 + 680/49 * 11^{(1/2)} - (34/7 + 10/7 * 11^{(1/2)})^2) * 5^{(1/2)} * \operatorname{arcsinh} \\
& (5^{(1/2)} / (250/49 + 34/49 * 11^{(1/2)} - 1/20 * (34/7 + 10/7 * 11^{(1/2)})^2)^{(1/2)} * (x + 1/5)) \\
& ) + 1/7 * (250/49 + 34/49 * 11^{(1/2)}) * (1/7 * (245 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + 49 * (34/7 + 10/ \\
& 7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250 + 34 * 11^{(1/2)})^{(1/2)} + 1/10 * (34/7 + 10/7 * 11^{(1/2)} \\
& (1/2)) * 5^{(1/2)} * \operatorname{arcsinh}(5^{(1/2)} / (250/49 + 34/49 * 11^{(1/2)} - 1/20 * (34/7 + 10/7 * 11^{(1/2)} \\
& (1/2))^{(1/2)} * (x + 1/5)) - 7 * (250/49 + 34/49 * 11^{(1/2)}) / (250 + 34 * 11^{(1/2)})^{(1/2)} * \operatorname{ar} \\
& \operatorname{ctanh}(49/2 * (500/49 + 68/49 * 11^{(1/2)} + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)})) \\
& ) / (250 + 34 * 11^{(1/2)})^{(1/2)} / (245 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + 49 * (34/7 + 10/7 * 11^{(1/2)} \\
& )) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250 + 34 * 11^{(1/2)})^{(1/2)})) - 21/968 * (61 + 13 * 11^{(1/2)}) * 1 \\
& 1^{(1/2)} * (-1/686 / (250/49 + 34/49 * 11^{(1/2)}) / (x - 2/7 - 1/7 * 11^{(1/2)})^2 * (5 * (x - 2/7 - 1/ \\
& 7 * 11^{(1/2)})^2 + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)} \\
& (1/2))^{(5/2)} + 1/1372 * (34/7 + 10/7 * 11^{(1/2)}) / (250/49 + 34/49 * 11^{(1/2)}) * (-1 / (250/49 + 3 \\
& 4/49 * 11^{(1/2)}) / (x - 2/7 - 1/7 * 11^{(1/2)}) * (5 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + (34/7 + 10/7 * 11 \\
& ^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(5/2)} + 3/2 * (34/7 + 10/7 * 11 \\
& ^{(1/2)}) / (250/49 + 34/49 * 11^{(1/2)}) * (1/3 * (5 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + (34/7 + 10/7 * 1 \\
& 1^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(3/2)} + 1/2 * (34/7 + 10/7 * 1 \\
& 1^{(1/2)}) * (1/20 * (10 * x + 2) * (5 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2 \\
& /7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(1/2)} + 1/200 * (5000/49 + 680/49 * 11^{(1/2)} \\
& ) - (34/7 + 10/7 * 11^{(1/2)})^2) * 5^{(1/2)} * \operatorname{arcsinh}(5^{(1/2)} / (250/49 + 34/49 * 11^{(1/2)} - 1/ \\
& 20 * (34/7 + 10/7 * 11^{(1/2)})^2)^{(1/2)} * (x + 1/5))) + (250/49 + 34/49 * 11^{(1/2)}) * (1/7 * (24 \\
& 5 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + 49 * (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250 + 3 \\
& 4 * 11^{(1/2)})^{(1/2)} + 1/10 * (34/7 + 10/7 * 11^{(1/2)}) * 5^{(1/2)} * \operatorname{arcsinh}(5^{(1/2)} / (250/49 \\
& + 34/49 * 11^{(1/2)} - 1/20 * (34/7 + 10/7 * 11^{(1/2)})^2)^{(1/2)} * (x + 1/5)) - 7 * (250/49 + 34/49 \\
& * 11^{(1/2)}) / (250 + 34 * 11^{(1/2)})^{(1/2)} * \operatorname{arctanh}(49/2 * (500/49 + 68/49 * 11^{(1/2)} + (34/ \\
& 7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)})) / (250 + 34 * 11^{(1/2)})^{(1/2)} / (245 * (x - 2/7 - \\
& 1/7 * 11^{(1/2)})^2 + 49 * (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250 + 34 * 11^{(1/2)} \\
& ))^{(1/2)})) + 20 / (250/49 + 34/49 * 11^{(1/2)}) * (1/40 * (10 * x + 2) * (5 * (x - 2/7 - 1/7 * 11^{(1/2)} \\
& ))^2 + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(3/2)} \\
& + 3/80 * (5000/49 + 680/49 * 11^{(1/2)} - (34/7 + 10/7 * 11^{(1/2)})^2) * (1/20 * (10 * x + 2) * (5 * (x \\
& - 2/7 - 1/7 * 11^{(1/2)})^2 + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 \\
& * 11^{(1/2)})^{(1/2)} + 1/200 * (5000/49 + 680/49 * 11^{(1/2)} - (34/7 + 10/7 * 11^{(1/2)})^2) * 5^{(1/2)} * \operatorname{arcsinh}(5^{(1/2)} / (250/49 + 34 \\
& /49 * 11^{(1/2)} - 1/20 * (34/7 + 10/7 * 11^{(1/2)})^2)^{(1/2)} * (x + 1/5))) + 15/686 / (250/49 + 34/49 * 11^{(1/2)}) * (1/3 * (5 * (x - 2/7 - 1/7 * 11^{(1/2)} \\
& ))^2 + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(3/2)} + 1 \\
& /2 * (34/7 + 10/7 * 11^{(1/2)}) * (1/20 * (10 * x + 2) * (5 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + (34/7 + 10/7 \\
& * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)}) + 250/49 + 34/49 * 11^{(1/2)})^{(1/2)} + 1/200 * (5000/49 \\
& + 680/49 * 11^{(1/2)} - (34/7 + 10/7 * 11^{(1/2)})^2) * 5^{(1/2)} * \operatorname{arcsinh}(5^{(1/2)} / (250/49 + 34 \\
& /49 * 11^{(1/2)} - 1/20 * (34/7 + 10/7 * 11^{(1/2)})^2)^{(1/2)} * (x + 1/5))) + (250/49 + 34/49 * 11^{(1/2)} \\
& (1/2)) * (1/7 * (245 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + 49 * (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * \\
& 11^{(1/2)}) + 250 + 34 * 11^{(1/2)})^{(1/2)} + 1/10 * (34/7 + 10/7 * 11^{(1/2)}) * 5^{(1/2)} * \operatorname{arcsinh}( \\
& 5^{(1/2)} / (250/49 + 34/49 * 11^{(1/2)} - 1/20 * (34/7 + 10/7 * 11^{(1/2)})^2)^{(1/2)} * (x + 1/5)) - \\
& 7 * (250/49 + 34/49 * 11^{(1/2)}) / (250 + 34 * 11^{(1/2)})^{(1/2)} * \operatorname{arctanh}(49/2 * (500/49 + 68/4 \\
& 9 * 11^{(1/2)} + (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)})) / (250 + 34 * 11^{(1/2)})^{(1/2)} / (245 * (x - 2/7 - 1/7 * 11^{(1/2)})^2 + 49 * (34/7 + 10/7 * 11^{(1/2)}) * (x - 2/7 - 1/7 * 11^{(1/2)})
\end{aligned}$$



+250+34\*11^(1/2))^((1/2))))))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(5x^2 + 2x + 3)^{\frac{3}{2}}(x^2 + 5x + 2)}{(7x^2 - 4x - 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)\*(5\*x^2+2\*x+3)^(3/2)/(-7\*x^2+4\*x+1)^3,x, algorithm="maxima")

[Out] -integrate((5\*x^2 + 2\*x + 3)^(3/2)\*(x^2 + 5\*x + 2)/(7\*x^2 - 4\*x - 1)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^2 + 5x + 2)(5x^2 + 2x + 3)^{3/2}}{(-7x^2 + 4x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2))/(4\*x - 7\*x^2 + 1)^3,x)

[Out] int(((5\*x + x^2 + 2)\*(2\*x + 5\*x^2 + 3)^(3/2))/(4\*x - 7\*x^2 + 1)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx - \int \frac{19x\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx - \int \frac{23x^2\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx - \int \frac{27x^3\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx - \int \frac{5x^4\sqrt{5x^2 + 2x + 3}}{343x^6 - 588x^5 + 189x^4 + 104x^3 - 27x^2 - 12x - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+5\*x+2)\*(5\*x\*\*2+2\*x+3)\*\*(3/2)/(-7\*x\*\*2+4\*x+1)\*\*3,x)

[Out] -Integral(6\*sqrt(5\*x\*\*2 + 2\*x + 3)/(343\*x\*\*6 - 588\*x\*\*5 + 189\*x\*\*4 + 104\*x\*\*3 - 27\*x\*\*2 - 12\*x - 1), x) - Integral(19\*x\*sqrt(5\*x\*\*2 + 2\*x + 3)/(343\*x\*\*6 - 588\*x\*\*5 + 189\*x\*\*4 + 104\*x\*\*3 - 27\*x\*\*2 - 12\*x - 1), x) - Integral(23\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3)/(343\*x\*\*6 - 588\*x\*\*5 + 189\*x\*\*4 + 104\*x\*\*3 - 27\*x\*\*2 - 12\*x - 1), x) - Integral(27\*x\*\*3\*sqrt(5\*x\*\*2 + 2\*x + 3)/(343\*x\*\*6 - 588\*x\*\*5 + 189\*x\*\*4 + 104\*x\*\*3 - 27\*x\*\*2 - 12\*x - 1), x) - Integral(5\*x\*\*4\*sqrt(5\*x\*\*2 + 2\*x + 3)/(343\*x\*\*6 - 588\*x\*\*5 + 189\*x\*\*4 + 104\*x\*\*3 - 27\*x\*\*2 - 12\*x - 1), x)

$$3.364 \quad \int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$$

Optimal. Leaf size=185

$$\frac{40722851\sqrt{5x^2+2x+3}x^2}{750000} + \frac{5793077\sqrt{5x^2+2x+3}x}{75000} - \frac{16515809\sqrt{5x^2+2x+3}}{156250} - \frac{343}{40}\sqrt{5x^2+2x+3}x^7 - \frac{1141}{40}\sqrt{5x^2+2x+3}$$

**Rubi [A]** time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1661, 640, 619, 215}

$$-\frac{343}{40}\sqrt{5x^2+2x+3}x^7 - \frac{1141}{40}\sqrt{5x^2+2x+3}x^6 + \frac{26159}{300}\sqrt{5x^2+2x+3}x^5 - \frac{47807\sqrt{5x^2+2x+3}x^4}{3750} - \frac{5160533\sqrt{5x^2+2x+3}x^3}{50000} + \frac{40722851\sqrt{5x^2+2x+3}x^2}{750000} + \frac{5793077\sqrt{5x^2+2x+3}x}{75000} - \frac{16515809\sqrt{5x^2+2x+3}}{156250} - \frac{77513689 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-16515809\*Sqrt[3 + 2\*x + 5\*x^2])/156250 + (5793077\*x\*Sqrt[3 + 2\*x + 5\*x^2])/75000 + (40722851\*x^2\*Sqrt[3 + 2\*x + 5\*x^2])/750000 - (5160533\*x^3\*Sqrt[3 + 2\*x + 5\*x^2])/50000 - (47807\*x^4\*Sqrt[3 + 2\*x + 5\*x^2])/3750 + (26159\*x^5\*Sqrt[3 + 2\*x + 5\*x^2])/300 - (1141\*x^6\*Sqrt[3 + 2\*x + 5\*x^2])/40 - (343\*x^7\*Sqrt[3 + 2\*x + 5\*x^2])/40 - (77513689\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(625000\*Sqrt[5])

Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1 + 4x - 7x^2)^3 (2 + 5x + x^2)}{\sqrt{3 + 2x + 5x^2}} dx &= -\frac{343}{40} x^7 \sqrt{3 + 2x + 5x^2} + \frac{1}{40} \int \frac{80 + 1160x + 4600x^2 - 2440x^3 - 34840x^4 + 28000x^5 + 28000x^6 + 161000x^7}{\sqrt{3 + 2x + 5x^2}} dx \\
&= -\frac{1141}{40} x^6 \sqrt{3 + 2x + 5x^2} - \frac{343}{40} x^7 \sqrt{3 + 2x + 5x^2} + \int \frac{2800 + 40600x + 161000x^2}{\sqrt{3 + 2x + 5x^2}} dx \\
&= \frac{26159}{300} x^5 \sqrt{3 + 2x + 5x^2} - \frac{1141}{40} x^6 \sqrt{3 + 2x + 5x^2} - \frac{343}{40} x^7 \sqrt{3 + 2x + 5x^2} \\
&= -\frac{47807x^4 \sqrt{3 + 2x + 5x^2}}{3750} + \frac{26159}{300} x^5 \sqrt{3 + 2x + 5x^2} - \frac{1141}{40} x^6 \sqrt{3 + 2x + 5x^2} \\
&= -\frac{5160533x^3 \sqrt{3 + 2x + 5x^2}}{50000} - \frac{47807x^4 \sqrt{3 + 2x + 5x^2}}{3750} + \frac{26159}{300} x^5 \sqrt{3 + 2x + 5x^2} \\
&= \frac{40722851x^2 \sqrt{3 + 2x + 5x^2}}{750000} - \frac{5160533x^3 \sqrt{3 + 2x + 5x^2}}{50000} - \frac{47807x^4 \sqrt{3 + 2x + 5x^2}}{3750} \\
&= \frac{5793077x \sqrt{3 + 2x + 5x^2}}{75000} + \frac{40722851x^2 \sqrt{3 + 2x + 5x^2}}{750000} - \frac{5160533x^3 \sqrt{3 + 2x + 5x^2}}{50000} \\
&= -\frac{16515809 \sqrt{3 + 2x + 5x^2}}{156250} + \frac{5793077x \sqrt{3 + 2x + 5x^2}}{75000} + \frac{40722851x^2 \sqrt{3 + 2x + 5x^2}}{750000} \\
&= -\frac{16515809 \sqrt{3 + 2x + 5x^2}}{156250} + \frac{5793077x \sqrt{3 + 2x + 5x^2}}{75000} + \frac{40722851x^2 \sqrt{3 + 2x + 5x^2}}{750000} \\
&= -\frac{16515809 \sqrt{3 + 2x + 5x^2}}{156250} + \frac{5793077x \sqrt{3 + 2x + 5x^2}}{75000} + \frac{40722851x^2 \sqrt{3 + 2x + 5x^2}}{750000}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 75, normalized size = 0.41

$$\frac{-5\sqrt{5x^2+2x+3} (32156250x^7 + 106968750x^6 - 326987500x^5 + 47807000x^4 + 387039975x^3 - 203614255x^2 - 289653850x + 396379416) - 465082134\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{18750000}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-5\*Sqrt[3 + 2\*x + 5\*x^2]\*(396379416 - 289653850\*x - 203614255\*x^2 + 387039975\*x^3 + 47807000\*x^4 - 326987500\*x^5 + 106968750\*x^6 + 32156250\*x^7) - 465082134\*Sqrt[5]\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/18750000

**IntegrateAlgebraic [A]** time = 0.94, size = 89, normalized size = 0.48

$$\frac{77513689 \log(\sqrt{5}\sqrt{5x^2+2x+3}-5x-1)}{625000\sqrt{5}} + \frac{\sqrt{5x^2+2x+3}(-32156250x^7-106968750x^6+326987500x^5-47807000x^4-387039975x^3+203614255x^2+289653850x-396379416)}{3750000}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(-396379416 + 289653850\*x + 203614255\*x^2 - 387039975\*x^3 - 47807000\*x^4 + 326987500\*x^5 - 106968750\*x^6 - 32156250\*x^7))/3750000 + (77513689\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(625000\*Sqrt[5])

**fricas [A]** time = 0.99, size = 87, normalized size = 0.47

$$-\frac{1}{3750000} (32156250x^7 + 106968750x^6 - 326987500x^5 + 47807000x^4 + 387039975x^3 - 203614255x^2 - 289653850x + 396379416)\sqrt{5x^2+2x+3} + \frac{77513689}{6250000} \sqrt{5} \log(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1) - 25x^2 - 10x - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2), x, algorithm="fricas")

[Out] -1/3750000\*(32156250\*x^7 + 106968750\*x^6 - 326987500\*x^5 + 47807000\*x^4 + 387039975\*x^3 - 203614255\*x^2 - 289653850\*x + 396379416)\*sqrt(5\*x^2 + 2\*x + 3) + 77513689/6250000\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.25, size = 82, normalized size = 0.44

$$-\frac{1}{3750000} (5((5(10(175(15(49x+163)x-7474)x+191228)x+15481599)x-40722851)x-57930770)x+396379416)\sqrt{5x^2+2x+3} + \frac{77513689}{3125000} \sqrt{5} \log(-\sqrt{5}(\sqrt{5x-\sqrt{5x^2+2x+3}})-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2),x, algorithm="giac")

[Out] -1/3750000\*(5\*((5\*(10\*(175\*(15\*(49\*x + 163)\*x - 7474)\*x + 191228)\*x + 15481599)\*x - 40722851)\*x - 57930770)\*x + 396379416)\*sqrt(5\*x^2 + 2\*x + 3) + 77513689/3125000\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)

**maple [A]** time = 0.03, size = 147, normalized size = 0.79

$$\frac{-343\sqrt{5x^2+2x+3}x^7}{40} - \frac{1141\sqrt{5x^2+2x+3}x^6}{40} + \frac{26159\sqrt{5x^2+2x+3}x^5}{300} - \frac{47807\sqrt{5x^2+2x+3}x^4}{3750} - \frac{5160533\sqrt{5x^2+2x+3}x^3}{50000} + \frac{40722851\sqrt{5x^2+2x+3}x^2}{750000} + \frac{5793077\sqrt{5x^2+2x+3}x}{75000} - \frac{77513689\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}(x+1)}{14}\right)}{3125000} - \frac{16515809\sqrt{5x^2+2x+3}}{156250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2),x)

[Out] -16515809/156250\*(5\*x^2+2\*x+3)^(1/2)-77513689/3125000\*5^(1/2)\*arcsinh(5/14\*14^(1/2)\*(x+1/5))-343/40\*x^7\*(5\*x^2+2\*x+3)^(1/2)-1141/40\*x^6\*(5\*x^2+2\*x+3)^(1/2)+26159/300\*x^5\*(5\*x^2+2\*x+3)^(1/2)-47807/3750\*x^4\*(5\*x^2+2\*x+3)^(1/2)-5160533/50000\*x^3\*(5\*x^2+2\*x+3)^(1/2)+40722851/750000\*x^2\*(5\*x^2+2\*x+3)^(1/2)+5793077/75000\*x\*(5\*x^2+2\*x+3)^(1/2)

**maxima [A]** time = 0.99, size = 148, normalized size = 0.80

$$\frac{-343\sqrt{5x^2+2x+3}x^7}{40} - \frac{1141\sqrt{5x^2+2x+3}x^6}{40} + \frac{26159\sqrt{5x^2+2x+3}x^5}{300} - \frac{47807\sqrt{5x^2+2x+3}x^4}{3750} - \frac{5160533\sqrt{5x^2+2x+3}x^3}{50000} + \frac{40722851\sqrt{5x^2+2x+3}x^2}{750000} + \frac{5793077\sqrt{5x^2+2x+3}x}{75000} - \frac{77513689\sqrt{5} \operatorname{arcsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{3125000} - \frac{16515809\sqrt{5x^2+2x+3}}{156250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2),x, algorithm="maxima")

[Out] -343/40\*sqrt(5\*x^2 + 2\*x + 3)\*x^7 - 1141/40\*sqrt(5\*x^2 + 2\*x + 3)\*x^6 + 26159/300\*sqrt(5\*x^2 + 2\*x + 3)\*x^5 - 47807/3750\*sqrt(5\*x^2 + 2\*x + 3)\*x^4 - 5160533/50000\*sqrt(5\*x^2 + 2\*x + 3)\*x^3 + 40722851/750000\*sqrt(5\*x^2 + 2\*x + 3)\*x^2 + 5793077/75000\*sqrt(5\*x^2 + 2\*x + 3)\*x - 77513689/3125000\*sqrt(5)\*arcsinh(1/14\*sqrt(14)\*(5\*x + 1)) - 16515809/156250\*sqrt(5\*x^2 + 2\*x + 3)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2)(-7x^2 + 4x + 1)^3}{\sqrt{5x^2 + 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(4\*x - 7\*x^2 + 1)^3)/(2\*x + 5\*x^2 + 3)^(1/2),x)

[Out] int(((5\*x + x^2 + 2)\*(4\*x - 7\*x^2 + 1)^3)/(2\*x + 5\*x^2 + 3)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(\frac{29x}{\sqrt{5x^2+2x+3}}\right)dx - \int\left(\frac{-115x^2}{\sqrt{5x^2+2x+3}}\right)dx - \int\frac{61x^3}{\sqrt{5x^2+2x+3}}dx - \int\frac{871x^4}{\sqrt{5x^2+2x+3}}dx - \int\left(\frac{-127x^5}{\sqrt{5x^2+2x+3}}\right)dx - \int\left(\frac{-2065x^6}{\sqrt{5x^2+2x+3}}\right)dx - \int\frac{1127x^7}{\sqrt{5x^2+2x+3}}dx - \int\frac{343x^8}{\sqrt{5x^2+2x+3}}dx - \int\left(\frac{2}{\sqrt{5x^2+2x+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x\*\*2+4\*x+1)\*\*3\*(x\*\*2+5\*x+2)/(5\*x\*\*2+2\*x+3)\*\*(1/2),x)

[Out] -Integral(-29\*x/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(-115\*x\*\*2/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(61\*x\*\*3/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(871\*x\*\*4/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(-127\*x\*\*5/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(-2065\*x\*\*6/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(1127\*x\*\*7/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(343\*x\*\*8/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(-2/sqrt(5\*x\*\*2 + 2\*x + 3), x)

$$3.365 \quad \int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$$

Optimal. Leaf size=143

$$-\frac{207427\sqrt{5x^2+2x+3}x^2}{37500} + \frac{36073\sqrt{5x^2+2x+3}x}{1875} - \frac{22053\sqrt{5x^2+2x+3}}{31250} + \frac{49}{30}\sqrt{5x^2+2x+3}x^5 + \frac{5131}{750}\sqrt{5x^2+2x+3}x^4$$

**Rubi [A]** time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1661, 640, 619, 215}

$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5 + \frac{5131}{750}\sqrt{5x^2+2x+3}x^4 - \frac{33259\sqrt{5x^2+2x+3}x^3}{2500} - \frac{207427\sqrt{5x^2+2x+3}x^2}{37500} + \frac{36073\sqrt{5x^2+2x+3}x}{1875} - \frac{22053\sqrt{5x^2+2x+3}}{31250} - \frac{1719097 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{31250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (-22053\*Sqrt[3 + 2\*x + 5\*x^2])/31250 + (36073\*x\*Sqrt[3 + 2\*x + 5\*x^2])/1875 - (207427\*x^2\*Sqrt[3 + 2\*x + 5\*x^2])/37500 - (33259\*x^3\*Sqrt[3 + 2\*x + 5\*x^2])/2500 + (5131\*x^4\*Sqrt[3 + 2\*x + 5\*x^2])/750 + (49\*x^5\*Sqrt[3 + 2\*x + 5\*x^2])/30 - (1719097\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(31250\*Sqrt[5])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x +

```

c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(1 + 4x - 7x^2)^2 (2 + 5x + x^2)}{\sqrt{3 + 2x + 5x^2}} dx &= \frac{49}{30} x^5 \sqrt{3 + 2x + 5x^2} + \frac{1}{30} \int \frac{60 + 630x + 1350x^2 - 2820x^3 - 6135x^4 + 5131x^5}{\sqrt{3 + 2x + 5x^2}} dx \\
&= \frac{5131}{750} x^4 \sqrt{3 + 2x + 5x^2} + \frac{49}{30} x^5 \sqrt{3 + 2x + 5x^2} + \frac{1}{750} \int \frac{1500 + 15750x + 15750x^2 - 207427x^3 - 33259x^4 + 36073x^5}{\sqrt{3 + 2x + 5x^2}} dx \\
&= -\frac{33259x^3 \sqrt{3 + 2x + 5x^2}}{2500} + \frac{5131}{750} x^4 \sqrt{3 + 2x + 5x^2} + \frac{49}{30} x^5 \sqrt{3 + 2x + 5x^2} \\
&= -\frac{207427x^2 \sqrt{3 + 2x + 5x^2}}{37500} - \frac{33259x^3 \sqrt{3 + 2x + 5x^2}}{2500} + \frac{5131}{750} x^4 \sqrt{3 + 2x + 5x^2} \\
&= \frac{36073x \sqrt{3 + 2x + 5x^2}}{1875} - \frac{207427x^2 \sqrt{3 + 2x + 5x^2}}{37500} - \frac{33259x^3 \sqrt{3 + 2x + 5x^2}}{2500} \\
&= -\frac{22053 \sqrt{3 + 2x + 5x^2}}{31250} + \frac{36073x \sqrt{3 + 2x + 5x^2}}{1875} - \frac{207427x^2 \sqrt{3 + 2x + 5x^2}}{37500} \\
&= -\frac{22053 \sqrt{3 + 2x + 5x^2}}{31250} + \frac{36073x \sqrt{3 + 2x + 5x^2}}{1875} - \frac{207427x^2 \sqrt{3 + 2x + 5x^2}}{37500} \\
&= -\frac{22053 \sqrt{3 + 2x + 5x^2}}{31250} + \frac{36073x \sqrt{3 + 2x + 5x^2}}{1875} - \frac{207427x^2 \sqrt{3 + 2x + 5x^2}}{37500}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 65, normalized size = 0.45

$$\frac{5\sqrt{5x^2 + 2x + 3} (306250x^5 + 1282750x^4 - 2494425x^3 - 1037135x^2 + 3607300x - 132318) - 10314582\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{937500}$$

Antiderivative was successfully verified.

```

[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2], x]

```



[Out]  $(5\sqrt{3 + 2x + 5x^2}) \cdot (-132318 + 3607300x - 1037135x^2 - 2494425x^3 + 1282750x^4 + 306250x^5) - 10314582\sqrt{5} \operatorname{ArcSinh}[(1 + 5x)/\sqrt{14}]) / 937500$

**IntegrateAlgebraic [A]** time = 0.67, size = 79, normalized size = 0.55

$$\frac{1719097 \log(\sqrt{5}\sqrt{5x^2 + 2x + 3} - 5x - 1)}{31250\sqrt{5}} + \frac{\sqrt{5x^2 + 2x + 3} (306250x^5 + 1282750x^4 - 2494425x^3 - 1037135x^2 + 3607300x - 132318)}{187500}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out]  $(\sqrt{3 + 2x + 5x^2}) \cdot (-132318 + 3607300x - 1037135x^2 - 2494425x^3 + 1282750x^4 + 306250x^5) / 187500 + (1719097 \cdot \operatorname{Log}[-1 - 5x + \sqrt{5} \sqrt{3 + 2x + 5x^2}]) / (31250 \sqrt{5})$

**fricas [A]** time = 0.69, size = 77, normalized size = 0.54

$$\frac{1}{187500} (306250x^5 + 1282750x^4 - 2494425x^3 - 1037135x^2 + 3607300x - 132318) \sqrt{5x^2 + 2x + 3} + \frac{1719097}{312500} \sqrt{5} \log(\sqrt{5}\sqrt{5x^2 + 2x + 3}(5x + 1) - 25x^2 - 10x - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2), x, algorithm="fricas")

[Out]  $1/187500 \cdot (306250x^5 + 1282750x^4 - 2494425x^3 - 1037135x^2 + 3607300x - 132318) \cdot \sqrt{5x^2 + 2x + 3} + 1719097/312500 \cdot \sqrt{5} \cdot \log(\sqrt{5} \cdot \sqrt{5x^2 + 2x + 3} \cdot (5x + 1) - 25x^2 - 10x - 8)$

**giac [A]** time = 0.39, size = 72, normalized size = 0.50

$$\frac{1}{187500} (5((5(70(175x + 733)x - 99777)x - 207427)x + 721460)x - 132318) \sqrt{5x^2 + 2x + 3} + \frac{1719097}{156250} \sqrt{5} \log(-\sqrt{5}(\sqrt{5x - \sqrt{5x^2 + 2x + 3}}) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2), x, algorithm="giac")

[Out]  $1/187500 \cdot (5 \cdot ((5 \cdot (70 \cdot (175x + 733)x - 99777)x - 207427)x + 721460)x - 132318) \cdot \sqrt{5x^2 + 2x + 3} + 1719097/156250 \cdot \sqrt{5} \cdot \log(-\sqrt{5} \cdot (\sqrt{5x - \sqrt{5x^2 + 2x + 3}}) - 1)$

**maple [A]** time = 0.01, size = 113, normalized size = 0.79

$$\frac{49\sqrt{5x^2 + 2x + 3} x^5}{30} + \frac{5131\sqrt{5x^2 + 2x + 3} x^4}{750} - \frac{33259\sqrt{5x^2 + 2x + 3} x^3}{2500} - \frac{207427\sqrt{5x^2 + 2x + 3} x^2}{37500} + \frac{36073\sqrt{5x^2 + 2x + 3} x}{1875} - \frac{1719097\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250} - \frac{22053\sqrt{5x^2 + 2x + 3}}{31250}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)`

[Out]  $-22053/31250*(5*x^2+2*x+3)^{(1/2)}-1719097/156250*5^{(1/2)}*\operatorname{arcsinh}(5/14*14^{(1/2)}*(x+1/5))+49/30*(5*x^2+2*x+3)^{(1/2)}*x^5+5131/750*(5*x^2+2*x+3)^{(1/2)}*x^4-33259/2500*(5*x^2+2*x+3)^{(1/2)}*x^3-207427/37500*(5*x^2+2*x+3)^{(1/2)}*x^2+36073/1875*(5*x^2+2*x+3)^{(1/2)}*x$

**maxima** [A] time = 0.97, size = 114, normalized size = 0.80

$\frac{49}{30}\sqrt{5x^2+2x+3}x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}x^4-\frac{33259}{2500}\sqrt{5x^2+2x+3}x^3-\frac{207427}{37500}\sqrt{5x^2+2x+3}x^2+\frac{36073}{1875}\sqrt{5x^2+2x+3}x-\frac{1719097}{156250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{22053}{31250}\sqrt{5x^2+2x+3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")`

[Out]  $49/30*\operatorname{sqrt}(5*x^2+2*x+3)*x^5+5131/750*\operatorname{sqrt}(5*x^2+2*x+3)*x^4-33259/2500*\operatorname{sqrt}(5*x^2+2*x+3)*x^3-207427/37500*\operatorname{sqrt}(5*x^2+2*x+3)*x^2+36073/1875*\operatorname{sqrt}(5*x^2+2*x+3)*x-1719097/156250*\operatorname{sqrt}(5)*\operatorname{arcsinh}(1/14*\operatorname{sqrt}(14)*(5*x+1))-22053/31250*\operatorname{sqrt}(5*x^2+2*x+3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2)(-7x^2 + 4x + 1)^2}{\sqrt{5x^2 + 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)`

[Out] `int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 5x + 2)(7x^2 - 4x - 1)^2}{\sqrt{5x^2 + 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)`

[Out] `Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/sqrt(5*x**2 + 2*x + 3), x)`

$$3.366 \quad \int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$$

**Optimal.** Leaf size=101

$$-\frac{571}{300}\sqrt{5x^2+2x+3}x^2 + \frac{59}{30}\sqrt{5x^2+2x+3}x + \frac{463}{125}\sqrt{5x^2+2x+3} - \frac{7}{20}\sqrt{5x^2+2x+3}x^3 - \frac{1901 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

**Rubi [A]** time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$ , Rules used = {1661, 640, 619, 215}

$$-\frac{7}{20}\sqrt{5x^2+2x+3}x^3 - \frac{571}{300}\sqrt{5x^2+2x+3}x^2 + \frac{59}{30}\sqrt{5x^2+2x+3}x + \frac{463}{125}\sqrt{5x^2+2x+3} - \frac{1901 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (463\*Sqrt[3 + 2\*x + 5\*x^2])/125 + (59\*x\*Sqrt[3 + 2\*x + 5\*x^2])/30 - (571\*x^2\*Sqrt[3 + 2\*x + 5\*x^2])/300 - (7\*x^3\*Sqrt[3 + 2\*x + 5\*x^2])/20 - (1901\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(250\*Sqrt[5])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1661

Int[(Pq\_)\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e\*x^(q - 1)\*(a + b\*x +

$c*x^2)^{(p+1)}/(c*(q+2*p+1)), x] + \text{Dist}[1/(c*(q+2*p+1)), \text{Int}[(a + b*x + c*x^2)^p \text{ExpandToSum}[c*(q+2*p+1)*Pq - a*e*(q-1)*x^{(q-2)} - b*e*(q+p)*x^{(q-1)} - c*e*(q+2*p+1)*x^q, x], x]] /; \text{FreeQ}[a, b, c, p], x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{LeQ}[p, -1]$

Rubi steps

$$\begin{aligned} \int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx &= -\frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{1}{20} \int \frac{40+260x+203x^2-571x^3}{\sqrt{3+2x+5x^2}} dx \\ &= -\frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{1}{300} \int \frac{600+7326x+5903x^2-571x^3}{\sqrt{3+2x+5x^2}} dx \\ &= \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{\int \frac{-11}{\sqrt{3+2x+5x^2}} dx}{300} \\ &= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} \\ &= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} \\ &= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 55, normalized size = 0.54

$$\frac{-5\sqrt{5x^2+2x+3}(525x^3+2855x^2-2950x-5556) - 11406\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{7500}$$

Antiderivative was successfully verified.

[In] Integrate[((1+4\*x-7\*x^2)\*(2+5\*x+x^2))/Sqrt[3+2\*x+5\*x^2],x]

[Out] (-5\*Sqrt[3+2\*x+5\*x^2]\*(-5556-2950\*x+2855\*x^2+525\*x^3)-11406\*Sqrt[5]\*ArcSinh[(1+5\*x)/Sqrt[14]])/7500

**IntegrateAlgebraic [A]** time = 0.56, size = 69, normalized size = 0.68

$$\frac{1901 \log\left(\sqrt{5}\sqrt{5x^2+2x+3}-5x-1\right)}{250\sqrt{5}} + \frac{\sqrt{5x^2+2x+3}(-525x^3-2855x^2+2950x+5556)}{1500}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2))/Sqrt[3 + 2\*x + 5\*x^2], x]

[Out] (Sqrt[3 + 2\*x + 5\*x^2]\*(5556 + 2950\*x - 2855\*x^2 - 525\*x^3))/1500 + (1901\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(250\*Sqrt[5])

**fricas** [A] time = 0.79, size = 67, normalized size = 0.66

$$-\frac{1}{1500} (525x^3 + 2855x^2 - 2950x - 5556)\sqrt{5x^2 + 2x + 3} + \frac{1901}{2500} \sqrt{5} \log\left(\sqrt{5}\sqrt{5x^2 + 2x + 3}(5x + 1) - 25x^2 - 10x - 8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2), x, algorithm="fricas")

[Out] -1/1500\*(525\*x^3 + 2855\*x^2 - 2950\*x - 5556)\*sqrt(5\*x^2 + 2\*x + 3) + 1901/2500\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac** [A] time = 0.22, size = 62, normalized size = 0.61

$$-\frac{1}{1500} (5((105x + 571)x - 590)x - 5556)\sqrt{5x^2 + 2x + 3} + \frac{1901}{1250} \sqrt{5} \log\left(-\sqrt{5}\left(\sqrt{5}x - \sqrt{5x^2 + 2x + 3}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2), x, algorithm="giac")

[Out] -1/1500\*(5\*((105\*x + 571)\*x - 590)\*x - 5556)\*sqrt(5\*x^2 + 2\*x + 3) + 1901/1250\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1)

**maple** [A] time = 0.01, size = 79, normalized size = 0.78

$$-\frac{7\sqrt{5x^2 + 2x + 3} x^3}{20} - \frac{571\sqrt{5x^2 + 2x + 3} x^2}{300} + \frac{59\sqrt{5x^2 + 2x + 3} x}{30} - \frac{1901\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250} + \frac{463\sqrt{5x^2 + 2x + 3}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2), x)

[Out] -7/20\*(5\*x^2+2\*x+3)^(1/2)\*x^3-571/300\*(5\*x^2+2\*x+3)^(1/2)\*x^2+59/30\*(5\*x^2+2\*x+3)^(1/2)\*x+463/125\*(5\*x^2+2\*x+3)^(1/2)-1901/1250\*5^(1/2)\*arcsinh(5/14\*14^(1/2)\*(x+1/5))

**maxima** [A] time = 0.96, size = 80, normalized size = 0.79

$$-\frac{7}{20} \sqrt{5x^2 + 2x + 3} x^3 - \frac{571}{300} \sqrt{5x^2 + 2x + 3} x^2 + \frac{59}{30} \sqrt{5x^2 + 2x + 3} x - \frac{1901}{1250} \sqrt{5} \operatorname{arsinh}\left(\frac{1}{14} \sqrt{14}(5x+1)\right) + \frac{463}{125} \sqrt{5x^2 + 2x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(1/2),x, algorithm="maxima")

[Out] -7/20\*sqrt(5\*x^2 + 2\*x + 3)\*x^3 - 571/300\*sqrt(5\*x^2 + 2\*x + 3)\*x^2 + 59/30\*sqrt(5\*x^2 + 2\*x + 3)\*x - 1901/1250\*sqrt(5)\*arcsinh(1/14\*sqrt(14)\*(5\*x + 1)) + 463/125\*sqrt(5\*x^2 + 2\*x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2)(-7x^2 + 4x + 1)}{\sqrt{5x^2 + 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x^2 + 2)\*(4\*x - 7\*x^2 + 1))/(2\*x + 5\*x^2 + 3)^(1/2),x)

[Out] int(((5\*x + x^2 + 2)\*(4\*x - 7\*x^2 + 1))/(2\*x + 5\*x^2 + 3)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{13x}{\sqrt{5x^2 + 2x + 3}}\right) dx - \int \left(-\frac{7x^2}{\sqrt{5x^2 + 2x + 3}}\right) dx - \int \frac{31x^3}{\sqrt{5x^2 + 2x + 3}} dx - \int \frac{7x^4}{\sqrt{5x^2 + 2x + 3}} dx - \int \left(-\frac{2}{\sqrt{5x^2 + 2x + 3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x\*\*2+4\*x+1)\*(x\*\*2+5\*x+2)/(5\*x\*\*2+2\*x+3)\*\*(1/2),x)

[Out] -Integral(-13\*x/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(-7\*x\*\*2/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(31\*x\*\*3/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(7\*x\*\*4/sqrt(5\*x\*\*2 + 2\*x + 3), x) - Integral(-2/sqrt(5\*x\*\*2 + 2\*x + 3), x)

$$3.367 \quad \int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}} dx$$

Optimal. Leaf size=164

$$-\frac{3}{14} \sqrt{\frac{4091-1055\sqrt{11}}{2794}} \tanh^{-1} \left( \frac{(17-5\sqrt{11})x - \sqrt{11} + 23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}} \right) + \frac{3}{14} \sqrt{\frac{4091+1055\sqrt{11}}{2794}} \tanh^{-1} \left( \frac{(17+5\sqrt{11})x + \sqrt{11} + 23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}} \right)$$

**Rubi [A]** time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$ , Rules used = {1076, 619, 215, 1032, 724, 206}

$$-\frac{3}{14} \sqrt{\frac{4091-1055\sqrt{11}}{2794}} \tanh^{-1} \left( \frac{(17-5\sqrt{11})x - \sqrt{11} + 23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}} \right) + \frac{3}{14} \sqrt{\frac{4091+1055\sqrt{11}}{2794}} \tanh^{-1} \left( \frac{(17+5\sqrt{11})x + \sqrt{11} + 23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}} \right) - \frac{\sinh^{-1} \left( \frac{5x+1}{\sqrt{14}} \right)}{7\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)\*Sqrt[3 + 2\*x + 5\*x^2]),x]

[Out] -ArcSinh[(1 + 5\*x)/Sqrt[14]]/(7\*Sqrt[5]) - (3\*Sqrt[(4091 - 1055\*Sqrt[11])/2794]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/14 + (3\*Sqrt[(4091 + 1055\*Sqrt[11])/2794]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/14

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol]
:> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

### Rubi steps

$$\int \frac{2 + 5x + x^2}{(1 + 4x - 7x^2)\sqrt{3 + 2x + 5x^2}} dx = -\left(\frac{1}{7} \int \frac{1}{\sqrt{3 + 2x + 5x^2}} dx\right) - \frac{1}{7} \int \frac{-15 - 39x}{(1 + 4x - 7x^2)\sqrt{3 + 2x + 5x^2}} dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{56}}} dx, x, 2 + 10x\right)}{14\sqrt{70}} + \frac{1}{77} \left(3(143 - 61\sqrt{11})\right) \int \frac{1}{(4 - 2\sqrt{11})\sqrt{3 + 2x + 5x^2}} dx$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{1}{77} \left(6(143 - 61\sqrt{11})\right) \text{Subst}\left(\int \frac{1}{2352 + 112(4 - 2\sqrt{11})\sqrt{3 + 2x + 5x^2}} dx, x, 2 + 10x\right)$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{3}{14} \sqrt{\frac{4091 - 1055\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{23 - \sqrt{11} + (17 - 5\sqrt{11})x}{\sqrt{2(125 - 17\sqrt{11})}\sqrt{3 + 2x + 5x^2}}\right)$$



**Mathematica [A]** time = 0.47, size = 157, normalized size = 0.96

$$\frac{3 \left( \sqrt{4091 - 1055\sqrt{11}} \tanh^{-1} \left( \frac{-5\sqrt{11}x + 17x - \sqrt{11} + 23}{\sqrt{250 - 34\sqrt{11}} \sqrt{5x^2 + 2x + 3}} \right) - \sqrt{4091 + 1055\sqrt{11}} \tanh^{-1} \left( \frac{5\sqrt{11}x + 17x + \sqrt{11} + 23}{\sqrt{250 + 34\sqrt{11}} \sqrt{5x^2 + 2x + 3}} \right) \right)}{14\sqrt{2794}} - \frac{\sinh^{-1} \left( \frac{5x+1}{\sqrt{14}} \right)}{7\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)\*Sqrt[3 + 2\*x + 5\*x^2]),x]

[Out] -1/7\*ArcSinh[(1 + 5\*x)/Sqrt[14]]/Sqrt[5] - (3\*(Sqrt[4091 - 1055\*Sqrt[11]]\*ArcTanH[(23 - Sqrt[11] + 17\*x - 5\*Sqrt[11]\*x)/(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])] - Sqrt[4091 + 1055\*Sqrt[11]]\*ArcTanH[(23 + Sqrt[11] + 17\*x + 5\*Sqrt[11]\*x)/(Sqrt[250 + 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])]))/(14\*Sqrt[2794])

**IntegrateAlgebraic [C]** time = 0.46, size = 211, normalized size = 1.29

$$\frac{3}{14} \text{RootSum} \left[ 7\#1^4 + 8\sqrt{5}\#1^3 - 70\#1^2 - 16\sqrt{5}\#1 + 83\&, \frac{-13\#1^2 \log(-\#1 + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) + 10\sqrt{5}\#1 \log(-\#1 + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) + 29 \log(-\#1 + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x)}{7\#1^3 + 6\sqrt{5}\#1^2 - 35\#1 - 4\sqrt{5}} \right] + \frac{\log(\sqrt{5}\sqrt{5x^2 + 2x + 3} - 5x - 1)}{7\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)\*Sqrt[3 + 2\*x + 5\*x^2]),x]

[Out] Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]]/(7\*Sqrt[5]) + (3\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 &, (29\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 10\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 - 13\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/14

**fricas [B]** time = 0.41, size = 297, normalized size = 1.81

$$\frac{-3\sqrt{78232}\sqrt{2794}\sqrt{1055}\sqrt{11} + 4091\log(3(\sqrt{2794}\sqrt{5x^2 + 2x + 3}\sqrt{1055}\sqrt{11} + 4091)(172\sqrt{11} - 715) + 185801\sqrt{11}(x + 3) + 557403x - 929005)/x + 3\sqrt{78232}\sqrt{2794}\sqrt{1055}\sqrt{11} + 4091\log(-3(\sqrt{2794}\sqrt{5x^2 + 2x + 3}\sqrt{1055}\sqrt{11} + 4091)(172\sqrt{11} - 715) - 185801\sqrt{11}(x + 3) - 557403x + 929005)/x - 1/78232\sqrt{2794}\sqrt{-9495}\sqrt{11} + 36819\log(-(\sqrt{2794}\sqrt{5x^2 + 2x + 3})(172\sqrt{11} + 715)\sqrt{-9495}\sqrt{11} + 36819) + 557403\sqrt{2794}\sqrt{1055}\sqrt{11}}{14\sqrt{2794}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)/(5\*x^2+2\*x+3)^(1/2),x, algorithm="fricas")

[Out] -3/78232\*sqrt(2794)\*sqrt(1055\*sqrt(11) + 4091)\*log(3\*(sqrt(2794)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(1055\*sqrt(11) + 4091)\*(172\*sqrt(11) - 715) + 185801\*sqrt(11)\*(x + 3) + 557403\*x - 929005)/x) + 3/78232\*sqrt(2794)\*sqrt(1055\*sqrt(11) + 4091)\*log(-3\*(sqrt(2794)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(1055\*sqrt(11) + 4091)\*(172\*sqrt(11) - 715) - 185801\*sqrt(11)\*(x + 3) - 557403\*x + 929005)/x) - 1/78232\*sqrt(2794)\*sqrt(-9495\*sqrt(11) + 36819)\*log(-(\sqrt(2794)\*sqrt(5\*x^2 + 2\*x + 3)\*(172\*sqrt(11) + 715)\*sqrt(-9495\*sqrt(11) + 36819) + 557403\*sqrt(2794)\*sqrt(1055\*sqrt(11) + 4091))

11)\*(x + 3) - 1672209\*x + 2787015)/x) + 1/78232\*sqrt(2794)\*sqrt(-9495\*sqrt(11) + 36819)\*log((sqrt(2794)\*sqrt(5\*x^2 + 2\*x + 3)\*(172\*sqrt(11) + 715)\*sqrt(-9495\*sqrt(11) + 36819) - 557403\*sqrt(11)\*(x + 3) + 1672209\*x - 2787015)/x) + 1/70\*sqrt(5)\*log(sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8)

**giac [A]** time = 0.27, size = 125, normalized size = 0.76

$\frac{1}{35} \sqrt{5} \log(-5\sqrt{5}x - \sqrt{5} + 5\sqrt{5x^2 + 2x + 3}) + 0.353184817631429 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} + 4.41924736459000) - 0.0986339689905714 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} + 1.25295163054000) - 0.353184817631429 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} - 1.02258038113000) + 0.0986339689905714 \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3} - 2.09411235400000)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)/(5\*x^2+2\*x+3)^(1/2),x, algorithm="giac")

[Out] 1/35\*sqrt(5)\*log(-5\*sqrt(5)\*x - sqrt(5) + 5\*sqrt(5\*x^2 + 2\*x + 3)) + 0.353184817631429\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 4.41924736459000) - 0.0986339689905714\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 1.25295163054000) - 0.353184817631429\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 1.02258038113000) + 0.0986339689905714\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 2.09411235400000)

**maple [A]** time = 0.02, size = 204, normalized size = 1.24

$$\frac{\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{35} + \frac{3(-61+13\sqrt{11})\sqrt{11} \operatorname{arctanh}\left(\frac{250-34\sqrt{11} + \frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}} \sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2 + 49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)} + 250-34\sqrt{11}}\right)}{154\sqrt{250-34\sqrt{11}}} + \frac{3\sqrt{11}(61+13\sqrt{11}) \operatorname{arctanh}\left(\frac{250+34\sqrt{11} + \frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}} \sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2 + 49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)} + 250+34\sqrt{11}}\right)}{154\sqrt{250+34\sqrt{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5\*x+2)/(-7\*x^2+4\*x+1)/(5\*x^2+2\*x+3)^(1/2),x)

[Out] -1/35\*5^(1/2)\*arcsinh(5/14\*14^(1/2)\*(x+1/5))+3/154\*(-61+13\*11^(1/2))\*11^(1/2)/(250-34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49-68/49\*11^(1/2)+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2)))/(250-34\*11^(1/2))^(1/2)/(245\*(x-2/7+1/7\*11^(1/2))^2+49\*(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250-34\*11^(1/2))^(1/2))+3/154\*11^(1/2)\*(61+13\*11^(1/2))/(250+34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49+68/49\*11^(1/2)+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2)))/(250+34\*11^(1/2))^(1/2)/(245\*(x-2/7-1/7\*11^(1/2))^2+49\*(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250+34\*11^(1/2))^(1/2))

**maxima [B]** time = 1.14, size = 465, normalized size = 2.84

$$\frac{1}{10780} \sqrt{11} \left( 28 \sqrt{11} \sqrt{5} \operatorname{arcsinh}\left(\frac{5}{14} \sqrt{7} x + \frac{1}{14} \sqrt{7} \sqrt{5}\right) - \frac{1365 \sqrt{11} \sqrt{2} \operatorname{arcsinh}\left(\frac{5 \sqrt{11} \sqrt{5} \sqrt{2}}{14 \sqrt{14+2 \sqrt{11}}}\right) + \frac{17 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{\sqrt{11} \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{23 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}}}{\sqrt{17 \sqrt{11} + 125}} \right) + \frac{290 \sqrt{11} \operatorname{arcsinh}\left(\frac{5 \sqrt{11} \sqrt{5} \sqrt{2}}{14 \sqrt{14+2 \sqrt{11}}}\right) - \frac{17 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{\sqrt{11} \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{23 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}}}{\sqrt{17 \sqrt{11} + 125}} \right) + \frac{6480 \sqrt{2} \operatorname{arcsinh}\left(\frac{5 \sqrt{11} \sqrt{5} \sqrt{2}}{14 \sqrt{14+2 \sqrt{11}}}\right) + \frac{17 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{\sqrt{11} \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{23 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}}}{\sqrt{17 \sqrt{11} + 125}} \right) + \frac{1830 \operatorname{arcsinh}\left(\frac{5 \sqrt{11} \sqrt{5} \sqrt{2}}{14 \sqrt{14+2 \sqrt{11}}}\right) - \frac{17 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{\sqrt{11} \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}} + \frac{23 \sqrt{5} \sqrt{2}}{7 \sqrt{14+2 \sqrt{11}}}}{\sqrt{17 \sqrt{11} + 125}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



$$3.368 \quad \int \frac{2+5x+x^2}{(1+4x-7x^2)^2 \sqrt{3+2x+5x^2}} dx$$

Optimal. Leaf size=178

$$\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588(-7x^2+4x+1)} - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{11176} + \frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{11176}$$

**Rubi [A]** time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1060, 1032, 724, 206}

$$\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588(-7x^2+4x+1)} - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{11176} + \frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{11176}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^2\*Sqrt[3 + 2\*x + 5\*x^2]), x]

[Out] (-3\*(40 - 371\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(5588\*(1 + 4\*x - 7\*x^2)) - (Sqrt[(3027900955 + 14035681\*Sqrt[11])/2794]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])]/11176 + (Sqrt[(3027900955 - 14035681\*Sqrt[11])/2794]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])]/11176

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

#### Rule 1032

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x],

$x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

### Rule 1060

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{(p_.)} * ((A_.) + (B_.)*(x_.) + (C_.)*(x_.)^2) * ((d_.) + (e_.)*(x_.) + (f_.)*(x_.)^2)^{(q_.)}, x\_Symbol] :> \text{Simp}[(a + b*x + c*x^2)^{(p + 1)} * (d + e*x + f*x^2)^{(q + 1)} * ((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f))) * x] / ((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), x] + \text{Dist}[1/((b^2 - 4*a*c) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)} * (d + e*x + f*x^2)^q * \text{Simp}[(b*B - 2*A*c - 2*a*C) * ((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)) * (p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))] * (a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))) * (p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))) * (p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))) * (b*f*(p + 1) - c*e*(2*p + q + 4))] * x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f))) * (2*p + 2*q + 5)*x^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, q\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \ \&\& \ !( \text{IntegerQ}[p] \ \&\& \ \text{ILtQ}[q, -1]) \ \&\& \ ! \ \text{IGtQ}[q, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{2 + 5x + x^2}{(1 + 4x - 7x^2)^2 \sqrt{3 + 2x + 5x^2}} dx &= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{5588(1 + 4x - 7x^2)} - \frac{\int \frac{-52136 - 29544x}{(1 + 4x - 7x^2)\sqrt{3 + 2x + 5x^2}} dx}{44704} \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{5588(1 + 4x - 7x^2)} - \frac{(-40623 + 53005\sqrt{11}) \int \frac{1}{(4 - 2\sqrt{11} - 14x)\sqrt{3 + 2x + 5x^2}} dx}{61468} \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{5588(1 + 4x - 7x^2)} - \frac{(40623 - 53005\sqrt{11}) \operatorname{Subst}\left(\int \frac{1}{2352 + 112x} dx\right)}{61468} \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{5588(1 + 4x - 7x^2)} - \frac{\sqrt{\frac{3027900955 + 14035681\sqrt{11}}{2794}} \operatorname{tanh}^{-1}\left(\frac{23 - 11x}{\sqrt{2(125 - 17\sqrt{11})}}\right)}{11176}
\end{aligned}$$

**Mathematica [A]** time = 1.00, size = 313, normalized size = 1.76

$$\frac{48972\sqrt{3+2x+5x^2}}{245872} + \frac{5280\sqrt{3+2x+5x^2}}{44704} + 53005 \sqrt{\frac{22}{125+17\sqrt{11}}} \log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + (55+17\sqrt{11})x + 23\sqrt{11} + 11\right) + 40623 \sqrt{\frac{2}{125+17\sqrt{11}}} \log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + (55+17\sqrt{11})x + 23\sqrt{11} + 11\right) + \sqrt{\frac{2}{125+17\sqrt{11}}} (53005\sqrt{11} - 40623) \operatorname{tanh}^{-1}\left(\frac{\sqrt{2(125-17\sqrt{11})}\sqrt{3+2x+5x^2}}{23-11x}\right) - \sqrt{\frac{2}{125+17\sqrt{11}}} (40623 + 53005\sqrt{11}) \log(-7x + \sqrt{11} + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^2\*Sqrt[3 + 2\*x + 5\*x^2]),x]

[Out] ((48972\*x\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2) + (5280\*Sqrt[3 + 2\*x + 5\*x^2])/(-1 - 4\*x + 7\*x^2) + Sqrt[2/(125 - 17\*Sqrt[11])]\*(-40623 + 53005\*Sqrt[11])\*ArcTanh[(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])/(-23 + Sqrt[11] + (-17 + 5\*Sqrt[11])\*x)] - Sqrt[2/(125 + 17\*Sqrt[11])]\*(40623 + 53005\*Sqrt[11])\*Log[2 + Sqrt[11] - 7\*x] + 40623\*Sqrt[2/(125 + 17\*Sqrt[11])]\*Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]] + 53005\*Sqrt[22/(125 + 17\*Sqrt[11])]\*Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]])/245872

**IntegrateAlgebraic [C]** time = 0.62, size = 352, normalized size = 1.98

$$\frac{1}{49} \operatorname{RootSum}\left[7\theta^4 + 8\sqrt{5}\theta^3 - 70\theta^2 - 16\sqrt{5}\theta + 836, \frac{7\sqrt{5}\theta \log(-\theta + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) - 397 \log(-\theta + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x)}{7\theta^3 + 6\sqrt{5}\theta^2 - 35\theta - 4\sqrt{5}}\right] + \frac{3 \operatorname{RootSum}\left[7\theta^4 + 8\sqrt{5}\theta^3 - 70\theta^2 - 16\sqrt{5}\theta + 836, \frac{-60319\theta^2 \log(-\theta + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) - 23896\sqrt{5}\theta \log(-\theta + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) - 151089 \log(-\theta + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x)}{7\theta^3 + 6\sqrt{5}\theta^2 - 35\theta - 4\sqrt{5}}\right]}{547624} - \frac{3\sqrt{5x^2 + 2x + 3}(371x - 40)}{5588(7x^2 - 4x - 1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^2\*Sqrt[3 + 2\*x + 5\*x^2]),x]

```
[Out] (-3*(-40 + 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(-1 - 4*x + 7*x^2)) - RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-397*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/49 + (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-1510889*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 238966*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 60319*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/547624
```

**fricas** [B] time = 0.58, size = 330, normalized size = 1.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 265381033753*sqrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) - 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) + 33528*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

**giac** [B] time = 0.28, size = 276, normalized size = 1.55

$$\frac{3(1231(\sqrt{5}-\sqrt{5x^2+2x+3})^2+1735\sqrt{5}(\sqrt{5}-\sqrt{5x^2+2x+3})^2-3913\sqrt{5}-3989\sqrt{5}-3913\sqrt{5x^2+2x+3})}{2794(\sqrt{5}-\sqrt{5x^2+2x+3})^2-8\sqrt{5}(\sqrt{5}-\sqrt{5x^2+2x+3})-70(\sqrt{5}-\sqrt{5x^2+2x+3})^2+10\sqrt{5}(\sqrt{5}-\sqrt{5x^2+2x+3})+83}+0.0924207110453\log(\sqrt{5x^2+2x+3}+4.4192473649000)-0.093860803464765\log(\sqrt{5x^2+2x+3}+1.2529163564000)-0.0924207110453\log(\sqrt{5x^2+2x+3}-1.022803813000)+0.093860803464765\log(\sqrt{5x^2+2x+3}-2.694112540000)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(5*(x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2
```

+ 16\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) + 83) + 0.0924287071106453  
 \*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 4.41924736459000) - 0.09386080346  
 04765\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 1.25295163054000) - 0.092428  
 7071106453\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 1.02258038113000) + 0.0  
 938608034604765\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 2.09411235400000)

**maple [B]** time = 0.02, size = 510, normalized size = 2.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{5x^2+2x+3}-\sqrt{5}x}{\sqrt{5x^2+2x+3}+\sqrt{5}x}\right)}{\sqrt{5x^2+2x+3}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{5x^2+2x+3}+\sqrt{5}x}{\sqrt{5x^2+2x+3}-\sqrt{5}x}\right)}{\sqrt{5x^2+2x+3}} + \frac{\left(\frac{\sqrt{5}\sqrt{5x^2+2x+3}-\sqrt{5}x}{\sqrt{5x^2+2x+3}+\sqrt{5}x}\right)\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{5x^2+2x+3}-\sqrt{5}x}{\sqrt{5x^2+2x+3}+\sqrt{5}x}\right)}{\sqrt{5x^2+2x+3}} + \frac{\left(\frac{\sqrt{5}\sqrt{5x^2+2x+3}+\sqrt{5}x}{\sqrt{5x^2+2x+3}-\sqrt{5}x}\right)\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{5x^2+2x+3}+\sqrt{5}x}{\sqrt{5x^2+2x+3}-\sqrt{5}x}\right)}{\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^2/(5\*x^2+2\*x+3)^(1/2), x)

[Out] -161/484\*11^(1/2)/(250-34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49-68/49\*11^(1/2)+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))))/(250-34\*11^(1/2))^(1/2)/(245\*(x-2/7+1/7\*11^(1/2))^2+49\*(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250-34\*11^(1/2))^(1/2)+(183/44-39/44\*11^(1/2))\*(-1/49/(250/49-34/49\*11^(1/2)))/(x-2/7+1/7\*11^(1/2))\*(5\*(x-2/7+1/7\*11^(1/2))^2+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250/49-34/49\*11^(1/2))^(1/2)+1/14\*(34/7-10/7\*11^(1/2))/(250/49-34/49\*11^(1/2))/(250-34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49-68/49\*11^(1/2)+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))))/(250-34\*11^(1/2))^(1/2)/(245\*(x-2/7+1/7\*11^(1/2))^2+49\*(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250-34\*11^(1/2))^(1/2))+161/484\*11^(1/2)/(250+34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49+68/49\*11^(1/2)+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))))/(250+34\*11^(1/2))^(1/2)/(245\*(x-2/7-1/7\*11^(1/2))^2+49\*(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250+34\*11^(1/2))^(1/2)+(183/44+39/44\*11^(1/2))\*(-1/49/(250/49+34/49\*11^(1/2)))/(x-2/7-1/7\*11^(1/2))\*(5\*(x-2/7-1/7\*11^(1/2))^2+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250/49+34/49\*11^(1/2))^(1/2)+1/14\*(34/7+10/7\*11^(1/2))/(250/49+34/49\*11^(1/2))/(250+34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49+68/49\*11^(1/2)+(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))))/(250+34\*11^(1/2))^(1/2)/(245\*(x-2/7-1/7\*11^(1/2))^2+49\*(34/7+10/7\*11^(1/2))\*(x-2/7-1/7\*11^(1/2))+250+34\*11^(1/2))^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5x + 2}{(7x^2 - 4x - 1)^2 \sqrt{5x^2 + 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^2/(5\*x^2+2\*x+3)^(1/2), x, algorithm="maxima")

[Out] integrate((x^2 + 5\*x + 2)/((7\*x^2 - 4\*x - 1)^2\*sqrt(5\*x^2 + 2\*x + 3)), x)



mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 + 5x + 2}{\sqrt{5x^2 + 2x + 3} (-7x^2 + 4x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x + x^2 + 2)/((2\*x + 5\*x^2 + 3)^(1/2)\*(4\*x - 7\*x^2 + 1)^2), x)

[Out] int((5\*x + x^2 + 2)/((2\*x + 5\*x^2 + 3)^(1/2)\*(4\*x - 7\*x^2 + 1)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5x + 2}{\sqrt{5x^2 + 2x + 3} (7x^2 - 4x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+5\*x+2)/(-7\*x\*\*2+4\*x+1)\*\*2/(5\*x\*\*2+2\*x+3)\*\*(1/2), x)

[Out] Integral((x\*\*2 + 5\*x + 2)/(sqrt(5\*x\*\*2 + 2\*x + 3)\*(7\*x\*\*2 - 4\*x - 1)\*\*2), x)

$$3.369 \quad \int \frac{2+5x+x^2}{(1+4x-7x^2)^3 \sqrt{3+2x+5x^2}} dx$$

Optimal. Leaf size=227

$$\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{62451488(-7x^2+4x+1)} - \frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176(-7x^2+4x+1)^2} - \frac{7(39370231-2538725\sqrt{11})\tanh^{-1}\left(\frac{1}{\sqrt{2}}\right)}{124902976\sqrt{22(125-17\sqrt{11})}}$$

Rubi [A] time = 0.27, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1060, 1032, 724, 206}

$$\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{62451488(-7x^2+4x+1)} - \frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176(-7x^2+4x+1)^2} - \frac{7(39370231-2538725\sqrt{11})\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{124902976\sqrt{22(125-17\sqrt{11})}} + \frac{7(39370231+2538725\sqrt{11})\tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{124902976\sqrt{22(125+17\sqrt{11})}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^3\*Sqrt[3 + 2\*x + 5\*x^2]), x]

[Out] (-3\*(40 - 371\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(11176\*(1 + 4\*x - 7\*x^2)^2) - (7\*(409769 - 1189370\*x)\*Sqrt[3 + 2\*x + 5\*x^2])/(62451488\*(1 + 4\*x - 7\*x^2)) - (7\*(39370231 - 2538725\*Sqrt[11])\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])/(124902976\*Sqrt[22\*(125 - 17\*Sqrt[11])]) + (7\*(39370231 + 2538725\*Sqrt[11])\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11]])\*Sqrt[3 + 2\*x + 5\*x^2]])/(124902976\*Sqrt[22\*(125 + 17\*Sqrt[11])])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 1032

Int[((g\_.) + (h\_.)\*(x\_))/(((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)\*Sqrt[(d\_.) + (e\_.)\*(x\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dis

```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 1060

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((A_) + (B_)*(x_) + (C_)*(x_)
)^2)*((d_) + (e_)*(x_) + (f_)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && !LtQ[q, -1]) && !
IGtQ[q, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{2 + 5x + x^2}{(1 + 4x - 7x^2)^3 \sqrt{3 + 2x + 5x^2}} dx &= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{11176(1 + 4x - 7x^2)^2} - \frac{\int \frac{-130024 - 81000x - 89040x^2}{(1 + 4x - 7x^2)^2 \sqrt{3 + 2x + 5x^2}} dx}{89408} \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{11176(1 + 4x - 7x^2)^2} - \frac{7(409769 - 1189370x)\sqrt{3 + 2x + 5x^2}}{62451488(1 + 4x - 7x^2)} + \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{11176(1 + 4x - 7x^2)^2} - \frac{7(409769 - 1189370x)\sqrt{3 + 2x + 5x^2}}{62451488(1 + 4x - 7x^2)} + \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{11176(1 + 4x - 7x^2)^2} - \frac{7(409769 - 1189370x)\sqrt{3 + 2x + 5x^2}}{62451488(1 + 4x - 7x^2)} \\
&= -\frac{3(40 - 371x)\sqrt{3 + 2x + 5x^2}}{11176(1 + 4x - 7x^2)^2} - \frac{7(409769 - 1189370x)\sqrt{3 + 2x + 5x^2}}{62451488(1 + 4x - 7x^2)}
\end{aligned}$$

**Mathematica [A]** time = 1.17, size = 371, normalized size = 1.63

$$\frac{\frac{7347311072 \sqrt{3+2x+5x^2}}{11176(1+4x-7x^2)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488(1+4x-7x^2)}}{1} + \frac{551183234 \sqrt{22/(125+17\sqrt{11})} \log(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + (55+17\sqrt{11})x + 23\sqrt{11} + 11)}{549573094} + \frac{390963650 \sqrt{2/(125+17\sqrt{11})} \log(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + (55+17\sqrt{11})x + 23\sqrt{11} + 11)}{549573094} + \frac{14 \sqrt{2/(125+17\sqrt{11})} (\log(2070231\sqrt{11} - 27925975) \operatorname{arctanh}(\frac{\sqrt{250-34\sqrt{11}}\sqrt{3+2x+5x^2}}{\sqrt{11}(1+4x-7x^2)})) - 14 \sqrt{2/(125+17\sqrt{11})} (\log(27925975 + 39370231\sqrt{11}) \log(-7x + \sqrt{11} + 2))}{549573094}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^3\*Sqrt[3 + 2\*x + 5\*x^2]),x]

[Out] ((-59009280\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2)^2 + (547311072\*x\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2)^2 + (732651920\*x\*Sqrt[3 + 2\*x + 5\*x^2])/(1 + 4\*x - 7\*x^2) + (252417704\*Sqrt[3 + 2\*x + 5\*x^2])/(-1 - 4\*x + 7\*x^2) + 14\*Sqrt[2/(125 - 17\*Sqrt[11])] \* (-27925975 + 39370231\*Sqrt[11]) \* ArcTanh[(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])/(-23 + Sqrt[11] + (-17 + 5\*Sqrt[11])\*x)] - 14\*Sqrt[2/(125 + 17\*Sqrt[11])] \* (27925975 + 39370231\*Sqrt[11]) \* Log[2 + Sqrt[11] - 7\*x] + 390963650\*Sqrt[2/(125 + 17\*Sqrt[11])] \* Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]] + 551183234\*Sqrt[22/(125 + 17\*Sqrt[11])] \* Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]])/5495730944

**IntegrateAlgebraic [C]** time = 0.95, size = 437, normalized size = 1.93

$$\frac{15272376 \operatorname{RootSum}\left[7x^4 + 8\sqrt{5}x^3 - 70x^2 - 16\sqrt{5}x + 836, \frac{\log\left(\frac{4x + \sqrt{2750 + 374\sqrt{11}}\sqrt{5x^2 + 2x + 3}}{\sqrt{11}(1 + 4x - 7x^2)} - \sqrt{5}\right)}{20\sqrt{5}x^2 - 150x + 2}\right]}{13480296} + \frac{\operatorname{RootSum}\left[7x^4 + 8\sqrt{5}x^3 - 70x^2 - 16\sqrt{5}x + 836, \frac{\operatorname{arctanh}\left(\frac{\sqrt{250 - 34\sqrt{11}}\sqrt{3 + 2x + 5x^2}}{\sqrt{11}(1 + 4x - 7x^2)}\right)}{20\sqrt{5}x^2 - 150x + 2}\right]}{131848224} + \frac{\operatorname{RootSum}\left[7x^4 + 8\sqrt{5}x^3 - 70x^2 - 16\sqrt{5}x + 836, \frac{\log\left(\frac{4x + \sqrt{2750 + 374\sqrt{11}}\sqrt{5x^2 + 2x + 3}}{\sqrt{11}(1 + 4x - 7x^2)} - \sqrt{5}\right)}{20\sqrt{5}x^2 - 150x + 2}\right]}{14894710223424} + \frac{\operatorname{RootSum}\left[7x^4 + 8\sqrt{5}x^3 - 70x^2 - 16\sqrt{5}x + 836, \frac{\log\left(\frac{4x + \sqrt{2750 + 374\sqrt{11}}\sqrt{5x^2 + 2x + 3}}{\sqrt{11}(1 + 4x - 7x^2)} - \sqrt{5}\right)}{20\sqrt{5}x^2 - 150x + 2}\right]}{14894710223424} + \frac{\sqrt{22/(125 + 17\sqrt{11})} (-58259130x^2 + 53383481x + 30715024 - 3538948)}{62451488(7x^2 - 4x - 1)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-3538943 + 3071502*x + 53381041*x^2 - 58279130*x^3)
)/(62451488*(-1 - 4*x + 7*x^2)^2) - (15272337*RootSum[83 - 16*Sqrt[5]*#1 -
70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x
^2] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/124902976 + R
ootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (10486671
792*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 6928653865*
Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 +
6*Sqrt[5]*#1^2 + 7*#1^3) & ]/1314845224 - (3*RootSum[83 - 16*Sqrt[5]*#1 - 7
0*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (36376673721218*Sqrt[5]*Log[-(Sqrt[5]*
x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 26508461599305*Log[-(Sqrt[5]*x) + Sqr
t[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^
3) & ])/14694710223424
```

**fricas** [B] time = 0.85, size = 390, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(128397
3697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x
+ 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt(11
) + 2940638404) + 7232150972206110797*sqrt(11)*(x + 3) - 216964529166183323
91*x + 36160754861030553985)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x
+ 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2794)*s
qrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(3
58684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x + 3) + 21
696452916618332391*x - 36160754861030553985)/x) + sqrt(2794)*(49*x^4 - 56*x
^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425)*
log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqr
t(-1283973697005131*sqrt(11) + 82616280769148425) + 7232150972206110797*sqr
t(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)/x) - sqrt(27
94)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 8
2616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11
) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) - 7232
150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x + 361607548610305
53985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*x + 3538943)*sqrt(
5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

**giac [B]** time = 0.32, size = 378, normalized size = 1.67

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^3/(5\*x^2+2\*x+3)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{31225744} \cdot (124397525 \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^7 + 26796567 \cdot \sqrt{5} \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^6 - 3595807617 \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^5 - 1719888775 \cdot \sqrt{5} \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^4 + 17096132999 \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^3 + 8328401413 \cdot \sqrt{5} \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^2 - 16383202915 \cdot \sqrt{5} \cdot x - 7800623485 \cdot \sqrt{5} + 16383202915 \cdot \sqrt{5x^2 + 2x + 3}) / (7 \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^4 - 8 \cdot \sqrt{5} \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^3 - 70 \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})^2 + 16 \cdot \sqrt{5} \cdot (\sqrt{5}x - \sqrt{5x^2 + 2x + 3})) + 83)^2 + 0.0423989586659649 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) + 4.41924736459000) - 0.0446437606656958 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) + 1.25295163054000) - 0.0423989586659649 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) - 1.02258038113000) + 0.0446437606656958 \cdot \log(-\sqrt{5}x + \sqrt{5x^2 + 2x + 3}) - 2.09411235400000)$

**maple [B]** time = 0.02, size = 1194, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^3/(5\*x^2+2\*x+3)^(1/2),x)

[Out] 
$$\begin{aligned} & -3535/21296 \cdot 11^{(1/2)} / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(49/2 \cdot (500/49 - 68/49 \cdot 11^{(1/2)} + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250 - 34 \cdot 11^{(1/2)})^{(1/2)} - 21/968 \cdot (-61 + 13 \cdot 11^{(1/2)}) \cdot 11^{(1/2)} \cdot (-1/686 / (250/49 - 34/49 \cdot 11^{(1/2)}) / (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 \cdot (5 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250/49 - 34/49 \cdot 11^{(1/2)})^{(1/2)} - 3/1372 \cdot (34/7 - 10/7 \cdot 11^{(1/2)}) / (250/49 - 34/49 \cdot 11^{(1/2)}) \cdot (-1 / (250/49 - 34/49 \cdot 11^{(1/2)}) / (x - 2/7 + 1/7 \cdot 11^{(1/2)}) \cdot (5 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250/49 - 34/49 \cdot 11^{(1/2)})^{(1/2)} + 7/2 \cdot (34/7 - 10/7 \cdot 11^{(1/2)}) / (250/49 - 34/49 \cdot 11^{(1/2)}) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(49/2 \cdot (500/49 - 68/49 \cdot 11^{(1/2)} + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250 - 34 \cdot 11^{(1/2)})^{(1/2)} + 5/98 / (250/49 - 34/49 \cdot 11^{(1/2)}) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} \cdot \operatorname{arctanh}(49/2 \cdot (500/49 - 68/49 \cdot 11^{(1/2)} + (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})) / (250 - 34 \cdot 11^{(1/2)})^{(1/2)} / (245 \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)})^2 + 49 \cdot (34/7 - 10/7 \cdot 11^{(1/2)}) \cdot (x - 2/7 + 1/7 \cdot 11^{(1/2)}) + 250 - 34 \cdot 11^{(1/2)})^{(1/2)}) - (-3535/1936 + 273/1936 \cdot 11^{(1/2)}) \cdot (-1/49 / (250/ \end{aligned}$$

$$\frac{49-34/49*11^{(1/2)}}{(x-2/7+1/7*11^{(1/2)})}*(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}+1/14*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}))+3535/21296*11^{(1/2)}/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))-(-3535/1936-273/1936*11^{(1/2)})*(-1/49/(250/49+34/49*11^{(1/2)})/(x-2/7-1/7*11^{(1/2)})*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}+1/14*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)})*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))-21/968*(61+13*11^{(1/2)})*11^{(1/2)}*(-1/686/(250/49+34/49*11^{(1/2)})/(x-2/7-1/7*11^{(1/2)})^2*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-3/1372*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(-1/(250/49+34/49*11^{(1/2)})/(x-2/7-1/7*11^{(1/2)})*(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}+7/2*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)})*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))+5/98/(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 5x + 2}{(7x^2 - 4x - 1)^3 \sqrt{5x^2 + 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^3/(5\*x^2+2\*x+3)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 + 5\*x + 2)/((7\*x^2 - 4\*x - 1)^3\*sqrt(5\*x^2 + 2\*x + 3)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 + 5x + 2}{\sqrt{5x^2 + 2x + 3} (-7x^2 + 4x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 2x + 2}{(2x^2 + 5x + 3)^{1/2} (4x^2 - 7x + 1)^3} dx - \int \frac{5x^2 + 2x + 2}{(2x^2 + 5x + 3)^{1/2} (4x^2 - 7x + 1)^3} dx - \int \frac{2}{(2x^2 + 5x + 3)^{1/2} (4x^2 - 7x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2), x)
```

```
[Out] -Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```



$$3.370 \quad \int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx$$

**Optimal.** Leaf size=166

$$\frac{2583293\sqrt{5x^2+2x+3}x^2}{187500} - \frac{3192602\sqrt{5x^2+2x+3}x}{46875} + \frac{15715799\sqrt{5x^2+2x+3}}{156250} + \frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

**Rubi [A]** time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{343\sqrt{5x^2+2x+3}x^5}{150} - \frac{25921\sqrt{5x^2+2x+3}x^4}{3750} + \frac{393659\sqrt{5x^2+2x+3}x^3}{12500} - \frac{2583293\sqrt{5x^2+2x+3}x^2}{187500} - \frac{3192602\sqrt{5x^2+2x+3}x}{46875} + \frac{15715799\sqrt{5x^2+2x+3}}{156250} + \frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}} + \frac{50047657 \sinh^{-1}\left(\frac{5x+1}{\sqrt{5}}\right)}{156250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (16\*(6122807 - 5338217\*x))/(546875\*sqrt[3 + 2\*x + 5\*x^2]) + (15715799\*sqrt[3 + 2\*x + 5\*x^2])/156250 - (3192602\*x\*sqrt[3 + 2\*x + 5\*x^2])/46875 - (2583293\*x^2\*sqrt[3 + 2\*x + 5\*x^2])/187500 + (393659\*x^3\*sqrt[3 + 2\*x + 5\*x^2])/12500 - (25921\*x^4\*sqrt[3 + 2\*x + 5\*x^2])/3750 - (343\*x^5\*sqrt[3 + 2\*x + 5\*x^2])/150 + (50047657\*ArcSinh[(1 + 5\*x)/sqrt[14]])/(156250\*sqrt[5])

Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(1 + 4x - 7x^2)^3 (2 + 5x + x^2)}{(3 + 2x + 5x^2)^{3/2}} dx &= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} + \frac{1}{28} \int \frac{\frac{473724104}{78125} + \frac{94462228x}{15625} - \frac{40822404x^2}{3125} - \frac{121}{\sqrt{3 + 2x + 5x^2}}}{\sqrt{3 + 2x + 5x^2}} dx \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} - \frac{343}{150}x^5\sqrt{3 + 2x + 5x^2} + \frac{1}{840} \int \frac{\frac{2842344624}{15625} + \dots}{\sqrt{3 + 2x + 5x^2}} dx \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} - \frac{25921x^4\sqrt{3 + 2x + 5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3 + 2x + 5x^2} + \dots \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} + \frac{393659x^3\sqrt{3 + 2x + 5x^2}}{12500} - \frac{25921x^4\sqrt{3 + 2x + 5x^2}}{3750} + \dots \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} - \frac{2583293x^2\sqrt{3 + 2x + 5x^2}}{187500} + \frac{393659x^3\sqrt{3 + 2x + 5x^2}}{12500} + \dots \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} - \frac{3192602x\sqrt{3 + 2x + 5x^2}}{46875} - \frac{2583293x^2\sqrt{3 + 2x + 5x^2}}{187500} + \dots \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} + \frac{15715799\sqrt{3 + 2x + 5x^2}}{156250} - \frac{3192602x\sqrt{3 + 2x + 5x^2}}{46875} + \dots \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} + \frac{15715799\sqrt{3 + 2x + 5x^2}}{156250} - \frac{3192602x\sqrt{3 + 2x + 5x^2}}{46875} + \dots \\
&= \frac{16(6122807 - 5338217x)}{546875\sqrt{3 + 2x + 5x^2}} + \frac{15715799\sqrt{3 + 2x + 5x^2}}{156250} - \frac{3192602x\sqrt{3 + 2x + 5x^2}}{46875} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.43, size = 75, normalized size = 0.45

$$\frac{2102001594\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) - \frac{5(75031250x^7 + 256821250x^6 - 897612625x^5 + 174819575x^4 + 1795638985x^3 - 2135143465x^2 + 1045703388x - 3155769618)}{\sqrt{5x^2+2x+3}}}{32812500}$$

Antiderivative was successfully verified.

[In] Integrate[(((1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out]  $((-5*(-3155769618 + 1045703388*x - 2135143465*x^2 + 1795638985*x^3 + 174819575*x^4 - 897612625*x^5 + 256821250*x^6 + 75031250*x^7))/\text{Sqrt}[3 + 2*x + 5*x^2] + 2102001594*\text{Sqrt}[5]*\text{ArcSinh}[(1 + 5*x)/\text{Sqrt}[14]])/32812500$

**IntegrateAlgebraic [A]** time = 0.71, size = 89, normalized size = 0.54

$$\frac{-75031250x^7 - 256821250x^6 + 897612625x^5 - 174819575x^4 - 1795638985x^3 + 2135143465x^2 - 1045703388x + 3155769618}{6562500\sqrt{5x^2 + 2x + 3}} - \frac{50047657 \log(\sqrt{5}\sqrt{5x^2 + 2x + 3} - 5x - 1)}{156250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 4\*x - 7\*x^2)^3\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out]  $(3155769618 - 1045703388*x + 2135143465*x^2 - 1795638985*x^3 - 174819575*x^4 + 897612625*x^5 - 256821250*x^6 - 75031250*x^7)/(6562500*\text{Sqrt}[3 + 2*x + 5*x^2]) - (50047657*\text{Log}[-1 - 5*x + \text{Sqrt}[5]*\text{Sqrt}[3 + 2*x + 5*x^2]])/(156250*\text{qrt}[5])$

**fricas [A]** time = 0.84, size = 112, normalized size = 0.67

$$\frac{1051000797\sqrt{5}(5x^2 + 2x + 3)\log(-\sqrt{5}\sqrt{5x^2 + 2x + 3}(5x + 1) - 25x^2 - 10x - 8) - 5(75031250x^7 + 256821250x^6 - 897612625x^5 + 174819575x^4 + 1795638985x^3 - 2135143465x^2 + 1045703388x - 3155769618)\sqrt{5x^2 + 2x + 3}}{32812500(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2), x, algorithm="fricas")

[Out]  $1/32812500*(1051000797*\text{sqrt}(5)*(5*x^2 + 2*x + 3)*\log(-\text{sqrt}(5)*\text{sqrt}(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6 - 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 1045703388*x - 3155769618)*\text{sqrt}(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)$

**giac [A]** time = 0.26, size = 81, normalized size = 0.49

$$-\frac{50047657}{781250}\sqrt{5}\log(-\sqrt{5}(\sqrt{5}x - \sqrt{5x^2 + 2x + 3}) - 1) - \frac{(35((5(35(70(175x + 599)x - 146549)x + 998969)x + 51303971)x - 61004099)x + 1045703388)x - 3155769618)}{6562500\sqrt{5x^2 + 2x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^3\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2), x, algorithm="giac")

[Out]  $-50047657/781250*\text{sqrt}(5)*\log(-\text{sqrt}(5)*(5*x^2 + 2*x + 3) - 1) - 1/6562500*((35*((5(35(70(175*x + 599)*x - 146549)*x + 998969)*x + 51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/\text{sqrt}(5*x^2 + 2*x + 3)$

**maple [A]** time = 0.03, size = 166, normalized size = 1.00

$$\frac{343x^7}{30\sqrt{5x^2+2x+3}} - \frac{29351x^6}{750\sqrt{5x^2+2x+3}} + \frac{1025843x^5}{7500\sqrt{5x^2+2x+3}} - \frac{998969x^4}{37500\sqrt{5x^2+2x+3}} - \frac{51303971x^3}{187500\sqrt{5x^2+2x+3}} + \frac{61004099x^2}{187500\sqrt{5x^2+2x+3}} - \frac{50047657x}{156250\sqrt{5x^2+2x+3}} + \frac{50047657\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}(x+1)}{14}\right)}{781250} + \frac{176049701x}{1093750\sqrt{5x^2+2x+3}} + \frac{176049701}{5468750\sqrt{5x^2+2x+3}} + \frac{175268451}{390625\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2), x)`

[Out]  $50047657/781250*5^{(1/2)}*\operatorname{arcsinh}(5/14*14^{(1/2)}*(x+1/5))+176049701/10937500*(10*x+2)/(5*x^2+2*x+3)^{(1/2)}-998969/37500*x^4/(5*x^2+2*x+3)^{(1/2)}-51303971/187500*x^3/(5*x^2+2*x+3)^{(1/2)}+61004099/187500*x^2/(5*x^2+2*x+3)^{(1/2)}-50047657/156250*x/(5*x^2+2*x+3)^{(1/2)}-343/30*x^7/(5*x^2+2*x+3)^{(1/2)}-29351/750*x^6/(5*x^2+2*x+3)^{(1/2)}+1025843/7500*x^5/(5*x^2+2*x+3)^{(1/2)}+175268451/390625/(5*x^2+2*x+3)^{(1/2)}$

**maxima [A]** time = 0.99, size = 148, normalized size = 0.89

$$\frac{343x^7}{30\sqrt{5x^2+2x+3}} - \frac{29351x^6}{750\sqrt{5x^2+2x+3}} + \frac{1025843x^5}{7500\sqrt{5x^2+2x+3}} - \frac{998969x^4}{37500\sqrt{5x^2+2x+3}} - \frac{51303971x^3}{187500\sqrt{5x^2+2x+3}} + \frac{61004099x^2}{187500\sqrt{5x^2+2x+3}} + \frac{50047657\sqrt{5} \operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)}{781250} - \frac{87141949x}{546875\sqrt{5x^2+2x+3}} + \frac{525961603}{1093750\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2), x, algorithm="maxima")`

[Out]  $-343/30*x^7/\operatorname{sqrt}(5*x^2 + 2*x + 3) - 29351/750*x^6/\operatorname{sqrt}(5*x^2 + 2*x + 3) + 1025843/7500*x^5/\operatorname{sqrt}(5*x^2 + 2*x + 3) - 998969/37500*x^4/\operatorname{sqrt}(5*x^2 + 2*x + 3) - 51303971/187500*x^3/\operatorname{sqrt}(5*x^2 + 2*x + 3) + 61004099/187500*x^2/\operatorname{sqrt}(5*x^2 + 2*x + 3) + 50047657/781250*\operatorname{sqrt}(5)*\operatorname{arcsinh}(1/14*\operatorname{sqrt}(14)*(5*x + 1)) - 87141949/546875*x/\operatorname{sqrt}(5*x^2 + 2*x + 3) + 525961603/1093750/\operatorname{sqrt}(5*x^2 + 2*x + 3)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2)(-7x^2 + 4x + 1)^3}{(5x^2 + 2x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)`

[Out] `int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

-----

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x\*\*2+4\*x+1)\*\*3\*(x\*\*2+5\*x+2)/(5\*x\*\*2+2\*x+3)\*\*(3/2),x)

[Out] -Integral(-29\*x/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(-115\*x\*\*2/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(61\*x\*\*3/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(871\*x\*\*4/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(-127\*x\*\*5/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(-2065\*x\*\*6/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(1127\*x\*\*7/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(343\*x\*\*8/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(-2/(5\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) + 2\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) + 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x)

$$3.371 \quad \int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx$$

**Optimal.** Leaf size=124

$$\frac{203}{100} \sqrt{5x^2 + 2x + 3} x^2 - \frac{8749 \sqrt{5x^2 + 2x + 3} x}{1250} - \frac{5086 \sqrt{5x^2 + 2x + 3}}{3125} - \frac{8(136602x + 12983)}{21875 \sqrt{5x^2 + 2x + 3}} + \frac{49}{100} \sqrt{5x^2 + 2x + 3}$$

**Rubi [A]** time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {1660, 1661, 640, 619, 215}

$$\frac{49}{100} \sqrt{5x^2 + 2x + 3} x^3 + \frac{203}{100} \sqrt{5x^2 + 2x + 3} x^2 - \frac{8749 \sqrt{5x^2 + 2x + 3} x}{1250} - \frac{5086 \sqrt{5x^2 + 2x + 3}}{3125} - \frac{8(136602x + 12983)}{21875 \sqrt{5x^2 + 2x + 3}} + \frac{89583 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-8\*(12983 + 136602\*x))/(21875\*Sqrt[3 + 2\*x + 5\*x^2]) - (5086\*Sqrt[3 + 2\*x + 5\*x^2])/3125 - (8749\*x\*Sqrt[3 + 2\*x + 5\*x^2])/1250 + (203\*x^2\*Sqrt[3 + 2\*x + 5\*x^2])/100 + (49\*x^3\*Sqrt[3 + 2\*x + 5\*x^2])/100 + (89583\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(1250\*Sqrt[5])

### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

### Rule 619

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

### Rule 640

Int[((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

### Rule 1660

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]

```

### Rule 1661

```

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx &= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{1}{28} \int \frac{\frac{4291112}{3125} - \frac{296716x}{625} - \frac{194012x^2}{125} + \frac{23716x^3}{25} + 1}{\sqrt{3+2x+5x^2}} dx \\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} + \frac{1}{560} \int \frac{\frac{17164448}{625} - \frac{1186}{12}}{\sqrt{3+2x+5x^2}} dx \\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} \\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} \\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} \\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} \\
&= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250}
\end{aligned}$$

**Mathematica [A]** time = 0.27, size = 65, normalized size = 0.52

$$\frac{5(42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536)}{\sqrt{5x^2+2x+3}} + 1254162\sqrt{5} \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

87500

Antiderivative was successfully verified.

```
[In] Integrate[(((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2)), x]
[Out] ((5*(-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5))/Sqrt[3 + 2*x + 5*x^2] + 1254162*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/87500
```

**IntegrateAlgebraic [A]** time = 0.95, size = 79, normalized size = 0.64

$$\frac{42875x^5 + 194775x^4 - 515655x^3 - 280805x^2 - 1298674x - 168536}{17500\sqrt{5x^2 + 2x + 3}} - \frac{89583 \log(\sqrt{5}\sqrt{5x^2 + 2x + 3} - 5x - 1)}{1250\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 4\*x - 7\*x^2)^2\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2),x]

[Out] (-168536 - 1298674\*x - 280805\*x^2 - 515655\*x^3 + 194775\*x^4 + 42875\*x^5)/(17500\*sqrt[3 + 2\*x + 5\*x^2]) - (89583\*Log[-1 - 5\*x + sqrt[5]\*sqrt[3 + 2\*x + 5\*x^2]])/(1250\*sqrt[5])

**fricas** [A] time = 0.87, size = 102, normalized size = 0.82

$$\frac{627081\sqrt{5}(5x^2+2x+3)\log(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8)+5(42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536)\sqrt{5x^2+2x+3}}{87500(5x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2),x, algorithm="fricas")

[Out] 1/87500\*(627081\*sqrt(5)\*(5\*x^2 + 2\*x + 3)\*log(-sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8) + 5\*(42875\*x^5 + 194775\*x^4 - 515655\*x^3 - 280805\*x^2 - 1298674\*x - 168536)\*sqrt(5\*x^2 + 2\*x + 3))/(5\*x^2 + 2\*x + 3)

**giac** [A] time = 0.25, size = 71, normalized size = 0.57

$$-\frac{89583}{6250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)+\frac{(35((35(35x+159)x-14733)x-8023)x-1298674)x-168536)}{17500\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2),x, algorithm="giac")

[Out] -89583/6250\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1) + 1/17500\*((35\*((35\*(35\*x + 159)\*x - 14733)\*x - 8023)\*x - 1298674)\*x - 168536)/sqrt(5\*x^2 + 2\*x + 3)

**maple** [A] time = 0.01, size = 132, normalized size = 1.06

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}} + \frac{1113x^4}{100\sqrt{5x^2+2x+3}} - \frac{14733x^3}{500\sqrt{5x^2+2x+3}} - \frac{8023x^2}{500\sqrt{5x^2+2x+3}} - \frac{89583x}{1250\sqrt{5x^2+2x+3}} + \frac{89583\sqrt{5}\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{6250} - \frac{5564(10x+2)}{21875\sqrt{5x^2+2x+3}} - \frac{28506}{3125\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-7\*x^2+4\*x+1)^2\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2),x)

[Out] 89583/6250\*5^(1/2)\*arcsinh(5/14\*14^(1/2)\*(x+1/5))-5564/21875\*(10\*x+2)/(5\*x^2+2\*x+3)^(1/2)+1113/100/(5\*x^2+2\*x+3)^(1/2)\*x^4-14733/500/(5\*x^2+2\*x+3)^(1/2)

2)\*x<sup>3</sup>-8023/500/(5\*x<sup>2</sup>+2\*x+3)<sup>(1/2)</sup>\*x<sup>2</sup>-89583/1250/(5\*x<sup>2</sup>+2\*x+3)<sup>(1/2)</sup>\*x+49/20/(5\*x<sup>2</sup>+2\*x+3)<sup>(1/2)</sup>\*x<sup>5</sup>-28506/3125/(5\*x<sup>2</sup>+2\*x+3)<sup>(1/2)</sup>

**maxima** [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}} + \frac{1113x^4}{100\sqrt{5x^2+2x+3}} - \frac{14733x^3}{500\sqrt{5x^2+2x+3}} - \frac{8023x^2}{500\sqrt{5x^2+2x+3}} + \frac{89583}{6250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{649337x}{8750\sqrt{5x^2+2x+3}} - \frac{42134}{4375\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x<sup>2</sup>+4\*x+1)<sup>2</sup>\*(x<sup>2</sup>+5\*x+2)/(5\*x<sup>2</sup>+2\*x+3)<sup>(3/2)</sup>,x, algorithm="maxima")

[Out] 49/20\*x<sup>5</sup>/sqrt(5\*x<sup>2</sup> + 2\*x + 3) + 1113/100\*x<sup>4</sup>/sqrt(5\*x<sup>2</sup> + 2\*x + 3) - 14733/500\*x<sup>3</sup>/sqrt(5\*x<sup>2</sup> + 2\*x + 3) - 8023/500\*x<sup>2</sup>/sqrt(5\*x<sup>2</sup> + 2\*x + 3) + 89583/6250\*sqrt(5)\*arcsinh(1/14\*sqrt(14)\*(5\*x + 1)) - 649337/8750\*x/sqrt(5\*x<sup>2</sup> + 2\*x + 3) - 42134/4375/sqrt(5\*x<sup>2</sup> + 2\*x + 3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2)(-7x^2 + 4x + 1)^2}{(5x^2 + 2x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5\*x + x<sup>2</sup> + 2)\*(4\*x - 7\*x<sup>2</sup> + 1)<sup>2</sup>)/(2\*x + 5\*x<sup>2</sup> + 3)<sup>(3/2)</sup>,x)

[Out] int(((5\*x + x<sup>2</sup> + 2)\*(4\*x - 7\*x<sup>2</sup> + 1)<sup>2</sup>)/(2\*x + 5\*x<sup>2</sup> + 3)<sup>(3/2)</sup>, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 5x + 2)(7x^2 - 4x - 1)^2}{(5x^2 + 2x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x\*\*2+4\*x+1)\*\*2\*(x\*\*2+5\*x+2)/(5\*x\*\*2+2\*x+3)\*\*(3/2),x)

[Out] Integral((x\*\*2 + 5\*x + 2)\*(7\*x\*\*2 - 4\*x - 1)\*\*2/(5\*x\*\*2 + 2\*x + 3)\*\*(3/2), x)

$$3.372 \quad \int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}} dx$$

**Optimal.** Leaf size=82

$$-\frac{7}{50}\sqrt{5x^2+2x+3}x - \frac{261}{250}\sqrt{5x^2+2x+3} - \frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}} + \frac{149 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

**Rubi [A]** time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$ , Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{7}{50}\sqrt{5x^2+2x+3}x - \frac{261}{250}\sqrt{5x^2+2x+3} - \frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}} + \frac{149 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-2\*(2321 + 2449\*x))/(875\*Sqrt[3 + 2\*x + 5\*x^2]) - (261\*Sqrt[3 + 2\*x + 5\*x^2])/250 - (7\*x\*Sqrt[3 + 2\*x + 5\*x^2])/50 + (149\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(25\*Sqrt[5])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 619

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[1/(2\*c\*((-4\*c)/(b^2 - 4\*a\*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4\*a\*c), x]^p, x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4\*a - b^2/c, 0]

#### Rule 640

Int[((d\_) + (e\_)\*(x\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(a + b\*x + c\*x^2)^(p + 1))/(2\*c\*(p + 1)), x] + Dist[(2\*c\*d - b\*e)/(2\*c), Int[(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[p, -1]

#### Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

### Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

### Rubi steps

$$\int \frac{(1 + 4x - 7x^2)(2 + 5x + x^2)}{(3 + 2x + 5x^2)^{3/2}} dx = -\frac{2(2321 + 2449x)}{875\sqrt{3 + 2x + 5x^2}} + \frac{1}{28} \int \frac{\frac{15736}{125} - \frac{3948x}{25} - \frac{196x^2}{5}}{\sqrt{3 + 2x + 5x^2}} dx$$

$$= -\frac{2(2321 + 2449x)}{875\sqrt{3 + 2x + 5x^2}} - \frac{7}{50}x\sqrt{3 + 2x + 5x^2} + \frac{1}{280} \int \frac{\frac{34412}{25} - \frac{7308x}{5}}{\sqrt{3 + 2x + 5x^2}} dx$$

$$= -\frac{2(2321 + 2449x)}{875\sqrt{3 + 2x + 5x^2}} - \frac{261}{250}\sqrt{3 + 2x + 5x^2} - \frac{7}{50}x\sqrt{3 + 2x + 5x^2} + \frac{149}{25} \int \frac{1}{\sqrt{3 + 2x + 5x^2}} dx$$

$$= -\frac{2(2321 + 2449x)}{875\sqrt{3 + 2x + 5x^2}} - \frac{261}{250}\sqrt{3 + 2x + 5x^2} - \frac{7}{50}x\sqrt{3 + 2x + 5x^2} + \frac{149}{25} \operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)$$

$$= -\frac{2(2321 + 2449x)}{875\sqrt{3 + 2x + 5x^2}} - \frac{261}{250}\sqrt{3 + 2x + 5x^2} - \frac{7}{50}x\sqrt{3 + 2x + 5x^2} + \frac{149}{25} \operatorname{arcsinh}\left(\frac{5x+1}{\sqrt{14}}\right)$$

**Mathematica [A]** time = 0.15, size = 55, normalized size = 0.67

$$\frac{149 \sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}} - \frac{245x^3 + 1925x^2 + 2837x + 2953}{350\sqrt{5x^2 + 2x + 3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] -1/350\*(2953 + 2837\*x + 1925\*x^2 + 245\*x^3)/Sqrt[3 + 2\*x + 5\*x^2] + (149\*ArcSinh[(1 + 5\*x)/Sqrt[14]])/(25\*Sqrt[5])

**IntegrateAlgebraic [A]** time = 0.67, size = 69, normalized size = 0.84

$$\frac{-245x^3 - 1925x^2 - 2837x - 2953}{350\sqrt{5x^2 + 2x + 3}} - \frac{149 \log\left(\sqrt{5}\sqrt{5x^2 + 2x + 3} - 5x - 1\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + 4\*x - 7\*x^2)\*(2 + 5\*x + x^2))/(3 + 2\*x + 5\*x^2)^(3/2), x]

[Out] (-2953 - 2837\*x - 1925\*x^2 - 245\*x^3)/(350\*Sqrt[3 + 2\*x + 5\*x^2]) - (149\*Log[-1 - 5\*x + Sqrt[5]\*Sqrt[3 + 2\*x + 5\*x^2]])/(25\*Sqrt[5])

**fricas [A]** time = 0.70, size = 92, normalized size = 1.12

$$\frac{1043\sqrt{5}(5x^2 + 2x + 3)\log\left(-\sqrt{5}\sqrt{5x^2 + 2x + 3}(5x + 1) - 25x^2 - 10x - 8\right) - 5(245x^3 + 1925x^2 + 2837x + 2953)\sqrt{5x^2 + 2x + 3}}{1750(5x^2 + 2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2), x, algorithm="fricas")

[Out] 1/1750\*(1043\*sqrt(5)\*(5\*x^2 + 2\*x + 3)\*log(-sqrt(5)\*sqrt(5\*x^2 + 2\*x + 3)\*(5\*x + 1) - 25\*x^2 - 10\*x - 8) - 5\*(245\*x^3 + 1925\*x^2 + 2837\*x + 2953)\*sqrt(5\*x^2 + 2\*x + 3))/(5\*x^2 + 2\*x + 3)

**giac [A]** time = 0.22, size = 62, normalized size = 0.76

$$-\frac{149}{125}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x - \sqrt{5x^2 + 2x + 3}\right) - 1\right) - \frac{(35(7x + 55)x + 2837)x + 2953}{350\sqrt{5x^2 + 2x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-7\*x^2+4\*x+1)\*(x^2+5\*x+2)/(5\*x^2+2\*x+3)^(3/2), x, algorithm="giac")

[Out] -149/125\*sqrt(5)\*log(-sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) - 1) - 1/350\*((35\*(7\*x + 55)\*x + 2837)\*x + 2953)/sqrt(5\*x^2 + 2\*x + 3)

**maple [A]** time = 0.01, size = 98, normalized size = 1.20

$$\frac{7x^3}{10\sqrt{5x^2+2x+3}} - \frac{11x^2}{2\sqrt{5x^2+2x+3}} - \frac{149x}{25\sqrt{5x^2+2x+3}} + \frac{149\sqrt{5} \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125} - \frac{1001}{125\sqrt{5x^2+2x+3}} - \frac{751(10x+2)}{3500\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2), x)`

[Out] `-7/10/(5*x^2+2*x+3)^(1/2)*x^3-11/2/(5*x^2+2*x+3)^(1/2)*x^2-149/25/(5*x^2+2*x+3)^(1/2)*x-1001/125/(5*x^2+2*x+3)^(1/2)-751/3500*(10*x+2)/(5*x^2+2*x+3)^(1/2)+149/125*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))`

**maxima [A]** time = 0.96, size = 80, normalized size = 0.98

$$-\frac{7x^3}{10\sqrt{5x^2+2x+3}} - \frac{11x^2}{2\sqrt{5x^2+2x+3}} + \frac{149}{125}\sqrt{5} \operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{2837x}{350\sqrt{5x^2+2x+3}} - \frac{2953}{350\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2), x, algorithm="maxima")`

[Out] `-7/10*x^3/sqrt(5*x^2+2*x+3) - 11/2*x^2/sqrt(5*x^2+2*x+3) + 149/125*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x+1)) - 2837/350*x/sqrt(5*x^2+2*x+3) - 2953/350/sqrt(5*x^2+2*x+3)`

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2 + 5x + 2)(-7x^2 + 4x + 1)}{(5x^2 + 2x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)`

[Out] `int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}} dx - \int \frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}} dx - \int \frac{31x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}} dx - \int \frac{7x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}} dx - \int \frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2), x)`

```
[Out] -Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) +
3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 + 2*x +
3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral
(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*
x**2 + 2*x + 3)), x)
```



$$3.373 \quad \int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}} dx$$

**Optimal.** Leaf size=166

$$\frac{131 - 605x}{3556\sqrt{5x^2 + 2x + 3}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016} + \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}$$

**Rubi [A]** time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1060, 1032, 724, 206}

$$\frac{131 - 605x}{3556\sqrt{5x^2 + 2x + 3}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016} + \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)\*(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out] -(131 - 605\*x)/(3556\*Sqrt[3 + 2\*x + 5\*x^2]) - (3\*Sqrt[(281693 - 25015\*Sqrt[11])/1397]\*ArcTanh[(23 - Sqrt[11] + (17 - 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 - 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/1016 + (3\*Sqrt[(281693 + 25015\*Sqrt[11])/1397]\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[2\*(125 + 17\*Sqrt[11])]\*Sqrt[3 + 2\*x + 5\*x^2])])/1016

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/Sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 1032

Int[((g\_) + (h\_)\*(x\_))/(((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)\*Sqrt[(d\_) + (e\_)\*(x\_) + (f\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c\*g - h\*(b - q))/q, Int[1/((b - q) + 2\*c\*x)\*Sqrt[d + e\*x + f\*x^2]), x],

$x] - \text{Dist}[(2*c*g - h*(b + q))/q, \text{Int}[1/((b + q + 2*c*x)*\text{Sqrt}[d + e*x + f*x^2]), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

### Rule 1060

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}*((A_.) + (B_)*(x_) + (C_)*(x_)^2)*((d_ + (e_)*(x_) + (f_)*(x_)^2)^{(q_)}, x\_Symbol] :> \text{Simp}[(a + b*x + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^{(q + 1)}*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x]/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + \text{Dist}[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)}*(d + e*x + f*x^2)^q*\text{Simp}[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, q\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[e^2 - 4*d*f, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] \ \&\& \ !( !\text{IntegerQ}[p] \ \&\& \ \text{ILtQ}[q, -1]) \ \&\& \ !\text{IGtQ}[q, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{2 + 5x + x^2}{(1 + 4x - 7x^2)(3 + 2x + 5x^2)^{3/2}} dx &= -\frac{131 - 605x}{3556\sqrt{3 + 2x + 5x^2}} + \frac{\int \frac{13776 + 14112x}{(1 + 4x - 7x^2)\sqrt{3 + 2x + 5x^2}} dx}{28448} \\
&= -\frac{131 - 605x}{3556\sqrt{3 + 2x + 5x^2}} + \frac{(21(66 - 53\sqrt{11})) \int \frac{1}{(4 - 2\sqrt{11} - 14x)\sqrt{3 + 2x + 5x^2}} dx}{2794} \\
&= -\frac{131 - 605x}{3556\sqrt{3 + 2x + 5x^2}} - \frac{(21(66 - 53\sqrt{11})) \operatorname{Subst}\left(\int \frac{1}{2352 + 112(4 - 2\sqrt{11})x} dx\right)}{2794} \\
&= -\frac{131 - 605x}{3556\sqrt{3 + 2x + 5x^2}} - \frac{3\sqrt{\frac{281693 - 25015\sqrt{11}}{1397}} \tanh^{-1}\left(\frac{23 - \sqrt{11} + (17 - 5\sqrt{11})x}{\sqrt{2(125 - 17\sqrt{11}) + 3(3 + 2x + 5x^2)}}\right)}{1016}
\end{aligned}$$

**Mathematica [A]** time = 1.13, size = 174, normalized size = 1.05

$$\frac{\frac{2794(605x - 131)}{\sqrt{5x^2 + 2x + 3}} - 21\sqrt{127(125 + 17\sqrt{11})} (53\sqrt{11} - 66) \tanh^{-1}\left(\frac{-5\sqrt{11}x + 17x - \sqrt{11} + 23}{\sqrt{250 - 34\sqrt{11}} \sqrt{5x^2 + 2x + 3}}\right) + 21\sqrt{127(125 - 17\sqrt{11})} (66 + 53\sqrt{11}) \tanh^{-1}\left(\frac{(17 + 5\sqrt{11})x + \sqrt{11} + 23}{\sqrt{250 + 34\sqrt{11}} \sqrt{5x^2 + 2x + 3}}\right)}{9935464}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)\*(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out] ((2794\*(-131 + 605\*x))/Sqrt[3 + 2\*x + 5\*x^2] - 21\*Sqrt[127\*(125 + 17\*Sqrt[11])]\*(-66 + 53\*Sqrt[11])\*ArcTanh[(23 - Sqrt[11] + 17\*x - 5\*Sqrt[11]\*x)/(Sqrt[250 - 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])] + 21\*Sqrt[127\*(125 - 17\*Sqrt[11])]\*(66 + 53\*Sqrt[11])\*ArcTanh[(23 + Sqrt[11] + (17 + 5\*Sqrt[11])\*x)/(Sqrt[250 + 34\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2])])/9935464

**IntegrateAlgebraic [C]** time = 0.62, size = 199, normalized size = 1.20

$$\frac{3}{254} \operatorname{RootSum}\left[7\#1^4 + 8\sqrt{5}\#1^3 - 70\#1^2 - 16\sqrt{5}\#1 + 83\&, \frac{-21\#1^2 \log(-\#1 + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) + 41\sqrt{5}\#1 \log(-\#1 + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x) + 22 \log(-\#1 + \sqrt{5x^2 + 2x + 3} - \sqrt{5}x)}{7\#1^3 + 6\sqrt{5}\#1^2 - 35\#1 - 4\sqrt{5}}\right] + \frac{605x - 131}{3556\sqrt{5x^2 + 2x + 3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)\*(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out] (-131 + 605\*x)/(3556\*Sqrt[3 + 2\*x + 5\*x^2]) + (3\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 &, (22\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 41\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1

]#1 - 21\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/254

**fricas** [B] time = 0.78, size = 333, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)/(5\*x^2+2\*x+3)^(3/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/19870928*(21*\sqrt{1397}*(5*x^2 + 2*x + 3)*\sqrt{25015*\sqrt{11} + 281693})* \\ & \log(3*(\sqrt{1397})*\sqrt{5*x^2 + 2*x + 3}*\sqrt{25015*\sqrt{11} + 281693}*(1335 \\ & *\sqrt{11} - 8173) + 23596727*\sqrt{11}*(x + 3) + 70790181*x - 117983635)/x) \\ & - 21*\sqrt{1397}*(5*x^2 + 2*x + 3)*\sqrt{25015*\sqrt{11} + 281693}*\log(-3*(\sqrt{1397})*\sqrt{5*x^2 + 2*x + 3}*\sqrt{25015*\sqrt{11} + 281693}*(1335*\sqrt{11} \\ & - 8173) - 23596727*\sqrt{11}*(x + 3) - 70790181*x + 117983635)/x) + 7*\sqrt{1397}*(1 \\ & 397)*(5*x^2 + 2*x + 3)*\sqrt{-225135*\sqrt{11} + 2535237}*\log(-(\sqrt{1397})*\sqrt{5*x^2 + 2*x + 3}*(1335*\sqrt{11} + 8173)*\sqrt{-225135*\sqrt{11} + 2535237} \\ & + 70790181*\sqrt{11}*(x + 3) - 212370543*x + 353950905)/x) - 7*\sqrt{1397}*( \\ & 5*x^2 + 2*x + 3)*\sqrt{-225135*\sqrt{11} + 2535237}*\log((\sqrt{1397})*\sqrt{5*x^2 + 2*x + 3}*(1335*\sqrt{11} + 8173)*\sqrt{-225135*\sqrt{11} + 2535237} - 7079 \\ & 0181*\sqrt{11}*(x + 3) + 212370543*x - 353950905)/x) - 5588*\sqrt{5*x^2 + 2*x + 3}*(605*x - 131)/(5*x^2 + 2*x + 3) \end{aligned}$$

**giac** [A] time = 0.25, size = 112, normalized size = 0.67

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)/(5\*x^2+2\*x+3)^(3/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/3556*(605*x - 131)/\sqrt{5*x^2 + 2*x + 3} + 0.0477059376663667*\log(-\sqrt{5} \\ & )*x + \sqrt{5*x^2 + 2*x + 3} + 4.41924736459000) - 0.0352174957838020*\log(-\sqrt{5} \\ & )*x + \sqrt{5*x^2 + 2*x + 3} + 1.25295163054000) - 0.0477059376663667*1 \\ & \log(-\sqrt{5})*x + \sqrt{5*x^2 + 2*x + 3} - 1.02258038113000) + 0.0352174957838 \\ & 020*\log(-\sqrt{5})*x + \sqrt{5*x^2 + 2*x + 3} - 2.09411235400000) \end{aligned}$$

**maple** [B] time = 0.02, size = 489, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^2+5x+2)/(-7x^2+4x+1)/(5x^2+2x+3)^{(3/2)}, x)$

[Out] 
$$\begin{aligned} & -1/196*(10*x+2)/(5*x^2+2*x+3)^{(1/2)}-3/154*(-61+13*11^{(1/2)})*11^{(1/2)}*(1/7/( \\ & 250/49-34/49*11^{(1/2)})/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/ \\ & 7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-1/7*(34/7-10/7*11^{(1/2)})/(250/ \\ & 49-34/49*11^{(1/2)})*(10*x+2)/(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2 \\ & )/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/4 \\ & 9-34/49*11^{(1/2)})^{(1/2)}-1/(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*a \\ & rctanh(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)} \\ & ))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)} \\ & ))*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}-3/154*(61+13*11^{(1/2)})*11 \\ & ^{(1/2)}*(1/7/(250/49+34/49*11^{(1/2)})/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11 \\ & ^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-1/7*(34/7+10/7*11 \\ & ^{(1/2)})/(250/49+34/49*11^{(1/2)})*(10*x+2)/(5000/49+680/49*11^{(1/2)}-(34/7+10/ \\ & 7*11^{(1/2)})^2)/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11 \\ & ^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-1/(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)} \\ & )^{(1/2)}*arctanh(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7 \\ & -1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/ \\ & 7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)} \end{aligned}$$

**maxima [B]** time = 1.16, size = 777, normalized size = 4.68

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Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((x^2+5x+2)/(-7x^2+4x+1)/(5x^2+2x+3)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & -1/4312*\text{sqrt}(11)*(20*\text{sqrt}(11)*x/\text{sqrt}(5*x^2 + 2*x + 3) - 7890*\text{sqrt}(11)*x/(17 \\ & * \text{sqrt}(11)*\text{sqrt}(5*x^2 + 2*x + 3) + 125*\text{sqrt}(5*x^2 + 2*x + 3)) + 7890*\text{sqrt}(11 \\ & )*x/(17*\text{sqrt}(11)*\text{sqrt}(5*x^2 + 2*x + 3) - 125*\text{sqrt}(5*x^2 + 2*x + 3)) - 13377 \\ & * \text{sqrt}(11)*\text{sqrt}(2)*\text{arcsinh}(5/7*\text{sqrt}(11)*\text{sqrt}(7)*\text{sqrt}(2)*x/\text{abs}(14*x - 2*\text{sqrt}( \\ & 11) - 4) + 17/7*\text{sqrt}(7)*\text{sqrt}(2)*x/\text{abs}(14*x - 2*\text{sqrt}(11) - 4) + 1/7*\text{sqrt}(11) \\ & * \text{sqrt}(7)*\text{sqrt}(2)/\text{abs}(14*x - 2*\text{sqrt}(11) - 4) + 23/7*\text{sqrt}(7)*\text{sqrt}(2)/\text{abs}(14*x \\ & - 2*\text{sqrt}(11) - 4))/(17*\text{sqrt}(11) + 125)^{(3/2)} + 4*\text{sqrt}(11)/\text{sqrt}(5*x^2 + 2*x \\ & + 3) - 26280*x/(17*\text{sqrt}(11)*\text{sqrt}(5*x^2 + 2*x + 3) + 125*\text{sqrt}(5*x^2 + 2*x + \\ & 3)) - 26280*x/(17*\text{sqrt}(11)*\text{sqrt}(5*x^2 + 2*x + 3) - 125*\text{sqrt}(5*x^2 + 2*x + \\ & 3)) + 156*\text{sqrt}(11)*\text{arcsinh}(5/7*\text{sqrt}(11)*\text{sqrt}(7)*\text{sqrt}(2)*x/\text{abs}(14*x + 2*\text{sqrt} \\ & (11) - 4) - 17/7*\text{sqrt}(7)*\text{sqrt}(2)*x/\text{abs}(14*x + 2*\text{sqrt}(11) - 4) + 1/7*\text{sqrt}(11) \\ & )*\text{sqrt}(7)*\text{sqrt}(2)/\text{abs}(14*x + 2*\text{sqrt}(11) - 4) - 23/7*\text{sqrt}(7)*\text{sqrt}(2)/\text{abs}(14* \\ & x + 2*\text{sqrt}(11) - 4))/(-34/49*\text{sqrt}(11) + 250/49)^{(3/2)} - 62769*\text{sqrt}(2)*\text{arcsi} \\ & nh(5/7*\text{sqrt}(11)*\text{sqrt}(7)*\text{sqrt}(2)*x/\text{abs}(14*x - 2*\text{sqrt}(11) - 4) + 17/7*\text{sqrt}(7) \\ & * \text{sqrt}(2)*x/\text{abs}(14*x - 2*\text{sqrt}(11) - 4) + 1/7*\text{sqrt}(11)*\text{sqrt}(7)*\text{sqrt}(2)/\text{abs}(14 \\ & *x - 2*\text{sqrt}(11) - 4) + 23/7*\text{sqrt}(7)*\text{sqrt}(2)/\text{abs}(14*x - 2*\text{sqrt}(11) - 4))/(17 \\ & * \text{sqrt}(11) + 125)^{(3/2)} + 2244*\text{sqrt}(11)/(17*\text{sqrt}(11)*\text{sqrt}(5*x^2 + 2*x + 3) + \end{aligned}$$

$125\sqrt{5x^2 + 2x + 3}) - 2244\sqrt{11}/(17\sqrt{11})\sqrt{5x^2 + 2x + 3} - 125\sqrt{5x^2 + 2x + 3}) - 732\operatorname{arcsinh}(5/7\sqrt{11})\sqrt{7})\sqrt{2})$   
 $*x/\operatorname{abs}(14*x + 2*\sqrt{11} - 4) - 17/7*\sqrt{7})\sqrt{2})x/\operatorname{abs}(14*x + 2*\sqrt{11}) - 4) + 1/7*\sqrt{11})\sqrt{7})\sqrt{2})/\operatorname{abs}(14*x + 2*\sqrt{11}) - 4) - 23/7*\sqrt{7})\sqrt{2})/\operatorname{abs}(14*x + 2*\sqrt{11}) - 4))/(-34/49*\sqrt{11}) + 250/49)^{(3/2)} +$   
 $12678/(17*\sqrt{11})\sqrt{5*x^2 + 2*x + 3) + 125*\sqrt{5*x^2 + 2*x + 3}) + 12678/(17*\sqrt{11})\sqrt{5*x^2 + 2*x + 3) - 125*\sqrt{5*x^2 + 2*x + 3}))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 + 5x + 2}{(5x^2 + 2x + 3)^{3/2} (-7x^2 + 4x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)`

[Out] `int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x}{35x^4\sqrt{5x^2+2x+3} - 6x^3\sqrt{5x^2+2x+3} + 8x^2\sqrt{5x^2+2x+3} - 14x\sqrt{5x^2+2x+3} - 3\sqrt{5x^2+2x+3}} dx - \int \frac{x^2}{35x^4\sqrt{5x^2+2x+3} - 6x^3\sqrt{5x^2+2x+3} + 8x^2\sqrt{5x^2+2x+3} - 14x\sqrt{5x^2+2x+3} - 3\sqrt{5x^2+2x+3}} dx - \int \frac{2}{35x^4\sqrt{5x^2+2x+3} - 6x^3\sqrt{5x^2+2x+3} + 8x^2\sqrt{5x^2+2x+3} - 14x\sqrt{5x^2+2x+3} - 3\sqrt{5x^2+2x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2), x)`

[Out] `-Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)`

$$3.374 \quad \int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}} dx$$

**Optimal.** Leaf size=215

$$\frac{3(40 - 371x)}{5588(-7x^2 + 4x + 1)\sqrt{5x^2 + 2x + 3}} - \frac{22755x + 76567}{19870928\sqrt{5x^2 + 2x + 3}} - \frac{7(541543 - 5144\sqrt{11}) \tanh^{-1}\left(\frac{(17-5\sqrt{11})}{\sqrt{2(125-17\sqrt{11})}}\right)}{2838704\sqrt{22(125 - 17\sqrt{11})}}$$

**Rubi [A]** time = 0.32, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1060, 1032, 724, 206}

$$\frac{3(40 - 371x)}{5588(-7x^2 + 4x + 1)\sqrt{5x^2 + 2x + 3}} - \frac{22755x + 76567}{19870928\sqrt{5x^2 + 2x + 3}} - \frac{7(541543 - 5144\sqrt{11}) \tanh^{-1}\left(\frac{(17-5\sqrt{11})x - \sqrt{11} + 23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2838704\sqrt{22(125 - 17\sqrt{11})}} + \frac{7(541543 + 5144\sqrt{11}) \tanh^{-1}\left(\frac{(17+5\sqrt{11})x + \sqrt{11} + 23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2838704\sqrt{22(125 + 17\sqrt{11})}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^2\*(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out] -(76567 + 22755\*x)/(19870928\*sqrt[3 + 2\*x + 5\*x^2]) - (3\*(40 - 371\*x))/(5588\*(1 + 4\*x - 7\*x^2)\*sqrt[3 + 2\*x + 5\*x^2]) - (7\*(541543 - 5144\*sqrt[11])\*ArcTanh[(23 - sqrt[11] + (17 - 5\*sqrt[11])\*x)/(sqrt[2\*(125 - 17\*sqrt[11])]\*sqrt[3 + 2\*x + 5\*x^2])])/(2838704\*sqrt[22\*(125 - 17\*sqrt[11])]) + (7\*(541543 + 5144\*sqrt[11])\*ArcTanh[(23 + sqrt[11] + (17 + 5\*sqrt[11])\*x)/(sqrt[2\*(125 + 17\*sqrt[11])]\*sqrt[3 + 2\*x + 5\*x^2])])/(2838704\*sqrt[22\*(125 + 17\*sqrt[11])])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_) + (e\_)\*(x\_))\*sqrt[(a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 1032

```

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]

```

### Rule 1060

```

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]

```

### Rubi steps



$$\int \frac{2 + 5x + x^2}{(1 + 4x - 7x^2)^2 (3 + 2x + 5x^2)^{3/2}} dx = -\frac{3(40 - 371x)}{5588 (1 + 4x - 7x^2) \sqrt{3 + 2x + 5x^2}} - \frac{\int \frac{-50216 - 37752x - 89040x^2}{(1 + 4x - 7x^2)(3 + 2x + 5x^2)^{3/2}} dx}{44704}$$

$$= -\frac{76567 + 22755x}{19870928 \sqrt{3 + 2x + 5x^2}} - \frac{3(40 - 371x)}{5588 (1 + 4x - 7x^2) \sqrt{3 + 2x + 5x^2}}$$

$$= -\frac{76567 + 22755x}{19870928 \sqrt{3 + 2x + 5x^2}} - \frac{3(40 - 371x)}{5588 (1 + 4x - 7x^2) \sqrt{3 + 2x + 5x^2}}$$

$$= -\frac{76567 + 22755x}{19870928 \sqrt{3 + 2x + 5x^2}} - \frac{3(40 - 371x)}{5588 (1 + 4x - 7x^2) \sqrt{3 + 2x + 5x^2}}$$

$$= -\frac{76567 + 22755x}{19870928 \sqrt{3 + 2x + 5x^2}} - \frac{3(40 - 371x)}{5588 (1 + 4x - 7x^2) \sqrt{3 + 2x + 5x^2}}$$

**Mathematica [A]** time = 1.14, size = 351, normalized size = 1.63

$$\frac{12968296 \sqrt{3+2x+5x^2} + 24422640 x \sqrt{3+2x+5x^2} + 5084772 x \sqrt{3+2x+5x^2} \operatorname{ArcTanh}\left(\frac{\sqrt{250-34\sqrt{11}} \sqrt{3+2x+5x^2}}{-23+\sqrt{11}+(-17+5\sqrt{11})x}\right) + 1672044 \sqrt{3+2x+5x^2} \operatorname{Log}\left(\frac{2+\sqrt{11}-7x}{1+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}\right) + 14 \sqrt{2/(125-17\sqrt{11})} \operatorname{Log}\left(\frac{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}\right) + 14 \sqrt{2/(125+17\sqrt{11})} \operatorname{Log}\left(\frac{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}\right)}{124902976}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)), x]
[Out] (12968296/(7*sqrt(3 + 2*x + 5*x^2))) + (24422640*x)/(7*sqrt(3 + 2*x + 5*x^2))
+ (5084772*x*sqrt(3 + 2*x + 5*x^2))/(1 + 4*x - 7*x^2) + (1672044*sqrt(3 +
2*x + 5*x^2))/(-1 - 4*x + 7*x^2) + 14*sqrt(2/(125 - 17*sqrt(11)))*(-56584
+ 541543*sqrt(11))*ArcTanh[(sqrt(250 - 34*sqrt(11))*sqrt(3 + 2*x + 5*x^2))/
(-23 + sqrt(11) + (-17 + 5*sqrt(11))*x)] - 14*sqrt(2/(125 + 17*sqrt(11)))*
(56584 + 541543*sqrt(11))*Log[2 + sqrt(11) - 7*x] + 792176*sqrt(2/(125 + 17*
sqrt(11)))*Log[11 + 23*sqrt(11) + (55 + 17*sqrt(11))*x + sqrt(2750 + 374*sqrt(11))*sqrt(3 +
2*x + 5*x^2)] + 7581602*sqrt(22/(125 + 17*sqrt(11)))*Log[11 + 23*sqrt(11) + (55 + 17*sqrt(11))*x + sqrt(2750 + 374*sqrt(11))*sqrt(3 +
2*x + 5*x^2)]/124902976
```

**IntegrateAlgebraic [C]** time = 0.95, size = 416, normalized size = 1.93

$$\frac{12968296 \sqrt{3+2x+5x^2} + 24422640 x \sqrt{3+2x+5x^2} + 5084772 x \sqrt{3+2x+5x^2} \operatorname{ArcTanh}\left(\frac{\sqrt{250-34\sqrt{11}} \sqrt{3+2x+5x^2}}{-23+\sqrt{11}+(-17+5\sqrt{11})x}\right) + 1672044 \sqrt{3+2x+5x^2} \operatorname{Log}\left(\frac{2+\sqrt{11}-7x}{1+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}\right) + 14 \sqrt{2/(125-17\sqrt{11})} \operatorname{Log}\left(\frac{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}\right) + 14 \sqrt{2/(125+17\sqrt{11})} \operatorname{Log}\left(\frac{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}{11+23\sqrt{11}+(55+17\sqrt{11})x+\sqrt{2750+374\sqrt{11}}\sqrt{3+2x+5x^2}}\right)}{124902976}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (503287 - 3628805*x - 444949*x^2 - 159285*x^3)/(19870928*Sqrt[3 + 2*x + 5*x^2]*(-1 - 4*x + 7*x^2)) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (116685*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 205710*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 8351*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(258064*Sqrt[5]) - (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (746007*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1016580*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 42623*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/(2838704*Sqrt[5])
```

**fricas [B]** time = 0.62, size = 392, normalized size = 1.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294093814065*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) - 2865029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) + 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) + 2865029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) - 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) + 5588*(159285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*x + 3))/(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)
```

**giac [A]** time = 0.27, size = 295, normalized size = 1.37

$$\frac{20281x + 13397}{90324\sqrt{5x^2+2x+3}} - \frac{9(4261(\sqrt{5x^2+2x+3})^2 + 7792\sqrt{5x^2+2x+3}) - 27981\sqrt{5x^2+2x+3} - 20960\sqrt{5x^2+2x+3}}{20626(9(\sqrt{5x^2+2x+3})^2 - 8\sqrt{5x^2+2x+3}) - 20(9(\sqrt{5x^2+2x+3})^2 + 16\sqrt{5x^2+2x+3}) - 80} + 0.0218058276254033 \log(-\sqrt{5x^2+2x+3}) + 1.01027649000 - 0.0332874364433911 \log(-\sqrt{5x^2+2x+3} + 1.25295163054000) - 0.0218058276254033 \log(-\sqrt{5x^2+2x+3} - 1.02258038113000) + 0.0332874364433911 \log(-\sqrt{5x^2+2x+3} - 2.09411235400000)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^2/(5\*x^2+2\*x+3)^(3/2),x, algorithm="giac")

[Out] 1/903224\*(25230\*x + 13397)/sqrt(5\*x^2 + 2\*x + 3) + 3/709676\*(42623\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^3 + 77302\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^2 - 275511\*sqrt(5)\*x - 219860\*sqrt(5) + 275511\*sqrt(5\*x^2 + 2\*x + 3))/(7\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^4 - 8\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^3 - 70\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3))^2 + 16\*sqrt(5)\*(sqrt(5)\*x - sqrt(5\*x^2 + 2\*x + 3)) + 83) + 0.0218058276254033\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 4.41924736459000) - 0.0332874364433911\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) + 1.25295163054000) - 0.0218058276254033\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 1.02258038113000) + 0.0332874364433911\*log(-sqrt(5)\*x + sqrt(5\*x^2 + 2\*x + 3) - 2.09411235400000)

**maple [B]** time = 0.02, size = 1214, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^2/(5\*x^2+2\*x+3)^(3/2),x)

[Out] 161/484\*11^(1/2)\*(1/7/(250/49-34/49\*11^(1/2)))/(5\*(x-2/7+1/7\*11^(1/2))^2+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250/49-34/49\*11^(1/2))^(1/2)-1/7\*(34/7-10/7\*11^(1/2))/(250/49-34/49\*11^(1/2))\*(10\*x+2)/(5000/49-680/49\*11^(1/2)-(34/7-10/7\*11^(1/2))^2)/(5\*(x-2/7+1/7\*11^(1/2))^2+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250/49-34/49\*11^(1/2))^(1/2)-1/(250/49-34/49\*11^(1/2))/(250-34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49-68/49\*11^(1/2)+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2)))/(250-34\*11^(1/2))^(1/2)/(245\*(x-2/7+1/7\*11^(1/2))^2+49\*(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250-34\*11^(1/2))^(1/2))+1/83/44-39/44\*11^(1/2))\*(-1/49/(250/49-34/49\*11^(1/2)))/(x-2/7+1/7\*11^(1/2))/(5\*(x-2/7+1/7\*11^(1/2))^2+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250/49-34/49\*11^(1/2))^(1/2)-3/98\*(34/7-10/7\*11^(1/2))/(250/49-34/49\*11^(1/2))\*(1/(250/49-34/49\*11^(1/2)))/(5\*(x-2/7+1/7\*11^(1/2))^2+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250/49-34/49\*11^(1/2))^(1/2)-(34/7-10/7\*11^(1/2))/(250/49-34/49\*11^(1/2))\*(10\*x+2)/(5000/49-680/49\*11^(1/2)-(34/7-10/7\*11^(1/2))^2)/(5\*(x-2/7+1/7\*11^(1/2))^2+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250/49-34/49\*11^(1/2))^(1/2)-7/(250/49-34/49\*11^(1/2))/(250-34\*11^(1/2))^(1/2)\*arctanh(49/2\*(500/49-68/49\*11^(1/2)+(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2)))/(250-34\*11^(1/2))^(1/2)/(245\*(x-2/7+1/7\*11^(1/2))^2+49\*(34/7-10/7\*11^(1/2))\*(x-2/7+1/7\*11^(1/2))+250-34\*11^(1/2))^(1/2))-20/49/(250/49-34/49\*11^(1/2))

$$\frac{(10x+2)/(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}+(183/44+39/44*11^{(1/2)})*(-1/49/(250/49+34/49*11^{(1/2)}))/(x-2/7-1/7*11^{(1/2)})/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-3/98*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(1/(250/49+34/49*11^{(1/2)}))/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(10*x+2)/(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-7/(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}-20/49/(250/49+34/49*11^{(1/2)})*(10*x+2)/(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-161/484*11^{(1/2)}*(1/7/(250/49+34/49*11^{(1/2)}))/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-1/7*(34/7+10/7*11^{(1/2)})/(250/49+34/49*11^{(1/2)})*(10*x+2)/(5000/49+680/49*11^{(1/2)}-(34/7+10/7*11^{(1/2)})^2)/(5*(x-2/7-1/7*11^{(1/2)})^2+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250/49+34/49*11^{(1/2)})^{(1/2)}-1/(250/49+34/49*11^{(1/2)})/(250+34*11^{(1/2)})^{(1/2)}*\operatorname{arctanh}(49/2*(500/49+68/49*11^{(1/2)}+(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})))/(250+34*11^{(1/2)})^{(1/2)}/(245*(x-2/7-1/7*11^{(1/2)})^2+49*(34/7+10/7*11^{(1/2)})*(x-2/7-1/7*11^{(1/2)})+250+34*11^{(1/2)})^{(1/2)}))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5x + 2}{(7x^2 - 4x - 1)^2 (5x^2 + 2x + 3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^2/(5\*x^2+2\*x+3)^(3/2),x, algorithm="maxima")

[Out] integrate((x^2 + 5\*x + 2)/((7\*x^2 - 4\*x - 1)^2\*(5\*x^2 + 2\*x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 + 5x + 2}{(5x^2 + 2x + 3)^{3/2} (-7x^2 + 4x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x + x^2 + 2)/((2\*x + 5\*x^2 + 3)^(3/2)\*(4\*x - 7\*x^2 + 1)^2),x)

[Out] `int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 5x + 2}{(5x^2 + 2x + 3)^{\frac{3}{2}} (7x^2 - 4x - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2), x)`

[Out] `Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2), x)`

$$3.375 \quad \int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}} dx$$

Optimal. Leaf size=250

$$\frac{2701733 - 9148874x}{62451488(-7x^2 + 4x + 1)\sqrt{5x^2 + 2x + 3}} - \frac{5(1118731375x + 461370781)}{222077491328\sqrt{5x^2 + 2x + 3}} - \frac{3(40 - 371x)}{11176(-7x^2 + 4x + 1)^2\sqrt{5x^2 + 2x + 3}}$$

**Rubi [A]** time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {1060, 1032, 724, 206}

$$\frac{2701733 - 9148874x}{62451488(-7x^2 + 4x + 1)\sqrt{5x^2 + 2x + 3}} - \frac{5(1118731375x + 461370781)}{222077491328\sqrt{5x^2 + 2x + 3}} - \frac{3(40 - 371x)}{11176(-7x^2 + 4x + 1)^2\sqrt{5x^2 + 2x + 3}} - \frac{7(2792860024 - 84865895\sqrt{11})\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{31725355904\sqrt{22(125-17\sqrt{11})}} + \frac{7(2792860024 + 84865895\sqrt{11})\tanh^{-1}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{31725355904\sqrt{22(125+17\sqrt{11})}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^3\*(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out] (-5\*(461370781 + 1118731375\*x))/(222077491328\*sqrt[3 + 2\*x + 5\*x^2]) - (3\*(40 - 371\*x))/(11176\*(1 + 4\*x - 7\*x^2)^2\*sqrt[3 + 2\*x + 5\*x^2]) - (2701733 - 9148874\*x)/(62451488\*(1 + 4\*x - 7\*x^2)\*sqrt[3 + 2\*x + 5\*x^2]) - (7\*(2792860024 - 84865895\*sqrt[11])\*ArcTanh[(23 - sqrt[11] + (17 - 5\*sqrt[11])\*x)/(sqrt[2\*(125 - 17\*sqrt[11])]\*sqrt[3 + 2\*x + 5\*x^2])])/(31725355904\*sqrt[22\*(125 - 17\*sqrt[11])]) + (7\*(2792860024 + 84865895\*sqrt[11])\*ArcTanh[(23 + sqrt[11] + (17 + 5\*sqrt[11])\*x)/(sqrt[2\*(125 + 17\*sqrt[11])]\*sqrt[3 + 2\*x + 5\*x^2])])/(31725355904\*sqrt[22\*(125 + 17\*sqrt[11])])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 724

Int[1/(((d\_.) + (e\_.)\*(x\_))\*sqrt[(a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Dist[-2, Subst[Int[1/(4\*c\*d^2 - 4\*b\*d\*e + 4\*a\*e^2 - x^2), x], x, (2\*a\*e - b\*d - (2\*c\*d - b\*e)\*x)/sqrt[a + b\*x + c\*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 1032

```
Int[((g_.) + (h_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]], x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

### Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && (!(IntegerQ[p] && ILtQ[q, -1])) && !
IGtQ[q, 0]
```

### Rubi steps





+ 14\*Sqrt[22/(125 + 17\*Sqrt[11])]\*(2792860024 + 84865895\*Sqrt[11])\*Log[11 + 23\*Sqrt[11] + (55 + 17\*Sqrt[11])\*x + Sqrt[2750 + 374\*Sqrt[11]]\*Sqrt[3 + 2\*x + 5\*x^2]]/1395915659776

**IntegrateAlgebraic [C]** time = 1.32, size = 611, normalized size = 2.44

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 5\*x + x^2)/((1 + 4\*x - 7\*x^2)^3\*(3 + 2\*x + 5\*x^2)^(3/2)), x]

[Out] (-14298727813 - 7828199499\*x + 148022158802\*x^2 - 109737266678\*x^3 + 200208943655\*x^4 - 274089186875\*x^5)/(222077491328\*Sqrt[3 + 2\*x + 5\*x^2]\*(-1 - 4\*x + 7\*x^2)^2) + RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (-4989740\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 3790865\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 400449\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ]/(32774128\*Sqrt[5]) + RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (-3200991286865\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 18470877323690\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 2296522946389\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ]/(3462389978432\*Sqrt[5]) - (9\*RootSum[83 - 16\*Sqrt[5]\*#1 - 70\*#1^2 + 8\*Sqrt[5]\*#1^3 + 7\*#1^4 & , (-8189062651053\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1] + 39132066594240\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1 + 5875617407695\*Sqrt[5]\*Log[-(Sqrt[5]\*x) + Sqrt[3 + 2\*x + 5\*x^2] - #1]\*#1^2)/(-4\*Sqrt[5] - 35\*#1 + 6\*Sqrt[5]\*#1^2 + 7\*#1^3) & ])/(76172579525504\*Sqrt[5])

**fricas [B]** time = 1.17, size = 452, normalized size = 1.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^3/(5\*x^2+2\*x+3)^(3/2), x, algorithm="fricas")

[Out] -1/1240969021540864\*(7\*sqrt(1397)\*(245\*x^6 - 182\*x^5 + 45\*x^4 - 124\*x^3 + 27\*x^2 + 26\*x + 3)\*sqrt(74693314710639641467\*sqrt(11) + 896266498377233657855)\*log(-(sqrt(1397)\*sqrt(5\*x^2 + 2\*x + 3)\*sqrt(74693314710639641467\*sqrt(11) + 896266498377233657855)\*(37271563201\*sqrt(11) + 407780707037) + 75502120686844055144479\*sqrt(11)\*(x + 3) - 226506362060532165433437\*x + 377510603434220275722395)/x) - 7\*sqrt(1397)\*(245\*x^6 - 182\*x^5 + 45\*x^4 - 124\*x^3 + 27\*x^2 + 26\*x + 3)\*sqrt(74693314710639641467\*sqrt(11) + 896266498377233657855

$$\begin{aligned} & ) * \log((\sqrt{1397}) * \sqrt{5x^2 + 2x + 3}) * \sqrt{74693314710639641467} * \sqrt{11} \\ & + 896266498377233657855) * (37271563201 * \sqrt{11} + 407780707037) - 7550212068 \\ & 6844055144479 * \sqrt{11} * (x + 3) + 226506362060532165433437 * x - 3775106034342 \\ & 20275722395) / x + 7 * \sqrt{1397} * (245 * x^6 - 182 * x^5 + 45 * x^4 - 124 * x^3 + 27 * x \\ & ^2 + 26 * x + 3) * \sqrt{-74693314710639641467} * \sqrt{11} + 896266498377233657855) \\ & * \log((\sqrt{1397}) * \sqrt{5x^2 + 2x + 3}) * (37271563201 * \sqrt{11} - 407780707037) \\ & ) * \sqrt{-74693314710639641467} * \sqrt{11} + 896266498377233657855) + 7550212068 \\ & 6844055144479 * \sqrt{11} * (x + 3) + 226506362060532165433437 * x - 3775106034342 \\ & 20275722395) / x - 7 * \sqrt{1397} * (245 * x^6 - 182 * x^5 + 45 * x^4 - 124 * x^3 + 27 * x \\ & ^2 + 26 * x + 3) * \sqrt{-74693314710639641467} * \sqrt{11} + 896266498377233657855) \\ & * \log(-(\sqrt{1397}) * \sqrt{5x^2 + 2x + 3}) * (37271563201 * \sqrt{11} - 407780707037) \\ & ) * \sqrt{-74693314710639641467} * \sqrt{11} + 896266498377233657855) - 7550212068 \\ & 6844055144479 * \sqrt{11} * (x + 3) - 226506362060532165433437 * x + 3775106034342 \\ & 220275722395) / x + 5588 * (274089186875 * x^5 - 200208943655 * x^4 + 109737266678 \\ & * x^3 - 148022158802 * x^2 + 7828199499 * x + 14298727813) * \sqrt{5x^2 + 2x + 3} \\ & ) / (245 * x^6 - 182 * x^5 + 45 * x^4 - 124 * x^3 + 27 * x^2 + 26 * x + 3) \end{aligned}$$

**giac [B]** time = 0.32, size = 397, normalized size = 1.59

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^3/(5\*x^2+2\*x+3)^(3/2),x, algorithm="giac")

[Out]  $\frac{1}{458837792} * (501205 * x + 1702037) / \sqrt{5x^2 + 2x + 3} + \frac{1}{7931338976} * (6871871279 * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^7 + 4012856750 * \sqrt{5} * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^6 - 223088535693 * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^5 - 100577598176 * \sqrt{5} * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^4 + 1255097956673 * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^3 + 566810398070 * \sqrt{5} * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^2 - 1246245909011 * \sqrt{5} * x - 561299654796 * \sqrt{5} + 1246245909011 * \sqrt{5x^2 + 2x + 3}) / (7 * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^4 - 8 * \sqrt{5} * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^3 - 70 * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3})^2 + 16 * \sqrt{5} * (\sqrt{5} * x - \sqrt{5x^2 + 2x + 3}) + 83)^2 + 0.0107382277384513 * \log(-\sqrt{5} * x + \sqrt{5x^2 + 2x + 3}) + 4.41924736459000) - 0.0142619066316905 * \log(-\sqrt{5} * x + \sqrt{5x^2 + 2x + 3}) + 1.25295163054000) - 0.0107382277384513 * \log(-\sqrt{5} * x + \sqrt{5x^2 + 2x + 3}) - 1.02258038113000) + 0.0142619066316905 * \log(-\sqrt{5} * x + \sqrt{5x^2 + 2x + 3}) - 2.09411235400000)$

**maple [B]** time = 0.02, size = 2600, normalized size = 10.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^{(3/2)}, x)$

[Out] 
$$\begin{aligned} & 3535/21296*11^{(1/2)}*(1/7/(250/49-34/49*11^{(1/2)})/(5*(x-2/7+1/7*11^{(1/2)})^2+ \\ & (34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-1/7* \\ & (34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(10*x+2)/(5000/49-680/49*11^{(1/2)}- \\ & (34/7-10/7*11^{(1/2)})^2)/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})* \\ & (x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-1/(250/49-34/49*11^{(1/2)}) \\ & /((250-34*11^{(1/2)})^{(1/2)}*\text{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)}) \\ & (1/2))* (x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)}) \\ & )^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)}) \\ & -21/968*(-61+13*11^{(1/2)})*11^{(1/2)}*(-1/686/(250/49-34/49*11^{(1/2)})/(x-2/7+1 \\ & /7*11^{(1/2)})^2/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11 \\ & ^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-5/1372*(34/7-10/7*11^{(1/2)})/(250/49-34 \\ & /49*11^{(1/2)})*(-1/(250/49-34/49*11^{(1/2)})/(x-2/7+1/7*11^{(1/2)})/(5*(x-2/7+1/ \\ & 7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)}) \\ & )^{(1/2)}-3/2*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(1/(250/49-34/49 \\ & *11^{(1/2)})/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}) \\ & )+250/49-34/49*11^{(1/2)})^{(1/2)}-(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)}) \\ & )*(10*x+2)/(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)/(5*(x-2/7+1/7* \\ & 11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)}) \\ & )^{(1/2)}-7/(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)}*\text{arctanh}(49/2*(500 \\ & /49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)}) \\ & )^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*1 \\ & 1^{(1/2)})+250-34*11^{(1/2)})^{(1/2)})-20/(250/49-34/49*11^{(1/2)})*(10*x+2)/(5000 \\ & /49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)})^2)/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7 \\ & -10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-15/686/( \\ & 250/49-34/49*11^{(1/2)})*(1/(250/49-34/49*11^{(1/2)})/(5*(x-2/7+1/7*11^{(1/2)})^2 \\ & +(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-(34 \\ & /7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)})*(10*x+2)/(5000/49-680/49*11^{(1/2)} \\ & -(34/7-10/7*11^{(1/2)})^2)/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x- \\ & 2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-7/(250/49-34/49*11^{(1/2)})/(2 \\ & 50-34*11^{(1/2)})^{(1/2)}*\text{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)}) \\ & (1/2))* (x-2/7+1/7*11^{(1/2)})))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)}) \\ & )^2+49*(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)})- \\ & (-3535/1936+273/1936*11^{(1/2)})*(-1/49/(250/49-34/49*11^{(1/2)})/(x-2/7+1/7*11^{(1/2)}) \\ & (1/2))/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+ \\ & 250/49-34/49*11^{(1/2)})^{(1/2)}-3/98*(34/7-10/7*11^{(1/2)})/(250/49-34/49*11^{(1/2)}) \\ & )*(1/(250/49-34/49*11^{(1/2)})/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)}) \\ & )*(x-2/7+1/7*11^{(1/2)})+250/49-34/49*11^{(1/2)})^{(1/2)}-(34/7-10/7*11^{(1/2)})/( \\ & 250/49-34/49*11^{(1/2)})*(10*x+2)/(5000/49-680/49*11^{(1/2)}-(34/7-10/7*11^{(1/2)}) \\ & )^2)/(5*(x-2/7+1/7*11^{(1/2)})^2+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+2 \\ & 50/49-34/49*11^{(1/2)})^{(1/2)}-7/(250/49-34/49*11^{(1/2)})/(250-34*11^{(1/2)})^{(1/2)} \\ & )*\text{arctanh}(49/2*(500/49-68/49*11^{(1/2)}+(34/7-10/7*11^{(1/2)})*(x-2/7+1/7*11^{(1/2)}) \\ & (1/2)))/(250-34*11^{(1/2)})^{(1/2)}/(245*(x-2/7+1/7*11^{(1/2)})^2+49*(34/7-10/7*11 \\ & ^{(1/2)})*(x-2/7+1/7*11^{(1/2)})+250-34*11^{(1/2)})^{(1/2)})-20/49/(250/49-34/49*1 \end{aligned}$$



$$\frac{8/49 \cdot 11^{1/2} + (34/7 + 10/7 \cdot 11^{1/2}) \cdot (x - 2/7 - 1/7 \cdot 11^{1/2})}{(250 + 34 \cdot 11^{1/2})^{1/2} / (245 \cdot (x - 2/7 - 1/7 \cdot 11^{1/2})^2 + 49 \cdot (34/7 + 10/7 \cdot 11^{1/2}) \cdot (x - 2/7 - 1/7 \cdot 11^{1/2}) + 250 + 34 \cdot 11^{1/2})^{1/2}}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 5x + 2}{(7x^2 - 4x - 1)^3 (5x^2 + 2x + 3)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5\*x+2)/(-7\*x^2+4\*x+1)^3/(5\*x^2+2\*x+3)^(3/2),x, algorithm="maxima")

[Out] -integrate((x^2 + 5\*x + 2)/((7\*x^2 - 4\*x - 1)^3\*(5\*x^2 + 2\*x + 3)^(3/2)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 + 5x + 2}{(5x^2 + 2x + 3)^{3/2} (-7x^2 + 4x + 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5\*x + x^2 + 2)/((2\*x + 5\*x^2 + 3)^(3/2)\*(4\*x - 7\*x^2 + 1)^3),x)

[Out] int((5\*x + x^2 + 2)/((2\*x + 5\*x^2 + 3)^(3/2)\*(4\*x - 7\*x^2 + 1)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x\*\*2+5\*x+2)/(-7\*x\*\*2+4\*x+1)\*\*3/(5\*x\*\*2+2\*x+3)\*\*(3/2),x)

[Out] -Integral(5\*x/(1715\*x\*\*8\*sqrt(5\*x\*\*2 + 2\*x + 3) - 2254\*x\*\*7\*sqrt(5\*x\*\*2 + 2\*x + 3) + 798\*x\*\*6\*sqrt(5\*x\*\*2 + 2\*x + 3) - 866\*x\*\*5\*sqrt(5\*x\*\*2 + 2\*x + 3) + 640\*x\*\*4\*sqrt(5\*x\*\*2 + 2\*x + 3) + 198\*x\*\*3\*sqrt(5\*x\*\*2 + 2\*x + 3) - 110\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) - 38\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) - 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(x\*\*2/(1715\*x\*\*8\*sqrt(5\*x\*\*2 + 2\*x + 3) - 2254\*x\*\*7\*sqrt(5\*x\*\*2 + 2\*x + 3) + 798\*x\*\*6\*sqrt(5\*x\*\*2 + 2\*x + 3) - 866\*x\*\*5\*sqrt(5\*x\*\*2 + 2\*x + 3) + 640\*x\*\*4\*sqrt(5\*x\*\*2 + 2\*x + 3) + 198\*x\*\*3\*sqrt(5\*x\*\*2 + 2\*x + 3) - 110\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) - 38\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) - 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x) - Integral(2/(1715\*x\*\*8\*sqrt(5\*x\*\*2 + 2\*x + 3) - 2254\*x\*\*7\*sqrt(5\*x\*\*2 + 2\*x + 3) + 798\*x\*\*6\*sqrt(5\*x\*\*2 + 2\*x + 3) - 866\*x\*\*5\*sqrt(5\*x\*\*2 + 2\*x + 3) + 640\*x\*\*4\*sqrt(5\*x\*\*2 + 2\*x + 3) + 198\*x\*\*3\*sqrt(5\*x\*\*2 + 2\*x + 3) - 110\*x\*\*2\*sqrt(5\*x\*\*2 + 2\*x + 3) - 38\*x\*sqrt(5\*x\*\*2 + 2\*x + 3) - 3\*sqrt(5\*x\*\*2 + 2\*x + 3)), x)

$$\begin{aligned} & 3) - 2254x^7\sqrt{5x^2 + 2x + 3} + 798x^6\sqrt{5x^2 + 2x + 3} - \\ & 866x^5\sqrt{5x^2 + 2x + 3} + 640x^4\sqrt{5x^2 + 2x + 3} + 198x^3\sqrt{5x^2 + 2x + 3} - \\ & 110x^2\sqrt{5x^2 + 2x + 3} - 38x\sqrt{5x^2 + 2x + 3} - 3\sqrt{5x^2 + 2x + 3}), x \end{aligned}$$

# Chapter 4

# Appendix

## Local contents

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## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica\_syntax\_CAS\_integration\_elementary\_version.zip

**Maple and Mupad format** Maple\_syntax\_CAS\_integration\_elementary\_version.zip

**Sympy format** SYMPY\_syntax\_CAS\_integration\_elementary\_version.zip

**Sage math format** SAGE\_syntax\_CAS\_integration\_elementary\_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
```



```

If[ExpnType[result]<=ExpnType[optimal],
  If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
    If[LeafCount[result]<=2*LeafCount[optimal],
      "A",
      "B"],
    "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],

```

```

    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

## 4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```



```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```